Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.7/61-1.1.3.7-a

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**115**]. This is test number [61].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (115)	0.00 (0)
Mathematica	100.00 (115)	0.00 (0)
Sympy	96.52 (111)	3.48 (4)
Maple	85.22 (98)	14.78 (17)
Fricas	85.22 (98)	14.78 (17)
Mupad	58.26 (67)	41.74 (48)
Giac	44.35 (51)	55.65 (64)
Reduce	44.35 (51)	55.65 (64)
Maxima	38.26 (44)	61.74 (71)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

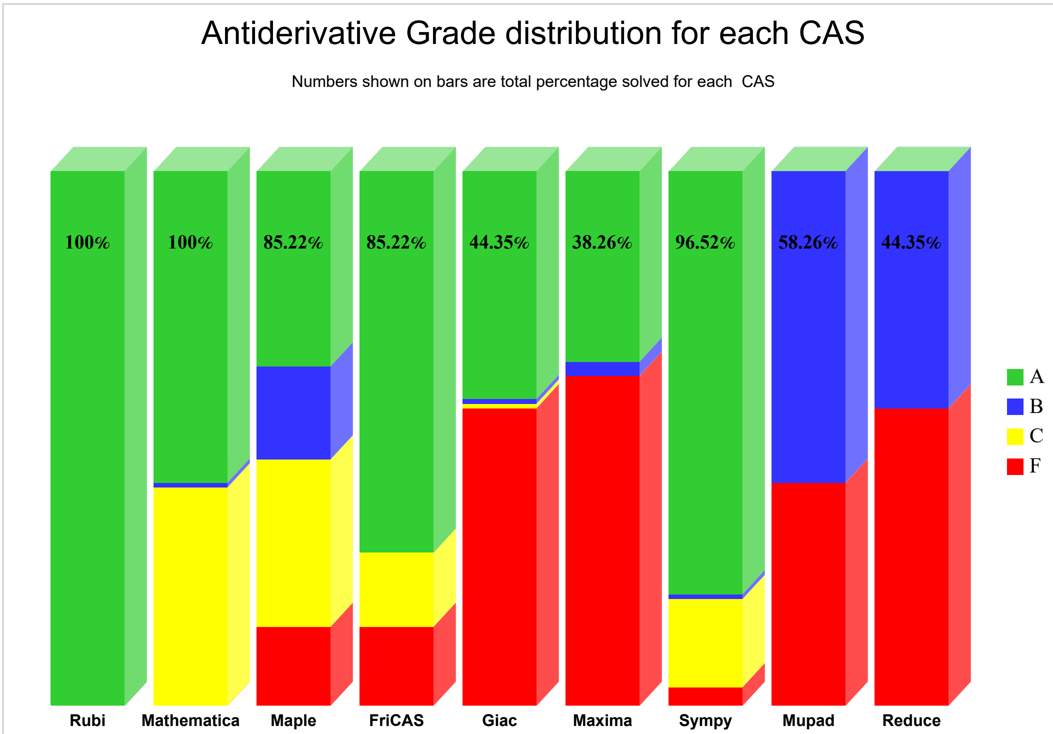
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

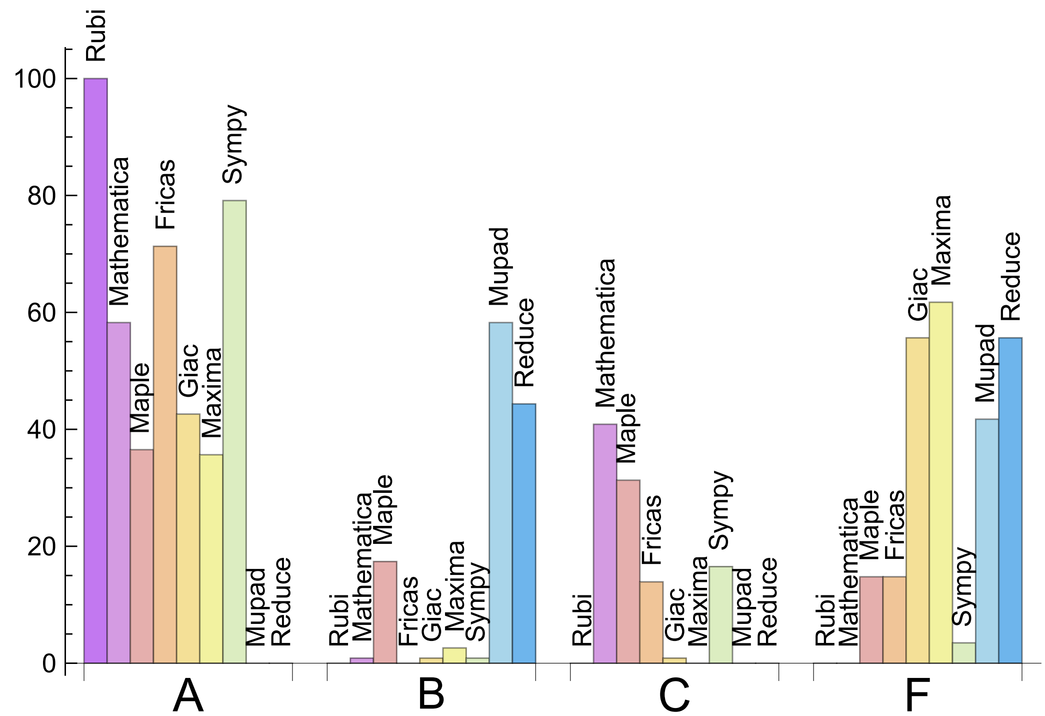
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Sympy	79.130	0.870	16.522	3.478
Fricas	71.304	0.000	13.913	14.783
Mathematica	58.261	0.870	40.870	0.000
Giac	42.609	0.870	0.870	55.652
Maple	36.522	17.391	31.304	14.783
Maxima	35.652	2.609	0.000	61.739
Mupad	0.000	58.261	0.000	41.739
Reduce	0.000	44.348	0.000	55.652

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	4	0.00	100.00	0.00
Fricas	17	88.24	11.76	0.00
Maple	17	100.00	0.00	0.00
Mupad	48	0.00	100.00	0.00
Giac	64	100.00	0.00	0.00
Reduce	64	100.00	0.00	0.00
Maxima	71	90.14	0.00	9.86

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.10
Giac	0.13
Reduce	0.15
Maple	0.28
Rubi	0.63
Fricas	1.13
Mupad	3.42
Mathematica	5.32
Sympy	7.15

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	85.61	1.30	41.50	0.90
Mathematica	100.86	0.71	89.00	0.77
Giac	103.18	0.97	50.00	0.91
Sympy	118.22	0.89	97.00	0.64
Reduce	154.39	1.09	36.00	0.86
Rubi	226.37	1.01	172.00	1.00
Mupad	234.27	1.45	124.00	1.15
Maple	318.93	1.21	65.00	0.88
Fricas	1372.38	4.47	48.50	0.74

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

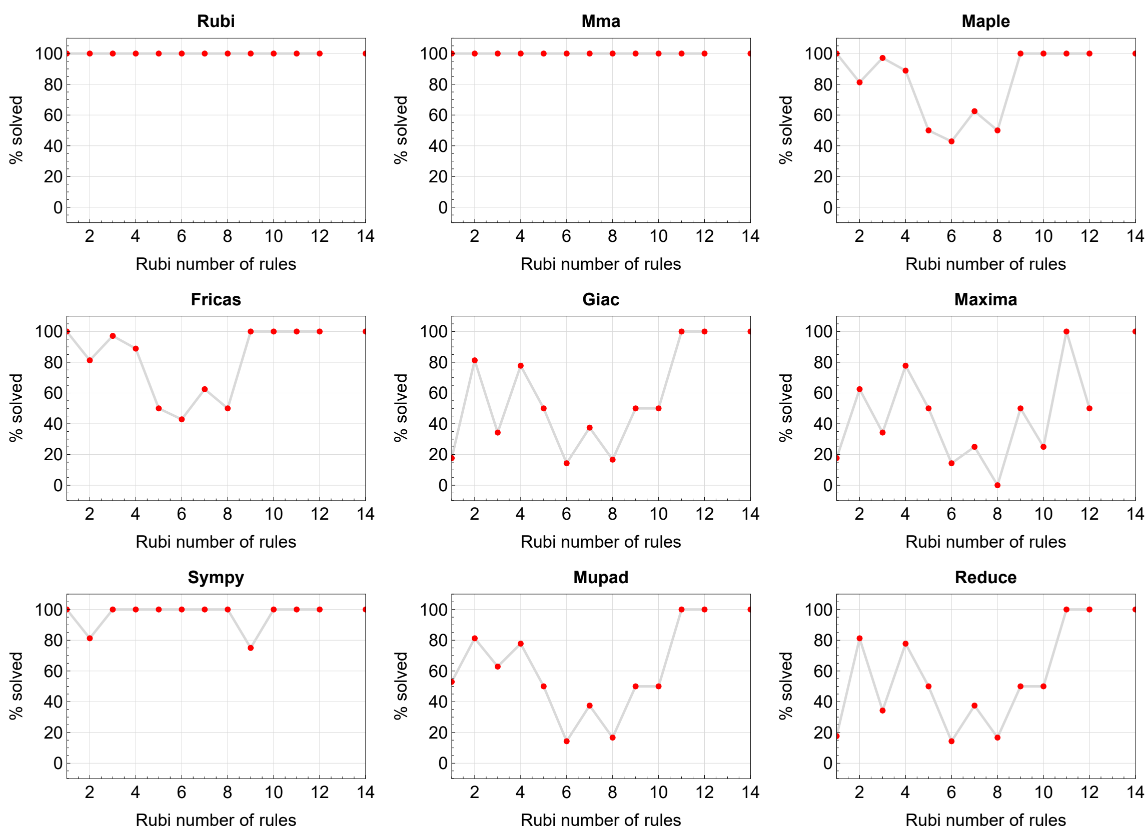


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

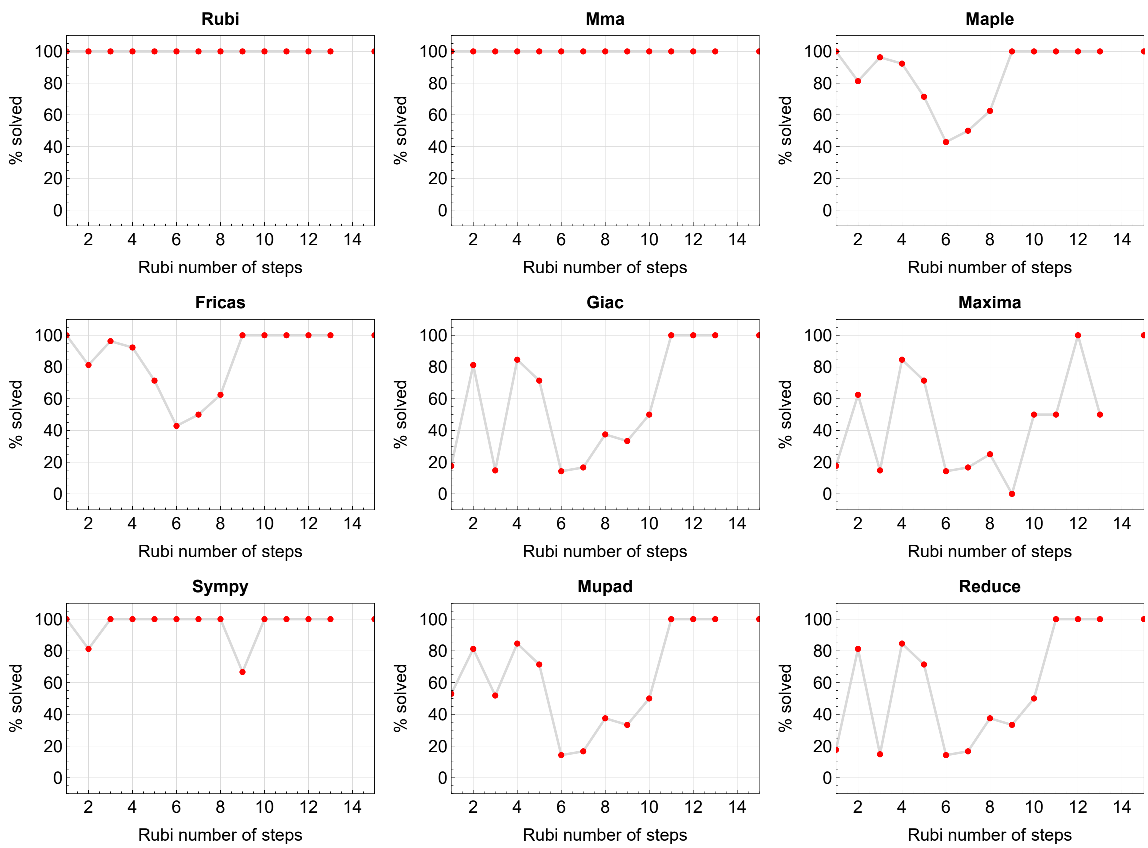


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

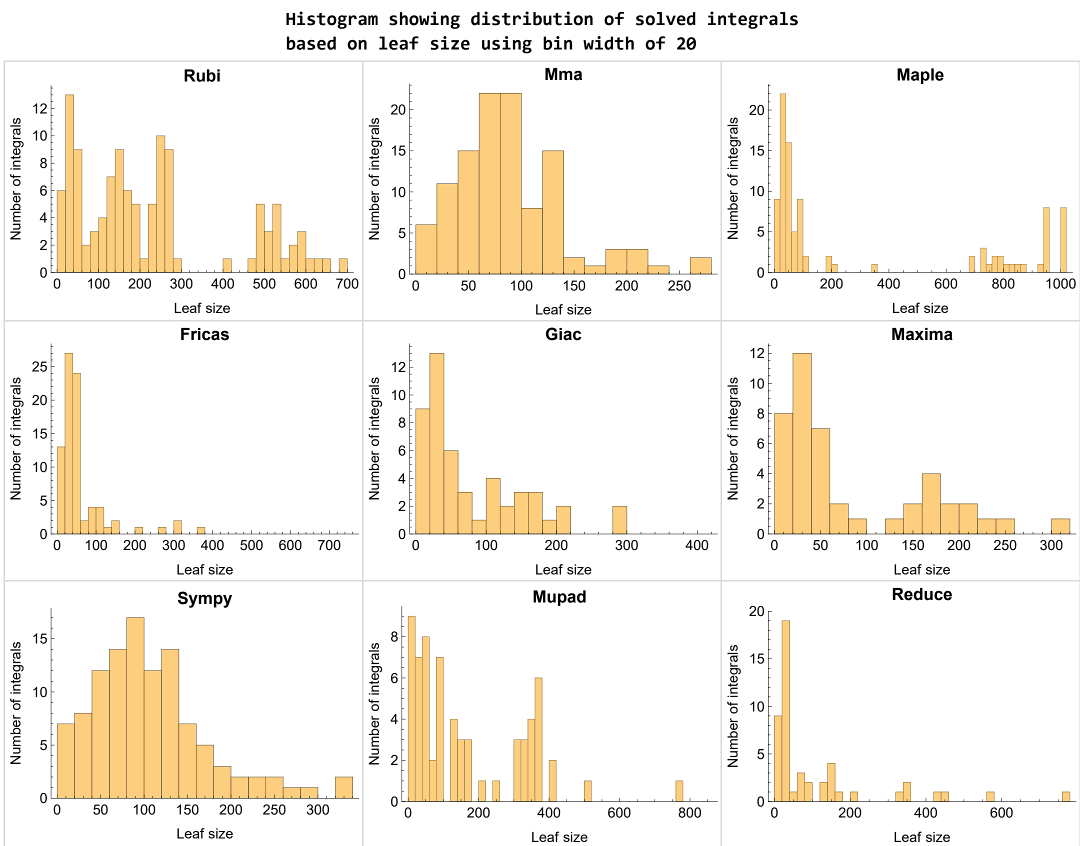


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

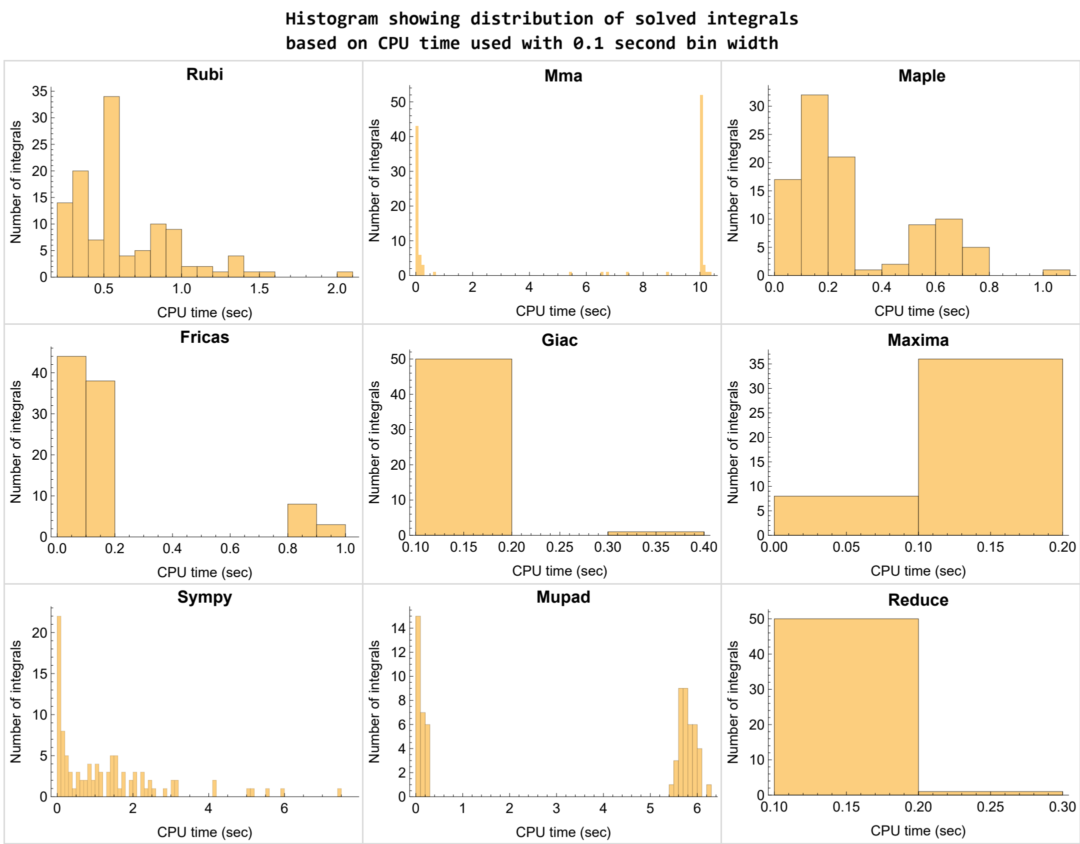


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

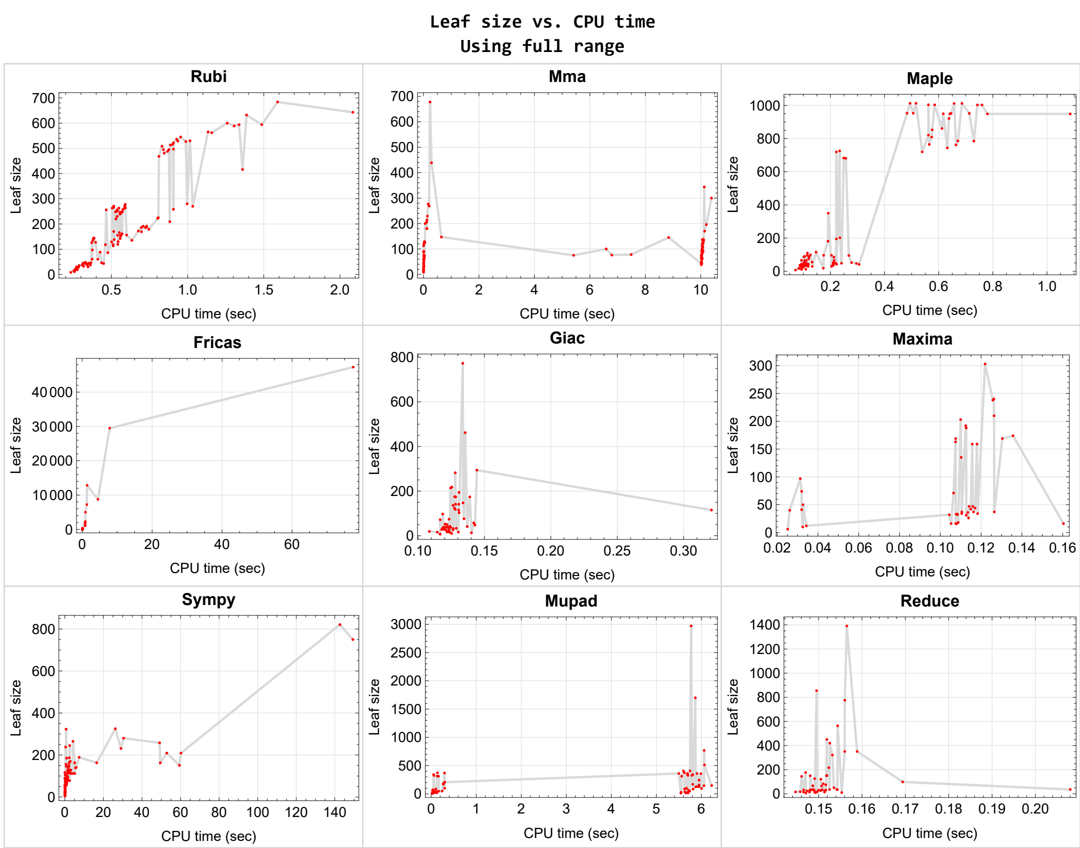


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{26, 27, 28, 29, 65, 77, 81, 97\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

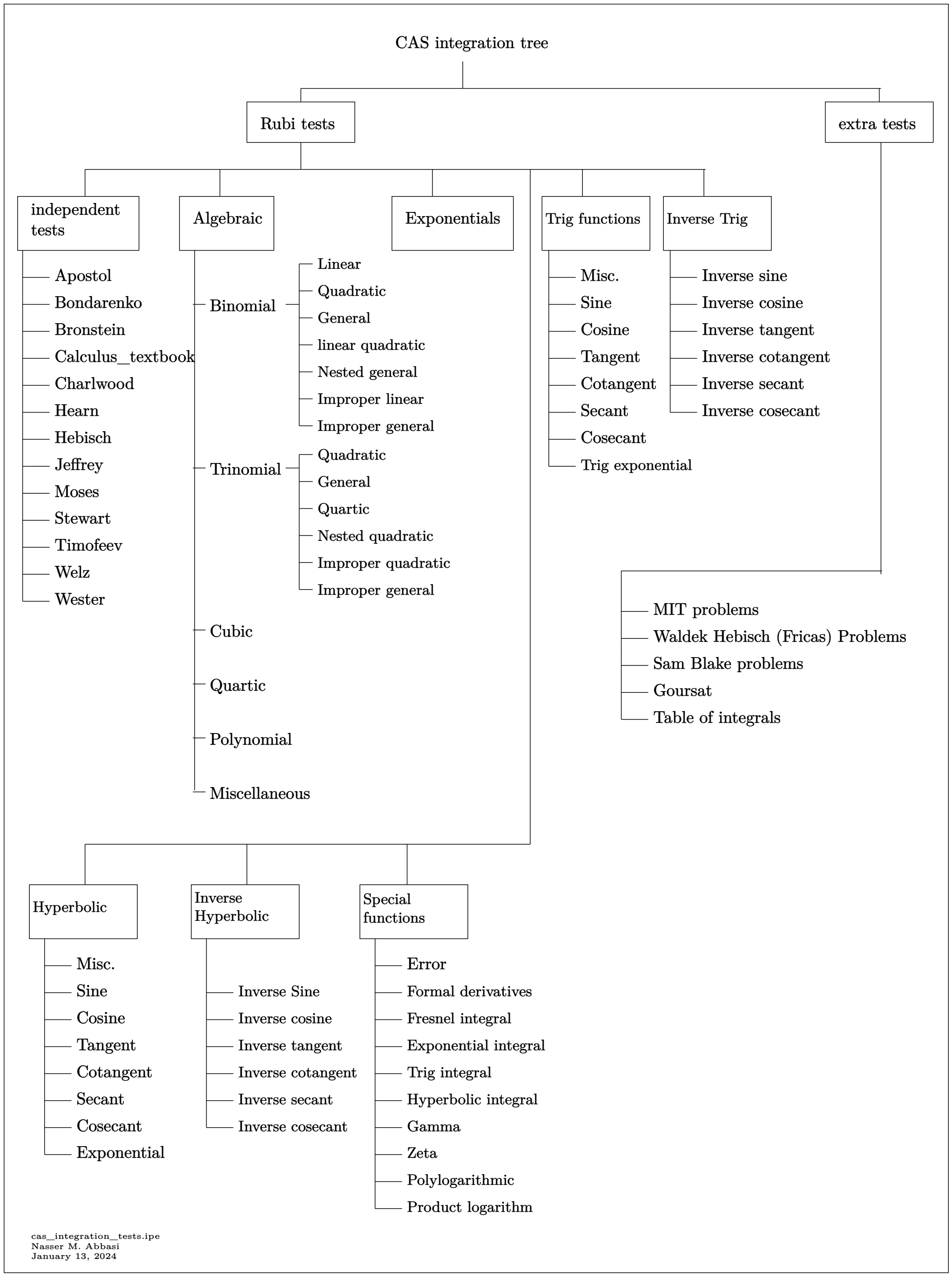
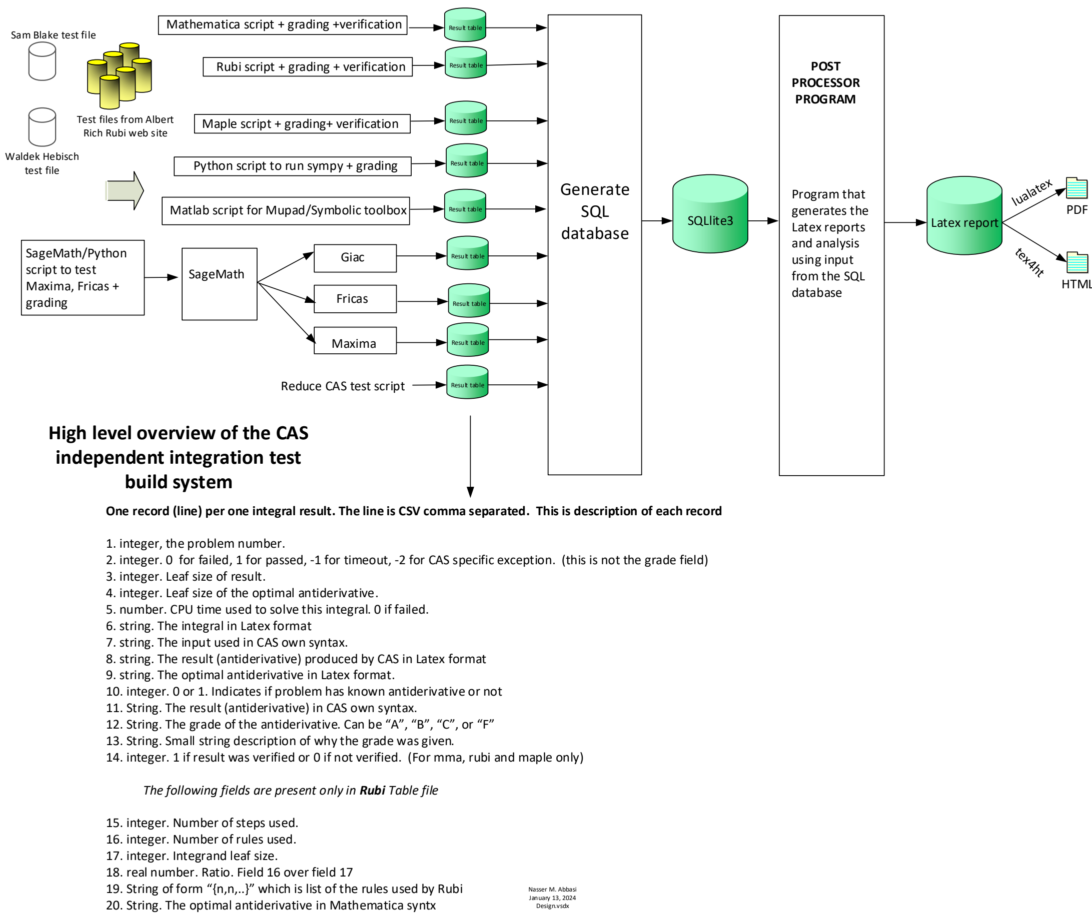


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

B grade { 22 }

C grade { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 91, 92, 93, 94 }

B grade { 21, 22, 33, 34, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 90 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 49, 63, 64, 65, 66, 75, 76, 77, 78, 79, 80, 81, 82, 95, 96, 97, 98 }

F normal fail { 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98 }

B grade { }

C grade { 1, 2, 3, 4, 5, 7, 8, 9, 23, 24, 25, 28, 29, 30, 48, 49 }

F normal fail { 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115 }

F(-1) timedout fail { 100, 108 }

F(-2) exception fail { }

Maxima

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 26, 27, 28, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

B grade { 21, 22, 37 }

C grade { }

F normal fail { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 9, 23, 24, 25, 29, 30 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

B grade { 33 }

C grade { 34 }

F normal fail { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 63, 64, 65, 66, 75, 76, 77, 78, 79, 80, 81, 82, 95, 96, 97, 98 }

C grade { }

F normal fail { }

F(-1) timedout fail { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 26, 27, 28, 29, 30, 33, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 110, 111, 112, 113, 115 }

B grade { 114 }

C grade { 6, 12, 13, 19, 20, 21, 22, 31, 32, 34, 39, 99, 100, 101, 102, 103, 104, 105, 106 }

F normal fail { }

F(-1) timedout fail { 23, 24, 25, 62 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

C grade { }

F normal fail { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	156	125	32	0	1961	76	141	151	127
N.S.	1	0.97	0.78	0.20	0.00	12.18	0.47	0.88	0.94	0.79
time (sec)	N/A	0.559	0.042	0.092	0.000	0.868	0.394	0.131	0.152	5.948

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	179	200	43	192	5014	156	175	217	357
N.S.	1	0.96	1.08	0.23	1.03	26.96	0.84	0.94	1.17	1.92
time (sec)	N/A	0.745	0.061	0.093	0.112	0.953	0.594	0.128	0.152	5.990

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	214	66	240	7245	245	214	321	370
N.S.	1	1.00	0.96	0.30	1.08	32.64	1.10	0.96	1.45	1.67
time (sec)	N/A	0.803	0.126	0.093	0.126	1.350	2.492	0.124	0.153	0.297

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	277	90	303	8787	325	294	420	513
N.S.	1	1.00	0.99	0.32	1.08	31.38	1.16	1.05	1.50	1.83
time (sec)	N/A	0.997	0.172	0.105	0.122	4.525	26.154	0.144	0.153	6.068

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	191	180	65	169	2088	105	174	351	169
N.S.	1	1.01	0.95	0.34	0.89	11.05	0.56	0.92	1.86	0.89
time (sec)	N/A	0.731	0.125	0.111	0.108	0.913	0.578	0.128	0.159	0.276

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	86	72	87	71	103	238	72	177	97
N.S.	1	1.09	0.91	1.10	0.90	1.30	3.01	0.91	2.24	1.23
time (sec)	N/A	0.476	0.042	0.102	0.107	0.112	0.371	0.117	0.147	5.668

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	225	205	86	203	2215	146	194	564	206
N.S.	1	1.05	0.95	0.40	0.94	10.30	0.68	0.90	2.62	0.96
time (sec)	N/A	0.808	0.110	0.121	0.110	0.897	0.715	0.131	0.154	0.300

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	258	229	110	238	2308	185	218	775	241
N.S.	1	1.08	0.95	0.46	0.99	9.62	0.77	0.91	3.23	1.00
time (sec)	N/A	0.907	0.142	0.115	0.126	0.907	0.862	0.126	0.156	5.949

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	154	125	34	0	1905	78	136	150	124
N.S.	1	0.96	0.78	0.21	0.00	11.83	0.48	0.84	0.93	0.77
time (sec)	N/A	0.563	0.039	0.112	0.000	0.877	0.394	0.126	0.148	5.873

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	15	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.79	0.84
time (sec)	N/A	0.264	0.008	0.085	0.105	0.095	0.056	0.124	0.146	0.033

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	16	16	26	16	15	16
N.S.	1	1.00	1.00	0.89	0.84	0.84	1.37	0.84	0.79	0.84
time (sec)	N/A	0.261	0.004	0.085	0.160	0.082	0.054	0.115	0.151	5.704

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	34	28	54	28	25	28
N.S.	1	1.00	1.07	1.00	1.17	0.97	1.86	0.97	0.86	0.97
time (sec)	N/A	0.283	0.009	0.133	0.110	0.124	0.095	0.118	0.148	0.061

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	33	26	53	26	25	28
N.S.	1	1.00	1.00	1.00	1.14	0.90	1.83	0.90	0.86	0.97
time (sec)	N/A	0.285	0.008	0.102	0.108	0.105	0.090	0.126	0.149	0.049

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	17	16	16
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.77	0.73	0.73
time (sec)	N/A	0.276	0.005	0.096	0.108	0.098	0.049	0.123	0.150	0.067

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	19	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.86	0.82	0.82
time (sec)	N/A	0.273	0.005	0.086	0.109	0.078	0.045	0.109	0.146	0.124

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	35	41	35	34	42	34	37
N.S.	1	1.00	1.00	0.80	0.93	0.80	0.77	0.95	0.77	0.84
time (sec)	N/A	0.320	0.007	0.107	0.032	0.086	0.142	0.124	0.149	5.705

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	40	34	34	41	36	36
N.S.	1	1.00	1.00	0.84	0.91	0.77	0.77	0.93	0.82	0.82
time (sec)	N/A	0.313	0.007	0.095	0.026	0.102	0.153	0.137	0.154	5.631

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	41	33	32	32	44	33	31	46
N.S.	1	1.02	1.00	0.80	0.78	0.78	1.07	0.80	0.76	1.12
time (sec)	N/A	0.346	0.008	0.105	0.104	0.095	0.078	0.119	0.149	0.167

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	29	34	28	54	28	25	28
N.S.	1	1.00	1.07	1.00	1.17	0.97	1.86	0.97	0.86	0.97
time (sec)	N/A	0.278	0.010	0.118	0.118	0.087	0.103	0.119	0.147	5.551

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	29	33	26	53	26	25	28
N.S.	1	1.00	1.00	1.00	1.14	0.90	1.83	0.90	0.86	0.97
time (sec)	N/A	0.270	0.007	0.096	0.108	0.087	0.089	0.121	0.151	0.048

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	35	35	195	163	107	88	48	27	49
N.S.	1	0.90	0.90	5.00	4.18	2.74	2.26	1.23	0.69	1.26
time (sec)	N/A	0.288	0.013	0.222	0.107	0.110	0.142	0.143	0.148	0.244

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	37	129	202	174	114	105	57	27	49
N.S.	1	0.90	3.15	4.93	4.24	2.78	2.56	1.39	0.66	1.20
time (sec)	N/A	0.308	0.037	0.235	0.136	0.117	0.175	0.142	0.150	5.732

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	269	85	0	12827	0	282	451	769
N.S.	1	1.00	1.00	0.31	0.00	47.51	0.00	1.04	1.67	2.85
time (sec)	N/A	1.034	0.202	0.213	0.000	1.422	0.000	0.128	0.152	6.060

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	439	181	0	29479	0	462	855	1700
N.S.	1	1.00	1.06	0.44	0.00	70.86	0.00	1.11	2.06	4.09
time (sec)	N/A	1.360	0.289	0.191	0.000	7.855	0.000	0.135	0.150	5.866

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	678	350	0	47284	0	773	1391	2971
N.S.	1	1.00	1.05	0.54	0.00	73.54	0.00	1.20	2.16	4.62
time (sec)	N/A	2.084	0.234	0.192	0.000	77.466	0.000	0.134	0.156	5.773

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	47	159	310	26	103	65	98
N.S.	1	1.00	0.76	0.40	1.35	2.63	0.22	0.87	0.55	0.83
time (sec)	N/A	0.511	0.014	0.112	0.116	0.122	0.092	0.131	0.151	0.299

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	90	49	159	305	22	115	67	96
N.S.	1	1.00	0.76	0.42	1.35	2.58	0.19	0.97	0.57	0.81
time (sec)	N/A	0.461	0.010	0.104	0.118	0.101	0.095	0.131	0.149	6.004

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	52	188	1961	76	147	145	127
N.S.	1	1.00	0.77	0.32	1.17	12.18	0.47	0.91	0.90	0.79
time (sec)	N/A	0.570	0.031	0.102	0.113	0.867	0.529	0.134	0.146	5.894

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	133	135	122	36	0	1344	75	118	121	158
N.S.	1	1.02	0.92	0.27	0.00	10.11	0.56	0.89	0.91	1.19
time (sec)	N/A	0.634	0.020	0.101	0.000	0.870	0.219	0.128	0.150	5.948

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	130	123	36	0	1040	70	124	126	178
N.S.	1	0.97	0.92	0.27	0.00	7.76	0.52	0.93	0.94	1.33
time (sec)	N/A	0.544	0.025	0.100	0.000	0.869	0.290	0.127	0.149	0.266

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	72	42	42	36	60	37	33	84
N.S.	1	1.00	1.95	1.14	1.14	0.97	1.62	1.00	0.89	2.27
time (sec)	N/A	0.362	0.019	0.119	0.115	0.118	0.172	0.125	0.147	5.815

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	71	38	44	36	60	38	34	86
N.S.	1	1.00	1.82	0.97	1.13	0.92	1.54	0.97	0.87	2.21
time (sec)	N/A	0.338	0.020	0.116	0.117	0.117	0.208	0.122	0.151	0.104

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	76	115	47	134	58	115	34	147
N.S.	1	1.00	1.58	2.40	0.98	2.79	1.21	2.40	0.71	3.06
time (sec)	N/A	0.330	0.017	0.147	0.116	0.107	0.205	0.321	0.148	6.227

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	43	72	96	36	40	85	76	34	145
N.S.	1	0.91	1.53	2.04	0.77	0.85	1.81	1.62	0.72	3.09
time (sec)	N/A	0.332	0.020	0.115	0.113	0.121	0.171	0.135	0.148	6.058

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	46	50	32	31	31	42	32	30	46
N.S.	1	1.15	1.25	0.80	0.78	0.78	1.05	0.80	0.75	1.15
time (sec)	N/A	0.366	0.009	0.092	0.113	0.115	0.082	0.127	0.150	0.159

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	12	12	12	12	7	13	11	12
N.S.	1	1.00	1.09	1.09	1.09	1.09	0.64	1.18	1.00	1.09
time (sec)	N/A	0.258	0.001	0.109	0.034	0.075	0.050	0.140	0.155	5.543

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	210	17	20	16	15	15
N.S.	1	1.00	1.00	0.86	10.00	0.81	0.95	0.76	0.71	0.71
time (sec)	N/A	0.264	0.002	0.174	0.126	0.099	0.094	0.130	0.148	5.656

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	28	26	26	24	27	35	35
N.S.	1	1.00	0.97	0.88	0.81	0.81	0.75	0.84	1.09	1.09
time (sec)	N/A	0.318	0.013	0.100	0.114	0.088	0.294	0.123	0.154	0.059

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	62	55	47	47	323	52	84	87
N.S.	1	1.09	1.13	1.00	0.85	0.85	5.87	0.95	1.53	1.58
time (sec)	N/A	0.411	0.028	0.132	0.114	0.109	0.640	0.120	0.151	5.627

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.75
time (sec)	N/A	0.233	0.001	0.071	0.025	0.072	0.031	0.117	0.147	0.024

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	32	32	5	33	31	63
N.S.	1	1.00	1.00	1.03	1.07	1.07	0.17	1.10	1.03	2.10
time (sec)	N/A	0.346	0.008	0.097	0.108	0.087	0.085	0.129	0.152	5.716

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	15	17	16	16
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.94	0.89	0.89
time (sec)	N/A	0.276	0.005	0.083	0.108	0.074	0.063	0.122	0.145	0.043

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	37	37	48	38	36	49
N.S.	1	1.00	1.00	0.86	0.84	0.84	1.09	0.86	0.82	1.11
time (sec)	N/A	0.351	0.008	0.099	0.126	0.085	0.098	0.123	0.208	0.079

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	98	97	97	117	97	99	97
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.04	0.86	0.88	0.86
time (sec)	N/A	0.512	0.004	0.126	0.031	0.083	0.033	0.119	0.169	5.669

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	74	74	90	74	75	74
N.S.	1	1.00	1.00	0.85	0.84	0.84	1.02	0.84	0.85	0.84
time (sec)	N/A	0.426	0.003	0.120	0.032	0.076	0.029	0.123	0.152	0.037

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	51	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.85	0.83
time (sec)	N/A	0.374	0.002	0.105	0.033	0.074	0.025	0.122	0.154	0.030

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	10	10	8	10	10	10
N.S.	1	1.00	0.86	0.79	0.71	0.71	0.57	0.71	0.71	0.71
time (sec)	N/A	0.253	0.001	0.093	0.033	0.076	0.038	0.125	0.149	0.019

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	156	124	32	135	1931	76	141	151	127
N.S.	1	0.97	0.77	0.20	0.84	11.99	0.47	0.88	0.94	0.79
time (sec)	N/A	0.600	0.040	0.102	0.110	0.844	0.490	0.128	0.152	5.944

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	191	180	65	169	2088	105	174	351	169
N.S.	1	1.01	0.95	0.34	0.89	11.05	0.56	0.92	1.86	0.89
time (sec)	N/A	0.709	0.113	0.105	0.130	0.877	0.679	0.139	0.156	5.806

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	38	37	37	44	38	36	49
N.S.	1	1.00	1.26	0.88	0.86	0.86	1.02	0.88	0.84	1.14
time (sec)	N/A	0.449	0.011	0.092	0.111	0.084	0.094	0.119	0.153	5.741

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	54	36	37	37	46	38	36	51
N.S.	1	1.00	1.17	0.78	0.80	0.80	1.00	0.83	0.78	1.11
time (sec)	N/A	0.435	0.009	0.087	0.116	0.098	0.127	0.121	0.149	0.096

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	594	78	786	0	117	265	0	152	0
N.S.	1	1.02	0.13	1.34	0.00	0.20	0.45	0.00	0.26	0.00
time (sec)	N/A	1.338	7.489	0.671	0.000	0.099	4.193	0.000	0.206	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	562	76	762	0	93	170	0	112	0
N.S.	1	1.01	0.14	1.37	0.00	0.17	0.31	0.00	0.20	0.00
time (sec)	N/A	1.159	6.789	0.664	0.000	0.102	2.830	0.000	0.191	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	530	75	744	0	69	163	0	76	0
N.S.	1	1.01	0.14	1.42	0.00	0.13	0.31	0.00	0.14	0.00
time (sec)	N/A	1.015	5.413	0.632	0.000	0.098	2.233	0.000	0.180	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	495	75	720	0	43	78	0	46	0
N.S.	1	1.01	0.15	1.47	0.00	0.09	0.16	0.00	0.09	0.00
time (sec)	N/A	0.877	10.031	0.539	0.000	0.104	2.441	0.000	0.166	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	527	96	765	0	94	163	0	68	0
N.S.	1	1.01	0.18	1.47	0.00	0.18	0.31	0.00	0.13	0.00
time (sec)	N/A	0.988	10.046	0.565	0.000	0.144	5.094	0.000	0.180	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	565	123	809	0	155	163	0	90	0
N.S.	1	1.02	0.22	1.46	0.00	0.28	0.29	0.00	0.16	0.00
time (sec)	N/A	1.134	10.076	0.574	0.000	0.096	16.500	0.000	0.201	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	600	138	853	0	214	163	0	112	0
N.S.	1	1.03	0.24	1.47	0.00	0.37	0.28	0.00	0.19	0.00
time (sec)	N/A	1.260	10.089	0.576	0.000	0.100	49.480	0.000	0.218	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	590	594	135	785	0	87	187	0	141	0
N.S.	1	1.01	0.23	1.33	0.00	0.15	0.32	0.00	0.24	0.00
time (sec)	N/A	1.487	10.102	0.730	0.000	0.128	2.293	0.000	0.210	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	589	130	821	0	153	189	0	360	0
N.S.	1	1.02	0.23	1.43	0.00	0.27	0.33	0.00	0.63	0.00
time (sec)	N/A	1.306	10.093	0.562	0.000	0.100	7.406	0.000	0.256	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	632	170	861	0	261	209	0	677	0
N.S.	1	1.01	0.27	1.38	0.00	0.42	0.33	0.00	1.08	0.00
time (sec)	N/A	1.387	10.139	0.612	0.000	0.101	60.202	0.000	0.320	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	672	684	196	921	0	373	0	0	1082	0
N.S.	1	1.02	0.29	1.37	0.00	0.56	0.00	0.00	1.61	0.00
time (sec)	N/A	1.590	10.202	0.638	0.000	0.120	0.000	0.000	0.405	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	47	47	0	21	92	0	53	312
N.S.	1	1.00	0.20	0.20	0.00	0.09	0.40	0.00	0.23	1.36
time (sec)	N/A	0.537	10.026	0.296	0.000	0.088	0.957	0.000	0.157	0.162

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	43	41	0	23	97	0	62	342
N.S.	1	1.00	0.17	0.16	0.00	0.09	0.38	0.00	0.24	1.33
time (sec)	N/A	0.574	10.035	0.306	0.000	0.110	1.377	0.000	0.154	5.806

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	63	95	0	21	82	0	55	326
N.S.	1	1.00	0.44	0.66	0.00	0.15	0.57	0.00	0.38	2.26
time (sec)	N/A	0.386	10.027	0.268	0.000	0.155	1.392	0.000	0.156	5.790

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	67	52	0	23	99	0	56	360
N.S.	1	1.00	0.50	0.39	0.00	0.17	0.73	0.00	0.41	2.67
time (sec)	N/A	0.378	10.034	0.278	0.000	0.102	1.199	0.000	0.155	5.903

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	90	1003	0	49	122	0	76	0
N.S.	1	1.00	0.19	2.14	0.00	0.10	0.26	0.00	0.16	0.00
time (sec)	N/A	0.812	10.051	0.760	0.000	0.099	2.093	0.000	0.175	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	481	91	949	0	55	128	0	83	0
N.S.	1	1.00	0.19	1.97	0.00	0.11	0.27	0.00	0.17	0.00
time (sec)	N/A	0.847	10.056	1.086	0.000	0.111	3.019	0.000	0.180	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	92	952	0	48	112	0	87	0
N.S.	1	1.00	0.34	3.51	0.00	0.18	0.41	0.00	0.32	0.00
time (sec)	N/A	0.515	10.038	0.713	0.000	0.091	3.195	0.000	0.173	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	93	1012	0	56	129	0	79	0
N.S.	1	1.00	0.35	3.80	0.00	0.21	0.48	0.00	0.30	0.00
time (sec)	N/A	0.515	10.046	0.657	0.000	0.094	2.262	0.000	0.176	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	516	89	1004	0	53	124	0	80	0
N.S.	1	0.99	0.17	1.93	0.00	0.10	0.24	0.00	0.15	0.00
time (sec)	N/A	0.903	10.051	0.562	0.000	0.126	1.505	0.000	0.173	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	529	89	950	0	56	129	0	87	0
N.S.	1	0.99	0.17	1.78	0.00	0.11	0.24	0.00	0.16	0.00
time (sec)	N/A	0.936	10.044	0.641	0.000	0.114	1.781	0.000	0.177	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	90	953	0	52	114	0	91	0
N.S.	1	1.00	0.35	3.72	0.00	0.20	0.45	0.00	0.36	0.00
time (sec)	N/A	0.537	10.059	0.483	0.000	0.091	1.941	0.000	0.177	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	92	1013	0	57	131	0	82	0
N.S.	1	1.00	0.37	4.04	0.00	0.23	0.52	0.00	0.33	0.00
time (sec)	N/A	0.540	10.033	0.494	0.000	0.098	1.559	0.000	0.178	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	49	48	0	21	92	0	54	313
N.S.	1	1.00	0.39	0.38	0.00	0.17	0.72	0.00	0.43	2.46
time (sec)	N/A	0.396	10.024	0.241	0.000	0.097	1.032	0.000	0.196	0.135

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	45	42	0	23	97	0	61	343
N.S.	1	1.00	0.32	0.30	0.00	0.16	0.68	0.00	0.43	2.42
time (sec)	N/A	0.384	10.020	0.223	0.000	0.101	1.463	0.000	0.200	5.690

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	63	96	0	21	82	0	56	327
N.S.	1	1.00	0.24	0.36	0.00	0.08	0.31	0.00	0.21	1.24
time (sec)	N/A	0.592	10.026	0.204	0.000	0.125	1.534	0.000	0.159	5.700

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	67	52	0	23	97	0	59	361
N.S.	1	1.00	0.27	0.21	0.00	0.09	0.39	0.00	0.24	1.46
time (sec)	N/A	0.558	10.024	0.210	0.000	0.106	1.133	0.000	0.159	5.692

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	47	48	0	21	92	0	57	312
N.S.	1	1.00	0.37	0.38	0.00	0.17	0.73	0.00	0.45	2.48
time (sec)	N/A	0.371	10.028	0.219	0.000	0.111	1.437	0.000	0.155	5.563

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	43	42	0	23	97	0	62	342
N.S.	1	1.00	0.30	0.29	0.00	0.16	0.68	0.00	0.43	2.39
time (sec)	N/A	0.382	10.015	0.210	0.000	0.093	1.356	0.000	0.158	0.054

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	63	96	0	21	82	0	55	326
N.S.	1	1.00	0.24	0.37	0.00	0.08	0.31	0.00	0.21	1.24
time (sec)	N/A	0.549	10.019	0.175	0.000	0.118	1.046	0.000	0.158	0.063

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	67	52	0	23	97	0	56	360
N.S.	1	1.00	0.27	0.21	0.00	0.09	0.39	0.00	0.23	1.45
time (sec)	N/A	0.572	10.019	0.211	0.000	0.102	1.437	0.000	0.155	5.499

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	90	1003	0	48	122	0	77	0
N.S.	1	1.00	0.35	3.92	0.00	0.19	0.48	0.00	0.30	0.00
time (sec)	N/A	0.466	10.061	0.743	0.000	0.091	2.370	0.000	0.172	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	90	949	0	56	128	0	84	0
N.S.	1	1.00	0.34	3.61	0.00	0.21	0.49	0.00	0.32	0.00
time (sec)	N/A	0.505	10.082	0.780	0.000	0.096	3.020	0.000	0.177	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	91	952	0	49	112	0	86	0
N.S.	1	1.00	0.18	1.92	0.00	0.10	0.23	0.00	0.17	0.00
time (sec)	N/A	0.907	10.052	0.646	0.000	0.136	3.142	0.000	0.174	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	488	93	1012	0	55	128	0	80	0
N.S.	1	1.00	0.19	2.07	0.00	0.11	0.26	0.00	0.16	0.00
time (sec)	N/A	0.868	10.058	0.686	0.000	0.096	2.533	0.000	0.175	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	89	1004	0	52	124	0	81	0
N.S.	1	1.00	0.37	4.17	0.00	0.22	0.51	0.00	0.34	0.00
time (sec)	N/A	0.560	10.043	0.585	0.000	0.084	1.443	0.000	0.184	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	89	950	0	57	129	0	88	0
N.S.	1	1.00	0.36	3.83	0.00	0.23	0.52	0.00	0.35	0.00
time (sec)	N/A	0.527	10.043	0.617	0.000	0.081	1.619	0.000	0.179	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	545	90	953	0	53	114	0	90	0
N.S.	1	0.99	0.16	1.74	0.00	0.10	0.21	0.00	0.16	0.00
time (sec)	N/A	0.954	10.039	0.506	0.000	0.087	1.543	0.000	0.179	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	536	92	1013	0	56	129	0	83	0
N.S.	1	0.99	0.17	1.88	0.00	0.10	0.24	0.00	0.15	0.00
time (sec)	N/A	0.928	10.033	0.516	0.000	0.085	1.460	0.000	0.176	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	495	75	720	0	43	78	0	46	0
N.S.	1	1.01	0.15	1.47	0.00	0.09	0.16	0.00	0.09	0.00
time (sec)	N/A	0.842	0.014	0.222	0.000	0.101	0.995	0.000	0.170	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	508	75	681	0	47	82	0	50	0
N.S.	1	1.01	0.15	1.35	0.00	0.09	0.16	0.00	0.10	0.00
time (sec)	N/A	0.832	10.027	0.257	0.000	0.085	1.073	0.000	0.169	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	522	76	683	0	43	73	0	54	0
N.S.	1	1.01	0.15	1.33	0.00	0.08	0.14	0.00	0.10	0.00
time (sec)	N/A	0.907	10.024	0.249	0.000	0.083	1.110	0.000	0.166	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	513	78	726	0	47	83	0	49	0
N.S.	1	1.01	0.15	1.43	0.00	0.09	0.16	0.00	0.10	0.00
time (sec)	N/A	0.887	10.024	0.234	0.000	0.088	1.096	0.000	0.168	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	247	42	33	0	18	61	0	38	373
N.S.	1	1.00	0.17	0.13	0.00	0.07	0.25	0.00	0.15	1.52
time (sec)	N/A	0.569	10.021	0.207	0.000	0.083	0.766	0.000	0.154	0.140

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	272	38	29	0	18	65	0	44	406
N.S.	1	1.00	0.14	0.11	0.00	0.07	0.24	0.00	0.16	1.50
time (sec)	N/A	0.587	10.023	0.204	0.000	0.106	0.879	0.000	0.153	5.748

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	275	278	58	65	0	18	56	0	38	374
N.S.	1	1.01	0.21	0.24	0.00	0.07	0.20	0.00	0.14	1.36
time (sec)	N/A	0.591	10.034	0.214	0.000	0.112	0.820	0.000	0.154	5.625

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	262	62	36	0	18	66	0	41	405
N.S.	1	1.00	0.24	0.14	0.00	0.07	0.25	0.00	0.16	1.55
time (sec)	N/A	0.584	10.056	0.211	0.000	0.100	0.889	0.000	0.155	5.601

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	189	100	0	0	0	124	0	120	0
N.S.	1	0.98	0.52	0.00	0.00	0.00	0.64	0.00	0.62	0.00
time (sec)	N/A	0.732	6.589	0.000	0.000	0.000	1.908	0.000	0.189	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	220	300	0	0	0	119	0	46	0
N.S.	1	1.35	1.84	0.00	0.00	0.00	0.73	0.00	0.28	0.00
time (sec)	N/A	0.530	10.385	0.000	0.000	0.000	1.742	0.000	0.176	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	119	0	0	0	119	0	46	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.77	0.00	0.30	0.00
time (sec)	N/A	0.540	10.052	0.000	0.000	0.000	1.570	0.000	0.155	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	166	100	0	0	0	112	0	103	0
N.S.	1	1.08	0.65	0.00	0.00	0.00	0.73	0.00	0.67	0.00
time (sec)	N/A	0.552	10.051	0.000	0.000	0.000	4.168	0.000	0.183	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	97	100	0	0	0	112	0	103	0
N.S.	1	0.97	1.00	0.00	0.00	0.00	1.12	0.00	1.03	0.00
time (sec)	N/A	0.375	10.051	0.000	0.000	0.000	5.152	0.000	0.156	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	129	83	0	0	0	231	0	163	0
N.S.	1	1.11	0.72	0.00	0.00	0.00	1.99	0.00	1.41	0.00
time (sec)	N/A	0.502	10.055	0.000	0.000	0.000	29.093	0.000	0.205	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	138	125	0	0	0	209	0	163	0
N.S.	1	1.08	0.98	0.00	0.00	0.00	1.63	0.00	1.27	0.00
time (sec)	N/A	0.519	10.077	0.000	0.000	0.000	52.880	0.000	0.185	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	172	107	0	0	0	750	0	223	0
N.S.	1	1.18	0.73	0.00	0.00	0.00	5.14	0.00	1.53	0.00
time (sec)	N/A	0.677	10.073	0.000	0.000	0.000	149.379	0.000	0.226	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	219	209	145	0	0	0	150	0	149	0
N.S.	1	0.95	0.66	0.00	0.00	0.00	0.68	0.00	0.68	0.00
time (sec)	N/A	0.883	8.841	0.000	0.000	0.000	2.068	0.000	0.191	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	239	344	0	0	0	144	0	63	0
N.S.	1	1.31	1.89	0.00	0.00	0.00	0.79	0.00	0.35	0.00
time (sec)	N/A	0.556	10.124	0.000	0.000	0.000	2.013	0.000	0.186	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	169	136	0	0	0	143	0	65	0
N.S.	1	0.99	0.80	0.00	0.00	0.00	0.84	0.00	0.38	0.00
time (sec)	N/A	0.697	10.054	0.000	0.000	0.000	1.747	0.000	0.159	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	186	110	0	0	0	139	0	139	0
N.S.	1	1.09	0.64	0.00	0.00	0.00	0.81	0.00	0.81	0.00
time (sec)	N/A	0.696	10.058	0.000	0.000	0.000	5.505	0.000	0.232	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	110	0	0	0	141	0	139	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.18	0.00	1.17	0.00
time (sec)	N/A	0.542	10.046	0.000	0.000	0.000	5.906	0.000	0.190	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	142	97	0	0	0	280	0	219	0
N.S.	1	1.05	0.72	0.00	0.00	0.00	2.07	0.00	1.62	0.00
time (sec)	N/A	0.557	10.062	0.000	0.000	0.000	30.439	0.000	0.221	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	151	137	0	0	0	258	0	219	0
N.S.	1	1.03	0.93	0.00	0.00	0.00	1.76	0.00	1.49	0.00
time (sec)	N/A	0.560	10.083	0.000	0.000	0.000	49.148	0.000	0.188	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	185	120	0	0	0	821	0	299	0
N.S.	1	1.12	0.73	0.00	0.00	0.00	4.98	0.00	1.81	0.00
time (sec)	N/A	0.724	10.077	0.000	0.000	0.000	142.643	0.000	0.253	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	170	147	0	0	0	151	0	0	0
N.S.	1	1.07	0.92	0.00	0.00	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.515	0.646	0.000	0.000	0.000	59.426	0.000	0.184	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [8] had the largest ratio of [.9333329999999999968]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	8	0.97	15	0.533
2	A	12	11	0.96	17	0.647
3	A	2	2	1.00	17	0.118
4	A	2	2	1.00	17	0.118
5	A	11	10	1.01	15	0.667
6	A	10	9	1.09	13	0.692
7	A	13	12	1.05	15	0.800
8	A	15	14	1.08	15	0.933
9	A	8	7	0.96	16	0.438
10	A	4	3	1.00	11	0.273
11	A	4	3	1.00	15	0.200
12	A	4	3	1.00	19	0.158
13	A	4	3	1.00	21	0.143
14	A	4	4	1.00	13	0.308
15	A	3	3	1.00	13	0.231
16	A	5	5	1.00	20	0.250
17	A	4	4	1.00	20	0.200
18	A	8	7	1.02	15	0.467
19	A	4	3	1.00	19	0.158
20	A	4	3	1.00	21	0.143
21	A	4	3	0.90	31	0.097

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	3	0.90	36	0.083
23	A	2	2	1.00	22	0.091
24	A	2	2	1.00	22	0.091
25	A	2	2	1.00	22	0.091
26	A	1	1	1.00	35	0.029
27	A	1	1	1.00	33	0.030
28	A	1	1	1.00	36	0.028
29	A	13	12	1.02	19	0.632
30	A	11	10	0.97	18	0.556
31	A	5	4	1.00	27	0.148
32	A	5	4	1.00	28	0.143
33	A	5	4	1.00	24	0.167
34	A	5	4	0.91	24	0.167
35	A	8	7	1.15	13	0.538
36	A	2	2	1.00	31	0.065
37	A	2	2	1.00	42	0.048
38	A	3	3	1.00	20	0.150
39	A	7	6	1.09	20	0.300
40	A	2	2	1.00	16	0.125
41	A	6	5	1.00	20	0.250
42	A	4	4	1.00	18	0.222
43	A	2	2	1.00	18	0.111
44	A	2	2	1.00	30	0.067
45	A	2	2	1.00	30	0.067
46	A	2	2	1.00	28	0.071
47	A	2	2	1.00	30	0.067
48	A	10	9	0.97	30	0.300
49	A	12	11	1.01	30	0.367
50	A	3	3	1.00	17	0.176
51	A	3	3	1.00	19	0.158
52	A	10	10	1.02	32	0.312
53	A	8	8	1.01	32	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	6	1.01	32	0.188
55	A	4	4	1.01	32	0.125
56	A	6	6	1.01	32	0.188
57	A	8	8	1.02	32	0.250
58	A	10	10	1.03	32	0.312
59	A	9	9	1.01	32	0.281
60	A	7	7	1.02	32	0.219
61	A	7	7	1.01	32	0.219
62	A	9	9	1.02	32	0.281
63	A	3	3	1.00	18	0.167
64	A	3	3	1.00	22	0.136
65	A	1	1	1.00	20	0.050
66	A	1	1	1.00	20	0.050
67	A	3	3	1.00	33	0.091
68	A	3	3	1.00	35	0.086
69	A	1	1	1.00	36	0.028
70	A	1	1	1.00	36	0.028
71	A	3	3	0.99	30	0.100
72	A	3	3	0.99	32	0.094
73	A	1	1	1.00	33	0.030
74	A	1	1	1.00	33	0.030
75	A	1	1	1.00	20	0.050
76	A	1	1	1.00	24	0.042
77	A	3	3	1.00	22	0.136
78	A	3	3	1.00	22	0.136
79	A	1	1	1.00	20	0.050
80	A	1	1	1.00	20	0.050
81	A	3	3	1.00	18	0.167
82	A	3	3	1.00	22	0.136
83	A	1	1	1.00	35	0.029
84	A	1	1	1.00	37	0.027
85	A	3	3	1.00	38	0.079

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.00	38	0.079
87	A	1	1	1.00	32	0.031
88	A	1	1	1.00	34	0.029
89	A	3	3	0.99	35	0.086
90	A	3	3	0.99	35	0.086
91	A	3	3	1.01	17	0.176
92	A	3	3	1.01	18	0.167
93	A	3	3	1.01	19	0.158
94	A	3	3	1.01	20	0.150
95	A	3	3	1.00	15	0.200
96	A	3	3	1.00	17	0.176
97	A	3	3	1.01	15	0.200
98	A	3	3	1.00	17	0.176
99	A	6	6	0.98	22	0.273
100	A	2	2	1.35	22	0.091
101	A	3	3	1.00	22	0.136
102	A	4	4	1.08	22	0.182
103	A	5	5	0.97	22	0.227
104	A	6	6	1.11	22	0.273
105	A	7	7	1.08	22	0.318
106	A	8	8	1.18	22	0.364
107	A	8	8	0.95	27	0.296
108	A	2	2	1.31	27	0.074
109	A	5	5	0.99	27	0.185
110	A	6	6	1.09	27	0.222
111	A	7	7	1.00	27	0.259
112	A	6	6	1.05	27	0.222
113	A	7	7	1.03	27	0.259
114	A	8	8	1.12	27	0.296
115	A	2	2	1.07	25	0.080

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{a+bx}{d+ex^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 161

$$\int \frac{a+bx}{d+ex^3}\,dx = -\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} - \frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}}$$

output
```
-1/3*(b*d^(1/3)+a*e^(1/3))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3
))*3^(1/2)/d^(2/3)/e^(2/3)-1/3*(b*d^(1/3)-a*e^(1/3))*ln(d^(1/3)+e^(1/3)*x)
/d^(2/3)/e^(2/3)-1/6*(a-b*d^(1/3)/e^(1/3))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^
(2/3)*x^2)/d^(2/3)/e^(1/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{a+bx}{d+ex^3}\,dx$$

$$= \frac{-2\sqrt{3}\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+\right.\right.}{6d^{2/3}e^{2/3}}$$

input
```
Integrate[(a + b*x)/(d + e*x^3),x]
```

output
```
(-2*Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqr
t[3]] - (b*d^(1/3) - a*e^(1/3))*(2*Log[d^(1/3) + e^(1/3)*x] - Log[d^(2/3)
- d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{d+ex^3}\,dx$$

$$\Big\downarrow \text{2399}$$

$$\frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}a+b\sqrt[3]{d}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{e}x+\sqrt[3]{d}}dx}{3d^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{\sqrt[3]{d}\left(2\sqrt[3]{e}a+b\sqrt[3]{d}\right)+\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{d}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int -\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3}{2}\sqrt[3]{d}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3}{2}\sqrt[3]{d}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\sqrt[3]{e}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}\sqrt[3]{e}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{3\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}}}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}\sqrt[3]{e}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{\sqrt{3}\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{3d^{2/3}\sqrt[3]{e}}+\frac{\left(a-\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

input
```
Int[(a + b*x)/(d + e*x^3),x]
```

output
```
((a - (b*d^(1/3))/e^(1/3))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(1/3)) +
 (-((Sqrt[3]*(b*d^(1/3) + a*e^(1/3))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sq
rt[3]])/e^(1/3)) - ((a - (b*d^(1/3))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3
)*x + e^(2/3)*x^2])/2)/(3*d^(2/3)*e^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^3+d\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^2}}{3e}$
default	$a\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\right.\right.}{6}\right.$

input
```
int((b*x+a)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/3/e*sum((_R*b+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 1961, normalized size of antiderivative = 12.18

$$\int \frac{a+bx}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)/(e*x^3+d),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^
3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d +
a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/4*((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(
1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2)
- (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3) - 2*
(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d
- a^3*e)/(d^2*e^2))^(1/3)))*a^2*d*e + 2*a*b^2*d + (b^3*d + a^3*e)*x) + 1/1
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e
)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3
*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^
2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)/(
d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*d*e + 16*a*b)/(d*e)))*log(
-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^
3*e)/(d^2*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d +
a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b^3*d + a^3*e)/(d^2*e^2) - (b^3*d - a^3*e)/(d^2
*e^2))^(1/3) - 2*(1/2)^(2/3)*a*b*(-I*sqrt(3) + 1)/(d*e*((b^3*d + a^3*e)...
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{a+bx}{d+ex^3}\,dx$$
$$= \text{RootSum}\left(27t^3d^2e^2 + 9tabde - a^3e + b^3d, \left(t \mapsto t\log\left(x + \frac{9t^2bd^2e + 3ta^2de + 2ab^2d}{a^3e + b^3d}\right)\right)\right)$$

input
```
integrate((b*x+a)/(e*x**3+d),x)
```

output
```
RootSum(27*_t**3*d**2*e**2 + 9*_t*a*b*d*e - a**3*e + b**3*d, Lambda(_t, _t
*log(x + (9*_t**2*b*d**2*e + 3*_t*a**2*d*e + 2*a*b**2*d)/(a**3*e + b**3*d)
)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{d+ex^3}\,dx = -\frac{\sqrt{3}\left(ae-\left(-de^2\right)^{\frac{1}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(ae+\left(-de^2\right)^{\frac{1}{3}}b\right)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(b\left(-\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d}$$

input
```
integrate((b*x+a)/(e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(a*e - (-d*e^2)^(1/3)*b)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/
3))/(-d/e)^(1/3))/(-d*e^2)^(2/3) - 1/6*(a*e + (-d*e^2)^(1/3)*b)*log(x^2 +
x*(-d/e)^(1/3) + (-d/e)^(2/3))/(-d*e^2)^(2/3) - 1/3*(b*(-d/e)^(1/3) + a)*(
-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/d
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{a+bx}{d+ex^3}\,dx = \sum_{k=1}^{3} \ln\left(e\left(a\,b+b^2\,x + \mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)^2\,d\,e\,9 + \mathrm{root}\left(27\,d^2\,e^2\,z^3+9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\mathrm{root}\left(27\,d^2\,e^2\,z^3 + 9\,a\,b\,d\,e\,z+b^3\,d-a^3\,e,z,k\right)$$

input
```
int((a + b*x)/(d + e*x^3),x)
```

output
```
symsum(log(e*(a*b + b^2*x + 9*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d -
a^3*e, z, k)^2*d*e + 3*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e,
z, k)*a*e*x))*root(27*d^2*e^2*z^3 + 9*a*b*d*e*z + b^3*d - a^3*e, z, k), k,
 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{a+bx}{d+ex^3}\,dx = \frac{-2e^{\frac{1}{3}}d^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)bd-e^{\frac{1}{3}}d^{\frac{2}{3}}\log\left(d^{\frac{2}{3}}-e^{\frac{1}{3}}d^{\frac{1}{3}}x+e^{\frac{2}{3}}x^2\right)a+2e^{\frac{1}{3}}d^{\frac{2}{3}}\log\left(\cdots\right.}{6e^{\frac{2}{3}}d^{\frac{4}{3}}}$$

input
```
int((b*x+a)/(e*x^3+d),x)
```

output
```
( - 2*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*s
qrt(3)))*a - 2*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*
b*d - e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2
)*a + 2*e**(1/3)*d**(2/3)*log(d**(1/3) + e**(1/3)*x)*a + log(d**(2/3) - e*
*(1/3)*d**(1/3)*x + e**(2/3)*x**2)*b*d - 2*log(d**(1/3) + e**(1/3)*x)*b*d)
/(6*e**(2/3)*d**(1/3)*d)
```

3.2 $\int \frac{(a+bx)^2}{c+dx^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\begin{aligned}\int \frac{(a+bx)^2}{c+dx^3} dx = &-\frac{a\left(2b\sqrt[3]{c}+a\sqrt[3]{d}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{2/3}} \\ &- \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}} \\ &+ \frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}\end{aligned}$$

output
```
-1/3*a*(2*b*c^(1/3)+a*d^(1/3))*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^
(1/3))*3^(1/2)/c^(2/3)/d^(2/3)-1/3*a*(2*b*c^(1/3)-a*d^(1/3))*ln(c^(1/3)+d^
(1/3)*x)/c^(2/3)/d^(2/3)+1/6*a*(2*b*c^(1/3)-a*d^(1/3))*ln(c^(2/3)-c^(1/3)*
d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/d^(2/3)+1/3*b^2*ln(d*x^3+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \frac{\left(2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\arctan\left(\frac{-\sqrt[3]{c}+2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}cd^{2/3}} + \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3cd^{2/3}} - \frac{\left(-2abc^{2/3}+a^2\sqrt[3]{c}\sqrt[3]{d}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6cd^{2/3}} + \frac{b^2\log\left(c+dx^3\right)}{3d}$$

input
```
Integrate[(a + b*x)^2/(c + d*x^3),x]
```

output
```
((2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1/3))*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c*d^(2/3)) + ((-2*a*b*c^(2/3) + a^2*c^(1/3)*d^(1
/3))*Log[c^(1/3) + d^(1/3)*x])/(3*c*d^(2/3)) - ((-2*a*b*c^(2/3) + a^2*c^(1
/3)*d^(1/3))*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c*d^(2/3))
 + (b^2*Log[c + d*x^3])/(3*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2410, 792, 2399, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx$$

$$\Big\downarrow \text{2410}$$

$$\int \frac{a^2+2bxa}{dx^3+c}dx+b^2\int \frac{x^2}{dx^3+c}dx$$

$$\Big\downarrow \text{ 792}$$

$$\int \frac{a^2+2bxa}{dx^3+c}dx+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{ 2399}$$

$$\frac{\int \frac{a\left(2\sqrt[3]{c}\left(\sqrt[3]{d}a+b\sqrt[3]{c}\right)+\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\int \frac{1}{\sqrt[3]{d}x+\sqrt[3]{c}}dx}{3c^{2/3}\sqrt[3]{d}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\int \frac{a\left(2\sqrt[3]{c}\left(\sqrt[3]{d}a+b\sqrt[3]{c}\right)+\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a\int \frac{2\sqrt[3]{c}\left(\sqrt[3]{d}a+b\sqrt[3]{c}\right)+\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{a\left(\frac{3}{2}\sqrt[3]{c}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{1}{2}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int -\frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{a\left(\frac{3}{2}\sqrt[3]{c}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{\sqrt[3]{d}\left(\sqrt[3]{c}-2\sqrt[3]{d}x\right)}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\left(\frac{3}{2}\sqrt[3]{c}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int \frac{1}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{1}{2}\sqrt[3]{d}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{1082}$$

$$\frac{a\left(\frac{1}{2}\sqrt[3]{d}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx+\frac{3\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{217}$$

$$\frac{a\left(\frac{1}{2}\sqrt[3]{d}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}dx-\frac{\sqrt{3}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{a\left(-\frac{\sqrt{3}\left(a\sqrt[3]{d}+2b\sqrt[3]{c}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{\sqrt[3]{d}}-\frac{1}{2}\left(a-\frac{2b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)\right)}{3c^{2/3}\sqrt[3]{d}}-\frac{a\left(2b\sqrt[3]{c}-a\sqrt[3]{d}\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{2/3}}+\frac{b^2\log\left(c+dx^3\right)}{3d}$$

input
```
Int[(a + b*x)^2/(c + d*x^3),x]
```

output
```
-1/3*(a*(2*b*c^(1/3) - a*d^(1/3))*Log[c^(1/3) + d^(1/3)*x])/(c^(2/3)*d^(2/
3)) + (a*(-((Sqrt[3]*(2*b*c^(1/3) + a*d^(1/3))*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/d^(1/3)) - ((a - (2*b*c^(1/3))/d^(1/3))*Log[c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2])/2))/(3*c^(2/3)*d^(1/3)) + (b^2*Log[c + d*x
^3])/(3*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}d+c\right)}\frac{\left(\_R^{2}b^{2}+2\_Rab+a^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3d}$
default	$a^{2}\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+2ab\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\right.}{}\right.$

input
```
int((b*x+a)^2/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*sum((_R^2*b^2+2*_R*a*b+a^2)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 5014, normalized size of antiderivative = 26.96

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2/(d*x^3+c),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx$$
$$=\text{RootSum}\left(27t^3c^2d^3-27t^2b^2c^2d^2+t\left(18a^3bcd^2+9b^4c^2d\right)-a^6d^2+2a^3b^3cd-b^6c^2,\left(t\mapsto t\log\left(x+\frac{18t\right.\right.\right.$$

input
```
integrate((b*x+a)**2/(d*x**3+c),x)
```

output
```
RootSum(27*_t**3*c**2*d**3 - 27*_t**2*b**2*c**2*d**2 + _t*(18*a**3*b*c*d**
2 + 9*b**4*c**2*d) - a**6*d**2 + 2*a**3*b**3*c*d - b**6*c**2, Lambda(_t, _
t*log(x + (18*_t**2*b*c**2*d**2 + 3*_t*a**3*c*d**2 - 12*_t*b**3*c**2*d + 7
*a**3*b**2*c*d + 2*b**5*c**2)/(a**5*d**2 + 8*a**2*b**3*c*d))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx =$$
$$-\frac{\sqrt{3}\left(2\,b^2c-\left(6\,ab\left(\frac{c}{d}\right)^{\frac{2}{3}}+3\,a^2\left(\frac{c}{d}\right)^{\frac{1}{3}}+\frac{2\,b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd}$$
$$+\frac{\left(2\,b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}+2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^2\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(b^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,ab\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^2/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*(2*b^2*c - (6*a*b*(c/d)^(2/3) + 3*a^2*(c/d)^(1/3) + 2*b^2*c/d
)*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/6*(2*b^
2*(c/d)^(2/3) + 2*a*b*(c/d)^(1/3) - a^2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(
2/3))/(d*(c/d)^(2/3)) + 1/3*(b^2*(c/d)^(2/3) - 2*a*b*(c/d)^(1/3) + a^2)*lo
g(x + (c/d)^(1/3))/(d*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \frac{b^2\log\left(|dx^3+c|\right)}{3\,d} - \frac{\sqrt{3}\left(a^2d-2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(a^2d+2\left(-cd^2\right)^{\frac{1}{3}}ab\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(2\,abd\left(-\frac{c}{d}\right)^{\frac{1}{3}}+a^2d\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

input
```
integrate((b*x+a)^2/(d*x^3+c),x, algorithm="giac")
```

output
```
1/3*b^2*log(abs(d*x^3 + c))/d - 1/3*sqrt(3)*(a^2*d - 2*(-c*d^2)^(1/3)*a*b)
*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c*d^2)^(2/3) - 1/
6*(a^2*d + 2*(-c*d^2)^(1/3)*a*b)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/
(-c*d^2)^(2/3) - 1/3*(2*a*b*d*(-c/d)^(1/3) + a^2*d)*(-c/d)^(1/3)*log(abs(x
 - (-c/d)^(1/3)))/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \sum_{k=1}^{3} \ln\Big(b^4\,c + \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)^2\,c\,d^2\,9 + 2\,a^3\,b\,d - \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)\,b^2\,c\,d\,6 + \mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)\,a^2\,d^2\,x\,3 + 3\,a^2\,b^2\,d\,x\Big)\,\mathrm{root}\big(27\,c^2\,d^3\,z^3 - 27\,b^2\,c^2\,d^2\,z^2 + 18\,a^3\,b\,c\,d^2\,z + 9\,b^4\,c^2\,d\,z + 2\,a^3\,b^3\,c\,d - b^6\,c^2 - a^6\,d^2, z, k\big)$$

input
```
int((a + b*x)^2/(c + d*x^3),x)
```

output
```
symsum(log(b^4*c + 9*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c
*d^2*z + 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)^2*c*d^2
+ 2*a^3*b*d - 6*root(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*
z + 9*b^4*c^2*d*z + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*b^2*c*d + 3*r
oot(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z
 + 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k)*a^2*d^2*x + 3*a^2*b^2*d*x)*roo
t(27*c^2*d^3*z^3 - 27*b^2*c^2*d^2*z^2 + 18*a^3*b*c*d^2*z + 9*b^4*c^2*d*z +
 2*a^3*b^3*c*d - b^6*c^2 - a^6*d^2, z, k), k, 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{c+dx^3}\,dx = \frac{-2d^{\frac{4}{3}}c^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^2 - 4\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)abcd - d^{\frac{4}{3}}c^{\frac{2}{3}}\log\left(c^{\frac{2}{3}} - d^{\frac{1}{3}}c^{\frac{1}{3}}x + d^{\frac{2}{3}}x^2\right)a^2 + 2d^{\frac{4}{3}}c^{\frac{2}{3}}\mathrm{lo}}{}$$

input
```
int((b*x+a)^2/(d*x^3+c),x)
```

output
```
( - 2*d**(1/3)*c**(2/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*s
qrt(3)))*a**2*d - 4*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*sqrt(
3)))*a*b*c*d - d**(1/3)*c**(2/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(
2/3)*x**2)*a**2*d + 2*d**(1/3)*c**(2/3)*log(c**(1/3) + d**(1/3)*x)*a**2*d
+ 2*d**(2/3)*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*
b**2*c + 2*d**(2/3)*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*b**2*c + 2*log(c**
(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b*c*d - 4*log(c**(1/3) + d*
*(1/3)*x)*a*b*c*d)/(6*d**(2/3)*c**(1/3)*c*d)
```

