Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.7/62-1.1.3.7-b

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May 19, 2024 Compiled on May 19, 2024 at 12:17am

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**122**]. This is test number [62].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (122)	0.00 (0)
Mathematica	100.00 (122)	0.00 (0)
Maple	95.08 (116)	4.92 (6)
Fricas	95.08 (116)	4.92 (6)
Sympy	87.70 (107)	12.30 (15)
Mupad	85.25 (104)	14.75 (18)
Giac	85.25 (104)	14.75 (18)
Maxima	85.25 (104)	14.75 (18)
Reduce	85.25 (104)	14.75 (18)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

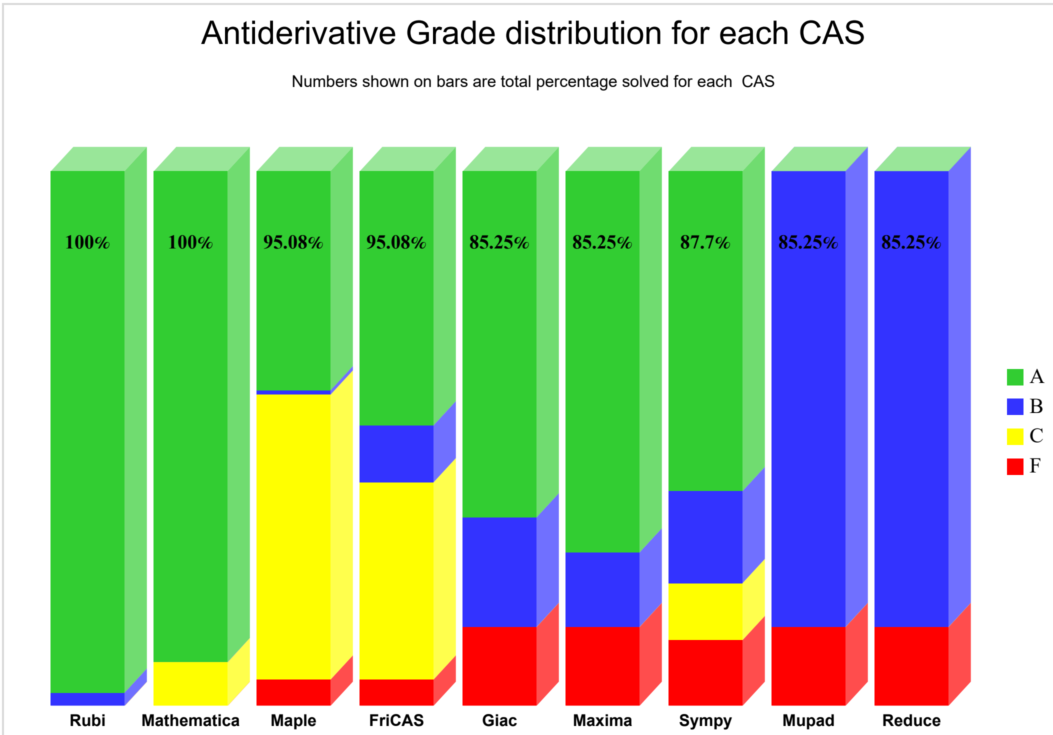
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

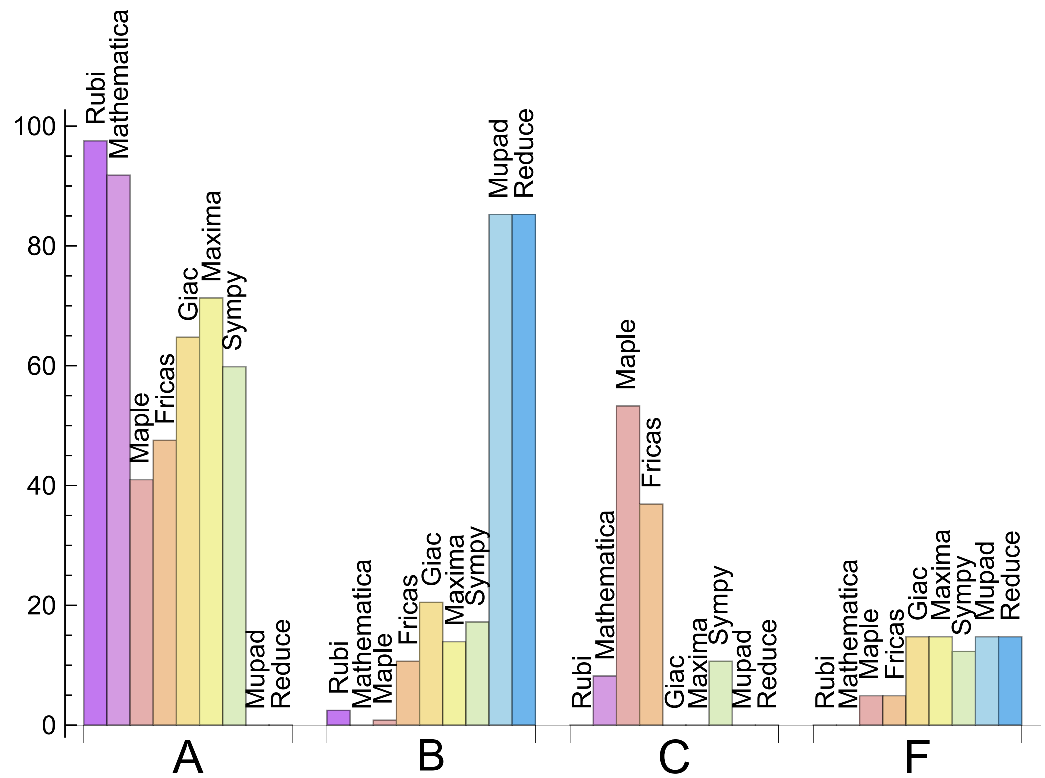
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.541	2.459	0.000	0.000
Mathematica	91.803	0.000	8.197	0.000
Maxima	71.311	13.934	0.000	14.754
Giac	64.754	20.492	0.000	14.754
Sympy	59.836	17.213	10.656	12.295
Fricas	47.541	10.656	36.885	4.918
Maple	40.984	0.820	53.279	4.918
Mupad	0.000	85.246	0.000	14.754
Reduce	0.000	85.246	0.000	14.754

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	6	100.00	0.00	0.00
Maple	6	100.00	0.00	0.00
Sympy	15	40.00	60.00	0.00
Mupad	18	0.00	100.00	0.00
Giac	18	100.00	0.00	0.00
Maxima	18	100.00	0.00	0.00
Reduce	18	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.13
Reduce	0.17
Maple	0.23
Rubi	0.53
Mathematica	1.50
Fricas	3.40
Mupad	3.68
Sympy	7.72

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	69.15	0.73	58.50	0.73
Mathematica	132.37	1.12	104.00	1.10
Rubi	136.99	1.19	117.00	1.00
Maxima	146.38	1.17	132.00	1.23
Giac	165.55	1.28	113.00	1.13
Sympy	175.50	1.66	114.00	1.11
Reduce	317.06	1.97	128.50	1.53
Mupad	534.19	3.36	122.50	1.23
Fricas	46326.47	230.48	102.50	1.20

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

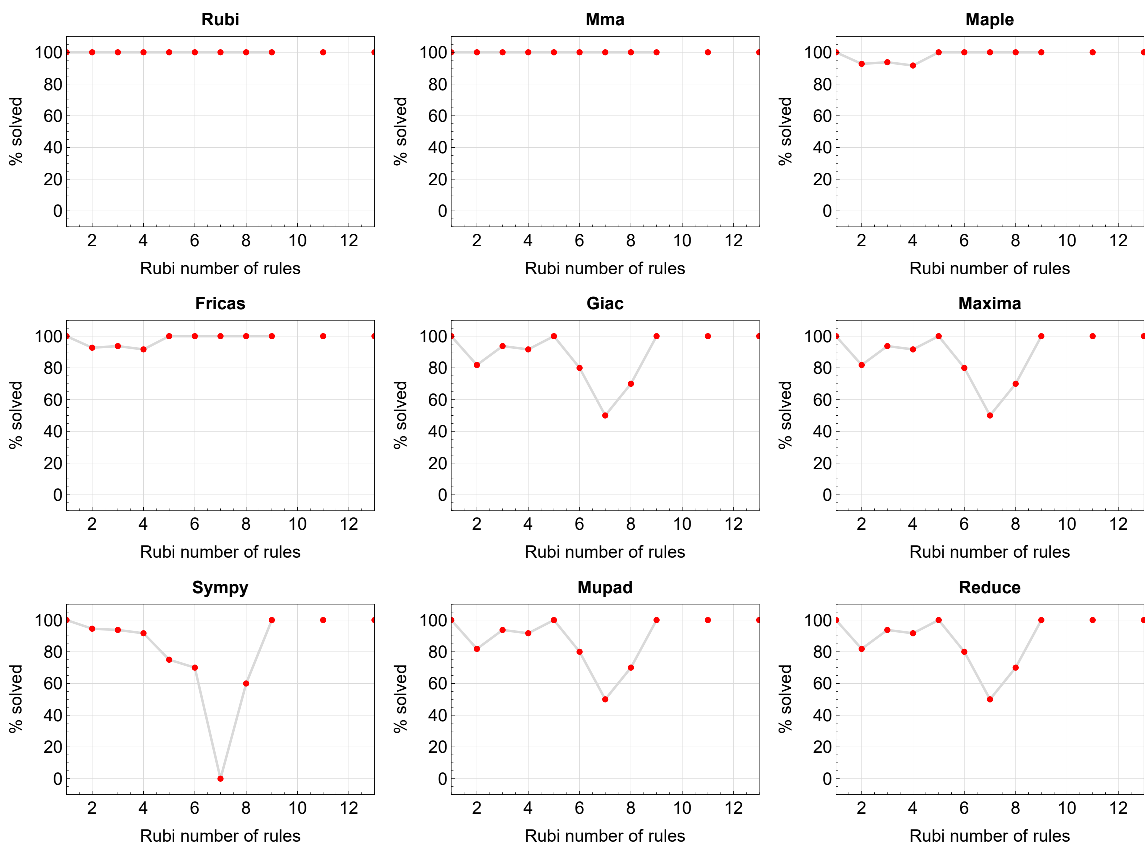


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

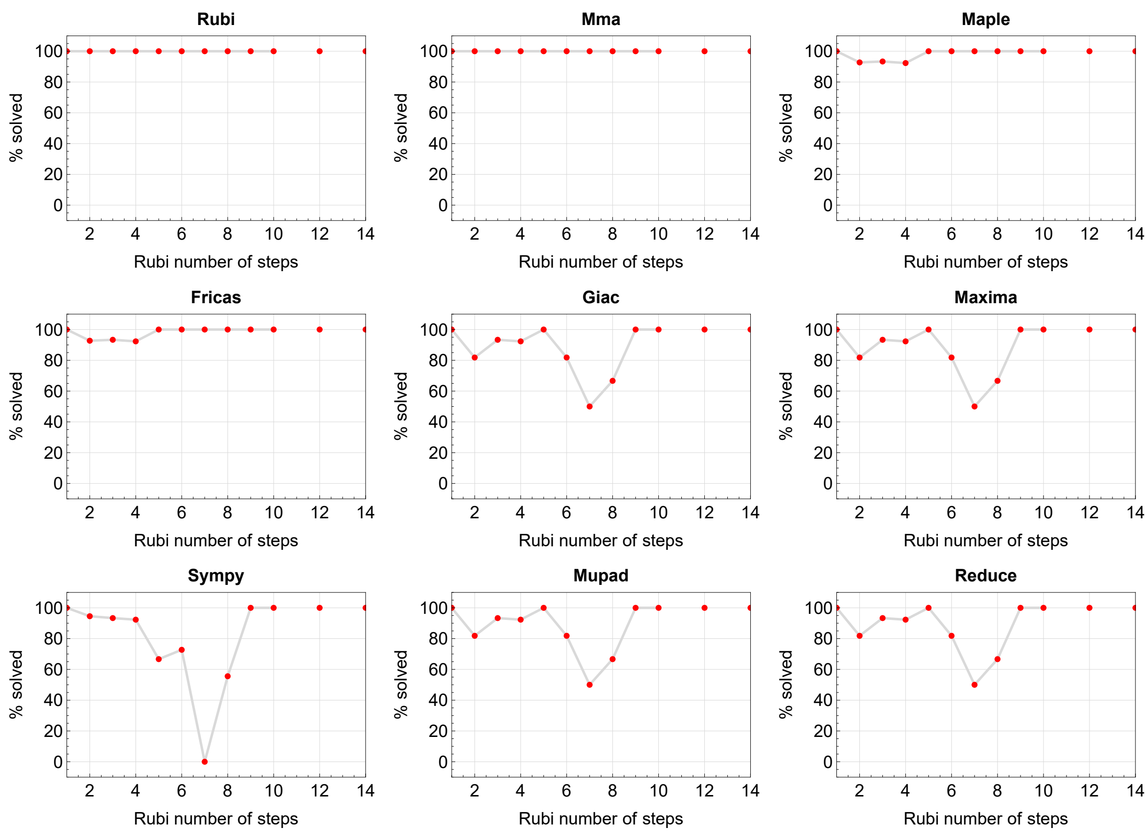


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

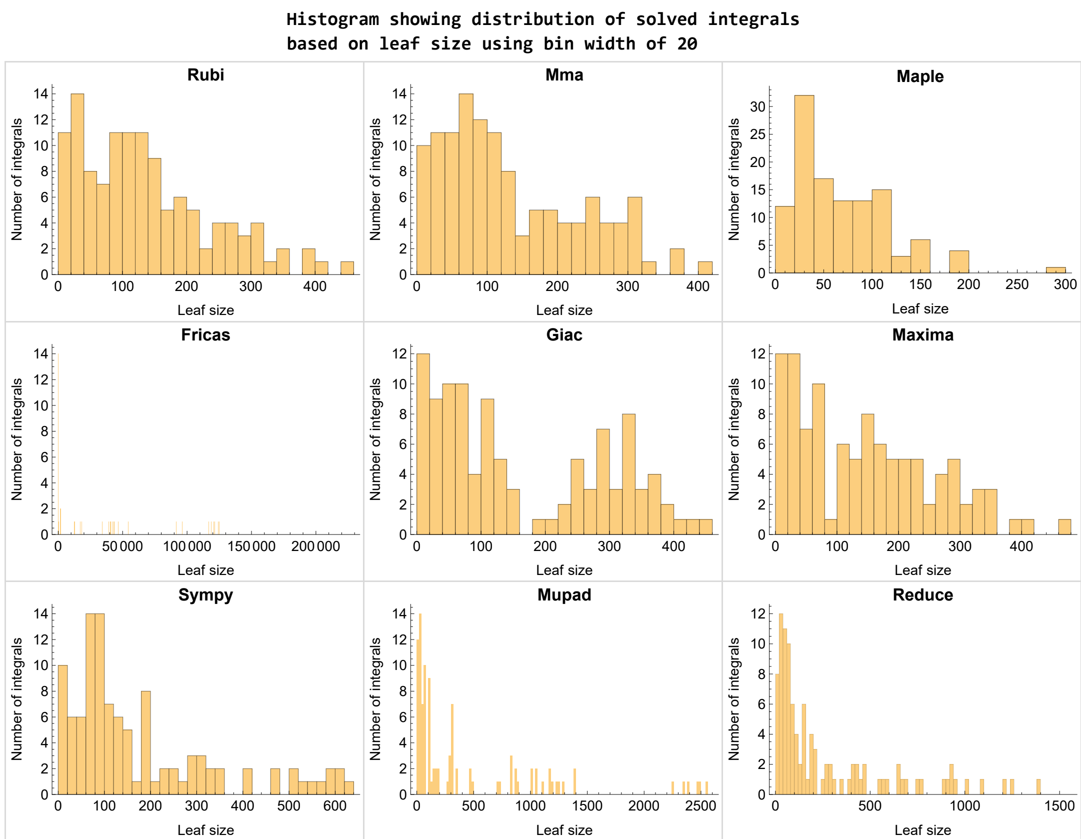


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

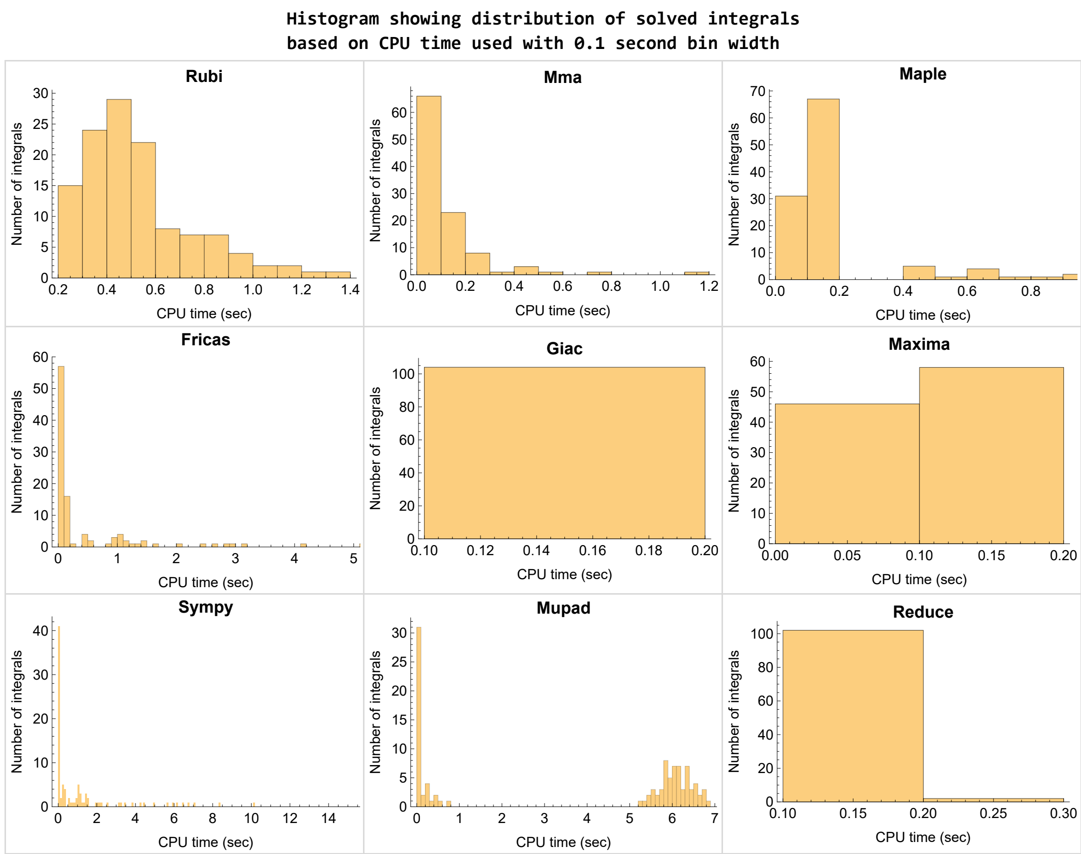


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

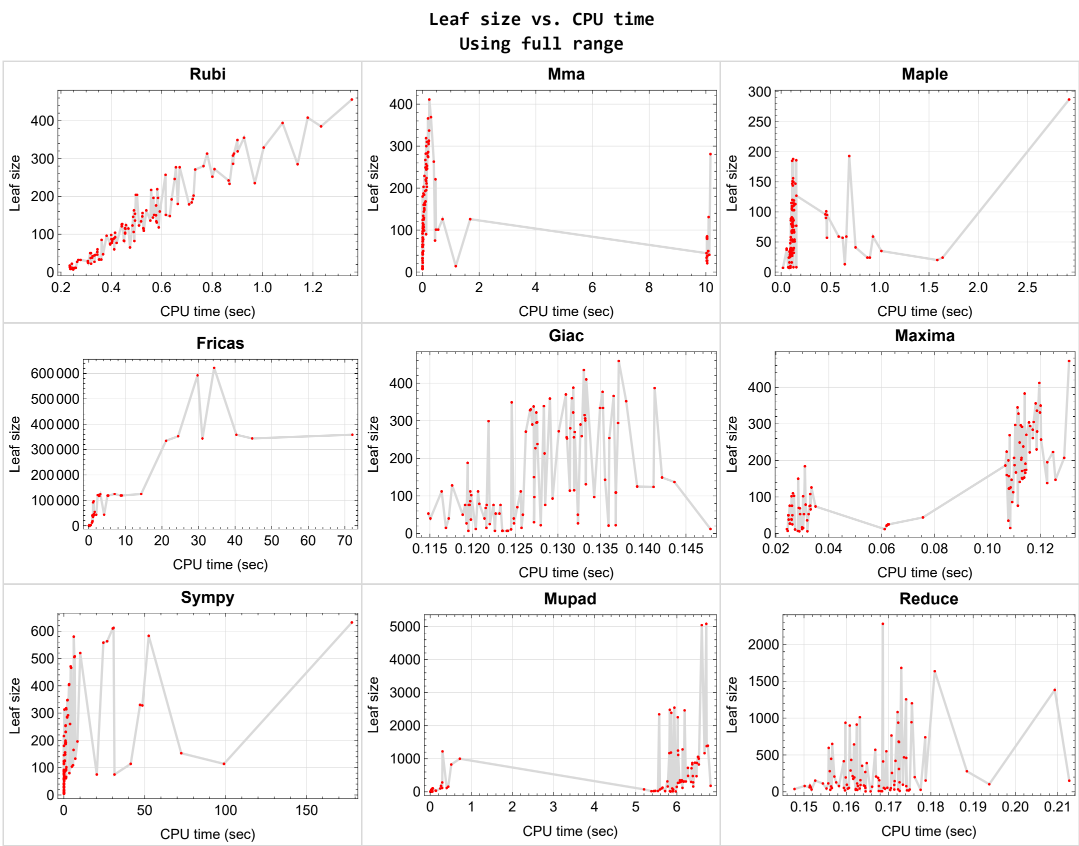


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{79, 85, 87\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

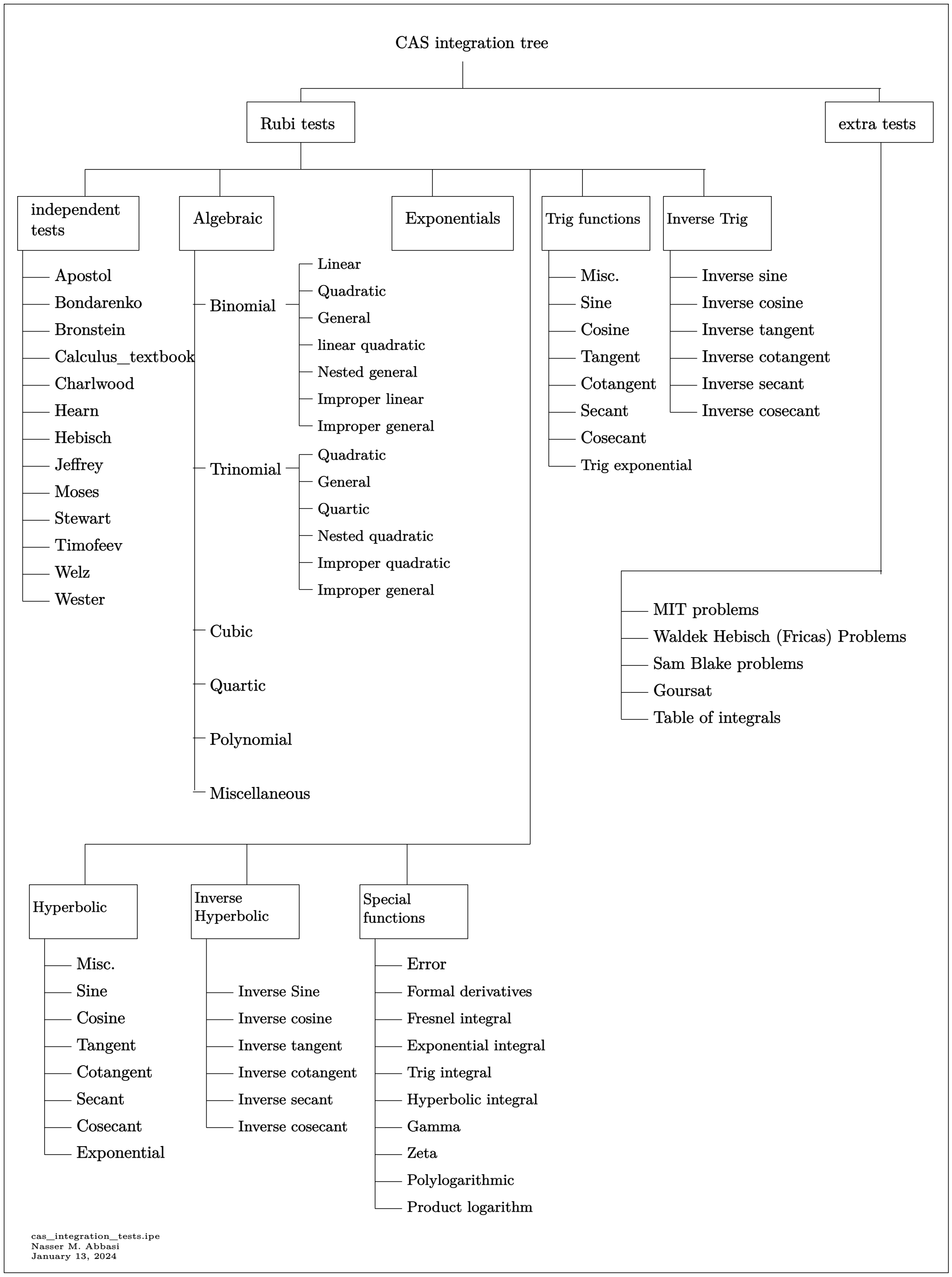
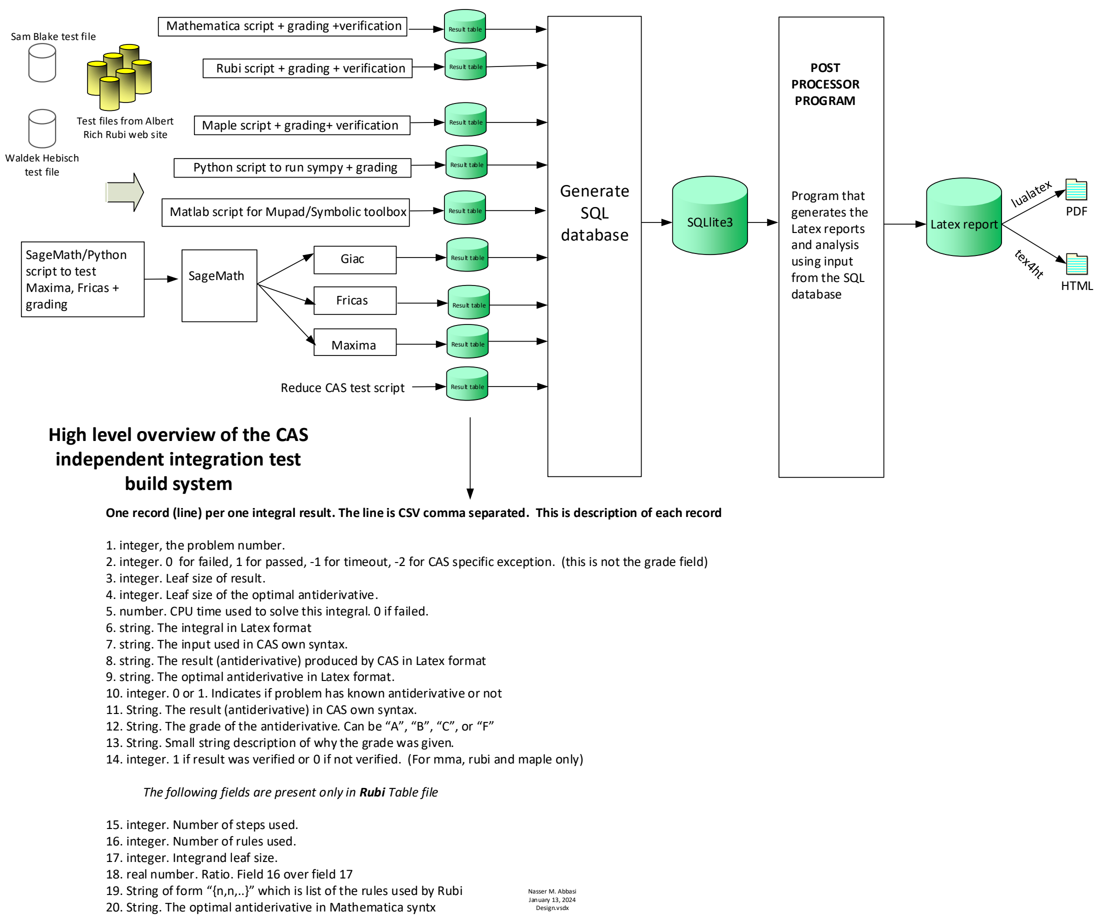


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 27
Mma 27
Maple 28
Fricas 28
Maxima 29
Giac 29
Mupad 29
Sympy 30
Reduce 30

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122 }

B grade { 16, 17, 18 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 121, 122 }

B grade { }

C grade { 13, 16, 46, 83, 111, 112, 113, 114, 115, 120 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 7, 8, 9, 22, 23, 24, 28, 29, 30, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 81, 83, 89, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 113, 114, 116, 117, 118, 119, 121, 122 }

B grade { 45 }

C grade { 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 26, 27, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 71, 72, 73, 74, 76, 77, 78, 79, 80, 82, 84, 85, 86, 87, 88, 90, 91, 92, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 115, 120 }

F normal fail { 19, 20, 21, 34, 35, 36 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 7, 8, 9, 22, 23, 24, 28, 29, 30, 45, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 74, 75, 77, 81, 82, 83, 85, 89, 90, 94, 95, 96, 97, 98, 104, 105, 106, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122 }

B grade { 4, 5, 6, 10, 11, 12, 13, 14, 15, 78, 86, 93, 99 }

C grade { 16, 17, 18, 25, 26, 27, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 71, 72, 73, 76, 79, 80, 84, 87, 88, 91, 92, 100, 101, 102, 103, 107, 108, 109, 110 }

F normal fail { 19, 20, 21, 34, 35, 36 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 81, 89, 90, 91, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 121, 122 }

B grade { 37, 74, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 92, 93, 99 }

C grade { }

F normal fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 34, 35, 36, 111, 112, 113, 114, 115, 120 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 7, 8, 9, 11, 12, 22, 23, 24, 28, 29, 30, 32, 33, 38, 40, 42, 44, 46, 48, 50, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 92, 94, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 121, 122 }

B grade { 4, 5, 6, 10, 25, 26, 27, 31, 37, 39, 41, 43, 45, 47, 49, 51, 53, 71, 72, 73, 83, 91, 93, 100, 101 }

C grade { }

F normal fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 34, 35, 36, 111, 112, 113, 114, 115, 120 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 121, 122 }

C grade { }

F normal fail { }

F(-1) timedout fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 34, 35, 36, 111, 112, 113, 114, 115, 120 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 46, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 81, 82, 84, 85, 86, 87, 89, 90, 91, 92, 94, 95, 96, 97, 98, 104, 105, 106, 113, 114, 118, 119, 120, 122 }

B grade { 4, 5, 6, 25, 26, 47, 48, 49, 50, 51, 52, 53, 54, 62, 71, 72, 73, 80, 88, 93, 99 }

C grade { 19, 20, 21, 34, 35, 45, 83, 111, 112, 115, 116, 117, 121 }

F normal fail { 13, 14, 15, 16, 17, 18 }

F(-1) timedout fail { 36, 100, 101, 102, 103, 107, 108, 109, 110 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 121, 122 }

C grade { }

F normal fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 34, 35, 36, 111, 112, 113, 114, 115, 120 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	110	110	110	124	112	73	106
N.S.	1	1.00	0.89	0.93	0.93	0.93	1.05	0.95	0.62	0.90
time (sec)	N/A	0.588	0.020	0.111	0.026	0.078	0.032	0.121	0.156	0.063

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	75	75	75	85	76	61	74
N.S.	1	1.00	0.99	0.90	0.90	0.90	1.02	0.92	0.73	0.89
time (sec)	N/A	0.450	0.008	0.118	0.033	0.088	0.027	0.122	0.152	0.040

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	40	39	39	42	40	31	40
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.89	0.85	0.66	0.85
time (sec)	N/A	0.367	0.005	0.099	0.026	0.072	0.022	0.121	0.152	0.027

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	160	53	141	1806	318	253	128	2256
N.S.	1	1.00	1.57	0.52	1.38	17.71	3.12	2.48	1.25	22.12
time (sec)	N/A	0.455	0.040	0.097	0.112	0.412	1.405	0.131	0.158	6.021

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	147	160	89	173	1933	348	294	280	2390
N.S.	1	1.16	1.26	0.70	1.36	15.22	2.74	2.31	2.20	18.82
time (sec)	N/A	0.523	0.089	0.100	0.114	0.446	2.272	0.137	0.156	5.865

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	192	203	120	222	2112	406	334	448	2464
N.S.	1	1.16	1.23	0.73	1.35	12.80	2.46	2.02	2.72	14.93
time (sec)	N/A	0.639	0.153	0.114	0.116	0.536	3.274	0.135	0.156	6.187

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	110	108	108	124	112	84	104
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.07	0.97	0.72	0.90
time (sec)	N/A	0.527	0.012	0.122	0.033	0.073	0.037	0.126	0.151	0.059

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	75	74	74	85	76	61	74
N.S.	1	1.00	1.00	0.91	0.90	0.90	1.04	0.93	0.74	0.90
time (sec)	N/A	0.451	0.008	0.114	0.035	0.072	0.029	0.120	0.151	5.201

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	38	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.83	0.85
time (sec)	N/A	0.344	0.005	0.097	0.026	0.089	0.023	0.115	0.148	0.024

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	271	246	48	270	1818	314	295	152	2345
N.S.	1	1.36	1.23	0.24	1.35	9.09	1.57	1.48	0.76	11.72
time (sec)	N/A	0.733	0.163	0.093	0.115	0.423	1.512	0.127	0.153	5.570

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	286	267	86	305	1941	347	322	596	2481
N.S.	1	1.27	1.18	0.38	1.35	8.59	1.54	1.42	2.64	10.98
time (sec)	N/A	0.883	0.101	0.098	0.117	0.464	2.160	0.127	0.156	5.828

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	329	310	115	356	2124	403	359	937	2546
N.S.	1	1.25	1.17	0.44	1.35	8.05	1.53	1.36	3.55	9.64
time (sec)	N/A	1.005	0.154	0.114	0.118	0.542	3.186	0.129	0.160	5.939

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	34	41	0	41	0	0	23	0
N.S.	1	1.32	1.36	1.64	0.00	1.64	0.00	0.00	0.92	0.00
time (sec)	N/A	0.353	10.025	0.755	0.000	0.108	0.000	0.000	0.158	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	41	57	0	41	0	0	23	0
N.S.	1	1.32	1.64	2.28	0.00	1.64	0.00	0.00	0.92	0.00
time (sec)	N/A	0.351	10.123	0.465	0.000	0.091	0.000	0.000	0.155	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	33	41	57	0	41	0	0	23	0
N.S.	1	1.32	1.64	2.28	0.00	1.64	0.00	0.00	0.92	0.00
time (sec)	N/A	0.360	10.101	0.624	0.000	0.083	0.000	0.000	0.161	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	217	45	59	0	42	0	0	21	0
N.S.	1	5.86	1.22	1.59	0.00	1.14	0.00	0.00	0.57	0.00
time (sec)	N/A	0.558	10.022	0.932	0.000	0.090	0.000	0.000	0.156	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	204	50	59	0	42	0	0	21	0
N.S.	1	5.10	1.25	1.48	0.00	1.05	0.00	0.00	0.52	0.00
time (sec)	N/A	0.497	10.084	0.584	0.000	0.093	0.000	0.000	0.157	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	204	50	59	0	42	0	0	21	0
N.S.	1	4.98	1.22	1.44	0.00	1.02	0.00	0.00	0.51	0.00
time (sec)	N/A	0.502	10.081	0.662	0.000	0.096	0.000	0.000	0.160	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	539	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	5.61	0.00
time (sec)	N/A	0.381	0.443	0.000	0.000	0.000	20.346	0.000	0.162	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	146	101	0	0	0	114	0	1477	0
N.S.	1	1.08	0.75	0.00	0.00	0.00	0.84	0.00	10.94	0.00
time (sec)	N/A	0.487	0.554	0.000	0.000	0.000	41.319	0.000	0.175	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	196	126	0	0	0	153	0	0	0
N.S.	1	1.11	0.72	0.00	0.00	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.585	0.708	0.000	0.000	0.000	72.537	0.000	0.194	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	104	103	103	103	114	112	113	104
N.S.	1	1.00	0.98	0.97	0.97	0.97	1.08	1.06	1.07	0.98
time (sec)	N/A	0.489	0.016	0.130	0.027	0.083	0.029	0.120	0.154	0.060

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	70	70	70	78	76	79	70
N.S.	1	1.00	0.97	0.93	0.93	0.93	1.04	1.01	1.05	0.93
time (sec)	N/A	0.395	0.009	0.129	0.029	0.076	0.025	0.119	0.150	0.035

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	38	38	39	40	45	38
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.89	0.91	1.02	0.86
time (sec)	N/A	0.333	0.007	0.056	0.025	0.073	0.022	0.117	0.155	0.046

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	104	144	69	138	1406	255	256	195	1116
N.S.	1	0.95	1.32	0.63	1.27	12.90	2.34	2.35	1.79	10.24
time (sec)	N/A	0.415	0.047	0.108	0.123	0.138	1.093	0.132	0.162	6.032

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	126	168	91	166	1509	286	290	415	1187
N.S.	1	0.96	1.28	0.69	1.27	11.52	2.18	2.21	3.17	9.06
time (sec)	N/A	0.441	0.108	0.109	0.114	0.119	1.986	0.127	0.159	5.855

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	163	195	119	207	1633	330	322	650	1247
N.S.	1	0.99	1.18	0.72	1.25	9.90	2.00	1.95	3.94	7.56
time (sec)	N/A	0.490	0.134	0.130	0.129	0.118	47.080	0.132	0.157	6.027

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	103	102	102	114	112	113	102
N.S.	1	1.00	1.00	0.99	0.98	0.98	1.10	1.08	1.09	0.98
time (sec)	N/A	0.455	0.010	0.130	0.026	0.110	0.029	0.116	0.163	5.630

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	78	76	79	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.07	1.04	1.08	0.96
time (sec)	N/A	0.395	0.007	0.122	0.029	0.085	0.028	0.123	0.158	5.588

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	45	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	1.07	0.90
time (sec)	N/A	0.326	0.004	0.056	0.032	0.087	0.019	0.125	0.163	0.046

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	252	257	67	261	1419	253	328	433	1170
N.S.	1	1.21	1.23	0.32	1.25	6.79	1.21	1.57	2.07	5.60
time (sec)	N/A	0.801	0.129	0.106	0.117	0.101	1.026	0.127	0.161	5.816

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	272	282	88	292	1519	284	349	899	1223
N.S.	1	1.19	1.24	0.39	1.28	6.66	1.25	1.53	3.94	5.36
time (sec)	N/A	0.809	0.116	0.113	0.113	0.109	2.060	0.125	0.161	0.300

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	308	300	115	331	1640	328	377	1382	1283
N.S.	1	1.18	1.15	0.44	1.26	6.26	1.25	1.44	5.27	4.90
time (sec)	N/A	0.883	0.136	0.104	0.120	0.114	48.506	0.135	0.209	6.138

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	90	0	0	0	75	0	301	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.88	0.00	3.54	0.00
time (sec)	N/A	0.361	0.128	0.000	0.000	0.000	31.285	0.000	0.181	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	156	101	0	0	0	114	0	990	0
N.S.	1	1.03	0.67	0.00	0.00	0.00	0.75	0.00	6.56	0.00
time (sec)	N/A	0.524	0.476	0.000	0.000	0.000	98.958	0.000	0.163	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	196	126	0	0	0	0	0	0	0
N.S.	1	0.75	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.567	1.683	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	134	34	126	39057	126	227	114	182
N.S.	1	1.00	1.54	0.39	1.45	448.93	1.45	2.61	1.31	2.09
time (sec)	N/A	0.400	0.024	0.100	0.109	0.981	0.550	0.127	0.156	6.820

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	219	184	32	207	41851	124	213	213	160
N.S.	1	1.31	1.10	0.19	1.24	250.60	0.74	1.28	1.28	0.96
time (sec)	N/A	0.582	0.059	0.129	0.110	1.030	0.579	0.128	0.161	6.398

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	168	69	157	40560	156	254	279	283
N.S.	1	1.05	1.53	0.63	1.43	368.73	1.42	2.31	2.54	2.57
time (sec)	N/A	0.468	0.107	0.121	0.108	1.001	0.761	0.136	0.189	0.294

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	246	224	66	238	43065	155	238	468	282
N.S.	1	1.30	1.19	0.35	1.26	227.86	0.82	1.26	2.48	1.49
time (sec)	N/A	0.651	0.143	0.132	0.114	1.250	0.679	0.128	0.174	6.065

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	150	193	89	186	40637	194	272	460	315
N.S.	1	1.10	1.42	0.65	1.37	298.80	1.43	2.00	3.38	2.32
time (sec)	N/A	0.577	0.096	0.119	0.107	1.095	0.991	0.130	0.174	6.170

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	280	249	86	269	43180	192	256	740	315
N.S.	1	1.31	1.16	0.40	1.26	201.78	0.90	1.20	3.46	1.47
time (sec)	N/A	0.766	0.136	0.127	0.108	2.046	1.195	0.131	0.179	6.119

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	184	217	113	223	40780	231	296	643	351
N.S.	1	1.14	1.34	0.70	1.38	251.73	1.43	1.83	3.97	2.17
time (sec)	N/A	0.719	0.126	0.145	0.125	1.442	1.087	0.128	0.162	6.120

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	313	274	110	304	43302	231	280	1012	350
N.S.	1	1.31	1.15	0.46	1.27	181.18	0.97	1.17	4.23	1.46
time (sec)	N/A	0.887	0.200	0.124	0.116	4.189	1.176	0.131	0.163	6.075

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	42	40	35	35	313	37	43	100
N.S.	1	1.00	1.75	1.67	1.46	1.46	13.04	1.54	1.79	4.17
time (sec)	N/A	0.308	0.012	0.129	0.108	0.090	0.363	0.120	0.160	0.246

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	98	99	27	86	34344	83	86	96	71
N.S.	1	1.29	1.30	0.36	1.13	451.89	1.09	1.13	1.26	0.93
time (sec)	N/A	0.399	0.056	0.111	0.109	1.156	0.336	0.120	0.160	5.910

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	187	39	153	120560	471	259	176	725
N.S.	1	1.01	1.63	0.34	1.33	1048.35	4.10	2.25	1.53	6.30
time (sec)	N/A	0.454	0.032	0.119	0.114	2.433	4.217	0.133	0.162	6.268

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	277	229	37	257	121386	466	271	353	712
N.S.	1	1.32	1.09	0.18	1.22	578.03	2.22	1.29	1.68	3.39
time (sec)	N/A	0.656	0.068	0.124	0.120	2.601	4.500	0.126	0.164	6.356

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	151	211	83	191	116982	508	306	412	477
N.S.	1	1.03	1.45	0.57	1.31	801.25	3.48	2.10	2.82	3.27
time (sec)	N/A	0.570	0.143	0.116	0.114	2.801	6.704	0.133	0.169	6.388

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	313	305	80	294	124258	505	301	764	472
N.S.	1	1.30	1.27	0.33	1.22	517.74	2.10	1.25	3.18	1.97
time (sec)	N/A	0.780	0.216	0.116	0.116	3.160	6.474	0.133	0.171	6.477

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	193	244	114	230	118710	563	334	675	826
N.S.	1	1.08	1.36	0.64	1.28	663.18	3.15	1.87	3.77	4.61
time (sec)	N/A	0.723	0.134	0.132	0.119	5.122	26.753	0.135	0.172	0.513

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	355	337	111	336	124787	558	330	1201	826
N.S.	1	1.30	1.23	0.41	1.23	457.10	2.04	1.21	4.40	3.03
time (sec)	N/A	0.927	0.233	0.130	0.119	7.061	24.411	0.127	0.176	6.534

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	233	276	150	279	118903	612	370	938	874
N.S.	1	1.10	1.31	0.71	1.32	563.52	2.90	1.75	4.45	4.14
time (sec)	N/A	0.869	0.186	0.125	0.119	8.734	30.697	0.131	0.172	6.486

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	394	369	147	383	124960	610	366	1636	873
N.S.	1	1.30	1.21	0.48	1.26	411.05	2.01	1.20	5.38	2.87
time (sec)	N/A	1.080	0.299	0.145	0.114	14.325	30.227	0.137	0.181	6.375

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	27	27	24	25	24	27	25	25	25
N.S.	1	0.96	0.96	0.86	0.89	0.86	0.96	0.89	0.89	0.89
time (sec)	N/A	0.264	0.003	0.078	0.026	0.081	0.020	0.122	0.170	5.895

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	32	32	27	27	27	29	27	27	27
N.S.	1	0.97	0.97	0.82	0.82	0.82	0.88	0.82	0.82	0.82
time (sec)	N/A	0.269	0.002	0.091	0.025	0.091	0.024	0.119	0.156	0.036

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	51	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.85	0.83
time (sec)	N/A	0.411	0.002	0.116	0.026	0.095	0.030	0.126	0.157	0.025

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	32	33	27	27	27	31	27	27	27
N.S.	1	0.97	1.00	0.82	0.82	0.82	0.94	0.82	0.82	0.82
time (sec)	N/A	0.278	0.001	0.112	0.025	0.084	0.024	0.125	0.162	0.040

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	60	50	53	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.00	0.83	0.88	0.83
time (sec)	N/A	0.345	0.002	0.125	0.025	0.073	0.029	0.132	0.159	0.026

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	61	53	53	53
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.94	0.82	0.82	0.82
time (sec)	N/A	0.474	0.003	0.114	0.029	0.081	0.051	0.123	0.161	0.029

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	77	76	76	90	76	77	76
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.98	0.83	0.84	0.83
time (sec)	N/A	0.415	0.003	0.125	0.033	0.082	0.028	0.120	0.174	0.043

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	16	15	27	29	15	26	26
N.S.	1	1.00	1.94	0.94	0.88	1.59	1.71	0.88	1.53	1.53
time (sec)	N/A	0.235	0.001	0.108	0.030	0.092	0.022	0.117	0.163	0.037

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	60	51	50	50	58	50	53	50
N.S.	1	1.00	1.33	1.13	1.11	1.11	1.29	1.11	1.18	1.11
time (sec)	N/A	0.336	0.003	0.138	0.025	0.073	0.029	0.119	0.165	0.026

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	60	53	55	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.20	1.06	1.10	1.06
time (sec)	N/A	0.316	0.004	0.136	0.032	0.086	0.038	0.115	0.170	0.031

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	88	76	77	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.14	0.99	1.00	0.99
time (sec)	N/A	0.421	0.004	0.154	0.025	0.072	0.027	0.122	0.164	0.039

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	65	54	53	53	61	53	53	53
N.S.	1	1.00	1.30	1.08	1.06	1.06	1.22	1.06	1.06	1.06
time (sec)	N/A	0.319	0.003	0.099	0.031	0.077	0.031	0.123	0.169	0.030

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	92	77	76	76	90	76	79	76
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.17	0.99	1.03	0.99
time (sec)	N/A	0.402	0.003	0.110	0.026	0.080	0.027	0.129	0.168	0.040

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	80	79	79	92	79	79	79
N.S.	1	1.00	1.18	0.98	0.96	0.96	1.12	0.96	0.96	0.96
time (sec)	N/A	0.491	0.004	0.103	0.031	0.068	0.027	0.121	0.163	0.043

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	102	102	121	102	103	102
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.94	0.94	0.94
time (sec)	N/A	0.529	0.005	0.114	0.030	0.073	0.026	0.120	0.194	5.887

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	150	150	180	150	151	150
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	0.99	1.00	0.99
time (sec)	N/A	0.616	0.004	0.116	0.028	0.076	0.030	0.127	0.213	5.997

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	160	220	89	200	117016	520	315	420	483
N.S.	1	0.98	1.34	0.54	1.22	713.51	3.17	1.92	2.56	2.95
time (sec)	N/A	0.592	0.113	0.115	0.108	2.911	10.116	0.133	0.172	6.272

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	202	253	120	249	118761	583	352	681	832
N.S.	1	1.03	1.28	0.61	1.26	602.85	2.96	1.79	3.46	4.22
time (sec)	N/A	0.726	0.160	0.115	0.112	5.354	52.453	0.138	0.172	6.374

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	242	286	156	297	118945	632	388	944	880
N.S.	1	1.06	1.25	0.68	1.30	519.41	2.76	1.69	4.12	3.84
time (sec)	N/A	0.865	0.201	0.122	0.110	9.044	177.987	0.132	0.175	6.439

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	134	78	25	123	72	88	97	94	36
N.S.	1	1.63	0.95	0.30	1.50	0.88	1.07	1.18	1.15	0.44
time (sec)	N/A	0.579	0.026	0.091	0.108	0.103	0.233	0.127	0.171	0.133

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	16	15	15	19	15	50	15
N.S.	1	1.00	1.00	0.73	0.68	0.68	0.86	0.68	2.27	0.68
time (sec)	N/A	0.243	0.009	0.084	0.109	0.085	0.059	0.126	0.171	5.900

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	123	107	29	147	12312	88	115	157	119
N.S.	1	1.18	1.03	0.28	1.41	118.38	0.85	1.11	1.51	1.14
time (sec)	N/A	0.484	0.046	0.090	0.126	0.870	0.368	0.132	0.168	6.136

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	134	78	25	123	79	88	97	92	32
N.S.	1	1.63	0.95	0.30	1.50	0.96	1.07	1.18	1.12	0.39
time (sec)	N/A	0.518	0.016	0.089	0.108	0.087	0.217	0.134	0.163	0.140

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	144	113	31	167	465	68	131	195	315
N.S.	1	1.38	1.09	0.30	1.61	4.47	0.65	1.26	1.88	3.03
time (sec)	N/A	0.565	0.034	0.093	0.111	0.097	0.285	0.133	0.160	5.769

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	104	123	99	33	147	12711	85	114	158	162
N.S.	1	1.18	0.95	0.32	1.41	122.22	0.82	1.10	1.52	1.56
time (sec)	N/A	0.510	0.029	0.090	0.109	0.915	0.391	0.131	0.163	5.777

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	163	129	34	187	46609	292	143	269	270
N.S.	1	1.29	1.02	0.27	1.48	369.91	2.32	1.13	2.13	2.14
time (sec)	N/A	0.539	0.050	0.085	0.110	1.418	2.572	0.136	0.164	6.025

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	11	11	10	11	39	9
N.S.	1	1.00	1.00	0.77	0.85	0.85	0.77	0.85	3.00	0.69
time (sec)	N/A	0.235	0.003	0.095	0.027	0.072	0.050	0.125	0.162	0.033

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	114	108	31	149	82	51	109	153	117
N.S.	1	1.20	1.14	0.33	1.57	0.86	0.54	1.15	1.61	1.23
time (sec)	N/A	0.456	0.026	0.094	0.113	0.087	0.189	0.137	0.179	0.292

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	37	65	28	113	27	53	93	96	25
N.S.	1	1.03	1.81	0.78	3.14	0.75	1.47	2.58	2.67	0.69
time (sec)	N/A	0.318	0.023	0.112	0.110	0.083	0.206	0.129	0.173	5.539

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	136	128	34	171	17085	199	125	207	307
N.S.	1	1.16	1.09	0.29	1.46	146.03	1.70	1.07	1.77	2.62
time (sec)	N/A	0.494	0.038	0.096	0.113	0.982	1.006	0.139	0.167	6.165

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	95	114	108	35	152	89	70	109	145	117
N.S.	1	1.20	1.14	0.37	1.60	0.94	0.74	1.15	1.53	1.23
time (sec)	N/A	0.528	0.018	0.090	0.113	0.088	0.166	0.137	0.171	0.403

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	154	148	36	195	481	148	137	256	286
N.S.	1	1.32	1.26	0.31	1.67	4.11	1.26	1.17	2.19	2.44
time (sec)	N/A	0.494	0.067	0.097	0.123	0.132	0.802	0.144	0.171	6.396

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	117	136	125	38	174	18086	189	124	199	300
N.S.	1	1.16	1.07	0.32	1.49	154.58	1.62	1.06	1.70	2.56
time (sec)	N/A	0.555	0.042	0.110	0.115	1.041	0.923	0.141	0.176	6.347

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	176	164	39	207	54479	580	149	310	1168
N.S.	1	1.27	1.18	0.28	1.49	391.94	4.17	1.07	2.23	8.40
time (sec)	N/A	0.578	0.071	0.099	0.113	1.622	6.115	0.142	0.172	6.672

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.75
time (sec)	N/A	0.237	0.001	0.078	0.032	0.078	0.024	0.123	0.166	0.025

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	31	76	58	73	70	86	156
N.S.	1	1.00	0.94	0.58	1.43	1.09	1.38	1.32	1.62	2.94
time (sec)	N/A	0.344	0.027	0.116	0.111	0.128	0.224	0.125	0.164	0.441

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	203	38	160	91748	187	290	207	312
N.S.	1	1.00	1.64	0.31	1.29	739.90	1.51	2.34	1.67	2.52
time (sec)	N/A	0.458	0.037	0.093	0.107	1.168	1.148	0.133	0.157	6.218

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	277	283	36	296	96349	187	270	390	305
N.S.	1	1.30	1.33	0.17	1.39	452.34	0.88	1.27	1.83	1.43
time (sec)	N/A	0.671	0.173	0.115	0.113	1.328	1.091	0.132	0.165	6.092

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	15	15	15	15	18	15
N.S.	1	1.00	0.82	0.73	1.36	1.36	1.36	1.36	1.64	1.36
time (sec)	N/A	0.259	0.002	0.072	0.030	0.077	0.034	0.122	0.175	0.032

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	14	8	12	12	8	12	11	11
N.S.	1	1.00	1.27	0.73	1.09	1.09	0.73	1.09	1.00	1.00
time (sec)	N/A	0.255	0.001	0.080	0.030	0.081	0.035	0.120	0.168	0.025

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	7	7	7	5	7	8	6
N.S.	1	1.00	0.82	0.64	0.64	0.64	0.45	0.64	0.73	0.55
time (sec)	N/A	0.247	0.001	0.019	0.024	0.077	0.035	0.124	0.168	0.020

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.75
time (sec)	N/A	0.244	0.001	0.094	0.029	0.088	0.037	0.120	0.165	0.002

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	8	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.14	1.00
time (sec)	N/A	0.249	0.001	0.095	0.028	0.084	0.054	0.124	0.168	0.033

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	12	12	10	7	12	7
N.S.	1	1.00	0.82	0.73	1.09	1.09	0.91	0.64	1.09	0.64
time (sec)	N/A	0.255	0.002	0.125	0.024	0.087	0.067	0.123	0.173	5.844

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	17	17	17	7	19	7
N.S.	1	1.00	0.82	0.73	1.55	1.55	1.55	0.64	1.73	0.64
time (sec)	N/A	0.258	0.002	0.155	0.030	0.083	0.072	0.124	0.160	0.055

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	249	65	202	592528	0	299	289	5082
N.S.	1	1.00	1.68	0.44	1.36	4003.57	0.00	2.02	1.95	34.34
time (sec)	N/A	0.632	0.104	0.110	0.113	29.731	0.000	0.122	0.161	6.716

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	180	221	108	224	334837	0	339	570	1393
N.S.	1	1.05	1.28	0.63	1.30	1946.73	0.00	1.97	3.31	8.10
time (sec)	N/A	0.664	0.458	0.111	0.107	21.129	0.000	0.128	0.167	6.753

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	235	263	147	284	343626	0	387	912	1002
N.S.	1	1.00	1.11	0.62	1.20	1456.04	0.00	1.64	3.86	4.25
time (sec)	N/A	0.969	0.399	0.113	0.117	31.067	0.000	0.141	0.162	6.532

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	285	313	188	345	343822	0	435	1255	1056
N.S.	1	1.01	1.11	0.67	1.23	1223.57	0.00	1.55	4.47	3.76
time (sec)	N/A	1.139	0.224	0.122	0.111	44.661	0.000	0.133	0.174	6.512

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	152	186	184	184	216	188	189	180
N.S.	1	1.00	0.85	1.04	1.03	1.03	1.21	1.05	1.06	1.01
time (sec)	N/A	0.709	0.063	0.154	0.031	0.095	0.037	0.119	0.168	0.265

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	111	127	126	126	146	128	129	126
N.S.	1	1.00	0.85	0.97	0.96	0.96	1.11	0.98	0.98	0.96
time (sec)	N/A	0.580	0.024	0.155	0.033	0.092	0.039	0.118	0.173	5.651

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	67	66	66	76	68	69	67
N.S.	1	1.00	0.94	0.82	0.80	0.80	0.93	0.83	0.84	0.82
time (sec)	N/A	0.403	0.016	0.132	0.033	0.080	0.023	0.122	0.166	0.041

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	319	311	58	328	622377	0	338	550	5042
N.S.	1	1.29	1.26	0.23	1.33	2519.74	0.00	1.37	2.23	20.41
time (sec)	N/A	0.901	0.221	0.098	0.112	34.261	0.000	0.127	0.169	6.605

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	349	319	105	350	352423	0	360	1082	1383
N.S.	1	1.30	1.19	0.39	1.30	1310.12	0.00	1.34	4.02	5.14
time (sec)	N/A	0.900	0.128	0.109	0.120	24.428	0.000	0.132	0.172	6.725

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	408	366	144	412	358509	0	410	1681	1001
N.S.	1	1.22	1.10	0.43	1.23	1073.38	0.00	1.23	5.03	3.00
time (sec)	N/A	1.179	0.196	0.119	0.120	40.344	0.000	0.133	0.173	0.721

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	456	411	185	472	358702	0	459	2279	1053
N.S.	1	1.21	1.09	0.49	1.25	951.46	0.00	1.22	6.05	2.79
time (sec)	N/A	1.355	0.240	0.112	0.131	72.053	0.000	0.137	0.169	6.520

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	72	0
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	0.60	0.00
time (sec)	N/A	0.442	10.042	0.449	0.000	0.125	1.261	0.000	0.169	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	85	101	0	82	66	0	57	0
N.S.	1	1.00	0.67	0.80	0.00	0.65	0.52	0.00	0.45	0.00
time (sec)	N/A	0.443	10.043	0.458	0.000	0.118	1.420	0.000	0.179	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	90	0	79	95	0	88	0
N.S.	1	1.00	0.93	1.03	0.00	0.91	1.09	0.00	1.01	0.00
time (sec)	N/A	0.406	10.040	0.455	0.000	0.114	1.451	0.000	0.239	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	95	0	73	90	0	79	0
N.S.	1	1.00	0.93	1.07	0.00	0.82	1.01	0.00	0.89	0.00
time (sec)	N/A	0.411	10.038	0.466	0.000	0.114	1.365	0.000	0.182	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	257	131	193	0	128	102	0	99	0
N.S.	1	1.00	0.51	0.75	0.00	0.50	0.40	0.00	0.38	0.00
time (sec)	N/A	0.615	10.097	0.691	0.000	0.112	1.561	0.000	0.188	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	80	12	20	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	5.71	0.86	1.43	0.86
time (sec)	N/A	0.237	1.174	0.646	0.061	0.087	3.421	0.148	0.174	5.970

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	27	24	25	34	104	22	31	23
N.S.	1	1.07	1.00	0.89	0.93	1.26	3.85	0.81	1.15	0.85
time (sec)	N/A	0.307	10.031	0.900	0.062	0.098	4.965	0.137	0.171	5.714

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	25	27	24	23	33	109	22	31	20
N.S.	1	0.74	0.79	0.71	0.68	0.97	3.21	0.65	0.91	0.59
time (sec)	N/A	0.307	10.033	0.875	0.062	0.091	5.654	0.128	0.175	5.830

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	38	38	35	44	44	133	30	42	29
N.S.	1	0.81	0.81	0.74	0.94	0.94	2.83	0.64	0.89	0.62
time (sec)	N/A	0.324	10.047	1.017	0.076	0.094	7.068	0.127	0.169	5.472

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	385	281	287	0	204	260	0	259	0
N.S.	1	0.99	0.72	0.74	0.00	0.53	0.67	0.00	0.67	0.00
time (sec)	N/A	1.232	10.162	2.921	0.000	0.237	3.833	0.000	0.216	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	22	21	20	21	21	165	21	27	19
N.S.	1	1.05	1.00	0.95	1.00	1.00	7.86	1.00	1.29	0.90
time (sec)	N/A	0.320	10.047	1.584	0.062	0.089	5.989	0.136	0.178	5.381

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	27	27	24	25	25	196	27	33	23
N.S.	1	0.71	0.71	0.63	0.66	0.66	5.16	0.71	0.87	0.61
time (sec)	N/A	0.333	10.046	1.638	0.063	0.086	8.400	0.132	0.172	5.437

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [74] had the largest ratio of [.8181819999999999965]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	23	0.087
2	A	2	2	1.00	23	0.087
3	A	2	2	1.00	21	0.095
4	A	2	2	1.00	23	0.087
5	A	4	4	1.16	23	0.174
6	A	6	6	1.16	23	0.261
7	A	2	2	1.00	22	0.091
8	A	2	2	1.00	22	0.091
9	A	2	2	1.00	20	0.100
10	A	2	2	1.36	22	0.091
11	A	12	11	1.27	22	0.500
12	A	14	13	1.25	22	0.591
13	A	8	8	1.32	21	0.381
14	A	8	8	1.32	19	0.421
15	A	8	8	1.32	26	0.308
16	B	7	7	5.86	19	0.368
17	B	6	6	5.10	17	0.353
18	B	6	6	4.98	22	0.273
19	A	2	2	1.00	17	0.118
20	A	2	2	1.08	22	0.091
21	A	2	2	1.11	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	23	0.087
23	A	2	2	1.00	23	0.087
24	A	2	2	1.00	21	0.095
25	A	6	6	0.95	23	0.261
26	A	5	5	0.96	23	0.217
27	A	5	5	0.99	23	0.217
28	A	2	2	1.00	22	0.091
29	A	2	2	1.00	22	0.091
30	A	2	2	1.00	20	0.100
31	A	12	11	1.21	22	0.500
32	A	12	11	1.19	22	0.500
33	A	12	11	1.18	22	0.500
34	A	3	3	1.00	17	0.176
35	A	4	4	1.03	22	0.182
36	A	2	2	0.75	27	0.074
37	A	2	2	1.00	16	0.125
38	A	2	2	1.31	15	0.133
39	A	4	4	1.05	16	0.250
40	A	4	4	1.30	15	0.267
41	A	6	6	1.10	16	0.375
42	A	6	6	1.31	15	0.400
43	A	8	8	1.14	16	0.500
44	A	8	8	1.31	15	0.533
45	A	2	2	1.00	15	0.133
46	A	2	2	1.29	13	0.154
47	A	2	2	1.01	21	0.095
48	A	2	2	1.32	20	0.100
49	A	4	4	1.03	21	0.190
50	A	4	4	1.30	20	0.200
51	A	6	6	1.08	21	0.286
52	A	6	6	1.30	20	0.300
53	A	8	8	1.10	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	8	1.30	20	0.400
55	A	3	3	0.96	11	0.273
56	A	3	3	0.97	12	0.250
57	A	2	2	1.00	15	0.133
58	A	3	3	0.97	14	0.214
59	A	2	2	1.00	17	0.118
60	A	3	3	1.00	19	0.158
61	A	2	2	1.00	20	0.100
62	A	2	2	1.00	14	0.143
63	A	4	4	1.00	17	0.235
64	A	4	4	1.00	19	0.211
65	A	3	3	1.00	20	0.150
66	A	4	4	1.00	21	0.190
67	A	3	3	1.00	22	0.136
68	A	4	4	1.00	24	0.167
69	A	3	3	1.00	25	0.120
70	A	3	3	1.00	25	0.120
71	A	4	4	0.98	26	0.154
72	A	6	6	1.03	26	0.231
73	A	8	8	1.06	26	0.308
74	A	10	9	1.63	11	0.818
75	A	4	3	1.00	12	0.250
76	A	2	2	1.18	15	0.133
77	A	10	9	1.63	14	0.643
78	A	9	8	1.38	17	0.471
79	A	3	3	1.18	19	0.158
80	A	2	2	1.29	20	0.100
81	A	2	2	1.00	14	0.143
82	A	2	2	1.20	17	0.118
83	A	6	5	1.03	19	0.263
84	A	2	2	1.16	20	0.100
85	A	3	3	1.20	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.32	22	0.091
87	A	3	3	1.16	24	0.125
88	A	2	2	1.27	25	0.080
89	A	2	2	1.00	19	0.105
90	A	2	2	1.00	17	0.118
91	A	2	2	1.00	20	0.100
92	A	2	2	1.30	19	0.105
93	A	2	2	1.00	21	0.095
94	A	2	2	1.00	21	0.095
95	A	2	2	1.00	19	0.105
96	A	2	2	1.00	19	0.105
97	A	2	2	1.00	21	0.095
98	A	2	2	1.00	21	0.095
99	A	2	2	1.00	21	0.095
100	A	2	2	1.00	31	0.065
101	A	3	3	1.05	31	0.097
102	A	5	5	1.00	31	0.161
103	A	7	7	1.01	31	0.226
104	A	3	3	1.00	30	0.100
105	A	3	3	1.00	30	0.100
106	A	2	2	1.00	28	0.071
107	A	2	2	1.29	30	0.067
108	A	4	4	1.30	30	0.133
109	A	6	6	1.22	30	0.200
110	A	8	8	1.21	30	0.267
111	A	2	2	1.00	17	0.118
112	A	2	2	1.00	20	0.100
113	A	2	2	1.00	18	0.111
114	A	2	2	1.00	19	0.105
115	A	2	2	1.00	22	0.091
116	A	1	1	1.00	23	0.043
117	A	1	1	1.07	26	0.038

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	1	1	0.74	28	0.036
119	A	1	1	0.81	31	0.032
120	A	2	2	0.99	42	0.048
121	A	1	1	1.05	37	0.027
122	A	1	1	0.71	44	0.023

CHAPTER 3

Listing of integrals

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3.1 $\int (a-cx^4)^3 (A+Bx^2+Cx^4)\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 118

$$\int (a-cx^4)^3 (A+Bx^2+Cx^4)\,dx = a^3Ax+\frac{1}{3}a^3Bx^3-\frac{1}{5}a^2(3Ac-aC)x^5-\frac{3}{7}a^2Bcx^7+\frac{1}{3}ac(Ac-aC)x^9+\frac{3}{11}aBc^2x^{11}-\frac{1}{13}c^2(Ac-3aC)x^{13}-\frac{1}{15}Bc^3x^{15}-\frac{1}{17}c^3Cx^{17}$$

output
```
a^3*A*x+1/3*a^3*B*x^3-1/5*a^2*(3*A*c-C*a)*x^5-3/7*a^2*B*c*x^7+1/3*a*c*(A*c
-C*a)*x^9+3/11*a*B*c^2*x^11-1/13*c^2*(A*c-3*C*a)*x^13-1/15*B*c^3*x^15-1/17
*c^3*C*x^17
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int (a-cx^4)^3 (A+Bx^2+Cx^4)\,dx = -\frac{1}{105}a^2cx^5(63A+45Bx^2+35Cx^4)+\frac{1}{429}ac^2x^9(143A+117Bx^2+99Cx^4)-\frac{c^3x^{13}(255A+221Bx^2+195Cx^4)}{3315}+a^3\left(Ax+\frac{Bx^3}{3}+\frac{Cx^5}{5}\right)$$

input
```
Integrate[(a - c*x^4)^3*(A + B*x^2 + C*x^4),x]
```

output
```
-1/105*(a^2*c*x^5*(63*A + 45*B*x^2 + 35*C*x^4)) + (a*c^2*x^9*(143*A + 117*
B*x^2 + 99*C*x^4))/429 - (c^3*x^13*(255*A + 221*B*x^2 + 195*C*x^4))/3315 +
 a^3*(A*x + (B*x^3)/3 + (C*x^5)/5)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{ 2200}$$

$$\int \left(a^3A+a^3Bx^2+a^2x^4(aC-3Ac)-3a^2Bcx^6-c^2x^{12}(Ac-3aC)-3acx^8(aC-Ac)+3aBc^2x^{10}-Bc^3x^{14}-$$

$$\Big\downarrow \text{ 2009}$$

$$a^3Ax+\frac{1}{3}a^3Bx^3-\frac{1}{5}a^2x^5(3Ac-aC)-\frac{3}{7}a^2Bcx^7-\frac{1}{13}c^2x^{13}(Ac-3aC)+\frac{1}{3}acx^9(Ac-aC)+\frac{3}{11}aBc^2x^{11}-\frac{1}{15}Bc^3x^{15}-\frac{1}{17}c^3Cx^{17}$$

input
```
Int[(a - c*x^4)^3*(A + B*x^2 + C*x^4),x]
```

output
```
a^3*A*x + (a^3*B*x^3)/3 - (a^2*(3*A*c - a*C)*x^5)/5 - (3*a^2*B*c*x^7)/7 +
(a*c*(A*c - a*C)*x^9)/3 + (3*a*B*c^2*x^11)/11 - (c^2*(A*c - 3*a*C)*x^13)/1
3 - (B*c^3*x^15)/15 - (c^3*C*x^17)/17
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

method	result
norman	$a^3Ax+\frac{a^3B\,x^3}{3}+\left(-\frac{3}{5}a^2Ac+\frac{1}{5}a^3C\right)x^5-\frac{3a^2Bc\,x^7}{7}+\left(\frac{1}{3}Aa\,c^2-\frac{1}{3}a^2cC\right)x^9+\frac{3aB\,c^2x^{11}}{11}+\left(-\right.$
default	$-\frac{c^3C\,x^{17}}{17}-\frac{B\,c^3x^{15}}{15}+\frac{\left(-A\,c^3+3a\,c^2C\right)x^{13}}{13}+\frac{3aB\,c^2x^{11}}{11}+\frac{\left(3Aa\,c^2-3a^2cC\right)x^9}{9}-\frac{3a^2Bc\,x^7}{7}+\frac{\left(-3a^2Ac+a^3C\right)x}{5}$
gosper	$a^3Ax+\frac{1}{3}a^3B\,x^3-\frac{3}{5}x^5a^2Ac+\frac{1}{5}x^5a^3C-\frac{3}{7}a^2Bc\,x^7+\frac{1}{3}x^9Aa\,c^2-\frac{1}{3}x^9a^2cC+\frac{3}{11}aB\,c^2x^{11}-$
risch	$a^3Ax+\frac{1}{3}a^3B\,x^3-\frac{3}{5}x^5a^2Ac+\frac{1}{5}x^5a^3C-\frac{3}{7}a^2Bc\,x^7+\frac{1}{3}x^9Aa\,c^2-\frac{1}{3}x^9a^2cC+\frac{3}{11}aB\,c^2x^{11}-$
parallelrisch	$a^3Ax+\frac{1}{3}a^3B\,x^3-\frac{3}{5}x^5a^2Ac+\frac{1}{5}x^5a^3C-\frac{3}{7}a^2Bc\,x^7+\frac{1}{3}x^9Aa\,c^2-\frac{1}{3}x^9a^2cC+\frac{3}{11}aB\,c^2x^{11}-$
orering	$\frac{x\left(-15015c^3C\,x^{16}-17017c^3B\,x^{14}-19635A\,c^3x^{12}+58905Ca\,c^2x^{12}+69615a\,c^2B\,x^{10}+85085Aa\,c^2x^8-85085C\,a^2c\,x^8-109395\right.}{255255}$

input
```
int((-c*x^4+a)^3*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a^3*A*x+1/3*a^3*B*x^3+(-3/5*a^2*A*c+1/5*a^3*C)*x^5-3/7*a^2*B*c*x^7+(1/3*A*
a*c^2-1/3*a^2*c*C)*x^9+3/11*a*B*c^2*x^11+(-1/13*A*c^3+3/13*a*c^2*C)*x^13-1
/15*B*c^3*x^15-1/17*c^3*C*x^17
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int \left(a-cx^4\right)^3 \left(A+Bx^2+Cx^4\right)\,dx=-\frac{1}{17}\,Cc^3x^{17}-\frac{1}{15}\,Bc^3x^{15}+\frac{3}{11}\,Bac^2x^{11}+\frac{1}{13}\,(3\,Cac^2-Ac^3)x^{13}-\frac{3}{7}\,Ba^2cx^7-\frac{1}{3}\,(Ca^2c-Aac^2)x^9+\frac{1}{3}\,Ba^3x^3+\frac{1}{5}\,(Ca^3-3\,Aa^2c)x^5+Aa^3x$$

input
```
integrate((-c*x^4+a)^3*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/17*C*c^3*x^17 - 1/15*B*c^3*x^15 + 3/11*B*a*c^2*x^11 + 1/13*(3*C*a*c^2 -
 A*c^3)*x^13 - 3/7*B*a^2*c*x^7 - 1/3*(C*a^2*c - A*a*c^2)*x^9 + 1/3*B*a^3*x
^3 + 1/5*(C*a^3 - 3*A*a^2*c)*x^5 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \left(a-cx^4\right)^3 \left(A+Bx^2+Cx^4\right)\,dx=Aa^3x+\frac{Ba^3x^3}{3}-\frac{3Ba^2cx^7}{7}+\frac{3Bac^2x^{11}}{11}-\frac{Bc^3x^{15}}{15}-\frac{Cc^3x^{17}}{17}+x^{13}\left(-\frac{Ac^3}{13}+\frac{3Cac^2}{13}\right)+x^9\left(\frac{Aac^2}{3}-\frac{Ca^2c}{3}\right)+x^5\left(-\frac{3Aa^2c}{5}+\frac{Ca^3}{5}\right)$$

input
```
integrate((-c*x**4+a)**3*(C*x**4+B*x**2+A),x)
```

output
```
A*a**3*x + B*a**3*x**3/3 - 3*B*a**2*c*x**7/7 + 3*B*a*c**2*x**11/11 - B*c**
3*x**15/15 - C*c**3*x**17/17 + x**13*(-A*c**3/13 + 3*C*a*c**2/13) + x**9*(
A*a*c**2/3 - C*a**2*c/3) + x**5*(-3*A*a**2*c/5 + C*a**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int (a-cx^4)^3 (A+Bx^2+Cx^4)\,dx = -\frac{1}{17}Cc^3x^{17} - \frac{1}{15}Bc^3x^{15} + \frac{3}{11}Bac^2x^{11} + \frac{1}{13}(3Cac^2 - Ac^3)x^{13} - \frac{3}{7}Ba^2cx^7 - \frac{1}{3}(Ca^2c - Aac^2)x^9 + \frac{1}{3}Ba^3x^3 + \frac{1}{5}(Ca^3 - 3Aa^2c)x^5 + Aa^3x$$

input
```
integrate((-c*x^4+a)^3*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
-1/17*C*c^3*x^17 - 1/15*B*c^3*x^15 + 3/11*B*a*c^2*x^11 + 1/13*(3*C*a*c^2 -
 A*c^3)*x^13 - 3/7*B*a^2*c*x^7 - 1/3*(C*a^2*c - A*a*c^2)*x^9 + 1/3*B*a^3*x
^3 + 1/5*(C*a^3 - 3*A*a^2*c)*x^5 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int (a-cx^4)^3 (A+Bx^2+Cx^4)\,dx = -\frac{1}{17}Cc^3x^{17} - \frac{1}{15}Bc^3x^{15} + \frac{3}{13}Cac^2x^{13} - \frac{1}{13}Ac^3x^{13} + \frac{3}{11}Bac^2x^{11} - \frac{1}{3}Ca^2cx^9 + \frac{1}{3}Aac^2x^9 - \frac{3}{7}Ba^2cx^7 + \frac{1}{5}Ca^3x^5 - \frac{3}{5}Aa^2cx^5 + \frac{1}{3}Ba^3x^3 + Aa^3x$$

input
```
integrate((-c*x^4+a)^3*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
-1/17*C*c^3*x^17 - 1/15*B*c^3*x^15 + 3/13*C*a*c^2*x^13 - 1/13*A*c^3*x^13 +
 3/11*B*a*c^2*x^11 - 1/3*C*a^2*c*x^9 + 1/3*A*a*c^2*x^9 - 3/7*B*a^2*c*x^7 +
 1/5*C*a^3*x^5 - 3/5*A*a^2*c*x^5 + 1/3*B*a^3*x^3 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\int \left(a-cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx = x^5 \left(\frac{C\,a^3}{5}-\frac{3\,A\,a^2\,c}{5}\right)-x^{13}\left(\frac{A\,c^3}{13}-\frac{3\,C\,a\,c^2}{13}\right)+\frac{B\,a^3\,x^3}{3}-\frac{B\,c^3\,x^{15}}{15}-\frac{C\,c^3\,x^{17}}{17}+A\,a^3\,x+\frac{a\,c\,x^9\,(A\,c-C\,a)}{3}-\frac{3\,B\,a^2\,c\,x^7}{7}+\frac{3\,B\,a\,c^2\,x^{11}}{11}$$

input
```
int((a - c*x^4)^3*(A + B*x^2 + C*x^4),x)
```

output
```
x^5*((C*a^3)/5 - (3*A*a^2*c)/5) - x^13*((A*c^3)/13 - (3*C*a*c^2)/13) + (B*
a^3*x^3)/3 - (B*c^3*x^15)/15 - (C*c^3*x^17)/17 + A*a^3*x + (a*c*x^9*(A*c -
 C*a))/3 - (3*B*a^2*c*x^7)/7 + (3*B*a*c^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \left(a-cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx = \frac{x(-15015c^4x^{16}-17017b\,c^3x^{14}+39270a\,c^3x^{12}+69615ab\,c^2x^{10}-109395a^2bc\,x^6-102102a^3c\,x^4+85085}{255255}$$

input
```
int((-c*x^4+a)^3*(C*x^4+B*x^2+A),x)
```

output
```
(x*(255255*a**4 + 85085*a**3*b*x**2 - 102102*a**3*c*x**4 - 109395*a**2*b*c
*x**6 + 69615*a*b*c**2*x**10 + 39270*a*c**3*x**12 - 17017*b*c**3*x**14 - 1
5015*c**4*x**16))/255255
```

3.2 $\int (a-cx^4)^2 (A+Bx^2+Cx^4) \, dx$

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Mathematica [A] (verified) . . . 77
Rubi [A] (verified) . . . 78
Maple [A] (verified) . . . 79
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Sympy [A] (verification not implemented) . . . 80
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Giac [A] (verification not implemented) . . . 81
Mupad [B] (verification not implemented) . . . 81
Reduce [B] (verification not implemented) . . . 82

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int (a-cx^4)^2 (A+Bx^2+Cx^4) \, dx = a^2Ax+\frac{1}{3}a^2Bx^3-\frac{1}{5}a(2Ac-aC)x^5-\frac{2}{7}aBcx^7 +\frac{1}{9}c(Ac-2aC)x^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{13}c^2Cx^{13}$$

output
```
a^2*A*x+1/3*a^2*B*x^3-1/5*a*(2*A*c-C*a)*x^5-2/7*a*B*c*x^7+1/9*c*(A*c-2*C*a
)*x^9+1/11*B*c^2*x^11+1/13*c^2*C*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int (a-cx^4)^2 (A+Bx^2+Cx^4) \, dx = a^2Ax+\frac{1}{3}a^2Bx^3+\frac{1}{5}a(-2Ac+aC)x^5-\frac{2}{7}aBcx^7 +\frac{1}{9}c(Ac-2aC)x^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{13}c^2Cx^{13}$$

input
```
Integrate[(a - c*x^4)^2*(A + B*x^2 + C*x^4),x]
```

output
```
a^2*A*x + (a^2*B*x^3)/3 + (a*(-2*A*c + a*C)*x^5)/5 - (2*a*B*c*x^7)/7 + (c*
(A*c - 2*a*C)*x^9)/9 + (B*c^2*x^11)/11 + (c^2*C*x^13)/13
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^4)^2 (A+Bx^2+Cx^4)\,dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(a^2A+a^2Bx^2+cx^8(Ac-2aC)+ax^4(aC-2Ac)-2aBcx^6+Bc^2x^{10}+c^2Cx^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{3}a^2Bx^3+\frac{1}{9}cx^9(Ac-2aC)-\frac{1}{5}ax^5(2Ac-aC)-\frac{2}{7}aBcx^7+\frac{1}{11}Bc^2x^{11}+\frac{1}{13}c^2Cx^{13}$$

input
```
Int[(a - c*x^4)^2*(A + B*x^2 + C*x^4),x]
```

output
```
a^2*A*x + (a^2*B*x^3)/3 - (a*(2*A*c - a*C)*x^5)/5 - (2*a*B*c*x^7)/7 + (c*(
A*c - 2*a*C)*x^9)/9 + (B*c^2*x^11)/11 + (c^2*C*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{11}}{11}+\frac{(A c^2-2acC)x^9}{9}-\frac{2aBc x^7}{7}+\frac{(-2Aac+a^2C)x^5}{5}+\frac{a^2 B x^3}{3}+a^2 Ax$
norman	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{11}}{11}+\left(\frac{1}{9}A c^2-\frac{2}{9}acC\right)x^9-\frac{2aBc x^7}{7}+\left(-\frac{2}{5}Aac+\frac{1}{5}a^2C\right)x^5+\frac{a^2 B x^3}{3}+a^2 Ax$
gosper	$\frac{1}{13}c^2C x^{13}+\frac{1}{11}B c^2x^{11}+\frac{1}{9}x^9A c^2-\frac{2}{9}x^9acC-\frac{2}{7}aBc x^7-\frac{2}{5}x^5Aac+\frac{1}{5}x^5a^2C+\frac{1}{3}a^2B x^3+$
risch	$\frac{1}{13}c^2C x^{13}+\frac{1}{11}B c^2x^{11}+\frac{1}{9}x^9A c^2-\frac{2}{9}x^9acC-\frac{2}{7}aBc x^7-\frac{2}{5}x^5Aac+\frac{1}{5}x^5a^2C+\frac{1}{3}a^2B x^3+$
parallelrisch	$\frac{1}{13}c^2C x^{13}+\frac{1}{11}B c^2x^{11}+\frac{1}{9}x^9A c^2-\frac{2}{9}x^9acC-\frac{2}{7}aBc x^7-\frac{2}{5}x^5Aac+\frac{1}{5}x^5a^2C+\frac{1}{3}a^2B x^3+$
orering	$\frac{x(3465c^2C x^{12}+4095c^2B x^{10}+5005A c^2x^8-10010Cac x^8-12870aBc x^6-18018Aac x^4+9009C a^2x^4+15015a^2B x^2+45045}{45045}$

input
```
int((-c*x^4+a)^2*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*c^2*C*x^13+1/11*B*c^2*x^11+1/9*(A*c^2-2*C*a*c)*x^9-2/7*a*B*c*x^7+1/5*
(-2*A*a*c+C*a^2)*x^5+1/3*a^2*B*x^3+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int (a-cx^4)^2 (A+Bx^2+Cx^4) dx = \frac{1}{13}Cc^2x^{13}+\frac{1}{11}Bc^2x^{11}-\frac{2}{7}Bacx^7 - \frac{1}{9}(2Cac-Ac^2)x^9+\frac{1}{3}Ba^2x^3 + \frac{1}{5}(Ca^2-2Aac)x^5+Aa^2x$$

input
```
integrate((-c*x^4+a)^2*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/13*C*c^2*x^13 + 1/11*B*c^2*x^11 - 2/7*B*a*c*x^7 - 1/9*(2*C*a*c - A*c^2)*
x^9 + 1/3*B*a^2*x^3 + 1/5*(C*a^2 - 2*A*a*c)*x^5 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int (a-cx^4)^2 (A+Bx^2+Cx^4)\,dx = Aa^2x + \frac{Ba^2x^3}{3} - \frac{2Bacx^7}{7} + \frac{Bc^2x^{11}}{11} + \frac{Cc^2x^{13}}{13} + x^9\left(\frac{Ac^2}{9} - \frac{2Cac}{9}\right) + x^5\left(-\frac{2Aac}{5} + \frac{Ca^2}{5}\right)$$

input
```
integrate((-c*x**4+a)**2*(C*x**4+B*x**2+A),x)
```

output
```
A*a**2*x + B*a**2*x**3/3 - 2*B*a*c*x**7/7 + B*c**2*x**11/11 + C*c**2*x**13
/13 + x**9*(A*c**2/9 - 2*C*a*c/9) + x**5*(-2*A*a*c/5 + C*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int (a-cx^4)^2 (A+Bx^2+Cx^4)\,dx = \frac{1}{13}\,Cc^2x^{13} + \frac{1}{11}\,Bc^2x^{11} - \frac{2}{7}\,Bacx^7 - \frac{1}{9}\,(2\,Cac - Ac^2)x^9 + \frac{1}{3}\,Ba^2x^3 + \frac{1}{5}\,(Ca^2 - 2\,Aac)x^5 + Aa^2x$$

input
```
integrate((-c*x^4+a)^2*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/13*C*c^2*x^13 + 1/11*B*c^2*x^11 - 2/7*B*a*c*x^7 - 1/9*(2*C*a*c - A*c^2)*
x^9 + 1/3*B*a^2*x^3 + 1/5*(C*a^2 - 2*A*a*c)*x^5 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int \left(a-cx^4\right)^2 \left(A+Bx^2+Cx^4\right) \, dx = \frac{1}{13}Cc^2x^{13}+\frac{1}{11}Bc^2x^{11}-\frac{2}{9}Cacx^9+\frac{1}{9}Ac^2x^9-\frac{2}{7}Bacx^7+\frac{1}{5}Ca^2x^5-\frac{2}{5}Aacx^5+\frac{1}{3}Ba^2x^3+Aa^2x$$

input
```
integrate((-c*x^4+a)^2*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/13*C*c^2*x^13 + 1/11*B*c^2*x^11 - 2/9*C*a*c*x^9 + 1/9*A*c^2*x^9 - 2/7*B*
a*c*x^7 + 1/5*C*a^2*x^5 - 2/5*A*a*c*x^5 + 1/3*B*a^2*x^3 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \left(a-cx^4\right)^2 \left(A+Bx^2+Cx^4\right) \, dx = x^5\left(\frac{C\,a^2}{5}-\frac{2\,A\,a\,c}{5}\right)+x^9\left(\frac{A\,c^2}{9}-\frac{2\,C\,a\,c}{9}\right)+\frac{B\,a^2\,x^3}{3}+\frac{B\,c^2\,x^{11}}{11}+\frac{C\,c^2\,x^{13}}{13}+A\,a^2\,x-\frac{2\,B\,a\,c\,x^7}{7}$$

input
```
int((a - c*x^4)^2*(A + B*x^2 + C*x^4),x)
```

output
```
x^5*((C*a^2)/5 - (2*A*a*c)/5) + x^9*((A*c^2)/9 - (2*C*a*c)/9) + (B*a^2*x^3
)/3 + (B*c^2*x^11)/11 + (C*c^2*x^13)/13 + A*a^2*x - (2*B*a*c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \left(a-cx^4\right)^2 \left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{x(3465c^3x^{12}+4095b\,c^2x^{10}-5005a\,c^2x^8-12870abc\,x^6-9009a^2c\,x^4+15015a^2b\,x^2+45045a^3)}{45045}$$

input
```
int((-c*x^4+a)^2*(C*x^4+B*x^2+A),x)
```

output
```
(x*(45045*a**3 + 15015*a**2*b*x**2 - 9009*a**2*c*x**4 - 12870*a*b*c*x**6 -
 5005*a*c**2*x**8 + 4095*b*c**2*x**10 + 3465*c**3*x**12))/45045
```

3.3 $\int (a-cx^4) (A+Bx^2+Cx^4) \, dx$

Optimal result 83
Mathematica [A] (verified) 83
Rubi [A] (verified) 84
Maple [A] (verified) 85
Fricas [A] (verification not implemented) 85
Sympy [A] (verification not implemented) 86
Maxima [A] (verification not implemented) 86
Giac [A] (verification not implemented) 86
Mupad [B] (verification not implemented) 87
Reduce [B] (verification not implemented) 87

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int (a-cx^4) (A+Bx^2+Cx^4) \, dx = aAx+\frac{1}{3}aBx^3-\frac{1}{5}(Ac-aC)x^5-\frac{1}{7}Bcx^7-\frac{1}{9}cCx^9$$

output
```
a*A*x+1/3*a*B*x^3-1/5*(A*c-C*a)*x^5-1/7*B*c*x^7-1/9*c*C*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (a-cx^4) (A+Bx^2+Cx^4) \, dx = aAx+\frac{1}{3}aBx^3+\frac{1}{5}(-Ac+aC)x^5-\frac{1}{7}Bcx^7-\frac{1}{9}cCx^9$$

input
```
Integrate[(a - c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
a*A*x + (a*B*x^3)/3 + ((-(A*c) + a*C)*x^5)/5 - (B*c*x^7)/7 - (c*C*x^9)/9
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^4)\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(-x^4(Ac-aC)+aA+aBx^2-Bcx^6-cCx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}x^5(Ac-aC)+aAx+\frac{1}{3}aBx^3-\frac{1}{7}Bcx^7-\frac{1}{9}cCx^9$$

input
```
Int[(a - c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
a*A*x + (a*B*x^3)/3 - ((A*c - a*C)*x^5)/5 - (B*c*x^7)/7 - (c*C*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{cC\,x^{9}}{9}-\frac{Bc\,x^{7}}{7}+\frac{\left(-Ac+Ca\right)x^{5}}{5}+\frac{Ba\,x^{3}}{3}+aAx$	40
norman	$-\frac{cC\,x^{9}}{9}-\frac{Bc\,x^{7}}{7}+\left(-\frac{Ac}{5}+\frac{Ca}{5}\right)x^{5}+\frac{Ba\,x^{3}}{3}+aAx$	40
gosper	$-\frac{1}{9}cC\,x^{9}-\frac{1}{7}Bc\,x^{7}-\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Ca+\frac{1}{3}Ba\,x^{3}+aAx$	41
risch	$-\frac{1}{9}cC\,x^{9}-\frac{1}{7}Bc\,x^{7}-\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Ca+\frac{1}{3}Ba\,x^{3}+aAx$	41
parallelrisch	$-\frac{1}{9}cC\,x^{9}-\frac{1}{7}Bc\,x^{7}-\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Ca+\frac{1}{3}Ba\,x^{3}+aAx$	41
orering	$\frac{x\left(-35Cc\,x^{8}-45Bc\,x^{6}-63Ac\,x^{4}+63Ca\,x^{4}+105Ba\,x^{2}+315Aa\right)}{315}$	44

input
```
int((-c*x^4+a)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
-1/9*c*C*x^9-1/7*B*c*x^7+1/5*(-A*c+C*a)*x^5+1/3*B*a*x^3+a*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \left(a-cx^4\right)\left(A+Bx^2+Cx^4\right)\,dx=-\frac{1}{9}\,Ccx^9-\frac{1}{7}\,Bcx^7+\frac{1}{5}\,(Ca-Ac)x^5+\frac{1}{3}\,Bax^3+Aax$$

input
```
integrate((-c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/9*C*c*x^9 - 1/7*B*c*x^7 + 1/5*(C*a - A*c)*x^5 + 1/3*B*a*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int (a-cx^4)(A+Bx^2+Cx^4)\,dx = Aax + \frac{Bax^3}{3} - \frac{Bcx^7}{7} - \frac{Ccx^9}{9} + x^5\left(-\frac{Ac}{5} + \frac{Ca}{5}\right)$$

input
```
integrate((-c*x**4+a)*(C*x**4+B*x**2+A),x)
```

output
```
A*a*x + B*a*x**3/3 - B*c*x**7/7 - C*c*x**9/9 + x**5*(-A*c/5 + C*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int (a-cx^4)(A+Bx^2+Cx^4)\,dx = -\frac{1}{9}Ccx^9 - \frac{1}{7}Bcx^7 + \frac{1}{5}(Ca-Ac)x^5 + \frac{1}{3}Bax^3 + Aax$$

input
```
integrate((-c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
-1/9*C*c*x^9 - 1/7*B*c*x^7 + 1/5*(C*a - A*c)*x^5 + 1/3*B*a*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int (a-cx^4)(A+Bx^2+Cx^4)\,dx = -\frac{1}{9}Ccx^9 - \frac{1}{7}Bcx^7 + \frac{1}{5}Cax^5 - \frac{1}{5}Acx^5 + \frac{1}{3}Bax^3 + Aax$$

input
```
integrate((-c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
-1/9*C*c*x^9 - 1/7*B*c*x^7 + 1/5*C*a*x^5 - 1/5*A*c*x^5 + 1/3*B*a*x^3 + A*a
*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \left(a-cx^4\right)\left(A+Bx^2+Cx^4\right)\,dx=-\frac{C\,c\,x^9}{9}-\frac{B\,c\,x^7}{7}+\left(\frac{C\,a}{5}-\frac{A\,c}{5}\right)\,x^5+\frac{B\,a\,x^3}{3}+A\,a\,x$$

input
```
int((a - c*x^4)*(A + B*x^2 + C*x^4),x)
```

output
```
A*a*x - x^5*((A*c)/5 - (C*a)/5) + (B*a*x^3)/3 - (B*c*x^7)/7 - (C*c*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \left(a-cx^4\right)\left(A+Bx^2+Cx^4\right)\,dx=\frac{x(-7c^2x^8-9bc\,x^6+21ab\,x^2+63a^2)}{63}$$

input
```
int((-c*x^4+a)*(C*x^4+B*x^2+A),x)
```

output
```
(x*(63*a**2 + 21*a*b*x**2 - 9*b*c*x**6 - 7*c**2*x**8))/63
```

3.4 $\int \frac{A+Bx^2+Cx^4}{a-cx^4} \, dx$

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Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4} \, dx = -\frac{Cx}{c} - \frac{\left(\sqrt{a}B\sqrt{c}-Ac-aC\right)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{5/4}} + \frac{\left(\sqrt{a}B\sqrt{c}+Ac+aC\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{5/4}}$$

output
```
-C*x/c-1/2*(a^(1/2)*B*c^(1/2)-A*c-C*a)*arctan(c^(1/4)*x/a^(1/4))/a^(3/4)/c
^(5/4)+1/2*(a^(1/2)*B*c^(1/2)+A*c+C*a)*arctanh(c^(1/4)*x/a^(1/4))/a^(3/4)/
c^(5/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4} \, dx = \frac{-4a^{3/4}\sqrt[4]{c}Cx + 2\left(-\sqrt{a}B\sqrt{c}+Ac+aC\right)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) - \left(\sqrt{a}B\sqrt{c}+Ac+aC\right)\log\left(\sqrt[4]{a}-\sqrt[4]{c}x\right) + \sqrt{}}{4a^{3/4}c^{5/4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a - c*x^4),x]
```

output
```
(-4*a^(3/4)*c^(1/4)*C*x + 2*(-(Sqrt[a]*B*Sqrt[c]) + A*c + a*C)*ArcTan[(c^(
1/4)*x)/a^(1/4)] - (Sqrt[a]*B*Sqrt[c] + A*c + a*C)*Log[a^(1/4) - c^(1/4)*x
] + Sqrt[a]*B*Sqrt[c]*Log[a^(1/4) + c^(1/4)*x] + A*c*Log[a^(1/4) + c^(1/4)
*x] + a*C*Log[a^(1/4) + c^(1/4)*x])/(4*a^(3/4)*c^(5/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4}dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{aC+Ac+Bcx^2}{c\left(a-cx^4\right)}-\frac{C}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)}{2a^{3/4}c^{5/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)}{2a^{3/4}c^{5/4}}-\frac{Cx}{c}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a - c*x^4),x]
```

output
```
-((C*x)/c) - ((Sqrt[a]*B*Sqrt[c] - A*c - a*C)*ArcTan[(c^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*c^(5/4)) + ((Sqrt[a]*B*Sqrt[c] + A*c + a*C)*ArcTanh[(c^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*c^(5/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

method	result	size
risch	$-\frac{Cx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}-a\right)}\frac{\left(-B\_R^{2}c-Ac-Ca\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4c^{2}}$	53
default	$-\frac{Cx}{c}+\frac{\frac{(Ac+Ca)\left(\frac{a}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{B\left(2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{c}$	119

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-C*x/c+1/4/c^2*sum((-B*_R^2*c-A*c-C*a)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. 2(73) = 146.