3.3 $\int \frac{(a+bx)^3}{c+dx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 222

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} + \frac{\left(b^3c-3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{\left(b^3c+3a^2b\sqrt[3]{c}d^{2/3}-a^3d\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c+dx^3\right)}{d}$$

output

```
b^3*x/d+1/3*(b^3*c-3*a^2*b*c^(1/3)*d^(2/3)-a^3*d)*arctan(1/3*(c^(1/3)-2*d^
(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2)/c^(2/3)/d^(4/3)-1/3*(b^3*c+3*a^2*b*c^(1/
3)*d^(2/3)-a^3*d)*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(4/3)+1/6*(b^3*c+3*a^2*b
*c^(1/3)*d^(2/3)-a^3*d)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)/
d^(4/3)+a*b^2*ln(d*x^3+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$
$$= \frac{6b^3c^{2/3}\sqrt[3]{d}x + 2\sqrt{3}\left(b^3c - 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) - 2\left(b^3c + 3a^2b\sqrt[3]{c}d^{2/3} - a^3d\right)\log\left(\sqrt[3]{c}\right.}{6c^{2/3}d^{4/3}}$$

input
```
Integrate[(a + b*x)^3/(c + d*x^3),x]
```

output
```
(6*b^3*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^
3*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] - 2*(b^3*c + 3*a^2*b*c^(1
/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x] + (b^3*c + 3*a^2*b*c^(1/3)*d
^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2] + 6*a*b^2*c
^(2/3)*d^(1/3)*Log[c + d*x^3])/(6*c^(2/3)*d^(4/3))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$
$$\Big\downarrow \; 2426$$
$$\int \left(\frac{b^3}{d} - \frac{a^3(-d) - 3a^2bdx - 3ab^2dx^2 + b^3c}{d\left(c+dx^3\right)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\left(a^3(-d) - 3a^2b\sqrt[3]{c}d^{2/3} + b^3c\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{\left(a^3(-d) + 3a^2b\sqrt[3]{c}d^{2/3} + b^3c\right)\log\left(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{\left(a^3(-d) + 3a^2b\sqrt[3]{c}d^{2/3} + b^3c\right)\log\left(\sqrt[3]{c} + \sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{ab^2\log\left(c + dx^3\right)}{d} + \frac{b^3x}{d}$$

input
```
Int[(a + b*x)^3/(c + d*x^3),x]
```

output
```
(b^3*x)/d + ((b^3*c - 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*ArcTan[(c^(1/3) - 2
*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(4/3)) - ((b^3*c + 3*a^
2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(4/3))
 + ((b^3*c + 3*a^2*b*c^(1/3)*d^(2/3) - a^3*d)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (a*b^2*Log[c + d*x^3])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.30

method	result
risch	$\frac{b^3 x}{d}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3 d+c\right)}\frac{\left(3\_R^2 a\,b^2 d+3\_R\,a^2 b d+a^3 d-b^3 c\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^2}$
default	$\frac{b^3 x}{d}+\frac{\left(a^3 d-b^3 c\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+3a^2bd\left(-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\right.\right.}{6}\right.}{d}$

input
```
int((b*x+a)^3/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
b^3*x/d+1/3/d^2*sum((3*_R^2*a*b^2*d+3*_R*a^2*b*d+a^3*d-b^3*c)/_R^2*ln(x-_R
),_R=RootOf(_Z^3*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 7245, normalized size of antiderivative = 32.64

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3/(d*x^3+c),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} + \text{RootSum}\left(27t^3c^2d^4 - 81t^2ab^2c^2d^3 + t\left(27a^5bcd^3 + 54a^2b^4c^2d^2\right) - a^9d^3 + 3a^6b^3cd^2 - 3a^3b^6c^2d + b^9c^3,\right.$$

input
```
integrate((b*x+a)**3/(d*x**3+c),x)
```

output
```
b**3*x/d + RootSum(27*_t**3*c**2*d**4 - 81*_t**2*a*b**2*c**2*d**3 + _t*(27
*a**5*b*c*d**3 + 54*a**2*b**4*c**2*d**2) - a**9*d**3 + 3*a**6*b**3*c*d**2
- 3*a**3*b**6*c**2*d + b**9*c**3, Lambda(_t, _t*log(x + (27*_t**2*a**2*b*c
**2*d**3 + 3*_t*a**6*c*d**3 - 60*_t*a**3*b**3*c**2*d**2 + 3*_t*b**6*c**3*d
 + 15*a**7*b**2*c*d**2 + 15*a**4*b**5*c**2*d - 3*a*b**8*c**3)/(a**9*d**3 +
 24*a**6*b**3*c*d**2 + 3*a**3*b**6*c**2*d - b**9*c**3))))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} - \frac{\sqrt{3}\left(\left(b^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + 2\,ab^2\right)c - \left(3\,a^2b\left(\frac{c}{d}\right)^{\frac{2}{3}} + a^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + \frac{2\,ab^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd} + \frac{\left(b^3c + \left(6\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}} + 3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}} - a^3\right)d\right)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{\left(b^3c - \left(3\,ab^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - 3\,a^2b\left(\frac{c}{d}\right)^{\frac{1}{3}} + a^3\right)d\right)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^3/(d*x^3+c),x, algorithm="maxima")
```

output
```
b^3*x/d - 1/3*sqrt(3)*((b^3*(c/d)^(1/3) + 2*a*b^2)*c - (3*a^2*b*(c/d)^(2/3
) + a^3*(c/d)^(1/3) + 2*a*b^2*c/d)*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3
))/(c/d)^(1/3))/(c*d) + 1/6*(b^3*c + (6*a*b^2*(c/d)^(2/3) + 3*a^2*b*(c/d)^
(1/3) - a^3)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) -
 1/3*(b^3*c - (3*a*b^2*(c/d)^(2/3) - 3*a^2*b*(c/d)^(1/3) + a^3)*d)*log(x +
 (c/d)^(1/3))/(d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx = \frac{b^3x}{d} + \frac{ab^2\log\left(|dx^3+c|\right)}{d} + \frac{\sqrt{3}\left(b^3c - a^3d + 3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}} + \frac{\left(b^3c - a^3d - 3\left(-cd^2\right)^{\frac{1}{3}}a^2b\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}} - \frac{\left(3a^2bd^3\left(-\frac{c}{d}\right)^{\frac{1}{3}} - b^3cd^2 + a^3d^3\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3cd^3}$$

input
```
integrate((b*x+a)^3/(d*x^3+c),x, algorithm="giac")
```

output
```
b^3*x/d + a*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(b^3*c - a^3*d + 3*(-c
*d^2)^(1/3)*a^2*b)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(
-c*d^2)^(2/3) + 1/6*(b^3*c - a^3*d - 3*(-c*d^2)^(1/3)*a^2*b)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(-c*d^2)^(2/3) - 1/3*(3*a^2*b*d^3*(-c/d)^(1/3)
 - b^3*c*d^2 + a^3*d^3)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\mathrm{root}\left(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-\right.\right.\right.$$
$$+x\left(6\,d\,a^4\,b^2+3\,c\,a\,b^5\right)+6\,a^2\,b^4\,c+3\,a^5\,b\,d\big)\ \mathrm{root}\big(27\,c^2\,d^4\,z^3-81\,a\,b^2\,c^2\,d^3\,z^2$$
$$\left.+54\,a^2\,b^4\,c^2\,d^2\,z+27\,a^5\,b\,c\,d^3\,z+3\,a^6\,b^3\,c\,d^2-3\,a^3\,b^6\,c^2\,d+b^9\,c^3-a^9\,d^3,z,k\big)\right)+\frac{b^3\,x}{d}$$

input
```
int((a + b*x)^3/(c + d*x^3),x)
```

output
```
symsum(log(root(27*c^2*d^4*z^3 - 81*a*b^2*c^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2
*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3*a^3*b^6*c^2*d + b^9*c^3 - a^9*
d^3, z, k)*(x*(3*a^3*d^2 - 3*b^3*c*d) + 9*root(27*c^2*d^4*z^3 - 81*a*b^2*c
^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^2 - 3
*a^3*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k)*c*d^2 - 18*a*b^2*c*d) + x*(6*a^4
*b^2*d + 3*a*b^5*c) + 6*a^2*b^4*c + 3*a^5*b*d)*root(27*c^2*d^4*z^3 - 81*a*
b^2*c^2*d^3*z^2 + 54*a^2*b^4*c^2*d^2*z + 27*a^5*b*c*d^3*z + 3*a^6*b^3*c*d^
2 - 3*a^3*b^6*c^2*d + b^9*c^3 - a^9*d^3, z, k), k, 1, 3) + (b^3*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^3}{c+dx^3}\,dx$$
$$= \frac{-2d^{\frac{4}{3}}c^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^3+2d^{\frac{1}{3}}c^{\frac{5}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)b^3-6\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^2bcd-d^{\frac{4}{3}}c^{\frac{2}{3}}\log\left(c^{\frac{2}{3}}\right.}{}$$

input
```
int((b*x+a)^3/(d*x^3+c),x)
```

output
```
( - 2*d**(1/3)*c**(2/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*s
qrt(3)))*a**3*d + 2*d**(1/3)*c**(2/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*
x)/(c**(1/3)*sqrt(3)))*b**3*c - 6*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(
c**(1/3)*sqrt(3)))*a**2*b*c*d - d**(1/3)*c**(2/3)*log(c**(2/3) - d**(1/3)*
c**(1/3)*x + d**(2/3)*x**2)*a**3*d + d**(1/3)*c**(2/3)*log(c**(2/3) - d**(
1/3)*c**(1/3)*x + d**(2/3)*x**2)*b**3*c + 2*d**(1/3)*c**(2/3)*log(c**(1/3)
 + d**(1/3)*x)*a**3*d - 2*d**(1/3)*c**(2/3)*log(c**(1/3) + d**(1/3)*x)*b**
3*c + 6*d**(2/3)*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x*
*2)*a*b**2*c + 6*d**(2/3)*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*a*b**2*c + 6
*d**(2/3)*c**(1/3)*b**3*c*x + 3*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2
/3)*x**2)*a**2*b*c*d - 6*log(c**(1/3) + d**(1/3)*x)*a**2*b*c*d)/(6*d**(2/3
)*c**(1/3)*c*d)
```

3.4 $\int \frac{(a+bx)^4}{c+dx^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 280

$$\int \frac{(a+bx)^4}{c+dx^3} dx = \frac{4ab^3x}{d} + \frac{b^4x^2}{2d} + \frac{\left(b^4c^{4/3} + 4ab^3c\sqrt[3]{d} - 4a^3b\sqrt[3]{c}d - a^4d^{4/3}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}} + \frac{\left(b\sqrt[3]{c}(b^3c-4a^3d) - a\sqrt[3]{d}(4b^3c-a^3d)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}} + \frac{\left(4ab^3c - a^4d - \frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} + \frac{2a^2b^2\log(c+dx^3)}{d}$$

output
```
4*a*b^3*x/d+1/2*b^4*x^2/d+1/3*(b^4*c^(4/3)+4*a*b^3*c*d^(1/3)-4*a^3*b*c^(1/
3)*d-a^4*d^(4/3))*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)*3^(1/2)/c^(1/3))*3^(1/2
)/c^(2/3)/d^(5/3)+1/3*(b*c^(1/3)*(-4*a^3*d+b^3*c)-a*d^(1/3)*(-a^3*d+4*b^3*
c))*ln(c^(1/3)+d^(1/3)*x)/c^(2/3)/d^(5/3)+1/6*(4*a*b^3*c-a^4*d-b*c^(1/3)*(
-4*a^3*d+b^3*c)/d^(1/3))*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(2/3)
/d^(4/3)+2*a^2*b^2*ln(d*x^3+c)/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$

$$= \frac{24ab^3d^{2/3}x + 3b^4d^{2/3}x^2 + \frac{2\sqrt{3}\left(b^4c^{4/3}+4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}d-a^4d^{4/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}} + \frac{2\left(b^4c^{4/3}-4ab^3c\sqrt[3]{d}-4a^3b\sqrt[3]{c}\right.}{c^{2/}}}{6}$$

input
```
Integrate[(a + b*x)^4/(c + d*x^3),x]
```

output
```
(24*a*b^3*d^(2/3)*x + 3*b^4*d^(2/3)*x^2 + (2*Sqrt[3]*(b^4*c^(4/3) + 4*a*b^
3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/c^(2/3) + (2*(b^4*c^(4/3) - 4*a*b^3*c*d^(1/3) - 4*a^3*b*
c^(1/3)*d + a^4*d^(4/3))*Log[c^(1/3) + d^(1/3)*x])/c^(2/3) - ((b^4*c^(4/3)
 - 4*a*b^3*c*d^(1/3) - 4*a^3*b*c^(1/3)*d + a^4*d^(4/3))*Log[c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3) + 12*a^2*b^2*d^(2/3)*Log[c + d*x^3])
/(6*d^(5/3))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$

$$\Big\downarrow \; 2426$$

$$\int \left(-\frac{a^4(-d)+bx\left(b^3c-4a^3d\right)-6a^2b^2dx^2+4ab^3c}{d\left(c+dx^3\right)}+\frac{4ab^3}{d}+\frac{b^4x}{d}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{2a^2b^2\log(c+dx^3)}{d}+\frac{\left(a^4\left(-d^{4/3}\right)-4a^3b\sqrt[3]{c}d+4ab^3c\sqrt[3]{d}+b^4c^{4/3}\right)\arctan\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{5/3}}+\frac{\left(a^4(-d)-\frac{b\sqrt[3]{c}(b^3c-4a^3d)}{\sqrt[3]{d}}+4ab^3c\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}+\frac{\left(b\sqrt[3]{c}(b^3c-4a^3d)-\sqrt[3]{d}(4ab^3c-a^4d)\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{5/3}}+\frac{4ab^3x}{d}+\frac{b^4x^2}{2d}$$

input
```
Int[(a + b*x)^4/(c + d*x^3),x]
```

output
```
(4*a*b^3*x)/d + (b^4*x^2)/(2*d) + ((b^4*c^(4/3) + 4*a*b^3*c*d^(1/3) - 4*a^
3*b*c^(1/3)*d - a^4*d^(4/3))*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/
3))])/(Sqrt[3]*c^(2/3)*d^(5/3)) + ((b*c^(1/3)*(b^3*c - 4*a^3*d) - d^(1/3)*
(4*a*b^3*c - a^4*d))*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(5/3)) + ((4*a
*b^3*c - a^4*d - (b*c^(1/3)*(b^3*c - 4*a^3*d))/d^(1/3))*Log[c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3)) + (2*a^2*b^2*Log[c + d*x
^3])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.32

method	result
risch	$\frac{b^4x^2}{2d}+\frac{4ab^3x}{d}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3d+c)}\frac{\left(6a^2b^2d\_R^2+b\left(4a^3d-b^3c\right)\_R+a^4d-4a\,b^3c\right)\ln\left(x-\_R\right)}{\_R^2}}{3d^2}$
default	$\frac{b^3\left(\frac{1}{2}b\,x^2+4ax\right)}{d}+\frac{\left(a^4d-4a\,b^3c\right)\left(\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3d\left(\frac{c}{d}\right)^{\frac{2}{3}}}\right)+\left(4a^3bd-b^4c\right)\left(-\frac{\ln(}{}\right.}{d}$

input
```
int((b*x+a)^4/(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^4*x^2/d+4*a*b^3*x/d+1/3/d^2*sum((6*a^2*b^2*d*_R^2+b*(4*a^3*d-b^3*c)*
_R+a^4*d-4*a*b^3*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.52 (sec) , antiderivative size = 8787, normalized size of antiderivative = 31.38

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^4/(d*x^3+c),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \frac{4ab^3x}{d} + \frac{b^4x^2}{2d}$$
$$+\mathrm{RootSum}\Big(27t^3c^2d^5 - 162t^2a^2b^2c^2d^4 + t\big(36a^7bcd^4 + 171a^4b^4c^2d^3 + 36ab^7c^3d^2\big) - a^{12}d^4 + 4a^9b^3cd^3 -$$

input
```
integrate((b*x+a)**4/(d*x**3+c),x)
```

output
```
4*a*b**3*x/d + b**4*x**2/(2*d) + RootSum(27*_t**3*c**2*d**5 - 162*_t**2*a*
*2*b**2*c**2*d**4 + _t*(36*a**7*b*c*d**4 + 171*a**4*b**4*c**2*d**3 + 36*a*
b**7*c**3*d**2) - a**12*d**4 + 4*a**9*b**3*c*d**3 - 6*a**6*b**6*c**2*d**2
+ 4*a**3*b**9*c**3*d - b**12*c**4, Lambda(_t, _t*log(x + (36*_t**2*a**3*b*
c**2*d**4 - 9*_t**2*b**4*c**3*d**3 + 3*_t*a**8*c*d**4 - 168*_t*a**5*b**3*c
**2*d**3 + 84*_t*a**2*b**6*c**3*d**2 + 26*a**10*b**2*c*d**3 + 48*a**7*b**5
*c**2*d**2 - 66*a**4*b**8*c**3*d - 8*a*b**11*c**4)/(a**12*d**4 + 52*a**9*b
**3*c*d**3 - 52*a**3*b**9*c**3*d - b**12*c**4))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx =$$
$$-\frac{\sqrt{3}\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{2}{3}} + 4\,ab^3\left(\frac{c}{d}\right)^{\frac{1}{3}} + 4\,a^2b^2\right)c - \left(4\,a^3b\left(\frac{c}{d}\right)^{\frac{2}{3}} + a^4\left(\frac{c}{d}\right)^{\frac{1}{3}} + \frac{4\,a^2b^2c}{d}\right)d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd}$$
$$+\frac{b^4x^2 + 8\,ab^3x}{2\,d}$$
$$-\frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}} - 4\,ab^3\right)c - \left(12\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}} + 4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}} - a^4\right)d\right)\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(\left(b^4\left(\frac{c}{d}\right)^{\frac{1}{3}} - 4\,ab^3\right)c + \left(6\,a^2b^2\left(\frac{c}{d}\right)^{\frac{2}{3}} - 4\,a^3b\left(\frac{c}{d}\right)^{\frac{1}{3}} + a^4\right)d\right)\log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*x+a)^4/(d*x^3+c),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*((b^4*(c/d)^(2/3) + 4*a*b^3*(c/d)^(1/3) + 4*a^2*b^2)*c - (4*a
^3*b*(c/d)^(2/3) + a^4*(c/d)^(1/3) + 4*a^2*b^2*c/d)*d)*arctan(1/3*sqrt(3)*
(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d) + 1/2*(b^4*x^2 + 8*a*b^3*x)/d - 1/6
*((b^4*(c/d)^(1/3) - 4*a*b^3)*c - (12*a^2*b^2*(c/d)^(2/3) + 4*a^3*b*(c/d)^
(1/3) - a^4)*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(d^2*(c/d)^(2/3)) +
 1/3*((b^4*(c/d)^(1/3) - 4*a*b^3)*c + (6*a^2*b^2*(c/d)^(2/3) - 4*a^3*b*(c/
d)^(1/3) + a^4)*d)*log(x + (c/d)^(1/3))/(d^2*(c/d)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \frac{2\,a^2b^2\log\left(|dx^3+c|\right)}{d} + \frac{\sqrt{3}\left(4\,ab^3cd - a^4d^2 - \left(-cd^2\right)^{\frac{1}{3}}b^4c + 4\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}d} + \frac{\left(4\,ab^3cd - a^4d^2 + \left(-cd^2\right)^{\frac{1}{3}}b^4c - 4\left(-cd^2\right)^{\frac{1}{3}}a^3bd\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}d} + \frac{b^4dx^2 + 8\,ab^3dx}{2\,d^2} + \frac{\left(b^4cd^4\left(-\frac{c}{d}\right)^{\frac{1}{3}} - 4\,a^3bd^5\left(-\frac{c}{d}\right)^{\frac{1}{3}} + 4\,ab^3cd^4 - a^4d^5\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^5}$$

input
```
integrate((b*x+a)^4/(d*x^3+c),x, algorithm="giac")
```

output
```
2*a^2*b^2*log(abs(d*x^3 + c))/d + 1/3*sqrt(3)*(4*a*b^3*c*d - a^4*d^2 - (-c
*d^2)^(1/3)*b^4*c + 4*(-c*d^2)^(1/3)*a^3*b*d)*arctan(1/3*sqrt(3)*(2*x + (-
c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) + 1/6*(4*a*b^3*c*d - a^4*d^2
+ (-c*d^2)^(1/3)*b^4*c - 4*(-c*d^2)^(1/3)*a^3*b*d)*log(x^2 + x*(-c/d)^(1/3
) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) + 1/2*(b^4*d*x^2 + 8*a*b^3*d*x)/d^2 +
 1/3*(b^4*c*d^4*(-c/d)^(1/3) - 4*a^3*b*d^5*(-c/d)^(1/3) + 4*a*b^3*c*d^4 -
a^4*d^5)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^5)
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx$$
$$= \left(\sum_{k=1}^{3}\ln\left(\mathrm{root}\left(27\,c^2\,d^5\,z^3-162\,a^2\,b^2\,c^2\,d^4\,z^2+171\,a^4\,b^4\,c^2\,d^3\,z+36\,a\,b^7\,c^3\,d^2\,z+36\,a^7\,b\,c\,d^4\,z-6\,a^6\,b^6\,c^2\,d^2+\right.\right.\right.$$
$$+\frac{4\,a^7\,b\,d^2+19\,a^4\,b^4\,c\,d+4\,a\,b^7\,c^2}{d}$$
$$\left.+\frac{x\,(10\,a^6\,b^2\,d^2+16\,a^3\,b^5\,c\,d+b^8\,c^2)}{d}\right)\mathrm{root}\left(27\,c^2\,d^5\,z^3-162\,a^2\,b^2\,c^2\,d^4\,z^2\right.$$
$$+171\,a^4\,b^4\,c^2\,d^3\,z+36\,a\,b^7\,c^3\,d^2\,z+36\,a^7\,b\,c\,d^4\,z-6\,a^6\,b^6\,c^2\,d^2+4\,a^9\,b^3\,c\,d^3$$
$$\left.\left.+4\,a^3\,b^9\,c^3\,d-b^{12}\,c^4-a^{12}\,d^4,z,k\right)\right)+\frac{b^4\,x^2}{2\,d}+\frac{4\,a\,b^3\,x}{d}$$

input
```
int((a + b*x)^4/(c + d*x^3),x)
```

output
```
symsum(log(root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 171*a^4*b^4*c^2
*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^2*d^2 + 4*a^9
*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*((x*(3*a^4*d^3 -
 12*a*b^3*c*d^2))/d + 9*root(27*c^2*d^5*z^3 - 162*a^2*b^2*c^2*d^4*z^2 + 17
1*a^4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7*b*c*d^4*z - 6*a^6*b^6*c^
2*d^2 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*c^4 - a^12*d^4, z, k)*c*d
^2 - 36*a^2*b^2*c*d) + (4*a*b^7*c^2 + 4*a^7*b*d^2 + 19*a^4*b^4*c*d)/d + (x
*(b^8*c^2 + 10*a^6*b^2*d^2 + 16*a^3*b^5*c*d))/d)*root(27*c^2*d^5*z^3 - 162
*a^2*b^2*c^2*d^4*z^2 + 171*a^4*b^4*c^2*d^3*z + 36*a*b^7*c^3*d^2*z + 36*a^7
*b*c*d^4*z - 6*a^6*b^6*c^2*d^2 + 4*a^9*b^3*c*d^3 + 4*a^3*b^9*c^3*d - b^12*
c^4 - a^12*d^4, z, k), k, 1, 3) + (b^4*x^2)/(2*d) + (4*a*b^3*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx)^4}{c+dx^3}\,dx = \frac{-2d^{\frac{4}{3}}c^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^4+8d^{\frac{1}{3}}c^{\frac{5}{3}}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a\,b^3-8\sqrt{3}\,atan\left(\frac{c^{\frac{1}{3}}-2d^{\frac{1}{3}}x}{c^{\frac{1}{3}}\sqrt{3}}\right)a^3bcd+2\sqrt{3}\,atan(}{}$$

input
```
int((b*x+a)^4/(d*x^3+c),x)
```

output
```
( - 2*d**(1/3)*c**(2/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)/(c**(1/3)*s
qrt(3)))*a**4*d + 8*d**(1/3)*c**(2/3)*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*
x)/(c**(1/3)*sqrt(3)))*a*b**3*c - 8*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)
/(c**(1/3)*sqrt(3)))*a**3*b*c*d + 2*sqrt(3)*atan((c**(1/3) - 2*d**(1/3)*x)
/(c**(1/3)*sqrt(3)))*b**4*c**2 - d**(1/3)*c**(2/3)*log(c**(2/3) - d**(1/3)
*c**(1/3)*x + d**(2/3)*x**2)*a**4*d + 4*d**(1/3)*c**(2/3)*log(c**(2/3) - d
**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a*b**3*c + 2*d**(1/3)*c**(2/3)*log(c**
(1/3) + d**(1/3)*x)*a**4*d - 8*d**(1/3)*c**(2/3)*log(c**(1/3) + d**(1/3)*x
)*a*b**3*c + 12*d**(2/3)*c**(1/3)*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**
(2/3)*x**2)*a**2*b**2*c + 12*d**(2/3)*c**(1/3)*log(c**(1/3) + d**(1/3)*x)*
a**2*b**2*c + 24*d**(2/3)*c**(1/3)*a*b**3*c*x + 3*d**(2/3)*c**(1/3)*b**4*c
*x**2 + 4*log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*a**3*b*c*d -
 log(c**(2/3) - d**(1/3)*c**(1/3)*x + d**(2/3)*x**2)*b**4*c**2 - 8*log(c**
(1/3) + d**(1/3)*x)*a**3*b*c*d + 2*log(c**(1/3) + d**(1/3)*x)*b**4*c**2)/(
6*d**(2/3)*c**(1/3)*c*d)
```

3.5 $\int \frac{c+dx}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 189

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

output
```
1/3*x*(d*x+c)/a/(b*x^3+a)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(2/3)+1/9*(2*b^(1/3)*c-a^(
1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$$

$$=\frac{\frac{6ax(c+dx)}{a+bx^3}-\frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\mathrm{l}}{}}{18a^2}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^2,x]
```

output
```
((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)
*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b
^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(
1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3
))/(18*a^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2394, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\left(a+bx^3\right)^2}\,dx$$

$$\Big\downarrow \; 2394$$

$$\frac{x(c+dx)}{3a\left(a+bx^3\right)}-\frac{\int -\frac{2c+dx}{bx^3+a}dx}{3a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2c+dx}{bx^3+a}dx}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2399$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\frac{-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

input
```
Int[(c + d*x)/(a + b*x^3)^2,x]
```

output
```
(x*(c + d*x))/(3*a*(a + b*x^3)) + (((2*c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(2*b^(1/3)*c + a^(1/3)*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((2*c - (a^(1/3)
*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)
*b^(1/3)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{d\,x^2}{3a}+\frac{cx}{3a}}{b\,x^3+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3 b+a\right)}\frac{\left(d\_R+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b\,x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b\,x^3+a)}+\frac{-\frac{\ln\left(x+\right.}{3b\left(\right.}}{}\right.$

input
```
int((d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*d/a*x^2+1/3*c/a*x)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 2088, normalized size of antiderivative = 11.05

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/
(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)
/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2
))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(
3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))
^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3
 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((
8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt
(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^
3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*
b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3)...
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.56

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \text{RootSum}\left(729t^3a^5b^2+54ta^2bcd+ad^3-8bc^3,\left(t\mapsto t\log\left(x+\frac{81t^2a^4bd+36ta^2bc^2+4acd^2}{ad^3+8bc^3}\right)\right)\right) + \frac{cx+dx^2}{3a^2+3abx^3}$$

input
```
integrate((d*x+c)/(b*x**3+a)**2,x)
```

output
```
RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda
(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d*
*3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = \frac{dx^2+cx}{3\,(abx^3+a^2)} + \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(
d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/
3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(2\,bc-\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(2\,bc+\left(-ab^2\right)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{2}{3}}\,a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2+cx}{3\,(bx^3+a)a}$$

input
```
integrate((d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)
^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c
*x)/((b*x^3 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{b\left(2\,c\,d+d^2\,x+\mathrm{root}(729\,a^5\,b^2\,z^3+54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k\right)^2\,a^3\,b\,81+\mathrm{root}(729\,a^5\,b^2\,z}{a^2\,9}\right.\right.$$
$$\left.\left. +54\,a^2\,b\,c\,d\,z-8\,b\,c^3+a\,d^3,z,k\right)\right)+\frac{\frac{d\,x^2}{3\,a}+\frac{c\,x}{3\,a}}{b\,x^3+a}$$

input
```
int((c + d*x)/(a + b*x^3)^2,x)
```

output
```
symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z
- 8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*
b*c*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))
/(a + b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.86

$$\int \frac{c+dx}{(a+bx^3)^2}\,dx$$
$$= \frac{-4b^{\frac{1}{3}}a^{\frac{5}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)c-4b^{\frac{4}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)c\,x^3-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2d-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}}{a}\right.}{}$$

input
```
int((d*x+c)/(b*x^3+a)^2,x)
```

output
```
( - 4*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a*c - 4*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/
(a**(1/3)*sqrt(3)))*b*c*x**3 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a
**(1/3)*sqrt(3)))*a**2*d - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1
/3)*sqrt(3)))*a*b*d*x**3 - 2*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**
(1/3)*x + b**(2/3)*x**2)*a*c - 2*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)
*a**(1/3)*x + b**(2/3)*x**2)*b*c*x**3 + 4*b**(1/3)*a**(2/3)*log(a**(1/3) +
 b**(1/3)*x)*a*c + 4*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*b*c*x**3
 + 6*b**(2/3)*a**(1/3)*a*c*x + 6*b**(2/3)*a**(1/3)*a*d*x**2 + log(a**(2/3)
 - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*d + log(a**(2/3) - b**(1/3)*a
**(1/3)*x + b**(2/3)*x**2)*a*b*d*x**3 - 2*log(a**(1/3) + b**(1/3)*x)*a**2*
d - 2*log(a**(1/3) + b**(1/3)*x)*a*b*d*x**3)/(18*b**(2/3)*a**(1/3)*a**2*(a
 + b*x**3))
```

3.6 $\int \frac{a+bx}{(1+x^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{x(a+bx)}{3(1+x^3)} - \frac{(2a+b)\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{9}(2a-b)\log(1+x) - \frac{1}{18}(2a-b)\log\left(1-x+x^2\right)$$

output
```
x*(b*x+a)/(3*x^3+3)-1/9*(2*a+b)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/9*(2
*a-b)*ln(1+x)-1/18*(2*a-b)*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{1}{18}\left(\frac{6x(a+bx)}{1+x^3} + 2\sqrt{3}(2a+b)\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 2(2a-b)\log(1+x) + (-2a+b)\log\left(1-x+x^2\right)\right)$$

input
```
Integrate[(a + b*x)/(1 + x^3)^2,x]
```

output
```
((6*x*(a + b*x))/(1 + x^3) + 2*Sqrt[3]*(2*a + b)*ArcTan[(-1 + 2*x)/Sqrt[3]
] + 2*(2*a - b)*Log[1 + x] + (-2*a + b)*Log[1 - x + x^2])/18
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2394, 25, 2399, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(x^3+1)^2}dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(a+bx)}{3(x^3+1)}-\frac{1}{3}\int -\frac{2a+bx}{x^3+1}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{2a+bx}{x^3+1}dx+\frac{x(a+bx)}{3(x^3+1)}$$

$$\Big\downarrow \text{2399}$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{4a+b-(2a-b)x}{x^2-x+1}dx+\frac{1}{3}(2a-b)\int \frac{1}{x+1}dx\right)+\frac{x(a+bx)}{3(x^3+1)}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{3}\int \frac{4a+b-(2a-b)x}{x^2-x+1}dx+\frac{1}{3}(2a-b)\log(x+1)\right)+\frac{x(a+bx)}{3(x^3+1)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}(2a+b)\int \frac{1}{x^2-x+1}dx-\frac{1}{2}(2a-b)\int -\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}(2a-b)\log(x+1)\right)+\frac{x(a+bx)}{3(x^3+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{3}{2}(2a+b)\int\frac{1}{x^2-x+1}dx+\frac{1}{2}(2a-b)\int\frac{1-2x}{x^2-x+1}dx\right)+\frac{1}{3}(2a-b)\log(x+1)\right)+\frac{x(a+bx)}{3(x^3+1)}$$

$\Big\downarrow$ 1083

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}(2a-b)\int\frac{1-2x}{x^2-x+1}dx-3(2a+b)\int\frac{1}{-(2x-1)^2-3}d(2x-1)\right)+\frac{1}{3}(2a-b)\log(x+1)\right)+\frac{x(a+bx)}{3(x^3+1)}$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(\frac{1}{3}\left(\frac{1}{2}(2a-b)\int\frac{1-2x}{x^2-x+1}dx+\sqrt{3}(2a+b)\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)+\frac{1}{3}(2a-b)\log(x+1)\right)+\frac{x(a+bx)}{3(x^3+1)}$$

$\Big\downarrow$ 1103

$$\frac{1}{3}\left(\frac{1}{3}\left(\sqrt{3}(2a+b)\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{1}{2}(2a-b)\log\left(x^2-x+1\right)\right)+\frac{1}{3}(2a-b)\log(x+1)\right)+\frac{x(a+bx)}{3(x^3+1)}$$

input
```
Int[(a + b*x)/(1 + x^3)^2,x]
```

output
```
(x*(a + b*x))/(3*(1 + x^3)) + (((2*a - b)*Log[1 + x])/3 + (Sqrt[3]*(2*a +
b)*ArcTan[(-1 + 2*x)/Sqrt[3]] - ((2*a - b)*Log[1 - x + x^2])/2)/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

method	result
risch	$\frac{\frac{1}{3} b\, x^{2}+\frac{1}{3} a x}{x^{3}+1}-\frac{\ln \left(x +1\right) b}{9}+\frac{2 \ln \left(x +1\right) a}{9}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\_Z^{2}+\left(2 a -b \right) \_Z +4 a^{2}+2 a b +b^{2}\right)}{\_R \ln \left(\left(2 \_R a +b^{2}\right) x +\_R^{2}+2 a b \right)}\right.}{9}$
default	$-\frac{\left(-2 b -a \right) x +b -a}{9 \left(x^{2}-x +1\right)}-\frac{\left(2 a -b \right) \ln \left(x^{2}-x +1\right)}{18}-\frac{2 \left(-3 a -\frac{3 b}{2}\right) \sqrt{3}\, \arctan \left(\frac{\left(2 x -1\right) \sqrt{3}}{3}\right)}{27}+\left(-\frac{b}{9}+\frac{2 a}{9}\right) \ln \left(x +1\right)-\frac{-\frac{b}{9}+\frac{a}{9}}{x +1}$
meijerg	$\frac{b \left(\frac{3 x^{2}}{3 x^{3}+3}-\frac{x^{2} \ln \left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{3 \left(x^{3}\right)^{\frac{2}{3}}}+\frac{x^{2} \ln \left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{6 \left(x^{3}\right)^{\frac{2}{3}}}+\frac{x^{2} \sqrt{3}\, \arctan \left(\frac{\sqrt{3}\, \left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3 \left(x^{3}\right)^{\frac{2}{3}}}\right)}{3}+\frac{a \left(\frac{3 x}{3 x^{3}+3}+\frac{2 x \ln \left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{3 \left(x^{3}\right)^{\frac{1}{3}}}-\frac{x \ln \left(1-\left(x\right.\right.}{3\left(\right.}\right.}{}$

input
```
int((b*x+a)/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/3*b*x^2+1/3*a*x)/(x^3+1)-1/9*ln(x+1)*b+2/9*ln(x+1)*a+1/9*sum(_R*ln((2*_
R*a+b^2)*x+_R^2+2*a*b),_R=RootOf(_Z^2+(2*a-b)*_Z+4*a^2+2*a*b+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.30

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{6\,bx^2+2\,\sqrt{3}((2\,a+b)x^3+2\,a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+6\,ax-((2\,a-b)x^3+2\,a-b)\log\left(x^2-x+\right.}{18\,(x^3+1)}$$

input
```
integrate((b*x+a)/(x^3+1)^2,x, algorithm="fricas")
```

output
```
1/18*(6*b*x^2 + 2*sqrt(3)*((2*a + b)*x^3 + 2*a + b)*arctan(1/3*sqrt(3)*(2*
x - 1)) + 6*a*x - ((2*a - b)*x^3 + 2*a - b)*log(x^2 - x + 1) + 2*((2*a - b
)*x^3 + 2*a - b)*log(x + 1))/(x^3 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.01

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{(2a-b)\log\left(x+\frac{4a^2\cdot(2a-b)+4ab^2+b(2a-b)^2}{8a^3+b^3}\right)}{9} + \left(-\frac{a}{9}+\frac{b}{18} - \frac{\sqrt{3}i(2a+b)}{18}\right)\log\left(x+\frac{36a^2\left(-\frac{a}{9}+\frac{b}{18}-\frac{\sqrt{3}i(2a+b)}{18}\right)+4ab^2+81b\left(-\frac{a}{9}+\frac{b}{18}-\frac{\sqrt{3}i(2a+b)}{18}\right)^2}{8a^3+b^3}\right) + \left(-\frac{a}{9}+\frac{b}{18} + \frac{\sqrt{3}i(2a+b)}{18}\right)\log\left(x+\frac{36a^2\left(-\frac{a}{9}+\frac{b}{18}+\frac{\sqrt{3}i(2a+b)}{18}\right)+4ab^2+81b\left(-\frac{a}{9}+\frac{b}{18}+\frac{\sqrt{3}i(2a+b)}{18}\right)^2}{8a^3+b^3}\right) + \frac{ax+bx^2}{3x^3+3}$$

input
```
integrate((b*x+a)/(x**3+1)**2,x)
```

output
```
(2*a - b)*log(x + (4*a**2*(2*a - b) + 4*a*b**2 + b*(2*a - b)**2)/(8*a**3 +
 b**3))/9 + (-a/9 + b/18 - sqrt(3)*I*(2*a + b)/18)*log(x + (36*a**2*(-a/9
+ b/18 - sqrt(3)*I*(2*a + b)/18) + 4*a*b**2 + 81*b*(-a/9 + b/18 - sqrt(3)*
I*(2*a + b)/18)**2)/(8*a**3 + b**3)) + (-a/9 + b/18 + sqrt(3)*I*(2*a + b)/
18)*log(x + (36*a**2*(-a/9 + b/18 + sqrt(3)*I*(2*a + b)/18) + 4*a*b**2 + 8
1*b*(-a/9 + b/18 + sqrt(3)*I*(2*a + b)/18)**2)/(8*a**3 + b**3)) + (a*x + b
*x**2)/(3*x**3 + 3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{1}{9}\sqrt{3}(2\,a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{18}\,(2\,a-b)\log\left(x^2-x+1\right) + \frac{1}{9}\,(2\,a-b)\log\left(x+1\right) + \frac{bx^2+ax}{3\,(x^3+1)}$$

input

```
integrate((b*x+a)/(x^3+1)^2,x, algorithm="maxima")
```

output

```
1/9*sqrt(3)*(2*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/18*(2*a - b)*log(x
^2 - x + 1) + 1/9*(2*a - b)*log(x + 1) + 1/3*(b*x^2 + a*x)/(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{1}{9}\sqrt{3}(2\,a+b)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) - \frac{1}{18}\,(2\,a-b)\log\left(x^2-x+1\right) + \frac{1}{9}\,(2\,a-b)\log\left(|x+1|\right) + \frac{bx^2+ax}{3\,(x^3+1)}$$

input

```
integrate((b*x+a)/(x^3+1)^2,x, algorithm="giac")
```

output

```
1/9*sqrt(3)*(2*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/18*(2*a - b)*log(x
^2 - x + 1) + 1/9*(2*a - b)*log(abs(x + 1)) + 1/3*(b*x^2 + a*x)/(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{\frac{b\,x^2}{3}+\frac{a\,x}{3}}{x^3+1} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{9}-\frac{b}{18}+\frac{\sqrt{3}\,a\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,b\,1\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{b}{18}-\frac{a}{9}+\frac{\sqrt{3}\,a\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,b\,1\mathrm{i}}{18}\right) + \ln(x+1)\left(\frac{2\,a}{9}-\frac{b}{9}\right)$$

input
```
int((a + b*x)/(x^3 + 1)^2,x)
```

output
```
((a*x)/3 + (b*x^2)/3)/(x^3 + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*(a/9 - b/1
8 + (3^(1/2)*a*1i)/9 + (3^(1/2)*b*1i)/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*
(b/18 - a/9 + (3^(1/2)*a*1i)/9 + (3^(1/2)*b*1i)/18) + log(x + 1)*((2*a)/9
- b/9)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.24

$$\int \frac{a+bx}{(1+x^3)^2}\,dx = \frac{4\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)a\,x^3+4\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)a+2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)b\,x^3+2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)b-2\log(x^2-x}{}$$

input
```
int((b*x+a)/(x^3+1)^2,x)
```

output
```
(4*sqrt(3)*atan((2*x - 1)/sqrt(3))*a*x**3 + 4*sqrt(3)*atan((2*x - 1)/sqrt(
3))*a + 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*b*x**3 + 2*sqrt(3)*atan((2*x - 1
)/sqrt(3))*b - 2*log(x**2 - x + 1)*a*x**3 - 2*log(x**2 - x + 1)*a + log(x*
*2 - x + 1)*b*x**3 + log(x**2 - x + 1)*b + 4*log(x + 1)*a*x**3 + 4*log(x +
 1)*a - 2*log(x + 1)*b*x**3 - 2*log(x + 1)*b + 6*a*x + 6*b*x**2)/(18*(x**3
 + 1))
```

3.7 $\int \frac{c+dx}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 215

$$\int \frac{c+dx}{(a+bx^3)^3} \, dx = \frac{x(c+dx)}{6a(a+bx^3)^2} + \frac{x(5c+4dx)}{18a^2(a+bx^3)} - \frac{\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{2/3}} + \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{2/3}} - \frac{\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{2/3}}$$

output
```
1/6*x*(d*x+c)/a/(b*x^3+a)^2+1/18*x*(4*d*x+5*c)/a^2/(b*x^3+a)-1/27*(5*b^(1/
3)*c+2*a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2
)/a^(8/3)/b^(2/3)+1/27*(5*b^(1/3)*c-2*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(
8/3)/b^(2/3)-1/54*(5*b^(1/3)*c-2*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/a^(8/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx$$

$$= \frac{\frac{9a^2x(c+dx)}{(a+bx^3)^2} + \frac{3ax(5c+4dx)}{a+bx^3} - \frac{2\sqrt{3}\sqrt[3]{a}\left(5\sqrt[3]{b}c+2\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{2\left(5\sqrt[3]{a}\sqrt[3]{b}c-2a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{(-\cdots}{}}{54a^3}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^3,x]
```

output
```
((9*a^2*x*(c + d*x))/(a + b*x^3)^2 + (3*a*x*(5*c + 4*d*x))/(a + b*x^3) - (
2*Sqrt[3]*a^(1/3)*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]])/b^(2/3) + (2*(5*a^(1/3)*b^(1/3)*c - 2*a^(2/3)*d)*Log[a^(1
/3) + b^(1/3)*x])/b^(2/3) + ((-5*a^(1/3)*b^(1/3)*c + 2*a^(2/3)*d)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(54*a^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {2394, 25, 2394, 27, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{x(c+dx)}{6a(a+bx^3)^2} - \frac{\int -\frac{5c+4dx}{(bx^3+a)^2}dx}{6a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5c+4dx}{\left(bx^3+a\right)^2}dx}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(5c+4dx)}{3a(a+bx^3)}-\frac{\int -\frac{2(5c+2dx)}{bx^3+a}dx}{3a}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{5c+2dx}{bx^3+a}dx}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2399$$

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}\left(5\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(5\sqrt[3]{b}c-2\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+}{6a}$$

$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\cdots}{6a}$$

$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}}{6a}$$

$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}}{6a}$$

$$\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{2\left(\frac{\frac{1}{2}\sqrt[3]{b}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{2\left(\frac{-\frac{1}{2}\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(2\sqrt[3]{a}d+5\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(5c-\frac{2\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x(5c+4dx)}{3a(a+bx^3)}}{6a}+\frac{x(c+dx)}{6a\left(a+bx^3\right)^2}$$

input

```
Int[(c + d*x)/(a + b*x^3)^3,x]
```

output

```
(x*(c + d*x))/(6*a*(a + b*x^3)^2) + ((x*(5*c + 4*d*x))/(3*a*(a + b*x^3)) +
 (2*(((5*c - (2*a^(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b
^(1/3)) + (-((Sqrt[3]*(5*b^(1/3)*c + 2*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((5*c - (2*a^(1/3)*d)/b^(1/3))*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))))/(3*a))/(6*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{2bd\,x^5}{9a^2}+\frac{5bc\,x^4}{18a^2}+\frac{7d\,x^2}{18a}+\frac{4cx}{9a}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3b+a)}\frac{(2d\_R+5c)\ln(x-\_R)}{\_R^2}}{27b\,a^2}$
default	$c\left(\frac{x}{6a(b\,x^3+a)^2}+\frac{\frac{5x}{18a(b\,x^3+a)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}\right)+d\left(\frac{}{6a(b\right.$

input
```
int((d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2/9*b*d/a^2*x^5+5/18*b/a^2*c*x^4+7/18*d/a*x^2+4/9*c/a*x)/(b*x^3+a)^2+1/27
/b/a^2*sum((2*_R*d+5*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 2215, normalized size of antiderivative = 10.30

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/108*(24*b*d*x^5 + 30*b*c*x^4 + 42*a*d*x^2 + 48*a*c*x - 2*(a^2*b^2*x^6 +
2*a^3*b*x^3 + a^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^
8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*s
qrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3
)/(a^8*b^2))^(1/3)))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*
a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)
*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3
 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^6*b*d - 25/2*((1/2)^(1/3)*(I*sqrt(3) +
1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/
3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^
8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))*a^3*b*c^2 + 40*a*c*d^2 +
 (125*b*c^3 + 8*a*d^3)*x) + ((a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*((1/2)^(1/3
)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)
/(a^8*b^2))^(1/3) - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3
 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3))) + 3*sqrt(
1/3)*(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)
*((125*b*c^3 + 8*a*d^3)/(a^8*b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)
 - 20*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^5*b*((125*b*c^3 + 8*a*d^3)/(a^8*
b^2) + (125*b*c^3 - 8*a*d^3)/(a^8*b^2))^(1/3)))^2*a^5*b + 160*c*d)/(a^5*b)
))*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((125*b*c^3 + 8*a*d^3)/(a^8*b^...
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.68

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx$$
$$= \text{RootSum}\left(19683t^3a^8b^2 + 810ta^3bcd + 8ad^3 - 125bc^3, \left(t \mapsto t\log\left(x + \frac{1458t^2a^6bd + 675ta^3bc^2 + 40acd}{8ad^3 + 125bc^3}\right.\right.\right.$$
$$+ \frac{8acx + 7adx^2 + 5bcx^4 + 4bdx^5}{18a^4 + 36a^3bx^3 + 18a^2b^2x^6}$$

input
```
integrate((d*x+c)/(b*x**3+a)**3,x)
```

output
```
RootSum(19683*_t**3*a**8*b**2 + 810*_t*a**3*b*c*d + 8*a*d**3 - 125*b*c**3,
 Lambda(_t, _t*log(x + (1458*_t**2*a**6*b*d + 675*_t*a**3*b*c**2 + 40*a*c*
d**2)/(8*a*d**3 + 125*b*c**3)))) + (8*a*c*x + 7*a*d*x**2 + 5*b*c*x**4 + 4*
b*d*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \frac{4\,bdx^5 + 5\,bcx^4 + 7\,adx^2 + 8\,acx}{18\,(a^2b^2x^6 + 2\,a^3bx^3 + a^4)} + \frac{\sqrt{3}\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} + 5\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 5\,c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(2\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 5\,c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/(a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4) + 1/27*sqrt(3)*(2*d*(a/b)^(1/3) + 5*c)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) + 1/54*(2*d*(a/b)^(1/3) -
5*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) - 1/27*(2*
d*(a/b)^(1/3) - 5*c)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.90

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(5\,bc-2\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(5\,bc+2\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,(-ab^2)^{\frac{2}{3}}\,a^2} - \frac{\left(2\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+5\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{4\,bdx^5+5\,bcx^4+7\,adx^2+8\,acx}{18\,(bx^3+a)^2a^2}$$

input
```
integrate((d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*sqrt(3)*(5*b*c - 2*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(5*b*c + 2*(-a*b^2)^(1/
3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27
*(2*d*(-a/b)^(1/3) + 5*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/
18*(4*b*d*x^5 + 5*b*c*x^4 + 7*a*d*x^2 + 8*a*c*x)/((b*x^3 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \frac{\frac{7\,d\,x^2}{18\,a}+\frac{4\,c\,x}{9\,a}+\frac{5\,b\,c\,x^4}{18\,a^2}+\frac{2\,b\,d\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$
$$+\left(\sum_{k=1}^{3}\ln\left(\frac{b\left(10\,c\,d+4\,d^2\,x+\mathrm{root}(19683\,a^8\,b^2\,z^3+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)^2\,a^5\,b\,729+\mathrm{ro}\right.}{a^4\,81}\right.\right.$$
$$\left.+810\,a^3\,b\,c\,d\,z-125\,b\,c^3+8\,a\,d^3,z,k)\right)$$

input
```
int((c + d*x)/(a + b*x^3)^3,x)
```

output
```
((7*d*x^2)/(18*a) + (4*c*x)/(9*a) + (5*b*c*x^4)/(18*a^2) + (2*b*d*x^5)/(9*
a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + symsum(log((b*(10*c*d + 4*d^2*x + 729*
root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3, z, k)^2*a^
5*b + 135*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125*b*c^3 + 8*a*d^3,
z, k)*a^2*b*c*x))/(81*a^4))*root(19683*a^8*b^2*z^3 + 810*a^3*b*c*d*z - 125
*b*c^3 + 8*a*d^3, z, k), k, 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.62

$$\int \frac{c+dx}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)/(b*x^3+a)^3,x)
```

output

```
( - 10*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*a**2*c - 20*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3
)*x)/(a**(1/3)*sqrt(3)))*a*b*c*x**3 - 10*b**(1/3)*a**(2/3)*sqrt(3)*atan((a
**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*c*x**6 - 4*sqrt(3)*atan((
a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3*d - 8*sqrt(3)*atan((a**(
1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b*d*x**3 - 4*sqrt(3)*atan((a
**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*d*x**6 - 5*b**(1/3)*a**
(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*c - 10*b**(
1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*c*x*
*3 - 5*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*b**2*c*x**6 + 10*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*a**2*c +
20*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*a*b*c*x**3 + 10*b**(1/3)*a
**(2/3)*log(a**(1/3) + b**(1/3)*x)*b**2*c*x**6 + 24*b**(2/3)*a**(1/3)*a**2
*c*x + 21*b**(2/3)*a**(1/3)*a**2*d*x**2 + 15*b**(2/3)*a**(1/3)*a*b*c*x**4
+ 12*b**(2/3)*a**(1/3)*a*b*d*x**5 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x +
 b**(2/3)*x**2)*a**3*d + 4*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x
**2)*a**2*b*d*x**3 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)
*a*b**2*d*x**6 - 4*log(a**(1/3) + b**(1/3)*x)*a**3*d - 8*log(a**(1/3) + b*
*(1/3)*x)*a**2*b*d*x**3 - 4*log(a**(1/3) + b**(1/3)*x)*a*b**2*d*x**6)/(54*
b**(2/3)*a**(1/3)*a**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.8 $\int \frac{c+dx}{(a+bx^3)^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 240

$$\begin{aligned}\int \frac{c+dx}{(a+bx^3)^4} \, dx = {} & \frac{x(c+dx)}{9a(a+bx^3)^3}+\frac{x(8c+7dx)}{54a^2(a+bx^3)^2}+\frac{2x(10c+7dx)}{81a^3(a+bx^3)} \\ & -\frac{2\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{81\sqrt{3}a^{11/3}b^{2/3}} \\ & +\frac{2\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{243a^{11/3}b^{2/3}} \\ & -\frac{\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{243a^{11/3}b^{2/3}}\end{aligned}$$

output
```
1/9*x*(d*x+c)/a/(b*x^3+a)^3+1/54*x*(7*d*x+8*c)/a^2/(b*x^3+a)^2+2/81*x*(7*d
*x+10*c)/a^3/(b*x^3+a)-2/243*(20*b^(1/3)*c+7*a^(1/3)*d)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(11/3)/b^(2/3)+2/243*(20*b^(1/3)
*c-7*a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(2/3)-1/243*(20*b^(1/3)*c
-7*a^(1/3)*d)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$

$$= \frac{\frac{54a^3x(c+dx)}{(a+bx^3)^3}+\frac{9a^2x(8c+7dx)}{(a+bx^3)^2}+\frac{12ax(10c+7dx)}{a+bx^3}-\frac{4\sqrt{3}\sqrt[3]{a}\left(20\sqrt[3]{b}c+7\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{4\left(20\sqrt[3]{a}\sqrt[3]{b}c-7a^{2/3}d\right)\log}{b^{2/3}}}{486a^4}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^4,x]
```

output
```
((54*a^3*x*(c + d*x))/(a + b*x^3)^3 + (9*a^2*x*(8*c + 7*d*x))/(a + b*x^3)^
2 + (12*a*x*(10*c + 7*d*x))/(a + b*x^3) - (4*Sqrt[3]*a^(1/3)*(20*b^(1/3)*c
 + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*
(20*a^(1/3)*b^(1/3)*c - 7*a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (
2*(-20*a^(1/3)*b^(1/3)*c + 7*a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(2/3))/(486*a^4)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {2394, 25, 2394, 27, 2394, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{9a(a+bx^3)^3}-\frac{\int -\frac{8c+7dx}{(bx^3+a)^3}dx}{9a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8c+7dx}{(bx^3+a)^3}dx}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(8c+7dx)}{6a(a+bx^3)^2}-\frac{\int -\frac{4(10c+7dx)}{\left(bx^3+a\right)^2}dx}{6a}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\int \frac{10c+7dx}{\left(bx^3+a\right)^2}dx}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{2\left(\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}-\frac{\int -\frac{20c+7dx}{bx^3+a}dx}{3a}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\left(\frac{\int \frac{20c+7dx}{bx^3+a}dx}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{2399}$$

$$\frac{\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{a}\left(40\sqrt[3]{b}c+7\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{a}\left(40\sqrt[3]{b}c+7\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(20\sqrt[3]{b}c-7\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}+\frac{x(8c+7dx)}{6a(a+bx^3)^2}}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\cdots\right)}{3a}\cdots}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\cdots\right)}{3a}\cdots}{9a}+\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\cdots}{3a}\cdots}{3a}\cdots\right.$$

$$9a$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 1082

$$2\left(\frac{\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\cdots\right.}{3a^{2/3}\sqrt[3]{b}}\cdots}{3a}\cdots}{3a}\cdots\right.$$

$$9a$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$\Big\downarrow$ 217

$$2\left(\frac{\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx\cdots}{3a\left(a+bx^3\cdots\right.}\cdots}{3a}\cdots\right.$$

$$9a$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

$$\downarrow 1103$$

$$\frac{\frac{2\left(\frac{\frac{-\frac{1}{2}\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(7\sqrt[3]{a}d+20\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(20c-\frac{7\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(10c+7dx)}{3a\left(a+bx^3\right)}\right)}{3a}}{9a}$$

$$\frac{x(c+dx)}{9a\left(a+bx^3\right)^3}$$

input
```
Int[(c + d*x)/(a + b*x^3)^4,x]
```

output
```
(x*(c + d*x))/(9*a*(a + b*x^3)^3) + ((x*(8*c + 7*d*x))/(6*a*(a + b*x^3)^2)
 + (2*((x*(10*c + 7*d*x))/(3*a*(a + b*x^3)) + (((20*c - (7*a^(1/3)*d)/b^(1
/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(20*b^(1/
3)*c + 7*a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3))
- ((20*c - (7*a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/2)/(3*a^(2/3)*b^(1/3)))/(3*a)))/(3*a))/(9*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{14 d b^{2} x^{8}}{81 a^{3}}+\frac{20 c b^{2} x^{7}}{81 a^{3}}+\frac{77 b d x^{5}}{162 a^{2}}+\frac{52 b c x^{4}}{81 a^{2}}+\frac{67 d x^{2}}{162 a}+\frac{41 c x}{81 a}}{\left(b x^{3}+a\right)^{3}}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3} b+a\right)}\frac{\left(7 d \_R+20 c\right)\ln\left(x-\_R\right)}{\_R^{2}}\right)}{243 a^{3} b}$
default	$c\left(\frac{x}{9 a\left(b x^{3}+a\right)^{3}}+\frac{\frac{4 x}{27 a\left(b x^{3}+a\right)^{2}}+\frac{8\left(\frac{5 x}{18 a\left(b x^{3}+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6 a}\right)}{9 a}}{a}\right)$

input

```
int((d*x+c)/(b*x^3+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(14/81*d/a^3*b^2*x^8+20/81*c/a^3*b^2*x^7+77/162*b*d/a^2*x^5+52/81*b/a^2*c*
x^4+67/162*d/a*x^2+41/81*c/a*x)/(b*x^3+a)^3+2/243/a^3/b*sum((7*_R*d+20*c)/
_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 2308, normalized size of antiderivative = 9.62

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="fricas")
```

output
```
1/972*(168*b^2*d*x^8 + 240*b^2*c*x^7 + 462*a*b*d*x^5 + 624*a*b*c*x^4 + 402
*a^2*d*x^2 + 492*a^2*c*x - 2*(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 +
a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000
*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(
a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^1
1*b^2))^(1/3)))*log(7/4*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d
*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3)))^2*a^8*b*d - 400*(4^(1/3)*(I*sqrt(3) + 1
)*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^
2))^(1/3) - 140*4^(2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d
^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3)))*a^4*b*c^2 +
7840*a*c*d^2 + 4*(8000*b*c^3 + 343*a*d^3)*x) + ((a^3*b^3*x^9 + 3*a^4*b^2*x
^6 + 3*a^5*b*x^3 + a^6)*(4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 343*a*d^3)
/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(2/3)*c*d
*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) + (8000*b*c^
3 - 343*a*d^3)/(a^11*b^2))^(1/3))) + 3*sqrt(1/3)*(a^3*b^3*x^9 + 3*a^4*b^2*
x^6 + 3*a^5*b*x^3 + a^6)*sqrt(-((4^(1/3)*(I*sqrt(3) + 1)*((8000*b*c^3 + 34
3*a*d^3)/(a^11*b^2) + (8000*b*c^3 - 343*a*d^3)/(a^11*b^2))^(1/3) - 140*4^(
2/3)*c*d*(-I*sqrt(3) + 1)/(a^7*b*((8000*b*c^3 + 343*a*d^3)/(a^11*b^2) +...
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$
$$= \text{RootSum}\left(14348907t^3a^{11}b^2 + 408240ta^4bcd + 2744ad^3 - 64000bc^3, \left(t \mapsto t\log\left(x + \frac{413343t^2a^8bd + 19}{1372ad}\right.\right.\right.$$
$$+ \frac{82a^2cx + 67a^2dx^2 + 104abcx^4 + 77abdx^5 + 40b^2cx^7 + 28b^2dx^8}{162a^6 + 486a^5bx^3 + 486a^4b^2x^6 + 162a^3b^3x^9}$$

input
```
integrate((d*x+c)/(b*x**3+a)**4,x)
```

output
```
RootSum(14348907*_t**3*a**11*b**2 + 408240*_t*a**4*b*c*d + 2744*a*d**3 - 6
4000*b*c**3, Lambda(_t, _t*log(x + (413343*_t**2*a**8*b*d + 194400*_t*a**4
*b*c**2 + 7840*a*c*d**2)/(1372*a*d**3 + 32000*b*c**3)))) + (82*a**2*c*x +
67*a**2*d*x**2 + 104*a*b*c*x**4 + 77*a*b*d*x**5 + 40*b**2*c*x**7 + 28*b**2
*d*x**8)/(162*a**6 + 486*a**5*b*x**3 + 486*a**4*b**2*x**6 + 162*a**3*b**3*
x**9)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \frac{28\,b^2dx^8 + 40\,b^2cx^7 + 77\,abdx^5 + 104\,abcx^4 + 67\,a^2dx^2 + 82\,a^2cx}{162\,(a^3b^3x^9 + 3\,a^4b^2x^6 + 3\,a^5bx^3 + a^6)}$$
$$+ \frac{2\sqrt{3}\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} + 20\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{2\left(7\,d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 20\,c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{243\,a^3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="maxima")
```

output
```
1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67*a^2
*d*x^2 + 82*a^2*c*x)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6) + 2
/243*sqrt(3)*(7*d*(a/b)^(1/3) + 20*c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) + 1/243*(7*d*(a/b)^(1/3) - 20*c)*log(x
^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) - 2/243*(7*d*(a/b)^(
1/3) - 20*c)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = -\frac{2\sqrt{3}\left(20\,bc-7\,(-ab^2)^{\frac{1}{3}}\,d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{\left(20\,bc+7\,(-ab^2)^{\frac{1}{3}}\,d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{243\,(-ab^2)^{\frac{2}{3}}\,a^3} - \frac{2\left(7\,d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+20\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{243\,a^4} + \frac{28\,b^2dx^8+40\,b^2cx^7+77\,abdx^5+104\,abcx^4+67\,a^2dx^2+82\,a^2cx}{162\,(bx^3+a)^3a^3}$$

input
```
integrate((d*x+c)/(b*x^3+a)^4,x, algorithm="giac")
```

output
```
-2/243*sqrt(3)*(20*b*c - 7*(-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a
/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a^3) - 1/243*(20*b*c + 7*(-a*b^2)
^(1/3)*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^3) -
2/243*(7*d*(-a/b)^(1/3) + 20*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^
4 + 1/162*(28*b^2*d*x^8 + 40*b^2*c*x^7 + 77*a*b*d*x^5 + 104*a*b*c*x^4 + 67
*a^2*d*x^2 + 82*a^2*c*x)/((b*x^3 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{b\left(560\,c\,d+196\,d^2\,x+\mathrm{root}(14348907\,a^{11}\,b^2\,z^3+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k\right.}{}\right.\right.$$
$$\left.+408240\,a^4\,b\,c\,d\,z-64000\,b\,c^3+2744\,a\,d^3,z,k)\right)$$
$$+\frac{\frac{67\,d\,x^2}{162\,a}+\frac{41\,c\,x}{81\,a}+\frac{20\,b^2\,c\,x^7}{81\,a^3}+\frac{14\,b^2\,d\,x^8}{81\,a^3}+\frac{52\,b\,c\,x^4}{81\,a^2}+\frac{77\,b\,d\,x^5}{162\,a^2}}{a^3+3\,a^2\,b\,x^3+3\,a\,b^2\,x^6+b^3\,x^9}$$

input
```
int((c + d*x)/(a + b*x^3)^4,x)
```

output
```
symsum(log((b*(560*c*d + 196*d^2*x + 59049*root(14348907*a^11*b^2*z^3 + 40
8240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)^2*a^7*b + 9720*root(143
48907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z - 64000*b*c^3 + 2744*a*d^3, z, k)*
a^3*b*c*x))/(6561*a^6))*root(14348907*a^11*b^2*z^3 + 408240*a^4*b*c*d*z -
64000*b*c^3 + 2744*a*d^3, z, k), k, 1, 3) + ((67*d*x^2)/(162*a) + (41*c*x)
/(81*a) + (20*b^2*c*x^7)/(81*a^3) + (14*b^2*d*x^8)/(81*a^3) + (52*b*c*x^4)
/(81*a^2) + (77*b*d*x^5)/(162*a^2))/(a^3 + b^3*x^9 + 3*a^2*b*x^3 + 3*a*b^2
*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 775, normalized size of antiderivative = 3.23

$$\int \frac{c+dx}{(a+bx^3)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)/(b*x^3+a)^4,x)
```

output

```
( - 80*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*a**3*c - 240*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/
3)*x)/(a**(1/3)*sqrt(3)))*a**2*b*c*x**3 - 240*b**(1/3)*a**(2/3)*sqrt(3)*at
an((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*c*x**6 - 80*b**(1/
3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**
3*c*x**9 - 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a
**4*d - 84*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**3
*b*d*x**3 - 84*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2*b**2*d*x**6 - 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqr
t(3)))*a*b**3*d*x**9 - 40*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/
3)*x + b**(2/3)*x**2)*a**3*c - 120*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/
3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*c*x**3 - 120*b**(1/3)*a**(2/3)*log(a
**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*c*x**6 - 40*b**(1/3)
*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*c*x**9
+ 80*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*a**3*c + 240*b**(1/3)*a*
*(2/3)*log(a**(1/3) + b**(1/3)*x)*a**2*b*c*x**3 + 240*b**(1/3)*a**(2/3)*lo
g(a**(1/3) + b**(1/3)*x)*a*b**2*c*x**6 + 80*b**(1/3)*a**(2/3)*log(a**(1/3)
 + b**(1/3)*x)*b**3*c*x**9 + 246*b**(2/3)*a**(1/3)*a**3*c*x + 201*b**(2/3)
*a**(1/3)*a**3*d*x**2 + 312*b**(2/3)*a**(1/3)*a**2*b*c*x**4 + 231*b**(2/3)
*a**(1/3)*a**2*b*d*x**5 + 120*b**(2/3)*a**(1/3)*a*b**2*c*x**7 + 84*b**(...
```

3.9 $\int \frac{a+bx}{d-ex^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 161

$$\int \frac{a+bx}{d-ex^3} \, dx = -\frac{\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{2/3}} - \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}e^{2/3}} + \frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{2/3}}$$

output

```
-1/3*(b*d^(1/3)-a*e^(1/3))*arctan(1/3*(d^(1/3)+2*e^(1/3)*x)*3^(1/2)/d^(1/3
))*3^(1/2)/d^(2/3)/e^(2/3)-1/3*(b*d^(1/3)+a*e^(1/3))*ln(d^(1/3)-e^(1/3)*x)
/d^(2/3)/e^(2/3)+1/6*(b*d^(1/3)+a*e^(1/3))*ln(d^(2/3)+d^(1/3)*e^(1/3)*x+e^
(2/3)*x^2)/d^(2/3)/e^(2/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{a+bx}{d-ex^3}\,dx$$

$$= \frac{-2\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)-\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\left(2\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+\right.\right.}{6d^{2/3}e^{2/3}}$$

input
```
Integrate[(a + b*x)/(d - e*x^3),x]
```

output
```
(-2*Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqr
t[3]] - (b*d^(1/3) + a*e^(1/3))*(2*Log[d^(1/3) - e^(1/3)*x] - Log[d^(2/3)
+ d^(1/3)*e^(1/3)*x + e^(2/3)*x^2]))/(6*d^(2/3)*e^(2/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2400, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{d-ex^3}\,dx$$

$$\Big\downarrow \text{2400}$$

$$\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}dx}{3d^{2/3}}-\frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(\sqrt[3]{e}a+b\sqrt[3]{d}\right)\sqrt[3]{e}x}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\int \frac{\sqrt[3]{d}\left(b\sqrt[3]{d}-2a\sqrt[3]{e}\right)-\left(\sqrt[3]{e}a+b\sqrt[3]{d}\right)\sqrt[3]{e}x}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \ 1142$$

$$-\frac{\frac{3}{2}\sqrt[3]{d}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int \frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{\sqrt[3]{e}\left(2\sqrt[3]{e}x+\sqrt[3]{d}\right)}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{3}{2}\sqrt[3]{d}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int \frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\sqrt[3]{e}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \ 1082$$

$$-\frac{-\frac{1}{2}\sqrt[3]{e}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{3\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\int \frac{1}{-\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{e}}}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\sqrt[3]{e}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

$$\Big\downarrow \ 1103$$

$$-\frac{\frac{\sqrt{3}\left(b\sqrt[3]{d}-a\sqrt[3]{e}\right)\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{3d^{2/3}\sqrt[3]{e}}-\frac{\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}$$

input
```
Int[(a + b*x)/(d - e*x^3),x]
```

output
```
-1/3*((a + (b*d^(1/3))/e^(1/3))*Log[d^(1/3) - e^(1/3)*x])/(d^(2/3)*e^(1/3)
) - ((Sqrt[3]*(b*d^(1/3) - a*e^(1/3))*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/S
qrt[3]])/e^(1/3) - ((a + (b*d^(1/3))/e^(1/3))*Log[d^(2/3) + d^(1/3)*e^(1/3
)*x + e^(2/3)*x^2])/2)/(3*d^(2/3)*e^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.21

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^{3}-d\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3e}$
default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+b\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}\right.}{}\right.$

input
```
int((b*x+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e*sum((_R*b+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e-d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 1905, normalized size of antiderivative = 11.83

$$\int \frac{a+bx}{d-ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)/(-e*x^3+d),x, algorithm="fricas")
```

output
```
-1/18*(9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d -
a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)
/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(1/36*(9*(I*sqrt(3
) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*
(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e - 1/6*(9*(I*sqrt(3) + 1)*(-1/
54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b
*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^
3*e)/(d^2*e^2))^(1/3)))*a^2*d*e - 2*a*b^2*d - (b^3*d - a^3*e)*x) + 1/36*(9
*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(
d^2*e^2))^(1/3) + 3*sqrt(1/3)*sqrt(-((9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^
3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) +
 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2)
)^(1/3)))^2*d*e - 144*a*b)/(d*e)) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*
d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))*log(-1/36*(
9*(I*sqrt(3) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/
(d^2*e^2))^(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e
^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^(1/3)))^2*b*d^2*e + 1/6*(9*(I*sqrt(3
) + 1)*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/54*(b^3*d - a^3*e)/(d^2*e^2))^
(1/3) + a*b*(-I*sqrt(3) + 1)/(d*e*(-1/54*(b^3*d + a^3*e)/(d^2*e^2) - 1/...
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.48

$$\int \frac{a+bx}{d-ex^3}\,dx = -\operatorname{RootSum}\left(27t^3d^2e^2-9tabde-a^3e-b^3d,\left(t\mapsto t\log\left(x+\frac{9t^2bd^2e-3ta^2de-2ab^2d}{a^3e-b^3d}\right)\right)\right)$$

input
```
integrate((b*x+a)/(-e*x**3+d),x)
```

output
```
-RootSum(27*_t**3*d**2*e**2 - 9*_t*a*b*d*e - a**3*e - b**3*d, Lambda(_t, _
t*log(x + (9*_t**2*b*d**2*e - 3*_t*a**2*d*e - 2*a*b**2*d)/(a**3*e - b**3*d
))))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx}{d-ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)/(-e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{a+bx}{d-ex^3}\,dx = \frac{\left(ae+(de^2)^{\frac{1}{3}}b\right)\log\left(x^2+x\left(\frac{d}{e}\right)^{\frac{1}{3}}+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(de^2)^{\frac{2}{3}}} - \frac{\left(b\left(\frac{d}{e}\right)^{\frac{1}{3}}+a\right)\left(\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} + \frac{\sqrt{3}\left((de^2)^{\frac{1}{3}}ae-(de^2)^{\frac{2}{3}}b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,de^2}$$

input

```
integrate((b*x+a)/(-e*x^3+d),x, algorithm="giac")
```

output

```
1/6*(a*e + (d*e^2)^(1/3)*b)*log(x^2 + x*(d/e)^(1/3) + (d/e)^(2/3))/(d*e^2)
^(2/3) - 1/3*(b*(d/e)^(1/3) + a)*(d/e)^(1/3)*log(abs(x - (d/e)^(1/3)))/d +
 1/3*sqrt(3)*((d*e^2)^(1/3)*a*e - (d*e^2)^(2/3)*b)*arctan(1/3*sqrt(3)*(2*x
 + (d/e)^(1/3))/(d/e)^(1/3))/(d*e^2)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{a+bx}{d-ex^3}\,dx = \sum_{k=1}^{3} \ln\left(e\left(a\,b+b^2\,x - \operatorname{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)^2 d\,e\,9 - \operatorname{root}\left(27\,d^2\,e^2\,z^3-9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)\,a\,e\,x\,3\right)\right)\operatorname{root}\left(27\,d^2\,e^2\,z^3 - 9\,a\,b\,d\,e\,z+b^3\,d+a^3\,e,z,k\right)$$

input

```
int((a + b*x)/(d - e*x^3),x)
```

output
```
symsum(log(e*(a*b + b^2*x - 9*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d +
a^3*e, z, k)^2*d*e - 3*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e,
z, k)*a*e*x))*root(27*d^2*e^2*z^3 - 9*a*b*d*e*z + b^3*d + a^3*e, z, k), k,
 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int \frac{a+bx}{d-ex^3}\,dx = \frac{2e^{\frac{1}{3}}d^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}+2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}+2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)bd+e^{\frac{1}{3}}d^{\frac{2}{3}}\log\left(d^{\frac{2}{3}}+e^{\frac{1}{3}}d^{\frac{1}{3}}x+e^{\frac{2}{3}}x^2\right)a-2e^{\frac{1}{3}}d^{\frac{2}{3}}\log\left(d^{\frac{1}{3}}\right.}{6e^{\frac{2}{3}}d^{\frac{4}{3}}}$$

input
```
int((b*x+a)/(-e*x^3+d),x)
```

output
```
(2*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) + 2*e**(1/3)*x)/(d**(1/3)*sqrt
(3)))*a - 2*sqrt(3)*atan((d**(1/3) + 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*b*d
 + e**(1/3)*d**(2/3)*log(d**(2/3) + e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a
 - 2*e**(1/3)*d**(2/3)*log(d**(1/3) - e**(1/3)*x)*a + log(d**(2/3) + e**(1
/3)*d**(1/3)*x + e**(2/3)*x**2)*b*d - 2*log(d**(1/3) - e**(1/3)*x)*b*d)/(6
*e**(2/3)*d**(1/3)*d)
```

3.10 $\int \frac{1+x}{1+x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1+x}{1+x^3} \, dx = -\frac{2 \arctan \left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
-2/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{1+x^3} \, dx = \frac{2 \arctan \left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + x)/(1 + x^3),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^3+1}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{x^2-x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(2x-1)^2-3}d(2x-1)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 + x)/(1 + x^3),x]
```

output
```
(2*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
meijerg	$-\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{2}{3}}}+\frac{x^2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{2}{3}}}+\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3\right.}{6(x^3)^{\frac{1}{3}}}$

input
```
int((x+1)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

input
```
integrate((1+x)/(x^3+1),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1+x)/(x**3+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

input
```
integrate((1+x)/(x^3+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

input
```
integrate((1+x)/(x^3+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,x-1)}{3}\right)}{3}$$

input
```
int((x + 1)/(x^3 + 1),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*(2*x - 1))/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1+x}{1+x^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3}$$

input
```
int((1+x)/(x^3+1),x)
```

output
```
(2*sqrt(3)*atan((2*x - 1)/sqrt(3)))/3
```

3.11 $\int \frac{1-x}{1-x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 - x)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{1-x^3} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{x^2+x+1}dx$$

$$\Big\downarrow \text{1083}$$

$$-2 \int \frac{1}{-(2x+1)^2-3}d(2x+1)$$

$$\Big\downarrow \text{217}$$

$$\frac{2 \arctan \left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(1 - x)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result
default	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
risch	$\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\mathrm{ar}\right.}{3(x^3)^{\frac{2}{3}}}$

input
```
int((1-x)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx=\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

input
```
integrate((1-x)/(-x^3+1),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((1-x)/(-x**3+1),x)
```

output
```
2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

input
```
integrate((1-x)/(-x^3+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)$$

input
```
integrate((1-x)/(-x^3+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1))
```

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,x+1)}{3}\right)}{3}$$

input
```
int((x - 1)/(x^3 - 1),x)
```

output
```
(2*3^(1/2)*atan((3^(1/2)*(2*x + 1))/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1-x}{1-x^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3}$$

input
```
int((1-x)/(-x^3+1),x)
```

output
```
(2*sqrt(3)*atan((2*x + 1)/sqrt(3)))/3
```

3.12 $\int \frac{A+Bx}{A^3+B^3x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx = -\frac{2\arctan\left(\frac{A-2Bx}{\sqrt{3}A}\right)}{\sqrt{3}AB}$$

output
```
-2/3*arctan(1/3*(-2*B*x+A)*3^(1/2)/A)*3^(1/2)/A/B
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx = \frac{2\arctan\left(\frac{-A+2Bx}{\sqrt{3}A}\right)}{\sqrt{3}AB}$$

input
```
Integrate[(A + B*x)/(A^3 + B^3*x^3),x]
```

output
```
(2*ArcTan[(-A + 2*B*x)/(Sqrt[3]*A)])/(Sqrt[3]*A*B)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{A^2-ABx+B^2x^2}dx$$
$$\Big\downarrow \text{1082}$$
$$\frac{2\int \frac{1}{-\left(1-\frac{2Bx}{A}\right)^2-3}d\left(1-\frac{2Bx}{A}\right)}{AB}$$
$$\Big\downarrow \text{217}$$
$$-\frac{2\arctan\left(\frac{1-\frac{2Bx}{A}}{\sqrt{3}}\right)}{\sqrt{3}AB}$$

input
```
Int[(A + B*x)/(A^3 + B^3*x^3),x]
```

output
```
(-2*ArcTan[(1 - (2*B*x)/A)/Sqrt[3]])/(Sqrt[3]*A*B)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2B\sqrt{3}\,x}{3A}-\frac{\sqrt{3}}{3}\right)}{3BA}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2B^{2}x-BA\right)\sqrt{3}}{3BA}\right)}{3BA}$	35

input
```
int((B*x+A)/(B^3*x^3+A^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)/B/A*arctan(2/3*B*3^(1/2)/A*x-1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx=\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,Bx-A)}{3\,A}\right)}{3\,AB}$$

input
```
integrate((B*x+A)/(B^3*x^3+A^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*B*x - A)/A)/(A*B)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx = \frac{-\frac{\sqrt{3}i\log\left(x+\frac{-A-\sqrt{3}iA}{2B}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-A+\sqrt{3}iA}{2B}\right)}{3}}{AB}$$

input
```
integrate((B*x+A)/(B**3*x**3+A**3),x)
```

output
```
(-sqrt(3)*I*log(x + (-A - sqrt(3)*I*A)/(2*B))/3 + sqrt(3)*I*log(x + (-A +
sqrt(3)*I*A)/(2*B))/3)/(A*B)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2B^2x-AB)}{3AB}\right)}{3AB}$$

input
```
integrate((B*x+A)/(B^3*x^3+A^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*B^2*x - A*B)/(A*B))/(A*B)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2Bx-A)}{3A}\right)}{3AB}$$

input
```
integrate((B*x+A)/(B^3*x^3+A^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*B*x - A)/A)/(A*B)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx=-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,B\,x}{3\,A}\right)}{3\,A\,B}$$

input
```
int((A + B*x)/(A^3 + B^3*x^3),x)
```

output
```
-(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*B*x)/(3*A)))/(3*A*B)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{A^3+B^3x^3}\,dx=-\frac{2\sqrt{3}\,atan\left(\frac{-2bx+a}{\sqrt{3}\,a}\right)}{3ab}$$

input
```
int((B*x+A)/(B^3*x^3+A^3),x)
```

output
```
( - 2*sqrt(3)*atan((a - 2*b*x)/(sqrt(3)*a)))/(3*a*b)
```

3.13 $\int \frac{A-Bx}{A^3-B^3x^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{A-Bx}{A^3-B^3x^3} \, dx = \frac{2\arctan\left(\frac{A+2Bx}{\sqrt{3}A}\right)}{\sqrt{3}AB}$$

output
```
2/3*arctan(1/3*(2*B*x+A)*3^(1/2)/A)*3^(1/2)/A/B
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{A-Bx}{A^3-B^3x^3} \, dx = \frac{2\arctan\left(\frac{A+2Bx}{\sqrt{3}A}\right)}{\sqrt{3}AB}$$

input
```
Integrate[(A - B*x)/(A^3 - B^3*x^3),x]
```

output
```
(2*ArcTan[(A + 2*B*x)/(Sqrt[3]*A)])/(Sqrt[3]*A*B)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A-Bx}{A^3-B^3x^3}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{A^2+ABx+B^2x^2}dx$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{2\int \frac{1}{-\left(\frac{2Bx}{A}+1\right)^2-3}d\left(\frac{2Bx}{A}+1\right)}{AB}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\arctan\left(\frac{\frac{2Bx}{A}+1}{\sqrt{3}}\right)}{\sqrt{3}AB}$$

input
```
Int[(A - B*x)/(A^3 - B^3*x^3),x]
```

output
```
(2*ArcTan[(1 + (2*B*x)/A)/Sqrt[3]])/(Sqrt[3]*A*B)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2B\sqrt{3}\,x}{3A}+\frac{\sqrt{3}}{3}\right)}{3BA}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2B^{2}x+BA\right)\sqrt{3}}{3BA}\right)}{3BA}$	34

input
```
int((-B*x+A)/(-B^3*x^3+A^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)/B/A*arctan(2/3*B*3^(1/2)/A*x+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{A-Bx}{A^3-B^3x^3}\,dx=\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,Bx+A)}{3\,A}\right)}{3\,AB}$$

input
```
integrate((-B*x+A)/(-B^3*x^3+A^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*B*x + A)/A)/(A*B)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{A-Bx}{A^3-B^3x^3}\,dx=\frac{-\frac{\sqrt{3}i\log\left(x+\frac{A-\sqrt{3}iA}{2B}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{A+\sqrt{3}iA}{2B}\right)}{3}}{AB}$$

input
```
integrate((-B*x+A)/(-B**3*x**3+A**3),x)
```

output
```
(-sqrt(3)*I*log(x + (A - sqrt(3)*I*A)/(2*B))/3 + sqrt(3)*I*log(x + (A + sq
rt(3)*I*A)/(2*B))/3)/(A*B)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{A-Bx}{A^3-B^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2B^2x+AB)}{3AB}\right)}{3AB}$$

input
```
integrate((-B*x+A)/(-B^3*x^3+A^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*B^2*x + A*B)/(A*B))/(A*B)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{A-Bx}{A^3-B^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2Bx+A)}{3A}\right)}{3AB}$$

input
```
integrate((-B*x+A)/(-B^3*x^3+A^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*B*x + A)/A)/(A*B)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{A-Bx}{A^3-B^3x^3}\,dx=\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\,\sqrt{3}\,B\,x}{3\,A}\right)}{3\,A\,B}$$

input
```
int((A - B*x)/(A^3 - B^3*x^3),x)
```

output
```
(2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*B*x)/(3*A)))/(3*A*B)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{A-Bx}{A^3-B^3x^3}\,dx=\frac{2\sqrt{3}\,atan\left(\frac{2bx+a}{\sqrt{3}\,a}\right)}{3ab}$$

input
```
int((-B*x+A)/(-B^3*x^3+A^3),x)
```

output
```
(2*sqrt(3)*atan((a + 2*b*x)/(sqrt(3)*a)))/(3*a*b)
```

3.14 $\int \frac{1+x}{1-x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1+x}{1-x^3} \, dx = -\frac{2}{3} \log(1-x) + \frac{1}{3} \log \left(1+x+x^2\right)$$

output
```
-2/3*ln(1-x)+1/3*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{1-x^3} \, dx = -\frac{2}{3} \log(1-x) + \frac{1}{3} \log \left(1+x+x^2\right)$$

input
```
Integrate[(1 + x)/(1 - x^3),x]
```

output
```
(-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2400, 16, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{1-x^3} \, dx$$

$$\Big\downarrow \text{2400}$$

$$\frac{2}{3}\int \frac{1}{1-x}dx-\frac{1}{3}\int -\frac{2x+1}{x^2+x+1}dx$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{3}\int -\frac{2x+1}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{2x+1}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log(1-x)$$

input
```
Int[(1 + x)/(1 - x^3),x]
```

output
```
(-2*Log[1 - x])/3 + Log[1 + x + x^2]/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result
default	$\frac{\ln(x^2+x+1)}{3}-\frac{2\ln(x-1)}{3}$
norman	$\frac{\ln(x^2+x+1)}{3}-\frac{2\ln(x-1)}{3}$
risch	$\frac{\ln(x^2+x+1)}{3}-\frac{2\ln(x-1)}{3}$
parallelrisch	$\frac{\ln(x^2+x+1)}{3}-\frac{2\ln(x-1)}{3}$
meijerg	$-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\cdots\right)}{3(x^3)^{\frac{1}{3}}}$

input
```
int((x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x^2+x+1)-2/3*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((1+x)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{1-x^3}\,dx = -\frac{2\log\left(x-1\right)}{3} + \frac{\log\left(x^2+x+1\right)}{3}$$

input
```
integrate((1+x)/(-x**3+1),x)
```

output
```
-2*log(x - 1)/3 + log(x**2 + x + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((1+x)/(-x^3+1),x, algorithm="maxima")
```

output
```
1/3*log(x^2 + x + 1) - 2/3*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1+x}{1-x^3}\,dx = \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(|x-1|\right)$$

input
```
integrate((1+x)/(-x^3+1),x, algorithm="giac")
```

output
```
1/3*log(x^2 + x + 1) - 2/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{\ln\left(x^2+x+1\right)}{3} - \frac{2\,\ln\left(x-1\right)}{3}$$

input
```
int(-(x + 1)/(x^3 - 1),x)
```

output
```
log(x + x^2 + 1)/3 - (2*log(x - 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{1-x^3}\,dx = \frac{\log(x^2+x+1)}{3} - \frac{2\log(x-1)}{3}$$

input
```
int((1+x)/(-x^3+1),x)
```

output
```
(log(x**2 + x + 1) - 2*log(x - 1))/3
```

3.15 $\int \frac{1-x}{1+x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1-x}{1+x^3}\,dx = \frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
2/3*ln(1+x)-1/3*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1+x^3}\,dx = \frac{2}{3}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 - x)/(1 + x^3),x]
```

output
```
(2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2399, 16, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{x^3+1} \, dx$$
$$\Big\downarrow \text{ 2399}$$
$$\frac{1}{3} \int \frac{1-2x}{x^2-x+1}dx+\frac{2}{3} \int \frac{1}{x+1}dx$$
$$\Big\downarrow \text{ 16}$$
$$\frac{1}{3} \int \frac{1-2x}{x^2-x+1}dx+\frac{2}{3} \log(x+1)$$
$$\Big\downarrow \text{ 1103}$$
$$\frac{2}{3} \log(x+1)-\frac{1}{3} \log \left(x^2-x+1\right)$$

input
```
Int[(1 - x)/(1 + x^3),x]
```

output
```
(2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result
default	$\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$
norman	$\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$
risch	$\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$
parallelrisch	$\frac{2\ln(x+1)}{3}-\frac{\ln(x^2-x+1)}{3}$
meijerg	$\frac{x\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{1}{3}}}-\frac{x\ln\left(1-(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{6(x^3)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2-(x^3)^{\frac{1}{3}}}\right)}{3(x^3)^{\frac{1}{3}}}+\frac{x^2\ln\left(1+(x^3)^{\frac{1}{3}}\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x^2\ln\left(1-(x^3)^{\frac{1}{3}}+(\right.}{6(x^3)^{\frac{2}{3}}}$

input
```
int((1-x)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(x+1)-1/3*ln(x^2-x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3}\,dx=-\frac{1}{3}\log\left(x^2-x+1\right)+\frac{2}{3}\log\left(x+1\right)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1-x}{1+x^3} \, dx = \frac{2\log(x+1)}{3} - \frac{\log(x^2-x+1)}{3}$$

input
```
integrate((1-x)/(x**3+1),x)
```

output
```
2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3} \, dx = -\frac{1}{3} \log\left(x^2-x+1\right) + \frac{2}{3} \log(x+1)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{1+x^3} \, dx = -\frac{1}{3} \log\left(x^2-x+1\right) + \frac{2}{3} \log(|x+1|)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3}\,dx = \frac{2\,\ln\left(x+1\right)}{3} - \frac{\ln\left(x^2-x+1\right)}{3}$$

input
```
int(-(x - 1)/(x^3 + 1),x)
```

output
```
(2*log(x + 1))/3 - log(x^2 - x + 1)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1-x}{1+x^3}\,dx = -\frac{\log(x^2-x+1)}{3} + \frac{2\log(x+1)}{3}$$

input
```
int((1-x)/(x^3+1),x)
```

output
```
( - log(x**2 - x + 1) + 2*log(x + 1))/3
```

3.16 $\int \frac{A+Bx}{A^3-B^3x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{A+Bx}{A^3-B^3x^3} \, dx = -\frac{2\log(A-Bx)}{3AB}+\frac{\log\left(A^2+ABx+B^2x^2\right)}{3AB}$$

output
```
-2/3*ln(-B*x+A)/A/B+1/3*ln(B^2*x^2+A*B*x+A^2)/A/B
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{A^3-B^3x^3} \, dx = -\frac{2\log(A-Bx)}{3AB}+\frac{\log\left(A^2+ABx+B^2x^2\right)}{3AB}$$

input
```
Integrate[(A + B*x)/(A^3 - B^3*x^3),x]
```

output
```
(-2*Log[A - B*x])/(3*A*B) + Log[A^2 + A*B*x + B^2*x^2]/(3*A*B)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2400, 16, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{A^3-B^3x^3}dx$$

$$\Big\downarrow \text{2400}$$

$$\frac{2\int \frac{1}{A-Bx}dx}{3A}-\frac{\int -\frac{AB(A+2Bx)}{A^2+BxA+B^2x^2}dx}{3A^2B}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\int -\frac{AB(A+2Bx)}{A^2+BxA+B^2x^2}dx}{3A^2B}-\frac{2\log(A-Bx)}{3AB}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{AB(A+2Bx)}{A^2+BxA+B^2x^2}dx}{3A^2B}-\frac{2\log(A-Bx)}{3AB}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+2Bx}{A^2+BxA+B^2x^2}dx}{3A}-\frac{2\log(A-Bx)}{3AB}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log\left(A^2+ABx+B^2x^2\right)}{3AB}-\frac{2\log(A-Bx)}{3AB}$$

input
```
Int[(A + B*x)/(A^3 - B^3*x^3),x]
```

output
```
(-2*Log[A - B*x])/(3*A*B) + Log[A^2 + A*B*x + B^2*x^2]/(3*A*B)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{-2\ln(-Bx+A)+\ln(B^2x^2+ABx+A^2)}{3BA}$	35
default	$-\frac{2\ln(-Bx+A)}{3AB}+\frac{\ln(B^2x^2+ABx+A^2)}{3AB}$	41
norman	$-\frac{2\ln(-Bx+A)}{3AB}+\frac{\ln(B^2x^2+ABx+A^2)}{3AB}$	41
risch	$-\frac{2\ln(Bx-A)}{3AB}+\frac{\ln(B^2x^2+ABx+A^2)}{3AB}$	42

input
```
int((B*x+A)/(-B^3*x^3+A^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-2*ln(-B*x+A)+ln(B^2*x^2+A*B*x+A^2))/B/A
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{A^3-B^3x^3}\,dx=\frac{\log\left(B^2x^2+ABx+A^2\right)-2\,\log\left(Bx-A\right)}{3\,AB}$$

input
```
integrate((B*x+A)/(-B^3*x^3+A^3),x, algorithm="fricas")
```

output
```
1/3*(log(B^2*x^2 + A*B*x + A^2) - 2*log(B*x - A))/(A*B)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{A^3-B^3x^3}\,dx=-\frac{2\log\left(-\frac{A}{B}+x\right)}{3AB}+\frac{\log\left(\frac{A^2}{B^2}+\frac{Ax}{B}+x^2\right)}{3AB}$$

input
```
integrate((B*x+A)/(-B**3*x**3+A**3),x)
```

output
```
-2*log(-A/B + x)/(3*A*B) + log(A**2/B**2 + A*x/B + x**2)/(3*A*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{A^3-B^3x^3}\,dx=\frac{\log\left(B^2x^2+ABx+A^2\right)}{3\,AB}-\frac{2\,\log\left(Bx-A\right)}{3\,AB}$$

input
```
integrate((B*x+A)/(-B^3*x^3+A^3),x, algorithm="maxima")
```

output
```
1/3*log(B^2*x^2 + A*B*x + A^2)/(A*B) - 2/3*log(B*x - A)/(A*B)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{A^3-B^3x^3}\,dx=\frac{\log\left(B^2x^2+ABx+A^2\right)}{3\,AB}-\frac{2\,\log\left(|Bx-A|\right)}{3\,AB}$$

input
```
integrate((B*x+A)/(-B^3*x^3+A^3),x, algorithm="giac")
```

output
```
1/3*log(B^2*x^2 + A*B*x + A^2)/(A*B) - 2/3*log(abs(B*x - A))/(A*B)
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{A^3-B^3x^3}\,dx=-\frac{2\,\ln\left(B\,x-A\right)-\ln\left(A^2+A\,B\,x+B^2\,x^2\right)}{3\,A\,B}$$

input
```
int((A + B*x)/(A^3 - B^3*x^3),x)
```

output
```
-(2*log(B*x - A) - log(A^2 + B^2*x^2 + A*B*x))/(3*A*B)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{A^3-B^3x^3}\,dx=\frac{\log(b^2x^2+abx+a^2)-2\log(-bx+a)}{3ab}$$

input
```
int((B*x+A)/(-B^3*x^3+A^3),x)
```

output
```
(log(a**2 + a*b*x + b**2*x**2) - 2*log(a - b*x))/(3*a*b)
```

3.17 $\int \frac{A-Bx}{A^3+B^3x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{A-Bx}{A^3+B^3x^3} \, dx = \frac{2\log(A+Bx)}{3AB} - \frac{\log\left(A^2-ABx+B^2x^2\right)}{3AB}$$

output
```
2/3*ln(B*x+A)/A/B-1/3*ln(B^2*x^2-A*B*x+A^2)/A/B
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{A-Bx}{A^3+B^3x^3} \, dx = \frac{2\log(A+Bx)}{3AB} - \frac{\log\left(A^2-ABx+B^2x^2\right)}{3AB}$$

input
```
Integrate[(A - B*x)/(A^3 + B^3*x^3),x]
```

output
```
(2*Log[A + B*x])/(3*A*B) - Log[A^2 - A*B*x + B^2*x^2]/(3*A*B)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2399, 16, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A-Bx}{A^3+B^3x^3}\,dx$$

$$\Big\downarrow \ 2399$$

$$\frac{\int \frac{AB(A-2Bx)}{A^2-BxA+B^2x^2}dx}{3A^2B}+\frac{2\int \frac{1}{A+Bx}dx}{3A}$$

$$\Big\downarrow \ 16$$

$$\frac{\int \frac{AB(A-2Bx)}{A^2-BxA+B^2x^2}dx}{3A^2B}+\frac{2\log(A+Bx)}{3AB}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{A-2Bx}{A^2-BxA+B^2x^2}dx}{3A}+\frac{2\log(A+Bx)}{3AB}$$

$$\Big\downarrow \ 1103$$

$$\frac{2\log(A+Bx)}{3AB}-\frac{\log\left(A^2-ABx+B^2x^2\right)}{3AB}$$

input
```
Int[(A - B*x)/(A^3 + B^3*x^3),x]
```

output
```
(2*Log[A + B*x])/(3*A*B) - Log[A^2 - A*B*x + B^2*x^2]/(3*A*B)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{2\ln(Bx+A)-\ln(B^2x^2-ABx+A^2)}{3BA}$	37
default	$\frac{2\ln(Bx+A)}{3AB}-\frac{\ln(B^2x^2-ABx+A^2)}{3AB}$	41
norman	$\frac{2\ln(Bx+A)}{3AB}-\frac{\ln(B^2x^2-ABx+A^2)}{3AB}$	41
risch	$\frac{2\ln(Bx+A)}{3AB}-\frac{\ln(B^2x^2-ABx+A^2)}{3AB}$	41

input
```
int((-B*x+A)/(B^3*x^3+A^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*ln(B*x+A)-ln(B^2*x^2-A*B*x+A^2))/B/A
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{A-Bx}{A^3+B^3x^3}\,dx = -\frac{\log\left(B^2x^2-ABx+A^2\right)-2\,\log\left(Bx+A\right)}{3\,AB}$$

input
```
integrate((-B*x+A)/(B^3*x^3+A^3),x, algorithm="fricas")
```

output
```
-1/3*(log(B^2*x^2 - A*B*x + A^2) - 2*log(B*x + A))/(A*B)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{A-Bx}{A^3+B^3x^3}\,dx = \frac{2\log\left(\frac{A}{B}+x\right)}{3AB} - \frac{\log\left(\frac{A^2}{B^2}-\frac{Ax}{B}+x^2\right)}{3AB}$$

input
```
integrate((-B*x+A)/(B**3*x**3+A**3),x)
```

output
```
2*log(A/B + x)/(3*A*B) - log(A**2/B**2 - A*x/B + x**2)/(3*A*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{A-Bx}{A^3+B^3x^3}\,dx = -\frac{\log\left(B^2x^2-ABx+A^2\right)}{3\,AB} + \frac{2\,\log\left(Bx+A\right)}{3\,AB}$$

input
```
integrate((-B*x+A)/(B^3*x^3+A^3),x, algorithm="maxima")
```

output
```
-1/3*log(B^2*x^2 - A*B*x + A^2)/(A*B) + 2/3*log(B*x + A)/(A*B)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{A-Bx}{A^3+B^3x^3}\,dx = -\frac{\log\left(B^2x^2-ABx+A^2\right)}{3\,AB}+\frac{2\,\log\left(|Bx+A|\right)}{3\,AB}$$

input
```
integrate((-B*x+A)/(B^3*x^3+A^3),x, algorithm="giac")
```

output
```
-1/3*log(B^2*x^2 - A*B*x + A^2)/(A*B) + 2/3*log(abs(B*x + A))/(A*B)
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{A-Bx}{A^3+B^3x^3}\,dx = \frac{2\,\ln\left(A+B\,x\right)-\ln\left(A^2-A\,B\,x+B^2\,x^2\right)}{3\,A\,B}$$

input
```
int((A - B*x)/(A^3 + B^3*x^3),x)
```

output
```
(2*log(A + B*x) - log(A^2 + B^2*x^2 - A*B*x))/(3*A*B)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{A-Bx}{A^3+B^3x^3}\,dx = \frac{-\log(b^2x^2-abx+a^2)+2\log(bx+a)}{3ab}$$

input
```
int((-B*x+A)/(B^3*x^3+A^3),x)
```

output
```
( - log(a**2 - a*b*x + b**2*x**2) + 2*log(a + b*x))/(3*a*b)
```

3.18 $\int \frac{3-x}{1-x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{3-x}{1-x^3} dx = \frac{4 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1-x) + \frac{1}{3}\log\left(1+x+x^2\right)$$

output
```
4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/3*ln(1-x)+1/3*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{3-x}{1-x^3} dx = \frac{4 \arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2}{3}\log(1-x) + \frac{1}{3}\log\left(1+x+x^2\right)$$

input
```
Integrate[(3 - x)/(1 - x^3),x]
```

output
```
(4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 - x])/3 + Log[1 + x + x^2
]/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2400, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-x}{1-x^3} \, dx$$

$$\Big\downarrow \ 2400$$

$$\frac{2}{3}\int \frac{1}{1-x}dx-\frac{1}{3}\int -\frac{2x+7}{x^2+x+1}dx$$

$$\Big\downarrow \ 16$$

$$-\frac{1}{3}\int -\frac{2x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{3}\int \frac{2x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{3}\left(6\int \frac{1}{x^2+x+1}dx+\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{3}\left(\int \frac{2x+1}{x^2+x+1}dx-12\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{3}\left(\int \frac{2x+1}{x^2+x+1}dx+4\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{3}\left(4\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\log\left(x^2+x+1\right)\right)-\frac{2}{3}\log(1-x)$$

input
```
Int[(3 - x)/(1 - x^3),x]
```

output
```
(-2*Log[1 - x])/3 + (4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[1 + x + x^2
])/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result
default	$\frac{\ln(x^2+x+1)}{3}+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{2\ln(x-1)}{3}$
risch	$\frac{\ln(4x^2+4x+4)}{3}+\frac{4\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\frac{2\ln(x-1)}{3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\text{ ar}\right.}{3(x^3)^{\frac{2}{3}}}$

input
```
int((3-x)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/3*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{3-x}{1-x^3}\,dx=\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((3-x)/(-x^3+1),x, algorithm="fricas")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log
(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{3-x}{1-x^3}\,dx = -\frac{2\log(x-1)}{3} + \frac{\log(x^2+x+1)}{3} + \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((3-x)/(-x**3+1),x)
```

output
```
-2*log(x - 1)/3 + log(x**2 + x + 1)/3 + 4*sqrt(3)*atan(2*sqrt(3)*x/3 + sqr
t(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((3-x)/(-x^3+1),x, algorithm="maxima")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log
(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{3-x}{1-x^3}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{2}{3}\log\left(|x-1|\right)$$

input
```
integrate((3-x)/(-x^3+1),x, algorithm="giac")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*log(x^2 + x + 1) - 2/3*log
(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{3-x}{1-x^3} dx = -\frac{2 \ln(x-1)}{3} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

input
```
int((x - 3)/(x^3 - 1),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*2i)/3 + 1/3) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*2i)/3 - 1/3) - (2*log(x - 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{3-x}{1-x^3} dx = \frac{4\sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} + \frac{\log(x^2+x+1)}{3} - \frac{2\log(x-1)}{3}$$

input
```
int((3-x)/(-x^3+1),x)
```

output
```
(4*sqrt(3)*atan((2*x + 1)/sqrt(3)) + log(x**2 + x + 1) - 2*log(x - 1))/3
```

3.19 $\int \frac{c+dx}{c^3+d^3x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = -\frac{2\arctan\left(\frac{c-2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

output
```
-2/3*arctan(1/3*(-2*d*x+c)*3^(1/2)/c)*3^(1/2)/c/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\arctan\left(\frac{-c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

input
```
Integrate[(c + d*x)/(c^3 + d^3*x^3),x]
```

output
```
(2*ArcTan[(-c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{c^2-cdx+d^2x^2}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\int \frac{1}{-\left(1-\frac{2dx}{c}\right)^2-3}d\left(1-\frac{2dx}{c}\right)}{cd}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\arctan\left(\frac{1-\frac{2dx}{c}}{\sqrt{3}}\right)}{\sqrt{3}cd}$$

input
```
Int[(c + d*x)/(c^3 + d^3*x^3),x]
```

output
```
(-2*ArcTan[(1 - (2*d*x)/c)/Sqrt[3]])/(Sqrt[3]*c*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}-\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2d^{2}x-cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	35

input
```
int((d*x+c)/(d^3*x^3+c^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)/d/c*arctan(2/3*d*3^(1/2)/c*x-1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx=\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

input
```
integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{-\frac{\sqrt{3}i\log\left(x+\frac{-c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

input
```
integrate((d*x+c)/(d**3*x**3+c**3),x)
```

output
```
(-sqrt(3)*I*log(x + (-c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (-c +
sqrt(3)*I*c)/(2*d))/3)/(c*d)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,d^2x-cd)}{3\,cd}\right)}{3\,cd}$$

input
```
integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x - c*d)/(c*d))/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx-c)}{3\,c}\right)}{3\,cd}$$

input
```
integrate((d*x+c)/(d^3*x^3+c^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x - c)/c)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx=-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

input
```
int((c + d*x)/(c^3 + d^3*x^3),x)
```

output
```
-(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{c^3+d^3x^3}\,dx=-\frac{2\sqrt{3}\,atan\left(\frac{-2dx+c}{\sqrt{3}\,c}\right)}{3cd}$$

input
```
int((d*x+c)/(d^3*x^3+c^3),x)
```

output
```
( - 2*sqrt(3)*atan((c - 2*d*x)/(sqrt(3)*c)))/(3*c*d)
```

3.20 $\int \frac{c-dx}{c^3-d^3x^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\arctan\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

output
```
2/3*arctan(1/3*(2*d*x+c)*3^(1/2)/c)*3^(1/2)/c/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\arctan\left(\frac{c+2dx}{\sqrt{3}c}\right)}{\sqrt{3}cd}$$

input
```
Integrate[(c - d*x)/(c^3 - d^3*x^3),x]
```

output
```
(2*ArcTan[(c + 2*d*x)/(Sqrt[3]*c)])/(Sqrt[3]*c*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{c^3-d^3x^3}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{c^2+cdx+d^2x^2}dx$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2\int \frac{1}{-\left(\frac{2dx}{c}+1\right)^2-3}d\left(\frac{2dx}{c}+1\right)}{cd}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{\frac{2dx}{c}+1}{\sqrt{3}}\right)}{\sqrt{3}cd}$$

input
```
Int[(c - d*x)/(c^3 - d^3*x^3),x]
```

output
```
(2*ArcTan[(1 + (2*d*x)/c)/Sqrt[3]])/(Sqrt[3]*c*d)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{2d\sqrt{3}\,x}{3c}+\frac{\sqrt{3}}{3}\right)}{3dc}$	29
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2d^{2}x+cd\right)\sqrt{3}}{3cd}\right)}{3cd}$	34

input
```
int((-d*x+c)/(-d^3*x^3+c^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)/d/c*arctan(2/3*d*3^(1/2)/c*x+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,dx+c)}{3\,c}\right)}{3\,cd}$$

input
```
integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{-\frac{\sqrt{3}i\log\left(x+\frac{c-\sqrt{3}ic}{2d}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{c+\sqrt{3}ic}{2d}\right)}{3}}{cd}$$

input
```
integrate((-d*x+c)/(-d**3*x**3+c**3),x)
```

output
```
(-sqrt(3)*I*log(x + (c - sqrt(3)*I*c)/(2*d))/3 + sqrt(3)*I*log(x + (c + sq
rt(3)*I*c)/(2*d))/3)/(c*d)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2d^2x+cd)}{3cd}\right)}{3cd}$$

input
```
integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d^2*x + c*d)/(c*d))/(c*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2dx+c)}{3c}\right)}{3cd}$$

input
```
integrate((-d*x+c)/(-d^3*x^3+c^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*d*x + c)/c)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\,d\,x}{3\,c}\right)}{3\,c\,d}$$

input
```
int((c - d*x)/(c^3 - d^3*x^3),x)
```

output
```
(2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*d*x)/(3*c)))/(3*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{c-dx}{c^3-d^3x^3}\,dx=\frac{2\sqrt{3}\,atan\left(\frac{2dx+c}{\sqrt{3}\,c}\right)}{3cd}$$

input
```
int((-d*x+c)/(-d^3*x^3+c^3),x)
```

output
```
(2*sqrt(3)*atan((c + 2*d*x)/(sqrt(3)*c)))/(3*c*d)
```

3.21 $\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 39

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3} dx = -\frac{2B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

output
```
-2/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B+b^{2/3}Bx}{a+bx^3} dx = -\frac{2B\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

input
```
Integrate[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
(-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{\frac{a^{2/3}}{\sqrt[3]{b}B} - \frac{\sqrt[3]{a}x}{B} + \frac{\sqrt[3]{b}x^2}{B}}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{2B\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2B\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

input
```
Int[(a^(1/3)*b^(1/3)*B + b^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
(-2*B*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(28) = 56$.

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 5.00

method	result
default	$B\,b^{\frac{1}{3}}\left(a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+b^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.\right.$

input
```
int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
B*b^(1/3)*(a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1)))+b^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1
/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b)^(1/
3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\, dx = \left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3 - 3\,a^{\frac{2}{3}}b^{\frac{1}{3}}x + 3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}b^{\frac{2}{3}}x^2 + ab^{\frac{1}{3}}x - a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}} -}{bx^3 + a}\right.\right.$$

input
```
integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 - 3*a^(2/3)*b^(1/3)*x + 3*sqrt(
1/3)*(2*a^(2/3)*b^(2/3)*x^2 + a*b^(1/3)*x - a^(4/3))*sqrt(-1/a^(2/3)) - a)
/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*b^(1/3)*x - a^(1/3))/a^(1
/3))/a^(1/3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\, dx = \frac{B\left(-\frac{\sqrt{3}i\log\left(x + \frac{-B\sqrt[3]{a} - \sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3} + \frac{\sqrt{3}i\log\left(x + \frac{-B\sqrt[3]{a} + \sqrt{3}iB\sqrt[3]{a}}{2B\sqrt[3]{b}}\right)}{3}\right)}{\sqrt[3]{a}}$$

input
```
integrate((a**(1/3)*b**(1/3)*B+b**(2/3)*B*x)/(b*x**3+a),x)
```

output
```
B*(-sqrt(3)*I*log(x + (-B*a**(1/3) - sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3)))
/3 + sqrt(3)*I*log(x + (-B*a**(1/3) + sqrt(3)*I*B*a**(1/3))/(2*B*b**(1/3))
)/3)/a**(1/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.18

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a + bx^3}\,dx = \frac{\sqrt{3}\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(Bb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}b^{\frac{1}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(B*b^(2/3)*(a/b)^(1/3) + B*a^(1/3)*b^(1/3))*arctan(1/3*sqrt(3)
*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(B*b^(2/3)*(a/b)^(
1/3) - B*a^(1/3)*b^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^
(2/3)) - 1/3*(B*b^(2/3)*(a/b)^(1/3) - B*a^(1/3)*b^(1/3))*log(x + (a/b)^(1/
3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3}\,dx = \frac{2\sqrt{3}Bb^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,b^{\frac{2}{3}}x-a^{\frac{1}{3}}b^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}b^{\frac{2}{3}}}}$$

input
```
integrate((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*B*b^(1/3)*arctan(1/3*sqrt(3)*(2*b^(2/3)*x - a^(1/3)*b^(1/3))/s
qrt(a^(2/3)*b^(2/3)))/sqrt(a^(2/3)*b^(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3}\,dx = \frac{2\sqrt{3}\,B\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{b}}{3\,\sqrt{-b}}-\frac{2\,\sqrt{3}\,b^{5/6}\,x}{3\,a^{1/3}\,\sqrt{-b}}\right)}{3\,a^{1/3}\,\sqrt{-b}}$$

input
```
int((B*a^(1/3)*b^(1/3) + B*b^(2/3)*x)/(a + b*x^3),x)
```

output
```
(2*3^(1/2)*B*b^(1/2)*atanh((3^(1/2)*b^(1/2))/(3*(-b)^(1/2)) - (2*3^(1/2)*b
^(5/6)*x)/(3*a^(1/3)*(-b)^(1/2))))/(3*a^(1/3)*(-b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt[3]{a}\sqrt[3]{b}B + b^{2/3}Bx}{a+bx^3}\,dx = -\frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b}{3a^{\frac{1}{3}}}$$

input
```
int((a^(1/3)*b^(1/3)*B+b^(2/3)*B*x)/(b*x^3+a),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b)/(3*a**
(1/3))
```

3.22 $\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 41

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=\frac{2B\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{-b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

output
```
2/3*B*arctan(1/3*(a^(1/3)+2*(-b)^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(41) = 82.

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=\frac{\sqrt[3]{-b}B\left(2\sqrt{3}\left(\sqrt[3]{-b}-\sqrt[3]{b}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{-b}+\sqrt[3]{b}\right)\left(2\log\left(\sqrt[3]{\right.\right.}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
((-b)^(1/3)*B*(2*Sqrt[3]*((-b)^(1/3) - b^(1/3))*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]] + ((-b)^(1/3) + b^(1/3))*(2*Log[a^(1/3) + b^(1/3)*x] - L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])))/(6*a^(1/3)*b^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2019, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{-\frac{a^{2/3}(-b)^{2/3}}{bB}+\frac{\sqrt[3]{a}x}{B}+\frac{\sqrt[3]{-b}x^2}{B}}dx$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2B\int \frac{1}{-\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2B\arctan\left(\frac{\frac{2\sqrt[3]{-b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

input
```
Int[(a^(1/3)*(-b)^(1/3)*B - (-b)^(2/3)*B*x)/(a + b*x^3),x]
```

output
```
(2*B*ArcTan[(1 + (2*(-b)^(1/3)*x)/a^(1/3))/Sqrt[3]])/(Sqrt[3]*a^(1/3))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(30) = 60$.

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.93

method	result
default	$-B\,b^{\frac{1}{3}}\left(-a^{\frac{1}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+(-b)^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\right.\right.}{3b\left(\frac{a}{b}\right.}\right.\right.$

input
```
int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x,method=_RETURNVERBOS
E)
```

output
```
-B*b^(1/3)*(-a^(1/3)*(1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/b/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1)))+(-b)^(1/3)*(-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*3^(1/2)/b/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))))*(-1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - (-b)^{2/3}Bx}{a + bx^3}\,dx = \left[\sqrt{\frac{1}{3}}B\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3 + 3\,a^{\frac{2}{3}}(-b)^{\frac{1}{3}}\,x - 3\,\sqrt{\frac{1}{3}}\left(2\,a^{\frac{2}{3}}(-b)^{\frac{2}{3}}\,x^2 - a(-b)^{\frac{1}{3}}\right.}{bx^3 + a}\right.\right.$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="fr
icas")
```

output
```
[sqrt(1/3)*B*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*a^(2/3)*(-b)^(1/3)*x - 3*sq
rt(1/3)*(2*a^(2/3)*(-b)^(2/3)*x^2 - a*(-b)^(1/3)*x - a^(4/3))*sqrt(-1/a^(2
/3)) - a)/(b*x^3 + a)), 2*sqrt(1/3)*B*arctan(sqrt(1/3)*(2*(-b)^(1/3)*x + a
^(1/3))/a^(1/3))/a^(1/3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - (-b)^{2/3}Bx}{a + bx^3}\,dx = -\frac{B\left(-\frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} - \frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + x\right)}{3} + \frac{\sqrt{3}i\log\left(-\frac{\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + \frac{\sqrt{3}i\sqrt[3]{a}(-b)^{\frac{2}{3}}}{2b} + x\right)}{3}\right)}{\sqrt[3]{a}}$$

input
```
integrate((a**(1/3)*(-b)**(1/3)*B-(-b)**(2/3)*B*x)/(b*x**3+a),x)
```

output
```
-B*(-sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) - sqrt(3)*I*a**(1/3)*(-b)**
(2/3)/(2*b) + x)/3 + sqrt(3)*I*log(-a**(1/3)*(-b)**(2/3)/(2*b) + sqrt(3)*I
*a**(1/3)*(-b)**(2/3)/(2*b) + x)/3)/a**(1/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(30) = 60$.

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.24

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B - (-b)^{2/3}Bx}{a+bx^3}\,dx = -\frac{\sqrt{3}\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(B(-b)^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ba^{\frac{1}{3}}(-b)^{\frac{1}{3}}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="ma
xima")
```

output
```
-1/3*sqrt(3)*(B*(-b)^(2/3)*(a/b)^(1/3) - B*a^(1/3)*(-b)^(1/3))*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2/3)) - 1/6*(B*(-b)^(2/
3)*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/
3))/(b*(a/b)^(2/3)) + 1/3*(B*(-b)^(2/3)*(a/b)^(1/3) + B*a^(1/3)*(-b)^(1/3)
)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=-\frac{2\sqrt{3}B(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\left(2\,(-b)^{\frac{2}{3}}x+a^{\frac{1}{3}}\,(-b)^{\frac{1}{3}}\right)}{3\sqrt{a^{\frac{2}{3}}\,(-b)^{\frac{2}{3}}}}\right)}{3\sqrt{a^{\frac{2}{3}}\,(-b)^{\frac{2}{3}}}}$$

input
```
integrate((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x, algorithm="gi
ac")
```

output
```
-2/3*sqrt(3)*B*(-b)^(1/3)*arctan(-1/3*sqrt(3)*(2*(-b)^(2/3)*x + a^(1/3)*(-
b)^(1/3))/sqrt(a^(2/3)*(-b)^(2/3)))/sqrt(a^(2/3)*(-b)^(2/3))
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=-\frac{2\sqrt{3}\,B\,\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sqrt{-b}}{3\,\sqrt{b}}-\frac{2\,\sqrt{3}\,\sqrt{b}\,x}{3\,a^{1/3}\,(-b)^{1/6}}\right)}{3\,a^{1/3}\,\sqrt{b}}$$

input
```
int(-(B*(-b)^(2/3)*x - B*a^(1/3)*(-b)^(1/3))/(a + b*x^3),x)
```

output
```
-(2*3^(1/2)*B*(-b)^(1/2)*atanh((3^(1/2)*(-b)^(1/2))/(3*b^(1/2)) - (2*3^(1/
2)*b^(1/2)*x)/(3*a^(1/3)*(-b)^(1/6))))/(3*a^(1/3)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[3]{a}\sqrt[3]{-b}B-(-b)^{2/3}Bx}{a+bx^3}\,dx=\frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b}{3a^{\frac{1}{3}}}$$

input
```
int((a^(1/3)*(-b)^(1/3)*B-(-b)^(2/3)*B*x)/(b*x^3+a),x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b)/(3*a**(1/
3))
```

3.23 $\int \frac{(a+bx+cx^2)^2}{d+ex^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3} \, dx = \frac{2bcx}{e} + \frac{c^2x^2}{2e} + \frac{\left(c^2d^{4/3} + 2bcd\sqrt[3]{e} - a\left(2b\sqrt[3]{d} + a\sqrt[3]{e}\right)e\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{5/3}} - \frac{\left(\sqrt[3]{e}(2bcd-a^2e) - \sqrt[3]{d}(c^2d-2abe)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{5/3}} + \frac{\left(2bcd - a^2e - \frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}\right)\log\left(d^{2/3} - \sqrt[3]{d}\sqrt[3]{e}x + e^{2/3}x^2\right)}{6d^{2/3}e^{4/3}} + \frac{(b^2+2ac)\log(d+ex^3)}{3e}$$

output
```
2*b*c*x/e+1/2*c^2*x^2/e+1/3*(c^2*d^(4/3)+2*b*c*d*e^(1/3)-a*(2*b*d^(1/3)+a*
e^(1/3))*e)*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(2
/3)/e^(5/3)-1/3*(e^(1/3)*(-a^2*e+2*b*c*d)-d^(1/3)*(-2*a*b*e+c^2*d))*ln(d^(
1/3)+e^(1/3)*x)/d^(2/3)/e^(5/3)+1/6*(2*b*c*d-a^2*e-d^(1/3)*(-2*a*b*e+c^2*d
)/e^(1/3))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(4/3)+1/3*(
2*a*c+b^2)*ln(e*x^3+d)/e
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$$
$$= \frac{12bce^{2/3}x + 3c^2e^{2/3}x^2 + \frac{2\sqrt{3}(cd^{2/3}-ae^{2/3})\left(cd^{2/3}+2b\sqrt[3]{d}\sqrt[3]{e}+ae^{2/3}\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}} + \frac{2\left(c^2d^{4/3}-2bcd\sqrt[3]{e}+a\left(-2b\sqrt[3]{\ldots}\right.\right.}{d^{2\ldots}}}{\ldots}$$

input
```
Integrate[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

output
```
(12*b*c*e^(2/3)*x + 3*c^2*e^(2/3)*x^2 + (2*Sqrt[3]*(c*d^(2/3) - a*e^(2/3))
*(c*d^(2/3) + 2*b*d^(1/3)*e^(1/3) + a*e^(2/3))*ArcTan[(1 - (2*e^(1/3)*x)/d
^(1/3))/Sqrt[3]])/d^(2/3) + (2*(c^2*d^(4/3) - 2*b*c*d*e^(1/3) + a*(-2*b*d^
(1/3) + a*e^(1/3))*e)*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - ((c^2*d^(4/3) -
2*b*c*d*e^(1/3) + a*(-2*b*d^(1/3) + a*e^(1/3))*e)*Log[d^(2/3) - d^(1/3)*e^
(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 2*(b^2 + 2*a*c)*e^(2/3)*Log[d + e*x^3])/
(6*e^(5/3))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$$
$$\Big\downarrow \; 2426$$
$$\int \left(-\frac{a^2(-e)-ex^2(2ac+b^2)+x(c^2d-2abe)+2bcd}{e(d+ex^3)}+\frac{2bc}{e}+\frac{c^2x}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^2(-e)-\frac{\sqrt[3]{d}(c^2d-2abe)}{\sqrt[3]{e}}+2bcd\right)}{6d^{2/3}e^{4/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}(2bcd-a^2e)-\sqrt[3]{d}(c^2d-2abe)\right)}{3d^{2/3}e^{5/3}}+\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-ae\left(a\sqrt[3]{e}+2b\sqrt[3]{d}\right)+2bcd\sqrt[3]{e}+c^2d^{4/3}\right)}{\sqrt{3}d^{2/3}e^{5/3}}+\frac{(2ac+b^2)\log\left(d+ex^3\right)}{3e}+\frac{2bcx}{e}+\frac{c^2x^2}{2e}$$

input
```
Int[(a + b*x + c*x^2)^2/(d + e*x^3),x]
```

output
```
(2*b*c*x)/e + (c^2*x^2)/(2*e) + ((c^2*d^(4/3) + 2*b*c*d*e^(1/3) - a*(2*b*d
^(1/3) + a*e^(1/3))*e)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/
(Sqrt[3]*d^(2/3)*e^(5/3)) - ((e^(1/3)*(2*b*c*d - a^2*e) - d^(1/3)*(c^2*d -
 2*a*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(5/3)) + ((2*b*c*d - a^2
*e - (d^(1/3)*(c^2*d - 2*a*b*e))/e^(1/3))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x
+ e^(2/3)*x^2])/(6*d^(2/3)*e^(4/3)) + ((b^2 + 2*a*c)*Log[d + e*x^3])/(3*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.31

method	result
risch	$\frac{c^2x^2}{2e}+\frac{2bcx}{e}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^3+d\right)}\frac{\left(e\left(2ac+b^2\right)\_R^2+\left(2abe-c^2d\right)\_R+a^2e-2dbc\right)\ln\left(x-\_R\right)}{\_R^2}}{3e^2}$
default	$\frac{c\left(\frac{1}{2}cx^2+2bx\right)}{e}+\frac{(a^2e-2dbc)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)+(2abe-c^2d)\left(-\frac{\ln\left(x+\left(\dots\right.\right.}{3e\left(\dots\right.}\right.}{e}$

input
```
int((c*x^2+b*x+a)^2/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2*x^2/e+2*b*c*x/e+1/3/e^2*sum((e*(2*a*c+b^2)*_R^2+(2*a*b*e-c^2*d)*_R
+a^2*e-2*d*b*c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 12827, normalized size of antiderivative = 47.51

$$\int \frac{\left(a+bx+cx^2\right)^2}{d+ex^3}\,dx=\text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**2/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$$
$$= \frac{(b^2+2\,ac)\log(|ex^3+d|)}{3\,e}$$
$$+ \frac{\sqrt{3}\left(2\,bcde-a^2e^2-(-de^2)^{\frac{1}{3}}\,c^2d+2\,(-de^2)^{\frac{1}{3}}\,abe\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,(-de^2)^{\frac{2}{3}}\,e}$$
$$+ \frac{\left(2\,bcde-a^2e^2+(-de^2)^{\frac{1}{3}}\,c^2d-2\,(-de^2)^{\frac{1}{3}}\,abe\right)\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(-de^2)^{\frac{2}{3}}\,e}$$
$$+ \frac{c^2ex^2+4\,bcex}{2\,e^2}$$
$$+ \frac{\left(c^2de^4\left(-\frac{d}{e}\right)^{\frac{1}{3}}-2\,abe^5\left(-\frac{d}{e}\right)^{\frac{1}{3}}+2\,bcde^4-a^2e^5\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,de^5}$$

input
```
integrate((c*x^2+b*x+a)^2/(e*x^3+d),x, algorithm="giac")
```

output
```
1/3*(b^2 + 2*a*c)*log(abs(e*x^3 + d))/e + 1/3*sqrt(3)*(2*b*c*d*e - a^2*e^2
 - (-d*e^2)^(1/3)*c^2*d + 2*(-d*e^2)^(1/3)*a*b*e)*arctan(1/3*sqrt(3)*(2*x
+ (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) + 1/6*(2*b*c*d*e - a^2*e^
2 + (-d*e^2)^(1/3)*c^2*d - 2*(-d*e^2)^(1/3)*a*b*e)*log(x^2 + x*(-d/e)^(1/3
) + (-d/e)^(2/3))/((-d*e^2)^(2/3)*e) + 1/2*(c^2*e*x^2 + 4*b*c*e*x)/e^2 + 1
/3*(c^2*d*e^4*(-d/e)^(1/3) - 2*a*b*e^5*(-d/e)^(1/3) + 2*b*c*d*e^4 - a^2*e^
5)*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d*e^5)
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.85

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx = \left(\sum_{k=1}^{3}\ln\left(\frac{2\,a^3\,b\,e^2+3\,a^2\,c^2\,d\,e+b^4\,d\,e+2\,b\,c^3\,d^2}{e} + \frac{x\,(-2\,a^3\,c\,e^2+3\,a^2\,b^2\,e^2+2\,b^3\,c\,d\,e+c^4\,d^2)}{e} - \mathrm{root}\big(27\,d^2\,e^5\,z^3-54\,a\,c\,d^2\,e^4\,z^2-27\,b^2\,d^2\,e^4\,z^2+27\,a^2\,c^2\,d^2\,e^3\,z+18\,b\,c^3\,d^3\,e^2\,z+18\,a^3\,b\,d\,e^4\,z+9\,b^4\,d^2\,e^3 - 54\,a\,c\,d^2\,e^4\,z^2-27\,b^2\,d^2\,e^4\,z^2+27\,a^2\,c^2\,d^2\,e^3\,z+18\,b\,c^3\,d^3\,e^2\,z+18\,a^3\,b\,d\,e^4\,z + 9\,b^4\,d^2\,e^3\,z+6\,a\,b^4\,c\,d^2\,e^2-9\,a^2\,b^2\,c^2\,d^2\,e^2-6\,a^4\,b\,c\,d\,e^3-6\,a\,b\,c^4\,d^3\,e-2\,a^3\,c^3\,d^2\,e^2 + 2\,b^3\,c^3\,d^3\,e+2\,a^3\,b^3\,d\,e^3-b^6\,d^2\,e^2-c^6\,d^4-a^6\,e^4,z,k\big)\right)+\frac{c^2\,x^2}{2\,e}+\frac{2\,b\,c\,x}{e}$$

input

```
int((a + b*x + c*x^2)^2/(d + e*x^3),x)
```

output

```
symsum(log((2*a^3*b*e^2 + 2*b*c^3*d^2 + b^4*d*e + 3*a^2*c^2*d*e)/e + (x*(c
^4*d^2 - 2*a^3*c*e^2 + 3*a^2*b^2*e^2 + 2*b^3*c*d*e))/e - 3*root(27*d^2*e^5
*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2 + 27*a^2*c^2*d^2*e^3*z + 18
*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2*e^3*z + 6*a*b^4*c*d^2*e^2
- 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a*b*c^4*d^3*e - 2*a^3*c^3*d^
2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6*d^2*e^2 - c^6*d^4 - a^6*e^
4, z, k)*e*(2*b^2*d - 3*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*
d^2*e^4*z^2 + 27*a^2*c^2*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z
 + 9*b^4*d^2*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c
*d*e^3 - 6*a*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3
*d*e^3 - b^6*d^2*e^2 - c^6*d^4 - a^6*e^4, z, k)*d*e + 4*a*c*d - a^2*e*x +
2*b*c*d*x))*root(27*d^2*e^5*z^3 - 54*a*c*d^2*e^4*z^2 - 27*b^2*d^2*e^4*z^2
+ 27*a^2*c^2*d^2*e^3*z + 18*b*c^3*d^3*e^2*z + 18*a^3*b*d*e^4*z + 9*b^4*d^2
*e^3*z + 6*a*b^4*c*d^2*e^2 - 9*a^2*b^2*c^2*d^2*e^2 - 6*a^4*b*c*d*e^3 - 6*a
*b*c^4*d^3*e - 2*a^3*c^3*d^2*e^2 + 2*b^3*c^3*d^3*e + 2*a^3*b^3*d*e^3 - b^6
*d^2*e^2 - c^6*d^4 - a^6*e^4, z, k), k, 1, 3) + (c^2*x^2)/(2*e) + (2*b*c*x
)/e
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx+cx^2)^2}{d+ex^3}\,dx$$

$$= \frac{-2e^{\frac{4}{3}}d^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)a^2+4e^{\frac{1}{3}}d^{\frac{5}{3}}\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)bc-4\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)abde+2\sqrt{3}\,atan\left(\right.}{}$$

input
```
int((c*x^2+b*x+a)^2/(e*x^3+d),x)
```

output
```
( - 2*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*s
qrt(3)))*a**2*e + 4*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*
x)/(d**(1/3)*sqrt(3)))*b*c*d - 4*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d
**(1/3)*sqrt(3)))*a*b*d*e + 2*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(
1/3)*sqrt(3)))*c**2*d**2 - e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)*d**(1
/3)*x + e**(2/3)*x**2)*a**2*e + 2*e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3
)*d**(1/3)*x + e**(2/3)*x**2)*b*c*d + 2*e**(1/3)*d**(2/3)*log(d**(1/3) + e
**(1/3)*x)*a**2*e - 4*e**(1/3)*d**(2/3)*log(d**(1/3) + e**(1/3)*x)*b*c*d +
 4*e**(2/3)*d**(1/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a
*c*d + 2*e**(2/3)*d**(1/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x
**2)*b**2*d + 4*e**(2/3)*d**(1/3)*log(d**(1/3) + e**(1/3)*x)*a*c*d + 2*e**
(2/3)*d**(1/3)*log(d**(1/3) + e**(1/3)*x)*b**2*d + 12*e**(2/3)*d**(1/3)*b*
c*d*x + 3*e**(2/3)*d**(1/3)*c**2*d*x**2 + 2*log(d**(2/3) - e**(1/3)*d**(1/
3)*x + e**(2/3)*x**2)*a*b*d*e - log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2
/3)*x**2)*c**2*d**2 - 4*log(d**(1/3) + e**(1/3)*x)*a*b*d*e + 2*log(d**(1/3
) + e**(1/3)*x)*c**2*d**2)/(6*e**(2/3)*d**(1/3)*d*e)
```

3.24 $\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 416

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = -\frac{(c^3d-b^3e-6abce)\,x}{e^2}+\frac{3c(b^2+ac)\,x^2}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}$$
$$-\frac{\left(c^3d^2-3b^2cd^{4/3}e^{2/3}-3ac^2d^{4/3}e^{2/3}-b^3de-6abcde+3a^2b\sqrt[3]{d}e^{5/3}+a^3e^2\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}}$$
$$+\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}}$$
$$-\frac{\left(c^3d^2-6abcde-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}(b^2cd+ac^2d-a^2be)\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}e^{7/3}}$$
$$-\frac{(bc^2d-ab^2e-a^2ce)\log(d+ex^3)}{e^2}$$

output

```
-(-6*a*b*c*e-b^3*e+c^3*d)*x/e^2+3/2*c*(a*c+b^2)*x^2/e+b*c^2*x^3/e+1/4*c^3*
x^4/e-1/3*(c^3*d^2-3*b^2*c*d^(4/3)*e^(2/3)-3*a*c^2*d^(4/3)*e^(2/3)-b^3*d*e
-6*a*b*c*d*e+3*a^2*b*d^(1/3)*e^(5/3)+a^3*e^2)*arctan(1/3*(d^(1/3)-2*e^(1/3
)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(2/3)/e^(7/3)+1/3*(c^3*d^2-6*a*b*c*d*e-e*(
-a^3*e+b^3*d)+3*d^(1/3)*e^(2/3)*(-a^2*b*e+a*c^2*d+b^2*c*d))*ln(d^(1/3)+e^(
1/3)*x)/d^(2/3)/e^(7/3)-1/6*(c^3*d^2-6*a*b*c*d*e-e*(-a^3*e+b^3*d)+3*d^(1/3
)*e^(2/3)*(-a^2*b*e+a*c^2*d+b^2*c*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)
*x^2)/d^(2/3)/e^(7/3)-(-a^2*c*e-a*b^2*e+b*c^2*d)*ln(e*x^3+d)/e^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$$

$$= \frac{12\sqrt[3]{e}(-c^3d+b^3e+6abce)\,x+18c(b^2+ac)\,e^{4/3}x^2+12bc^2e^{4/3}x^3+3c^3e^{4/3}x^4-\frac{4\sqrt{3}\left(c^3d^2-3ac^2d^{4/3}e^{2/3}+e\left(-\cdots\right.\right.}{\cdots}\cdots}{\cdots}$$

input
```
Integrate[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

output
```
(12*e^(1/3)*(-(c^3*d) + b^3*e + 6*a*b*c*e)*x + 18*c*(b^2 + a*c)*e^(4/3)*x^
2 + 12*b*c^2*e^(4/3)*x^3 + 3*c^3*e^(4/3)*x^4 - (4*Sqrt[3]*(c^3*d^2 - 3*a*c
^2*d^(4/3)*e^(2/3) + e*(-(b^3*d) + 3*a^2*b*d^(1/3)*e^(2/3) + a^3*e) - 3*c*
(b^2*d^(4/3)*e^(2/3) + 2*a*b*d*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt
[3]])/d^(2/3) + (4*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c^2*d^(4/3)*e^
(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3*e^2)*Log[d^(
1/3) + e^(1/3)*x])/d^(2/3) - (2*(c^3*d^2 + 3*b^2*c*d^(4/3)*e^(2/3) + 3*a*c
^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e - 3*a^2*b*d^(1/3)*e^(5/3) + a^3
*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + 12*e^(1/3)
*(-(b*c^2*d) + a*b^2*e + a^2*c*e)*Log[d + e*x^3])/(12*e^(7/3))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx$$

$$\Big\downarrow \; 2426$$

$$\int \left(\frac{-e(b^3d-a^3e)-3ex^2\left(a^2(-c)e-ab^2e+bc^2d\right)-3ex\left(a^2(-b)e+ac^2d+b^2cd\right)-6abcde+c^3d^2}{e^2\left(d+ex^3\right)}-\frac{-6abce+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log\left(d+ex^3\right)\left(a^2(-c)e-ab^2e+bc^2d\right)}{e^2}-$$

$$\frac{\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(a^3e^2+3a^2b\sqrt[3]{d}e^{5/3}-6abcde-3ac^2d^{4/3}e^{2/3}-b^3de-3b^2cd^{4/3}e^{2/3}+c^3d^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}-$$

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-6abcde+c^3d^2\right)}{6d^{2/3}e^{7/3}}+$$

$$\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(-e(b^3d-a^3e)+3\sqrt[3]{d}e^{2/3}\left(a^2(-b)e+ac^2d+b^2cd\right)-6abcde+c^3d^2\right)}{3d^{2/3}e^{7/3}}-$$

$$\frac{x\left(-6abce+b^3(-e)+c^3d\right)}{e^2}+\frac{3cx^2\left(ac+b^2\right)}{2e}+\frac{bc^2x^3}{e}+\frac{c^3x^4}{4e}$$

input
```
Int[(a + b*x + c*x^2)^3/(d + e*x^3),x]
```

output
```
-(((c^3*d - b^3*e - 6*a*b*c*e)*x)/e^2) + (3*c*(b^2 + a*c)*x^2)/(2*e) + (b*
c^2*x^3)/e + (c^3*x^4)/(4*e) - ((c^3*d^2 - 3*b^2*c*d^(4/3)*e^(2/3) - 3*a*c
^2*d^(4/3)*e^(2/3) - b^3*d*e - 6*a*b*c*d*e + 3*a^2*b*d^(1/3)*e^(5/3) + a^3
*e^2)*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*
e^(7/3)) + ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)
*(b^2*c*d + a*c^2*d - a^2*b*e))*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/
3)) - ((c^3*d^2 - 6*a*b*c*d*e - e*(b^3*d - a^3*e) + 3*d^(1/3)*e^(2/3)*(b^2
*c*d + a*c^2*d - a^2*b*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])
/(6*d^(2/3)*e^(7/3)) - ((b*c^2*d - a*b^2*e - a^2*c*e)*Log[d + e*x^3])/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
 + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.44

method	result
risch	$\frac{c^3x^4}{4e}+\frac{b\,c^2x^3}{e}+\frac{3a\,c^2x^2}{2e}+\frac{3b^2c\,x^2}{2e}+\frac{6abcx}{e}+\frac{b^3x}{e}-\frac{c^3dx}{e^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^3+d\right)}\left(3e\left(a^2ce+a\,b^2e-b\,c^2d\right)\_R^2+3e\left(a^2\right.\right.}{3}$
default	$\frac{\frac{1}{4}c^3x^4e+b\,c^2x^3e+\frac{3}{2}a\,c^2e\,x^2+\frac{3}{2}b^2ce\,x^2+6abcex+b^3ex-c^3dx}{e^2}+\frac{\left(a^3e^2-6abcde-b^3de+c^3d^2\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right.\right.}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right.}{}$

input
```
int((c*x^2+b*x+a)^3/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^3*x^4/e+b*c^2*x^3/e+3/2/e*a*c^2*x^2+3/2/e*b^2*c*x^2+6/e*a*b*c*x+1/e*
b^3*x-1/e^2*c^3*d*x+1/3/e^3*sum((3*e*(a^2*c*e+a*b^2*e-b*c^2*d)*_R^2+3*e*(a
^2*b*e-a*c^2*d-b^2*c*d)*_R+a^3*e^2-6*a*b*c*d*e-b^3*d*e+c^3*d^2)/_R^2*ln(x-
_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.86 (sec) , antiderivative size = 29479, normalized size of antiderivative = 70.86

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**3/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx =$$

$$-\frac{\sqrt{3}\left(c^3d^2-b^3de-6\,abcde+a^3e^2+3\,(-de^2)^{\frac{1}{3}}\,b^2cd+3\,(-de^2)^{\frac{1}{3}}\,ac^2d-3\,(-de^2)^{\frac{1}{3}}\,a^2be\right)\arctan\left(\frac{\sqrt{3}\left(2\right.}{3}\right.}{3\,(-de^2)^{\frac{2}{3}}\,e}$$

$$-\frac{\left(c^3d^2-b^3de-6\,abcde+a^3e^2-3\,(-de^2)^{\frac{1}{3}}\,b^2cd-3\,(-de^2)^{\frac{1}{3}}\,ac^2d+3\,(-de^2)^{\frac{1}{3}}\,a^2be\right)\log\left(x^2+x\left(-\frac{d}{e}\right)\right.}{6\,(-de^2)^{\frac{2}{3}}\,e}$$

$$-\frac{(bc^2d-ab^2e-a^2ce)\log\left(|ex^3+d|\right)}{e^2}$$

$$+\frac{c^3e^3x^4+4\,bc^2e^3x^3+6\,b^2ce^3x^2+6\,ac^2e^3x^2-4\,c^3de^2x+4\,b^3e^3x+24\,abce^3x}{4\,e^4}$$

$$+\frac{\left(3\,b^2cde^8\left(-\frac{d}{e}\right)^{\frac{1}{3}}+3\,ac^2de^8\left(-\frac{d}{e}\right)^{\frac{1}{3}}-3\,a^2be^9\left(-\frac{d}{e}\right)^{\frac{1}{3}}-c^3d^2e^7+b^3de^8+6\,abcde^8-a^3e^9\right)\left(-\frac{d}{e}\right)^{\frac{1}{3}}\log\left(\left|x\right.\right.}{3\,de^9}$$

input
```
integrate((c*x^2+b*x+a)^3/(e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(c^3*d^2 - b^3*d*e - 6*a*b*c*d*e + a^3*e^2 + 3*(-d*e^2)^(1/3)
*b^2*c*d + 3*(-d*e^2)^(1/3)*a*c^2*d - 3*(-d*e^2)^(1/3)*a^2*b*e)*arctan(1/3
*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/((-d*e^2)^(2/3)*e) - 1/6*(c^3*
d^2 - b^3*d*e - 6*a*b*c*d*e + a^3*e^2 - 3*(-d*e^2)^(1/3)*b^2*c*d - 3*(-d*e
^2)^(1/3)*a*c^2*d + 3*(-d*e^2)^(1/3)*a^2*b*e)*log(x^2 + x*(-d/e)^(1/3) + (
-d/e)^(2/3))/((-d*e^2)^(2/3)*e) - (b*c^2*d - a*b^2*e - a^2*c*e)*log(abs(e*
x^3 + d))/e^2 + 1/4*(c^3*e^3*x^4 + 4*b*c^2*e^3*x^3 + 6*b^2*c*e^3*x^2 + 6*a
*c^2*e^3*x^2 - 4*c^3*d*e^2*x + 4*b^3*e^3*x + 24*a*b*c*e^3*x)/e^4 + 1/3*(3*
b^2*c*d*e^8*(-d/e)^(1/3) + 3*a*c^2*d*e^8*(-d/e)^(1/3) - 3*a^2*b*e^9*(-d/e)
^(1/3) - c^3*d^2*e^7 + b^3*d*e^8 + 6*a*b*c*d*e^8 - a^3*e^9)*(-d/e)^(1/3)*l
og(abs(x - (-d/e)^(1/3)))/(d*e^9)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 1700, normalized size of antiderivative = 4.09