Time = 0.41 (sec) , antiderivative size = 1806, normalized size of antiderivative = 17.71

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(c*sqrt((a*c^2*sqrt((C^4*a^4 + A^4*c^4 + 2*(B^2*C^2 + 2*A*C^3)*a^3*c
+ (B^4 + 4*A*B^2*C + 6*A^2*C^2)*a^2*c^2 + 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^
3*c^5)) + 2*B*C*a + 2*A*B*c)/(a*c^2))*log((C^4*a^4 + 4*A*C^3*a^3*c + 4*A^3
*C*a*c^3 + A^4*c^4 - (B^4 - 6*A^2*C^2)*a^2*c^2)*x + (B*a^3*c^4*sqrt((C^4*a
^4 + A^4*c^4 + 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C + 6*A^2*C^2)
*a^2*c^2 + 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5)) - C^3*a^4*c - A^3*a*c^4
 - (B^2*C + 3*A*C^2)*a^3*c^2 - (A*B^2 + 3*A^2*C)*a^2*c^3)*sqrt((a*c^2*sqrt
((C^4*a^4 + A^4*c^4 + 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C + 6*A
^2*C^2)*a^2*c^2 + 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5)) + 2*B*C*a + 2*A*
B*c)/(a*c^2))) - c*sqrt((a*c^2*sqrt((C^4*a^4 + A^4*c^4 + 2*(B^2*C^2 + 2*A*
C^3)*a^3*c + (B^4 + 4*A*B^2*C + 6*A^2*C^2)*a^2*c^2 + 2*(A^2*B^2 + 2*A^3*C)
*a*c^3)/(a^3*c^5)) + 2*B*C*a + 2*A*B*c)/(a*c^2))*log((C^4*a^4 + 4*A*C^3*a^
3*c + 4*A^3*C*a*c^3 + A^4*c^4 - (B^4 - 6*A^2*C^2)*a^2*c^2)*x - (B*a^3*c^4*
sqrt((C^4*a^4 + A^4*c^4 + 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C +
 6*A^2*C^2)*a^2*c^2 + 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5)) - C^3*a^4*c
- A^3*a*c^4 - (B^2*C + 3*A*C^2)*a^3*c^2 - (A*B^2 + 3*A^2*C)*a^2*c^3)*sqrt(
(a*c^2*sqrt((C^4*a^4 + A^4*c^4 + 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*
B^2*C + 6*A^2*C^2)*a^2*c^2 + 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5)) + 2*B
*C*a + 2*A*B*c)/(a*c^2))) - c*sqrt(-(a*c^2*sqrt((C^4*a^4 + A^4*c^4 + 2*(B^
2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C + 6*A^2*C^2)*a^2*c^2 + 2*(A^2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(99) = 198.

Time = 1.41 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.12

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4}\,dx = -\frac{Cx}{c}$$
$$-\operatorname{RootSum}\Big(256t^4a^3c^5+t^2\left(-64ABa^2c^4-64BCa^3c^3\right)-A^4c^4-4A^3Cac^3+2A^2B^2ac^3-6A^2C^2a^2c^2+$$

input
```
integrate((C*x**4+B*x**2+A)/(-c*x**4+a),x)
```

output
```
-C*x/c - RootSum(256*_t**4*a**3*c**5 + _t**2*(-64*A*B*a**2*c**4 - 64*B*C*a
**3*c**3) - A**4*c**4 - 4*A**3*C*a*c**3 + 2*A**2*B**2*a*c**3 - 6*A**2*C**2
*a**2*c**2 + 4*A*B**2*C*a**2*c**2 - 4*A*C**3*a**3*c - B**4*a**2*c**2 + 2*B
**2*C**2*a**3*c - C**4*a**4, Lambda(_t, _t*log(x + (64*_t**3*B*a**3*c**4 -
 4*_t*A**3*a*c**4 - 12*_t*A**2*C*a**2*c**3 - 12*_t*A*B**2*a**2*c**3 - 12*_
t*A*C**2*a**3*c**2 - 12*_t*B**2*C*a**3*c**2 - 4*_t*C**3*a**4*c)/(A**4*c**4
 + 4*A**3*C*a*c**3 + 6*A**2*C**2*a**2*c**2 + 4*A*C**3*a**3*c - B**4*a**2*c
**2 + C**4*a**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4}\,dx = -\frac{Cx}{c}+\frac{\frac{2\left(Ca\sqrt{c}-\left(B\sqrt{a}-A\sqrt{c}\right)c\right)\arctan\left(\frac{\sqrt{c}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\left(Ca\sqrt{c}+\left(B\sqrt{a}+A\sqrt{c}\right)c\right)\log\left(\frac{\sqrt{c}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{c}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{4\,c}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-C*x/c + 1/4*(2*(C*a*sqrt(c) - (B*sqrt(a) - A*sqrt(c))*c)*arctan(sqrt(c)*x
/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - (C*a*sqr
t(c) + (B*sqrt(a) + A*sqrt(c))*c)*log((sqrt(c)*x - sqrt(sqrt(a)*sqrt(c)))/
(sqrt(c)*x + sqrt(sqrt(a)*sqrt(c))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c
)))/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.48

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4}\,dx = -\frac{\sqrt{2}\left(Cac+Ac^2+\sqrt{-ac}Bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\left(-ac^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(Cac+Ac^2-\sqrt{-ac}Bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\left(-ac^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}\left(Cac+Ac^2-\sqrt{-ac}Bc\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{8\left(-ac^3\right)^{\frac{3}{4}}} + \frac{\sqrt{2}\left(Cac+Ac^2-\sqrt{-ac}Bc\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{8\left(-ac^3\right)^{\frac{3}{4}}} - \frac{Cx}{c}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(C*a*c + A*c^2 + sqrt(-a*c)*B*c)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(-a/c)^(1/4))/(-a/c)^(1/4))/(-a*c^3)^(3/4) - 1/4*sqrt(2)*(C*a*c + A*
c^2 - sqrt(-a*c)*B*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/c)^(1/4))/(-a/
c)^(1/4))/(-a*c^3)^(3/4) - 1/8*sqrt(2)*(C*a*c + A*c^2 - sqrt(-a*c)*B*c)*lo
g(x^2 + sqrt(2)*x*(-a/c)^(1/4) + sqrt(-a/c))/(-a*c^3)^(3/4) + 1/8*sqrt(2)*
(C*a*c + A*c^2 - sqrt(-a*c)*B*c)*log(x^2 - sqrt(2)*x*(-a/c)^(1/4) + sqrt(-
a/c))/(-a*c^3)^(3/4) - C*x/c
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 2256, normalized size of antiderivative = 22.12

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2 + C*x^4)/(a - c*x^4),x)
```

output
```
2*atanh((8*A^2*c^3*x*((B*C)/(8*c^2) + (A*B)/(8*a*c) + (A^2*(a^3*c^5)^(1/2)
)/(16*a^3*c^3) + (B^2*(a^3*c^5)^(1/2))/(16*a^2*c^4) + (C^2*(a^3*c^5)^(1/2)
)/(16*a*c^5) + (A*C*(a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*A^3*(a^3*c^5)
^(1/2))/a^2 + 2*B*C^2*a^2 + 2*A^2*B*c^2 + 2*B^3*a*c + (6*A*C^2*(a^3*c^5)^(
1/2))/c^2 + (2*B^2*C*(a^3*c^5)^(1/2))/c^2 + (2*C^3*a*(a^3*c^5)^(1/2))/c^3
+ (2*A*B^2*(a^3*c^5)^(1/2))/(a*c) + (6*A^2*C*(a^3*c^5)^(1/2))/(a*c) + 4*A*
B*C*a*c) + (8*B^2*a*c^2*x*((B*C)/(8*c^2) + (A*B)/(8*a*c) + (A^2*(a^3*c^5)^
(1/2))/(16*a^3*c^3) + (B^2*(a^3*c^5)^(1/2))/(16*a^2*c^4) + (C^2*(a^3*c^5)^
(1/2))/(16*a*c^5) + (A*C*(a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*A^3*(a^3
*c^5)^(1/2))/a^2 + 2*B*C^2*a^2 + 2*A^2*B*c^2 + 2*B^3*a*c + (6*A*C^2*(a^3*c
^5)^(1/2))/c^2 + (2*B^2*C*(a^3*c^5)^(1/2))/c^2 + (2*C^3*a*(a^3*c^5)^(1/2))
/c^3 + (2*A*B^2*(a^3*c^5)^(1/2))/(a*c) + (6*A^2*C*(a^3*c^5)^(1/2))/(a*c) +
 4*A*B*C*a*c) + (8*C^2*a^2*c*x*((B*C)/(8*c^2) + (A*B)/(8*a*c) + (A^2*(a^3*
c^5)^(1/2))/(16*a^3*c^3) + (B^2*(a^3*c^5)^(1/2))/(16*a^2*c^4) + (C^2*(a^3*
c^5)^(1/2))/(16*a*c^5) + (A*C*(a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*A^3
*(a^3*c^5)^(1/2))/a^2 + 2*B*C^2*a^2 + 2*A^2*B*c^2 + 2*B^3*a*c + (6*A*C^2*(
a^3*c^5)^(1/2))/c^2 + (2*B^2*C*(a^3*c^5)^(1/2))/c^2 + (2*C^3*a*(a^3*c^5)^(
1/2))/c^3 + (2*A*B^2*(a^3*c^5)^(1/2))/(a*c) + (6*A^2*C*(a^3*c^5)^(1/2))/(a
*c) + 4*A*B*C*a*c) + (16*A*C*a*c^2*x*((B*C)/(8*c^2) + (A*B)/(8*a*c) + (A^2
*(a^3*c^5)^(1/2))/(16*a^3*c^3) + (B^2*(a^3*c^5)^(1/2))/(16*a^2*c^4) + (...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2+Cx^4}{a-cx^4}\,dx = \frac{-2c^{\frac{1}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)b+4c^{\frac{3}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)-c^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}-c^{\frac{1}{4}}x\right)b+c^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}+c^{\frac{1}{4}}x\right)b-2c^{\frac{3}{4}}a^{\frac{5}{4}}\log}{4ac}$$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a),x)
```

output
```
( - 2*c**(1/4)*a**(3/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*b + 4*c**(3/
4)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a - c**(1/4)*a**(3/4)*lo
g(a**(1/4) - c**(1/4)*x)*b + c**(1/4)*a**(3/4)*log(a**(1/4) + c**(1/4)*x)*
b - 2*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a + 2*c**(3/4)*a**(1/4)
*log(a**(1/4) + c**(1/4)*x)*a - 4*a*c*x)/(4*a*c)
```

3.5 $\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^2}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^2}\,dx = \frac{x\left(A+\frac{aC}{c}+Bx^2\right)}{4a\,(a-cx^4)} - \frac{\left(\sqrt{a}B\sqrt{c}-3Ac+aC\right)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8a^{7/4}c^{5/4}} + \frac{\left(\sqrt{a}B\sqrt{c}+3Ac-aC\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8a^{7/4}c^{5/4}}$$

output
```
1/4*x*(A+a*C/c+B*x^2)/a/(-c*x^4+a)-1/8*(a^(1/2)*B*c^(1/2)-3*A*c+C*a)*arcta
n(c^(1/4)*x/a^(1/4))/a^(7/4)/c^(5/4)+1/8*(a^(1/2)*B*c^(1/2)+3*A*c-C*a)*arc
tanh(c^(1/4)*x/a^(1/4))/a^(7/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^2}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{c}x(Ac+aC+Bcx^2)}{a-cx^4} - 2\left(\sqrt{a}B\sqrt{c}-3Ac+aC\right)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + \left(-\sqrt{a}B\sqrt{c}-3Ac+aC\right)\log\left(\sqrt[4]{a}-\right.}{16a^{7/4}c^{5/4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a - c*x^4)^2,x]
```

output
```
((4*a^(3/4)*c^(1/4)*x*(A*c + a*C + B*c*x^2))/(a - c*x^4) - 2*(Sqrt[a]*B*Sq
rt[c] - 3*A*c + a*C)*ArcTan[(c^(1/4)*x)/a^(1/4)] + (-(Sqrt[a]*B*Sqrt[c]) -
 3*A*c + a*C)*Log[a^(1/4) - c^(1/4)*x] - (-(Sqrt[a]*B*Sqrt[c]) - 3*A*c + a
*C)*Log[a^(1/4) + c^(1/4)*x])/(16*a^(7/4)*c^(5/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2397, 1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\left(a-cx^4\right)^2}dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{Bcx^2+3Ac-aC}{a-cx^4}dx}{4ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{4ac\left(a-cx^4\right)}$$

$$\Big\downarrow \text{1481}$$

$$\frac{\frac{1}{2}\sqrt{c}\left(B\sqrt{c}-\frac{3Ac-aC}{\sqrt{a}}\right)\int \frac{1}{-cx^2-\sqrt{a}\sqrt{c}}dx+\frac{1}{2}\sqrt{c}\left(\frac{3Ac-aC}{\sqrt{a}}+B\sqrt{c}\right)\int \frac{1}{\sqrt{a}\sqrt{c}-cx^2}dx}{4ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{4ac\left(a-cx^4\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{1}{2}\sqrt{c}\left(\frac{3Ac-aC}{\sqrt{a}}+B\sqrt{c}\right)\int \frac{1}{\sqrt{a}\sqrt{c}-cx^2}dx-\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(B\sqrt{c}-\frac{3Ac-aC}{\sqrt{a}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}}}{4ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{4ac\left(a-cx^4\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\frac{3Ac-aC}{\sqrt{a}}+B\sqrt{c}\right)}{2\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(B\sqrt{c}-\frac{3Ac-aC}{\sqrt{a}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}}}{4ac}+\frac{x(aC+Ac+Bcx^2)}{4ac\,(a-cx^4)}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a - c*x^4)^2,x]
```

output
```
(x*(A*c + a*C + B*c*x^2))/(4*a*c*(a - c*x^4)) + (-1/2*((B*Sqrt[c] - (3*A*c
 - a*C)/Sqrt[a])*ArcTan[(c^(1/4)*x)/a^(1/4)])/(a^(1/4)*c^(1/4)) + ((B*Sqrt
[c] + (3*A*c - a*C)/Sqrt[a])*ArcTanh[(c^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*c^(1
/4)))/(4*a*c)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\frac{x^3 B}{4a}+\frac{(Ac+Ca)x}{4ac}}{-c\,x^4+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4-a\right)}\frac{\left(B\_R^2+\frac{3Ac-Ca}{c}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$	89
default	$\frac{\frac{x^3 B}{4a}+\frac{(Ac+Ca)x}{4ac}}{-c\,x^4+a}+\frac{\frac{(3Ac-Ca)\left(\frac{a}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{B\left(2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{4ac}$	155

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*x^3*B+1/4*(A*c+C*a)/a/c*x)/(-c*x^4+a)-1/16/a/c*sum((B*_R^2+1/c*(3*A
*c-C*a))/_R^3*ln(x-_R),_R=RootOf(_Z^4*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1933 vs. 2(98) = 196.

Time = 0.45 (sec) , antiderivative size = 1933, normalized size of antiderivative = 15.22

$$\int \frac{A+Bx^2+Cx^4}{\left(a-cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*B*c*x^3 - (a*c^2*x^4 - a^2*c)*sqrt(-(a^3*c^2*sqrt((C^4*a^4 + 81*A
^4*c^4 + 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2
*c^2 + 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a - 6*A*B*c)/(a^3*
c^2))*log((C^4*a^4 - 12*A*C^3*a^3*c - 108*A^3*C*a*c^3 + 81*A^4*c^4 - (B^4
- 54*A^2*C^2)*a^2*c^2)*x + (B*a^6*c^4*sqrt((C^4*a^4 + 81*A^4*c^4 + 2*(B^2*
C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2*c^2 + 18*(A^2*B
^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) - C^3*a^5*c + 27*A^3*a^2*c^4 - (B^2*C - 9*
A*C^2)*a^4*c^2 + 3*(A*B^2 - 9*A^2*C)*a^3*c^3)*sqrt(-(a^3*c^2*sqrt((C^4*a^4
 + 81*A^4*c^4 + 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C
^2)*a^2*c^2 + 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a - 6*A*B*c
)/(a^3*c^2))) + (a*c^2*x^4 - a^2*c)*sqrt(-(a^3*c^2*sqrt((C^4*a^4 + 81*A^4*
c^4 + 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2*c^
2 + 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a - 6*A*B*c)/(a^3*c^2
))*log((C^4*a^4 - 12*A*C^3*a^3*c - 108*A^3*C*a*c^3 + 81*A^4*c^4 - (B^4 - 5
4*A^2*C^2)*a^2*c^2)*x - (B*a^6*c^4*sqrt((C^4*a^4 + 81*A^4*c^4 + 2*(B^2*C^2
 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2*c^2 + 18*(A^2*B^2
- 6*A^3*C)*a*c^3)/(a^7*c^5)) - C^3*a^5*c + 27*A^3*a^2*c^4 - (B^2*C - 9*A*C
^2)*a^4*c^2 + 3*(A*B^2 - 9*A^2*C)*a^3*c^3)*sqrt(-(a^3*c^2*sqrt((C^4*a^4 +
81*A^4*c^4 + 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)
*a^2*c^2 + 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a - 6*A*B*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(121) = 242$.

Time = 2.27 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.74

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^2}\, dx$$
$$= \operatorname{RootSum}\left(65536t^4a^7c^5 + t^2\left(-3072ABa^4c^4 + 1024BCa^5c^3\right) - 81A^4c^4 + 108A^3Cac^3 + 18A^2B^2ac^3 - 54\right.$$
$$+ \frac{-Bcx^3 + x(-Ac - Ca)}{-4a^2c + 4ac^2x^4}$$

input
```
integrate((C*x**4+B*x**2+A)/(-c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**5 + _t**2*(-3072*A*B*a**4*c**4 + 1024*B*C*a**5
*c**3) - 81*A**4*c**4 + 108*A**3*C*a*c**3 + 18*A**2*B**2*a*c**3 - 54*A**2*
C**2*a**2*c**2 - 12*A*B**2*C*a**2*c**2 + 12*A*C**3*a**3*c - B**4*a**2*c**2
 + 2*B**2*C**2*a**3*c - C**4*a**4, Lambda(_t, _t*log(x + (-4096*_t**3*B*a*
*6*c**4 + 432*_t*A**3*a**2*c**4 - 432*_t*A**2*C*a**3*c**3 + 144*_t*A*B**2*
a**3*c**3 + 144*_t*A*C**2*a**4*c**2 - 48*_t*B**2*C*a**4*c**2 - 16*_t*C**3*
a**5*c)/(81*A**4*c**4 - 108*A**3*C*a*c**3 + 54*A**2*C**2*a**2*c**2 - 12*A*
C**3*a**3*c - B**4*a**2*c**2 + C**4*a**4)))) + (-B*c*x**3 + x*(-A*c - C*a)
)/(-4*a**2*c + 4*a*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^2}\,dx = -\frac{Bcx^3+(Ca+Ac)x}{4\,(ac^2x^4-a^2c)} - \frac{\frac{2\,(Ca\sqrt{c}+(B\sqrt{a}-3\,A\sqrt{c})c)\arctan\left(\frac{\sqrt{c}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{(Ca\sqrt{c}-(B\sqrt{a}+3\,A\sqrt{c})c)\log\left(\frac{\sqrt{c}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{c}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{16\,ac}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(B*c*x^3 + (C*a + A*c)*x)/(a*c^2*x^4 - a^2*c) - 1/16*(2*(C*a*sqrt(c)
+ (B*sqrt(a) - 3*A*sqrt(c))*c)*arctan(sqrt(c)*x/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - (C*a*sqrt(c) - (B*sqrt(a) + 3*A*sqr
t(c))*c)*log((sqrt(c)*x - sqrt(sqrt(a)*sqrt(c)))/(sqrt(c)*x + sqrt(sqrt(a)
*sqrt(c))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)))/(a*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(98) = 196$.

Time = 0.14 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.31

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^2}\,dx = \frac{\sqrt{2}\left(Cac-3\,Ac^2-\sqrt{-ac}Bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\left(-ac^3\right)^{\frac{3}{4}}a} + \frac{\sqrt{2}\left(Cac-3\,Ac^2+\sqrt{-ac}Bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\left(-ac^3\right)^{\frac{3}{4}}a} + \frac{\sqrt{2}\left(Cac-3\,Ac^2+\sqrt{-ac}Bc\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{32\left(-ac^3\right)^{\frac{3}{4}}a} - \frac{\sqrt{2}\left(Cac-3\,Ac^2+\sqrt{-ac}Bc\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{32\left(-ac^3\right)^{\frac{3}{4}}a} - \frac{Bcx^3+Cax+Acx}{4\left(cx^4-a\right)ac}$$

input

```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^2,x, algorithm="giac")
```

output

```
1/16*sqrt(2)*(C*a*c - 3*A*c^2 - sqrt(-a*c)*B*c)*arctan(1/2*sqrt(2)*(2*x +
sqrt(2)*(-a/c)^(1/4))/(-a/c)^(1/4))/((-a*c^3)^(3/4)*a) + 1/16*sqrt(2)*(C*a
*c - 3*A*c^2 + sqrt(-a*c)*B*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/c)^(1
/4))/(-a/c)^(1/4))/((-a*c^3)^(3/4)*a) + 1/32*sqrt(2)*(C*a*c - 3*A*c^2 + sq
rt(-a*c)*B*c)*log(x^2 + sqrt(2)*x*(-a/c)^(1/4) + sqrt(-a/c))/((-a*c^3)^(3/
4)*a) - 1/32*sqrt(2)*(C*a*c - 3*A*c^2 + sqrt(-a*c)*B*c)*log(x^2 - sqrt(2)*
x*(-a/c)^(1/4) + sqrt(-a/c))/((-a*c^3)^(3/4)*a) - 1/4*(B*c*x^3 + C*a*x + A
*c*x)/((c*x^4 - a)*a*c)
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 2390, normalized size of antiderivative = 18.82

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^2,x)
```

output
```
2*atanh((C^2*c*x*((3*A*B)/(128*a^3*c) - (B*C)/(128*a^2*c^2) - (9*A^2*(a^7*
c^5)^(1/2))/(256*a^7*c^3) - (B^2*(a^7*c^5)^(1/2))/(256*a^6*c^4) - (C^2*(a^
7*c^5)^(1/2))/(256*a^5*c^5) + (3*A*C*(a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2)
)/(2*((B*C^2)/(32*a) - (27*A^3*(a^7*c^5)^(1/2))/(32*a^7) + (B^3*c)/(32*a^2
) + (9*A^2*B*c^2)/(32*a^3) + (C^3*(a^7*c^5)^(1/2))/(32*a^4*c^3) - (3*A*B^2
*(a^7*c^5)^(1/2))/(32*a^6*c) - (9*A*C^2*(a^7*c^5)^(1/2))/(32*a^5*c^2) + (2
7*A^2*C*(a^7*c^5)^(1/2))/(32*a^6*c) + (B^2*C*(a^7*c^5)^(1/2))/(32*a^5*c^2)
 - (3*A*B*C*c)/(16*a^2))) + (9*A^2*c^3*x*((3*A*B)/(128*a^3*c) - (B*C)/(128
*a^2*c^2) - (9*A^2*(a^7*c^5)^(1/2))/(256*a^7*c^3) - (B^2*(a^7*c^5)^(1/2))/
(256*a^6*c^4) - (C^2*(a^7*c^5)^(1/2))/(256*a^5*c^5) + (3*A*C*(a^7*c^5)^(1/
2))/(128*a^6*c^4))^(1/2))/(2*((B^3*c)/32 - (27*A^3*(a^7*c^5)^(1/2))/(32*a^
5) + (B*C^2*a)/32 - (3*A*B*C*c)/16 + (9*A^2*B*c^2)/(32*a) + (C^3*(a^7*c^5)
^(1/2))/(32*a^2*c^3) - (3*A*B^2*(a^7*c^5)^(1/2))/(32*a^4*c) - (9*A*C^2*(a^
7*c^5)^(1/2))/(32*a^3*c^2) + (27*A^2*C*(a^7*c^5)^(1/2))/(32*a^4*c) + (B^2*
C*(a^7*c^5)^(1/2))/(32*a^3*c^2))) + (B^2*c^2*x*((3*A*B)/(128*a^3*c) - (B*C
)/(128*a^2*c^2) - (9*A^2*(a^7*c^5)^(1/2))/(256*a^7*c^3) - (B^2*(a^7*c^5)^(
1/2))/(256*a^6*c^4) - (C^2*(a^7*c^5)^(1/2))/(256*a^5*c^5) + (3*A*C*(a^7*c^
5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((B*C^2)/32 - (27*A^3*(a^7*c^5)^(1/2))/
(32*a^6) + (B^3*c)/(32*a) + (9*A^2*B*c^2)/(32*a^2) + (C^3*(a^7*c^5)^(1/2))
/(32*a^3*c^3) - (3*A*B^2*(a^7*c^5)^(1/2))/(32*a^5*c) - (9*A*C^2*(a^7*c^...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.20

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^2}\,dx$$
$$= \frac{-2c^{\frac{1}{4}}a^{\frac{7}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)b+2c^{\frac{5}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)b\,x^4+4c^{\frac{3}{4}}a^{\frac{9}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)-4c^{\frac{7}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)x^4-c^{\frac{1}{4}}a^{\frac{7}{4}}\log}{}$$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^2,x)
```

output
```
( - 2*c**(1/4)*a**(3/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*b + 2*c**(
1/4)*a**(3/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*b*c*x**4 + 4*c**(3/4)*
a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**2 - 4*c**(3/4)*a**(1/4)*
atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*c*x**4 - c**(1/4)*a**(3/4)*log(a**
(1/4) - c**(1/4)*x)*a*b + c**(1/4)*a**(3/4)*log(a**(1/4) - c**(1/4)*x)*b*c
*x**4 + c**(1/4)*a**(3/4)*log(a**(1/4) + c**(1/4)*x)*a*b - c**(1/4)*a**(3/
4)*log(a**(1/4) + c**(1/4)*x)*b*c*x**4 - 2*c**(3/4)*a**(1/4)*log(a**(1/4)
- c**(1/4)*x)*a**2 + 2*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a*c*x*
*4 + 2*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*a**2 - 2*c**(3/4)*a**(
1/4)*log(a**(1/4) + c**(1/4)*x)*a*c*x**4 + 8*a**2*c*x + 4*a*b*c*x**3)/(16*
a**2*c*(a - c*x**4))
```

3.6 $\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^3}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^3}\,dx = \frac{x\left(A+\frac{aC}{c}+Bx^2\right)}{8a\,(a-cx^4)^2}+\frac{x\left(7A-\frac{aC}{c}+5Bx^2\right)}{32a^2\,(a-cx^4)} - \frac{\left(5\sqrt{a}B\sqrt{c}-21Ac+3aC\right)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64a^{11/4}c^{5/4}} + \frac{\left(5\sqrt{a}B\sqrt{c}+21Ac-3aC\right)\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64a^{11/4}c^{5/4}}$$

output
```
1/8*x*(A+a*C/c+B*x^2)/a/(-c*x^4+a)^2+1/32*x*(7*A-a*C/c+5*B*x^2)/a^2/(-c*x^
4+a)-1/64*(5*a^(1/2)*B*c^(1/2)-21*A*c+3*C*a)*arctan(c^(1/4)*x/a^(1/4))/a^(
11/4)/c^(5/4)+1/64*(5*a^(1/2)*B*c^(1/2)+21*A*c-3*C*a)*arctanh(c^(1/4)*x/a^
(1/4))/a^(11/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^3}\,dx = \frac{\frac{16a^{7/4}\sqrt[4]{c}x(Ac+aC+Bcx^2)}{(a-cx^4)^2}-\frac{4a^{3/4}\sqrt[4]{c}(-7Acx+aCx-5Bcx^3)}{a-cx^4}-2(5\sqrt{a}B\sqrt{c}-21Ac+3aC)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+(-5}{128a^{11/4}c^{5/4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a - c*x^4)^3,x]
```

output
```
((16*a^(7/4)*c^(1/4)*x*(A*c + a*C + B*c*x^2))/(a - c*x^4)^2 - (4*a^(3/4)*c
^(1/4)*(-7*A*c*x + a*C*x - 5*B*c*x^3))/(a - c*x^4) - 2*(5*Sqrt[a]*B*Sqrt[c
] - 21*A*c + 3*a*C)*ArcTan[(c^(1/4)*x)/a^(1/4)] + (-5*Sqrt[a]*B*Sqrt[c] -
21*A*c + 3*a*C)*Log[a^(1/4) - c^(1/4)*x] - (-5*Sqrt[a]*B*Sqrt[c] - 21*A*c
+ 3*a*C)*Log[a^(1/4) + c^(1/4)*x])/(128*a^(11/4)*c^(5/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2397, 1493, 25, 1481, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^3}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{5Bcx^2+7Ac-aC}{(a-cx^4)^2}dx}{8ac}+\frac{x(aC+Ac+Bcx^2)}{8ac\,(a-cx^4)^2}$$

$$\Big\downarrow \text{1493}$$

$$\frac{\frac{x(-aC+7Ac+5Bcx^2)}{4a(a-cx^4)}-\frac{\int -\frac{5Bcx^2+3(7Ac-aC)}{a-cx^4}dx}{4a}}{8ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{8ac\left(a-cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{5Bcx^2+3(7Ac-aC)}{a-cx^4}dx}{4a}+\frac{x(-aC+7Ac+5Bcx^2)}{4a(a-cx^4)}}{8ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{8ac\left(a-cx^4\right)^2}$$

$$\Big\downarrow \; 1481$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(5B\sqrt{c}-\frac{3(7Ac-aC)}{\sqrt{a}}\right)\int \frac{1}{-cx^2-\sqrt{a}\sqrt{c}}dx+\frac{1}{2}\sqrt{c}\left(\frac{3(7Ac-aC)}{\sqrt{a}}+5B\sqrt{c}\right)\int \frac{1}{\sqrt{a}\sqrt{c}-cx^2}dx}{4a}+\frac{x(-aC+7Ac+5Bcx^2)}{4a(a-cx^4)}}{8ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{8ac\left(a-cx^4\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{1}{2}\sqrt{c}\left(\frac{3(7Ac-aC)}{\sqrt{a}}+5B\sqrt{c}\right)\int \frac{1}{\sqrt{a}\sqrt{c}-cx^2}dx-\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5B\sqrt{c}-\frac{3(7Ac-aC)}{\sqrt{a}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}}}{4a}+\frac{x(-aC+7Ac+5Bcx^2)}{4a(a-cx^4)}}{8ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{8ac\left(a-cx^4\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\frac{3(7Ac-aC)}{\sqrt{a}}+5B\sqrt{c}\right)}{2\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5B\sqrt{c}-\frac{3(7Ac-aC)}{\sqrt{a}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}}}{4a}+\frac{x(-aC+7Ac+5Bcx^2)}{4a(a-cx^4)}}{8ac}+\frac{x\left(aC+Ac+Bcx^2\right)}{8ac\left(a-cx^4\right)^2}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a - c*x^4)^3,x]
```

output
```
(x*(A*c + a*C + B*c*x^2))/(8*a*c*(a - c*x^4)^2) + ((x*(7*A*c - a*C + 5*B*c
*x^2))/(4*a*(a - c*x^4)) + (-1/2*((5*B*Sqrt[c] - (3*(7*A*c - a*C))/Sqrt[a]
)*ArcTan[(c^(1/4)*x)/a^(1/4)])/(a^(1/4)*c^(1/4)) + ((5*B*Sqrt[c] + (3*(7*A
*c - a*C))/Sqrt[a])*ArcTanh[(c^(1/4)*x)/a^(1/4)])/(2*a^(1/4)*c^(1/4)))/(4*
a))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1481
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Simp[(e/2 + c*(d/(2*q)))   Int[1/(-q + c*x^2), x], x] + Simp[(
e/2 - c*(d/(2*q)))   Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] &
& NeQ[c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

method	result
risch	$\frac{-\frac{5Bc x^7}{32a^2}-\frac{(7Ac-Ca)x^5}{32a^2}+\frac{9x^3B}{32a}+\frac{(11Ac+3Ca)x}{32ac}}{(-c x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4-a\right)}\frac{\left(5B\_R^2+\frac{21Ac-3Ca}{c}\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2c}$
default	$\frac{-\frac{5Bc x^7}{32a^2}-\frac{(7Ac-Ca)x^5}{32a^2}+\frac{9x^3B}{32a}+\frac{(11Ac+3Ca)x}{32ac}}{(-c x^4+a)^2}+\frac{\frac{(21Ac-3Ca)\left(\frac{a}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{5B\left(2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)-l\right.}{4\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{32a^2c}$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*B*c/a^2*x^7-1/32*(7*A*c-C*a)/a^2*x^5+9/32/a*x^3*B+1/32*(11*A*c+3*C*
a)/a/c*x)/(-c*x^4+a)^2-1/128/a^2/c*sum((5*B*_R^2+3/c*(7*A*c-C*a))/_R^3*ln(
x-_R),_R=RootOf(_Z^4*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2112 vs. $2(135) = 270$.

Time = 0.54 (sec) , antiderivative size = 2112, normalized size of antiderivative = 12.80

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(20*B*c^2*x^7 - 36*B*a*c*x^3 - 4*(C*a*c - 7*A*c^2)*x^5 - (a^2*c^3*x
^8 - 2*a^3*c^2*x^4 + a^4*c)*sqrt(-(a^5*c^2*sqrt((81*C^4*a^4 + 194481*A^4*c
^4 + 18*(25*B^2*C^2 - 126*A*C^3)*a^3*c + (625*B^4 - 6300*A*B^2*C + 23814*A
^2*C^2)*a^2*c^2 + 882*(25*A^2*B^2 - 126*A^3*C)*a*c^3)/(a^11*c^5)) + 30*B*C
*a - 210*A*B*c)/(a^5*c^2))*log((81*C^4*a^4 - 2268*A*C^3*a^3*c - 111132*A^3
*C*a*c^3 + 194481*A^4*c^4 - (625*B^4 - 23814*A^2*C^2)*a^2*c^2)*x + (5*B*a^
9*c^4*sqrt((81*C^4*a^4 + 194481*A^4*c^4 + 18*(25*B^2*C^2 - 126*A*C^3)*a^3*
c + (625*B^4 - 6300*A*B^2*C + 23814*A^2*C^2)*a^2*c^2 + 882*(25*A^2*B^2 - 1
26*A^3*C)*a*c^3)/(a^11*c^5)) - 27*C^3*a^6*c + 9261*A^3*a^3*c^4 - 3*(25*B^2
*C - 189*A*C^2)*a^5*c^2 + 21*(25*A*B^2 - 189*A^2*C)*a^4*c^3)*sqrt(-(a^5*c^
2*sqrt((81*C^4*a^4 + 194481*A^4*c^4 + 18*(25*B^2*C^2 - 126*A*C^3)*a^3*c +
(625*B^4 - 6300*A*B^2*C + 23814*A^2*C^2)*a^2*c^2 + 882*(25*A^2*B^2 - 126*A
^3*C)*a*c^3)/(a^11*c^5)) + 30*B*C*a - 210*A*B*c)/(a^5*c^2))) + (a^2*c^3*x^
8 - 2*a^3*c^2*x^4 + a^4*c)*sqrt(-(a^5*c^2*sqrt((81*C^4*a^4 + 194481*A^4*c^
4 + 18*(25*B^2*C^2 - 126*A*C^3)*a^3*c + (625*B^4 - 6300*A*B^2*C + 23814*A^
2*C^2)*a^2*c^2 + 882*(25*A^2*B^2 - 126*A^3*C)*a*c^3)/(a^11*c^5)) + 30*B*C*
a - 210*A*B*c)/(a^5*c^2))*log((81*C^4*a^4 - 2268*A*C^3*a^3*c - 111132*A^3*
C*a*c^3 + 194481*A^4*c^4 - (625*B^4 - 23814*A^2*C^2)*a^2*c^2)*x - (5*B*a^9
*c^4*sqrt((81*C^4*a^4 + 194481*A^4*c^4 + 18*(25*B^2*C^2 - 126*A*C^3)*a^3*c
 + (625*B^4 - 6300*A*B^2*C + 23814*A^2*C^2)*a^2*c^2 + 882*(25*A^2*B^2 -...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(158) = 316$.

Time = 3.27 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.46

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^3}\,dx =$$

$$-\operatorname{RootSum}\left(268435456t^4a^{11}c^5 + t^2\left(-6881280ABa^6c^4 + 983040BCa^7c^3\right) - 194481A^4c^4 + 111132A^3C\right.$$

$$-\frac{-9Bacx^3 + 5Bc^2x^7 + x^5\cdot(7Ac^2 - Cac) + x(-11Aac - 3Ca^2)}{32a^4c - 64a^3c^2x^4 + 32a^2c^3x^8}$$

input
```
integrate((C*x**4+B*x**2+A)/(-c*x**4+a)**3,x)
```

output
```
-RootSum(268435456*_t**4*a**11*c**5 + _t**2*(-6881280*A*B*a**6*c**4 + 9830
40*B*C*a**7*c**3) - 194481*A**4*c**4 + 111132*A**3*C*a*c**3 + 22050*A**2*B
**2*a*c**3 - 23814*A**2*C**2*a**2*c**2 - 6300*A*B**2*C*a**2*c**2 + 2268*A*
C**3*a**3*c - 625*B**4*a**2*c**2 + 450*B**2*C**2*a**3*c - 81*C**4*a**4, La
mbda(_t, _t*log(x + (10485760*_t**3*B*a**9*c**4 - 1185408*_t*A**3*a**3*c**
4 + 508032*_t*A**2*C*a**4*c**3 - 201600*_t*A*B**2*a**4*c**3 - 72576*_t*A*C
**2*a**5*c**2 + 28800*_t*B**2*C*a**5*c**2 + 3456*_t*C**3*a**6*c)/(194481*A
**4*c**4 - 111132*A**3*C*a*c**3 + 23814*A**2*C**2*a**2*c**2 - 2268*A*C**3*
a**3*c - 625*B**4*a**2*c**2 + 81*C**4*a**4)))) - (-9*B*a*c*x**3 + 5*B*c**2
*x**7 + x**5*(7*A*c**2 - C*a*c) + x*(-11*A*a*c - 3*C*a**2))/(32*a**4*c - 6
4*a**3*c**2*x**4 + 32*a**2*c**3*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2+Cx^4}{(a-cx^4)^3}\,dx = -\frac{5\,Bc^2x^7-9\,Bacx^3-(Cac-7\,Ac^2)x^5-(3\,Ca^2+11\,Aac)x}{32\,(a^2c^3x^8-2\,a^3c^2x^4+a^4c)} - \frac{\frac{2\,(3\,Ca\sqrt{c}+(5\,B\sqrt{a}-21\,A\sqrt{c})c)\arctan\left(\frac{\sqrt{c}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{(3\,Ca\sqrt{c}-(5\,B\sqrt{a}+21\,A\sqrt{c})c)\log\left(\frac{\sqrt{c}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{c}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{128\,a^2c}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(5*B*c^2*x^7 - 9*B*a*c*x^3 - (C*a*c - 7*A*c^2)*x^5 - (3*C*a^2 + 11*A
*a*c)*x)/(a^2*c^3*x^8 - 2*a^3*c^2*x^4 + a^4*c) - 1/128*(2*(3*C*a*sqrt(c) +
 (5*B*sqrt(a) - 21*A*sqrt(c))*c)*arctan(sqrt(c)*x/sqrt(sqrt(a)*sqrt(c)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - (3*C*a*sqrt(c) - (5*B*sqrt(a) + 2
1*A*sqrt(c))*c)*log((sqrt(c)*x - sqrt(sqrt(a)*sqrt(c)))/(sqrt(c)*x + sqrt(
sqrt(a)*sqrt(c))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)))/(a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(135) = 270$.

Time = 0.13 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.02

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\left(3\,Cac - 21\,Ac^2 - 5\,\sqrt{-ac}Bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,\left(-ac^3\right)^{\frac{3}{4}}a^2}$$
$$+ \frac{\sqrt{2}\left(3\,Cac - 21\,Ac^2 + 5\,\sqrt{-ac}Bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,\left(-ac^3\right)^{\frac{3}{4}}a^2}$$
$$+ \frac{\sqrt{2}\left(3\,Cac - 21\,Ac^2 + 5\,\sqrt{-ac}Bc\right)\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{c}}\right)}{256\,\left(-ac^3\right)^{\frac{3}{4}}a^2}$$
$$- \frac{\sqrt{2}\left(3\,Cac - 21\,Ac^2 + 5\,\sqrt{-ac}Bc\right)\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{c}}\right)}{256\,\left(-ac^3\right)^{\frac{3}{4}}a^2}$$
$$- \frac{5\,Bc^2x^7 - Cacx^5 + 7\,Ac^2x^5 - 9\,Bacx^3 - 3\,Ca^2x - 11\,Aacx}{32\,\left(cx^4 - a\right)^2a^2c}$$

input
```
integrate((C*x^4+B*x^2+A)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(3*C*a*c - 21*A*c^2 - 5*sqrt(-a*c)*B*c)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(-a/c)^(1/4))/(-a/c)^(1/4))/((-a*c^3)^(3/4)*a^2) + 1/128*sqr
t(2)*(3*C*a*c - 21*A*c^2 + 5*sqrt(-a*c)*B*c)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(-a/c)^(1/4))/(-a/c)^(1/4))/((-a*c^3)^(3/4)*a^2) + 1/256*sqrt(2)*(3*C
*a*c - 21*A*c^2 + 5*sqrt(-a*c)*B*c)*log(x^2 + sqrt(2)*x*(-a/c)^(1/4) + sqr
t(-a/c))/((-a*c^3)^(3/4)*a^2) - 1/256*sqrt(2)*(3*C*a*c - 21*A*c^2 + 5*sqrt
(-a*c)*B*c)*log(x^2 - sqrt(2)*x*(-a/c)^(1/4) + sqrt(-a/c))/((-a*c^3)^(3/4)
*a^2) - 1/32*(5*B*c^2*x^7 - C*a*c*x^5 + 7*A*c^2*x^5 - 9*B*a*c*x^3 - 3*C*a^
2*x - 11*A*a*c*x)/((c*x^4 - a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 2464, normalized size of antiderivative = 14.93

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^2 + C*x^4)/(a - c*x^4)^3,x)
```

output
```
((9*B*x^3)/(32*a) - (x^5*(7*A*c - C*a))/(32*a^2) - (5*B*c*x^7)/(32*a^2) +
(x*(11*A*c + 3*C*a))/(32*a*c))/(a^2 + c^2*x^8 - 2*a*c*x^4) + atan((((x*(44
1*A^2*c^3 + 25*B^2*a*c^2 + 9*C^2*a^2*c - 126*A*C*a*c^2))/(256*a^4) - ((344
064*A*a^5*c^3 - 49152*C*a^6*c^2)*(-(441*A^2*c^2*(a^11*c^5)^(1/2) + 9*C^2*a
^2*(a^11*c^5)^(1/2) + 25*B^2*a*c*(a^11*c^5)^(1/2) - 210*A*B*a^6*c^4 + 30*B
*C*a^7*c^3 - 126*A*C*a*c*(a^11*c^5)^(1/2))/(16384*a^11*c^5))^(1/2))/(32768
*a^6))*(-(441*A^2*c^2*(a^11*c^5)^(1/2) + 9*C^2*a^2*(a^11*c^5)^(1/2) + 25*B
^2*a*c*(a^11*c^5)^(1/2) - 210*A*B*a^6*c^4 + 30*B*C*a^7*c^3 - 126*A*C*a*c*(
a^11*c^5)^(1/2))/(16384*a^11*c^5))^(1/2)*1i + ((x*(441*A^2*c^3 + 25*B^2*a*
c^2 + 9*C^2*a^2*c - 126*A*C*a*c^2))/(256*a^4) + ((344064*A*a^5*c^3 - 49152
*C*a^6*c^2)*(-(441*A^2*c^2*(a^11*c^5)^(1/2) + 9*C^2*a^2*(a^11*c^5)^(1/2) +
 25*B^2*a*c*(a^11*c^5)^(1/2) - 210*A*B*a^6*c^4 + 30*B*C*a^7*c^3 - 126*A*C*
a*c*(a^11*c^5)^(1/2))/(16384*a^11*c^5))^(1/2))/(32768*a^6))*(-(441*A^2*c^2
*(a^11*c^5)^(1/2) + 9*C^2*a^2*(a^11*c^5)^(1/2) + 25*B^2*a*c*(a^11*c^5)^(1/
2) - 210*A*B*a^6*c^4 + 30*B*C*a^7*c^3 - 126*A*C*a*c*(a^11*c^5)^(1/2))/(163
84*a^11*c^5))^(1/2)*1i)/((45*B*C^2*a^2 + 2205*A^2*B*c^2 - 125*B^3*a*c - 63
0*A*B*C*a*c)/(16384*a^6) + ((x*(441*A^2*c^3 + 25*B^2*a*c^2 + 9*C^2*a^2*c -
 126*A*C*a*c^2))/(256*a^4) - ((344064*A*a^5*c^3 - 49152*C*a^6*c^2)*(-(441*
A^2*c^2*(a^11*c^5)^(1/2) + 9*C^2*a^2*(a^11*c^5)^(1/2) + 25*B^2*a*c*(a^11*c
^5)^(1/2) - 210*A*B*a^6*c^4 + 30*B*C*a^7*c^3 - 126*A*C*a*c*(a^11*c^5)^(...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.72

$$\int \frac{A + Bx^2 + Cx^4}{(a - cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(-c*x^4+a)^3,x)
```

output
```
( - 10*c**(1/4)*a**(3/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**2*b + 20
*c**(1/4)*a**(3/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*b*c*x**4 - 10*c
**(1/4)*a**(3/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*b*c**2*x**8 + 36*c*
*(3/4)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**3 - 72*c**(3/4)*a
**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**2*c*x**4 + 36*c**(3/4)*a*
*(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*c**2*x**8 - 5*c**(1/4)*a**(
3/4)*log(a**(1/4) - c**(1/4)*x)*a**2*b + 10*c**(1/4)*a**(3/4)*log(a**(1/4)
 - c**(1/4)*x)*a*b*c*x**4 - 5*c**(1/4)*a**(3/4)*log(a**(1/4) - c**(1/4)*x)
*b*c**2*x**8 + 5*c**(1/4)*a**(3/4)*log(a**(1/4) + c**(1/4)*x)*a**2*b - 10*
c**(1/4)*a**(3/4)*log(a**(1/4) + c**(1/4)*x)*a*b*c*x**4 + 5*c**(1/4)*a**(3
/4)*log(a**(1/4) + c**(1/4)*x)*b*c**2*x**8 - 18*c**(3/4)*a**(1/4)*log(a**(
1/4) - c**(1/4)*x)*a**3 + 36*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*
a**2*c*x**4 - 18*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a*c**2*x**8
+ 18*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*a**3 - 36*c**(3/4)*a**(1
/4)*log(a**(1/4) + c**(1/4)*x)*a**2*c*x**4 + 18*c**(3/4)*a**(1/4)*log(a**(
1/4) + c**(1/4)*x)*a*c**2*x**8 + 56*a**3*c*x + 36*a**2*b*c*x**3 - 24*a**2*
c**2*x**5 - 20*a*b*c**2*x**7)/(128*a**3*c*(a**2 - 2*a*c*x**4 + c**2*x**8))
```

3.7 $\int (a+cx^4)^3 (A+Bx^2+Cx^4)\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 116

$$\begin{aligned}\int (a+cx^4)^3 (A+Bx^2+Cx^4)\,dx &= a^3Ax+\frac{1}{3}a^3Bx^3+\frac{1}{5}a^2(3Ac+aC)x^5\\&\quad+\frac{3}{7}a^2Bcx^7+\frac{1}{3}ac(Ac+aC)x^9+\frac{3}{11}aBc^2x^{11}\\&\quad+\frac{1}{13}c^2(Ac+3aC)x^{13}+\frac{1}{15}Bc^3x^{15}+\frac{1}{17}c^3Cx^{17}\end{aligned}$$

output
```
a^3*A*x+1/3*a^3*B*x^3+1/5*a^2*(3*A*c+C*a)*x^5+3/7*a^2*B*c*x^7+1/3*a*c*(A*c
+C*a)*x^9+3/11*a*B*c^2*x^11+1/13*c^2*(A*c+3*C*a)*x^13+1/15*B*c^3*x^15+1/17
*c^3*C*x^17
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+cx^4)^3 (A+Bx^2+Cx^4)\,dx &= a^3Ax+\frac{1}{3}a^3Bx^3+\frac{1}{5}a^2(3Ac+aC)x^5\\&\quad+\frac{3}{7}a^2Bcx^7+\frac{1}{3}ac(Ac+aC)x^9+\frac{3}{11}aBc^2x^{11}\\&\quad+\frac{1}{13}c^2(Ac+3aC)x^{13}+\frac{1}{15}Bc^3x^{15}+\frac{1}{17}c^3Cx^{17}\end{aligned}$$

input
```
Integrate[(a + c*x^4)^3*(A + B*x^2 + C*x^4),x]
```

output
```
a^3*A*x + (a^3*B*x^3)/3 + (a^2*(3*A*c + a*C)*x^5)/5 + (3*a^2*B*c*x^7)/7 +
(a*c*(A*c + a*C)*x^9)/3 + (3*a*B*c^2*x^11)/11 + (c^2*(A*c + 3*a*C)*x^13)/1
3 + (B*c^3*x^15)/15 + (c^3*C*x^17)/17
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx$$

$$\Big\downarrow \text{ 2200}$$

$$\int \left(a^3A+a^3Bx^2+a^2x^4(aC+3Ac)+3a^2Bcx^6+c^2x^{12}(3aC+Ac)+3acx^8(aC+Ac)+3aBc^2x^{10}+Bc^3x^{14}+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$a^3Ax+\frac{1}{3}a^3Bx^3+\frac{1}{5}a^2x^5(aC+3Ac)+\frac{3}{7}a^2Bcx^7+\frac{1}{13}c^2x^{13}(3aC+Ac)+\frac{1}{3}acx^9(aC+Ac)+\frac{3}{11}aBc^2x^{11}+\frac{1}{15}Bc^3x^{15}+\frac{1}{17}c^3Cx^{17}$$

input
```
Int[(a + c*x^4)^3*(A + B*x^2 + C*x^4),x]
```

output
```
a^3*A*x + (a^3*B*x^3)/3 + (a^2*(3*A*c + a*C)*x^5)/5 + (3*a^2*B*c*x^7)/7 +
(a*c*(A*c + a*C)*x^9)/3 + (3*a*B*c^2*x^11)/11 + (c^2*(A*c + 3*a*C)*x^13)/1
3 + (B*c^3*x^15)/15 + (c^3*C*x^17)/17
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95

method	result
norman	$a^3Ax+\frac{a^3B\,x^3}{3}+\left(\frac{3}{5}a^2Ac+\frac{1}{5}a^3C\right)x^5+\frac{3a^2Bc\,x^7}{7}+\left(\frac{1}{3}Aa\,c^2+\frac{1}{3}a^2cC\right)x^9+\frac{3aB\,c^2x^{11}}{11}+\left(\frac{1}{13}A\right.$
default	$\frac{c^3C\,x^{17}}{17}+\frac{B\,c^3x^{15}}{15}+\frac{\left(A\,c^3+3a\,c^2C\right)x^{13}}{13}+\frac{3aB\,c^2x^{11}}{11}+\frac{\left(3Aa\,c^2+3a^2cC\right)x^9}{9}+\frac{3a^2Bc\,x^7}{7}+\frac{\left(3a^2Ac+a^3C\right)x^5}{5}+$
gosper	$a^3Ax+\frac{1}{3}a^3B\,x^3+\frac{3}{5}x^5a^2Ac+\frac{1}{5}x^5a^3C+\frac{3}{7}a^2Bc\,x^7+\frac{1}{3}x^9Aa\,c^2+\frac{1}{3}x^9a^2cC+\frac{3}{11}aB\,c^2x^{11}+$
risch	$a^3Ax+\frac{1}{3}a^3B\,x^3+\frac{3}{5}x^5a^2Ac+\frac{1}{5}x^5a^3C+\frac{3}{7}a^2Bc\,x^7+\frac{1}{3}x^9Aa\,c^2+\frac{1}{3}x^9a^2cC+\frac{3}{11}aB\,c^2x^{11}+$
parallelrisch	$a^3Ax+\frac{1}{3}a^3B\,x^3+\frac{3}{5}x^5a^2Ac+\frac{1}{5}x^5a^3C+\frac{3}{7}a^2Bc\,x^7+\frac{1}{3}x^9Aa\,c^2+\frac{1}{3}x^9a^2cC+\frac{3}{11}aB\,c^2x^{11}+$
orering	$\frac{x\left(15015c^3C\,x^{16}+17017c^3B\,x^{14}+19635A\,c^3x^{12}+58905Ca\,c^2x^{12}+69615a\,c^2B\,x^{10}+85085Aa\,c^2x^8+85085C\,a^2c\,x^8+109395a\right.}{255255}$

input
```
int((c*x^4+a)^3*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a^3*A*x+1/3*a^3*B*x^3+(3/5*a^2*A*c+1/5*a^3*C)*x^5+3/7*a^2*B*c*x^7+(1/3*A*a
*c^2+1/3*a^2*c*C)*x^9+3/11*a*B*c^2*x^11+(1/13*A*c^3+3/13*a*c^2*C)*x^13+1/1
5*B*c^3*x^15+1/17*c^3*C*x^17
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \left(a+cx^4\right)^3 \left(A+Bx^2+Cx^4\right)\,dx = \frac{1}{17}\,Cc^3x^{17} + \frac{1}{15}\,Bc^3x^{15} + \frac{3}{11}\,Bac^2x^{11} + \frac{1}{13}\,\left(3\,Cac^2+Ac^3\right)x^{13} + \frac{3}{7}\,Ba^2cx^7 + \frac{1}{3}\,\left(Ca^2c+Aac^2\right)x^9 + \frac{1}{3}\,Ba^3x^3 + \frac{1}{5}\,\left(Ca^3+3\,Aa^2c\right)x^5 + Aa^3x$$

input
```
integrate((c*x^4+a)^3*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/17*C*c^3*x^17 + 1/15*B*c^3*x^15 + 3/11*B*a*c^2*x^11 + 1/13*(3*C*a*c^2 +
A*c^3)*x^13 + 3/7*B*a^2*c*x^7 + 1/3*(C*a^2*c + A*a*c^2)*x^9 + 1/3*B*a^3*x^
3 + 1/5*(C*a^3 + 3*A*a^2*c)*x^5 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \left(a+cx^4\right)^3 \left(A+Bx^2+Cx^4\right)\,dx = Aa^3x + \frac{Ba^3x^3}{3} + \frac{3Ba^2cx^7}{7} + \frac{3Bac^2x^{11}}{11} + \frac{Bc^3x^{15}}{15} + \frac{Cc^3x^{17}}{17} + x^{13}\left(\frac{Ac^3}{13} + \frac{3Cac^2}{13}\right) + x^9\left(\frac{Aac^2}{3} + \frac{Ca^2c}{3}\right) + x^5\cdot\left(\frac{3Aa^2c}{5} + \frac{Ca^3}{5}\right)$$

input
```
integrate((c*x**4+a)**3*(C*x**4+B*x**2+A),x)
```

output
```
A*a**3*x + B*a**3*x**3/3 + 3*B*a**2*c*x**7/7 + 3*B*a*c**2*x**11/11 + B*c**
3*x**15/15 + C*c**3*x**17/17 + x**13*(A*c**3/13 + 3*C*a*c**2/13) + x**9*(A
*a*c**2/3 + C*a**2*c/3) + x**5*(3*A*a**2*c/5 + C*a**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \left(a+cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx = \frac{1}{17} Cc^3x^{17} + \frac{1}{15} Bc^3x^{15} + \frac{3}{11} Bac^2x^{11} + \frac{1}{13} \left(3 \, Cac^2 + Ac^3\right)x^{13} + \frac{3}{7} Ba^2cx^7 + \frac{1}{3} \left(Ca^2c + Aac^2\right)x^9 + \frac{1}{3} Ba^3x^3 + \frac{1}{5} \left(Ca^3 + 3 \, Aa^2c\right)x^5 + Aa^3x$$

input

```
integrate((c*x^4+a)^3*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output

```
1/17*C*c^3*x^17 + 1/15*B*c^3*x^15 + 3/11*B*a*c^2*x^11 + 1/13*(3*C*a*c^2 +
A*c^3)*x^13 + 3/7*B*a^2*c*x^7 + 1/3*(C*a^2*c + A*a*c^2)*x^9 + 1/3*B*a^3*x^
3 + 1/5*(C*a^3 + 3*A*a^2*c)*x^5 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \left(a+cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx = \frac{1}{17} Cc^3x^{17} + \frac{1}{15} Bc^3x^{15} + \frac{3}{13} Cac^2x^{13} + \frac{1}{13} Ac^3x^{13} + \frac{3}{11} Bac^2x^{11} + \frac{1}{3} Ca^2cx^9 + \frac{1}{3} Aac^2x^9 + \frac{3}{7} Ba^2cx^7 + \frac{1}{5} Ca^3x^5 + \frac{3}{5} Aa^2cx^5 + \frac{1}{3} Ba^3x^3 + Aa^3x$$

input

```
integrate((c*x^4+a)^3*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output

```
1/17*C*c^3*x^17 + 1/15*B*c^3*x^15 + 3/13*C*a*c^2*x^13 + 1/13*A*c^3*x^13 +
3/11*B*a*c^2*x^11 + 1/3*C*a^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a^2*c*x^7 +
1/5*C*a^3*x^5 + 3/5*A*a^2*c*x^5 + 1/3*B*a^3*x^3 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \left(a+cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx = x^5\left(\frac{C\,a^3}{5}+\frac{3\,A\,c\,a^2}{5}\right)+x^{13}\left(\frac{A\,c^3}{13}+\frac{3\,C\,a\,c^2}{13}\right) + \frac{B\,a^3\,x^3}{3}+\frac{B\,c^3\,x^{15}}{15}+\frac{C\,c^3\,x^{17}}{17}+A\,a^3\,x + \frac{a\,c\,x^9\,(A\,c+C\,a)}{3}+\frac{3\,B\,a^2\,c\,x^7}{7}+\frac{3\,B\,a\,c^2\,x^{11}}{11}$$

input
```
int((a + c*x^4)^3*(A + B*x^2 + C*x^4),x)
```

output
```
x^5*((C*a^3)/5 + (3*A*a^2*c)/5) + x^13*((A*c^3)/13 + (3*C*a*c^2)/13) + (B*
a^3*x^3)/3 + (B*c^3*x^15)/15 + (C*c^3*x^17)/17 + A*a^3*x + (a*c*x^9*(A*c +
 C*a))/3 + (3*B*a^2*c*x^7)/7 + (3*B*a*c^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \left(a+cx^4\right)^3 \left(A+Bx^2+Cx^4\right) \, dx = \frac{x(15015c^4x^{16}+17017b\,c^3x^{14}+78540a\,c^3x^{12}+69615ab\,c^2x^{10}+170170a^2c^2x^8+109395a^2bc\,x^6+204204}{255255}$$

input
```
int((c*x^4+a)^3*(C*x^4+B*x^2+A),x)
```

output
```
(x*(255255*a**4 + 85085*a**3*b*x**2 + 204204*a**3*c*x**4 + 109395*a**2*b*c
*x**6 + 170170*a**2*c**2*x**8 + 69615*a*b*c**2*x**10 + 78540*a*c**3*x**12
+ 17017*b*c**3*x**14 + 15015*c**4*x**16))/255255
```

3.8 $\int (a+cx^4)^2 (A+Bx^2+Cx^4)\,dx$

Optimal result 120
Mathematica [A] (verified) 120
Rubi [A] (verified) 121
Maple [A] (verified) 122
Fricas [A] (verification not implemented) 122
Sympy [A] (verification not implemented) 123
Maxima [A] (verification not implemented) 123
Giac [A] (verification not implemented) 124
Mupad [B] (verification not implemented) 124
Reduce [B] (verification not implemented) 125

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int (a+cx^4)^2 (A+Bx^2+Cx^4)\,dx = a^2Ax+\frac{1}{3}a^2Bx^3+\frac{1}{5}a(2Ac+aC)x^5+\frac{2}{7}aBcx^7 +\frac{1}{9}c(Ac+2aC)x^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{13}c^2Cx^{13}$$

output
```
a^2*A*x+1/3*a^2*B*x^3+1/5*a*(2*A*c+C*a)*x^5+2/7*a*B*c*x^7+1/9*c*(A*c+2*C*a
)*x^9+1/11*B*c^2*x^11+1/13*c^2*C*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+cx^4)^2 (A+Bx^2+Cx^4)\,dx = a^2Ax+\frac{1}{3}a^2Bx^3+\frac{1}{5}a(2Ac+aC)x^5+\frac{2}{7}aBcx^7 +\frac{1}{9}c(Ac+2aC)x^9+\frac{1}{11}Bc^2x^{11}+\frac{1}{13}c^2Cx^{13}$$

input
```
Integrate[(a + c*x^4)^2*(A + B*x^2 + C*x^4),x]
```

output
```
a^2*A*x + (a^2*B*x^3)/3 + (a*(2*A*c + a*C)*x^5)/5 + (2*a*B*c*x^7)/7 + (c*(
A*c + 2*a*C)*x^9)/9 + (B*c^2*x^11)/11 + (c^2*C*x^13)/13
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)^2 (A+Bx^2+Cx^4)\, dx$$

$$\Big\downarrow \; 2200$$

$$\int \left(a^2A+a^2Bx^2+cx^8(2aC+Ac)+ax^4(aC+2Ac)+2aBcx^6+Bc^2x^{10}+c^2Cx^{12}\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2Ax+\frac{1}{3}a^2Bx^3+\frac{1}{9}cx^9(2aC+Ac)+\frac{1}{5}ax^5(aC+2Ac)+\frac{2}{7}aBcx^7+\frac{1}{11}Bc^2x^{11}+\frac{1}{13}c^2Cx^{13}$$

input
```
Int[(a + c*x^4)^2*(A + B*x^2 + C*x^4),x]
```

output
```
a^2*A*x + (a^2*B*x^3)/3 + (a*(2*A*c + a*C)*x^5)/5 + (2*a*B*c*x^7)/7 + (c*(
A*c + 2*a*C)*x^9)/9 + (B*c^2*x^11)/11 + (c^2*C*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

method	result
default	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{11}}{11}+\frac{(A c^2+2acC)x^9}{9}+\frac{2aBc x^7}{7}+\frac{(2Aac+a^2C)x^5}{5}+\frac{a^2 B x^3}{3}+a^2 A x$
norman	$\frac{c^2 C x^{13}}{13}+\frac{B c^2 x^{11}}{11}+\left(\frac{1}{9}A c^2+\frac{2}{9}acC\right)x^9+\frac{2aBc x^7}{7}+\left(\frac{2}{5}Aac+\frac{1}{5}a^2C\right)x^5+\frac{a^2 B x^3}{3}+a^2 A x$
gosper	$\frac{1}{13}c^2 C x^{13}+\frac{1}{11}B c^2 x^{11}+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 acC+\frac{2}{7}aBc x^7+\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 C+\frac{1}{3}a^2 B x^3+$
risch	$\frac{1}{13}c^2 C x^{13}+\frac{1}{11}B c^2 x^{11}+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 acC+\frac{2}{7}aBc x^7+\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 C+\frac{1}{3}a^2 B x^3+$
parallelrisch	$\frac{1}{13}c^2 C x^{13}+\frac{1}{11}B c^2 x^{11}+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 acC+\frac{2}{7}aBc x^7+\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 C+\frac{1}{3}a^2 B x^3+$
orering	$\frac{x(3465c^2 C x^{12}+4095c^2 B x^{10}+5005A c^2 x^8+10010Cac x^8+12870aBc x^6+18018Aac x^4+9009C a^2 x^4+15015a^2 B x^2+45045}{45045}$

input
```
int((c*x^4+a)^2*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*c^2*C*x^13+1/11*B*c^2*x^11+1/9*(A*c^2+2*C*a*c)*x^9+2/7*a*B*c*x^7+1/5*
(2*A*a*c+C*a^2)*x^5+1/3*a^2*B*x^3+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \left(a+cx^4\right)^2\left(A+Bx^2+Cx^4\right)\,dx &= \frac{1}{13}\,Cc^2x^{13}+\frac{1}{11}\,Bc^2x^{11}+\frac{2}{7}\,Bacx^7\\&\quad+\frac{1}{9}\left(2\,Cac+Ac^2\right)x^9+\frac{1}{3}\,Ba^2x^3\\&\quad+\frac{1}{5}\left(Ca^2+2\,Aac\right)x^5+Aa^2x\end{aligned}$$

input
```
integrate((c*x^4+a)^2*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/13*C*c^2*x^13 + 1/11*B*c^2*x^11 + 2/7*B*a*c*x^7 + 1/9*(2*C*a*c + A*c^2)*
x^9 + 1/3*B*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*c)*x^5 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int (a+cx^4)^2 (A+Bx^2+Cx^4)\, dx = Aa^2x + \frac{Ba^2x^3}{3} + \frac{2Bacx^7}{7} + \frac{Bc^2x^{11}}{11} + \frac{Cc^2x^{13}}{13} + x^9\left(\frac{Ac^2}{9} + \frac{2Cac}{9}\right) + x^5 \cdot \left(\frac{2Aac}{5} + \frac{Ca^2}{5}\right)$$

input
```
integrate((c*x**4+a)**2*(C*x**4+B*x**2+A),x)
```

output
```
A*a**2*x + B*a**2*x**3/3 + 2*B*a*c*x**7/7 + B*c**2*x**11/11 + C*c**2*x**13
/13 + x**9*(A*c**2/9 + 2*C*a*c/9) + x**5*(2*A*a*c/5 + C*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int (a+cx^4)^2 (A+Bx^2+Cx^4)\, dx = \frac{1}{13}\,Cc^2x^{13} + \frac{1}{11}\,Bc^2x^{11} + \frac{2}{7}\,Bacx^7 + \frac{1}{9}\,(2\,Cac + Ac^2)x^9 + \frac{1}{3}\,Ba^2x^3 + \frac{1}{5}\,(Ca^2 + 2\,Aac)x^5 + Aa^2x$$

input
```
integrate((c*x^4+a)^2*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/13*C*c^2*x^13 + 1/11*B*c^2*x^11 + 2/7*B*a*c*x^7 + 1/9*(2*C*a*c + A*c^2)*
x^9 + 1/3*B*a^2*x^3 + 1/5*(C*a^2 + 2*A*a*c)*x^5 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int (a+cx^4)^2 (A+Bx^2+Cx^4)\ dx = \frac{1}{13}Cc^2x^{13}+\frac{1}{11}Bc^2x^{11}+\frac{2}{9}Cacx^9+\frac{1}{9}Ac^2x^9+\frac{2}{7}Bacx^7+\frac{1}{5}Ca^2x^5+\frac{2}{5}Aacx^5+\frac{1}{3}Ba^2x^3+Aa^2x$$

input
```
integrate((c*x^4+a)^2*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/13*C*c^2*x^13 + 1/11*B*c^2*x^11 + 2/9*C*a*c*x^9 + 1/9*A*c^2*x^9 + 2/7*B*
a*c*x^7 + 1/5*C*a^2*x^5 + 2/5*A*a*c*x^5 + 1/3*B*a^2*x^3 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int (a+cx^4)^2 (A+Bx^2+Cx^4)\ dx = x^5\left(\frac{C\,a^2}{5}+\frac{2\,A\,c\,a}{5}\right)+x^9\left(\frac{A\,c^2}{9}+\frac{2\,C\,a\,c}{9}\right)+\frac{B\,a^2\,x^3}{3}+\frac{B\,c^2\,x^{11}}{11}+\frac{C\,c^2\,x^{13}}{13}+A\,a^2\,x+\frac{2\,B\,a\,c\,x^7}{7}$$

input
```
int((a + c*x^4)^2*(A + B*x^2 + C*x^4),x)
```

output
```
x^5*((C*a^2)/5 + (2*A*a*c)/5) + x^9*((A*c^2)/9 + (2*C*a*c)/9) + (B*a^2*x^3
)/3 + (B*c^2*x^11)/11 + (C*c^2*x^13)/13 + A*a^2*x + (2*B*a*c*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int (a + cx^4)^2 (A + Bx^2 + Cx^4) \, dx$$
$$= \frac{x(1155c^3x^{12} + 1365b\,c^2x^{10} + 5005a\,c^2x^8 + 4290abc\,x^6 + 9009a^2c\,x^4 + 5005a^2b\,x^2 + 15015a^3)}{15015}$$

input
```
int((c*x^4+a)^2*(C*x^4+B*x^2+A),x)
```

output
```
(x*(15015*a**3 + 5005*a**2*b*x**2 + 9009*a**2*c*x**4 + 4290*a*b*c*x**6 + 5
005*a*c**2*x**8 + 1365*b*c**2*x**10 + 1155*c**3*x**12))/15015
```

3.9 $\int (a+cx^4)(A+Bx^2+Cx^4)\,dx$

Optimal result 126
Mathematica [A] (verified) 126
Rubi [A] (verified) 127
Maple [A] (verified) 128
Fricas [A] (verification not implemented) 128
Sympy [A] (verification not implemented) 129
Maxima [A] (verification not implemented) 129
Giac [A] (verification not implemented) 129
Mupad [B] (verification not implemented) 130
Reduce [B] (verification not implemented) 130

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int (a+cx^4)\left(A+Bx^2+Cx^4\right)dx = aAx+\frac{1}{3}aBx^3+\frac{1}{5}(Ac+aC)x^5+\frac{1}{7}Bcx^7+\frac{1}{9}cCx^9$$

output
```
a*A*x+1/3*a*B*x^3+1/5*(A*c+C*a)*x^5+1/7*B*c*x^7+1/9*c*C*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+cx^4)\left(A+Bx^2+Cx^4\right)dx = aAx+\frac{1}{3}aBx^3+\frac{1}{5}(Ac+aC)x^5+\frac{1}{7}Bcx^7+\frac{1}{9}cCx^9$$

input
```
Integrate[(a + c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
a*A*x + (a*B*x^3)/3 + ((A*c + a*C)*x^5)/5 + (B*c*x^7)/7 + (c*C*x^9)/9
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)(A+Bx^2+Cx^4)\,dx$$

$$\Big\downarrow \text{2200}$$

$$\int (x^4(aC+Ac)+aA+aBx^2+Bcx^6+cCx^8)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aC+Ac)+aAx+\frac{1}{3}aBx^3+\frac{1}{7}Bcx^7+\frac{1}{9}cCx^9$$

input
```
Int[(a + c*x^4)*(A + B*x^2 + C*x^4),x]
```

output
```
a*A*x + (a*B*x^3)/3 + ((A*c + a*C)*x^5)/5 + (B*c*x^7)/7 + (c*C*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{B a \,x^{3}}{3}+\frac{\left(A c +C a \right) x^{5}}{5}+\frac{B c \,x^{7}}{7}+\frac{c C \,x^{9}}{9}$	39
norman	$\frac{c C \,x^{9}}{9}+\frac{B c \,x^{7}}{7}+\left(\frac{A c}{5}+\frac{C a}{5}\right) x^{5}+\frac{B a \,x^{3}}{3}+aAx$	40
gosper	$\frac{1}{9} c C \,x^{9}+\frac{1}{7} B c \,x^{7}+\frac{1}{5} x^{5} A c +\frac{1}{5} x^{5} C a +\frac{1}{3} B a \,x^{3}+aAx$	41
risch	$\frac{1}{9} c C \,x^{9}+\frac{1}{7} B c \,x^{7}+\frac{1}{5} x^{5} A c +\frac{1}{5} x^{5} C a +\frac{1}{3} B a \,x^{3}+aAx$	41
parallelrisch	$\frac{1}{9} c C \,x^{9}+\frac{1}{7} B c \,x^{7}+\frac{1}{5} x^{5} A c +\frac{1}{5} x^{5} C a +\frac{1}{3} B a \,x^{3}+aAx$	41
orering	$\frac{x \left(35 C c \,x^{8}+45 B c \,x^{6}+63 A c \,x^{4}+63 C a \,x^{4}+105 B a \,x^{2}+315 A a \right)}{315}$	44

input
```
int((c*x^4+a)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/3*B*a*x^3+1/5*(A*c+C*a)*x^5+1/7*B*c*x^7+1/9*c*C*x^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+cx^4\right) \left(A+Bx^2+Cx^4\right) \, dx=\frac{1}{9} \, C c x^{9} + \frac{1}{7} \, B c x^{7} + \frac{1}{5} \, {\left(C a + A c\right)} x^{5} + \frac{1}{3} \, B a x^{3} + A a x$$

input
```
integrate((c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
1/9*C*c*x^9 + 1/7*B*c*x^7 + 1/5*(C*a + A*c)*x^5 + 1/3*B*a*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+cx^4\right)\left(A+Bx^2+Cx^4\right)\,dx = Aax + \frac{Bax^3}{3} + \frac{Bcx^7}{7} + \frac{Ccx^9}{9} + x^5\left(\frac{Ac}{5}+\frac{Ca}{5}\right)$$

input
```
integrate((c*x**4+a)*(C*x**4+B*x**2+A),x)
```

output
```
A*a*x + B*a*x**3/3 + B*c*x**7/7 + C*c*x**9/9 + x**5*(A*c/5 + C*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+cx^4\right)\left(A+Bx^2+Cx^4\right)\,dx = \frac{1}{9}\,Ccx^9 + \frac{1}{7}\,Bcx^7 + \frac{1}{5}\,(Ca+Ac)x^5 + \frac{1}{3}\,Bax^3 + Aax$$

input
```
integrate((c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*C*c*x^9 + 1/7*B*c*x^7 + 1/5*(C*a + A*c)*x^5 + 1/3*B*a*x^3 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \left(a+cx^4\right)\left(A+Bx^2+Cx^4\right)\,dx = \frac{1}{9}\,Ccx^9 + \frac{1}{7}\,Bcx^7 + \frac{1}{5}\,Cax^5 + \frac{1}{5}\,Acx^5 + \frac{1}{3}\,Bax^3 + Aax$$

input
```
integrate((c*x^4+a)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
1/9*C*c*x^9 + 1/7*B*c*x^7 + 1/5*C*a*x^5 + 1/5*A*c*x^5 + 1/3*B*a*x^3 + A*a*
x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int (a+cx^4) (A+Bx^2+Cx^4) \, dx = \frac{C\,c\,x^9}{9} + \frac{B\,c\,x^7}{7} + \left(\frac{A\,c}{5} + \frac{C\,a}{5}\right) x^5 + \frac{B\,a\,x^3}{3} + A\,a\,x$$

input
```
int((a + c*x^4)*(A + B*x^2 + C*x^4),x)
```

output
```
x^5*((A*c)/5 + (C*a)/5) + A*a*x + (B*a*x^3)/3 + (B*c*x^7)/7 + (C*c*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+cx^4) (A+Bx^2+Cx^4) \, dx = \frac{x(35c^2x^8 + 45bc\,x^6 + 126ac\,x^4 + 105ab\,x^2 + 315a^2)}{315}$$

input
```
int((c*x^4+a)*(C*x^4+B*x^2+A),x)
```

output
```
(x*(315*a**2 + 105*a*b*x**2 + 126*a*c*x**4 + 45*b*c*x**6 + 35*c**2*x**8))/
315
```

3.10 $\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx$

Optimal result 131
Mathematica [A] (verified) 132
Rubi [A] (verified) 132
Maple [C] (verified) 133
Fricas [B] (verification not implemented) 134
Sympy [A] (verification not implemented) 135
Maxima [A] (verification not implemented) 136
Giac [B] (verification not implemented) 137
Mupad [B] (verification not implemented) 138
Reduce [B] (verification not implemented) 138

Optimal result

Integrand size = 22, antiderivative size = 200

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx =& \frac{Cx}{c} - \frac{\left(\sqrt{a}B\sqrt{c}+Ac-aC\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} \\ &+ \frac{\left(\sqrt{a}B\sqrt{c}+Ac-aC\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{5/4}} \\ &- \frac{\left(\sqrt{a}B\sqrt{c}-Ac+aC\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{5/4}}\end{aligned}$$

output
```
C*x/c+1/4*(a^(1/2)*B*c^(1/2)+A*c-C*a)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))
*2^(1/2)/a^(3/4)/c^(5/4)+1/4*(a^(1/2)*B*c^(1/2)+A*c-C*a)*arctan(1+2^(1/2)*
c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(5/4)-1/4*(a^(1/2)*B*c^(1/2)-A*c+C*a)
*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/
c^(5/4)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx = \frac{8a^{3/4}\sqrt[4]{c}Cx+2\sqrt{2}(-\sqrt{a}B\sqrt{c}-Ac+aC)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-2\sqrt{2}(-\sqrt{a}B\sqrt{c}-Ac+aC)\arctan}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + c*x^4),x]
```

output
```
(8*a^(3/4)*c^(1/4)*C*x + 2*Sqrt[2]*(-(Sqrt[a]*B*Sqrt[c]) - A*c + a*C)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*(-(Sqrt[a]*B*Sqrt[c]) - A*
c + a*C)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*(Sqrt[a]*B*Sqrt
[c] - A*c + a*C)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] -
Sqrt[2]*(Sqrt[a]*B*Sqrt[c] - A*c + a*C)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{-aC+Ac+Bcx^2}{c(a+cx^4)}+\frac{C}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}B\sqrt{c}-aC+Ac\right)}{2\sqrt{2}a^{3/4}c^{5/4}}+\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B\sqrt{c}+aC-Ac\right)}{4\sqrt{2}a^{3/4}c^{5/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B\sqrt{c}+aC-Ac\right)}{4\sqrt{2}a^{3/4}c^{5/4}}+\frac{Cx}{c}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + c*x^4),x]
```

output
```
(C*x)/c - ((Sqrt[a]*B*Sqrt[c] + A*c - a*C)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((Sqrt[a]*B*Sqrt[c] + A*c - a*C)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((Sq
rt[a]*B*Sqrt[c] - A*c + a*C)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) - ((Sqrt[a]*B*Sqrt[c] - A*c + a*C)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*c^(5/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.24

method	result
risch	$\frac{Cx}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(B\_R^{2}c+Ac-Ca\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4c^{2}}$
default	$\frac{Cx}{c}+\frac{\frac{(Ac-Ca)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{B\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+\cdots\right.}{\cdots}}{c}$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
C*x/c+1/4/c^2*sum((B*_R^2*c+A*c-C*a)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1818 vs. $2(143) = 286$.