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)^3/(d + e*x^3),x)
```

output
```
x*((b^3 + 6*a*b*c)/e - (c^3*d)/e^2) + symsum(log(root(27*d^2*e^7*z^3 + 81*
b*c^2*d^3*e^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b
^2*c*d^2*e^5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*
c^4*d^4*e^3*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3
*e^4*z - 27*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18
*a^4*b*c^4*d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2
*c^2*d^2*e^4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^
3*c^3*d^3*e^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*
a^6*c^3*d^2*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e
+ 3*a^6*b^3*d*e^5 + b^9*d^3*e^3 - c^9*d^6 - a^9*e^6, z, k)*((3*x*(a^3*e^4
- b^3*d*e^3 + c^3*d^2*e^2 - 6*a*b*c*d*e^3))/e^2 - (3*(6*a*b^2*d*e^3 - 6*b*
c^2*d^2*e^2 + 6*a^2*c*d*e^3))/e^2 + 9*root(27*d^2*e^7*z^3 + 81*b*c^2*d^3*e
^5*z^2 - 81*a^2*c*d^2*e^6*z^2 - 81*a*b^2*d^2*e^6*z^2 - 27*a^3*b^2*c*d^2*e^
5*z + 27*a^2*b*c^3*d^3*e^4*z + 27*a*b^3*c^2*d^3*e^4*z + 54*b^2*c^4*d^4*e^3
*z + 54*a^4*c^2*d^2*e^5*z + 54*a^2*b^4*d^2*e^5*z + 27*b^5*c*d^3*e^4*z - 27
*a*c^5*d^4*e^3*z + 27*a^5*b*d*e^6*z + 18*a^4*b^4*c*d^2*e^4 - 18*a^4*b*c^4*
d^3*e^3 + 18*a*b^4*c^4*d^4*e^2 - 9*a*b^7*c*d^3*e^3 - 27*a^5*b^2*c^2*d^2*e^
4 + 27*a^2*b^5*c^2*d^3*e^3 - 27*a^2*b^2*c^5*d^4*e^2 - 21*a^3*b^3*c^3*d^3*e
^3 - 9*a^7*b*c*d*e^5 - 9*a*b*c^7*d^5*e - 3*b^6*c^3*d^4*e^2 - 3*a^6*c^3*d^2
*e^4 - 3*a^3*c^6*d^4*e^2 - 3*a^3*b^6*d^2*e^4 + 3*b^3*c^6*d^5*e + 3*a^6*...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx+cx^2)^3}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^3/(e*x^3+d),x)
```

output
```
( - 4*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*s
qrt(3)))*a**3*e**2 + 24*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1
/3)*x)/(d**(1/3)*sqrt(3)))*a*b*c*d*e + 4*e**(1/3)*d**(2/3)*sqrt(3)*atan((d
**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*b**3*d*e - 4*e**(1/3)*d**(2/3)
*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*c**3*d**2 - 12
*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a**2*b*d*e**2
+ 12*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a*c**2*d**
2*e + 12*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*b**2*c
*d**2*e - 2*e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3
)*x**2)*a**3*e**2 + 12*e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*
x + e**(2/3)*x**2)*a*b*c*d*e + 2*e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)
*d**(1/3)*x + e**(2/3)*x**2)*b**3*d*e - 2*e**(1/3)*d**(2/3)*log(d**(2/3) -
 e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*c**3*d**2 + 4*e**(1/3)*d**(2/3)*log(
d**(1/3) + e**(1/3)*x)*a**3*e**2 - 24*e**(1/3)*d**(2/3)*log(d**(1/3) + e**
(1/3)*x)*a*b*c*d*e - 4*e**(1/3)*d**(2/3)*log(d**(1/3) + e**(1/3)*x)*b**3*d
*e + 4*e**(1/3)*d**(2/3)*log(d**(1/3) + e**(1/3)*x)*c**3*d**2 + 12*e**(2/3
)*d**(1/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a**2*c*d*e
+ 12*e**(2/3)*d**(1/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)
*a*b**2*d*e - 12*e**(2/3)*d**(1/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e*
*(2/3)*x**2)*b*c**2*d**2 + 12*e**(2/3)*d**(1/3)*log(d**(1/3) + e**(1/3)...
```

3.25 $\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 643

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$$
$$= -\frac{2(3b^2c^2d+2ac^3d-2ab^3e-6a^2bce)\,x}{e^2} - \frac{(4bc^3d-b^4e-12ab^2ce-6a^2c^2e)\,x^2}{2e^2}$$
$$-\frac{c(c^3d-4b^3e-12abce)\,x^3}{3e^2} + \frac{c^2(3b^2+2ac)\,x^4}{2e} + \frac{4bc^3x^5}{5e} + \frac{c^4x^6}{6e}$$
$$-\frac{\left(b\sqrt[3]{d}+a\sqrt[3]{e}\right)\left(4c^3d^2+6c^2\left(bd^{5/3}\sqrt[3]{e}-ad^{4/3}e^{2/3}\right)-12abcde-e\left(b^3d+3ab^2d^{2/3}\sqrt[3]{e}-3a^2b\sqrt[3]{d}e^{2/3}-\right.\right.}{\sqrt{3}d^{2/3}e^{8/3}}$$
$$+\frac{\left(\sqrt[3]{e}(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2)+\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3\right.}{3d^{2/3}e^{8/3}}$$
$$-\frac{\left(6b^2c^2d^2+4ac^3d^2-4ab^3de-12a^2bcde+a^4e^2+\frac{\sqrt[3]{d}(b^4de+12ab^2cde+6a^2c^2de-4b(c^3d^2+a^3e^2))}{\sqrt[3]{e}}\right)\log\left(d^{2/3}-\right.}{6d^{2/3}e^{7/3}}$$
$$+\frac{(c^4d^2-12abc^2de+6a^2b^2e^2-4ce(b^3d-a^3e))\log\left(d+ex^3\right)}{3e^3}$$

output
```
-2*(-6*a^2*b*c*e-2*a*b^3*e+2*a*c^3*d+3*b^2*c^2*d)*x/e^2-1/2*(-6*a^2*c^2*e-
12*a*b^2*c*e-b^4*e+4*b*c^3*d)*x^2/e^2-1/3*c*(-12*a*b*c*e-4*b^3*e+c^3*d)*x^
3/e^2+1/2*c^2*(2*a*c+3*b^2)*x^4/e+4/5*b*c^3*x^5/e+1/6*c^4*x^6/e-1/3*(b*d^(
1/3)+a*e^(1/3))*(4*c^3*d^2+6*c^2*(b*d^(5/3)*e^(1/3)-a*d^(4/3)*e^(2/3))-12*
a*b*c*d*e-e*(b^3*d+3*a*b^2*d^(2/3)*e^(1/3)-3*a^2*b*d^(1/3)*e^(2/3)-a^3*e))
*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(2/3)/e^(8/3)
+1/3*(e^(1/3)*(a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^
2)+d^(1/3)*(b^4*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e-4*b*(a^3*e^2+c^3*d^2)))*l
n(d^(1/3)+e^(1/3)*x)/d^(2/3)/e^(8/3)-1/6*(6*b^2*c^2*d^2+4*a*c^3*d^2-4*a*b^
3*d*e-12*a^2*b*c*d*e+a^4*e^2+d^(1/3)*(b^4*d*e+12*a*b^2*c*d*e+6*a^2*c^2*d*e
-4*b*(a^3*e^2+c^3*d^2))/e^(1/3))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)
/d^(2/3)/e^(7/3)+1/3*(c^4*d^2-12*a*b*c^2*d*e+6*a^2*b^2*e^2-4*c*e*(-a^3*e+b
^3*d))*ln(e*x^3+d)/e^3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx$$

$$= \frac{60e^{2/3}(-3b^2c^2d-2ac^3d+2ab^3e+6a^2bce)\,x+15e^{2/3}(-4bc^3d+b^4e+12ab^2ce+6a^2c^2e)\,x^2+10ce^{2/3}(-}{}$$

input
```
Integrate[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

output
```
(60*e^(2/3)*(-3*b^2*c^2*d - 2*a*c^3*d + 2*a*b^3*e + 6*a^2*b*c*e)*x + 15*e^
(2/3)*(-4*b*c^3*d + b^4*e + 12*a*b^2*c*e + 6*a^2*c^2*e)*x^2 + 10*c*e^(2/3)
*(-(c^3*d) + 4*b^3*e + 12*a*b*c*e)*x^3 + 15*c^2*(3*b^2 + 2*a*c)*e^(5/3)*x^
4 + 24*b*c^3*e^(5/3)*x^5 + 5*c^4*e^(5/3)*x^6 + (10*Sqrt[3]*(b*d^(1/3) + a*
e^(1/3))*(-4*c^3*d^2 + c^2*(-6*b*d^(5/3)*e^(1/3) + 6*a*d^(4/3)*e^(2/3)) +
12*a*b*c*d*e + e*(b^3*d + 3*a*b^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3
) - a^3*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (10*(4*
a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a*b^3*d*e^(4/3
) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) - 4*b*(c^3*d^(
7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(1/3) + e^(1/3)*x])/d^(
2/3) - (5*(4*a*c^3*d^2*e^(1/3) + b^4*d^(4/3)*e + 6*a^2*c^2*d^(4/3)*e - 4*a
*b^3*d*e^(4/3) + a^4*e^(7/3) + 6*b^2*(c^2*d^2*e^(1/3) + 2*a*c*d^(4/3)*e) -
 4*b*(c^3*d^(7/3) + 3*a^2*c*d*e^(4/3) + a^3*d^(1/3)*e^2))*Log[d^(2/3) - d^
(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3) + (10*(c^4*d^2 - 12*a*b*c^2*d*e +
6*a^2*b^2*e^2 + 4*c*e*(-(b^3*d) + a^3*e))*Log[d + e*x^3])/e^(1/3))/(30*e^(
8/3))
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3} \, dx$$

$$\Big\downarrow \text{ 2426}$$

$$\int \left(-\frac{x(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d)}{e^2}-\frac{2(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d)}{e^2}+\frac{a^4e^2-12a^2bcde-x}{\quad}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x^2\left(-6a^2c^2e-12ab^2ce+b^4(-e)+4bc^3d\right)}{2e^2}-\frac{2x\left(-6a^2bce-2ab^3e+2ac^3d+3b^2c^2d\right)}{e^2}-$$

$$\frac{\left(a\sqrt[3]{e}+b\sqrt[3]{d}\right)\arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(-e\left(a^3(-e)-3a^2b\sqrt[3]{d}e^{2/3}+3ab^2d^{2/3}\sqrt[3]{e}+b^3d\right)+6c^2\left(bd^{5/3}\sqrt[3]{e}-ad^{4/3}e^{2/}\right.\right.}{\sqrt{3}d^{2/3}e^{8/3}}$$

$$\frac{\log\left(d+ex^3\right)\left(-4ce\left(b^3d-a^3e\right)+6a^2b^2e^2-12abc^2de+c^4d^2\right)}{3e^3}-$$

$$\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(a^4e^2-12a^2bcde+\frac{\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+12ab^2cde+b^4de\right)}{\sqrt[3]{e}}-4ab^3de+4ac^3d^2\right.}{6d^{2/3}e^{7/3}}$$

$$\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(\sqrt[3]{e}\left(a^4e^2-12a^2bcde-4ab^3de+4ac^3d^2+6b^2c^2d^2\right)+\sqrt[3]{d}\left(-4b\left(a^3e^2+c^3d^2\right)+6a^2c^2de+12a\right.\right.}{3d^{2/3}e^{8/3}}$$

$$\frac{cx^3\left(-12abce-4b^3e+c^3d\right)}{3e^2}+\frac{c^2x^4\left(2ac+3b^2\right)}{2e}+\frac{4bc^3x^5}{5e}+\frac{c^4x^6}{6e}$$

input
```
Int[(a + b*x + c*x^2)^4/(d + e*x^3),x]
```

output
```
(-2*(3*b^2*c^2*d + 2*a*c^3*d - 2*a*b^3*e - 6*a^2*b*c*e)*x)/e^2 - ((4*b*c^3
*d - b^4*e - 12*a*b^2*c*e - 6*a^2*c^2*e)*x^2)/(2*e^2) - (c*(c^3*d - 4*b^3*
e - 12*a*b*c*e)*x^3)/(3*e^2) + (c^2*(3*b^2 + 2*a*c)*x^4)/(2*e) + (4*b*c^3*
x^5)/(5*e) + (c^4*x^6)/(6*e) - ((b*d^(1/3) + a*e^(1/3))*(4*c^3*d^2 + 6*c^2
*(b*d^(5/3)*e^(1/3) - a*d^(4/3)*e^(2/3)) - 12*a*b*c*d*e - e*(b^3*d + 3*a*b
^2*d^(2/3)*e^(1/3) - 3*a^2*b*d^(1/3)*e^(2/3) - a^3*e))*ArcTan[(d^(1/3) - 2
*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(8/3)) + ((e^(1/3)*(6*b
^2*c^2*d^2 + 4*a*c^3*d^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2) + d^(1/
3)*(b^4*d*e + 12*a*b^2*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))*L
og[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(8/3)) - ((6*b^2*c^2*d^2 + 4*a*c^3*d
^2 - 4*a*b^3*d*e - 12*a^2*b*c*d*e + a^4*e^2 + (d^(1/3)*(b^4*d*e + 12*a*b^2
*c*d*e + 6*a^2*c^2*d*e - 4*b*(c^3*d^2 + a^3*e^2)))/e^(1/3))*Log[d^(2/3) -
d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3)*e^(7/3)) + ((c^4*d^2 - 12*a*b
*c^2*d*e + 6*a^2*b^2*e^2 - 4*c*e*(b^3*d - a^3*e))*Log[d + e*x^3])/(3*e^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.54

method	result
risch	$\frac{c^4x^6}{6e}+\frac{4b\,c^3x^5}{5e}+\frac{a\,c^3x^4}{e}+\frac{3b^2c^2x^4}{2e}+\frac{4ab\,c^2x^3}{e}+\frac{4b^3c\,x^3}{3e}-\frac{c^4d\,x^3}{3e^2}+\frac{3a^2c^2x^2}{e}+\frac{6a\,b^2c\,x^2}{e}+\frac{b^4x^2}{2e}-\frac{2b\,c^3d\,x^2}{e^2}+$
default	$\frac{\frac{1}{6}c^4x^6e+\frac{4}{5}c^3b\,x^5e+a\,c^3e\,x^4+\frac{3}{2}b^2c^2e\,x^4+4ab\,c^2e\,x^3+\frac{4}{3}b^3ce\,x^3-\frac{1}{3}c^4d\,x^3+3a^2c^2e\,x^2+6a\,b^2ce\,x^2+\frac{1}{2}b^4e\,x^2-2b\,c^3d\,x^2+12a^2bcex+4a}{e^2}$

input
```
int((c*x^2+b*x+a)^4/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*c^4*x^6/e+4/5*b*c^3*x^5/e+1/e*a*c^3*x^4+3/2/e*b^2*c^2*x^4+4/e*a*b*c^2*
x^3+4/3/e*b^3*c*x^3-1/3/e^2*c^4*d*x^3+3/e*a^2*c^2*x^2+6/e*a*b^2*c*x^2+1/2/
e*b^4*x^2-2/e^2*b*c^3*d*x^2+12/e*a^2*b*c*x+4/e*a*b^3*x-4/e^2*a*c^3*d*x-6/e
^2*b^2*c^2*d*x+1/3/e^3*sum(((4*a^3*c*e^2+6*a^2*b^2*e^2-12*a*b*c^2*d*e-4*b^
3*c*d*e+c^4*d^2)*_R^2+(4*a^3*b*e^2-6*a^2*c^2*d*e-12*a*b^2*c*d*e-b^4*d*e+4*
b*c^3*d^2)*_R+a^4*e^2-12*a^2*b*c*d*e-4*a*b^3*d*e+4*a*c^3*d^2+6*b^2*c^2*d^2
)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.47 (sec) , antiderivative size = 47284, normalized size of antiderivative = 73.54

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**4/(e*x**3+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^4/(e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e - 4*a*b^3*d*e^2 - 12*a^2*b*c
*d*e^2 + a^4*e^3 - 4*(-d*e^2)^(1/3)*b*c^3*d^2 + (-d*e^2)^(1/3)*b^4*d*e + 1
2*(-d*e^2)^(1/3)*a*b^2*c*d*e + 6*(-d*e^2)^(1/3)*a^2*c^2*d*e - 4*(-d*e^2)^(
1/3)*a^3*b*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/((-d
*e^2)^(2/3)*e^2) - 1/6*(6*b^2*c^2*d^2*e + 4*a*c^3*d^2*e - 4*a*b^3*d*e^2 -
12*a^2*b*c*d*e^2 + a^4*e^3 + 4*(-d*e^2)^(1/3)*b*c^3*d^2 - (-d*e^2)^(1/3)*b
^4*d*e - 12*(-d*e^2)^(1/3)*a*b^2*c*d*e - 6*(-d*e^2)^(1/3)*a^2*c^2*d*e + 4*
(-d*e^2)^(1/3)*a^3*b*e^2)*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/((-d*e^
2)^(2/3)*e^2) + 1/3*(c^4*d^2 - 4*b^3*c*d*e - 12*a*b*c^2*d*e + 6*a^2*b^2*e^
2 + 4*a^3*c*e^2)*log(abs(e*x^3 + d))/e^3 + 1/30*(5*c^4*e^5*x^6 + 24*b*c^3*
e^5*x^5 + 45*b^2*c^2*e^5*x^4 + 30*a*c^3*e^5*x^4 - 10*c^4*d*e^4*x^3 + 40*b^
3*c*e^5*x^3 + 120*a*b*c^2*e^5*x^3 - 60*b*c^3*d*e^4*x^2 + 15*b^4*e^5*x^2 +
180*a*b^2*c*e^5*x^2 + 90*a^2*c^2*e^5*x^2 - 180*b^2*c^2*d*e^4*x - 120*a*c^3
*d*e^4*x + 120*a*b^3*e^5*x + 360*a^2*b*c*e^5*x)/e^6 - 1/3*(4*b*c^3*d^2*e^1
1*(-d/e)^(1/3) - b^4*d*e^12*(-d/e)^(1/3) - 12*a*b^2*c*d*e^12*(-d/e)^(1/3)
- 6*a^2*c^2*d*e^12*(-d/e)^(1/3) + 4*a^3*b*e^13*(-d/e)^(1/3) + 6*b^2*c^2*d^
2*e^11 + 4*a*c^3*d^2*e^11 - 4*a*b^3*d*e^12 - 12*a^2*b*c*d*e^12 + a^4*e^13)
*(-d/e)^(1/3)*log(abs(x - (-d/e)^(1/3)))/(d*e^13)
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 2971, normalized size of antiderivative = 4.62

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)^4/(d + e*x^3),x)
```

output
```
x^2*((b^4 + 6*a^2*c^2 + 12*a*b^2*c)/(2*e) - (2*b*c^3*d)/e^2) - x^3*((c^4*d
)/(3*e^2) - (4*b*c*(3*a*c + b^2))/(3*e)) + symsum(log(root(27*d^2*e^9*z^3
+ 324*a*b*c^2*d^3*e^7*z^2 + 108*b^3*c*d^3*e^7*z^2 - 108*a^3*c*d^2*e^8*z^2
- 162*a^2*b^2*d^2*e^8*z^2 - 27*c^4*d^4*e^6*z^2 - 72*a*b*c^6*d^5*e^4*z + 21
6*a^2*b^2*c^4*d^4*e^5*z + 144*a^3*b^3*c^2*d^3*e^6*z - 108*a^5*b^2*c*d^2*e^
7*z + 108*a^2*b^5*c*d^3*e^6*z - 36*a^4*b*c^3*d^3*e^6*z + 36*a*b^4*c^3*d^4*
e^5*z + 144*b^3*c^5*d^5*e^4*z + 90*b^6*c^2*d^4*e^5*z - 144*a^3*c^5*d^4*e^5
*z + 90*a^6*c^2*d^2*e^7*z + 171*a^4*b^4*d^2*e^7*z + 36*a*b^7*d^3*e^6*z + 3
6*a^7*b*d*e^8*z + 9*c^8*d^6*e^3*z + 36*a^7*b^4*c*d^2*e^6 - 36*a^7*b*c^4*d^
3*e^5 - 36*a^4*b^7*c*d^3*e^5 - 36*a^4*b*c^7*d^5*e^3 - 36*a*b^7*c^4*d^5*e^3
 + 36*a*b^4*c^7*d^6*e^2 + 12*a*b^10*c*d^4*e^4 + 108*a^5*b^5*c^2*d^3*e^5 -
108*a^5*b^2*c^5*d^4*e^4 + 108*a^2*b^5*c^5*d^5*e^3 - 96*a^6*b^3*c^3*d^3*e^5
 + 96*a^3*b^6*c^3*d^4*e^4 - 96*a^3*b^3*c^6*d^5*e^3 - 54*a^8*b^2*c^2*d^2*e^
6 - 54*a^2*b^8*c^2*d^4*e^4 - 54*a^2*b^2*c^8*d^6*e^2 - 9*a^4*b^4*c^4*d^4*e^
4 - 12*a^10*b*c*d*e^7 - 12*a*b*c^10*d^7*e - 6*b^6*c^6*d^6*e^2 + 4*b^9*c^3*
d^5*e^3 - 6*a^6*c^6*d^4*e^4 - 4*a^9*c^3*d^2*e^6 - 4*a^3*c^9*d^6*e^2 - 6*a^
6*b^6*d^2*e^6 + 4*a^3*b^9*d^3*e^5 + 4*b^3*c^9*d^7*e + 4*a^9*b^3*d*e^7 - b^
12*d^4*e^4 - c^12*d^8 - a^12*e^8, z, k)*((x*(3*a^4*e^5 + 12*a*c^3*d^2*e^3
+ 18*b^2*c^2*d^2*e^3 - 12*a*b^3*d*e^4 - 36*a^2*b*c*d*e^4))/e^3 - (6*c^4*d^
3*e^3 + 36*a^2*b^2*d*e^5 - 24*b^3*c*d^2*e^4 + 24*a^3*c*d*e^5 - 72*a*b*c...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1391, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx+cx^2)^4}{d+ex^3}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^4/(e*x^3+d),x)
```

output
```
( - 10*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*
sqrt(3)))*a**4*e**3 + 120*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**
(1/3)*x)/(d**(1/3)*sqrt(3)))*a**2*b*c*d*e**2 + 40*e**(1/3)*d**(2/3)*sqrt(3
)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a*b**3*d*e**2 - 40*e*
*(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))
*a*c**3*d**2*e - 60*e**(1/3)*d**(2/3)*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*
x)/(d**(1/3)*sqrt(3)))*b**2*c**2*d**2*e - 40*sqrt(3)*atan((d**(1/3) - 2*e*
*(1/3)*x)/(d**(1/3)*sqrt(3)))*a**3*b*d*e**3 + 60*sqrt(3)*atan((d**(1/3) -
2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a**2*c**2*d**2*e**2 + 120*sqrt(3)*atan((
d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a*b**2*c*d**2*e**2 + 10*sqrt(
3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*b**4*d**2*e**2 - 40*
sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*b*c**3*d**3*e -
 5*e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a
**4*e**3 + 60*e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2
/3)*x**2)*a**2*b*c*d*e**2 + 20*e**(1/3)*d**(2/3)*log(d**(2/3) - e**(1/3)*d
**(1/3)*x + e**(2/3)*x**2)*a*b**3*d*e**2 - 20*e**(1/3)*d**(2/3)*log(d**(2/
3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a*c**3*d**2*e - 30*e**(1/3)*d**(
2/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*b**2*c**2*d**2*e
+ 10*e**(1/3)*d**(2/3)*log(d**(1/3) + e**(1/3)*x)*a**4*e**3 - 120*e**(1/3)
*d**(2/3)*log(d**(1/3) + e**(1/3)*x)*a**2*b*c*d*e**2 - 40*e**(1/3)*d**(...
```

3.26 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx$

Optimal result

Integrand size = 35, antiderivative size = 118

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx = -\frac{B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
-1/3*B*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b
^(2/3)-1/3*B*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*B*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right) dx = \frac{B\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

output
```
(B*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3)
 + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(1/3
)*b^(2/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{Bx+Cx^2}{a+bx^3}-\frac{Cx^2}{a+bx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{B\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{B\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[-((C*x^2)/(a + b*x^3)) + (B*x + C*x^2)/(a + b*x^3),x]
```

output
```
-((B*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b
^(2/3))) - (B*Log[a^(1/3) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)) + (B*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

method	result	size
risch	$-\frac{C\ln\left(b\,x^{3}+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b+a\right)}\frac{\left(C\_R+B\right)\ln\left(x-\_R\right)}{\_R}}{3b}$	47
default	$-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	94

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b*ln(b*x^3+a)+1/3/b*sum(1/_R*(C*_R+B)*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.63

$$\int\left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)dx$$

$$=\left[\frac{3\sqrt{\frac{1}{3}}Bab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}a\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3\,(-ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+(-ab^2)^{\frac{2}{3}}B\log\left(\right.}{6\,ab^2}\right.$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*B*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sq
rt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1
/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*B*log(b^2*x^2 +
 (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*B*log(b*x - (-a*b
^2)^(1/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*B*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^
(2/3)*B*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2
/3)*B*log(b*x - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = B\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

input
```
integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x)/(b*x**3+a),x)
```

output
```
B*RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = -\frac{C\log\left(bx^3+a\right)}{3\,b}+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{1}{3}}+B\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{1}{3}}-B\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*C*log(b*x^3 + a)/b + 1/6*(2*C*(a/b)^(1/3) + B)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b*(a/b)^(1/3)) + 1/3*(C*(a/b)^(1/3) - B)*log(x + (a/b)^(
1/3))/(b*(a/b)^(1/3)) - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 2*C*a/b)*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx = \frac{\sqrt{3}B\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}}-\frac{B\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*B*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a*b^
2)^(1/3) - 1/6*B*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(1/3) -
 1/3*B*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx=-\frac{B\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B-\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}+\frac{\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(B+\sqrt{3}\,B\,1\mathrm{i}\right)}{6\,a^{1/3}\,b^{2/3}}$$

input
```
int((B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

output
```
(log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*(B - 3^(1/2)*B*1i))/(6*
a^(1/3)*b^(2/3)) - (B*log(b^(1/3)*x + a^(1/3)))/(3*a^(1/3)*b^(2/3)) + (log
(3^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*(B + 3^(1/2)*B*1i))/(6*a^(1
/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{Bx+Cx^2}{a+bx^3}\right)\,dx=\frac{b^{\frac{1}{3}}\left(-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\right)}{6a^{\frac{1}{3}}}$$

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+B*x)/(b*x^3+a),x)
```

output
```
(b*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + log(
a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*log(a**(1/3) + b**(1/3
)*x)))/(6*b**(2/3)*a**(1/3))
```

3.27 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx$

Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\frac{A\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
-1/3*A*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b
^(1/3)+1/3*A*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)-1/6*A*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = -\frac{A\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}}$$

input
```
Integrate[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

output
```
-1/6*(A*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(
1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(2
/3)*b^(1/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{A+Cx^2}{a+bx^3}-\frac{Cx^2}{a+bx^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{A\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}-\frac{A\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{A\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[-((C*x^2)/(a + b*x^3)) + (A + C*x^2)/(a + b*x^3),x]
```

output
```
-((A*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b
^(1/3))) + (A*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) - (A*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

method	result	size
risch	$-\frac{C\ln\left(b\,x^{3}+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b+a\right)}\frac{\left(C\_R^{2}+A\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b}$	49
default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	94

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b*ln(b*x^3+a)+1/3/b*sum((C*_R^2+A)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.58

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = \left[\frac{3\sqrt{\frac{1}{3}}Aab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}A\log\left(abx\right.}{6\,a^2b}\right.$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*A*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^
(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)
*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*A*log(a*b*x^2 - (
a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*A*log(a*b*x + (a^2*b)^
(2/3)))/(a^2*b), 1/6*(6*sqrt(1/3)*A*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(
1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a
^2*b)^(2/3)*A*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)
^(2/3)*A*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)\,dx = A\,\mathrm{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

input
```
integrate(-C*x**2/(b*x**3+a)+(C*x**2+A)/(b*x**3+a),x)
```

output
```
A*RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)dx = -\frac{C\log\left(bx^3+a\right)}{3\,b} - \frac{\sqrt{3}\left(2\,Ca-\left(3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab} + \frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input

```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="maxima")
```

output

```
-1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*A*(a/b)^(1/3) + 2*C*a/b)
*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(2*C*(
a/b)^(2/3) - A)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1
/3*(C*(a/b)^(2/3) + A)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right)dx = -\frac{A\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}A\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{\left(-ab^2\right)^{\frac{1}{3}}A\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

input

```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*A*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1
/3)*A*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-
a*b^2)^(1/3)*A*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = \frac{A \ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A-\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}} - \frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}-\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(A+\sqrt{3}\,A\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

input
```
int((A + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

output
```
(A*log(b^(1/3)*x + a^(1/3)))/(3*a^(2/3)*b^(1/3)) - (log(a^(1/3) - 2*b^(1/3
)*x - 3^(1/2)*a^(1/3)*1i)*(A - 3^(1/2)*A*1i))/(6*a^(2/3)*b^(1/3)) - (log(2
*b^(1/3)*x - 3^(1/2)*a^(1/3)*1i - a^(1/3))*(A + 3^(1/2)*A*1i))/(6*a^(2/3)*
b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Cx^2}{a+bx^3}\right) dx = \frac{a^{\frac{1}{3}}\left(-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\right)}{6b^{\frac{1}{3}}}$$

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+A)/(b*x^3+a),x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
 - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + 2*log(a**(1/3) +
b**(1/3)*x)))/(6*b**(1/3))
```

3.28 $\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) dx$

Optimal result

Integrand size = 36, antiderivative size = 161

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right) dx = -\frac{\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output

```
-1/3*(A*b^(1/3)+a^(1/3)*B)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3
))*3^(1/2)/a^(2/3)/b^(2/3)+1/3*(A*b^(1/3)-a^(1/3)*B)*ln(a^(1/3)+b^(1/3)*x)
/a^(2/3)/b^(2/3)-1/6*(A-a^(1/3)*B/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/a^(2/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)dx$$

$$=\frac{-2\sqrt{3}\left(A\sqrt[3]{b}+\sqrt[3]{a}B\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.\right.}{6a^{2/3}b^{2/3}}$$

input
```
Integrate[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

output
```
(-2*Sqrt[3]*(A*b^(1/3) + a^(1/3)*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + (A*b^(1/3) - a^(1/3)*B)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{A+Bx+Cx^2}{a+bx^3}-\frac{Cx^2}{a+bx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\sqrt[3]{a}B+A\sqrt[3]{b}\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}}-\frac{\left(A-\frac{\sqrt[3]{a}B}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}+\frac{\left(A\sqrt[3]{b}-\sqrt[3]{a}B\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}}$$

input
```
Int[-((C*x^2)/(a + b*x^3)) + (A + B*x + C*x^2)/(a + b*x^3),x]
```

output
```
-(((A*b^(1/3) + a^(1/3)*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(2/3)*b^(2/3))) + ((A*b^(1/3) - a^(1/3)*B)*Log[a^(1/3) + b^
(1/3)*x])/(3*a^(2/3)*b^(2/3)) - ((A - (a^(1/3)*B)/b^(1/3))*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

method	result
risch	$-\frac{C\ln\left(b\,x^{3}+a\right)}{3b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}b+a\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3b}$
default	$\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{A\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{B\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{B\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C/b*ln(b*x^3+a)+1/3/b*sum((C*_R^2+B*_R+A)/_R^2*ln(x-_R),_R=RootOf(_Z^
3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 1961, normalized size of antiderivative = 12.18

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx=\text{Too large to display}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^
3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a +
A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)
*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(
1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2)
- (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b - 1/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3) - 2*
(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a
- A^3*b)/(a^2*b^2))^(1/3)))*A^2*a*b + 2*A*B^2*a + (B^3*a + A^3*b)*x) + 1/1
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b
)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3
*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^
2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)/(
a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*a*b + 16*A*B)/(a*b)))*log(
-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^
3*b)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a +
A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2*b^2))^(1/3)))^2*B*a^2*b + 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((B^3*a + A^3*b)/(a^2*b^2) - (B^3*a - A^3*b)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*A*B*(-I*sqrt(3) + 1)/(a*b*((B^3*a + A^3*b)...
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx$$
$$=\operatorname{RootSum}\left(27t^3a^2b^2+9tABab-A^3b+B^3a,\left(t\mapsto t\log\left(x+\frac{9t^2Ba^2b+3tA^2ab+2AB^2a}{A^3b+B^3a}\right)\right)\right)$$

input
```
integrate(-C*x**2/(b*x**3+a)+(C*x**2+B*x+A)/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*b**2 + 9*_t*A*B*a*b - A**3*b + B**3*a, Lambda(_t, _t
*log(x + (9*_t**2*B*a**2*b + 3*_t*A**2*a*b + 2*A*B**2*a)/(A**3*b + B**3*a)
)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx$$
$$=-\frac{C\log\left(bx^3+a\right)}{3\,b}$$
$$-\frac{\sqrt{3}\left(2\,Ca-\left(3\,B\left(\frac{a}{b}\right)^{\frac{2}{3}}+3\,A\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2\,Ca}{b}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$
$$+\frac{\left(2\,C\left(\frac{a}{b}\right)^{\frac{2}{3}}+B\left(\frac{a}{b}\right)^{\frac{1}{3}}-A\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(C\left(\frac{a}{b}\right)^{\frac{2}{3}}-B\left(\frac{a}{b}\right)^{\frac{1}{3}}+A\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*C*log(b*x^3 + a)/b - 1/9*sqrt(3)*(2*C*a - (3*B*(a/b)^(2/3) + 3*A*(a/b
)^(1/3) + 2*C*a/b)*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/
(a*b) + 1/6*(2*C*(a/b)^(2/3) + B*(a/b)^(1/3) - A)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*(C*(a/b)^(2/3) - B*(a/b)^(1/3) + A)*l
og(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx = -\frac{\sqrt{3}\left(Ab-\left(-ab^2\right)^{\frac{1}{3}}B\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(Ab+\left(-ab^2\right)^{\frac{1}{3}}B\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(Bb\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

input
```
integrate(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(A*b - (-a*b^2)^(1/3)*B)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(A*b + (-a*b^2)^(1/3)*B)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(B*b*(-a/b)^(1/3) + A*
b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx = \sum_{k=1}^{3}\ln\left(b\left(B^2\,x+A\,B+\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)^2\,a\,b\,9+A\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)\,b\,x\,3\right)\right)\,\mathrm{root}\left(27\,a^2\,b^2\,z^3+9\,A\,B\,a\,b\,z+B^3\,a-A^3\,b,z,k\right)$$

input
```
int((A + B*x + C*x^2)/(a + b*x^3) - (C*x^2)/(a + b*x^3),x)
```

output
```
symsum(log(b*(B^2*x + A*B + 9*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a -
A^3*b, z, k)^2*a*b + 3*A*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b
, z, k)*b*x))*root(27*a^2*b^2*z^3 + 9*A*B*a*b*z + B^3*a - A^3*b, z, k), k,
 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \left(-\frac{Cx^2}{a+bx^3}+\frac{A+Bx+Cx^2}{a+bx^3}\right)\,dx = \frac{-2b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b-b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{1}{3}}+\right.}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(-C*x^2/(b*x^3+a)+(C*x^2+B*x+A)/(b*x^3+a),x)
```

output
```
( - 2*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3))) - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b
- b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) +
2*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x) + log(a**(2/3) - b**(1/3)*a
**(1/3)*x + b**(2/3)*x**2)*b - 2*log(a**(1/3) + b**(1/3)*x)*b)/(6*b**(2/3)
*a**(1/3))
```

3.29 $\int \frac{bx+cx^2}{d+ex^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{bx+cx^2}{d+ex^3} \, dx = -\frac{b \arctan\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\sqrt[3]{d}e^{2/3}} + \frac{\left(c-\frac{b\sqrt[3]{e}}{\sqrt[3]{d}}\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3e} + \frac{\left(2c+\frac{b\sqrt[3]{e}}{\sqrt[3]{d}}\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6e}$$

output
```
-1/3*b*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(1/3)/e
^(2/3)+1/3*(c-b*e^(1/3)/d^(1/3))*ln(d^(1/3)+e^(1/3)*x)/e+1/6*(2*c+b*e^(1/3
)/d^(1/3))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/e
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{bx+cx^2}{d+ex^3} \, dx$$

$$= \frac{-2\sqrt{3}b\sqrt[3]{e}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - 2b\sqrt[3]{e}\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right) + b\sqrt[3]{e}\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right) + 2c\sqrt[3]{d}\,\mathrm{lo}}{6\sqrt[3]{d}e}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x^3),x]
```

output
```
(-2*Sqrt[3]*b*e^(1/3)*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*b*e^
(1/3)*Log[d^(1/3) + e^(1/3)*x] + b*e^(1/3)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x
 + e^(2/3)*x^2] + 2*c*d^(1/3)*Log[d + e*x^3])/(6*d^(1/3)*e)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {2027, 2410, 27, 792, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{d+ex^3} \, dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(b+cx)}{d+ex^3}dx$$

$$\Big\downarrow \text{2410}$$

$$\int \frac{bx}{ex^3+d}dx+c\int \frac{x^2}{ex^3+d}dx$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{x}{ex^3+d}dx+c\int \frac{x^2}{ex^3+d}dx$$

$$\Big\downarrow \text{792}$$

$$b\int \frac{x}{ex^3+d}dx+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \text{821}$$

$$b\left(\frac{\int \frac{\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\int \frac{1}{\sqrt[3]{e}x+\sqrt[3]{d}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \ 16$$

$$b\left(\frac{\int \frac{\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \ 1142$$

$$b\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \ 25$$

$$b\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\int \frac{\sqrt[3]{e}\left(\sqrt[3]{d}-2\sqrt[3]{e}x\right)}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \ 27$$

$$b\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$$\Big\downarrow \ 1082$$

$$b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}\right)}{\sqrt[3]{e}}-\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$\downarrow$ 217

$$b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{e^{2/3}x^2-\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

$\downarrow$ 1103

$$b\left(\frac{\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{2\sqrt[3]{e}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3\sqrt[3]{d}\sqrt[3]{e}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3\sqrt[3]{d}e^{2/3}}\right)+\frac{c\log\left(d+ex^3\right)}{3e}$$

input
```
Int[(b*x + c*x^2)/(d + e*x^3),x]
```

output
```
b*(-1/3*Log[d^(1/3) + e^(1/3)*x]/(d^(1/3)*e^(2/3)) + (-((Sqrt[3]*ArcTan[(1
 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/e^(1/3)) + Log[d^(2/3) - d^(1/3)*e^(1/
3)*x + e^(2/3)*x^2]/(2*e^(1/3)))/(3*d^(1/3)*e^(1/3))) + (c*Log[d + e*x^3])
/(3*e)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^{3} e+d\right)}\frac{\left(\_R^{2} c+\_R b\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3e}$	36
default	$b\left(-\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)+\frac{c\ln\left(x^{3}e+d\right)}{3e}$	108

input
```
int((c*x^2+b*x)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
1/3/e*sum((_R^2*c+_R*b)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e+d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 1344, normalized size of antiderivative = 10.11

$$\int \frac{bx + cx^2}{d + ex^3} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="fricas")
```

output
```
-1/12*(2*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*
d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*e*log(1/4*(3*(I*sqrt(3) + 1)*(-1/54*c^3
/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*d
*e^2 + (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d
- b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + b^2*e*x + c^2*d) - ((3*(I*sqrt(3)
 + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1
/3) - 2*c/e)*e + 3*sqrt(1/3)*e*sqrt(-((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 +
1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)^2*e^2 + 4*
(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e
)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2) + 6*c)*log(-1/4*(3*(I*sqrt(3)
+ 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/
3) - 2*c/e)^2*d*e^2 - (3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2)
 + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*d*e + 2*b^2*e*x - c^2*d
+ 3/4*sqrt(1/3)*((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/
54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*d*e^2 + 2*c*d*e)*sqrt(-((3*(I*s
qrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^
3))^(1/3) - 2*c/e)^2*e^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/
(d*e^2) + 1/54*(c^3*d - b^3*e)/(d*e^3))^(1/3) - 2*c/e)*c*e + 4*c^2)/e^2))
- ((3*(I*sqrt(3) + 1)*(-1/54*c^3/e^3 + 1/54*b^3/(d*e^2) + 1/54*(c^3*d - b^
3*e)/(d*e^3))^(1/3) - 2*c/e)*e - 3*sqrt(1/3)*e*sqrt(-((3*(I*sqrt(3) + 1...
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

$$\int \frac{bx + cx^2}{d + ex^3}\, dx$$
$$= \text{RootSum}\left(27t^3de^3 - 27t^2cde^2 + 9tc^2de + b^3e - c^3d, \left(t \mapsto t\log\left(x + \frac{9t^2de^2 - 6tcde + c^2d}{b^2e}\right)\right)\right)$$

input
```
integrate((c*x**2+b*x)/(e*x**3+d),x)
```

output
```
RootSum(27*_t**3*d*e**3 - 27*_t**2*c*d*e**2 + 9*_t*c**2*d*e + b**3*e - c**
3*d, Lambda(_t, _t*log(x + (9*_t**2*d*e**2 - 6*_t*c*d*e + c**2*d)/(b**2*e)
)))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{bx + cx^2}{d + ex^3}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,(-de^2)^{\frac{1}{3}}} - \frac{b\log\left(x^2+x\left(-\frac{d}{e}\right)^{\frac{1}{3}}+\left(-\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,(-de^2)^{\frac{1}{3}}} - \frac{b\left(-\frac{d}{e}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} + \frac{c\log\left(|ex^3+d|\right)}{3\,e}$$

input
```
integrate((c*x^2+b*x)/(e*x^3+d),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x + (-d/e)^(1/3))/(-d/e)^(1/3))/(-d*e^
2)^(1/3) - 1/6*b*log(x^2 + x*(-d/e)^(1/3) + (-d/e)^(2/3))/(-d*e^2)^(1/3) -
 1/3*b*(-d/e)^(2/3)*log(abs(x - (-d/e)^(1/3)))/d + 1/3*c*log(abs(e*x^3 + d
))/e
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int \frac{bx+cx^2}{d+ex^3}\,dx = \sum_{k=1}^{3} \ln\left(-\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)\,\left(6\,c\,d\,e - \mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)\,d\,e^2\,9\right) + c^2\,d+b^2\,e\,x\right)\,\mathrm{root}\left(27\,d\,e^3\,z^3-27\,c\,d\,e^2\,z^2+9\,c^2\,d\,e\,z+b^3\,e-c^3\,d,z,k\right)$$

input
```
int((b*x + c*x^2)/(d + e*x^3),x)
```

output
```
symsum(log(c^2*d - root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d*e*z + b^3*
e - c^3*d, z, k)*(6*c*d*e - 9*root(27*d*e^3*z^3 - 27*c*d*e^2*z^2 + 9*c^2*d
*e*z + b^3*e - c^3*d, z, k)*d*e^2) + b^2*e*x)*root(27*d*e^3*z^3 - 27*c*d*e
^2*z^2 + 9*c^2*d*e*z + b^3*e - c^3*d, z, k), k, 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{bx+cx^2}{d+ex^3}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}-2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)be+2e^{\frac{2}{3}}d^{\frac{1}{3}}\log\left(d^{\frac{2}{3}}-e^{\frac{1}{3}}d^{\frac{1}{3}}x+e^{\frac{2}{3}}x^2\right)c+2e^{\frac{2}{3}}d^{\frac{1}{3}}\log\left(d^{\frac{1}{3}}+e^{\frac{1}{3}}x\right)c+\log\left(d^{\frac{2}{3}}-e^{\frac{1}{3}}d^{\frac{1}{3}}\right.}{6e^{\frac{5}{3}}d^{\frac{1}{3}}}$$

input
```
int((c*x^2+b*x)/(e*x^3+d),x)
```

output
```
( - 2*sqrt(3)*atan((d**(1/3) - 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*b*e + 2*e
**(2/3)*d**(1/3)*log(d**(2/3) - e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*c + 2
*e**(2/3)*d**(1/3)*log(d**(1/3) + e**(1/3)*x)*c + log(d**(2/3) - e**(1/3)*
d**(1/3)*x + e**(2/3)*x**2)*b*e - 2*log(d**(1/3) + e**(1/3)*x)*b*e)/(6*e**
(2/3)*d**(1/3)*e)
```

3.30 $\int \frac{a+cx^2}{d-ex^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 134

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \frac{a\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}\sqrt[3]{e}} - \frac{a\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{a\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{6d^{2/3}\sqrt[3]{e}} - \frac{c\log\left(d-ex^3\right)}{3e}$$

output
```
1/3*a*arctan(1/3*(d^(1/3)+2*e^(1/3)*x)*3^(1/2)/d^(1/3))*3^(1/2)/d^(2/3)/e^
(1/3)-1/3*a*ln(d^(1/3)-e^(1/3)*x)/d^(2/3)/e^(1/3)+1/6*a*ln(d^(2/3)+d^(1/3)
*e^(1/3)*x+e^(2/3)*x^2)/d^(2/3)/e^(1/3)-1/3*c*ln(-e*x^3+d)/e
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{a+cx^2}{d-ex^3}\,dx$$
$$= \frac{2\sqrt{3}ae^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) - 2ae^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right) + ae^{2/3}\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right) - 2cd^{2/3}}{6d^{2/3}e}$$

input
```
Integrate[(a + c*x^2)/(d - e*x^3),x]
```

output
```
(2*Sqrt[3]*a*e^(2/3)*ArcTan[(1 + (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] - 2*a*e^(
2/3)*Log[d^(1/3) - e^(1/3)*x] + a*e^(2/3)*Log[d^(2/3) + d^(1/3)*e^(1/3)*x
+ e^(2/3)*x^2] - 2*c*d^(2/3)*Log[d - e*x^3])/(6*d^(2/3)*e)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {2410, 27, 750, 16, 792, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{d-ex^3}dx$$

$$\Big\downarrow \text{ 2410}$$

$$\int \frac{a}{d-ex^3}dx+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$a\int \frac{1}{d-ex^3}dx+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{ 750}$$

$$a\left(\frac{\int \frac{\sqrt[3]{e}x+2\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e}x}dx}{3d^{2/3}}\right)+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{ 16}$$

$$a\left(\frac{\int \frac{\sqrt[3]{e}x+2\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}-\frac{\log \left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)+c\int \frac{x^2}{d-ex^3}dx$$

$$\Big\downarrow \text{ 792}$$

$$a\left(\frac{\int \frac{\sqrt[3]{e}x+2\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}-\frac{\log \left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log \left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 1142

$$a\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\int \frac{\sqrt[3]{e}\left(2\sqrt[3]{e}x+\sqrt[3]{d}\right)}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{2\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log \left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log \left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 27

$$a\left(\frac{\frac{3}{2}\sqrt[3]{d}\int \frac{1}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx}{3d^{2/3}}-\frac{\log \left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log \left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 1082

$$a\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log \left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log \left(d-ex^3\right)}{3e}$$

$\Big\downarrow$ 217

$$a\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{e}x+\sqrt[3]{d}}{e^{2/3}x^2+\sqrt[3]{d}\sqrt[3]{e}x+d^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

$$\Big\downarrow \text{1103}$$

$$a\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{e}}+\frac{\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)}{2\sqrt[3]{e}}}{3d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\sqrt[3]{e}x\right)}{3d^{2/3}\sqrt[3]{e}}\right)-\frac{c\log\left(d-ex^3\right)}{3e}$$

input
```
Int[(a + c*x^2)/(d - e*x^3),x]
```

output
```
a*(-1/3*Log[d^(1/3) - e^(1/3)*x]/(d^(2/3)*e^(1/3)) + ((Sqrt[3]*ArcTan[(1 +
 (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/e^(1/3) + Log[d^(2/3) + d^(1/3)*e^(1/3)*
x + e^(2/3)*x^2]/(2*e^(1/3)))/(3*d^(2/3))) - (c*Log[d - e*x^3])/(3*e)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}e-d\right)}\frac{\left(\_R^{2}c+a\right)\ln\left(x-\_R\right)}{\_R^{2}}}{3e}$	36
default	$a\left(-\frac{\ln\left(x-\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^{2}+\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)-\frac{c\ln\left(-x^{3}e+d\right)}{3e}$	110

input
```
int((c*x^2+a)/(-e*x^3+d),x,method=_RETURNVERBOSE)
```

output
```
-1/3/e*sum((_R^2*c+a)/_R^2*ln(x-_R),_R=RootOf(_Z^3*e-d))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 1040, normalized size of antiderivative = 7.76

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="fricas")
```

output
```
-1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 +
a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e*log(-1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*
e + a*e*x + c*d) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) -
(c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e + 3*sqrt(1/3)*e*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^
2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a
^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*c^2)/e^
2) - 6*c)*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c
^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e + 2*a*e*x + 3/2*sqrt(1/3)*
d*e*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2
+ a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1
)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c
*e + 4*c^2)/e^2) - c*d) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^
2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*e - 3*sqrt(1/3)*e*sqr
t(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e
^2)/(d^2*e^3))^(1/3) + 2*c/e)^2*e^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/
e^3 + a^3/(d^2*e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*c*e + 4*
c^2)/e^2) - 6*c)*log(1/2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(c^3/e^3 + a^3/(d^2*
e) - (c^3*d^2 + a^3*e^2)/(d^2*e^3))^(1/3) + 2*c/e)*d*e + 2*a*e*x - 3/2*...
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int \frac{a+cx^2}{d-ex^3}\,dx = -\operatorname{RootSum}\left(27t^3d^2e^3-27t^2cd^2e^2+9tc^2d^2e-a^3e^2-c^3d^2,\left(t\mapsto t\log\left(x+\frac{-3tde+cd}{ae}\right)\right)\right)$$

input
```
integrate((c*x**2+a)/(-e*x**3+d),x)
```

output
```
-RootSum(27*_t**3*d**2*e**3 - 27*_t**2*c*d**2*e**2 + 9*_t*c**2*d**2*e - a*
*3*e**2 - c**3*d**2, Lambda(_t, _t*log(x + (-3*_t*d*e + c*d)/(a*e))))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + cx^2}{d - ex^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{a + cx^2}{d - ex^3} \, dx = -\frac{a\left(\frac{d}{e}\right)^{\frac{1}{3}} \log\left(\left|x - \left(\frac{d}{e}\right)^{\frac{1}{3}}\right|\right)}{3\,d} - \frac{c\log\left(|ex^3 - d|\right)}{3\,e} + \frac{\sqrt{3}(de^2)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{e}\right)^{\frac{1}{3}}}\right)}{3\,de} + \frac{(de^2)^{\frac{1}{3}}\,a\log\left(x^2 + x\left(\frac{d}{e}\right)^{\frac{1}{3}} + \left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6\,de}$$

input
```
integrate((c*x^2+a)/(-e*x^3+d),x, algorithm="giac")
```

output
```
-1/3*a*(d/e)^(1/3)*log(abs(x - (d/e)^(1/3)))/d - 1/3*c*log(abs(e*x^3 - d))
/e + 1/3*sqrt(3)*(d*e^2)^(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (d/e)^(1/3))/(d
/e)^(1/3))/(d*e) + 1/6*(d*e^2)^(1/3)*a*log(x^2 + x*(d/e)^(1/3) + (d/e)^(2/
3))/(d*e)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.33

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \sum_{k=1}^{3} \ln\Big(-\big(c + \mathrm{root}\big(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\big)\;e\,3\big)\;\big(c\,d + \mathrm{root}\big(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\big)\;d\,e\,3 + a\,e\,x\big)\Big)\;\mathrm{root}\big(27\,d^2\,e^3\,z^3+27\,c\,d^2\,e^2\,z^2+9\,c^2\,d^2\,e\,z+c^3\,d^2+a^3\,e^2,z,k\big)$$

input
```
int((a + c*x^2)/(d - e*x^3),x)
```

output
```
symsum(log(-(c + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z
+ c^3*d^2 + a^3*e^2, z, k)*e)*(c*d + 3*root(27*d^2*e^3*z^3 + 27*c*d^2*e^2*
z^2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k)*d*e + a*e*x))*root(27*d^2*e
^3*z^3 + 27*c*d^2*e^2*z^2 + 9*c^2*d^2*e*z + c^3*d^2 + a^3*e^2, z, k), k, 1
, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.94

$$\int \frac{a+cx^2}{d-ex^3}\,dx = \frac{2d^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{d^{\frac{1}{3}}+2e^{\frac{1}{3}}x}{d^{\frac{1}{3}}\sqrt{3}}\right)ae + d^{\frac{1}{3}}\log\left(d^{\frac{2}{3}}+e^{\frac{1}{3}}d^{\frac{1}{3}}x+e^{\frac{2}{3}}x^2\right)ae - 2d^{\frac{1}{3}}\log\left(d^{\frac{1}{3}}-e^{\frac{1}{3}}x\right)ae - 2e^{\frac{1}{3}}\log\left(d^{\frac{2}{3}}+e^{\frac{1}{3}}d\right.}{6e^{\frac{4}{3}}d}$$

input
```
int((c*x^2+a)/(-e*x^3+d),x)
```

output
```
(2*d**(1/3)*sqrt(3)*atan((d**(1/3) + 2*e**(1/3)*x)/(d**(1/3)*sqrt(3)))*a*e
 + d**(1/3)*log(d**(2/3) + e**(1/3)*d**(1/3)*x + e**(2/3)*x**2)*a*e - 2*d*
*(1/3)*log(d**(1/3) - e**(1/3)*x)*a*e - 2*e**(1/3)*log(d**(2/3) + e**(1/3)
*d**(1/3)*x + e**(2/3)*x**2)*c*d - 2*e**(1/3)*log(d**(1/3) - e**(1/3)*x)*c
*d)/(6*e**(1/3)*d*e)
```

3.31 $\int \frac{2a^2+b^2x^2}{a^3+b^3x^3} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 37

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3} \, dx = -\frac{2\arctan\left(\frac{a-2bx}{\sqrt{3}a}\right)}{\sqrt{3}b} + \frac{\log(a+bx)}{b}$$

output
```
-2/3*arctan(1/3*(-2*b*x+a)*3^(1/2)/a)*3^(1/2)/b+ln(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3} \, dx = \frac{2\sqrt{3}\arctan\left(\frac{-a+2bx}{\sqrt{3}a}\right) + 2\log(a+bx) - \log\left(a^2-abx+b^2x^2\right) + \log\left(a^3+b^3x^3\right)}{3b}$$

input
```
Integrate[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-a + 2*b*x)/(Sqrt[3]*a)] + 2*Log[a + b*x] - Log[a^2 - a
*b*x + b^2*x^2] + Log[a^3 + b^3*x^3])/(3*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx$$

$$\Big\downarrow \ 2407$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{xa}{b}+x^2}dx}{b^2}+\frac{\int \frac{1}{\frac{a}{b}+x}dx}{b}$$

$$\Big\downarrow \ 16$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}-\frac{xa}{b}+x^2}dx}{b^2}+\frac{\log(a+bx)}{b}$$

$$\Big\downarrow \ 1082$$

$$\frac{2\int \frac{1}{-\left(1-\frac{2bx}{a}\right)^2-3}d\left(1-\frac{2bx}{a}\right)}{b}+\frac{\log(a+bx)}{b}$$

$$\Big\downarrow \ 217$$

$$\frac{\log(a+bx)}{b}-\frac{2\arctan\left(\frac{1-\frac{2bx}{a}}{\sqrt{3}}\right)}{\sqrt{3}b}$$

input
```
Int[(2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x]
```

output
```
(-2*ArcTan[(1 - (2*b*x)/a)/Sqrt[3]])/(Sqrt[3]*b) + Log[a + b*x]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
risch	$\frac{\ln(bx+a)}{b}+\left(\sum_{\_R=\mathrm{RootOf}(3\_Z^2b^2+1)}\_R\ln\left(3ab\_R+2bx-a\right)\right)$	42
default	$\frac{\ln(bx+a)}{b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^2x-ab\right)\sqrt{3}}{3ab}\right)}{3b}$	43

input
```
int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x+a)/b+sum(_R*ln(3*_R*a*b+2*b*x-a),_R=RootOf(3*_Z^2*b^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx-a)}{3\,a}\right)+3\,\log\left(bx+a\right)}{3\,b}$$

input
```
integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a) + 3*log(b*x + a))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{-\frac{\sqrt{3}i\log\left(x+\frac{-a-\sqrt{3}ia}{2b}\right)}{3}+\frac{\sqrt{3}i\log\left(x+\frac{-a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(\frac{a}{b}+x\right)}{b}$$

input
```
integrate((b**2*x**2+2*a**2)/(b**3*x**3+a**3),x)
```

output
```
(-sqrt(3)*I*log(x + (-a - sqrt(3)*I*a)/(2*b))/3 + sqrt(3)*I*log(x + (-a +
sqrt(3)*I*a)/(2*b))/3 + log(a/b + x))/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,b^2x-ab)}{3\,ab}\right)}{3\,b}+\frac{\log\left(bx+a\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x - a*b)/(a*b))/b + log(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx-a)}{3\,a}\right)}{3\,b}+\frac{\log\left(|bx+a|\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x - a)/a)/b + log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{\ln\left(a+b\,x\right)}{b}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4+4\,x\,a^2\,b^5}-\frac{4\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4+4\,x\,a^2\,b^5}\right)}{3\,b}$$

input
```
int((2*a^2 + b^2*x^2)/(a^3 + b^3*x^3),x)
```

output
```
log(a + b*x)/b - (2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 + 4*a^2*b^
5*x) - (4*3^(1/2)*a^2*b^5*x)/(4*a^3*b^4 + 4*a^2*b^5*x)))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{2a^2+b^2x^2}{a^3+b^3x^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{-2bx+a}{\sqrt{3}\,a}\right)+3\log(bx+a)}{3b}$$

input
```
int((b^2*x^2+2*a^2)/(b^3*x^3+a^3),x)
```

output
```
( - 2*sqrt(3)*atan((a - 2*b*x)/(sqrt(3)*a)) + 3*log(a + b*x))/(3*b)
```

3.32 $\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 39

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx=\frac{2\arctan\left(\frac{a+2bx}{\sqrt{3}a}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

output
```
2/3*arctan(1/3*(2*b*x+a)*3^(1/2)/a)*3^(1/2)/b-ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.82

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx =\frac{2\sqrt{3}\arctan\left(\frac{a+2bx}{\sqrt{3}a}\right)-2\log(a-bx)+\log\left(a^2+abx+b^2x^2\right)-\log\left(a^3-b^3x^3\right)}{3b}$$

input
```
Integrate[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(a + 2*b*x)/(Sqrt[3]*a)] - 2*Log[a - b*x] + Log[a^2 + a*
b*x + b^2*x^2] - Log[a^3 - b^3*x^3])/(3*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2407, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}dx$$

$$\Big\downarrow \text{ 2407}$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}+\frac{xa}{b}+x^2}dx}{b^2}-\frac{\int \frac{1}{x-\frac{a}{b}}dx}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{a\int \frac{1}{\frac{a^2}{b^2}+\frac{xa}{b}+x^2}dx}{b^2}-\frac{\log(a-bx)}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{2\int \frac{1}{-\left(\frac{2bx}{a}+1\right)^2-3}d\left(\frac{2bx}{a}+1\right)}{b}-\frac{\log(a-bx)}{b}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\arctan\left(\frac{\frac{2bx}{a}+1}{\sqrt{3}}\right)}{\sqrt{3}b}-\frac{\log(a-bx)}{b}$$

input
```
Int[(2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x]
```

output
```
(2*ArcTan[(1 + (2*b*x)/a)/Sqrt[3]])/(Sqrt[3]*b) - Log[a - b*x]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2407
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = Rt[a/b, 3]}, Simp[C/b
Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x], x]]
 /; EqQ[A - Rt[a/b, 3]*B - 2*Rt[a/b, 3]^2*C, 0]] /; FreeQ[{a, b}, x] && Pol
yQ[P2, x, 2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{2 \arctan\left(\frac{(2bx+a)\sqrt{3}}{3a}\right)\sqrt{3}}{3b}-\frac{\ln(bx-a)}{b}$	38
default	$-\frac{\ln(-bx+a)}{b}+\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2b^{2}x+ab\right)\sqrt{3}}{3ab}\right)}{3b}$	44

input
```
int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(1/3*(2*b*x+a)*3^(1/2)/a)*3^(1/2)/b-1/b*ln(b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)-3\,\log\left(bx-a\right)}{3\,b}$$

input
```
integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a) - 3*log(b*x - a))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = -\frac{\frac{\sqrt{3}i\log\left(x+\frac{a-\sqrt{3}ia}{2b}\right)}{3}-\frac{\sqrt{3}i\log\left(x+\frac{a+\sqrt{3}ia}{2b}\right)}{3}+\log\left(-\frac{a}{b}+x\right)}{b}$$

input
```
integrate((b**2*x**2+2*a**2)/(-b**3*x**3+a**3),x)
```

output
```
-(sqrt(3)*I*log(x + (a - sqrt(3)*I*a)/(2*b))/3 - sqrt(3)*I*log(x + (a + sq
rt(3)*I*a)/(2*b))/3 + log(-a/b + x))/b
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,b^2x+ab)}{3\,ab}\right)}{3\,b}-\frac{\log\left(bx-a\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b^2*x + a*b)/(a*b))/b - log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(2\,bx+a)}{3\,a}\right)}{3\,b} - \frac{\log\left(|bx-a|\right)}{b}$$

input
```
integrate((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*b*x + a)/a)/b - log(abs(b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.21

$$\int \frac{2a^2+b^2x^2}{a^3-b^3x^3}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}\,a^3\,b^4}{4\,a^3\,b^4-4\,a^2\,b^5\,x}+\frac{4\sqrt{3}\,a^2\,b^5\,x}{4\,a^3\,b^4-4\,a^2\,b^5\,x}\right)}{3\,b} - \frac{\ln\left(a-b\,x\right)}{b}$$

input
```
int((2*a^2 + b^2*x^2)/(a^3 - b^3*x^3),x)
```

output
```
(2*3^(1/2)*atan((4*3^(1/2)*a^3*b^4)/(4*a^3*b^4 - 4*a^2*b^5*x) + (4*3^(1/2)
*a^2*b^5*x)/(4*a^3*b^4 - 4*a^2*b^5*x)))/(3*b) - log(a - b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{2a^2 + b^2x^2}{a^3 - b^3x^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2bx+a}{\sqrt{3}\,a}\right) - 3\log(-bx + a)}{3b}$$

input
```
int((b^2*x^2+2*a^2)/(-b^3*x^3+a^3),x)
```

output
```
(2*sqrt(3)*atan((a + 2*b*x)/(sqrt(3)*a)) - 3*log(a - b*x))/(3*b)
```

3.33 $\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx=-\frac{2C\arctan\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{C\log\left(2+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

output
```
-2/3*C*arctan(1/3*(1-b^(1/3)*x)*3^(1/2))*3^(1/2)/b^(1/3)+C*ln(2+b^(1/3)*x)
/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{8C+b^{2/3}Cx^2}{8+bx^3}\,dx=\frac{C\left(2\sqrt{3}\arctan\left(\frac{-1+\sqrt[3]{b}x}{\sqrt{3}}\right)+2\log\left(2+\sqrt[3]{b}x\right)-\log\left(4-2\sqrt[3]{b}x+b^{2/3}x^2\right)+\log(8\right.}{3\sqrt[3]{b}}$$

input
```
Integrate[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

output
```
(C*(2*Sqrt[3]*ArcTan[(-1 + b^(1/3)*x)/Sqrt[3]] + 2*Log[2 + b^(1/3)*x] - Lo
g[4 - 2*b^(1/3)*x + b^(2/3)*x^2] + Log[8 + b*x^3]))/(3*b^(1/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2402, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b^{2/3}Cx^2+8C}{bx^3+8}dx$$

$$\Big\downarrow \text{2402}$$

$$\frac{2C\int \frac{1}{x^2-\frac{2x}{\sqrt[3]{b}}+\frac{4}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\int \frac{1}{x+\frac{2}{\sqrt[3]{b}}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{16}$$

$$\frac{2C\int \frac{1}{x^2-\frac{2x}{\sqrt[3]{b}}+\frac{4}{b^{2/3}}}dx}{b^{2/3}}+\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2C\int \frac{1}{-\left(1-\sqrt[3]{b}x\right)^2-3}d\left(1-\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{C\log\left(\sqrt[3]{b}x+2\right)}{\sqrt[3]{b}}-\frac{2C\arctan\left(\frac{1-\sqrt[3]{b}x}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

input
```
Int[(8*C + b^(2/3)*C*x^2)/(8 + b*x^3),x]
```

output
```
(-2*C*ArcTan[(1 - b^(1/3)*x)/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (C*Log[2 + b^(1
/3)*x])/b^(1/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(37) = 74$.

Time $= 0.15$ (sec) , antiderivative size $= 115$, normalized size of antiderivative $= 2.40$

method	result	siz
default	$C\left(\frac{8^{\frac{1}{3}}\ln\left(x+8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}-\frac{8^{\frac{1}{3}}\ln\left(x^{2}-8^{\frac{1}{3}}\left(\frac{1}{b}\right)^{\frac{1}{3}}x+8^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{8^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{8^{\frac{2}{3}}x}{4\left(\frac{1}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{1}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(b\,x^{3}+8\right)}{3b^{\frac{1}{3}}}\right)$	115
meijerg	$\frac{2C\left(\frac{b^{\frac{1}{3}}x\ln\left(1+\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}\right)}{\left(b\,x^{3}\right)^{\frac{1}{3}}}-\frac{b^{\frac{1}{3}}x\ln\left(1-\frac{\left(b\,x^{3}\right)^{\frac{1}{3}}}{2}+\frac{\left(b\,x^{3}\right)^{\frac{2}{3}}}{4}\right)}{2\left(b\,x^{3}\right)^{\frac{1}{3}}}+\frac{b^{\frac{1}{3}}x\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b\,x^{3}\right)^{\frac{1}{3}}}{4-\left(b\,x^{3}\right)^{\frac{1}{3}}}\right)}{\left(b\,x^{3}\right)^{\frac{1}{3}}}\right)}{3b^{\frac{1}{3}}}+\frac{C\ln\left(1+\frac{b\,x^{3}}{8}\right)}{3b^{\frac{1}{3}}}$	123

input
```
int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x,method=_RETURNVERBOSE)
```

output
```
C*(1/3/b*8^(1/3)/(1/b)^(2/3)*ln(x+8^(1/3)*(1/b)^(1/3))-1/6/b*8^(1/3)/(1/b)
^(2/3)*ln(x^2-8^(1/3)*(1/b)^(1/3)*x+8^(2/3)*(1/b)^(2/3))+1/3/b*8^(1/3)/(1/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/4*8^(2/3)/(1/b)^(1/3)*x-1))+1/3/b^(
1/3)*ln(b*x^3+8))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.79

$$\int\frac{8C+b^{2/3}Cx^{2}}{8+bx^{3}}\,dx=\left[\frac{\sqrt{\frac{1}{3}}Cb\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{bx^{3}+6\sqrt{\frac{1}{3}}\left(bx^{2}+b^{\frac{2}{3}}x-2\,b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-6\,b^{\frac{1}{3}}x-4}{bx^{3}+8}\right)+Cb^{\frac{2}{3}}\log\left(bx+2\,b^{\frac{2}{3}}\right)}{b},\right.$$

input
```
integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="fricas")
```

output
```
[(sqrt(1/3)*C*b*sqrt(-1/b^(2/3))*log((b*x^3 + 6*sqrt(1/3)*(b*x^2 + b^(2/3)
*x - 2*b^(1/3))*sqrt(-1/b^(2/3)) - 6*b^(1/3)*x - 4)/(b*x^3 + 8)) + C*b^(2/
3)*log(b*x + 2*b^(2/3)))/b, (2*sqrt(1/3)*C*b^(2/3)*arctan(sqrt(1/3)*(b^(2/
3)*x - b^(1/3))/b^(1/3)) + C*b^(2/3)*log(b*x + 2*b^(2/3)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{8C + b^{2/3}Cx^2}{8 + bx^3}\,dx = \operatorname{RootSum}\left(3t^3b^{\frac{5}{3}} - 3t^2Cb^{\frac{4}{3}} + tC^2b - C^3b^{\frac{2}{3}}, \left(t \mapsto t\log\left(x + \frac{3t\sqrt[3]{b} - C}{C\sqrt[3]{b}}\right)\right)\right)$$

input
```
integrate((8*C+b**(2/3)*C*x**2)/(b*x**3+8),x)
```

output
```
RootSum(3*_t**3*b**(5/3) - 3*_t**2*C*b**(4/3) + _t*C**2*b - C**3*b**(2/3),
 Lambda(_t, _t*log(x + (3*_t*b**(1/3) - C)/(C*b**(1/3)))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{8C + b^{2/3}Cx^2}{8 + bx^3}\,dx = \frac{2\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(b^{\frac{2}{3}}x - b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}} + \frac{C\log\left(\frac{b^{\frac{1}{3}}x + 2}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}$$

input
```
integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*C*arctan(1/3*sqrt(3)*(b^(2/3)*x - b^(1/3))/b^(1/3))/b^(1/3) +
C*log((b^(1/3)*x + 2)/b^(1/3))/b^(1/3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(36) = 72$.

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.40

$$\int \frac{8C + b^{2/3}Cx^2}{8 + bx^3}\,dx = \frac{2}{3}\sqrt{3}C\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(x + \left(-\frac{1}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{b}\right)^{\frac{1}{3}}}\right) - \frac{1}{3}\left(Cb^{\frac{2}{3}}\left(-\frac{1}{b}\right)^{\frac{2}{3}} + 2C\right)\left(-\frac{1}{b}\right)^{\frac{1}{3}}\log\left(\left|x - 2\left(-\frac{1}{b}\right)^{\frac{1}{3}}\right|\right) + \frac{1}{3}\left(C\left(-\frac{1}{b}\right)^{\frac{1}{3}} + \frac{C}{b^{\frac{1}{3}}}\right)\log\left(x^2 + 2x\left(-\frac{1}{b}\right)^{\frac{1}{3}} + 4\left(-\frac{1}{b}\right)^{\frac{2}{3}}\right)$$

input
```
integrate((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*C*(-1/b)^(1/3)*arctan(1/3*sqrt(3)*(x + (-1/b)^(1/3))/(-1/b)^(1
/3)) - 1/3*(C*b^(2/3)*(-1/b)^(2/3) + 2*C)*(-1/b)^(1/3)*log(abs(x - 2*(-1/b
)^(1/3))) + 1/3*(C*(-1/b)^(1/3) + C/b^(1/3))*log(x^2 + 2*x*(-1/b)^(1/3) +
4*(-1/b)^(2/3))
```

Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{8C + b^{2/3}Cx^2}{8 + bx^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{\left(C - \mathrm{root}\left(27\,b^3\,z^3 - 27\,C\,b^{8/3}\,z^2 + 9\,C^2\,b^{7/3}\,z - 9\,C^3\,b^2, z, k\right)\,b^{1/3}\,3\right)\,\left(-C\right.}{}\right.$$

input
```
int((8*C + C*b^(2/3)*x^2)/(b*x^3 + 8),x)
```

output
```
symsum(log(-(8*(C - 3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z
 - 9*C^3*b^2, z, k)*b^(1/3))*(3*root(27*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2
*b^(7/3)*z - 9*C^3*b^2, z, k)*b^(1/3) - C + C*b^(1/3)*x))/b^(5/3))*root(27
*b^3*z^3 - 27*C*b^(8/3)*z^2 + 9*C^2*b^(7/3)*z - 9*C^3*b^2, z, k), k, 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int \frac{8C + b^{2/3}Cx^2}{8 + bx^3}\,dx = \frac{c\left(2\sqrt{3}\,atan\left(\frac{b^{\frac{1}{3}}x-1}{\sqrt{3}}\right) + 3\log\left(b^{\frac{1}{3}}x + 2\right)\right)}{3b^{\frac{1}{3}}}$$

input
```
int((8*C+b^(2/3)*C*x^2)/(b*x^3+8),x)
```

output
```
(c*(2*sqrt(3)*atan((b**(1/3)*x - 1)/sqrt(3)) + 3*log(b**(1/3)*x + 2)))/(3*
b**(1/3))
```

3.34 $\int \frac{a^{2/3}C+2Cx^2}{a+8x^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3} dx = -\frac{C \arctan\left(\frac{\sqrt[3]{a}-4x}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}} + \frac{1}{4} C \log\left(\sqrt[3]{a} + 2x\right)$$

output
```
-1/6*C*arctan(1/3*(a^(1/3)-4*x)*3^(1/2)/a^(1/3))*3^(1/2)+1/4*C*ln(a^(1/3)+
2*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3} dx = \frac{1}{12} C \left(-2\sqrt{3} \arctan\left(\frac{1 - \frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2\log\left(\sqrt[3]{a} + 2x\right) - \log\left(a^{2/3} - 2\sqrt[3]{a}x + 4x^2\right) + \log\left(a + 8x^3\right)\right)$$

input
```
Integrate[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

output
```
(C*(-2*Sqrt[3]*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]] + 2*Log[a^(1/3) + 2*x]
- Log[a^(2/3) - 2*a^(1/3)*x + 4*x^2] + Log[a + 8*x^3]))/12
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2402, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3} \, dx$$

$$\Big\downarrow \text{ 2402}$$

$$\frac{1}{8}\sqrt[3]{a}C \int \frac{1}{x^2 - \frac{\sqrt[3]{a}x}{2} + \frac{a^{2/3}}{4}}dx + \frac{1}{4}C \int \frac{1}{x + \frac{\sqrt[3]{a}}{2}}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{8}\sqrt[3]{a}C \int \frac{1}{x^2 - \frac{\sqrt[3]{a}x}{2} + \frac{a^{2/3}}{4}}dx + \frac{1}{4}C \log\left(\sqrt[3]{a} + 2x\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}C \int \frac{1}{-\left(1 - \frac{4x}{\sqrt[3]{a}}\right)^2 - 3}d\left(1 - \frac{4x}{\sqrt[3]{a}}\right) + \frac{1}{4}C \log\left(\sqrt[3]{a} + 2x\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{4}C \log\left(\sqrt[3]{a} + 2x\right) - \frac{C \arctan\left(\frac{1 - \frac{4x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[(a^(2/3)*C + 2*C*x^2)/(a + 8*x^3),x]
```

output
```
-1/2*(C*ArcTan[(1 - (4*x)/a^(1/3))/Sqrt[3]])/Sqrt[3] + (C*Log[a^(1/3) + 2*
x])/4
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

method	result	si
default	$C\left(a^{\frac{2}{3}}\left(\frac{8^{\frac{2}{3}}\ln\left(x+\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}}{8}\right)}{24a^{\frac{2}{3}}}-\frac{8^{\frac{2}{3}}\ln\left(x^{2}-\frac{8^{\frac{2}{3}}a^{\frac{1}{3}}x}{8}+\frac{8^{\frac{1}{3}}a^{\frac{2}{3}}}{8}\right)}{48a^{\frac{2}{3}}}+\frac{8^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,8^{\frac{1}{3}}x}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{24a^{\frac{2}{3}}}\right)+\frac{\ln(8x^{3}+a)}{12}\right)$	96

input
```
int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
C*(a^(2/3)*(1/24*8^(2/3)/a^(2/3)*ln(x+1/8*8^(2/3)*a^(1/3))-1/48*8^(2/3)/a^
(2/3)*ln(x^2-1/8*8^(2/3)*a^(1/3)*x+1/8*8^(1/3)*a^(2/3))+1/24*8^(2/3)/a^(2/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*8^(1/3)/a^(1/3)*x-1)))+1/12*ln(8*x^3+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = \frac{1}{6}\sqrt{3}C\arctan\left(\frac{4\sqrt{3}a^{\frac{2}{3}}x - \sqrt{3}a}{3\,a}\right) + \frac{1}{4}\,C\log\left(2\,x + a^{\frac{1}{3}}\right)$$

input
```
integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*C*arctan(1/3*(4*sqrt(3)*a^(2/3)*x - sqrt(3)*a)/a) + 1/4*C*log(
2*x + a^(1/3))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = C\left(\frac{\log\left(\frac{\sqrt[3]{a}}{2} + x\right)}{4} - \frac{\sqrt{3}i\log\left(x + \frac{-C\sqrt[3]{a} - \sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12} + \frac{\sqrt{3}i\log\left(x + \frac{-C\sqrt[3]{a} + \sqrt{3}iC\sqrt[3]{a}}{4C}\right)}{12}\right)$$

input
```
integrate((a**(2/3)*C+2*C*x**2)/(8*x**3+a),x)
```

output
```
C*(log(a**(1/3)/2 + x)/4 - sqrt(3)*I*log(x + (-C*a**(1/3) - sqrt(3)*I*C*a*
*(1/3))/(4*C))/12 + sqrt(3)*I*log(x + (-C*a**(1/3) + sqrt(3)*I*C*a**(1/3))
/(4*C))/12)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3}\,dx = \frac{1}{6}\sqrt{3}C\arctan\left(\frac{\sqrt{3}\left(4\,x - a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) + \frac{1}{4}\,C\log\left(x + \frac{1}{2}\,a^{\frac{1}{3}}\right)$$

input
```
integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="maxima")
```

output
```
1/6*sqrt(3)*C*arctan(1/3*sqrt(3)*(4*x - a^(1/3))/a^(1/3)) + 1/4*C*log(x +
1/2*a^(1/3))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = -\frac{\sqrt{3}(-i\sqrt{3}|a|-a)C\arctan\left(\frac{\sqrt{3}\left(4x+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{12a} - \frac{\left(C(-a)^{\frac{2}{3}}+2Ca^{\frac{2}{3}}\right)(-a)^{\frac{1}{3}}\log\left(\left|x-\frac{1}{2}(-a)^{\frac{1}{3}}\right|\right)}{12a}$$

input
```
integrate((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*(-I*sqrt(3)*abs(a) - a)*C*arctan(1/3*sqrt(3)*(4*x + (-a)^(1/
3))/(-a)^(1/3))/a - 1/12*(C*(-a)^(2/3) + 2*C*a^(2/3))*(-a)^(1/3)*log(abs(x
 - 1/2*(-a)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.09

$$\int \frac{a^{2/3}C+2Cx^2}{a+8x^3}\,dx = \sum_{k=1}^{3}\ln\left(-\frac{a^{2/3}\left(C-12\,\mathrm{root}(1728\,a^2\,z^3-432\,C\,a^2\,z^2+36\,C^2\,a^2\,z-9\,C^3\,a^2,z,k)\right)\,(\right.$$

input
```
int((C*a^(2/3) + 2*C*x^2)/(a + 8*x^3),x)
```

output
```
symsum(log(-(a^(2/3)*(C - 12*root(1728*a^2*z^3 - 432*C*a^2*z^2 + 36*C^2*a^
2*z - 9*C^3*a^2, z, k))*(4*C*x - C*a^(1/3) + 12*root(1728*a^2*z^3 - 432*C*
a^2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k)*a^(1/3)))/128)*root(1728*a^2*z^3
 - 432*C*a^2*z^2 + 36*C^2*a^2*z - 9*C^3*a^2, z, k), k, 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{a^{2/3}C + 2Cx^2}{a + 8x^3} \, dx = \frac{c\left(-2\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}} - 4x}{a^{\frac{1}{3}}\sqrt{3}}\right) + 3\log\left(a^{\frac{1}{3}} + 2x\right)\right)}{12}$$

input
```
int((a^(2/3)*C+2*C*x^2)/(8*x^3+a),x)
```

output
```
(c*( - 2*sqrt(3)*atan((a**(1/3) - 4*x)/(a**(1/3)*sqrt(3))) + 3*log(a**(1/3
) + 2*x)))/12
```

3.35 $\int \frac{-3+x^2}{-1+x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{-3+x^2}{-1+x^3} dx = \sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - \frac{2}{3}\log(1-x) + \frac{5}{6}\log\left(1+x+x^2\right)$$

output
```
arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/3*ln(1-x)+5/6*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{-3+x^2}{-1+x^3} dx = \sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - \log(1-x) + \frac{1}{2}\log\left(1+x+x^2\right) + \frac{1}{3}\log\left(1-x^3\right)$$

input
```
Integrate[(-3 + x^2)/(-1 + x^3),x]
```

output
```
Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[1 - x] + Log[1 + x + x^2]/2 + Log[
1 - x^3]/3
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2414, 16, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-3}{x^3-1}dx$$

$$\Big\downarrow \text{2414}$$

$$\frac{2}{3}\int \frac{1}{1-x}dx-\frac{1}{3}\int -\frac{5x+7}{x^2+x+1}dx$$

$$\Big\downarrow \text{16}$$

$$-\frac{1}{3}\int -\frac{5x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{5x+7}{x^2+x+1}dx-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{9}{2}\int \frac{1}{x^2+x+1}dx+\frac{5}{2}\int \frac{2x+1}{x^2+x+1}dx\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{5}{2}\int \frac{2x+1}{x^2+x+1}dx-9\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{5}{2}\int \frac{2x+1}{x^2+x+1}dx+3\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{2}{3}\log(1-x)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(3\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{5}{2}\log\left(x^2+x+1\right)\right)-\frac{2}{3}\log(1-x)$$

input
```
Int[(-3 + x^2)/(-1 + x^3),x]
```

output
```
(-2*Log[1 - x])/3 + (3*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + (5*Log[1 + x + 
x^2])/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, 
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B 
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 
2] && LtQ[a/b, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{5\ln(x^2+x+1)}{6}+\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}-\frac{2\ln(x-1)}{3}$	32
risch	$-\frac{2\ln(x-1)}{3}+\frac{5\ln(9x^2+9x+9)}{6}+\sqrt{3}\ \arctan\left(\frac{2\left(\frac{3}{2}+3x\right)\sqrt{3}}{9}\right)$	36
meijerg	$-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}+\frac{\ln(-x^3+1)}{3}$	73

input
```
int((x^2-3)/(x^3-1),x,method=_RETURNVERBOSE)
```

output
```
5/6*ln(x^2+x+1)+arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-2/3*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{-3+x^2}{-1+x^3}\,dx=\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{5}{6}\log\left(x^2+x+1\right)-\frac{2}{3}\log\left(x-1\right)$$

input
```
integrate((x^2-3)/(x^3-1),x, algorithm="fricas")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{-3+x^2}{-1+x^3}\,dx = -\frac{2\log(x-1)}{3} + \frac{5\log(x^2+x+1)}{6} + \sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)$$

input
```
integrate((x**2-3)/(x**3-1),x)
```

output
```
-2*log(x - 1)/3 + 5*log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqr
t(3)/3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{-3+x^2}{-1+x^3}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{6}\log\left(x^2+x+1\right) - \frac{2}{3}\log(x-1)$$

input
```
integrate((x^2-3)/(x^3-1),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(x -
 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{-3+x^2}{-1+x^3}\,dx = \sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{5}{6}\log\left(x^2+x+1\right) - \frac{2}{3}\log(|x-1|)$$

input
```
integrate((x^2-3)/(x^3-1),x, algorithm="giac")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/6*log(x^2 + x + 1) - 2/3*log(abs
(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{-3+x^2}{-1+x^3} \, dx = -\frac{2 \ln (x-1)}{3} - \ln \left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) \left(-\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + \ln \left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) \left(\frac{5}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

input
```
int((x^2 - 3)/(x^3 - 1),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/2 + 5/6) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*1i)/2 - 5/6) - (2*log(x - 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{-3+x^2}{-1+x^3} \, dx = \sqrt{3}\, atan\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{5 \log(x^2+x+1)}{6} - \frac{2 \log(x-1)}{3}$$

input
```
int((x^2-3)/(x^3-1),x)
```

output
```
(6*sqrt(3)*atan((2*x + 1)/sqrt(3)) + 5*log(x**2 + x + 1) - 4*log(x - 1))/6
```

3.36 $\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 11

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx=\frac{\log(B-Cx)}{C}$$

output
```
ln(-C*x+B)/C
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2+BCx+C^2x^2}{-B^3+C^3x^3}\,dx=\frac{\log(-B+Cx)}{C}$$

input
```
Integrate[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

output
```
Log[-B + C*x]/C
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B^2 + BCx + C^2x^2}{C^3x^3 - B^3}\,dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{Cx - B}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(B - Cx)}{C}$$

input
```
Int[(B^2 + B*C*x + C^2*x^2)/(-B^3 + C^3*x^3),x]
```

output
```
Log[B - C*x]/C
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\ln(-Cx+B)}{C}$	12
norman	$\frac{\ln(-Cx+B)}{C}$	12
risch	$\frac{\ln(-Cx+B)}{C}$	12
parallelrisch	$\frac{\ln(-Cx+B)}{C}$	12

input
```
int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x,method=_RETURNVERBOSE)
```

output
```
ln(-C*x+B)/C
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\log\left(Cx - B\right)}{C}$$

input
```
integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="fricas")
```

output
```
log(C*x - B)/C
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3}\,dx = \frac{\log\left(-B + Cx\right)}{C}$$

input
```
integrate((C**2*x**2+B*C*x+B**2)/(C**3*x**3-B**3),x)
```

output
```
log(-B + C*x)/C
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} \, dx = \frac{\log (Cx - B)}{C}$$

input
```
integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="maxima")
```

output
```
log(C*x - B)/C
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} \, dx = \frac{\log (|Cx - B|)}{C}$$

input
```
integrate((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x, algorithm="giac")
```

output
```
log(abs(C*x - B))/C
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} \, dx = \frac{\ln (C\,x - B)}{C}$$

input
```
int(-(B^2 + C^2*x^2 + B*C*x)/(B^3 - C^3*x^3),x)
```

output
```
log(C*x - B)/C
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{B^2 + BCx + C^2x^2}{-B^3 + C^3x^3} \, dx = \frac{\log(-cx + b)}{c}$$

input
```
int((C^2*x^2+B*C*x+B^2)/(C^3*x^3-B^3),x)
```

output
```
log(b - c*x)/c
```

3.37 $\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 21

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

output
```
C*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

input
```
Integrate[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
(C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx$$

$$\Big\downarrow \ 2019$$

$$\int \frac{1}{\frac{\sqrt[3]{a}}{C}+\frac{\sqrt[3]{b}x}{C}}dx$$

$$\Big\downarrow \ 16$$

$$\frac{C\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}$$

input
```
Int[(a^(2/3)*C - a^(1/3)*b^(1/3)*C*x + b^(2/3)*C*x^2)/(a + b*x^3),x]
```

output
```
(C*Log[a^(1/3) + b^(1/3)*x])/b^(1/3)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result
risch	$\frac{C\ln\left(a^{\frac{1}{3}}b^{\frac{2}{3}}+bx\right)}{b^{\frac{1}{3}}}$
default	$C\left(a^{\frac{2}{3}}\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)-a^{\frac{1}{3}}b^{\frac{1}{3}}\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\right.\right.$

input
```
int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x,method=_RETU
RNVERBOSE)
```

output
```
C/b^(1/3)*ln(a^(1/3)*b^(2/3)+b*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{a^{2/3}C-\sqrt[3]{a}\sqrt[3]{b}Cx+b^{2/3}Cx^2}{a+bx^3}\,dx=\frac{C\log\left(bx+a^{\frac{1}{3}}b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}$$

input
```
integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algor
ithm="fricas")
```

output
```
C*log(b*x + a^(1/3)*b^(2/3))/b^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\log\left(\sqrt[3]{a}b^{\frac{2}{3}} + bx\right)}{\sqrt[3]{b}}$$

input
```
integrate((a**(2/3)*C-a**(1/3)*b**(1/3)*C*x+b**(2/3)*C*x**2)/(b*x**3+a),x)
```

output
```
C*log(a**(1/3)*b**(2/3) + b*x)/b**(1/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(15) = 30.

Time = 0.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 10.00

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx =$$

$$-\frac{\sqrt{3}\left(2\,Cab^{\frac{2}{3}} + \left(3\,Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - 3\,Ca^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - \frac{2\,Ca}{b^{\frac{1}{3}}}\right)b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab}$$
$$+\frac{\left(2\,Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} - Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} - Ca^{\frac{2}{3}}\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+\frac{\left(Cb^{\frac{2}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}} + Ca^{\frac{1}{3}}b^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + Ca^{\frac{2}{3}}\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algor
ithm="maxima")
```

output
```
-1/9*sqrt(3)*(2*C*a*b^(2/3) + (3*C*a^(1/3)*b^(1/3)*(a/b)^(2/3) - 3*C*a^(2/
3)*(a/b)^(1/3) - 2*C*a/b^(1/3))*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/
(a/b)^(1/3))/(a*b) + 1/6*(2*C*b^(2/3)*(a/b)^(2/3) - C*a^(1/3)*b^(1/3)*(a/b
)^(1/3) - C*a^(2/3))*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)
) + 1/3*(C*b^(2/3)*(a/b)^(2/3) + C*a^(1/3)*b^(1/3)*(a/b)^(1/3) + C*a^(2/3)
)*log(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\log\left(\left|b^{\frac{1}{3}}x + a^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

input
```
integrate((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x, algor
ithm="giac")
```

output
```
C*log(abs(b^(1/3)*x + a^(1/3)))/b^(1/3)
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{C\,\ln\left(x + \frac{a^{1/3}}{b^{1/3}}\right)}{b^{1/3}}$$

input
```
int((C*a^(2/3) + C*b^(2/3)*x^2 - C*a^(1/3)*b^(1/3)*x)/(a + b*x^3),x)
```

output
```
(C*log(x + a^(1/3)/b^(1/3)))/b^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a^{2/3}C - \sqrt[3]{a}\sqrt[3]{b}Cx + b^{2/3}Cx^2}{a + bx^3}\,dx = \frac{\log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)c}{b^{\frac{1}{3}}}$$

input
```
int((a^(2/3)*C-a^(1/3)*b^(1/3)*C*x+b^(2/3)*C*x^2)/(b*x^3+a),x)
```

output
```
(b**(2/3)*log(a**(1/3) + b**(1/3)*x)*c)/b
```

3.38 $\int \frac{a+ax+cx^2}{1-x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{a+ax+cx^2}{1-x^3} \, dx = -\frac{1}{3}(2a+c)\log(1-x)+\frac{1}{3}(a-c)\log\left(1+x+x^2\right)$$

output
```
-1/3*(2*a+c)*ln(1-x)+1/3*(a-c)*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{a+ax+cx^2}{1-x^3} \, dx = \frac{1}{3}\left(-((2a+c)\log(1-x))+(a-c)\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[(a + a*x + c*x^2)/(1 - x^3),x]
```

output
```
(-((2*a + c)*Log[1 - x]) + (a - c)*Log[1 + x + x^2])/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2414, 16, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+a+cx^2}{1-x^3}dx$$

$$\Big\downarrow \text{2414}$$

$$\frac{1}{3}\int \frac{a-c+2(a-c)x}{x^2+x+1}dx+\frac{1}{3}(2a+c)\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \frac{a-c+2(a-c)x}{x^2+x+1}dx-\frac{1}{3}(2a+c)\log(1-x)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}(a-c)\log\left(x^2+x+1\right)-\frac{1}{3}(2a+c)\log(1-x)$$

input
```
Int[(a + a*x + c*x^2)/(1 - x^3),x]
```

output
```
-1/3*((2*a + c)*Log[1 - x]) + ((a - c)*Log[1 + x + x^2])/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result
default	$\frac{(a-c)\ln(x^2+x+1)}{3}+\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln\left(x-1\right)$
norman	$\left(-\frac{2a}{3}-\frac{c}{3}\right)\ln\left(x-1\right)+\left(\frac{a}{3}-\frac{c}{3}\right)\ln\left(x^2+x+1\right)$
parallelrisch	$-\frac{2\ln(x-1)a}{3}-\frac{\ln(x-1)c}{3}+\frac{\ln(x^2+x+1)a}{3}-\frac{\ln(x^2+x+1)c}{3}$
risch	$-\frac{2\ln(x-1)a}{3}-\frac{\ln(x-1)c}{3}+\frac{\ln(-x^2-x-1)a}{3}-\frac{\ln(-x^2-x-1)c}{3}$
meijerg	$-\frac{c\ln(-x^3+1)}{3}-\frac{a x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{ax\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\ln\ldots\right.}{}$

input
```
int((c*x^2+a*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a-c)*ln(x^2+x+1)+(-2/3*a-1/3*c)*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a+ax+cx^2}{1-x^3}\,dx=\frac{1}{3}\left(a-c\right)\log\left(x^2+x+1\right)-\frac{1}{3}\left(2\,a+c\right)\log\left(x-1\right)$$

input
```
integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{a + ax + cx^2}{1 - x^3}\, dx = \frac{(a - c)\log\left(x^2 + x + 1\right)}{3} - \frac{(2a + c)\log\left(x - 1\right)}{3}$$

input
```
integrate((c*x**2+a*x+a)/(-x**3+1),x)
```

output
```
(a - c)*log(x**2 + x + 1)/3 - (2*a + c)*log(x - 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a + ax + cx^2}{1 - x^3}\, dx = \frac{1}{3}\,(a - c)\log\left(x^2 + x + 1\right) - \frac{1}{3}\,(2\,a + c)\log\left(x - 1\right)$$

input
```
integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="maxima")
```

output
```
1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{a + ax + cx^2}{1 - x^3}\, dx = \frac{1}{3}\,(a - c)\log\left(x^2 + x + 1\right) - \frac{1}{3}\,(2\,a + c)\log\left(|x - 1|\right)$$

input
```
integrate((c*x^2+a*x+a)/(-x^3+1),x, algorithm="giac")
```

output
```
1/3*(a - c)*log(x^2 + x + 1) - 1/3*(2*a + c)*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{a + ax + cx^2}{1 - x^3} \, dx = \frac{a \ln(x^2 + x + 1)}{3} - \frac{c \ln(x - 1)}{3} - \frac{2a \ln(x - 1)}{3} - \frac{c \ln(x^2 + x + 1)}{3}$$

input
```
int(-(a + a*x + c*x^2)/(x^3 - 1),x)
```

output
```
(a*log(x + x^2 + 1))/3 - (c*log(x - 1))/3 - (2*a*log(x - 1))/3 - (c*log(x
+ x^2 + 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{a + ax + cx^2}{1 - x^3} \, dx = \frac{\log(x^2 + x + 1)\, a}{3} - \frac{\log(x^2 + x + 1)\, c}{3} - \frac{2\log(x - 1)\, a}{3} - \frac{\log(x - 1)\, c}{3}$$

input
```
int((c*x^2+a*x+a)/(-x^3+1),x)
```

output
```
(log(x**2 + x + 1)*a - log(x**2 + x + 1)*c - 2*log(x - 1)*a - log(x - 1)*c
)/3
```

3.39 $\int \frac{a+bx+cx^2}{1-x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{1-x^3} \, dx = \frac{(a-b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}(a+b+c)\log(1-x) + \frac{1}{6}(a+b-2c)\log\left(1+x+x^2\right)$$

output
```
1/3*(a-b)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/3*(a+b+c)*ln(1-x)+1/6*(a+b
-2*c)*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{a+bx+cx^2}{1-x^3} \, dx = \frac{1}{6}\left(2\sqrt{3}(a-b)\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2(a+b)\log(1-x) + (a+b)\log\left(1+x+x^2\right) - 2c\log\left(1-x^3\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(1 - x^3),x]
```

output
```
(2*Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*(a + b)*Log[1 - x] + (a +
b)*Log[1 + x + x^2] - 2*c*Log[1 - x^3])/6
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2414, 16, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{1-x^3}dx$$

$$\Big\downarrow \text{2414}$$

$$\frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{x^2+x+1}dx+\frac{1}{3}(a+b+c)\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \frac{2a-b-c+(a+b-2c)x}{x^2+x+1}dx-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{1}{2}(a+b-2c)\int \frac{2x+1}{x^2+x+1}dx+\frac{3}{2}(a-b)\int \frac{1}{x^2+x+1}dx\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(\frac{1}{2}(a+b-2c)\int \frac{2x+1}{x^2+x+1}dx-3(a-b)\int \frac{1}{-(2x+1)^2-3}d(2x+1)\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{2}(a+b-2c)\int \frac{2x+1}{x^2+x+1}dx+\sqrt{3}(a-b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\sqrt{3}(a-b)\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x^2+x+1\right)(a+b-2c)\right)-\frac{1}{3}\log(1-x)(a+b+c)$$

input
```
Int[(a + b*x + c*x^2)/(1 - x^3),x]
```

output
```
-1/3*((a + b + c)*Log[1 - x]) + (Sqrt[3]*(a - b)*ArcTan[(1 + 2*x)/Sqrt[3]]
 + ((a + b - 2*c)*Log[1 + x + x^2])/2)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result
default	$\frac{(a+b-2c)\ln(x^2+x+1)}{6}+\frac{2\left(\frac{3a}{2}-\frac{3b}{2}\right)\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\left(-\frac{c}{3}-\frac{b}{3}-\frac{a}{3}\right)\ln(x-1)$
meijerg	$-\frac{c\ln(-x^3+1)}{3}-\frac{b x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-$ $ax\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\ln\left(1+(x^3\right.\right.$
risch	$-\frac{\ln(x-1)a}{3}-\frac{\ln(x-1)c}{3}+\frac{a\ln(4a^2x^2+4ab\,x^2+4b^2x^2+4a^2x+4abx+4b^2x+4a^2+4ab+4b^2)}{6}+$ $b\ln(4a^2x^2+4ab\,x^2+4b^2x^2+4a^2$

input
```
int((c*x^2+b*x+a)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/6*(a+b-2*c)*ln(x^2+x+1)+2/9*(3/2*a-3/2*b)*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)+(-1/3*c-1/3*b-1/3*a)*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right) - \frac{1}{3}(a+b+c)\log(x-1)$$

input
```
integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(
x^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.87

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx$$
$$= -\frac{(a+b+c)\log\left(x+\frac{a^2c-a^2(a+b+c)-2ab^2+bc^2-2bc(a+b+c)+b(a+b+c)^2}{a^3-b^3}\right)}{3} - \left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$
$$\left.-\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}-\frac{\sqrt{3}i(a-b)}{6}\right.}{a^3-b^3}\right.$$
$$-\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}\right.$$
$$\left.+\frac{\sqrt{3}i(a-b)}{6}\right)\log\left(x+\frac{a^2c-3a^2\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right)-2ab^2+bc^2-6bc\left(-\frac{a}{6}-\frac{b}{6}+\frac{c}{3}+\frac{\sqrt{3}i(a-b)}{6}\right.}{a^3-b^3}\right.$$

input
```
integrate((c*x**2+b*x+a)/(-x**3+1),x)
```

output
```
-(a + b + c)*log(x + (a**2*c - a**2*(a + b + c) - 2*a*b**2 + b*c**2 - 2*b*
c*(a + b + c) + b*(a + b + c)**2)/(a**3 - b**3))/3 - (-a/6 - b/6 + c/3 - s
qrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 - b/6 + c/3 - sqrt(3)*I
*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a -
 b)/6) + 9*b*(-a/6 - b/6 + c/3 - sqrt(3)*I*(a - b)/6)**2)/(a**3 - b**3)) -
 (-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6)*log(x + (a**2*c - 3*a**2*(-a/6 -
 b/6 + c/3 + sqrt(3)*I*(a - b)/6) - 2*a*b**2 + b*c**2 - 6*b*c*(-a/6 - b/6
+ c/3 + sqrt(3)*I*(a - b)/6) + 9*b*(-a/6 - b/6 + c/3 + sqrt(3)*I*(a - b)/6
)**2)/(a**3 - b**3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\sqrt{3}(a-b)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right) - \frac{1}{3}(a+b+c)\log\left(x-1\right)$$

input
```
integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(a - b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2*c)*log(
x^2 + x + 1) - 1/3*(a + b + c)*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{1}{3}\left(\sqrt{3}a-\sqrt{3}b\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{1}{6}(a+b-2c)\log\left(x^2+x+1\right) - \frac{1}{3}(a+b+c)\log\left(|x-1|\right)$$

input
```
integrate((c*x^2+b*x+a)/(-x^3+1),x, algorithm="giac")
```

output
```
1/3*(sqrt(3)*a - sqrt(3)*b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*(a + b - 2
*c)*log(x^2 + x + 1) - 1/3*(a + b + c)*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}-\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{a}{6}+\frac{b}{6}-\frac{c}{3}+\frac{\sqrt{3}\,a\,1\mathrm{i}}{6}-\frac{\sqrt{3}\,b\,1\mathrm{i}}{6}\right) - \ln(x-1)\left(\frac{a}{3}+\frac{b}{3}+\frac{c}{3}\right)$$

input
```
int(-(a + b*x + c*x^2)/(x^3 - 1),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 - (3^(1/2)*a*1i)/6 + (3^(1/
2)*b*1i)/6) + log(x + (3^(1/2)*1i)/2 + 1/2)*(a/6 + b/6 - c/3 + (3^(1/2)*a*
1i)/6 - (3^(1/2)*b*1i)/6) - log(x - 1)*(a/3 + b/3 + c/3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{a+bx+cx^2}{1-x^3}\,dx = \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)a}{3} - \frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)b}{3} + \frac{\log(x^2+x+1)\,a}{6} + \frac{\log(x^2+x+1)\,b}{6} - \frac{\log(x^2+x+1)\,c}{3} - \frac{\log(x-1)\,a}{3} - \frac{\log(x-1)\,b}{3} - \frac{\log(x-1)\,c}{3}$$

input
```
int((c*x^2+b*x+a)/(-x^3+1),x)
```

output
```
(2*sqrt(3)*atan((2*x + 1)/sqrt(3))*a - 2*sqrt(3)*atan((2*x + 1)/sqrt(3))*b
 + log(x**2 + x + 1)*a + log(x**2 + x + 1)*b - 2*log(x**2 + x + 1)*c - 2*l
og(x - 1)*a - 2*log(x - 1)*b - 2*log(x - 1)*c)/6
```

3.40 $\int \frac{1+x+x^2}{1-x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 8

$$\int \frac{1+x+x^2}{1-x^3} dx = -\log(1-x)$$

output
```
-ln(1-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{1-x^3} dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2)/(1 - x^3),x]
```

output
```
-Log[1 - x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{1-x^3} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$-\log(1-x)$$

input
```
Int[(1 + x + x^2)/(1 - x^3),x]
```

output
```
-Log[1 - x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln(x-1)$
norman	$-\ln(x-1)$
risch	$-\ln(x-1)$
parallelrisch	$-\ln(x-1)$
meijerg	$-\frac{\ln(-x^3+1)}{3}-\frac{x^2\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{3(x^3)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(\right.}{}\right.}{}$

input
```
int((x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx=-\log(x-1)$$

input
```
integrate((x^2+x+1)/(-x^3+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log{(x-1)}$$

input
```
integrate((x**2+x+1)/(-x**3+1),x)
```

output
```
-log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log{(x-1)}$$

input
```
integrate((x^2+x+1)/(-x^3+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log{(|x-1|)}$$

input
```
integrate((x^2+x+1)/(-x^3+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\ln\left(x-1\right)$$

input
```
int(-(x + x^2 + 1)/(x^3 - 1),x)
```

output
```
-log(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2}{1-x^3}\,dx = -\log(x-1)$$

input
```
int((x^2+x+1)/(-x^3+1),x)
```

output
```
 - log(x - 1)
```

3.41 $\int \frac{1-x+3x^2}{1-x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int \frac{1-x+3x^2}{1-x^3} \, dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

output
```
2/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-ln(-x^3+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1-x+3x^2}{1-x^3} \, dx = \frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1-x^3\right)$$

input
```
Integrate[(1 - x + 3*x^2)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2410, 792, 2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-x+1}{1-x^3}dx$$
$$\Big\downarrow \text{ 2410}$$
$$\int \frac{1-x}{1-x^3}dx+3\int \frac{x^2}{1-x^3}dx$$
$$\Big\downarrow \text{ 792}$$
$$\int \frac{1-x}{1-x^3}dx-\log\left(1-x^3\right)$$
$$\Big\downarrow \text{ 2019}$$
$$\int \frac{1}{x^2+x+1}dx-\log\left(1-x^3\right)$$
$$\Big\downarrow \text{ 1083}$$
$$-2\int \frac{1}{-(2x+1)^2-3}d(2x+1)-\log\left(1-x^3\right)$$
$$\Big\downarrow \text{ 217}$$
$$\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1-x^3\right)$$

input
```
Int[(1 - x + 3*x^2)/(1 - x^3),x]
```

output
```
(2*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] - Log[1 - x^3]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result
risch	$-\ln\left(x-1\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\ln\left(x^{2}+x+1\right)$
default	$-\ln\left(x^{2}+x+1\right)+\frac{2\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(x-1\right)$
meijerg	$-\ln\left(-x^{3}+1\right)+\frac{x^{2}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x\right.\right.}{}\right.}{}$

input
```
int((3*x^2-x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x-1)+2/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \log\left(x^2+x+1\right) - \log\left(x-1\right)$$

input
```
integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="fricas")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = -\log\left(x-1\right)$$

input
```
integrate((3*x**2-x+1)/(-x**3+1),x)
```

output
```
-log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \log\left(x^2+x+1\right) - \log\left(x-1\right)$$

input
```
integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

input
```
integrate((3*x^2-x+1)/(-x^3+1),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - log(x^2 + x + 1) - log(abs(x -
 1))
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = -\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - \ln\left(x-1\right) - \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{3}\,\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{3}$$

input
```
int(-(3*x^2 - x + 1)/(x^3 - 1),x)
```

output
```
(3^(1/2)*log(x + (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x + (3^(1/2)*1i)/2 + 1/
2) - log(x - 1) - (3^(1/2)*log(x - (3^(1/2)*1i)/2 + 1/2)*1i)/3 - log(x - (
3^(1/2)*1i)/2 + 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{1-x+3x^2}{1-x^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3} - \log\left(x^2+x+1\right) - \log(x-1)$$

input
```
int((3*x^2-x+1)/(-x^3+1),x)
```

output
```
(2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3*log(x**2 + x + 1) - 3*log(x - 1))/3
```

3.42 $\int \frac{1+x+4x^2}{1-x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1+x+4x^2}{1-x^3} dx = -2\log(1-x) - \log\left(1+x+x^2\right)$$

output
```
-2*ln(1-x)-ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{1-x^3} dx = -2\log(1-x) - \log\left(1+x+x^2\right)$$

input
```
Integrate[(1 + x + 4*x^2)/(1 - x^3),x]
```

output
```
-2*Log[1 - x] - Log[1 + x + x^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2414, 16, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+x+1}{1-x^3}dx$$

$$\Big\downarrow \text{ 2414}$$

$$\frac{1}{3}\int -\frac{3(2x+1)}{x^2+x+1}dx+2\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{3}\int -\frac{3(2x+1)}{x^2+x+1}dx-2\log(1-x)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{2x+1}{x^2+x+1}dx-2\log(1-x)$$

$$\Big\downarrow \text{ 1103}$$

$$-\log\left(x^2+x+1\right)-2\log(1-x)$$

input
```
Int[(1 + x + 4*x^2)/(1 - x^3),x]
```

output
```
-2*Log[1 - x] - Log[1 + x + x^2]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2414
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-a/b)^(1/3)}, Simp[q*((A + B*q
 + C*q^2)/(3*a))   Int[1/(q - x), x], x] + Simp[q/(3*a)   Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result
default	$-\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$
norman	$-\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$
risch	$-\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$
parallelrisch	$-\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$
meijerg	$-\frac{4\ln\left(-x^3+1\right)}{3}-\frac{x^2\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^3\right)^{\frac{1}{3}}+\left(x^3\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^3\right)^{\frac{1}{3}}}{2+\left(x^3\right)^{\frac{1}{3}}}\right)\right)}{3\left(x^3\right)^{\frac{2}{3}}}-\frac{x\left(\ln\left(1-\left(x^3\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\right.}{}\right.}{}$

input
```
int((4*x^2+x+1)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x^2+x+1)-2*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right)-2\,\log\left(x-1\right)$$

input
```
integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="fricas")
```

output
```
-log(x^2 + x + 1) - 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -2\log\left(x-1\right)-\log\left(x^2+x+1\right)$$

input
```
integrate((4*x**2+x+1)/(-x**3+1),x)
```

output
```
-2*log(x - 1) - log(x**2 + x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right)-2\,\log\left(x-1\right)$$

input
```
integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="maxima")
```

output
```
-log(x^2 + x + 1) - 2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right)-2\log\left(|x-1|\right)$$

input
```
integrate((4*x^2+x+1)/(-x^3+1),x, algorithm="giac")
```

output
```
-log(x^2 + x + 1) - 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\ln\left(x^2+x+1\right)-2\ln\left(x-1\right)$$

input
```
int(-(x + 4*x^2 + 1)/(x^3 - 1),x)
```

output
```
- log(x + x^2 + 1) - 2*log(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x+4x^2}{1-x^3}\,dx = -\log\left(x^2+x+1\right)-2\log(x-1)$$

input
```
int((4*x^2+x+1)/(-x^3+1),x)
```

output
```
- log(x**2 + x + 1) - 2*log(x - 1)
```

3.43 $\int \frac{1-x+4x^3}{1+x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{1-x+4x^3}{1+x^3} \, dx = 4x+\frac{4\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

output
```
4*x+4/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-2/3*ln(1+x)+1/3*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1-x+4x^3}{1+x^3} \, dx = 4x-\frac{4\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\log(1+x)+\frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 - x + 4*x^3)/(1 + x^3),x]
```

output
```
4*x - (4*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 -
x + x^2]/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^3-x+1}{x^3+1}\,dx$$

$$\Big\downarrow \ 2426$$

$$\int \left(4-\frac{x+3}{x^3+1}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{4\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log\left(x^2-x+1\right)+4x-\frac{2}{3}\log(x+1)$$

input
```
Int[(1 - x + 4*x^3)/(1 + x^3),x]
```

output
```
4*x + (4*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] - (2*Log[1 + x])/3 + Log[1 - x
 + x^2]/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
default	$4x+\frac{\ln\left(x^{2}-x+1\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}-\frac{2\ln\left(x+1\right)}{3}$
risch	$4x-\frac{2\ln\left(x+1\right)}{3}+\frac{\ln\left(16x^{2}-16x+16\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{(4x-2)\sqrt{3}}{6}\right)}{3}$
meijerg	$4x-\frac{4x\left(\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}-\frac{\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2\left(x^{3}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3}+\frac{x^{2}\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x^{2}\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{6\left(x^{3}\right)^{\frac{2}{3}}}$

input

```
int((4*x^3-x+1)/(x^3+1),x,method=_RETURNVERBOSE)
```

output

```
4*x+1/3*ln(x^2-x+1)-4/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))-2/3*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1-x+4x^3}{1+x^3}\,dx=-\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+4\,x+\frac{1}{3}\,\log\left(x^2-x+1\right)-\frac{2}{3}\,\log\left(x+1\right)$$

input

```
integrate((4*x^3-x+1)/(x^3+1),x, algorithm="fricas")
```

output

```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) -
2/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4x - \frac{2\log{(x+1)}}{3} + \frac{\log{(x^2-x+1)}}{3} - \frac{4\sqrt{3}\operatorname{atan}{\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}}{3}$$

input
```
integrate((4*x**3-x+1)/(x**3+1),x)
```

output
```
4*x - 2*log(x + 1)/3 + log(x**2 - x + 1)/3 - 4*sqrt(3)*atan(2*sqrt(3)*x/3
- sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 4x + \frac{1}{3}\log\left(x^2-x+1\right) - \frac{2}{3}\log\left(x+1\right)$$

input
```
integrate((4*x^3-x+1)/(x^3+1),x, algorithm="maxima")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) -
2/3*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + 4x + \frac{1}{3}\log\left(x^2-x+1\right) - \frac{2}{3}\log\left(|x+1|\right)$$

input
```
integrate((4*x^3-x+1)/(x^3+1),x, algorithm="giac")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 4*x + 1/3*log(x^2 - x + 1) -
2/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = 4\,x - \frac{2\,\ln(x+1)}{3} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3} + \frac{\sqrt{3}\,2\mathrm{i}}{3}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3} + \frac{\sqrt{3}\,2\mathrm{i}}{3}\right)$$

input
```
int((4*x^3 - x + 1)/(x^3 + 1),x)
```

output
```
4*x - (2*log(x + 1))/3 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 + 1
/3) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*2i)/3 - 1/3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1-x+4x^3}{1+x^3}\,dx = -\frac{4\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} + \frac{\log(x^2-x+1)}{3} - \frac{2\log(x+1)}{3} + 4x$$

input
```
int((4*x^3-x+1)/(x^3+1),x)
```

output
```
( - 4*sqrt(3)*atan((2*x - 1)/sqrt(3)) + log(x**2 - x + 1) - 2*log(x + 1) +
 12*x)/3
```

3.44 $\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5 +\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10} +\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

output

```
a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^
2*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*
x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx = a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5 +\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10} +\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

input
```
Integrate[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x
^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^3 \left(ac+adx+bcx^3+bdx^4\right) dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(a^4c+a^4dx+4a^3bcx^3+4a^3bdx^4+6a^2b^2cx^6+6a^2b^2dx^7+4ab^3cx^9+4ab^3dx^{10}+b^4cx^{12}+b^4dx^{13}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^4cx+\frac{1}{2}a^4dx^2+a^3bcx^4+\frac{4}{5}a^3bdx^5+\frac{6}{7}a^2b^2cx^7+\frac{3}{4}a^2b^2dx^8+\frac{2}{5}ab^3cx^{10}+\frac{4}{11}ab^3dx^{11}+\frac{1}{13}b^4cx^{13}+\frac{1}{14}b^4dx^{14}$$

input
```
Int[(a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + a^3*b*c*x^4 + (4*a^3*b*d*x^5)/5 + (6*a^2*b^2*c*x
^7)/7 + (3*a^2*b^2*d*x^8)/4 + (2*a*b^3*c*x^10)/5 + (4*a*b^3*d*x^11)/11 + (
b^4*c*x^13)/13 + (b^4*d*x^14)/14
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

method	result
default	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}a\,b^3d\,x^{11}+$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}a\,b^3d\,x^{11}+$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}a\,b^3d\,x^{11}+$
parallelrisch	$a^4cx+\frac{1}{2}a^4d\,x^2+a^3bc\,x^4+\frac{4}{5}a^3bd\,x^5+\frac{6}{7}a^2b^2c\,x^7+\frac{3}{4}a^2b^2d\,x^8+\frac{2}{5}a\,b^3c\,x^{10}+\frac{4}{11}a\,b^3d\,x^{11}+$
gosper	$\frac{x(1430b^4d\,x^{13}+1540b^4c\,x^{12}+7280a\,b^3d\,x^{10}+8008a\,b^3c\,x^9+15015a^2b^2d\,x^7+17160a^2b^2c\,x^6+16016a^3bd\,x^4+20020a^3bc\,x^3+1}{20020}$
orering	$\frac{x(1430b^4d\,x^{13}+1540b^4c\,x^{12}+7280a\,b^3d\,x^{10}+8008a\,b^3c\,x^9+15015a^2b^2d\,x^7+17160a^2b^2c\,x^6+16016a^3bd\,x^4+20020a^3bc\,x^3+1}{20020(dx+c)(b\,x^3+a)}$

input
```
int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
a^4*c*x+1/2*a^4*d*x^2+a^3*b*c*x^4+4/5*a^3*b*d*x^5+6/7*a^2*b^2*c*x^7+3/4*a^
2*b^2*d*x^8+2/5*a*b^3*c*x^10+4/11*a*b^3*d*x^11+1/13*b^4*c*x^13+1/14*b^4*d*
x^14
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\ dx = \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

input
```
integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

output
```
1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1
/2*a^4*d*x^2 + a^4*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4)\ dx = a^4cx + \frac{a^4dx^2}{2} + a^3bcx^4 + \frac{4a^3bdx^5}{5} + \frac{6a^2b^2cx^7}{7} + \frac{3a^2b^2dx^8}{4} + \frac{2ab^3cx^{10}}{5} + \frac{4ab^3dx^{11}}{11} + \frac{b^4cx^{13}}{13} + \frac{b^4dx^{14}}{14}$$

input
```
integrate((b*x**3+a)**3*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**4*c*x + a**4*d*x**2/2 + a**3*b*c*x**4 + 4*a**3*b*d*x**5/5 + 6*a**2*b**2
*c*x**7/7 + 3*a**2*b**2*d*x**8/4 + 2*a*b**3*c*x**10/5 + 4*a*b**3*d*x**11/1
1 + b**4*c*x**13/13 + b**4*d*x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx = \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

input
```
integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

output
```
1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1
/2*a^4*d*x^2 + a^4*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int (a+bx^3)^3 (ac+adx+bcx^3+bdx^4) \, dx = \frac{1}{14}b^4dx^{14} + \frac{1}{13}b^4cx^{13} + \frac{4}{11}ab^3dx^{11} + \frac{2}{5}ab^3cx^{10} + \frac{3}{4}a^2b^2dx^8 + \frac{6}{7}a^2b^2cx^7 + \frac{4}{5}a^3bdx^5 + a^3bcx^4 + \frac{1}{2}a^4dx^2 + a^4cx$$

input
```
integrate((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
1/14*b^4*d*x^14 + 1/13*b^4*c*x^13 + 4/11*a*b^3*d*x^11 + 2/5*a*b^3*c*x^10 +
 3/4*a^2*b^2*d*x^8 + 6/7*a^2*b^2*c*x^7 + 4/5*a^3*b*d*x^5 + a^3*b*c*x^4 + 1
/2*a^4*d*x^2 + a^4*c*x
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{4\,d\,a^3\,b\,x^5}{5}+c\,a^3\,b\,x^4+\frac{3\,d\,a^2\,b^2\,x^8}{4}+\frac{6\,c\,a^2\,b^2\,x^7}{7}+\frac{4\,d\,a\,b^3\,x^{11}}{11}+\frac{2\,c\,a\,b^3\,x^{10}}{5}+\frac{d\,b^4\,x^{14}}{14}+\frac{c\,b^4\,x^{13}}{13}$$

input
```
int((a + b*x^3)^3*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
(a^4*d*x^2)/2 + (b^4*c*x^13)/13 + (b^4*d*x^14)/14 + a^4*c*x + (6*a^2*b^2*c
*x^7)/7 + (3*a^2*b^2*d*x^8)/4 + a^3*b*c*x^4 + (2*a*b^3*c*x^10)/5 + (4*a^3*
b*d*x^5)/5 + (4*a*b^3*d*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \left(a+bx^3\right)^3 \left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{x(1430b^4d\,x^{13}+1540b^4c\,x^{12}+7280a\,b^3d\,x^{10}+8008a\,b^3c\,x^9+15015a^2b^2d\,x^7+17160a^2b^2c\,x^6+16016a^3}{20020}$$

input
```
int((b*x^3+a)^3*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

output
```
(x*(20020*a**4*c + 10010*a**4*d*x + 20020*a**3*b*c*x**3 + 16016*a**3*b*d*x
**4 + 17160*a**2*b**2*c*x**6 + 15015*a**2*b**2*d*x**7 + 8008*a*b**3*c*x**9
 + 7280*a*b**3*d*x**10 + 1540*b**4*c*x**12 + 1430*b**4*d*x**13))/20020
```

3.45 $\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4)\, dx$

Optimal result

Integrand size = 30, antiderivative size = 88

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4)\, dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*
a*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4)\, dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7 + \frac{3}{8}ab^2dx^8 + \frac{1}{10}b^3cx^{10} + \frac{1}{11}b^3dx^{11}$$

input
```
Integrate[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2
*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4)\, dx$$

$$\Big\downarrow \text{2389}$$

$$\int (a^3c+a^3dx+3a^2bcx^3+3a^2bdx^4+3ab^2cx^6+3ab^2dx^7+b^3cx^9+b^3dx^{10})\, dx$$

$$\Big\downarrow \text{2009}$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{3}{4}a^2bcx^4+\frac{3}{5}a^2bdx^5+\frac{3}{7}ab^2cx^7+\frac{3}{8}ab^2dx^8+\frac{1}{10}b^3cx^{10}+\frac{1}{11}b^3dx^{11}$$

input
```
Int[(a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (3*a^2*b*c*x^4)/4 + (3*a^2*b*d*x^5)/5 + (3*a*b^2
*c*x^7)/7 + (3*a*b^2*d*x^8)/8 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

method	result
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}a^2bd\,x^5+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}a\,b^2d\,x^8+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}a^2bd\,x^5+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}a\,b^2d\,x^8+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}a^2bd\,x^5+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}a\,b^2d\,x^8+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{3}{4}a^2bc\,x^4+\frac{3}{5}a^2bd\,x^5+\frac{3}{7}a\,b^2c\,x^7+\frac{3}{8}a\,b^2d\,x^8+\frac{1}{10}b^3c\,x^{10}+\frac{1}{11}b^3d\,x^{11}$
gosper	$\frac{x\left(280b^3d\,x^{10}+308b^3c\,x^9+1155a\,b^2d\,x^7+1320a\,b^2c\,x^6+1848a^2bd\,x^4+2310a^2bc\,x^3+1540a^3dx+3080c\,a^3\right)}{3080}$
orering	$\frac{x\left(280b^3d\,x^{10}+308b^3c\,x^9+1155a\,b^2d\,x^7+1320a\,b^2c\,x^6+1848a^2bd\,x^4+2310a^2bc\,x^3+1540a^3dx+3080c\,a^3\right)\left(db\,x^4+bc\,x^3+adx+\right.}{3080(dx+c)(b\,x^3+a)}$

input
```
int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+3/4*a^2*b*c*x^4+3/5*a^2*b*d*x^5+3/7*a*b^2*c*x^7+3/8*
a*b^2*d*x^8+1/10*b^3*c*x^10+1/11*b^3*d*x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

input
```
integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

output
```
1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/
5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx = a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bcx^4}{4}+\frac{3a^2bdx^5}{5}+\frac{3ab^2cx^7}{7}+\frac{3ab^2dx^8}{8}+\frac{b^3cx^{10}}{10}+\frac{b^3dx^{11}}{11}$$

input
```
integrate((b*x**3+a)**2*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + 3*a**2*b*c*x**4/4 + 3*a**2*b*d*x**5/5 + 3*a*b**
2*c*x**7/7 + 3*a*b**2*d*x**8/8 + b**3*c*x**10/10 + b**3*d*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{1}{11}\,b^3dx^{11}+\frac{1}{10}\,b^3cx^{10}+\frac{3}{8}\,ab^2dx^8+\frac{3}{7}\,ab^2cx^7+\frac{3}{5}\,a^2bdx^5+\frac{3}{4}\,a^2bcx^4+\frac{1}{2}\,a^3dx^2+a^3cx$$

input
```
integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

output
```
1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/
5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx = \frac{1}{11}b^3dx^{11} + \frac{1}{10}b^3cx^{10} + \frac{3}{8}ab^2dx^8 + \frac{3}{7}ab^2cx^7 + \frac{3}{5}a^2bdx^5 + \frac{3}{4}a^2bcx^4 + \frac{1}{2}a^3dx^2 + a^3cx$$

input
```
integrate((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
1/11*b^3*d*x^11 + 1/10*b^3*c*x^10 + 3/8*a*b^2*d*x^8 + 3/7*a*b^2*c*x^7 + 3/
5*a^2*b*d*x^5 + 3/4*a^2*b*c*x^4 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int (a+bx^3)^2 (ac+adx+bcx^3+bdx^4) \, dx = \frac{d\,a^3\,x^2}{2} + c\,a^3\,x + \frac{3\,d\,a^2\,b\,x^5}{5} + \frac{3\,c\,a^2\,b\,x^4}{4} + \frac{3\,d\,a\,b^2\,x^8}{8} + \frac{3\,c\,a\,b^2\,x^7}{7} + \frac{d\,b^3\,x^{11}}{11} + \frac{c\,b^3\,x^{10}}{10}$$

input
```
int((a + b*x^3)^2*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
(a^3*d*x^2)/2 + (b^3*c*x^10)/10 + (b^3*d*x^11)/11 + a^3*c*x + (3*a^2*b*c*x
^4)/4 + (3*a*b^2*c*x^7)/7 + (3*a^2*b*d*x^5)/5 + (3*a*b^2*d*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \left(a+bx^3\right)^2 \left(ac+adx+bcx^3+bdx^4\right)\,dx$$
$$=\frac{x(280b^3d\,x^{10}+308b^3c\,x^9+1155a\,b^2d\,x^7+1320a\,b^2c\,x^6+1848a^2bd\,x^4+2310a^2bc\,x^3+1540a^3dx+308}{3080}$$

input
```
int((b*x^3+a)^2*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

output
```
(x*(3080*a**3*c + 1540*a**3*d*x + 2310*a**2*b*c*x**3 + 1848*a**2*b*d*x**4
+ 1320*a*b**2*c*x**6 + 1155*a*b**2*d*x**7 + 308*b**3*c*x**9 + 280*b**3*d*x
**10))/3080
```

3.46 $\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 60

$$\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx = \frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8+\frac{a^2(c+dx)^2}{2d}$$

output
```
1/2*a*b*c*x^4+2/5*a*b*d*x^5+1/7*b^2*c*x^7+1/8*b^2*d*x^8+1/2*a^2*(d*x+c)^2/
d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx^3)(ac+adx+bcx^3+bdx^4)\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

input
```
Integrate[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7
+ (b^2*d*x^8)/8
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(a^2c+a^2dx+2abcx^3+2abdx^4+b^2cx^6+b^2dx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{2}abcx^4+\frac{2}{5}abdx^5+\frac{1}{7}b^2cx^7+\frac{1}{8}b^2dx^8$$

input
```
Int[(a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output

```
a^2*c*x + (a^2*d*x^2)/2 + (a*b*c*x^4)/2 + (2*a*b*d*x^5)/5 + (b^2*c*x^7)/7
+ (b^2*d*x^8)/8
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{1}{8}d\,b^{2}x^{8}+\frac{1}{7}b^{2}c\,x^{7}+\frac{2}{5}x^{5}adb+\frac{1}{2}abc\,x^{4}+\frac{1}{2}a^{2}d\,x^{2}+a^{2}cx$	51
norman	$\frac{1}{8}d\,b^{2}x^{8}+\frac{1}{7}b^{2}c\,x^{7}+\frac{2}{5}x^{5}adb+\frac{1}{2}abc\,x^{4}+\frac{1}{2}a^{2}d\,x^{2}+a^{2}cx$	51
risch	$\frac{1}{8}d\,b^{2}x^{8}+\frac{1}{7}b^{2}c\,x^{7}+\frac{2}{5}x^{5}adb+\frac{1}{2}abc\,x^{4}+\frac{1}{2}a^{2}d\,x^{2}+a^{2}cx$	51
parallelrisch	$\frac{1}{8}d\,b^{2}x^{8}+\frac{1}{7}b^{2}c\,x^{7}+\frac{2}{5}x^{5}adb+\frac{1}{2}abc\,x^{4}+\frac{1}{2}a^{2}d\,x^{2}+a^{2}cx$	51
gosper	$\frac{x\left(35b^{2}d\,x^{7}+40b^{2}c\,x^{6}+112abd\,x^{4}+140abc\,x^{3}+140a^{2}dx+280a^{2}c\right)}{280}$	52
orering	$\frac{x\left(35b^{2}d\,x^{7}+40b^{2}c\,x^{6}+112abd\,x^{4}+140abc\,x^{3}+140a^{2}dx+280a^{2}c\right)\left(db\,x^{4}+bc\,x^{3}+adx+ac\right)}{280(dx+c)(b\,x^{3}+a)}$	88

input
```
int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
1/8*d*b^2*x^8+1/7*b^2*c*x^7+2/5*x^5*a*d*b+1/2*a*b*c*x^4+1/2*a^2*d*x^2+a^2*
c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{1}{8}\,b^2dx^8+\frac{1}{7}\,b^2cx^7+\frac{2}{5}\,abdx^5+\frac{1}{2}\,abcx^4+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas")
```

output
```
1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*
x^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx = a^2cx+\frac{a^2dx^2}{2}+\frac{abcx^4}{2}+\frac{2abdx^5}{5}+\frac{b^2cx^7}{7}+\frac{b^2dx^8}{8}$$

input
```
integrate((b*x**3+a)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a*b*c*x**4/2 + 2*a*b*d*x**5/5 + b**2*c*x**7/7 +
 b**2*d*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx = \frac{1}{8}b^2dx^8+\frac{1}{7}b^2cx^7+\frac{2}{5}abdx^5 + \frac{1}{2}abcx^4+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima")
```

output
```
1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*
x^2 + a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^3) (ac+adx+bcx^3+bdx^4) \, dx = \frac{1}{8}b^2dx^8+\frac{1}{7}b^2cx^7+\frac{2}{5}abdx^5 + \frac{1}{2}abcx^4+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
1/8*b^2*d*x^8 + 1/7*b^2*c*x^7 + 2/5*a*b*d*x^5 + 1/2*a*b*c*x^4 + 1/2*a^2*d*
x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{2\,d\,a\,b\,x^5}{5}+\frac{c\,a\,b\,x^4}{2}+\frac{d\,b^2\,x^8}{8}+\frac{c\,b^2\,x^7}{7}$$

input
```
int((a + b*x^3)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*c*x^7)/7 + (b^2*d*x^8)/8 + a^2*c*x + (a*b*c*x^4)/2 +
(2*a*b*d*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \left(a+bx^3\right)\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{x(35b^2d\,x^7+40b^2c\,x^6+112abd\,x^4+140abc\,x^3+140a^2dx+280a^2c)}{280}$$

input
```
int((b*x^3+a)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

output
```
(x*(280*a**2*c + 140*a**2*d*x + 140*a*b*c*x**3 + 112*a*b*d*x**4 + 40*b**2*
c*x**6 + 35*b**2*d*x**7))/280
```

3.47 $\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 14

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} \, dx = \frac{(c+dx)^2}{2d}$$

output
```
1/2*(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3} \, dx = cx + \frac{dx^2}{2}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

output
```
c*x + (d*x^2)/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}dx$$
$$\Big\downarrow \text{2019}$$
$$\int (c+dx)dx$$
$$\Big\downarrow \text{17}$$
$$\frac{(c+dx)^2}{2d}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x]
```

output
```
(c + d*x)^2/(2*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$\frac{1}{2}d\,x^2+cx$	11
norman	$\frac{1}{2}d\,x^2+cx$	11
risch	$\frac{1}{2}d\,x^2+cx$	11
parallelrisch	$\frac{1}{2}d\,x^2+cx$	11
parts	$\frac{1}{2}d\,x^2+cx$	11
orering	$\frac{x(dx+2c)\left(db\,x^4+bc\,x^3+adx+ac\right)}{2(dx+c)\left(b\,x^3+a\right)}$	47

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/2*d*x^2 + c*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} \, dx = cx + \frac{dx^2}{2}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a),x)
```

output
```
c*x + d*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} \, dx = \frac{1}{2} \, dx^2 + cx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*d*x^2 + c*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac + adx + bcx^3 + bdx^4}{a + bx^3} \, dx = \frac{1}{2} \, dx^2 + cx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x, algorithm="giac")
```

output
```
1/2*d*x^2 + c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{d\,x^2}{2}+c\,x$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3),x)
```

output
```
c*x + (d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ac+adx+bcx^3+bdx^4}{a+bx^3}\,dx=\frac{x(dx+2c)}{2}$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a),x)
```

output
```
(x*(2*c + d*x))/2
```

3.48 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 161

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = -\frac{\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{2/3}} + \frac{\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{2/3}} - \frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
-1/3*(b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3
))*3^(1/2)/a^(2/3)/b^(2/3)+1/3*(b^(1/3)*c-a^(1/3)*d)*ln(a^(1/3)+b^(1/3)*x)
/a^(2/3)/b^(2/3)-1/6*(c-a^(1/3)*d/b^(1/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/a^(2/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$$
$$=\frac{-2\sqrt{3}\left(\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)\left(2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.\right.}{6a^{2/3}b^{2/3}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

output
```
(-2*Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] + (b^(1/3)*c - a^(1/3)*d)*(2*Log[a^(1/3) + b^(1/3)*x] - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*a^(2/3)*b^(2/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2019, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx$$
$$\Big\downarrow \; 2019$$
$$\int \frac{c+dx}{a+bx^3}dx$$
$$\Big\downarrow \; 2399$$
$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}$$

$$\Big\downarrow \ 16$$

$$\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{1}{2}\sqrt[3]{b}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{1}{2}\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x]
```

output
```
((c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) +
 (-((Sqrt[3]*(b^(1/3)*c + a^(1/3)*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sq
rt[3]])/b^(1/3)) - ((c - (a^(1/3)*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)*b^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^3 b+a\right)}\frac{\left(d\_R+c\right)\ln\left(x-\_R\right)}{\_R^2}}{3b}$
default	$c\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)+d\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{6b\left(\frac{a}{b}\right.}\right.$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/b*sum((_R*d+c)/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 1931, normalized size of antiderivative = 11.99

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/6*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*
d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 +
a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)
*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(
1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2)
+ (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d - 1/2*((1/2)^(1/3)*(I*sqrt(
3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3) - 2*
(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3
- a*d^3)/(a^2*b^2))^(1/3)))*a*b*c^2 + 2*a*c*d^2 + (b*c^3 + a*d^3)*x) + 1/1
2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3
)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d
^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)) + 3*sqrt(1/3)*sqrt(-(((1
/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^
2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)/(
a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a*b + 16*c*d)/(a*b)))*log(
-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*
d^3)/(a^2*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 +
a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2*b^2))^(1/3)))^2*a^2*b*d + 1/2*((1/
2)^(1/3)*(I*sqrt(3) + 1)*((b*c^3 + a*d^3)/(a^2*b^2) + (b*c^3 - a*d^3)/(a^2
*b^2))^(1/3) - 2*(1/2)^(2/3)*c*d*(-I*sqrt(3) + 1)/(a*b*((b*c^3 + a*d^3)...
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2}\,dx = \text{RootSum}\left(27t^3a^2b^2 + 9tabcd + ad^3 - bc^3, \left(t \mapsto t\log\left(x + \frac{9t^2a^2bd + 3tabc^2 + 2acd^2}{ad^3 + bc^3}\right)\right)\right)$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**2,x)
```

output
```
RootSum(27*_t**3*a**2*b**2 + 9*_t*a*b*c*d + a*d**3 - b*c**3, Lambda(_t, _t
*log(x + (9*_t**2*a**2*b*d + 3*_t*a*b*c**2 + 2*a*c*d**2)/(a*d**3 + b*c**3)
)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.84

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}}-c\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*(d*(a/b)^(1/3) + c)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/
b)^(1/3))/(b*(a/b)^(2/3)) + 1/6*(d*(a/b)^(1/3) - c)*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*(d*(a/b)^(1/3) - c)*log(x + (a/b)^(
1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^2}\,dx = -\frac{\sqrt{3}\left(bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) - 1/6*(b*c + (-a*b^2)^(1/3)*d)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) - 1/3*(d*(-a/b)^(1/3) + c)*(
-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2} dx = \sum_{k=1}^{3} \ln\left(b\left(c\,d + d^2\,x + \mathrm{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z + a\,d^3 - b\,c^3, z, k\right)^2 a\,b\,9 + \mathrm{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z + a\,d^3 - b\,c^3, z, k\right)\,b\,c\,x\,3\right)\right)\mathrm{root}\left(27\,a^2\,b^2\,z^3 + 9\,a\,b\,c\,d\,z + a\,d^3 - b\,c^3, z, k\right)$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^2,x)
```

output
```
symsum(log(b*(c*d + d^2*x + 9*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 -
b*c^3, z, k)^2*a*b + 3*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3,
z, k)*b*c*x))*root(27*a^2*b^2*z^3 + 9*a*b*c*d*z + a*d^3 - b*c^3, z, k), k,
 1, 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^2} dx = \frac{-2b^{\frac{1}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)c - 2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)ad - b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)c + 2b^{\frac{1}{3}}a^{\frac{2}{3}}\log\left(a\right.}{6b^{\frac{2}{3}}a^{\frac{4}{3}}}$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^2,x)
```

output
```
( - 2*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*c - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a*d - b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2
)*c + 2*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*c + log(a**(2/3) - b*
*(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*d - 2*log(a**(1/3) + b**(1/3)*x)*a*d)
/(6*b**(2/3)*a**(1/3)*a)
```

3.49 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 189

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = \frac{x(c+dx)}{3a(a+bx^3)} - \frac{\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{2/3}} + \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{2/3}} - \frac{\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{2/3}}$$

output
```
1/3*x*(d*x+c)/a/(b*x^3+a)-1/9*(2*b^(1/3)*c+a^(1/3)*d)*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(2/3)+1/9*(2*b^(1/3)*c-a^(
1/3)*d)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(2/3)-1/18*(2*b^(1/3)*c-a^(1/3)*d)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$$

$$=\frac{\frac{6ax(c+dx)}{a+bx^3}-\frac{2\sqrt{3}\sqrt[3]{a}\left(2\sqrt[3]{b}c+\sqrt[3]{a}d\right)\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{2\left(2\sqrt[3]{a}\sqrt[3]{b}c-a^{2/3}d\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{\left(-2\sqrt[3]{a}\sqrt[3]{b}c+a^{2/3}d\right)\mathrm{l}}{}}{18a^2}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

output
```
((6*a*x*(c + d*x))/(a + b*x^3) - (2*Sqrt[3]*a^(1/3)*(2*b^(1/3)*c + a^(1/3)
*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(2*a^(1/3)*b
^(1/3)*c - a^(2/3)*d)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-2*a^(1/3)*b^(
1/3)*c + a^(2/3)*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3
))/(18*a^2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.367$, Rules used = {2019, 2394, 25, 2399, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{c+dx}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{3a\left(a+bx^3\right)}-\frac{\int -\frac{2c+dx}{bx^3+a}dx}{3a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2c+dx}{bx^3+a}dx}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 2399$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\int \frac{\sqrt[3]{a}\left(4\sqrt[3]{b}c+\sqrt[3]{a}d\right)-\sqrt[3]{b}\left(2\sqrt[3]{b}c-\sqrt[3]{a}d\right)x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{\frac{1}{2}\sqrt[3]{b}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{-\frac{1}{2}\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)\left(\sqrt[3]{a}d+2\sqrt[3]{b}c\right)}{\sqrt[3]{b}}}{3a^{2/3}\sqrt[3]{b}}+\frac{\left(2c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3a}+\frac{x(c+dx)}{3a\left(a+bx^3\right)}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x]
```

output
```
(x*(c + d*x))/(3*a*(a + b*x^3)) + (((2*c - (a^(1/3)*d)/b^(1/3))*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*(2*b^(1/3)*c + a^(1/3)*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - ((2*c - (a^(1/3)
*d)/b^(1/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/2)/(3*a^(2/3)
*b^(1/3)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result
risch	$\frac{\frac{d x^2}{3a}+\frac{cx}{3a}}{b x^3+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(\_Z^3 b+a\right)}\frac{\left(d\_R+2c\right)\ln\left(x-\_R\right)}{\_R^2}}{9ba}$
default	$c\left(\frac{x}{3a(b x^3+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}\right)+d\left(\frac{x^2}{3a(b x^3+a)}+\frac{-\frac{\ln(x+}{3b(}}{}\right.$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3*d/a*x^2+1/3*c/a*x)/(b*x^3+a)+1/9/b/a*sum((_R*d+2*c)/_R^2*ln(x-_R),_R=
RootOf(_Z^3*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 2088, normalized size of antiderivative = 11.05

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^3} \, dx = \text{Too large to display}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/36*(12*d*x^2 - 2*(a*b*x^3 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3
+ a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*
d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/
(a^5*b^2))^(1/3)))*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)
/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqr
t(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2
))^(1/3)))^2*a^4*b*d - 2*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(
a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(
3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))
^(1/3)))*a^2*b*c^2 + 4*a*c*d^2 + (8*b*c^3 + a*d^3)*x) + 12*c*x + ((a*b*x^3
 + a^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c
^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((
8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3))) + 3*sqrt
(1/3)*(a*b*x^3 + a^2)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*((8*b*c^3 + a*d^
3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4*(1/2)^(2/3)*c*d*(I*s
qrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b
^2))^(1/3)))^2*a^3*b + 32*c*d)/(a^3*b)))*log(-1/4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*b*c^3 - a*d^3)/(a^5*b^2))^(1/3) + 4
*(1/2)^(2/3)*c*d*(I*sqrt(3) - 1)/(a^3*b*((8*b*c^3 + a*d^3)/(a^5*b^2) + (8*
b*c^3 - a*d^3)/(a^5*b^2))^(1/3)))^2*a^4*b*d + 2*((1/2)^(1/3)*(I*sqrt(3)...
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.56

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^3} \, dx$$
$$= \operatorname{RootSum}\left(729t^3a^5b^2 + 54ta^2bcd + ad^3 - 8bc^3, \left(t \mapsto t\log\left(x + \frac{81t^2a^4bd + 36ta^2bc^2 + 4acd^2}{ad^3 + 8bc^3}\right)\right)\right)$$
$$+ \frac{cx + dx^2}{3a^2 + 3abx^3}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**3,x)
```

output
```
RootSum(729*_t**3*a**5*b**2 + 54*_t*a**2*b*c*d + a*d**3 - 8*b*c**3, Lambda
(_t, _t*log(x + (81*_t**2*a**4*b*d + 36*_t*a**2*b*c**2 + 4*a*c*d**2)/(a*d*
*3 + 8*b*c**3)))) + (c*x + d*x**2)/(3*a**2 + 3*a*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^3} \, dx = \frac{dx^2 + cx}{3\,(abx^3 + a^2)}$$
$$+ \frac{\sqrt{3}\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}} + 2\,c\right)\arctan\left(\frac{\sqrt{3}\left(2\,x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$+ \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2\,c\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$
$$- \frac{\left(d\left(\frac{a}{b}\right)^{\frac{1}{3}} - 2\,c\right)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/3*(d*x^2 + c*x)/(a*b*x^3 + a^2) + 1/9*sqrt(3)*(d*(a/b)^(1/3) + 2*c)*arct
an(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) + 1/18*(
d*(a/b)^(1/3) - 2*c)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*(a/b)^(2/
3)) - 1/9*(d*(a/b)^(1/3) - 2*c)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^3}\,dx = -\frac{\sqrt{3}\left(2\,bc-\left(-ab^2\right)^{\frac{1}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(2\,bc+\left(-ab^2\right)^{\frac{1}{3}}d\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a} - \frac{\left(d\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,c\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{dx^2+cx}{3\,(bx^3+a)a}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/9*sqrt(3)*(2*b*c - (-a*b^2)^(1/3)*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + (-a*b^2)^(1/3)*d)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(d*(-a/b)
^(1/3) + 2*c)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*(d*x^2 + c
*x)/((b*x^3 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.89

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^3}\,dx$$
$$= \left(\sum_{k=1}^{3} \ln\left(\frac{b\left(2\,c\,d + d^2\,x + \mathrm{root}(729\,a^5\,b^2\,z^3 + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k)^2\,a^3\,b\,81 + \mathrm{root}(729\,a^5\,b^2\,z\right.}{a^2\,9}\right.\right.$$
$$\left.\left. + 54\,a^2\,b\,c\,d\,z - 8\,b\,c^3 + a\,d^3, z, k\right)\right) + \frac{\frac{d\,x^2}{3\,a} + \frac{c\,x}{3\,a}}{b\,x^3 + a}$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^3,x)
```

output
```
symsum(log((b*(2*c*d + d^2*x + 81*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z -
8*b*c^3 + a*d^3, z, k)^2*a^3*b + 18*root(729*a^5*b^2*z^3 + 54*a^2*b*c*d*z
- 8*b*c^3 + a*d^3, z, k)*a*b*c*x))/(9*a^2))*root(729*a^5*b^2*z^3 + 54*a^2*
b*c*d*z - 8*b*c^3 + a*d^3, z, k), k, 1, 3) + ((d*x^2)/(3*a) + (c*x)/(3*a))
/(a + b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.86

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^3}\,dx$$
$$= \frac{-4b^{\frac{1}{3}}a^{\frac{5}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)c - 4b^{\frac{4}{3}}a^{\frac{2}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)c\,x^3 - 2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2d - 2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}\cdots}{a\cdots}\right.}{\cdots}$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^3,x)
```

output
```
( - 4*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*s
qrt(3)))*a*c - 4*b**(1/3)*a**(2/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/
(a**(1/3)*sqrt(3)))*b*c*x**3 - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a
**(1/3)*sqrt(3)))*a**2*d - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1
/3)*sqrt(3)))*a*b*d*x**3 - 2*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)*a**
(1/3)*x + b**(2/3)*x**2)*a*c - 2*b**(1/3)*a**(2/3)*log(a**(2/3) - b**(1/3)
*a**(1/3)*x + b**(2/3)*x**2)*b*c*x**3 + 4*b**(1/3)*a**(2/3)*log(a**(1/3) +
 b**(1/3)*x)*a*c + 4*b**(1/3)*a**(2/3)*log(a**(1/3) + b**(1/3)*x)*b*c*x**3
 + 6*b**(2/3)*a**(1/3)*a*c*x + 6*b**(2/3)*a**(1/3)*a*d*x**2 + log(a**(2/3)
 - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*d + log(a**(2/3) - b**(1/3)*a
**(1/3)*x + b**(2/3)*x**2)*a*b*d*x**3 - 2*log(a**(1/3) + b**(1/3)*x)*a**2*
d - 2*log(a**(1/3) + b**(1/3)*x)*a*b*d*x**3)/(18*b**(2/3)*a**(1/3)*a**2*(a
 + b*x**3))
```