Time = 0.42 (sec) , antiderivative size = 1818, normalized size of antiderivative = 9.09

$$\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(c*sqrt((a*c^2*sqrt(-(C^4*a^4 + A^4*c^4 - 2*(B^2*C^2 + 2*A*C^3)*a^3*c
 + (B^4 + 4*A*B^2*C + 6*A^2*C^2)*a^2*c^2 - 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a
^3*c^5)) + 2*B*C*a - 2*A*B*c)/(a*c^2))*log((C^4*a^4 - 4*A*C^3*a^3*c - 4*A^
3*C*a*c^3 + A^4*c^4 - (B^4 - 6*A^2*C^2)*a^2*c^2)*x + (B*a^3*c^4*sqrt(-(C^4
*a^4 + A^4*c^4 - 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C + 6*A^2*C^
2)*a^2*c^2 - 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5)) + C^3*a^4*c - A^3*a*c
^4 - (B^2*C + 3*A*C^2)*a^3*c^2 + (A*B^2 + 3*A^2*C)*a^2*c^3)*sqrt((a*c^2*sq
rt(-(C^4*a^4 + A^4*c^4 - 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C +
6*A^2*C^2)*a^2*c^2 - 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5)) + 2*B*C*a - 2
*A*B*c)/(a*c^2))) - c*sqrt((a*c^2*sqrt(-(C^4*a^4 + A^4*c^4 - 2*(B^2*C^2 +
2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C + 6*A^2*C^2)*a^2*c^2 - 2*(A^2*B^2 + 2*A^
3*C)*a*c^3)/(a^3*c^5)) + 2*B*C*a - 2*A*B*c)/(a*c^2))*log((C^4*a^4 - 4*A*C^
3*a^3*c - 4*A^3*C*a*c^3 + A^4*c^4 - (B^4 - 6*A^2*C^2)*a^2*c^2)*x - (B*a^3*
c^4*sqrt(-(C^4*a^4 + A^4*c^4 - 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^
2*C + 6*A^2*C^2)*a^2*c^2 - 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5)) + C^3*a
^4*c - A^3*a*c^4 - (B^2*C + 3*A*C^2)*a^3*c^2 + (A*B^2 + 3*A^2*C)*a^2*c^3)*
sqrt((a*c^2*sqrt(-(C^4*a^4 + A^4*c^4 - 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4
+ 4*A*B^2*C + 6*A^2*C^2)*a^2*c^2 - 2*(A^2*B^2 + 2*A^3*C)*a*c^3)/(a^3*c^5))
 + 2*B*C*a - 2*A*B*c)/(a*c^2))) - c*sqrt(-(a*c^2*sqrt(-(C^4*a^4 + A^4*c^4
- 2*(B^2*C^2 + 2*A*C^3)*a^3*c + (B^4 + 4*A*B^2*C + 6*A^2*C^2)*a^2*c^2 -...
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx = \frac{Cx}{c}$$

$$+\text{RootSum}\Big(256t^4a^3c^5 + t^2\cdot\left(64ABa^2c^4 - 64BCa^3c^3\right) + A^4c^4 - 4A^3Cac^3 + 2A^2B^2ac^3 + 6A^2C^2a^2c^2 -$$

input
```
integrate((C*x**4+B*x**2+A)/(c*x**4+a),x)
```

output
```
C*x/c + RootSum(256*_t**4*a**3*c**5 + _t**2*(64*A*B*a**2*c**4 - 64*B*C*a**
3*c**3) + A**4*c**4 - 4*A**3*C*a*c**3 + 2*A**2*B**2*a*c**3 + 6*A**2*C**2*a
**2*c**2 - 4*A*B**2*C*a**2*c**2 - 4*A*C**3*a**3*c + B**4*a**2*c**2 + 2*B**
2*C**2*a**3*c + C**4*a**4, Lambda(_t, _t*log(x + (-64*_t**3*B*a**3*c**4 +
4*_t*A**3*a*c**4 - 12*_t*A**2*C*a**2*c**3 - 12*_t*A*B**2*a**2*c**3 + 12*_t
*A*C**2*a**3*c**2 + 12*_t*B**2*C*a**3*c**2 - 4*_t*C**3*a**4*c)/(A**4*c**4
- 4*A**3*C*a*c**3 + 6*A**2*C**2*a**2*c**2 - 4*A*C**3*a**3*c - B**4*a**2*c*
*2 + C**4*a**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx = \frac{Cx}{c} - \frac{\frac{2\sqrt{2}(Ca\sqrt{c}-(B\sqrt{a}+A\sqrt{c})c)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}(Ca\sqrt{c}-(B\sqrt{a}+A\sqrt{c})c)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}(C}{}}{8c}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="maxima")
```

output
```
C*x/c - 1/8*(2*sqrt(2)*(C*a*sqrt(c) - (B*sqrt(a) + A*sqrt(c))*c)*arctan(1/
2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(C*a*sqrt(c) - (B*sqrt(
a) + A*sqrt(c))*c)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(
2)*(C*a*sqrt(c) + (B*sqrt(a) - A*sqrt(c))*c)*log(sqrt(c)*x^2 + sqrt(2)*a^(
1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(C*a*sqrt(c) + (B*sq
rt(a) - A*sqrt(c))*c)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(3/4)))/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(143) = 286$.

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.48

$$\int \frac{A + Bx^2 + Cx^4}{a + cx^4}\,dx$$
$$= \frac{Cx}{c} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Cac - (ac^3)^{\frac{1}{4}}\,Ac^2 - (ac^3)^{\frac{3}{4}}\,B\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}$$
$$- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Cac - (ac^3)^{\frac{1}{4}}\,Ac^2 - (ac^3)^{\frac{3}{4}}\,B\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}$$
$$- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Cac - (ac^3)^{\frac{1}{4}}\,Ac^2 + (ac^3)^{\frac{3}{4}}\,B\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$
$$+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Cac - (ac^3)^{\frac{1}{4}}\,Ac^2 + (ac^3)^{\frac{3}{4}}\,B\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a),x, algorithm="giac")
```

output
```
C*x/c - 1/4*sqrt(2)*((a*c^3)^(1/4)*C*a*c - (a*c^3)^(1/4)*A*c^2 - (a*c^3)^(
3/4)*B)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3
) - 1/4*sqrt(2)*((a*c^3)^(1/4)*C*a*c - (a*c^3)^(1/4)*A*c^2 - (a*c^3)^(3/4)
*B)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) -
1/8*sqrt(2)*((a*c^3)^(1/4)*C*a*c - (a*c^3)^(1/4)*A*c^2 + (a*c^3)^(3/4)*B)*
log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/8*sqrt(2)*((a*c^3
)^(1/4)*C*a*c - (a*c^3)^(1/4)*A*c^2 + (a*c^3)^(3/4)*B)*log(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 2345, normalized size of antiderivative = 11.72

$$\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2 + C*x^4)/(a + c*x^4),x)
```

output
```
2*atanh((8*A^2*c^3*x*((B*C)/(8*c^2) - (A*B)/(8*a*c) + (A^2*(-a^3*c^5)^(1/2
))/(16*a^3*c^3) - (B^2*(-a^3*c^5)^(1/2))/(16*a^2*c^4) + (C^2*(-a^3*c^5)^(1
/2))/(16*a*c^5) - (A*C*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*A^3*(-a^3
*c^5)^(1/2))/a^2 - 2*B*C^2*a^2 - 2*A^2*B*c^2 + 2*B^3*a*c + (6*A*C^2*(-a^3*
c^5)^(1/2))/c^2 + (2*B^2*C*(-a^3*c^5)^(1/2))/c^2 - (2*C^3*a*(-a^3*c^5)^(1/
2))/c^3 - (2*A*B^2*(-a^3*c^5)^(1/2))/(a*c) - (6*A^2*C*(-a^3*c^5)^(1/2))/(a
*c) + 4*A*B*C*a*c) - (8*B^2*a*c^2*x*((B*C)/(8*c^2) - (A*B)/(8*a*c) + (A^2*
(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (B^2*(-a^3*c^5)^(1/2))/(16*a^2*c^4) + (C^
2*(-a^3*c^5)^(1/2))/(16*a*c^5) - (A*C*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2)
)/((2*A^3*(-a^3*c^5)^(1/2))/a^2 - 2*B*C^2*a^2 - 2*A^2*B*c^2 + 2*B^3*a*c +
(6*A*C^2*(-a^3*c^5)^(1/2))/c^2 + (2*B^2*C*(-a^3*c^5)^(1/2))/c^2 - (2*C^3*a
*(-a^3*c^5)^(1/2))/c^3 - (2*A*B^2*(-a^3*c^5)^(1/2))/(a*c) - (6*A^2*C*(-a^3
*c^5)^(1/2))/(a*c) + 4*A*B*C*a*c) + (8*C^2*a^2*c*x*((B*C)/(8*c^2) - (A*B)/
(8*a*c) + (A^2*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (B^2*(-a^3*c^5)^(1/2))/(16
*a^2*c^4) + (C^2*(-a^3*c^5)^(1/2))/(16*a*c^5) - (A*C*(-a^3*c^5)^(1/2))/(8*
a^2*c^4))^(1/2))/((2*A^3*(-a^3*c^5)^(1/2))/a^2 - 2*B*C^2*a^2 - 2*A^2*B*c^2
 + 2*B^3*a*c + (6*A*C^2*(-a^3*c^5)^(1/2))/c^2 + (2*B^2*C*(-a^3*c^5)^(1/2))
/c^2 - (2*C^3*a*(-a^3*c^5)^(1/2))/c^3 - (2*A*B^2*(-a^3*c^5)^(1/2))/(a*c) -
 (6*A^2*C*(-a^3*c^5)^(1/2))/(a*c) + 4*A*B*C*a*c) - (16*A*C*a*c^2*x*((B*C)/
(8*c^2) - (A*B)/(8*a*c) + (A^2*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (B^2*(-...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^2+Cx^4}{a+cx^4}\,dx = \frac{-2c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)b+2c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)b+c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}\right.}{8ac}$$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+a),x)
```

output
```
( - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b + c*
*(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqr
t(c)*x**2)*b - c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x +
 sqrt(a) + sqrt(c)*x**2)*b + 8*a*c*x)/(8*a*c)
```

3.11 $\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 226

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx &= \frac{x\left(A-\frac{aC}{c}+Bx^2\right)}{4a\,(a+cx^4)} \\ &- \frac{\left(\sqrt{a}B\sqrt{c}+3Ac+aC\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} \\ &+ \frac{\left(\sqrt{a}B\sqrt{c}+3Ac+aC\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{5/4}} \\ &- \frac{\left(\sqrt{a}B\sqrt{c}-3Ac-aC\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{5/4}}\end{aligned}$$

output
```
1/4*x*(A-a*C/c+B*x^2)/a/(c*x^4+a)+1/16*(a^(1/2)*B*c^(1/2)+3*A*c+C*a)*arcta
n(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(5/4)+1/16*(a^(1/2)*B*c^
(1/2)+3*A*c+C*a)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(5/
4)-1/16*(a^(1/2)*B*c^(1/2)-3*A*c-C*a)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a
^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx$$
$$= \frac{\frac{8a^{3/4}\sqrt[4]{c}x(Ac-aC+Bcx^2)}{a+cx^4} - 2\sqrt{2}\left(\sqrt{a}B\sqrt{c}+3Ac+aC\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt{2}\left(\sqrt{a}B\sqrt{c}+3Ac+aC\right)}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + c*x^4)^2,x]
```

output
```
((8*a^(3/4)*c^(1/4)*x*(A*c - a*C + B*c*x^2))/(a + c*x^4) - 2*Sqrt[2]*(Sqrt
[a]*B*Sqrt[c] + 3*A*c + a*C)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*S
qrt[2]*(Sqrt[a]*B*Sqrt[c] + 3*A*c + a*C)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^
(1/4)] + Sqrt[2]*(Sqrt[a]*B*Sqrt[c] - 3*A*c - a*C)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(-(Sqrt[a]*B*Sqrt[c]) + 3*A*c +
a*C)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*c
^(5/4))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2397, 25, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{2397}$$
$$\frac{x(-aC+Ac+Bcx^2)}{4ac(a+cx^4)} - \frac{\int -\frac{Bcx^2+3Ac+aC}{cx^4+a}dx}{4ac}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Bcx^2+3Ac+aC}{cx^4+a}dx}{4ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1482}$$

$$\frac{\frac{\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}}{4ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\frac{\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{a}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{a}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)}{2\sqrt{a}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$\Big\downarrow$ 1479

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)}{2\sqrt{a}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\cdots\right)}{2\sqrt{a}}}{4ac}$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)}{2\sqrt{a}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\cdots\right)}{2\sqrt{a}}}{4ac}$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)\left(\sqrt{a}B\sqrt{c}+aC+3Ac\right)}{2\sqrt{a}}-\frac{\left(\sqrt{a}B\sqrt{c}-aC-3Ac\right)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\cdots\right)}{2\sqrt{a}}}{4ac}$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(\sqrt{a}B\sqrt{c}+aC+3Ac)}{2\sqrt{a}}-\frac{\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(\sqrt{a}}{2\sqrt{a}}}{4ac}$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{4ac\left(a+cx^4\right)}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + c*x^4)^2,x]
```

output
```
(x*(A*c - a*C + B*c*x^2))/(4*a*c*(a + c*x^4)) + (((Sqrt[a]*B*Sqrt[c] + 3*A
*c + a*C)*(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1
/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))
)/(2*Sqrt[a]) - ((Sqrt[a]*B*Sqrt[c] - 3*A*c - a*C)*(-1/2*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))
))/(2*Sqrt[a]))/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\frac{x^3 B}{4a}+\frac{(Ac-Ca)x}{4ac}}{c x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(B\_R^2+\frac{3Ac+Ca}{c}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$\frac{\frac{x^3 B}{4a}+\frac{(Ac-Ca)x}{4ac}}{c x^4+a}+\frac{\frac{(3Ac+Ca)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{B\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right.}{x^2+\left(\frac{a}{c}\right.}\right.\right.}{}}{4ac}$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a*x^3*B+1/4*(A*c-C*a)/a/c*x)/(c*x^4+a)+1/16/a/c*sum((B*_R^2+1/c*(3*A*
c+C*a))/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1941 vs. $2(165) = 330$.

Time = 0.46 (sec) , antiderivative size = 1941, normalized size of antiderivative = 8.59

$$\int \frac{A+Bx^2+Cx^4}{\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(4*B*c*x^3 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(C^4*a^4 + 81*A
^4*c^4 - 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2
*c^2 - 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a + 6*A*B*c)/(a^3*
c^2))*log((C^4*a^4 + 12*A*C^3*a^3*c + 108*A^3*C*a*c^3 + 81*A^4*c^4 - (B^4
- 54*A^2*C^2)*a^2*c^2)*x + (B*a^6*c^4*sqrt(-(C^4*a^4 + 81*A^4*c^4 - 2*(B^2
*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2*c^2 - 18*(A^2*
B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + C^3*a^5*c + 27*A^3*a^2*c^4 - (B^2*C - 9
*A*C^2)*a^4*c^2 - 3*(A*B^2 - 9*A^2*C)*a^3*c^3)*sqrt(-(a^3*c^2*sqrt(-(C^4*a
^4 + 81*A^4*c^4 - 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2
*C^2)*a^2*c^2 - 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a + 6*A*B
*c)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*c^2*sqrt(-(C^4*a^4 + 81*A
^4*c^4 - 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2
*c^2 - 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a + 6*A*B*c)/(a^3*
c^2))*log((C^4*a^4 + 12*A*C^3*a^3*c + 108*A^3*C*a*c^3 + 81*A^4*c^4 - (B^4
- 54*A^2*C^2)*a^2*c^2)*x - (B*a^6*c^4*sqrt(-(C^4*a^4 + 81*A^4*c^4 - 2*(B^2
*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2*C^2)*a^2*c^2 - 18*(A^2*
B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + C^3*a^5*c + 27*A^3*a^2*c^4 - (B^2*C - 9
*A*C^2)*a^4*c^2 - 3*(A*B^2 - 9*A^2*C)*a^3*c^3)*sqrt(-(a^3*c^2*sqrt(-(C^4*a
^4 + 81*A^4*c^4 - 2*(B^2*C^2 - 6*A*C^3)*a^3*c + (B^4 - 12*A*B^2*C + 54*A^2
*C^2)*a^2*c^2 - 18*(A^2*B^2 - 6*A^3*C)*a*c^3)/(a^7*c^5)) + 2*B*C*a + 6*...
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx$$
$$= \operatorname{RootSum}\left(65536t^4a^7c^5 + t^2\cdot\left(3072ABa^4c^4 + 1024BCa^5c^3\right) + 81A^4c^4 + 108A^3Cac^3 + 18A^2B^2ac^3 + 54\right.$$
$$+ \frac{Bcx^3 + x(Ac - Ca)}{4a^2c + 4ac^2x^4}$$

input
```
integrate((C*x**4+B*x**2+A)/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**5 + _t**2*(3072*A*B*a**4*c**4 + 1024*B*C*a**5*
c**3) + 81*A**4*c**4 + 108*A**3*C*a*c**3 + 18*A**2*B**2*a*c**3 + 54*A**2*C
**2*a**2*c**2 + 12*A*B**2*C*a**2*c**2 + 12*A*C**3*a**3*c + B**4*a**2*c**2
+ 2*B**2*C**2*a**3*c + C**4*a**4, Lambda(_t, _t*log(x + (-4096*_t**3*B*a**
6*c**4 + 432*_t*A**3*a**2*c**4 + 432*_t*A**2*C*a**3*c**3 - 144*_t*A*B**2*a
**3*c**3 + 144*_t*A*C**2*a**4*c**2 - 48*_t*B**2*C*a**4*c**2 + 16*_t*C**3*a
**5*c)/(81*A**4*c**4 + 108*A**3*C*a*c**3 + 54*A**2*C**2*a**2*c**2 + 12*A*C
**3*a**3*c - B**4*a**2*c**2 + C**4*a**4)))) + (B*c*x**3 + x*(A*c - C*a))/(
4*a**2*c + 4*a*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx = \frac{Bcx^3-(Ca-Ac)x}{4\,(ac^2x^4+a^2c)} + \frac{\frac{2\sqrt{2}(Ca\sqrt{c}+(B\sqrt{a}+3\,A\sqrt{c})c)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}(Ca\sqrt{c}+(B\sqrt{a}+3\,A\sqrt{c})c)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \cdots}{32\,ac}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(B*c*x^3 - (C*a - A*c)*x)/(a*c^2*x^4 + a^2*c) + 1/32*(2*sqrt(2)*(C*a*s
qrt(c) + (B*sqrt(a) + 3*A*sqrt(c))*c)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sq
rt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))*sqrt(c)) + 2*sqrt(2)*(C*a*sqrt(c) + (B*sqrt(a) + 3*A*sqrt(c))*c)*arctan
(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(C*a*sqrt(c) - (B*sqrt
(a) - 3*A*sqrt(c))*c)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(3/4)) - sqrt(2)*(C*a*sqrt(c) - (B*sqrt(a) - 3*A*sqrt(c))*c)
*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))
/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx$$
$$= \frac{Bcx^3-Cax+Acx}{4(cx^4+a)ac} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}Cac+3(ac^3)^{\frac{1}{4}}Ac^2+(ac^3)^{\frac{3}{4}}B\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}Cac+3(ac^3)^{\frac{1}{4}}Ac^2+(ac^3)^{\frac{3}{4}}B\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^2c^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}Cac+3(ac^3)^{\frac{1}{4}}Ac^2-(ac^3)^{\frac{3}{4}}B\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}Cac+3(ac^3)^{\frac{1}{4}}Ac^2-(ac^3)^{\frac{3}{4}}B\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^2c^3}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(B*c*x^3 - C*a*x + A*c*x)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*((a*c^3)^(1
/4)*C*a*c + 3*(a*c^3)^(1/4)*A*c^2 + (a*c^3)^(3/4)*B)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)*((a*c^3)^(
1/4)*C*a*c + 3*(a*c^3)^(1/4)*A*c^2 + (a*c^3)^(3/4)*B)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(2)*((a*c^3)^
(1/4)*C*a*c + 3*(a*c^3)^(1/4)*A*c^2 - (a*c^3)^(3/4)*B)*log(x^2 + sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*((a*c^3)^(1/4)*C*a*c +
3*(a*c^3)^(1/4)*A*c^2 - (a*c^3)^(3/4)*B)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 2481, normalized size of antiderivative = 10.98

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2 + C*x^4)/(a + c*x^4)^2,x)
```

output
```
2*atanh((C^2*c*x*((9*A^2*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (B*C)/(128*a^2*
c^2) - (3*A*B)/(128*a^3*c) - (B^2*(-a^7*c^5)^(1/2))/(256*a^6*c^4) + (C^2*(
-a^7*c^5)^(1/2))/(256*a^5*c^5) + (3*A*C*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(
1/2))/(2*((27*A^3*(-a^7*c^5)^(1/2))/(32*a^7) - (B*C^2)/(32*a) + (B^3*c)/(3
2*a^2) - (9*A^2*B*c^2)/(32*a^3) + (C^3*(-a^7*c^5)^(1/2))/(32*a^4*c^3) - (3
*A*B^2*(-a^7*c^5)^(1/2))/(32*a^6*c) + (9*A*C^2*(-a^7*c^5)^(1/2))/(32*a^5*c
^2) + (27*A^2*C*(-a^7*c^5)^(1/2))/(32*a^6*c) - (B^2*C*(-a^7*c^5)^(1/2))/(3
2*a^5*c^2) - (3*A*B*C*c)/(16*a^2))) + (9*A^2*c^3*x*((9*A^2*(-a^7*c^5)^(1/2
))/(256*a^7*c^3) - (B*C)/(128*a^2*c^2) - (3*A*B)/(128*a^3*c) - (B^2*(-a^7*
c^5)^(1/2))/(256*a^6*c^4) + (C^2*(-a^7*c^5)^(1/2))/(256*a^5*c^5) + (3*A*C*
(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((B^3*c)/32 + (27*A^3*(-a^7*c^5
)^(1/2))/(32*a^5) - (B*C^2*a)/32 - (3*A*B*C*c)/16 - (9*A^2*B*c^2)/(32*a) +
 (C^3*(-a^7*c^5)^(1/2))/(32*a^2*c^3) - (3*A*B^2*(-a^7*c^5)^(1/2))/(32*a^4*
c) + (9*A*C^2*(-a^7*c^5)^(1/2))/(32*a^3*c^2) + (27*A^2*C*(-a^7*c^5)^(1/2))
/(32*a^4*c) - (B^2*C*(-a^7*c^5)^(1/2))/(32*a^3*c^2))) + (B^2*c^2*x*((9*A^2
*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (B*C)/(128*a^2*c^2) - (3*A*B)/(128*a^3*
c) - (B^2*(-a^7*c^5)^(1/2))/(256*a^6*c^4) + (C^2*(-a^7*c^5)^(1/2))/(256*a^
5*c^5) + (3*A*C*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((B*C^2)/32 - (
27*A^3*(-a^7*c^5)^(1/2))/(32*a^6) - (B^3*c)/(32*a) + (9*A^2*B*c^2)/(32*a^2
) - (C^3*(-a^7*c^5)^(1/2))/(32*a^3*c^3) + (3*A*B^2*(-a^7*c^5)^(1/2))/(3...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.64

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+a)^2,x)
```

output
```
( - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*b - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((
c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b*c*
x**4 - 8*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 8*c**(3/4)*a**(1/4)*sqrt(2)*at
an((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*
a*c*x**4 + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*b + 2*c**(1/4)*a**(3/4)*sqrt(2)
*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)
))*b*c*x**4 + 8*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 8*c**(3/4)*a**(1/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*a*c*x**4 + c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(
2)*x + sqrt(a) + sqrt(c)*x**2)*a*b + c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(
1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*b*c*x**4 - c**(1/4)*a**(
3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*b
 - c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + s
qrt(c)*x**2)*b*c*x**4 - 4*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/
4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 4*c**(3/4)*a**(1/4)*sqrt(2)*
log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 +...
```

3.12 $\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 264

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3}\,dx &= \frac{x\left(A-\frac{aC}{c}+Bx^2\right)}{8a\,(a+cx^4)^2}+\frac{x\left(7A+\frac{aC}{c}+5Bx^2\right)}{32a^2\,(a+cx^4)}\\&-\frac{\left(5\sqrt{a}B\sqrt{c}+21Ac+3aC\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{5/4}}\\&+\frac{\left(5\sqrt{a}B\sqrt{c}+21Ac+3aC\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{5/4}}\\&-\frac{\left(5\sqrt{a}B\sqrt{c}-21Ac-3aC\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{11/4}c^{5/4}}\end{aligned}$$

output
```
1/8*x*(A-a*C/c+B*x^2)/a/(c*x^4+a)^2+1/32*x*(7*A+a*C/c+5*B*x^2)/a^2/(c*x^4+
a)+1/128*(5*a^(1/2)*B*c^(1/2)+21*A*c+3*C*a)*arctan(-1+2^(1/2)*c^(1/4)*x/a^
(1/4))*2^(1/2)/a^(11/4)/c^(5/4)+1/128*(5*a^(1/2)*B*c^(1/2)+21*A*c+3*C*a)*a
rctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/c^(5/4)-1/128*(5*a^(1/
2)*B*c^(1/2)-21*A*c-3*C*a)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1
/2)*x^2))*2^(1/2)/a^(11/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3}\,dx = \frac{\frac{32a^{7/4}\sqrt[4]{c}x(Ac-aC+Bcx^2)}{(a+cx^4)^2}+\frac{8a^{3/4}\sqrt[4]{c}x(7Ac+aC+5Bcx^2)}{a+cx^4}-2\sqrt{2}(5\sqrt{a}B\sqrt{c}+21Ac+3aC)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + c*x^4)^3,x]
```

output
```
((32*a^(7/4)*c^(1/4)*x*(A*c - a*C + B*c*x^2))/(a + c*x^4)^2 + (8*a^(3/4)*c
^(1/4)*x*(7*A*c + a*C + 5*B*c*x^2))/(a + c*x^4) - 2*Sqrt[2]*(5*Sqrt[a]*B*S
qrt[c] + 21*A*c + 3*a*C)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[
2]*(5*Sqrt[a]*B*Sqrt[c] + 21*A*c + 3*a*C)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)] + Sqrt[2]*(5*Sqrt[a]*B*Sqrt[c] - 21*A*c - 3*a*C)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(-5*Sqrt[a]*B*Sqrt[c] + 21
*A*c + 3*a*C)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(256
*a^(11/4)*c^(5/4))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2397, 25, 1493, 25, 1482, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \; 2397$$

$$\frac{x(-aC+Ac+Bcx^2)}{8ac\,(a+cx^4)^2}-\frac{\int -\frac{5Bcx^2+7Ac+aC}{(cx^4+a)^2}dx}{8ac}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5Bcx^2+7Ac+aC}{(cx^4+a)^2}dx}{8ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1493}$$

$$\frac{\frac{x(aC+7Ac+5Bcx^2)}{4a(a+cx^4)}-\frac{\int -\frac{5Bcx^2+3(7Ac+aC)}{cx^4+a}dx}{4a}}{8ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5Bcx^2+3(7Ac+aC)}{cx^4+a}dx}{4a}+\frac{x(aC+7Ac+5Bcx^2)}{4a(a+cx^4)}}{8ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1482}$$

$$\frac{\frac{\frac{(5\sqrt{a}B\sqrt{c}+3aC+21Ac)\int \frac{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\int \frac{\sqrt{c}\left(\sqrt{a}-\sqrt{c}x^2\right)}{cx^4+a}dx}{2\sqrt{a}\sqrt{c}}}{4a}+\frac{x(aC+7Ac+5Bcx^2)}{4a(a+cx^4)}}{8ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{(5\sqrt{a}B\sqrt{c}+3aC+21Ac)\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4a}+\frac{x(aC+7Ac+5Bcx^2)}{4a(a+cx^4)}}{8ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\frac{(5\sqrt{a}B\sqrt{c}+3aC+21Ac)\left(\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}\right)}{2\sqrt{a}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4a}+\frac{x(aC+7Ac+5Bcx^2)}{4a(a+cx^4)}}{8ac}+\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{(5\sqrt{a}B\sqrt{c}+3aC+21Ac)\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)}{2\sqrt{a}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4a} \cdots}{8ac} \cdots$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$\big\downarrow$ 217

$$\frac{\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(5\sqrt{a}B\sqrt{c}+3aC+21Ac)}{2\sqrt{a}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4a}+\frac{x(aC+7Ac+5Bcx^2)}{4a(a+cx^4)}}{8ac}+$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1479

$$\frac{\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(5\sqrt{a}B\sqrt{c}+3aC+21Ac)}{2\sqrt{a}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\left(-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\cdots\right)}{\sqrt[4]{c}\left(x^2+\cdots\right)}\cdots}{2\sqrt{2}\cdots}\cdots\right.}{2\sqrt{a}}}{4a}\cdots}{8ac}\cdots$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(5\sqrt{a}B\sqrt{c}+3aC+21Ac)}{2\sqrt{a}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x\cdots\right.}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right.}}{2\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right.}{2\sqrt{a}}\cdots}{4a}\cdots}{8ac}$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(5\sqrt{a}B\sqrt{c}+3aC+21Ac)}{2\sqrt{a}}-\frac{(5\sqrt{a}B\sqrt{c}-3aC-21Ac)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}\right)}{2\sqrt{a}}\cdots}{4a}\cdots}{8ac}$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\frac{\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(5\sqrt{a}B\sqrt{c}+3aC+21Ac)}{2\sqrt{a}}-\frac{\left(\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}\right)(5\sqrt{a}B\cdots}{2\sqrt{a}}\cdots}{4a}\cdots}{8ac}$$

$$\frac{x\left(-aC+Ac+Bcx^2\right)}{8ac\left(a+cx^4\right)^2}$$

input

```
Int[(A + B*x^2 + C*x^4)/(a + c*x^4)^3,x]
```

output
```
(x*(A*c - a*C + B*c*x^2))/(8*a*c*(a + c*x^4)^2) + ((x*(7*A*c + a*C + 5*B*c
*x^2))/(4*a*(a + c*x^4)) + (((5*Sqrt[a]*B*Sqrt[c] + 21*A*c + 3*a*C)*(-(Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[a]) -
((5*Sqrt[a]*B*Sqrt[c] - 21*A*c - 3*a*C)*(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4))))/(2*Sqrt[
a]))/(4*a))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

rule 1493
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Simp[1/(4*a*(p + 1)
)   Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x]
/; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && Integer
Q[2*p]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{5Bc\,x^7}{32a^2}+\frac{(7Ac+Ca)x^5}{32a^2}+\frac{9x^3B}{32a}+\frac{(11Ac-3Ca)x}{32ac}}{(c\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(5B\_R^2+\frac{21Ac+3Ca}{c}\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2c}$
default	$\frac{\frac{5Bc\,x^7}{32a^2}+\frac{(7Ac+Ca)x^5}{32a^2}+\frac{9x^3B}{32a}+\frac{(11Ac-3Ca)x}{32ac}}{(c\,x^4+a)^2}+\frac{\frac{(21Ac+3Ca)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{c}\right.}\right.\right.}{8a}}{}$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*B*c/a^2*x^7+1/32*(7*A*c+C*a)/a^2*x^5+9/32/a*x^3*B+1/32*(11*A*c-3*C*a
)/a/c*x)/(c*x^4+a)^2+1/128/a^2/c*sum((5*B*_R^2+3/c*(7*A*c+C*a))/_R^3*ln(x-
_R),_R=RootOf(_Z^4*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2124 vs. $2(201) = 402$.

Time = 0.54 (sec) , antiderivative size = 2124, normalized size of antiderivative = 8.05

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(20*B*c^2*x^7 + 36*B*a*c*x^3 + 4*(C*a*c + 7*A*c^2)*x^5 + (a^2*c^3*x^
8 + 2*a^3*c^2*x^4 + a^4*c)*sqrt(-(a^5*c^2*sqrt(-(81*C^4*a^4 + 194481*A^4*c
^4 - 18*(25*B^2*C^2 - 126*A*C^3)*a^3*c + (625*B^4 - 6300*A*B^2*C + 23814*A
^2*C^2)*a^2*c^2 - 882*(25*A^2*B^2 - 126*A^3*C)*a*c^3)/(a^11*c^5)) + 30*B*C
*a + 210*A*B*c)/(a^5*c^2))*log((81*C^4*a^4 + 2268*A*C^3*a^3*c + 111132*A^3
*C*a*c^3 + 194481*A^4*c^4 - (625*B^4 - 23814*A^2*C^2)*a^2*c^2)*x + (5*B*a^
9*c^4*sqrt(-(81*C^4*a^4 + 194481*A^4*c^4 - 18*(25*B^2*C^2 - 126*A*C^3)*a^3
*c + (625*B^4 - 6300*A*B^2*C + 23814*A^2*C^2)*a^2*c^2 - 882*(25*A^2*B^2 -
126*A^3*C)*a*c^3)/(a^11*c^5)) + 27*C^3*a^6*c + 9261*A^3*a^3*c^4 - 3*(25*B^
2*C - 189*A*C^2)*a^5*c^2 - 21*(25*A*B^2 - 189*A^2*C)*a^4*c^3)*sqrt(-(a^5*c
^2*sqrt(-(81*C^4*a^4 + 194481*A^4*c^4 - 18*(25*B^2*C^2 - 126*A*C^3)*a^3*c
+ (625*B^4 - 6300*A*B^2*C + 23814*A^2*C^2)*a^2*c^2 - 882*(25*A^2*B^2 - 126
*A^3*C)*a*c^3)/(a^11*c^5)) + 30*B*C*a + 210*A*B*c)/(a^5*c^2))) - (a^2*c^3*
x^8 + 2*a^3*c^2*x^4 + a^4*c)*sqrt(-(a^5*c^2*sqrt(-(81*C^4*a^4 + 194481*A^4
*c^4 - 18*(25*B^2*C^2 - 126*A*C^3)*a^3*c + (625*B^4 - 6300*A*B^2*C + 23814
*A^2*C^2)*a^2*c^2 - 882*(25*A^2*B^2 - 126*A^3*C)*a*c^3)/(a^11*c^5)) + 30*B
*C*a + 210*A*B*c)/(a^5*c^2))*log((81*C^4*a^4 + 2268*A*C^3*a^3*c + 111132*A
^3*C*a*c^3 + 194481*A^4*c^4 - (625*B^4 - 23814*A^2*C^2)*a^2*c^2)*x - (5*B*
a^9*c^4*sqrt(-(81*C^4*a^4 + 194481*A^4*c^4 - 18*(25*B^2*C^2 - 126*A*C^3)*a
^3*c + (625*B^4 - 6300*A*B^2*C + 23814*A^2*C^2)*a^2*c^2 - 882*(25*A^2*B...
```

Sympy [A] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.53

$$\int \frac{A + Bx^2 + Cx^4}{(a + cx^4)^3}\,dx$$
$$= \operatorname{RootSum}\Big(268435456t^4a^{11}c^5 + t^2 \cdot \left(6881280ABa^6c^4 + 983040BCa^7c^3\right) + 194481A^4c^4 + 111132A^3Cac$$
$$+ \frac{9Bacx^3 + 5Bc^2x^7 + x^5 \cdot (7Ac^2 + Cac) + x(11Aac - 3Ca^2)}{32a^4c + 64a^3c^2x^4 + 32a^2c^3x^8}$$

input
```
integrate((C*x**4+B*x**2+A)/(c*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*c**5 + _t**2*(6881280*A*B*a**6*c**4 + 983040
*B*C*a**7*c**3) + 194481*A**4*c**4 + 111132*A**3*C*a*c**3 + 22050*A**2*B**
2*a*c**3 + 23814*A**2*C**2*a**2*c**2 + 6300*A*B**2*C*a**2*c**2 + 2268*A*C*
*3*a**3*c + 625*B**4*a**2*c**2 + 450*B**2*C**2*a**3*c + 81*C**4*a**4, Lamb
da(_t, _t*log(x + (-10485760*_t**3*B*a**9*c**4 + 1185408*_t*A**3*a**3*c**4
 + 508032*_t*A**2*C*a**4*c**3 - 201600*_t*A*B**2*a**4*c**3 + 72576*_t*A*C*
*2*a**5*c**2 - 28800*_t*B**2*C*a**5*c**2 + 3456*_t*C**3*a**6*c)/(194481*A*
*4*c**4 + 111132*A**3*C*a*c**3 + 23814*A**2*C**2*a**2*c**2 + 2268*A*C**3*a
**3*c - 625*B**4*a**2*c**2 + 81*C**4*a**4)))) + (9*B*a*c*x**3 + 5*B*c**2*x
**7 + x**5*(7*A*c**2 + C*a*c) + x*(11*A*a*c - 3*C*a**2))/(32*a**4*c + 64*a
**3*c**2*x**4 + 32*a**2*c**3*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.35

$$\int \frac{A + Bx^2 + Cx^4}{(a + cx^4)^3} \, dx = \frac{5 \, Bc^2x^7 + 9 \, Bacx^3 + (Cac + 7 \, Ac^2)x^5 - (3 \, Ca^2 - 11 \, Aac)x}{32 \, (a^2c^3x^8 + 2 \, a^3c^2x^4 + a^4c)} + \frac{\frac{2 \, \sqrt{2}(3 \, Ca\sqrt{c} + (5 \, B\sqrt{a} + 21 \, A\sqrt{c})c) \arctan\left(\frac{\sqrt{2}\left(2 \, \sqrt{c}x + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2 \, \sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2 \, \sqrt{2}(3 \, Ca\sqrt{c} + (5 \, B\sqrt{a} + 21 \, A\sqrt{c})c) \arctan\left(\frac{\sqrt{2}\left(2 \, \sqrt{c}x - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2 \, \sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}}{256 \, a}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*B*c^2*x^7 + 9*B*a*c*x^3 + (C*a*c + 7*A*c^2)*x^5 - (3*C*a^2 - 11*A*
a*c)*x)/(a^2*c^3*x^8 + 2*a^3*c^2*x^4 + a^4*c) + 1/256*(2*sqrt(2)*(3*C*a*sq
rt(c) + (5*B*sqrt(a) + 21*A*sqrt(c))*c)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))*sqrt(c)) + 2*sqrt(2)*(3*C*a*sqrt(c) + (5*B*sqrt(a) + 21*A*sqrt(c))*c)
*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*C*a*sqrt(c)
- (5*B*sqrt(a) - 21*A*sqrt(c))*c)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4
)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*C*a*sqrt(c) - (5*B*sqrt(a) -
 21*A*sqrt(c))*c)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(
a^(3/4)*c^(3/4)))/(a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,Cac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2+5\,(ac^3)^{\frac{3}{4}}\,B\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,Cac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2+5\,(ac^3)^{\frac{3}{4}}\,B\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,Cac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2-5\,(ac^3)^{\frac{3}{4}}\,B\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}$$
$$- \frac{\sqrt{2}\left(3\,(ac^3)^{\frac{1}{4}}\,Cac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2-5\,(ac^3)^{\frac{3}{4}}\,B\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}$$
$$+ \frac{5\,Bc^2x^7+Cacx^5+7\,Ac^2x^5+9\,Bacx^3-3\,Ca^2x+11\,Aacx}{32\,(cx^4+a)^2a^2c}$$

input
```
integrate((C*x^4+B*x^2+A)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(3*(a*c^3)^(1/4)*C*a*c + 21*(a*c^3)^(1/4)*A*c^2 + 5*(a*c^3)^
(3/4)*B)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*
c^3) + 1/128*sqrt(2)*(3*(a*c^3)^(1/4)*C*a*c + 21*(a*c^3)^(1/4)*A*c^2 + 5*(
a*c^3)^(3/4)*B)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a^3*c^3) + 1/256*sqrt(2)*(3*(a*c^3)^(1/4)*C*a*c + 21*(a*c^3)^(1/4)*A*c^
2 - 5*(a*c^3)^(3/4)*B)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c
^3) - 1/256*sqrt(2)*(3*(a*c^3)^(1/4)*C*a*c + 21*(a*c^3)^(1/4)*A*c^2 - 5*(a
*c^3)^(3/4)*B)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/
32*(5*B*c^2*x^7 + C*a*c*x^5 + 7*A*c^2*x^5 + 9*B*a*c*x^3 - 3*C*a^2*x + 11*A
*a*c*x)/((c*x^4 + a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 2546, normalized size of antiderivative = 9.64

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^2 + C*x^4)/(a + c*x^4)^3,x)
```

output
```
((9*B*x^3)/(32*a) + (x^5*(7*A*c + C*a))/(32*a^2) + (5*B*c*x^7)/(32*a^2) +
(x*(11*A*c - 3*C*a))/(32*a*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) - 2*atanh((9*C^
2*c*x*((25*B^2*(-a^11*c^5)^(1/2))/(16384*a^10*c^4) - (15*B*C)/(8192*a^4*c^
2) - (441*A^2*(-a^11*c^5)^(1/2))/(16384*a^11*c^3) - (105*A*B)/(8192*a^5*c)
 - (9*C^2*(-a^11*c^5)^(1/2))/(16384*a^9*c^5) - (63*A*C*(-a^11*c^5)^(1/2))/
(8192*a^10*c^4))^(1/2))/(128*((9261*A^3*(-a^11*c^5)^(1/2))/(16384*a^10) +
(45*B*C^2)/(16384*a^2) - (125*B^3*c)/(16384*a^3) + (2205*A^2*B*c^2)/(16384
*a^4) + (27*C^3*(-a^11*c^5)^(1/2))/(16384*a^7*c^3) - (525*A*B^2*(-a^11*c^5
)^(1/2))/(16384*a^9*c) + (567*A*C^2*(-a^11*c^5)^(1/2))/(16384*a^8*c^2) + (
3969*A^2*C*(-a^11*c^5)^(1/2))/(16384*a^9*c) - (75*B^2*C*(-a^11*c^5)^(1/2))
/(16384*a^8*c^2) + (315*A*B*C*c)/(8192*a^3))) - (25*B^2*c^2*x*((25*B^2*(-a
^11*c^5)^(1/2))/(16384*a^10*c^4) - (15*B*C)/(8192*a^4*c^2) - (441*A^2*(-a^
11*c^5)^(1/2))/(16384*a^11*c^3) - (105*A*B)/(8192*a^5*c) - (9*C^2*(-a^11*c
^5)^(1/2))/(16384*a^9*c^5) - (63*A*C*(-a^11*c^5)^(1/2))/(8192*a^10*c^4))^(
1/2))/(128*((9261*A^3*(-a^11*c^5)^(1/2))/(16384*a^9) + (45*B*C^2)/(16384*a
) - (125*B^3*c)/(16384*a^2) + (2205*A^2*B*c^2)/(16384*a^3) + (27*C^3*(-a^1
1*c^5)^(1/2))/(16384*a^6*c^3) - (525*A*B^2*(-a^11*c^5)^(1/2))/(16384*a^8*c
) + (567*A*C^2*(-a^11*c^5)^(1/2))/(16384*a^7*c^2) + (3969*A^2*C*(-a^11*c^5
)^(1/2))/(16384*a^8*c) - (75*B^2*C*(-a^11*c^5)^(1/2))/(16384*a^7*c^2) + (3
15*A*B*C*c)/(8192*a^2))) + (441*A^2*c^3*x*((25*B^2*(-a^11*c^5)^(1/2))/(...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 937, normalized size of antiderivative = 3.55

$$\int \frac{A+Bx^2+Cx^4}{(a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((C*x^4+B*x^2+A)/(c*x^4+a)^3,x)
```

output

```
( - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*b - 20*c**(1/4)*a**(3/4)*sqrt(2)*a
tan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))
*a*b*c*x**4 - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b*c**2*x**8 - 48*c**(3/4)*a**
(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*a**3 - 96*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4
)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*x**4 - 48*c**
(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**
(1/4)*a**(1/4)*sqrt(2)))*a*c**2*x**8 + 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((
c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2
*b + 20*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*b*c*x**4 + 10*c**(1/4)*a**(3/4)*sqrt
(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt
(2)))*b*c**2*x**8 + 48*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 96*c**(3/4)*a**(
1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(
1/4)*sqrt(2)))*a**2*c*x**4 + 48*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**2*x**8 +
5*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a...
```

3.13 $\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx$

Optimal result . . . 165
Mathematica [C] (verified) . . . 165
Rubi [A] (verified) . . . 166
Maple [C] (verified) . . . 168
Fricas [B] (verification not implemented) . . . 169
Sympy [F] . . . 169
Maxima [F] . . . 169
Giac [F] . . . 170
Mupad [F(-1)] . . . 170
Reduce [F] . . . 170

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \sqrt{\frac{1}{1+x^2}}\sqrt{1+x^2}E(\arctan(x)|2)$$

output
```
(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticE(x/(x^2+1)^(1/2),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \frac{x}{\sqrt{1-x^4}} - \frac{2}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},x^4\right)$$

input
```
Integrate[(1 - x^2)^2/(1 - x^4)^(3/2),x]
```

output
```
x/Sqrt[1 - x^4] - (2*x^3*Hypergeometric2F1[3/4, 3/2, 7/4, x^4])/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1388, 314, 25, 335, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{1-x^2}}{(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 314}$$

$$\frac{x\sqrt{1-x^2}}{\sqrt{x^2+1}} - \int -\frac{x^2}{\sqrt{1-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{x^2+1}}dx + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 335}$$

$$\int \frac{x^2}{\sqrt{1-x^4}}dx + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 836}$$

$$-\int \frac{1}{\sqrt{1-x^4}}dx + \int \frac{x^2+1}{\sqrt{1-x^4}}dx + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 762}$$

$$\int \frac{x^2+1}{\sqrt{1-x^4}}dx - \text{EllipticF}(\arcsin(x),-1) + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx - \text{EllipticF}(\arcsin(x),-1) + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 327}$$

$$-\text{EllipticF}(\arcsin(x),-1)+E(\arcsin(x)|-1)+\frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

input
```
Int[(1 - x^2)^2/(1 - x^4)^(3/2),x]
```

output
```
(x*Sqrt[1 - x^2])/Sqrt[1 + x^2] + EllipticE[ArcSin[x], -1] - EllipticF[Arc
Sin[x], -1]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

method	result	size
meijerg	$x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], x^4\right)+\frac{x^5 \operatorname{hypergeom}\left(\left[\frac{5}{4}, \frac{3}{2}\right],\left[\frac{9}{4}\right], x^4\right)}{5}-\frac{2 x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right],\left[\frac{7}{4}\right], x^4\right)}{3}$	41
risch	$-\frac{x(x^2-1)}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	57
elliptic	$\frac{(-x^2+1)x}{\sqrt{(-x^2+1)(x^2+1)}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	64
default	$\frac{x}{\sqrt{-x^4+1}}-\frac{x^3}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	65

input
```
int((-x^2+1)^2/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,3/2],[5/4],x^4)+1/5*x^5*hypergeom([5/4,3/2],[9/4],x^4)-2/
3*x^3*hypergeom([3/4,3/2],[7/4],x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \frac{(x^2+1)E(\arcsin(x)\,|-1) - (x^2+1)F(\arcsin(x)\,|-1) + \sqrt{-x^4+1}x}{x^2+1}$$

input
```
integrate((-x^2+1)^2/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
((x^2 + 1)*elliptic_e(arcsin(x), -1) - (x^2 + 1)*elliptic_f(arcsin(x), -1)
 + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \int \frac{(x-1)^2(x+1)^2}{(-(x-1)(x+1)(x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-x**2+1)**2/(-x**4+1)**(3/2),x)
```

output
```
Integral((x - 1)**2*(x + 1)**2/(-(x - 1)*(x + 1)*(x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \int \frac{(x^2-1)^2}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-x^2+1)^2/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^2/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \int \frac{(x^2-1)^2}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-x^2+1)^2/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^2/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \int \frac{(x^2-1)^2}{(1-x^4)^{3/2}}\,dx$$

input
```
int((x^2 - 1)^2/(1 - x^4)^(3/2),x)
```

output
```
int((x^2 - 1)^2/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(1-x^2)^2}{(1-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^4+2x^2+1}dx$$

input
```
int((-x^2+1)^2/(-x^4+1)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 1)/(x**4 + 2*x**2 + 1),x)
```

3.14 $\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx$

Optimal result . . . 171
Mathematica [A] (verified) . . . 171
Rubi [A] (verified) . . . 172
Maple [C] (verified) . . . 174
Fricas [B] (verification not implemented) . . . 174
Sympy [F] . . . 175
Maxima [F] . . . 175
Giac [F] . . . 176
Mupad [F(-1)] . . . 176
Reduce [F] . . . 176

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx = \sqrt{\frac{1}{1+x^2}}\sqrt{1+x^2}E(\arctan(x)|2)$$

output
```
(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticE(x/(x^2+1)^(1/2),2^(1/2))
```

Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx = E(\arcsin(x)|-1) - \frac{-x+x^3+\sqrt{1-x^4}\,\mathrm{EllipticF}(\arcsin(x),-1)}{\sqrt{1-x^4}}$$

input
```
Integrate[Sqrt[1 - x^4]/(1 + x^2)^2,x]
```

output
```
EllipticE[ArcSin[x], -1] - (-x + x^3 + Sqrt[1 - x^4]*EllipticF[ArcSin[x],
-1])/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1388, 314, 25, 335, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^4}}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{1-x^2}}{(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{314}$$

$$\frac{x\sqrt{1-x^2}}{\sqrt{x^2+1}} - \int -\frac{x^2}{\sqrt{1-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{25}$$

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{x^2+1}}dx + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{335}$$

$$\int \frac{x^2}{\sqrt{1-x^4}}dx + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{836}$$

$$-\int \frac{1}{\sqrt{1-x^4}}dx + \int \frac{x^2+1}{\sqrt{1-x^4}}dx + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{762}$$

$$\int \frac{x^2+1}{\sqrt{1-x^4}}dx - \text{EllipticF}(\arcsin(x),-1) + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx - \text{EllipticF}(\arcsin(x),-1) + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{327}$$

$$-\text{EllipticF}(\arcsin(x), -1) + E(\arcsin(x)|-1) + \frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

input
```
Int[Sqrt[1 - x^4]/(1 + x^2)^2,x]
```

output
```
(x*Sqrt[1 - x^2])/Sqrt[1 + x^2] + EllipticE[ArcSin[x], -1] - EllipticF[Arc
Sin[x], -1]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

method	result	size
risch	$-\frac{x(x^2-1)}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	57
default	$\frac{(-x^2+1)x}{\sqrt{(-x^2+1)(x^2+1)}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	64
elliptic	$\frac{(-x^2+1)x}{\sqrt{(-x^2+1)(x^2+1)}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	64

input
```
int((-x^4+1)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x*(x^2-1)/(-x^4+1)^(1/2)-(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ell
ipticF(x,I)-EllipticE(x,I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(13) = 26.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx=\frac{(x^2+1)E(\arcsin{(x)}\,|-1)-(x^2+1)F(\arcsin{(x)}\,|-1)+\sqrt{-x^4+1}x}{x^2+1}$$

input
```
integrate((-x^4+1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
((x^2 + 1)*elliptic_e(arcsin(x), -1) - (x^2 + 1)*elliptic_f(arcsin(x), -1)
 + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{(x^2+1)^2}\,dx$$

input
```
integrate((-x**4+1)**(1/2)/(x**2+1)**2,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/(x**2 + 1)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{-x^4+1}}{(x^2+1)^2}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 1)/(x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{-x^4+1}}{(x^2+1)^2}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 1)/(x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{1-x^4}}{(x^2+1)^2}\,dx$$

input
```
int((1 - x^4)^(1/2)/(x^2 + 1)^2,x)
```

output
```
int((1 - x^4)^(1/2)/(x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{-x^4+1}}{x^4+2x^2+1}dx$$

input
```
int((-x^4+1)^(1/2)/(x^2+1)^2,x)
```

output
```
int(sqrt( - x**4 + 1)/(x**4 + 2*x**2 + 1),x)
```

3.15 $\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

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Reduce [F] . . . 182

Optimal result

Integrand size = 26, antiderivative size = 25

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \sqrt{\frac{1}{1+x^2}}\sqrt{1+x^2}E(\arctan(x)|2)$$

output
```
(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticE(x/(x^2+1)^(1/2),2^(1/2))
```

Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = E(\arcsin(x)|-1) - \frac{-x+x^3+\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{1-x^4}}$$

input
```
Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
EllipticE[ArcSin[x], -1] - (-x + x^3 + Sqrt[1 - x^4]*EllipticF[ArcSin[x],
-1])/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1388, 314, 25, 335, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2+1)\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{1-x^2}}{(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 314}$$

$$\frac{x\sqrt{1-x^2}}{\sqrt{x^2+1}}-\int -\frac{x^2}{\sqrt{1-x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{x^2+1}}dx+\frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 335}$$

$$\int \frac{x^2}{\sqrt{1-x^4}}dx+\frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 836}$$

$$-\int \frac{1}{\sqrt{1-x^4}}dx+\int \frac{x^2+1}{\sqrt{1-x^4}}dx+\frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 762}$$

$$\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\text{EllipticF}(\arcsin(x),-1)+\frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx-\text{EllipticF}(\arcsin(x),-1)+\frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 327}$$

$$-\text{EllipticF}(\arcsin(x),-1)+E(\arcsin(x)|-1)+\frac{\sqrt{1-x^2}x}{\sqrt{x^2+1}}$$

input
```
Int[(1 - x^2)/((1 + x^2)*Sqrt[1 - x^4]),x]
```

output
```
(x*Sqrt[1 - x^2])/Sqrt[1 + x^2] + EllipticE[ArcSin[x], -1] - EllipticF[Arc
Sin[x], -1]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 335
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(e*x)^m*(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]
))
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

method	result	size
risch	$-\frac{x(x^2-1)}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	57
default	$\frac{(-x^2+1)x}{\sqrt{(-x^2+1)(x^2+1)}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	64
elliptic	$\frac{(-x^2+1)x}{\sqrt{(-x^2+1)(x^2+1)}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	64

input
```
int((-x^2+1)/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*(x^2-1)/(-x^4+1)^(1/2)-(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(Ell
ipticF(x,I)-EllipticE(x,I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{(x^2+1)E(\arcsin(x)\,|-1)-(x^2+1)F(\arcsin(x)\,|-1)+\sqrt{-x^4+1}x}{x^2+1}$$

input
```
integrate((-x^2+1)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
((x^2 + 1)*elliptic_e(arcsin(x), -1) - (x^2 + 1)*elliptic_f(arcsin(x), -1)
 + sqrt(-x^4 + 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\int \frac{x^2}{x^2\sqrt{1-x^4}+\sqrt{1-x^4}}\,dx - \int \left(-\frac{1}{x^2\sqrt{1-x^4}+\sqrt{1-x^4}}\right)dx$$

input
```
integrate((-x**2+1)/(x**2+1)/(-x**4+1)**(1/2),x)
```

output
```
-Integral(x**2/(x**2*sqrt(1 - x**4) + sqrt(1 - x**4)), x) - Integral(-1/(x
**2*sqrt(1 - x**4) + sqrt(1 - x**4)), x)
```

Maxima [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int -\frac{x^2-1}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = -\int \frac{x^2-1}{(x^2+1)\sqrt{1-x^4}}\,dx$$

input
```
int(-(x^2 - 1)/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

output
```
-int((x^2 - 1)/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1-x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^4+2x^2+1}dx$$

input
```
int((-x^2+1)/(x^2+1)/(-x^4+1)^(1/2),x)
```

output
```
int(sqrt( - x**4 + 1)/(x**4 + 2*x**2 + 1),x)
```

3.16 $\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx$

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Rubi [B] (verified) 184
Maple [C] (verified) 187
Fricas [C] (verification not implemented) 187
Sympy [F] 188
Maxima [F] 188
Giac [F] 188
Mupad [F(-1)] 189
Reduce [F] 189

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = -\frac{\sqrt{1-x^2}E(\arctan(x)|2)}{\sqrt{\frac{1}{1+x^2}}\sqrt{-1+x^4}}$$

output
```
-(-x^2+1)^(1/2)*EllipticE(x/(x^2+1)^(1/2),2^(1/2))/(1/(x^2+1))^(1/2)/(x^4-
1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = \frac{x\left(-3+2x^2\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},x^4\right)\right)}{3\sqrt{-1+x^4}}$$

input
```
Integrate[(1 - x^2)^2/(-1 + x^4)^(3/2),x]
```

output
```
(x*(-3 + 2*x^2*Sqrt[1 - x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, x^4]))/(3*Sq
rt[-1 + x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. 2(37) = 74.

Time = 0.56 (sec) , antiderivative size = 217, normalized size of antiderivative = 5.86, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1396, 314, 25, 344, 835, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^2)^2}{(x^4-1)^{3/2}}\,dx$$

$$\Big\downarrow \; 1396$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\int \frac{\sqrt{1-x^2}}{(-x^2-1)^{3/2}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \; 314$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\left(\int -\frac{x^2}{\sqrt{-x^2-1}\sqrt{1-x^2}}dx-\frac{x\sqrt{1-x^2}}{\sqrt{-x^2-1}}\right)}{\sqrt{x^4-1}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\left(-\int \frac{x^2}{\sqrt{-x^2-1}\sqrt{1-x^2}}dx-\frac{\sqrt{1-x^2}x}{\sqrt{-x^2-1}}\right)}{\sqrt{x^4-1}}$$

$$\Big\downarrow \; 344$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\left(-\frac{\sqrt{x^4-1}\int \frac{x^2}{\sqrt{x^4-1}}dx}{\sqrt{-x^2-1}\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}x}{\sqrt{-x^2-1}}\right)}{\sqrt{x^4-1}}$$

$$\Big\downarrow \; 835$$

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\left(-\frac{\sqrt{x^4-1}\left(\int \frac{1}{\sqrt{x^4-1}}dx-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)}{\sqrt{-x^2-1}\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}x}{\sqrt{-x^2-1}}\right)}{\sqrt{x^4-1}}$$

$\big\downarrow$ 763

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\left(-\frac{\sqrt{x^4-1}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)}{\sqrt{-x^2-1}\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}x}{\sqrt{-x^2-1}}\right)}{\sqrt{x^4-1}}$$

$\big\downarrow$ 1499

$$\frac{\sqrt{-x^2-1}\sqrt{1-x^2}\left(-\frac{\sqrt{x^4-1}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}\right)}{\sqrt{-x^2-1}\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}x}{\sqrt{-x^2-1}}\right)}{\sqrt{x^4-1}}$$

input

```
Int[(1 - x^2)^2/(-1 + x^4)^(3/2),x]
```

output

```
(Sqrt[-1 - x^2]*Sqrt[1 - x^2]*(-((x*Sqrt[1 - x^2])/Sqrt[-1 - x^2]) - (Sqrt
[-1 + x^4]*((x*(1 + x^2))/Sqrt[-1 + x^4] - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1
+ x^2]*EllipticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4]
+ (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2
]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])))/(Sqrt[-1 - x^2]*Sqrt[1 - x^2])))/Sqrt
[-1 + x^4]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

method	result
risch	$\frac{x(x^2-1)}{\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\text{EllipticF}(ix,i)-\text{EllipticE}(ix,i))}{\sqrt{x^4-1}}$
elliptic	$\frac{(x^2-1)x}{\sqrt{(x^2-1)(x^2+1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\text{EllipticF}(ix,i)-\text{EllipticE}(ix,i))}{\sqrt{x^4-1}}$
default	$-\frac{x}{\sqrt{x^4-1}}+\frac{x^3}{\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\text{EllipticF}(ix,i)-\text{EllipticE}(ix,i))}{\sqrt{x^4-1}}$
meijerg	$\frac{(-\,\text{signum}(x^4-1))^{\frac{3}{2}}x\,\text{hypergeom}([\frac{1}{4},\frac{3}{2}],[\frac{5}{4}],x^4)}{\text{signum}(x^4-1)^{\frac{3}{2}}}+\frac{(-\,\text{signum}(x^4-1))^{\frac{3}{2}}x^5\,\text{hypergeom}([\frac{5}{4},\frac{3}{2}],[\frac{9}{4}],x^4)}{5\,\text{signum}(x^4-1)^{\frac{3}{2}}}-\frac{2(-\,\text{signum}(x^4-1))}{3\,\text{si}}$

input
```
int((-x^2+1)^2/(x^4-1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*(x^2-1)/(x^4-1)^(1/2)+I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Elli
pticF(I*x,I)-EllipticE(I*x,I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = \frac{(i\,x^2+i)E(\arcsin{(x)}\mid -1)+(-i\,x^2-i)F(\arcsin{(x)}\mid -1)+\sqrt{x^4-1}x}{x^2+1}$$

input
```
integrate((-x^2+1)^2/(x^4-1)^(3/2),x, algorithm="fricas")
```

output
```
((I*x^2 + I)*elliptic_e(arcsin(x), -1) + (-I*x^2 - I)*elliptic_f(arcsin(x)
, -1) + sqrt(x^4 - 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = \int \frac{(x-1)^2\,(x+1)^2}{((x-1)\,(x+1)\,(x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((-x**2+1)**2/(x**4-1)**(3/2),x)
```

output
```
Integral((x - 1)**2*(x + 1)**2/((x - 1)*(x + 1)*(x**2 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = \int \frac{(x^2-1)^2}{(x^4-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-x^2+1)^2/(x^4-1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)^2/(x^4 - 1)^(3/2), x)
```

Giac [F]

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = \int \frac{(x^2-1)^2}{(x^4-1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-x^2+1)^2/(x^4-1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^2 - 1)^2/(x^4 - 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = \int \frac{(x^2-1)^2}{(x^4-1)^{3/2}}\,dx$$

input
```
int((x^2 - 1)^2/(x^4 - 1)^(3/2),x)
```

output
```
int((x^2 - 1)^2/(x^4 - 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{(1-x^2)^2}{(-1+x^4)^{3/2}}\,dx = \int \frac{\sqrt{x^4-1}}{x^4+2x^2+1}dx$$

input
```
int((-x^2+1)^2/(x^4-1)^(3/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**4 + 2*x**2 + 1),x)
```

3.17 $\int \frac{\sqrt{-1+x^4}}{\left(1+x^2\right)^2}\,dx$

Optimal result . . . 190
Mathematica [A] (verified) . . . 190
Rubi [B] (verified) . . . 191
Maple [C] (verified) . . . 193
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Sympy [F] . . . 194
Maxima [F] . . . 195
Giac [F] . . . 195
Mupad [F(-1)] . . . 195
Reduce [F] . . . 196

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{\sqrt{-1+x^4}}{\left(1+x^2\right)^2}\,dx=\frac{\sqrt{-1+x^4}E(\arctan(x)|2)}{\sqrt{\frac{1-x^2}{1+x^2}}\left(1+x^2\right)}$$

output
```
(x^4-1)^(1/2)*EllipticE(x/(x^2+1)^(1/2),2^(1/2))/((-x^2+1)/(x^2+1))^(1/2)/
(x^2+1)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{-1+x^4}}{\left(1+x^2\right)^2}\,dx =\frac{-x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)+\sqrt{1-x^4}\operatorname{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

input
```
Integrate[Sqrt[-1 + x^4]/(1 + x^2)^2,x]
```

output
```
(-x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + Sqrt[1 - x^4]*Ellipti
cF[ArcSin[x], -1])/Sqrt[-1 + x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 204 vs. $2(40) = 80$.

Time = 0.50 (sec) , antiderivative size = 204, normalized size of antiderivative = 5.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1396, 314, 344, 835, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4-1}}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{ 1396}$$

$$\frac{\sqrt{x^4-1}\int \frac{\sqrt{x^2-1}}{(x^2+1)^{3/2}}dx}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 314}$$

$$\frac{\sqrt{x^4-1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\int \frac{x^2}{\sqrt{x^2-1}\sqrt{x^2+1}}dx\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 344}$$

$$\frac{\sqrt{x^4-1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\int \frac{x^2}{\sqrt{x^4-1}}dx}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 835}$$

$$\frac{\sqrt{x^4-1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\left(\int \frac{1}{\sqrt{x^4-1}}dx-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{ 763}$$

$$\frac{\sqrt{x^4-1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\int\frac{1-x^2}{\sqrt{x^4-1}}dx\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

$$\Big\downarrow \text{1499}$$

$$\frac{\sqrt{x^4-1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}$$

input
```
Int[Sqrt[-1 + x^4]/(1 + x^2)^2,x]
```

output
```
(Sqrt[-1 + x^4]*((x*Sqrt[-1 + x^2])/Sqrt[1 + x^2] - (Sqrt[-1 + x^4]*((x*(1
 + x^2))/Sqrt[-1 + x^4] - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticE[
ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] + (Sqrt[-1 + x^2]
*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2
]*Sqrt[-1 + x^4])))/(Sqrt[-1 + x^2]*Sqrt[1 + x^2])))/(Sqrt[-1 + x^2]*Sqrt[
1 + x^2])
```

Defintions of rubi rules used

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

method	result	size
risch	$\frac{x(x^2-1)}{\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	59
default	$\frac{(x^2-1)x}{\sqrt{(x^2-1)(x^2+1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	65
elliptic	$\frac{(x^2-1)x}{\sqrt{(x^2-1)(x^2+1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	65

input
```
int((x^4-1)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(x^2-1)/(x^4-1)^(1/2)+I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Elli
pticF(I*x,I)-EllipticE(I*x,I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{-1+x^4}}{(1+x^2)^2}\,dx = \frac{(i\,x^2+i)E(\arcsin{(x)}\mid -1)+(-i\,x^2-i)F(\arcsin{(x)}\mid -1)+\sqrt{x^4-1}x}{x^2+1}$$

input
```
integrate((x^4-1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
((I*x^2 + I)*elliptic_e(arcsin(x), -1) + (-I*x^2 - I)*elliptic_f(arcsin(x)
, -1) + sqrt(x^4 - 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{(x-1)(x+1)(x^2+1)}}{(x^2+1)^2}\,dx$$

input
```
integrate((x**4-1)**(1/2)/(x**2+1)**2,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1)*(x**2 + 1))/(x**2 + 1)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4-1}}{(x^2+1)^2}\,dx$$

input
```
integrate((x^4-1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 - 1)/(x^2 + 1)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{-1+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4-1}}{(x^2+1)^2}\,dx$$

input
```
integrate((x^4-1)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(x^4 - 1)/(x^2 + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4-1}}{(x^2+1)^2}\,dx$$

input
```
int((x^4 - 1)^(1/2)/(x^2 + 1)^2,x)
```

output
```
int((x^4 - 1)^(1/2)/(x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{-1+x^4}}{(1+x^2)^2}\,dx = \int \frac{\sqrt{x^4-1}}{x^4+2x^2+1}dx$$

input
```
int((x^4-1)^(1/2)/(x^2+1)^2,x)
```

output
```
int(sqrt(x**4 - 1)/(x**4 + 2*x**2 + 1),x)
```

3.18 $\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx$

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Mathematica [A] (verified) 197
Rubi [B] (verified) 198
Maple [C] (verified) 200
Fricas [C] (verification not implemented) 201
Sympy [F] 201
Maxima [F] 202
Giac [F] 202
Mupad [F(-1)] 202
Reduce [F] 203

Optimal result

Integrand size = 22, antiderivative size = 41

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx=-\frac{(1-x^2)\,E(\arctan(x)|2)}{\sqrt{\frac{1-x^2}{1+x^2}}\sqrt{-1+x^4}}$$

output
```
-(-x^2+1)*EllipticE(x/(x^2+1)^(1/2),2^(1/2))/((-x^2+1)/(x^2+1))^(1/2)/(x^4
-1)^(1/2)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{-x+x^3-\sqrt{1-x^4}E(\arcsin(x)|-1)+\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

input
```
Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

output
```
(-x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + Sqrt[1 - x^4]*Ellipti
cF[ArcSin[x], -1])/Sqrt[-1 + x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 204 vs. $2(41) = 82$.

Time = 0.50 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1396, 314, 344, 835, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{(x^2+1)\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1396}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\int \frac{\sqrt{x^2-1}}{(x^2+1)^{3/2}}dx}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{314}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\int \frac{x^2}{\sqrt{x^2-1}\sqrt{x^2+1}}dx\right)}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{344}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\int \frac{x^2}{\sqrt{x^4-1}}dx}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{835}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\left(\int \frac{1}{\sqrt{x^4-1}}dx-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{763}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\int\frac{1-x^2}{\sqrt{x^4-1}}dx\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^4-1}}$$

$$\Big\downarrow \text{1499}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\left(\frac{x\sqrt{x^2-1}}{\sqrt{x^2+1}}-\frac{\sqrt{x^4-1}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{x\left(x^2+1\right)}{\sqrt{x^4-1}}\right)}{\sqrt{x^2-1}\sqrt{x^2+1}}\right)}{\sqrt{x^4-1}}$$

input
```
Int[(-1 + x^2)/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

output
```
(Sqrt[-1 + x^2]*Sqrt[1 + x^2]*((x*Sqrt[-1 + x^2])/Sqrt[1 + x^2] - (Sqrt[-1
 + x^4]*((x*(1 + x^2))/Sqrt[-1 + x^4] - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x
^2]*EllipticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] + (
Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]],
 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])))/(Sqrt[-1 + x^2]*Sqrt[1 + x^2])))/Sqrt[-1
 + x^4]
```

Defintions of rubi rules used

rule 314
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] + Simp[1/(2*a*
(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*p + 3) + d
*(2*(p + q + 1) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, 2, p, q,
 x]
```

rule 344
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]*((c + d*x^2)^FracPart[p]/(a*c
+ b*d*x^4)^FracPart[p])   Int[(e*x)^m*(a*c + b*d*x^4)^p, x], x] /; FreeQ[{a
, b, c, d, e, m, p}, x] && EqQ[b*c + a*d, 0] &&  !IntegerQ[p]
```

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 1396
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_)*((d_) + (e_.)*(x_)^(n_))^(q_.), x
_Symbol] :> Simp[(a + c*x^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d
+ c*(x^n/e))^FracPart[p])   Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p,
x], x] /; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*
e^2, 0] &&  !IntegerQ[p] &&  !(EqQ[q, 1] && EqQ[n, 2])
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

method	result	size
risch	$\frac{x(x^2-1)}{\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	59
default	$\frac{(x^2-1)x}{\sqrt{(x^2-1)(x^2+1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	65
elliptic	$\frac{(x^2-1)x}{\sqrt{(x^2-1)(x^2+1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	65

input
```
int((x^2-1)/(x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(x^2-1)/(x^4-1)^(1/2)+I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*(Elli
pticF(I*x,I)-EllipticE(I*x,I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{(i\,x^2+i)E(\arcsin{(x)}\mid -1)+(-i\,x^2-i)F(\arcsin{(x)}\mid -1)+\sqrt{x^4-1}x}{x^2+1}$$

input
```
integrate((x^2-1)/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
((I*x^2 + I)*elliptic_e(arcsin(x), -1) + (-I*x^2 - I)*elliptic_f(arcsin(x)
, -1) + sqrt(x^4 - 1)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{(x-1)(x+1)}{\sqrt{(x-1)(x+1)(x^2+1)}(x^2+1)}\,dx$$

input
```
integrate((x**2-1)/(x**2+1)/(x**4-1)**(1/2),x)
```

output
```
Integral((x - 1)*(x + 1)/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2-1}{\sqrt{x^4-1}(x^2+1)}\,dx$$

input
```
integrate((x^2-1)/(x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2-1}{\sqrt{x^4-1}(x^2+1)}\,dx$$

input
```
integrate((x^2-1)/(x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 - 1)/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{x^2-1}{(x^2+1)\sqrt{x^4-1}}\,dx$$

input
```
int((x^2 - 1)/((x^2 + 1)*(x^4 - 1)^(1/2)),x)
```

output
```
int((x^2 - 1)/((x^2 + 1)*(x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^4+2x^2+1}dx$$

input
```
int((x^2-1)/(x^2+1)/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**4 + 2*x**2 + 1),x)
```

3.19 $\int (A+Bx^2)(a+bx^4)^p\,dx$

Optimal result . . . 204
Mathematica [A] (verified) . . . 204
Rubi [A] (verified) . . . 205
Maple [F] . . . 206
Fricas [F] . . . 206
Sympy [C] (verification not implemented) . . . 207
Maxima [F] . . . 207
Giac [F] . . . 208
Mupad [F(-1)] . . . 208
Reduce [F] . . . 208

Optimal result

Integrand size = 17, antiderivative size = 96

$$\int (A+Bx^2)(a+bx^4)^p\,dx = Ax(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}Bx^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

output
```
A*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*B*
x^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (A+Bx^2)(a+bx^4)^p\,dx = \frac{1}{3}x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(3A\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+Bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(A + B*x^2)*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(3*A*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + B*x^
2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx^2) (a + bx^4)^p \, dx$$

$$\Big\downarrow \text{ 1516}$$

$$\int \left(A(a + bx^4)^p + Bx^2 (a + bx^4)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$Ax(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -\frac{bx^4}{a}\right) + \frac{1}{3}Bx^3(a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -\frac{bx^4}{a}\right)$$

input
```
Int[(A + B*x^2)*(a + b*x^4)^p,x]
```

output
```
(A*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*
x^4)/a)^p + (B*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)
/a)])/(3*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int (B\,x^2+A)\,(b\,x^4+a)^p\,dx$$

input
```
int((B*x^2+A)*(b*x^4+a)^p,x)
```

output
```
int((B*x^2+A)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (A+Bx^2)\,(a+bx^4)^p\;dx=\int (Bx^2+A)\,(bx^4+a)^p\,dx$$

input
```
integrate((B*x^2+A)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (A+Bx^2)(a+bx^4)^p\,dx = \frac{Aa^p x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{Ba^p x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**4+a)**p,x)
```

output
```
A*a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + B*a**p*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int (A+Bx^2)(a+bx^4)^p\,dx = \int (Bx^2+A)(bx^4+a)^p\,dx$$

input
```
integrate((B*x^2+A)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(A+Bx^2\right)\left(a+bx^4\right)^p\,dx=\int \left(Bx^2+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((B*x^2+A)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx^2\right)\left(a+bx^4\right)^p\,dx=\int \left(B\,x^2+A\right)\,\left(b\,x^4+a\right)^p\,dx$$

input
```
int((A + B*x^2)*(a + b*x^4)^p,x)
```

output
```
int((A + B*x^2)*(a + b*x^4)^p, x)
```

Reduce [F]

$$\int \left(A+Bx^2\right)\left(a+bx^4\right)^p\,dx$$
$$=\frac{4(b\,x^4+a)^p\,apx+3(b\,x^4+a)^p\,ax+4(b\,x^4+a)^p\,bp\,x^3+(b\,x^4+a)^p\,b\,x^3+256\left(\int \frac{(b\,x^4+a)^p}{16b\,p^2x^4+16bp\,x^4+3b\,x^4+16a}\right.}{}$$

input
```
int((B*x^2+A)*(b*x^4+a)^p,x)
```

output
```
(4*(a + b*x**4)**p*a*p*x + 3*(a + b*x**4)**p*a*x + 4*(a + b*x**4)**p*b*p*x
**3 + (a + b*x**4)**p*b*x**3 + 256*int((a + b*x**4)**p/(16*a*p**2 + 16*a*p
 + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a**2*p**4 + 448*int((
a + b*x**4)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 +
3*b*x**4),x)*a**2*p**3 + 240*int((a + b*x**4)**p/(16*a*p**2 + 16*a*p + 3*a
 + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a**2*p**2 + 36*int((a + b*x
**4)**p/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**
4),x)*a**2*p + 256*int(((a + b*x**4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a +
16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a*b*p**4 + 320*int(((a + b*x**
4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b
*x**4),x)*a*b*p**3 + 112*int(((a + b*x**4)**p*x**2)/(16*a*p**2 + 16*a*p +
3*a + 16*b*p**2*x**4 + 16*b*p*x**4 + 3*b*x**4),x)*a*b*p**2 + 12*int(((a +
b*x**4)**p*x**2)/(16*a*p**2 + 16*a*p + 3*a + 16*b*p**2*x**4 + 16*b*p*x**4
+ 3*b*x**4),x)*a*b*p)/(16*p**2 + 16*p + 3)
```

3.20 $\int (a+bx^4)^p (A+Bx^2+Cx^4)\,dx$

Optimal result . . . 210
Mathematica [A] (verified) . . . 211
Rubi [A] (verified) . . . 211
Maple [F] . . . 212
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Sympy [C] (verification not implemented) . . . 213
Maxima [F] . . . 214
Giac [F] . . . 214
Mupad [F(-1)] . . . 214
Reduce [F] . . . 215

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int (a+bx^4)^p (A+Bx^2+Cx^4)\,dx = \frac{Cx(a+bx^4)^{1+p}}{b(5+4p)} + \left(A-\frac{aC}{5b+4bp}\right)x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + \frac{1}{3}Bx^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

output
```
C*x*(b*x^4+a)^(p+1)/b/(5+4*p)+(A-a*C/(4*b*p+5*b))*x*(b*x^4+a)^p*hypergeom(
[1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*B*x^3*(b*x^4+a)^p*hypergeom(
[3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.75

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\frac{1}{15}x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(15A\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+3Cx^4\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(a + b*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(a + b*x^4)^p*(15*A*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + 5*B
*x^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*C*x^4*Hypergeometri
c2F1[5/4, -p, 9/4, -((b*x^4)/a)]))/(15*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(A\left(a+bx^4\right)^p+Bx^2\left(a+bx^4\right)^p+Cx^4\left(a+bx^4\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$Ax(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{3}Bx^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+\frac{1}{5}Cx^5(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(a + b*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(A*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*
x^4)/a)^p + (B*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)
/a)])/(3*(1 + (b*x^4)/a)^p) + (C*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4,
-p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p}\left(C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((b*x^4+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
int((b*x^4+a)^p*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 41.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\frac{Aa^p x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{Ba^p x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{Ca^p x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**p*(C*x**4+B*x**2+A),x)
```

output
```
A*a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + B*a**p*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4)) + C*a**p*x**5*gamma(5/4)*hyper((5/4, -p), (9/
4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(b\,x^4+a\right)^p\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a + b*x^4)^p*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + b*x^4)^p*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (a+bx^4)^p (A+Bx^2+Cx^4)\, dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
(16*(a + b*x**4)**p*a*b*p**2*x + 32*(a + b*x**4)**p*a*b*p*x + 15*(a + b*x*
*4)**p*a*b*x + 16*(a + b*x**4)**p*a*c*p**2*x + 12*(a + b*x**4)**p*a*c*p*x
+ 16*(a + b*x**4)**p*b**2*p**2*x**3 + 24*(a + b*x**4)**p*b**2*p*x**3 + 5*(
a + b*x**4)**p*b**2*x**3 + 16*(a + b*x**4)**p*b*c*p**2*x**5 + 16*(a + b*x*
*4)**p*b*c*p*x**5 + 3*(a + b*x**4)**p*b*c*x**5 + 4096*int((a + b*x**4)**p/
(64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4
 + 92*b*p*x**4 + 15*b*x**4),x)*a**2*b*p**6 + 17408*int((a + b*x**4)**p/(64
*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4 +
92*b*p*x**4 + 15*b*x**4),x)*a**2*b*p**5 + 28160*int((a + b*x**4)**p/(64*a*
p**3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*
b*p*x**4 + 15*b*x**4),x)*a**2*b*p**4 + 21376*int((a + b*x**4)**p/(64*a*p**
3 + 144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p
*x**4 + 15*b*x**4),x)*a**2*b*p**3 + 7440*int((a + b*x**4)**p/(64*a*p**3 +
144*a*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**
4 + 15*b*x**4),x)*a**2*b*p**2 + 900*int((a + b*x**4)**p/(64*a*p**3 + 144*a
*p**2 + 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 1
5*b*x**4),x)*a**2*b*p - 1024*int((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 +
 92*a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**
4),x)*a**2*c*p**5 - 3072*int((a + b*x**4)**p/(64*a*p**3 + 144*a*p**2 + 92*
a*p + 15*a + 64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4...
```

3.21 $\int (a+bx^4)^p (A+Bx^2+Cx^4+Dx^6)\, dx$

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Optimal result

Integrand size = 27, antiderivative size = 176

$$\int (a+bx^4)^p (A+Bx^2+Cx^4+Dx^6)\, dx = \frac{Cx(a+bx^4)^{1+p}}{b(5+4p)} + \frac{Dx^3(a+bx^4)^{1+p}}{b(7+4p)} + \left(A-\frac{aC}{5b+4bp}\right)x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + \frac{1}{3}\left(B-\frac{3aD}{7b+4bp}\right)x^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

output

```
C*x*(b*x^4+a)^(p+1)/b/(5+4*p)+D*x^3*(b*x^4+a)^(p+1)/b/(7+4*p)+(A-a*C/(4*b*
p+5*b))*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+
1/3*(B-3*a*D/(4*b*p+7*b))*x^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4
/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int (a+bx^4)^p\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\frac{1}{105}x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(105A\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+35Bx^2\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+21Cx^4\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+15Dx^6\,\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(a + b*x^4)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(a + b*x^4)^p*(105*A*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + 35
*B*x^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 21*C*x^4*Hypergeome
tric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 15*D*x^6*Hypergeometric2F1[7/4, -p,
11/4, -((b*x^4)/a)]))/(105*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p\left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$\Big\downarrow \ 2432$$
$$\int \left(A(a+bx^4)^p+Bx^2(a+bx^4)^p+Cx^4(a+bx^4)^p+Dx^6(a+bx^4)^p\right)dx$$

$$\downarrow \text{2009}$$

$$Ax(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + \frac{1}{3}Bx^3(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right) + \frac{1}{5}Cx^5(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right) + \frac{1}{7}Dx^7(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(a + b*x^4)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(A*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*
x^4)/a)^p + (B*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)
/a)])/(3*(1 + (b*x^4)/a)^p) + (C*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4,
-p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p) + (D*x^7*(a + b*x^4)^p*Hyper
geometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p} \left(Dx^{6}+C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int (a+bx^4)^p (A+Bx^2+Cx^4+Dx^6)\ dx = \int (Dx^6+Cx^4+Bx^2+A)(bx^4+a)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 72.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

$$\int (a+bx^4)^p (A+Bx^2+Cx^4+Dx^6)\ dx = \frac{Aa^p x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{Ba^p x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{Ca^p x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{Da^p x^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix}\frac{7}{4},-p\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((b*x**4+a)**p*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
A*a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + B*a**p*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(7/4)) + C*a**p*x**5*gamma(5/4)*hyper((5/4, -p), (9/
4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + D*a**p*x**7*gamma(7/4)*hyp
er((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4))
```

Maxima [F]

$$\int (a+bx^4)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (a+bx^4)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx = \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \int \left(b\,x^4+a\right)^p \left(A+B\,x^2+C\,x^4+x^6\,D\right) \, dx$$

input
```
int((a + b*x^4)^p*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((a + b*x^4)^p*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) \, dx = \text{too large to display}$$

input
```
int((b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output

```
(64*(a + b*x**4)**p*a*b*p**3*x + 240*(a + b*x**4)**p*a*b*p**2*x + 284*(a +
 b*x**4)**p*a*b*p*x + 105*(a + b*x**4)**p*a*b*x + 64*(a + b*x**4)**p*a*c*p
**3*x + 160*(a + b*x**4)**p*a*c*p**2*x + 84*(a + b*x**4)**p*a*c*p*x + 64*(
a + b*x**4)**p*a*d*p**3*x**3 + 96*(a + b*x**4)**p*a*d*p**2*x**3 + 20*(a +
b*x**4)**p*a*d*p*x**3 + 64*(a + b*x**4)**p*b**2*p**3*x**3 + 208*(a + b*x**
4)**p*b**2*p**2*x**3 + 188*(a + b*x**4)**p*b**2*p*x**3 + 35*(a + b*x**4)**
p*b**2*x**3 + 64*(a + b*x**4)**p*b*c*p**3*x**5 + 176*(a + b*x**4)**p*b*c*p
**2*x**5 + 124*(a + b*x**4)**p*b*c*p*x**5 + 21*(a + b*x**4)**p*b*c*x**5 +
64*(a + b*x**4)**p*b*d*p**3*x**7 + 144*(a + b*x**4)**p*b*d*p**2*x**7 + 92*
(a + b*x**4)**p*b*d*p*x**7 + 15*(a + b*x**4)**p*b*d*x**7 + 65536*int((a +
b*x**4)**p/(256*a*p**4 + 1024*a*p**3 + 1376*a*p**2 + 704*a*p + 105*a + 256
*b*p**4*x**4 + 1024*b*p**3*x**4 + 1376*b*p**2*x**4 + 704*b*p*x**4 + 105*b*
x**4),x)*a**2*b*p**8 + 507904*int((a + b*x**4)**p/(256*a*p**4 + 1024*a*p**
3 + 1376*a*p**2 + 704*a*p + 105*a + 256*b*p**4*x**4 + 1024*b*p**3*x**4 + 1
376*b*p**2*x**4 + 704*b*p*x**4 + 105*b*x**4),x)*a**2*b*p**7 + 1626112*int(
(a + b*x**4)**p/(256*a*p**4 + 1024*a*p**3 + 1376*a*p**2 + 704*a*p + 105*a
+ 256*b*p**4*x**4 + 1024*b*p**3*x**4 + 1376*b*p**2*x**4 + 704*b*p*x**4 + 1
05*b*x**4),x)*a**2*b*p**6 + 2771968*int((a + b*x**4)**p/(256*a*p**4 + 1024
*a*p**3 + 1376*a*p**2 + 704*a*p + 105*a + 256*b*p**4*x**4 + 1024*b*p**3*x*
*4 + 1376*b*p**2*x**4 + 704*b*p*x**4 + 105*b*x**4),x)*a**2*b*p**5 + 269...
```

3.22 $\int (a-cx^4)^3 (A+Bx^4+Cx^8) \, dx$

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Optimal result

Integrand size = 23, antiderivative size = 106

$$\int (a-cx^4)^3 \left(A+Bx^4+Cx^8\right) \, dx = a^3Ax+\frac{1}{5}a^2(aB-3Ac)x^5 - \frac{1}{9}a\left(3aBc-3Ac^2-a^2C\right)x^9 + \frac{1}{13}c\left(3aBc-Ac^2-3a^2C\right)x^{13} - \frac{1}{17}c^2(Bc-3aC)x^{17}-\frac{1}{21}c^3Cx^{21}$$

output
```
a^3*A*x+1/5*a^2*(-3*A*c+B*a)*x^5-1/9*a*(-3*A*c^2+3*B*a*c-C*a^2)*x^9+1/13*c
*(-A*c^2+3*B*a*c-3*C*a^2)*x^13-1/17*c^2*(B*c-3*C*a)*x^17-1/21*c^3*C*x^21
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int (a-cx^4)^3 (A+Bx^4+Cx^8)\, dx = a^3Ax+\frac{1}{5}a^2(aB-3Ac)x^5 +\frac{1}{9}a(-3aBc+3Ac^2+a^2C)\,x^9 -\frac{1}{13}c(-3aBc+Ac^2+3a^2C)\,x^{13} -\frac{1}{17}c^2(Bc-3aC)x^{17}-\frac{1}{21}c^3Cx^{21}$$

input

```
Integrate[(a - c*x^4)^3*(A + B*x^4 + C*x^8),x]
```

output

```
a^3*A*x + (a^2*(a*B - 3*A*c)*x^5)/5 + (a*(-3*a*B*c + 3*A*c^2 + a^2*C)*x^9)
/9 - (c*(-3*a*B*c + A*c^2 + 3*a^2*C)*x^13)/13 - (c^2*(B*c - 3*a*C)*x^17)/1
7 - (c^3*C*x^21)/21
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^4)^3 (A+Bx^4+Cx^8)\, dx$$

$$\Big\downarrow \text{1737}$$

$$\int (a^3A-cx^{12}(3a^2C-3aBc+Ac^2)+ax^8(a^2C-3aBc+3Ac^2)+a^2x^4(aB-3Ac)-c^2x^{16}(Bc-3aC)-c^3C$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax + \frac{1}{13}cx^{13}(-3a^2C + 3aBc - Ac^2) - \frac{1}{9}ax^9(a^2(-C) + 3aBc - 3Ac^2) + \frac{1}{5}a^2x^5(aB - 3Ac) - \frac{1}{17}c^2x^{17}(Bc - 3aC) - \frac{1}{21}c^3Cx^{21}$$

input
```
Int[(a - c*x^4)^3*(A + B*x^4 + C*x^8),x]
```

output
```
a^3*A*x + (a^2*(a*B - 3*A*c)*x^5)/5 - (a*(3*a*B*c - 3*A*c^2 - a^2*C)*x^9)/
9 + (c*(3*a*B*c - A*c^2 - 3*a^2*C)*x^13)/13 - (c^2*(B*c - 3*a*C)*x^17)/17
- (c^3*C*x^21)/21
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

method	result
norman	$a^3Ax + \left(-\frac{3}{5}a^2Ac + \frac{1}{5}B\,a^3\right)x^5 + \left(\frac{1}{3}Aa\,c^2 - \frac{1}{3}a^2cB + \frac{1}{9}a^3C\right)x^9 + \left(-\frac{1}{13}A\,c^3 + \frac{3}{13}a\,c^2B - \frac{3}{13}\right.$
default	$-\frac{c^3C\,x^{21}}{21} + \frac{(-c^3B+3a\,c^2C)x^{17}}{17} + \frac{(-A\,c^3+3a\,c^2B-3a^2cC)x^{13}}{13} + \frac{(3Aa\,c^2-3a^2cB+a^3C)x^9}{9} + \frac{(-3a^2Ac+B\,a^3)x^5}{5}$
gosper	$a^3Ax - \frac{3}{5}x^5a^2Ac + \frac{1}{5}x^5B\,a^3 + \frac{1}{3}x^9Aa\,c^2 - \frac{1}{3}x^9a^2cB + \frac{1}{9}x^9a^3C - \frac{1}{13}x^{13}A\,c^3 + \frac{3}{13}x^{13}a\,c^2B -$
risch	$a^3Ax - \frac{3}{5}x^5a^2Ac + \frac{1}{5}x^5B\,a^3 + \frac{1}{3}x^9Aa\,c^2 - \frac{1}{3}x^9a^2cB + \frac{1}{9}x^9a^3C - \frac{1}{13}x^{13}A\,c^3 + \frac{3}{13}x^{13}a\,c^2B -$
parallelrisch	$a^3Ax - \frac{3}{5}x^5a^2Ac + \frac{1}{5}x^5B\,a^3 + \frac{1}{3}x^9Aa\,c^2 - \frac{1}{3}x^9a^2cB + \frac{1}{9}x^9a^3C - \frac{1}{13}x^{13}A\,c^3 + \frac{3}{13}x^{13}a\,c^2B -$
orering	$\frac{x(-3315c^3C\,x^{20}-4095B\,c^3x^{16}+12285Ca\,c^2x^{16}-5355A\,c^3x^{12}+16065Ba\,c^2x^{12}-16065C\,a^2c\,x^{12}+23205Aa\,c^2x^8-23205B\,a}{69615}$

input
```
int((-c*x^4+a)^3*(C*x^8+B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
a^3*A*x+(-3/5*a^2*A*c+1/5*B*a^3)*x^5+(1/3*A*a*c^2-1/3*a^2*c*B+1/9*a^3*C)*x
^9+(-1/13*A*c^3+3/13*a*c^2*B-3/13*a^2*c*C)*x^13+(-1/17*c^3*B+3/17*a*c^2*C)
*x^17-1/21*c^3*C*x^21
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \left(a-cx^4\right)^3 \left(A+Bx^4+Cx^8\right)\,dx = -\frac{1}{21}\,Cc^3x^{21}+\frac{1}{17}\,(3\,Cac^2-Bc^3)x^{17} -\frac{1}{13}\,(3\,Ca^2c-3\,Bac^2+Ac^3)x^{13} +\frac{1}{9}\,(Ca^3-3\,Ba^2c+3\,Aac^2)x^9 +\frac{1}{5}\,(Ba^3-3\,Aa^2c)x^5+Aa^3x$$

input
```
integrate((-c*x^4+a)^3*(C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
-1/21*C*c^3*x^21 + 1/17*(3*C*a*c^2 - B*c^3)*x^17 - 1/13*(3*C*a^2*c - 3*B*a
*c^2 + A*c^3)*x^13 + 1/9*(C*a^3 - 3*B*a^2*c + 3*A*a*c^2)*x^9 + 1/5*(B*a^3
- 3*A*a^2*c)*x^5 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08

$$\int \left(a-cx^4\right)^3 \left(A+Bx^4+Cx^8\right)\,dx = Aa^3x-\frac{Cc^3x^{21}}{21}+x^{17}\left(-\frac{Bc^3}{17}+\frac{3Cac^2}{17}\right) +x^{13}\left(-\frac{Ac^3}{13}+\frac{3Bac^2}{13}-\frac{3Ca^2c}{13}\right) +x^9\left(\frac{Aac^2}{3}-\frac{Ba^2c}{3}+\frac{Ca^3}{9}\right) +x^5\left(-\frac{3Aa^2c}{5}+\frac{Ba^3}{5}\right)$$

input
```
integrate((-c*x**4+a)**3*(C*x**8+B*x**4+A),x)
```

output
```
A*a**3*x - C*c**3*x**21/21 + x**17*(-B*c**3/17 + 3*C*a*c**2/17) + x**13*(-
A*c**3/13 + 3*B*a*c**2/13 - 3*C*a**2*c/13) + x**9*(A*a*c**2/3 - B*a**2*c/3
 + C*a**3/9) + x**5*(-3*A*a**2*c/5 + B*a**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int (a-cx^4)^3 (A+Bx^4+Cx^8)\, dx = -\frac{1}{21}\, Cc^3x^{21} + \frac{1}{17}\,(3\, Cac^2 - Bc^3)x^{17} - \frac{1}{13}\,(3\, Ca^2c - 3\, Bac^2 + Ac^3)x^{13} + \frac{1}{9}\,(Ca^3 - 3\, Ba^2c + 3\, Aac^2)x^9 + \frac{1}{5}\,(Ba^3 - 3\, Aa^2c)x^5 + Aa^3x$$

input
```
integrate((-c*x^4+a)^3*(C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
-1/21*C*c^3*x^21 + 1/17*(3*C*a*c^2 - B*c^3)*x^17 - 1/13*(3*C*a^2*c - 3*B*a
*c^2 + A*c^3)*x^13 + 1/9*(C*a^3 - 3*B*a^2*c + 3*A*a*c^2)*x^9 + 1/5*(B*a^3
- 3*A*a^2*c)*x^5 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int (a-cx^4)^3 (A+Bx^4+Cx^8)\, dx = -\frac{1}{21}\, Cc^3x^{21} + \frac{3}{17}\, Cac^2x^{17} - \frac{1}{17}\, Bc^3x^{17} - \frac{3}{13}\, Ca^2cx^{13} + \frac{3}{13}\, Bac^2x^{13} - \frac{1}{13}\, Ac^3x^{13} + \frac{1}{9}\, Ca^3x^9 - \frac{1}{3}\, Ba^2cx^9 + \frac{1}{3}\, Aac^2x^9 + \frac{1}{5}\, Ba^3x^5 - \frac{3}{5}\, Aa^2cx^5 + Aa^3x$$

input
```
integrate((-c*x^4+a)^3*(C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
-1/21*C*c^3*x^21 + 3/17*C*a*c^2*x^17 - 1/17*B*c^3*x^17 - 3/13*C*a^2*c*x^13
 + 3/13*B*a*c^2*x^13 - 1/13*A*c^3*x^13 + 1/9*C*a^3*x^9 - 1/3*B*a^2*c*x^9 +
 1/3*A*a*c^2*x^9 + 1/5*B*a^3*x^5 - 3/5*A*a^2*c*x^5 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int \left(a-cx^4\right)^3 \left(A+Bx^4+Cx^8\right)\,dx = x^9\left(\frac{C\,a^3}{9}-\frac{B\,a^2\,c}{3}+\frac{A\,a\,c^2}{3}\right) - x^{13}\left(\frac{3\,C\,a^2\,c}{13}-\frac{3\,B\,a\,c^2}{13}+\frac{A\,c^3}{13}\right) + x^5\left(\frac{B\,a^3}{5}-\frac{3\,A\,a^2\,c}{5}\right) - x^{17}\left(\frac{B\,c^3}{17}-\frac{3\,C\,a\,c^2}{17}\right) - \frac{C\,c^3\,x^{21}}{21} + A\,a^3\,x$$

input
```
int((a - c*x^4)^3*(A + B*x^4 + C*x^8),x)
```

output
```
x^9*((C*a^3)/9 + (A*a*c^2)/3 - (B*a^2*c)/3) - x^13*((A*c^3)/13 - (3*B*a*c^
2)/13 + (3*C*a^2*c)/13) + x^5*((B*a^3)/5 - (3*A*a^2*c)/5) - x^17*((B*c^3)/
17 - (3*C*a*c^2)/17) - (C*c^3*x^21)/21 + A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int \left(a-cx^4\right)^3 \left(A+Bx^4+Cx^8\right)\,dx = \frac{x(-3315c^4x^{20}+12285a\,c^3x^{16}-4095b\,c^3x^{16}-16065a^2c^2x^{12}+16065ab\,c^2x^{12}-5355a\,c^3x^{12}+7735a^3c\,x}{69615}$$

input
```
int((-c*x^4+a)^3*(C*x^8+B*x^4+A),x)
```

output

```
(x*(69615*a**4 + 13923*a**3*b*x**4 + 7735*a**3*c*x**8 - 41769*a**3*c*x**4
- 23205*a**2*b*c*x**8 - 16065*a**2*c**2*x**12 + 23205*a**2*c**2*x**8 + 160
65*a*b*c**2*x**12 + 12285*a*c**3*x**16 - 5355*a*c**3*x**12 - 4095*b*c**3*x
**16 - 3315*c**4*x**20))/69615
```

3.23 $\int (a-cx^4)^2 (A+Bx^4+Cx^8) \, dx$

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Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (a-cx^4)^2 (A+Bx^4+Cx^8) \, dx = a^2Ax+\frac{1}{5}a(aB-2Ac)x^5-\frac{1}{9}(2aBc-Ac^2-a^2C)\,x^9 + \frac{1}{13}c(Bc-2aC)x^{13}+\frac{1}{17}c^2Cx^{17}$$

output
```
a^2*A*x+1/5*a*(-2*A*c+B*a)*x^5-1/9*(-A*c^2+2*B*a*c-C*a^2)*x^9+1/13*c*(B*c-
2*C*a)*x^13+1/17*c^2*C*x^17
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (a-cx^4)^2 (A+Bx^4+Cx^8) \, dx = a^2Ax+\frac{1}{5}a(aB-2Ac)x^5+\frac{1}{9}(-2aBc+Ac^2+a^2C)\,x^9 + \frac{1}{13}c(Bc-2aC)x^{13}+\frac{1}{17}c^2Cx^{17}$$

input
```
Integrate[(a - c*x^4)^2*(A + B*x^4 + C*x^8),x]
```

output
```
a^2*A*x + (a*(a*B - 2*A*c)*x^5)/5 + ((-2*a*B*c + A*c^2 + a^2*C)*x^9)/9 + (
c*(B*c - 2*a*C)*x^13)/13 + (c^2*C*x^17)/17
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^4\right)^2 \left(A+Bx^4+Cx^8\right) \, dx$$

$$\Big\downarrow \ 1737$$

$$\int \left(x^8\left(a^2C-2aBc+Ac^2\right)+a^2A+ax^4(aB-2Ac)+cx^{12}(Bc-2aC)+c^2Cx^{16}\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{9}x^9\left(a^2(-C)+2aBc-Ac^2\right)+a^2Ax+\frac{1}{5}ax^5(aB-2Ac)+\frac{1}{13}cx^{13}(Bc-2aC)+\frac{1}{17}c^2Cx^{17}$$

input
```
Int[(a - c*x^4)^2*(A + B*x^4 + C*x^8),x]
```

output
```
a^2*A*x + (a*(a*B - 2*A*c)*x^5)/5 - ((2*a*B*c - A*c^2 - a^2*C)*x^9)/9 + (c
*(B*c - 2*a*C)*x^13)/13 + (c^2*C*x^17)/17
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

method	result
default	$\frac{c^2 C x^{17}}{17}+\frac{(c^2 B-2acC)x^{13}}{13}+\frac{(A c^2-2aBc+a^2 C)x^9}{9}+\frac{(-2Aac+a^2 B)x^5}{5}+a^2 A x$
norman	$a^2 A x+\left(-\frac{2}{5}Aac+\frac{1}{5}a^2 B\right)x^5+\left(\frac{1}{9}A c^2-\frac{2}{9}aBc+\frac{1}{9}a^2 C\right)x^9+\left(\frac{1}{13}c^2 B-\frac{2}{13}acC\right)x^{13}+\frac{c^2 C x}{17}$
gosper	$a^2 A x-\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 B+\frac{1}{9}x^9 A c^2-\frac{2}{9}x^9 aBc+\frac{1}{9}x^9 a^2 C+\frac{1}{13}x^{13}c^2 B-\frac{2}{13}x^{13}acC+\frac{1}{17}c^2$
risch	$a^2 A x-\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 B+\frac{1}{9}x^9 A c^2-\frac{2}{9}x^9 aBc+\frac{1}{9}x^9 a^2 C+\frac{1}{13}x^{13}c^2 B-\frac{2}{13}x^{13}acC+\frac{1}{17}c^2$
parallelrisch	$a^2 A x-\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 B+\frac{1}{9}x^9 A c^2-\frac{2}{9}x^9 aBc+\frac{1}{9}x^9 a^2 C+\frac{1}{13}x^{13}c^2 B-\frac{2}{13}x^{13}acC+\frac{1}{17}c^2$
orering	$\frac{x(585c^2 C x^{16}+765B c^2 x^{12}-1530Cac x^{12}+1105A c^2 x^8-2210Bac x^8+1105C a^2 x^8-3978Aac x^4+1989B a^2 x^4+9945a^2 A)}{9945}$

input
```
int((-c*x^4+a)^2*(C*x^8+B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/17*c^2*C*x^17+1/13*(B*c^2-2*C*a*c)*x^13+1/9*(A*c^2-2*B*a*c+C*a^2)*x^9+1/
5*(-2*A*a*c+B*a^2)*x^5+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int (a-cx^4)^2 (A+Bx^4+Cx^8)\,dx = \frac{1}{17}Cc^2x^{17}-\frac{1}{13}(2Cac-Bc^2)x^{13}+\frac{1}{9}(Ca^2-2Bac+Ac^2)x^9+\frac{1}{5}(Ba^2-2Aac)x^5+Aa^2x$$

input
```
integrate((-c*x^4+a)^2*(C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
1/17*C*c^2*x^17 - 1/13*(2*C*a*c - B*c^2)*x^13 + 1/9*(C*a^2 - 2*B*a*c + A*c
^2)*x^9 + 1/5*(B*a^2 - 2*A*a*c)*x^5 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \left(a-cx^4\right)^2 \left(A+Bx^4+Cx^8\right)\,dx = Aa^2x+\frac{Cc^2x^{17}}{17}+x^{13}\left(\frac{Bc^2}{13}-\frac{2Cac}{13}\right)+x^9\left(\frac{Ac^2}{9}-\frac{2Bac}{9}+\frac{Ca^2}{9}\right)+x^5\left(-\frac{2Aac}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate((-c*x**4+a)**2*(C*x**8+B*x**4+A),x)
```

output
```
A*a**2*x + C*c**2*x**17/17 + x**13*(B*c**2/13 - 2*C*a*c/13) + x**9*(A*c**2
/9 - 2*B*a*c/9 + C*a**2/9) + x**5*(-2*A*a*c/5 + B*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \left(a-cx^4\right)^2 \left(A+Bx^4+Cx^8\right)\,dx = \frac{1}{17}\,Cc^2x^{17}-\frac{1}{13}\,(2\,Cac-Bc^2)x^{13}+\frac{1}{9}\,(Ca^2-2\,Bac+Ac^2)x^9+\frac{1}{5}\,(Ba^2-2\,Aac)x^5+Aa^2x$$

input
```
integrate((-c*x^4+a)^2*(C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
1/17*C*c^2*x^17 - 1/13*(2*C*a*c - B*c^2)*x^13 + 1/9*(C*a^2 - 2*B*a*c + A*c
^2)*x^9 + 1/5*(B*a^2 - 2*A*a*c)*x^5 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int (a-cx^4)^2 (A+Bx^4+Cx^8)\,dx = \frac{1}{17}Cc^2x^{17} - \frac{2}{13}Cacx^{13} + \frac{1}{13}Bc^2x^{13} + \frac{1}{9}Ca^2x^9 - \frac{2}{9}Bacx^9 + \frac{1}{9}Ac^2x^9 + \frac{1}{5}Ba^2x^5 - \frac{2}{5}Aacx^5 + Aa^2x$$

input
```
integrate((-c*x^4+a)^2*(C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
1/17*C*c^2*x^17 - 2/13*C*a*c*x^13 + 1/13*B*c^2*x^13 + 1/9*C*a^2*x^9 - 2/9*
B*a*c*x^9 + 1/9*A*c^2*x^9 + 1/5*B*a^2*x^5 - 2/5*A*a*c*x^5 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int (a-cx^4)^2 (A+Bx^4+Cx^8)\,dx = x^5\left(\frac{B\,a^2}{5} - \frac{2\,A\,a\,c}{5}\right) + x^{13}\left(\frac{B\,c^2}{13} - \frac{2\,C\,a\,c}{13}\right) + x^9\left(\frac{C\,a^2}{9} - \frac{2\,B\,a\,c}{9} + \frac{A\,c^2}{9}\right) + \frac{C\,c^2\,x^{17}}{17} + A\,a^2\,x$$

input
```
int((a - c*x^4)^2*(A + B*x^4 + C*x^8),x)
```

output
```
x^5*((B*a^2)/5 - (2*A*a*c)/5) + x^13*((B*c^2)/13 - (2*C*a*c)/13) + x^9*((A
*c^2)/9 + (C*a^2)/9 - (2*B*a*c)/9) + (C*c^2*x^17)/17 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int (a - cx^4)^2 (A + Bx^4 + Cx^8) \, dx$$
$$= \frac{x(585c^3x^{16} - 1530a\,c^2x^{12} + 765b\,c^2x^{12} + 1105a^2c\,x^8 - 2210abc\,x^8 + 1105a\,c^2x^8 + 1989a^2b\,x^4 - 3978a^2c}{9945}$$

input

```
int((-c*x^4+a)^2*(C*x^8+B*x^4+A),x)
```

output

```
(x*(9945*a**3 + 1989*a**2*b*x**4 + 1105*a**2*c*x**8 - 3978*a**2*c*x**4 - 2
210*a*b*c*x**8 - 1530*a*c**2*x**12 + 1105*a*c**2*x**8 + 765*b*c**2*x**12 +
 585*c**3*x**16))/9945
```

3.24 $\int (a-cx^4)(A+Bx^4+Cx^8)\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx=aAx+\frac{1}{5}(aB-Ac)x^5-\frac{1}{9}(Bc-aC)x^9-\frac{1}{13}cCx^{13}$$

output
```
a*A*x+1/5*(-A*c+B*a)*x^5-1/9*(B*c-C*a)*x^9-1/13*c*C*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx=aAx+\frac{1}{5}(aB-Ac)x^5+\frac{1}{9}(-Bc+aC)x^9-\frac{1}{13}cCx^{13}$$

input
```
Integrate[(a - c*x^4)*(A + B*x^4 + C*x^8),x]
```

output
```
a*A*x + ((a*B - A*c)*x^5)/5 + ((-(B*c) + a*C)*x^9)/9 - (c*C*x^13)/13
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^4)\left(A+Bx^4+Cx^8\right)\,dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(x^4(aB-Ac)+aA-x^8(Bc-aC)-cCx^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aB-Ac)+aAx-\frac{1}{9}x^9(Bc-aC)-\frac{1}{13}cCx^{13}$$

input
```
Int[(a - c*x^4)*(A + B*x^4 + C*x^8),x]
```

output
```
a*A*x + ((a*B - A*c)*x^5)/5 - ((B*c - a*C)*x^9)/9 - (c*C*x^13)/13
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{cC\,x^{13}}{13}+\frac{(-Bc+Ca)x^{9}}{9}+\frac{(-Ac+Ba)x^{5}}{5}+aAx$	39
norman	$-\frac{cC\,x^{13}}{13}+\left(-\frac{Bc}{9}+\frac{Ca}{9}\right)x^{9}+\left(-\frac{Ac}{5}+\frac{Ba}{5}\right)x^{5}+aAx$	39
gosper	$-\frac{1}{13}cC\,x^{13}-\frac{1}{9}x^{9}Bc+\frac{1}{9}x^{9}Ca-\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Ba+aAx$	41
risch	$-\frac{1}{13}cC\,x^{13}-\frac{1}{9}x^{9}Bc+\frac{1}{9}x^{9}Ca-\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Ba+aAx$	41
parallelrisch	$-\frac{1}{13}cC\,x^{13}-\frac{1}{9}x^{9}Bc+\frac{1}{9}x^{9}Ca-\frac{1}{5}x^{5}Ac+\frac{1}{5}x^{5}Ba+aAx$	41
orering	$\frac{x\left(-45Cc\,x^{12}-65Bc\,x^{8}+65Ca\,x^{8}-117Ac\,x^{4}+117Ba\,x^{4}+585Aa\right)}{585}$	44

input
```
int((-c*x^4+a)*(C*x^8+B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
-1/13*c*C*x^13+1/9*(-B*c+C*a)*x^9+1/5*(-A*c+B*a)*x^5+a*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx=-\frac{1}{13}\,Ccx^{13}+\frac{1}{9}\,(Ca-Bc)x^9+\frac{1}{5}\,(Ba-Ac)x^5+Aax$$

input
```
integrate((-c*x^4+a)*(C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
-1/13*C*c*x^13 + 1/9*(C*a - B*c)*x^9 + 1/5*(B*a - A*c)*x^5 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx = Aax-\frac{Ccx^{13}}{13}+x^9\left(-\frac{Bc}{9}+\frac{Ca}{9}\right)+x^5\left(-\frac{Ac}{5}+\frac{Ba}{5}\right)$$

input
```
integrate((-c*x**4+a)*(C*x**8+B*x**4+A),x)
```

output
```
A*a*x - C*c*x**13/13 + x**9*(-B*c/9 + C*a/9) + x**5*(-A*c/5 + B*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx = -\frac{1}{13}Ccx^{13}+\frac{1}{9}(Ca-Bc)x^9+\frac{1}{5}(Ba-Ac)x^5+Aax$$

input
```
integrate((-c*x^4+a)*(C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
-1/13*C*c*x^13 + 1/9*(C*a - B*c)*x^9 + 1/5*(B*a - A*c)*x^5 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx = -\frac{1}{13}Ccx^{13}+\frac{1}{9}Cax^9-\frac{1}{9}Bcx^9+\frac{1}{5}Bax^5-\frac{1}{5}Acx^5+Aax$$

input
```
integrate((-c*x^4+a)*(C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
-1/13*C*c*x^13 + 1/9*C*a*x^9 - 1/9*B*c*x^9 + 1/5*B*a*x^5 - 1/5*A*c*x^5 + A
*a*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx=-\frac{C\,c\,x^{13}}{13}+\left(\frac{C\,a}{9}-\frac{B\,c}{9}\right)\,x^9+\left(\frac{B\,a}{5}-\frac{A\,c}{5}\right)\,x^5+A\,a\,x$$

input
```
int((a - c*x^4)*(A + B*x^4 + C*x^8),x)
```

output
```
x^5*((B*a)/5 - (A*c)/5) + x^9*((C*a)/9 - (B*c)/9) + A*a*x - (C*c*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \left(a-cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx = \frac{x(-45c^2x^{12}+65ac\,x^8-65bc\,x^8+117ab\,x^4-117ac\,x^4+585a^2)}{585}$$

input
```
int((-c*x^4+a)*(C*x^8+B*x^4+A),x)
```

output
```
(x*(585*a**2 + 117*a*b*x**4 + 65*a*c*x**8 - 117*a*c*x**4 - 65*b*c*x**8 - 4
5*c**2*x**12))/585
```

3.25 $\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx$

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Reduce [B] (verification not implemented) 248

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx = -\frac{(Bc+aC)x}{c^2}-\frac{Cx^5}{5c}+\frac{(aBc+Ac^2+a^2C)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{9/4}}+\frac{(aBc+Ac^2+a^2C)\,\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{9/4}}$$

output
```
-(B*c+C*a)*x/c^2-1/5*C*x^5/c+1/2*(A*c^2+B*a*c+C*a^2)*arctan(c^(1/4)*x/a^(1
/4))/a^(3/4)/c^(9/4)+1/2*(A*c^2+B*a*c+C*a^2)*arctanh(c^(1/4)*x/a^(1/4))/a^
(3/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx = \frac{-20a^{3/4}\sqrt[4]{c}(Bc+aC)x-4a^{3/4}c^{5/4}Cx^5+10(aBc+Ac^2+a^2C)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-5(aBc+Ac^2+a^2C)}{20a^{3/4}c^{9/4}}$$

input
```
Integrate[(A + B*x^4 + C*x^8)/(a - c*x^4),x]
```

output
```
(-20*a^(3/4)*c^(1/4)*(B*c + a*C)*x - 4*a^(3/4)*c^(5/4)*C*x^5 + 10*(a*B*c +
 A*c^2 + a^2*C)*ArcTan[(c^(1/4)*x)/a^(1/4)] - 5*(a*B*c + A*c^2 + a^2*C)*Lo
g[a^(1/4) - c^(1/4)*x] + 5*(a*B*c + A*c^2 + a^2*C)*Log[a^(1/4) + c^(1/4)*x
])/(20*a^(3/4)*c^(9/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1741, 27, 913, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}dx$$

$$\Big\downarrow \text{1741}$$

$$-\frac{\int -\frac{5\left((Bc+aC)x^4+Ac\right)}{a-cx^4}dx}{5c}-\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(Bc+aC)x^4+Ac}{a-cx^4}dx}{c}-\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\int \frac{1}{a-cx^4}dx}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\int \frac{1}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{a}}dx}{2\sqrt{a}}\right)}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\int \frac{1}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}\right)}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}\right)}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^5}{5c}$$

input
```
Int[(A + B*x^4 + C*x^8)/(a - c*x^4),x]
```

output
```
-1/5*(C*x^5)/c + (-(((B*c + a*C)*x)/c) + ((a*B*c + A*c^2 + a^2*C)*(ArcTan[
(c^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*c^(1/4)) + ArcTanh[(c^(1/4)*x)/a^(1/4)]/(2
*a^(3/4)*c^(1/4))))/c)/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result	size
risch	$-\frac{C\,x^{5}}{5c}-\frac{Bx}{c}-\frac{Cax}{c^{2}}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}-a\right)}\frac{\left(A\,c^{2}+aBc+a^{2}C\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4c^{3}}$	69
default	$-\frac{\frac{1}{5}C\,x^{5}c+Bcx+Cax}{c^{2}}-\frac{\left(-A\,c^{2}-aBc-a^{2}C\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{4c^{2}a}$	93

input
```
int((C*x^8+B*x^4+A)/(-c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*C*x^5/c-1/c*B*x-1/c^2*C*a*x-1/4/c^3*sum((A*c^2+B*a*c+C*a^2)/_R^3*ln(x
-_R),_R=RootOf(_Z^4*c-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 1406, normalized size of antiderivative = 12.90

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx = \text{Too large to display}$$

input

```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a),x, algorithm="fricas")
```

output

```
-1/20*(4*C*c*x^5 - 5*c^2*((C^4*a^8 + 4*B*C^3*a^7*c + 4*A^3*B*a*c^7 + A^4*c
^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 + 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^
4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^4 + 4*(A*B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3
*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*c^9))^(1/4)*log(a*c^2*((C^4*a^8 + 4*B*C^
3*a^7*c + 4*A^3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 + 4*(B
^3*C + 3*A*B*C^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^4 + 4*(A*
B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*c^9))^(1/
4) + (C*a^2 + B*a*c + A*c^2)*x) - 5*I*c^2*((C^4*a^8 + 4*B*C^3*a^7*c + 4*A^
3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 + 4*(B^3*C + 3*A*B*C
^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^4 + 4*(A*B^3 + 3*A^2*B*
C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*c^9))^(1/4)*log(I*a*c^2
*((C^4*a^8 + 4*B*C^3*a^7*c + 4*A^3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2*A*
C^3)*a^6*c^2 + 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^2*C
^2)*a^4*c^4 + 4*(A*B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a^2*
c^6)/(a^3*c^9))^(1/4) + (C*a^2 + B*a*c + A*c^2)*x) + 5*I*c^2*((C^4*a^8 + 4
*B*C^3*a^7*c + 4*A^3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 +
 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^4 +
4*(A*B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*c^9)
)^(1/4)*log(-I*a*c^2*((C^4*a^8 + 4*B*C^3*a^7*c + 4*A^3*B*a*c^7 + A^4*c^8 +
 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 + 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^4...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(109) = 218$.

Time = 1.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.34

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx = -\frac{Cx^5}{5c} - x\left(\frac{B}{c}+\frac{Ca}{c^2}\right)$$
$$-\text{RootSum}\left(256t^4a^3c^9 - A^4c^8 - 4A^3Bac^7 - 4A^3Ca^2c^6 - 6A^2B^2a^2c^6 - 12A^2BCa^3c^5 - 6A^2C^2a^4c^4 - 4\right.$$

input
```
integrate((C*x**8+B*x**4+A)/(-c*x**4+a),x)
```

output
```
-C*x**5/(5*c) - x*(B/c + C*a/c**2) - RootSum(256*_t**4*a**3*c**9 - A**4*c*
*8 - 4*A**3*B*a*c**7 - 4*A**3*C*a**2*c**6 - 6*A**2*B**2*a**2*c**6 - 12*A**
2*B*C*a**3*c**5 - 6*A**2*C**2*a**4*c**4 - 4*A*B**3*a**3*c**5 - 12*A*B**2*C
*a**4*c**4 - 12*A*B*C**2*a**5*c**3 - 4*A*C**3*a**6*c**2 - B**4*a**4*c**4 -
 4*B**3*C*a**5*c**3 - 6*B**2*C**2*a**6*c**2 - 4*B*C**3*a**7*c - C**4*a**8,
 Lambda(_t, _t*log(-4*_t*a*c**2/(A*c**2 + B*a*c + C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx = -\frac{Ccx^5+5\,(Ca+Bc)x}{5\,c^2} + \frac{\frac{2\,(Ca^2+Bac+Ac^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} - \frac{(Ca^2+Bac+Ac^2)\log\left(\frac{\sqrt{c}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{c}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}}{4\,c^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a),x, algorithm="maxima")
```

output
```
-1/5*(C*c*x^5 + 5*(C*a + B*c)*x)/c^2 + 1/4*(2*(C*a^2 + B*a*c + A*c^2)*arct
an(sqrt(c)*x/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) - (C*a
^2 + B*a*c + A*c^2)*log((sqrt(c)*x - sqrt(sqrt(a)*sqrt(c)))/(sqrt(c)*x + s
qrt(sqrt(a)*sqrt(c))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))))/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(87) = 174$.

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.35

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx = -\frac{\sqrt{2}(Ca^2+Bac+Ac^2)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\left(-ac^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}(Ca^2+Bac+Ac^2)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\left(-ac^3\right)^{\frac{3}{4}}} - \frac{\sqrt{2}(Ca^2+Bac+Ac^2)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{8\left(-ac^3\right)^{\frac{3}{4}}} + \frac{\sqrt{2}(Ca^2+Bac+Ac^2)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{8\left(-ac^3\right)^{\frac{3}{4}}} - \frac{Cc^4x^5+5\,Cac^3x+5\,Bc^4x}{5\,c^5}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(C*a^2 + B*a*c + A*c^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/c)^(1/4))/(-a/c)^(1/4))/(-a*c^3)^(3/4) - 1/4*sqrt(2)*(C*a^2 + B*a*c + A*c
^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/c)^(1/4))/(-a/c)^(1/4))/(-a*c^3)
^(3/4) - 1/8*sqrt(2)*(C*a^2 + B*a*c + A*c^2)*log(x^2 + sqrt(2)*x*(-a/c)^(1
/4) + sqrt(-a/c))/(-a*c^3)^(3/4) + 1/8*sqrt(2)*(C*a^2 + B*a*c + A*c^2)*log
(x^2 - sqrt(2)*x*(-a/c)^(1/4) + sqrt(-a/c))/(-a*c^3)^(3/4) - 1/5*(C*c^4*x^
5 + 5*C*a*c^3*x + 5*B*c^4*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 1116, normalized size of antiderivative = 10.24

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4 + C*x^8)/(a - c*x^4),x)
```

output
```
(atan((((4*x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 + 2*A*B*a*c^3 + 2*B*C*a^3*c
 + 2*A*C*a^2*c^2))/c - ((16*B*a^2*c^2 + 16*A*a*c^3 + 16*C*a^3*c)*(A*c^2 +
C*a^2 + B*a*c))/(4*a^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 + B*a*c)*1i)/(4*a^(3/4
)*c^(9/4)) + (((4*x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 + 2*A*B*a*c^3 + 2*B*C
*a^3*c + 2*A*C*a^2*c^2))/c + ((16*B*a^2*c^2 + 16*A*a*c^3 + 16*C*a^3*c)*(A*
c^2 + C*a^2 + B*a*c))/(4*a^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 + B*a*c)*1i)/(4*
a^(3/4)*c^(9/4)))/((((4*x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 + 2*A*B*a*c^3 +
 2*B*C*a^3*c + 2*A*C*a^2*c^2))/c - ((16*B*a^2*c^2 + 16*A*a*c^3 + 16*C*a^3*
c)*(A*c^2 + C*a^2 + B*a*c))/(4*a^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 + B*a*c))/
(4*a^(3/4)*c^(9/4)) - (((4*x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 + 2*A*B*a*c^
3 + 2*B*C*a^3*c + 2*A*C*a^2*c^2))/c + ((16*B*a^2*c^2 + 16*A*a*c^3 + 16*C*a
^3*c)*(A*c^2 + C*a^2 + B*a*c))/(4*a^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 + B*a*c
))/(4*a^(3/4)*c^(9/4))))*(A*c^2 + C*a^2 + B*a*c)*1i)/(2*a^(3/4)*c^(9/4)) -
 (C*x^5)/(5*c) - x*(B/c + (C*a)/c^2) + (atan((((4*x*(A^2*c^4 + C^2*a^4 +
B^2*a^2*c^2 + 2*A*B*a*c^3 + 2*B*C*a^3*c + 2*A*C*a^2*c^2))/c - ((16*B*a^2*c
^2 + 16*A*a*c^3 + 16*C*a^3*c)*(A*c^2 + C*a^2 + B*a*c)*1i)/(4*a^(3/4)*c^(9/
4)))*(A*c^2 + C*a^2 + B*a*c))/(4*a^(3/4)*c^(9/4)) + (((4*x*(A^2*c^4 + C^2*
a^4 + B^2*a^2*c^2 + 2*A*B*a*c^3 + 2*B*C*a^3*c + 2*A*C*a^2*c^2))/c + ((16*B
*a^2*c^2 + 16*A*a*c^3 + 16*C*a^3*c)*(A*c^2 + C*a^2 + B*a*c)*1i)/(4*a^(3/4)
*c^(9/4)))*(A*c^2 + C*a^2 + B*a*c))/(4*a^(3/4)*c^(9/4)))/((((4*x*(A^2*c...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{A+Bx^4+Cx^8}{a-cx^4}\,dx$$
$$= \frac{10c^{\frac{3}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right) + 10c^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)b + 10c^{\frac{7}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right) - 5c^{\frac{3}{4}}a^{\frac{5}{4}}\log\left(a^{\frac{1}{4}} - c^{\frac{1}{4}}x\right) - 5c^{\frac{3}{4}}a^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
int((C*x^8+B*x^4+A)/(-c*x^4+a),x)
```

output
```
(10*c**(3/4)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a + 10*c**(3/4
)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*b + 10*c**(3/4)*a**(1/4)*
atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*c - 5*c**(3/4)*a**(1/4)*log(a**(1/4)
 - c**(1/4)*x)*a - 5*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*b - 5*c*
*(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*c + 5*c**(3/4)*a**(1/4)*log(a**
(1/4) + c**(1/4)*x)*a + 5*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*b +
 5*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*c - 20*a*c*x - 20*b*c*x -
4*c**2*x**5)/(20*c**2)
```

3.26 $\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^2}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^2}\,dx = \frac{Cx}{c^2}+\frac{(aBc+Ac^2+a^2C)\,x}{4ac^2\,(a-cx^4)} - \frac{(aBc-3Ac^2+5a^2C)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8a^{7/4}c^{9/4}} - \frac{(aBc-3Ac^2+5a^2C)\,\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8a^{7/4}c^{9/4}}$$

output
```
C*x/c^2+1/4*(A*c^2+B*a*c+C*a^2)*x/a/c^2/(-c*x^4+a)-1/8*(-3*A*c^2+B*a*c+5*C
*a^2)*arctan(c^(1/4)*x/a^(1/4))/a^(7/4)/c^(9/4)-1/8*(-3*A*c^2+B*a*c+5*C*a^
2)*arctanh(c^(1/4)*x/a^(1/4))/a^(7/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^2}\,dx$$

$$=\frac{16\sqrt[4]{c}Cx+\frac{4\sqrt[4]{c}(aBc+Ac^2+a^2C)x}{a(a-cx^4)}-\frac{2(aBc-3Ac^2+5a^2C)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{(aBc-3Ac^2+5a^2C)\log\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{a^{7/4}}-\frac{(aBc-3}{}}{16c^{9/4}}$$

input
```
Integrate[(A + B*x^4 + C*x^8)/(a - c*x^4)^2,x]
```

output
```
(16*c^(1/4)*C*x + (4*c^(1/4)*(a*B*c + A*c^2 + a^2*C)*x)/(a*(a - c*x^4)) -
(2*(a*B*c - 3*A*c^2 + 5*a^2*C)*ArcTan[(c^(1/4)*x)/a^(1/4)])/a^(7/4) + ((a*
B*c - 3*A*c^2 + 5*a^2*C)*Log[a^(1/4) - c^(1/4)*x])/a^(7/4) - ((a*B*c - 3*A
*c^2 + 5*a^2*C)*Log[a^(1/4) + c^(1/4)*x])/a^(7/4))/(16*c^(9/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1739, 913, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^2}\,dx$$

$$\Big\downarrow \ 1739$$

$$\frac{x(a^2C+aBc+Ac^2)}{4ac^2(a-cx^4)}-\frac{\int \frac{4acCx^4-3Ac^2+aBc+a^2C}{a-cx^4}dx}{4ac^2}$$

$$\Big\downarrow \ 913$$

$$\frac{x(a^2C+aBc+Ac^2)}{4ac^2(a-cx^4)}-\frac{(5a^2C+aBc-3Ac^2)\int \frac{1}{a-cx^4}dx-4aCx}{4ac^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{x(a^2C+aBc+Ac^2)}{4ac^2(a-cx^4)}-\frac{(5a^2C+aBc-3Ac^2)\left(\frac{\int \frac{1}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{a}}dx}{2\sqrt{a}}\right)-4aCx}{4ac^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x(a^2C+aBc+Ac^2)}{4ac^2(a-cx^4)}-\frac{(5a^2C+aBc-3Ac^2)\left(\frac{\int \frac{1}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}\right)-4aCx}{4ac^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{x(a^2C+aBc+Ac^2)}{4ac^2(a-cx^4)}-\frac{(5a^2C+aBc-3Ac^2)\left(\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}\right)-4aCx}{4ac^2}$$

input
```
Int[(A + B*x^4 + C*x^8)/(a - c*x^4)^2,x]
```

output
```
((a*B*c + A*c^2 + a^2*C)*x)/(4*a*c^2*(a - c*x^4)) - (-4*a*C*x + (a*B*c - 3
*A*c^2 + 5*a^2*C)*(ArcTan[(c^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*c^(1/4)) + ArcTa
nh[(c^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*c^(1/4))))/(4*a*c^2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{Cx}{c^2}+\frac{(A\,c^2+aBc+a^2C)x}{4a\,c^2(-c\,x^4+a)}-\frac{\sum_{\_R=\operatorname{RootOf}(c\_Z^4-a)}\frac{(3A\,c^2-aBc-5a^2C)\ln(x-\_R)}{\_R^3}}{16c^3a}$	91
default	$\frac{Cx}{c^2}+\frac{\frac{(A\,c^2+aBc+a^2C)x}{4a(-c\,x^4+a)}+\frac{(3A\,c^2-aBc-5a^2C)\left(\frac{a}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}}{c^2}$	111

input
```
int((C*x^8+B*x^4+A)/(-c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x/c^2+1/4*(A*c^2+B*a*c+C*a^2)*x/a/c^2/(-c*x^4+a)-1/16/c^3/a*sum((3*A*c^2
-B*a*c-5*C*a^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 1509, normalized size of antiderivative = 11.52

$$\int \frac{A + Bx^4 + Cx^8}{(a - cx^4)^2} dx = \text{Too large to display}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(16*C*a*c*x^5 + (a*c^3*x^4 - a^2*c^2)*((625*C^4*a^8 + 500*B*C^3*a^7*c
 - 108*A^3*B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*c^2 + 20*(B
^3*C - 45*A*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)*a^4*c^4 -
12*(A*B^3 - 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c^6)/(a^7*c^
9))^(1/4)*log(a^2*c^2*((625*C^4*a^8 + 500*B*C^3*a^7*c - 108*A^3*B*a*c^7 +
81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*c^2 + 20*(B^3*C - 45*A*B*C^2)*a^
5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)*a^4*c^4 - 12*(A*B^3 - 45*A^2*B*
C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c^6)/(a^7*c^9))^(1/4) - (5*C*a^2
+ B*a*c - 3*A*c^2)*x) - (-I*a*c^3*x^4 + I*a^2*c^2)*((625*C^4*a^8 + 500*B*C
^3*a^7*c - 108*A^3*B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*c^2
 + 20*(B^3*C - 45*A*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)*a^
4*c^4 - 12*(A*B^3 - 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c^6)
/(a^7*c^9))^(1/4)*log(I*a^2*c^2*((625*C^4*a^8 + 500*B*C^3*a^7*c - 108*A^3*
B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*c^2 + 20*(B^3*C - 45*A
*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)*a^4*c^4 - 12*(A*B^3 -
 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c^6)/(a^7*c^9))^(1/4) -
 (5*C*a^2 + B*a*c - 3*A*c^2)*x) - (I*a*c^3*x^4 - I*a^2*c^2)*((625*C^4*a^8
+ 500*B*C^3*a^7*c - 108*A^3*B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3
)*a^6*c^2 + 20*(B^3*C - 45*A*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^
2*C^2)*a^4*c^4 - 12*(A*B^3 - 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(134) = 268$.

Time = 1.99 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.18

$$\int \frac{A + Bx^4 + Cx^8}{(a - cx^4)^2}\, dx = \frac{Cx}{c^2} + \frac{x(-Ac^2 - Bac - Ca^2)}{-4a^2c^2 + 4ac^3x^4} + \mathrm{RootSum}\left(65536t^4a^7c^9 - 81A^4c^8 + 108A^3Bac^7 + 540A^3Ca^2c^6 - 54A^2B^2a^2c^6 - 540A^2BCa^3c^5 - 135\right.$$

input
```
integrate((C*x**8+B*x**4+A)/(-c*x**4+a)**2,x)
```

output
```
C*x/c**2 + x*(-A*c**2 - B*a*c - C*a**2)/(-4*a**2*c**2 + 4*a*c**3*x**4) + R
ootSum(65536*_t**4*a**7*c**9 - 81*A**4*c**8 + 108*A**3*B*a*c**7 + 540*A**3
*C*a**2*c**6 - 54*A**2*B**2*a**2*c**6 - 540*A**2*B*C*a**3*c**5 - 1350*A**2
*C**2*a**4*c**4 + 12*A*B**3*a**3*c**5 + 180*A*B**2*C*a**4*c**4 + 900*A*B*C
**2*a**5*c**3 + 1500*A*C**3*a**6*c**2 - B**4*a**4*c**4 - 20*B**3*C*a**5*c*
*3 - 150*B**2*C**2*a**6*c**2 - 500*B*C**3*a**7*c - 625*C**4*a**8, Lambda(_
t, _t*log(-16*_t*a**2*c**2/(-3*A*c**2 + B*a*c + 5*C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.27

$$\int \frac{A + Bx^4 + Cx^8}{(a - cx^4)^2}\, dx = -\frac{(Ca^2 + Bac + Ac^2)x}{4\,(ac^3x^4 - a^2c^2)} + \frac{Cx}{c^2} - \frac{\frac{2\,(5\,Ca^2 + Bac - 3\,Ac^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} - \frac{(5\,Ca^2 + Bac - 3\,Ac^2)\log\left(\frac{\sqrt{c}x - \sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{c}x + \sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}}{16\,ac^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(C*a^2 + B*a*c + A*c^2)*x/(a*c^3*x^4 - a^2*c^2) + C*x/c^2 - 1/16*(2*(
5*C*a^2 + B*a*c - 3*A*c^2)*arctan(sqrt(c)*x/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(c))) - (5*C*a^2 + B*a*c - 3*A*c^2)*log((sqrt(c)*x - sq
rt(sqrt(a)*sqrt(c)))/(sqrt(c)*x + sqrt(sqrt(a)*sqrt(c))))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(c))))/(a*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(110) = 220$.

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.21

$$\int \frac{A+Bx^4+Cx^8}{\left(a-cx^4\right)^2}\,dx = \frac{\sqrt{2}(5\,Ca^2+Bac-3\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\left(-ac^3\right)^{\frac{3}{4}}a} + \frac{\sqrt{2}(5\,Ca^2+Bac-3\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\left(-ac^3\right)^{\frac{3}{4}}a} + \frac{\sqrt{2}(5\,Ca^2+Bac-3\,Ac^2)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{32\left(-ac^3\right)^{\frac{3}{4}}a} - \frac{\sqrt{2}(5\,Ca^2+Bac-3\,Ac^2)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{c}}\right)}{32\left(-ac^3\right)^{\frac{3}{4}}a} + \frac{Cx}{c^2} - \frac{Ca^2x+Bacx+Ac^2x}{4\left(cx^4-a\right)ac^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(5*C*a^2 + B*a*c - 3*A*c^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(-a/c)^(1/4))/(-a/c)^(1/4))/((-a*c^3)^(3/4)*a) + 1/16*sqrt(2)*(5*C*a^2 +
B*a*c - 3*A*c^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/c)^(1/4))/(-a/c)^(1
/4))/((-a*c^3)^(3/4)*a) + 1/32*sqrt(2)*(5*C*a^2 + B*a*c - 3*A*c^2)*log(x^2
 + sqrt(2)*x*(-a/c)^(1/4) + sqrt(-a/c))/((-a*c^3)^(3/4)*a) - 1/32*sqrt(2)*
(5*C*a^2 + B*a*c - 3*A*c^2)*log(x^2 - sqrt(2)*x*(-a/c)^(1/4) + sqrt(-a/c))
/((-a*c^3)^(3/4)*a) + C*x/c^2 - 1/4*(C*a^2*x + B*a*c*x + A*c^2*x)/((c*x^4
- a)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 1187, normalized size of antiderivative = 9.06

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4 + C*x^8)/(a - c*x^4)^2,x)
```

output
```
(C*x)/c^2 - (atan(((((x*(9*A^2*c^4 + 25*C^2*a^4 + B^2*a^2*c^2 - 6*A*B*a*c^
3 + 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c) - ((4*B*a*c^2 - 12*A*c^3 + 2
0*C*a^2*c)*(5*C*a^2 - 3*A*c^2 + B*a*c))/(16*a^(7/4)*c^(9/4)))*(5*C*a^2 - 3
*A*c^2 + B*a*c)*1i)/(16*a^(7/4)*c^(9/4)) + (((x*(9*A^2*c^4 + 25*C^2*a^4 +
B^2*a^2*c^2 - 6*A*B*a*c^3 + 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c) + ((
4*B*a*c^2 - 12*A*c^3 + 20*C*a^2*c)*(5*C*a^2 - 3*A*c^2 + B*a*c))/(16*a^(7/4
)*c^(9/4)))*(5*C*a^2 - 3*A*c^2 + B*a*c)*1i)/(16*a^(7/4)*c^(9/4)))/((((x*(9
*A^2*c^4 + 25*C^2*a^4 + B^2*a^2*c^2 - 6*A*B*a*c^3 + 10*B*C*a^3*c - 30*A*C*
a^2*c^2))/(4*a^2*c) - ((4*B*a*c^2 - 12*A*c^3 + 20*C*a^2*c)*(5*C*a^2 - 3*A*
c^2 + B*a*c))/(16*a^(7/4)*c^(9/4)))*(5*C*a^2 - 3*A*c^2 + B*a*c))/(16*a^(7/
4)*c^(9/4)) - (((x*(9*A^2*c^4 + 25*C^2*a^4 + B^2*a^2*c^2 - 6*A*B*a*c^3 + 1
0*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c) + ((4*B*a*c^2 - 12*A*c^3 + 20*C*a
^2*c)*(5*C*a^2 - 3*A*c^2 + B*a*c))/(16*a^(7/4)*c^(9/4)))*(5*C*a^2 - 3*A*c^
2 + B*a*c))/(16*a^(7/4)*c^(9/4))))*(5*C*a^2 - 3*A*c^2 + B*a*c)*1i)/(8*a^(7
/4)*c^(9/4)) - (atan(((((x*(9*A^2*c^4 + 25*C^2*a^4 + B^2*a^2*c^2 - 6*A*B*a
*c^3 + 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c) - ((4*B*a*c^2 - 12*A*c^3
+ 20*C*a^2*c)*(5*C*a^2 - 3*A*c^2 + B*a*c)*1i)/(16*a^(7/4)*c^(9/4)))*(5*C*a
^2 - 3*A*c^2 + B*a*c))/(16*a^(7/4)*c^(9/4)) + (((x*(9*A^2*c^4 + 25*C^2*a^4
 + B^2*a^2*c^2 - 6*A*B*a*c^3 + 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c) +
 ((4*B*a*c^2 - 12*A*c^3 + 20*C*a^2*c)*(5*C*a^2 - 3*A*c^2 + B*a*c)*1i)/(...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.17

$$\int \frac{A + Bx^4 + Cx^8}{(a - cx^4)^2} \, dx$$
$$= \frac{-10c^{\frac{3}{4}}a^{\frac{9}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right) - 2c^{\frac{3}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)b + 10c^{\frac{7}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right)x^4 + 6c^{\frac{7}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}}\right) + 2c^{\frac{7}{4}}a^{\frac{1}{4}}atan}{}$$

input
```
int((C*x^8+B*x^4+A)/(-c*x^4+a)^2,x)
```

output
```
( - 10*c**(3/4)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**2 - 2*c*
*(3/4)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*b + 10*c**(3/4)*a*
*(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*c*x**4 + 6*c**(3/4)*a**(1/4
)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*c + 2*c**(3/4)*a**(1/4)*atan((sq
rt(c)*x)/(c**(1/4)*a**(1/4)))*b*c*x**4 - 6*c**(3/4)*a**(1/4)*atan((sqrt(c)
*x)/(c**(1/4)*a**(1/4)))*c**2*x**4 + 5*c**(3/4)*a**(1/4)*log(a**(1/4) - c*
*(1/4)*x)*a**2 + c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a*b - 5*c**(
3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a*c*x**4 - 3*c**(3/4)*a**(1/4)*lo
g(a**(1/4) - c**(1/4)*x)*a*c - c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x
)*b*c*x**4 + 3*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*c**2*x**4 - 5*
c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*a**2 - c**(3/4)*a**(1/4)*log(
a**(1/4) + c**(1/4)*x)*a*b + 5*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x
)*a*c*x**4 + 3*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*a*c + c**(3/4)
*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*b*c*x**4 - 3*c**(3/4)*a**(1/4)*log(a*
*(1/4) + c**(1/4)*x)*c**2*x**4 + 20*a**2*c*x + 4*a*b*c*x - 16*a*c**2*x**5
 + 4*a*c**2*x)/(16*a*c**2*(a - c*x**4))
```

3.27 $\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^3}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^3}\,dx = \frac{(aBc+Ac^2+a^2C)\,x}{8ac^2\,(a-cx^4)^2}-\frac{(aBc-7Ac^2+9a^2C)\,x}{32a^2c^2\,(a-cx^4)} - \frac{(3aBc-21Ac^2-5a^2C)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64a^{11/4}c^{9/4}} - \frac{(3aBc-21Ac^2-5a^2C)\,\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64a^{11/4}c^{9/4}}$$

output
```
1/8*(A*c^2+B*a*c+C*a^2)*x/a/c^2/(-c*x^4+a)^2-1/32*(-7*A*c^2+B*a*c+9*C*a^2)
*x/a^2/c^2/(-c*x^4+a)-1/64*(-21*A*c^2+3*B*a*c-5*C*a^2)*arctan(c^(1/4)*x/a^
(1/4))/a^(11/4)/c^(9/4)-1/64*(-21*A*c^2+3*B*a*c-5*C*a^2)*arctanh(c^(1/4)*x
/a^(1/4))/a^(11/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^3}\,dx = \frac{\frac{16a^{7/4}\sqrt[4]{c}(aBc+Ac^2+a^2C)x}{(a-cx^4)^2}+\frac{4a^{3/4}\sqrt[4]{c}(aBc-7Ac^2+9a^2C)x}{-a+cx^4}+2(-3aBc+21Ac^2+5a^2C)\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-(-3a}{128a^{11/4}c^{9/4}}$$

input
```
Integrate[(A + B*x^4 + C*x^8)/(a - c*x^4)^3,x]
```

output
```
((16*a^(7/4)*c^(1/4)*(a*B*c + A*c^2 + a^2*C)*x)/(a - c*x^4)^2 + (4*a^(3/4)
*c^(1/4)*(a*B*c - 7*A*c^2 + 9*a^2*C)*x)/(-a + c*x^4) + 2*(-3*a*B*c + 21*A*
c^2 + 5*a^2*C)*ArcTan[(c^(1/4)*x)/a^(1/4)] - (-3*a*B*c + 21*A*c^2 + 5*a^2*
C)*Log[a^(1/4) - c^(1/4)*x] + (-3*a*B*c + 21*A*c^2 + 5*a^2*C)*Log[a^(1/4)
+ c^(1/4)*x])/(128*a^(11/4)*c^(9/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1739, 910, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^3}\,dx$$

$$\Big\downarrow \text{1739}$$

$$\frac{x(a^2C+aBc+Ac^2)}{8ac^2(a-cx^4)^2}-\frac{\int \frac{8acCx^4-7Ac^2+aBc+a^2C}{(a-cx^4)^2}dx}{8ac^2}$$

$$\Big\downarrow \text{910}$$

$$\frac{x(a^2C+aBc+Ac^2)}{8ac^2(a-cx^4)^2}-\frac{\frac{(-5a^2C+3aBc-21Ac^2)\int \frac{1}{a-cx^4}dx}{4a}+\frac{x(9a^2C+aBc-7Ac^2)}{4a(a-cx^4)}}{8ac^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{x(a^2C+aBc+Ac^2)}{8ac^2(a-cx^4)^2}-\frac{\frac{(-5a^2C+3aBc-21Ac^2)\left(\frac{\int\frac{1}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\int\frac{1}{\sqrt{c}x^2+\sqrt{a}}dx}{2\sqrt{a}}\right)}{4a}+\frac{x(9a^2C+aBc-7Ac^2)}{4a(a-cx^4)}}{8ac^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x(a^2C+aBc+Ac^2)}{8ac^2(a-cx^4)^2}-\frac{\frac{(-5a^2C+3aBc-21Ac^2)\left(\frac{\int\frac{1}{\sqrt{a}-\sqrt{c}x^2}dx}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}\right)}{4a}+\frac{x(9a^2C+aBc-7Ac^2)}{4a(a-cx^4)}}{8ac^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{x(a^2C+aBc+Ac^2)}{8ac^2(a-cx^4)^2}-\frac{\frac{x(9a^2C+aBc-7Ac^2)}{4a(a-cx^4)}+\frac{(-5a^2C+3aBc-21Ac^2)\left(\frac{\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{c}}\right)}{4a}}{8ac^2}$$

input
```
Int[(A + B*x^4 + C*x^8)/(a - c*x^4)^3,x]
```

output
```
((a*B*c + A*c^2 + a^2*C)*x)/(8*a*c^2*(a - c*x^4)^2) - (((a*B*c - 7*A*c^2 +
 9*a^2*C)*x)/(4*a*(a - c*x^4)) + ((3*a*B*c - 21*A*c^2 - 5*a^2*C)*(ArcTan[(
c^(1/4)*x)/a^(1/4)]/(2*a^(3/4)*c^(1/4)) + ArcTanh[(c^(1/4)*x)/a^(1/4)]/(2*
a^(3/4)*c^(1/4))))/(4*a))/(8*a*c^2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.72

method	result	siz
risch	$\frac{-\frac{\left(7Ac^2-aBc-9a^2C\right)x^5}{32a^2c}+\frac{\left(11Ac^2+3aBc-5a^2C\right)x}{32ac^2}}{\left(-cx^4+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4-a\right)}\frac{\left(21Ac^2-3aBc+5a^2C\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2c^3}$	119
default	$\frac{-\frac{\left(7Ac^2-aBc-9a^2C\right)x^5}{32a^2c}+\frac{\left(11Ac^2+3aBc-5a^2C\right)x}{32ac^2}}{\left(-cx^4+a\right)^2}+\frac{\left(21Ac^2-3aBc+5a^2C\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{c}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)\right)}{128a^3c^2}$	140

input
```
int((C*x^8+B*x^4+A)/(-c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/32*(7*A*c^2-B*a*c-9*C*a^2)/a^2/c*x^5+1/32*(11*A*c^2+3*B*a*c-5*C*a^2)/a
/c^2*x)/(-c*x^4+a)^2-1/128/a^2/c^3*sum((21*A*c^2-3*B*a*c+5*C*a^2)/_R^3*ln(
x-_R),_R=RootOf(_Z^4*c-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 1633, normalized size of antiderivative = 9.90

$$\int \frac{A + Bx^4 + Cx^8}{(a - cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(4*(9*C*a^2*c + B*a*c^2 - 7*A*c^3)*x^5 + (a^2*c^4*x^8 - 2*a^3*c^3*x^
4 + a^4*c^2)*((625*C^4*a^8 - 1500*B*C^3*a^7*c - 111132*A^3*B*a*c^7 + 19448
1*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 - 540*(B^3*C + 35*A*B*C^2)*
a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 - 2268*(A*B^3 +
35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^11*c^9))^(1/
4)*log(a^3*c^2*((625*C^4*a^8 - 1500*B*C^3*a^7*c - 111132*A^3*B*a*c^7 + 194
481*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 - 540*(B^3*C + 35*A*B*C^2
)*a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 - 2268*(A*B^3
+ 35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^11*c^9))^(
1/4) + (5*C*a^2 - 3*B*a*c + 21*A*c^2)*x) - (-I*a^2*c^4*x^8 + 2*I*a^3*c^3*x
^4 - I*a^4*c^2)*((625*C^4*a^8 - 1500*B*C^3*a^7*c - 111132*A^3*B*a*c^7 + 19
4481*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 - 540*(B^3*C + 35*A*B*C^
2)*a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 - 2268*(A*B^3
 + 35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^11*c^9))^
(1/4)*log(I*a^3*c^2*((625*C^4*a^8 - 1500*B*C^3*a^7*c - 111132*A^3*B*a*c^7
+ 194481*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 - 540*(B^3*C + 35*A*
B*C^2)*a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 - 2268*(A
*B^3 + 35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^11*c^
9))^(1/4) + (5*C*a^2 - 3*B*a*c + 21*A*c^2)*x) - (I*a^2*c^4*x^8 - 2*I*a^3*c
^3*x^4 + I*a^4*c^2)*((625*C^4*a^8 - 1500*B*C^3*a^7*c - 111132*A^3*B*a*c...
```

Sympy [A] (verification not implemented)

Time = 47.08 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^3}\,dx = -\frac{x^5\cdot(7Ac^3-Bac^2-9Ca^2c)+x(-11Aac^2-3Ba^2c+5Ca^3)}{32a^4c^2-64a^3c^3x^4+32a^2c^4x^8} - \mathrm{RootSum}\Big(268435456t^4a^{11}c^9-194481A^4c^8+111132A^3Bac^7-185220A^3Ca^2c^6-23814A^2B^2a^2c^6+$$

input
```
integrate((C*x**8+B*x**4+A)/(-c*x**4+a)**3,x)
```

output
```
-(x**5*(7*A*c**3 - B*a*c**2 - 9*C*a**2*c) + x*(-11*A*a*c**2 - 3*B*a**2*c +
 5*C*a**3))/(32*a**4*c**2 - 64*a**3*c**3*x**4 + 32*a**2*c**4*x**8) - RootS
um(268435456*_t**4*a**11*c**9 - 194481*A**4*c**8 + 111132*A**3*B*a*c**7 -
185220*A**3*C*a**2*c**6 - 23814*A**2*B**2*a**2*c**6 + 79380*A**2*B*C*a**3*
c**5 - 66150*A**2*C**2*a**4*c**4 + 2268*A*B**3*a**3*c**5 - 11340*A*B**2*C*
a**4*c**4 + 18900*A*B*C**2*a**5*c**3 - 10500*A*C**3*a**6*c**2 - 81*B**4*a*
*4*c**4 + 540*B**3*C*a**5*c**3 - 1350*B**2*C**2*a**6*c**2 + 1500*B*C**3*a*
*7*c - 625*C**4*a**8, Lambda(_t, _t*log(-128*_t*a**3*c**2/(21*A*c**2 - 3*B
*a*c + 5*C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^3}\,dx = \frac{(9\,Ca^2c+Bac^2-7\,Ac^3)x^5-(5\,Ca^3-3\,Ba^2c-11\,Aac^2)x}{32\,(a^2c^4x^8-2\,a^3c^3x^4+a^4c^2)} + \frac{\frac{2\,(5\,Ca^2-3\,Bac+21\,Ac^2)\arctan\left(\frac{\sqrt{c}x}{\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} - \frac{(5\,Ca^2-3\,Bac+21\,Ac^2)\log\left(\frac{\sqrt{c}x-\sqrt{\sqrt{a}\sqrt{c}}}{\sqrt{c}x+\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}}{128\,a^2c^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*((9*C*a^2*c + B*a*c^2 - 7*A*c^3)*x^5 - (5*C*a^3 - 3*B*a^2*c - 11*A*a*
c^2)*x)/(a^2*c^4*x^8 - 2*a^3*c^3*x^4 + a^4*c^2) + 1/128*(2*(5*C*a^2 - 3*B*
a*c + 21*A*c^2)*arctan(sqrt(c)*x/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(c))) - (5*C*a^2 - 3*B*a*c + 21*A*c^2)*log((sqrt(c)*x - sqrt(sqrt(
a)*sqrt(c)))/(sqrt(c)*x + sqrt(sqrt(a)*sqrt(c))))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(c))))/(a^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(143) = 286$.

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.95

$$\int \frac{A + Bx^4 + Cx^8}{(a - cx^4)^3}\,dx$$
$$= -\frac{\sqrt{2}(5\,Ca^2 - 3\,Bac + 21\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,(-ac^3)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}(5\,Ca^2 - 3\,Bac + 21\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(-\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,(-ac^3)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}(5\,Ca^2 - 3\,Bac + 21\,Ac^2)\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{c}}\right)}{256\,(-ac^3)^{\frac{3}{4}}\,a^2} + \frac{\sqrt{2}(5\,Ca^2 - 3\,Bac + 21\,Ac^2)\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{c}}\right)}{256\,(-ac^3)^{\frac{3}{4}}\,a^2} + \frac{9\,Ca^2cx^5 + Bac^2x^5 - 7\,Ac^3x^5 - 5\,Ca^3x + 3\,Ba^2cx + 11\,Aac^2x}{32\,(cx^4 - a)^2a^2c^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(-c*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/128*sqrt(2)*(5*C*a^2 - 3*B*a*c + 21*A*c^2)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(-a/c)^(1/4))/(-a/c)^(1/4))/((-a*c^3)^(3/4)*a^2) - 1/128*sqrt(2)*(5*
C*a^2 - 3*B*a*c + 21*A*c^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/c)^(1/4)
)/(-a/c)^(1/4))/((-a*c^3)^(3/4)*a^2) - 1/256*sqrt(2)*(5*C*a^2 - 3*B*a*c +
21*A*c^2)*log(x^2 + sqrt(2)*x*(-a/c)^(1/4) + sqrt(-a/c))/((-a*c^3)^(3/4)*a
^2) + 1/256*sqrt(2)*(5*C*a^2 - 3*B*a*c + 21*A*c^2)*log(x^2 - sqrt(2)*x*(-a
/c)^(1/4) + sqrt(-a/c))/((-a*c^3)^(3/4)*a^2) + 1/32*(9*C*a^2*c*x^5 + B*a*c
^2*x^5 - 7*A*c^3*x^5 - 5*C*a^3*x + 3*B*a^2*c*x + 11*A*a*c^2*x)/((c*x^4 - a
)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 1247, normalized size of antiderivative = 7.56

$$\int \frac{A+Bx^4+Cx^8}{(a-cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4 + C*x^8)/(a - c*x^4)^3,x)
```

output
```
((x^5*(9*C*a^2 - 7*A*c^2 + B*a*c))/(32*a^2*c) + (x*(11*A*c^2 - 5*C*a^2 + 3
*B*a*c))/(32*a*c^2))/(a^2 + c^2*x^8 - 2*a*c*x^4) + (atan(((((x*(441*A^2*c^
4 + 25*C^2*a^4 + 9*B^2*a^2*c^2 - 126*A*B*a*c^3 - 30*B*C*a^3*c + 210*A*C*a^
2*c^2))/(256*a^4*c) - ((21*A*c^3 - 3*B*a*c^2 + 5*C*a^2*c)*(21*A*c^2 + 5*C*
a^2 - 3*B*a*c))/(256*a^(15/4)*c^(9/4)))*(21*A*c^2 + 5*C*a^2 - 3*B*a*c)*1i)
/(128*a^(11/4)*c^(9/4)) + (((x*(441*A^2*c^4 + 25*C^2*a^4 + 9*B^2*a^2*c^2 -
 126*A*B*a*c^3 - 30*B*C*a^3*c + 210*A*C*a^2*c^2))/(256*a^4*c) + ((21*A*c^3
 - 3*B*a*c^2 + 5*C*a^2*c)*(21*A*c^2 + 5*C*a^2 - 3*B*a*c))/(256*a^(15/4)*c^
(9/4)))*(21*A*c^2 + 5*C*a^2 - 3*B*a*c)*1i)/(128*a^(11/4)*c^(9/4)))/((((x*(
441*A^2*c^4 + 25*C^2*a^4 + 9*B^2*a^2*c^2 - 126*A*B*a*c^3 - 30*B*C*a^3*c +
210*A*C*a^2*c^2))/(256*a^4*c) - ((21*A*c^3 - 3*B*a*c^2 + 5*C*a^2*c)*(21*A*
c^2 + 5*C*a^2 - 3*B*a*c))/(256*a^(15/4)*c^(9/4)))*(21*A*c^2 + 5*C*a^2 - 3*
B*a*c))/(128*a^(11/4)*c^(9/4)) - (((x*(441*A^2*c^4 + 25*C^2*a^4 + 9*B^2*a^
2*c^2 - 126*A*B*a*c^3 - 30*B*C*a^3*c + 210*A*C*a^2*c^2))/(256*a^4*c) + ((2
1*A*c^3 - 3*B*a*c^2 + 5*C*a^2*c)*(21*A*c^2 + 5*C*a^2 - 3*B*a*c))/(256*a^(1
5/4)*c^(9/4)))*(21*A*c^2 + 5*C*a^2 - 3*B*a*c))/(128*a^(11/4)*c^(9/4))))*(2
1*A*c^2 + 5*C*a^2 - 3*B*a*c)*1i)/(64*a^(11/4)*c^(9/4)) + (atan(((((x*(441*
A^2*c^4 + 25*C^2*a^4 + 9*B^2*a^2*c^2 - 126*A*B*a*c^3 - 30*B*C*a^3*c + 210*
A*C*a^2*c^2))/(256*a^4*c) - ((21*A*c^3 - 3*B*a*c^2 + 5*C*a^2*c)*(21*A*c^2
+ 5*C*a^2 - 3*B*a*c)*1i)/(256*a^(15/4)*c^(9/4)))*(21*A*c^2 + 5*C*a^2 - ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.94

$$\int \frac{A + Bx^4 + Cx^8}{(a - cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((C*x^8+B*x^4+A)/(-c*x^4+a)^3,x)
```

output
```
(10*c**(3/4)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**3 - 6*c**(3
/4)*a**(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**2*b - 20*c**(3/4)*a*
*(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**2*c*x**4 + 42*c**(3/4)*a**
(1/4)*atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a**2*c + 12*c**(3/4)*a**(1/4)*
atan((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*b*c*x**4 + 10*c**(3/4)*a**(1/4)*at
an((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*c**2*x**8 - 84*c**(3/4)*a**(1/4)*ata
n((sqrt(c)*x)/(c**(1/4)*a**(1/4)))*a*c**2*x**4 - 6*c**(3/4)*a**(1/4)*atan(
(sqrt(c)*x)/(c**(1/4)*a**(1/4)))*b*c**2*x**8 + 42*c**(3/4)*a**(1/4)*atan((
sqrt(c)*x)/(c**(1/4)*a**(1/4)))*c**3*x**8 - 5*c**(3/4)*a**(1/4)*log(a**(1/
4) - c**(1/4)*x)*a**3 + 3*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a**
2*b + 10*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a**2*c*x**4 - 21*c**
(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a**2*c - 6*c**(3/4)*a**(1/4)*log
(a**(1/4) - c**(1/4)*x)*a*b*c*x**4 - 5*c**(3/4)*a**(1/4)*log(a**(1/4) - c*
*(1/4)*x)*a*c**2*x**8 + 42*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*a*
c**2*x**4 + 3*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*b*c**2*x**8 - 2
1*c**(3/4)*a**(1/4)*log(a**(1/4) - c**(1/4)*x)*c**3*x**8 + 5*c**(3/4)*a**(
1/4)*log(a**(1/4) + c**(1/4)*x)*a**3 - 3*c**(3/4)*a**(1/4)*log(a**(1/4) +
c**(1/4)*x)*a**2*b - 10*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*a**2*
c*x**4 + 21*c**(3/4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*a**2*c + 6*c**(3/
4)*a**(1/4)*log(a**(1/4) + c**(1/4)*x)*a*b*c*x**4 + 5*c**(3/4)*a**(1/4)...
```

3.28 $\int (a+cx^4)^3 (A+Bx^4+Cx^8)\, dx$

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Optimal result

Integrand size = 22, antiderivative size = 104

$$\int (a+cx^4)^3 (A+Bx^4+Cx^8)\, dx = a^3Ax + \frac{1}{5}a^2(aB+3Ac)x^5 + \frac{1}{9}a\left(3aBc+3Ac^2+a^2C\right)x^9 + \frac{1}{13}c\left(3aBc+Ac^2+3a^2C\right)x^{13} + \frac{1}{17}c^2(Bc+3aC)x^{17} + \frac{1}{21}c^3Cx^{21}$$

output
```
a^3*A*x+1/5*a^2*(3*A*c+B*a)*x^5+1/9*a*(3*A*c^2+3*B*a*c+C*a^2)*x^9+1/13*c*(
A*c^2+3*B*a*c+3*C*a^2)*x^13+1/17*c^2*(B*c+3*C*a)*x^17+1/21*c^3*C*x^21
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \left(a+cx^4\right)^3 \left(A+Bx^4+Cx^8\right) \, dx = a^3Ax+\frac{1}{5}a^2(aB+3Ac)x^5+\frac{1}{9}a\left(3aBc+3Ac^2+a^2C\right)x^9+\frac{1}{13}c\left(3aBc+Ac^2+3a^2C\right)x^{13}+\frac{1}{17}c^2(Bc+3aC)x^{17}+\frac{1}{21}c^3Cx^{21}$$

input
```
Integrate[(a + c*x^4)^3*(A + B*x^4 + C*x^8),x]
```

output
```
a^3*A*x + (a^2*(a*B + 3*A*c)*x^5)/5 + (a*(3*a*B*c + 3*A*c^2 + a^2*C)*x^9)/
9 + (c*(3*a*B*c + A*c^2 + 3*a^2*C)*x^13)/13 + (c^2*(B*c + 3*a*C)*x^17)/17
+ (c^3*C*x^21)/21
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^3 \left(A+Bx^4+Cx^8\right) \, dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(a^3A+cx^{12}\left(3a^2C+3aBc+Ac^2\right)+ax^8\left(a^2C+3aBc+3Ac^2\right)+a^2x^4(aB+3Ac)+c^2x^{16}(3aC+Bc)+c^3C$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax + \frac{1}{13}cx^{13}\left(3a^2C + 3aBc + Ac^2\right) + \frac{1}{9}ax^9\left(a^2C + 3aBc + 3Ac^2\right) + \frac{1}{5}a^2x^5(aB + 3Ac) + \frac{1}{17}c^2x^{17}(3aC + Bc) + \frac{1}{21}c^3Cx^{21}$$

input
```
Int[(a + c*x^4)^3*(A + B*x^4 + C*x^8),x]
```

output
```
a^3*A*x + (a^2*(a*B + 3*A*c)*x^5)/5 + (a*(3*a*B*c + 3*A*c^2 + a^2*C)*x^9)/
9 + (c*(3*a*B*c + A*c^2 + 3*a^2*C)*x^13)/13 + (c^2*(B*c + 3*a*C)*x^17)/17
+ (c^3*C*x^21)/21
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

method	result
default	$\frac{c^3C\,x^{21}}{21} + \frac{(c^3B+3a\,c^2C)x^{17}}{17} + \frac{(A\,c^3+3a\,c^2B+3a^2cC)x^{13}}{13} + \frac{(3Aa\,c^2+3a^2cB+a^3C)x^9}{9} + \frac{(3a^2Ac+B\,a^3)x^5}{5} + a^3A$
norman	$a^3Ax + \left(\frac{3}{5}a^2Ac + \frac{1}{5}B\,a^3\right)x^5 + \left(\frac{1}{3}Aa\,c^2 + \frac{1}{3}a^2cB + \frac{1}{9}a^3C\right)x^9 + \left(\frac{1}{13}A\,c^3 + \frac{3}{13}a\,c^2B + \frac{3}{13}a^2c\right.$
gosper	$a^3Ax + \frac{3}{5}x^5a^2Ac + \frac{1}{5}x^5B\,a^3 + \frac{1}{3}x^9Aa\,c^2 + \frac{1}{3}x^9a^2cB + \frac{1}{9}x^9a^3C + \frac{1}{13}x^{13}A\,c^3 + \frac{3}{13}x^{13}a\,c^2B +$
risch	$a^3Ax + \frac{3}{5}x^5a^2Ac + \frac{1}{5}x^5B\,a^3 + \frac{1}{3}x^9Aa\,c^2 + \frac{1}{3}x^9a^2cB + \frac{1}{9}x^9a^3C + \frac{1}{13}x^{13}A\,c^3 + \frac{3}{13}x^{13}a\,c^2B +$
parallelrisch	$a^3Ax + \frac{3}{5}x^5a^2Ac + \frac{1}{5}x^5B\,a^3 + \frac{1}{3}x^9Aa\,c^2 + \frac{1}{3}x^9a^2cB + \frac{1}{9}x^9a^3C + \frac{1}{13}x^{13}A\,c^3 + \frac{3}{13}x^{13}a\,c^2B +$
orering	$\frac{x(3315c^3C\,x^{20}+4095B\,c^3x^{16}+12285Ca\,c^2x^{16}+5355A\,c^3x^{12}+16065Ba\,c^2x^{12}+16065C\,a^2c\,x^{12}+23205Aa\,c^2x^8+23205B\,a^2c}{69615}$

input
```
int((c*x^4+a)^3*(C*x^8+B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/21*c^3*C*x^21+1/17*(B*c^3+3*C*a*c^2)*x^17+1/13*(A*c^3+3*B*a*c^2+3*C*a^2*
c)*x^13+1/9*(3*A*a*c^2+3*B*a^2*c+C*a^3)*x^9+1/5*(3*A*a^2*c+B*a^3)*x^5+a^3*
A*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \left(a+cx^4\right)^3 \left(A+Bx^4+Cx^8\right) \, dx = \frac{1}{21}\, Cc^3x^{21} + \frac{1}{17}\left(3\, Cac^2 + Bc^3\right)x^{17} + \frac{1}{13}\left(3\, Ca^2c + 3\, Bac^2 + Ac^3\right)x^{13} + \frac{1}{9}\left(Ca^3 + 3\, Ba^2c + 3\, Aac^2\right)x^9 + \frac{1}{5}\left(Ba^3 + 3\, Aa^2c\right)x^5 + Aa^3x$$

input
```
integrate((c*x^4+a)^3*(C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
1/21*C*c^3*x^21 + 1/17*(3*C*a*c^2 + B*c^3)*x^17 + 1/13*(3*C*a^2*c + 3*B*a*
c^2 + A*c^3)*x^13 + 1/9*(C*a^3 + 3*B*a^2*c + 3*A*a*c^2)*x^9 + 1/5*(B*a^3 +
 3*A*a^2*c)*x^5 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \left(a+cx^4\right)^3 \left(A+Bx^4+Cx^8\right) \, dx = Aa^3x + \frac{Cc^3x^{21}}{21} + x^{17}\left(\frac{Bc^3}{17} + \frac{3Cac^2}{17}\right) + x^{13}\left(\frac{Ac^3}{13} + \frac{3Bac^2}{13} + \frac{3Ca^2c}{13}\right) + x^9\left(\frac{Aac^2}{3} + \frac{Ba^2c}{3} + \frac{Ca^3}{9}\right) + x^5 \cdot \left(\frac{3Aa^2c}{5} + \frac{Ba^3}{5}\right)$$

input
```
integrate((c*x**4+a)**3*(C*x**8+B*x**4+A),x)
```

output
```
A*a**3*x + C*c**3*x**21/21 + x**17*(B*c**3/17 + 3*C*a*c**2/17) + x**13*(A*
c**3/13 + 3*B*a*c**2/13 + 3*C*a**2*c/13) + x**9*(A*a*c**2/3 + B*a**2*c/3 +
 C*a**3/9) + x**5*(3*A*a**2*c/5 + B*a**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int (a+cx^4)^3 (A+Bx^4+Cx^8) \, dx = \frac{1}{21} Cc^3x^{21} + \frac{1}{17} (3 Cac^2 + Bc^3)x^{17} + \frac{1}{13} (3 Ca^2c + 3 Bac^2 + Ac^3)x^{13} + \frac{1}{9} (Ca^3 + 3 Ba^2c + 3 Aac^2)x^9 + \frac{1}{5} (Ba^3 + 3 Aa^2c)x^5 + Aa^3x$$

input
```
integrate((c*x^4+a)^3*(C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
1/21*C*c^3*x^21 + 1/17*(3*C*a*c^2 + B*c^3)*x^17 + 1/13*(3*C*a^2*c + 3*B*a*
c^2 + A*c^3)*x^13 + 1/9*(C*a^3 + 3*B*a^2*c + 3*A*a*c^2)*x^9 + 1/5*(B*a^3 +
 3*A*a^2*c)*x^5 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int (a+cx^4)^3 (A+Bx^4+Cx^8) \, dx = \frac{1}{21} Cc^3x^{21} + \frac{3}{17} Cac^2x^{17} + \frac{1}{17} Bc^3x^{17} + \frac{3}{13} Ca^2cx^{13} + \frac{3}{13} Bac^2x^{13} + \frac{1}{13} Ac^3x^{13} + \frac{1}{9} Ca^3x^9 + \frac{1}{3} Ba^2cx^9 + \frac{1}{3} Aac^2x^9 + \frac{1}{5} Ba^3x^5 + \frac{3}{5} Aa^2cx^5 + Aa^3x$$

input
```
integrate((c*x^4+a)^3*(C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
1/21*C*c^3*x^21 + 3/17*C*a*c^2*x^17 + 1/17*B*c^3*x^17 + 3/13*C*a^2*c*x^13
+ 3/13*B*a*c^2*x^13 + 1/13*A*c^3*x^13 + 1/9*C*a^3*x^9 + 1/3*B*a^2*c*x^9 +
1/3*A*a*c^2*x^9 + 1/5*B*a^3*x^5 + 3/5*A*a^2*c*x^5 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int (a+cx^4)^3 (A+Bx^4+Cx^8)\ dx = x^9\left(\frac{C\,a^3}{9}+\frac{B\,a^2\,c}{3}+\frac{A\,a\,c^2}{3}\right) + x^{13}\left(\frac{3\,C\,a^2\,c}{13}+\frac{3\,B\,a\,c^2}{13}+\frac{A\,c^3}{13}\right) + x^5\left(\frac{B\,a^3}{5}+\frac{3\,A\,c\,a^2}{5}\right) + x^{17}\left(\frac{B\,c^3}{17}+\frac{3\,C\,a\,c^2}{17}\right)+\frac{C\,c^3\,x^{21}}{21}+A\,a^3\,x$$

input
```
int((a + c*x^4)^3*(A + B*x^4 + C*x^8),x)
```

output
```
x^9*((C*a^3)/9 + (A*a*c^2)/3 + (B*a^2*c)/3) + x^13*((A*c^3)/13 + (3*B*a*c^
2)/13 + (3*C*a^2*c)/13) + x^5*((B*a^3)/5 + (3*A*a^2*c)/5) + x^17*((B*c^3)/
17 + (3*C*a*c^2)/17) + (C*c^3*x^21)/21 + A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int (a+cx^4)^3 (A+Bx^4+Cx^8)\ dx = \frac{x(3315c^4x^{20}+12285a\,c^3x^{16}+4095b\,c^3x^{16}+16065a^2c^2x^{12}+16065ab\,c^2x^{12}+5355a\,c^3x^{12}+7735a^3c\,x^8 -}{69615}$$

input
```
int((c*x^4+a)^3*(C*x^8+B*x^4+A),x)
```

output
```
(x*(69615*a**4 + 13923*a**3*b*x**4 + 7735*a**3*c*x**8 + 41769*a**3*c*x**4
+ 23205*a**2*b*c*x**8 + 16065*a**2*c**2*x**12 + 23205*a**2*c**2*x**8 + 160
65*a*b*c**2*x**12 + 12285*a*c**3*x**16 + 5355*a*c**3*x**12 + 4095*b*c**3*x
**16 + 3315*c**4*x**20))/69615
```

3.29 $\int (a+cx^4)^2 (A+Bx^4+Cx^8) \, dx$

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Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (a+cx^4)^2 (A+Bx^4+Cx^8) \, dx = a^2Ax + \frac{1}{5}a(aB+2Ac)x^5 + \frac{1}{9}(2aBc+Ac^2+a^2C)x^9 + \frac{1}{13}c(Bc+2aC)x^{13} + \frac{1}{17}c^2Cx^{17}$$

output
```
a^2*A*x+1/5*a*(2*A*c+B*a)*x^5+1/9*(A*c^2+2*B*a*c+C*a^2)*x^9+1/13*c*(B*c+2*
C*a)*x^13+1/17*c^2*C*x^17
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (a+cx^4)^2 (A+Bx^4+Cx^8) \, dx = a^2Ax + \frac{1}{5}a(aB+2Ac)x^5 + \frac{1}{9}(2aBc+Ac^2+a^2C)x^9 + \frac{1}{13}c(Bc+2aC)x^{13} + \frac{1}{17}c^2Cx^{17}$$

input
```
Integrate[(a + c*x^4)^2*(A + B*x^4 + C*x^8),x]
```

output
```
a^2*A*x + (a*(a*B + 2*A*c)*x^5)/5 + ((2*a*B*c + A*c^2 + a^2*C)*x^9)/9 + (c
*(B*c + 2*a*C)*x^13)/13 + (c^2*C*x^17)/17
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 \left(A+Bx^4+Cx^8\right) \, dx$$

$$\Big\downarrow \; 1737$$

$$\int \left(x^8 \left(a^2 C+2 a B c+A c^2\right)+a^2 A+a x^4 (a B+2 A c)+c x^{12} (2 a C+B c)+c^2 C x^{16}\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{9} x^9 \left(a^2 C+2 a B c+A c^2\right)+a^2 A x+\frac{1}{5} a x^5 (a B+2 A c)+\frac{1}{13} c x^{13} (2 a C+B c)+\frac{1}{17} c^2 C x^{17}$$

input
```
Int[(a + c*x^4)^2*(A + B*x^4 + C*x^8),x]
```

output

```
a^2*A*x + (a*(a*B + 2*A*c)*x^5)/5 + ((2*a*B*c + A*c^2 + a^2*C)*x^9)/9 + (c
*(B*c + 2*a*C)*x^13)/13 + (c^2*C*x^17)/17
```


Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
default	$\frac{c^2 C x^{17}}{17}+\frac{(c^2 B+2acC)x^{13}}{13}+\frac{(A c^2+2aBc+a^2 C)x^9}{9}+\frac{(2Aac+a^2 B)x^5}{5}+a^2 A x$
norman	$a^2 A x+\left(\frac{2}{5}Aac+\frac{1}{5}a^2 B\right)x^5+\left(\frac{1}{9}A c^2+\frac{2}{9}aBc+\frac{1}{9}a^2 C\right)x^9+\left(\frac{1}{13}c^2 B+\frac{2}{13}acC\right)x^{13}+\frac{c^2 C x^{17}}{17}$
gosper	$a^2 A x+\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 B+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 aBc+\frac{1}{9}x^9 a^2 C+\frac{1}{13}x^{13}c^2 B+\frac{2}{13}x^{13}acC+\frac{1}{17}c^2$
risch	$a^2 A x+\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 B+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 aBc+\frac{1}{9}x^9 a^2 C+\frac{1}{13}x^{13}c^2 B+\frac{2}{13}x^{13}acC+\frac{1}{17}c^2$
parallelrisch	$a^2 A x+\frac{2}{5}x^5 Aac+\frac{1}{5}x^5 a^2 B+\frac{1}{9}x^9 A c^2+\frac{2}{9}x^9 aBc+\frac{1}{9}x^9 a^2 C+\frac{1}{13}x^{13}c^2 B+\frac{2}{13}x^{13}acC+\frac{1}{17}c^2$
orering	$\frac{x(585c^2 C x^{16}+765B c^2 x^{12}+1530Cac x^{12}+1105A c^2 x^8+2210Bac x^8+1105C a^2 x^8+3978Aac x^4+1989B a^2 x^4+9945a^2 A)}{9945}$

input
```
int((c*x^4+a)^2*(C*x^8+B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/17*c^2*C*x^17+1/13*(B*c^2+2*C*a*c)*x^13+1/9*(A*c^2+2*B*a*c+C*a^2)*x^9+1/
5*(2*A*a*c+B*a^2)*x^5+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (a+cx^4)^2 (A+Bx^4+Cx^8)\,dx = \frac{1}{17}Cc^2x^{17}+\frac{1}{13}(2\,Cac+Bc^2)x^{13}+\frac{1}{9}(Ca^2+2\,Bac+Ac^2)x^9+\frac{1}{5}(Ba^2+2\,Aac)x^5+Aa^2x$$

input
```
integrate((c*x^4+a)^2*(C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
1/17*C*c^2*x^17 + 1/13*(2*C*a*c + B*c^2)*x^13 + 1/9*(C*a^2 + 2*B*a*c + A*c
^2)*x^9 + 1/5*(B*a^2 + 2*A*a*c)*x^5 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int \left(a+cx^4\right)^2 \left(A+Bx^4+Cx^8\right)\,dx = Aa^2x+\frac{Cc^2x^{17}}{17}+x^{13}\left(\frac{Bc^2}{13}+\frac{2Cac}{13}\right) +x^9\left(\frac{Ac^2}{9}+\frac{2Bac}{9}+\frac{Ca^2}{9}\right)+x^5\cdot\left(\frac{2Aac}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate((c*x**4+a)**2*(C*x**8+B*x**4+A),x)
```

output
```
A*a**2*x + C*c**2*x**17/17 + x**13*(B*c**2/13 + 2*C*a*c/13) + x**9*(A*c**2
/9 + 2*B*a*c/9 + C*a**2/9) + x**5*(2*A*a*c/5 + B*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(a+cx^4\right)^2 \left(A+Bx^4+Cx^8\right)\,dx = \frac{1}{17}\,Cc^2x^{17}+\frac{1}{13}\,(2\,Cac+Bc^2)x^{13} +\frac{1}{9}\,(Ca^2+2\,Bac+Ac^2)x^9 +\frac{1}{5}\,(Ba^2+2\,Aac)x^5+Aa^2x$$

input
```
integrate((c*x^4+a)^2*(C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
1/17*C*c^2*x^17 + 1/13*(2*C*a*c + B*c^2)*x^13 + 1/9*(C*a^2 + 2*B*a*c + A*c
^2)*x^9 + 1/5*(B*a^2 + 2*A*a*c)*x^5 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \left(a+cx^4\right)^2 \left(A+Bx^4+Cx^8\right)\,dx=\frac{1}{17}Cc^2x^{17}+\frac{2}{13}Cacx^{13}+\frac{1}{13}Bc^2x^{13}+\frac{1}{9}Ca^2x^9+\frac{2}{9}Bacx^9+\frac{1}{9}Ac^2x^9+\frac{1}{5}Ba^2x^5+\frac{2}{5}Aacx^5+Aa^2x$$

input
```
integrate((c*x^4+a)^2*(C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
1/17*C*c^2*x^17 + 2/13*C*a*c*x^13 + 1/13*B*c^2*x^13 + 1/9*C*a^2*x^9 + 2/9*
B*a*c*x^9 + 1/9*A*c^2*x^9 + 1/5*B*a^2*x^5 + 2/5*A*a*c*x^5 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(a+cx^4\right)^2 \left(A+Bx^4+Cx^8\right)\,dx=x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,c\,a}{5}\right)+x^{13}\left(\frac{B\,c^2}{13}+\frac{2\,C\,a\,c}{13}\right)+x^9\left(\frac{C\,a^2}{9}+\frac{2\,B\,a\,c}{9}+\frac{A\,c^2}{9}\right)+\frac{C\,c^2\,x^{17}}{17}+A\,a^2\,x$$

input
```
int((a + c*x^4)^2*(A + B*x^4 + C*x^8),x)
```

output
```
x^5*((B*a^2)/5 + (2*A*a*c)/5) + x^13*((B*c^2)/13 + (2*C*a*c)/13) + x^9*((A
*c^2)/9 + (C*a^2)/9 + (2*B*a*c)/9) + (C*c^2*x^17)/17 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int (a+cx^4)^2 (A+Bx^4+Cx^8)\, dx$$
$$= \frac{x(585c^3x^{16}+1530a\,c^2x^{12}+765b\,c^2x^{12}+1105a^2c\,x^8+2210abc\,x^8+1105a\,c^2x^8+1989a^2b\,x^4+3978a^2c}{9945}$$

input
```
int((c*x^4+a)^2*(C*x^8+B*x^4+A),x)
```

output
```
(x*(9945*a**3 + 1989*a**2*b*x**4 + 1105*a**2*c*x**8 + 3978*a**2*c*x**4 + 2
210*a*b*c*x**8 + 1530*a*c**2*x**12 + 1105*a*c**2*x**8 + 765*b*c**2*x**12 +
 585*c**3*x**16))/9945
```

3.30 $\int (a+cx^4)(A+Bx^4+Cx^8)\,dx$

Optimal result 282
Mathematica [A] (verified) 282
Rubi [A] (verified) 283
Maple [A] (verified) 284
Fricas [A] (verification not implemented) 284
Sympy [A] (verification not implemented) 285
Maxima [A] (verification not implemented) 285
Giac [A] (verification not implemented) 285
Mupad [B] (verification not implemented) 286
Reduce [B] (verification not implemented) 286

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int (a+cx^4)(A+Bx^4+Cx^8)\,dx = aAx+\frac{1}{5}(aB+Ac)x^5+\frac{1}{9}(Bc+aC)x^9+\frac{1}{13}cCx^{13}$$

output
```
a*A*x+1/5*(A*c+B*a)*x^5+1/9*(B*c+C*a)*x^9+1/13*c*C*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (a+cx^4)(A+Bx^4+Cx^8)\,dx = aAx+\frac{1}{5}(aB+Ac)x^5+\frac{1}{9}(Bc+aC)x^9+\frac{1}{13}cCx^{13}$$

input
```
Integrate[(a + c*x^4)*(A + B*x^4 + C*x^8),x]
```

output
```
a*A*x + ((a*B + A*c)*x^5)/5 + ((B*c + a*C)*x^9)/9 + (c*C*x^13)/13
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4) (A+Bx^4+Cx^8) \, dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(x^4(aB+Ac)+aA+x^8(aC+Bc)+cCx^{12}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aB+Ac)+aAx+\frac{1}{9}x^9(aC+Bc)+\frac{1}{13}cCx^{13}$$

input
```
Int[(a + c*x^4)*(A + B*x^4 + C*x^8),x]
```

output
```
a*A*x + ((a*B + A*c)*x^5)/5 + ((B*c + a*C)*x^9)/9 + (c*C*x^13)/13
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$aAx+\frac{(Ac+Ba)x^5}{5}+\frac{(Bc+Ca)x^9}{9}+\frac{cC\,x^{13}}{13}$	37
norman	$\frac{cC\,x^{13}}{13}+\left(\frac{Bc}{9}+\frac{Ca}{9}\right)x^9+\left(\frac{Ac}{5}+\frac{Ba}{5}\right)x^5+aAx$	39
gosper	$\frac{1}{13}cC\,x^{13}+\frac{1}{9}x^9Bc+\frac{1}{9}x^9Ca+\frac{1}{5}x^5Ac+\frac{1}{5}x^5Ba+aAx$	41
risch	$\frac{1}{13}cC\,x^{13}+\frac{1}{9}x^9Bc+\frac{1}{9}x^9Ca+\frac{1}{5}x^5Ac+\frac{1}{5}x^5Ba+aAx$	41
parallelrisch	$\frac{1}{13}cC\,x^{13}+\frac{1}{9}x^9Bc+\frac{1}{9}x^9Ca+\frac{1}{5}x^5Ac+\frac{1}{5}x^5Ba+aAx$	41
orering	$\frac{x(45Cc\,x^{12}+65Bc\,x^8+65Ca\,x^8+117Ac\,x^4+117Ba\,x^4+585Aa)}{585}$	44

input
```
int((c*x^4+a)*(C*x^8+B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/5*(A*c+B*a)*x^5+1/9*(B*c+C*a)*x^9+1/13*c*C*x^13
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int(a+cx^4)\left(A+Bx^4+Cx^8\right)\,dx=\frac{1}{13}Ccx^{13}+\frac{1}{9}(Ca+Bc)x^9+\frac{1}{5}(Ba+Ac)x^5+Aax$$

input
```
integrate((c*x^4+a)*(C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
1/13*C*c*x^13 + 1/9*(C*a + B*c)*x^9 + 1/5*(B*a + A*c)*x^5 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int (a+cx^4)\left(A+Bx^4+Cx^8\right)dx = Aax + \frac{Ccx^{13}}{13} + x^9\left(\frac{Bc}{9}+\frac{Ca}{9}\right) + x^5\left(\frac{Ac}{5}+\frac{Ba}{5}\right)$$

input
```
integrate((c*x**4+a)*(C*x**8+B*x**4+A),x)
```

output
```
A*a*x + C*c*x**13/13 + x**9*(B*c/9 + C*a/9) + x**5*(A*c/5 + B*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (a+cx^4)\left(A+Bx^4+Cx^8\right)dx = \frac{1}{13}Ccx^{13} + \frac{1}{9}(Ca+Bc)x^9 + \frac{1}{5}(Ba+Ac)x^5 + Aax$$

input
```
integrate((c*x^4+a)*(C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
1/13*C*c*x^13 + 1/9*(C*a + B*c)*x^9 + 1/5*(B*a + A*c)*x^5 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int (a+cx^4)\left(A+Bx^4+Cx^8\right)dx = \frac{1}{13}Ccx^{13} + \frac{1}{9}Cax^9 + \frac{1}{9}Bcx^9 + \frac{1}{5}Bax^5 + \frac{1}{5}Acx^5 + Aax$$

input
```
integrate((c*x^4+a)*(C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
1/13*C*c*x^13 + 1/9*C*a*x^9 + 1/9*B*c*x^9 + 1/5*B*a*x^5 + 1/5*A*c*x^5 + A*
a*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \left(a+cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx=\frac{C\,c\,x^{13}}{13}+\left(\frac{C\,a}{9}+\frac{B\,c}{9}\right)\,x^9+\left(\frac{B\,a}{5}+\frac{A\,c}{5}\right)\,x^5+A\,a\,x$$

input
```
int((a + c*x^4)*(A + B*x^4 + C*x^8),x)
```

output
```
x^5*((B*a)/5 + (A*c)/5) + x^9*((C*a)/9 + (B*c)/9) + A*a*x + (C*c*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \left(a+cx^4\right)\left(A+Bx^4+Cx^8\right)\,dx$$
$$=\frac{x(45c^2x^{12}+65ac\,x^8+65bc\,x^8+117ab\,x^4+117ac\,x^4+585a^2)}{585}$$

input
```
int((c*x^4+a)*(C*x^8+B*x^4+A),x)
```

output
```
(x*(585*a**2 + 117*a*b*x**4 + 65*a*c*x**8 + 117*a*c*x**4 + 65*b*c*x**8 + 4
5*c**2*x**12))/585
```

3.31 $\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx$

Optimal result 287
Mathematica [A] (verified) 288
Rubi [A] (verified) 288
Maple [C] (verified) 293
Fricas [C] (verification not implemented) 293
Sympy [A] (verification not implemented) 294
Maxima [A] (verification not implemented) 295
Giac [B] (verification not implemented) 295
Mupad [B] (verification not implemented) 297
Reduce [B] (verification not implemented) 297

Optimal result

Integrand size = 22, antiderivative size = 209

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx = \frac{(Bc-aC)x}{c^2}+\frac{Cx^5}{5c} + \frac{(aBc-Ac^2-a^2C)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} - \frac{(aBc-Ac^2-a^2C)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{9/4}} - \frac{(aBc-Ac^2-a^2C)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{9/4}}$$

output

```
(B*c-C*a)*x/c^2+1/5*C*x^5/c-1/4*(-A*c^2+B*a*c-C*a^2)*arctan(-1+2^(1/2)*c^(
1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(9/4)-1/4*(-A*c^2+B*a*c-C*a^2)*arctan(1+
2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(9/4)-1/4*(-A*c^2+B*a*c-C*a^2
)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)
/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx$$
$$= \frac{40\sqrt[4]{c}(Bc-aC)x+8c^{5/4}Cx^5-\frac{10\sqrt{2}(-aBc+Ac^2+a^2C)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{10\sqrt{2}(-aBc+Ac^2+a^2C)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{a}}\right)}{a^{3/4}}}{40c^{9/4}}$$

input
```
Integrate[(A + B*x^4 + C*x^8)/(a + c*x^4),x]
```

output
```
(40*c^(1/4)*(B*c - a*C)*x + 8*c^(5/4)*C*x^5 - (10*Sqrt[2]*(-(a*B*c) + A*c^
2 + a^2*C)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (10*Sqrt[2]*
(-(a*B*c) + A*c^2 + a^2*C)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4
) - (5*Sqrt[2]*(-(a*B*c) + A*c^2 + a^2*C)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (5*Sqrt[2]*(-(a*B*c) + A*c^2 + a^2*C)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(40*c^(9/4)
)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1741, 27, 913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx$$
$$\Big\downarrow \text{1741}$$
$$\frac{\int \frac{5((Bc-aC)x^4+Ac)}{cx^4+a}dx}{5c}+\frac{Cx^5}{5c}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(Bc-aC)x^4+Ac}{cx^4+a}dx}{c}+\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{913}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\int \frac{1}{cx^4+a}dx}{c}}{c}+\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{c}}{c}+\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{c}}{c}+\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}}{c}+\frac{Cx^5}{5c}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}}{c}+\frac{Cx^5}{5c}$$

$\Big\downarrow$ 1479

$$\frac{\frac{x(Bc-aC)}{c}-\frac{(a^2(-C)+aBc-Ac^2)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{a}}\cdots\right.}{c}}{c}$$

$$\frac{Cx^5}{5c}$$

$\Big\downarrow$ 25

$$\frac{\frac{x(Bc-aC)}{c}-\frac{(a^2(-C)+aBc-Ac^2)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\cdots\right.}{c}}{c}$$

$$\frac{Cx^5}{5c}$$

$\Big\downarrow$ 27

$$\frac{\frac{x(Bc-aC)}{c}-\frac{(a^2(-C)+aBc-Ac^2)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}}{c}+$$

$$\frac{Cx^5}{5c}$$

$\Big\downarrow$ 1103

$$\frac{\frac{x(Bc-aC)}{c}-\frac{(a^2(-C)+aBc-Ac^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right.}{c}}{c}$$

$$\frac{Cx^5}{5c}$$

input
```
Int[(A + B*x^4 + C*x^8)/(a + c*x^4),x]
```

output
```
(C*x^5)/(5*c) + (((B*c - a*C)*x)/c - ((a*B*c - A*c^2 - a^2*C)*((-(ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^
(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]
*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.32

method	result
risch	$\frac{C x^5}{5c}+\frac{Bx}{c}-\frac{Cax}{c^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(A c^2-aBc+a^2C\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^3}$
default	$\frac{\frac{1}{5}C x^5 c+Bcx-Cax}{c^2}+\frac{(A c^2-aBc+a^2C)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2a}$

input
```
int((C*x^8+B*x^4+A)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*C*x^5/c+1/c*B*x-1/c^2*C*a*x+1/4/c^3*sum((A*c^2-B*a*c+C*a^2)/_R^3*ln(x-
_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 1419, normalized size of antiderivative = 6.79

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/20*(4*C*c*x^5 + 5*c^2*(-(C^4*a^8 - 4*B*C^3*a^7*c - 4*A^3*B*a*c^7 + A^4*c
^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 - 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^
4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^4 - 4*(A*B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3
*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*c^9))^(1/4)*log(a*c^2*(-(C^4*a^8 - 4*B*C
^3*a^7*c - 4*A^3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 - 4*(
B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^4 - 4*(A
*B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*c^9))^(1
/4) + (C*a^2 - B*a*c + A*c^2)*x) + 5*I*c^2*(-(C^4*a^8 - 4*B*C^3*a^7*c - 4*
A^3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 - 4*(B^3*C + 3*A*B
*C^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^4 - 4*(A*B^3 + 3*A^2*
B*C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*c^9))^(1/4)*log(I*a*c
^2*(-(C^4*a^8 - 4*B*C^3*a^7*c - 4*A^3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2
*A*C^3)*a^6*c^2 - 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^
2*C^2)*a^4*c^4 - 4*(A*B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a
^2*c^6)/(a^3*c^9))^(1/4) + (C*a^2 - B*a*c + A*c^2)*x) - 5*I*c^2*(-(C^4*a^8
 - 4*B*C^3*a^7*c - 4*A^3*B*a*c^7 + A^4*c^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c
^2 - 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 + (B^4 + 12*A*B^2*C + 6*A^2*C^2)*a^4*c^
4 - 4*(A*B^3 + 3*A^2*B*C)*a^3*c^5 + 2*(3*A^2*B^2 + 2*A^3*C)*a^2*c^6)/(a^3*
c^9))^(1/4)*log(-I*a*c^2*(-(C^4*a^8 - 4*B*C^3*a^7*c - 4*A^3*B*a*c^7 + A^4*
c^8 + 2*(3*B^2*C^2 + 2*A*C^3)*a^6*c^2 - 4*(B^3*C + 3*A*B*C^2)*a^5*c^3 +...
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx = \frac{Cx^5}{5c}+x\left(\frac{B}{c}-\frac{Ca}{c^2}\right)$$
$$+\text{RootSum}\left(256t^4a^3c^9+A^4c^8-4A^3Bac^7+4A^3Ca^2c^6+6A^2B^2a^2c^6-12A^2BCa^3c^5+6A^2C^2a^4c^4-4\right.$$

input
```
integrate((C*x**8+B*x**4+A)/(c*x**4+a),x)
```

output
```
C*x**5/(5*c) + x*(B/c - C*a/c**2) + RootSum(256*_t**4*a**3*c**9 + A**4*c**
8 - 4*A**3*B*a*c**7 + 4*A**3*C*a**2*c**6 + 6*A**2*B**2*a**2*c**6 - 12*A**2
*B*C*a**3*c**5 + 6*A**2*C**2*a**4*c**4 - 4*A*B**3*a**3*c**5 + 12*A*B**2*C*
a**4*c**4 - 12*A*B*C**2*a**5*c**3 + 4*A*C**3*a**6*c**2 + B**4*a**4*c**4 -
4*B**3*C*a**5*c**3 + 6*B**2*C**2*a**6*c**2 - 4*B*C**3*a**7*c + C**4*a**8,
Lambda(_t, _t*log(4*_t*a*c**2/(A*c**2 - B*a*c + C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx = \frac{Ccx^5-5\,(Ca-Bc)x}{5\,c^2} + \frac{\frac{2\sqrt{2}(Ca^2-Bac+Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}(Ca^2-Bac+Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}(Ca^2-Bac+Ac}{}}{8\,c^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/5*(C*c*x^5 - 5*(C*a - B*c)*x)/c^2 + 1/8*(2*sqrt(2)*(C*a^2 - B*a*c + A*c^
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)
*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*(C*a^2 - B*a*c + A*
c^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(
a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*(C*a^2 - B*a*c + A*
c^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/
4)) - sqrt(2)*(C*a^2 - B*a*c + A*c^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^
(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(158) = 316$.