3.50 $\int \frac{2x^2+x^4}{1+x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \frac{x^2}{2}+\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(1+x)+\frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
1/2*x^2+1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+ln(1+x)+1/2*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \frac{1}{6}\left(3x^2-2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-\log\left(1-x+x^2\right)+4\log\left(1+x^3\right)\right)$$

input
```
Integrate[(2*x^2 + x^4)/(1 + x^3),x]
```

output
```
(3*x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Log[1 + x] - Log[1 - x +
 x^2] + 4*Log[1 + x^3])/6
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2027, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+2x^2}{x^3+1}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2(x^2+2)}{x^3+1}dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{(2x-1)x}{x^3+1}+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^2}{2}+\frac{1}{2}\log\left(x^2-x+1\right)+\log(x+1)$$

input
```
Int[(2*x^2 + x^4)/(1 + x^3),x]
```

output
```
x^2/2 + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 + x] + Log[1 - x + x^2]/
2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{x^{2}}{2}+\frac{\ln \left(x^{2}-x+1\right)}{2}-\frac{\sqrt{3}\, \arctan \left(\frac{(2 x-1) \sqrt{3}}{3}\right)}{3}+\ln \left(x+1\right)$	38
risch	$\frac{x^{2}}{2}-\frac{\sqrt{3}\, \arctan \left(\frac{(2 x-1) \sqrt{3}}{3}\right)}{3}+\frac{\ln \left(4 x^{2}-4 x+4\right)}{2}+\ln \left(x+1\right)$	40
meijerg	$\frac{x^{2}}{2}-\frac{x^{2}\left(-\frac{\ln \left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{\left(x^{3}\right)^{\frac{2}{3}}}+\frac{\ln \left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2\left(x^{3}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\, \arctan \left(\frac{\sqrt{3}\,\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{\left(x^{3}\right)^{\frac{2}{3}}}\right)}{3}+\frac{2 \ln \left(x^{3}+1\right)}{3}$	89

input
```
int((x^4+2*x^2)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{1}{2}\,x^{2}-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^{2}-x+1\right)+\log\left(x+1\right)$$

input
```
integrate((x^4+2*x^2)/(x^3+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
 + log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{x^2}{2}+\log{(x+1)}+\frac{\log{(x^2-x+1)}}{2}-\frac{\sqrt{3}\operatorname{atan}{\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}}{3}$$

input
```
integrate((x**4+2*x**2)/(x**3+1),x)
```

output
```
x**2/2 + log(x + 1) + log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(x+1\right)$$

input
```
integrate((x^4+2*x^2)/(x^3+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
 + log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{2x^2+x^4}{1+x^3}\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(|x+1|\right)$$

input
```
integrate((x^4+2*x^2)/(x^3+1),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = \ln(x+1) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{x^2}{2}$$

input
```
int((2*x^2 + x^4)/(x^3 + 1),x)
```

output
```
log(x + 1) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 + 1/2) - log(x
+ (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/6 - 1/2) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{2x^2+x^4}{1+x^3}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3}+\frac{\log(x^2-x+1)}{2}+\log(x+1)+\frac{x^2}{2}$$

input
```
int((x^4+2*x^2)/(x^3+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 3*log(x**2 - x + 1) + 6*log(x + 1)
 + 3*x**2)/6
```

3.51 $\int \frac{2x^2+x^4}{1-x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{2x^2+x^4}{1-x^3} \, dx = -\frac{x^2}{2} - \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \log(1-x) - \frac{1}{2}\log\left(1+x+x^2\right)$$

output
```
-1/2*x^2-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-ln(1-x)-1/2*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \frac{2x^2+x^4}{1-x^3} \, dx = \frac{1}{6}\left(-3x^2 - 2\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\log(1-x) + \log\left(1+x+x^2\right) - 4\log\left(1-x^3\right)\right)$$

input
```
Integrate[(2*x^2 + x^4)/(1 - x^3),x]
```

output
```
(-3*x^2 - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + Log[1 + x + 
 x^2] - 4*Log[1 - x^3])/6
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2027, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+2x^2}{1-x^3}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2(x^2+2)}{1-x^3}dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{x(2x+1)}{1-x^3}-x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+x+1\right)-\log(1-x)$$

input
```
Int[(2*x^2 + x^4)/(1 - x^3),x]
```

output
```
-1/2*x^2 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 - x] - Log[1 + x + x^
2]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{x^{2}}{2}-\ln\left(x-1\right)-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}-\frac{\ln\left(x^{2}+x+1\right)}{2}$	36
default	$-\frac{x^{2}}{2}-\frac{\ln\left(x^{2}+x+1\right)}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{3}-\ln\left(x-1\right)$	38
meijerg	$\frac{(-1)^{\frac{1}{3}}\left(\frac{3x^{2}(-1)^{\frac{2}{3}}}{2}+\frac{x^{2}(-1)^{\frac{2}{3}}\left(\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}\right)-\frac{\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2+\left(x^{3}\right)^{\frac{1}{3}}}\right)\right)}{\left(x^{3}\right)^{\frac{2}{3}}}\right)}{3}-\frac{2\ln\left(-x^{3}+1\right)}{3}$	90

input
```
int((x^4+2*x^2)/(-x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-ln(x-1)-1/3*3^(1/2)*arctan(2/3*(x+1/2)*3^(1/2))-1/2*ln(x^2+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{2x^2+x^4}{1-x^3}\,dx=-\frac{1}{2}\,x^{2}-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{1}{2}\,\log\left(x^{2}+x+1\right)-\log\left(x-1\right)$$

input
```
integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="fricas")
```

output
```
-1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1
) - log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{x^2}{2} - \log(x-1) - \frac{\log(x^2+x+1)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**4+2*x**2)/(-x**3+1),x)
```

output
```
-x**2/2 - log(x - 1) - log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) - \log(x-1)$$

input
```
integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1
) - log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{2}\,\log\left(x^2+x+1\right) - \log\left(|x-1|\right)$$

input
```
integrate((x^4+2*x^2)/(-x^3+1),x, algorithm="giac")
```

output
```
-1/2*x^2 - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/2*log(x^2 + x + 1
) - log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\ln(x-1)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\frac{x^2}{2}$$

input
```
int(-(2*x^2 + x^4)/(x^3 - 1),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/2) - log(x - 1) - log(x
+ (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/2) - x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{2x^2+x^4}{1-x^3}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{3}-\frac{\log(x^2+x+1)}{2}-\log(x-1)-\frac{x^2}{2}$$

input
```
int((x^4+2*x^2)/(-x^3+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x + 1)/sqrt(3)) - 3*log(x**2 + x + 1) - 6*log(x - 1)
 - 3*x**2)/6
```

3.52 $\int (a+bx^3)^{3/2} (ac+adx+bcx^3+bdx^4) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 585

$$\int (a+bx^3)^{3/2} (ac+adx+bcx^3+bdx^4) \, dx = \frac{810a^3d\sqrt{a+bx^3}}{1729b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{54a^2(1729cx+935dx^2)\sqrt{a+bx^3}}{323323} + \frac{30a(247cx+187dx^2)(a+bx^3)^{3/2}}{46189} + \frac{2}{323}(19cx+17dx^2)(a+bx^3)^{5/2} - \frac{405\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\right.\right.}{1729b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
810/1729*a^3*d*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+54/
323323*a^2*(935*d*x^2+1729*c*x)*(b*x^3+a)^(1/2)+30/46189*a*(187*d*x^2+247*
c*x)*(b*x^3+a)^(3/2)+2/323*(17*d*x^2+19*c*x)*(b*x^3+a)^(5/2)-405/1729*3^(1
/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(10/3)*d*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Ellip
ticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(
1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+54/323323*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*
a^3*(1729*b^(1/3)*c-935*(1-3^(1/2))*a^(1/3)*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)
*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)
,I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+
b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.13

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) \, dx = \frac{a^2x\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{3},\frac{5}{3},-\right.\right.}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
(a^2*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-5/2, 1/3, 4/3, -((b*x^3)/a)
] + d*x*Hypergeometric2F1[-5/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x
^3)/a])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2391, 2392, 27, 2392, 27, 2392, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right) \, dx$$

$$\Big\downarrow \text{2391}$$

$$\int \left(a+bx^3\right)^{5/2} (c+dx)dx$$

$$\Big\downarrow \text{2392}$$

$$\frac{15}{2}a\int \frac{2}{323}(19c+17dx)\left(bx^3+a\right)^{3/2}dx+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{15}{323}a\int (19c+17dx)\left(bx^3+a\right)^{3/2}dx+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{2392}$$

$$\frac{15}{323}a\left(\frac{9}{2}a\int \frac{2}{143}(247c+187dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(247cx+187dx^2\right)\right)+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{15}{323}a\left(\frac{9}{143}a\int (247c+187dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(247cx+187dx^2\right)\right)+\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{2392}$$

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{2}a\int \frac{2(1729c+935dx)}{35\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(1729cx+935dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(247cx+187d\right.\right.$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\int\frac{1729c+935dx}{\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(1729cx+935dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(247cx+187dx^2\right.\right.$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$\downarrow$ 2417

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\left(\left(1729c-\frac{935\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{bx^3+a}}dx+\frac{935d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{35}\sqrt{a+b}\right.\right.$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$\downarrow$ 759

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{935d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(1729c-\frac{935(}{}\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{}\right.}{\left(\left(1+\sqrt{3}\right)\right.}}}\right.\right.\right.$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

$\downarrow$ 2416

$$\frac{15}{323}a\left(\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(1729c-\frac{935\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.\right.$$
$$\frac{2}{323}\left(a+bx^3\right)^{5/2}\left(19cx+17dx^2\right)$$

input
```
Int[(a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
(2*(19*c*x + 17*d*x^2)*(a + b*x^3)^(5/2))/323 + (15*a*((2*(247*c*x + 187*d
*x^2)*(a + b*x^3)^(3/2))/143 + (9*a*((2*(1729*c*x + 935*d*x^2)*Sqrt[a + b*
x^3])/35 + (3*a*((935*d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(1729*c - (935*(1 - Sqrt[3])*a^(1/3)*
d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/35))/143))/323
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2391
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.34

method	result	size
risch	Expression too large to display	786
elliptic	Expression too large to display	830
default	Expression too large to display	1618

input
```
int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
2/323323*x*(17017*b^2*d*x^7+19019*b^2*c*x^6+53669*a*b*d*x^4+63973*a*b*c*x^
3+61897*a^2*d*x+91637*a^2*c)*(b*x^3+a)^(1/2)+81/323323*a^3*(-3458/3*I*c*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1870/3
*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.20

$$\int (a+bx^3)^{3/2} (ac+adx+bcx^3+bdx^4)\, dx = \frac{2\left(140049\, a^3\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-75735\, a^3\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weie}\right.\right.}{}$$

input
```
integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas
")
```

output
```
2/323323*(140049*a^3*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 75735*a
^3*sqrt(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + (17017*b^3*d*x^8 + 19019*b^3*c*x^7 + 53669*a*b^2*d*x^5 + 63973*a*b^2*c*
x^4 + 61897*a^2*b*d*x^2 + 91637*a^2*b*c*x)*sqrt(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.45

$$\int \left(a+bx^3\right)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{a^{\frac{5}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{5}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{2a^{\frac{3}{2}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{3}{2}}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{\sqrt{a}b^2cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{\sqrt{a}b^2dx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**(5/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(5/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 2*a**(3/2)*b*c*x**4*gamma(4/3)*
hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a*
*(3/2)*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(8/3)) + sqrt(a)*b**2*c*x**7*gamma(7/3)*hyper((-1/2, 7/3), (
10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + sqrt(a)*b**2*d*x**8*ga
mma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11
/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\, dx = \int \left(bdx^4+bcx^3+adx+ac\right)\left(bx^3+a\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\, dx = \int \left(bdx^4+bcx^3+adx+ac\right)\left(bx^3+a\right)^{\frac{3}{2}}\, dx$$

input
```
integrate((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^{3/2} \left(ac+adx+bcx^3+bdx^4\right)\, dx = \int \left(b\,x^3+a\right)^{3/2}\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right)\, dx$$

input
```
int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
int((a + b*x^3)^(3/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

Reduce [F]

$$\int (a+bx^3)^{3/2}\left(ac+adx+bcx^3+bdx^4\right)dx = \frac{106\sqrt{b\,x^3+a}\,a^2cx}{187} + \frac{662\sqrt{b\,x^3+a}\,a^2d\,x^2}{1729} + \frac{74\sqrt{b\,x^3+a}\,abc\,x^4}{187} + \frac{82\sqrt{b\,x^3+a}\,abd\,x^5}{247} + \frac{2\sqrt{b\,x^3+a}\,b^2c\,x^7}{17} + \frac{2\sqrt{b\,x^3+a}\,b^2d\,x^8}{19} + \frac{81\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^3c}{187} + \frac{405\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^3d}{1729}$$

input
```
int((b*x^3+a)^(3/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

output
```
(183274*sqrt(a + b*x**3)*a**2*c*x + 123794*sqrt(a + b*x**3)*a**2*d*x**2 +
127946*sqrt(a + b*x**3)*a*b*c*x**4 + 107338*sqrt(a + b*x**3)*a*b*d*x**5 +
38038*sqrt(a + b*x**3)*b**2*c*x**7 + 34034*sqrt(a + b*x**3)*b**2*d*x**8 +
140049*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a**3*c + 75735*int((sqrt(a + b
*x**3)*x)/(a + b*x**3),x)*a**3*d)/323323
```

3.53 $\int \sqrt{a+bx^3}(ac+adx+bcx^3+bdx^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 556

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{54a^2d\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{18a(91cx+55dx^2)\sqrt{a+bx^3}}{5005}+\frac{2}{143}\left(13cx+11dx^2\right)\left(a+bx^3\right)^{3/2}-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(91\sqrt[3]{b}c-55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\operatorname{arcsi}\right.}{5005b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
54/91*a^2*d*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+18/500
5*a*(55*d*x^2+91*c*x)*(b*x^3+a)^(1/2)+2/143*(11*d*x^2+13*c*x)*(b*x^3+a)^(3
/2)-27/91*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(7/3)*d*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2
)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a
^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+18/5005*3^(3/4)*(1/2*6^(1/2)+1/
2*2^(1/2))*a^2*(91*b^(1/3)*c-55*(1-3^(1/2))*a^(1/3)*d)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.14

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$$
$$=\frac{ax\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
(a*x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3
)/a])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2391, 2392, 27, 2392, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$$

$$\Big\downarrow \text{2391}$$

$$\int \left(a+bx^3\right)^{3/2}(c+dx)dx$$

$$\Big\downarrow \text{2392}$$

$$\frac{9}{2}a\int \frac{2}{143}(13c+11dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{9}{143}a\int (13c+11dx)\sqrt{bx^3+a}dx+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \text{2392}$$

$$\frac{9}{143}a\left(\frac{3}{2}a\int \frac{2(91c+55dx)}{35\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(91cx+55dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{9}{143}a\left(\frac{3}{35}a\int \frac{91c+55dx}{\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(91cx+55dx^2\right)\right)+\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$$\Big\downarrow \text{2417}$$

$$\frac{9}{143}a\left(\frac{3}{35}a\left(\left(91c-\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{55d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{35}\sqrt{a+bx^3}(91cx+5\right.$$
$$\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$\Big\downarrow$ 759

$$\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{55d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91c-\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right.}}}\right.\right.$$

$$\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

$\Big\downarrow$ 2416

$$\frac{9}{143}a\left(\frac{3}{35}a\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(91c-\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+(}{\sqrt[3]{b}x+(}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.$$

$$\frac{2}{143}\left(a+bx^3\right)^{3/2}\left(13cx+11dx^2\right)$$

input
```
Int[Sqrt[a + b*x^3]*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x]
```

output
```
(2*(13*c*x + 11*d*x^2)*(a + b*x^3)^(3/2))/143 + (9*a*((2*(91*c*x + 55*d*x^
2)*Sqrt[a + b*x^3])/35 + (3*a*((55*d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(91*c - (55*(1 - Sqrt[3]
)*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/35))/143
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2391
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[PolynomialQuoti
ent[Pq, a + b*x^n, x]*(a + b*x^n)^(p + 1), x] /; FreeQ[{a, b, p}, x] && Pol
yQ[Pq, x] && IGtQ[n, 0] && GeQ[Expon[Pq, x], n] && EqQ[PolynomialRemainder[
Pq, a + b*x^n, x], 0]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.37

method	result	size
risch	Expression too large to display	762
elliptic	Expression too large to display	788
default	Expression too large to display	1546

input
```
int((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
2/5005*x*(385*b*d*x^4+455*b*c*x^3+880*a*d*x+1274*a*c)*(b*x^3+a)^(1/2)+27/5
005*a^2*(-182/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*
b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))-110/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/
b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.17

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx$$
$$=\frac{2\left(2457\,a^2\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-1485\,a^2\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInvers}\right.\right.}{5005\,b}$$

input
```
integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="fricas
")
```

output
```
2/5005*(2457*a^2*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 1485*a^2*sq
rt(b)*d*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (3
85*b^2*d*x^5 + 455*b^2*c*x^4 + 880*a*b*d*x^2 + 1274*a*b*c*x)*sqrt(b*x^3 +
a))/b
```

Sympy [A] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.31

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{3}{2}}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)*(b*d*x**4+b*c*x**3+a*d*x+a*c),x)
```

output
```
a**(3/2)*c*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(3/2)*d*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + sqrt(a)*b*c*x**4*gamma(4/3)*hyp
er((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + sqrt(a)
*b*d*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(8/3))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \left(bdx^4+bcx^3+adx+ac\right)\sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)*sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx=\int \sqrt{b\,x^3+a}\,\left(b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c\right)\,dx$$

input
```
int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4),x)
```

output
```
int((a + b*x^3)^(1/2)*(a*c + a*d*x + b*c*x^3 + b*d*x^4), x)
```

Reduce [F]

$$\int \sqrt{a+bx^3}\left(ac+adx+bcx^3+bdx^4\right)\,dx = \frac{28\sqrt{b\,x^3+a}\,acx}{55}+\frac{32\sqrt{b\,x^3+a}\,ad\,x^2}{91}+\frac{2\sqrt{b\,x^3+a}\,bc\,x^4}{11}+\frac{2\sqrt{b\,x^3+a}\,bd\,x^5}{13}+\frac{27\left(\int\frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2c}{55}+\frac{27\left(\int\frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2d}{91}$$

input
```
int((b*x^3+a)^(1/2)*(b*d*x^4+b*c*x^3+a*d*x+a*c),x)
```

output
```
(2548*sqrt(a + b*x**3)*a*c*x + 1760*sqrt(a + b*x**3)*a*d*x**2 + 910*sqrt(a
 + b*x**3)*b*c*x**4 + 770*sqrt(a + b*x**3)*b*d*x**5 + 2457*int(sqrt(a + b*
x**3)/(a + b*x**3),x)*a**2*c + 1485*int((sqrt(a + b*x**3)*x)/(a + b*x**3),
x)*a**2*d)/5005
```

3.54 $\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 525

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx=\frac{6ad\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{35}\left(7cx+5dx^2\right)\sqrt{a+bx^3}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(7\sqrt[3]{b}c-5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\left(\ldots\right.\right.\right.}{35b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
6/7*a*d*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+2/35*(5*d*
x^2+7*c*x)*(b*x^3+a)^(1/2)-3/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*d
*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/35*3^(3/
4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*(7*b^(1/3)*c-5*(1-3^(1/2))*a^(1/3)*d)*(a^(1
/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.14

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[a + b*x^3]*(2*c*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)] +
d*x*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[1 + (b*x^3)/
a])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2019, 2392, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \sqrt{a+bx^3}(c+dx)dx$$

$$\Big\downarrow \text{2392}$$

$$\frac{3}{2}a\int \frac{2(7c+5dx)}{35\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{35}a\int \frac{7c+5dx}{\sqrt{bx^3+a}}dx+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \text{2417}$$

$$\frac{3}{35}a\left(\left(7c-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{5d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \text{759}$$

$$\frac{3}{35}a\left(\frac{5d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7c-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx}}\right.$$

$$\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

$$\Big\downarrow \text{2416}$$

$$\frac{3}{35}a\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(7c-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{2}{35}\sqrt{a+bx^3}\left(7cx+5dx^2\right)$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/Sqrt[a + b*x^3],x]
```

output
```
(2*(7*c*x + 5*d*x^2)*Sqrt[a + b*x^3])/35 + (3*a*((5*d*((2*Sqrt[a + b*x^3])
/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]
]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(7*c -
(5*(1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/35
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.42

method	result
risch	$\frac{2x(5dx+7c)\sqrt{b\,x^3+a}}{35}+\frac{3a\left(-\frac{14ic\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\right.}{}\right.}{}}}{}\right.}{2iac\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\ldots}}$
elliptic	$\frac{2d\,x^2\sqrt{b\,x^3+a}}{7}+\frac{2cx\sqrt{b\,x^3+a}}{5}-$
default	Expression too large to display

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*x*(5*d*x+7*c)*(b*x^3+a)^(1/2)+3/35*a*(-14/3*I*c*3^(1/2)/b*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-10/3*I*d*3^(1/2)/b*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.13

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}}\, dx$$
$$= \frac{2 \left(21\, a\sqrt{b}c\text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) - 15\, a\sqrt{b}d\text{weierstrassZeta}\left(0, -\frac{4\,a}{b}, \text{weierstrassPInverse}\left(0, -\right.\right.\right.}{35\, b}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="fricas
")
```

output
```
2/35*(21*a*sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - 15*a*sqrt(b)*d*we
ierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b*d*x^2 +
 7*b*c*x)*sqrt(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}}\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + sqrt(a)*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((1/2, 4/3
), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + b*d*x**5*gam
ma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gam
ma(8/3))
```

Maxima [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}} \, dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{\sqrt{bx^3 + a}} \, dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}} \, dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{\sqrt{b\,x^3 + a}} \, dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{\sqrt{a + bx^3}}\,dx = \frac{2\sqrt{b\,x^3 + a}\,cx}{5} + \frac{2\sqrt{b\,x^3 + a}\,d\,x^2}{7} + \frac{3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)ac}{5} + \frac{3\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)ad}{7}$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(1/2),x)
```

output
```
(14*sqrt(a + b*x**3)*c*x + 10*sqrt(a + b*x**3)*d*x**2 + 21*int(sqrt(a + b*
x**3)/(a + b*x**3),x)*a*c + 15*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a*
d)/35
```

3.55 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 490

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*d*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*a^(1/3)*d*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^
(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b
^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)/(b*x^3+a)^(1/2)+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*c-(1-3^(1/2))*a^
(1/3)*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2}\right.}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3]
)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2019, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{c+dx}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \; 2417$$

$$\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \; 759$$

$$\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2416$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{d\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x]
```

output
```
(d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3
^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sq
rt[2 + Sqrt[3]]*(c - ((1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.47

method	result
elliptic	$-\frac{2ic\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^{3}+a}}$
default	Expression too large to display

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{wei}\right.\right.}{b}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="fricas
")
```

output
```
2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.16

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx=\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(3/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{3/2}} \, dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{3/2}} \,dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{3/2}} \, dx = \left(\int \frac{\sqrt{b\,x^3 + a}}{b\,x^3 + a}dx\right) c + \left(\int \frac{\sqrt{b\,x^3 + a}\,x}{b\,x^3 + a}dx\right) d$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a + b*x**3),x)*c + int((sqrt(a + b*x**3)*x)/(a + b*x
**3),x)*d
```

3.56 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 522

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2x(c+dx)}{3a\sqrt{a+bx^3}} - \frac{2d\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{\sqrt{2-\sqrt{3}}\,d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{\cdots}}\right.\right.}{3\sqrt[4]{3}ab^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x*(d*x+c)/a/(b*x^3+a)^(1/2)-2/3*d*(b*x^3+a)^(1/2)/a/b^(2/3)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x)+1/3*(1/2*6^(1/2)-1/2*2^(1/2))*d*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2
)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(2/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/9*(1/2*6^(1/2
)+1/2*2^(1/2))*(b^(1/3)*c+(1-3^(1/2))*a^(1/3)*d)*(a^(1/3)+b^(1/3)*x)*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.18

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{x\left(4c+2c\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+3dx\sqrt{1+\frac{bx^3}{a}}\,\text{H}\right.}{6a\sqrt{a+bx^3}}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

output
```
(x*(4*c + 2*c*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^
3)/a)] + 3*d*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x
^3)/a)]))/(6*a*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2019, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx$$

$\Big\downarrow$ 2019

$$\int \frac{c+dx}{(a+bx^3)^{3/2}}dx$$

$\Big\downarrow$ 2394

$$\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{c-dx}{2\sqrt{bx^3+a}}dx}{3a}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{c-dx}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

$\Big\downarrow$ 2417

$$\frac{\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

$\Big\downarrow$ 759

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{d\int \frac{\sqrt[3]{b}x+(1}{\sqrt{bx}}}{\sqrt[3]{}}}{3a}$$

$$\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

$\Big\downarrow$ 2416

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{d\left(\frac{2\cdots}{\sqrt[3]{b}\left(\left(1+\cdots\right.\right.}\cdots\right.}{\cdots}}{3a\cdots}$$

$$\frac{2x(c+dx)}{3a\sqrt{a+bx^3}}$$

input

```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x]
```

output

```
(2*x*(c + d*x))/(3*a*Sqrt[a + b*x^3]) + (-((d*((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(c + ((1 - Sqr
t[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.47

method	result
elliptic	$-\frac{2b\left(-\frac{d\,x^{2}}{3ab}-\frac{cx}{3ab}\right)}{\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2ic\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{\left(\right.}}}{9a}$
default	Expression too large to display

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/3*d/a/b*x^2-1/3*c/a/b*x)/((x^3+a/b)*b)^(1/2)-2/9*I*c/a*3^(1/2)/b*
(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/9*I*d/a*3^(1
/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.18

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(\left(bcx^3+ac\right)\sqrt{b}\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(bdx^3+ad\right)\sqrt{b}\,\text{weierst}\right.}{3\,(ab^2}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="fricas
")
```

output
```
2/3*((b*c*x^3 + a*c)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (b*d*x^3
+ a*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x
)) + (b*d*x^2 + b*c*x)*sqrt(b*x^3 + a))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.31

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(5/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 5
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(7/3)) + b*d*x**5*
gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)
*gamma(8/3))
```

Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{5/2}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \int \frac{bdx^4 + bcx^3 + adx + ac}{(bx^3 + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \int \frac{b\,d\,x^4 + b\,c\,x^3 + a\,d\,x + a\,c}{(b\,x^3 + a)^{5/2}} \, dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{5/2}} \, dx = \left(\int \frac{\sqrt{b\,x^3 + a}}{b^2x^6 + 2ab\,x^3 + a^2}dx\right) c + \left(\int \frac{\sqrt{b\,x^3 + a}\,x}{b^2x^6 + 2ab\,x^3 + a^2}dx\right) d$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(5/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*c + int((sqrt(a +
b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*d
```

3.57 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 554

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}} \, dx = \frac{2x(c+dx)}{9a(a+bx^3)^{3/2}} + \frac{2x(7c+5dx)}{27a^2\sqrt{a+bx^3}} - \frac{10d\sqrt{a+bx^3}}{27a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{5\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}c+5\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{27\sqrt[4]{3}a^2b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*x*(d*x+c)/a/(b*x^3+a)^(3/2)+2/27*x*(5*d*x+7*c)/a^2/(b*x^3+a)^(1/2)-10/
27*d*(b*x^3+a)^(1/2)/a^2/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+5/27*(1/2
*6^(1/2)-1/2*2^(1/2))*d*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))
*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)
/a^(5/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/81*(1/2*6^(1/2)+1/2*2^(1/2))*(7*b^(1/3)*c+5
*(1-3^(1/2))*a^(1/3)*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))
*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)
/a^2/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.22

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{4cx(10a+7bx^3)+14cx(a+bx^3)\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cdot\right.}{54a^2\,(a+\cdot}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(4*c*x*(10*a + 7*b*x^3) + 14*c*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeom
etric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 27*d*x^2*(a + b*x^3)*Sqrt[1 + (b*x
^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(54*a^2*(a + b*x^3)
^(3/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2019, 2394, 27, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{c+dx}{(a+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}-\frac{2\int -\frac{7c+5dx}{2(bx^3+a)^{3/2}}dx}{9a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7c+5dx}{(bx^3+a)^{3/2}}dx}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{2x(7c+5dx)}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{7c-5dx}{2\sqrt{bx^3+a}}dx}{3a}}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{7c-5dx}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(7c+5dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{2417}$$

$$\frac{\frac{\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+7c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{5d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(7c+5dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(c+dx)}{9a(a+bx^3)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+7c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5d\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt{bx^3+a}}}{\sqrt[3]{b}}}{3a}$$

$$9a$$

$$\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+7c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-5d\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}\right.}{3a}$$

$$9a$$

$$\frac{2x(c+dx)}{9a\left(a+bx^3\right)^{3/2}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(2*x*(c + d*x))/(9*a*(a + b*x^3)^(3/2)) + ((2*x*(7*c + 5*d*x))/(3*a*Sqrt[a
 + b*x^3]) + ((-5*d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))
)/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(7*c + (5*(1 - Sqrt[3])*a^(1/3)*d)/b^(1/3
))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a))/(9*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 809, normalized size of antiderivative = 1.46

method	result	size
elliptic	Expression too large to display	809
default	Expression too large to display	1782

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/9*d/a/b^2*x^2+2/9*c/a/b^2*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-5/27/b/a
^2*d*x^2-7/27/b/a^2*c*x)/((x^3+a/b)*b)^(1/2)-14/81*I*c/a^2*3^(1/2)/b*(-a*b
^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+10/81*I*d/a^2*3^(1/
2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+
a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.28

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{7/2}}\, dx = \frac{2\left(7\left(b^2cx^6 + 2\,abcx^3 + a^2c\right)\sqrt{b}\text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right) + 5\left(b^2dx^6\right.\right.}{}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="fricas
")
```

output
```
2/27*(7*(b^2*c*x^6 + 2*a*b*c*x^3 + a^2*c)*sqrt(b)*weierstrassPInverse(0, -
4*a/b, x) + 5*(b^2*d*x^6 + 2*a*b*d*x^3 + a^2*d)*sqrt(b)*weierstrassZeta(0,
 -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + (5*b^2*d*x^5 + 7*b^2*c*x^4 +
 8*a*b*d*x^2 + 10*a*b*c*x)*sqrt(b*x^3 + a))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 +
 a^4*b)
```

Sympy [A] (verification not implemented)

Time = 16.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.29

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{7}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(7/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 7/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(5/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 7
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)*gamma(7/3)) + b*d*x**5*
gamma(5/3)*hyper((5/3, 7/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(7/2)
*gamma(8/3))
```

Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx=\int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx=\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{(b\,x^3+a)^{7/2}}\,dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(7/2), x)
```

Reduce [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{7/2}}\,dx=\left(\int \frac{\sqrt{b\,x^3+a}}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx\right)c+\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx\right)d$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(7/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)
*c + int((sqrt(a + b*x**3)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3
*x**9),x)*d
```

3.58 $\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 581

$$
\begin{aligned}
\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx &= \frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}+\frac{2x(13c+11dx)}{135a^2\,(a+bx^3)^{3/2}} \\
&+\frac{2x(91c+55dx)}{405a^3\sqrt{a+bx^3}}-\frac{22d\sqrt{a+bx^3}}{81a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&+\frac{11\sqrt{2-\sqrt{3}}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&+\frac{2\sqrt{2+\sqrt{3}}\left(91\sqrt[3]{b}c+55\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{405\sqrt[4]{3}a^3b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}
$$

output
```
2/15*x*(d*x+c)/a/(b*x^3+a)^(5/2)+2/135*x*(11*d*x+13*c)/a^2/(b*x^3+a)^(3/2)
+2/405*x*(55*d*x+91*c)/a^3/(b*x^3+a)^(1/2)-22/81*d*(b*x^3+a)^(1/2)/a^3/b^(
2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+11/81*(1/2*6^(1/2)-1/2*2^(1/2))*d*(a^
(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(8/3)/b^(2/3)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2
)+2/1215*(1/2*6^(1/2)+1/2*2^(1/2))*(91*b^(1/3)*c+55*(1-3^(1/2))*a^(1/3)*d)
*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^3/b^(2/3)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.24

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx = \frac{4cx(157a^2+221abx^3+91b^2x^6)+182cx(a+bx^3)^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeome}}{8}$$

input
```
Integrate[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

output
```
(4*c*x*(157*a^2 + 221*a*b*x^3 + 91*b^2*x^6) + 182*c*x*(a + b*x^3)^2*Sqrt[1
 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 405*d*x^2*(
a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 7/2, 5/3, -((b*x^3
)/a)])/(810*a^3*(a + b*x^3)^(5/2))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2019, 2394, 27, 2394, 27, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{c+dx}{(a+bx^3)^{7/2}}dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}-\frac{2\int -\frac{13c+11dx}{2(bx^3+a)^{5/2}}dx}{15a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{13c+11dx}{(bx^3+a)^{5/2}}dx}{15a}+\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}-\frac{2\int -\frac{91c+55dx}{2\left(bx^3+a\right)^{3/2}}dx}{9a}}{15a}+\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{91c+55dx}{\left(bx^3+a\right)^{3/2}}dx}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{2x(91c+55dx)}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{91c-55dx}{2\sqrt{bx^3+a}}dx}{3a}}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\,(a+bx^3)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{91c-55dx}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(91c+55dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \; 2417$$

$$\frac{\frac{\frac{\left(\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+91c\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{55d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(91c+55dx)}{3a\sqrt{a+bx^3}}}{9a}+\frac{2x(13c+11dx)}{9a(a+bx^3)^{3/2}}}{15a}+\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \; 759$$

$$\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+91c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{55d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\ldots}{\sqrt{bx^3+a}}\ldots}{\sqrt[3]{b}}\ldots}{3a}\ldots}{9a}\ldots}{15a}\ldots\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

$$\Big\downarrow \; 2416$$

$$\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{55\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+91c\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{55d\left(\frac{2\sqrt{a+\cdots}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}\cdots\right.}{\cdots}}{3a}\cdots}{9a}\cdots$$

$$\frac{2x(c+dx)}{15a\left(a+bx^3\right)^{5/2}}$$

input
```
Int[(a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x]
```

output
```
(2*x*(c + d*x))/(15*a*(a + b*x^3)^(5/2)) + ((2*x*(13*c + 11*d*x))/(9*a*(a
 + b*x^3)^(3/2)) + ((2*x*(91*c + 55*d*x))/(3*a*Sqrt[a + b*x^3]) + ((-55*d*(
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2
 + Sqrt[3]]*(91*c + (55*(1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]))/(3*a))/(9*a))/(15*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.47

method	result	size
elliptic	Expression too large to display	853
default	Expression too large to display	1902

input

```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
(2/15*d/a/b^3*x^2+2/15*c/a/b^3*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^3+(22/135/b^2/
a^2*d*x^2+26/135/b^2/a^2*c*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-11/81/b/a^
3*d*x^2-91/405/b/a^3*c*x)/((x^3+a/b)*b)^(1/2)-182/1215*I*c/a^3*3^(1/2)/b*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+22/243*I/a^3*d*
3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.37

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}}\, dx = \frac{2\left(91\left(b^3cx^9 + 3\,ab^2cx^6 + 3\,a^2bcx^3 + a^3c\right)\sqrt{b}\text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, \right.\right.}{}$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="fricas
")
```

output
```
2/405*(91*(b^3*c*x^9 + 3*a*b^2*c*x^6 + 3*a^2*b*c*x^3 + a^3*c)*sqrt(b)*weie
rstrassPInverse(0, -4*a/b, x) + 55*(b^3*d*x^9 + 3*a*b^2*d*x^6 + 3*a^2*b*d*
x^3 + a^3*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (55*b^3*d*x^8 + 91*b^3*c*x^7 + 143*a*b^2*d*x^5 + 221*a*b^2*c*x^
4 + 115*a^2*b*d*x^2 + 157*a^2*b*c*x)*sqrt(b*x^3 + a))/(a^3*b^4*x^9 + 3*a^4
*b^3*x^6 + 3*a^5*b^2*x^3 + a^6*b)
```

Sympy [A] (verification not implemented)

Time = 49.48 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.28

$$\int \frac{ac + adx + bcx^3 + bdx^4}{(a + bx^3)^{9/2}}\, dx = \frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{9}{2} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{9}{2} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{9}{2} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{bdx^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{5}{3}, \frac{9}{2} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{9}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*d*x**4+b*c*x**3+a*d*x+a*c)/(b*x**3+a)**(9/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 9/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
7/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 9/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(7/2)*gamma(5/3)) + b*c*x**4*gamma(4/3)*hyper((4/3, 9
/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)*gamma(7/3)) + b*d*x**5*
gamma(5/3)*hyper((5/3, 9/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(9/2)
*gamma(8/3))
```

Maxima [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx=\int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="maxima
")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Giac [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx=\int \frac{bdx^4+bcx^3+adx+ac}{(bx^3+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*d*x^4 + b*c*x^3 + a*d*x + a*c)/(b*x^3 + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx=\int \frac{b\,d\,x^4+b\,c\,x^3+a\,d\,x+a\,c}{(b\,x^3+a)^{9/2}}\,dx$$

input
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2),x)
```

output
```
int((a*c + a*d*x + b*c*x^3 + b*d*x^4)/(a + b*x^3)^(9/2), x)
```

Reduce [F]

$$\int \frac{ac+adx+bcx^3+bdx^4}{(a+bx^3)^{9/2}}\,dx=\left(\int \frac{\sqrt{b\,x^3+a}}{b^4x^{12}+4a\,b^3x^9+6a^2b^2x^6+4a^3b\,x^3+a^4}dx\right)c+\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^4x^{12}+4a\,b^3x^9+6a^2b^2x^6+4a^3b\,x^3+a^4}dx\right)d$$

input
```
int((b*d*x^4+b*c*x^3+a*d*x+a*c)/(b*x^3+a)^(9/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x
**9 + b**4*x**12),x)*c + int((sqrt(a + b*x**3)*x)/(a**4 + 4*a**3*b*x**3 +
6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x)*d
```

3.59 $\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 590

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \frac{2e\sqrt{a+bx^3}}{3b}+\frac{2fx\sqrt{a+bx^3}}{5b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}+\frac{2(7bd-4ag)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-\right.}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{Ellipt}}{35\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*e*(b*x^3+a)^(1/2)/b+2/5*f*x*(b*x^3+a)^(1/2)/b+2/7*g*x^2*(b*x^3+a)^(1/2
)/b+2/7*(-4*a*g+7*b*d)*(b*x^3+a)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)-1/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(-4*a*g+7*b*d)*(a^(1/3)
+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+
b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((
1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/105*(1/2*6^(1/2)+
1/2*2^(1/2))*(7*b^(1/3)*(-2*a*f+5*b*c)-5*(1-3^(1/2))*a^(1/3)*(-4*a*g+7*b*d
))*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(5/3)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{4(a+bx^3)\left(35e+3x(7f+5gx)\right)+42(5bc-2af)x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+15(7}{210b\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

output
```
(4*(a + b*x^3)*(35*e + 3*x*(7*f + 5*g*x)) + 42*(5*b*c - 2*a*f)*x*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 15*(7*b*d - 4*
a*g)*x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
])/(210*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2427, 27, 2427, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{2\int \frac{7bfx^3+7bex^2+(7bd-4ag)x+7bc}{2\sqrt{bx^3+a}}dx}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{7bfx^3+7bex^2+(7bd-4ag)x+7bc}{\sqrt{bx^3+a}}dx}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{\frac{2\int \frac{35b^2ex^2+5b(7bd-4ag)x+7b(5bc-2af)}{2\sqrt{bx^3+a}}dx}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{35b^2ex^2+5b(7bd-4ag)x+7b(5bc-2af)}{\sqrt{bx^3+a}}dx}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{ 2425}$$

$$\frac{\frac{35b^2e\int \frac{x^2}{\sqrt{bx^3+a}}dx+\int \frac{7b(5bc-2af)+5b(7bd-4ag)x}{\sqrt{bx^3+a}}dx}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\frac{\int \frac{7b(5bc-2af)+5b(7bd-4ag)x}{\sqrt{bx^3+a}}dx+\frac{70}{3}be\sqrt{a+bx^3}}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}+\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$$\Big\downarrow \text{ 2417}$$

$$\frac{\frac{b^{2/3}\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)\int \frac{1}{\sqrt{bx^3+a}}dx+5b^{2/3}(7bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{70}{3}be\sqrt{a+bx^3}}{5b}+\frac{14}{5}fx\sqrt{a+bx^3}}{7b}$$

$$\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$\Big\downarrow$ 759

$$\frac{\frac{5b^{2/3}(7bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{5b}}{7b}$$

$$\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

$\Big\downarrow$ 2416

$$\frac{\frac{\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(7\sqrt[3]{b}(5bc-2af)-5\left(1-\sqrt{3}\right)\sqrt[3]{a}(7bd-4ag)\right)}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{}}{}$$

$$\frac{2gx^2\sqrt{a+bx^3}}{7b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/Sqrt[a + b*x^3],x]
```

output
```
(2*g*x^2*Sqrt[a + b*x^3])/(7*b) + ((14*f*x*Sqrt[a + b*x^3])/5 + ((70*b*e*S
qrt[a + b*x^3])/3 + 5*b^(2/3)*(7*b*d - 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3
)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b
^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])) + (2*Sqrt[2 + Sqrt[3]]*b^(1/3)*(7*b^(1/3)*(5*b*
c - 2*a*f) - 5*(1 - Sqrt[3])*a^(1/3)*(7*b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3]))/(5*b))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	785
risch	Expression too large to display	1043
default	Expression too large to display	1491

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*g*x^2*(b*x^3+a)^(1/2)/b+2/5*f*x*(b*x^3+a)^(1/2)/b+2/3*e*(b*x^3+a)^(1/2
)/b-2/3*I*(c-2/5*a/b*f)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2))-2/3*I*(d-4/7*a/b*g)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.15

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = \frac{2\left(21\,(5\,bc-2\,af)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-15\,(7\,bd-4\,ag)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weier}\right.\right.}{105\,b^2}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(21*(5*b*c - 2*a*f)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - 15*(
7*b*d - 4*a*g)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x)) + (15*b*g*x^2 + 21*b*f*x + 35*b*e)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.32

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx = e\left(\begin{cases} \frac{x^3}{3\sqrt{a}} & \text{for } b=0 \\ \frac{2\sqrt{a+bx^3}}{3b} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
e*Piecewise((x**3/(3*sqrt(a)), Eq(b, 0)), (2*sqrt(a + b*x**3)/(3*b), True)
) + c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
sqrt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((1/2, 4
/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(7/3)) + g*x**5*gam
ma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gam
ma(8/3))
```

Maxima [F]

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\sqrt{a + bx^3}}\, dx = \int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}\, dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\sqrt{a + bx^3}}\, dx = \int \frac{gx^4 + fx^3 + ex^2 + dx + c}{\sqrt{bx^3 + a}}\, dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{\sqrt{a + bx^3}}\, dx = \int \frac{g\,x^4 + f\,x^3 + e\,x^2 + d\,x + c}{\sqrt{b\,x^3 + a}}\, dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{70\sqrt{b\,x^3+a}\,e+42\sqrt{b\,x^3+a}\,fx+30\sqrt{b\,x^3+a}\,g\,x^2-42\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)af+105\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)bc-60(}{105b}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(1/2),x)
```

output
```
(70*sqrt(a + b*x**3)*e + 42*sqrt(a + b*x**3)*f*x + 30*sqrt(a + b*x**3)*g*x
**2 - 42*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a*f + 105*int(sqrt(a + b*x**
3)/(a + b*x**3),x)*b*c - 60*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a*g +
 105*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*b*d)/(105*b)
```

3.60 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 575

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = -\frac{2(ae-(bc-af)x-(bd-ag)x^2)}{3ab\sqrt{a+bx^3}} - \frac{2(bd-4ag)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt{2-\sqrt{3}}(bd-4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{ar}\right.}{3\sqrt[4]{3}ab^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/3*(-2*a*e+2*(-a*f+b*c)*x+2*(-a*g+b*d)*x^2)/a/b/(b*x^3+a)^(1/2)-2/3*(-4*a
*g+b*d)*(b*x^3+a)^(1/2)/a/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+1/3*(1/2
*6^(1/2)-1/2*2^(1/2))*(-4*a*g+b*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE((
(1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2
*I)*3^(1/4)/a^(2/3)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(
1/3)*(2*a*f+b*c)+(1-3^(1/2))*a^(1/3)*(-4*a*g+b*d))*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{4bcx-4a(e+x(f-3gx))+2(bc+2af)x\sqrt{1+\frac{bx^3}{a}}\text{ Hypergeometric2F}}{6a}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

output
```
(4*b*c*x - 4*a*(e + x*(f - 3*g*x)) + 2*(b*c + 2*a*f)*x*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)] + 3*(b*d - 4*a*g)*x^2*Sqrt
[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(6*a*b*Sqr
t[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2397, 27, 2425, 793, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 2397$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}-\frac{2\int -\frac{-3b^2ex^2-b(bd-4ag)x+b(bc+2af)}{2\sqrt{bx^3+a}}dx}{3ab^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-3b^2ex^2-b(bd-4ag)x+b(bc+2af)}{\sqrt{bx^3+a}}dx}{3ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2425$$

$$\frac{\int \frac{b(bc+2af)-b(bd-4ag)x}{\sqrt{bx^3+a}}dx-3b^2e\int \frac{x^2}{\sqrt{bx^3+a}}dx}{3ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 793$$

$$\frac{\int \frac{b(bc+2af)-b(bd-4ag)x}{\sqrt{bx^3+a}}dx-2be\sqrt{a+bx^3}}{3ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2417$$

$$\frac{b^{2/3}\left(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd-4ag)\right)\int \frac{1}{\sqrt{bx^3+a}}dx-b^{2/3}(bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx-2be\sqrt{a+bx^3}}{3ab^2}$$
$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 759$$

$$-b^{2/3}(bd-4ag)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx+\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

$$3ab^2$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\sqrt[3]{b}(2af+bc)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(bd\right.}{\sqrt[4]{3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{3ab\sqrt{a+bx^3}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(3*a*b*Sqrt[a + b*x^3]) + (-2*
b*e*Sqrt[a + b*x^3] - b^(2/3)*(b*d - 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])) + (2*Sqrt[2 + Sqrt[3]]*b^(1/3)*(b^(1/3)*(b*c + 2*
a*f) + (1 - Sqrt[3])*a^(1/3)*(b*d - 4*a*g))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a*
b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.43

method	result	size
elliptic	Expression too large to display	821
default	Expression too large to display	1547

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/3*(a*g-b*d)/a/b^2*x^2+1/3*(a*f-b*c)/b^2/a*x+1/3*e/b^2)/((x^3+a/b)*
b)^(1/2)-2/3*I*(f/b-1/3*(a*f-b*c)/a/b)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*(g/b+1/3*(a*g-b*d)/a/b)*3^(1/2)/b
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(((b^2c+2\,abf)x^3+abc+2\,a^2f)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\right.}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(((b^2*c + 2*a*b*f)*x^3 + a*b*c + 2*a^2*f)*sqrt(b)*weierstrassPInverse
(0, -4*a/b, x) + ((b^2*d - 4*a*b*g)*x^3 + a*b*d - 4*a^2*g)*sqrt(b)*weierst
rassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) - sqrt(b*x^3 + a)*(
a*b*e - (b^2*d - a*b*g)*x^2 - (b^2*c - a*b*f)*x))/(a*b^3*x^3 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 7.41 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = e\left(\begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(3/2),x)
```

output
```
e*Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), Tru
e)) + c*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*a**(3/2)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3)) + f*x**4*gamma(4/3)*hyper((4/
3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(7/3)) + g*x**
5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,e-6\sqrt{b\,x^3+a}\,fx+6\sqrt{b\,x^3+a}\,g\,x^2+6\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}\,d\right.}{}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*e - 6*sqrt(a + b*x**3)*f*x + 6*sqrt(a + b*x**3)*g*x
**2 + 6*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**2*f + 3
*int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a*b*c + 6*int(sqr
t(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a*b*f*x**3 + 3*int(sqrt(a
 + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*b**2*c*x**3 - 12*int((sqrt(a
 + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**2*g + 3*int((sqrt(a +
b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a*b*d - 12*int((sqrt(a + b*x
**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a*b*g*x**3 + 3*int((sqrt(a + b*
x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*b**2*d*x**3)/(3*b*(a + b*x**3)
)
```

3.61 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 624

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = -\frac{2(ae-(bc-af)x-(bd-ag)x^2)}{9ab\,(a+bx^3)^{3/2}} + \frac{2x(7bc+2af+(5bd+4ag)x)}{27a^2b\sqrt{a+bx^3}} - \frac{2(5bd+4ag)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt{2-\sqrt{3}}(5bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{9\,3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}(7bc+2af)+\left(1-\sqrt{3}\right)\sqrt[3]{a}(5bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\right.}{27\sqrt[4]{3}a^2b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/9*(-2*a*e+2*(-a*f+b*c)*x+2*(-a*g+b*d)*x^2)/a/b/(b*x^3+a)^(3/2)+2/27*x*(7
*b*c+2*a*f+(4*a*g+5*b*d)*x)/a^2/b/(b*x^3+a)^(1/2)-2/27*(4*a*g+5*b*d)*(b*x^
3+a)^(1/2)/a^2/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+1/27*(1/2*6^(1/2)-1
/2*2^(1/2))*(4*a*g+5*b*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2
))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/
4)/a^(5/3)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/
3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/81*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*(2*
a*f+7*b*c)+(1-3^(1/2))*a^(1/3)*(4*a*g+5*b*d))*(a^(1/3)+b^(1/3)*x)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*
EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),
I*3^(1/2)+2*I)*3^(3/4)/a^2/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.27

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{140b^2cx^4+40abx(5c+fx^3)-4a^2(15e+x(5f+27gx))+10(7bc+2a$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

output
```
(140*b^2*c*x^4 + 40*a*b*x*(5*c + f*x^3) - 4*a^2*(15*e + x*(5*f + 27*g*x))
+ 10*(7*b*c + 2*a*f)*x*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/3, 1/2, 4/3, -((b*x^3)/a)] + 27*(5*b*d + 4*a*g)*x^2*(a + b*x^3)*Sqrt[1 +
(b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(270*a^2*b*(a +
 b*x^3)^(3/2))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2397, 27, 2393, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \ 2397$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\,(a+bx^3)^{3/2}}-\frac{2\int -\frac{3b^2ex^2+b(5bd+4ag)x+b(7bc+2af)}{2(bx^3+a)^{3/2}}dx}{9ab^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3b^2ex^2+b(5bd+4ag)x+b(7bc+2af)}{(bx^3+a)^{3/2}}dx}{9ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \ 2393$$

$$\frac{-\frac{2\int -\frac{b(7bc+2af-(5bd+4ag)x)}{2\sqrt{bx^3+a}}dx}{3a}-\frac{2(3abe-bx(x(4ag+5bd)+2af+7bc))}{3a\sqrt{a+bx^3}}}{9ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b\int \frac{7bc+2af-(5bd+4ag)x}{\sqrt{bx^3+a}}dx}{3a}-\frac{2(3abe-bx(x(4ag+5bd)+2af+7bc))}{3a\sqrt{a+bx^3}}}{9ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \ 2417$$

$$\frac{\frac{b\left(\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+5bd)}{\sqrt[3]{b}}+2af+7bc\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{(4ag+5bd)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3a}-\frac{2(3abe-bx(x(4ag+5bd)+2af+7bc))}{3a\sqrt{a+bx^3}}}{9ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\,(a+bx^3)^{3/2}}$$

$$\Big\downarrow \ 759$$

$$\frac{b\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+5bd)}{\sqrt[3]{b}}+2af+7bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{(4ag+}{}\right)}{3a}$$

$$9ab^2$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}}$$

$\downarrow$ 2416

$$\frac{b\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+5bd)}{\sqrt[3]{b}}+2af+7bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{(4ag+}{}\right)}{3a}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{9ab\left(a+bx^3\right)^{3/2}}$$

input

```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x]
```

output
```
(2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(9*a*b*(a + b*x^3)^(3/2)) + ((
-2*(3*a*b*e - b*x*(7*b*c + 2*a*f + (5*b*d + 4*a*g)*x)))/(3*a*Sqrt[a + b*x^
3]) + (b*(-(((5*b*d + 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])))/b^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(7*b*c + 2*a*f + ((1 - Sqrt[3])
*a^(1/3)*(5*b*d + 4*a*g))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*a)
)/(9*a*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.38

method	result	size
elliptic	Expression too large to display	861
default	Expression too large to display	1673

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/9/a/b^3*(a*g-b*d)*x^2-2/9/a/b^3*(a*f-b*c)*x-2/9/b^3*e)*(b*x^3+a)^(1/2)
/(x^3+a/b)^2-2*b*(-1/27/a^2/b^2*(4*a*g+5*b*d)*x^2-1/27/a^2/b^2*(2*a*f+7*b*
c)*x)/((x^3+a/b)*b)^(1/2)-2/81*I/b^2/a^2*(2*a*f+7*b*c)*3^(1/2)*(-a*b^2)^(1
/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+2/81*I/b^2/a^2*(4*a*g+5*b
*d)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*El
lipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \frac{2\left(((7\,b^3c+2\,ab^2f)x^6+7\,a^2bc+2\,a^3f+2\,(7\,ab^2c+2\,a^2bf)x^3)\sqrt{b}\mathrm{weier}\right.}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

output
```
2/27*(((7*b^3*c + 2*a*b^2*f)*x^6 + 7*a^2*b*c + 2*a^3*f + 2*(7*a*b^2*c + 2*
a^2*b*f)*x^3)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + ((5*b^3*d + 4*a*
b^2*g)*x^6 + 5*a^2*b*d + 4*a^3*g + 2*(5*a*b^2*d + 4*a^2*b*g)*x^3)*sqrt(b)*
weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) + ((5*b^3*d
+ 4*a*b^2*g)*x^5 + (7*b^3*c + 2*a*b^2*f)*x^4 - 3*a^2*b*e + (8*a*b^2*d + a^
2*b*g)*x^2 + (10*a*b^2*c - a^2*b*f)*x)*sqrt(b*x^3 + a))/(a^2*b^4*x^6 + 2*a
^3*b^3*x^3 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 60.20 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = e\left(\begin{cases} -\frac{2}{9ab\sqrt{a+bx^3}+9b^2x^3\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^3}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{fx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{gx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(5/2),x)
```

output
```
e*Piecewise((-2/(9*a*b*sqrt(a + b*x**3) + 9*b**2*x**3*sqrt(a + b*x**3)), N
e(b, 0)), (x**3/(3*a**(5/2)), True)) + c*x*gamma(1/3)*hyper((1/3, 5/2), (4
/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(4/3)) + d*x**2*gamma(2/3
)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(5/
3)) + f*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a
)/(3*a**(5/2)*gamma(7/3)) + g*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{5/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{5/2}}\,dx=\frac{-70\sqrt{b\,x^3+a}\,e-90\sqrt{b\,x^3+a}\,fx-126\sqrt{b\,x^3+a}\,g\,x^2+90\left(\int \frac{\sqrt{}}{b^3x^9+3a\,b}\right.}{}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(5/2),x)
```

output
```
( - 70*sqrt(a + b*x**3)*e - 90*sqrt(a + b*x**3)*f*x - 126*sqrt(a + b*x**3)
*g*x**2 + 90*int(sqrt(a + b*x**3)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 +
b**3*x**9),x)*a**3*f + 315*int(sqrt(a + b*x**3)/(a**3 + 3*a**2*b*x**3 + 3*
a*b**2*x**6 + b**3*x**9),x)*a**2*b*c + 180*int(sqrt(a + b*x**3)/(a**3 + 3*
a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**2*b*f*x**3 + 630*int(sqrt(a
 + b*x**3)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a*b**2*c*
x**3 + 90*int(sqrt(a + b*x**3)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**
3*x**9),x)*a*b**2*f*x**6 + 315*int(sqrt(a + b*x**3)/(a**3 + 3*a**2*b*x**3
+ 3*a*b**2*x**6 + b**3*x**9),x)*b**3*c*x**6 + 252*int((sqrt(a + b*x**3)*x)
/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**3*g + 315*int((s
qrt(a + b*x**3)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a
**2*b*d + 504*int((sqrt(a + b*x**3)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x*
*6 + b**3*x**9),x)*a**2*b*g*x**3 + 630*int((sqrt(a + b*x**3)*x)/(a**3 + 3*
a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a*b**2*d*x**3 + 252*int((sqrt(
a + b*x**3)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a*b**
2*g*x**6 + 315*int((sqrt(a + b*x**3)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x
**6 + b**3*x**9),x)*b**3*d*x**6)/(315*b*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.62 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 672

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = -\frac{2(ae-(bc-af)x-(bd-ag)x^2)}{15ab\,(a+bx^3)^{5/2}}$$
$$+\frac{2x(13bc+2af+(11bd+4ag)x)}{135a^2b\,(a+bx^3)^{3/2}}$$
$$+\frac{2x(7(13bc+2af)+5(11bd+4ag)x)}{405a^3b\sqrt{a+bx^3}}-\frac{2(11bd+4ag)\sqrt{a+bx^3}}{81a^3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt{2-\sqrt{3}}(11bd+4ag)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{27\ 3^{3/4}a^{8/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(7\sqrt[3]{b}(13bc+2af)+5\left(1-\sqrt{3}\right)\sqrt[3]{a}(11bd+4ag)\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{Ellipti}}{405\sqrt[4]{3}a^3b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/15*(-2*a*e+2*(-a*f+b*c)*x+2*(-a*g+b*d)*x^2)/a/b/(b*x^3+a)^(5/2)+2/135*x*
(13*b*c+2*a*f+(4*a*g+11*b*d)*x)/a^2/b/(b*x^3+a)^(3/2)+2/405*x*(14*a*f+91*b
*c+5*(4*a*g+11*b*d)*x)/a^3/b/(b*x^3+a)^(1/2)-2/81*(4*a*g+11*b*d)*(b*x^3+a)
^(1/2)/a^3/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+1/81*(1/2*6^(1/2)-1/2*2
^(1/2))*(4*a*g+11*b*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/
a^(8/3)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/1215*(1/2*6^(1/2)+1/2*2^(1/2))*(7*b^(1/3)*(2
*a*f+13*b*c)+5*(1-3^(1/2))*a^(1/3)*(4*a*g+11*b*d))*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x),I*3^(1/2)+2*I)*3^(3/4)/a^3/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.29

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \frac{4004b^3cx^7+44ab^2x^4(221c+14fx^3)+44a^2bx(157c+34fx^3)-4a^3(29}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(4004*b^3*c*x^7 + 44*a*b^2*x^4*(221*c + 14*f*x^3) + 44*a^2*b*x*(157*c + 34
*f*x^3) - 4*a^3*(297*e + x*(77*f + 405*g*x)) + 154*(13*b*c + 2*a*f)*x*(a +
 b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a
)] + 405*(11*b*d + 4*a*g)*x^2*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[2/3, 7/2, 5/3, -((b*x^3)/a)])/(8910*a^3*b*(a + b*x^3)^(5/2))
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2397, 27, 2393, 27, 2394, 27, 2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{2x(x(bd-ag)-af+bc+bex^2)}{15ab(a+bx^3)^{5/2}}-\frac{2\int -\frac{9b^2ex^2+b(11bd+4ag)x+b(13bc+2af)}{2(bx^3+a)^{5/2}}dx}{15ab^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{9b^2ex^2+b(11bd+4ag)x+b(13bc+2af)}{(bx^3+a)^{5/2}}dx}{15ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{15ab(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{ 2393}$$

$$\frac{-\frac{2\int -\frac{b(7(13bc+2af)+5(11bd+4ag)x)}{2\left(bx^3+a\right)^{3/2}}dx}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{15ab(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{b\int \frac{7(13bc+2af)+5(11bd+4ag)x}{\left(bx^3+a\right)^{3/2}}dx}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{15ab(a+bx^3)^{5/2}}$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{\frac{b\left(\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{3a\sqrt{a+bx^3}}-\frac{2\int -\frac{7(13bc+2af)-5(11bd+4ag)x}{2\sqrt{bx^3+a}}dx}{3a}\right)}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x(x(bd-ag)-af+bc+bex^2)}{15ab(a+bx^3)^{5/2}}$$

$$\downarrow \; 27$$

$$\frac{\frac{b\left(\frac{\int \frac{7(13bc+2af)-5(11bd+4ag)x}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{3a\sqrt{a+bx^3}}\right)}{9a}-\frac{2(9abe-bx(x(4ag+11bd)+2af+13bc))}{9a(a+bx^3)^{3/2}}}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

$$\downarrow \; 2417$$

$$\frac{\frac{b\left(\frac{\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+11bd)}{\sqrt[3]{b}}+14af+91bc\right)\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{5(4ag+11bd)\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}+\frac{2x(7(2af+13bc)+5x(4ag+11bd))}{3a\sqrt{a+bx^3}}\right)}{9a}-\frac{2(9abe-bx(x\ldots}{9\ldots}}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

$$\downarrow \; 759$$

$$\frac{\frac{b\left(\frac{\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+11bd)}{\sqrt[3]{b}}+14af+91bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\ldots}{3a}\ldots\right)}{9a}\ldots}{15ab^2}+\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

$$\downarrow \; 2416$$

$$\frac{b\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)\left(\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(4ag+11bd)}{\sqrt[3]{b}}+14af+91bc\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \cdots}{\cdots}\right)}{\cdots}$$

$$\frac{2x\left(x(bd-ag)-af+bc+bex^2\right)}{15ab\left(a+bx^3\right)^{5/2}}$$

input

```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x]
```

output
```
(2*x*(b*c - a*f + (b*d - a*g)*x + b*e*x^2))/(15*a*b*(a + b*x^3)^(5/2)) + (
(-2*(9*a*b*e - b*x*(13*b*c + 2*a*f + (11*b*d + 4*a*g)*x)))/(9*a*(a + b*x^3
)^(3/2)) + (b*((2*x*(7*(13*b*c + 2*a*f) + 5*(11*b*d + 4*a*g)*x))/(3*a*Sqrt
[a + b*x^3]) + ((-5*(11*b*d + 4*a*g)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])))/b^(1/3) + (2*Sqrt[2 + Sqrt[3]]*(91*b*c + 14*a*f + (5*(1
 - Sqrt[3])*a^(1/3)*(11*b*d + 4*a*g))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x
^3]))/(3*a)))/(9*a))/(15*a*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.37

method	result	size
elliptic	Expression too large to display	921
default	Expression too large to display	1793

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/15/b^4/a*(a*g-b*d)*x^2-2/15/b^4/a*(a*f-b*c)*x-2/15/b^4*e)*(b*x^3+a)^(1
/2)/(x^3+a/b)^3+(2/135/a^2/b^3*(4*a*g+11*b*d)*x^2+2/135/a^2/b^3*(2*a*f+13*
b*c)*x)*(b*x^3+a)^(1/2)/(x^3+a/b)^2-2*b*(-1/81*(4*a*g+11*b*d)/a^3/b^2*x^2-
7/405*(2*a*f+13*b*c)/a^3/b^2*x)/((x^3+a/b)*b)^(1/2)-14/1215*I*(2*a*f+13*b*
c)/a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+2/243*I*(4*a*g+11*b*d)/a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.56

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^3)^{7/2}} \, dx = \frac{2\left(7\left((13\,b^4c + 2\,ab^3f)x^9 + 3\,(13\,ab^3c + 2\,a^2b^2f)x^6 + 13\,a^3bc + 2\,a^4f + \right.\right.}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="fricas")
```

output
```
2/405*(7*((13*b^4*c + 2*a*b^3*f)*x^9 + 3*(13*a*b^3*c + 2*a^2*b^2*f)*x^6 +
13*a^3*b*c + 2*a^4*f + 3*(13*a^2*b^2*c + 2*a^3*b*f)*x^3)*sqrt(b)*weierstra
ssPInverse(0, -4*a/b, x) + 5*((11*b^4*d + 4*a*b^3*g)*x^9 + 3*(11*a*b^3*d +
 4*a^2*b^2*g)*x^6 + 11*a^3*b*d + 4*a^4*g + 3*(11*a^2*b^2*d + 4*a^3*b*g)*x^
3)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)) +
 (5*(11*b^4*d + 4*a*b^3*g)*x^8 + 7*(13*b^4*c + 2*a*b^3*f)*x^7 + 13*(11*a*b
^3*d + 4*a^2*b^2*g)*x^5 - 27*a^3*b*e + 17*(13*a*b^3*c + 2*a^2*b^2*f)*x^4 +
 5*(23*a^2*b^2*d + a^3*b*g)*x^2 + (157*a^2*b^2*c - 7*a^3*b*f)*x)*sqrt(b*x^
3 + a))/(a^3*b^5*x^9 + 3*a^4*b^4*x^6 + 3*a^5*b^3*x^3 + a^6*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**3+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx=\int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{gx^4+fx^3+ex^2+dx+c}{(bx^3+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((g*x^4 + f*x^3 + e*x^2 + d*x + c)/(b*x^3 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \int \frac{g\,x^4+f\,x^3+e\,x^2+d\,x+c}{(b\,x^3+a)^{7/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^3)^(7/2), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^3)^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^3+a)^(7/2),x)
```

output
```
( - 286*sqrt(a + b*x**3)*e - 330*sqrt(a + b*x**3)*f*x - 390*sqrt(a + b*x**
3)*g*x**2 + 330*int(sqrt(a + b*x**3)/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x
**6 + 4*a*b**3*x**9 + b**4*x**12),x)*a**4*f + 2145*int(sqrt(a + b*x**3)/(a
**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x)*a*
*3*b*c + 990*int(sqrt(a + b*x**3)/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6
 + 4*a*b**3*x**9 + b**4*x**12),x)*a**3*b*f*x**3 + 6435*int(sqrt(a + b*x**3
)/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x
)*a**2*b**2*c*x**3 + 990*int(sqrt(a + b*x**3)/(a**4 + 4*a**3*b*x**3 + 6*a*
*2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x)*a**2*b**2*f*x**6 + 6435*int(
sqrt(a + b*x**3)/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9
+ b**4*x**12),x)*a*b**3*c*x**6 + 330*int(sqrt(a + b*x**3)/(a**4 + 4*a**3*b
*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x)*a*b**3*f*x**9 +
2145*int(sqrt(a + b*x**3)/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b
**3*x**9 + b**4*x**12),x)*b**4*c*x**9 + 780*int((sqrt(a + b*x**3)*x)/(a**4
 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x)*a**4*
g + 2145*int((sqrt(a + b*x**3)*x)/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6
 + 4*a*b**3*x**9 + b**4*x**12),x)*a**3*b*d + 2340*int((sqrt(a + b*x**3)*x)
/(a**4 + 4*a**3*b*x**3 + 6*a**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x)
*a**3*b*g*x**3 + 6435*int((sqrt(a + b*x**3)*x)/(a**4 + 4*a**3*b*x**3 + 6*a
**2*b**2*x**6 + 4*a*b**3*x**9 + b**4*x**12),x)*a**2*b**2*d*x**3 + 2340*...
```

3.63 $\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 230

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+4*3^(1/4)*(1/2*6^(1/2
)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1
/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(1+\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometr
ic2F1[1/2, 2/3, 5/3, -x^3])/2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2417}$$

$$2\sqrt{3}\int \frac{1}{\sqrt{x^3+1}}dx+\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

input
```
Int[(1 + Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 + x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.20

method	result
meijerg	$x \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], -x^3\right) + \sqrt{3}\, x \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], -x^3\right) + \frac{x^2 \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{5}{3}\right], -x\right.}{2}$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + \frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-}{-\frac{3}{2}-}}}{}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + \frac{2\sqrt{3}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

input
```
int((1+3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],-x^3)+3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3
)+1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.09

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = 2\left(\sqrt{3}+1\right)\text{weierstrassPInverse}(0,-4,x) - 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) + 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4, w
eierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.40

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gam
ma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I
*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.36

$$\begin{aligned}
&\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx \\
&= \sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right) \\
&\quad - \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} \\
&\quad + \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}
\end{aligned}$$

input
```
int((x + 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) - (6*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) + (6*((x + (3^(1/2)*
1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elliptic
F(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)+\int \frac{\sqrt{x^3+1}}{x^3+1}dx+\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx$$

input
```
int((1+3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(x**3 + 1)/(x**3 + 1),x) + int(sqrt(x**3 + 1)/(x**3 + 1),x
) + int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)
```

3.64 $\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 257

$$\begin{aligned}\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx &= -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\\ &\quad-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\end{aligned}$$

output
```
-2*(-x^3+1)^(1/2)/(1+3^(1/2)-x)+3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((
x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticE((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^
(1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)-4*3^(1/4)*(1/2*6^(
1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(
1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \left(1+\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) - \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, x^3])/2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 2417$$

$$2\sqrt{3}\int \frac{1}{\sqrt{1-x^3}}dx+\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 759$$

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{ 2416}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

input
```
Int[(1 + Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)],
-7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^{3}\right)+\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^{3}\right)-\frac{x^{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{3}\right)}{2}$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2i\sqrt{3}}{}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\sqrt{i\left(x+\frac{1}{2}-i\right.}}{}$

input
```
int((1+3^(1/2)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],x^3)+3^(1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)-
1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -2\left(i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,4,x) - 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) + I)*weierstrassPInverse(0, 4, x) - 2*I*weierstrassZeta(0, 4
, weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.38

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)-x)/(-x**3+1)**(1/2),x)
```

output
```
-x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamm
a(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(
3*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_po
lar(2*I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.33

$$\begin{aligned}
&\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx \\
&= \sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right) \\
&\quad + \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} \\
&\quad - \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}
\end{aligned}$$

input
```
int((3^(1/2) - x + 1)/(1 - x^3)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)-\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)+\int \frac{\sqrt{-x^3+1}\,x}{x^3-1}dx$$

input
```
int((1+3^(1/2)-x)/(-x^3+1)^(1/2),x)
```

output
```
 - sqrt(3)*int(sqrt( - x**3 + 1)/(x**3 - 1),x) - int(sqrt( - x**3 + 1)/(x*
*3 - 1),x) + int((sqrt( - x**3 + 1)*x)/(x**3 - 1),x)
```

3.65 $\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx=\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2*(x^3-1)^(1/2)/(1-3^(1/2)-x)-3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^
2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticE((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*
3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\frac{x\sqrt{1-x^3}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - x^3]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x
^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2418}$$

$$\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(1 + Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)
]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}+\frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x \cdots}{2\sqrt{\operatorname{si}\cdots}}$
elliptic	$\frac{2\left(1+\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\sqrt{3}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}\cdots}{\frac{3}{2}-\cdots}}\cdots}{\cdots}$

input
```
int((1+3^(1/2)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)+3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/
2],[4/3],x^3)-1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom
([1/2,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = 2\left(\sqrt{3}+1\right)\text{weierstrassPInverse}(0,4,x) + 2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) + 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, wei
erstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)-x)/(x**3-1)**(1/2),x)
```

output
```
I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3)
*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*x*gamma
(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.26

$$\begin{aligned}&\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx\\&=\frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}}\\&\quad+\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\\&\quad-\frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\end{aligned}$$

input
```
int((3^(1/2) - x + 1)/(x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2
) + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) + x^3)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^
(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx=\sqrt{3}\left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right)+\int \frac{\sqrt{x^3-1}}{x^3-1}dx-\left(\int \frac{\sqrt{x^3-1}\,x}{x^3-1}dx\right)$$

input
```
int((1+3^(1/2)-x)/(x^3-1)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(x**3 - 1)/(x**3 - 1),x) + int(sqrt(x**3 - 1)/(x**3 - 1),x
) - int((sqrt(x**3 - 1)*x)/(x**3 - 1),x)
```

3.66 $\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 135

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx=-\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((
x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-
I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx=\frac{x\sqrt{1+x^3}\left(2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output
```
(x*Sqrt[1 + x^3]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] +
 x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

input
```
Int[(1 + Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x
)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

method	result
meijerg	$-ix\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-i\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{ix^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right]\ldots\right.}{2}$
elliptic	$-\frac{2i\left(1+\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\,\sqrt{i\ldots}}{\ldots}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\ldots\right.}}{\ldots}$

input
```
int((1+3^(1/2)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*x*hypergeom([1/3,1/2],[4/3],-x^3)-I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3]
,-x^3)-1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -2\left(i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,-4,x) + 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) + I)*weierstrassPInverse(0, -4, x) + 2*I*weierstrassZeta(0,
-4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)+x)/(-x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamm
a(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
I*pi))/(3*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_
polar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.67

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}} - \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} + \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int((x + 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(
1/2) - (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 -
1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) + (6*(x^3 + 1)^(1/2)*(
(x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -i\left(\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)+\int \frac{\sqrt{x^3+1}}{x^3+1}dx+\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)$$

input
```
int((1+3^(1/2)+x)/(-x^3-1)^(1/2),x)
```

output
```
 - i*(sqrt(3)*int(sqrt(x**3 + 1)/(x**3 + 1),x) + int(sqrt(x**3 + 1)/(x**3
+ 1),x) + int((sqrt(x**3 + 1)*x)/(x**3 + 1),x))
```

3.67 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 468

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b*x^3+a)^(1/2)/b^(1/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2
))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(1/
3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(
b*x^3+a)^(1/2)+4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3
)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.19

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right.\right.}{2\sqrt{a+bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\sqrt{a+b x^3}} \, dx$$

$$\Big\downarrow \; 2417$$

$$2 \sqrt{3} \sqrt[3]{a} \int \frac{1}{\sqrt{b x^3+a}}dx+\int \frac{\sqrt[3]{b} x+\left(1-\sqrt{3}\right) \sqrt[3]{a}}{\sqrt{b x^3+a}}dx$$

$$\Big\downarrow \; 759$$

$$\int \frac{\sqrt[3]{b} x+\left(1-\sqrt{3}\right) \sqrt[3]{a}}{\sqrt{b x^3+a}}dx+\frac{4 \sqrt[4]{3} \sqrt{2+\sqrt{3}} \sqrt[3]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right)^2}} \text{EllipticF}\left(\arcsin \left(\frac{\sqrt[3]{b} x+\left(1-\sqrt{3}\right) \sqrt[3]{a}}{\sqrt[3]{b} x+\left(1+\sqrt{3}\right) \sqrt[3]{a}}\right),-7-4 \sqrt{3}\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right)^2}} \sqrt{a+b x^3}}$$

$$\Big\downarrow \; 2416$$

$$\frac{4 \sqrt[4]{3} \sqrt{2+\sqrt{3}} \sqrt[3]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right)^2}} \text{EllipticF}\left(\arcsin \left(\frac{\sqrt[3]{b} x+\left(1-\sqrt{3}\right) \sqrt[3]{a}}{\sqrt[3]{b} x+\left(1+\sqrt{3}\right) \sqrt[3]{a}}\right),-7-4 \sqrt{3}\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right)^2}} \sqrt{a+b x^3}}-\frac{\sqrt[4]{3} \sqrt{2-\sqrt{3}} \sqrt[3]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a} \sqrt[3]{b} x+b^{2/3} x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right)^2}} E\left(\arcsin \left(\frac{\sqrt[3]{b} x+\left(1-\sqrt{3}\right) \sqrt[3]{a}}{\sqrt[3]{b} x+\left(1+\sqrt{3}\right) \sqrt[3]{a}}\right)|-7-4 \sqrt{3}\right)}{\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right)^2}} \sqrt{a+b x^3}}+\frac{2 \sqrt{a+b x^3}}{\sqrt[3]{b} \left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right)}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (4*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(346) = 692.

Time = 0.76 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.14

method	result	size
default	Expression too large to display	1003

input

```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output

```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))-2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\right.\right.\right.}{b}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - b^(5/
6)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.26

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}+\frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1+3**(1/2))*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)$$

input
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) + a**(1/3)*int(sqrt(
a + b*x**3)/(a + b*x**3),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(a + b*x**
3),x)
```

3.68 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 481

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(-b*x^3+a)^(1/2)/b^(1/3)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)+3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1
/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x),I*3^(1/2)+2*I)/b^(
1/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)
/(-b*x^3+a)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(
1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1
/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)-b^(1/3)*x),I*3^(1/2)+2*I)/b^(1/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^
(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.19

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$
$$=\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/(2*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\sqrt{a-b x^3}} dx$$

$$\Big\downarrow \text{2417}$$

$$2\sqrt{3}\sqrt[3]{a} \int \frac{1}{\sqrt{a-b x^3}}dx+\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\sqrt{a-b x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\sqrt{a-b x^3}}dx-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right)^2}} \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-b x^3}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right)^2}} \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-b x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right)^2}} E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-b x^3}}-\frac{2\sqrt{a-b x^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
(-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3]) - (4*3^(1/4)*Sqrt[2
+ Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(359) = 718.

Time = 1.09 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.97

method	result	size
default	Expression too large to display	949

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3)
)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2
)/(-b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b
^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I
*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2
)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*
b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)*3^
(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/
3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)
*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInvers}\right.\right.}{b}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

output
```
-2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt
(-b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/
b
```

Sympy [A] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.27

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1+3**(1/2))*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x*
*3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=-\int \frac{b^{1/3}\,x-a^{1/3}\,(\sqrt{3}+1)}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2),x)
```

output
```
-int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right) - b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b\,x^3+a}dx\right)$$

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
a**(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(a - b*x**3),x) + a**(1/3)*int(sqrt(
a - b*x**3)/(a - b*x**3),x) - b**(1/3)*int((sqrt(a - b*x**3)*x)/(a - b*x**
3),x)
```

3.69 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 271

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b*x^3-a)^(1/2)/b^(1/3)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)-3^(1/4)*(1/2*6^(
1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticE(((1+3^(1/2
))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),2*I-I*3^(1/2))/b^(1/
3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/
(b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.34

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)-\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, (b*x^3)/a] - b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a])
)/(2*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
(2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. 2(206) = 412.

Time = 0.71 (sec) , antiderivative size = 952, normalized size of antiderivative = 3.51

method	result	size
default	Expression too large to display	952

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*I/b^(2/3)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3
))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*Ellipti
cF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3
^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b
^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*
(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1
/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right.\right.\right.}{b}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

output
```
2*(a^(1/3)*sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + b^(5/6
)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.41

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1+3**(1/2))*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*
gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(
3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(-(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\int \frac{b^{1/3}\,x-a^{1/3}\,(\sqrt{3}+1)}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int(-(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2),x)
```

output
```
-int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/(b*x^3 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+\sqrt{3})\,\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right) - a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right) + b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b\,x^3+a}dx\right)$$

input
```
int(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
 - a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(a - b*x**3),x) - a**(1/3)*int
(sqrt( - a + b*x**3)/(a - b*x**3),x) + b**(1/3)*int((sqrt( - a + b*x**3)*x
)/(a - b*x**3),x)
```

3.70 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 266

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(-b*x^3-a)^(1/2)/b^(1/3)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)+3^(1/4)*(1/2*6
^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x),2*I-I*3^(1/2))/b^(
1/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.35

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1/2
, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3
)/a)]))/(2*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$$
$$\Big\downarrow \; 2418$$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(201) = 402.

Time = 0.66 (sec) , antiderivative size = 1012, normalized size of antiderivative = 3.80

method	result	size
default	Expression too large to display	1012

input
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))-2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)
^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b
*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}+1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInv}\right.\right.}{b}$$

input
```
integrate(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="f
ricas")
```

output
```
-2*(a^(1/3)*sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqr
t(-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)
))/b
```

Sympy [A] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.48

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1+3**(1/2))*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4
/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) - I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/sqrt(-b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}+1\right)}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/(- a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -i\left(a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)\right)$$

input
```
int(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
 - i*(a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) + a**(1/3)*int
(sqrt(a + b*x**3)/(a + b*x**3),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(a +
 b*x**3),x))
```

3.71 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 520

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\operatorname{arcsi}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b/a)^(1/3)*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-3^(1
/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1
/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*((1+3
^(1/2))*b^(1/3)-(1-3^(1/2))*a^(1/3)*(b/a)^(1/3))*(a^(1/3)+b^(1/3)*x)*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2)
)*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right.}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -
((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2417}$$

$$\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{\sqrt[3]{b}}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
((b/a)^(1/3)*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/
3) + (2*Sqrt[2 + Sqrt[3]]*(1 + Sqrt[3] - ((1 - Sqrt[3])*a^(1/3)*(b/a)^(1/3
))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(386) = 772$.

Time = 0.56 (sec) , antiderivative size = 1004, normalized size of antiderivative = 1.93

method	result	size
default	Expression too large to display	1004

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I
*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(\right.\right.\right.}{b}$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*(b/a)
^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.24

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)+(b/a)**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x
**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper
((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx=\frac{a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)}{a^{\frac{1}{3}}}$$

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
(a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) + a**(1/3)*int(sqrt
(a + b*x**3)/(a + b*x**3),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(a + b*x*
*3),x))/a**(1/3)
```

3.72 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 533

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{b}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcsi}\ldots\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(b/a)^(1/3)*(-b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)+3^
(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*((
a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^
(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/
3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*((
1+3^(1/2))*b^(1/3)-(1-3^(1/2))*a^(1/3)*(b/a)^(1/3))*(a^(1/3)-b^(1/3)*x)*((
a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^
(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/
3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.17

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[a - b*x^3]
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{ 2417}$$

$$\left(-\frac{(1-\sqrt{3})\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}\right)}{\sqrt[3]{b}}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}+\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)\ldots\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
((b/a)^(1/3)*((-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/
3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^
(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])))/b^(1
/3) - (2*Sqrt[2 + Sqrt[3]]*(1 + Sqrt[3] - ((1 - Sqrt[3])*a^(1/3)*(b/a)^(1/
3))/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(399) = 798$.

Time = 0.64 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.78

method	result	size
default	Expression too large to display	950

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^
(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2)/(-b*x^3+a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/
3))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^
2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2
/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*3^(1/2)/b*(a*b^
2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1
/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b
*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/
2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.11

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInvers}\right.\right.}{b}$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

output
```
-2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(-b)*(b/
a)^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)-(b/a)**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2
*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*
hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

output
```
-integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx=\int \frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2),x)
```

output
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \frac{a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right)-b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b\,x^3+a}dx\right)}{a^{\frac{1}{3}}}$$

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
(a**(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(a - b*x**3),x) + a**(1/3)*int(sqrt
(a - b*x**3)/(a - b*x**3),x) - b**(1/3)*int((sqrt(a - b*x**3)*x)/(a - b*x*
*3),x))/a**(1/3)
```

3.73 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 256

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a+bx^3}}{b\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b/a)^(2/3)*(b*x^3-a)^(1/2)/b/(1-3^(1/2)-(b/a)^(1/3)*x)-3^(1/4)*(1/2*6^(
1/2)+1/2*2^(1/2))*(1-(b/a)^(1/3)*x)*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1-
3^(1/2)-(b/a)^(1/3)*x)^2)^(1/2)*EllipticE((1+3^(1/2)-(b/a)^(1/3)*x)/(1-3^(
1/2)-(b/a)^(1/3)*x),2*I-I*3^(1/2))/(b/a)^(1/3)/(-(1-(b/a)^(1/3)*x)/(1-3^(1
/2)-(b/a)^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(-2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(-2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[-a + b*x^3]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{bx^3-a}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
(2*(b/a)^(2/3)*Sqrt[-a + b*x^3])/(b*(1 - Sqrt[3] - (b/a)^(1/3)*x)) - (3^(1
/4)*Sqrt[2 + Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^
(2/3)*x^2)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
- (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^(
1/3)*Sqrt[-((1 - (b/a)^(1/3)*x)/(1 - Sqrt[3] - (b/a)^(1/3)*x)^2)]*Sqrt[-a
+ b*x^3])
```

Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(213) = 426$.

Time = 0.48 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.72

method	result	size
default	Expression too large to display	953

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^
(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1
/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))
^(1/2)/(b*x^3-a)^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2
)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/
b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*E
llipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1
/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*3^(1/2)/b*(a*b^2
)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*
b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)*EllipticF(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2
)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.20

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\sqrt{b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b}\right.\right.\right.}{b}$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*(sqrt(3) + 1)*weierstrassPInverse(0, 4*a/b, x) + sqrt(b)*(b/a)^
(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)-(b/a)**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt
(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/
a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**
3/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x*(b/a)^(1/3) - sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int \frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2),x)
```

output
```
int((3^(1/2) - x*(b/a)^(1/3) + 1)/(b*x^3 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{-a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right)-a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b\,x^3+a}dx\right)}{a^{\frac{1}{3}}}$$

input
```
int((1+3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
( - a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(a - b*x**3),x) - a**(1/3)*in
t(sqrt( - a + b*x**3)/(a - b*x**3),x) + b**(1/3)*int((sqrt( - a + b*x**3)*
x)/(a - b*x**3),x))/a**(1/3)
```

3.74 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 251

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(b/a)^(2/3)*(-b*x^3-a)^(1/2)/b/(1-3^(1/2)+(b/a)^(1/3)*x)+3^(1/4)*(1/2*6
^(1/2)+1/2*2^(1/2))*(1+(b/a)^(1/3)*x)*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(
1-3^(1/2)+(b/a)^(1/3)*x)^2)^(1/2)*EllipticE((1+3^(1/2)+(b/a)^(1/3)*x)/(1-3
^(1/2)+(b/a)^(1/3)*x),2*I-I*3^(1/2))/(b/a)^(1/3)/(-(1+(b/a)^(1/3)*x)/(1-3^
(1/2)+(b/a)^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.37

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2\left(1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right.}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*(1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -
((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)
]))/(2*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\sqrt{-a-bx^3}}\,dx$$

$$\Big\downarrow \; 2418$$

$$\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{-\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)^2}}\sqrt{-a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{-a-bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(-2*(b/a)^(2/3)*Sqrt[-a - b*x^3])/(b*(1 - Sqrt[3] + (b/a)^(1/3)*x)) + (3^(
1/4)*Sqrt[2 + Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)
^(2/3)*x^2)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)], -7 + 4*Sqrt[3]])/((b/a)^
(1/3)*Sqrt[-((1 + (b/a)^(1/3)*x)/(1 - Sqrt[3] + (b/a)^(1/3)*x)^2)]*Sqrt[-a
 - b*x^3])
```

Defintions of rubi rules used

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(208) = 416.

Time = 0.49 (sec) , antiderivative size = 1013, normalized size of antiderivative = 4.04

method	result	size
default	Expression too large to display	1013

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))-2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(-
b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3
*I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}(\sqrt{3}+1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPIn}\right.\right.}{b}$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas"
)
```

output
```
-2*(sqrt(-b)*(sqrt(3) + 1)*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*(b
/a)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.52

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1+3**(1/2)+(b/a)**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2)
, (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) - I*x*gamma(1/3
)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Giac [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/(- a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$$
$$= -\frac{i\left(a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)\right)}{a^{\frac{1}{3}}}$$

input
```
int((1+3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
( - i*(a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) + a**(1/3)*in
t(sqrt(a + b*x**3)/(a + b*x**3),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(a
+ b*x**3),x)))/a**(1/3)
```

3.75 $\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \left(1-\sqrt{3}\right)x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (x^2*Hypergeometr
ic2F1[1/2, 2/3, 5/3, -x^3])/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[(1 - Sqrt[3] + x)/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 + x^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x\cdots\right.}{2}$
elliptic	$\frac{2\left(1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\cdots}{-\frac{3}{2}-\cdots}}\cdots}{\cdots}$
default	$-\frac{2\sqrt{3}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\cdots}{-\frac{3}{2}-\cdots}}\cdots}{\cdots}$

input
```
int((1-3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],-x^3)-3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^3
)+1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.17

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = -2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x) - 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) - 2*weierstrassZeta(0, -4,
weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi)
)/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I
*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.46

$$\begin{aligned}
&\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx \\
&= -\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right) \\
&\quad - \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad + \frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int((x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

output
```
(6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
))^(1/2) - (6*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((
x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2) - 3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx = -\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)+\int \frac{\sqrt{x^3+1}}{x^3+1}dx+\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx$$

input
```
int((1-3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
- sqrt(3)*int(sqrt(x**3 + 1)/(x**3 + 1),x) + int(sqrt(x**3 + 1)/(x**3 + 1
),x) + int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)
```

3.76 $\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}} dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}} dx = -\frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2*(-x^3+1)^(1/2)/(1+3^(1/2)-x)+3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((
x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticE((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^
(1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}} dx = \left(1-\sqrt{3}\right) x \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) - \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(1 - Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] - (x^2*Hypergeometri
c2F1[1/2, 2/3, 5/3, x^3])/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx$$
$$\Big\downarrow \; 2416$$
$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

input
```
Int[(1 - Sqrt[3] - x)/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*S
qrt[1 - x^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

method	result
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)-\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\cdots}{}$
default	$\frac{2i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\cdots\right.}}{}$

input
```
int((1-3^(1/2)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],x^3)-3^(1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)-
1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.16

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -2\left(-i\sqrt{3}+i\right)\text{weierstrassPInverse}(0,4,x) - 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1-3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(3) + I)*weierstrassPInverse(0, 4, x) - 2*I*weierstrassZeta(0,
4, weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)-x)/(-x**3+1)**(1/2),x)
```

output
```
-x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamm
a(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*
I*pi))/(3*gamma(4/3)) + x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_po
lar(2*I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.42

$$\begin{aligned}
&\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx \\
&= -\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right) \\
&\quad + \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad - \frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int(-(x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

output
```
(6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - 3^(1/2)*x*hypergeom([1/3,
1/2], 4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{1-x^3}}\,dx = \sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right) - \left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right) + \int \frac{\sqrt{-x^3+1}\,x}{x^3-1}dx$$

input
```
int((1-3^(1/2)-x)/(-x^3+1)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt( - x**3 + 1)/(x**3 - 1),x) - int(sqrt( - x**3 + 1)/(x**3
- 1),x) + int((sqrt( - x**3 + 1)*x)/(x**3 - 1),x)
```

3.77 $\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 264

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2*(x^3-1)^(1/2)/(1-3^(1/2)-x)-3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^
2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticE((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*
3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)+4*3^(1/4)*(1/2*6^(1/
2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/
2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.24

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x
^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{ 2419}$$

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-2\sqrt{3}\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} - \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}} + \frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

input
```
Int[(1 - Sqrt[3] - x)/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)
]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2
)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x
)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}([\frac{1}{3},\frac{1}{2}],[\frac{4}{3}],x^3)}{\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}([\frac{1}{3},\frac{1}{2}],[\frac{4}{3}],x^3)}{\sqrt{\operatorname{signum}(x^3-1)}} - \frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x}{2\sqrt{\operatorname{si}}}$
elliptic	$\frac{2(1-\sqrt{3})\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{}$
default	$-\frac{2\sqrt{3}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x-}}{}$

input
```
int((1-3^(1/2)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)-3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/
2],[4/3],x^3)-1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom
([1/2,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.08

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -2\left(\sqrt{3}-1\right)\mathrm{weierstrassPInverse}(0,4,x) + 2\,\mathrm{weierstrassZeta}(0,4,\mathrm{weierstrassPInverse}(0,4,x))$$

input
```
integrate((1-3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) + 2*weierstrassZeta(0, 4, we
ierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)-x)/(x**3-1)**(1/2),x)
```

output
```
I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - I*x*gam
ma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + sqrt(3)*I*x*gamma
(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((1-3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.24

$$\begin{aligned}
&\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx \\
&= -\frac{\sqrt{3}\,x\sqrt{1-x^3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}} \\
&\quad + \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} \\
&\quad - \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}
\end{aligned}$$

input
```
int(-(x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

output
```
(6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/
2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2) - (3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/
(x^3 - 1)^(1/2) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^
(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx = -\sqrt{3}\left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right) + \int \frac{\sqrt{x^3-1}}{x^3-1}dx - \left(\int \frac{\sqrt{x^3-1}\,x}{x^3-1}dx\right)$$

input
```
int((1-3^(1/2)-x)/(x^3-1)^(1/2),x)
```

output
```
 - sqrt(3)*int(sqrt(x**3 - 1)/(x**3 - 1),x) + int(sqrt(x**3 - 1)/(x**3 - 1
),x) - int((sqrt(x**3 - 1)*x)/(x**3 - 1),x)
```

3.78 $\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx &= -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\\ &\quad-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

output
```
-2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((
x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-
I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)-4*3^(1/4)*(1/2*6^
(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+
3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3
-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output
```
(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
 + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \text{2419}$$

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-2\sqrt{3}\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

input

```
Int[(1 - Sqrt[3] + x)/Sqrt[-1 - x^3],x]
```

output

```
(-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x
)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^
2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] +
x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]
)
```

Defintions of rubi rules used

rule 760

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

method	result
meijerg	$-ix\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+i\sqrt{3}\,x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{ix^2\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right.\right.}{2}$
elliptic	$-\frac{2i\left(1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\,\sqrt{i}}{}$
default	$\frac{2i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3-1}}-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

input
```
int((1-3^(1/2)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*x*hypergeom([1/3,1/2],[4/3],-x^3)+I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3]
,-x^3)-1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -2\left(-i\sqrt{3}+i\right)\mathrm{weierstrassPInverse}(0,-4,x) + 2i\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((1-3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(3) + I)*weierstrassPInverse(0, -4, x) + 2*I*weierstrassZeta(0,
 -4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)+x)/(-x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamm
a(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(
3*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_
polar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1-3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.46

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}}$$
$$-\frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$+\frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

output
```
(6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2))^(1/2)) - (3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3,
-x^3))/(- x^3 - 1)^(1/2)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+x}{\sqrt{-1-x^3}}\,dx = i\left(\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)-\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)-\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)\right)$$

input
```
int((1-3^(1/2)+x)/(-x^3-1)^(1/2),x)
```

output
```
i*(sqrt(3)*int(sqrt(x**3 + 1)/(x**3 + 1),x) - int(sqrt(x**3 + 1)/(x**3 + 1
),x) - int((sqrt(x**3 + 1)*x)/(x**3 + 1),x))
```

3.79 $\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}} \, dx = -\frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2*(x^3+1)^(1/2)/(1+x+3^(1/2))+3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(
1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}} \, dx = \left(-1+\sqrt{3}\right) x \operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) - \frac{1}{2}x^2 \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

output
```
(-1 + Sqrt[3])*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] - (x^2*Hypergeomet
ric2F1[1/2, 2/3, 5/3, -x^3])/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}-1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

input
```
Int[(-1 + Sqrt[3] - x)/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.38

method	result
meijerg	$-x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+\sqrt{3}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],\ldots\right)}{2}$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ldots}{\ldots}$
default	$\frac{2\sqrt{3}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\ldots}{\ldots}$

input
```
int((-1+3^(1/2)-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*hypergeom([1/3,1/2],[4/3],-x^3)+3^(1/2)*x*hypergeom([1/3,1/2],[4/3],-x^
3)-1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.17

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = 2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,-4,x) + 2\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((-1+3^(1/2)-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) - 1)*weierstrassPInverse(0, -4, x) + 2*weierstrassZeta(0, -4, w
eierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.73

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = -\frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1+3**(1/2)-x)/(x**3+1)**(1/2),x)
```

output
```
-x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(
5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*ga
mma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(
I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx=\int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((-1+3^(1/2)-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx=\int -\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((-1+3^(1/2)-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) + 1)/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.48

$$\begin{aligned}
&\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx \\
&=\sqrt{3}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right) \\
&\quad+\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad-\frac{6\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int(-(x - 3^(1/2) + 1)/(x^3 + 1)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, -x^3) + (6*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (6*((x + (3^(1/2)*
1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elliptic
F(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{1+x^3}}\,dx = \sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right) - \left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right) - \left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)$$

input
```
int((-1+3^(1/2)-x)/(x^3+1)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(x**3 + 1)/(x**3 + 1),x) - int(sqrt(x**3 + 1)/(x**3 + 1),x
) - int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)
```

3.80 $\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}} \, dx = \frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
2*(-x^3+1)^(1/2)/(1+3^(1/2)-x)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x
^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticE((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(
1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.30

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}} \, dx = \frac{1}{2}x\left(2\left(-1+\sqrt{3}\right) \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) + x \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)$$

input
```
Integrate[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

output
```
(x*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + x*Hypergeomet
ric2F1[1/2, 2/3, 5/3, x^3]))/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{2416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[(-1 + Sqrt[3] + x)/Sqrt[1 - x^3],x]
```

output
```
(2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[1 - x^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result
meijerg	$-x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^{3}\right)+\sqrt{3}\,x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^{3}\right)+\frac{x^{2}\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{3}\right)}{2}$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}-\frac{2i\sqrt{3}}{}$
default	$-\frac{2i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}}-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-i\right.}}{}$

input
```
int((-1+3^(1/2)+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*hypergeom([1/3,1/2],[4/3],x^3)+3^(1/2)*x*hypergeom([1/3,1/2],[4/3],x^3)
+1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.16

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = -2\left(i\sqrt{3}-i\right)\text{weierstrassPInverse}(0,4,x) + 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((-1+3^(1/2)+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) - I)*weierstrassPInverse(0, 4, x) + 2*I*weierstrassZeta(0, 4
, weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1+3**(1/2)+x)/(-x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3)) - x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3
*gamma(4/3)) + sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_pol
ar(2*I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((-1+3^(1/2)+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((-1+3^(1/2)+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.39

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx$$
$$= \sqrt{3}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)$$
$$-\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$
$$+\frac{6\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int((x + 3^(1/2) - 1)/(1 - x^3)^(1/2),x)
```

output
```
3^(1/2)*x*hypergeom([1/3, 1/2], 4/3, x^3) - (6*(x^3 - 1)^(1/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)) + (6*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2
)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{1-x^3}}\,dx = -\sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)+\int \frac{\sqrt{-x^3+1}}{x^3-1}dx-\left(\int \frac{\sqrt{-x^3+1}\,x}{x^3-1}dx\right)$$

input
```
int((-1+3^(1/2)+x)/(-x^3+1)^(1/2),x)
```

output
```
 - sqrt(3)*int(sqrt( - x**3 + 1)/(x**3 - 1),x) + int(sqrt( - x**3 + 1)/(x*
*3 - 1),x) - int((sqrt( - x**3 + 1)*x)/(x**3 - 1),x)
```

3.81 $\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 263

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2*(x^3-1)^(1/2)/(1-3^(1/2)-x)+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x
^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticE((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I
*3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-4*3^(1/4)*(1/2*6^(1
/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1
/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.24

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \frac{x\sqrt{1-x^3}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

output
```
(x*Sqrt[1 - x^3]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] +
 x*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \; 2419$$

$$2\sqrt{3}\int \frac{1}{\sqrt{x^3-1}}dx-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \; 760$$

$$-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

input
```
Int[(-1 + Sqrt[3] + x)/Sqrt[-1 + x^3],x]
```

output
```
(-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x
)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2
)]*Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^
2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] -
x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]
)
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.37

method	result
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}\left([\frac{1}{3},\frac{1}{2}],[\frac{4}{3}],x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}+\frac{\sqrt{3}\,\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}\left([\frac{1}{3},\frac{1}{2}],[\frac{4}{3}],x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}+\frac{\sqrt{-\operatorname{signum}(x^3-1}}{2\sqrt{}}$
elliptic	$\frac{2\left(\sqrt{3}-1\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{}}{}$
default	$\frac{2\sqrt{3}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}-}}}{}$

input
```
int((-1+3^(1/2)+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],
x^3)+3^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1
/2],[4/3],x^3)+1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeo
m([1/2,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.08

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = 2\left(\sqrt{3}-1\right)\text{weierstrassPInverse}(0,4,x) - 2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((-1+3^(1/2)+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3) - 1)*weierstrassPInverse(0, 4, x) - 2*weierstrassZeta(0, 4, wei
erstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.31

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1+3**(1/2)+x)/(x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3)) - sqrt(3
)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + I*x*gamm
a(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((-1+3^(1/2)+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((-1+3^(1/2)+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) - 1)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.24

$$\begin{aligned}
&\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx \\
&= \frac{\sqrt{3}\,x\,\sqrt{1-x^3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ x^3\right)}{\sqrt{x^3-1}} \\
&\quad - \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} \\
&\quad + \frac{6\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}
\end{aligned}$$

input
```
int((x + 3^(1/2) - 1)/(x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(1 - x^3)^(1/2)*hypergeom([1/3, 1/2], 4/3, x^3))/(x^3 - 1)^(1/2
) - (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) + x^3)^(1/2) + (6*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^
(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{-1+\sqrt{3}+x}{\sqrt{-1+x^3}}\,dx = \sqrt{3}\left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right) - \left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right) + \int \frac{\sqrt{x^3-1}\,x}{x^3-1}dx$$

input
```
int((-1+3^(1/2)+x)/(x^3-1)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt(x**3 - 1)/(x**3 - 1),x) - int(sqrt(x**3 - 1)/(x**3 - 1),x
) + int((sqrt(x**3 - 1)*x)/(x**3 - 1),x)
```

3.82 $\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 248

$$\begin{aligned}\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx &= \frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\\&\quad+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

output
```
2*(-x^3-1)^(1/2)/(1+x-3^(1/2))-3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x
^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I
*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)+4*3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3
^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-
1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = -\frac{x\sqrt{1+x^3}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

output
```
-1/2*(x*Sqrt[1 + x^3]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
-x^3] + x*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/Sqrt[-1 - x^3]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}-1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2419}$$

$$2\sqrt{3}\int \frac{1}{\sqrt{-x^3-1}}dx-\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\int\frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2418}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

input
```
Int[(-1 + Sqrt[3] - x)/Sqrt[-1 - x^3],x]
```

output
```
(2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)
]*Sqrt[-1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2
)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x
)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

method	result
meijerg	$ix \,\mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], -x^3\right) - i\sqrt{3}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], -x^3\right) + \frac{ix^2 \,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{5}{3}\right],\right.}{2}$
elliptic	$-\frac{2i\left(\sqrt{3}-1\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}} + \frac{2i\sqrt{3}\,\sqrt{i}}{}$
default	$-\frac{2i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3-1}} + \frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$

input
```
int((-1+3^(1/2)-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*x*hypergeom([1/3,1/2],[4/3],-x^3)-I*3^(1/2)*x*hypergeom([1/3,1/2],[4/3],
-x^3)+1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = -2\left(i\sqrt{3}-i\right)\text{weierstrassPInverse}(0,-4,x) - 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((-1+3^(1/2)-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(3) - I)*weierstrassPInverse(0, -4, x) - 2*I*weierstrassZeta(0,
-4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \frac{ix^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((-1+3**(1/2)-x)/(-x**3-1)**(1/2),x)
```

output
```
I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma
(5/3)) - sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I
*pi))/(3*gamma(4/3)) + I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_p
olar(I*pi))/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((-1+3^(1/2)-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \int -\frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((-1+3^(1/2)-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) + 1)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.45

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,x\,\sqrt{x^3+1}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -x^3\right)}{\sqrt{-x^3-1}} + \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{6\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(-(x - 3^(1/2) + 1)/(- x^3 - 1)^(1/2),x)
```

output
```
(3^(1/2)*x*(x^3 + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -x^3))/(- x^3 - 1)^(
1/2) + (6*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 -
1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (6*(x^3 + 1)^(1/2)*(
(x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{-1+\sqrt{3}-x}{\sqrt{-1-x^3}}\,dx = i\left(-\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)+\int \frac{\sqrt{x^3+1}}{x^3+1}dx+\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)$$

input
```
int((-1+3^(1/2)-x)/(-x^3-1)^(1/2),x)
```

output
```
i*( - sqrt(3)*int(sqrt(x**3 + 1)/(x**3 + 1),x) + int(sqrt(x**3 + 1)/(x**3
+ 1),x) + int((sqrt(x**3 + 1)*x)/(x**3 + 1),x))
```

3.83 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 256

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b*x^3+a)^(1/2)/b^(1/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2
))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(1/
3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(
b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$=\frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\right.\right.}{2\sqrt{a+bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x
^3)/a)]))/(2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{2416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx$$
$$\Big\downarrow \; 2416$$

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. 2(189) = 378.

Time = 0.74 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.92

method	result	size
default	Expression too large to display	1003

input
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(
-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(
-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))+2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fr
icas")
```

output
```
-2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + b^(5
/6)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.48

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyper((1
/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx=\int \frac{b^{1/3}\,x-a^{1/3}\,(\sqrt{3}-1)}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx = -a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right) + a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right) + b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)$$

input
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
 - a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) + a**(1/3)*int(sq
rt(a + b*x**3)/(a + b*x**3),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(a + b*
x**3),x)
```

3.84 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 263

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(-b*x^3+a)^(1/2)/b^(1/3)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)+3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1
/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x),I*3^(1/2)+2*I)/b^(
1/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)
/(-b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.34

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/
3, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^
3)/a]))/Sqrt[a - b*x^3]
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$

$$\Big\downarrow \ 2416$$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
(-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 948 vs. $2(196) = 392$.

Time = 0.78 (sec) , antiderivative size = 949, normalized size of antiderivative = 3.61

method	result	size
default	Expression too large to display	949

input
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)
/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3
)*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)
^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(
1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3
^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/
2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*
b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-
I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^
(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx$$
$$= \frac{2\left(a^{\frac{1}{3}}\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="f
ricas")
```

output
```
2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(
-b)*b^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.49

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -\frac{\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2*I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/3
,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + x*gamma(1/3)*hyp
er((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="m
axima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{b^{1/3}\,x+a^{1/3}\,(\sqrt{3}-1)}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2),x)
```

output
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}\,dx = -a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right) + a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right) - b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b\,x^3+a}dx\right)$$

input

```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output

```
- a**(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(a - b*x**3),x) + a**(1/3)*int(sq
rt(a - b*x**3)/(a - b*x**3),x) - b**(1/3)*int((sqrt(a - b*x**3)*x)/(a - b*
x**3),x)
```

3.85 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 497

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt{-a+bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{}\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b*x^3-a)^(1/2)/b^(1/3)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)-3^(1/4)*(1/2*6^(
1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticE(((1+3^(1/2
))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),2*I-I*3^(1/2))/b^(1/
3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/
(b*x^3-a)^(1/2)+4*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)-b^(1/
3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3
)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/
3)-b^(1/3)*x),2*I-I*3^(1/2))/b^(1/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(
1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.18

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\right.\right.}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/
3, 1/2, 4/3, (b*x^3)/a] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^
3)/a]))/Sqrt[-a + b*x^3]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2419}$$

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx-2\sqrt{3}\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{760}$$

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
(2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/
((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3]) + (4*3^(1/4)*Sqr
t[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)]
, -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(375) = 750$.

Time = 0.65 (sec) , antiderivative size = 952, normalized size of antiderivative = 1.92

method	result	size
default	Expression too large to display	952

input
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-2*I*a^(1/3)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-
a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/
(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I/b^(2/3)
*3^(1/2)*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/
2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(
a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*a^(1/3)*3^(1/2)/b*(a*b^2)^(1/3)*(-I*
(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1
/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = -\frac{2\left(a^{\frac{1}{3}}\sqrt{b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-b^{\frac{5}{6}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b}\right.\right.\right.}{b}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fr
icas")
```

output
```
-2*(a^(1/3)*sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - b^(5/
6)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.23

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = \frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}-\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1-3**(1/2))*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*
gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*a**(1/
6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a
)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="ma
xima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate(-(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2),x)
```

output
```
int(-(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/(b*x^3 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{-a+bx^3}}\,dx = a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right)-a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right) + b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b\,x^3+a}dx\right)$$

input
```
int(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(a - b*x**3),x) - a**(1/3)*int(sq
rt( - a + b*x**3)/(a - b*x**3),x) + b**(1/3)*int((sqrt( - a + b*x**3)*x)/(
a - b*x**3),x)
```

3.86 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 488

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{}\right.}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(-b*x^3-a)^(1/2)/b^(1/3)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)+3^(1/4)*(1/2*6
^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x),2*I-I*3^(1/2))/b^(
1/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)/(-b*x^3-a)^(1/2)-4*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^
(1/3)+b^(1/3)*x),2*I-I*3^(1/2))/b^(1/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.19

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{b}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\right.\right.}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*a^(1/3)*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)] + b^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x
^3)/a)]))/(2*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{ 2419}$$

$$\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx-2\sqrt{3}\sqrt[3]{a}\int \frac{1}{\sqrt{-bx^3-a}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{ 2418}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(
1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3]) - (4*3^(1/4)*Sq
rt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1011 vs. 2(366) = 732.

Time = 0.69 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.07

method	result	size
default	Expression too large to display	1012

input
```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3*I*a^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^
(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^
(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))-2/3*I/b^(2/3)*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*
(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))+2*I*a^(1/3)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = \frac{2\left(a^{\frac{1}{3}}\sqrt{-b}\left(\sqrt{3}-1\right)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}b^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInver}\right.\right.}{b}$$

input
```
integrate(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="f
ricas")
```

output
```
2*(a^(1/3)*sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt
(-b)*b^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
)/b
```

Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.26

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = -\frac{i\sqrt[3]{b}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[6]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(((1-3**(1/2))*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-I*b**(1/3)*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*h
yper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/6)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Giac [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/sqrt(-b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2),x)
```

output
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/(- a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}}\,dx = i\left(a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)-a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)-b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)\right)$$

input

```
int(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output

```
i*(a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) - a**(1/3)*int(sq
rt(a + b*x**3)/(a + b*x**3),x) - b**(1/3)*int((sqrt(a + b*x**3)*x)/(a + b*
x**3),x))
```

3.87 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 241

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\,E\left(\arcsin\left(\frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b/a)^(2/3)*(b*x^3+a)^(1/2)/b/(1+3^(1/2)+(b/a)^(1/3)*x)-3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*(1+(b/a)^(1/3)*x)*((1-(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(1+
3^(1/2)+(b/a)^(1/3)*x)^2)^(1/2)*EllipticE((1-3^(1/2)+(b/a)^(1/3)*x)/(1+3^(
1/2)+(b/a)^(1/3)*x),I*3^(1/2)+2*I)/(b/a)^(1/3)/((1+(b/a)^(1/3)*x)/(1+3^(1/
2)+(b/a)^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.37

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx$$
$$= \frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2(-1+\sqrt{3})\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\right.\right.}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
 -((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a+bx^3}}{b\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(x\sqrt[3]{\frac{b}{a}}+1\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}-x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{x\sqrt[3]{\frac{b}{a}}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[a + b*x^3],x]
```

output
```
(2*(b/a)^(2/3)*Sqrt[a + b*x^3])/(b*(1 + Sqrt[3] + (b/a)^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*(1 + (b/a)^(1/3)*x)*Sqrt[(1 - (b/a)^(1/3)*x + (b/a)^(
2/3)*x^2)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
 (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(1
/3)*Sqrt[(1 + (b/a)^(1/3)*x)/(1 + Sqrt[3] + (b/a)^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(196) = 392.

Time = 0.58 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.17

method	result	size
default	Expression too large to display	1004

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-2/3*I*
(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.22

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(\sqrt{b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInvers}\right.\right.}{b}$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(b)*(b/a
)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.51

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)+(b/a)**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2), (4/
3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*hyper
((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{a+bx^3}}\,dx = \frac{-a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)}{a^{\frac{1}{3}}}$$

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
( - a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) + a**(1/3)*int(s
qrt(a + b*x**3)/(a + b*x**3),x) + b**(1/3)*int((sqrt(a + b*x**3)*x)/(a + b
*x**3),x))/a**(1/3)
```

3.88 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 248

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx=-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{\frac{1+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{2/3}x^2}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2*(b/a)^(2/3)*(-b*x^3+a)^(1/2)/b/(1+3^(1/2)-(b/a)^(1/3)*x)+3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*(1-(b/a)^(1/3)*x)*((1+(b/a)^(1/3)*x+(b/a)^(2/3)*x^2)/(
1+3^(1/2)-(b/a)^(1/3)*x)^2)^(1/2)*EllipticE((1-3^(1/2)-(b/a)^(1/3)*x)/(1+3
^(1/2)-(b/a)^(1/3)*x),I*3^(1/2)+2*I)/(b/a)^(1/3)/((1-(b/a)^(1/3)*x)/(1+3^(
1/2)-(b/a)^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.36

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[a - b*x^3]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(1-x\sqrt[3]{\frac{b}{a}}\right)\sqrt{\frac{x^2\left(\frac{b}{a}\right)^{2/3}+x\sqrt[3]{\frac{b}{a}}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-\sqrt[3]{\frac{b}{a}}x-\sqrt{3}+1}{-\sqrt[3]{\frac{b}{a}}x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{\frac{b}{a}}\sqrt{\frac{1-x\sqrt[3]{\frac{b}{a}}}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)^2}}\sqrt{a-bx^3}}-\frac{2\left(\frac{b}{a}\right)^{2/3}\sqrt{a-bx^3}}{b\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)}$$

input
```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[a - b*x^3],x]
```

output
```
(-2*(b/a)^(2/3)*Sqrt[a - b*x^3])/(b*(1 + Sqrt[3] - (b/a)^(1/3)*x)) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*(1 - (b/a)^(1/3)*x)*Sqrt[(1 + (b/a)^(1/3)*x + (b/a)^
(2/3)*x^2)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*EllipticE[ArcSin[(1 - Sqrt[3]
- (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)], -7 - 4*Sqrt[3]])/((b/a)^(
1/3)*Sqrt[(1 - (b/a)^(1/3)*x)/(1 + Sqrt[3] - (b/a)^(1/3)*x)^2]*Sqrt[a - b*
x^3])
```

Defintions of rubi rules used

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(203) = 406$.

Time = 0.62 (sec) , antiderivative size = 950, normalized size of antiderivative = 3.83

method	result	size
default	Expression too large to display	950

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1
/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2
)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)
^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b
*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^
(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/
2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(
a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^
2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b
*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/
2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \frac{2\left(\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(\right.\right.\right.}{b}$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

output
```
2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*(b/a
)^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.52

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = -\frac{x^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt{3}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)-(b/a)**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(2
*I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - sqrt(3)*x*gamma(1/3)*hyper((1/3, 1/2),
(4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + x*gamma(1/3)*
hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

output
```
-integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx=\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}\,dx = \frac{-a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right)+a^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right)-b^{\frac{1}{3}}\left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b\,x^3+a}dx\right)}{a^{\frac{1}{3}}}$$

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
( - a**(1/3)*sqrt(3)*int(sqrt(a - b*x**3)/(a - b*x**3),x) + a**(1/3)*int(s
qrt(a - b*x**3)/(a - b*x**3),x) - b**(1/3)*int((sqrt(a - b*x**3)*x)/(a - b
*x**3),x))/a**(1/3)
```

3.89 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 549

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
2*(b/a)^(1/3)*(b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)-3^(1
/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(b/a)^(1/3)*(a^(1/3)-b^(1/3)*x)*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1
/2)*EllipticE(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)
*x),2*I-I*3^(1/2))/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1
/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)-2/3*(1/2*6^(1/2)-1/2*2^(1/2))*((1-
3^(1/2))*b^(1/3)-(1+3^(1/2))*a^(1/3)*(b/a)^(1/3))*(a^(1/3)-b^(1/3)*x)*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1
/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)
*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/
2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.16

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx =$$
$$-\frac{x\sqrt{1-\frac{bx^3}{a}}\left(2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output
```
-1/2*(x*Sqrt[1 - (b*x^3)/a]*(2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2,
4/3, (b*x^3)/a] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a
]))/Sqrt[-a + b*x^3]
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 2419}$$

$$\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}\right)}{\sqrt[3]{b}}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right)\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input

```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/Sqrt[-a + b*x^3],x]
```

output

```
((b/a)^(1/3)*((2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^
(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3)
- b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])))/
b^(1/3) - (2*Sqrt[2 - Sqrt[3]]*(1 - Sqrt[3] - ((1 + Sqrt[3])*a^(1/3)*(b/a)
^(1/3))/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 +
 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(415) = 830$.

Time = 0.51 (sec) , antiderivative size = 953, normalized size of antiderivative = 1.74

method	result	size
default	Expression too large to display	953

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1
/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)
*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))-2/3*I*(b/a)^(1/3)*3^(
1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1
/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2)
*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1/
2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*
b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)
^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))+2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)
*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt{b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0\right.\right.\right.}{b}$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(b)*(sqrt(3) - 1)*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*(b/a)
^(1/3)*weierstrassZeta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.21

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)-(b/a)**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt
(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sq
rt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**
3/a)/(3*sqrt(a)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx = \int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x*(b/a)^(1/3) + sqrt(3) - 1)/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx=\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/(b*x^3 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a+bx^3}}\,dx=\frac{a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right)-a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right)+b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b\,x^3+a}dx\right)}{a^{\frac{1}{3}}}$$

input
```
int((1-3^(1/2)-(b/a)^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
(a**(1/3)*sqrt(3)*int(sqrt( - a + b*x**3)/(a - b*x**3),x) - a**(1/3)*int(s
qrt( - a + b*x**3)/(a - b*x**3),x) + b**(1/3)*int((sqrt( - a + b*x**3)*x)/
(a - b*x**3),x))/a**(1/3)
```

3.90 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 540

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{2\sqrt[3]{\frac{b}{a}}\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$+\frac{2\sqrt{2-\sqrt{3}}\left(\left(1-\sqrt{3}\right)\sqrt[3]{b}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*(b/a)^(1/3)*(-b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)+3^
(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(b/a)^(1/3)*(a^(1/3)+b^(1/3)*x)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^
(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/
3)*x),2*I-I*3^(1/2))/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(
(1-3^(1/2))*b^(1/3)-(1+3^(1/2))*a^(1/3)*(b/a)^(1/3))*(a^(1/3)+b^(1/3)*x)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)
^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1
/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.17

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx$$
$$=\frac{x\sqrt{1+\frac{bx^3}{a}}\left(-2\left(-1+\sqrt{3}\right)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+\sqrt[3]{\frac{b}{a}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\right.\right.}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(-2*(-1 + Sqrt[3])*Hypergeometric2F1[1/3, 1/2, 4/3,
 -((b*x^3)/a)] + (b/a)^(1/3)*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/
a)]))/(2*Sqrt[-a - b*x^3])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2419}$$

$$\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{\frac{b}{a}}}{\sqrt[3]{b}}-\sqrt{3}+1\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{\sqrt[3]{\frac{b}{a}}\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right.}\right.}{\sqrt[3]{b}}$$

input

```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/Sqrt[-a - b*x^3],x]
```

output

```
((b/a)^(1/3)*((-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])))
/b^(1/3) + (2*Sqrt[2 - Sqrt[3]]*(1 - Sqrt[3] - ((1 + Sqrt[3])*a^(1/3)*(b/a
)^(1/3))/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
+ 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(406) = 812$.

Time = 0.52 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.88

method	result	size
default	Expression too large to display	1013

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))-2/3*I*(b/a)^(1/3)*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-
a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
))+2*I/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.10

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt{-b}(\sqrt{3}-1)\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{-b}\left(\frac{b}{a}\right)^{\frac{1}{3}}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInve}\right.\right.}{b}$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas"
)
```

output
```
2*(sqrt(-b)*(sqrt(3) - 1)*weierstrassPInverse(0, -4*a/b, x) + sqrt(-b)*(b/
a)^(1/3)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.24

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = -\frac{ix^2\sqrt[3]{\frac{b}{a}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)} - \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt{3}ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((1-3**(1/2)+(b/a)**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-I*x**2*(b/a)**(1/3)*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar
(I*pi)/a)/(3*sqrt(a)*gamma(5/3)) - I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3)) + sqrt(3)*I*x*gamma(1/3
)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(4/3
))
```

Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Giac [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/sqrt(-b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/(- a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\sqrt{-a-bx^3}}\,dx =\frac{i\left(a^{\frac{1}{3}}\sqrt{3}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)-a^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)-b^{\frac{1}{3}}\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)\right)}{a^{\frac{1}{3}}}$$

input
```
int((1-3^(1/2)+(b/a)^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
(i*(a**(1/3)*sqrt(3)*int(sqrt(a + b*x**3)/(a + b*x**3),x) - a**(1/3)*int(s
qrt(a + b*x**3)/(a + b*x**3),x) - b**(1/3)*int((sqrt(a + b*x**3)*x)/(a + b
*x**3),x)))/a**(1/3)
```

3.91 $\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 490

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{2d\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c-\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*d*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*a^(1/3)*d*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^
(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b
^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/
2)/(b*x^3+a)^(1/2)+2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*c-(1-3^(1/2))*a^
(1/3)*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{a+bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[a + b*x^3]
)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2417}$$

$$\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{d\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{d\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[a + b*x^3],x]
```

output
```
(d*((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3
^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/b^(1/3) + (2*Sq
rt[2 + Sqrt[3]]*(c - ((1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.47

method	result
default	$-\frac{2 i c \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^{3}+a}}$
elliptic	$-\frac{2 i c \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3b\sqrt{b\,x^{3}+a}}$

input
```
int((d*x+c)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b
^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(b)*d*weierstrassZeta
(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3+a)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c + d*x)/(a + b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{a+bx^3}}\,dx = \left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)c + \left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)d$$

input
```
int((d*x+c)/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a + b*x**3),x)*c + int((sqrt(a + b*x**3)*x)/(a + b*x
**3),x)*d
```

3.92 $\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 503

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{2d\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}c+\left(1-\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
2*d*(-b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)-3^(1/4)*(1/2*
6^(1/2)-1/2*2^(1/2))*a^(1/3)*d*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3
^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x),I*3^(1/2)+2*I)/
b^(2/3)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1
/2)/(-b*x^3+a)^(1/2)-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(b^(1/3)*c+(1-3^(1/2))*
a^(1/3)*d)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((
1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1
/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(2/3)/(a^(
1/3)*(a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{a-bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[a - b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] +
d*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[a - b*x^3])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx$$

$\downarrow$ 2417

$$\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\int \frac{1}{\sqrt{a-bx^3}}dx-\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}$$

$\downarrow$ 759

$$-\frac{d\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{a-bx^3}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$\downarrow$ 2416

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\frac{d\left(\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt{a-bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}\right)}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[a - b*x^3],x]
```

output
```
-((d*((-2*Sqrt[a - b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) +
 (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])))/b^(1/3)) - (
2*Sqrt[2 + Sqrt[3]]*(c + ((1 - Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*S
qrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]
*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.35

method	result
default	$\frac{2ic\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^2\right)^{\frac{1}{3}}}}\,\mathrm{EllipticF}\left(\right.}{3b\sqrt{-b\,x^3+a}}$
elliptic	$\frac{2ic\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^2\right)^{\frac{1}{3}}}}\,\mathrm{EllipticF}\left(\right.}{3b\sqrt{-b\,x^3+a}}$

input
```
int((d*x+c)/(-b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1
/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3
^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^
(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1
/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^
(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/
(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2
)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b
^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}c\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{-b}d\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(-b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(-b)*d*weierstrassZe
ta(0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{2i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-b*x**3+a)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(2*I*pi)/a)/(3*sq
rt(a)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp
_polar(2*I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{a-b\,x^3}}\,dx$$

input
```
int((c + d*x)/(a - b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{a-bx^3}}\,dx = \left(\int \frac{\sqrt{-b\,x^3+a}}{-b\,x^3+a}dx\right)c + \left(\int \frac{\sqrt{-b\,x^3+a}\,x}{-b\,x^3+a}dx\right)d$$

input
```
int((d*x+c)/(-b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**3)/(a - b*x**3),x)*c + int((sqrt(a - b*x**3)*x)/(a - b*x
**3),x)*d
```

3.93 $\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 515

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{2d\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}c+\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2*d*(b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)+3^(1/4)*(1/2*
6^(1/2)+1/2*2^(1/2))*a^(1/3)*d*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticE(((1+3
^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),2*I-I*3^(1/2))/
b^(2/3)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(
1/2)/(b*x^3-a)^(1/2)-2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(b^(1/3)*c+(1+3^(1/2))*
a^(1/3)*d)*(a^(1/3)-b^(1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((
1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1
/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-a^
(1/3)*(a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \frac{x\sqrt{1-\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\frac{bx^3}{a}\right)\right)}{2\sqrt{-a+bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a + b*x^3],x]
```

output
```
(x*Sqrt[1 - (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, (b*x^3)/a] +
d*x*Hypergeometric2F1[1/2, 2/3, 5/3, (b*x^3)/a]))/(2*Sqrt[-a + b*x^3])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{bx^3-a}}dx$$

$\big\downarrow$ 2419

$$\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\int \frac{1}{\sqrt{bx^3-a}}dx-\frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}$$

$\big\downarrow$ 760

$$-\frac{d\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt{bx^3-a}}dx}{\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$\big\downarrow$ 2418

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\frac{d\left(\frac{2\sqrt{bx^3-a}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}\right.}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a + b*x^3],x]
```

output
```
-((d*((2*Sqrt[-a + b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) -
 (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) +
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*El
lipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])))/b^(1/3))
 - (2*Sqrt[2 - Sqrt[3]]*(c + ((1 + Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/
3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.33

method	result
default	$\frac{2ic\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\mathrm{EllipticF}\left(\right.}{3b\sqrt{b\,x^{3}-a}}$
elliptic	$\frac{2ic\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\left(x+\frac{\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(a\,b^{2}\right)^{\frac{1}{3}}}}\,\mathrm{EllipticF}\left(\right.}{3b\sqrt{b\,x^{3}-a}}$

input
```
int((d*x+c)/(b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*c*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/
2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x
^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/
3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+2/3*I*d*3^
(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(
1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*(a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(b*x^3-a)^(1/2
)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*3^(1
/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a
*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.08

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \frac{2\left(\sqrt{b}c\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)-\sqrt{b}d\text{weierstrassZeta}\left(0,\frac{4a}{b},\text{weierstrassPInverse}\left(0,\frac{4a}{b},x\right)\right)\right)}{b}$$

input
```
integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(b)*c*weierstrassPInverse(0, 4*a/b, x) - sqrt(b)*d*weierstrassZeta(
0, 4*a/b, weierstrassPInverse(0, 4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.14

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3-a)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3/a)/(3*sqrt(a)*gamma(4/3
)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3/a)/(3*sqrt(a)*ga
mma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx=\int \frac{c+d\,x}{\sqrt{b\,x^3-a}}\,dx$$

input
```
int((c + d*x)/(b*x^3 - a)^(1/2),x)
```

output
```
int((c + d*x)/(b*x^3 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^3}}\,dx=-\left(\int \frac{\sqrt{b\,x^3-a}}{-b\,x^3+a}dx\right)c-\left(\int \frac{\sqrt{b\,x^3-a}\,x}{-b\,x^3+a}dx\right)d$$

input
```
int((d*x+c)/(b*x^3-a)^(1/2),x)
```

output
```
- (int(sqrt( - a + b*x**3)/(a - b*x**3),x)*c + int((sqrt( - a + b*x**3)*x
)/(a - b*x**3),x)*d)
```

3.94 $\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 508

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{2d\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$

$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}d\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}c-\left(1+\sqrt{3}\right)\sqrt[3]{a}d\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2*d*(-b*x^3-a)^(1/2)/b^(2/3)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)+3^(1/4)*(1/2
*6^(1/2)+1/2*2^(1/2))*a^(1/3)*d*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x),2*I-I*3^(1/2))
/b^(2/3)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^
(1/2)/(-b*x^3-a)^(1/2)+2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(b^(1/3)*c-(1+3^(1/2)
)*a^(1/3)*d)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^
(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/b^(2/3)/(-
a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x
^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.15

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \frac{x\sqrt{1+\frac{bx^3}{a}}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt{-a-bx^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a - b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
 + d*x*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(2*Sqrt[-a - b*x^3
])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2419}$$

$$\left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{d\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$\frac{d\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{-bx^3-a}}dx}{\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(c-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+\right.}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{d\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt{-a-bx^3}}{\sqrt[3]{b}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)}{\sqrt[3]{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a - b*x^3],x]
```

output
```
(d*((-2*Sqrt[-a - b*x^3])/(b^(1/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) +
(3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)
*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])))/b^(1/3) +
 (2*Sqrt[2 - Sqrt[3]]*(c - ((1 + Sqrt[3])*a^(1/3)*d)/b^(1/3))*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(1/3)
*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.43

method	result
default	$-\frac{2 i c \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3 b \sqrt{-b\,x^{3}-a}}$
elliptic	$-\frac{2 i c \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{3 b \sqrt{-b\,x^{3}-a}}$

input
```
int((d*x+c)/(-b*x^3-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*c*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)/(-b*x^3-a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))-2/3*I*d*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(
1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)/(-b*x^3-a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a
*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(
1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.09

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{2\left(\sqrt{-b}c\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{-b}d\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b}\right.\right.\right.}{b}$$

input
```
integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(-b)*c*weierstrassPInverse(0, -4*a/b, x) - sqrt(-b)*d*weierstrassZ
eta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.16

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-b*x**3-a)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*s
qrt(a)*gamma(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx = \int \frac{dx+c}{\sqrt{-bx^3-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^3 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx=\int \frac{c+d\,x}{\sqrt{-b\,x^3-a}}\,dx$$

input
```
int((c + d*x)/(- a - b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(- a - b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^3}}\,dx=-i\left(\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)c+\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)d\right)$$

input
```
int((d*x+c)/(-b*x^3-a)^(1/2),x)
```

output
```
- i*(int(sqrt(a + b*x**3)/(a + b*x**3),x)*c + int((sqrt(a + b*x**3)*x)/(a
+ b*x**3),x)*d)
```

3.95 $\int \frac{c+dx}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 246

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \frac{2d\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*d*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d*(1+x)*
((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*
3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+2/3*(1/2*6^(1/2)+
1/2*2^(1/2))*(c-(1-3^(1/2))*d)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*Ell
ipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = cx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right) + \frac{1}{2}dx^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[(c + d*x)/Sqrt[1 + x^3],x]
```

output
```
c*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + (d*x^2*Hypergeometric2F1[1/2,
 2/3, 5/3, -x^3])/2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2417}$$

$$\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{x^3+1}}dx+d\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$d\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+d\left(\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)$$

input
```
Int[(c + d*x)/Sqrt[1 + x^3],x]
```

output
```
d*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] +
x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]
*Sqrt[1 + x^3])) + (2*Sqrt[2 + Sqrt[3]]*(c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt
[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
+ Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^
2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.13

method	result
meijerg	$\frac{d\,x^2\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}+cx\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)$
default	$\frac{2c\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+2d\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$
elliptic	$\frac{2c\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+2d\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}$

input
```
int((d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2*hypergeom([1/2,2/3],[5/3],-x^3)+c*x*hypergeom([1/3,1/2],[4/3],-x
^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = 2\,c\text{weierstrassPInverse}(0,-4,x) - 2\,d\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*c*weierstrassPInverse(0, -4, x) - 2*d*weierstrassZeta(0, -4, weierstrass
PInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.25

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(x**3+1)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3
*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.52

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)/(x^3 + 1)^(1/2),x)
```

output
```
(2*c*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF
(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2
)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{1+x^3}}\,dx = \left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)c + \left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)d$$

input
```
int((d*x+c)/(x^3+1)^(1/2),x)
```

output
```
int(sqrt(x**3 + 1)/(x**3 + 1),x)*c + int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)*
d
```

3.96 $\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 271

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \frac{2d\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
2*d*(-x^3+1)^(1/2)/(1+3^(1/2)-x)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*d*(1-x)
*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticE((1-3^(1/2)-x)/(1+3^(1/2)-x),I
*3^(1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)-2/3*(1/2*6^(1/2
)+1/2*2^(1/2))*(c+d-3^(1/2)*d)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*Ell
ipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2
)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = cx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right) + \frac{1}{2}dx^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[(c + d*x)/Sqrt[1 - x^3],x]
```

output
```
c*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + (d*x^2*Hypergeometric2F1[1/2,
2/3, 5/3, x^3])/2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2417, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{ 2417}$$

$$\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-x^3}}dx - d\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 759}$$

$$-d\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-d\left(\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}\right)$$

input
```
Int[(c + d*x)/Sqrt[1 - x^3],x]
```

output
```
-(d*((-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1
- x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3]
 - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)
^2]*Sqrt[1 - x^3]))) - (2*Sqrt[2 + Sqrt[3]]*(c + d - Sqrt[3]*d)*(1 - x)*Sq
rt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

method	result
meijerg	$\frac{d\,x^2 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}+c x \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)$
default	$-\frac{2 i c\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2 i d\sqrt{3}\,\sqrt{i\left(x-\right.}}{}$
elliptic	$-\frac{2 i c\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{i\sqrt{3}}{2}+\frac{1}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2 i d\sqrt{3}\,\sqrt{i\left(x-\right.}}{}$

input
```
int((d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2*hypergeom([1/2,2/3],[5/3],x^3)+c*x*hypergeom([1/3,1/2],[4/3],x^3
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -2i\,c\text{weierstrassPInverse}(0,4,x) + 2i\,d\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*c*weierstrassPInverse(0, 4, x) + 2*I*d*weierstrassZeta(0, 4, weierstr
assPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(
4/3)) + d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.50

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)/(1 - x^3)^(1/2),x)
```

output
```
- (2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) -
 (2*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1
i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 -
1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1
/2))
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)c - \left(\int \frac{\sqrt{-x^3+1}\,x}{x^3-1}dx\right)d$$

input
```
int((d*x+c)/(-x^3+1)^(1/2),x)
```

output
```
 - (int(sqrt( - x**3 + 1)/(x**3 - 1),x)*c + int((sqrt( - x**3 + 1)*x)/(x**
3 - 1),x)*d)
```

3.97 $\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 275

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{2d\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2-\sqrt{3}}(c+d+\sqrt{3}d)\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2*d*(x^3-1)^(1/2)/(1-3^(1/2)-x)+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*d*(1-x)
*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticE((1+3^(1/2)-x)/(1-3^(1/2)-x),2
*I-I*3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-2/3*(1/2*6^(1/2
)-1/2*2^(1/2))*(c+d+3^(1/2)*d)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*Ell
ipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/
2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \frac{x\sqrt{1-x^3}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-1 + x^3],x]
```

output
```
(x*Sqrt[1 - x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, x^3] + d*x*Hypergeo
metric2F1[1/2, 2/3, 5/3, x^3]))/(2*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2419}$$

$$\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{x^3-1}}dx-d\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$-d\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-$$
$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-$$
$$d\left(\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right)$$

input
```
Int[(c + d*x)/Sqrt[-1 + x^3],x]
```

output
```
-(d*((2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
 - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3]))) - (2*Sqrt[2 - Sqrt[3]]*(c + d + Sqrt[3]*d)*(1 - x
)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

method	result
meijerg	$\frac{d\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, x^{2} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^{3}\right)}{2\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}+\frac{c\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, x \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^{3}\right)}{\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}$
default	$\frac{2c\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\frac{2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-}{\frac{3}{2}-}}}{}$
elliptic	$\frac{2c\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{i\sqrt{3}}{2}+\frac{1}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\frac{2d\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-}{\frac{3}{2}-}}}{}$

input
```
int((d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3],[
5/3],x^3)+c/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/
2],[4/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = 2\,c\text{weierstrassPInverse}(0,4,x) - 2\,d\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*c*weierstrassPInverse(0, 4, x) - 2*d*weierstrassZeta(0, 4, weierstrassPI
nverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\,x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\,x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(x**3-1)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - I*d*x**
2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \int \frac{dx+c}{\sqrt{x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.36

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)/(x^3 - 1)^(1/2),x)
```

output
```
- (2*c*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*d*(((3^(1/2)*1i)/2 - 1/2)*elli
pticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
x^3)^(1/2)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{-1+x^3}}\,dx = \left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right)c+\left(\int \frac{\sqrt{x^3-1}\,x}{x^3-1}dx\right)d$$

input
```
int((d*x+c)/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**3 - 1),x)*c + int((sqrt(x**3 - 1)*x)/(x**3 - 1),x)*
d
```

3.98 $\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 261

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -\frac{2d\sqrt{-1-x^3}}{1-\sqrt{3}+x}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}d(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{2\sqrt{2-\sqrt{3}}\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output

```
-2*d*(-x^3-1)^(1/2)/(1+x-3^(1/2))+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*d*(1+x
)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),
2*I-I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)+2/3*(1/2*6^(1
/2)-1/2*2^(1/2))*(c-(1+3^(1/2))*d)*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)
*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x
-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.24

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[(c + d*x)/Sqrt[-1 - x^3],x]
```

output
```
(x*Sqrt[1 + x^3]*(2*c*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3] + d*x*Hyperge
ometric2F1[1/2, 2/3, 5/3, -x^3]))/(2*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \text{ 2419}$$

$$\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{-x^3-1}}dx+d\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$d\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+d\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}\right)$$

input
```
Int[(c + d*x)/Sqrt[-1 - x^3],x]
```

output
```
d*((-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 +
 x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3]
+ x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x
)^2)]*Sqrt[-1 - x^3])) + (2*Sqrt[2 - Sqrt[3]]*(c - (1 + Sqrt[3])*d)*(1 + x
)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] +
x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[
3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.14

method	result
meijerg	$-\frac{i d\, x^{2} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{2}{3}\right], \left[\frac{5}{3}\right], -x^{3}\right)}{2}-i c x \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], -x^{3}\right)$
default	$-\frac{2 i c\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2 i d\sqrt{3}\sqrt{i\left(x-\frac{1}{2}\right.}}{}$
elliptic	$-\frac{2 i c\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2 i d\sqrt{3}\sqrt{i\left(x-\frac{1}{2}\right.}}{}$

input
```
int((d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*x^2*hypergeom([1/2,2/3],[5/3],-x^3)-I*c*x*hypergeom([1/3,1/2],[4/
3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -2i\,c\text{weierstrassPInverse}(0,-4,x) + 2i\,d\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*c*weierstrassPInverse(0, -4, x) + 2*I*d*weierstrassZeta(0, -4, weiers
trassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.25

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -\frac{icx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{idx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
-I*c*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma
(4/3)) - I*d*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi
))/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \int \frac{dx+c}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate((d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.55

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

input
```
int((c + d*x)/(- x^3 - 1)^(1/2),x)
```

output
```
(2*c*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1
/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2
*d*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1
/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{-1-x^3}}\,dx = -i\left(\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)c+\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)d\right)$$

input
```
int((d*x+c)/(-x^3-1)^(1/2),x)
```

output
```
 - i*(int(sqrt(x**3 + 1)/(x**3 + 1),x)*c + int((sqrt(x**3 + 1)*x)/(x**3 +
1),x)*d)
```

3.99 $\int \sqrt[3]{a+bx^3}(A+Bx+Dx^3)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 193

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Dx^3\right)\,dx = \frac{aDx\sqrt[3]{a+bx^3}}{10b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15Ax+10Bx^2+6Dx^4\right)-\frac{aB\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{a(5Ab-aD)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b\left(a+bx^3\right)^{2/3}}-\frac{aB\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}$$

output

```
1/10*a*D*x*(b*x^3+a)^(1/3)/b+1/30*(b*x^3+a)^(1/3)*(6*D*x^4+10*B*x^2+15*A*x
)-1/9*a*B*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2
/3)+1/10*a*(5*A*b-D*a)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x
^3/a)/b/(b*x^3+a)^(2/3)-1/6*a*B*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 6.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.52

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Dx^3\right)\,dx = \frac{x\sqrt[3]{a+bx^3}\left(4A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+2Bx\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+D\right.}{4\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(A + B*x + D*x^3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(4*A*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)]
+ 2*B*x*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + D*x^3*Hypergeome
tric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)]))/(4*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2392, 27, 2427, 27, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2392}$$

$$a\int \frac{6Dx^3+10Bx+15A}{30\left(bx^3+a\right)^{2/3}}dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15Ax+10Bx^2+6Dx^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}a\int \frac{6Dx^3+10Bx+15A}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15Ax+10Bx^2+6Dx^4\right)$$

$$\Big\downarrow \text{2427}$$

$$\frac{1}{30}a\left(\frac{\int \frac{2(3(5Ab-aD)+10bBx)}{(bx^3+a)^{2/3}}dx}{2b}+\frac{3Dx\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{30}\sqrt[3]{a+bx^3}(15Ax+10Bx^2+6Dx^4)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}a\left(\frac{\int \frac{3(5Ab-aD)+10bBx}{(bx^3+a)^{2/3}}dx}{b}+\frac{3Dx\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{30}\sqrt[3]{a+bx^3}(15Ax+10Bx^2+6Dx^4)$$

$$\Big\downarrow \text{2432}$$

$$\frac{1}{30}a\left(\frac{\int \left(\frac{3(5Ab-aD)}{(bx^3+a)^{2/3}}+\frac{10bBx}{(bx^3+a)^{2/3}}\right)dx}{b}+\frac{3Dx\sqrt[3]{a+bx^3}}{b}\right)+$$
$$\frac{1}{30}\sqrt[3]{a+bx^3}(15Ax+10Bx^2+6Dx^4)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{30}a\left(\frac{\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}(5Ab-aD)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-\frac{10\sqrt[3]{b}B\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-5\sqrt[3]{b}B\log\left(\sqrt[3]{b}x-\sqrt[3]{a}\right.}{b}\right.$$
$$\frac{1}{30}\sqrt[3]{a+bx^3}(15Ax+10Bx^2+6Dx^4)$$

input
```
Int[(a + b*x^3)^(1/3)*(A + B*x + D*x^3),x]
```

output
```
((a + b*x^3)^(1/3)*(15*A*x + 10*B*x^2 + 6*D*x^4))/30 + (a*((3*D*x*(a + b*x
^3)^(1/3))/b + ((-10*b^(1/3)*B*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3)
)/Sqrt[3]])/Sqrt[3] + (3*(5*A*b - a*D)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - 5*b^(1/3)*B*Log[b
^(1/3)*x - (a + b*x^3)^(1/3)])/b))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(D x^{3}+B x +A \right)dx$$

input
```
int((b*x^3+a)^(1/3)*(D*x^3+B*x+A),x)
```

output
```
int((b*x^3+a)^(1/3)*(D*x^3+B*x+A),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Dx^3\right)\,dx=\int \left(Dx^3+Bx+A\right)\left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(D*x^3+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + B*x + A)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Dx^3\right)\,dx=\frac{A\sqrt[3]{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt[3]{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{D\sqrt[3]{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(D*x**3+B*x+A),x)
```

output
```
A*a**(1/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + B*a**(1/3)*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + D*a**(1/3)*x**4*gamma(4/3)*hype
r((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}(A+Bx+Dx^3)\ dx = \int (Dx^3+Bx+A)\left(bx^3+a\right)^{\frac{1}{3}} dx$$

input
```
integrate((b*x^3+a)^(1/3)*(D*x^3+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + B*x + A)*(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}(A+Bx+Dx^3)\ dx = \int (Dx^3+Bx+A)\left(bx^3+a\right)^{\frac{1}{3}} dx$$

input
```
integrate((b*x^3+a)^(1/3)*(D*x^3+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)*(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}(A+Bx+Dx^3)\ dx = \int \left(b\,x^3+a\right)^{1/3}\left(A+B\,x+x^3\,D\right) dx$$

input
```
int((a + b*x^3)^(1/3)*(A + B*x + x^3*D),x)
```

output
```
int((a + b*x^3)^(1/3)*(A + B*x + x^3*D), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Dx^3\right)\,dx$$
$$=\frac{15(b\,x^3+a)^{\frac{1}{3}}\,abx+3(b\,x^3+a)^{\frac{1}{3}}\,adx+10(b\,x^3+a)^{\frac{1}{3}}\,b^2x^2+6(b\,x^3+a)^{\frac{1}{3}}\,bd\,x^4+15\left(\int\frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2b}{30b}$$

input
```
int((b*x^3+a)^(1/3)*(D*x^3+B*x+A),x)
```

output
```
(15*(a + b*x**3)**(1/3)*a*b*x + 3*(a + b*x**3)**(1/3)*a*d*x + 10*(a + b*x*
*3)**(1/3)*b**2*x**2 + 6*(a + b*x**3)**(1/3)*b*d*x**4 + 15*int((a + b*x**3
)**(1/3)/(a + b*x**3),x)*a**2*b - 3*int((a + b*x**3)**(1/3)/(a + b*x**3),x
)*a**2*d + 10*int(((a + b*x**3)**(1/3)*x)/(a + b*x**3),x)*a*b**2)/(30*b)
```