Time = 0.13 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx$$
$$= \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac+(ac^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}$$
$$+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac+(ac^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3}$$
$$+ \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac+(ac^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$
$$- \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac+(ac^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$
$$+ \frac{Cc^4x^5-5\,Cac^3x+5\,Bc^4x}{5\,c^5}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4)*B*a*c + (a*c^3)^(1/4)*A*c
^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) +
1/4*sqrt(2)*((a*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4)*B*a*c + (a*c^3)^(1/4)*A*c
^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) +
1/8*sqrt(2)*((a*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4)*B*a*c + (a*c^3)^(1/4)*A*c
^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a
*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4)*B*a*c + (a*c^3)^(1/4)*A*c^2)*log(x^2 - s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/5*(C*c^4*x^5 - 5*C*a*c^3*x +
 5*B*c^4*x)/c^5
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 1170, normalized size of antiderivative = 5.60

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4 + C*x^8)/(a + c*x^4),x)
```

output
```
x*(B/c - (C*a)/c^2) + (C*x^5)/(5*c) + (atan((((4*x*(A^2*c^4 + C^2*a^4 + B
^2*a^2*c^2 - 2*A*B*a*c^3 - 2*B*C*a^3*c + 2*A*C*a^2*c^2))/c - ((16*A*a*c^3
- 16*B*a^2*c^2 + 16*C*a^3*c)*(A*c^2 + C*a^2 - B*a*c))/(4*(-a)^(3/4)*c^(9/4
)))*(A*c^2 + C*a^2 - B*a*c)*1i)/(4*(-a)^(3/4)*c^(9/4)) + (((4*x*(A^2*c^4 +
 C^2*a^4 + B^2*a^2*c^2 - 2*A*B*a*c^3 - 2*B*C*a^3*c + 2*A*C*a^2*c^2))/c + (
(16*A*a*c^3 - 16*B*a^2*c^2 + 16*C*a^3*c)*(A*c^2 + C*a^2 - B*a*c))/(4*(-a)^
(3/4)*c^(9/4)))*(A*c^2 + C*a^2 - B*a*c)*1i)/(4*(-a)^(3/4)*c^(9/4)))/((((4*
x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 - 2*A*B*a*c^3 - 2*B*C*a^3*c + 2*A*C*a^2
*c^2))/c - ((16*A*a*c^3 - 16*B*a^2*c^2 + 16*C*a^3*c)*(A*c^2 + C*a^2 - B*a*
c))/(4*(-a)^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 - B*a*c))/(4*(-a)^(3/4)*c^(9/4)
) - (((4*x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 - 2*A*B*a*c^3 - 2*B*C*a^3*c +
2*A*C*a^2*c^2))/c + ((16*A*a*c^3 - 16*B*a^2*c^2 + 16*C*a^3*c)*(A*c^2 + C*a
^2 - B*a*c))/(4*(-a)^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 - B*a*c))/(4*(-a)^(3/4
)*c^(9/4))))*(A*c^2 + C*a^2 - B*a*c)*1i)/(2*(-a)^(3/4)*c^(9/4)) + (atan(((
((4*x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 - 2*A*B*a*c^3 - 2*B*C*a^3*c + 2*A*C
*a^2*c^2))/c - ((16*A*a*c^3 - 16*B*a^2*c^2 + 16*C*a^3*c)*(A*c^2 + C*a^2 -
B*a*c)*1i)/(4*(-a)^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 - B*a*c))/(4*(-a)^(3/4)*
c^(9/4)) + (((4*x*(A^2*c^4 + C^2*a^4 + B^2*a^2*c^2 - 2*A*B*a*c^3 - 2*B*C*a
^3*c + 2*A*C*a^2*c^2))/c + ((16*A*a*c^3 - 16*B*a^2*c^2 + 16*C*a^3*c)*(A*c^
2 + C*a^2 - B*a*c)*1i)/(4*(-a)^(3/4)*c^(9/4)))*(A*c^2 + C*a^2 - B*a*c))...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.07

$$\int \frac{A+Bx^4+Cx^8}{a+cx^4}\,dx = \frac{-10c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)b-10c^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)}{}$$

input
```
int((C*x^8+B*x^4+A)/(c*x^4+a),x)
```

output
```
( - 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((
c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b -
10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*c + 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 10*c
**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c
**(1/4)*a**(1/4)*sqrt(2)))*b + 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)
*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c - 5*c**(3/
4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)
*x**2)*a + 5*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x
+ sqrt(a) + sqrt(c)*x**2)*b - 5*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c + 5*c**(3/4)*a**(1/4)*sqrt(
2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a - 5*c**(3/4
)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**
2)*b + 5*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(
a) + sqrt(c)*x**2)*c - 40*a*c*x + 40*b*c*x + 8*c**2*x**5)/(40*c**2)
```

3.32 $\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2} \, dx$

Optimal result 299
Mathematica [A] (verified) 300
Rubi [A] (verified) 300
Maple [C] (verified) 305
Fricas [C] (verification not implemented) 306
Sympy [A] (verification not implemented) 307
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Giac [A] (verification not implemented) 308
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Reduce [B] (verification not implemented) 310

Optimal result

Integrand size = 22, antiderivative size = 228

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2} \, dx = \frac{Cx}{c^2} - \frac{(aBc-Ac^2-a^2C)\,x}{4ac^2\,(a+cx^4)} - \frac{(aBc+3Ac^2-5a^2C)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{9/4}} + \frac{(aBc+3Ac^2-5a^2C)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{9/4}} + \frac{(aBc+3Ac^2-5a^2C)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{9/4}}$$

output
```
C*x/c^2-1/4*(-A*c^2+B*a*c-C*a^2)*x/a/c^2/(c*x^4+a)+1/16*(3*A*c^2+B*a*c-5*C
*a^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(9/4)+1/16*(3
*A*c^2+B*a*c-5*C*a^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/
c^(9/4)+1/16*(3*A*c^2+B*a*c-5*C*a^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^
(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2}\,dx = \frac{32\sqrt[4]{c}Cx+\frac{8\sqrt[4]{c}(-aBc+Ac^2+a^2C)x}{a(a+cx^4)}+\frac{2\sqrt{2}(-aBc-3Ac^2+5a^2C)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}-\frac{2\sqrt{2}(-aBc-3Ac^2+5a^2C)\arctan\left(1+\cdots\right.}{a^{7/4}}\cdots}{32c^{9/4}\cdots}$$

input
```
Integrate[(A + B*x^4 + C*x^8)/(a + c*x^4)^2,x]
```

output
```
(32*c^(1/4)*C*x + (8*c^(1/4)*(-(a*B*c) + A*c^2 + a^2*C)*x)/(a*(a + c*x^4))
 + (2*Sqrt[2]*(-(a*B*c) - 3*A*c^2 + 5*a^2*C)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*(-(a*B*c) - 3*A*c^2 + 5*a^2*C)*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-(a*B*c) - 3*A*c^2 + 5*
a^2*C)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - (
Sqrt[2]*(-(a*B*c) - 3*A*c^2 + 5*a^2*C)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/
4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*c^(9/4))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1739, 25, 913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \ 1739$$

$$-\frac{\int -\frac{4acCx^4+3Ac^2+aBc-a^2C}{cx^4+a}dx}{4ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4acCx^4+3Ac^2+aBc-a^2C}{cx^4+a}dx}{4ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$\Big\downarrow$ 913

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\int \frac{1}{cx^4+a}dx+4aCx}{4ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$\Big\downarrow$ 755

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)+4aCx}{4ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)+4aCx}{4ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$\Big\downarrow$ 1082

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)+4aC}{4ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$\Big\downarrow$ 217

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)+4aCx}{4ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{a}}\cdots\right)\cdots}{4ac^2}\cdots$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\cdots\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\cdots\right)\cdots}{4ac^2}\cdots$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)+4aCx}{4ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{\left(-5a^2C+aBc+3Ac^2\right)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{4ac^2\left(a+cx^4\right)}$$

input

```
Int[(A + B*x^4 + C*x^8)/(a + c*x^4)^2,x]
```

output

```
-1/4*((a*B*c - A*c^2 - a^2*C)*x)/(a*c^2*(a + c*x^4)) + (4*a*C*x + (a*B*c +
 3*A*c^2 - 5*a^2*C)*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4
)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/(4*a*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.39

method	result
risch	$\frac{C x}{c^{2}}+\frac{\left(A\,c^{2}-a B c+a^{2} C\right) x}{4 a\,c^{2} \left(c\,x^{4}+a\right)}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c\,\_Z^{4}+a\right)}\frac{\left(3 A\,c^{2}+a B c-5 a^{2} C\right) \ln \left(x-\_R \right)}{\_R^{3}}}{16 c^{3} a}$
default	$\frac{C x}{c^{2}}+\frac{\frac{\left(A\,c^{2}-a B c+a^{2} C\right) x}{4 a \left(c\,x^{4}+a\right)}+\frac{\left(3 A\,c^{2}+a B c-5 a^{2} C\right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32 a^{2}}}{c^{2}}$

input
```
int((C*x^8+B*x^4+A)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x/c^2+1/4*(A*c^2-B*a*c+C*a^2)/a*x/c^2/(c*x^4+a)+1/16/c^3/a*sum((3*A*c^2+
B*a*c-5*C*a^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1519, normalized size of antiderivative = 6.66

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(16*C*a*c*x^5 + (a*c^3*x^4 + a^2*c^2)*(-(625*C^4*a^8 - 500*B*C^3*a^7*
c + 108*A^3*B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*c^2 - 20*(
B^3*C - 45*A*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)*a^4*c^4 +
 12*(A*B^3 - 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c^6)/(a^7*c
^9))^(1/4)*log(a^2*c^2*(-(625*C^4*a^8 - 500*B*C^3*a^7*c + 108*A^3*B*a*c^7
+ 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*c^2 - 20*(B^3*C - 45*A*B*C^2)*
a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)*a^4*c^4 + 12*(A*B^3 - 45*A^2*
B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c^6)/(a^7*c^9))^(1/4) - (5*C*a^
2 - B*a*c - 3*A*c^2)*x) - (-I*a*c^3*x^4 - I*a^2*c^2)*(-(625*C^4*a^8 - 500*
B*C^3*a^7*c + 108*A^3*B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*
c^2 - 20*(B^3*C - 45*A*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)
*a^4*c^4 + 12*(A*B^3 - 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c
^6)/(a^7*c^9))^(1/4)*log(I*a^2*c^2*(-(625*C^4*a^8 - 500*B*C^3*a^7*c + 108*
A^3*B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*A*C^3)*a^6*c^2 - 20*(B^3*C -
45*A*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 1350*A^2*C^2)*a^4*c^4 + 12*(A*B
^3 - 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 10*A^3*C)*a^2*c^6)/(a^7*c^9))^(1/
4) - (5*C*a^2 - B*a*c - 3*A*c^2)*x) - (I*a*c^3*x^4 + I*a^2*c^2)*(-(625*C^4
*a^8 - 500*B*C^3*a^7*c + 108*A^3*B*a*c^7 + 81*A^4*c^8 + 150*(B^2*C^2 - 10*
A*C^3)*a^6*c^2 - 20*(B^3*C - 45*A*B*C^2)*a^5*c^3 + (B^4 - 180*A*B^2*C + 13
50*A^2*C^2)*a^4*c^4 + 12*(A*B^3 - 45*A^2*B*C)*a^3*c^5 + 54*(A^2*B^2 - 1...
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2}\,dx=\frac{Cx}{c^2}+\frac{x(Ac^2-Bac+Ca^2)}{4a^2c^2+4ac^3x^4}$$
$$+\text{RootSum}\Big(65536t^4a^7c^9+81A^4c^8+108A^3Bac^7-540A^3Ca^2c^6+54A^2B^2a^2c^6-540A^2BCa^3c^5+135$$

input
```
integrate((C*x**8+B*x**4+A)/(c*x**4+a)**2,x)
```

output
```
C*x/c**2 + x*(A*c**2 - B*a*c + C*a**2)/(4*a**2*c**2 + 4*a*c**3*x**4) + Roo
tSum(65536*_t**4*a**7*c**9 + 81*A**4*c**8 + 108*A**3*B*a*c**7 - 540*A**3*C
*a**2*c**6 + 54*A**2*B**2*a**2*c**6 - 540*A**2*B*C*a**3*c**5 + 1350*A**2*C
**2*a**4*c**4 + 12*A*B**3*a**3*c**5 - 180*A*B**2*C*a**4*c**4 + 900*A*B*C**
2*a**5*c**3 - 1500*A*C**3*a**6*c**2 + B**4*a**4*c**4 - 20*B**3*C*a**5*c**3
 + 150*B**2*C**2*a**6*c**2 - 500*B*C**3*a**7*c + 625*C**4*a**8, Lambda(_t,
 _t*log(-16*_t*a**2*c**2/(-3*A*c**2 - B*a*c + 5*C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2}\,dx=\frac{(Ca^2-Bac+Ac^2)x}{4\,(ac^3x^4+a^2c^2)}+\frac{Cx}{c^2}$$
$$-\frac{\frac{2\sqrt{2}(5\,Ca^2-Bac-3\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}(5\,Ca^2-Bac-3\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}(5\,Ca^2-B}{}}{32\,ac^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(C*a^2 - B*a*c + A*c^2)*x/(a*c^3*x^4 + a^2*c^2) + C*x/c^2 - 1/32*(2*sq
rt(2)*(5*C*a^2 - B*a*c - 3*A*c^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2
)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c)))
+ 2*sqrt(2)*(5*C*a^2 - B*a*c - 3*A*c^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))) + sqrt(2)*(5*C*a^2 - B*a*c - 3*A*c^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*(5*C*a^2 - B*a*c - 3*
A*c^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
1/4)))/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^2}\,dx = \frac{Cx}{c^2} - \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac-3\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} - \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac-3\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} - \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac-3\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} + \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2-(ac^3)^{\frac{1}{4}}\,Bac-3\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} + \frac{Ca^2x-Bacx+Ac^2x}{4\,(cx^4+a)ac^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
C*x/c^2 - 1/16*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4)*B*a*c - 3*(a
*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1
/4))/(a^2*c^3) - 1/16*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4)*B*a*c
 - 3*(a*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a^2*c^3) - 1/32*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4
)*B*a*c - 3*(a*c^3)^(1/4)*A*c^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(a^2*c^3) + 1/32*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 - (a*c^3)^(1/4)*B*a*c
- 3*(a*c^3)^(1/4)*A*c^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2
*c^3) + 1/4*(C*a^2*x - B*a*c*x + A*c^2*x)/((c*x^4 + a)*a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1223, normalized size of antiderivative = 5.36

$$\int \frac{A + Bx^4 + Cx^8}{(a + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^4 + C*x^8)/(a + c*x^4)^2,x)
```

output
```
(C*x)/c^2 + (x*(A*c^2 + C*a^2 - B*a*c))/(4*a*(a*c^2 + c^3*x^4)) + (atan(((
(((12*A*c^3 + 4*B*a*c^2 - 20*C*a^2*c)*(3*A*c^2 - 5*C*a^2 + B*a*c))/(16*(-a
)^(7/4)*c^(9/4)) - (x*(9*A^2*c^4 + 25*C^2*a^4 + B^2*a^2*c^2 + 6*A*B*a*c^3
- 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c))*(3*A*c^2 - 5*C*a^2 + B*a*c)*1
i)/(16*(-a)^(7/4)*c^(9/4)) - ((((12*A*c^3 + 4*B*a*c^2 - 20*C*a^2*c)*(3*A*c
^2 - 5*C*a^2 + B*a*c))/(16*(-a)^(7/4)*c^(9/4)) + (x*(9*A^2*c^4 + 25*C^2*a^
4 + B^2*a^2*c^2 + 6*A*B*a*c^3 - 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c))
*(3*A*c^2 - 5*C*a^2 + B*a*c)*1i)/(16*(-a)^(7/4)*c^(9/4)))/(((((12*A*c^3 +
4*B*a*c^2 - 20*C*a^2*c)*(3*A*c^2 - 5*C*a^2 + B*a*c))/(16*(-a)^(7/4)*c^(9/4
)) - (x*(9*A^2*c^4 + 25*C^2*a^4 + B^2*a^2*c^2 + 6*A*B*a*c^3 - 10*B*C*a^3*c
 - 30*A*C*a^2*c^2))/(4*a^2*c))*(3*A*c^2 - 5*C*a^2 + B*a*c))/(16*(-a)^(7/4)
*c^(9/4)) + ((((12*A*c^3 + 4*B*a*c^2 - 20*C*a^2*c)*(3*A*c^2 - 5*C*a^2 + B*
a*c))/(16*(-a)^(7/4)*c^(9/4)) + (x*(9*A^2*c^4 + 25*C^2*a^4 + B^2*a^2*c^2 +
 6*A*B*a*c^3 - 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*c))*(3*A*c^2 - 5*C*a
^2 + B*a*c))/(16*(-a)^(7/4)*c^(9/4))))*(3*A*c^2 - 5*C*a^2 + B*a*c)*1i)/(8*
(-a)^(7/4)*c^(9/4)) + (atan((((((12*A*c^3 + 4*B*a*c^2 - 20*C*a^2*c)*(3*A*c
^2 - 5*C*a^2 + B*a*c)*1i)/(16*(-a)^(7/4)*c^(9/4)) - (x*(9*A^2*c^4 + 25*C^2
*a^4 + B^2*a^2*c^2 + 6*A*B*a*c^3 - 10*B*C*a^3*c - 30*A*C*a^2*c^2))/(4*a^2*
c))*(3*A*c^2 - 5*C*a^2 + B*a*c))/(16*(-a)^(7/4)*c^(9/4)) - ((((12*A*c^3 +
4*B*a*c^2 - 20*C*a^2*c)*(3*A*c^2 - 5*C*a^2 + B*a*c)*1i)/(16*(-a)^(7/4)*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.94

$$\int \frac{A + Bx^4 + Cx^8}{(a + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((C*x^8+B*x^4+A)/(c*x^4+a)^2,x)
```

output
```
(10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c
**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*b +
 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
*c - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b*c*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*
atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))
)*c**2*x**4 - 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 2*c**(3/4)*a**(1/4)*sq
rt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sq
rt(2)))*a*b - 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 6*c**(3/4)*a**(1/4
)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4
)*sqrt(2)))*a*c + 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b*c*x**4 + 6*c**(3/4)*a**(
1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(
1/4)*sqrt(2)))*c**2*x**4 + 5*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**
(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - c**(3/4)*a**(1/4)*sqrt...
```

3.33 $\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^3}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 262

$$\begin{aligned}\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^3}\,dx &= -\frac{(aBc-Ac^2-a^2C)\,x}{8ac^2\,(a+cx^4)^2}+\frac{(aBc+7Ac^2-9a^2C)\,x}{32a^2c^2\,(a+cx^4)} \\ &\quad -\frac{(3aBc+21Ac^2+5a^2C)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{9/4}} \\ &\quad +\frac{(3aBc+21Ac^2+5a^2C)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{9/4}} \\ &\quad +\frac{(3aBc+21Ac^2+5a^2C)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{11/4}c^{9/4}}\end{aligned}$$

output
```
-1/8*(-A*c^2+B*a*c-C*a^2)*x/a/c^2/(c*x^4+a)^2+1/32*(7*A*c^2+B*a*c-9*C*a^2)
*x/a^2/c^2/(c*x^4+a)+1/128*(21*A*c^2+3*B*a*c+5*C*a^2)*arctan(-1+2^(1/2)*c^
(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/c^(9/4)+1/128*(21*A*c^2+3*B*a*c+5*C*a^2)
*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/c^(9/4)+1/128*(21*A*
c^2+3*B*a*c+5*C*a^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^
2))*2^(1/2)/a^(11/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^3}\,dx$$
$$= \frac{\frac{32a^{7/4}\sqrt[4]{c}(-aBc+Ac^2+a^2C)x}{(a+cx^4)^2} - \frac{8a^{3/4}\sqrt[4]{c}(-aBc-7Ac^2+9a^2C)x}{a+cx^4} - 2\sqrt{2}(3aBc+21Ac^2+5a^2C)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right.}{}$$

input
```
Integrate[(A + B*x^4 + C*x^8)/(a + c*x^4)^3,x]
```

output
```
((32*a^(7/4)*c^(1/4)*(-(a*B*c) + A*c^2 + a^2*C)*x)/(a + c*x^4)^2 - (8*a^(3
/4)*c^(1/4)*(-(a*B*c) - 7*A*c^2 + 9*a^2*C)*x)/(a + c*x^4) - 2*Sqrt[2]*(3*a
*B*c + 21*A*c^2 + 5*a^2*C)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqr
t[2]*(3*a*B*c + 21*A*c^2 + 5*a^2*C)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
] - Sqrt[2]*(3*a*B*c + 21*A*c^2 + 5*a^2*C)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(3*a*B*c + 21*A*c^2 + 5*a^2*C)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(256*a^(11/4)*c^(9/4))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1739, 25, 910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{1739}$$
$$-\frac{\int -\frac{8acCx^4+7Ac^2+aBc-a^2C}{(cx^4+a)^2}dx}{8ac^2} - \frac{x(a^2(-C)+aBc-Ac^2)}{8ac^2(a+cx^4)^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8acCx^4+7Ac^2+aBc-a^2C}{(cx^4+a)^2}dx}{8ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{8ac^2\left(a+cx^4\right)^2}$$

$\downarrow$ 910

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\int\frac{1}{cx^4+a}dx}{4a}+\frac{x(-9a^2C+aBc+7Ac^2)}{4a(a+cx^4)}}{8ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{8ac^2\left(a+cx^4\right)^2}$$

$\downarrow$ 755

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int\frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x(-9a^2C+aBc+7Ac^2)}{4a(a+cx^4)}}{8ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{8ac^2\left(a+cx^4\right)^2}$$

$\downarrow$ 1476

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x(-9a^2C+aBc+7Ac^2)}{4a(a+cx^4)}}{8ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{8ac^2\left(a+cx^4\right)^2}$$

$\downarrow$ 1082

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x(-9a^2C+aBc-}{4a(a+cx^4}}{8ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{8ac^2\left(a+cx^4\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x(-9a^2C+aBc+7Ac^2)}{4a(a+cx^4)}}{8ac^2}-\frac{x(a^2(-C)+aBc-Ac^2)}{8ac^2(a+cx^4)^2}$$

$\downarrow$ 1479

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\cdots}{8ac^2}$$

$$\frac{x(a^2(-C)+aBc-Ac^2)}{8ac^2(a+cx^4)^2}$$

$\downarrow$ 25

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x(\cdots}{\cdots}}{8ac^2}$$

$$\frac{x(a^2(-C)+aBc-Ac^2)}{8ac^2(a+cx^4)^2}$$

$\downarrow$ 27

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x(-9a^2C+aBc+}{4a(a+cx^4)}}{8ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{8ac^2\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{(5a^2C+3aBc+21Ac^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x(-}{}}{8ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{8ac^2\left(a+cx^4\right)^2}$$

input
```
Int[(A + B*x^4 + C*x^8)/(a + c*x^4)^3,x]
```

output
```
-1/8*((a*B*c - A*c^2 - a^2*C)*x)/(a*c^2*(a + c*x^4)^2) + (((a*B*c + 7*A*c^
2 - 9*a^2*C)*x)/(4*a*(a + c*x^4)) + ((3*a*B*c + 21*A*c^2 + 5*a^2*C)*((-(Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[
1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) +
(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1
/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*S
qrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/(4*a))/(8*a*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{\left(7A\,c^{2}+aBc-9a^{2}C\right)x^{5}}{32a^{2}c}+\frac{\left(11A\,c^{2}-3aBc-5a^{2}C\right)x}{32a\,c^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(21A\,c^{2}+3aBc+5a^{2}C\right)\ln\left(x-\_R\right)}{\_R^{3}}}{128a^{2}c^{3}}$
default	$\frac{\frac{\left(7A\,c^{2}+aBc-9a^{2}C\right)x^{5}}{32a^{2}c}+\frac{\left(11A\,c^{2}-3aBc-5a^{2}C\right)x}{32a\,c^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{\left(21A\,c^{2}+3aBc+5a^{2}C\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.\right.}{256a^{3}c^{2}}$

input
```
int((C*x^8+B*x^4+A)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/32*(7*A*c^2+B*a*c-9*C*a^2)/a^2/c*x^5+1/32*(11*A*c^2-3*B*a*c-5*C*a^2)/a/
c^2*x)/(c*x^4+a)^2+1/128/a^2/c^3*sum((21*A*c^2+3*B*a*c+5*C*a^2)/_R^3*ln(x-
_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1640, normalized size of antiderivative = 6.26

$$\int \frac{A + Bx^4 + Cx^8}{(a + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(4*(9*C*a^2*c - B*a*c^2 - 7*A*c^3)*x^5 - (a^2*c^4*x^8 + 2*a^3*c^3*x
^4 + a^4*c^2)*(-(625*C^4*a^8 + 1500*B*C^3*a^7*c + 111132*A^3*B*a*c^7 + 194
481*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 + 540*(B^3*C + 35*A*B*C^2
)*a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 + 2268*(A*B^3
+ 35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^11*c^9))^(
1/4)*log(a^3*c^2*(-(625*C^4*a^8 + 1500*B*C^3*a^7*c + 111132*A^3*B*a*c^7 +
194481*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 + 540*(B^3*C + 35*A*B*
C^2)*a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 + 2268*(A*B
^3 + 35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^11*c^9)
)^(1/4) + (5*C*a^2 + 3*B*a*c + 21*A*c^2)*x) + (-I*a^2*c^4*x^8 - 2*I*a^3*c^
3*x^4 - I*a^4*c^2)*(-(625*C^4*a^8 + 1500*B*C^3*a^7*c + 111132*A^3*B*a*c^7
+ 194481*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 + 540*(B^3*C + 35*A*
B*C^2)*a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 + 2268*(A
*B^3 + 35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^11*c^
9))^(1/4)*log(I*a^3*c^2*(-(625*C^4*a^8 + 1500*B*C^3*a^7*c + 111132*A^3*B*a
*c^7 + 194481*A^4*c^8 + 150*(9*B^2*C^2 + 70*A*C^3)*a^6*c^2 + 540*(B^3*C +
35*A*B*C^2)*a^5*c^3 + 27*(3*B^4 + 420*A*B^2*C + 2450*A^2*C^2)*a^4*c^4 + 22
68*(A*B^3 + 35*A^2*B*C)*a^3*c^5 + 2646*(9*A^2*B^2 + 70*A^3*C)*a^2*c^6)/(a^
11*c^9))^(1/4) + (5*C*a^2 + 3*B*a*c + 21*A*c^2)*x) + (I*a^2*c^4*x^8 + 2*I*
a^3*c^3*x^4 + I*a^4*c^2)*(-(625*C^4*a^8 + 1500*B*C^3*a^7*c + 111132*A^3...
```

Sympy [A] (verification not implemented)

Time = 48.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^3}\,dx = \frac{x^5\cdot(7Ac^3+Bac^2-9Ca^2c)+x(11Aac^2-3Ba^2c-5Ca^3)}{32a^4c^2+64a^3c^3x^4+32a^2c^4x^8}$$
$$+\mathrm{RootSum}\Big(268435456t^4a^{11}c^9+194481A^4c^8+111132A^3Bac^7+185220A^3Ca^2c^6+23814A^2B^2a^2c^6+$$

input
```
integrate((C*x**8+B*x**4+A)/(c*x**4+a)**3,x)
```

output
```
(x**5*(7*A*c**3 + B*a*c**2 - 9*C*a**2*c) + x*(11*A*a*c**2 - 3*B*a**2*c - 5
*C*a**3))/(32*a**4*c**2 + 64*a**3*c**3*x**4 + 32*a**2*c**4*x**8) + RootSum
(268435456*_t**4*a**11*c**9 + 194481*A**4*c**8 + 111132*A**3*B*a*c**7 + 18
5220*A**3*C*a**2*c**6 + 23814*A**2*B**2*a**2*c**6 + 79380*A**2*B*C*a**3*c*
*5 + 66150*A**2*C**2*a**4*c**4 + 2268*A*B**3*a**3*c**5 + 11340*A*B**2*C*a*
*4*c**4 + 18900*A*B*C**2*a**5*c**3 + 10500*A*C**3*a**6*c**2 + 81*B**4*a**4
*c**4 + 540*B**3*C*a**5*c**3 + 1350*B**2*C**2*a**6*c**2 + 1500*B*C**3*a**7
*c + 625*C**4*a**8, Lambda(_t, _t*log(128*_t*a**3*c**2/(21*A*c**2 + 3*B*a*
c + 5*C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^3}\,dx = -\frac{(9\,Ca^2c-Bac^2-7\,Ac^3)x^5+(5\,Ca^3+3\,Ba^2c-11\,Aac^2)x}{32\,(a^2c^4x^8+2\,a^3c^3x^4+a^4c^2)}$$
$$+\frac{\frac{2\sqrt{2}(5\,Ca^2+3\,Bac+21\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}(5\,Ca^2+3\,Bac+21\,Ac^2)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}(5\,C}{}}{256\,a^2c^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*((9*C*a^2*c - B*a*c^2 - 7*A*c^3)*x^5 + (5*C*a^3 + 3*B*a^2*c - 11*A*a
*c^2)*x)/(a^2*c^4*x^8 + 2*a^3*c^3*x^4 + a^4*c^2) + 1/256*(2*sqrt(2)*(5*C*a
^2 + 3*B*a*c + 21*A*c^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)
*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(
2)*(5*C*a^2 + 3*B*a*c + 21*A*c^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2
)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c)))
+ sqrt(2)*(5*C*a^2 + 3*B*a*c + 21*A*c^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*(5*C*a^2 + 3*B*a*c + 21*
A*c^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
1/4)))/(a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^4+Cx^8}{(a+cx^4)^3}\,dx = \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2+3\,(ac^3)^{\frac{1}{4}}\,Bac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2+3\,(ac^3)^{\frac{1}{4}}\,Bac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2+3\,(ac^3)^{\frac{1}{4}}\,Bac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} - \frac{\sqrt{2}\left(5\,(ac^3)^{\frac{1}{4}}\,Ca^2+3\,(ac^3)^{\frac{1}{4}}\,Bac+21\,(ac^3)^{\frac{1}{4}}\,Ac^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} - \frac{9\,Ca^2cx^5-Bac^2x^5-7\,Ac^3x^5+5\,Ca^3x+3\,Ba^2cx-11\,Aac^2x}{32\,(cx^4+a)^2a^2c^2}$$

input
```
integrate((C*x^8+B*x^4+A)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 + 3*(a*c^3)^(1/4)*B*a*c + 21*(a*c^3)^
(1/4)*A*c^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(
a^3*c^3) + 1/128*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 + 3*(a*c^3)^(1/4)*B*a*c +
21*(a*c^3)^(1/4)*A*c^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/
c)^(1/4))/(a^3*c^3) + 1/256*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 + 3*(a*c^3)^(1/
4)*B*a*c + 21*(a*c^3)^(1/4)*A*c^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a^3*c^3) - 1/256*sqrt(2)*(5*(a*c^3)^(1/4)*C*a^2 + 3*(a*c^3)^(1/4)*B
*a*c + 21*(a*c^3)^(1/4)*A*c^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^3*c^3) - 1/32*(9*C*a^2*c*x^5 - B*a*c^2*x^5 - 7*A*c^3*x^5 + 5*C*a^3*x
+ 3*B*a^2*c*x - 11*A*a*c^2*x)/((c*x^4 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 1283, normalized size of antiderivative = 4.90

$$\int \frac{A+Bx^4+Cx^8}{\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4 + C*x^8)/(a + c*x^4)^3,x)
```

output
```
((x^5*(7*A*c^2 - 9*C*a^2 + B*a*c))/(32*a^2*c) - (x*(5*C*a^2 - 11*A*c^2 + 3
*B*a*c))/(32*a*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (atan((((((21*A*c^3 + 3
*B*a*c^2 + 5*C*a^2*c)*(21*A*c^2 + 5*C*a^2 + 3*B*a*c))/(256*(-a)^(15/4)*c^(
9/4)) - (x*(441*A^2*c^4 + 25*C^2*a^4 + 9*B^2*a^2*c^2 + 126*A*B*a*c^3 + 30*
B*C*a^3*c + 210*A*C*a^2*c^2))/(256*a^4*c))*(21*A*c^2 + 5*C*a^2 + 3*B*a*c)*
1i)/(128*(-a)^(11/4)*c^(9/4)) - ((((21*A*c^3 + 3*B*a*c^2 + 5*C*a^2*c)*(21*
A*c^2 + 5*C*a^2 + 3*B*a*c))/(256*(-a)^(15/4)*c^(9/4)) + (x*(441*A^2*c^4 +
25*C^2*a^4 + 9*B^2*a^2*c^2 + 126*A*B*a*c^3 + 30*B*C*a^3*c + 210*A*C*a^2*c^
2))/(256*a^4*c))*(21*A*c^2 + 5*C*a^2 + 3*B*a*c)*1i)/(128*(-a)^(11/4)*c^(9/
4)))/(((((21*A*c^3 + 3*B*a*c^2 + 5*C*a^2*c)*(21*A*c^2 + 5*C*a^2 + 3*B*a*c)
)/(256*(-a)^(15/4)*c^(9/4)) - (x*(441*A^2*c^4 + 25*C^2*a^4 + 9*B^2*a^2*c^2
 + 126*A*B*a*c^3 + 30*B*C*a^3*c + 210*A*C*a^2*c^2))/(256*a^4*c))*(21*A*c^2
 + 5*C*a^2 + 3*B*a*c))/(128*(-a)^(11/4)*c^(9/4)) + ((((21*A*c^3 + 3*B*a*c^
2 + 5*C*a^2*c)*(21*A*c^2 + 5*C*a^2 + 3*B*a*c))/(256*(-a)^(15/4)*c^(9/4)) +
 (x*(441*A^2*c^4 + 25*C^2*a^4 + 9*B^2*a^2*c^2 + 126*A*B*a*c^3 + 30*B*C*a^3
*c + 210*A*C*a^2*c^2))/(256*a^4*c))*(21*A*c^2 + 5*C*a^2 + 3*B*a*c))/(128*(
-a)^(11/4)*c^(9/4))))*(21*A*c^2 + 5*C*a^2 + 3*B*a*c)*1i)/(64*(-a)^(11/4)*c
^(9/4)) - (atan((((((21*A*c^3 + 3*B*a*c^2 + 5*C*a^2*c)*(21*A*c^2 + 5*C*a^2
 + 3*B*a*c)*1i)/(256*(-a)^(15/4)*c^(9/4)) - (x*(441*A^2*c^4 + 25*C^2*a^4 +
 9*B^2*a^2*c^2 + 126*A*B*a*c^3 + 30*B*C*a^3*c + 210*A*C*a^2*c^2))/(256*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1382, normalized size of antiderivative = 5.27

$$\int \frac{A + Bx^4 + Cx^8}{(a + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((C*x^8+B*x^4+A)/(c*x^4+a)^3,x)
```

output
```
( - 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*
*2*b - 20*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*x**4 - 42*c**(3/4)*a**(1/4)*s
qrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*s
qrt(2)))*a**2*c - 12*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*b*c*x**4 - 10*c**(3/4)*
a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*a*c**2*x**8 - 84*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/
4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**2*x**
4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*b*c**2*x**8 - 42*c**(3/4)*a**(1/4)*sqrt(
2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*c**3*x**8 + 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3 + 6*c**(3/4)*a**(1/4)
*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)
*sqrt(2)))*a**2*b + 20*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*x**4 + 42*c**(3/
4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**...
```

3.34 $\int (a+bx^4)^p (A+Bx^4)\, dx$

Optimal result . . . 325
Mathematica [A] (verified) . . . 325
Rubi [A] (verified) . . . 326
Maple [F] . . . 327
Fricas [F] . . . 327
Sympy [C] (verification not implemented) . . . 328
Maxima [F] . . . 328
Giac [F] . . . 329
Mupad [F(-1)] . . . 329
Reduce [F] . . . 329

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int (a+bx^4)^p (A+Bx^4)\, dx = \frac{Bx(a+bx^4)^{1+p}}{b(5+4p)} + \left(A-\frac{aB}{5b+4bp}\right) x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

output
```
B*x*(b*x^4+a)^(p+1)/b/(5+4*p)+(A-a*B/(4*b*p+5*b))*x*(b*x^4+a)^p*hypergeom(
[1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06

$$\int (a+bx^4)^p (A+Bx^4)\, dx = \frac{x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(B(a+bx^4)\left(1+\frac{bx^4}{a}\right)^p + (-aB+Ab(5+4p))\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},\right.\right.}{b(5+4p)}$$

input
```
Integrate[(a + b*x^4)^p*(A + B*x^4),x]
```

output
```
(x*(a + b*x^4)^p*(B*(a + b*x^4)*(1 + (b*x^4)/a)^p + (-(a*B) + A*b*(5 + 4*p
))*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)]))/(b*(5 + 4*p)*(1 + (b*x^
4)/a)^p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx^4\right)\left(a+bx^4\right)^p\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(A-\frac{aB}{4bp+5b}\right)\int \left(bx^4+a\right)^p dx+\frac{Bx\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{779}$$

$$\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(A-\frac{aB}{4bp+5b}\right)\int \left(\frac{bx^4}{a}+1\right)^p dx+\frac{Bx\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

$$\Big\downarrow \text{778}$$

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(A-\frac{aB}{4bp+5b}\right)\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{Bx\left(a+bx^4\right)^{p+1}}{b(4p+5)}$$

input
```
Int[(a + b*x^4)^p*(A + B*x^4),x]
```

output
```
(B*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((A - (a*B)/(5*b + 4*b*p))*x*(a
+ b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^
p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (b\,x^{4}+a)^{p} \left(B\,x^{4}+A\right) dx$$

input
```
int((b*x^4+a)^p*(B*x^4+A),x)
```

output
```
int((b*x^4+a)^p*(B*x^4+A),x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^4\right)\, dx = \int \left(Bx^{4}+A\right)\left(bx^{4}+a\right)^{p}\, dx$$

input
```
integrate((b*x^4+a)^p*(B*x^4+A),x, algorithm="fricas")
```

output
```
integral((B*x^4 + A)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \left(a+bx^4\right)^p\left(A+Bx^4\right)\,dx=\frac{Aa^px\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{Ba^px^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**p*(B*x**4+A),x)
```

output
```
A*a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + B*a**p*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)^p\left(A+Bx^4\right)\,dx=\int \left(Bx^4+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(B*x^4+A),x, algorithm="maxima")
```

output
```
integrate((B*x^4 + A)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (a + bx^4)^p (A + Bx^4) \, dx = \int (Bx^4 + A)(bx^4 + a)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(B*x^4+A),x, algorithm="giac")
```

output
```
integrate((B*x^4 + A)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + bx^4)^p (A + Bx^4) \, dx = \int (B x^4 + A) \left(b x^4 + a\right)^p \, dx$$

input
```
int((A + B*x^4)*(a + b*x^4)^p,x)
```

output
```
int((A + B*x^4)*(a + b*x^4)^p, x)
```

Reduce [F]

$$\int (a + bx^4)^p (A + Bx^4) \, dx$$
$$= \frac{8(b\,x^4 + a)^p\,apx + 5(b\,x^4 + a)^p\,ax + 4(b\,x^4 + a)^p\,bp\,x^5 + (b\,x^4 + a)^p\,b\,x^5 + 256\Big(\int \frac{(b\,x^4+a)^p}{16b\,p^2x^4+24bp\,x^4+5b\,x^4+16a}}{}$$

input
```
int((b*x^4+a)^p*(B*x^4+A),x)
```

output
```
(8*(a + b*x**4)**p*a*p*x + 5*(a + b*x**4)**p*a*x + 4*(a + b*x**4)**p*b*p*x
**5 + (a + b*x**4)**p*b*x**5 + 256*int((a + b*x**4)**p/(16*a*p**2 + 24*a*p
 + 5*a + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*p**4 + 640*int((
a + b*x**4)**p/(16*a*p**2 + 24*a*p + 5*a + 16*b*p**2*x**4 + 24*b*p*x**4 +
5*b*x**4),x)*a**2*p**3 + 464*int((a + b*x**4)**p/(16*a*p**2 + 24*a*p + 5*a
 + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*p**2 + 80*int((a + b*x
**4)**p/(16*a*p**2 + 24*a*p + 5*a + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**
4),x)*a**2*p)/(16*p**2 + 24*p + 5)
```

3.35 $\int (a+bx^4)^p (A+Bx^4+Cx^8)\,dx$

Optimal result 331
Mathematica [A] (verified) 332
Rubi [A] (verified) 332
Maple [F] 334
Fricas [F] 335
Sympy [C] (verification not implemented) 335
Maxima [F] 336
Giac [F] 336
Mupad [F(-1)] 336
Reduce [F] 337

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int (a+bx^4)^p (A+Bx^4+Cx^8)\,dx = -\frac{(5aC-bB(9+4p))x(a+bx^4)^{1+p}}{b^2(5+4p)(9+4p)} + \frac{Cx^5(a+bx^4)^{1+p}}{b(9+4p)} + \left(A+\frac{a(5aC-bB(9+4p))}{b^2(5+4p)(9+4p)}\right)x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

output
```
-(5*C*a-b*B*(9+4*p))*x*(b*x^4+a)^(p+1)/b^2/(5+4*p)/(9+4*p)+C*x^5*(b*x^4+a)
^(p+1)/b/(9+4*p)+(A+a*(5*C*a-b*B*(9+4*p))/b^2/(5+4*p)/(9+4*p))*x*(b*x^4+a)
^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.67

$$\int (a+bx^4)^p (A+Bx^4+Cx^8)\,dx = \frac{1}{45}x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(45A\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+9Bx^4\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+5Cx^8\,\text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(a + b*x^4)^p*(A + B*x^4 + C*x^8),x]
```

output
```
(x*(a + b*x^4)^p*(45*A*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + 9*B
*x^4*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*C*x^8*Hypergeometri
c2F1[9/4, -p, 13/4, -((b*x^4)/a)]))/(45*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1741, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p (A+Bx^4+Cx^8)\,dx$$

$$\Big\downarrow \text{ 1741}$$

$$\frac{\int \left(bx^4+a\right)^p \left(Ab(4p+9)-(5aC-bB(4p+9))x^4\right)dx}{b(4p+9)}+\frac{Cx^5\left(a+bx^4\right)^{p+1}}{b(4p+9)}$$

$$\Big\downarrow \; 913$$

$$\frac{\left(\frac{a(5aC-bB(4p+9))}{b(4p+5)}+Ab(4p+9)\right)\int \left(bx^4+a\right)^p dx-\frac{x\left(a+bx^4\right)^{p+1}(5aC-bB(4p+9))}{b(4p+5)}}{b(4p+9)}+\frac{Cx^5\left(a+bx^4\right)^{p+1}}{b(4p+9)}$$

$$\Big\downarrow \; 779$$

$$\frac{\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(\frac{a(5aC-bB(4p+9))}{b(4p+5)}+Ab(4p+9)\right)\int \left(\frac{bx^4}{a}+1\right)^p dx-\frac{x\left(a+bx^4\right)^{p+1}(5aC-bB(4p+9))}{b(4p+5)}}{b(4p+9)}+\frac{Cx^5\left(a+bx^4\right)^{p+1}}{b(4p+9)}$$

$$\Big\downarrow \; 778$$

$$\frac{x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)\left(\frac{a(5aC-bB(4p+9))}{b(4p+5)}+Ab(4p+9)\right)-\frac{x\left(a+bx^4\right)^{p+1}(5a}{b(4p+}}{b(4p+9)}$$

$$\frac{Cx^5\left(a+bx^4\right)^{p+1}}{b(4p+9)}$$

input
```
Int[(a + b*x^4)^p*(A + B*x^4 + C*x^8),x]
```

output
```
(C*x^5*(a + b*x^4)^(1 + p))/(b*(9 + 4*p)) + (-(((5*a*C - b*B*(9 + 4*p))*x*
(a + b*x^4)^(1 + p))/(b*(5 + 4*p))) + ((A*b*(9 + 4*p) + (a*(5*a*C - b*B*(9
 + 4*p)))/(b*(5 + 4*p)))*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -
((b*x^4)/a)])/(1 + (b*x^4)/a)^p)/(b*(9 + 4*p))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

Maple [F]

$$\int (b\,x^{4}+a)^{p} \left(C\,x^{8}+B\,x^{4}+A\right) dx$$

input
```
int((b*x^4+a)^p*(C*x^8+B*x^4+A),x)
```

output
```
int((b*x^4+a)^p*(C*x^8+B*x^4+A),x)
```

Fricas [F]

$$\int (a+bx^4)^p (A+Bx^4+Cx^8)\,dx = \int (Cx^8+Bx^4+A)(bx^4+a)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
integral((C*x^8 + B*x^4 + A)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 98.96 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int (a+bx^4)^p (A+Bx^4+Cx^8)\,dx = \frac{Aa^p x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{Ba^p x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, -p \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{Ca^p x^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{9}{4}, -p \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**4+a)**p*(C*x**8+B*x**4+A),x)
```

output
```
A*a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + B*a**p*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*gamma(9/4)) + C*a**p*x**9*gamma(9/4)*hyper((9/4, -p), (13
/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^4+Cx^8\right) \, dx=\int \left(Cx^8+Bx^4+A\right)\left(bx^4+a\right)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
integrate((C*x^8 + B*x^4 + A)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^4+Cx^8\right) \, dx=\int \left(Cx^8+Bx^4+A\right)\left(bx^4+a\right)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
integrate((C*x^8 + B*x^4 + A)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(A+Bx^4+Cx^8\right) \, dx=\int \left(b\,x^4+a\right)^p\,\left(C\,x^8+B\,x^4+A\right) \,d x$$

input
```
int((a + b*x^4)^p*(A + B*x^4 + C*x^8),x)
```

output
```
int((a + b*x^4)^p*(A + B*x^4 + C*x^8), x)
```

Reduce [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^4+Cx^8\right)\,dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^p*(C*x^8+B*x^4+A),x)
```

output
```
( - 20*(a + b*x**4)**p*a**2*c*p*x + 32*(a + b*x**4)**p*a*b**2*p**2*x + 92*
(a + b*x**4)**p*a*b**2*p*x + 45*(a + b*x**4)**p*a*b**2*x + 16*(a + b*x**4)
**p*a*b*c*p**2*x**5 + 4*(a + b*x**4)**p*a*b*c*p*x**5 + 16*(a + b*x**4)**p*
b**3*p**2*x**5 + 40*(a + b*x**4)**p*b**3*p*x**5 + 9*(a + b*x**4)**p*b**3*x
**5 + 16*(a + b*x**4)**p*b**2*c*p**2*x**9 + 24*(a + b*x**4)**p*b**2*c*p*x*
*9 + 5*(a + b*x**4)**p*b**2*c*x**9 + 1280*int((a + b*x**4)**p/(64*a*p**3 +
 240*a*p**2 + 236*a*p + 45*a + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*
x**4 + 45*b*x**4),x)*a**3*c*p**4 + 4800*int((a + b*x**4)**p/(64*a*p**3 + 2
40*a*p**2 + 236*a*p + 45*a + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x*
*4 + 45*b*x**4),x)*a**3*c*p**3 + 4720*int((a + b*x**4)**p/(64*a*p**3 + 240
*a*p**2 + 236*a*p + 45*a + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4
 + 45*b*x**4),x)*a**3*c*p**2 + 900*int((a + b*x**4)**p/(64*a*p**3 + 240*a*
p**2 + 236*a*p + 45*a + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 +
45*b*x**4),x)*a**3*c*p + 4096*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**2
+ 236*a*p + 45*a + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*
x**4),x)*a**2*b**2*p**6 + 28672*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p**
2 + 236*a*p + 45*a + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*
b*x**4),x)*a**2*b**2*p**5 + 74240*int((a + b*x**4)**p/(64*a*p**3 + 240*a*p
**2 + 236*a*p + 45*a + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 4
5*b*x**4),x)*a**2*b**2*p**4 + 86528*int((a + b*x**4)**p/(64*a*p**3 + 24...
```

3.36 $\int (a+bx^4)^p (A+Bx^4+Cx^8+Dx^{12})\,dx$

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Maxima [F] . . . 341
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Reduce [F] . . . 342

Optimal result

Integrand size = 27, antiderivative size = 262

$$\int (a+bx^4)^p (A+Bx^4+Cx^8+Dx^{12})\,dx = \frac{(45a^2D-5abC(13+4p)+b^2B(117+88p+16p^2))\,x(a+bx^4)^{1+p}}{b^3(5+4p)(9+4p)(13+4p)} - \frac{(9aD-bC(13+4p))x^5(a+bx^4)^{1+p}}{b^2(9+4p)(13+4p)} + \frac{Dx^9(a+bx^4)^{1+p}}{b(13+4p)} + \frac{\left(Ab^3(117+88p+16p^2)-\frac{a(45a^2D-5abC(13+4p)+b^2B(117+88p+16p^2))}{5+4p}\right)x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeom}}{b^3(9+4p)(13+4p)}$$

output
```
(45*a^2*D-5*a*b*C*(13+4*p)+b^2*B*(16*p^2+88*p+117))*x*(b*x^4+a)^(p+1)/b^3/
(5+4*p)/(9+4*p)/(13+4*p)-(9*a*D-b*C*(13+4*p))*x^5*(b*x^4+a)^(p+1)/b^2/(9+4
*p)/(13+4*p)+D*x^9*(b*x^4+a)^(p+1)/b/(13+4*p)+(A*b^3*(16*p^2+88*p+117)-a*(
45*a^2*D-5*a*b*C*(13+4*p)+b^2*B*(16*p^2+88*p+117))/(5+4*p))*x*(b*x^4+a)^p*
hypergeom([1/4, -p],[5/4],-b*x^4/a)/b^3/(9+4*p)/(13+4*p)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.48

$$\int (a+bx^4)^p (A+Bx^4+Cx^8+Dx^{12})\,dx = \frac{1}{585}x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(585A\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + 117Bx^4\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right) + 65Cx^8\,\text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^4}{a}\right) + 45Dx^{12}\,\text{Hypergeometric2F1}\left(\frac{13}{4},-p,\frac{17}{4},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(a + b*x^4)^p*(A + B*x^4 + C*x^8 + D*x^12),x]
```

output
```
(x*(a + b*x^4)^p*(585*A*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + 11
7*B*x^4*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 65*C*x^8*Hypergeom
etric2F1[9/4, -p, 13/4, -((b*x^4)/a)] + 45*D*x^12*Hypergeometric2F1[13/4,
-p, 17/4, -((b*x^4)/a)]))/(585*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p (A+Bx^4+Cx^8+Dx^{12})\,dx$$

$$\Big\downarrow \; 2432$$

$$\int \left(A(a+bx^4)^p + Bx^4(a+bx^4)^p + Cx^8(a+bx^4)^p + Dx^{12}(a+bx^4)^p\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$Ax(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{5}Bx^5(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+\frac{1}{9}Cx^9(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^4}{a}\right)+\frac{1}{13}Dx^{13}(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{13}{4},-p,\frac{17}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(a + b*x^4)^p*(A + B*x^4 + C*x^8 + D*x^12),x]
```

output
```
(A*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*
x^4)/a)^p + (B*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^4)
/a)])/(5*(1 + (b*x^4)/a)^p) + (C*x^9*(a + b*x^4)^p*Hypergeometric2F1[9/4,
-p, 13/4, -((b*x^4)/a)])/(9*(1 + (b*x^4)/a)^p) + (D*x^13*(a + b*x^4)^p*Hyp
ergeometric2F1[13/4, -p, 17/4, -((b*x^4)/a)])/(13*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p}\left(Dx^{12}+C\,x^{8}+B\,x^{4}+A\right)dx$$

input
```
int((b*x^4+a)^p*(D*x^12+C*x^8+B*x^4+A),x)
```

output
```
int((b*x^4+a)^p*(D*x^12+C*x^8+B*x^4+A),x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^4+Cx^8+Dx^{12}\right)\,dx=\int \left(Dx^{12}+Cx^8+Bx^4+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(D*x^12+C*x^8+B*x^4+A),x, algorithm="fricas")
```

output
```
integral((D*x^12 + C*x^8 + B*x^4 + A)*(b*x^4 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^4\right)^p \left(A+Bx^4+Cx^8+Dx^{12}\right)\,dx=\text{Timed out}$$

input
```
integrate((b*x**4+a)**p*(D*x**12+C*x**8+B*x**4+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^4\right)^p \left(A+Bx^4+Cx^8+Dx^{12}\right)\,dx=\int \left(Dx^{12}+Cx^8+Bx^4+A\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((b*x^4+a)^p*(D*x^12+C*x^8+B*x^4+A),x, algorithm="maxima")
```

output
```
integrate((D*x^12 + C*x^8 + B*x^4 + A)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (a+bx^4)^p (A+Bx^4+Cx^8+Dx^{12}) \, dx = \int (Dx^{12}+Cx^8+Bx^4+A)(bx^4+a)^p \, dx$$

input
```
integrate((b*x^4+a)^p*(D*x^12+C*x^8+B*x^4+A),x, algorithm="giac")
```

output
```
integrate((D*x^12 + C*x^8 + B*x^4 + A)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^4)^p (A+Bx^4+Cx^8+Dx^{12}) \, dx = \int (b\,x^4+a)^p \left(A+B\,x^4+C\,x^8+x^{12}\,D\right) \, dx$$

input
```
int((a + b*x^4)^p*(A + B*x^4 + C*x^8 + x^12*D),x)
```

output
```
int((a + b*x^4)^p*(A + B*x^4 + C*x^8 + x^12*D), x)
```

Reduce [F]

$$\int (a+bx^4)^p (A+Bx^4+Cx^8+Dx^{12}) \, dx = \text{too large to display}$$

input
```
int((b*x^4+a)^p*(D*x^12+C*x^8+B*x^4+A),x)
```

output

```
(180*(a + b*x**4)**p*a**3*d*p*x - 80*(a + b*x**4)**p*a**2*b*c*p**2*x - 260
*(a + b*x**4)**p*a**2*b*c*p*x - 144*(a + b*x**4)**p*a**2*b*d*p**2*x**5 - 3
6*(a + b*x**4)**p*a**2*b*d*p*x**5 + 128*(a + b*x**4)**p*a*b**3*p**3*x + 78
4*(a + b*x**4)**p*a*b**3*p**2*x + 1376*(a + b*x**4)**p*a*b**3*p*x + 585*(a
 + b*x**4)**p*a*b**3*x + 64*(a + b*x**4)**p*a*b**2*c*p**3*x**5 + 224*(a +
b*x**4)**p*a*b**2*c*p**2*x**5 + 52*(a + b*x**4)**p*a*b**2*c*p*x**5 + 64*(a
 + b*x**4)**p*a*b**2*d*p**3*x**9 + 96*(a + b*x**4)**p*a*b**2*d*p**2*x**9 +
 20*(a + b*x**4)**p*a*b**2*d*p*x**9 + 64*(a + b*x**4)**p*b**4*p**3*x**5 +
368*(a + b*x**4)**p*b**4*p**2*x**5 + 556*(a + b*x**4)**p*b**4*p*x**5 + 117
*(a + b*x**4)**p*b**4*x**5 + 64*(a + b*x**4)**p*b**3*c*p**3*x**9 + 304*(a
+ b*x**4)**p*b**3*c*p**2*x**9 + 332*(a + b*x**4)**p*b**3*c*p*x**9 + 65*(a
+ b*x**4)**p*b**3*c*x**9 + 64*(a + b*x**4)**p*b**3*d*p**3*x**13 + 240*(a +
 b*x**4)**p*b**3*d*p**2*x**13 + 236*(a + b*x**4)**p*b**3*d*p*x**13 + 45*(a
 + b*x**4)**p*b**3*d*x**13 - 46080*int((a + b*x**4)**p/(256*a*p**4 + 1792*
a*p**3 + 4064*a*p**2 + 3248*a*p + 585*a + 256*b*p**4*x**4 + 1792*b*p**3*x*
*4 + 4064*b*p**2*x**4 + 3248*b*p*x**4 + 585*b*x**4),x)*a**4*d*p**5 - 32256
0*int((a + b*x**4)**p/(256*a*p**4 + 1792*a*p**3 + 4064*a*p**2 + 3248*a*p +
 585*a + 256*b*p**4*x**4 + 1792*b*p**3*x**4 + 4064*b*p**2*x**4 + 3248*b*p*
x**4 + 585*b*x**4),x)*a**4*d*p**4 - 731520*int((a + b*x**4)**p/(256*a*p**4
 + 1792*a*p**3 + 4064*a*p**2 + 3248*a*p + 585*a + 256*b*p**4*x**4 + 179...
```

3.37 $\int \frac{c+dx}{a-bx^4} dx$

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Rubi [A] (verified) 345
Maple [C] (verified) 346
Fricas [C] (verification not implemented) 346
Sympy [A] (verification not implemented) 347
Maxima [B] (verification not implemented) 347
Giac [B] (verification not implemented) 348
Mupad [B] (verification not implemented) 349
Reduce [B] (verification not implemented) 349

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{c+dx}{a-bx^4} dx = \frac{c \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}} + \frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

output
```
1/2*c*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)+1/2*c*arctanh(b^(1/4)*x/a^
(1/4))/a^(3/4)/b^(1/4)+1/2*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$\int \frac{c+dx}{a-bx^4} dx$$
$$= \frac{2\sqrt[4]{b}c \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(\sqrt[4]{b}c + \sqrt[4]{a}d\right) \log\left(\sqrt[4]{a} - \sqrt[4]{b}x\right) + \sqrt[4]{b}c \log\left(\sqrt[4]{a} + \sqrt[4]{b}x\right) - \sqrt[4]{a}d \log\left(\sqrt[4]{a} + \sqrt[4]{b}x\right.}{4a^{3/4}\sqrt{b}}$$

input
```
Integrate[(c + d*x)/(a - b*x^4),x]
```

output
```
(2*b^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)] - (b^(1/4)*c + a^(1/4)*d)*Log[a^(
1/4) - b^(1/4)*x] + b^(1/4)*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*d*Log[a^(
1/4) + b^(1/4)*x] + a^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*a^(3/4)*Sqrt[
b])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a-bx^4} \, dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{c}{a-bx^4}+\frac{dx}{a-bx^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x)/(a - b*x^4),x]
```

output
```
(c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (c*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*S
qrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4-a\right)}\frac{\left(d\_R+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	34
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}$	87

input
```
int((d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 39057, normalized size of antiderivative = 448.93

$$\int \frac{c+dx}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{c+dx}{a-bx^4}\,dx =$$
$$-\operatorname{RootSum}\left(256t^4a^3b^2-32t^2a^2bd^2-16tabc^2d+ad^4-bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2+16t^2a^2bc^2}{4ac}\right.\right.\right.$$

input
```
integrate((d*x+c)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a
*d**4 - b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 + 16*_t**2*a
**2*b*c**2*d + 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4
 + b*c**5))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(57) = 114.

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{c+dx}{a-bx^4}\,dx = \frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}$$

input
```
integrate((d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b
))) + 1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(
b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*c*log((sqrt(b)*x - sqrt(sqrt(a)*
sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(57) = 114$.

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.61

$$\int \frac{c+dx}{a-bx^4}\,dx = \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd-(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}bd-(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b)
)/(a*b) - 1/8*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) +
sqrt(-a/b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d - (-a*b^3)^(1/4)*b
*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
- 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b*d - (-a*b^3)^(1/4)*b*c)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.82 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.09

$$\int \frac{c+dx}{a-bx^4}\,dx$$
$$= \begin{cases} \frac{2c+3dx}{6bx^3} \\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}(-b)^{1/4}x}{a^{1/4}}-1\right)\left(2a^{1/4}d+\sqrt{2}(-b)^{1/4}c\right)}{4a^{3/4}\sqrt{-b}} - \frac{\operatorname{atan}\left(\frac{\sqrt{2}(-b)^{1/4}x}{a^{1/4}}+1\right)\left(4a^{1/4}d-2\sqrt{2}(-b)^{1/4}c\right)}{8a^{3/4}\sqrt{-b}} + \frac{\sqrt{2}c\ln\left(\frac{\sqrt{-b}x^2+\sqrt{a}+\sqrt{2}a^{1/4}}{\sqrt{-b}x^2+\sqrt{a}-\sqrt{2}a^{1/4}}\right.}{8a^{3/4}(-b)^{1/4}} \end{cases}$$

input
```
int((c + d*x)/(a - b*x^4),x)
```

output
```
piecewise(a == 0, (2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*(-b)^(1/
4)*x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*(-b)^(1/4)*c))/(4*a^(3/4)*(-b)^(
1/2)) - (atan((2^(1/2)*(-b)^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)
*(-b)^(1/4)*c))/(8*a^(3/4)*(-b)^(1/2)) + (2^(1/2)*c*log(((-b)^(1/2)*x^2 +
a^(1/2) + 2^(1/2)*a^(1/4)*(-b)^(1/4)*x)/((-b)^(1/2)*x^2 + a^(1/2) - 2^(1/2
)*a^(1/4)*(-b)^(1/4)*x)))/(8*a^(3/4)*(-b)^(1/4)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{c+dx}{a-bx^4}\,dx$$
$$= \frac{\sqrt{b}\left(2b^{\frac{1}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c - b^{\frac{1}{4}}a^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)c + b^{\frac{1}{4}}a^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}x\right)c - \sqrt{a}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)d - \sqrt{a}\,\mathrm{l}\right.}{4ab}$$

input
```
int((d*x+c)/(-b*x^4+a),x)
```

output
```
(sqrt(b)*(2*b**(1/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*c - b*
*(1/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*c + b**(1/4)*a**(1/4)*log(a**(1
/4) + b**(1/4)*x)*c - sqrt(a)*log(a**(1/4) - b**(1/4)*x)*d - sqrt(a)*log(a
**(1/4) + b**(1/4)*x)*d + sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*d))/(4*a*b)
```

3.38 $\int \frac{c+dx}{a+bx^4}\,dx$

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Optimal result

Integrand size = 15, antiderivative size = 167

$$\int \frac{c+dx}{a+bx^4}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}} + \frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

output
```
1/2*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)+1/4*c*arctan(-1+2^(1/2)*
b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)+1/4*c*arctan(1+2^(1/2)*b^(1/4)*
x/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)+1/4*c*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x
/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

$$\int \frac{c+dx}{a+bx^4}\,dx = \frac{-2\left(\sqrt{2}\sqrt[4]{b}c+2\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{b}c-2\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{b}c\left(-\mathrm{l}\right.}{8a^{3/4}\sqrt{b}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4),x]
```

output
```
(-2*(Sqrt[2]*b^(1/4)*c + 2*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)] + 2*(Sqrt[2]*b^(1/4)*c - 2*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)] + Sqrt[2]*b^(1/4)*c*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a
^(3/4)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+bx^4}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{c}{a+bx^4}+\frac{dx}{a+bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x)/(a + b*x^4),x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (c*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (c*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)
*b^(1/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.19

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(d\_R+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	32
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}$	124

input
```
int((d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 41851, normalized size of antiderivative = 250.60

$$\int \frac{c+dx}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{c+dx}{a+bx^4}\,dx$$
$$=\operatorname{RootSum}\left(256t^4a^3b^2+32t^2a^2bd^2-16tabc^2d+ad^4+bc^4,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3bd^2-16t^2a^2bc^2d-}{4acd^4}\right.\right.\right.$$

input
```
integrate((d*x+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b*d**2 - 16*_t*a*b*c**2*d + a*
d**4 + b*c**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*b*d**2 - 16*_t**2*a*
*2*b*c**2*d - 8*_t*a**2*d**4 - 4*_t*a*b*c**4 + 5*a*c**2*d**3)/(4*a*c*d**4
- b*c**5))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.24

$$\int \frac{c+dx}{a+bx^4}\,dx=\frac{\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-2\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}+\frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c+2\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}$$

input
```
integrate((d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3
/4)*b^(1/4)) - 1/8*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/(a^(3/4)*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(1/4)*c - 2*sqrt(a)*d
)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*b
^(1/4)*c + 2*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*
b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.28

$$\int \frac{c+dx}{a+bx^4}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd-(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}bd-(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}$$

input
```
integrate((d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(
a*b) - 1/8*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt
(2)*(sqrt(2)*sqrt(a*b)*b*d - (a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{a+bx^4}\,dx$$
$$= \begin{cases} -\frac{2c+3dx}{6bx^3} & \text{if} \\ \frac{\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,d+\sqrt{2}\,b^{1/4}\,c\right)}{4\,a^{3/4}\,\sqrt{b}}-\frac{\operatorname{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,d-2\,\sqrt{2}\,b^{1/4}\,c\right)}{8\,a^{3/4}\,\sqrt{b}}+\frac{\sqrt{2}\,c\,\ln\left(\frac{\sqrt{a}+\sqrt{b}\,x^2+\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}{\sqrt{a}+\sqrt{b}\,x^2-\sqrt{2}\,a^{1/4}\,b^{1/4}\,x}\right)}{8\,a^{3/4}\,b^{1/4}} & \text{if} \end{cases}$$

input
```
int((c + d*x)/(a + b*x^4),x)
```

output
```
piecewise(a == 0, -(2*c + 3*d*x)/(6*b*x^3), a ~= 0, (atan((2^(1/2)*b^(1/4)
*x)/a^(1/4) - 1)*(2*a^(1/4)*d + 2^(1/2)*b^(1/4)*c))/(4*a^(3/4)*b^(1/2)) -
(atan((2^(1/2)*b^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*d - 2*2^(1/2)*b^(1/4)*c)
)/(8*a^(3/4)*b^(1/2)) + (2^(1/2)*c*log((a^(1/2) + b^(1/2)*x^2 + 2^(1/2)*a^
(1/4)*b^(1/4)*x)/(a^(1/2) + b^(1/2)*x^2 - 2^(1/2)*a^(1/4)*b^(1/4)*x)))/(8*
a^(3/4)*b^(1/4)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.28

$$\int \frac{c+dx}{a+bx^4}\,dx$$
$$= \frac{\sqrt{b}\left(-2b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-4\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-$$

input
```
int((d*x+c)/(b*x^4+a),x)
```

output
```
(sqrt(b)*( - 2*b**(1/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 4*sqrt(a)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + 2*b**(1/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*c - 4*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d - b**(1/4)*a**(1/4)*sqrt(2)*log( - b*
*(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c + b**(1/4)*a**(1/4)*
sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c))/(8*a
*b)
```

3.39 $\int \frac{c+dx}{(a-bx^4)^2}\,dx$

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Rubi [A] (verified) 359
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Giac [B] (verification not implemented) 363
Mupad [B] (verification not implemented) 364
Reduce [B] (verification not implemented) 364

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx = \frac{x(c+dx)}{4a(a-bx^4)} + \frac{3c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{3c\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}\sqrt[4]{b}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*x*(d*x+c)/a/(-b*x^4+a)+3/8*c*arctan(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)
+3/8*c*arctanh(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(1/4)+1/4*d*arctanh(b^(1/2)*x^
2/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= \frac{\frac{4ax(c+dx)}{a-bx^4} + \frac{6\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c+2\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{\left(3\sqrt[4]{a}\sqrt[4]{b}c-2\sqrt{a}d\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{\sqrt{b}} + 2\sqrt{a}\ldots}{16a^2}$$

input
```
Integrate[(c + d*x)/(a - b*x^4)^2,x]
```

output
```
((4*a*x*(c + d*x))/(a - b*x^4) + (6*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])
/b^(1/4) - ((3*a^(1/4)*b^(1/4)*c + 2*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/
Sqrt[b] + ((3*a^(1/4)*b^(1/4)*c - 2*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/S
qrt[b] + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a-bx^4)^2}dx$$

$$\Big\downarrow \ 2394$$

$$\frac{x(c+dx)}{4a(a-bx^4)}-\frac{\int -\frac{3c+2dx}{a-bx^4}dx}{4a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3c+2dx}{a-bx^4}dx}{4a}+\frac{x(c+dx)}{4a(a-bx^4)}$$

$$\Big\downarrow \ 2415$$

$$\frac{\int \left(\frac{3c}{a-bx^4}+\frac{2dx}{a-bx^4}\right)dx}{4a}+\frac{x(c+dx)}{4a(a-bx^4)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{3c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{3c\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(c+dx)}{4a(a-bx^4)}$$

input
```
Int[(c + d*x)/(a - b*x^4)^2,x]
```

output
```
(x*(c + d*x))/(4*a*(a - b*x^4)) + ((3*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^
(3/4)*b^(1/4)) + (3*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) +
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\frac{d\,x^2}{4a}+\frac{cx}{4a}}{-b\,x^4+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4-a\right)}\frac{\left(2d\_R+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$	69
default	$c\left(\frac{x}{4a(-b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b\,x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)$	128

input
```
int((d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*d/a*x^2+1/4*c/a*x)/(-b*x^4+a)-1/16/b/a*sum((2*_R*d+3*c)/_R^3*ln(x-_R)
,_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 40560, normalized size of antiderivative = 368.73

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= \mathrm{RootSum}\left(65536t^4a^7b^2 - 2048t^2a^4bd^2 + 1152ta^2bc^2d + 16ad^4 - 81bc^4, \left(t \mapsto t\log\left(x + \frac{32768t^3a^6bd^2 +\right.\right.\right.$$
$$+ \frac{-cx-dx^2}{-4a^2+4abx^4}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**2 - 2048*_t**2*a**4*b*d**2 + 1152*_t*a**2*b*c*
*2*d + 16*a*d**4 - 81*b*c**4, Lambda(_t, _t*log(x + (32768*_t**3*a**6*b*d*
*2 + 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 + 1296*_t*a**2*b*c**4 + 3
60*a*c**2*d**3)/(192*a*c*d**4 + 243*b*c**5)))) + (-c*x - d*x**2)/(-4*a**2
+ 4*a*b*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= -\frac{dx^2+cx}{4\,(abx^4-a^2)} + \frac{\frac{6\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{3\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{16\,a}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(6*c*arctan(sqrt(b)*x/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*d*log(sqrt(b)*x^2 + sqrt(
a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) -
 3*c*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqr
t(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(79) = 158$.

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.31

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx = \frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{3\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,a^2b} - \frac{dx^2+cx}{4\,(bx^4-a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}bd+3\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{-ab}bd+3\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b
))/(a^2*b) - 3/32*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^(1/4
) + sqrt(-a/b))/(a^2*b) - 1/4*(d*x^2 + c*x)/((b*x^4 - a)*a) + 1/16*sqrt(2)
*(2*sqrt(2)*sqrt(-a*b)*b*d + 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(-a*b)*b*d + 3*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(-a/b)^(1/4))/(-a/b)^(1/4))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.57

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(3\,c\,d^2+2\,d^3\,x+\mathrm{root}(65536\,a^7\,b^2\,z^4-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4\right.}{}\right.\right.$$
$$\left.\left.-2048\,a^4\,b\,d^2\,z^2+1152\,a^2\,b\,c^2\,d\,z-81\,b\,c^4+16\,a\,d^4,z,k)\right)+\frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

input
```
int((c + d*x)/(a - b*x^4)^2,x)
```

output
```
symsum(log(-(b^2*(3*c*d^2 + 2*d^3*x + 192*root(65536*a^7*b^2*z^4 - 2048*a^
4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c -
128*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*
b*c^4 + 16*a*d^4, z, k)^2*a^3*b*d*x + 36*root(65536*a^7*b^2*z^4 - 2048*a^4
*b*d^2*z^2 + 1152*a^2*b*c^2*d*z - 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(
16*a^3))*root(65536*a^7*b^2*z^4 - 2048*a^4*b*d^2*z^2 + 1152*a^2*b*c^2*d*z
- 81*b*c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a
- b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.54

$$\int \frac{c+dx}{(a-bx^4)^2}\,dx$$
$$= \frac{6b^{\frac{3}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c-6b^{\frac{7}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c\,x^4-3b^{\frac{3}{4}}a^{\frac{5}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)c+3b^{\frac{7}{4}}a^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)c\,x^4+3b}{}$$

input
```
int((d*x+c)/(-b*x^4+a)^2,x)
```

output
```
(6*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*c - 6*b**(3/4
)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b*c*x**4 - 3*b**(3/4)*a**
(1/4)*log(a**(1/4) - b**(1/4)*x)*a*c + 3*b**(3/4)*a**(1/4)*log(a**(1/4) -
b**(1/4)*x)*b*c*x**4 + 3*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*c
- 3*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*b*c*x**4 - 2*sqrt(b)*sqrt
(a)*log(a**(1/4) - b**(1/4)*x)*a*d + 2*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(
1/4)*x)*b*d*x**4 - 2*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a*d + 2*sq
rt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*b*d*x**4 + 2*sqrt(b)*sqrt(a)*log(
sqrt(a) + sqrt(b)*x**2)*a*d - 2*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2
)*b*d*x**4 + 4*a*b*c*x + 4*a*b*d*x**2)/(16*a**2*b*(a - b*x**4))
```

3.40 $\int \frac{c+dx}{(a+bx^4)^2} dx$

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Mathematica [A] (verified) 367
Rubi [A] (verified) 367
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Giac [A] (verification not implemented) 371
Mupad [B] (verification not implemented) 372
Reduce [B] (verification not implemented) 372

Optimal result

Integrand size = 15, antiderivative size = 189

$$\int \frac{c+dx}{(a+bx^4)^2} dx = \frac{x(c+dx)}{4a(a+bx^4)} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{3c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3c\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

output
```
1/4*x*(d*x+c)/a/(b*x^4+a)+1/4*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(3/2)/b^(1/2
)+3/16*c*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(1/4)+3/16
*c*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(1/4)+3/16*c*arct
anh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(1/
4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$= \frac{\frac{8a^{3/4}x(c+dx)}{a+bx^4} - \frac{2\left(3\sqrt{2}\sqrt[4]{b}c+4\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{2\left(3\sqrt{2}\sqrt[4]{b}c-4\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} - \frac{3\sqrt{2}c\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\right.}}{\sqrt[4]{b}}}{32a^{7/4}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4)^2,x]
```

output
```
((8*a^(3/4)*x*(c + d*x))/(a + b*x^4) - (2*(3*Sqrt[2]*b^(1/4)*c + 4*a^(1/4)
*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] + (2*(3*Sqrt[2]*b^(1/
4)*c - 4*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (3*
Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4)
+ (3*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(
1/4))/(32*a^(7/4))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{ 2394}$$
$$\frac{x(c+dx)}{4a(a+bx^4)} - \frac{\int -\frac{3c+2dx}{bx^4+a}dx}{4a}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3c+2dx}{bx^4+a}dx}{4a}+\frac{x(c+dx)}{4a\,(a+bx^4)}$$

$\Big\downarrow$ 2415

$$\frac{\int \left(\frac{3c}{bx^4+a}+\frac{2dx}{bx^4+a}\right)dx}{4a}+\frac{x(c+dx)}{4a\,(a+bx^4)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{3c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{3c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{3c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{3c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{d\,\mathrm{ar}}{}}{4a}$$
$$\frac{x(c+dx)}{4a\,(a+bx^4)}$$

input
```
Int[(c + d*x)/(a + b*x^4)^2,x]
```

output
```
(x*(c + d*x))/(4*a*(a + b*x^4)) + ((d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt
[a]*Sqrt[b]) - (3*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*b^(1/4)) + (3*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(1/4)) - (3*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (3*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{d\,x^2}{4a}+\frac{cx}{4a}}{b\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^4+a\right)}\frac{\left(2d\,\_R+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b\,x^4+a}\right.$

input
```
int((d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*d/a*x^2+1/4*c/a*x)/(b*x^4+a)+1/16/b/a*sum((2*_R*d+3*c)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 43065, normalized size of antiderivative = 227.86

$$\int\frac{c+dx}{\left(a+bx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.82

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$= \mathrm{RootSum}\left(65536t^4a^7b^2+2048t^2a^4bd^2-1152ta^2bc^2d+16ad^4+81bc^4, \left(t\mapsto t\log\left(x+\frac{-32768t^3a^6bd^2}{}\right.\right.\right.$$
$$+\frac{cx+dx^2}{4a^2+4abx^4}$$

input
```
integrate((d*x+c)/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**2 + 2048*_t**2*a**4*b*d**2 - 1152*_t*a**2*b*c*
*2*d + 16*a*d**4 + 81*b*c**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*b*d
**2 - 4608*_t**2*a**4*b*c**2*d - 512*_t*a**3*d**4 - 1296*_t*a**2*b*c**4 +
360*a*c**2*d**3)/(192*a*c*d**4 - 243*b*c**5)))) + (c*x + d*x**2)/(4*a**2 +
 4*a*b*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.26

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \frac{dx^2+cx}{4\,(abx^4+a^2)}$$
$$+\frac{\frac{3\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{3\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-4\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}+}{32\,a}$$

input
```
integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(3*sqrt(2)*c*log(sqrt(b)*x^2 + sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 3*sqrt(2)*c*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(3*s
qrt(2)*a^(1/4)*b^(1/4)*c - 4*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(b))*b^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(1/4)*c + 4*sqrt(a)*d)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.26

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b} - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b} + \frac{dx^2+cx}{4(bx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd+3(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}bd+3(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^2}$$

input
```
integrate((d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b) - 3/32*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^2*b) + 1/4*(d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sq
rt(2)*sqrt(a*b)*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)
*b*d + 3*(a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))
/(a/b)^(1/4))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.49

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\frac{b^2\left(3\,c\,d^2+2\,d^3\,x-\mathrm{root}(65536\,a^7\,b^2\,z^4+2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,\,z\right.}{}\right.\right.$$
$$\left.\left. +2048\,a^4\,b\,d^2\,z^2-1152\,a^2\,b\,c^2\,d\,z+81\,b\,c^4+16\,a\,d^4,z,k)\right)\right)+\frac{\frac{d\,x^2}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

input
```
int((c + d*x)/(a + b*x^4)^2,x)
```

output
```
symsum(log((b^2*(3*c*d^2 + 2*d^3*x - 192*root(65536*a^7*b^2*z^4 + 2048*a^4
*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)^2*a^3*b*c + 1
28*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b
*c^4 + 16*a*d^4, z, k)^2*a^3*b*d*x - 36*root(65536*a^7*b^2*z^4 + 2048*a^4*
b*d^2*z^2 - 1152*a^2*b*c^2*d*z + 81*b*c^4 + 16*a*d^4, z, k)*a*b*c^2*x))/(1
6*a^3))*root(65536*a^7*b^2*z^4 + 2048*a^4*b*d^2*z^2 - 1152*a^2*b*c^2*d*z +
 81*b*c^4 + 16*a*d^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) + (c*x)/(4*a))/(a +
 b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.48

$$\int \frac{c+dx}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)/(b*x^4+a)^2,x)
```

output

```
( - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c*
x**4 - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b
**(1/4)*a**(1/4)*sqrt(2)))*a*d - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d*x**4 + 6*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)
*a**(1/4)*sqrt(2)))*a*c + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c*x**4 - 8*sqrt(
b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*d - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*s
qrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d*x**4 - 3*b**(3/4)*a**(1/4)*sqrt
(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*c - 3*b
**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sq
rt(b)*x**2)*b*c*x**4 + 3*b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*s
qrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*c + 3*b**(3/4)*a**(1/4)*sqrt(2)*log(b
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c*x**4 + 8*a*b*c*x
+ 8*a*b*d*x**2)/(32*a**2*b*(a + b*x**4))
```

3.41 $\int \frac{c+dx}{(a-bx^4)^3}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{x(c+dx)}{8a(a-bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a-bx^4)} + \frac{21c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{21c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}\sqrt[4]{b}} + \frac{3d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/8*x*(d*x+c)/a/(-b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(-b*x^4+a)+21/64*c*arc
tan(b^(1/4)*x/a^(1/4))/a^(11/4)/b^(1/4)+21/64*c*arctanh(b^(1/4)*x/a^(1/4))
/a^(11/4)/b^(1/4)+3/16*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{\frac{16a^2x(c+dx)}{(a-bx^4)^2} + \frac{4ax(7c+6dx)}{a-bx^4} + \frac{42\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c+4\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \frac{3\left(7\sqrt[4]{a}\sqrt[4]{b}c-4\sqrt{a}d\right)\log\left(\right.}{\sqrt{b}}}{128a^3}$$

input
```
Integrate[(c + d*x)/(a - b*x^4)^3,x]
```

output
```
((16*a^2*x*(c + d*x))/(a - b*x^4)^2 + (4*a*x*(7*c + 6*d*x))/(a - b*x^4) +
(42*a^(1/4)*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(1/4) - (3*(7*a^(1/4)*b^(1/4)
*c + 4*Sqrt[a]*d)*Log[a^(1/4) - b^(1/4)*x])/Sqrt[b] + (3*(7*a^(1/4)*b^(1/4
)*c - 4*Sqrt[a]*d)*Log[a^(1/4) + b^(1/4)*x])/Sqrt[b] + (12*Sqrt[a]*d*Log[S
qrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a-bx^4)^3}dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{8a(a-bx^4)^2}-\frac{\int -\frac{7c+6dx}{(a-bx^4)^2}dx}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7c+6dx}{(a-bx^4)^2}dx}{8a}+\frac{x(c+dx)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(7c+6dx)}{4a(a-bx^4)}-\frac{\int -\frac{3(7c+4dx)}{a-bx^4}dx}{4a}}{8a}+\frac{x(c+dx)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{7c+4dx}{a-bx^4}dx}{4a}+\frac{x(7c+6dx)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{3\int \left(\frac{7c}{a-bx^4}+\frac{4dx}{a-bx^4}\right)dx}{4a}+\frac{x(7c+6dx)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3\left(\frac{7c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{7c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{2d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x(7c+6dx)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a-bx^4\right)^2}$$

input
```
Int[(c + d*x)/(a - b*x^4)^3,x]
```

output
```
(x*(c + d*x))/(8*a*(a - b*x^4)^2) + ((x*(7*c + 6*d*x))/(4*a*(a - b*x^4)) +
 (3*((7*c*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (7*c*ArcTanh[
(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (2*d*ArcTanh[(Sqrt[b]*x^2)/Sqr
t[a]])/(Sqrt[a]*Sqrt[b])))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

method	result
risch	$\frac{-\frac{3bd\,x^6}{16a^2}-\frac{7bc\,x^5}{32a^2}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{\left(-b\,x^4+a\right)^2}-\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(4d\_R+7c\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{128b\,a^2}$
default	$c\left(\frac{x}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{7x}{32a\left(-b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^2}}{a}\right)+d\left(\frac{x^2}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{3x^2}{16a\left(-b\,x^4+a\right)}+\right.$

input
```
int((d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/16*b*d/a^2*x^6-7/32*b/a^2*c*x^5+5/16*d/a*x^2+11/32*c/a*x)/(-b*x^4+a)^2
-3/128/b/a^2*sum((4*_R*d+7*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 40637, normalized size of antiderivative = 298.80

$$\int \frac{c+dx}{\left(a-bx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = -\operatorname{RootSum}\left(268435456t^4a^{11}b^2 - 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 - 194481bc^4, \left(t \mapsto t\,\mathrm{l}\right.\right. - \frac{-11acx - 10adx^2 + 7bcx^5 + 6bdx^6}{32a^4 - 64a^3bx^4 + 32a^2b^2x^8}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**3,x)
```

output
```
-RootSum(268435456*_t**4*a**11*b**2 - 4718592*_t**2*a**6*b*d**2 - 2709504*
_t*a**3*b*c**2*d + 20736*a*d**4 - 194481*b*c**4, Lambda(_t, _t*log(x + (-6
7108864*_t**3*a**9*b*d**2 + 9633792*_t**2*a**6*b*c**2*d + 589824*_t*a**4*d
**4 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 + 4537
89*b*c**5)))) - (-11*a*c*x - 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a*
*4 - 64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.37

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = -\frac{6\,bdx^6 + 7\,bcx^5 - 10\,adx^2 - 11\,acx}{32\,(a^2b^2x^8 - 2\,a^3bx^4 + a^4)} + \frac{3\left(\frac{14\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{4\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{4\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{7\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right)}{128\,a^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/(a^2*b^2*x^8 - 2*a^3
*b*x^4 + a^4) + 3/128*(14*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))) + 4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b
)) - 4*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 7*c*log((sqrt(b)*x
 - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(b))))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(104) = 208$.

Time = 0.13 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.00

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{21\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\sqrt{2}(-ab^3)^{\frac{1}{4}}\,c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,a^3b} - \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd-7\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} - \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{-ab}bd-7\,(-ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} - \frac{6\,bdx^6+7\,bcx^5-10\,adx^2-11\,acx}{32\,(bx^4-a)^2a^2}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(a^3*b) - 21/256*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b)^
(1/4) + sqrt(-a/b))/(a^3*b) - 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d - 7*
(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/(a^3*b^2) - 3/128*sqrt(2)*(4*sqrt(2)*sqrt(-a*b)*b*d - 7*(-a*b^3)^(
1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(a
^3*b^2) - 1/32*(6*b*d*x^6 + 7*b*c*x^5 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a
)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.32

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}-\frac{7\,b\,c\,x^5}{32\,a^2}-\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2-2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(63\,c\,d^2+36\,d^3\,x+\mathrm{root}(268435456\,a^{11}\,b^2\,z^4-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z\right.}{}\right.\right.$$
$$\left.\left.-4718592\,a^6\,b\,d^2\,z^2+2709504\,a^3\,b\,c^2\,d\,z-194481\,b\,c^4+20736\,a\,d^4,z,k\right)\right)$$

input
```
int((c + d*x)/(a - b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a^2) - (3*b*d*x^6)/(
16*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(-(3*b^2*(63*c*d^2 + 36*d
^3*x + 7168*root(268435456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*
a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c + 1176*root(26
8435456*a^11*b^2*z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194
481*b*c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x - 4096*root(268435456*a^11*b^2*
z^4 - 4718592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736
*a*d^4, z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 - 4718
592*a^6*b*d^2*z^2 + 2709504*a^3*b*c^2*d*z - 194481*b*c^4 + 20736*a*d^4, z,
 k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 460, normalized size of antiderivative = 3.38

$$\int \frac{c+dx}{(a-bx^4)^3}\,dx$$
$$= \frac{42b^{\frac{3}{4}}a^{\frac{9}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c-84b^{\frac{7}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c\,x^4+42b^{\frac{11}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c\,x^8-21b^{\frac{3}{4}}a^{\frac{9}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)c+}{}$$

input
```
int((d*x+c)/(-b*x^4+a)^3,x)
```

output
```
(42*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*c - 84*b*
*(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*c*x**4 + 42*b**(
3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**2*c*x**8 - 21*b**(3
/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a**2*c + 42*b**(3/4)*a**(1/4)*log(
a**(1/4) - b**(1/4)*x)*a*b*c*x**4 - 21*b**(3/4)*a**(1/4)*log(a**(1/4) - b*
*(1/4)*x)*b**2*c*x**8 + 21*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*
*2*c - 42*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*b*c*x**4 + 21*b**
(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*b**2*c*x**8 - 12*sqrt(b)*sqrt(a)
*log(a**(1/4) - b**(1/4)*x)*a**2*d + 24*sqrt(b)*sqrt(a)*log(a**(1/4) - b**
(1/4)*x)*a*b*d*x**4 - 12*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*b**2*d
*x**8 - 12*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a**2*d + 24*sqrt(b)*
sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a*b*d*x**4 - 12*sqrt(b)*sqrt(a)*log(a**
(1/4) + b**(1/4)*x)*b**2*d*x**8 + 12*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)
*x**2)*a**2*d - 24*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*a*b*d*x**4
+ 12*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*b**2*d*x**8 + 44*a**2*b*c
*x + 40*a**2*b*d*x**2 - 28*a*b**2*c*x**5 - 24*a*b**2*d*x**6)/(128*a**3*b*(
a**2 - 2*a*b*x**4 + b**2*x**8))
```

3.42 $\int \frac{c+dx}{(a+bx^4)^3}\,dx$

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Optimal result

Integrand size = 15, antiderivative size = 214

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{x(c+dx)}{8a(a+bx^4)^2} + \frac{x(7c+6dx)}{32a^2(a+bx^4)} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} - \frac{21c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21c\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

output
```
1/8*x*(d*x+c)/a/(b*x^4+a)^2+1/32*x*(6*d*x+7*c)/a^2/(b*x^4+a)+3/16*d*arctan
(b^(1/2)*x^2/a^(1/2))/a^(5/2)/b^(1/2)+21/128*c*arctan(-1+2^(1/2)*b^(1/4)*x
/a^(1/4))*2^(1/2)/a^(11/4)/b^(1/4)+21/128*c*arctan(1+2^(1/2)*b^(1/4)*x/a^(
1/4))*2^(1/2)/a^(11/4)/b^(1/4)+21/128*c*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/
(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(11/4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.16

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{32a^{7/4}x(c+dx)}{(a+bx^4)^2} + \frac{8a^{3/4}x(7c+6dx)}{a+bx^4} - \frac{6\left(7\sqrt{2}\sqrt[4]{b}c+8\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(7\sqrt{2}\sqrt[4]{b}c-8\sqrt[4]{a}d\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} -}{256a^{11/4}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4)^3,x]
```

output
```
((32*a^(7/4)*x*(c + d*x))/(a + b*x^4)^2 + (8*a^(3/4)*x*(7*c + 6*d*x))/(a +
 b*x^4) - (6*(7*Sqrt[2]*b^(1/4)*c + 8*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/Sqrt[b] + (6*(7*Sqrt[2]*b^(1/4)*c - 8*a^(1/4)*d)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[b] - (21*Sqrt[2]*c*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (21*Sqrt[2]*c*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(256*a^(11/4))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{8a(a+bx^4)^2} - \frac{\int -\frac{7c+6dx}{(bx^4+a)^2}dx}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7c+6dx}{(bx^4+a)^2}dx}{8a}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(7c+6dx)}{4a(a+bx^4)}-\frac{\int -\frac{3(7c+4dx)}{bx^4+a}dx}{4a}}{8a}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{7c+4dx}{bx^4+a}dx}{4a}+\frac{x(7c+6dx)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{3\int \left(\frac{7c}{bx^4+a}+\frac{4dx}{bx^4+a}\right)dx}{4a}+\frac{x(7c+6dx)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3\left(-\frac{7c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{7c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{7c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{7c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{2d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}}{8a}$$

$$\frac{x(c+dx)}{8a\left(a+bx^4\right)^2}$$

input
```
Int[(c + d*x)/(a + b*x^4)^3,x]
```

output
```
(x*(c + d*x))/(8*a*(a + b*x^4)^2) + ((x*(7*c + 6*d*x))/(4*a*(a + b*x^4)) +
 (3*((2*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - (7*c*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (7*c*ArcTan
[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (7*c*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(
1/4)) + (7*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(1/4))))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{3bd\,x^{6}}{16a^{2}}+\frac{7bc\,x^{5}}{32a^{2}}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}}{\left(b\,x^{4}+a\right)^{2}}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(4d\_R+7c\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128b\,a^{2}}$
default	$c\left(\frac{x}{8a(b\,x^{4}+a)^{2}}+\frac{\frac{7x}{32a\left(b\,x^{4}+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}\right)+d\Bigg($

input
```
int((d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/16*b*d/a^2*x^6+7/32*b/a^2*c*x^5+5/16*d/a*x^2+11/32*c/a*x)/(b*x^4+a)^2+3
/128/b/a^2*sum((4*_R*d+7*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 43180, normalized size of antiderivative = 201.78

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx$$
$$= \operatorname{RootSum}\left(268435456t^4a^{11}b^2 + 4718592t^2a^6bd^2 - 2709504ta^3bc^2d + 20736ad^4 + 194481bc^4, \left(t \mapsto t\log\right.\right.$$
$$+ \frac{11acx + 10adx^2 + 7bcx^5 + 6bdx^6}{32a^4 + 64a^3bx^4 + 32a^2b^2x^8}$$

input
```
integrate((d*x+c)/(b*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*b**2 + 4718592*_t**2*a**6*b*d**2 - 2709504*_
t*a**3*b*c**2*d + 20736*a*d**4 + 194481*b*c**4, Lambda(_t, _t*log(x + (-67
108864*_t**3*a**9*b*d**2 - 9633792*_t**2*a**6*b*c**2*d - 589824*_t*a**4*d*
*4 - 2765952*_t*a**3*b*c**4 + 423360*a*c**2*d**3)/(193536*a*c*d**4 - 45378
9*b*c**5)))) + (11*a*c*x + 10*a*d*x**2 + 7*b*c*x**5 + 6*b*d*x**6)/(32*a**4
 + 64*a**3*b*x**4 + 32*a**2*b**2*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.26

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\,(a^2b^2x^8+2\,a^3bx^4+a^4)} + \frac{3\left(\frac{7\,\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{7\,\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-8\,\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}\right.}{256\,a^2}$$

input
```
integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(6*b*d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/(a^2*b^2*x^8 + 2*a^3*
b*x^4 + a^4) + 3/256*(7*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4
)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 7*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*b^(1/
4)*c - 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(7
*sqrt(2)*a^(1/4)*b^(1/4)*c + 8*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
- sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(b))*b^(1/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{c+dx}{\left(a+bx^4\right)^3}\,dx = \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} - \frac{21\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\,(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ab}bd+7\,(ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^2} + \frac{6\,bdx^6+7\,bcx^5+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2}$$

input

```
integrate((d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output

```
21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(a^3*b) - 21/256*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/4)
 + sqrt(a/b))/(a^3*b) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)
^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b*d + 7*(a*b^3)^(1/4)*b*c)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) + 1/32
*(6*b*d*x^6 + 7*b*c*x^5 + 10*a*d*x^2 + 11*a*c*x)/((b*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \frac{\frac{5\,d\,x^2}{16\,a}+\frac{11\,c\,x}{32\,a}+\frac{7\,b\,c\,x^5}{32\,a^2}+\frac{3\,b\,d\,x^6}{16\,a^2}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(\frac{b^2\left(63\,c\,d^2+36\,d^3\,x-\mathrm{root}(268435456\,a^{11}\,b^2\,z^4+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+\right.}{}\right.\right.$$
$$\left.+4718592\,a^6\,b\,d^2\,z^2-2709504\,a^3\,b\,c^2\,d\,z+194481\,b\,c^4+20736\,a\,d^4,z,k)\right)$$

input
```
int((c + d*x)/(a + b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a^2) + (3*b*d*x^6)/(
16*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log((3*b^2*(63*c*d^2 + 36*d^
3*x - 7168*root(268435456*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a
^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z, k)^2*a^5*b*c - 1176*root(268
435456*a^11*b^2*z^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 1944
81*b*c^4 + 20736*a*d^4, z, k)*a^2*b*c^2*x + 4096*root(268435456*a^11*b^2*z
^4 + 4718592*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*
a*d^4, z, k)^2*a^5*b*d*x))/(2048*a^6))*root(268435456*a^11*b^2*z^4 + 47185
92*a^6*b*d^2*z^2 - 2709504*a^3*b*c^2*d*z + 194481*b*c^4 + 20736*a*d^4, z,
k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.46

$$\int \frac{c+dx}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)/(b*x^4+a)^3,x)
```

output
```
( - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*c - 84*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b*c*x**4 - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c*x**8 - 48*sqrt(b)*sqrt
(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt
(2)))*a**2*d - 96*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt
(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**4 - 48*sqrt(b)*sqrt(a)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2
*d*x**8 + 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*c + 84*b**(3/4)*a**(1/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqr
t(2)))*a*b*c*x**4 + 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c*x**8 - 48*sqrt(b
)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4
)*sqrt(2)))*a**2*d - 96*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**4 - 48*sqrt(b)*sqrt(a)*
atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))
)*b**2*d*x**8 - 21*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt
(2)*x + sqrt(a) + sqrt(b)*x**2)*a**2*c - 42*b**(3/4)*a**(1/4)*sqrt(2)*l...
```

3.43 $\int \frac{c+dx}{(a-bx^4)^4}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \frac{x(c+dx)}{12a\,(a-bx^4)^3} + \frac{x(11c+10dx)}{96a^2\,(a-bx^4)^2} + \frac{x(77c+60dx)}{384a^3\,(a-bx^4)} + \frac{77c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{77c\,\mathrm{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}\sqrt[4]{b}} + \frac{5d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/12*x*(d*x+c)/a/(-b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*
x*(60*d*x+77*c)/a^3/(-b*x^4+a)+77/256*c*arctan(b^(1/4)*x/a^(1/4))/a^(15/4)
/b^(1/4)+77/256*c*arctanh(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(1/4)+5/32*d*arcta
nh(b^(1/2)*x^2/a^(1/2))/a^(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.34

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$

$$= \frac{\frac{128a^3x(c+dx)}{(a-bx^4)^3} + \frac{16a^2x(11c+10dx)}{(a-bx^4)^2} + \frac{4ax(77c+60dx)}{a-bx^4} + \frac{462\sqrt[4]{a}c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} - \frac{3\left(77\sqrt[4]{a}\sqrt[4]{b}c+40\sqrt{a}d\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{\sqrt{b}} + \cdots}{1536a^4}$$

input
```
Integrate[(c + d*x)/(a - b*x^4)^4,x]
```

output
```
((128*a^3*x*(c + d*x))/(a - b*x^4)^3 + (16*a^2*x*(11*c + 10*d*x))/(a - b*x
^4)^2 + (4*a*x*(77*c + 60*d*x))/(a - b*x^4) + (462*a^(1/4)*c*ArcTan[(b^(1/
4)*x)/a^(1/4)])/b^(1/4) - (3*(77*a^(1/4)*b^(1/4)*c + 40*Sqrt[a]*d)*Log[a^(
1/4) - b^(1/4)*x])/Sqrt[b] + (3*(77*a^(1/4)*b^(1/4)*c - 40*Sqrt[a]*d)*Log[
a^(1/4) + b^(1/4)*x])/Sqrt[b] + (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])
/Sqrt[b])/(1536*a^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a-bx^4)^4}dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{12a(a-bx^4)^3}-\frac{\int -\frac{11c+10dx}{(a-bx^4)^3}dx}{12a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{11c+10dx}{(a-bx^4)^3}dx}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(11c+10dx)}{8a(a-bx^4)^2}-\frac{\int -\frac{77c+60dx}{(a-bx^4)^2}dx}{8a}}{12a}+\frac{x(c+dx)}{12a(a-bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{77c+60dx}{(a-bx^4)^2}dx}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\,(a-bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x(77c+60dx)}{4a(a-bx^4)}-\frac{\int -\frac{3(77c+40dx)}{a-bx^4}dx}{4a}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\,(a-bx^4)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{77c+40dx}{a-bx^4}dx}{4a}+\frac{x(77c+60dx)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\,(a-bx^4)^3}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\frac{3\int \left(\frac{77c}{a-bx^4}+\frac{40dx}{a-bx^4}\right)dx}{4a}+\frac{x(77c+60dx)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\,(a-bx^4)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{3\left(\frac{77c\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{77c\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{20d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x(77c+60dx)}{4a(a-bx^4)}}{8a}+\frac{x(11c+10dx)}{8a(a-bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\,(a-bx^4)^3}$$

input
```
Int[(c + d*x)/(a - b*x^4)^4,x]
```

output
```
(x*(c + d*x))/(12*a*(a - b*x^4)^3) + ((x*(11*c + 10*d*x))/(8*a*(a - b*x^4)
^2) + ((x*(77*c + 60*d*x))/(4*a*(a - b*x^4)) + (3*((77*c*ArcTan[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(1/4)) + (77*c*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a
^(3/4)*b^(1/4)) + (20*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))
)/(4*a))/(8*a))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{\frac{5 d \,b^{2} x^{10}}{32 a^{3}}+\frac{77 c \,b^{2} x^{9}}{384 a^{3}}-\frac{5 b d \,x^{6}}{12 a^{2}}-\frac{33 b c \,x^{5}}{64 a^{2}}+\frac{11 d \,x^{2}}{32 a}+\frac{51 c x}{128 a}}{\left(-b \,x^{4}+a \right)^{3}}-\frac{\sum_{\textit{\_R} =\operatorname{RootOf}\left(b \,\textit{\_Z}^{4}-a \right)}\frac{\left(40 d \textit{\_R} +77 c \right) \ln \left(x -\textit{\_R} \right)}{\textit{\_R}^{3}}}{512 b \,a^{3}}$	113
default	$\frac{\frac{5 d \,b^{2} x^{10}}{32 a^{3}}+\frac{77 c \,b^{2} x^{9}}{384 a^{3}}-\frac{5 b d \,x^{6}}{12 a^{2}}-\frac{33 b c \,x^{5}}{64 a^{2}}+\frac{11 d \,x^{2}}{32 a}+\frac{51 c x}{128 a}}{\left(-b \,x^{4}+a \right)^{3}}+\frac{\frac{77 c \left(\frac{a}{b}\right)^{\frac{1}{4}} \left(\ln \left(\frac{x +\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x -\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2 \arctan \left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4 a}+\frac{10 d \ln \left(\frac{a +x^{2} \sqrt{a b}}{a -x^{2} \sqrt{a b}}\right)}{\sqrt{a b}}}{128 a^{3}}$	165

input
```
int((d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9-5/12*b*d/a^2*x^6-33/64*b/a^2*c*x
^5+11/32*d/a*x^2+51/128*c/a*x)/(-b*x^4+a)^3-1/512/b/a^3*sum((40*_R*d+77*c)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 40780, normalized size of antiderivative = 251.73

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$
$$= \text{RootSum}\Big(68719476736t^4a^{15}b^2 - 838860800t^2a^8bd^2 + 485703680ta^4bc^2d + 2560000ad^4 - 35153041bc^4,$$
$$+ \frac{-153a^2cx - 132a^2dx^2 + 198abcx^5 + 160abdx^6 - 77b^2cx^9 - 60b^2dx^{10}}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**4,x)
```

output
```
RootSum(68719476736*_t**4*a**15*b**2 - 838860800*_t**2*a**8*b*d**2 + 48570
3680*_t*a**4*b*c**2*d + 2560000*a*d**4 - 35153041*b*c**4, Lambda(_t, _t*lo
g(x + (429496729600*_t**3*a**12*b*d**2 + 62170071040*_t**2*a**8*b*c**2*d -
 2621440000*_t*a**5*d**4 + 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2*
d**3)/(788480000*a*c*d**4 + 2706784157*b*c**5)))) + (-153*a**2*c*x - 132*a
**2*d*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 - 77*b**2*c*x**9 - 60*b**2*d*
x**10)/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3*b**3
*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.38

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = -\frac{60\,b^2dx^{10}+77\,b^2cx^9-160\,abdx^6-198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6)} + \frac{\frac{154\,c\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{77\,c\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{512\,a^3}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 132
*a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^
6) + 1/512*(154*c*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(b))) + 40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d
*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) - 77*c*log((sqrt(b)*x - sqrt
(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(129) = 258$.

Time = 0.13 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.83

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$
$$= \frac{77\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b} - \frac{77\sqrt{2}(-ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,a^4b} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}bd+77\,(-ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{-ab}bd+77\,(-ab^3)^{\frac{1}{4}}bc\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} - \frac{60\,b^2dx^{10}+77\,b^2cx^9-160\,abdx^6-198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4-a)^3a^3}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-
a/b))/(a^4*b) - 77/1024*sqrt(2)*(-a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(-a/b
)^(1/4) + sqrt(-a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d +
 77*(-a*b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-
a/b)^(1/4))/(a^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(-a*b)*b*d + 77*(-a*
b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/(a^4*b^2) - 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 - 160*a*b*d*x^6 - 198*
a*b*c*x^5 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.17

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx$$
$$= \left(\sum_{k=1}^{4}\ln\left(-\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x+\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4-838860800\,a^8\,b\,d^2\,z^2+485703680\,a\right.}{}\right.\right.$$
$$\left.-838860800\,a^8\,b\,d^2\,z^2+485703680\,a^4\,b\,c^2\,d\,z-35153041\,b\,c^4+2560000\,a\,d^4,z,k)\right)$$
$$+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}-\frac{33\,b\,c\,x^5}{64\,a^2}-\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3-3\,a^2\,b\,x^4+3\,a\,b^2\,x^8-b^3\,x^{12}}$$

input
```
int((c + d*x)/(a - b*x^4)^4,x)
```

output
```
symsum(log(-(b^2*(1925*c*d^2 + 1000*d^3*x + 315392*root(68719476736*a^15*b
^2*z^4 - 838860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^
4 + 2560000*a*d^4, z, k)^2*a^7*b*c + 47432*root(68719476736*a^15*b^2*z^4 -
 838860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560
000*a*d^4, z, k)*a^3*b*c^2*x - 163840*root(68719476736*a^15*b^2*z^4 - 8388
60800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*a
*d^4, z, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 - 838
860800*a^8*b*d^2*z^2 + 485703680*a^4*b*c^2*d*z - 35153041*b*c^4 + 2560000*
a*d^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c
*x^9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) - (33*b*c*x^5)/(64*a^2) - (5*b*d
*x^6)/(12*a^2))/(a^3 - b^3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 643, normalized size of antiderivative = 3.97

$$\int \frac{c+dx}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)/(-b*x^4+a)^4,x)
```

output
```
(462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*c - 1386
*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*c*x**4 + 1
386*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*c*x**8
- 462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**3*c*x**12
 - 231*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a**3*c + 693*b**(3/4)*
a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a**2*b*c*x**4 - 693*b**(3/4)*a**(1/4)*
log(a**(1/4) - b**(1/4)*x)*a*b**2*c*x**8 + 231*b**(3/4)*a**(1/4)*log(a**(1
/4) - b**(1/4)*x)*b**3*c*x**12 + 231*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(
1/4)*x)*a**3*c - 693*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a**2*b*c
*x**4 + 693*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*b**2*c*x**8 - 2
31*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*b**3*c*x**12 - 120*sqrt(b)
*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*a**3*d + 360*sqrt(b)*sqrt(a)*log(a**(1
/4) - b**(1/4)*x)*a**2*b*d*x**4 - 360*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1
/4)*x)*a*b**2*d*x**8 + 120*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*b**3
*d*x**12 - 120*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a**3*d + 360*sqr
t(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a**2*b*d*x**4 - 360*sqrt(b)*sqrt(a
)*log(a**(1/4) + b**(1/4)*x)*a*b**2*d*x**8 + 120*sqrt(b)*sqrt(a)*log(a**(1
/4) + b**(1/4)*x)*b**3*d*x**12 + 120*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)
*x**2)*a**3*d - 360*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*a**2*b*d*x
**4 + 360*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*a*b**2*d*x**8 - 1...
```

3.44 $\int \frac{c+dx}{(a+bx^4)^4}\,dx$

Optimal result 400
Mathematica [A] (verified) 401
Rubi [A] (verified) 401
Maple [C] (verified) 404
Fricas [C] (verification not implemented) 404
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Reduce [B] (verification not implemented) 408

Optimal result

Integrand size = 15, antiderivative size = 239

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \frac{x(c+dx)}{12a(a+bx^4)^3} + \frac{x(11c+10dx)}{96a^2(a+bx^4)^2} + \frac{x(77c+60dx)}{384a^3(a+bx^4)} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} - \frac{77c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{77c\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

output

```
1/12*x*(d*x+c)/a/(b*x^4+a)^3+1/96*x*(10*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*
(60*d*x+77*c)/a^3/(b*x^4+a)+5/32*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(7/2)/b^(
1/2)+77/512*c*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(15/4)/b^(1/4
)+77/512*c*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(15/4)/b^(1/4)+77
/512*c*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^
(15/4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.15

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{256a^{11/4}x(c+dx)}{(a+bx^4)^3} + \frac{32a^{7/4}x(11c+10dx)}{(a+bx^4)^2} + \frac{8a^{3/4}x(77c+60dx)}{a+bx^4} - \frac{6\left(77\sqrt{2}\sqrt[4]{b}c+80\sqrt[4]{a}d\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{b}} + \frac{6\left(77\sqrt{2}\sqrt[4]{b}c-80\right.}{}}{3072a^{15/4}}$$

input
```
Integrate[(c + d*x)/(a + b*x^4)^4,x]
```

output
```
((256*a^(11/4)*x*(c + d*x))/(a + b*x^4)^3 + (32*a^(7/4)*x*(11*c + 10*d*x))
/(a + b*x^4)^2 + (8*a^(3/4)*x*(77*c + 60*d*x))/(a + b*x^4) - (6*(77*Sqrt[2
]*b^(1/4)*c + 80*a^(1/4)*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/Sqrt[
b] + (6*(77*Sqrt[2]*b^(1/4)*c - 80*a^(1/4)*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/Sqrt[b] - (231*Sqrt[2]*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/b^(1/4) + (231*Sqrt[2]*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(3072*a^(15/4))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(c+dx)}{12a(a+bx^4)^3} - \frac{\int -\frac{11c+10dx}{(bx^4+a)^3}dx}{12a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{11c+10dx}{(bx^4+a)^3}dx}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(11c+10dx)}{8a(a+bx^4)^2}-\frac{\int -\frac{77c+60dx}{\left(bx^4+a\right)^2}dx}{8a}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{77c+60dx}{\left(bx^4+a\right)^2}dx}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{\frac{x(77c+60dx)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3(77c+40dx)}{bx^4+a}dx}{4a}}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{77c+40dx}{bx^4+a}dx}{4a}+\frac{x(77c+60dx)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{77c}{bx^4+a}+\frac{40dx}{bx^4+a}\right)dx}{4a}+\frac{x(77c+60dx)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx)}{8a(a+bx^4)^2}}{12a}+\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{3\bigg(-\frac{77c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{77c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{77c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{77c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{20d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}}{8a}}{12a}$$

$$\frac{x(c+dx)}{12a\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x)/(a + b*x^4)^4,x]
```

output
```
(x*(c + d*x))/(12*a*(a + b*x^4)^3) + ((x*(11*c + 10*d*x))/(8*a*(a + b*x^4)
^2) + ((x*(77*c + 60*d*x))/(4*a*(a + b*x^4)) + (3*((20*d*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - (77*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) + (77*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (77*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + (77*c*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))
)/(4*a))/(8*a))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.46

method	result
risch	$\frac{\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(b\,x^{4}+a\right)^{3}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(40d\_R+77c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{512b\,a^{3}}$
default	$\frac{\frac{5d\,b^{2}x^{10}}{32a^{3}}+\frac{77c\,b^{2}x^{9}}{384a^{3}}+\frac{5bd\,x^{6}}{12a^{2}}+\frac{33bc\,x^{5}}{64a^{2}}+\frac{11d\,x^{2}}{32a}+\frac{51cx}{128a}}{\left(b\,x^{4}+a\right)^{3}}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)\cdots}\right.\right.}{8a}\cdots}{128a^{3}}$

input
```
int((d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9+5/12*b*d/a^2*x^6+33/64*b/a^2*c*x
^5+11/32*d/a*x^2+51/128*c/a*x)/(b*x^4+a)^3+1/512/b/a^3*sum((40*_R*d+77*c)/
_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 43302, normalized size of antiderivative = 181.18

$$\int \frac{c+dx}{\left(a+bx^4\right)^4}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.97

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \operatorname{RootSum}\Bigg(68719476736t^4a^{15}b^2 + 838860800t^2a^8bd^2 - 485703680ta^4bc^2d + 2560000ad^4 + 35153041bc^4, \\ + \frac{153a^2cx + 132a^2dx^2 + 198abcx^5 + 160abdx^6 + 77b^2cx^9 + 60b^2dx^{10}}{384a^6 + 1152a^5bx^4 + 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

input

```
integrate((d*x+c)/(b*x**4+a)**4,x)
```

output

```
RootSum(68719476736*_t**4*a**15*b**2 + 838860800*_t**2*a**8*b*d**2 - 48570
3680*_t*a**4*b*c**2*d + 2560000*a*d**4 + 35153041*b*c**4, Lambda(_t, _t*lo
g(x + (-429496729600*_t**3*a**12*b*d**2 - 62170071040*_t**2*a**8*b*c**2*d
- 2621440000*_t*a**5*d**4 - 17998356992*_t*a**4*b*c**4 + 1897280000*a*c**2
*d**3)/(788480000*a*c*d**4 - 2706784157*b*c**5)))) + (153*a**2*c*x + 132*a
**2*d*x**2 + 198*a*b*c*x**5 + 160*a*b*d*x**6 + 77*b**2*c*x**9 + 60*b**2*d*
x**10)/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b**3*
x**12)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.27

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \frac{60\,b^2dx^{10} + 77\,b^2cx^9 + 160\,abdx^6 + 198\,abcx^5 + 132\,a^2dx^2 + 153\,a^2cx}{384\,(a^3b^3x^{12} + 3\,a^4b^2x^8 + 3\,a^5bx^4 + a^6)} \\ + \frac{\frac{77\,\sqrt{2}c\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{77\,\sqrt{2}c\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} + \frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}c-80\,\sqrt{a}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{1}{4}}}}{1024\,a^3}$$

input

```
integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 132*
a^2*d*x^2 + 153*a^2*c*x)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6
) + 1/1024*(77*sqrt(2)*c*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqr
t(a))/(a^(3/4)*b^(1/4)) - 77*sqrt(2)*c*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b
^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(1/4)*c -
80*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/s
qrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(1/4)) + 2*(77*sqrt
(2)*a^(1/4)*b^(1/4)*c + 80*sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b
))*b^(1/4)))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.17

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b} - \frac{77\sqrt{2}(ab^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} + \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}bd+77\,(ab^3)^{\frac{1}{4}}\,bc\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^2} + \frac{60\,b^2dx^{10}+77\,b^2cx^9+160\,abdx^6+198\,abcx^5+132\,a^2dx^2+153\,a^2cx}{384\,(bx^4+a)^3a^3}$$

input
```
integrate((d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b
))/(a^4*b) - 77/1024*sqrt(2)*(a*b^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^4*b) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*
b^3)^(1/4)*b*c)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a^4*b^2) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b*d + 77*(a*b^3)^(1/4)*b
*c)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^2)
+ 1/384*(60*b^2*d*x^10 + 77*b^2*c*x^9 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 13
2*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.46

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\frac{b^2\left(1925\,c\,d^2+1000\,d^3\,x-\mathrm{root}(68719476736\,a^{15}\,b^2\,z^4+838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\right.}{}\right.\right.$$
$$\left.\left. +838860800\,a^8\,b\,d^2\,z^2-485703680\,a^4\,b\,c^2\,d\,z+35153041\,b\,c^4+2560000\,a\,d^4,z,k)\right)\right)$$
$$+\frac{\frac{11\,d\,x^2}{32\,a}+\frac{51\,c\,x}{128\,a}+\frac{77\,b^2\,c\,x^9}{384\,a^3}+\frac{5\,b^2\,d\,x^{10}}{32\,a^3}+\frac{33\,b\,c\,x^5}{64\,a^2}+\frac{5\,b\,d\,x^6}{12\,a^2}}{a^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^8+b^3\,x^{12}}$$

input
```
int((c + d*x)/(a + b*x^4)^4,x)
```

output
```
symsum(log((b^2*(1925*c*d^2 + 1000*d^3*x - 315392*root(68719476736*a^15*b^
2*z^4 + 838860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4
 + 2560000*a*d^4, z, k)^2*a^7*b*c - 47432*root(68719476736*a^15*b^2*z^4 +
838860800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 25600
00*a*d^4, z, k)*a^3*b*c^2*x + 163840*root(68719476736*a^15*b^2*z^4 + 83886
0800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a*
d^4, z, k)^2*a^7*b*d*x))/(32768*a^9))*root(68719476736*a^15*b^2*z^4 + 8388
60800*a^8*b*d^2*z^2 - 485703680*a^4*b*c^2*d*z + 35153041*b*c^4 + 2560000*a
*d^4, z, k), k, 1, 4) + ((11*d*x^2)/(32*a) + (51*c*x)/(128*a) + (77*b^2*c*
x^9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (33*b*c*x^5)/(64*a^2) + (5*b*d*
x^6)/(12*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.23

$$\int \frac{c+dx}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)/(b*x^4+a)^4,x)
```

output
```
( - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt
(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*c - 1386*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*b*c*x**4 - 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*c*x**8 - 462*b
**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b
**(1/4)*a**(1/4)*sqrt(2)))*b**3*c*x**12 - 480*sqrt(b)*sqrt(a)*atan((b**(1/
4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*d - 1
440*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*a**2*b*d*x**4 - 1440*sqrt(b)*sqrt(a)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*d*x**8
 - 480*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**
(1/4)*a**(1/4)*sqrt(2)))*b**3*d*x**12 + 462*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*3*c + 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c*x**4 + 1386*b**(3/4)*a**(
1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(
1/4)*sqrt(2)))*a*b**2*c*x**8 + 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*c*x**1
2 - 480*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/...
```

3.45 $\int \frac{c+dx}{1-x^4} dx$

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Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{c+dx}{1-x^4} dx = \frac{1}{2}c\arctan(x) + \frac{1}{2}c\,\text{arctanh}(x) + \frac{1}{2}d\,\text{arctanh}\left(x^2\right)$$

output
```
1/2*c*arctan(x)+1/2*c*arctanh(x)+1/2*d*arctanh(x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{c+dx}{1-x^4} dx = \frac{1}{4}\left(2c\arctan(x) - (c+d)\log(1-x) + c\log(1+x) - d\log(1+x) + d\log\left(1+x^2\right)\right)$$

input
```
Integrate[(c + d*x)/(1 - x^4),x]
```

output
```
(2*c*ArcTan[x] - (c + d)*Log[1 - x] + c*Log[1 + x] - d*Log[1 + x] + d*Log[
1 + x^2])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{1-x^4}dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{c}{1-x^4}+\frac{dx}{1-x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c\arctan(x)+\frac{1}{2}c\text{arctanh}(x)+\frac{1}{2}d\text{arctanh}\left(x^2\right)$$

input
```
Int[(c + d*x)/(1 - x^4),x]
```

output
```
(c*ArcTan[x])/2 + (c*ArcTanh[x])/2 + (d*ArcTanh[x^2])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result
default	$\frac{(-c-d)\ln(x-1)}{4}-\frac{(-c+d)\ln(x+1)}{4}+\frac{d\ln(x^2+1)}{4}+\frac{c\arctan(x)}{2}$
meijerg	$\frac{d\,\mathrm{arctanh}(x^2)}{2}-\frac{cx\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}$
parallelrisch	$-\frac{\ln(x-1)c}{4}-\frac{\ln(x-1)d}{4}+\frac{\ln(x+1)c}{4}-\frac{\ln(x+1)d}{4}+\frac{\ln(x-i)d}{4}-\frac{i\ln(x-i)c}{4}+\frac{\ln(x+i)d}{4}+\frac{i\ln(x+i)c}{4}$
risch	$-\frac{\ln(x-1)c}{4}-\frac{\ln(x-1)d}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^2-2d\_Z+c^2+d^2\right)}\_R\ln\left(\left(\_R^2d-\_Rc^2-d^3\right)x-\_R^2c-cd^2\right)\right)}{4}+\ln$

input
```
int((d*x+c)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-c-d)*ln(x-1)-1/4*(-c+d)*ln(x+1)+1/4*d*ln(x^2+1)+1/2*c*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{c+dx}{1-x^4}\,dx=\frac{1}{2}\,c\arctan(x)+\frac{1}{4}\,d\log\left(x^2+1\right)+\frac{1}{4}\,(c-d)\log(x+1)-\frac{1}{4}\,(c+d)\log(x-1)$$

input
```
integrate((d*x+c)/(-x^4+1),x, algorithm="fricas")
```

output
```
1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d
)*log(x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 313, normalized size of antiderivative = 13.04

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{(c-d)\log\left(x+\frac{c^4(c-d)+5c^2d^3+c^2d(c-d)^2-2d^4(c-d)+2d^2(c-d)^3}{c^5+4cd^4}\right)}{4} - \frac{(c+d)\log\left(x+\frac{-c^4(c+d)+5c^2d^3+c^2d(c+d)^2+2d^4(c+d)-2d^2(c+d)^3}{c^5+4cd^4}\right)}{4} - \left(-\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(-\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(-\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(-\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(-\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right) - \left(\frac{ic}{4}-\frac{d}{4}\right)\log\left(x+\frac{-4c^4\left(\frac{ic}{4}-\frac{d}{4}\right)+5c^2d^3+16c^2d\left(\frac{ic}{4}-\frac{d}{4}\right)^2+8d^4\left(\frac{ic}{4}-\frac{d}{4}\right)-128d^2\left(\frac{ic}{4}-\frac{d}{4}\right)^3}{c^5+4cd^4}\right)$$

input
```
integrate((d*x+c)/(-x**4+1),x)
```

output
```
(c - d)*log(x + (c**4*(c - d) + 5*c**2*d**3 + c**2*d*(c - d)**2 - 2*d**4*(
c - d) + 2*d**2*(c - d)**3)/(c**5 + 4*c*d**4))/4 - (c + d)*log(x + (-c**4*
(c + d) + 5*c**2*d**3 + c**2*d*(c + d)**2 + 2*d**4*(c + d) - 2*d**2*(c + d
)**3)/(c**5 + 4*c*d**4))/4 - (-I*c/4 - d/4)*log(x + (-4*c**4*(-I*c/4 - d/4
) + 5*c**2*d**3 + 16*c**2*d*(-I*c/4 - d/4)**2 + 8*d**4*(-I*c/4 - d/4) - 12
8*d**2*(-I*c/4 - d/4)**3)/(c**5 + 4*c*d**4)) - (I*c/4 - d/4)*log(x + (-4*c
**4*(I*c/4 - d/4) + 5*c**2*d**3 + 16*c**2*d*(I*c/4 - d/4)**2 + 8*d**4*(I*c
/4 - d/4) - 128*d**2*(I*c/4 - d/4)**3)/(c**5 + 4*c*d**4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}\,c\arctan\left(x\right) + \frac{1}{4}\,d\log\left(x^2+1\right) + \frac{1}{4}\,(c-d)\log\left(x+1\right) - \frac{1}{4}\,(c+d)\log\left(x-1\right)$$

input
```
integrate((d*x+c)/(-x^4+1),x, algorithm="maxima")
```

output
```
1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(x + 1) - 1/4*(c + d
)*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{1}{2}\,c\arctan\left(x\right) + \frac{1}{4}\,d\log\left(x^2+1\right) + \frac{1}{4}\,(c-d)\log\left(|x+1|\right) - \frac{1}{4}\,(c+d)\log\left(|x-1|\right)$$

input
```
integrate((d*x+c)/(-x^4+1),x, algorithm="giac")
```

output
```
1/2*c*arctan(x) + 1/4*d*log(x^2 + 1) + 1/4*(c - d)*log(abs(x + 1)) - 1/4*(
c + d)*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.17

$$\int \frac{c+dx}{1-x^4}\,dx = -\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{3/4}\,\sqrt{2}\,x+1\right)\,\left(\sqrt{2}\,c+2\,(-1)^{1/4}\,d\right)}{4} - \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{3/4}\,\sqrt{2}\,x-1\right)\,\left(2\,\sqrt{2}\,c-4\,(-1)^{1/4}\,d\right)}{8} + \frac{(-1)^{1/4}\,\sqrt{2}\,c\,\ln\left(\frac{x^2+(-1)^{1/4}\,\sqrt{2}\,x+1\mathrm{i}}{x^2-(-1)^{1/4}\,\sqrt{2}\,x+1\mathrm{i}}\right)}{8}$$

input
```
int(-(c + d*x)/(x^4 - 1),x)
```

output
```
((-1)^(1/4)*2^(1/2)*c*log((x^2 + (-1)^(1/4)*2^(1/2)*x + 1i)/(x^2 - (-1)^(1
/4)*2^(1/2)*x + 1i)))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x - 1)*(2*2^
(1/2)*c - 4*(-1)^(1/4)*d))/8 - ((-1)^(1/4)*atan((-1)^(3/4)*2^(1/2)*x + 1)*
(2^(1/2)*c + 2*(-1)^(1/4)*d))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{c+dx}{1-x^4}\,dx = \frac{atan(x)\,c}{2} + \frac{\log(x^2+1)\,d}{4} - \frac{\log(x-1)\,c}{4} - \frac{\log(x-1)\,d}{4} + \frac{\log(x+1)\,c}{4} - \frac{\log(x+1)\,d}{4}$$

input
```
int((d*x+c)/(-x^4+1),x)
```

output
```
(2*atan(x)*c + log(x**2 + 1)*d - log(x - 1)*c - log(x - 1)*d + log(x + 1)*
c - log(x + 1)*d)/4
```

3.46 $\int \frac{c+dx}{1+x^4} dx$

Optimal result 415
Mathematica [C] (verified) 415
Rubi [A] (verified) 416
Maple [C] (verified) 417
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Maxima [A] (verification not implemented) 419
Giac [A] (verification not implemented) 419
Mupad [B] (verification not implemented) 420
Reduce [B] (verification not implemented) 420

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{c+dx}{1+x^4} dx = \frac{1}{2}d\arctan\left(x^2\right) - \frac{c\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}} + \frac{c\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}} + \frac{c\text{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{2\sqrt{2}}$$

output
```
1/2*d*arctan(x^2)+1/4*c*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*c*arctan(1+x*2^(1
/2))*2^(1/2)+1/4*c*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int \frac{c+dx}{1+x^4} dx = \frac{1}{4}\left(-\left(\left(\sqrt[4]{-1}c+id\right)\log\left(\sqrt[4]{-1}-x\right)\right) + \left(-(-1)^{3/4}c + id\right)\log\left((-1)^{3/4}-x\right) + \left(\sqrt[4]{-1}c-id\right)\log\left(\sqrt[4]{-1}+x\right) + \left((-1)^{3/4}c+id\right)\log\left((-1)^{3/4}+x\right)\right)$$

input
```
Integrate[(c + d*x)/(1 + x^4),x]
```

output
```
(-(((-1)^(1/4)*c + I*d)*Log[(-1)^(1/4) - x]) + (-((-1)^(3/4)*c) + I*d)*Log
[(-1)^(3/4) - x] + ((-1)^(1/4)*c - I*d)*Log[(-1)^(1/4) + x] + ((-1)^(3/4)*
c + I*d)*Log[(-1)^(3/4) + x])/4
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^4+1}\,dx$$

$$\Big\downarrow \; 2415$$

$$\int \left(\frac{c}{x^4+1}+\frac{dx}{x^4+1}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{c\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{c\arctan\left(\sqrt{2}x+1\right)}{2\sqrt{2}}+\frac{1}{2}d\arctan\left(x^2\right)-\frac{c\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{c\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}$$

input
```
Int[(c + d*x)/(1 + x^4),x]
```

output
```
(d*ArcTan[x^2])/2 - (c*ArcTan[1 - Sqrt[2]*x])/(2*Sqrt[2]) + (c*ArcTan[1 +
Sqrt[2]*x])/(2*Sqrt[2]) - (c*Log[1 - Sqrt[2]*x + x^2])/(4*Sqrt[2]) + (c*Lo
g[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+1\right)}\frac{\left(d\_R+c\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4}$
default	$\frac{c\sqrt{2}\left(\ln\left(\frac{x^{2}+x\sqrt{2}+1}{x^{2}-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}+\frac{d\arctan\left(x^{2}\right)}{2}$
meijerg	$\frac{d\arctan\left(x^{2}\right)}{2}+\frac{c\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}+\sqrt{x^{4}}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^{4}\right)^{\frac{1}{4}}+\sqrt{x^{4}}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{4}\right.}{2+\sqrt{2}\left(\right.}\right.}{\left(x^{4}\right)^{\frac{1}{4}}}\right.}{4}$

input
```
int((d*x+c)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 34344, normalized size of antiderivative = 451.89

$$\int \frac{c+dx}{1+x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(x^4+1),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{1+x^4}\,dx$$
$$=\operatorname{RootSum}\left(256t^4+32t^2d^2-16tc^2d+c^4+d^4,\left(t\mapsto t\log\left(x+\frac{128t^3d^2+16t^2c^2d+4tc^4+8td^4-5c^2d^3}{c^5-4cd^4}\right.\right.\right.$$

input
```
integrate((d*x+c)/(x**4+1),x)
```

output
```
RootSum(256*_t**4 + 32*_t**2*d**2 - 16*_t*c**2*d + c**4 + d**4, Lambda(_t,
 _t*log(x + (128*_t**3*d**2 + 16*_t**2*c**2*d + 4*_t*c**4 + 8*_t*d**4 - 5*
c**2*d**3)/(c**5 - 4*c*d**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{c+dx}{1+x^4}\,dx = \frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right) + \frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

input
```
integrate((d*x+c)/(x^4+1),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
 + 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(s
qrt(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{c+dx}{1+x^4}\,dx = \frac{1}{8}\sqrt{2}c\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{8}\sqrt{2}c\log\left(x^2-\sqrt{2}x+1\right) + \frac{1}{4}\left(\sqrt{2}c-2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}c+2\,d\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\sqrt{2}\right)\right)$$

input
```
integrate((d*x+c)/(x^4+1),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*c*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*c*log(x^2 - sqrt(2)*x
 + 1) + 1/4*(sqrt(2)*c - 2*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(s
qrt(2)*c + 2*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{1+x^4}\,dx = \operatorname{atan}\left(\sqrt{2}\,x-1\right)\left(\frac{d}{2}+\frac{\sqrt{2}\,c}{4}\right) - \operatorname{atan}\left(\sqrt{2}\,x+1\right)\left(\frac{d}{2}-\frac{\sqrt{2}\,c}{4}\right)+\frac{\sqrt{2}\,c\,\ln\left(\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right)}{8}$$

input
```
int((c + d*x)/(x^4 + 1),x)
```

output
```
atan(2^(1/2)*x - 1)*(d/2 + (2^(1/2)*c)/4) - atan(2^(1/2)*x + 1)*(d/2 - (2^
(1/2)*c)/4) + (2^(1/2)*c*log((2^(1/2)*x + x^2 + 1)/(x^2 - 2^(1/2)*x + 1)))
/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int \frac{c+dx}{1+x^4}\,dx = -\frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)c}{4}-\frac{atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)d}{2}+\frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)c}{4}-\frac{atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)d}{2}-\frac{\sqrt{2}\log\left(-\sqrt{2}\,x+x^2+1\right)c}{8}+\frac{\sqrt{2}\log\left(\sqrt{2}\,x+x^2+1\right)c}{8}$$

input
```
int((d*x+c)/(x^4+1),x)
```

output
```
( - 2*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*c - 4*atan((sqrt(2) - 2*x)/sqr
t(2))*d + 2*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*c - 4*atan((sqrt(2) + 2*
x)/sqrt(2))*d - sqrt(2)*log( - sqrt(2)*x + x**2 + 1)*c + sqrt(2)*log(sqrt(
2)*x + x**2 + 1)*c)/8
```

3.47 $\int \frac{c+dx+ex^2}{a-bx^4}\,dx$

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Rubi [A] (verified) 422
Maple [C] (verified) 423
Fricas [C] (verification not implemented) 423
Sympy [B] (verification not implemented) 424
Maxima [A] (verification not implemented) 425
Giac [B] (verification not implemented) 425
Mupad [B] (verification not implemented) 427
Reduce [B] (verification not implemented) 428

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx=\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

output
```
1/2*(c-a^(1/2)*e/b^(1/2))*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)+1/2*(b
^(1/2)*c+a^(1/2)*e)*arctanh(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(3/4)+1/2*d*arcta
nh(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.63

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx$$
$$=\frac{2\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}c+\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{a}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)+\sqrt{b}c\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)-}{4a^{3/4}b^{3/4}}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4),x]
```

output
```
(2*(Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + a^(1
/4)*b^(1/4)*d + Sqrt[a]*e)*Log[a^(1/4) - b^(1/4)*x] + Sqrt[b]*c*Log[a^(1/4
) + b^(1/4)*x] - a^(1/4)*b^(1/4)*d*Log[a^(1/4) + b^(1/4)*x] + Sqrt[a]*e*Lo
g[a^(1/4) + b^(1/4)*x] + a^(1/4)*b^(1/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(4*
a^(3/4)*b^(3/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx$$
$$\Big\downarrow \ 2415$$
$$\int \left(\frac{c+ex^2}{a-bx^4}+\frac{dx}{a-bx^4}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x + e*x^2)/(a - b*x^4),x]
```

output
```
((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4
)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.34

method	result	size
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^2e+d\_R+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	39
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	139

input
```
int((e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 120560, normalized size of antiderivative = 1048.35

$$\int\frac{c+dx+ex^2}{a-bx^4}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(109) = 218$.

Time = 4.22 (sec) , antiderivative size = 471, normalized size of antiderivative = 4.10

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = -\operatorname{RootSum}\left(256t^4a^3b^3+t^2\left(-64a^2b^2ce-32a^2b^2d^2\right)+t\left(-16a^2bde^2-16ab^2c^2d\right)-a^2e^4+2abc^2e^2-4a\right.$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**3 + _t**2*(-64*a**2*b**2*c*e - 32*a**2*b**2*d**
2) + _t*(-16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) - a**2*e**4 + 2*a*b*c**2*e*
*2 - 4*a*b*c*d**2*e + a*b*d**4 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t*
*3*a**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2
 + 48*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e - 16*_t**2*a**2
*b**3*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 + 16*_t*a**2*b
**2*c**3*e**2 - 36*_t*a**2*b**2*c**2*d**2*e - 8*_t*a**2*b**2*c*d**4 + 4*_t
*a*b**3*c**5 + 3*a**3*d*e**5 - 5*a**2*b*c*d**3*e**2 + 2*a**2*b*d**5*e + 5*
a*b**2*c**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 + a**2*b*c**2*e**4 - 8*a*
*2*b*c*d**2*e**3 + 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d
**2*e - 4*a*b**2*c**2*d**4 - b**3*c**6))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \frac{d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} - \frac{d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}\sqrt{b}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 1/4*d*log(sqrt(b)*x^2
 - sqrt(a))/(sqrt(a)*sqrt(b)) + 1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)
*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(s
qrt(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x +
 sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.25

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx$$
$$= -\frac{\sqrt{2}\left(b^2c-\sqrt{2}(-ab^3)^{\frac{1}{4}}bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c+\sqrt{2}(-ab^3)^{\frac{1}{4}}bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$
$$+\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/
4*sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*s
qrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 725, normalized size of antiderivative = 6.30

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx = \sum_{k=1}^{4}\ln\Bigl(-b^2\,c\,d^2+b^2\,c^2\,e-b^2\,d^3\,x-a\,b\,e^3$$

$$-\mathrm{root}\bigl(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e$$

$$-\,\mathrm{root}\bigl(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$

$$-\,4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\bigr)\;b^3\,c^2\,x\,4$$

$$+\mathrm{root}\bigl(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e$$

$$-\,\mathrm{root}\bigl(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$

$$-\,4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\bigr)\;a\,b^2\,e^2\,x\,4$$

$$+\,\mathrm{root}\bigl(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z+16\,a\,b^2\,c^2\,d\,z$$

$$-\,4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\bigr)\;a\,b^2\,d\,e\,8$$

$$+\,2\,b^2\,c\,d\,e\,x\Bigr)\;\mathrm{root}\bigl(256\,a^3\,b^3\,z^4-64\,a^2\,b^2\,c\,e\,z^2-32\,a^2\,b^2\,d^2\,z^2+16\,a^2\,b\,d\,e^2\,z$$

$$+\,16\,a\,b^2\,c^2\,d\,z-4\,a\,b\,c\,d^2\,e+2\,a\,b\,c^2\,e^2+a\,b\,d^4-a^2\,e^4-b^2\,c^4,z,k\bigr)$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4),x)
```

output
```
symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b
^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a
*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4
, z, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c
^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^
3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*
b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4,
 z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*
b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*
c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k)*a*b^2*e^2*x + 8*root(256*a^3*
b^3*z^4 - 64*a^2*b^2*c*e*z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*
a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^
4, z, k)*a*b^2*d*e + 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 - 64*a^2*b^2*c*e*
z^2 - 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z + 16*a*b^2*c^2*d*z - 4*a*b*c*d
^2*e + 2*a*b*c^2*e^2 + a*b*d^4 - a^2*e^4 - b^2*c^4, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.53

$$\int \frac{c+dx+ex^2}{a-bx^4}\,dx$$
$$= \frac{-2b^{\frac{1}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)e+2b^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c-b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)e+b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}x\right)e-b^{\frac{3}{4}}a^{\frac{1}{4}}\log}{}$$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*e + 2*b**(3/
4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*c - b**(1/4)*a**(3/4)*lo
g(a**(1/4) - b**(1/4)*x)*e + b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*
e - b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*c + b**(3/4)*a**(1/4)*log
(a**(1/4) + b**(1/4)*x)*c - sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*d -
 sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*d + sqrt(b)*sqrt(a)*log(sqrt(a
) + sqrt(b)*x**2)*d)/(4*a*b)
```

3.48 $\int \frac{c+dx+ex^2}{a+bx^4} dx$

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Optimal result

Integrand size = 20, antiderivative size = 210

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}$$

output

```
1/2*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)+1/4*(b^(1/2)*c+a^(1/2)*e
)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4)+1/4*(b^(1/2
)*c+a^(1/2)*e)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4)
+1/4*(b^(1/2)*c-a^(1/2)*e)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1
/2)*x^2))*2^(1/2)/a^(3/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$
$$= \frac{-2\left(\sqrt{2}\sqrt{b}c+2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt{b}c-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1\right.}{8a^{3/4}}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4),x]
```

output
```
(-2*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1
 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)
*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*
(Sqrt[b]*c - Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(8*a^(3/4
)*b^(3/4))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$
$$\Big\downarrow \text{ 2415}$$
$$\int \left(\frac{c+ex^2}{a+bx^4}+\frac{dx}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4),x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c
 - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(3/4)*b^(3/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(\_R^{2}e+d\_R+c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^{2}-}{x^{2}+}\right.\right.}{}$

input
```
int((e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 121386, normalized size of antiderivative = 578.03

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(197) = 394.

Time = 4.50 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.22

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^3 + t^2\cdot\left(64a^2b^2ce + 32a^2b^2d^2\right) + t\left(16a^2bde^2 - 16ab^2c^2d\right) + a^2e^4 + 2abc^2e^2 - 4abcd^2\right.$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**3 + _t**2*(64*a**2*b**2*c*e + 32*a**2*b**2*d**2)
 + _t*(16*a**2*b*d*e**2 - 16*a*b**2*c**2*d) + a**2*e**4 + 2*a*b*c**2*e**2
- 4*a*b*c*d**2*e + a*b*d**4 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a
**4*b**2*e**3 - 64*_t**3*a**3*b**3*c**2*e + 128*_t**3*a**3*b**3*c*d**2 + 4
8*_t**2*a**3*b**2*c*d*e**2 - 32*_t**2*a**3*b**2*d**3*e + 16*_t**2*a**2*b**
3*c**3*d + 12*_t*a**3*b*c*e**4 + 12*_t*a**3*b*d**2*e**3 - 16*_t*a**2*b**2*
c**3*e**2 + 36*_t*a**2*b**2*c**2*d**2*e + 8*_t*a**2*b**2*c*d**4 + 4*_t*a*b
**3*c**5 + 3*a**3*d*e**5 + 5*a**2*b*c*d**3*e**2 - 2*a**2*b*d**5*e + 5*a*b*
*2*c**4*d*e - 5*a*b**2*c**3*d**3)/(a**3*e**6 - a**2*b*c**2*e**4 + 8*a**2*b
*c*d**2*e**3 - 4*a**2*b*d**4*e**2 - a*b**2*c**4*e**2 + 8*a*b**2*c**3*d**2*
e - 4*a*b**2*c**2*d**4 + b**3*c**6))))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.22

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-2\,\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e+2\,\sqrt{a}\sqrt{b}d\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{3}{4}}}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*e)*l
og(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) +
1/4*(sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 2*sqrt(a)*sqr
t(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 1/4*(sqrt(2)*a^(
1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 2*sqrt(a)*sqrt(b)*d)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.29

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) -
 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4
)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) +
 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^
3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 712, normalized size of antiderivative = 3.39

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(b^2\,c\,d^2 - b^2\,c^2\,e + b^2\,d^3\,x - a\,b\,e^3$$

$$-\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e$$

$$-\,\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z$$
$$-\,4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\;b^3\,c^2\,x\,4$$

$$+\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e$$

$$+\,\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z$$
$$-\,4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\;a\,b^2\,e^2\,x\,4$$

$$-\,\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z - 16\,a\,b^2\,c^2\,d\,z$$
$$-\,4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)\;a\,b^2\,d\,e\,8$$

$$-\,2\,b^2\,c\,d\,e\,x\Big)\;\mathrm{root}\big(256\,a^3\,b^3\,z^4 + 64\,a^2\,b^2\,c\,e\,z^2 + 32\,a^2\,b^2\,d^2\,z^2 + 16\,a^2\,b\,d\,e^2\,z$$
$$-\,16\,a\,b^2\,c^2\,d\,z - 4\,a\,b\,c\,d^2\,e + 2\,a\,b\,c^2\,e^2 + a\,b\,d^4 + a^2\,e^4 + b^2\,c^4, z, k\big)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4),x)
```

output
```
symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - 16*root(256*a^3*b
^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a
*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4
, z, k)^2*a*b^3*c - 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b
^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c
^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*b^3*c^2*x + 16*root(256*a^3*b^
3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*
b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4,
 z, k)^2*a*b^3*d*x + 4*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*
b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*
c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k)*a*b^2*e^2*x - 8*root(256*a^3*
b^3*z^4 + 64*a^2*b^2*c*e*z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*
a*b^2*c^2*d*z - 4*a*b*c*d^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^
4, z, k)*a*b^2*d*e - 2*b^2*c*d*e*x)*root(256*a^3*b^3*z^4 + 64*a^2*b^2*c*e*
z^2 + 32*a^2*b^2*d^2*z^2 + 16*a^2*b*d*e^2*z - 16*a*b^2*c^2*d*z - 4*a*b*c*d
^2*e + 2*a*b*c^2*e^2 + a*b*d^4 + a^2*e^4 + b^2*c^4, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.68

$$\int \frac{c+dx+ex^2}{a+bx^4}\,dx$$

$$= \frac{-2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)e-2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2}{}$$

input
```
int((e*x^2+d*x+c)/(b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*e - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 4*
sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a
**(1/4)*sqrt(2)))*d + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*e + 2*b**(3/4)*a**(1/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4
)*sqrt(2)))*c - 4*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + b**(1/4)*a**(3/4)*sqrt(2)*log( - b
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*e - b**(1/4)*a**(3/4)
*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*e - b**
(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt
(b)*x**2)*c + b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x +
sqrt(a) + sqrt(b)*x**2)*c)/(8*a*b)
```

3.49 $\int \frac{c+dx+ex^2}{(a-bx^4)^2} dx$

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Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2} dx = \frac{x(c+dx+ex^2)}{4a(a-bx^4)} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*x*(e*x^2+d*x+c)/a/(-b*x^4+a)+1/8*(3*b^(1/2)*c-a^(1/2)*e)*arctan(b^(1/4
)*x/a^(1/4))/a^(7/4)/b^(3/4)+1/8*(3*b^(1/2)*c+a^(1/2)*e)*arctanh(b^(1/4)*x
/a^(1/4))/a^(7/4)/b^(3/4)+1/4*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(3/2)/b^(1/
2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.45

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$

$$= \frac{\frac{4ax(c+x(d+ex))}{a-bx^4} - \frac{2\sqrt[4]{a}\left(-3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{\left(3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{b^{3/4}} + \frac{\left(3\sqrt[4]{a}\sqrt{b}c-2\sqrt{a}\sqrt[4]{}\right.}{}}{16a^2}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

output
```
((4*a*x*(c + x*(d + e*x)))/(a - b*x^4) - (2*a^(1/4)*(-3*Sqrt[b]*c + Sqrt[a
]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((3*a^(1/4)*Sqrt[b]*c + 2*Sqrt
[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((3*a^(1/4)
*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^
(3/4) + (2*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(16*a^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$

$$\Big\downarrow \; 2394$$

$$\frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)} - \frac{\int -\frac{ex^2+2dx+3c}{a-bx^4}dx}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{ex^2+2dx+3c}{a-bx^4}dx}{4a} + \frac{x\left(c+dx+ex^2\right)}{4a\left(a-bx^4\right)}$$

$$\Big\downarrow \ 2415$$

$$\frac{\int \left(\frac{2dx}{a-bx^4}+\frac{ex^2+3c}{a-bx^4}\right)dx}{4a}+\frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(c+dx+ex^2)}{4a(a-bx^4)}$$

input
```
Int[(c + d*x + e*x^2)/(a - b*x^4)^2,x]
```

output
```
(x*(c + d*x + e*x^2))/(4*a*(a - b*x^4)) + (((3*Sqrt[b]*c - Sqrt[a]*e)*ArcT
an[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*
ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(Sqrt[b]*x^
2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^2e+2d\_R+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(-b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b\,x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\left(\frac{}{4a(-}\right.$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^3+1/4*d/a*x^2+1/4*c/a*x)/(-b*x^4+a)-1/16/b/a*sum((_R^2*e+2*_R*d
+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.80 (sec) , antiderivative size = 116982, normalized size of antiderivative = 801.25

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(134) = 268$.

Time = 6.70 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.48

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$
$$= \mathrm{RootSum}\Big(65536t^4a^7b^3 + t^2(-3072a^4b^2ce - 2048a^4b^2d^2) + t(128a^3bde^2 + 1152a^2b^2c^2d) - a^2e^4 + 18abc$$
$$+ \frac{-cx-dx^2-ex^3}{-4a^2+4abx^4}$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**
2*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*
a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _
t*log(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 9830
4*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5
*b**2*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_
t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*
d**2*e + 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
 - 120*a**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*
a*b**2*c**3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3
+ 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576
*a*b**2*c**2*d**4 - 729*b**3*c**6)))) + (-c*x - d*x**2 - e*x**3)/(-4*a**2
+ 4*a*b*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = -\frac{ex^3+dx^2+cx}{4\,(abx^4-a^2)} + \frac{\frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\,\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\,\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,a}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 - a^2) + 1/16*(2*d*log(sqrt(b)*x^2 + s
qrt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b
)) + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sqrt(a)*e)*log((sq
rt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(107) = 214$.

Time = 0.13 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.10

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx$$
$$= -\frac{\sqrt{2}\left(3\,b^2c-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}(3\,b^2c-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}(3\,b^2c-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{ex^3+dx^2+cx}{4\,(bx^4-a)a}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4
)*a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqr
t(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4
)*a) - 1/4*(e*x^3 + d*x^2 + c*x)/((b*x^4 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.27

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4} + \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\right.\right.\right.$$
$$\left. - \frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3} - \frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\right)\mathrm{root}\left(65536\,a^7\,b^3\,z^4\right.$$
$$-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$
$$\left.\left. +18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\right)\right)$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4)^2,x)
```

output
```
((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a - b*x^4) + symsum(log(- r
oot(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152
*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 1
6*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*z^4 - 3072*a^4
*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e
^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4
, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^2*e^2))/(16*a^
3) - (b^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(64*a^3) - (x*(
2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*
c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z
- 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z,
k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.82

$$\int \frac{c+dx+ex^2}{(a-bx^4)^2}\,dx = \frac{-2b^{\frac{1}{4}}a^{\frac{7}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)e+2b^{\frac{5}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)e\,x^4+6b^{\frac{3}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c-6b^{\frac{7}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c\,x^4-b^{\frac{1}{4}}a^{\frac{7}{4}}l}{}$$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

output

```
( - 2*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*e + 2*b**(
1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b*e*x**4 + 6*b**(3/4)*
a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*c - 6*b**(3/4)*a**(1/4)*a
tan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b*c*x**4 - b**(1/4)*a**(3/4)*log(a**(
1/4) - b**(1/4)*x)*a*e + b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*b*e*
x**4 + b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*e - b**(1/4)*a**(3/4
)*log(a**(1/4) + b**(1/4)*x)*b*e*x**4 - 3*b**(3/4)*a**(1/4)*log(a**(1/4) -
 b**(1/4)*x)*a*c + 3*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*b*c*x**4
 + 3*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*c - 3*b**(3/4)*a**(1/4
)*log(a**(1/4) + b**(1/4)*x)*b*c*x**4 - 2*sqrt(b)*sqrt(a)*log(a**(1/4) - b
**(1/4)*x)*a*d + 2*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*b*d*x**4 - 2
*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a*d + 2*sqrt(b)*sqrt(a)*log(a*
*(1/4) + b**(1/4)*x)*b*d*x**4 + 2*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x*
*2)*a*d - 2*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*b*d*x**4 + 4*a*b*c
*x + 4*a*b*d*x**2 + 4*a*b*e*x**3)/(16*a**2*b*(a - b*x**4))
```

3.50 $\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 240

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = \frac{x(c+dx+ex^2)}{4a(a+bx^4)} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}$$

output
```
1/4*x*(e*x^2+d*x+c)/a/(b*x^4+a)+1/4*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(3/2)/
b^(1/2)+1/16*(3*b^(1/2)*c+a^(1/2)*e)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*
2^(1/2)/a^(7/4)/b^(3/4)+1/16*(3*b^(1/2)*c+a^(1/2)*e)*arctan(1+2^(1/2)*b^(1
/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(3/4)+1/16*(3*b^(1/2)*c-a^(1/2)*e)*arctan
h(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.27

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$

$$= \frac{\frac{8ax(c+x(d+ex))}{a+bx^4} - \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{b}c+4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{b}c-4\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1+\right.}{b^{3/4}}}{32a^2}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

output
```
((8*a*x*(c + x*(d + e*x)))/(a + b*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c +
 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/b^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(1/4)*b^(1/4)*d +
 Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sq
rt[2]*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[b]*c - a^(3/4)*e
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(32*a^2
)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \ 2394$$

$$\frac{x(c+dx+ex^2)}{4a(a+bx^4)} - \frac{\int -\frac{ex^2+2dx+3c}{bx^4+a}dx}{4a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{ex^2+2dx+3c}{bx^4+a}dx}{4a}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}$$

$$\Big\downarrow \ 2415$$

$$\frac{\int \left(\frac{2dx}{bx^4+a}+\frac{ex^2+3c}{bx^4+a}\right)dx}{4a}+\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\cdots\right.}{\cdots}}{4a}$$

$$\frac{x\left(c+dx+ex^2\right)}{4a\left(a+bx^4\right)}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4)^2,x]
```

output
```
(x*(c + d*x + e*x^2))/(4*a*(a + b*x^4)) + ((d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]
])/(Sqrt[a]*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c + Sqrt[a]*e)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((3
*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)))/
(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^2 e+2d\_R+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b\,x^4+a}\right.$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^3+1/4*d/a*x^2+1/4*c/a*x)/(b*x^4+a)+1/16/b/a*sum((_R^2*e+2*_R*d+
3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 124258, normalized size of antiderivative = 517.74

$$\int \frac{c + dx + ex^2}{(a + bx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. 2(224) = 448.

Time = 6.47 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.10

$$\int \frac{c + dx + ex^2}{(a + bx^4)^2} \, dx$$
$$= \text{RootSum}\left(65536t^4a^7b^3 + t^2 \cdot \left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc\right.$$
$$+ \frac{cx + dx^2 + ex^3}{4a^2 + 4abx^4}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a
*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t
*log(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304
*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*
b**2*d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t
*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d
**2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
+ 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a
*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 -
 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*
a*b**2*c**2*d**4 + 729*b**3*c**6)))) + (c*x + d*x**2 + e*x**3)/(4*a**2 + 4
*a*b*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.22

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = \frac{ex^3+dx^2+cx}{4\,(abx^4+a^2)}$$
$$+\frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\ldots\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\ldots}}}{32\,a}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(e*x^3 + d*x^2 + c*x)/(a*b*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(b)*c - s
qrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(
1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)
*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(
3/4)*b^(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sq
rt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b
))*b^(3/4)))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.25

$$\begin{aligned}
&\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx \\
&= \frac{ex^3+dx^2+cx}{4\,(bx^4+a)a} \\
&+ \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac14}\,b^2c+(ab^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\,\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3} \\
&+ \frac{\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac14}\,b^2c+(ab^3)^{\frac34}\,e\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\,\left(\frac{a}{b}\right)^{\frac14}}\right)}{16\,a^2b^3} \\
&+ \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^2c-(ab^3)^{\frac34}\,e\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} \\
&- \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac14}\,b^2c-(ab^3)^{\frac34}\,e\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}
\end{aligned}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(e*x^3 + d*x^2 + c*x)/((b*x^4 + a)*a) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a
*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)
*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*
(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*
e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = \frac{\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a} + \left(\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48 \right.\right.\right. - \frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\Big)\,\mathrm{root}\big(65536\,a^7\,b^3\,z^4 + 3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e + 18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\big)\Bigg)$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^2,x)
```

output
```
((d*x^2)/(4*a) + (e*x^3)/(4*a) + (c*x)/(4*a))/(a + b*x^4) + symsum(log((x*
(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*c*d^2 + a*b*e^
3)/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2
*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*
a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root(65536*a^7*b^3
*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z
+ 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^
2*c^4 + a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 - 4*a^2*b
^2*e^2))/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*
e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z -
48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)
, k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 764, normalized size of antiderivative = 3.18

$$\int \frac{c+dx+ex^2}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*e - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*e*
x**4 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c - 6*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b
*c*x**4 - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)
/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d*x**4 + 2*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*e + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*e*x**4 + 6*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a*c + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c*x**4 - 8
*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*a*d - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d*x**4 + b**(1/4)*a**(3/4)*
sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*e +
 b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)...
```

3.51 $\int \frac{c+dx+ex^2}{(a-bx^4)^3} \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 179

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3} \, dx = \frac{x(c+dx+ex^2)}{8a(a-bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/8*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(-b*x^4+
a)+1/64*(21*b^(1/2)*c-5*a^(1/2)*e)*arctan(b^(1/4)*x/a^(1/4))/a^(11/4)/b^(3
/4)+1/64*(21*b^(1/2)*c+5*a^(1/2)*e)*arctanh(b^(1/4)*x/a^(1/4))/a^(11/4)/b^
(3/4)+3/16*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.36

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$$

$$= \frac{\frac{16a^2x(c+x(d+ex))}{(a-bx^4)^2} + \frac{4ax(7c+x(6d+5ex))}{a-bx^4} + \frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{\left(21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d+5a^{3/4}e\right)\log\left(\sqrt[4]{a}-\right.}{b^{3/4}}}{128a^3}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

output
```
((16*a^2*x*(c + x*(d + e*x)))/(a - b*x^4)^2 + (4*a*x*(7*c + x*(6*d + 5*e*x
)))/(a - b*x^4) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*
x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b^(1/4)*d + 5*a
^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*Sqrt[b]*c - 12*
Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (12*S
qrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2394, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x\left(c+dx+ex^2\right)}{8a\left(a-bx^4\right)^2} - \frac{\int -\frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{5ex^2+21c}{a-bx^4}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(21\sqrt{b}c-5\sqrt{a}e)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}e+21\sqrt{b}c)}{2a^{3/4}b^{3/4}}+\frac{6d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2)/(a - b*x^4)^3,x]
```

output
```
(x*(c + d*x + e*x^2))/(8*a*(a - b*x^4)^2) + ((x*(7*c + 6*d*x + 5*e*x^2))/(
4*a*(a - b*x^4)) + (((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/
4)])/(2*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh[(b^(1/4)*
x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (6*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(S
qrt[a]*Sqrt[b]))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

method	result
risch	$\frac{-\frac{5be\,x^{7}}{32a^{2}}-\frac{3bd\,x^{6}}{16a^{2}}-\frac{7bc\,x^{5}}{32a^{2}}+\frac{9e\,x^{3}}{32a}+\frac{5d\,x^{2}}{16a}+\frac{11cx}{32a}}{\left(-b\,x^{4}+a\right)^{2}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}-a\right)}\frac{\left(5\_R^{2}e+12d\_R+21c\right)\ln\left(x-\_R\right)}{\_R^{3}}}{128b\,a^{2}}$
default	$c\left(\frac{x}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(-b\,x^{4}+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^{2}}}{a}\right)+d\left(\frac{x^{2}}{8a\left(-b\,x^{4}+a\right)^{2}}+\frac{\frac{3x^{2}}{16a\left(-b\,x^{4}+a\right)}+\cdots\right.$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6-7/32*b/a^2*c*x^5+9/32*e/a*x^3+5/16*d/a
*x^2+11/32*c/a*x)/(-b*x^4+a)^2-1/128/b/a^2*sum((5*_R^2*e+12*_R*d+21*c)/_R^
3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.12 (sec) , antiderivative size = 118710, normalized size of antiderivative = 663.18

$$\int \frac{c + dx + ex^2}{(a - bx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(170) = 340.

Time = 26.75 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.15

$$\int \frac{c + dx + ex^2}{(a - bx^4)^3} \, dx = \\ - \operatorname{RootSum}\Big(268435456t^4a^{11}b^3 + t^2\left(-6881280a^6b^2ce - 4718592a^6b^2d^2\right) + t\left(-153600a^4bde^2 - 2709504\right. \\ - \frac{-11acx - 10adx^2 - 9aex^3 + 7bcx^5 + 6bdx^6 + 5bex^7}{32a^4 - 64a^3bx^4 + 32a^2b^2x^8}$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
-RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 4718
592*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d
) - 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d
**4 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2
*e**3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*
d**2 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3
*e - 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 691200
0*_t*a**5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a*
*4*b**2*c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b*
*3*c**5 + 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b
*d**5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**
3*e**6 + 275625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a*
*2*b*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e -
36578304*a*b**2*c**2*d**4 - 85766121*b**3*c**6))) - (-11*a*c*x - 10*a*d*x
**2 - 9*a*e*x**3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 - 64*a**
3*b*x**4 + 32*a**2*b**2*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = -\frac{5\,bex^7+6\,bdx^6+7\,bcx^5-9\,aex^3-10\,adx^2-11\,acx}{32\,(a^2b^2x^8-2\,a^3bx^4+a^4)} + \frac{\frac{12\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,\sqrt{b}c+5\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 - 9*a*e*x^3 - 10*a*d*x^2 - 11*a*c
*x)/(a^2*b^2*x^8 - 2*a^3*b*x^4 + a^4) + 1/128*(12*d*log(sqrt(b)*x^2 + sqrt
(a))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b))
 + 2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sqrt(b)*c + 5*sqrt(a)*e)*log
((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(139) = 278$.

Time = 0.14 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.87

$$\int \frac{c + dx + ex^2}{(a - bx^4)^3}\,dx$$
$$= -\frac{\sqrt{2}\left(21\,b^2c - 12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd + 5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,\left(-ab^3\right)^{\frac{3}{4}}a^2} - \frac{\sqrt{2}\left(21\,b^2c + 12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd - 5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,\left(-ab^3\right)^{\frac{3}{4}}a^2} - \frac{\sqrt{2}\left(21\,b^2c - 5\,\sqrt{-ab}be\right)\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\,\left(-ab^3\right)^{\frac{3}{4}}a^2} + \frac{\sqrt{2}\left(21\,b^2c - 5\,\sqrt{-ab}be\right)\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{256\,\left(-ab^3\right)^{\frac{3}{4}}a^2} - \frac{5\,bex^7 + 6\,bdx^6 + 7\,bcx^5 - 9\,aex^3 - 10\,adx^2 - 11\,acx}{32\,(bx^4 - a)^2a^2}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*
sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log
(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*s
qrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 -
 9*a*e*x^3 - 10*a*d*x^2 - 11*a*c*x)/((b*x^4 - a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 826, normalized size of antiderivative = 4.61

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) - (7*b*c*x^5)/(32*a
^2) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 - 2*a*b*
x^4) + symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^
3*x + 344064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 47185
92*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 6048
0*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b
^2*c^4, z, k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6
*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*
a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 62
5*a^2*e^4 - 194481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*roo
t(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z
^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e +
22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^
2*b^2*c^2*x - 196608*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2
 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 -
194481*b^2*c^4, z, k)^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 -
6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a
*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*r...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 675, normalized size of antiderivative = 3.77

$$\int \frac{c+dx+ex^2}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

output
```
( - 10*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*e + 20
*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*e*x**4 - 10*b
**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**2*e*x**8 + 42*b*
*(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*c - 84*b**(3/4)
*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*c*x**4 + 42*b**(3/4)*a
**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**2*c*x**8 - 5*b**(1/4)*a**
(3/4)*log(a**(1/4) - b**(1/4)*x)*a**2*e + 10*b**(1/4)*a**(3/4)*log(a**(1/4
) - b**(1/4)*x)*a*b*e*x**4 - 5*b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x
)*b**2*e*x**8 + 5*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a**2*e - 10
*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*b*e*x**4 + 5*b**(1/4)*a**(
3/4)*log(a**(1/4) + b**(1/4)*x)*b**2*e*x**8 - 21*b**(3/4)*a**(1/4)*log(a**
(1/4) - b**(1/4)*x)*a**2*c + 42*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*
x)*a*b*c*x**4 - 21*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*b**2*c*x**
8 + 21*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a**2*c - 42*b**(3/4)*a
**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*b*c*x**4 + 21*b**(3/4)*a**(1/4)*log(a
**(1/4) + b**(1/4)*x)*b**2*c*x**8 - 12*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(
1/4)*x)*a**2*d + 24*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*a*b*d*x**4
- 12*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*b**2*d*x**8 - 12*sqrt(b)*s
qrt(a)*log(a**(1/4) + b**(1/4)*x)*a**2*d + 24*sqrt(b)*sqrt(a)*log(a**(1/4)
 + b**(1/4)*x)*a*b*d*x**4 - 12*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*...
```

3.52 $\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 273

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx &= \frac{x(c+dx+ex^2)}{8a\,(a+bx^4)^2} + \frac{x(7c+6dx+5ex^2)}{32a^2\,(a+bx^4)} + \frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} \\ &\quad - \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} \\ &\quad + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} \\ &\quad + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}\end{aligned}$$

output

```
1/8*x*(e*x^2+d*x+c)/a/(b*x^4+a)^2+1/32*x*(5*e*x^2+6*d*x+7*c)/a^2/(b*x^4+a)
+3/16*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(5/2)/b^(1/2)+1/128*(21*b^(1/2)*c+5*
a^(1/2)*e)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/b^(3/4)+1
/128*(21*b^(1/2)*c+5*a^(1/2)*e)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2
)/a^(11/4)/b^(3/4)+1/128*(21*b^(1/2)*c-5*a^(1/2)*e)*arctanh(2^(1/2)*a^(1/4
)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(11/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.23

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx$$

$$= \frac{\frac{32a^2x(c+x(d+ex))}{(a+bx^4)^2}+\frac{8ax(7c+x(6d+5ex))}{a+bx^4}-\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c+24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c-24\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

output
```
((32*a^2*x*(c + x*(d + e*x)))/(a + b*x^4)^2 + (8*a*x*(7*c + x*(6*d + 5*e*x
)))/(a + b*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1/4)*b^(1/4)*d
+ 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) +
(2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqrt[2]*(-21*a^(1
/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3/4)*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(256*a^3)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2394, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{\left(a+bx^4\right)^3}\,dx$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{x\left(c+dx+ex^2\right)}{8a\left(a+bx^4\right)^2}-\frac{\int -\frac{5ex^2+6dx+7c}{\left(bx^4+a\right)^2}dx}{8a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{5ex^2+6dx+7c}{(bx^4+a)^2}dx}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\int \left(\frac{12dx}{bx^4+a}+\frac{5ex^2+21c}{bx^4+a}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}e+21\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(5\sqrt{a}e+21\sqrt{b}c)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{(21\sqrt{b}c-5\sqrt{a}e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{(21\sqrt{b}c-5\sqrt{a}e)\log\left(\sqrt{\ldots}\right.}{4\sqrt{2}a\ldots}}{4a}}{8a}$$

$$\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4)^3,x]
```

output
```
(x*(c + d*x + e*x^2))/(8*a*(a + b*x^4)^2) + ((x*(7*c + 6*d*x + 5*e*x^2))/(
4*a*(a + b*x^4)) + ((6*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
- ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((21*Sqrt[b]*c - 5
*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)))/(4*a))/(8
*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.41

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)}\frac{(5\_R^2e+12d\_R+21c)\ln(x-\_R)}{\_R^3}}{128b\,a^2}$
default	$c\left(\frac{x}{8a(b\,x^4+a)^2}+\frac{\frac{7x}{32a(b\,x^4+a)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+d\Bigg($

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+7/32*b/a^2*c*x^5+9/32*e/a*x^3+5/16*d/a*
x^2+11/32*c/a*x)/(b*x^4+a)^2+1/128/b/a^2*sum((5*_R^2*e+12*_R*d+21*c)/_R^3*
ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.06 (sec) , antiderivative size = 124787, normalized size of antiderivative = 457.10

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(262) = 524$.

Time = 24.41 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.04

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$$
$$= \operatorname{RootSum}\left(268435456t^4a^{11}b^3 + t^2\cdot\left(6881280a^6b^2ce + 4718592a^6b^2d^2\right) + t\left(153600a^4bde^2 - 2709504a^3b^2\right.\right.$$
$$+ \frac{11acx + 10adx^2 + 9aex^3 + 7bcx^5 + 6bdx^6 + 5bex^7}{32a^4 + 64a^3bx^4 + 32a^2b^2x^8}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 471859
2*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) +
 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4
 + 194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**
3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2
 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e +
 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t
*a**5*b*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b
**2*c**2*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c
**5 + 112500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**
5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e*
*6 - 275625*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b
*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 3657
8304*a*b**2*c**2*d**4 + 85766121*b**3*c**6))) + (11*a*c*x + 10*a*d*x**2 +
 9*a*e*x**3 + 7*b*c*x**5 + 6*b*d*x**6 + 5*b*e*x**7)/(32*a**4 + 64*a**3*b*x
**4 + 32*a**2*b**2*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.23

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \frac{5\,bex^7+6\,bdx^6+7\,bcx^5+9\,aex^3+10\,adx^2+11\,acx}{32\,(a^2b^2x^8+2\,a^3bx^4+a^4)}$$

$$+\frac{\frac{\sqrt{2}\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(21\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+5\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-\right.}{a}}{256\,a^2}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*e*x^3 + 10*a*d*x^2 + 11*a*c*
x)/(a^2*b^2*x^8 + 2*a^3*b*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqrt(b)*c - 5*sq
rt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(3/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3
/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/
4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sq
rt(2)*a^(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt
(a)*sqrt(b))*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.21

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx$$
$$= \frac{5\,bex^7+6\,bdx^6+7\,bcx^5+9\,aex^3+10\,adx^2+11\,acx}{32\,(bx^4+a)^2a^2}$$
$$+ \frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+ \frac{\sqrt{2}\left(12\,\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+ \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$- \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/32*(5*b*e*x^7 + 6*b*d*x^6 + 7*b*c*x^5 + 9*a*e*x^3 + 10*a*d*x^2 + 11*a*c*
x)/((b*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a
*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b
^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)
^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a^3*b^3) - 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e
)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.03

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) + (11*c*x)/(32*a) + (7*b*c*x^5)/(32*a
^2) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(32*a^2))/(a^2 + b^2*x^8 + 2*a*b*
x^4) + symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^
3*x + 344064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 47185
92*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 6048
0*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b
^2*c^4, z, k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6
*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*
a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 62
5*a^2*e^4 + 194481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*roo
t(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z
^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e +
22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^
2*b^2*c^2*x - 196608*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2
 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*
z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 +
194481*b^2*c^4, z, k)^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 +
6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*
z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a
*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*r...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1201, normalized size of antiderivative = 4.40

$$\int \frac{c+dx+ex^2}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

output

```
( - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*e - 20*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b*e*x**4 - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*e*x**8 - 42*b**(3/4)*a**
(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**
(1/4)*sqrt(2)))*a**2*c - 84*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c*x**4 - 42*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b**2*c*x**8 - 48*sqrt(b)*sqrt(a)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d - 96*s
qrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a*b*d*x**4 - 48*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*d*x**8 + 10*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*a**2*e + 20*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*e*x**4 +
 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*e*x**8 + 42*b**(3/4)*a**(1/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt...
```

3.53 $\int \frac{c+dx+ex^2}{(a-bx^4)^4} dx$

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Optimal result

Integrand size = 21, antiderivative size = 211

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4} dx = \frac{x(c+dx+ex^2)}{12a(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/12*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(-b*x
^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(-b*x^4+a)+1/256*(77*b^(1/2)*c-
15*a^(1/2)*e)*arctan(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(3/4)+1/256*(77*b^(1/2)
*c+15*a^(1/2)*e)*arctanh(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(3/4)+5/32*d*arctan
h(b^(1/2)*x^2/a^(1/2))/a^(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$

$$= \frac{\frac{128a^3x(c+x(d+ex))}{(a-bx^4)^3} + \frac{4ax(77c+15x(4d+3ex))}{a-bx^4} + \frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2} + \frac{6\sqrt[4]{a}\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} - \frac{3\left(77\sqrt[4]{a}\sqrt{b}\right.}{}}{1536a}$$

input
```
Integrate[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

output
```
((128*a^3*x*(c + x*(d + e*x)))/(a - b*x^4)^3 + (4*a*x*(77*c + 15*x*(4*d +
3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a - b*x^4)^2
+ (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/b^
(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Lo
g[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c - 40*Sqrt[a]*b^
(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) + (120*Sqrt[a]*d
*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3} - \frac{\int -\frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{\left(a-bx^4\right)^2}dx}{8a}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{\left(a-bx^4\right)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int -\frac{3\left(15ex^2+40dx+77c\right)}{a-bx^4}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{a-bx^4}dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{a-bx^4}+\frac{15ex^2+77c}{a-bx^4}\right)dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(77\sqrt{b}c-15\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{20d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a-bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2)/(a - b*x^4)^4,x]
```

output
```
(x*(c + d*x + e*x^2))/(12*a*(a - b*x^4)^3) + ((x*(11*c + 10*d*x + 9*e*x^2)
)/(8*a*(a - b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*(a - b*x^4))
+ (3*(((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/
4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(3/4)) + (20*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt
[b])))/(4*a))/(8*a))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(-b\,x^4+a)^3} - \frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4-a)} \frac{(15\_R^2e+40d\_R+\cdots}{\_R^3\cdots}}{512b\,a^3}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(-b\,x^4+a)^3} + \frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)\cdots}{4a}}{\cdots}$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/
a^2*x^7-5/12*b*d/a^2*x^6-33/64*b/a^2*c*x^5+113/384*e/a*x^3+11/32*d/a*x^2+5
1/128*c/a*x)/(-b*x^4+a)^3-1/512/b/a^3*sum((15*_R^2*e+40*_R*d+77*c)/_R^3*ln
(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.73 (sec) , antiderivative size = 118903, normalized size of antiderivative = 563.52

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(201) = 402$.

Time = 30.70 (sec) , antiderivative size = 612, normalized size of antiderivative = 2.90

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx$$
$$= \text{RootSum}\Big(68719476736t^4a^{15}b^3 + t^2(-1211105280a^8b^2ce - 838860800a^8b^2d^2) + t(18432000a^5bde^2 + 48$$
$$+ \frac{-153a^2cx - 132a^2dx^2 - 113a^2ex^3 + 198abcx^5 + 160abdx^6 + 126abex^7 - 77b^2cx^9 - 60b^2dx^{10} - 45b^2e}{-384a^6 + 1152a^5bx^4 - 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

input
```
integrate((e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(-1211105280*a**8*b**2*c*e -
838860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 + 485703680*a**4*b*
*2*c**2*d) - 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*
e + 2560000*a*b*d**4 - 35153041*b**2*c**4, Lambda(_t, _t*log(x + (45298483
2000*_t**3*a**13*b**2*e**3 + 11936653639680*_t**3*a**12*b**3*c**2*e - 3307
1248179200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2
 - 503316480000*_t**2*a**9*b**2*d**3*e - 4787095470080*_t**2*a**8*b**3*c**
3*d - 5987520000*_t*a**6*b*c*e**4 - 8294400000*_t*a**6*b*d**2*e**3 - 21037
0406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 2
01850880000*_t*a**5*b**2*c*d**4 - 1385873488384*_t*a**4*b**3*c**5 + 911250
00*a**3*d*e**5 - 5544000000*a**2*b*c*d**3*e**2 + 3072000000*a**2*b*d**5*e
+ 105459123000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*
a**3*e**6 + 300155625*a**2*b*c**2*e**4 - 3326400000*a**2*b*c*d**2*e**3 + 2
304000000*a**2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b
**2*c**3*d**2*e - 60712960000*a*b**2*c**2*d**4 - 208422380089*b**3*c**6)))
) + (-153*a**2*c*x - 132*a**2*d*x**2 - 113*a**2*e*x**3 + 198*a*b*c*x**5 +
160*a*b*d*x**6 + 126*a*b*e*x**7 - 77*b**2*c*x**9 - 60*b**2*d*x**10 - 45*b*
*2*e*x**11)/(-384*a**6 + 1152*a**5*b*x**4 - 1152*a**4*b**2*x**8 + 384*a**3
*b**3*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = -\frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9-126\,abex^7-160\,abdx^6-198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153}{384\,(a^3b^3x^{12}-3\,a^4b^2x^8+3\,a^5bx^4-a^6)} + \frac{\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,\sqrt{b}c+15\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 - 126*a*b*e*x^7 - 160
*a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/
(a^3*b^3*x^12 - 3*a^4*b^2*x^8 + 3*a^5*b*x^4 - a^6) + 1/512*(40*d*log(sqrt(
b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(sqrt(b)*x^2 - sqrt(a))/(sqr
t(a)*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*sqrt(b)*c + 15
*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt
(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(170) = 340$.

Time = 0.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.75

$$\int \frac{c + dx + ex^2}{(a - bx^4)^4}\, dx$$
$$= -\frac{\sqrt{2}\left(77\, b^2 c - 40\, \sqrt{2}(-ab^3)^{\frac{1}{4}}\, bd + 15\, \sqrt{-ab}be\right) \arctan\left(\frac{\sqrt{2}\left(2\, x + \sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\, (-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\, (-ab^3)^{\frac{3}{4}}\, a^3}$$
$$- \frac{\sqrt{2}\left(77\, b^2 c + 40\, \sqrt{2}(-ab^3)^{\frac{1}{4}}\, bd - 15\, \sqrt{-ab}be\right) \arctan\left(\frac{\sqrt{2}\left(2\, x - \sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\, (-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\, (-ab^3)^{\frac{3}{4}}\, a^3}$$
$$- \frac{\sqrt{2}(77\, b^2 c - 15\, \sqrt{-ab}be) \log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024\, (-ab^3)^{\frac{3}{4}}\, a^3}$$
$$+ \frac{\sqrt{2}(77\, b^2 c - 15\, \sqrt{-ab}be) \log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{1024\, (-ab^3)^{\frac{3}{4}}\, a^3}$$
$$- \frac{45\, b^2 ex^{11} + 60\, b^2 dx^{10} + 77\, b^2 cx^9 - 126\, abex^7 - 160\, abdx^6 - 198\, abcx^5 + 113\, a^2 ex^3 + 132\, a^2 dx^2 + 153}{384\, (bx^4 - a)^3 a^3}$$

input
```
integrate((e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
-1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 1
5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(
1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)
*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1
024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4
) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^1
0 + 77*b^2*c*x^9 - 126*a*b*e*x^7 - 160*a*b*d*x^6 - 198*a*b*c*x^5 + 113*a^2
*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 - a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.14

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2)/(a - b*x^4)^4,x)
```

output
```
((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^
9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) - (33*b
*c*x^5)/(64*a^2) - (5*b*d*x^6)/(12*a^2) - (21*b*e*x^7)/(64*a^2))/(a^3 - b^
3*x^12 - 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 + 123200*
b*c*d^2 - 88935*b*c^2*e + 64000*b*d^3*x + 20185088*root(68719476736*a^15*b
^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050
*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2
*a^7*b^2*c + 115200*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e
*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^
5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4
- 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^4*b*e^2*x - 92400*b*c*d*e*x +
3035648*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 83886
0800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z
- 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b
^2*c^4 - 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^
15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 4857
03680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 266
8050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z,
k)^2*a^7*b^2*d*x - 614400*root(68719476736*a^15*b^3*z^4 - 1211105280*a^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 938, normalized size of antiderivative = 4.45

$$\int \frac{c+dx+ex^2}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

output
```
( - 90*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*e + 27
0*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*e*x**4 -
270*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*e*x**8
+ 90*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**3*e*x**12
+ 462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*c - 138
6*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*c*x**4 +
1386*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*c*x**8
 - 462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**3*c*x**1
2 - 45*b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a**3*e + 135*b**(1/4)*
a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a**2*b*e*x**4 - 135*b**(1/4)*a**(3/4)*
log(a**(1/4) - b**(1/4)*x)*a*b**2*e*x**8 + 45*b**(1/4)*a**(3/4)*log(a**(1/
4) - b**(1/4)*x)*b**3*e*x**12 + 45*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/
4)*x)*a**3*e - 135*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a**2*b*e*x
**4 + 135*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*b**2*e*x**8 - 45*
b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*b**3*e*x**12 - 231*b**(3/4)*a
**(1/4)*log(a**(1/4) - b**(1/4)*x)*a**3*c + 693*b**(3/4)*a**(1/4)*log(a**(
1/4) - b**(1/4)*x)*a**2*b*c*x**4 - 693*b**(3/4)*a**(1/4)*log(a**(1/4) - b*
*(1/4)*x)*a*b**2*c*x**8 + 231*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)
*b**3*c*x**12 + 231*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a**3*c -
693*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a**2*b*c*x**4 + 693*b*...
```

3.54 $\int \frac{c+dx+ex^2}{(a+bx^4)^4} dx$

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Optimal result

Integrand size = 20, antiderivative size = 304

$$\begin{aligned}\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx &= \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3}+\frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2}+\frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} \\ &\quad+\frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}-\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\ &\quad+\frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\ &\quad+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}\end{aligned}$$

output

```
1/12*x*(e*x^2+d*x+c)/a/(b*x^4+a)^3+1/96*x*(9*e*x^2+10*d*x+11*c)/a^2/(b*x^4
+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(b*x^4+a)+5/32*d*arctan(b^(1/2)*x
^2/a^(1/2))/a^(7/2)/b^(1/2)+1/512*(77*b^(1/2)*c+15*a^(1/2)*e)*arctan(-1+2^
(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(15/4)/b^(3/4)+1/512*(77*b^(1/2)*c+15*a
^(1/2)*e)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(15/4)/b^(3/4)+1/5
12*(77*b^(1/2)*c-15*a^(1/2)*e)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+
b^(1/2)*x^2))*2^(1/2)/a^(15/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.21

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{256a^3x(c+x(d+ex))}{(a+bx^4)^3} + \frac{8ax(77c+15x(4d+3ex))}{a+bx^4} + \frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2} - \frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c+80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\right.}{b^{3/4}}}{}$$

input
```
Integrate[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

output
```
((256*a^3*x*(c + x*(d + e*x)))/(a + b*x^4)^3 + (8*a*x*(77*c + 15*x*(4*d +
3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10*d + 9*e*x)))/(a + b*x^4)^2
- (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqr
t[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (6*a^(1/4)*(77*
Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^(1/4)*Sqrt[b]*
c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2394, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$\Big\downarrow \ 2394$$

$$\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3} - \frac{\int -\frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{\left(bx^4+a\right)^2}dx}{8a}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{\left(bx^4+a\right)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3\left(15ex^2+40dx+77c\right)}{bx^4+a}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{bx^4+a}dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{bx^4+a}+\frac{15ex^2+77c}{bx^4+a}\right)dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}+\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)}{}}{4a}}{8a}}{12a}$$

$$\frac{x\left(c+dx+ex^2\right)}{12a\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2)/(a + b*x^4)^4,x]
```

output
```
(x*(c + d*x + e*x^2))/(12*a*(a + b*x^4)^3) + ((x*(11*c + 10*d*x + 9*e*x^2)
)/(8*a*(a + b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*(a + b*x^4))
+ (3*((20*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((77*Sqrt[b
]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((77*Sqrt[b]*c - 15*Sqrt[a]*e
)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/
4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))))/(4*a))/(8*a))/(12
*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(15\_R^2e+40d\_R+\cdots\right.}{\_R^3\cdots}}{512b\,a^3}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}}{(b\,x^4+a)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arct}\cdots\right.}{8a}\cdots}{\cdots}$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/
a^2*x^7+5/12*b*d/a^2*x^6+33/64*b/a^2*c*x^5+113/384*e/a*x^3+11/32*d/a*x^2+5
1/128*c/a*x)/(b*x^4+a)^3+1/512/b/a^3*sum((15*_R^2*e+40*_R*d+77*c)/_R^3*ln(
x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.33 (sec) , antiderivative size = 124960, normalized size of antiderivative = 411.05

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(292) = 584.

Time = 30.23 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.01

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$
$$= \text{RootSum}\Big(68719476736t^4a^{15}b^3 + t^2\cdot\left(1211105280a^8b^2ce + 838860800a^8b^2d^2\right) + t\big(18432000a^5bde^2 - 48$$
$$+ \frac{153a^2cx + 132a^2dx^2 + 113a^2ex^3 + 198abcx^5 + 160abdx^6 + 126abex^7 + 77b^2cx^9 + 60b^2dx^{10} + 45b^2ex^1}{384a^6 + 1152a^5bx^4 + 1152a^4b^2x^8 + 384a^3b^3x^{12}}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(1211105280*a**8*b**2*c*e + 8
38860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 - 485703680*a**4*b**
2*c**2*d) + 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e
 + 2560000*a*b*d**4 + 35153041*b**2*c**4, Lambda(_t, _t*log(x + (452984832
000*_t**3*a**13*b**2*e**3 - 11936653639680*_t**3*a**12*b**3*c**2*e + 33071
248179200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2
- 503316480000*_t**2*a**9*b**2*d**3*e + 4787095470080*_t**2*a**8*b**3*c**3
*d + 5987520000*_t*a**6*b*c*e**4 + 8294400000*_t*a**6*b*d**2*e**3 - 210370
406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 20
1850880000*_t*a**5*b**2*c*d**4 + 1385873488384*_t*a**4*b**3*c**5 + 9112500
0*a**3*d*e**5 + 5544000000*a**2*b*c*d**3*e**2 - 3072000000*a**2*b*d**5*e +
 105459123000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a
**3*e**6 - 300155625*a**2*b*c**2*e**4 + 3326400000*a**2*b*c*d**2*e**3 - 23
04000000*a**2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b*
*2*c**3*d**2*e - 60712960000*a*b**2*c**2*d**4 + 208422380089*b**3*c**6)))
 + (153*a**2*c*x + 132*a**2*d*x**2 + 113*a**2*e*x**3 + 198*a*b*c*x**5 + 16
0*a*b*d*x**6 + 126*a*b*e*x**7 + 77*b**2*c*x**9 + 60*b**2*d*x**10 + 45*b**2
*e*x**11)/(384*a**6 + 1152*a**5*b*x**4 + 1152*a**4*b**2*x**8 + 384*a**3*b*
*3*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.26

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$

$$= \frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9+126\,abex^7+160\,abdx^6+198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153\,a^2}{384\,(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6)}$$

$$+\frac{\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}}{}}{1024\,a^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(45*b^2*e*x^11 + 60*b^2*d*x^10 + 77*b^2*c*x^9 + 126*a*b*e*x^7 + 160*
a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/(
a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6) + 1/1024*(sqrt(2)*(77*sq
rt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt
(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)
*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(77*sqrt
(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e - 80*sqrt(a)*sqrt(b)*
d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)
*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)
*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*sqrt(a)*sqrt(b)*d)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.20

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$-\frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$+\frac{45\,b^2ex^{11}+60\,b^2dx^{10}+77\,b^2cx^9+126\,abex^7+160\,abdx^6+198\,abcx^5+113\,a^2ex^3+132\,a^2dx^2+153}{384\,(bx^4+a)^3a^3}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a
*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*
b^2*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))
/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b
^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/10
24*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^2*e*x^11 + 60*b^2*d*x^
10 + 77*b^2*c*x^9 + 126*a*b*e*x^7 + 160*a*b*d*x^6 + 198*a*b*c*x^5 + 113*a^
2*e*x^3 + 132*a^2*d*x^2 + 153*a^2*c*x)/((b*x^4 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.87

$$\int \frac{c+dx+ex^2}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^4,x)
```

output
```
((11*d*x^2)/(32*a) + (113*e*x^3)/(384*a) + (51*c*x)/(128*a) + (77*b^2*c*x^
9)/(384*a^3) + (5*b^2*d*x^10)/(32*a^3) + (15*b^2*e*x^11)/(128*a^3) + (33*b
*c*x^5)/(64*a^2) + (5*b*d*x^6)/(12*a^2) + (21*b*e*x^7)/(64*a^2))/(a^3 + b^
3*x^12 + 3*a^2*b*x^4 + 3*a*b^2*x^8) + symsum(log(-(b*(3375*a*e^3 - 123200*
b*c*d^2 + 88935*b*c^2*e - 64000*b*d^3*x + 20185088*root(68719476736*a^15*b
^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 48570368
0*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050
*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2
*a^7*b^2*c - 115200*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e
*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^
5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4
+ 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^4*b*e^2*x + 92400*b*c*d*e*x +
3035648*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 83886
0800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z
- 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b
^2*c^4 + 50625*a^2*e^4, z, k)*a^3*b^2*c^2*x - 10485760*root(68719476736*a^
15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 4857
03680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 266
8050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z,
k)^2*a^7*b^2*d*x + 614400*root(68719476736*a^15*b^3*z^4 + 1211105280*a^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1636, normalized size of antiderivative = 5.38

$$\int \frac{c + dx + ex^2}{(a + bx^4)^4} \, dx = \text{Too large to display}$$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

output
```
( - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*e - 270*b**(1/4)*a**(3/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**2*b*e*x**4 - 270*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*e*x**8 - 90*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1
/4)*a**(1/4)*sqrt(2)))*b**3*e*x**12 - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3
*c - 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c*x**4 - 1386*b**(3/4)*a**(1/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*b**2*c*x**8 - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*c*x**12
- 480*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**3*d - 1440*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*d*x**4 - 14
40*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4
)*a**(1/4)*sqrt(2)))*a*b**2*d*x**8 - 480*sqrt(b)*sqrt(a)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*d*x**12 +
90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b...
```

3.55 $\int a(e+fx^4)^2\,dx$

Optimal result 496
Mathematica [A] (verified) 496
Rubi [A] (verified) 497
Maple [A] (verified) 498
Fricas [A] (verification not implemented) 498
Sympy [A] (verification not implemented) 499
Maxima [A] (verification not implemented) 499
Giac [A] (verification not implemented) 499
Mupad [B] (verification not implemented) 500
Reduce [B] (verification not implemented) 500

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int a\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9$$

output
```
a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int a\left(e+fx^4\right)^2\,dx=a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

input
```
Integrate[a*(e + f*x^4)^2,x]
```

output
```
a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {27, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{27}$$

$$a\int \left(fx^4+e\right)^2dx$$

$$\Big\downarrow \text{747}$$

$$a\int \left(f^2x^8+2efx^4+e^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)$$

input
```
Int[a*(e + f*x^4)^2,x]
```

output
```
a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\left(\frac{1}{9}f^{2}x^{9}+\frac{2}{5}x^{5}ef+e^{2}x\right)a$	24
parallelrisch	$\left(\frac{1}{9}f^{2}x^{9}+\frac{2}{5}x^{5}ef+e^{2}x\right)a$	24
norman	$a\,e^{2}x+\frac{2}{5}aef\,x^{5}+\frac{1}{9}a\,f^{2}x^{9}$	25
risch	$a\,e^{2}x+\frac{2}{5}aef\,x^{5}+\frac{1}{9}a\,f^{2}x^{9}$	25
gosper	$\frac{x\left(5f^{2}x^{8}+18x^{4}ef+45e^{2}\right)a}{45}$	26
orering	$\frac{x\left(5f^{2}x^{8}+18x^{4}ef+45e^{2}\right)a}{45}$	26

input
```
int(a*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/9*f^2*x^9+2/5*x^5*e*f+e^2*x)*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int a\left(e+fx^{4}\right)^{2}\,dx=\frac{1}{9}\,af^{2}x^{9}+\frac{2}{5}\,aefx^{5}+ae^{2}x$$

input
```
integrate(a*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/9*a*f^2*x^9 + 2/5*a*e*f*x^5 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int a(e+fx^4)^2\,dx = ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9}$$

input
```
integrate(a*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx = \frac{1}{45}\,(5\,f^2x^9 + 18\,efx^5 + 45\,e^2x)a$$

input
```
integrate(a*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx = \frac{1}{45}\,(5\,f^2x^9 + 18\,efx^5 + 45\,e^2x)a$$

input
```
integrate(a*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/45*(5*f^2*x^9 + 18*e*f*x^5 + 45*e^2*x)*a
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx = \frac{a\,x\,(45\,e^2+18\,e\,f\,x^4+5\,f^2\,x^8)}{45}$$

input
```
int(a*(e + f*x^4)^2,x)
```

output
```
(a*x*(45*e^2 + 5*f^2*x^8 + 18*e*f*x^4))/45
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int a(e+fx^4)^2\,dx = \frac{ax(5f^2x^8+18ef\,x^4+45e^2)}{45}$$

input
```
int(a*(f*x^4+e)^2,x)
```

output
```
(a*x*(45*e**2 + 18*e*f*x**4 + 5*f**2*x**8))/45
```

3.56 $\int bx(e+fx^4)^2\,dx$

Optimal result 501
Mathematica [A] (verified) 501
Rubi [A] (verified) 502
Maple [A] (verified) 503
Fricas [A] (verification not implemented) 503
Sympy [A] (verification not implemented) 504
Maxima [A] (verification not implemented) 504
Giac [A] (verification not implemented) 504
Mupad [B] (verification not implemented) 505
Reduce [B] (verification not implemented) 505

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int bx\left(e+fx^4\right)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

output
```
1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int bx\left(e+fx^4\right)^2\,dx=b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

input
```
Integrate[b*x*(e + f*x^4)^2,x]
```

output
```
b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int bx\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{27}$$

$$b\int x\left(fx^4+e\right)^2 dx$$

$$\Big\downarrow \text{802}$$

$$b\int \left(f^2x^9+2efx^5+e^2x\right) dx$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)$$

input
```
Int[b*x*(e + f*x^4)^2,x]
```

output
```
b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$\left(\frac{1}{10}f^{2}x^{10}+\frac{1}{3}ef\,x^{6}+\frac{1}{2}e^{2}x^{2}\right)b$	27
parallelrisch	$\left(\frac{1}{10}f^{2}x^{10}+\frac{1}{3}ef\,x^{6}+\frac{1}{2}e^{2}x^{2}\right)b$	27
gosper	$\frac{x^{2}\left(3f^{2}x^{8}+10x^{4}ef+15e^{2}\right)b}{30}$	28
norman	$\frac{1}{2}b\,e^{2}x^{2}+\frac{1}{3}bef\,x^{6}+\frac{1}{10}b\,f^{2}x^{10}$	28
risch	$\frac{1}{2}b\,e^{2}x^{2}+\frac{1}{3}bef\,x^{6}+\frac{1}{10}b\,f^{2}x^{10}$	28
orering	$\frac{x^{2}\left(3f^{2}x^{8}+10x^{4}ef+15e^{2}\right)b}{30}$	28

input
```
int(b*x*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/10*f^2*x^10+1/3*e*f*x^6+1/2*e^2*x^2)*b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx\left(e+fx^{4}\right)^{2}\,dx=\frac{1}{10}\,bf^{2}x^{10}+\frac{1}{3}\,befx^{6}+\frac{1}{2}\,be^{2}x^{2}$$

input
```
integrate(b*x*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/10*b*f^2*x^10 + 1/3*b*e*f*x^6 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int bx(e+fx^4)^2\,dx = \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

input
```
integrate(b*x*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{1}{30}\left(3\,f^2x^{10} + 10\,efx^6 + 15\,e^2x^2\right)b$$

input
```
integrate(b*x*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx = \frac{1}{30}\left(3\,f^2x^{10} + 10\,efx^6 + 15\,e^2x^2\right)b$$

input
```
integrate(b*x*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/30*(3*f^2*x^10 + 10*e*f*x^6 + 15*e^2*x^2)*b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx=\frac{b\,x^2\,(15\,e^2+10\,e\,f\,x^4+3\,f^2\,x^8)}{30}$$

input
```
int(b*x*(e + f*x^4)^2,x)
```

output
```
(b*x^2*(15*e^2 + 3*f^2*x^8 + 10*e*f*x^4))/30
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int bx(e+fx^4)^2\,dx=\frac{b\,x^2(3f^2x^8+10ef\,x^4+15e^2)}{30}$$

input
```
int(b*x*(f*x^4+e)^2,x)
```

output
```
(b*x**2*(15*e**2 + 10*e*f*x**4 + 3*f**2*x**8))/30
```

3.57 $\int (a+bx) \left(e+fx^4\right)^2 dx$

Optimal result 506
Mathematica [A] (verified) 506
Rubi [A] (verified) 507
Maple [A] (verified) 508
Fricas [A] (verification not implemented) 508
Sympy [A] (verification not implemented) 509
Maxima [A] (verification not implemented) 509
Giac [A] (verification not implemented) 509
Mupad [B] (verification not implemented) 510
Reduce [B] (verification not implemented) 510

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int (a+bx) \left(e+fx^4\right)^2 dx = \frac{2}{5}aefx^5 + \frac{1}{3}befx^6 + \frac{1}{9}af^2x^9 + \frac{1}{10}bf^2x^{10} + \frac{e^2(a+bx)^2}{2b}$$

output
```
2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/2*e^2*(b*x+a)^
2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx) \left(e+fx^4\right)^2 dx = ae^2x + \frac{1}{2}be^2x^2 + \frac{2}{5}aefx^5 + \frac{1}{3}befx^6 + \frac{1}{9}af^2x^9 + \frac{1}{10}bf^2x^{10}$$

input
```
Integrate[(a + b*x)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9
+ (b*f^2*x^10)/10
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(ae^2+2aefx^4+af^2x^8+be^2x+2befx^5+bf^2x^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}$$

input
```
Int[(a + b*x)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9
+ (b*f^2*x^10)/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{2}b\,e^2x^2+a\,e^2x$	51
default	$\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{2}b\,e^2x^2+a\,e^2x$	51
norman	$\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{2}b\,e^2x^2+a\,e^2x$	51
risch	$\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{2}b\,e^2x^2+a\,e^2x$	51
parallelrisch	$\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{2}b\,e^2x^2+a\,e^2x$	51
orering	$\frac{x\left(9b\,f^2x^9+10a\,f^2x^8+30bef\,x^5+36aef\,x^4+45b\,e^2x+90a\,e^2\right)}{90}$	52

input
```
int((b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/3*b*e*f*x^6+2/5*a*e*f*x^5+1/2*b*e^2*x^2+a*
e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx=\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^
2*x^2 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int (a+bx)\left(e+fx^4\right)^2\,dx = ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{be^2x^2}{2} + \frac{befx^6}{3} + \frac{bf^2x^{10}}{10}$$

input
```
integrate((b*x+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
 b*f**2*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx = \frac{1}{10}\,bf^2x^{10} + \frac{1}{9}\,af^2x^9 + \frac{1}{3}\,befx^6 + \frac{2}{5}\,aefx^5 + \frac{1}{2}\,be^2x^2 + ae^2x$$

input
```
integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^
2*x^2 + a*e^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx)\left(e+fx^4\right)^2\,dx = \frac{1}{10}\,bf^2x^{10} + \frac{1}{9}\,af^2x^9 + \frac{1}{3}\,befx^6 + \frac{2}{5}\,aefx^5 + \frac{1}{2}\,be^2x^2 + ae^2x$$

input
```
integrate((b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/3*b*e*f*x^6 + 2/5*a*e*f*x^5 + 1/2*b*e^
2*x^2 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx) \left(e+fx^4\right)^2 \, dx = \frac{b\,e^2\,x^2}{2} + a\,e^2\,x + \frac{b\,e\,f\,x^6}{3} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{b\,f^2\,x^{10}}{10} + \frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + b*x),x)
```

output
```
(b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + a*e^2*x + (2*a*e*f*x^5)/
5 + (b*e*f*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int (a+bx) \left(e+fx^4\right)^2 \, dx = \frac{x(9b\,f^2x^9 + 10a\,f^2x^8 + 30bef\,x^5 + 36aef\,x^4 + 45b\,e^2x + 90a\,e^2)}{90}$$

input
```
int((b*x+a)*(f*x^4+e)^2,x)
```

output
```
(x*(90*a*e**2 + 36*a*e*f*x**4 + 10*a*f**2*x**8 + 45*b*e**2*x + 30*b*e*f*x*
*5 + 9*b*f**2*x**9))/90
```