3.100 $\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{Dx(a+bx^3)^{2/3}}{3b} + \frac{(3Ab-aD)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{(3Ab-aD)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output

```
1/3*D*x*(b*x^3+a)^(2/3)/b+1/9*(3*A*b-D*a)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3
+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)+1/2*B*x^2*(1+b*x^3/a)^(1/3)*hypergeom(
[1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/6*(3*A*b-D*a)*ln(-b^(1/3)*x+(
b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 10.38 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}\left(\frac{6aDx}{b\sqrt[3]{a+bx^3}}+\frac{6Dx^4}{\sqrt[3]{a+bx^3}} + \frac{2\sqrt{3}(3Ab-aD)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{4/3}} + \frac{9Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{6A\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} + \frac{2aD\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{b^{4/3}} + \frac{3A\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{\sqrt[3]{b}} - \frac{aD\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{b^{4/3}}\right)$$

input
```
Integrate[(A + B*x + D*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
((6*a*D*x)/(b*(a + b*x^3)^(1/3)) + (6*D*x^4)/(a + b*x^3)^(1/3) + (2*Sqrt[3
]*(3*A*b - a*D)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/b^(
4/3) + (9*B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((
b*x^3)/a)])/(a + b*x^3)^(1/3) - (6*A*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)
])/b^(1/3) + (2*a*D*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/b^(4/3) + (3*A
*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])
/b^(1/3) - (a*D*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a +
 b*x^3)^(1/3)])/b^(4/3))/18
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \; 2432$$

$$\int \left(\frac{A}{\sqrt[3]{a+bx^3}}+\frac{Bx}{\sqrt[3]{a+bx^3}}+\frac{Dx^3}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{A\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{A\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}-\frac{aD\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{aD\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}+\frac{Bx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}+\frac{Dx\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(A + B*x + D*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(D*x*(a + b*x^3)^(2/3))/(3*b) + (A*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (a*D*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*
x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) + (B*x^2*(1 + (b*x^3)/a)^(1/3)*H
ypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (A*
Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (a*D*Log[-(b^(1/3)*x)
 + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{D x^{3}+B x+A}{\left(b \,x^{3}+a \right)^{\frac{1}{3}}}d x$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(1/3),x)
```

output
```
int((D*x^3+B*x+A)/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{Dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+B*x+A)/(b*x**3+a)**(1/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + B*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + D*x**4*gamma(4/3)*hyper((1/3, 4/3
), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*A + integr
ate((D*x^3 + B*x)/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{A+B\,x+x^3\,D}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((A + B*x + x^3*D)/(a + b*x^3)^(1/3),x)
```

output
```
int((A + B*x + x^3*D)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)d+\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**3)**(1/3),x)*d + int(x/(a + b*x**3)**(1/3),x)*b + int(1
/(a + b*x**3)**(1/3),x)*a
```

3.101 $\int \frac{A+Bx+Dx^3}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{Dx\sqrt[3]{a+bx^3}}{2b} - \frac{B\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{(2Ab-aD)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} - \frac{B\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
1/2*D*x*(b*x^3+a)^(1/3)/b-1/3*B*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))*3^(1/2)/b^(2/3)+1/2*(2*A*b-D*a)*x*(1+b*x^3/a)^(2/3)*hypergeom([1
/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)-1/2*B*ln(b^(1/3)*x-(b*x^3+a)^(1
/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx+Dx^3}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{4Ax\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+2Bx^2\,\text{Hypergeometric2F1}}{4\left(a+bx^3\right)^{2}}$$

input
```
Integrate[(A + B*x + D*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(4*A*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
] + 2*B*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)] + D*x^4*(1
 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a
+ b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2427, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Dx^3}{\left(a+bx^3\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{\int \frac{2Ab+2Bxb-aD}{\left(bx^3+a\right)^{2/3}}dx}{2b}+\frac{D\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{ 2432}$$

$$\frac{\int \left(\frac{2Ab\left(1-\frac{aD}{2Ab}\right)}{\left(bx^3+a\right)^{2/3}}+\frac{2bBx}{\left(bx^3+a\right)^{2/3}}\right)dx}{2b}+\frac{D\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2Ab-aD)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-\frac{2\sqrt[3]{b}B\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\sqrt[3]{b}B\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b}+\frac{Dx\sqrt[3]{a+bx^3}}{2b}$$

input
```
Int[(A + B*x + D*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(D*x*(a + b*x^3)^(1/3))/(2*b) + ((-2*b^(1/3)*B*ArcTan[(1 + (2*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + ((2*A*b - a*D)*x*(1 + (b*x^3)/a)^(2/
3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - b^(
1/3)*B*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{Dx^3 + Bx + A}{(b\,x^3 + a)^{\frac{2}{3}}}dx$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(2/3),x)
```

output
```
int((D*x^3+B*x+A)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{2/3}}\,dx = \int \frac{Dx^3 + Bx + A}{(bx^3 + a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + B*x + A)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.77

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{2/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3}, \frac{2}{3} \\ \frac{4}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3}, \frac{2}{3} \\ \frac{5}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{Dx^4\Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+B*x+A)/(b*x**3+a)**(2/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(4/3)) + B*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + D*x**4*gamma(4/3)*hyper((2/3, 4/3
), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{A+B\,x+x^3\,D}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((A + B*x + x^3*D)/(a + b*x^3)^(2/3),x)
```

output
```
int((A + B*x + x^3*D)/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{2/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)d + \left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(2/3),x)
```

output
```
int(x**3/(a + b*x**3)**(2/3),x)*d + int(x/(a + b*x**3)**(2/3),x)*b + int(1
/(a + b*x**3)**(2/3),x)*a
```

3.102 $\int \frac{A+Bx+Dx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}\,dx=\frac{(Ab-aD)x}{ab\sqrt[3]{a+bx^3}}+\frac{D\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}+\frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt[3]{a+bx^3}}-\frac{D\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

output
```
(A*b-D*a)*x/a/b/(b*x^3+a)^(1/3)+1/3*D*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^
(1/3))*3^(1/2))*3^(1/2)/b^(4/3)+1/2*B*x^2*(1+b*x^3/a)^(1/3)*hypergeom([2/3
, 4/3],[5/3],-b*x^3/a)/a/(b*x^3+a)^(1/3)-1/2*D*ln(-b^(1/3)*x+(b*x^3+a)^(1/
3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{4Ax+2Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+Dx^4\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hype}}{4a\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x + D*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(4*A*x + 2*B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[2/3, 4/3, 5/3, -(
(b*x^3)/a)] + D*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3,
 -((b*x^3)/a)])/(4*a*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2397, 25, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(-aD+Ab+bBx)}{ab\sqrt[3]{a+bx^3}}-\frac{\int -\frac{aD-bBx}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aD-bBx}{\sqrt[3]{bx^3+a}}dx}{ab}+\frac{x(-aD+Ab+bBx)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{2432}$$

$$\frac{\int \left(\frac{aD}{\sqrt[3]{bx^3+a}} - \frac{bBx}{\sqrt[3]{bx^3+a}}\right)dx}{ab} + \frac{x(-aD+Ab+bBx)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(-aD+Ab+bBx)}{ab\sqrt[3]{a+bx^3}} + \frac{\frac{aD\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{bBx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{aD\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}}{ab}$$

input
```
Int[(A + B*x + D*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(x*(A*b - a*D + b*B*x))/(a*b*(a + b*x^3)^(1/3)) + ((a*D*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (b*B*x^2*(1 + (b*
x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(2*(a + b*x^
3)^(1/3)) - (a*D*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{D x^3 + Bx + A}{(b\,x^3 + a)^{\frac{4}{3}}} dx$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(4/3),x)
```

output
```
int((D*x^3+B*x+A)/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{4/3}}\,dx = \int \frac{D x^3 + Bx + A}{(bx^3 + a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + B*x + A)*(b*x^3 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{Dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+B*x+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + B*x**2*gamm
a(2/3)*hyper((2/3, 4/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gam
ma(5/3)) + D*x**4*gamma(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*
pi)/a)/(3*a**(4/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
A*x/((b*x^3 + a)^(1/3)*a) + integrate((D*x^3 + B*x)/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}\,dx=\int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}\,dx=\int \frac{A+B\,x+x^3\,D}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((A + B*x + x^3*D)/(a + b*x^3)^(4/3),x)
```

output
```
int((A + B*x + x^3*D)/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{4/3}}\,dx=\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)d+\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)b+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)a$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*d + int(x
/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*b + int(1/((a + b
*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*a
```

3.103 $\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}} \, dx = -\frac{Dx}{b(a+bx^3)^{2/3}} + \frac{Bx^2}{2a(a+bx^3)^{2/3}} + \frac{(Ab+aD)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{ab(a+bx^3)^{2/3}}$$

output
```
-D*x/b/(b*x^3+a)^(2/3)+1/2*B*x^2/a/(b*x^3+a)^(2/3)+(A*b+D*a)*x*(1+b*x^3/a)
^(2/3)*hypergeom([1/3, 5/3],[4/3],-b*x^3/a)/a/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}} \, dx = \frac{2Bx^2 + 4Ax\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right) + Dx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{4a(a+bx^3)^{2/3}}$$

input
```
Integrate[(A + B*x + D*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
(2*B*x^2 + 4*A*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -(
(b*x^3)/a)] + D*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[4/3, 5/3, 7/3,
 -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2397, 25, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(-aD+Ab+bBx)}{2ab\left(a+bx^3\right)^{2/3}}-\frac{\int -\frac{Ab+aD}{(bx^3+a)^{2/3}}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aD}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(-aD+Ab+bBx)}{2ab\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aD+Ab)\int \frac{1}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(-aD+Ab+bBx)}{2ab\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+Ab)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2ab\left(a+bx^3\right)^{2/3}}+\frac{x(-aD+Ab+bBx)}{2ab\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(-aD+Ab+bBx)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}$$

input
```
Int[(A + B*x + D*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
(x*(A*b - a*D + b*B*x))/(2*a*b*(a + b*x^3)^(2/3)) + ((A*b + a*D)*x*(1 + (b
*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a +
 b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{Dx^3+Bx+A}{(b\,x^3+a)^{\frac{5}{3}}}dx$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(5/3),x)
```

output
```
int((D*x^3+B*x+A)/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + B*x + A)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{4}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right)}{3a^{\frac{5}{3}}\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{Dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\ \frac{7}{3}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+B*x+A)/(b*x**3+a)**(5/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/3)*gamma(4/3)) + B*x**2*gamma(2/3)/(3*a**(5/3)*(1 + b*x**3/a)**(2/3)*gam
ma(5/3)) + D*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_polar(I*
pi)/a)/(3*a**(5/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}}\,dx = \int \frac{A+B\,x+x^3\,D}{(b\,x^3+a)^{5/3}}\,dx$$

input
```
int((A + B*x + x^3*D)/(a + b*x^3)^(5/3),x)
```

output
```
int((A + B*x + x^3*D)/(a + b*x^3)^(5/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{5/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)d + \left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)a$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(5/3),x)
```

output
```
int(x**3/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*d + int(x
/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*b + int(1/((a + b
*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*a
```

3.104 $\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{(Ab-aD)x}{4ab\,(a+bx^3)^{4/3}} + \frac{(3Ab+aD)x}{4a^2b\sqrt[3]{a+bx^3}} + \frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^2\sqrt[3]{a+bx^3}}$$

output
```
1/4*(A*b-D*a)*x/a/b/(b*x^3+a)^(4/3)+1/4*(3*A*b+D*a)*x/a^2/b/(b*x^3+a)^(1/3
)+1/2*B*x^2*(1+b*x^3/a)^(1/3)*hypergeom([2/3, 7/3],[5/3],-b*x^3/a)/a^2/(b*
x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx = \frac{4aAx+3Abx^4+aDx^4+2Bx^2(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},-\right.}{4a^2\,(a+bx^3)^{4/3}}$$

input
```
Integrate[(A + B*x + D*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(4*a*A*x + 3*A*b*x^4 + a*D*x^4 + 2*B*x^2*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3)
*Hypergeometric2F1[2/3, 7/3, 5/3, -((b*x^3)/a)])/(4*a^2*(a + b*x^3)^(4/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2397, 25, 2394, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Dx^3}{\left(a+bx^3\right)^{7/3}}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(-aD+Ab+bBx)}{4ab\left(a+bx^3\right)^{4/3}}-\frac{\int -\frac{3Ab+2Bxb+aD}{\left(bx^3+a\right)^{4/3}}dx}{4ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3Ab+2Bxb+aD}{\left(bx^3+a\right)^{4/3}}dx}{4ab}+\frac{x(-aD+Ab+bBx)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}-\frac{\int \frac{2bBx}{\sqrt[3]{bx^3+a}}dx}{a}}{4ab}+\frac{x(-aD+Ab+bBx)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}-\frac{2bB\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{a}}{4ab}+\frac{x(-aD+Ab+bBx)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\frac{x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}-\frac{2bB\sqrt[3]{\frac{bx^3}{a}+1}\int\frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{a\sqrt[3]{a+bx^3}}}{4ab}+\frac{x(-aD+Ab+bBx)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\frac{x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}-\frac{bBx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{a\sqrt[3]{a+bx^3}}}{4ab}+\frac{x(-aD+Ab+bBx)}{4ab\left(a+bx^3\right)^{4/3}}$$

input
```
Int[(A + B*x + D*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(x*(A*b - a*D + b*B*x))/(4*a*b*(a + b*x^3)^(4/3)) + ((x*(3*A*b + a*D + 2*b
*B*x))/(a*(a + b*x^3)^(1/3)) - (b*B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometr
ic2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a*(a + b*x^3)^(1/3)))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{D x^{3}+B x +A}{\left(b \,x^{3}+a \right)^{\frac{7}{3}}}d x$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(7/3),x)
```

output
```
int((D*x^3+B*x+A)/(b*x^3+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{Dx^{3}+Bx+A}{(bx^{3}+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + B*x + A)*(b*x^3 + a)^(2/3)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^
2*b*x^3 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx = A\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right) + \frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{Dx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+B*x+A)/(b*x**3+a)**(7/3),x)
```

output
```
A*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(
7/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**
(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)
**(1/3)*gamma(7/3))) + B*x**2*gamma(2/3)*hyper((2/3, 7/3), (5/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(7/3)*gamma(5/3)) + D*x**4*gamma(4/3)/(3*a**(7/3)
*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3
)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*A*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a^2) + integrate((D*
x^3 + B*x)/((b^2*x^6 + 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx=\int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx=\int \frac{A+B\,x+x^3\,D}{(b\,x^3+a)^{7/3}}\,dx$$

input
```
int((A + B*x + x^3*D)/(a + b*x^3)^(7/3),x)
```

output
```
int((A + B*x + x^3*D)/(a + b*x^3)^(7/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{7/3}}\,dx=\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)d$$
$$+\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)b$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)a$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(7/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a +
 b*x**3)**(1/3)*b**2*x**6),x)*d + int(x/((a + b*x**3)**(1/3)*a**2 + 2*(a +
 b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)**(1/3)*b**2*x**6),x)*b + int(1/((a
 + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)**(1
/3)*b**2*x**6),x)*a
```

3.105 $\int \frac{A+Bx+Dx^3}{\left(a+bx^3\right)^{8/3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{8/3}}\,dx = -\frac{Dx}{4b\,(a+bx^3)^{5/3}} + \frac{Bx^2}{5a\,(a+bx^3)^{5/3}} + \frac{3Bx^2}{10a^2\,(a+bx^3)^{2/3}} + \frac{(4Ab+aD)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a^2b\,(a+bx^3)^{2/3}}$$

output
```
-1/4*D*x/b/(b*x^3+a)^(5/3)+1/5*B*x^2/a/(b*x^3+a)^(5/3)+3/10*B*x^2/a^2/(b*x
^3+a)^(2/3)+1/4*(4*A*b+D*a)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 8/3],[4/3]
,-b*x^3/a)/a^2/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{8/3}}\,dx = \frac{2Bx^2(5a+3bx^3)+20Ax(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right.}{20a^2\,(a+bx^3)^{5}}$$

input
```
Integrate[(A + B*x + D*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
(2*B*x^2*(5*a + 3*b*x^3) + 20*A*x*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hyperg
eometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)] + 5*D*x^4*(a + b*x^3)*(1 + (b*x^3
)/a)^(2/3)*Hypergeometric2F1[4/3, 8/3, 7/3, -((b*x^3)/a)])/(20*a^2*(a + b*
x^3)^(5/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2397, 25, 2394, 25, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{8/3}}\,dx$$

$$\Big\downarrow \; 2397$$

$$\frac{x(-aD+Ab+bBx)}{5ab\,(a+bx^3)^{5/3}}-\frac{\int -\frac{4Ab+3Bxb+aD}{(bx^3+a)^{5/3}}dx}{5ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{4Ab+3Bxb+aD}{(bx^3+a)^{5/3}}dx}{5ab}+\frac{x(-aD+Ab+bBx)}{5ab\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}-\frac{\int -\frac{4Ab+aD}{\left(bx^3+a\right)^{2/3}}dx}{2a}}{5ab}+\frac{x(-aD+Ab+bBx)}{5ab\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{4Ab+aD}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x(-aD+Ab+bBx)}{5ab\,(a+bx^3)^{5/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(aD+4Ab)\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}+\frac{x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x(-aD+Ab+bBx)}{5ab\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+4Ab)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}+\frac{x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x(-aD+Ab+bBx)}{5ab\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+4Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x(-aD+Ab+bBx)}{5ab\left(a+bx^3\right)^{5/3}}$$

input
```
Int[(A + B*x + D*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
(x*(A*b - a*D + b*B*x))/(5*a*b*(a + b*x^3)^(5/3)) + ((x*(4*A*b + a*D + 3*b
*B*x))/(2*a*(a + b*x^3)^(2/3)) + ((4*A*b + a*D)*x*(1 + (b*x^3)/a)^(2/3)*Hy
pergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a + b*x^3)^(2/3)))/(5*
a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{D x^{3}+B x +A}{\left(b \,x^{3}+a \right)^{\frac{8}{3}}}d x$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(8/3),x)
```

output
```
int((D*x^3+B*x+A)/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{8/3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + B*x + A)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^
2*b*x^3 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 52.88 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{8/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{8}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)} + B\left(\frac{5ax^2\Gamma\left(\frac{2}{3}\right)}{9a^{\frac{11}{3}}\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)+9a^{\frac{8}{3}}bx^3\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)} + \frac{3bx^5\Gamma\left(\frac{2}{3}\right)}{9a^{\frac{11}{3}}\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)+9a^{\frac{8}{3}}bx^3\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}\right) + \frac{Dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{8}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+B*x+A)/(b*x**3+a)**(8/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
8/3)*gamma(4/3)) + B*(5*a*x**2*gamma(2/3)/(9*a**(11/3)*(1 + b*x**3/a)**(2/
3)*gamma(8/3) + 9*a**(8/3)*b*x**3*(1 + b*x**3/a)**(2/3)*gamma(8/3)) + 3*b*
x**5*gamma(2/3)/(9*a**(11/3)*(1 + b*x**3/a)**(2/3)*gamma(8/3) + 9*a**(8/3)
*b*x**3*(1 + b*x**3/a)**(2/3)*gamma(8/3))) + D*x**4*gamma(4/3)*hyper((4/3,
 8/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{8/3}}\,dx = \int \frac{Dx^3 + Bx + A}{(bx^3 + a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{8/3}}\,dx = \int \frac{Dx^3 + Bx + A}{(bx^3 + a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{8/3}}\,dx = \int \frac{A + B\,x + x^3\,D}{(b\,x^3 + a)^{8/3}}\,dx$$

input
```
int((A + B*x + x^3*D)/(a + b*x^3)^(8/3),x)
```

output
```
int((A + B*x + x^3*D)/(a + b*x^3)^(8/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{8/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)d + \left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)a$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(8/3),x)
```

output
```
int(x**3/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a +
 b*x**3)**(2/3)*b**2*x**6),x)*d + int(x/((a + b*x**3)**(2/3)*a**2 + 2*(a +
 b*x**3)**(2/3)*a*b*x**3 + (a + b*x**3)**(2/3)*b**2*x**6),x)*b + int(1/((a
 + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a + b*x**3)**(2
/3)*b**2*x**6),x)*a
```

3.106 $\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{(Ab-aD)x}{7ab\,(a+bx^3)^{7/3}} + \frac{(6Ab+aD)x}{28a^2b\,(a+bx^3)^{4/3}} + \frac{3(6Ab+aD)x}{28a^3b\sqrt[3]{a+bx^3}} + \frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{10}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^3\sqrt[3]{a+bx^3}}$$

output
```
1/7*(A*b-D*a)*x/a/b/(b*x^3+a)^(7/3)+1/28*(6*A*b+D*a)*x/a^2/b/(b*x^3+a)^(4/
3)+3/28*(6*A*b+D*a)*x/a^3/b/(b*x^3+a)^(1/3)+1/2*B*x^2*(1+b*x^3/a)^(1/3)*hy
pergeom([2/3, 10/3],[5/3],-b*x^3/a)/a^3/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{18Ab^2x^7+3abx^4(14A+Dx^3)+7a^2(4Ax+Dx^4)+14Bx^2(a+bx^3)^2\sqrt[3]{1+\frac{bx^3}{a}}\ \text{H}}{28a^3\,(a+bx^3)^{7/3}}$$

input
```
Integrate[(A + B*x + D*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(18*A*b^2*x^7 + 3*a*b*x^4*(14*A + D*x^3) + 7*a^2*(4*A*x + D*x^4) + 14*B*x^
2*(a + b*x^3)^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[2/3, 10/3, 5/3, -(
(b*x^3)/a)])/(28*a^3*(a + b*x^3)^(7/3))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2397, 25, 2394, 25, 2394, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(-aD+Ab+bBx)}{7ab(a+bx^3)^{7/3}}-\frac{\int -\frac{6Ab+5Bxb+aD}{(bx^3+a)^{7/3}}dx}{7ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6Ab+5Bxb+aD}{(bx^3+a)^{7/3}}dx}{7ab}+\frac{x(-aD+Ab+bBx)}{7ab(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}-\frac{\int -\frac{3(6Ab+aD)+10bBx}{(bx^3+a)^{4/3}}dx}{4a}}{7ab}+\frac{x(-aD+Ab+bBx)}{7ab(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3(6Ab+aD)+10bBx}{(bx^3+a)^{4/3}}dx}{4a}+\frac{x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x(-aD+Ab+bBx)}{7ab(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{\int \frac{10bBx}{\sqrt[3]{bx^3+a}}dx}{a}}{4a}+\frac{x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x(-aD+Ab+bBx)}{7ab\,(a+bx^3)^{7/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{10bB\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{a}}{4a}+\frac{x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x(-aD+Ab+bBx)}{7ab\,(a+bx^3)^{7/3}}$$

$$\Big\downarrow \; 889$$

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{10bB\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{a\sqrt[3]{a+bx^3}}}{4a}+\frac{x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x(-aD+Ab+bBx)}{7ab\,(a+bx^3)^{7/3}}$$

$$\Big\downarrow \; 888$$

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{5bBx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{a\sqrt[3]{a+bx^3}}}{4a}+\frac{x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x(-aD+Ab+bBx)}{7ab\,(a+bx^3)^{7/3}}$$

input

```
Int[(A + B*x + D*x^3)/(a + b*x^3)^(10/3),x]
```

output

```
(x*(A*b - a*D + b*B*x))/(7*a*b*(a + b*x^3)^(7/3)) + ((x*(6*A*b + a*D + 5*b
*B*x))/(4*a*(a + b*x^3)^(4/3)) + ((x*(3*(6*A*b + a*D) + 10*b*B*x))/(a*(a +
 b*x^3)^(1/3)) - (5*b*B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2
/3, 5/3, -((b*x^3)/a)])/(a*(a + b*x^3)^(1/3)))/(4*a))/(7*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{D x^3 + B x + A}{\left(b\,x^3 + a\right)^{\frac{10}{3}}}dx$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(10/3),x)
```

output
```
int((D*x^3+B*x+A)/(b*x^3+a)^(10/3),x)
```

Fricas [F]

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{10/3}}\,dx = \int \frac{Dx^3 + Bx + A}{(bx^3 + a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + B*x + A)*(b*x^3 + a)^(2/3)/(b^4*x^12 + 4*a*b^3*x^9 + 6*a
^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 149.38 (sec) , antiderivative size = 750, normalized size of antiderivative = 5.14

$$\int \frac{A + Bx + Dx^3}{(a + bx^3)^{10/3}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+B*x+A)/(b*x**3+a)**(10/3),x)
```

output
```
A*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) +
81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*
x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**
3/a)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*
x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gam
ma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a
**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*
gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*
b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*g
amma(10/3)) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**
(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3)
+ 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*
b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3))) + B*x**2*gamma(2/3)*hyper((2
/3, 10/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(10/3)*gamma(5/3)) + D*
(7*a*x**4*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 18*a
**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)*b**2*x**6*(
1 + b*x**3/a)**(1/3)*gamma(10/3)) + 3*b*x**7*gamma(4/3)/(9*a**(13/3)*(1 +
b*x**3/a)**(1/3)*gamma(10/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*g
amma(10/3) + 9*a**(7/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)))
```

Maxima [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}\,dx=\int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*A*x^7/((b*x^3 +
a)^(7/3)*a^3) + integrate((D*x^3 + B*x)/((b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*
x^3 + a^3)*(b*x^3 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{Dx^3+Bx+A}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+B*x+A)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + B*x + A)/(b*x^3 + a)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{A+B\,x+x^3\,D}{(b\,x^3+a)^{10/3}}\,dx$$

input
```
int((A + B*x + x^3*D)/(a + b*x^3)^(10/3),x)
```

output
```
int((A + B*x + x^3*D)/(a + b*x^3)^(10/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Dx^3}{(a+bx^3)^{10/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}d\right.$$
$$+\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)b$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)a$$

input
```
int((D*x^3+B*x+A)/(b*x^3+a)^(10/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3
*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*d + i
nt(x/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3*(a
+ b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*b + int(1
/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3*(a + b*
x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*a
```

3.107 $\int \sqrt[3]{a+bx^3}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 219

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{aC\sqrt[3]{a+bx^3}}{4b}+\frac{aDx\sqrt[3]{a+bx^3}}{10b}+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)-\frac{aB\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{a(5Ab-aD)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b\left(a+bx^3\right)^{2/3}}-\frac{aB\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}$$

output

```
1/4*a*C*(b*x^3+a)^(1/3)/b+1/10*a*D*x*(b*x^3+a)^(1/3)/b+1/60*(b*x^3+a)^(1/3
)*(12*D*x^4+15*C*x^3+20*B*x^2+30*A*x)-1/9*a*B*arctan(1/3*(1+2*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)+1/10*a*(5*A*b-D*a)*x*(1+b*x^3/a)^(
2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)-1/6*a*B*ln(b^(
1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 8.84 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.66

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{\sqrt[3]{a+bx^3}\left(aC\sqrt[3]{1+\frac{bx^3}{a}}+bCx^3\sqrt[3]{1+\frac{bx^3}{a}}+4Abx\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+2bBx^2\,\text{Hyp}\right.}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((a + b*x^3)^(1/3)*(a*C*(1 + (b*x^3)/a)^(1/3) + b*C*x^3*(1 + (b*x^3)/a)^(1
/3) + 4*A*b*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)] + 2*b*B*x^2*
Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + b*D*x^4*Hypergeometric2F
1[-1/3, 4/3, 7/3, -((b*x^3)/a)]))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2392, 27, 2427, 27, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2392}$$

$$a\int \frac{12Dx^3+15Cx^2+20Bx+30A}{60\left(bx^3+a\right)^{2/3}}dx+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{60}a\int \frac{12Dx^3+15Cx^2+20Bx+30A}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

$$\Big\downarrow \text{2427}$$

$$\frac{1}{60}a\left(\frac{\int \frac{2\left(15bCx^2+20bBx+6(5Ab-aD)\right)}{(bx^3+a)^{2/3}}dx}{2b}+\frac{6Dx\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{60}a\left(\frac{\int \frac{15bCx^2+20bBx+6(5Ab-aD)}{(bx^3+a)^{2/3}}dx}{b}+\frac{6Dx\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

$$\Big\downarrow \text{2425}$$

$$\frac{1}{60}a\left(\frac{\int \frac{6(5Ab-aD)+20bBx}{(bx^3+a)^{2/3}}dx+15bC\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{b}+\frac{6Dx\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{60}a\left(\frac{\int \frac{6(5Ab-aD)+20bBx}{(bx^3+a)^{2/3}}dx+15C\sqrt[3]{a+bx^3}}{b}+\frac{6Dx\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

$$\Big\downarrow \text{2432}$$

$$\frac{1}{60}a\left(\frac{\int \left(\frac{6(5Ab-aD)}{(bx^3+a)^{2/3}}+\frac{20bBx}{(bx^3+a)^{2/3}}\right)dx+15C\sqrt[3]{a+bx^3}}{b}+\frac{6Dx\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{60}a\left(\frac{\frac{6x\left(\frac{bx^3}{a}+1\right)^{2/3}(5Ab-aD)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-\frac{20\sqrt[3]{b}B\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}-10\sqrt[3]{b}B\log\left(\sqrt[3]{b}x-\sqrt[3]{\right.}}{b}\right.$$

$$\frac{1}{60}\sqrt[3]{a+bx^3}\left(30Ax+20Bx^2+15Cx^3+12Dx^4\right)$$

input
```
Int[(a + b*x^3)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((a + b*x^3)^(1/3)*(30*A*x + 20*B*x^2 + 15*C*x^3 + 12*D*x^4))/60 + (a*((6*
D*x*(a + b*x^3)^(1/3))/b + (15*C*(a + b*x^3)^(1/3) - (20*b^(1/3)*B*ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + (6*(5*A*b - a*D)
*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
a + b*x^3)^(2/3) - 10*b^(1/3)*B*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/b))/60
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(D x^{3}+C\,x^{2}+B x +A \right)dx$$

input
```
int((b*x^3+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x^3+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.68

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\sqrt[3]{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt[3]{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+C\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)+\frac{D\sqrt[3]{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**(1/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + B*a**(1/3)*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + C*Piecewise((a**(1/3)*x**3/3, E
q(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True)) + D*a**(1/3)*x**4*gamma(4/3)*
hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)\left(bx^3+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(b\,x^3+a\right)^{1/3}\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^3)^(1/3)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^3)^(1/3)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{30(b\,x^3+a)^{\frac{1}{3}}\,abx+15(b\,x^3+a)^{\frac{1}{3}}\,ac+6(b\,x^3+a)^{\frac{1}{3}}\,adx+20(b\,x^3+a)^{\frac{1}{3}}\,b^2x^2+15(b\,x^3+a)^{\frac{1}{3}}\,bc\,x^3+12\text{(}}{60b}$$

input
```
int((b*x^3+a)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(30*(a + b*x**3)**(1/3)*a*b*x + 15*(a + b*x**3)**(1/3)*a*c + 6*(a + b*x**3
)**(1/3)*a*d*x + 20*(a + b*x**3)**(1/3)*b**2*x**2 + 15*(a + b*x**3)**(1/3)
*b*c*x**3 + 12*(a + b*x**3)**(1/3)*b*d*x**4 + 30*int((a + b*x**3)**(1/3)/(
a + b*x**3),x)*a**2*b - 6*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**2*d +
 20*int(((a + b*x**3)**(1/3)*x)/(a + b*x**3),x)*a*b**2)/(60*b)
```

3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 182

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{C(a+bx^3)^{2/3}}{2b}+\frac{Dx(a+bx^3)^{2/3}}{3b} + \frac{(3Ab-aD)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{(3Ab-aD)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output

```
1/2*C*(b*x^3+a)^(2/3)/b+1/3*D*x*(b*x^3+a)^(2/3)/b+1/9*(3*A*b-D*a)*arctan(1
/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)+1/2*B*x^2*(1+b
*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/6*(3*
A*b-D*a)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{C(a+bx^3)^{2/3}}{2b}+\frac{Dx(a+bx^3)^{2/3}}{3b} + \frac{A\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{aD\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{A\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{aD\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}} + \frac{A\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}} - \frac{aD\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{18b^{4/3}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(C*(a + b*x^3)^(2/3))/(2*b) + (D*x*(a + b*x^3)^(2/3))/(3*b) + (A*ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (a*D*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) +
(B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)
])/(2*(a + b*x^3)^(1/3)) - (A*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b
^(1/3)) + (a*D*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*b^(4/3)) + (A*Lo
g[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6
*b^(1/3)) - (a*D*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a
+ b*x^3)^(1/3)])/(18*b^(4/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{A}{\sqrt[3]{a+bx^3}}+\frac{Bx}{\sqrt[3]{a+bx^3}}+\frac{Cx^2}{\sqrt[3]{a+bx^3}}+\frac{Dx^3}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{A\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}-\frac{aD\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{aD\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}+\frac{Bx^2\sqrt[3]{\frac{bx^3}{a}+1}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}+\frac{C\left(a+bx^3\right)^{2/3}}{2b}+\frac{Dx\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(1/3),x]
```

output
```
(C*(a + b*x^3)^(2/3))/(2*b) + (D*x*(a + b*x^3)^(2/3))/(3*b) + (A*ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (a*D*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) +
(B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)
])/(2*(a + b*x^3)^(1/3)) - (A*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^
(1/3)) + (a*D*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{Dx^{3}+C\,x^{2}+Bx+A}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(1/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}+C\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{Dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**3+a)**(1/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + B*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + C*Piecewise((x**3/(3*a**(1/3)), E
q(b, 0)), ((a + b*x**3)**(2/3)/(2*b), True)) + D*x**4*gamma(4/3)*hyper((1/
3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*A + integr
ate((D*x^3 + C*x^2 + B*x)/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(1/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt[3]{a+bx^3}}\,dx=\left(\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)d+\left(\int \frac{x^2}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)c+\left(\int \frac{x}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)a$$

input

```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(1/3),x)
```

output

```
int(x**3/(a + b*x**3)**(1/3),x)*d + int(x**2/(a + b*x**3)**(1/3),x)*c + in
t(x/(a + b*x**3)**(1/3),x)*b + int(1/(a + b*x**3)**(1/3),x)*a
```

3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}} dx$

Optimal result

Integrand size = 27, antiderivative size = 171

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}} dx = \frac{C\sqrt[3]{a+bx^3}}{b} + \frac{Dx\sqrt[3]{a+bx^3}}{2b} - \frac{B\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{(2Ab-aD)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}} - \frac{B\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
C*(b*x^3+a)^(1/3)/b+1/2*D*x*(b*x^3+a)^(1/3)/b-1/3*B*arctan(1/3*(1+2*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)+1/2*(2*A*b-D*a)*x*(1+b*x^3/a
)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)-1/2*B*ln(b^
(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{4aC+4bCx^3+4Abx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+2\ldots}{\ldots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(4*a*C + 4*b*C*x^3 + 4*A*b*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3,
2/3, 4/3, -((b*x^3)/a)] + 2*b*B*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)
/(a + b*x^3)] + b*D*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3,
7/3, -((b*x^3)/a)])/(4*b*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2427, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{\int \frac{2bCx^2+2bBx+2Ab-aD}{(bx^3+a)^{2/3}}dx}{2b}+\frac{Dx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{ 2425}$$

$$\frac{\int \frac{2Ab+2Bxb-aD}{(bx^3+a)^{2/3}}dx+2bC\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{2b}+\frac{Dx\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\int \frac{2Ab+2Bxb-aD}{(bx^3+a)^{2/3}}dx + 2C\sqrt[3]{a+bx^3}}{2b} + \frac{Dx\sqrt[3]{a+bx^3}}{2b}$$

$\Big\downarrow$ 2432

$$\frac{\int \left(\frac{2Ab\left(1-\frac{aD}{2Ab}\right)}{(bx^3+a)^{2/3}} + \frac{2bBx}{(bx^3+a)^{2/3}}\right)dx + 2C\sqrt[3]{a+bx^3}}{2b} + \frac{Dx\sqrt[3]{a+bx^3}}{2b}$$

$\Big\downarrow$ 2009

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2Ab-aD)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{2\sqrt[3]{b}B\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}} - \sqrt[3]{b}B\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right) -}{2b}$$
$$\frac{Dx\sqrt[3]{a+bx^3}}{2b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(2/3),x]
```

output
```
(D*x*(a + b*x^3)^(1/3))/(2*b) + (2*C*(a + b*x^3)^(1/3) - (2*b^(1/3)*B*ArcT
an[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/Sqrt[3] + ((2*A*b - a*D
)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/
(a + b*x^3)^(2/3) - b^(1/3)*B*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{D x^{3}+C \,x^{2}+B x +A}{\left(b \,x^{3}+a \right)^{\frac{2}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(2/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{D x^{3} + C x^{2} + B x + A}{{\left(b x^{3} + a\right)}^{\frac{2}{3}}} \,d x$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + C\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right) + \frac{Dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**3+a)**(2/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(4/3)) + B*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + C*Piecewise((x**3/(3*a**(2/3)), E
q(b, 0)), ((a + b*x**3)**(1/3)/b, True)) + D*x**4*gamma(4/3)*hyper((2/3, 4
/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(2/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{2/3}}\,dx=\frac{(b\,x^3+a)^{\frac{1}{3}}\,c+\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)bd+\left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b^2+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)}{b}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(1/3)*c + int(x**3/(a + b*x**3)**(2/3),x)*b*d + int(x/(a +
b*x**3)**(2/3),x)*b**2 + int(1/(a + b*x**3)**(2/3),x)*a*b)/b
```

3.110 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 171

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{4/3}}\,dx=-\frac{C}{b\sqrt[3]{a+bx^3}}+\frac{(Ab-aD)x}{ab\sqrt[3]{a+bx^3}}+\frac{D\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}+\frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt[3]{a+bx^3}}-\frac{D\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}$$

output
```
-C/b/(b*x^3+a)^(1/3)+(A*b-D*a)*x/a/b/(b*x^3+a)^(1/3)+1/3*D*arctan(1/3*(1+2
*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)+1/2*B*x^2*(1+b*x^3/a)
^(1/3)*hypergeom([2/3, 4/3],[5/3],-b*x^3/a)/a/(b*x^3+a)^(1/3)-1/2*D*ln(-b^
(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{4/3}}\,dx = \frac{-4aC+4Abx+2bBx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+bI}{4ab\sqrt[3]{a+bx^3}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(-4*a*C + 4*A*b*x + 2*b*B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[2/3,
 4/3, 5/3, -((b*x^3)/a)] + b*D*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1
[4/3, 4/3, 7/3, -((b*x^3)/a)])/(4*a*b*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2397, 25, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{x(-aD+Ab+bBx+bCx^2)}{ab\sqrt[3]{a+bx^3}} - \frac{\int -\frac{-2bCx^2-bBx+aD}{\sqrt[3]{bx^3+a}}dx}{ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-2bCx^2-bBx+aD}{\sqrt[3]{bx^3+a}}dx}{ab} + \frac{x(-aD+Ab+bBx+bCx^2)}{ab\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 2425}$$

$$\frac{\int \frac{aD-bBx}{\sqrt[3]{bx^3+a}}dx - 2bC\int \frac{x^2}{\sqrt[3]{bx^3+a}}dx}{ab} + \frac{x(-aD+Ab+bBx+bCx^2)}{ab\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 793

$$\frac{\int \frac{aD-bBx}{\sqrt[3]{bx^3+a}}dx - C(a+bx^3)^{2/3}}{ab} + \frac{x(-aD+Ab+bBx+bCx^2)}{ab\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 2432

$$\frac{\int \left(\frac{aD}{\sqrt[3]{bx^3+a}} - \frac{bBx}{\sqrt[3]{bx^3+a}}\right)dx - C(a+bx^3)^{2/3}}{ab} + \frac{x(-aD+Ab+bBx+bCx^2)}{ab\sqrt[3]{a+bx^3}}$$

$\Big\downarrow$ 2009

$$\frac{x(-aD+Ab+bBx+bCx^2)}{ab\sqrt[3]{a+bx^3}} + \frac{\frac{aD\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{bBx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - C(a+bx^3)^{2/3} - \frac{aD\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{\ldots}\right)}{2\sqrt[3]{b}}}{ab}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(4/3),x]
```

output
```
(x*(A*b - a*D + b*B*x + b*C*x^2))/(a*b*(a + b*x^3)^(1/3)) + (-(C*(a + b*x^
3)^(2/3)) + (a*D*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(S
qrt[3]*b^(1/3)) - (b*B*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/
3, 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (a*D*Log[-(b^(1/3)*x) + (a
+ b*x^3)^(1/3)])/(2*b^(1/3)))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{Dx^3 + C\,x^2 + Bx + A}{\left(b\,x^3 + a\right)^{\frac{4}{3}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(4/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^3)^{4/3}}\,dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{(bx^3 + a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^3
+ a^2), x)
```

Sympy [A] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^3)^{4/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1 + \frac{bx^3}{a}}\,\Gamma\left(\frac{4}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{4}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)} + C\left(\begin{cases}-\frac{1}{b\sqrt[3]{a + bx^3}} & \text{for}\ b \neq 0\\ \frac{x^3}{3a^{\frac{4}{3}}} & \text{otherwise}\end{cases}\right) + \frac{Dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3}, \frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**3+a)**(4/3),x)
```

output
```
A*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + B*x**2*gamm
a(2/3)*hyper((2/3, 4/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gam
ma(5/3)) + C*Piecewise((-1/(b*(a + b*x**3)**(1/3)), Ne(b, 0)), (x**3/(3*a*
*(4/3)), True)) + D*x**4*gamma(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*a**(4/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^3)^{4/3}}\,dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{(bx^3 + a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
A*x/((b*x^3 + a)^(1/3)*a) + integrate((D*x^3 + C*x^2 + B*x)/(b*x^3 + a)^(4
/3), x)
```

Giac [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^3)^{4/3}}\,dx = \int \frac{Dx^3 + Cx^2 + Bx + A}{(bx^3 + a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{4/3}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(4/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{4/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)d + \left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)c + \left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(4/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*d + int(x
**2/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*c + int(x/((a
+ b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*b + int(1/((a + b*x**3
)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)*a
```

3.111 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{5/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 119

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=-\frac{C}{2b\left(a+bx^3\right)^{2/3}}-\frac{Dx}{b\left(a+bx^3\right)^{2/3}}+\frac{Bx^2}{2a\left(a+bx^3\right)^{2/3}}+\frac{(Ab+aD)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{ab\left(a+bx^3\right)^{2/3}}$$

output
```
-1/2*C/b/(b*x^3+a)^(2/3)-D*x/b/(b*x^3+a)^(2/3)+1/2*B*x^2/a/(b*x^3+a)^(2/3)
+(A*b+D*a)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 5/3],[4/3],-b*x^3/a)/a/b/(b
*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{5/3}}\,dx=\frac{-2aC+2bBx^2+4Abx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+}{4ab\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
(-2*a*C + 2*b*B*x^2 + 4*A*b*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3,
 5/3, 4/3, -((b*x^3)/a)] + b*D*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[4/3, 5/3, 7/3, -((b*x^3)/a)])/(4*a*b*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2397, 25, 2425, 27, 779, 778, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{5/3}}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(-aD+Ab+bBx+bCx^2)}{2ab(a+bx^3)^{2/3}}-\frac{\int -\frac{-bCx^2+Ab+aD}{(bx^3+a)^{2/3}}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-bCx^2+Ab+aD}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(-aD+Ab+bBx+bCx^2)}{2ab(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{2425}$$

$$\frac{\int \frac{Ab+aD}{(bx^3+a)^{2/3}}dx-bC\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(-aD+Ab+bBx+bCx^2)}{2ab(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aD+Ab)\int \frac{1}{(bx^3+a)^{2/3}}dx-bC\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(-aD+Ab+bBx+bCx^2)}{2ab(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+Ab)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}-bC\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x(-aD+Ab+bBx+bCx^2)}{2ab(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-bC\int\frac{x^2}{(bx^3+a)^{2/3}}dx}{2ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{2ab\,(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{793}$$

$$\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{2ab\,(a+bx^3)^{2/3}}+\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-C\sqrt[3]{a+bx^3}}{2ab}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(5/3),x]
```

output
```
(x*(A*b - a*D + b*B*x + b*C*x^2))/(2*a*b*(a + b*x^3)^(2/3)) + (-(C*(a + b*
x^3)^(1/3)) + ((A*b + a*D)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3,
2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3))/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Maple [F]

$$\int \frac{Dx^{3}+C\,x^{2}+Bx+A}{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(5/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{5/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
+ a^2), x)
```

Sympy [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{5/3}}\,dx=\frac{Ax\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right)}{3a^{\frac{5}{3}}\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}+C\left(\begin{cases}-\frac{1}{2b(a+bx^3)^{\frac{2}{3}}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{5}{3}}} & \text{otherwise}\end{cases}\right)+\frac{Dx^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**3+a)**(5/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
5/3)*gamma(4/3)) + B*x**2*gamma(2/3)/(3*a**(5/3)*(1 + b*x**3/a)**(2/3)*gam
ma(5/3)) + C*Piecewise((-1/(2*b*(a + b*x**3)**(2/3)), Ne(b, 0)), (x**3/(3*
a**(5/3)), True)) + D*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{5/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(5/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{5/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{5/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^3+a)^{5/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(5/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(5/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{5/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)d + \left(\int \frac{x^2}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)c + \left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a+(b\,x^3+a)^{\frac{2}{3}}\,b\,x^3}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(5/3),x)
```

output
```
int(x**3/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*d + int(x
**2/((a + b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*c + int(x/((a
+ b*x**3)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*b + int(1/((a + b*x**3
)**(2/3)*a + (a + b*x**3)**(2/3)*b*x**3),x)*a
```

3.112 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 135

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}} \, dx = -\frac{C}{4b(a+bx^3)^{4/3}} + \frac{(Ab-aD)x}{4ab(a+bx^3)^{4/3}} + \frac{(3Ab+aD)x}{4a^2b\sqrt[3]{a+bx^3}} + \frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^2\sqrt[3]{a+bx^3}}$$

output
```
-1/4*C/b/(b*x^3+a)^(4/3)+1/4*(A*b-D*a)*x/a/b/(b*x^3+a)^(4/3)+1/4*(3*A*b+D*
a)*x/a^2/b/(b*x^3+a)^(1/3)+1/2*B*x^2*(1+b*x^3/a)^(1/3)*hypergeom([2/3, 7/3
],[5/3],-b*x^3/a)/a^2/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}} \, dx = \frac{-a^2C+3Ab^2x^4+abx(4A+Dx^3)+2bBx^2(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeon}}{4a^2b(a+bx^3)^{4/3}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(7/3),x]
```

output
```
(-(a^2*C) + 3*A*b^2*x^4 + a*b*x*(4*A + D*x^3) + 2*b*B*x^2*(a + b*x^3)*(1 +
 (b*x^3)/a)^(1/3)*Hypergeometric2F1[2/3, 7/3, 5/3, -((b*x^3)/a)])/(4*a^2*b
*(a + b*x^3)^(4/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2397, 25, 2393, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{7/3}}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{4ab\left(a+bx^3\right)^{4/3}}-\frac{\int -\frac{bCx^2+2bBx+3Ab+aD}{\left(bx^3+a\right)^{4/3}}dx}{4ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{bCx^2+2bBx+3Ab+aD}{\left(bx^3+a\right)^{4/3}}dx}{4ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{ 2393}$$

$$\frac{-\frac{\int \frac{2bBx}{\sqrt[3]{bx^3+a}}dx}{a}-\frac{aC-x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}}{4ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{2bB\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{a}-\frac{aC-x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}}{4ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{-\frac{2bB\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{a\sqrt[3]{a+bx^3}}-\frac{aC-x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}}{4ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{4ab\left(a+bx^3\right)^{4/3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{-\frac{aC-x(aD+3Ab+2bBx)}{a\sqrt[3]{a+bx^3}}-\frac{bBx^2\sqrt[3]{\frac{bx^3}{a}+1}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{a\sqrt[3]{a+bx^3}}}{4ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{4ab\left(a+bx^3\right)^{4/3}}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(7/3),x]
```

output

```
(x*(A*b - a*D + b*B*x + b*C*x^2))/(4*a*b*(a + b*x^3)^(4/3)) + (-((a*C - x*
(3*A*b + a*D + 2*b*B*x))/(a*(a + b*x^3)^(1/3))) - (b*B*x^2*(1 + (b*x^3)/a)
^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a*(a + b*x^3)^(1/3
)))/(4*a*b)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{D x^{3}+C \,x^{2}+B x +A}{\left(b \,x^{3}+a \right)^{\frac{7}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(7/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(2/3)/(b^3*x^9 + 3*a*b^2*x^
6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [A] (verification not implemented)

Time = 30.44 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}}\,dx=A\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right)+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{7}{3}}\Gamma\left(\frac{5}{3}\right)}+C\left(\begin{cases}-\frac{1}{4ab\sqrt[3]{a+bx^3}+4b^2x^3\sqrt[3]{a+bx^3}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{7}{3}}} & \text{otherwise}\end{cases}\right)+\frac{Dx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**3+a)**(7/3),x)
```

output
```
A*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(
7/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**
(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)
**(1/3)*gamma(7/3))) + B*x**2*gamma(2/3)*hyper((2/3, 7/3), (5/3,), b*x**3*
exp_polar(I*pi)/a)/(3*a**(7/3)*gamma(5/3)) + C*Piecewise((-1/(4*a*b*(a + b
*x**3)**(1/3) + 4*b**2*x**3*(a + b*x**3)**(1/3)), Ne(b, 0)), (x**3/(3*a**(
7/3)), True)) + D*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gamma(
7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*A*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a^2) + integrate((D*
x^3 + C*x^2 + B*x)/((b^2*x^6 + 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^3+a)^{7/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(7/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(7/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{7/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)d + \left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)c + \left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)b + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{1}{3}}\,b^2x^6}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(7/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a +
 b*x**3)**(1/3)*b**2*x**6),x)*d + int(x**2/((a + b*x**3)**(1/3)*a**2 + 2*(
a + b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)**(1/3)*b**2*x**6),x)*c + int(x/
((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)**(1/3)*a*b*x**3 + (a + b*x**3)*
*(1/3)*b**2*x**6),x)*b + int(1/((a + b*x**3)**(1/3)*a**2 + 2*(a + b*x**3)*
*(1/3)*a*b*x**3 + (a + b*x**3)**(1/3)*b**2*x**6),x)*a
```

3.113 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}} \, dx = -\frac{C}{5b(a+bx^3)^{5/3}} - \frac{Dx}{4b(a+bx^3)^{5/3}} + \frac{Bx^2}{5a(a+bx^3)^{5/3}} + \frac{3Bx^2}{10a^2(a+bx^3)^{2/3}} + \frac{(4Ab+aD)x\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{4a^2b(a+bx^3)^{2/3}}$$

output
```
-1/5*C/b/(b*x^3+a)^(5/3)-1/4*D*x/b/(b*x^3+a)^(5/3)+1/5*B*x^2/a/(b*x^3+a)^(
5/3)+3/10*B*x^2/a^2/(b*x^3+a)^(2/3)+1/4*(4*A*b+D*a)*x*(1+b*x^3/a)^(2/3)*hy
pergeom([1/3, 8/3],[4/3],-b*x^3/a)/a^2/b/(b*x^3+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}} \, dx = \frac{-4a^2C+10abBx^2+6b^2Bx^5+20Abx(a+bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3} \text{Hypergeomet}}{2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
(-4*a^2*C + 10*a*b*B*x^2 + 6*b^2*B*x^5 + 20*A*b*x*(a + b*x^3)*(1 + (b*x^3)
/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)] + 5*b*D*x^4*(a +
b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[4/3, 8/3, 7/3, -((b*x^3)/a)
])/(20*a^2*b*(a + b*x^3)^(5/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2397, 25, 2393, 25, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{x(-aD+Ab+bBx+bCx^2)}{5ab(a+bx^3)^{5/3}}-\frac{\int -\frac{2bCx^2+3bBx+4Ab+aD}{(bx^3+a)^{5/3}}dx}{5ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2bCx^2+3bBx+4Ab+aD}{(bx^3+a)^{5/3}}dx}{5ab}+\frac{x(-aD+Ab+bBx+bCx^2)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 2393}$$

$$\frac{-\frac{\int -\frac{4Ab+aD}{(bx^3+a)^{2/3}}dx}{2a}-\frac{2aC-x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x(-aD+Ab+bBx+bCx^2)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{4Ab+aD}{(bx^3+a)^{2/3}}dx}{2a}-\frac{2aC-x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x(-aD+Ab+bBx+bCx^2)}{5ab(a+bx^3)^{5/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{(aD+4Ab)\int \frac{1}{\left(bx^3+a\right)^{2/3}}dx}{2a}-\frac{2aC-x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{5ab\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \ 779$$

$$\frac{\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+4Ab)\int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{2a(a+bx^3)^{2/3}}-\frac{2aC-x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{5ab\left(a+bx^3\right)^{5/3}}$$

$$\Big\downarrow \ 778$$

$$\frac{\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(aD+4Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2a(a+bx^3)^{2/3}}-\frac{2aC-x(aD+4Ab+3bBx)}{2a(a+bx^3)^{2/3}}}{5ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{5ab\left(a+bx^3\right)^{5/3}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(8/3),x]
```

output
```
(x*(A*b - a*D + b*B*x + b*C*x^2))/(5*a*b*(a + b*x^3)^(5/3)) + (-1/2*(2*a*C
 - x*(4*A*b + a*D + 3*b*B*x))/(a*(a + b*x^3)^(2/3)) + ((4*A*b + a*D)*x*(1
+ (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*(a
 + b*x^3)^(2/3)))/(5*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{D x^{3}+C \,x^{2}+B x +A}{\left(b \,x^{3}+a \right)^{\frac{8}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(8/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x^
6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [A] (verification not implemented)

Time = 49.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}}\,dx=\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{8}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)}+B\left(\frac{5ax^2\Gamma\left(\frac{2}{3}\right)}{9a^{\frac{11}{3}}\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)+9a^{\frac{8}{3}}bx^3\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{3bx^5\Gamma\left(\frac{2}{3}\right)}{9a^{\frac{11}{3}}\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)+9a^{\frac{8}{3}}bx^3\left(1+\frac{bx^3}{a}\right)^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}\right)+C\left(\begin{cases}-\frac{1}{5ab(a+bx^3)^{\frac{2}{3}}+5b^2x^3(a+bx^3)^{\frac{2}{3}}} & \text{for } b\neq 0\\ \frac{x^3}{3a^{\frac{8}{3}}} & \text{otherwise}\end{cases}\right)+\frac{Dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{8}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**3+a)**(8/3),x)
```

output
```
A*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
8/3)*gamma(4/3)) + B*(5*a*x**2*gamma(2/3)/(9*a**(11/3)*(1 + b*x**3/a)**(2/
3)*gamma(8/3) + 9*a**(8/3)*b*x**3*(1 + b*x**3/a)**(2/3)*gamma(8/3)) + 3*b*
x**5*gamma(2/3)/(9*a**(11/3)*(1 + b*x**3/a)**(2/3)*gamma(8/3) + 9*a**(8/3)
*b*x**3*(1 + b*x**3/a)**(2/3)*gamma(8/3))) + C*Piecewise((-1/(5*a*b*(a + b
*x**3)**(2/3) + 5*b**2*x**3*(a + b*x**3)**(2/3)), Ne(b, 0)), (x**3/(3*a**(
8/3)), True)) + D*x**4*gamma(4/3)*hyper((4/3, 8/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{8/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\left(bx^3+a\right)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(8/3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{8/3}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\left(bx^3+a\right)^{\frac{8}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^3+a)^{8/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(8/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(8/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{8/3}}\,dx=\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)d$$
$$+\left(\int \frac{x^2}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)c$$
$$+\left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)b$$
$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,a^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,ab\,x^3+(b\,x^3+a)^{\frac{2}{3}}\,b^2x^6}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(8/3),x)
```

output
```
int(x**3/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a +
 b*x**3)**(2/3)*b**2*x**6),x)*d + int(x**2/((a + b*x**3)**(2/3)*a**2 + 2*(
a + b*x**3)**(2/3)*a*b*x**3 + (a + b*x**3)**(2/3)*b**2*x**6),x)*c + int(x/
((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)**(2/3)*a*b*x**3 + (a + b*x**3)*
*(2/3)*b**2*x**6),x)*b + int(1/((a + b*x**3)**(2/3)*a**2 + 2*(a + b*x**3)*
*(2/3)*a*b*x**3 + (a + b*x**3)**(2/3)*b**2*x**6),x)*a
```

3.114 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 165

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{10/3}}\,dx = -\frac{C}{7b\left(a+bx^3\right)^{7/3}} + \frac{(Ab-aD)x}{7ab\left(a+bx^3\right)^{7/3}} + \frac{(6Ab+aD)x}{28a^2b\left(a+bx^3\right)^{4/3}} + \frac{3(6Ab+aD)x}{28a^3b\sqrt[3]{a+bx^3}} + \frac{Bx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{10}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a^3\sqrt[3]{a+bx^3}}$$

output
```
-1/7*C/b/(b*x^3+a)^(7/3)+1/7*(A*b-D*a)*x/a/b/(b*x^3+a)^(7/3)+1/28*(6*A*b+D
*a)*x/a^2/b/(b*x^3+a)^(4/3)+3/28*(6*A*b+D*a)*x/a^3/b/(b*x^3+a)^(1/3)+1/2*B
*x^2*(1+b*x^3/a)^(1/3)*hypergeom([2/3, 10/3],[5/3],-b*x^3/a)/a^3/(b*x^3+a)
^(1/3)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{10/3}}\,dx = \frac{-4a^3C+18Ab^3x^7+7a^2bx(4A+Dx^3)+3ab^2x^4(14A+Dx^3)+14bBx^2(a}{28a^3b\,(a+bx^3)^{7/3}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(-4*a^3*C + 18*A*b^3*x^7 + 7*a^2*b*x*(4*A + D*x^3) + 3*a*b^2*x^4*(14*A + D
*x^3) + 14*b*B*x^2*(a + b*x^3)^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[2
/3, 10/3, 5/3, -((b*x^3)/a)])/(28*a^3*b*(a + b*x^3)^(7/3))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2397, 25, 2393, 25, 2394, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{10/3}}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\,(a+bx^3)^{7/3}}-\frac{\int -\frac{4bCx^2+5bBx+6Ab+aD}{(bx^3+a)^{7/3}}dx}{7ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4bCx^2+5bBx+6Ab+aD}{(bx^3+a)^{7/3}}dx}{7ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\,(a+bx^3)^{7/3}}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{3(6Ab+aD)+10bBx}{\left(bx^3+a\right)^{4/3}}dx}{4a}-\frac{4aC-x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\left(a+bx^3\right)^{7/3}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{3(6Ab+aD)+10bBx}{\left(bx^3+a\right)^{4/3}}dx}{4a}-\frac{4aC-x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\left(a+bx^3\right)^{7/3}}$$

$\Big\downarrow$ 2394

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{\int \frac{10bBx}{\sqrt[3]{bx^3+a}}dx}{a}}{4a}-\frac{4aC-x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\left(a+bx^3\right)^{7/3}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{10bB\int \frac{x}{\sqrt[3]{bx^3+a}}dx}{a}}{4a}-\frac{4aC-x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\left(a+bx^3\right)^{7/3}}$$

$\Big\downarrow$ 889

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{10bB\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{a\sqrt[3]{a+bx^3}}}{4a}-\frac{4aC-x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\left(a+bx^3\right)^{7/3}}$$

$\Big\downarrow$ 888

$$\frac{\frac{\frac{x(3(aD+6Ab)+10bBx)}{a\sqrt[3]{a+bx^3}}-\frac{5bBx^2\sqrt[3]{\frac{bx^3}{a}+1}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{a\sqrt[3]{a+bx^3}}}{4a}-\frac{4aC-x(aD+6Ab+5bBx)}{4a(a+bx^3)^{4/3}}}{7ab}+\frac{x\left(-aD+Ab+bBx+bCx^2\right)}{7ab\left(a+bx^3\right)^{7/3}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^3)^(10/3),x]
```

output
```
(x*(A*b - a*D + b*B*x + b*C*x^2))/(7*a*b*(a + b*x^3)^(7/3)) + (-1/4*(4*a*C
 - x*(6*A*b + a*D + 5*b*B*x))/(a*(a + b*x^3)^(4/3)) + ((x*(3*(6*A*b + a*D)
 + 10*b*B*x))/(a*(a + b*x^3)^(1/3)) - (5*b*B*x^2*(1 + (b*x^3)/a)^(1/3)*Hyp
ergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a*(a + b*x^3)^(1/3)))/(4*a))
/(7*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{D x^{3}+C \,x^{2}+B x +A}{\left(b \,x^{3}+a \right)^{\frac{10}{3}}}d x$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(10/3),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(10/3),x)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^3\right)^{10/3}}\,dx=\int \frac{Dx^{3}+Cx^{2}+Bx+A}{\left(bx^{3}+a\right)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^(2/3)/(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(146) = 292$.

Time = 142.64 (sec) , antiderivative size = 821, normalized size of antiderivative = 4.98

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx^3)^{10/3}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**3+a)**(10/3),x)
```

output
```
A*(28*a**5*x*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) +
81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*
x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**
3/a)**(1/3)*gamma(10/3)) + 70*a**4*b*x**4*gamma(1/3)/(27*a**(25/3)*(1 + b*
x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gam
ma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a
**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 60*a**3*b**2*x**7*
gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(22/3)*
b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 81*a**(19/3)*b**2*x**6*(1 + b*x
**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*b**3*x**9*(1 + b*x**3/a)**(1/3)*g
amma(10/3)) + 18*a**2*b**3*x**10*gamma(1/3)/(27*a**(25/3)*(1 + b*x**3/a)**
(1/3)*gamma(10/3) + 81*a**(22/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3)
+ 81*a**(19/3)*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 27*a**(16/3)*
b**3*x**9*(1 + b*x**3/a)**(1/3)*gamma(10/3))) + B*x**2*gamma(2/3)*hyper((2
/3, 10/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(10/3)*gamma(5/3)) + C*
Piecewise((-1/(7*a**2*b*(a + b*x**3)**(1/3) + 14*a*b**2*x**3*(a + b*x**3)*
*(1/3) + 7*b**3*x**6*(a + b*x**3)**(1/3)), Ne(b, 0)), (x**3/(3*a**(10/3)),
 True)) + D*(7*a*x**4*gamma(4/3)/(9*a**(13/3)*(1 + b*x**3/a)**(1/3)*gamma(
10/3) + 18*a**(10/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(10/3) + 9*a**(7/3)
*b**2*x**6*(1 + b*x**3/a)**(1/3)*gamma(10/3)) + 3*b*x**7*gamma(4/3)/(9*...
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

output
```
1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*A*x^7/((b*x^3 +
a)^(7/3)*a^3) + integrate((D*x^3 + C*x^2 + B*x)/((b^3*x^9 + 3*a*b^2*x^6 +
3*a^2*b*x^3 + a^3)*(b*x^3 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(bx^3+a)^{\frac{10}{3}}}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(b*x^3 + a)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{10/3}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(b\,x^3+a)^{10/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(10/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^3)^(10/3), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^3)^{10/3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)}\right.$$

$$+\left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)c$$

$$+\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)b$$

$$+\left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a^2b\,x^3+3\,(b\,x^3+a)^{\frac{1}{3}}\,a\,b^2x^6+(b\,x^3+a)^{\frac{1}{3}}\,b^3x^9}dx\right)a$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^3+a)^(10/3),x)
```

output
```
int(x**3/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3
*(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*d + i
nt(x**2/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3*
(a + b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*c + in
t(x/((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3*(a +
 b*x**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*b + int(1/
((a + b*x**3)**(1/3)*a**3 + 3*(a + b*x**3)**(1/3)*a**2*b*x**3 + 3*(a + b*x
**3)**(1/3)*a*b**2*x**6 + (a + b*x**3)**(1/3)*b**3*x**9),x)*a
```

3.115 $\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 159

$$\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3)\, dx = \frac{C(a+bx^3)^{1+p}}{3b(1+p)} + \frac{Dx(a+bx^3)^{1+p}}{b(4+3p)} + \left(A-\frac{aD}{4b+3bp}\right)x(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right) + \frac{1}{2}Bx^2(a+bx^3)^p\left(1+\frac{bx^3}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)$$

output
```
1/3*C*(b*x^3+a)^(p+1)/b/(p+1)+D*x*(b*x^3+a)^(p+1)/b/(4+3*p)+(A-a*D/(3*b*p+
4*b))*x*(b*x^3+a)^p*hypergeom([1/3, -p],[4/3],-b*x^3/a)/((1+b*x^3/a)^p)+1/
2*B*x^2*(b*x^3+a)^p*hypergeom([2/3, -p],[5/3],-b*x^3/a)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3)\, dx = \frac{1}{12}(a+bx^3)^p \left(\frac{4C(a+bx^3)}{b(1+p)} + 12Ax\left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right) + 6Bx^2\left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right) + 3Dx^4\left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)\right)$$

input
```
Integrate[(a + b*x^3)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((a + b*x^3)^p*((4*C*(a + b*x^3))/(b*(1 + p)) + (12*A*x*Hypergeometric2F1[
1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (6*B*x^2*Hypergeometric2F
1[2/3, -p, 5/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (3*D*x^4*Hypergeometric
2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p))/12
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3)\, dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(A(a+bx^3)^p + Bx(a+bx^3)^p + Cx^2(a+bx^3)^p + Dx^3(a+bx^3)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$Ax(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},-\frac{bx^3}{a}\right)+\frac{1}{2}Bx^2(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},-\frac{bx^3}{a}\right)+\frac{C(a+bx^3)^{p+1}}{3b(p+1)}+\frac{1}{4}Dx^4(a+bx^3)^p\left(\frac{bx^3}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{4}{3},-p,\frac{7}{3},-\frac{bx^3}{a}\right)$$

input
```
Int[(a + b*x^3)^p*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(C*(a + b*x^3)^(1 + p))/(3*b*(1 + p)) + (A*x*(a + b*x^3)^p*Hypergeometric2
F1[1/3, -p, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^p + (B*x^2*(a + b*x^3)^p*H
ypergeometric2F1[2/3, -p, 5/3, -((b*x^3)/a)])/(2*(1 + (b*x^3)/a)^p) + (D*x
^4*(a + b*x^3)^p*Hypergeometric2F1[4/3, -p, 7/3, -((b*x^3)/a)])/(4*(1 + (b
*x^3)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int (b\,x^3+a)^p\,(Dx^3+C\,x^2+Bx+A)\,dx$$

input
```
int((b*x^3+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x^3+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3)\ dx = \int (Dx^3+Cx^2+Bx+A)(bx^3+a)^p\,dx$$

input
```
integrate((b*x^3+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 59.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.95

$$\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3)\ dx = \frac{Aa^p x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{3}, -p \\ \frac{4}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{Ba^p x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix}\frac{2}{3}, -p \\ \frac{5}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + C\left(\begin{cases}\frac{a^p x^3}{3} & \text{for}\ b=0 \\ \frac{\begin{cases}\frac{(a+bx^3)^{p+1}}{p+1} & \text{for}\ p\neq -1 \\ \log\left(a+bx^3\right) & \text{otherwise}\end{cases}}{3b} & \text{otherwise}\end{cases}\right) + \frac{Da^p x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix}\frac{4}{3}, -p \\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**3+a)**p*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**p*x*gamma(1/3)*hyper((1/3, -p), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(4/3)) + B*a**p*x**2*gamma(2/3)*hyper((2/3, -p), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*gamma(5/3)) + C*Piecewise((a**p*x**3/3, Eq(b, 0)), (Piece
wise(((a + b*x**3)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**3), True))/
(3*b), True)) + D*a**p*x**4*gamma(4/3)*hyper((4/3, -p), (7/3,), b*x**3*exp
_polar(I*pi)/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3) \, dx = \int (Dx^3+Cx^2+Bx+A)(bx^3+a)^p \, dx$$

input
```
integrate((b*x^3+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^p, x)
```

Giac [F]

$$\int (a+bx^3)^p (A+Bx+Cx^2+Dx^3) \, dx = \int (Dx^3+Cx^2+Bx+A)(bx^3+a)^p \, dx$$

input
```
integrate((b*x^3+a)^p*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x^3 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^3\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx=\int {\left(b\,x^3+a\right)}^p\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x^3)^p*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x^3)^p*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int \left(a+bx^3\right)^p \left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{too large to display}$$

input
```
int((b*x^3+a)^p*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(27*(a + b*x**3)**p*a*b*p**3*x + 81*(a + b*x**3)**p*a*b*p**2*x + 78*(a + b
*x**3)**p*a*b*p*x + 24*(a + b*x**3)**p*a*b*x + 27*(a + b*x**3)**p*a*c*p**3
 + 63*(a + b*x**3)**p*a*c*p**2 + 42*(a + b*x**3)**p*a*c*p + 8*(a + b*x**3)
**p*a*c + 27*(a + b*x**3)**p*a*d*p**3*x + 45*(a + b*x**3)**p*a*d*p**2*x +
18*(a + b*x**3)**p*a*d*p*x + 27*(a + b*x**3)**p*b**2*p**3*x**2 + 72*(a + b
*x**3)**p*b**2*p**2*x**2 + 57*(a + b*x**3)**p*b**2*p*x**2 + 12*(a + b*x**3
)**p*b**2*x**2 + 27*(a + b*x**3)**p*b*c*p**3*x**3 + 63*(a + b*x**3)**p*b*c
*p**2*x**3 + 42*(a + b*x**3)**p*b*c*p*x**3 + 8*(a + b*x**3)**p*b*c*x**3 +
27*(a + b*x**3)**p*b*d*p**3*x**4 + 54*(a + b*x**3)**p*b*d*p**2*x**4 + 33*(
a + b*x**3)**p*b*d*p*x**4 + 6*(a + b*x**3)**p*b*d*x**4 + 2187*int((a + b*x
**3)**p/(27*a*p**3 + 63*a*p**2 + 42*a*p + 8*a + 27*b*p**3*x**3 + 63*b*p**2
*x**3 + 42*b*p*x**3 + 8*b*x**3),x)*a**2*b*p**7 + 11664*int((a + b*x**3)**p
/(27*a*p**3 + 63*a*p**2 + 42*a*p + 8*a + 27*b*p**3*x**3 + 63*b*p**2*x**3 +
 42*b*p*x**3 + 8*b*x**3),x)*a**2*b*p**6 + 25029*int((a + b*x**3)**p/(27*a*
p**3 + 63*a*p**2 + 42*a*p + 8*a + 27*b*p**3*x**3 + 63*b*p**2*x**3 + 42*b*p
*x**3 + 8*b*x**3),x)*a**2*b*p**5 + 27540*int((a + b*x**3)**p/(27*a*p**3 +
63*a*p**2 + 42*a*p + 8*a + 27*b*p**3*x**3 + 63*b*p**2*x**3 + 42*b*p*x**3 +
 8*b*x**3),x)*a**2*b*p**4 + 16308*int((a + b*x**3)**p/(27*a*p**3 + 63*a*p*
*2 + 42*a*p + 8*a + 27*b*p**3*x**3 + 63*b*p**2*x**3 + 42*b*p*x**3 + 8*b*x*
*3),x)*a**2*b*p**3 + 4896*int((a + b*x**3)**p/(27*a*p**3 + 63*a*p**2 + ...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
    if m:
      print ("func ", func , " is elementary_function")
    else:
      print ("func ", func , " is NOT elementary_function")


  return m

def is_special_function(func):
  #debug=False
  if debug:
    print ("type(func)=", type(func))

  m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
      'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
      'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
      'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
      'elliptic_pi','exp_integral_e','log_integral',
      'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
      'weierstrassPPrime','weierstrassSigma']

  if debug:
    print ("m=",m)
    if m:
      print ("func ", func ," is special_function")
    else:
      print ("func ", func ," is NOT special_function")


  return m


def is_hypergeometric_function(func):
  return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
  return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

  #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file