3.58 $\int cx^2(e+fx^4)^2\,dx$

Optimal result . . . 511
Mathematica [A] (verified) . . . 511
Rubi [A] (verified) . . . 512
Maple [A] (verified) . . . 513
Fricas [A] (verification not implemented) . . . 513
Sympy [A] (verification not implemented) . . . 514
Maxima [A] (verification not implemented) . . . 514
Giac [A] (verification not implemented) . . . 514
Mupad [B] (verification not implemented) . . . 515
Reduce [B] (verification not implemented) . . . 515

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int cx^2\left(e+fx^4\right)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

output
```
1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int cx^2\left(e+fx^4\right)^2\,dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Integrate[c*x^2*(e + f*x^4)^2,x]
```

output
```
(c*e^2*x^3)/3 + (2*c*e*f*x^7)/7 + (c*f^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int cx^2\left(e+fx^4\right)^2dx$$

$$\Big\downarrow \text{27}$$

$$c\int x^2\left(fx^4+e\right)^2dx$$

$$\Big\downarrow \text{802}$$

$$c\int \left(f^2x^{10}+2efx^6+e^2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{e^2x^3}{3}+\frac{2}{7}efx^7+\frac{f^2x^{11}}{11}\right)$$

input
```
Int[c*x^2*(e + f*x^4)^2,x]
```

output
```
c*((e^2*x^3)/3 + (2*e*f*x^7)/7 + (f^2*x^11)/11)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
default	$\left(\frac{1}{11}f^2x^{11}+\frac{2}{7}ef\,x^7+\frac{1}{3}x^3e^2\right)c$	27
parallelrisch	$\left(\frac{1}{11}f^2x^{11}+\frac{2}{7}ef\,x^7+\frac{1}{3}x^3e^2\right)c$	27
gosper	$\frac{x^3(21f^2x^8+66x^4ef+77e^2)c}{231}$	28
norman	$\frac{1}{3}c\,e^2x^3+\frac{2}{7}cef\,x^7+\frac{1}{11}c\,f^2x^{11}$	28
risch	$\frac{1}{3}c\,e^2x^3+\frac{2}{7}cef\,x^7+\frac{1}{11}c\,f^2x^{11}$	28
orering	$\frac{x^3(21f^2x^8+66x^4ef+77e^2)c}{231}$	28

input
```
int(c*x^2*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/11*f^2*x^11+2/7*e*f*x^7+1/3*x^3*e^2)*c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,ce^2x^3$$

input
```
integrate(c*x^2*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 2/7*c*e*f*x^7 + 1/3*c*e^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int cx^2(e+fx^4)^2\,dx=\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

input
```
integrate(c*x**2*(f*x**4+e)**2,x)
```

output
```
c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx=\frac{1}{231}\left(21\,f^2x^{11}+66\,efx^7+77\,e^2x^3\right)c$$

input
```
integrate(c*x^2*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2(e+fx^4)^2\,dx=\frac{1}{231}\left(21\,f^2x^{11}+66\,efx^7+77\,e^2x^3\right)c$$

input
```
integrate(c*x^2*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/231*(21*f^2*x^11 + 66*e*f*x^7 + 77*e^2*x^3)*c
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2\left(e+fx^4\right)^2\,dx=\frac{c\,x^3\,\left(77\,e^2+66\,e\,f\,x^4+21\,f^2\,x^8\right)}{231}$$

input
```
int(c*x^2*(e + f*x^4)^2,x)
```

output
```
(c*x^3*(77*e^2 + 21*f^2*x^8 + 66*e*f*x^4))/231
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int cx^2\left(e+fx^4\right)^2\,dx=\frac{c\,x^3(21f^2x^8+66ef\,x^4+77e^2)}{231}$$

input
```
int(c*x^2*(f*x^4+e)^2,x)
```

output
```
(c*x**3*(77*e**2 + 66*e*f*x**4 + 21*f**2*x**8))/231
```

3.59 $\int (a+cx^2) (e+fx^4)^2 \, dx$

Optimal result . . . 516
Mathematica [A] (verified) . . . 516
Rubi [A] (verified) . . . 517
Maple [A] (verified) . . . 518
Fricas [A] (verification not implemented) . . . 518
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Optimal result

Integrand size = 17, antiderivative size = 60

$$\int (a+cx^2) (e+fx^4)^2 \, dx = ae^2x + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{11}cf^2x^{11}$$

output
```
a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2
*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+cx^2) (e+fx^4)^2 \, dx = ae^2x + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{11}cf^2x^{11}$$

input
```
Integrate[(a + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/
9 + (c*f^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{1468}$$

$$\int \left(ae^2+2aefx^4+af^2x^8+ce^2x^2+2cefx^6+cf^2x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Int[(a + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/
9 + (c*f^2*x^11)/11
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
default	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
norman	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
risch	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
parallelrisch	$a\,e^{2} x+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{11} c\,f^{2} x^{11}$	51
orering	$\frac{x \left(315 c\,f^{2} x^{10}+385 a\,f^{2} x^{8}+990 c e f\,x^{6}+1386 a e f\,x^{4}+1155 c\,e^{2} x^{2}+3465 a\,e^{2}\right)}{3465}$	54

input
```
int((c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2
*x^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+ae^2x$$

input
```
integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^
2*x^3 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+cx^2)(e+fx^4)^2\,dx = ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11}$$

input
```
integrate((c*x**2+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
 + c*f**2*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+cx^2)(e+fx^4)^2\,dx = \frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{2}{7}cefx^7 + \frac{2}{5}aefx^5 + \frac{1}{3}ce^2x^3 + ae^2x$$

input
```
integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^
2*x^3 + a*e^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+cx^2)(e+fx^4)^2\,dx = \frac{1}{11}cf^2x^{11} + \frac{1}{9}af^2x^9 + \frac{2}{7}cefx^7 + \frac{2}{5}aefx^5 + \frac{1}{3}ce^2x^3 + ae^2x$$

input
```
integrate((c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 2/5*a*e*f*x^5 + 1/3*c*e^
2*x^3 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{c\,e^2\,x^3}{3}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((a + c*x^2)*(e + f*x^4)^2,x)
```

output
```
(a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (c*f^2*x^11)/11 + a*e^2*x + (2*a*e*f*x^5)/
5 + (2*c*e*f*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \left(a+cx^2\right)\left(e+fx^4\right)^2\,dx$$
$$=\frac{x(315c\,f^2x^{10}+385a\,f^2x^8+990cef\,x^6+1386aef\,x^4+1155c\,e^2x^2+3465a\,e^2)}{3465}$$

input
```
int((c*x^2+a)*(f*x^4+e)^2,x)
```

output
```
(x*(3465*a*e**2 + 1386*a*e*f*x**4 + 385*a*f**2*x**8 + 1155*c*e**2*x**2 + 9
90*c*e*f*x**6 + 315*c*f**2*x**10))/3465
```

3.60 $\int (bx+cx^2)(e+fx^4)^2\,dx$

Optimal result 521
Mathematica [A] (verified) 521
Rubi [A] (verified) 522
Maple [A] (verified) 523
Fricas [A] (verification not implemented) 523
Sympy [A] (verification not implemented) 524
Maxima [A] (verification not implemented) 524
Giac [A] (verification not implemented) 525
Mupad [B] (verification not implemented) 525
Reduce [B] (verification not implemented) 525

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int (bx+cx^2)\left(e+fx^4\right)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

output
```
1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/
11*c*f^2*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int (bx+cx^2)\left(e+fx^4\right)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}$$

input
```
Integrate[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x
^10)/10 + (c*f^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2027, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2027}$$

$$\int x(b+cx)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(be^2x+2befx^5+bf^2x^9+ce^2x^2+2cefx^6+cf^2x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Int[(b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x
^10)/10 + (c*f^2*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{2}\left(210 c\, f^{2} x^{9}+231 b\, f^{2} x^{8}+660 c e f\, x^{5}+770 b e f\, x^{4}+770 c\, e^{2} x +1155 b\, e^{2}\right)}{2310}$	54
default	$\frac{1}{2} b\, e^{2} x^{2}+\frac{1}{3} c\, e^{2} x^{3}+\frac{1}{3} b e f\, x^{6}+\frac{2}{7} c e f\, x^{7}+\frac{1}{10} b\, f^{2} x^{10}+\frac{1}{11} c\, f^{2} x^{11}$	54
norman	$\frac{1}{2} b\, e^{2} x^{2}+\frac{1}{3} c\, e^{2} x^{3}+\frac{1}{3} b e f\, x^{6}+\frac{2}{7} c e f\, x^{7}+\frac{1}{10} b\, f^{2} x^{10}+\frac{1}{11} c\, f^{2} x^{11}$	54
risch	$\frac{1}{2} b\, e^{2} x^{2}+\frac{1}{3} c\, e^{2} x^{3}+\frac{1}{3} b e f\, x^{6}+\frac{2}{7} c e f\, x^{7}+\frac{1}{10} b\, f^{2} x^{10}+\frac{1}{11} c\, f^{2} x^{11}$	54
parallelrisch	$\frac{1}{2} b\, e^{2} x^{2}+\frac{1}{3} c\, e^{2} x^{3}+\frac{1}{3} b e f\, x^{6}+\frac{2}{7} c e f\, x^{7}+\frac{1}{10} b\, f^{2} x^{10}+\frac{1}{11} c\, f^{2} x^{11}$	54
orering	$\frac{x \left(210 c\, f^{2} x^{9}+231 b\, f^{2} x^{8}+660 c e f\, x^{5}+770 b e f\, x^{4}+770 c\, e^{2} x +1155 b\, e^{2}\right) \left(c\, x^{2}+b x \right)}{2310 c x +2310 b}$	68

input
```
int((c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2310*x^2*(210*c*f^2*x^9+231*b*f^2*x^8+660*c*e*f*x^5+770*b*e*f*x^4+770*c*
e^2*x+1155*b*e^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right) \left(e+fx^4\right)^2 \, dx = \frac{1}{11} \, c f^{2} x^{11} + \frac{1}{10} \, b f^{2} x^{10} + \frac{2}{7} \, c e f x^{7} + \frac{1}{3} \, b e f x^{6} + \frac{1}{3} \, c e^{2} x^{3} + \frac{1}{2} \, b e^{2} x^{2}$$

input
```
integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*
e^2*x^3 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

input
```
integrate((c*x**2+b*x)*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x
**7/7 + c*f**2*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

input
```
integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*
e^2*x^3 + 1/2*b*e^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx + cx^2) (e + fx^4)^2 \, dx = \frac{1}{11} cf^2 x^{11} + \frac{1}{10} bf^2 x^{10} + \frac{2}{7} cefx^7 + \frac{1}{3} befx^6 + \frac{1}{3} ce^2 x^3 + \frac{1}{2} be^2 x^2$$

input
```
integrate((c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 2/7*c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/3*c*
e^2*x^3 + 1/2*b*e^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx + cx^2) (e + fx^4)^2 \, dx = \frac{c e^2 x^3}{3} + \frac{b e^2 x^2}{2} + \frac{2 c e f x^7}{7} + \frac{b e f x^6}{3} + \frac{c f^2 x^{11}}{11} + \frac{b f^2 x^{10}}{10}$$

input
```
int((b*x + c*x^2)*(e + f*x^4)^2,x)
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (b*e*f
*x^6)/3 + (2*c*e*f*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (bx + cx^2) (e + fx^4)^2 \, dx = \frac{x^2 (210 c f^2 x^9 + 231 b f^2 x^8 + 660 cef x^5 + 770 bef x^4 + 770 c e^2 x + 1155 b e^2)}{2310}$$

input
```
int((c*x^2+b*x)*(f*x^4+e)^2,x)
```

output

```
(x**2*(1155*b*e**2 + 770*b*e*f*x**4 + 231*b*f**2*x**8 + 770*c*e**2*x + 660
*c*e*f*x**5 + 210*c*f**2*x**9))/2310
```

3.61 $\int (a+bx+cx^2)(e+fx^4)^2 \, dx$

Optimal result 527
Mathematica [A] (verified) 527
Rubi [A] (verified) 528
Maple [A] (verified) 529
Fricas [A] (verification not implemented) 529
Sympy [A] (verification not implemented) 530
Maxima [A] (verification not implemented) 530
Giac [A] (verification not implemented) 531
Mupad [B] (verification not implemented) 531
Reduce [B] (verification not implemented) 532

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int (a+bx+cx^2)(e+fx^4)^2 \, dx = ae^2x + \frac{1}{2}be^2x^2 + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{1}{3}befx^6 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{10}bf^2x^{10} + \frac{1}{11}cf^2x^{11}$$

output
```
a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*
x^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)(e+fx^4)^2 \, dx = ae^2x + \frac{1}{2}be^2x^2 + \frac{1}{3}ce^2x^3 + \frac{2}{5}aefx^5 + \frac{1}{3}befx^6 + \frac{2}{7}cefx^7 + \frac{1}{9}af^2x^9 + \frac{1}{10}bf^2x^{10} + \frac{1}{11}cf^2x^{11}$$

input
```
Integrate[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3
+ (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx^4)^2 (a+bx+cx^2)\, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(ae^2+2aefx^4+af^2x^8+be^2x+2befx^5+bf^2x^9+ce^2x^2+2cefx^6+cf^2x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}$$

input
```
Int[(a + b*x + c*x^2)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3
+ (2*c*e*f*x^7)/7 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result
gosper	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{11} c\,f^{2} x$
default	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{11} c\,f^{2} x$
norman	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{11} c\,f^{2} x$
risch	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{11} c\,f^{2} x$
parallelrisch	$a\,e^{2} x+\frac{1}{2} b\,e^{2} x^{2}+\frac{1}{3} c\,e^{2} x^{3}+\frac{2}{5} a e f\,x^{5}+\frac{1}{3} b e f\,x^{6}+\frac{2}{7} c e f\,x^{7}+\frac{1}{9} a\,f^{2} x^{9}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{11} c\,f^{2} x$
orering	$\frac{x \left(630 c\,f^{2} x^{10}+693 b\,f^{2} x^{9}+770 a\,f^{2} x^{8}+1980 c e f\,x^{6}+2310 b e f\,x^{5}+2772 a e f\,x^{4}+2310 c\,e^{2} x^{2}+3465 b\,e^{2} x+6930 a\,e^{2}\right)}{6930}$

input
```
int((c*x^2+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
a*e^2*x+1/2*b*e^2*x^2+1/3*c*e^2*x^3+2/5*a*e*f*x^5+1/3*b*e*f*x^6+2/7*c*e*f*
x^7+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/11*c*f^2*x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}$$

input
```
integrate((c*x**2+b*x+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
 b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx = \frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{2}{5}aefx^5+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2+ae^2x$$

input
```
integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{9}\,af^2x^9+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{2}{5}\,aefx^5+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2+ae^2x$$

input
```
integrate((c*x^2+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 2/7*c*e*f*x^7 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \left(a+bx+cx^2\right)\left(e+fx^4\right)^2\,dx=\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + b*x + c*x^2),x)
```

output
```
(b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (c*f^2*x
^11)/11 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int (a+bx+cx^2)(e+fx^4)^2\,dx$$
$$= \frac{x(630c f^2x^{10}+693b f^2x^9+770a f^2x^8+1980cef x^6+2310bef x^5+2772aef x^4+2310c e^2x^2+3465b e}{6930}$$

input
```
int((c*x^2+b*x+a)*(f*x^4+e)^2,x)
```

output
```
(x*(6930*a*e**2 + 2772*a*e*f*x**4 + 770*a*f**2*x**8 + 3465*b*e**2*x + 2310
*b*e*f*x**5 + 693*b*f**2*x**9 + 2310*c*e**2*x**2 + 1980*c*e*f*x**6 + 630*c
*f**2*x**10))/6930
```

3.62 $\int dx^3(e+fx^4)^2\,dx$

Optimal result 533
Mathematica [A] (verified) 533
Rubi [A] (verified) 534
Maple [A] (verified) 535
Fricas [A] (verification not implemented) 535
Sympy [B] (verification not implemented) 536
Maxima [A] (verification not implemented) 536
Giac [A] (verification not implemented) 536
Mupad [B] (verification not implemented) 537
Reduce [B] (verification not implemented) 537

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int dx^3\left(e+fx^4\right)^2\,dx=\frac{d(e+fx^4)^3}{12f}$$

output
```
1/12*d*(f*x^4+e)^3/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int dx^3\left(e+fx^4\right)^2\,dx=\frac{1}{4}de^2x^4+\frac{1}{4}defx^8+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[d*x^3*(e + f*x^4)^2,x]
```

output
```
(d*e^2*x^4)/4 + (d*e*f*x^8)/4 + (d*f^2*x^12)/12
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {27, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int dx^3\left(e+fx^4\right)^2\,dx$$
$$\Big\downarrow \; 27$$
$$d\int x^3\left(fx^4+e\right)^2dx$$
$$\Big\downarrow \; 793$$
$$\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[d*x^3*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{d(f\,x^4+e)^3}{12f}$	16
gosper	$\frac{x^4(f^2x^8+3x^4ef+3e^2)d}{12}$	27
parallelrisch	$\left(\frac{1}{12}f^2x^{12}+\frac{1}{4}ef\,x^8+\frac{1}{4}e^2x^4\right)d$	27
orering	$\frac{x^4(f^2x^8+3x^4ef+3e^2)d}{12}$	27
norman	$\frac{1}{4}d\,e^2x^4+\frac{1}{12}d\,f^2x^{12}+\frac{1}{4}def\,x^8$	28
risch	$\frac{d\,f^2x^{12}}{12}+\frac{def\,x^8}{4}+\frac{d\,e^2x^4}{4}+\frac{d\,e^3}{12f}$	37

input
```
int(d*x^3*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*(f*x^4+e)^3/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int dx^3(e+fx^4)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{4}\,defx^8+\frac{1}{4}\,de^2x^4$$

input
```
integrate(d*x^3*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/4*d*e*f*x^8 + 1/4*d*e^2*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int dx^3(e+fx^4)^2\,dx = \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

input
```
integrate(d*x**3*(f*x**4+e)**2,x)
```

output
```
d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int dx^3(e+fx^4)^2\,dx = \frac{(fx^4+e)^3d}{12\,f}$$

input
```
integrate(d*x^3*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*(f*x^4 + e)^3*d/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int dx^3(e+fx^4)^2\,dx = \frac{(fx^4+e)^3d}{12\,f}$$

input
```
integrate(d*x^3*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*(f*x^4 + e)^3*d/f
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int dx^3 \left(e+fx^4\right)^2 \, dx = \frac{d\,x^4\,(3\,e^2+3\,e\,f\,x^4+f^2\,x^8)}{12}$$

input
```
int(d*x^3*(e + f*x^4)^2,x)
```

output
```
(d*x^4*(3*e^2 + f^2*x^8 + 3*e*f*x^4))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int dx^3 \left(e+fx^4\right)^2 \, dx = \frac{d\,x^4(f^2x^8+3ef\,x^4+3e^2)}{12}$$

input
```
int(d*x^3*(f*x^4+e)^2,x)
```

output
```
(d*x**4*(3*e**2 + 3*e*f*x**4 + f**2*x**8))/12
```

3.63 $\int (a+dx^3)(e+fx^4)^2 \, dx$

Optimal result 538
Mathematica [A] (verified) 538
Rubi [A] (verified) 539
Maple [A] (verified) 540
Fricas [A] (verification not implemented) 541
Sympy [A] (verification not implemented) 541
Maxima [A] (verification not implemented) 542
Giac [A] (verification not implemented) 542
Mupad [B] (verification not implemented) 542
Reduce [B] (verification not implemented) 543

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int (a+dx^3)(e+fx^4)^2 \, dx = ae^2x + \frac{2}{5}aefx^5 + \frac{1}{9}af^2x^9 + \frac{d(e+fx^4)^3}{12f}$$

output
```
a*e^2*x+2/5*a*e*f*x^5+1/9*a*f^2*x^9+1/12*d*(f*x^4+e)^3/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int (a+dx^3)(e+fx^4)^2 \, dx = ae^2x + \frac{1}{4}de^2x^4 + \frac{2}{5}aefx^5 + \frac{1}{4}defx^8 + \frac{1}{9}af^2x^9 + \frac{1}{12}df^2x^{12}$$

input
```
Integrate[(a + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9
+ (d*f^2*x^12)/12
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2017, 27, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+dx^3) \left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2017}$$

$$\int a\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{27}$$

$$a\int \left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{747}$$

$$a\int \left(f^2x^8+2efx^4+e^2\right)dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{2009}$$

$$a\left(e^2x+\frac{2}{5}efx^5+\frac{f^2x^9}{9}\right)+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(a + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f) + a*(e^2*x + (2*e*f*x^5)/5 + (f^2*x^9)/9)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
gosper	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,e^{2}x^{4}+a\,e^{2}x$	51
default	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,e^{2}x^{4}+a\,e^{2}x$	51
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,e^{2}x^{4}+a\,e^{2}x$	51
risch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,e^{2}x^{4}+a\,e^{2}x$	51
parallelrisch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{9}a\,f^{2}x^{9}+\frac{1}{4}def\,x^{8}+\frac{2}{5}aef\,x^{5}+\frac{1}{4}d\,e^{2}x^{4}+a\,e^{2}x$	51
orering	$\frac{x\left(15d\,f^{2}x^{11}+20a\,f^{2}x^{8}+45def\,x^{7}+72aef\,x^{4}+45d\,e^{2}x^{3}+180a\,e^{2}\right)}{180}$	54

input
```
int((d*x^3+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/5*a*e*f*x^5+1/4*d*e^2*x^4+a*
e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)(e+fx^4)^2 \, dx = \frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + ae^2x$$

input
```
integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^
2*x^4 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int (a+dx^3)(e+fx^4)^2 \, dx = ae^2x + \frac{2aefx^5}{5} + \frac{af^2x^9}{9} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + d*e**2*x**4/4 + d*e*f*x**8/4 +
 d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)\,(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + ae^2x$$

input
```
integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^
2*x^4 + a*e^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)\,(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12} + \frac{1}{9}af^2x^9 + \frac{1}{4}defx^8 + \frac{2}{5}aefx^5 + \frac{1}{4}de^2x^4 + ae^2x$$

input
```
integrate((d*x^3+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/5*a*e*f*x^5 + 1/4*d*e^
2*x^4 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int (a+dx^3)\,(e+fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4} + a\,e^2\,x + \frac{d\,e\,f\,x^8}{4} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{d\,f^2\,x^{12}}{12} + \frac{a\,f^2\,x^9}{9}$$

input
```
int((a + d*x^3)*(e + f*x^4)^2,x)
```

output
```
(a*f^2*x^9)/9 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + a*e^2*x + (2*a*e*f*x^5)/
5 + (d*e*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \left(a+dx^3\right)\left(e+fx^4\right)^2 dx = \frac{x(15d f^2x^{11}+20a f^2x^8+45def x^7+72aef x^4+45d e^2x^3+180a e^2)}{180}$$

input
```
int((d*x^3+a)*(f*x^4+e)^2,x)
```

output
```
(x*(180*a*e**2 + 72*a*e*f*x**4 + 20*a*f**2*x**8 + 45*d*e**2*x**3 + 45*d*e*
f*x**7 + 15*d*f**2*x**11))/180
```

3.64 $\int (bx + dx^3)(e + fx^4)^2 \, dx$

Optimal result 544
Mathematica [A] (verified) 544
Rubi [A] (verified) 545
Maple [A] (verified) 546
Fricas [A] (verification not implemented) 547
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Maxima [A] (verification not implemented) 548
Giac [A] (verification not implemented) 548
Mupad [B] (verification not implemented) 548
Reduce [B] (verification not implemented) 549

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int (bx + dx^3)(e + fx^4)^2 \, dx = \frac{1}{2}be^2x^2 + \frac{1}{3}befx^6 + \frac{1}{10}bf^2x^{10} + \frac{d(e + fx^4)^3}{12f}$$

output
```
1/2*b*e^2*x^2+1/3*b*e*f*x^6+1/10*b*f^2*x^10+1/12*d*(f*x^4+e)^3/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (bx+dx^3)(e+fx^4)^2 \, dx = \frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4 + (b*f^2*x^1
0)/10 + (d*f^2*x^12)/12
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2017, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+dx^3\right)\left(e+fx^4\right)^2dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int bx\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 27}$$

$$b\int x\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 802}$$

$$b\int \left(f^2x^9+2efx^5+e^2x\right)dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 2009}$$

$$b\left(\frac{e^2x^2}{2}+\frac{1}{3}efx^6+\frac{f^2x^{10}}{10}\right)+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f) + b*((e^2*x^2)/2 + (e*f*x^6)/3 + (f^2*x^10)/10)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{10}b\,f^{2}x^{10}+\frac{1}{4}def\,x^{8}+\frac{1}{3}bef\,x^{6}+\frac{1}{4}d\,e^{2}x^{4}+\frac{1}{2}b\,e^{2}x^{2}$	54
norman	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{10}b\,f^{2}x^{10}+\frac{1}{4}def\,x^{8}+\frac{1}{3}bef\,x^{6}+\frac{1}{4}d\,e^{2}x^{4}+\frac{1}{2}b\,e^{2}x^{2}$	54
risch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{10}b\,f^{2}x^{10}+\frac{1}{4}def\,x^{8}+\frac{1}{3}bef\,x^{6}+\frac{1}{4}d\,e^{2}x^{4}+\frac{1}{2}b\,e^{2}x^{2}$	54
parallelrisch	$\frac{1}{12}d\,f^{2}x^{12}+\frac{1}{10}b\,f^{2}x^{10}+\frac{1}{4}def\,x^{8}+\frac{1}{3}bef\,x^{6}+\frac{1}{4}d\,e^{2}x^{4}+\frac{1}{2}b\,e^{2}x^{2}$	54
gosper	$\frac{x^{2}\left(5d\,f^{2}x^{10}+6b\,f^{2}x^{8}+15def\,x^{6}+20bef\,x^{4}+15d\,e^{2}x^{2}+30b\,e^{2}\right)}{60}$	56
orering	$\frac{x\left(5d\,f^{2}x^{10}+6b\,f^{2}x^{8}+15def\,x^{6}+20bef\,x^{4}+15d\,e^{2}x^{2}+30b\,e^{2}\right)\left(d\,x^{3}+bx\right)}{60d\,x^{2}+60b}$	72

input
```
int((d*x^3+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/4*d*e*f*x^8+1/3*b*e*f*x^6+1/4*d*e^2*x^4+
1/2*b*e^2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2$$

input
```
integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*
e^2*x^4 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int (bx+dx^3)(e+fx^4)^2\,dx = \frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+b*x)*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**
8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2$$

input
```
integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*
e^2*x^4 + 1/2*b*e^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2$$

input
```
integrate((d*x^3+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 1/3*b*e*f*x^6 + 1/4*d*
e^2*x^4 + 1/2*b*e^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (bx+dx^3)(e+fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}$$

input
```
int((b*x + d*x^3)*(e + f*x^4)^2,x)
```

output
```
(b*e^2*x^2)/2 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x^12)/12 + (b*e*f
*x^6)/3 + (d*e*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \left(bx + dx^3\right) \left(e + fx^4\right)^2 \, dx = \frac{x^2(5d\,f^2x^{10} + 6b\,f^2x^8 + 15def\,x^6 + 20bef\,x^4 + 15d\,e^2x^2 + 30b\,e^2)}{60}$$

input
```
int((d*x^3+b*x)*(f*x^4+e)^2,x)
```

output
```
(x**2*(30*b*e**2 + 20*b*e*f*x**4 + 6*b*f**2*x**8 + 15*d*e**2*x**2 + 15*d*e
*f*x**6 + 5*d*f**2*x**10))/60
```

3.65 $\int (a+bx+dx^3)(e+fx^4)^2 \, dx$

Optimal result . . . 550
Mathematica [A] (verified) . . . 550
Rubi [A] (verified) . . . 551
Maple [A] (verified) . . . 552
Fricas [A] (verification not implemented) . . . 553
Sympy [A] (verification not implemented) . . . 553
Maxima [A] (verification not implemented) . . . 554
Giac [A] (verification not implemented) . . . 554
Mupad [B] (verification not implemented) . . . 554
Reduce [B] (verification not implemented) . . . 555

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int (a+bx+dx^3)(e+fx^4)^2 \, dx = \frac{2}{5}aefx^5+\frac{1}{3}befx^6+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10} + \frac{e^2(a+bx)^2}{2b}+\frac{d(e+fx^4)^3}{12f}$$

output
```
2/5*a*e*f*x^5+1/3*b*e*f*x^6+1/9*a*f^2*x^9+1/10*b*f^2*x^10+1/2*e^2*(b*x+a)^
2/b+1/12*d*(f*x^4+e)^3/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int (a+bx+dx^3)(e+fx^4)^2 \, dx = ae^2x+\frac{1}{2}be^2x^2+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{1}{3}befx^6 + \frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{10}bf^2x^{10}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3
+ (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*f^2*x^12)/12
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e+fx^4\right)^2 \left(a+bx+dx^3\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int (a+bx)\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(bf^2x^9+af^2x^8+2befx^5+2aefx^4+be^2x+ae^2\right)dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(a + b*x + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (b*e^2*x^2)/2 + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (a*f^2*x^9)/9
+ (b*f^2*x^10)/10 + (d*(e + f*x^4)^3)/(12*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e$
default	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e$
norman	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e$
risch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e$
parallelrisch	$\frac{1}{12}d\,f^2x^{12}+\frac{1}{10}b\,f^2x^{10}+\frac{1}{9}a\,f^2x^9+\frac{1}{4}def\,x^8+\frac{1}{3}bef\,x^6+\frac{2}{5}aef\,x^5+\frac{1}{4}d\,e^2x^4+\frac{1}{2}b\,e^2x^2+a\,e$
orering	$\frac{x\left(15d\,f^2x^{11}+18b\,f^2x^9+20a\,f^2x^8+45def\,x^7+60bef\,x^5+72aef\,x^4+45d\,e^2x^3+90b\,e^2x+180a\,e^2\right)}{180}$

input
```
int((d*x^3+b*x+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/10*b*f^2*x^10+1/9*a*f^2*x^9+1/4*d*e*f*x^8+1/3*b*e*f*x^6+
2/5*a*e*f*x^5+1/4*d*e^2*x^4+1/2*b*e^2*x^2+a*e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{2}{5}aefx^5+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2+ae^2x$$

input
```
integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \left(a+bx+dx^3\right)\left(e+fx^4\right)^2\,dx=ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+b*x+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + b*e**2*x**2/2 + b*e*f*x**6/3 +
 b*f**2*x**10/10 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{2}{5}aefx^5+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2+ae^2x$$

input
```
integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{1}{12}df^2x^{12}+\frac{1}{10}bf^2x^{10}+\frac{1}{9}af^2x^9+\frac{1}{4}defx^8+\frac{1}{3}befx^6+\frac{2}{5}aefx^5+\frac{1}{4}de^2x^4+\frac{1}{2}be^2x^2+ae^2x$$

input
```
integrate((d*x^3+b*x+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/10*b*f^2*x^10 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 1/3*b*
e*f*x^6 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/2*b*e^2*x^2 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+bx+dx^3)\left(e+fx^4\right)^2\,dx=\frac{d\,e^2\,x^4}{4}+\frac{b\,e^2\,x^2}{2}+a\,e^2\,x+\frac{d\,e\,f\,x^8}{4}+\frac{b\,e\,f\,x^6}{3}+\frac{2\,a\,e\,f\,x^5}{5}+\frac{d\,f^2\,x^{12}}{12}+\frac{b\,f^2\,x^{10}}{10}+\frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + b*x + d*x^3),x)
```

output
```
(b*e^2*x^2)/2 + (a*f^2*x^9)/9 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (d*f^2*x
^12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (b*e*f*x^6)/3 + (d*e*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int (a+bx+dx^3) (e+fx^4)^2 \, dx = \frac{x(15d f^2x^{11}+18b f^2x^9+20a f^2x^8+45def x^7+60bef x^5+72aef x^4+45d e^2x^3+90b e^2x+180a e^2)}{180}$$

input
```
int((d*x^3+b*x+a)*(f*x^4+e)^2,x)
```

output
```
(x*(180*a*e**2 + 72*a*e*f*x**4 + 20*a*f**2*x**8 + 90*b*e**2*x + 60*b*e*f*x
**5 + 18*b*f**2*x**9 + 45*d*e**2*x**3 + 45*d*e*f*x**7 + 15*d*f**2*x**11))/
180
```

3.66 $\int \left(cx^2+dx^3\right) \left(e+fx^4\right)^2 \, dx$

Optimal result . . . 556
Mathematica [A] (verified) . . . 556
Rubi [A] (verified) . . . 557
Maple [A] (verified) . . . 558
Fricas [A] (verification not implemented) . . . 559
Sympy [A] (verification not implemented) . . . 559
Maxima [A] (verification not implemented) . . . 560
Giac [A] (verification not implemented) . . . 560
Mupad [B] (verification not implemented) . . . 560
Reduce [B] (verification not implemented) . . . 561

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \left(cx^2+dx^3\right) \left(e+fx^4\right)^2 \, dx=\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

output
```
1/3*c*e^2*x^3+2/7*c*e*f*x^7+1/11*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \left(cx^2+dx^3\right) \left(e+fx^4\right)^2 \, dx=\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4 + (c*f^2*x
^11)/11 + (d*f^2*x^12)/12
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2017, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2 dx$$

$$\Big\downarrow \text{2017}$$

$$\int cx^2\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{27}$$

$$c\int x^2\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{802}$$

$$c\int \left(f^2x^{10}+2efx^6+e^2x^2\right)dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{e^2x^3}{3}+\frac{2}{7}efx^7+\frac{f^2x^{11}}{11}\right)+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(d*(e + f*x^4)^3)/(12*f) + c*((e^2*x^3)/3 + (2*e*f*x^7)/7 + (f^2*x^11)/11)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{x^{3} \left(77 d \,f^{2} x^{9}+84 c \,f^{2} x^{8}+231 d e f \,x^{5}+264 c e f \,x^{4}+231 d \,e^{2} x +308 c \,e^{2}\right)}{924}$	54
default	$\frac{1}{12} d \,f^{2} x^{12}+\frac{1}{11} c \,f^{2} x^{11}+\frac{1}{4} d e f \,x^{8}+\frac{2}{7} c e f \,x^{7}+\frac{1}{4} d \,e^{2} x^{4}+\frac{1}{3} c \,e^{2} x^{3}$	54
norman	$\frac{1}{12} d \,f^{2} x^{12}+\frac{1}{11} c \,f^{2} x^{11}+\frac{1}{4} d e f \,x^{8}+\frac{2}{7} c e f \,x^{7}+\frac{1}{4} d \,e^{2} x^{4}+\frac{1}{3} c \,e^{2} x^{3}$	54
risch	$\frac{1}{12} d \,f^{2} x^{12}+\frac{1}{11} c \,f^{2} x^{11}+\frac{1}{4} d e f \,x^{8}+\frac{2}{7} c e f \,x^{7}+\frac{1}{4} d \,e^{2} x^{4}+\frac{1}{3} c \,e^{2} x^{3}$	54
parallelrisch	$\frac{1}{12} d \,f^{2} x^{12}+\frac{1}{11} c \,f^{2} x^{11}+\frac{1}{4} d e f \,x^{8}+\frac{2}{7} c e f \,x^{7}+\frac{1}{4} d \,e^{2} x^{4}+\frac{1}{3} c \,e^{2} x^{3}$	54
orering	$\frac{x \left(77 d \,f^{2} x^{9}+84 c \,f^{2} x^{8}+231 d e f \,x^{5}+264 c e f \,x^{4}+231 d \,e^{2} x +308 c \,e^{2}\right) \left(d \,x^{3}+c \,x^{2}\right)}{924 d x +924 c}$	70

input
```
int((d*x^3+c*x^2)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/924*x^3*(77*d*f^2*x^9+84*c*f^2*x^8+231*d*e*f*x^5+264*c*e*f*x^4+231*d*e^2
*x+308*c*e^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12} + \frac{1}{11}cf^2x^{11} + \frac{1}{4}defx^8 + \frac{2}{7}cefx^7 + \frac{1}{4}de^2x^4 + \frac{1}{3}ce^2x^3$$

input
```
integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*
e^2*x^4 + 1/3*c*e^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int (cx^2+dx^3)(e+fx^4)^2\,dx = \frac{ce^2x^3}{3} + \frac{2cefx^7}{7} + \frac{cf^2x^{11}}{11} + \frac{de^2x^4}{4} + \frac{defx^8}{4} + \frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+c*x**2)*(f*x**4+e)**2,x)
```

output
```
c*e**2*x**3/3 + 2*c*e*f*x**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x
**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

input
```
integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*
e^2*x^4 + 1/3*c*e^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3$$

input
```
integrate((d*x^3+c*x^2)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/4*d*e*f*x^8 + 2/7*c*e*f*x^7 + 1/4*d*
e^2*x^4 + 1/3*c*e^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}$$

input
```
int((e + f*x^4)^2*(c*x^2 + d*x^3),x)
```

output
```
(c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12 + (2*c*e
*f*x^7)/7 + (d*e*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int (cx^2 + dx^3) (e + fx^4)^2 dx = \frac{x^3(77d f^2x^9 + 84c f^2x^8 + 231def x^5 + 264cef x^4 + 231d e^2x + 308c e^2)}{924}$$

input
```
int((d*x^3+c*x^2)*(f*x^4+e)^2,x)
```

output
```
(x**3*(308*c*e**2 + 264*c*e*f*x**4 + 84*c*f**2*x**8 + 231*d*e**2*x + 231*d
*e*f*x**5 + 77*d*f**2*x**9))/924
```

3.67 $\int (a+cx^2+dx^3)(e+fx^4)^2\,dx$

Optimal result . . . 562
Mathematica [A] (verified) . . . 562
Rubi [A] (verified) . . . 563
Maple [A] (verified) . . . 564
Fricas [A] (verification not implemented) . . . 565
Sympy [A] (verification not implemented) . . . 565
Maxima [A] (verification not implemented) . . . 566
Giac [A] (verification not implemented) . . . 566
Mupad [B] (verification not implemented) . . . 566
Reduce [B] (verification not implemented) . . . 567

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int (a+cx^2+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{1}{3}ce^2x^3+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

output
```
a*e^2*x+1/3*c*e^2*x^3+2/5*a*e*f*x^5+2/7*c*e*f*x^7+1/9*a*f^2*x^9+1/11*c*f^2
*x^11+1/12*d*(f*x^4+e)^3/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\int (a+cx^2+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{2}{5}aefx^5+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{9}af^2x^9+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/
7 + (d*e*f*x^8)/4 + (a*f^2*x^9)/9 + (c*f^2*x^11)/11 + (d*f^2*x^12)/12
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2017, 1468, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e+fx^4\right)^2 \left(a+cx^2+dx^3\right) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(cx^2+a\right)\left(fx^4+e\right)^2dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 1468}$$

$$\int \left(cf^2x^{10}+af^2x^8+2cefx^6+2aefx^4+ce^2x^2+ae^2\right)dx+\frac{d\left(e+fx^4\right)^3}{12f}$$

$$\Big\downarrow \text{ 2009}$$

$$ae^2x+\frac{2}{5}aefx^5+\frac{1}{9}af^2x^9+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d\left(e+fx^4\right)^3}{12f}$$

input
```
Int[(a + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
a*e^2*x + (c*e^2*x^3)/3 + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (a*f^2*x^9)/
9 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Defintions of rubi rules used

rule 1468
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{12}d f^2 x^{12}+\frac{1}{11}c f^2 x^{11}+\frac{1}{9}a f^2 x^9+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{2}{5}aef x^5+\frac{1}{4}d e^2 x^4+\frac{1}{3}c e^2 x^3+a e$
default	$\frac{1}{12}d f^2 x^{12}+\frac{1}{11}c f^2 x^{11}+\frac{1}{9}a f^2 x^9+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{2}{5}aef x^5+\frac{1}{4}d e^2 x^4+\frac{1}{3}c e^2 x^3+a e$
norman	$\frac{1}{12}d f^2 x^{12}+\frac{1}{11}c f^2 x^{11}+\frac{1}{9}a f^2 x^9+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{2}{5}aef x^5+\frac{1}{4}d e^2 x^4+\frac{1}{3}c e^2 x^3+a e$
risch	$\frac{1}{12}d f^2 x^{12}+\frac{1}{11}c f^2 x^{11}+\frac{1}{9}a f^2 x^9+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{2}{5}aef x^5+\frac{1}{4}d e^2 x^4+\frac{1}{3}c e^2 x^3+a e$
parallelrisch	$\frac{1}{12}d f^2 x^{12}+\frac{1}{11}c f^2 x^{11}+\frac{1}{9}a f^2 x^9+\frac{1}{4}def x^8+\frac{2}{7}cef x^7+\frac{2}{5}aef x^5+\frac{1}{4}d e^2 x^4+\frac{1}{3}c e^2 x^3+a e$
orering	$\frac{x(1155d f^2 x^{11}+1260c f^2 x^{10}+1540a f^2 x^8+3465def x^7+3960cef x^6+5544aef x^4+3465d e^2 x^3+4620c e^2 x^2+13860a e^2)}{13860}$

input
```
int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*d*f^2*x^12+1/11*c*f^2*x^11+1/9*a*f^2*x^9+1/4*d*e*f*x^8+2/7*c*e*f*x^7+
2/5*a*e*f*x^5+1/4*d*e^2*x^4+1/3*c*e^2*x^3+a*e^2*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{9}af^2x^9+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{2}{5}aefx^5+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+ae^2x$$

input
```
integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*
e*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int (a+cx^2+dx^3)(e+fx^4)^2\,dx = ae^2x+\frac{2aefx^5}{5}+\frac{af^2x^9}{9}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+c*x**2+a)*(f*x**4+e)**2,x)
```

output
```
a*e**2*x + 2*a*e*f*x**5/5 + a*f**2*x**9/9 + c*e**2*x**3/3 + 2*c*e*f*x**7/7
 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{9} af^2x^9 + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{2}{5} aefx^5 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3 + ae^2x$$

input
```
integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*
e*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{1}{12} df^2x^{12} + \frac{1}{11} cf^2x^{11} + \frac{1}{9} af^2x^9 + \frac{1}{4} defx^8 + \frac{2}{7} cefx^7 + \frac{2}{5} aefx^5 + \frac{1}{4} de^2x^4 + \frac{1}{3} ce^2x^3 + ae^2x$$

input
```
integrate((d*x^3+c*x^2+a)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/9*a*f^2*x^9 + 1/4*d*e*f*x^8 + 2/7*c*
e*f*x^7 + 2/5*a*e*f*x^5 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{d\,e^2\,x^4}{4} + \frac{c\,e^2\,x^3}{3} + a\,e^2\,x + \frac{d\,e\,f\,x^8}{4} + \frac{2\,c\,e\,f\,x^7}{7} + \frac{2\,a\,e\,f\,x^5}{5} + \frac{d\,f^2\,x^{12}}{12} + \frac{c\,f^2\,x^{11}}{11} + \frac{a\,f^2\,x^9}{9}$$

input
```
int((e + f*x^4)^2*(a + c*x^2 + d*x^3),x)
```

output
```
(a*f^2*x^9)/9 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (c*f^2*x^11)/11 + (d*f^2*x
^12)/12 + a*e^2*x + (2*a*e*f*x^5)/5 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int (a + cx^2 + dx^3) (e + fx^4)^2 \, dx = \frac{x(1155d f^2x^{11} + 1260c f^2x^{10} + 1540a f^2x^8 + 3465def x^7 + 3960cef x^6 + 5544aef x^4 + 3465d e^2x^3 + 46}{13860}$$

input
```
int((d*x^3+c*x^2+a)*(f*x^4+e)^2,x)
```

output
```
(x*(13860*a*e**2 + 5544*a*e*f*x**4 + 1540*a*f**2*x**8 + 4620*c*e**2*x**2 +
 3960*c*e*f*x**6 + 1260*c*f**2*x**10 + 3465*d*e**2*x**3 + 3465*d*e*f*x**7
+ 1155*d*f**2*x**11))/13860
```

3.68 $\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

output
```
1/2*b*e^2*x^2+1/3*c*e^2*x^3+1/3*b*e*f*x^6+2/7*c*e*f*x^7+1/10*b*f^2*x^10+1/
11*c*f^2*x^11+1/12*d*(f*x^4+e)^3/f
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{2}be^2x^2+\frac{1}{3}ce^2x^3+\frac{1}{4}de^2x^4+\frac{1}{3}befx^6+\frac{2}{7}cefx^7+\frac{1}{4}defx^8+\frac{1}{10}bf^2x^{10}+\frac{1}{11}cf^2x^{11}+\frac{1}{12}df^2x^{12}$$

input
```
Integrate[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (d*e^2*x^4)/4 + (b*e*f*x^6)/3 + (2*c*e*f*x
^7)/7 + (d*e*f*x^8)/4 + (b*f^2*x^10)/10 + (c*f^2*x^11)/11 + (d*f^2*x^12)/1
2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2017, 2027, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx^4)^2 (bx+cx^2+dx^3) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (cx^2+bx)(fx^4+e)^2 dx + \frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{2027}$$

$$\int x(b+cx)(fx^4+e)^2 dx + \frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{2123}$$

$$\int (cf^2x^{10}+bf^2x^9+2cefx^6+2befx^5+ce^2x^2+be^2x) \, dx + \frac{d(e+fx^4)^3}{12f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}be^2x^2+\frac{1}{3}befx^6+\frac{1}{10}bf^2x^{10}+\frac{1}{3}ce^2x^3+\frac{2}{7}cefx^7+\frac{1}{11}cf^2x^{11}+\frac{d(e+fx^4)^3}{12f}$$

input
```
Int[(b*x + c*x^2 + d*x^3)*(e + f*x^4)^2,x]
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (b*f^2*x
^10)/10 + (c*f^2*x^11)/11 + (d*(e + f*x^4)^3)/(12*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{x^{2}\left(385 d\,f^{2} x^{10}+420 c\,f^{2} x^{9}+462 b\,f^{2} x^{8}+1155 d e f\,x^{6}+1320 c e f\,x^{5}+1540 b e f\,x^{4}+1155 d\,e^{2} x^{2}+1540 c\,e^{2} x+2310 b\,e^{2}\right)}{4620}$
default	$\frac{1}{12} d\,f^{2} x^{12}+\frac{1}{11} c\,f^{2} x^{11}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{4} d e f\,x^{8}+\frac{2}{7} c e f\,x^{7}+\frac{1}{3} b e f\,x^{6}+\frac{1}{4} d\,e^{2} x^{4}+\frac{1}{3} c\,e^{2} x^{3}+\frac{1}{2}$
norman	$\frac{1}{12} d\,f^{2} x^{12}+\frac{1}{11} c\,f^{2} x^{11}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{4} d e f\,x^{8}+\frac{2}{7} c e f\,x^{7}+\frac{1}{3} b e f\,x^{6}+\frac{1}{4} d\,e^{2} x^{4}+\frac{1}{3} c\,e^{2} x^{3}+\frac{1}{2}$
risch	$\frac{1}{12} d\,f^{2} x^{12}+\frac{1}{11} c\,f^{2} x^{11}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{4} d e f\,x^{8}+\frac{2}{7} c e f\,x^{7}+\frac{1}{3} b e f\,x^{6}+\frac{1}{4} d\,e^{2} x^{4}+\frac{1}{3} c\,e^{2} x^{3}+\frac{1}{2}$
parallelrisch	$\frac{1}{12} d\,f^{2} x^{12}+\frac{1}{11} c\,f^{2} x^{11}+\frac{1}{10} b\,f^{2} x^{10}+\frac{1}{4} d e f\,x^{8}+\frac{2}{7} c e f\,x^{7}+\frac{1}{3} b e f\,x^{6}+\frac{1}{4} d\,e^{2} x^{4}+\frac{1}{3} c\,e^{2} x^{3}+\frac{1}{2}$
orering	$\frac{x\left(385 d\,f^{2} x^{10}+420 c\,f^{2} x^{9}+462 b\,f^{2} x^{8}+1155 d e f\,x^{6}+1320 c e f\,x^{5}+1540 b e f\,x^{4}+1155 d\,e^{2} x^{2}+1540 c\,e^{2} x+2310 b\,e^{2}\right)\left(d\,x^{3}+c\,x^{2}+\right.}{4620 d\,x^{2}+4620 c x+4620 b}$

input
```
int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4620*x^2*(385*d*f^2*x^10+420*c*f^2*x^9+462*b*f^2*x^8+1155*d*e*f*x^6+1320
*c*e*f*x^5+1540*b*e*f*x^4+1155*d*e^2*x^2+1540*c*e^2*x+2310*b*e^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{1}{12}\,df^2x^{12}+\frac{1}{11}\,cf^2x^{11}+\frac{1}{10}\,bf^2x^{10}+\frac{1}{4}\,defx^8+\frac{2}{7}\,cefx^7+\frac{1}{3}\,befx^6+\frac{1}{4}\,de^2x^4+\frac{1}{3}\,ce^2x^3+\frac{1}{2}\,be^2x^2$$

input
```
integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="fricas")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*
c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \left(bx+cx^2+dx^3\right)\left(e+fx^4\right)^2\,dx=\frac{be^2x^2}{2}+\frac{befx^6}{3}+\frac{bf^2x^{10}}{10}+\frac{ce^2x^3}{3}+\frac{2cefx^7}{7}+\frac{cf^2x^{11}}{11}+\frac{de^2x^4}{4}+\frac{defx^8}{4}+\frac{df^2x^{12}}{12}$$

input
```
integrate((d*x**3+c*x**2+b*x)*(f*x**4+e)**2,x)
```

output
```
b*e**2*x**2/2 + b*e*f*x**6/3 + b*f**2*x**10/10 + c*e**2*x**3/3 + 2*c*e*f*x
**7/7 + c*f**2*x**11/11 + d*e**2*x**4/4 + d*e*f*x**8/4 + d*f**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2$$

input
```
integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="maxima")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*
c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{1}{12}df^2x^{12}+\frac{1}{11}cf^2x^{11}+\frac{1}{10}bf^2x^{10}+\frac{1}{4}defx^8+\frac{2}{7}cefx^7+\frac{1}{3}befx^6+\frac{1}{4}de^2x^4+\frac{1}{3}ce^2x^3+\frac{1}{2}be^2x^2$$

input
```
integrate((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x, algorithm="giac")
```

output
```
1/12*d*f^2*x^12 + 1/11*c*f^2*x^11 + 1/10*b*f^2*x^10 + 1/4*d*e*f*x^8 + 2/7*
c*e*f*x^7 + 1/3*b*e*f*x^6 + 1/4*d*e^2*x^4 + 1/3*c*e^2*x^3 + 1/2*b*e^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx+cx^2+dx^3)(e+fx^4)^2\,dx = \frac{d\,e^2\,x^4}{4}+\frac{c\,e^2\,x^3}{3}+\frac{b\,e^2\,x^2}{2}+\frac{d\,e\,f\,x^8}{4}+\frac{2\,c\,e\,f\,x^7}{7}+\frac{b\,e\,f\,x^6}{3}+\frac{d\,f^2\,x^{12}}{12}+\frac{c\,f^2\,x^{11}}{11}+\frac{b\,f^2\,x^{10}}{10}$$

input
```
int((e + f*x^4)^2*(b*x + c*x^2 + d*x^3),x)
```

output
```
(b*e^2*x^2)/2 + (c*e^2*x^3)/3 + (b*f^2*x^10)/10 + (d*e^2*x^4)/4 + (c*f^2*x
^11)/11 + (d*f^2*x^12)/12 + (b*e*f*x^6)/3 + (2*c*e*f*x^7)/7 + (d*e*f*x^8)/
4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int (bx + cx^2 + dx^3) (e + fx^4)^2 \, dx$$
$$= \frac{x^2(385d f^2x^{10} + 420c f^2x^9 + 462b f^2x^8 + 1155def x^6 + 1320cef x^5 + 1540bef x^4 + 1155d e^2x^2 + 1540c}{4620}$$

input
```
int((d*x^3+c*x^2+b*x)*(f*x^4+e)^2,x)
```

output
```
(x**2*(2310*b*e**2 + 1540*b*e*f*x**4 + 462*b*f**2*x**8 + 1540*c*e**2*x + 1
320*c*e*f*x**5 + 420*c*f**2*x**9 + 1155*d*e**2*x**2 + 1155*d*e*f*x**6 + 38
5*d*f**2*x**10))/4620
```

3.69 $\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx$

Optimal result 574
Mathematica [A] (verified) 574
Rubi [A] (verified) 575
Maple [A] (verified) 576
Fricas [A] (verification not implemented) 577
Sympy [A] (verification not implemented) 577
Maxima [A] (verification not implemented) 578
Giac [A] (verification not implemented) 578
Mupad [B] (verification not implemented) 579
Reduce [B] (verification not implemented) 579

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*
x^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5
+ (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d
*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3)\, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int (ex^2+dx+c)(bx^4+a)^2 dx+\frac{f(a+bx^4)^3}{12b}$$

$$\Big\downarrow \text{ 2188}$$

$$\int (b^2ex^{10}+b^2dx^9+b^2cx^8+2abex^6+2abdx^5+2abcx^4+a^2ex^2+a^2dx+a^2c)\, dx+\frac{f(a+bx^4)^3}{12b}$$

$$\Big\downarrow \text{ 2009}$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f(a+bx^4)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3
+ (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f
*(a + b*x^4)^3)/(12*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a$
default	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a$
norman	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a$
risch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a$
parallelrisch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a$
orering	$\frac{x(1155f\,b^2x^{11}+1260e\,b^2x^{10}+1386b^2d\,x^9+1540b^2c\,x^8+3465fab\,x^7+3960abe\,x^6+4620x^5adb+5544abc\,x^4+3465f\,a^2x^3+462}{13860}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*f*x^
8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/
2*a^2*d*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = \frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9 + \frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5 + \frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4} + \frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4} + \frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5
+ a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x*
*10/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

input
```
int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x
^4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a
*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{x(1155b^2f\,x^{11}+1260b^2e\,x^{10}+1386b^2d\,x^9+1540b^2c\,x^8+3465abf\,x^7+3960abe\,x^6+4620abd\,x^5+5544}{13860}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

output
```
(x*(13860*a**2*c + 6930*a**2*d*x + 4620*a**2*e*x**2 + 3465*a**2*f*x**3 + 5
544*a*b*c*x**4 + 4620*a*b*d*x**5 + 3960*a*b*e*x**6 + 3465*a*b*f*x**7 + 154
0*b**2*c*x**8 + 1386*b**2*d*x**9 + 1260*b**2*e*x**10 + 1155*b**2*f*x**11))
/13860
```

3.70 $\int (c+dx+ex^2+fx^3)(a+bx^4)^3 \, dx$

Optimal result 580
Mathematica [A] (verified) 581
Rubi [A] (verified) 581
Maple [A] (verified) 583
Fricas [A] (verification not implemented) 583
Sympy [A] (verification not implemented) 584
Maxima [A] (verification not implemented) 584
Giac [A] (verification not implemented) 585
Mupad [B] (verification not implemented) 586
Reduce [B] (verification not implemented) 586

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3 \, dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^
2*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x
^13+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/
5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9
)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*
c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^3(c+dx+ex^2+fx^3)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int (ex^2+dx+c)(bx^4+a)^3dx+\frac{f(a+bx^4)^4}{16b}$$

$$\Big\downarrow \text{2188}$$

$$\int \left(b^3ex^{14}+b^3dx^{13}+b^3cx^{12}+3ab^2ex^{10}+3ab^2dx^9+3ab^2cx^8+3a^2bex^6+3a^2bdx^5+3a^2bcx^4+a^3ex^2+a^3dx\right.$$
$$\frac{f(a+bx^4)^4}{16b}$$
$$\Big\downarrow \text{2009}$$
$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f(a+bx^4)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6
)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2
*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a
+ b*x^4)^4)/(16*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

method	result
gosper	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c$
default	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c$
norman	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c$
risch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c$
parallelrisch	$a^3cx+\frac{1}{2}a^3d\,x^2+\frac{1}{3}a^3e\,x^3+\frac{1}{4}f\,a^3x^4+\frac{3}{5}a^2bc\,x^5+\frac{1}{2}a^2bd\,x^6+\frac{3}{7}a^2be\,x^7+\frac{3}{8}f\,a^2b\,x^8+\frac{1}{3}a\,b^2c$
orering	$\frac{x(15015f\,b^3x^{15}+16016e\,b^3x^{14}+17160b^3d\,x^{13}+18480b^3c\,x^{12}+60060fa\,b^2x^{11}+65520ea\,b^2x^{10}+72072da\,b^2x^9+80080a\,b^2c\,x^8}{2402}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*f*a^3*x^4+3/5*a^2*b*c*x^5+1/2*a^2*
b*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+
3/11*a*b^2*e*x^11+1/4*f*a*b^2*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^
3*e*x^15+1/16*f*b^3*x^16
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{16}\,b^3fx^{16}+\frac{1}{15}\,b^3ex^{15}+\frac{1}{14}\,b^3dx^{14}+\frac{1}{13}\,b^3cx^{13}+\frac{1}{4}\,ab^2fx^{12}+\frac{3}{11}\,ab^2ex^{11}+\frac{3}{10}\,ab^2dx^{10}+\frac{1}{3}\,ab^2cx^9+\frac{3}{8}\,a^2bfx^8+\frac{3}{7}\,a^2bex^7+\frac{1}{2}\,a^2bdx^6+\frac{3}{5}\,a^2bcx^5+\frac{1}{4}\,a^3fx^4+\frac{1}{3}\,a^3ex^3+\frac{1}{2}\,a^3dx^2+a^3cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx + \frac{a^3dx^2}{2} + \frac{a^3ex^3}{3} + \frac{a^3fx^4}{4} + \frac{3a^2bcx^5}{5} + \frac{a^2bdx^6}{2} + \frac{3a^2bex^7}{7} + \frac{3a^2bfx^8}{8} + \frac{ab^2cx^9}{3} + \frac{3ab^2dx^{10}}{10} + \frac{3ab^2ex^{11}}{11} + \frac{ab^2fx^{12}}{4} + \frac{b^3cx^{13}}{13} + \frac{b^3dx^{14}}{14} + \frac{b^3ex^{15}}{15} + \frac{b^3fx^{16}}{16}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5
/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x*
*9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3
*c*x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = \frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3 \, dx = \frac{1}{16}b^3fx^{16} + \frac{1}{15}b^3ex^{15} + \frac{1}{14}b^3dx^{14} + \frac{1}{13}b^3cx^{13} + \frac{1}{4}ab^2fx^{12} + \frac{3}{11}ab^2ex^{11} + \frac{3}{10}ab^2dx^{10} + \frac{1}{3}ab^2cx^9 + \frac{3}{8}a^2bfx^8 + \frac{3}{7}a^2bex^7 + \frac{1}{2}a^2bdx^6 + \frac{3}{5}a^2bcx^5 + \frac{1}{4}a^3fx^4 + \frac{1}{3}a^3ex^3 + \frac{1}{2}a^3dx^2 + a^3cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{f\,a^3\,x^4}{4}+\frac{e\,a^3\,x^3}{3}+\frac{d\,a^3\,x^2}{2}+c\,a^3\,x+\frac{3\,f\,a^2\,b\,x^8}{8}+\frac{3\,e\,a^2\,b\,x^7}{7}+\frac{d\,a^2\,b\,x^6}{2}+\frac{3\,c\,a^2\,b\,x^5}{5}+\frac{f\,a\,b^2\,x^{12}}{4}+\frac{3\,e\,a\,b^2\,x^{11}}{11}+\frac{3\,d\,a\,b^2\,x^{10}}{10}+\frac{c\,a\,b^2\,x^9}{3}+\frac{f\,b^3\,x^{16}}{16}+\frac{e\,b^3\,x^{15}}{15}+\frac{d\,b^3\,x^{14}}{14}+\frac{c\,b^3\,x^{13}}{13}$$

input
```
int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f
*x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5
+ (a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7
)/7 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx$$
$$=\frac{x(15015b^3f\,x^{15}+16016b^3e\,x^{14}+17160b^3d\,x^{13}+18480b^3c\,x^{12}+60060a\,b^2f\,x^{11}+65520a\,b^2e\,x^{10}+7207}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

output
```
(x*(240240*a**3*c + 120120*a**3*d*x + 80080*a**3*e*x**2 + 60060*a**3*f*x**
3 + 144144*a**2*b*c*x**4 + 120120*a**2*b*d*x**5 + 102960*a**2*b*e*x**6 + 9
0090*a**2*b*f*x**7 + 80080*a*b**2*c*x**8 + 72072*a*b**2*d*x**9 + 65520*a*b
**2*e*x**10 + 60060*a*b**2*f*x**11 + 18480*b**3*c*x**12 + 17160*b**3*d*x**
13 + 16016*b**3*e*x**14 + 15015*b**3*f*x**15))/240240
```

3.71 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$

Optimal result 587
Mathematica [A] (verified) 588
Rubi [A] (verified) 588
Maple [C] (verified) 590
Fricas [C] (verification not implemented) 590
Sympy [B] (verification not implemented) 591
Maxima [A] (verification not implemented) 592
Giac [B] (verification not implemented) 592
Mupad [B] (verification not implemented) 594
Reduce [B] (verification not implemented) 595

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx = \frac{f}{4b(a-bx^4)} + \frac{x(c+dx+ex^2)}{4a(a-bx^4)} + \frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{3/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*f/b/(-b*x^4+a)+1/4*x*(e*x^2+d*x+c)/a/(-b*x^4+a)+1/8*(3*b^(1/2)*c-a^(1/
2)*e)*arctan(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(3/4)+1/8*(3*b^(1/2)*c+a^(1/2)*e
)*arctanh(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(3/4)+1/4*d*arctanh(b^(1/2)*x^2/a^(
1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.34

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx = \frac{\frac{4a(af+bx(c+x(d+ex)))}{a-bx^4}-2\sqrt[4]{a}\sqrt[4]{b}\left(-3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\sqrt[4]{b}\left(3\sqrt[4]{a}\sqrt{b}c+2\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\right.}{16a^2b}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

output
```
((4*a*(a*f + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2*a^(1/4)*b^(1/4)*(-3*S
qrt[b]*c + Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)] - b^(1/4)*(3*a^(1/4)*Sqr
t[b]*c + 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x] + b^(1/
4)*(3*a^(1/4)*Sqrt[b]*c - 2*Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) + b
^(1/4)*x] + 2*Sqrt[a]*Sqrt[b]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^2*b)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$

$$\Big\downarrow \ 2393$$

$$\frac{af+bx(c+dx+ex^2)}{4ab(a-bx^4)}-\frac{\int -\frac{ex^2+2dx+3c}{a-bx^4}dx}{4a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{ex^2+2dx+3c}{a-bx^4}dx}{4a}+\frac{af+bx(c+dx+ex^2)}{4ab(a-bx^4)}$$

$$\Big\downarrow \; 2415$$

$$\frac{\int \left(\frac{2dx}{a-bx^4}+\frac{ex^2+3c}{a-bx^4}\right)dx}{4a}+\frac{af+bx(c+dx+ex^2)}{4ab(a-bx^4)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{af+bx(c+dx+ex^2)}{4ab(a-bx^4)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x]
```

output
```
(a*f + b*x*(c + d*x + e*x^2))/(4*a*b*(a - b*x^4)) + (((3*Sqrt[b]*c - Sqrt[
a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c + S
qrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (d*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}+\frac{f}{4b}}{-b\,x^4+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^2 e+2d\_R+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(-b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{16a^2}\right)+d\left(\frac{x^2}{4a(-b\,x^4+a)}+\frac{\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)+e\Bigg(\frac{\cdots}{4a(-\cdots}$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^3+1/4*d/a*x^2+1/4*c/a*x+1/4*f/b)/(-b*x^4+a)-1/16/b/a*sum((_R^2*
e+2*_R*d+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 117016, normalized size of antiderivative = 713.51

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(146) = 292$.

Time = 10.12 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.17

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^2}\,dx$$
$$= \text{RootSum}\left(65536t^4a^7b^3 + t^2(-3072a^4b^2ce - 2048a^4b^2d^2) + t(128a^3bde^2 + 1152a^2b^2c^2d) - a^2e^4 + 18abc\right.$$
$$+ \frac{-af - bcx - bdx^2 - bex^3}{-4a^2b + 4ab^2x^4}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(-3072*a**4*b**2*c*e - 2048*a**4*b**
2*d**2) + _t*(128*a**3*b*d*e**2 + 1152*a**2*b**2*c**2*d) - a**2*e**4 + 18*
a*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 - 81*b**2*c**4, Lambda(_t, _
t*log(x + (4096*_t**3*a**7*b**2*e**3 + 36864*_t**3*a**6*b**3*c**2*e - 9830
4*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5
*b**2*d**3*e - 13824*_t**2*a**4*b**3*c**3*d - 144*_t*a**4*b*c*e**4 - 192*_
t*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*
d**2*e + 1536*_t*a**3*b**2*c*d**4 - 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
 - 120*a**2*b*c*d**3*e**2 + 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*
a*b**2*c**3*d**3)/(a**3*e**6 + 9*a**2*b*c**2*e**4 - 96*a**2*b*c*d**2*e**3
+ 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576
*a*b**2*c**2*d**4 - 729*b**3*c**6)))) + (-a*f - b*c*x - b*d*x**2 - b*e*x**
3)/(-4*a**2*b + 4*a*b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.22

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^2}\,dx = -\frac{bex^3 + bdx^2 + bcx + af}{4\,(ab^2x^4 - a^2b)} + \frac{\frac{2\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} - \frac{2\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}} + \frac{2\left(3\,\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\,\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,a}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*f)/(a*b^2*x^4 - a^2*b) + 1/16*(2*d*log
(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 2*d*log(sqrt(b)*x^2 - sqrt(a))
/(sqrt(a)*sqrt(b)) + 2*(3*sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (3*sqrt(b)*c + sq
rt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)
*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(124) = 248$.

Time = 0.13 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.92

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$
$$= -\frac{\sqrt{2}\left(3\,b^2c-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}\left(3\,b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{bex^3+bdx^2+bcx+af}{4\,(bx^4-a)ab}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*b^2*c - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4
)*a) - 1/16*sqrt(2)*(3*b^2*c + 2*sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(
-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/32*sqrt(2)*(3*b^2*c - sqr
t(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4
)*a) - 1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*f)/((b*x^4 - a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.95

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(65536\,a^7\,b^3\,z^4-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\right.\right.\right.$$

$$\left. - \frac{-9\,b^2\,c^2\,e+12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3} - \frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\right)\mathrm{root}\big(65536\,a^7\,b^3\,z^4$$

$$-3072\,a^4\,b^2\,c\,e\,z^2-2048\,a^4\,b^2\,d^2\,z^2+1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$

$$\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4-81\,b^2\,c^4-a^2\,e^4,z,k\big)\right) + \frac{\frac{f}{4\,b}+\frac{d\,x^2}{4\,a}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{a-b\,x^4}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^2,x)
```

output
```
symsum(log(- root(65536*a^7*b^3*z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*
d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a
*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4 - a^2*e^4, z, k)*(root(65536*a^7*b^3*
z^4 - 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z +
 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2
*c^4 - a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3*c^2 + 4*a^2*b^
2*e^2))/(16*a^3) - (b^2*d*e)/a) - (12*b^2*c*d^2 - 9*b^2*c^2*e + a*b*e^3)/(
64*a^3) - (x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3))*root(65536*a^7*b^3*z^4 -
 3072*a^4*b^2*c*e*z^2 - 2048*a^4*b^2*d^2*z^2 + 1152*a^2*b^2*c^2*d*z + 128*
a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 - 81*b^2*c^4
- a^2*e^4, z, k), k, 1, 4) + (f/(4*b) + (d*x^2)/(4*a) + (e*x^3)/(4*a) + (c
*x)/(4*a))/(a - b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.56

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^2}\,dx$$
$$= \frac{-2b^{\frac{1}{4}}a^{\frac{7}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)e+2b^{\frac{5}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)e\,x^4+6b^{\frac{3}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c-6b^{\frac{7}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c\,x^4-b^{\frac{1}{4}}a^{\frac{7}{4}}\mathrm{l}}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*e + 2*b**(
1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b*e*x**4 + 6*b**(3/4)*
a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*c - 6*b**(3/4)*a**(1/4)*a
tan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b*c*x**4 - b**(1/4)*a**(3/4)*log(a**(
1/4) - b**(1/4)*x)*a*e + b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*b*e*
x**4 + b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*e - b**(1/4)*a**(3/4
)*log(a**(1/4) + b**(1/4)*x)*b*e*x**4 - 3*b**(3/4)*a**(1/4)*log(a**(1/4) -
 b**(1/4)*x)*a*c + 3*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*b*c*x**4
 + 3*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*c - 3*b**(3/4)*a**(1/4
)*log(a**(1/4) + b**(1/4)*x)*b*c*x**4 - 2*sqrt(b)*sqrt(a)*log(a**(1/4) - b
**(1/4)*x)*a*d + 2*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*b*d*x**4 - 2
*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a*d + 2*sqrt(b)*sqrt(a)*log(a*
*(1/4) + b**(1/4)*x)*b*d*x**4 + 2*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x*
*2)*a*d - 2*sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*b*d*x**4 + 4*a*b*c
*x + 4*a*b*d*x**2 + 4*a*b*e*x**3 + 4*a*b*f*x**4)/(16*a**2*b*(a - b*x**4))
```

3.72 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$

Optimal result 596
Mathematica [A] (verified) 597
Rubi [A] (verified) 597
Maple [C] (verified) 599
Fricas [C] (verification not implemented) 600
Sympy [B] (verification not implemented) 601
Maxima [A] (verification not implemented) 602
Giac [B] (verification not implemented) 602
Mupad [B] (verification not implemented) 604
Reduce [B] (verification not implemented) 604

Optimal result

Integrand size = 26, antiderivative size = 197

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = \frac{f}{8b(a-bx^4)^2}+\frac{x(c+dx+ex^2)}{8a(a-bx^4)^2}+\frac{x(7c+6dx+5ex^2)}{32a^2(a-bx^4)} + \frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}} + \frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{3/4}}+\frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/8*f/b/(-b*x^4+a)^2+1/8*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^2+1/32*x*(5*e*x^2+6*
d*x+7*c)/a^2/(-b*x^4+a)+1/64*(21*b^(1/2)*c-5*a^(1/2)*e)*arctan(b^(1/4)*x/a
^(1/4))/a^(11/4)/b^(3/4)+1/64*(21*b^(1/2)*c+5*a^(1/2)*e)*arctanh(b^(1/4)*x
/a^(1/4))/a^(11/4)/b^(3/4)+3/16*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(5/2)/b^(
1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx = \frac{\frac{4ax(7c+x(6d+5ex))}{a-bx^4}+\frac{16a^2(af+bx(c+x(d+ex)))}{b(a-bx^4)^2}+\frac{2\sqrt[4]{a}\left(21\sqrt{b}c-5\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{\left(21\sqrt[4]{a}\sqrt{b}c+12\sqrt{a}\sqrt[4]{b}d+5a^{3/4}e\right)\log\left(\right.}{b^{3/4}}}{128a^3}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

output
```
((4*a*x*(7*c + x*(6*d + 5*e*x)))/(a - b*x^4) + (16*a^2*(a*f + b*x*(c + x*(
d + e*x))))/(b*(a - b*x^4)^2) + (2*a^(1/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*Ar
cTan[(b^(1/4)*x)/a^(1/4)])/b^(3/4) - ((21*a^(1/4)*Sqrt[b]*c + 12*Sqrt[a]*b
^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + ((21*a^(1/4)*S
qrt[b]*c - 12*Sqrt[a]*b^(1/4)*d + 5*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b
^(3/4) + (12*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(128*a^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2393, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$

$$\Big\downarrow \text{2393}$$

$$\frac{af+bx\left(c+dx+ex^2\right)}{8ab\left(a-bx^4\right)^2}-\frac{\int -\frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5ex^2+6dx+7c}{(a-bx^4)^2}dx}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

$\Big\downarrow$ 2394

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{a-bx^4}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

$\Big\downarrow$ 2415

$$\frac{\frac{\int \left(\frac{12dx}{a-bx^4}+\frac{5ex^2+21c}{a-bx^4}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(21\sqrt{b}c-5\sqrt{a}e)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(5\sqrt{a}e+21\sqrt{b}c)}{2a^{3/4}b^{3/4}}+\frac{6d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a-bx^4)}}{8a}+\frac{af+bx(c+dx+ex^2)}{8ab\,(a-bx^4)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x]
```

output
```
(a*f + b*x*(c + d*x + e*x^2))/(8*a*b*(a - b*x^4)^2) + ((x*(7*c + 6*d*x + 5
*e*x^2))/(4*a*(a - b*x^4)) + (((21*Sqrt[b]*c - 5*Sqrt[a]*e)*ArcTan[(b^(1/4
)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTanh
[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (6*d*ArcTanh[(Sqrt[b]*x^2)/Sq
rt[a]])/(Sqrt[a]*Sqrt[b]))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.61

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}-\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}+\frac{f}{8b}}{\left(-b\,x^4+a\right)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(5\_R^2e+12d\_R+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128b\,a^2}$
default	$c\left(\frac{x}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{7x}{32a\left(-b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{128a^2}}{a}\right)+d\left(\frac{x^2}{8a\left(-b\,x^4+a\right)^2}+\frac{\frac{3x^2}{16a\left(-b\,x^4+a\right)}+\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6-7/32*b/a^2*c*x^5+9/32*e/a*x^3+5/16*d/a
*x^2+11/32*c/a*x+1/8*f/b)/(-b*x^4+a)^2-1/128/b/a^2*sum((5*_R^2*e+12*_R*d+2
1*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.35 (sec) , antiderivative size = 118761, normalized size of antiderivative = 602.85

$$\int \frac{c+dx+ex^2+fx^3}{\left(a-bx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(184) = 368$.

Time = 52.45 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.96

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx =$$

$$-\operatorname{RootSum}\Big(268435456t^4a^{11}b^3+t^2\left(-6881280a^6b^2ce-4718592a^6b^2d^2\right)+t\left(-153600a^4bde^2-2709504\right.$$

$$-\frac{-4a^2f-11abcx-10abdx^2-9abex^3+7b^2cx^5+6b^2dx^6+5b^2ex^7}{32a^4b-64a^3b^2x^4+32a^2b^3x^8}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
-RootSum(268435456*_t**4*a**11*b**3 + _t**2*(-6881280*a**6*b**2*c*e - 4718
592*a**6*b**2*d**2) + _t*(-153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d
) - 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d
**4 - 194481*b**2*c**4, Lambda(_t, _t*log(x + (-262144000*_t**3*a**10*b**2
*e**3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*
d**2 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3
*e - 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 691200
0*_t*a**5*b*d**2*e**3 + 118540800*_t*a**4*b**2*c**3*e**2 - 365783040*_t*a*
*4*b**2*c**2*d**2*e - 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b*
*3*c**5 + 112500*a**3*d*e**5 - 4536000*a**2*b*c*d**3*e**2 + 2488320*a**2*b
*d**5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**
3*e**6 + 275625*a**2*b*c**2*e**4 - 3024000*a**2*b*c*d**2*e**3 + 2073600*a*
*2*b*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e -
36578304*a*b**2*c**2*d**4 - 85766121*b**3*c**6)))) - (-4*a**2*f - 11*a*b*c
*x - 10*a*b*d*x**2 - 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2
*e*x**7)/(32*a**4*b - 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.26

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= -\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-9\,abex^3-10\,abdx^2-11\,abcx-4\,a^2f}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)}$$
$$+\frac{\frac{12\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{12\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(21\,\sqrt{b}c-5\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,\sqrt{b}c+5\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - 9*a*b*e*x^3 - 10*a*b*d*x^
2 - 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 - 2*a^3*b^2*x^4 + a^4*b) + 1/128*(1
2*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 12*d*log(sqrt(b)*x^2 -
sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(21*sqrt(b)*c - 5*sqrt(a)*e)*arctan(sqrt(b)
*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (21*sq
rt(b)*c + 5*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x
+ sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(156) = 312.

Time = 0.14 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.79

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3}\,dx$$
$$= -\frac{\sqrt{2}\left(21\,b^2c-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c-5\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$+\frac{\sqrt{2}\left(21\,b^2c-5\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-9\,abex^3-10\,abdx^2-11\,abcx-4\,a^2f}{32\,(bx^4-a)^2a^2b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/128*sqrt(2)*(21*b^2*c - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqrt(-a*b)*b*
e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)
^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c + 12*sqrt(2)*(-a*b^3)^(1/4)*b*d - 5*
sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/
4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log
(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) + 1/256*s
qrt(2)*(21*b^2*c - 5*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sq
rt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c
*x^5 - 9*a*b*e*x^3 - 10*a*b*d*x^2 - 11*a*b*c*x - 4*a^2*f)/((b*x^4 - a)^2*a
^2*b)
```

Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.22

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3} dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^3,x)
```

output
```
symsum(log(-(b*(125*a*e^3 + 3024*b*c*d^2 - 2205*b*c^2*e + 1728*b*d^3*x + 3
44064*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*
b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4,
 z, k)^2*a^5*b^2*c + 3200*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*
e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e
^4 - 194481*b^2*c^4, z, k)*a^3*b*e^2*x - 2520*b*c*d*e*x + 56448*root(26843
5456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 27
09504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a
*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^2*b^2*c
^2*x - 196608*root(268435456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718
592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 604
80*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 - 625*a^2*e^4 - 194481*
b^2*c^4, z, k)^2*a^5*b^2*d*x - 15360*root(268435456*a^11*b^3*z^4 - 6881280
*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 2709504*a^3*b^2*c^2*d*z + 153
600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4
- 625*a^2*e^4 - 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435
456*a^11*b^3*z^4 - 6881280*a^6*b^2*c*e*z^2 - 4718592*a^6*b^2*d^2*z^2 + 270
9504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 681, normalized size of antiderivative = 3.46

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^3} dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

output
```
( - 10*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*e + 20
*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*e*x**4 - 10*b
**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**2*e*x**8 + 42*b*
*(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*c - 84*b**(3/4)
*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*c*x**4 + 42*b**(3/4)*a
**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**2*c*x**8 - 5*b**(1/4)*a**
(3/4)*log(a**(1/4) - b**(1/4)*x)*a**2*e + 10*b**(1/4)*a**(3/4)*log(a**(1/4
) - b**(1/4)*x)*a*b*e*x**4 - 5*b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x
)*b**2*e*x**8 + 5*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a**2*e - 10
*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*b*e*x**4 + 5*b**(1/4)*a**(
3/4)*log(a**(1/4) + b**(1/4)*x)*b**2*e*x**8 - 21*b**(3/4)*a**(1/4)*log(a**
(1/4) - b**(1/4)*x)*a**2*c + 42*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*
x)*a*b*c*x**4 - 21*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*b**2*c*x**
8 + 21*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a**2*c - 42*b**(3/4)*a
**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*b*c*x**4 + 21*b**(3/4)*a**(1/4)*log(a
**(1/4) + b**(1/4)*x)*b**2*c*x**8 - 12*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(
1/4)*x)*a**2*d + 24*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*a*b*d*x**4
- 12*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*b**2*d*x**8 - 12*sqrt(b)*s
qrt(a)*log(a**(1/4) + b**(1/4)*x)*a**2*d + 24*sqrt(b)*sqrt(a)*log(a**(1/4)
 + b**(1/4)*x)*a*b*d*x**4 - 12*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*...
```

3.73 $\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$

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Optimal result

Integrand size = 26, antiderivative size = 229

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \frac{f}{12b(a-bx^4)^3} + \frac{x(c+dx+ex^2)}{12a(a-bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a-bx^4)^2} + \frac{x(77c+60dx+45ex^2)}{384a^3(a-bx^4)} + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{3/4}} + \frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/12*f/b/(-b*x^4+a)^3+1/12*x*(e*x^2+d*x+c)/a/(-b*x^4+a)^3+1/96*x*(9*e*x^2+
10*d*x+11*c)/a^2/(-b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(-b*x^4+a
)+1/256*(77*b^(1/2)*c-15*a^(1/2)*e)*arctan(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(
3/4)+1/256*(77*b^(1/2)*c+15*a^(1/2)*e)*arctanh(b^(1/4)*x/a^(1/4))/a^(15/4)
/b^(3/4)+5/32*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.25

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$= \frac{\frac{4ax(77c+15x(4d+3ex))}{a-bx^4}+\frac{16a^2x(11c+x(10d+9ex))}{(a-bx^4)^2}-\frac{128a^3(af+bx(c+x(d+ex)))}{b(-a+bx^4)^3}+\frac{6\sqrt[4]{a}\left(77\sqrt{b}c-15\sqrt{a}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{3\left(77\right.}{}}{153}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

output
```
((4*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a - b*x^4) + (16*a^2*x*(11*c + x*(10
*d + 9*e*x)))/(a - b*x^4)^2 - (128*a^3*(a*f + b*x*(c + x*(d + e*x))))/(b*(
-a + b*x^4)^3) + (6*a^(1/4)*(77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*
x)/a^(1/4)])/b^(3/4) - (3*(77*a^(1/4)*Sqrt[b]*c + 40*Sqrt[a]*b^(1/4)*d + 1
5*a^(3/4)*e)*Log[a^(1/4) - b^(1/4)*x])/b^(3/4) + (3*(77*a^(1/4)*Sqrt[b]*c
- 40*Sqrt[a]*b^(1/4)*d + 15*a^(3/4)*e)*Log[a^(1/4) + b^(1/4)*x])/b^(3/4) +
 (120*Sqrt[a]*d*Log[Sqrt[a] + Sqrt[b]*x^2])/Sqrt[b])/(1536*a^4)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2393, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$\Big\downarrow \text{ 2393}$$

$$\frac{af+bx(c+dx+ex^2)}{12ab(a-bx^4)^3}-\frac{\int -\frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{9ex^2+10dx+11c}{(a-bx^4)^3}dx}{12a}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{\left(a-bx^4\right)^2}dx}{8a}}{12a}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{\left(a-bx^4\right)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2394$$

$$\frac{\frac{\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int -\frac{3\left(15ex^2+40dx+77c\right)}{a-bx^4}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{a-bx^4}dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{a-bx^4}+\frac{15ex^2+77c}{a-bx^4}\right)dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(77\sqrt{b}c-15\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{20d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\right)}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a-bx^4)^2}}{12a}+\frac{af+bx\left(c+dx+ex^2\right)}{12ab\left(a-bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x]
```

output
```
(a*f + b*x*(c + d*x + e*x^2))/(12*a*b*(a - b*x^4)^3) + ((x*(11*c + 10*d*x
+ 9*e*x^2))/(8*a*(a - b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*(a
- b*x^4)) + (3*(((77*Sqrt[b]*c - 15*Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)]
)/(2*a^(3/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)
/a^(1/4)])/(2*a^(3/4)*b^(3/4)) + (20*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sq
rt[a]*Sqrt[b])))/(4*a))/(8*a))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.68

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{\left(-b\,x^4+a\right)^3}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(15\_R^2e+40d\_\right.}{}}{512b\,a^3}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}-\frac{21be\,x^7}{64a^2}-\frac{5bd\,x^6}{12a^2}-\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}+\frac{f}{12b}}{\left(-b\,x^4+a\right)^3}+\frac{\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)}\right.\right.}{4a}}{}$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9-21/64*b*e/
a^2*x^7-5/12*b*d/a^2*x^6-33/64*b/a^2*c*x^5+113/384*e/a*x^3+11/32*d/a*x^2+5
1/128*c/a*x+1/12*f/b)/(-b*x^4+a)^3-1/512/b/a^3*sum((15*_R^2*e+40*_R*d+77*c
)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.04 (sec) , antiderivative size = 118945, normalized size of antiderivative = 519.41

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(214) = 428.

Time = 177.99 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.76

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$
$$= \text{RootSum}\Big(68719476736t^4a^{15}b^3 + t^2\left(-1211105280a^8b^2ce - 838860800a^8b^2d^2\right) + t\left(18432000a^5bde^2 + 48\right.$$
$$+ \frac{-32a^3f - 153a^2bcx - 132a^2bdx^2 - 113a^2bex^3 + 198ab^2cx^5 + 160ab^2dx^6 + 126ab^2ex^7 - 77b^3cx^9 - 60}{-384a^6b + 1152a^5b^2x^4 - 1152a^4b^3x^8 + 384a^3b^4x^{12}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(-1211105280*a**8*b**2*c*e -
838860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 + 485703680*a**4*b*
*2*c**2*d) - 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*
e + 2560000*a*b*d**4 - 35153041*b**2*c**4, Lambda(_t, _t*log(x + (45298483
2000*_t**3*a**13*b**2*e**3 + 11936653639680*_t**3*a**12*b**3*c**2*e - 3307
1248179200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2
 - 503316480000*_t**2*a**9*b**2*d**3*e - 4787095470080*_t**2*a**8*b**3*c**
3*d - 5987520000*_t*a**6*b*c*e**4 - 8294400000*_t*a**6*b*d**2*e**3 - 21037
0406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 2
01850880000*_t*a**5*b**2*c*d**4 - 1385873488384*_t*a**4*b**3*c**5 + 911250
00*a**3*d*e**5 - 5544000000*a**2*b*c*d**3*e**2 + 3072000000*a**2*b*d**5*e
+ 105459123000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*
a**3*e**6 + 300155625*a**2*b*c**2*e**4 - 3326400000*a**2*b*c*d**2*e**3 + 2
304000000*a**2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b
**2*c**3*d**2*e - 60712960000*a*b**2*c**2*d**4 - 208422380089*b**3*c**6)))
) + (-32*a**3*f - 153*a**2*b*c*x - 132*a**2*b*d*x**2 - 113*a**2*b*e*x**3 +
 198*a*b**2*c*x**5 + 160*a*b**2*d*x**6 + 126*a*b**2*e*x**7 - 77*b**3*c*x**
9 - 60*b**3*d*x**10 - 45*b**3*e*x**11)/(-384*a**6*b + 1152*a**5*b**2*x**4
- 1152*a**4*b**3*x**8 + 384*a**3*b**4*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = -\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)} + \frac{\frac{40\,d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}-\frac{40\,d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}\sqrt{b}}+\frac{2\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,\sqrt{b}c+15\,\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{512\,a^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 126*a*b^2*e*x^7 - 1
60*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153
*a^2*b*c*x + 32*a^3*f)/(a^3*b^4*x^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6
*b) + 1/512*(40*d*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*sqrt(b)) - 40*d*log(
sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*sqrt(b)) + 2*(77*sqrt(b)*c - 15*sqrt(a)*e)
*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sq
rt(b)) - (77*sqrt(b)*c + 15*sqrt(a)*e)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(
b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(187) = 374$.

Time = 0.13 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.69

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx$$

$$= -\frac{\sqrt{2}\left(77\,b^2c-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}\left(77\,b^2c+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$+\frac{\sqrt{2}\left(77\,b^2c-15\,\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+}{384\,(bx^4-a)^3a^3b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
-1/512*sqrt(2)*(77*b^2*c - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sqrt(-a*b)*b
*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3
)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c + 40*sqrt(2)*(-a*b^3)^(1/4)*b*d - 1
5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(
1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)
*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) + 1/1
024*sqrt(2)*(77*b^2*c - 15*sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4
) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^3) - 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^1
0 + 77*b^3*c*x^9 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 1
13*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 32*a^3*f)/((b*x^4 - a)^
3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 880, normalized size of antiderivative = 3.84

$$\int \frac{c+dx+ex^2+fx^3}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4)^4,x)
```

output
```
symsum(log(-(b*(3375*a*e^3 + 123200*b*c*d^2 - 88935*b*c^2*e + 64000*b*d^3*
x + 20185088*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 -
838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e
^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153
041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*c + 115200*root(68719476736*a
^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485
703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 26
68050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z,
 k)*a^4*b*e^2*x - 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4
- 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b
^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e^4, z, k)*a^3*b^2*
c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^
2 - 838860800*a^8*b^2*d^2*z^2 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b
*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 3
5153041*b^2*c^4 - 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x - 614400*root(6871947
6736*a^15*b^3*z^4 - 1211105280*a^8*b^2*c*e*z^2 - 838860800*a^8*b^2*d^2*z^2
 + 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*
e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 - 35153041*b^2*c^4 - 50625*a^2*e
^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 - 12...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 944, normalized size of antiderivative = 4.12

$$\int \frac{c + dx + ex^2 + fx^3}{(a - bx^4)^4}\, dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

output

```
( - 90*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*e + 27
0*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*e*x**4 -
270*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*e*x**8
+ 90*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**3*e*x**12
+ 462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*c - 138
6*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*c*x**4 +
1386*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*c*x**8
 - 462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**3*c*x**1
2 - 45*b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a**3*e + 135*b**(1/4)*
a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a**2*b*e*x**4 - 135*b**(1/4)*a**(3/4)*
log(a**(1/4) - b**(1/4)*x)*a*b**2*e*x**8 + 45*b**(1/4)*a**(3/4)*log(a**(1/
4) - b**(1/4)*x)*b**3*e*x**12 + 45*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/
4)*x)*a**3*e - 135*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a**2*b*e*x
**4 + 135*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*b**2*e*x**8 - 45*
b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*b**3*e*x**12 - 231*b**(3/4)*a
**(1/4)*log(a**(1/4) - b**(1/4)*x)*a**3*c + 693*b**(3/4)*a**(1/4)*log(a**(
1/4) - b**(1/4)*x)*a**2*b*c*x**4 - 693*b**(3/4)*a**(1/4)*log(a**(1/4) - b*
*(1/4)*x)*a*b**2*c*x**8 + 231*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)
*b**3*c*x**12 + 231*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a**3*c -
693*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a**2*b*c*x**4 + 693*b*...
```

3.74 $\int \frac{a}{2+3x^4}\,dx$

Optimal result 617
Mathematica [A] (verified) 617
Rubi [A] (verified) 618
Maple [C] (verified) 621
Fricas [A] (verification not implemented) 622
Sympy [A] (verification not implemented) 622
Maxima [B] (verification not implemented) 623
Giac [A] (verification not implemented) 623
Mupad [B] (verification not implemented) 624
Reduce [B] (verification not implemented) 624

Optimal result

Integrand size = 11, antiderivative size = 82

$$\int \frac{a}{2+3x^4}\,dx=-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{4\sqrt[4]{6}}$$

output
```
1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/2
4*a*arctanh(2^(3/4)*3^(1/4)*x/(2^(1/2)+3^(1/2)*x^2))*6^(3/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{a}{2+3x^4}\,dx =\frac{a\left(-2\arctan\left(1-\sqrt[4]{6}x\right)+2\arctan\left(1+\sqrt[4]{6}x\right)-\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{8\sqrt[4]{6}}$$

input
```
Integrate[a/(2 + 3*x^4),x]
```

output
```
(a*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] - Log[2 - 2*6^(1/4)
*x + Sqrt[6]*x^2] + Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(1/4))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {27, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a}{3x^4+2}dx$$

$$\Big\downarrow \text{27}$$

$$a\int \frac{1}{3x^4+2}dx$$

$$\Big\downarrow \text{755}$$

$$a\left(\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}+\frac{\int \frac{\sqrt{3}x^2+\sqrt{2}}{3x^4+2}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1476}$$

$$a\left(\frac{\frac{\int \frac{1}{x^2-\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}+\frac{\int \frac{1}{x^2+\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$a\left(\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}+\frac{\frac{\int \frac{1}{-\left(1-\sqrt[4]{6}x\right)^2-1}d\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\int \frac{1}{-\left(\sqrt[4]{6}x+1\right)^2-1}d\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$a\left(\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1479}$$

$$a\left(\frac{-\frac{\int -\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\int -\frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{25}$$

$$a\left(\frac{\frac{\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{\int \frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{27}$$

$$a\left(\frac{\frac{\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt[4]{6}x+1}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1103}$$

$$a\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}+\frac{\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{2}}\right)$$

input
```
Int[a/(2 + 3*x^4),x]
```

output
```
a*((-(ArcTan[1 - 6^(1/4)*x]/(2^(3/4)*3^(1/4))) + ArcTan[1 + 6^(1/4)*x]/(2^
(3/4)*3^(1/4)))/(2*Sqrt[2]) + (-1/2*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(2^(3
/4)*3^(1/4)) + Log[Sqrt[6] + 6^(3/4)*x + 3*x^2]/(2*2^(3/4)*3^(1/4)))/(2*Sq
rt[2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.30

method	result
risch	$\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}$
meijerg	$\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}} \cdots}{8+3^{\frac{1}{4}} \cdots}\right)}{\left(x^4\right)^{\cdots}} \cdots\right.}{96}$

input
```
int(a/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*a*sum(1/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{a}{2+3x^4}\,dx = \frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(6^{\frac{1}{4}}x+1\right)+\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(6^{\frac{1}{4}}x-1\right)+\frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(3\,x^2+6^{\frac{3}{4}}x+\sqrt{6}\right)-\frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(3\,x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)$$

input
```
integrate(a/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/24*6^(3/4)*a*arctan(6^(1/4)*x + 1) + 1/24*6^(3/4)*a*arctan(6^(1/4)*x - 1
) + 1/48*6^(3/4)*a*log(3*x^2 + 6^(3/4)*x + sqrt(6)) - 1/48*6^(3/4)*a*log(3
*x^2 - 6^(3/4)*x + sqrt(6))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{a}{2+3x^4}\,dx = a\left(-\frac{6^{\frac{3}{4}}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{48}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x-1\right)}{24}+\frac{6^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{6}x+1\right)}{24}\right)$$

input
```
integrate(a/(3*x**4+2),x)
```

output
```
a*(-6**(3/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*log(x**2 +
 6**(3/4)*x/3 + sqrt(6)/3)/48 + 6**(3/4)*atan(6**(1/4)*x - 1)/24 + 6**(3/4
)*atan(6**(1/4)*x + 1)/24)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.50

$$\int \frac{a}{2+3x^4}\,dx$$
$$= \frac{1}{48}\left(2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+3^{\frac{3}{4}}2^{\frac{3}{4}}\right.$$

input
```
integrate(a/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/48*(2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*
2^(3/4))) + 2*3^(3/4)*2^(3/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^
(1/4)*2^(3/4))) + 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sq
rt(2)) - 3^(3/4)*2^(3/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{a}{2+3x^4}\,dx$$
$$= \frac{1}{48}\left(2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{3}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)\right.$$

input
```
integrate(a/(3*x^4+2),x, algorithm="giac")
```

output
```
1/48*(2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))
) + 2*6^(3/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))
+ 6^(3/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 6^(3/4)*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*a
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.44

$$\int \frac{a}{2+3x^4}\,dx = -\frac{(-1)^{1/4}\,6144^{3/4}\,a\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}+\mathrm{atanh}\left(\frac{(-1)^{1/4}\,6144^{1/4}\,x}{8}\right)\,1\mathrm{i}\right)}{3072}$$

input
```
int(a/(3*x^4 + 2),x)
```

output
```
-((-1)^(1/4)*6144^(3/4)*a*(atan(((-1)^(1/4)*6144^(1/4)*x)/8)*1i + atanh(((
-1)^(1/4)*6144^(1/4)*x)/8)*1i))/3072
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{a}{2+3x^4}\,dx = \frac{\sqrt{6}\,6^{\frac{1}{4}}a\left(-2atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)+2atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)-\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)+\log\left(\right.}{48}$$

input
```
int(a/(3*x^4+2),x)
```

output
```
(sqrt(6)*6**(1/4)*a*( - 2*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6
**(1/4))) + 2*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4))) -
log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2)) + log(sqrt(2)*6**(1/4)
*x + sqrt(3)*x**2 + sqrt(2))))/48
```

3.75 $\int \frac{bx}{2+3x^4} dx$

Optimal result 625
Mathematica [A] (verified) 625
Rubi [A] (verified) 626
Maple [A] (verified) 627
Fricas [A] (verification not implemented) 627
Sympy [A] (verification not implemented) 628
Maxima [A] (verification not implemented) 628
Giac [A] (verification not implemented) 628
Mupad [B] (verification not implemented) 629
Reduce [B] (verification not implemented) 629

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{bx}{2+3x^4} dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{bx}{2+3x^4} dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Integrate[(b*x)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {27, 807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx}{3x^4+2}dx$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{x}{3x^4+2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}b\int \frac{1}{3x^4+2}dx^2$$

$$\Big\downarrow \text{216}$$

$$\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Int[(b*x)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{b \arctan\left(\frac{\sqrt{6}\, x^{2}}{2}\right) \sqrt{6}}{12}$	16
risch	$\frac{b \arctan\left(\frac{\sqrt{6}\, x^{2}}{2}\right) \sqrt{6}}{12}$	16
meijerg	$\frac{\sqrt{6}\, b \arctan\left(\frac{x^{2} \sqrt{3}\, \sqrt{2}}{2}\right)}{12}$	19

input
```
int(b*x/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)$$

input
```
integrate(b*x/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{bx}{2+3x^4}\,dx = \frac{\sqrt{6}b\,\mathrm{atan}\left(\frac{\sqrt{6}x^2}{2}\right)}{12}$$

input
```
integrate(b*x/(3*x**4+2),x)
```

output
```
sqrt(6)*b*atan(sqrt(6)*x**2/2)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)$$

input
```
integrate(b*x/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)$$

input
```
integrate(b*x/(3*x^4+2),x, algorithm="giac")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2)
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{bx}{2+3x^4}\,dx = \frac{\sqrt{6}\,b\,\mathrm{atan}\left(\frac{\sqrt{6}\,x^2}{2}\right)}{12}$$

input
```
int((b*x)/(3*x^4 + 2),x)
```

output
```
(6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{bx}{2+3x^4}\,dx = -\frac{\sqrt{6}\,b\left(atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)+atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)\right)}{12}$$

input
```
int(b*x/(3*x^4+2),x)
```

output
```
( - sqrt(6)*b*(atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4))) +
 atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))))/12
```

3.76 $\int \frac{a+bx}{2+3x^4} \, dx$

Optimal result . . . 630
Mathematica [A] (verified) . . . 630
Rubi [A] (verified) . . . 631
Maple [C] (verified) . . . 632
Fricas [C] (verification not implemented) . . . 633
Sympy [A] (verification not implemented) . . . 633
Maxima [B] (verification not implemented) . . . 634
Giac [A] (verification not implemented) . . . 634
Mupad [B] (verification not implemented) . . . 635
Reduce [B] (verification not implemented) . . . 636

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{a+bx}{2+3x^4} \, dx = \frac{b \arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{a \arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a \operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{4\sqrt[4]{6}}$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)
+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/24*a*arctanh(2^(3/4)*3^(1/4)*x/(2^(1
/2)+3^(1/2)*x^2))*6^(3/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{a+bx}{2+3x^4} \, dx = \frac{-2\left(\sqrt[4]{6}a+2b\right) \arctan\left(1-\sqrt[4]{6}x\right) + 2\left(\sqrt[4]{6}a-2b\right) \arctan\left(1+\sqrt[4]{6}x\right) + \sqrt[4]{6}a\left(-\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x\right.\right.}{8\sqrt{6}}$$

input
```
Integrate[(a + b*x)/(2 + 3*x^4),x]
```

output
```
(-2*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*(6^(1/4)*a - 2*b)*ArcTan[1
 + 6^(1/4)*x] + 6^(1/4)*a*(-Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + Log[2 + 2
*6^(1/4)*x + Sqrt[6]*x^2]))/(8*Sqrt[6])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{3x^4+2}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{a}{3x^4+2}+\frac{bx}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Int[(a + b*x)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/
4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)\sqrt{6}}{12}$
meijerg	$\frac{\sqrt{6}\,b\arctan\left(\frac{x^2\sqrt{3}\sqrt{2}}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\sqrt{3}\cdots\right.}{2\left(x^4\right)^{\frac{1}{4}}}\right.}{96}$

input
```
int((b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 12312, normalized size of antiderivative = 118.38

$$\int \frac{a+bx}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{a+bx}{2+3x^4}\,dx$$
$$= \text{RootSum}\left(18432t^4+384t^2b^2-96ta^2b+3a^4+2b^4, \left(t \mapsto t\log\left(x+\frac{3072t^3b^2+192t^2a^2b+24ta^4+32t}{3a^5-8ab^4}\right.\right.\right.$$

input
```
integrate((b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(18432*_t**4 + 384*_t**2*b**2 - 96*_t*a**2*b + 3*a**4 + 2*b**4, Lam
bda(_t, _t*log(x + (3072*_t**3*b**2 + 192*_t**2*a**2*b + 24*_t*a**4 + 32*_
t*b**4 - 10*a**2*b**3)/(3*a**5 - 8*a*b**4))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(72) = 144$.

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.41

$$\int \frac{a+bx}{2+3x^4}\,dx = \frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) - \frac{1}{48}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) - 1/
48*3^(3/4)*2^(3/4)*a*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24
*sqrt(3)*(3^(1/4)*2^(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*s
qrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)
*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{a+bx}{2+3x^4}\,dx = \frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\cdot 6^{\frac{3}{4}}a\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/48*6^(3/4)*a*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*6^(3/4)
*a*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(3/4)*a - 2*sqrt
(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*
(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2
/3)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{a+bx}{2+3x^4}\,dx = \frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2+\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48} - \frac{2^{3/4}\,3^{3/4}\,a\,\ln\left(x^2-\frac{6^{3/4}\,x}{3}+\frac{\sqrt{6}}{3}\right)}{48} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{24} + \frac{2^{3/4}\,3^{3/4}\,a\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{24} + \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x-1\right)}{12} - \frac{\sqrt{2}\,\sqrt{3}\,b\,\mathrm{atan}\left(6^{1/4}\,x+1\right)}{12}$$

input
```
int((a + b*x)/(3*x^4 + 2),x)
```

output
```
(2^(3/4)*3^(3/4)*a*log((6^(3/4)*x)/3 + 6^(1/2)/3 + x^2))/48 - (2^(3/4)*3^(
3/4)*a*log(6^(1/2)/3 - (6^(3/4)*x)/3 + x^2))/48 + (2^(3/4)*3^(3/4)*a*atan(
6^(1/4)*x - 1))/24 + (2^(3/4)*3^(3/4)*a*atan(6^(1/4)*x + 1))/24 + (2^(1/2)
*3^(1/2)*b*atan(6^(1/4)*x - 1))/12 - (2^(1/2)*3^(1/2)*b*atan(6^(1/4)*x + 1
))/12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.51

$$\int \frac{a+bx}{2+3x^4}\,dx$$
$$= \frac{\sqrt{6}\left(-2\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a-4atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b+2\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a-4ata}{48}$$

input
```
int((b*x+a)/(3*x^4+2),x)
```

output
```
(sqrt(6)*( - 2*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**
(1/4)))*a - 4*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b
+ 2*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*a -
 4*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b - 6**(1/4)*
log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*a + 6**(1/4)*log(sqrt(
2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*a))/48
```

3.77 $\int \frac{cx^2}{2+3x^4}\,dx$

Optimal result 637
Mathematica [A] (verified) 637
Rubi [A] (verified) 638
Maple [C] (verified) 641
Fricas [A] (verification not implemented) 642
Sympy [A] (verification not implemented) 642
Maxima [B] (verification not implemented) 643
Giac [A] (verification not implemented) 643
Mupad [B] (verification not implemented) 644
Reduce [B] (verification not implemented) 644

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{cx^2}{2+3x^4}\,dx=-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}-\frac{c\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{2\ 6^{3/4}}$$

output
```
1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)-1/1
2*c*arctanh(2^(3/4)*3^(1/4)*x/(2^(1/2)+3^(1/2)*x^2))*6^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{cx^2}{2+3x^4}\,dx =\frac{c\left(-2\arctan\left(1-\sqrt[4]{6}x\right)+2\arctan\left(1+\sqrt[4]{6}x\right)+\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)\right)}{4\ 6^{3/4}}$$

input
```
Integrate[(c*x^2)/(2 + 3*x^4),x]
```

output
```
(c*(-2*ArcTan[1 - 6^(1/4)*x] + 2*ArcTan[1 + 6^(1/4)*x] + Log[2 - 2*6^(1/4)
*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(4*6^(3/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx^2}{3x^4+2}dx$$

$$\Big\downarrow \text{27}$$

$$c\int \frac{x^2}{3x^4+2}dx$$

$$\Big\downarrow \text{826}$$

$$c\left(\frac{\int \frac{\sqrt{3}x^2+\sqrt{2}}{3x^4+2}dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1476}$$

$$c\left(\frac{\frac{\int \frac{1}{x^2-\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}+\frac{\int \frac{1}{x^2+\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx}{2\sqrt{3}}}{2\sqrt{3}}-\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$c\left(\frac{\frac{\int \frac{1}{-\left(1-\sqrt[4]{6}x\right)^2-1}d\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\int \frac{1}{-\left(\sqrt[4]{6}x+1\right)^2-1}d\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\int \frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{217}$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\int\frac{\sqrt{2}-\sqrt{3}x^2}{3x^4+2}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1479}$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{-\frac{\int-\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\int-\frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{25}$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\frac{\int\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{\int\frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{27}$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\frac{\int\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}\sqrt[4]{3}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt[4]{6}x+1}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\sqrt{3}}\right)$$

$$\Big\downarrow \text{1103}$$

$$c\left(\frac{\frac{\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}\sqrt[4]{3}}-\frac{\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}-\frac{\frac{\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}-\frac{\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}\sqrt[4]{3}}}{2\sqrt{3}}\right)$$

input
```
Int[(c*x^2)/(2 + 3*x^4),x]
```

output
```
c*((-(ArcTan[1 - 6^(1/4)*x]/(2^(3/4)*3^(1/4))) + ArcTan[1 + 6^(1/4)*x]/(2^
(3/4)*3^(1/4)))/(2*Sqrt[3]) - (-1/2*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2]/(2^(3
/4)*3^(1/4)) + Log[Sqrt[6] + 6^(3/4)*x + 3*x^2]/(2*2^(3/4)*3^(1/4)))/(2*Sq
rt[3]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.30

method	result
risch	$\frac{c\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{4}+2\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{12}$
default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\ldots\right)}{\left(x^{4}\ldots\right.}\ldots\right)}{216}$

input
```
int(c*x^2/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*c*sum(1/_R*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{cx^2}{2+3x^4}\,dx = \frac{1}{432}\cdot 216^{\frac{3}{4}}c\arctan\left(\frac{1}{36}\cdot 216^{\frac{3}{4}}x+1\right)+\frac{1}{432}\cdot 216^{\frac{3}{4}}c\arctan\left(\frac{1}{36}\cdot 216^{\frac{3}{4}}x-1\right)-\frac{1}{864}\cdot 216^{\frac{3}{4}}c\log\left(18\,x^2+6\cdot 216^{\frac{1}{4}}x+6\sqrt{6}\right)+\frac{1}{864}\cdot 216^{\frac{3}{4}}c\log\left(18\,x^2-6\cdot 216^{\frac{1}{4}}x+6\sqrt{6}\right)$$

input
```
integrate(c*x^2/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/432*216^(3/4)*c*arctan(1/36*216^(3/4)*x + 1) + 1/432*216^(3/4)*c*arctan(
1/36*216^(3/4)*x - 1) - 1/864*216^(3/4)*c*log(18*x^2 + 6*216^(1/4)*x + 6*s
qrt(6)) + 1/864*216^(3/4)*c*log(18*x^2 - 6*216^(1/4)*x + 6*sqrt(6))
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{cx^2}{2+3x^4}\,dx = c\left(\frac{\sqrt[4]{6}\log\left(x^2-\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}-\frac{\sqrt[4]{6}\log\left(x^2+\frac{6^{\frac{3}{4}}x}{3}+\frac{\sqrt{6}}{3}\right)}{24}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}x-1\right)}{12}+\frac{\sqrt[4]{6}\,\mathrm{atan}\left(\sqrt[4]{6}x+1\right)}{12}\right)$$

input
```
integrate(c*x**2/(3*x**4+2),x)
```

output
```
c*(6**(1/4)*log(x**2 - 6**(3/4)*x/3 + sqrt(6)/3)/24 - 6**(1/4)*log(x**2 +
6**(3/4)*x/3 + sqrt(6)/3)/24 + 6**(1/4)*atan(6**(1/4)*x - 1)/12 + 6**(1/4)
*atan(6**(1/4)*x + 1)/12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.50

$$\int \frac{cx^2}{2+3x^4}\,dx$$
$$= \frac{1}{24}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)-3^{\frac{1}{4}}2^{\frac{1}{4}}$$

input
```
integrate(c*x^2/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*(2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*
2^(3/4))) + 2*3^(1/4)*2^(1/4)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^
(1/4)*2^(3/4))) - 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sq
rt(2)) + 3^(1/4)*2^(1/4)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)))*c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{cx^2}{2+3x^4}\,dx$$
$$= \frac{1}{24}\left(2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate(c*x^2/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))
) + 2*6^(1/4)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4)))
- 6^(1/4)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 6^(1/4)*log(x^2 -
 sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)))*c
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

$$\int \frac{cx^2}{2+3x^4}\,dx = \frac{(-1)^{1/4}\,24^{1/4}\,c\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\,24^{1/4}\,x}{2}\right)\right)}{12}$$

input
```
int((c*x^2)/(3*x^4 + 2),x)
```

output
```
((-1)^(1/4)*24^(1/4)*c*(atan(((-1)^(1/4)*24^(1/4)*x)/2) - atanh(((-1)^(1/4
)*24^(1/4)*x)/2)))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{cx^2}{2+3x^4}\,dx$$
$$= \frac{6^{\frac{1}{4}}c\left(-2atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)+2atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)+\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)-\log\left(\sqrt{2}\,6\right.}{24}$$

input
```
int(c*x^2/(3*x^4+2),x)
```

output
```
(6**(1/4)*c*( - 2*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4))
) + 2*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4))) + log( - s
qrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2)) - log(sqrt(2)*6**(1/4)*x + sqr
t(3)*x**2 + sqrt(2))))/24
```

3.78 $\int \frac{a+cx^2}{2+3x^4}\,dx$

Optimal result 645
Mathematica [A] (verified) 645
Rubi [A] (verified) 646
Maple [C] (verified) 649
Fricas [B] (verification not implemented) 650
Sympy [A] (verification not implemented) 651
Maxima [B] (verification not implemented) 651
Giac [A] (verification not implemented) 652
Mupad [B] (verification not implemented) 653
Reduce [B] (verification not implemented) 654

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int \frac{a+cx^2}{2+3x^4}\,dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\operatorname{arctanh}\left(\frac{2\sqrt[4]{6}x}{2+\sqrt{6}x^2}\right)}{4\ 6^{3/4}}$$

output
```
1/24*(6^(1/2)*a+2*c)*arctan(-1+6^(1/4)*x)*6^(1/4)+1/24*(6^(1/2)*a+2*c)*arc
tan(1+6^(1/4)*x)*6^(1/4)+1/24*(6^(1/2)*a-2*c)*arctanh(2*6^(1/4)*x/(2+6^(1/
2)*x^2))*6^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{-2\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)-\left(\sqrt{6}a-2c\right)\left(\log\left(2-2\sqrt[4]{6}x+\right.\right.}{8\ 6^{3/4}}$$

input
```
Integrate[(a + c*x^2)/(2 + 3*x^4),x]
```

output
```
(-2*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a + 2*c)*ArcTan[1
 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - Lo
g[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1482, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2}{3x^4+2}dx$$

$$\Big\downarrow \text{1482}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\int \frac{3x^2+\sqrt{6}}{3x^4+2}dx$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{1}{2}\int \frac{1}{x^2-\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx+\frac{1}{2}\int \frac{1}{x^2+\frac{2^{3/4}x}{\sqrt[4]{3}}+\sqrt{\frac{2}{3}}}dx\right)+\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx+\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\int \frac{1}{-\left(1-\sqrt[4]{6}x\right)^2-1}d\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\int \frac{1}{-\left(\sqrt[4]{6}x+1\right)^2-1}d\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\int \frac{\sqrt{6}-3x^2}{3x^4+2}dx+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$\Big\downarrow$ 1479

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(-\frac{\sqrt[4]{3}\int -\frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}-\frac{\sqrt[4]{3}\int -\frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(\frac{\sqrt[4]{3}\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}+\frac{\sqrt[4]{3}\int \frac{6^{3/4}\left(\sqrt[4]{6}x+1\right)}{3x^2+6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(\frac{\sqrt[4]{3}\int \frac{6^{3/4}-6x}{3x^2-6^{3/4}x+\sqrt{6}}dx}{2\ 2^{3/4}}+\frac{3}{2}\int \frac{\sqrt[4]{6}x+1}{3x^2+6^{3/4}x+\sqrt{6}}dx\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)$$

$\Big\downarrow$ 1103

$$\frac{1}{12}\left(\sqrt{6}a+2c\right)\left(\frac{\sqrt[4]{3}\arctan\left(\sqrt[4]{6}x+1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\arctan\left(1-\sqrt[4]{6}x\right)}{2^{3/4}}\right)+$$
$$\frac{1}{12}\left(\sqrt{6}a-2c\right)\left(\frac{\sqrt[4]{3}\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}}-\frac{\sqrt[4]{3}\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{2\ 2^{3/4}}\right)$$

input
```
Int[(a + c*x^2)/(2 + 3*x^4),x]
```

output
```
((Sqrt[6]*a + 2*c)*(-((3^(1/4)*ArcTan[1 - 6^(1/4)*x])/2^(3/4)) + (3^(1/4)*
ArcTan[1 + 6^(1/4)*x])/2^(3/4)))/12 + ((Sqrt[6]*a - 2*c)*(-1/2*(3^(1/4)*Lo
g[Sqrt[6] - 6^(3/4)*x + 3*x^2])/2^(3/4) + (3^(1/4)*Log[Sqrt[6] + 6^(3/4)*x
 + 3*x^2])/(2*2^(3/4))))/12
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 1482
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Simp[(d*q + a*e)/(2*a*c)   Int[(q + c*x^2)/(a + c*x^4), x], x] +
Simp[(d*q - a*e)/(2*a*c)   Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a
, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-
a)*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^2 c+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x}{3}}\right.\right.}{}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\right.}{\left(x^4\right.}\right.}{216}$

input
```
int((c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^2*c+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.47

$$\int \frac{a+cx^2}{2+3x^4}\,dx = -\frac{1}{4}\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.+6\left(3\,a^3-2\,ac^2-2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$
$$+\frac{1}{4}\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.-6\left(3\,a^3-2\,ac^2-2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$
$$-\frac{1}{4}\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.+6\left(3\,a^3-2\,ac^2+2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$
$$+\frac{1}{4}\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.-6\left(3\,a^3-2\,ac^2+2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$

input
```
integrate((c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-1/3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))*log(-(9*a^4
 - 4*c^4)*x + 6*(3*a^3 - 2*a*c^2 - 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)*
c)*sqrt(-1/3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))) + 1/4*sqrt(-
1/3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))*log(-(9*a^4 - 4*c^4)*x
 - 6*(3*a^3 - 2*a*c^2 - 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)*c)*sqrt(-1/
3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))) - 1/4*sqrt(-1/3*a*c - 1
/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))*log(-(9*a^4 - 4*c^4)*x + 6*(3*a^3
 - 2*a*c^2 + 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)*c)*sqrt(-1/3*a*c - 1/6
*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))) + 1/4*sqrt(-1/3*a*c - 1/6*sqrt(-3/
2*a^4 + 2*a^2*c^2 - 2/3*c^4))*log(-(9*a^4 - 4*c^4)*x - 6*(3*a^3 - 2*a*c^2
+ 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)*c)*sqrt(-1/3*a*c - 1/6*sqrt(-3/2*
a^4 + 2*a^2*c^2 - 2/3*c^4)))
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{a+cx^2}{2+3x^4}\,dx$$
$$=\text{RootSum}\left(55296t^4+2304t^2ac+9a^4+12a^2c^2+4c^4,\left(t\mapsto t\log\left(x+\frac{-4608t^3c+72ta^3-144tac^2}{9a^4-4c^4}\right)\right)\right)$$

input
```
integrate((c*x**2+a)/(3*x**4+2),x)
```

output
```
RootSum(55296*_t**4 + 2304*_t**2*a*c + 9*a**4 + 12*a**2*c**2 + 4*c**4, Lam
bda(_t, _t*log(x + (-4608*_t**3*c + 72*_t*a**3 - 144*_t*a*c**2)/(9*a**4 -
4*c**4))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(77) = 154.

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.61

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a+\sqrt{2}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) - \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

input
```
integrate((c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2
*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*3^(1/4)*2^(3/4)*(sqrt(3)*a + sqrt(2)
*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/48*3^(
1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x +
 sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 -
 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(
2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)
^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*log(x
^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4)*c)*l
og(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.03

$$\begin{aligned}\int \frac{a+cx^2}{2+3x^4}\,dx &= -2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}\right.\\&\quad\left.-\frac{144\,c^2\,x\,\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3+18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2-12\,c^3}\right)\sqrt{-\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}\\&\quad+2\,\mathrm{atanh}\left(\frac{216\,a^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}\right.\\&\quad\left.-\frac{144\,c^2\,x\,\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}}{9\mathrm{i}\,\sqrt{6}\,a^3-18\,a^2\,c-6\mathrm{i}\,\sqrt{6}\,a\,c^2+12\,c^3}\right)\sqrt{\frac{1\mathrm{i}\,\sqrt{6}\,a^2}{192}-\frac{a\,c}{48}-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{288}}\end{aligned}$$

input
```
int((a + c*x^2)/(3*x^4 + 2),x)
```

output
```
2*atanh((216*a^2*x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288
)^(1/2))/(6^(1/2)*a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^
2*x*((6^(1/2)*a^2*1i)/192 - (a*c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2))/(6^(1/
2)*a^3*9i - 18*a^2*c + 12*c^3 - 6^(1/2)*a*c^2*6i))*((6^(1/2)*a^2*1i)/192 -
 (a*c)/48 - (6^(1/2)*c^2*1i)/288)^(1/2) - 2*atanh((216*a^2*x*((6^(1/2)*c^2
*1i)/288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^
2*c - 12*c^3 - 6^(1/2)*a*c^2*6i) - (144*c^2*x*((6^(1/2)*c^2*1i)/288 - (6^(
1/2)*a^2*1i)/192 - (a*c)/48)^(1/2))/(6^(1/2)*a^3*9i + 18*a^2*c - 12*c^3 -
6^(1/2)*a*c^2*6i))*((6^(1/2)*c^2*1i)/288 - (6^(1/2)*a^2*1i)/192 - (a*c)/48
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.88

$$\int \frac{a+cx^2}{2+3x^4}\,dx$$

$$= \frac{6^{\frac{1}{4}}\left(-2\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a-4atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c+2\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a+4at}{}$$

input
```
int((c*x^2+a)/(3*x^4+2),x)
```

output
```
(6**(1/4)*( - 2*sqrt(6)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**
(1/4)))*a - 4*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c
+ 2*sqrt(6)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*a +
4*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c - sqrt(6)*lo
g( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*a + sqrt(6)*log(sqrt(2)*
6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*a + 2*log( - sqrt(2)*6**(1/4)*x + sqr
t(3)*x**2 + sqrt(2))*c - 2*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2)
)*c))/48
```

3.79 $\int \frac{bx+cx^2}{2+3x^4}\,dx$

Optimal result 655
Mathematica [A] (verified) 655
Rubi [A] (verified) 656
Maple [C] (warning: unable to verify) 657
Fricas [C] (verification not implemented) 658
Sympy [A] (verification not implemented) 658
Maxima [B] (verification not implemented) 659
Giac [A] (verification not implemented) 659
Mupad [B] (verification not implemented) 660
Reduce [B] (verification not implemented) 661

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}} - \frac{c\operatorname{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{2\ 6^{3/4}}$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)
+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)-1/12*c*arctanh(2^(3/4)*3^(1/4)*x/(2^(1
/2)+3^(1/2)*x^2))*6^(1/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \frac{-2\left(\sqrt[4]{6}b+c\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(-\sqrt[4]{6}b+c\right)\arctan\left(1+\sqrt[4]{6}x\right)+c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-c\,\mathrm{lo}}{4\ 6^{3/4}}$$

input
```
Integrate[(b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(-2*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*(-(6^(1/4)*b) + c)*ArcTan[1
+ 6^(1/4)*x] + c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - c*Log[2 + 2*6^(1/4)*
x + Sqrt[6]*x^2])/(4*6^(3/4))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2027, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{3x^4+2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(b+cx)}{3x^4+2}dx$$

$$\Big\downarrow \text{2370}$$

$$\int \left(\frac{bx}{3x^4+2}+\frac{cx^2}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}$$

input
```
Int[(b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/
4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^2c+\_Rb\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{b\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\ldots\right)}{\left(x^4\right.\ldots}\right.}{216}$

input
```
int((c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^2*c+_R*b)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 12711, normalized size of antiderivative = 122.22

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

$$\int \frac{bx+cx^2}{2+3x^4}\,dx$$
$$= \text{RootSum}\left(27648t^4+576t^2b^2+96tbc^2+3b^4+2c^4, \left(t \mapsto t\log\left(x+\frac{-1152t^3c^2+288t^2b^3-36tb^2c^2+3}{6b^4c-c^5}\right.\right.\right.$$

input
```
integrate((c*x**2+b*x)/(3*x**4+2),x)
```

output
```
RootSum(27648*_t**4 + 576*_t**2*b**2 + 96*_t*b*c**2 + 3*b**4 + 2*c**4, Lam
bda(_t, _t*log(x + (-1152*_t**3*c**2 + 288*_t**2*b**3 - 36*_t*b**2*c**2 +
3*b**5 - 3*b*c**4)/(6*b**4*c - c**5))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(72) = 144$.

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.41

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c-2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\sqrt{2}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}c+2\sqrt{3}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) - \frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

input
```
integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c - 2*sqrt(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*
(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/24*sqrt(2)*(3^(1/4)*2^(3/4)*c + 2*sqr
t(3)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) - 1/24
*3^(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*3
^(1/4)*2^(1/4)*c*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = -\frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{24}\cdot 6^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{12}\left(\sqrt{6}b-6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6}b+6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$

input
```
integrate((c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

output
```
-1/24*6^(1/4)*c*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*6^(1/4
)*c*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/12*(sqrt(6)*b - 6^(1/
4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/12*(
sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)
^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.56

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = \sum_{k=1}^{4} \ln\left(9\,b^3\,x-6\,c^3 - \mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,b\,c\,144 + \mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)^2\,b\,x\,864 + \mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)\,c^2\,x\,72\right)\,\mathrm{root}\left(z^4+\frac{b^2\,z^2}{48}+\frac{b\,c^2\,z}{288}+\frac{c^4}{13824}+\frac{b^4}{9216},z,k\right)$$

input
```
int((b*x + c*x^2)/(3*x^4 + 2),x)
```

output
```
symsum(log(9*b^3*x - 6*c^3 - 144*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)/288 +
 c^4/13824 + b^4/9216, z, k)*b*c + 864*root(z^4 + (b^2*z^2)/48 + (b*c^2*z)
/288 + c^4/13824 + b^4/9216, z, k)^2*b*x + 72*root(z^4 + (b^2*z^2)/48 + (b
*c^2*z)/288 + c^4/13824 + b^4/9216, z, k)*c^2*x)*root(z^4 + (b^2*z^2)/48 +
 (b*c^2*z)/288 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.52

$$\int \frac{bx+cx^2}{2+3x^4}\,dx = -\frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} + \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} + \frac{6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} - \frac{6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24}$$

input
```
int((c*x^2+b*x)/(3*x^4+2),x)
```

output
```
( - 2*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c
 - 2*sqrt(6)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b +
 2*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c -
2*sqrt(6)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b + 6*
*(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c - 6**(1/4)*lo
g(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c)/24
```

3.80 $\int \frac{a+bx+cx^2}{2+3x^4} dx$

Optimal result 662
Mathematica [A] (verified) 662
Rubi [A] (verified) 663
Maple [C] (verified) 664
Fricas [C] (verification not implemented) 665
Sympy [B] (verification not implemented) 665
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Giac [A] (verification not implemented) 667
Mupad [B] (verification not implemented) 668
Reduce [B] (verification not implemented) 669

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{a+bx+cx^2}{2+3x^4} dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\operatorname{arctanh}\left(\frac{2\sqrt[4]{6}x}{2+\sqrt{6}x^2}\right)}{4\ 6^{3/4}}$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/24*(6^(1/2)*a+2*c)*arctan(-1+6^(1
/4)*x)*6^(1/4)+1/24*(6^(1/2)*a+2*c)*arctan(1+6^(1/4)*x)*6^(1/4)+1/24*(6^(1
/2)*a-2*c)*arctanh(2*6^(1/4)*x/(2+6^(1/2)*x^2))*6^(1/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{2+3x^4} dx = \frac{-2\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)-\left(\sqrt{6}a-2c\right)}{8\ 6^{3/4}}$$

input
```
Integrate[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(-2*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*(Sqrt[6]*a -
 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - (Sqrt[6]*a - 2*c)*(Log[2 - 2*6
^(1/4)*x + Sqrt[6]*x^2] - Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2]))/(8*6^(3/4))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{3x^4+2}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{a+cx^2}{3x^4+2}+\frac{bx}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{\left(\sqrt{6}a+2c\right)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}-\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{\left(\sqrt{6}a-2c\right)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}$$

input
```
Int[(a + b*x + c*x^2)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1
/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)
) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((S
qrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^2c+\_Rb+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{\sqrt{6}x^2}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\cdots}{\cdots}$
meijerg	$\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\cdots\right)}{\left(x^4\cdots\right.}\cdots\right.}{216}\cdots$

input
```
int((c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^2*c+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 46609, normalized size of antiderivative = 369.91

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(110) = 220.

Time = 2.57 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.32

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx$$
$$= \text{RootSum}\Big(55296t^4 + t^2\cdot\left(2304ac+1152b^2\right) + t\left(-288a^2b+192bc^2\right) + 9a^4 + 12a^2c^2 - 24ab^2c + 6b^4 + 4$$

input
```
integrate((c*x**2+b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(55296*_t**4 + _t**2*(2304*a*c + 1152*b**2) + _t*(-288*a**2*b + 192
*b*c**2) + 9*a**4 + 12*a**2*c**2 - 24*a*b**2*c + 6*b**4 + 4*c**4, Lambda(_
t, _t*log(x + (-13824*_t**3*a**2*c + 27648*_t**3*a*b**2 + 9216*_t**3*c**3
+ 1728*_t**2*a**3*b + 3456*_t**2*a*b*c**2 - 2304*_t**2*b**3*c + 216*_t*a**
5 - 576*_t*a**3*c**2 + 1296*_t*a**2*b**2*c + 288*_t*a*b**4 + 288*_t*a*c**4
 + 288*_t*b**2*c**3 + 90*a**4*b*c - 90*a**3*b**3 + 60*a*b**3*c**2 - 24*b**
5*c + 24*b*c**5)/(27*a**6 - 18*a**4*c**2 + 144*a**3*b**2*c - 72*a**2*b**4
- 12*a**2*c**4 + 96*a*b**2*c**3 - 48*b**4*c**2 + 8*c**6)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(92) = 184$.

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.48

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) - \frac{1}{48}\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}\left(\sqrt{3}a-\sqrt{2}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a-2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}a+2\sqrt{3}\sqrt{2}b+2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(3)*x^2 + 3^(1/4)*2^(
3/4)*x + sqrt(2)) - 1/48*3^(1/4)*2^(3/4)*(sqrt(3)*a - sqrt(2)*c)*log(sqrt(
3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*(3^(3/4)*2^(3/4)*a - 2*sqrt(3
)*sqrt(2)*b + 2*3^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
 + 3^(1/4)*2^(3/4))) + 1/24*(3^(3/4)*2^(3/4)*a + 2*sqrt(3)*sqrt(2)*b + 2*3
^(1/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4
)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx$$
$$= \frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+ \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+ \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$
$$- \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^
(3/4)*a - 2*6^(1/4)*c)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48
*(6^(3/4)*a - 2*6^(1/4)*c)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.14

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = \sum_{k=1}^{4} \ln\left(9\,a\,b^2-9\,a^2\,c-\mathrm{root}\left(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296} - \frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216} + \frac{a^4}{6144},z,k\right)\,\left(\mathrm{root}\left(z^4+\frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}-\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216}\right. + 144\,b\,c+x\,\left(108\,a^2-72\,c^2\right)\right)-6\,c^3+x\,\left(9\,b^3-18\,a\,b\,c\right)\Bigg)\,\mathrm{root}\left(z^4 + \frac{z^2\,(2304\,a\,c+1152\,b^2)}{55296}-\frac{z\,(288\,a^2\,b-192\,b\,c^2)}{55296}-\frac{a\,b^2\,c}{2304}+\frac{a^2\,c^2}{4608}+\frac{c^4}{13824}+\frac{b^4}{9216} + \frac{a^4}{6144},z,k\right)$$

input

```
int((a + b*x + c*x^2)/(3*x^4 + 2),x)
```

output

```
symsum(log(9*a*b^2 - 9*a^2*c - root(z^4 + (z^2*(2304*a*c + 1152*b^2))/5529
6 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2)/4608 +
c^4/13824 + b^4/9216 + a^4/6144, z, k)*(root(z^4 + (z^2*(2304*a*c + 1152*b
^2))/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*c)/2304 + (a^2*c^2
)/4608 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) + 144*b*
c + x*(108*a^2 - 72*c^2)) - 6*c^3 + x*(9*b^3 - 18*a*b*c))*root(z^4 + (z^2*
(2304*a*c + 1152*b^2))/55296 - (z*(288*a^2*b - 192*b*c^2))/55296 - (a*b^2*
c)/2304 + (a^2*c^2)/4608 + c^4/13824 + b^4/9216 + a^4/6144, z, k), k, 1, 4
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.13

$$\int \frac{a+bx+cx^2}{2+3x^4}\,dx = -\frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} - \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} + \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} - \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} - \frac{6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24}$$

input
```
int((c*x^2+b*x+a)/(3*x^4+2),x)
```

output
```
( - 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(
1/4)))*a - 4*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1
/4)))*c - 4*sqrt(6)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4
)))*b + 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*
6**(1/4)))*a + 4*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6
**(1/4)))*c - 4*sqrt(6)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**
(1/4)))*b - sqrt(6)*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sq
rt(2))*a + sqrt(6)*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2
))*a + 2*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c -
2*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c)/48
```

3.81 $\int \frac{dx^3}{2+3x^4} \, dx$

Optimal result 670
Mathematica [A] (verified) 670
Rubi [A] (verified) 671
Maple [A] (verified) 672
Fricas [A] (verification not implemented) 672
Sympy [A] (verification not implemented) 673
Maxima [A] (verification not implemented) 673
Giac [A] (verification not implemented) 673
Mupad [B] (verification not implemented) 674
Reduce [B] (verification not implemented) 674

Optimal result

Integrand size = 14, antiderivative size = 13

$$\int \frac{dx^3}{2+3x^4} \, dx = \frac{1}{12} d \log \left(2+3x^4\right)$$

output
```
1/12*d*ln(3*x^4+2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{dx^3}{2+3x^4} \, dx = \frac{1}{12} d \log \left(2+3x^4\right)$$

input
```
Integrate[(d*x^3)/(2 + 3*x^4),x]
```

output
```
(d*Log[2 + 3*x^4])/12
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{dx^3}{3x^4+2} \, dx$$
$$\Big\downarrow \; 27$$
$$d \int \frac{x^3}{3x^4+2} dx$$
$$\Big\downarrow \; 792$$
$$\frac{1}{12} d \log \left(3x^4+2\right)$$

input
```
Int[(d*x^3)/(2 + 3*x^4),x]
```

output
```
(d*Log[2 + 3*x^4])/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{d \ln \left(x^{4}+\frac{2}{3}\right)}{12}$	10
derivativedivides	$\frac{d \ln \left(3 x^{4}+2\right)}{12}$	12
default	$\frac{d \ln \left(3 x^{4}+2\right)}{12}$	12
norman	$\frac{d \ln \left(3 x^{4}+2\right)}{12}$	12
meijerg	$\frac{d \ln \left(1+\frac{3 x^{4}}{2}\right)}{12}$	12
risch	$\frac{d \ln \left(3 x^{4}+2\right)}{12}$	12

input
```
int(d*x^3/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*d*ln(x^4+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx=\frac{1}{12}\,d\log\left(3\,x^{4}+2\right)$$

input
```
integrate(d*x^3/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/12*d*log(3*x^4 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{d\log(3x^4+2)}{12}$$

input
```
integrate(d*x**3/(3*x**4+2),x)
```

output
```
d*log(3*x**4 + 2)/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

input
```
integrate(d*x^3/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/12*d*log(3*x^4 + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

input
```
integrate(d*x^3/(3*x^4+2),x, algorithm="giac")
```

output
```
1/12*d*log(3*x^4 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{d\ \ln\left(x^4+\frac{2}{3}\right)}{12}$$

input
```
int((d*x^3)/(3*x^4 + 2),x)
```

output
```
(d*log(x^4 + 2/3))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.00

$$\int \frac{dx^3}{2+3x^4}\,dx = \frac{d\left(\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)+\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)\right)}{12}$$

input
```
int(d*x^3/(3*x^4+2),x)
```

output
```
(d*(log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2)) + log(sqrt(2)*6**(
1/4)*x + sqrt(3)*x**2 + sqrt(2))))/12
```

3.82 $\int \frac{a+dx^3}{2+3x^4}\,dx$

Optimal result . . . 675
Mathematica [A] (verified) . . . 675
Rubi [A] (verified) . . . 676
Maple [C] (verified) . . . 677
Fricas [A] (verification not implemented) . . . 678
Sympy [A] (verification not implemented) . . . 678
Maxima [B] (verification not implemented) . . . 679
Giac [A] (verification not implemented) . . . 679
Mupad [B] (verification not implemented) . . . 680
Reduce [B] (verification not implemented) . . . 680

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{a+dx^3}{2+3x^4}\,dx = -\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}} + \frac{a\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/2
4*a*arctanh(2^(3/4)*3^(1/4)*x/(2^(1/2)+3^(1/2)*x^2))*6^(3/4)+1/12*d*ln(3*x
^4+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{48}\Big(-26^{3/4}a\arctan\left(1-\sqrt[4]{6}x\right) +2\,6^{3/4}a\arctan\left(1+\sqrt[4]{6}x\right)-6^{3/4}a\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+6^{3/4}a\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+4d\log\left(2+3x$$

input
```
Integrate[(a + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(3/4)*a*ArcTan[1 - 6^(1/4)*x] + 2*6^(3/4)*a*ArcTan[1 + 6^(1/4)*x] -
6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)
*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+dx^3}{3x^4+2}\,dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{a}{3x^4+2}+\frac{dx^3}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + d*x^3)/(2 + 3*x^4),x]
```

output
```
-1/4*(a*ArcTan[1 - 6^(1/4)*x])/6^(1/4) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1
/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] +
 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) + (d*Log[2 + 3*x^4])/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{3}d+a\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{d\ln\left(3x^{4}+2\right)}{12}$
meijerg	$\frac{d\ln\left(1+\frac{3x^{4}}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{1}{4}}}\right.}{96}$

input
```
int((d*x^3+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(6^{\frac{1}{4}}x+1\right)+\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(6^{\frac{1}{4}}x-1\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a+4\,d\right)\log\left(3\,x^2+6^{\frac{3}{4}}x+\sqrt{6}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-4\,d\right)\log\left(3\,x^2-6^{\frac{3}{4}}x+\sqrt{6}\right)$$

input
```
integrate((d*x^3+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/24*6^(3/4)*a*arctan(6^(1/4)*x + 1) + 1/24*6^(3/4)*a*arctan(6^(1/4)*x - 1
) + 1/48*(6^(3/4)*a + 4*d)*log(3*x^2 + 6^(3/4)*x + sqrt(6)) - 1/48*(6^(3/4
)*a - 4*d)*log(3*x^2 - 6^(3/4)*x + sqrt(6))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \text{RootSum}\left(165888t^4-55296t^3d+6912t^2d^2-384td^3+27a^4+8d^4,\left(t\mapsto t\log\left(x+\frac{24t-2d}{3a}\right)\right)\right)$$

input
```
integrate((d*x**3+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + 6912*_t**2*d**2 - 384*_t*d**3 + 27*
a**4 + 8*d**4, Lambda(_t, _t*log(x + (24*_t - 2*d)/(3*a))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}a\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)+\frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

input
```
integrate((d*x^3+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/24*3^(3/4)*2^(3/4)*a*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
- 3^(1/4)*2^(3/4))) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*lo
g(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^
(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{24}\cdot 6^{\frac{3}{4}}a\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{48}\left(6^{\frac{3}{4}}a+4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)-\frac{1}{48}\left(6^{\frac{3}{4}}a-4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/24*6^(3/4)*a*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4
))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
- 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int \frac{a+dx^3}{2+3x^4}\,dx = \ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} - \frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right) + \ln\left(x + \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12} + \frac{6^{1/4}\,\sqrt{\frac{3}{4}\mathrm{i}}\,a}{12}\right) + \ln\left(x - \frac{(-\right.$$

input
```
int((a + d*x^3)/(3*x^4 + 2),x)
```

output
```
log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(3i/4)^(1/2)*a)/1
2) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(3i/4)^(1/2)
*a)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-3i/4)
^(1/2)*a)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(
-3i/4)^(1/2)*a)/12)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

$$\int \frac{a+dx^3}{2+3x^4}\,dx = -\frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} - \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12} + \frac{\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12}$$

input

```
int((d*x^3+a)/(3*x^4+2),x)
```

output

```
( - 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(
1/4)))*a + 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(
2)*6**(1/4)))*a - sqrt(6)*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**
2 + sqrt(2))*a + sqrt(6)*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 +
sqrt(2))*a + 4*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d + 4*l
og(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d)/48
```

3.83 $\int \frac{bx+dx^3}{2+3x^4}\,dx$

Optimal result 682
Mathematica [C] (verified) 682
Rubi [A] (verified) 683
Maple [A] (verified) 684
Fricas [A] (verification not implemented) 685
Sympy [C] (verification not implemented) 685
Maxima [B] (verification not implemented) 686
Giac [B] (verification not implemented) 686
Mupad [B] (verification not implemented) 687
Reduce [B] (verification not implemented) 687

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int \frac{bx+dx^3}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/12*d*ln(3*x^4+2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \frac{bx+dx^3}{2+3x^4}\,dx=\frac{1}{24}\left(i\sqrt{6}b+2d\right)\log\left(\sqrt{6}-3ix^2\right)+\frac{1}{24}\left(-i\sqrt{6}b+2d\right)\log\left(\sqrt{6}+3ix^2\right)$$

input
```
Integrate[(b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
((I*Sqrt[6]*b + 2*d)*Log[Sqrt[6] - (3*I)*x^2])/24 + (((-I)*Sqrt[6]*b + 2*d
)*Log[Sqrt[6] + (3*I)*x^2])/24
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2027, 1577, 452, 216, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+dx^3}{3x^4+2}\,dx$$

$$\Big\downarrow \; 2027$$

$$\int \frac{x(b+dx^2)}{3x^4+2}dx$$

$$\Big\downarrow \; 1577$$

$$\frac{1}{2}\int \frac{dx^2+b}{3x^4+2}dx^2$$

$$\Big\downarrow \; 452$$

$$\frac{1}{2}\left(b\int \frac{1}{3x^4+2}dx^2+d\int \frac{x^2}{3x^4+2}dx^2\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(d\int \frac{x^2}{3x^4+2}dx^2+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{\sqrt{6}}\right)$$

$$\Big\downarrow \; 240$$

$$\frac{1}{2}\left(\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{\sqrt{6}}+\frac{1}{6}d\log\left(3x^4+2\right)\right)$$

input
```
Int[(b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
((b*ArcTan[Sqrt[3/2]*x^2])/Sqrt[6] + (d*Log[2 + 3*x^4])/6)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{b \arctan\left(\frac{\sqrt{6}\,x^{2}}{2}\right)\sqrt{6}}{12}+\frac{d \ln\left(3 x^{4}+2\right)}{12}$	28
risch	$\frac{d \ln\left(9 x^{4}+6\right)}{12}+\frac{b \arctan\left(\frac{\sqrt{6}\,x^{2}}{2}\right)\sqrt{6}}{12}$	28
meijerg	$\frac{d \ln\left(1+\frac{3 x^{4}}{2}\right)}{12}+\frac{\sqrt{6}\,b \arctan\left(\frac{x^{2}\sqrt{3}\,\sqrt{2}}{2}\right)}{12}$	31

input
```
int((d*x^3+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/12*d*ln(3*x^4+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \frac{1}{12}\,\sqrt{6}b\arctan\left(\frac{1}{2}\,\sqrt{6}x^2\right)+\frac{1}{12}\,d\log\left(3\,x^4+2\right)$$

input
```
integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*b*arctan(1/2*sqrt(6)*x^2) + 1/12*d*log(3*x^4 + 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = \left(-\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2-\frac{\sqrt{6}i}{3}\right)+\left(\frac{\sqrt{6}ib}{24}+\frac{d}{12}\right)\log\left(x^2+\frac{\sqrt{6}i}{3}\right)$$

input
```
integrate((d*x**3+b*x)/(3*x**4+2),x)
```

output
```
(-sqrt(6)*I*b/24 + d/12)*log(x**2 - sqrt(6)*I/3) + (sqrt(6)*I*b/24 + d/12)
*log(x**2 + sqrt(6)*I/3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(27) = 54$.

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.14

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = -\frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{12}\sqrt{3}\sqrt{2}b\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{12}d\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{12}d\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right)$$

input
```
integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*
2^(3/4))) + 1/12*sqrt(3)*sqrt(2)*b*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x
 - 3^(1/4)*2^(3/4))) + 1/12*d*log(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2
)) + 1/12*d*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.58

$$\int \frac{bx+dx^3}{2+3x^4}\,dx = -\frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\sqrt{6}b\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}d\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{12}d\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+b*x)/(3*x^4+2),x, algorithm="giac")
```

output
```
-1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4))
) + 1/12*sqrt(6)*b*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/
4))) + 1/12*d*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/12*d*log(x^
2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{bx + dx^3}{2 + 3x^4} \, dx = \frac{d \ln\left(x^4 + \frac{2}{3}\right)}{12} + \frac{\sqrt{6}\, b \operatorname{atan}\left(\frac{\sqrt{6}\, x^2}{2}\right)}{12}$$

input
```
int((b*x + d*x^3)/(3*x^4 + 2),x)
```

output
```
(d*log(x^4 + 2/3))/12 + (6^(1/2)*b*atan((6^(1/2)*x^2)/2))/12
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{bx + dx^3}{2 + 3x^4} \, dx = -\frac{\sqrt{6}\, atan\left(\frac{\left(\sqrt{2}\, 6^{\frac{1}{4}} - 2\sqrt{3}\, x\right) 6^{\frac{3}{4}}}{6\sqrt{2}}\right) b}{12} - \frac{\sqrt{6}\, atan\left(\frac{\left(\sqrt{2}\, 6^{\frac{1}{4}} + 2\sqrt{3}\, x\right) 6^{\frac{3}{4}}}{6\sqrt{2}}\right) b}{12} + \frac{\log\left(-\sqrt{2}\, 6^{\frac{1}{4}} x + \sqrt{3}\, x^2 + \sqrt{2}\right) d}{12} + \frac{\log\left(\sqrt{2}\, 6^{\frac{1}{4}} x + \sqrt{3}\, x^2 + \sqrt{2}\right) d}{12}$$

input
```
int((d*x^3+b*x)/(3*x^4+2),x)
```

output
```
( - sqrt(6)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b -
sqrt(6)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b + log(
 - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d + log(sqrt(2)*6**(1/4)*x
 + sqrt(3)*x**2 + sqrt(2))*d)/12
```

3.84 $\int \frac{a+bx+dx^3}{2+3x^4}\,dx$

Optimal result . . . 688
Mathematica [A] (verified) . . . 688
Rubi [A] (verified) . . . 689
Maple [C] (verified) . . . 690
Fricas [C] (verification not implemented) . . . 691
Sympy [A] (verification not implemented) . . . 691
Maxima [B] (verification not implemented) . . . 692
Giac [A] (verification not implemented) . . . 692
Mupad [B] (verification not implemented) . . . 693
Reduce [B] (verification not implemented) . . . 694

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(1+\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{4\sqrt[4]{6}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/24*a*arctan(-1+6^(1/4)*x)*6^(3/4)
+1/24*a*arctan(1+6^(1/4)*x)*6^(3/4)+1/24*a*arctanh(2^(3/4)*3^(1/4)*x/(2^(1
/2)+3^(1/2)*x^2))*6^(3/4)+1/12*d*ln(3*x^4+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx=\frac{1}{48}\left(-2\sqrt{6}\left(\sqrt[4]{6}a+2b\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\sqrt{6}\left(\sqrt[4]{6}a-2b\right)\arctan\left(1+\sqrt[4]{6}x\right)-6^{3/4}a\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+6^{3/4}a\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+4d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*Sqrt[6]*(6^(1/4)*a + 2*b)*ArcTan[1 - 6^(1/4)*x] + 2*Sqrt[6]*(6^(1/4)*a
 - 2*b)*ArcTan[1 + 6^(1/4)*x] - 6^(3/4)*a*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^
2] + 6^(3/4)*a*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+dx^3}{3x^4+2}dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{a}{3x^4+2}+\frac{x(b+dx^2)}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\arctan\left(1-\sqrt[4]{6}x\right)}{4\sqrt[4]{6}}+\frac{a\arctan\left(\sqrt[4]{6}x+1\right)}{4\sqrt[4]{6}}-\frac{a\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{a\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\sqrt[4]{6}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + b*x + d*x^3)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (a*ArcTan[1 - 6^(1/4)*x])/(4*6^(1/
4)) + (a*ArcTan[1 + 6^(1/4)*x])/(4*6^(1/4)) - (a*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(8*6^(1/4)) + (a*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(1/4)) +
(d*Log[2 + 3*x^4])/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)} \frac{\left(\_R^3 d+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)\sqrt{6}}{12}+\frac{d\ln(}{}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{\sqrt{6}\,b\arctan\left(\frac{x^2\sqrt{3}\sqrt{2}}{2}\right)}{12}+\frac{24^{\frac{3}{4}}a\left(-\frac{x\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{1}{4}}}+\frac{x\sqrt{2}\ln(}{}\right.}{96}$

input
```
int((d*x^3+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 17085, normalized size of antiderivative = 146.03

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.70

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx$$

$$= \text{RootSum}\Big(165888t^4 - 55296t^3d + t^2\cdot\left(3456b^2+6912d^2\right) + t\left(-864a^2b-576b^2d-384d^3\right) + 27a^4 + 72a$$

input
```
integrate((d*x**3+b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(3456*b**2 + 6912*d**2) + _t*
(-864*a**2*b - 576*b**2*d - 384*d**3) + 27*a**4 + 72*a**2*b*d + 18*b**4 +
24*b**2*d**2 + 8*d**4, Lambda(_t, _t*log(x + (27648*_t**3*b**2 + 1728*_t**
2*a**2*b - 6912*_t**2*b**2*d + 216*_t*a**4 - 288*_t*a**2*b*d + 288*_t*b**4
 + 576*_t*b**2*d**2 - 18*a**4*d - 90*a**2*b**3 + 12*a**2*b*d**2 - 24*b**4*
d - 16*b**2*d**3)/(27*a**5 - 72*a*b**4))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(83) = 166.

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.46

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d+3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{144}\cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d-3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a-2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{24}\sqrt{3}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}a+2\sqrt{2}b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d + 3*a)*log(sqrt(3)*x^2 + 3^(1/4
)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(2*3^(1/4)*2^(1/4)*d - 3*a)
*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/24*sqrt(3)*(3^(1/4)*2^
(3/4)*a - 2*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2
^(3/4))) + 1/24*sqrt(3)*(3^(1/4)*2^(3/4)*a + 2*sqrt(2)*b)*arctan(1/6*3^(3/
4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a-2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\sqrt{6}b\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a+4\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-4\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a - 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(
2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b)*arctan(3/4*sqrt(2)*(2/3)
^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a + 4*d)*log(x^2 + sqr
t(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 4*d)*log(x^2 - sqrt(2)
*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.62

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = \sum_{k=1}^{4} \ln\Bigg(x\left(9\,a^2\,d+9\,b^3+6\,b\,d^2\right)+9\,a\,b^2-6\,a\,d^2 - \mathrm{root}\Bigg(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\Bigg)\,\Bigg(\mathrm{root}\Bigg(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912} -144\,a\,d+x\left(108\,a^2+144\,b\,d\right)\Bigg)\Bigg)\,\mathrm{root}\Bigg(z^4-\frac{d\,z^3}{3}+\frac{z^2\,\left(3456\,b^2+6912\,d^2\right)}{165888}-\frac{z\,\left(864\,a^2\,b+576\,b^2\,d+384\,d^3\right)}{165888}+\frac{a^2\,b\,d}{2304}+\frac{b^2\,d^2}{6912}+\frac{d^4}{20736}+\frac{b^4}{9216}+\frac{a^4}{6144},z,k\Bigg)$$

input
```
int((a + b*x + d*x^3)/(3*x^4 + 2),x)
```

output
```
symsum(log(x*(9*a^2*d + 6*b*d^2 + 9*b^3) + 9*a*b^2 - 6*a*d^2 - root(z^4 -
(d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/165888 - (z*(864*a^2*b + 576*b^2*d
 + 384*d^3))/165888 + (a^2*b*d)/2304 + (b^2*d^2)/6912 + d^4/20736 + b^4/92
16 + a^4/6144, z, k)*(root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 + 6912*d^2))/1
65888 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*d)/2304 + (b
^2*d^2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k)*(864*a - 864*b*x) -
144*a*d + x*(144*b*d + 108*a^2)))*root(z^4 - (d*z^3)/3 + (z^2*(3456*b^2 +
6912*d^2))/165888 - (z*(864*a^2*b + 576*b^2*d + 384*d^3))/165888 + (a^2*b*
d)/2304 + (b^2*d^2)/6912 + d^4/20736 + b^4/9216 + a^4/6144, z, k), k, 1, 4
)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.77

$$\int \frac{a+bx+dx^3}{2+3x^4}\,dx = -\frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} - \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12} + \frac{\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12}$$

input
```
int((d*x^3+b*x+a)/(3*x^4+2),x)
```

output
```
( - 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(
1/4)))*a - 4*sqrt(6)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/
4)))*b + 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)
*6**(1/4)))*a - 4*sqrt(6)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6
**(1/4)))*b - sqrt(6)*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 +
sqrt(2))*a + sqrt(6)*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt
(2))*a + 4*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d + 4*log(s
qrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d)/48
```

3.85 $\int \frac{cx^2+dx^3}{2+3x^4}\,dx$

Optimal result 696
Mathematica [A] (verified) 696
Rubi [A] (verified) 697
Maple [C] (warning: unable to verify) 698
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Sympy [A] (verification not implemented) 699
Maxima [B] (verification not implemented) 700
Giac [A] (verification not implemented) 701
Mupad [B] (verification not implemented) 701
Reduce [B] (verification not implemented) 702

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx=-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}-\frac{c\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{2\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)-1/1
2*c*arctanh(2^(3/4)*3^(1/4)*x/(2^(1/2)+3^(1/2)*x^2))*6^(1/4)+1/12*d*ln(3*x
^4+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx=\frac{1}{24}\left(-2\sqrt[4]{6}c\arctan\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}c\arctan\left(1+\sqrt[4]{6}x\right)+\sqrt[4]{6}c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\sqrt[4]{6}c\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+2d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*c*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*c*ArcTan[1 + 6^(1/4)*x] +
6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] - 6^(1/4)*c*Log[2 + 2*6^(1/4)
*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2027, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx^2+dx^3}{3x^4+2}\,dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2(c+dx)}{3x^4+2}dx$$

$$\Big\downarrow \text{2370}$$

$$\int \left(\frac{cx^2}{3x^4+2}+\frac{dx^3}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(\sqrt[4]{6}x+1\right)}{2\ 6^{3/4}}+\frac{c\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}-\frac{c\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{4\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
-1/2*(c*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3
/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] +
 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{3} d+\_R^{2} c\right) \ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}+\frac{d\ln\left(3x^{4}+2\right)}{12}$
meijerg	$\frac{d\ln\left(1+\frac{3x^{4}}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\,\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\,\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\,\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}}{216}$

input
```
int((d*x^3+c*x^2)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{cx^2+dx^3}{2+3x^4}\,dx =& \frac{1}{432}\cdot 216^{\frac{3}{4}}c\arctan\left(\frac{1}{36}\cdot 216^{\frac{3}{4}}x+1\right)\\&+\frac{1}{432}\cdot 216^{\frac{3}{4}}c\arctan\left(\frac{1}{36}\cdot 216^{\frac{3}{4}}x-1\right)\\&-\frac{1}{864}\left(216^{\frac{3}{4}}c-72\,d\right)\log\left(18\,x^2+6\cdot 216^{\frac{1}{4}}x+6\,\sqrt{6}\right)\\&+\frac{1}{864}\left(216^{\frac{3}{4}}c+72\,d\right)\log\left(18\,x^2-6\cdot 216^{\frac{1}{4}}x+6\,\sqrt{6}\right)\end{aligned}$$

input
```
integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/432*216^(3/4)*c*arctan(1/36*216^(3/4)*x + 1) + 1/432*216^(3/4)*c*arctan(
1/36*216^(3/4)*x - 1) - 1/864*(216^(3/4)*c - 72*d)*log(18*x^2 + 6*216^(1/4
)*x + 6*sqrt(6)) + 1/864*(216^(3/4)*c + 72*d)*log(18*x^2 - 6*216^(1/4)*x +
 6*sqrt(6))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx$$
$$=\operatorname{RootSum}\left(41472t^4-13824t^3d+1728t^2d^2-96td^3+3c^4+2d^4,\left(t\mapsto t\log\left(x+\frac{3456t^3-864t^2d+72t}{3c^3}\right.\right.\right.$$

input
```
integrate((d*x**3+c*x**2)/(3*x**4+2),x)
```

output
```
RootSum(41472*_t**4 - 13824*_t**3*d + 1728*_t**2*d**2 - 96*_t*d**3 + 3*c**
4 + 2*d**4, Lambda(_t, _t*log(x + (3456*_t**3 - 864*_t**2*d + 72*_t*d**2 -
 2*d**3)/(3*c**3))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(68) = 136$.

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{cx^2+dx^3}{2+3x^4}\,dx = {} & \frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d-\sqrt{3}c\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) \\ & +\frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d+\sqrt{3}c\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) \\ & +\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x+3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) \\ & +\frac{1}{12}\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}c\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x-3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)\end{aligned}$$

input
```
integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(
1/4)*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt
(3)*c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/12*3^(1/4)*2^(1/
4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/12*3^
(1/4)*2^(1/4)*c*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))
)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.15

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\cdot 6^{\frac{1}{4}}c\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c-2\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c+2\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2)*(2/3)^(1/4)))
 + 1/12*6^(1/4)*c*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4
))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
+ 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = \ln\left(x - \frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}+\frac{6^{1/4}\,\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x+\frac{(-1)^{1/4}\,2^{1/4}\,3^{3/4}}{3}\right)\left(\frac{d}{12}-\frac{6^{1/4}\,\sqrt{-\frac{1}{2}\mathrm{i}}\,c}{12}\right)+\ln\left(x-\right.$$

input
```
int((c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output
```
log(x - ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*(-1i/2)^(1/2)*c)/
12) + log(x + ((-1)^(1/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(-1i/2)^(1/
2)*c)/12) + log(x - ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 - (6^(1/4)*(1i/2
)^(1/2)*c)/12) + log(x + ((-1)^(3/4)*2^(1/4)*3^(3/4))/3)*(d/12 + (6^(1/4)*
(1i/2)^(1/2)*c)/12)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\int \frac{cx^2+dx^3}{2+3x^4}\,dx = -\frac{6^{\frac{1}{4}} atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} + \frac{6^{\frac{1}{4}} atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} + \frac{6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} - \frac{6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} + \frac{\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12} + \frac{\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12}$$

input
```
int((d*x^3+c*x^2)/(3*x^4+2),x)
```

output
```
( - 2*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c
 + 2*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c
+ 6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c - 6**(1/4
)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c + 2*log( - sqrt(2)*6*
*(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d + 2*log(sqrt(2)*6**(1/4)*x + sqrt(3)*
x**2 + sqrt(2))*d)/24
```

3.86 $\int \frac{a+cx^2+dx^3}{2+3x^4} \, dx$

Optimal result 703
Mathematica [A] (verified) 704
Rubi [A] (verified) 704
Maple [C] (verified) 705
Fricas [B] (verification not implemented) 706
Sympy [A] (verification not implemented) 708
Maxima [B] (verification not implemented) 709
Giac [A] (verification not implemented) 710
Mupad [B] (verification not implemented) 711
Reduce [B] (verification not implemented) 712

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{a+cx^2+dx^3}{2+3x^4} \, dx = -\frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\ 6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\text{arctanh}\left(\frac{2\sqrt[4]{6}x}{2+\sqrt{6}x^2}\right)}{4\ 6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/24*(6^(1/2)*a+2*c)*arctan(-1+6^(1/4)*x)*6^(1/4)+1/24*(6^(1/2)*a+2*c)*arc
tan(1+6^(1/4)*x)*6^(1/4)+1/24*(6^(1/2)*a-2*c)*arctanh(2*6^(1/4)*x/(2+6^(1/
2)*x^2))*6^(1/4)+1/12*d*ln(3*x^4+2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{48}\Big(-2\sqrt[4]{6}\big(\sqrt{6}a+2c\big)\arctan\big(1-\sqrt[4]{6}x\big) + 2\sqrt[4]{6}\big(\sqrt{6}a+2c\big)\arctan\big(1+\sqrt[4]{6}x\big) - \sqrt[4]{6}\big(\sqrt{6}a-2c\big)\log\big(2-2\sqrt[4]{6}x+\sqrt{6}x^2\big) + \sqrt[4]{6}\big(\sqrt{6}a-2c\big)\log\big(2+2\sqrt[4]{6}x+\sqrt{6}x^2\big) + 4d\log\big(2+3x^4\big)\Big)$$

input
```
Integrate[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*(Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(Sqrt[6]*a
 + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 - 2*6^(1/4
)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)*Log[2 + 2*6^(1/4)*x + Sqrt[
6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^2+dx^3}{3x^4+2}\,dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{a+cx^2}{3x^4+2}+\frac{dx^3}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(\sqrt{6}a+2c)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{(\sqrt{6}a+2c)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}-\frac{(\sqrt{6}a-2c)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{(\sqrt{6}a-2c)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
-1/4*((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1/4)*x])/6^(3/4) + ((Sqrt[6]*a + 2*c
)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^
(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x
 + 3*x^2])/(8*6^(3/4)) + (d*Log[2 + 3*x^4])/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.31

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3d+\_R^2c+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x}{3}}{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x}{3}}\right.\right.}{}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}\right.}{216}$

input
```
int((d*x^3+c*x^2+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(88) = 176$.

Time = 0.13 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.11

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx$$
$$= \frac{1}{12}\left(d-3\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.+6\left(3\,a^3-2\,ac^2-2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$
$$+\frac{1}{12}\left(d+3\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.-6\left(3\,a^3-2\,ac^2-2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac+\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$
$$+\frac{1}{12}\left(d-3\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.+6\left(3\,a^3-2\,ac^2+2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$
$$+\frac{1}{12}\left(d+3\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)\log\left(-\left(9\,a^4-4\,c^4\right)x\right.$$
$$\left.-6\left(3\,a^3-2\,ac^2+2\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}c\right)\sqrt{-\frac{1}{3}\,ac-\frac{1}{6}\sqrt{-\frac{3}{2}\,a^4+2\,a^2c^2-\frac{2}{3}\,c^4}}\right)$$

input
```
integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
1/12*(d - 3*sqrt(-1/3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)))*log
(-(9*a^4 - 4*c^4)*x + 6*(3*a^3 - 2*a*c^2 - 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2
/3*c^4)*c)*sqrt(-1/3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))) + 1/
12*(d + 3*sqrt(-1/3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)))*log(-
(9*a^4 - 4*c^4)*x - 6*(3*a^3 - 2*a*c^2 - 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3
*c^4)*c)*sqrt(-1/3*a*c + 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))) + 1/12
*(d - 3*sqrt(-1/3*a*c - 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)))*log(-(9
*a^4 - 4*c^4)*x + 6*(3*a^3 - 2*a*c^2 + 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c
^4)*c)*sqrt(-1/3*a*c - 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4))) + 1/12*(
d + 3*sqrt(-1/3*a*c - 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)))*log(-(9*a
^4 - 4*c^4)*x - 6*(3*a^3 - 2*a*c^2 + 2*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4
)*c)*sqrt(-1/3*a*c - 1/6*sqrt(-3/2*a^4 + 2*a^2*c^2 - 2/3*c^4)))
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx$$
$$=\text{RootSum}\Big(165888t^4-55296t^3d+t^2\cdot(6912ac+6912d^2)+t(-1152acd-384d^3)+27a^4+36a^2c^2+4$$

input
```
integrate((d*x**3+c*x**2+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 6912*d**2) + _t*(
-1152*a*c*d - 384*d**3) + 27*a**4 + 36*a**2*c**2 + 48*a*c*d**2 + 12*c**4 +
 8*d**4, Lambda(_t, _t*log(x + (-13824*_t**3*c + 3456*_t**2*c*d + 216*_t*a
**3 - 432*_t*a*c**2 - 288*_t*c*d**2 - 18*a**3*d + 36*a*c**2*d + 8*c*d**3)/
(27*a**4 - 12*c**4))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(88) = 176$.

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.67

$$\int \frac{a + cx^2 + dx^3}{2 + 3x^4} \, dx = -\frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(\sqrt{3}\sqrt{2}c - 2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d - 3\,a\right) \log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}} 2^{\frac{3}{4}} \left(\sqrt{3}\sqrt{2}c + 2 \cdot 3^{\frac{1}{4}} 2^{\frac{1}{4}} d - 3\,a\right) \log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}} 2^{\frac{3}{4}} x + \sqrt{2}\right) + \frac{1}{72}\sqrt{3}\left(3 \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} a + 2 \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} c\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x + 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right) + \frac{1}{72}\sqrt{3}\left(3 \cdot 3^{\frac{1}{4}} 2^{\frac{3}{4}} a + 2 \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} c\right) \arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}} 2^{\frac{1}{4}} \left(2\sqrt{3}x - 3^{\frac{1}{4}} 2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
-1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log
(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(
3)*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4
)*x + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*
arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)
*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c)*arctan(1/6*3^(3/4)*2^(1/4)*(2
*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{24}\left(6^{\frac{3}{4}}a+2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c+4\,d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right) - \frac{1}{48}\left(6^{\frac{3}{4}}a-2\cdot 6^{\frac{1}{4}}c-4\,d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(
2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)
^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^(3/4)*a - 2*6^(1/4)*c + 4*d)
*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3)) - 1/48*(6^(3/4)*a - 2*6^(1/4
)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.44

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = \ln\left(-2\,c+\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c-2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}+\frac{\sqrt{\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c-\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c-2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}-\frac{\sqrt{\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c-\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c+\sqrt{6}\,a\,1\mathrm{i}+x\,\sqrt{-3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c+2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}-\frac{\sqrt{-\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)+\ln\left(2\,c+\sqrt{6}\,a\,1\mathrm{i}-x\,\sqrt{-3\mathrm{i}\,\sqrt{6}\,a^2-12\,a\,c+2\mathrm{i}\,\sqrt{6}\,c^2}\right)\left(\frac{d}{12}+\frac{\sqrt{-\frac{3\mathrm{i}\,\sqrt{6}\,a^2}{4}-3\,a\,c+\frac{1\mathrm{i}\,\sqrt{6}\,c^2}{2}}}{12}\right)$$

input
```
int((a + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output
```
log(6^(1/2)*a*1i - 2*c + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2
))*(d/12 + ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + l
og(2*c - 6^(1/2)*a*1i + x*(6^(1/2)*a^2*3i - 12*a*c - 6^(1/2)*c^2*2i)^(1/2)
)*(d/12 - ((6^(1/2)*a^2*3i)/4 - 3*a*c - (6^(1/2)*c^2*1i)/2)^(1/2)/12) + lo
g(2*c + 6^(1/2)*a*1i + x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))
*(d/12 - ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12) + log
(2*c + 6^(1/2)*a*1i - x*(6^(1/2)*c^2*2i - 6^(1/2)*a^2*3i - 12*a*c)^(1/2))*
(d/12 + ((6^(1/2)*c^2*1i)/2 - (6^(1/2)*a^2*3i)/4 - 3*a*c)^(1/2)/12)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.19

$$\int \frac{a+cx^2+dx^3}{2+3x^4}\,dx = -\frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} - \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} + \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} - \frac{6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} + \frac{\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12} + \frac{\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12}$$

input
```
int((d*x^3+c*x^2+a)/(3*x^4+2),x)
```

output
```
( - 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(
1/4)))*a - 4*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1
/4)))*c + 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2
)*6**(1/4)))*a + 4*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)
*6**(1/4)))*c - sqrt(6)*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2
+ sqrt(2))*a + sqrt(6)*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sq
rt(2))*a + 2*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*
c - 2*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c + 4*log(
 - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d + 4*log(sqrt(2)*6**(1/4)
*x + sqrt(3)*x**2 + sqrt(2))*d)/48
```

3.87 $\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx$

Optimal result 714
Mathematica [A] (verified) 714
Rubi [A] (verified) 715
Maple [C] (warning: unable to verify) 716
Fricas [C] (verification not implemented) 717
Sympy [A] (verification not implemented) 718
Maxima [B] (verification not implemented) 718
Giac [A] (verification not implemented) 719
Mupad [B] (verification not implemented) 720
Reduce [B] (verification not implemented) 721

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx=\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}-\frac{c\arctan\left(1-\sqrt[4]{6}x\right)}{2\ 6^{3/4}}+\frac{c\arctan\left(1+\sqrt[4]{6}x\right)}{2\ 6^{3/4}}-\frac{c\,\text{arctanh}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{2}+\sqrt{3}x^2}\right)}{2\ 6^{3/4}}+\frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/12*c*arctan(-1+6^(1/4)*x)*6^(1/4)
+1/12*c*arctan(1+6^(1/4)*x)*6^(1/4)-1/12*c*arctanh(2^(3/4)*3^(1/4)*x/(2^(1
/2)+3^(1/2)*x^2))*6^(1/4)+1/12*d*ln(3*x^4+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx=\frac{1}{24}\left(-2\sqrt[4]{6}\left(\sqrt[4]{6}b+c\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}\left(-\sqrt[4]{6}b+c\right)\arctan\left(1+\sqrt[4]{6}x\right)+\sqrt[4]{6}c\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)-\sqrt[4]{6}c\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+2d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*(6^(1/4)*b + c)*ArcTan[1 - 6^(1/4)*x] + 2*6^(1/4)*(-(6^(1/4)*b
) + c)*ArcTan[1 + 6^(1/4)*x] + 6^(1/4)*c*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2
] - 6^(1/4)*c*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 2*d*Log[2 + 3*x^4])/24
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2028, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx + cx^2 + dx^3}{3x^4 + 2} \, dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x(b + cx + dx^2)}{3x^4 + 2} dx$$

$$\Big\downarrow \text{ 2370}$$

$$\int \left(\frac{x(b + dx^2)}{3x^4 + 2} + \frac{cx^2}{3x^4 + 2} \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b \arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{c \arctan\left(1 - \sqrt[4]{6}x\right)}{2\ 6^{3/4}} + \frac{c \arctan\left(\sqrt[4]{6}x + 1\right)}{2\ 6^{3/4}} + \frac{c \log\left(3x^2 - 6^{3/4}x + \sqrt{6}\right)}{4\ 6^{3/4}} - \frac{c \log\left(3x^2 + 6^{3/4}x + \sqrt{6}\right)}{4\ 6^{3/4}} + \frac{1}{12} d \log\left(3x^4 + 2\right)$$

input
```
Int[(b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - (c*ArcTan[1 - 6^(1/4)*x])/(2*6^(3/
4)) + (c*ArcTan[1 + 6^(1/4)*x])/(2*6^(3/4)) + (c*Log[Sqrt[6] - 6^(3/4)*x +
 3*x^2])/(4*6^(3/4)) - (c*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(4*6^(3/4)) +
(d*Log[2 + 3*x^4])/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.32

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^{4}+2\right)}\frac{\left(\_R^{3}d+\_R^{2}c+\_Rb\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{12}$
default	$\frac{b\arctan\left(\frac{\sqrt{6}\,x^{2}}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\,6^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^{2}+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{144}+\frac{d\ln(\cdots}{}$
meijerg	$\frac{d\ln\left(1+\frac{3x^{4}}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^{3}\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}+\frac{x^{3}\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^{4}\right)^{\frac{1}{4}}}\right)}{\left(x^{4}\right)^{\frac{3}{4}}}-\frac{x^{3}\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^{4}\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^{4}}}{2}\right)}{2\left(x^{4}\right)^{\frac{3}{4}}}\cdots\right)}{216}$

input
```
int((d*x^3+c*x^2+b*x)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c+_R*b)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 18086, normalized size of antiderivative = 154.58

$$\int\frac{bx+cx^2+dx^3}{2+3x^4}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.62

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4} \, dx$$
$$= \text{RootSum}\left(82944t^4 - 27648t^3d + t^2 \cdot \left(1728b^2 + 3456d^2\right) + t\left(-288b^2d + 288bc^2 - 192d^3\right) + 9b^4 + 12b^2d\right.$$

input
```
integrate((d*x**3+c*x**2+b*x)/(3*x**4+2),x)
```

output
```
RootSum(82944*_t**4 - 27648*_t**3*d + _t**2*(1728*b**2 + 3456*d**2) + _t*(
-288*b**2*d + 288*b*c**2 - 192*d**3) + 9*b**4 + 12*b**2*d**2 - 24*b*c**2*d
 + 6*c**4 + 4*d**4, Lambda(_t, _t*log(x + (-3456*_t**3*c**2 + 864*_t**2*b*
*3 + 864*_t**2*c**2*d - 144*_t*b**3*d - 108*_t*b**2*c**2 - 72*_t*c**2*d**2
 + 9*b**5 + 6*b**3*d**2 + 9*b**2*c**2*d - 9*b*c**4 + 2*c**2*d**3)/(18*b**4
*c - 3*c**5))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(83) = 166.

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{bx + cx^2 + dx^3}{2 + 3x^4} \, dx = {} & \frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c - 6\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) \\ & + \frac{1}{72}\sqrt{3}\sqrt{2}\left(3^{\frac{3}{4}}2^{\frac{3}{4}}c + 6\,b\right)\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) \\ & + \frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d - \sqrt{3}c\right)\log\left(\sqrt{3}x^2 + 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right) \\ & + \frac{1}{72}\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(3^{\frac{1}{4}}2^{\frac{3}{4}}d + \sqrt{3}c\right)\log\left(\sqrt{3}x^2 - 3^{\frac{1}{4}}2^{\frac{3}{4}}x + \sqrt{2}\right)\end{aligned}$$

input
```
integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="maxima")
```

output
```
1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c - 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*
(2*sqrt(3)*x + 3^(1/4)*2^(3/4))) + 1/72*sqrt(3)*sqrt(2)*(3^(3/4)*2^(3/4)*c
 + 6*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4))) + 1/72
*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d - sqrt(3)*c)*log(sqrt(3)*x^2 + 3^(1/4)
*2^(3/4)*x + sqrt(2)) + 1/72*3^(3/4)*2^(1/4)*(3^(1/4)*2^(3/4)*d + sqrt(3)*
c)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4)*x + sqrt(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4}\,dx = -\frac{1}{12}\left(\sqrt{6}b - 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) + \frac{1}{12}\left(\sqrt{6}b + 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2\,x - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\left(6^{\frac{1}{4}}c - 2\,d\right)\log\left(x^2 + \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right) + \frac{1}{24}\left(6^{\frac{1}{4}}c + 2\,d\right)\log\left(x^2 - \sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x + \sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2+b*x)/(3*x^4+2),x, algorithm="giac")
```

output
```
-1/12*(sqrt(6)*b - 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x + sqrt(2
)*(2/3)^(1/4))) + 1/12*(sqrt(6)*b + 6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3
/4)*(2*x - sqrt(2)*(2/3)^(1/4))) - 1/24*(6^(1/4)*c - 2*d)*log(x^2 + sqrt(2
)*(2/3)^(1/4)*x + sqrt(2/3)) + 1/24*(6^(1/4)*c + 2*d)*log(x^2 - sqrt(2)*(2
/3)^(1/4)*x + sqrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 6.35 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.56

$$\int \frac{bx + cx^2 + dx^3}{2 + 3x^4}\,dx$$
$$= \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(1728\,b^2 + 3456\,d^2)}{82944} - \frac{z\,(-288\,b\,c^2 + 288\,b^2\,d + 192\,d^3)}{82944} - \frac{b\,c^2\,d}{3456} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{c^4}{13824} + \frac{b^4}{9216}, z, k\right)\left(144\,b\,c + x\,(144\,b\,d - 72\,c^2) - \mathrm{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(1728\,b^2 + 3456\,d^2)}{82944} - \frac{z\,(-288\,b\,c^2 + 288\,b^2\,d + 192\,d^3)}{82944} - \frac{b\,c^2\,d}{3456} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{c^4}{13824}, z, k\right)\right) + x\,(9\,b^3 + 6\,b\,d^2 - 6\,c^2\,d) - 6\,c^3 + 12\,b\,c\,d\right)\,\mathrm{root}\left(z^4 - \frac{d\,z^3}{3} + \frac{z^2\,(1728\,b^2 + 3456\,d^2)}{82944} - \frac{z\,(-288\,b\,c^2 + 288\,b^2\,d + 192\,d^3)}{82944} - \frac{b\,c^2\,d}{3456} + \frac{b^2\,d^2}{6912} + \frac{d^4}{20736} + \frac{c^4}{13824} + \frac{b^4}{9216}, z, k\right)$$

input
```
int((b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output
```
symsum(log(x*(6*b*d^2 - 6*c^2*d + 9*b^3) - root(z^4 - (d*z^3)/3 + (z^2*(17
28*b^2 + 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944
- (b*c^2*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k
)*(144*b*c + x*(144*b*d - 72*c^2) - 864*root(z^4 - (d*z^3)/3 + (z^2*(1728*
b^2 + 3456*d^2))/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (
b*c^2*d)/3456 + (b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k)*b
*x) - 6*c^3 + 12*b*c*d)*root(z^4 - (d*z^3)/3 + (z^2*(1728*b^2 + 3456*d^2))
/82944 - (z*(- 288*b*c^2 + 288*b^2*d + 192*d^3))/82944 - (b*c^2*d)/3456 +
(b^2*d^2)/6912 + d^4/20736 + c^4/13824 + b^4/9216, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.70

$$\int \frac{bx+cx^2+dx^3}{2+3x^4}\,dx = -\frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} + \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} + \frac{6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} - \frac{6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} + \frac{\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12} + \frac{\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12}$$

input
```
int((d*x^3+c*x^2+b*x)/(3*x^4+2),x)
```

output
```
( - 2*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c
 - 2*sqrt(6)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b +
 2*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*c -
2*sqrt(6)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**(1/4)))*b + 6*
*(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c - 6**(1/4)*lo
g(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c + 2*log( - sqrt(2)*6**(1/
4)*x + sqrt(3)*x**2 + sqrt(2))*d + 2*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2
 + sqrt(2))*d)/24
```

3.88 $\int \frac{a+bx+cx^2+dx^3}{2+3x^4} dx$

Optimal result 722
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Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}} - \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1-\sqrt[4]{6}x\right)}{4\,6^{3/4}} + \frac{\left(\sqrt{6}a+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)}{4\,6^{3/4}} + \frac{\left(\sqrt{6}a-2c\right)\operatorname{arctanh}\left(\frac{2\sqrt[4]{6}x}{2+\sqrt{6}x^2}\right)}{4\,6^{3/4}} + \frac{1}{12}d\log\left(2+3x^4\right)$$

output
```
1/12*b*arctan(1/2*6^(1/2)*x^2)*6^(1/2)+1/24*(6^(1/2)*a+2*c)*arctan(-1+6^(1
/4)*x)*6^(1/4)+1/24*(6^(1/2)*a+2*c)*arctan(1+6^(1/4)*x)*6^(1/4)+1/24*(6^(1
/2)*a-2*c)*arctanh(2*6^(1/4)*x/(2+6^(1/2)*x^2))*6^(1/4)+1/12*d*ln(3*x^4+2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \frac{1}{48}\left(-2\sqrt[4]{6}\left(\sqrt{6}a+2\left(\sqrt[4]{6}b+c\right)\right)\arctan\left(1-\sqrt[4]{6}x\right)+2\sqrt[4]{6}\left(\sqrt{6}a-2\sqrt[4]{6}b+2c\right)\arctan\left(1+\sqrt[4]{6}x\right)-\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2-2\sqrt[4]{6}x+\sqrt{6}x^2\right)+\sqrt[4]{6}\left(\sqrt{6}a-2c\right)\log\left(2+2\sqrt[4]{6}x+\sqrt{6}x^2\right)+4d\log\left(2+3x^4\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(-2*6^(1/4)*(Sqrt[6]*a + 2*(6^(1/4)*b + c))*ArcTan[1 - 6^(1/4)*x] + 2*6^(1
/4)*(Sqrt[6]*a - 2*6^(1/4)*b + 2*c)*ArcTan[1 + 6^(1/4)*x] - 6^(1/4)*(Sqrt[
6]*a - 2*c)*Log[2 - 2*6^(1/4)*x + Sqrt[6]*x^2] + 6^(1/4)*(Sqrt[6]*a - 2*c)
*Log[2 + 2*6^(1/4)*x + Sqrt[6]*x^2] + 4*d*Log[2 + 3*x^4])/48
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2+dx^3}{3x^4+2}\,dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{a+cx^2}{3x^4+2}+\frac{x\left(b+dx^2\right)}{3x^4+2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(\sqrt{6}a+2c)\arctan\left(1-\sqrt[4]{6}x\right)}{4\ 6^{3/4}}+\frac{(\sqrt{6}a+2c)\arctan\left(\sqrt[4]{6}x+1\right)}{4\ 6^{3/4}}-\frac{(\sqrt{6}a-2c)\log\left(3x^2-6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{(\sqrt{6}a-2c)\log\left(3x^2+6^{3/4}x+\sqrt{6}\right)}{8\ 6^{3/4}}+\frac{b\arctan\left(\sqrt{\frac{3}{2}}x^2\right)}{2\sqrt{6}}+\frac{1}{12}d\log\left(3x^4+2\right)$$

input
```
Int[(a + b*x + c*x^2 + d*x^3)/(2 + 3*x^4),x]
```

output
```
(b*ArcTan[Sqrt[3/2]*x^2])/(2*Sqrt[6]) - ((Sqrt[6]*a + 2*c)*ArcTan[1 - 6^(1
/4)*x])/(4*6^(3/4)) + ((Sqrt[6]*a + 2*c)*ArcTan[1 + 6^(1/4)*x])/(4*6^(3/4)
) - ((Sqrt[6]*a - 2*c)*Log[Sqrt[6] - 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + ((S
qrt[6]*a - 2*c)*Log[Sqrt[6] + 6^(3/4)*x + 3*x^2])/(8*6^(3/4)) + (d*Log[2 +
 3*x^4])/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4+2\right)}\frac{\left(\_R^3 d+\_R^2 c+\_R b+a\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{12}$
default	$\frac{a\sqrt{3}\,6^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}{x^2-\frac{\sqrt{3}\,6^{\frac{1}{4}}x\sqrt{2}}{3}+\frac{\sqrt{6}}{3}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{3}\,6^{\frac{3}{4}}x}{6}-1\right)\right)}{48}+\frac{b\arctan\left(\frac{\sqrt{6}\,x^2}{2}\right)\sqrt{6}}{12}+\frac{c\sqrt{3}\cdots}{\cdots}$
meijerg	$\frac{d\ln\left(1+\frac{3x^4}{2}\right)}{12}+\frac{54^{\frac{3}{4}}c\left(\frac{x^3\sqrt{2}\ln\left(1-6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{8-3^{\frac{1}{4}}8^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+6^{\frac{1}{4}}\left(x^4\right)^{\frac{1}{4}}+\frac{\sqrt{3}\sqrt{2}\sqrt{x^4}}{2}\right)}{2\left(x^4\right)^{\frac{3}{4}}}\cdots\right.}{216}\cdots$

input
```
int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x,method=_RETURNVERBOSE)
```

output
```
1/12*sum((_R^3*d+_R^2*c+_R*b+a)/_R^3*ln(x-_R),_R=RootOf(3*_Z^4+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 54479, normalized size of antiderivative = 391.94

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx=\text{Too large to display}$$

input
```
integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(122) = 244$.

Time = 6.11 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.17

$$\int \frac{a + bx + cx^2 + dx^3}{2 + 3x^4}\,dx$$
$$= \text{RootSum}\left(165888t^4 - 55296t^3d + t^2 \cdot \left(6912ac + 3456b^2 + 6912d^2\right) + t\left(-864a^2b - 1152acd - 576b^2d + \right.\right.$$

input
```
integrate((d*x**3+c*x**2+b*x+a)/(3*x**4+2),x)
```

output
```
RootSum(165888*_t**4 - 55296*_t**3*d + _t**2*(6912*a*c + 3456*b**2 + 6912*
d**2) + _t*(-864*a**2*b - 1152*a*c*d - 576*b**2*d + 576*b*c**2 - 384*d**3)
 + 27*a**4 + 72*a**2*b*d + 36*a**2*c**2 - 72*a*b**2*c + 48*a*c*d**2 + 18*b
**4 + 24*b**2*d**2 - 48*b*c**2*d + 12*c**4 + 8*d**4, Lambda(_t, _t*log(x +
 (-41472*_t**3*a**2*c + 82944*_t**3*a*b**2 + 27648*_t**3*c**3 + 5184*_t**2
*a**3*b + 10368*_t**2*a**2*c*d - 20736*_t**2*a*b**2*d + 10368*_t**2*a*b*c*
*2 - 6912*_t**2*b**3*c - 6912*_t**2*c**3*d + 648*_t*a**5 - 864*_t*a**3*b*d
 - 1728*_t*a**3*c**2 + 3888*_t*a**2*b**2*c - 864*_t*a**2*c*d**2 + 864*_t*a
*b**4 + 1728*_t*a*b**2*d**2 - 1728*_t*a*b*c**2*d + 864*_t*a*c**4 + 1152*_t
*b**3*c*d + 864*_t*b**2*c**3 + 576*_t*c**3*d**2 - 54*a**5*d + 270*a**4*b*c
 - 270*a**3*b**3 + 36*a**3*b*d**2 + 144*a**3*c**2*d - 324*a**2*b**2*c*d +
24*a**2*c*d**3 - 72*a*b**4*d + 180*a*b**3*c**2 - 48*a*b**2*d**3 + 72*a*b*c
**2*d**2 - 72*a*c**4*d - 72*b**5*c - 48*b**3*c*d**2 - 72*b**2*c**3*d + 72*
b*c**5 - 16*c**3*d**3)/(81*a**6 - 54*a**4*c**2 + 432*a**3*b**2*c - 216*a**
2*b**4 - 36*a**2*c**4 + 288*a*b**2*c**3 - 144*b**4*c**2 + 24*c**6))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.49

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = -\frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c - 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3\,a\right)\log\left(\sqrt{3}x^2+3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{144} \cdot 3^{\frac{3}{4}}2^{\frac{3}{4}}\left(\sqrt{3}\sqrt{2}c + 2\cdot 3^{\frac{1}{4}}2^{\frac{1}{4}}d - 3\,a\right)\log\left(\sqrt{3}x^2-3^{\frac{1}{4}}2^{\frac{3}{4}}x+\sqrt{2}\right) + \frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a + 2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c - 6\sqrt{2}b\right)\arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x + 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right) + \frac{1}{72}\sqrt{3}\left(3\cdot 3^{\frac{1}{4}}2^{\frac{3}{4}}a + 2\cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}c + 6\sqrt{2}b\right)\arctan\left(\frac{1}{6} \cdot 3^{\frac{3}{4}}2^{\frac{1}{4}}\left(2\sqrt{3}x - 3^{\frac{1}{4}}2^{\frac{3}{4}}\right)\right)$$

input
```
integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="maxima")
```

output
```
-1/144*3^(3/4)*2^(3/4)*(sqrt(3)*sqrt(2)*c - 2*3^(1/4)*2^(1/4)*d - 3*a)*log
(sqrt(3)*x^2 + 3^(1/4)*2^(3/4)*x + sqrt(2)) + 1/144*3^(3/4)*2^(3/4)*(sqrt(
3)*sqrt(2)*c + 2*3^(1/4)*2^(1/4)*d - 3*a)*log(sqrt(3)*x^2 - 3^(1/4)*2^(3/4
)*x + sqrt(2)) + 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c -
 6*sqrt(2)*b)*arctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x + 3^(1/4)*2^(3/4)))
+ 1/72*sqrt(3)*(3*3^(1/4)*2^(3/4)*a + 2*3^(3/4)*2^(1/4)*c + 6*sqrt(2)*b)*a
rctan(1/6*3^(3/4)*2^(1/4)*(2*sqrt(3)*x - 3^(1/4)*2^(3/4)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.07

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx$$
$$= \frac{1}{24}\left(6^{\frac{3}{4}}a - 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+ \frac{1}{24}\left(6^{\frac{3}{4}}a + 2\sqrt{6}b + 2\cdot 6^{\frac{1}{4}}c\right)\arctan\left(\frac{3}{4}\sqrt{2}\left(\frac{2}{3}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}\right)\right)$$
$$+ \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c + 4d\right)\log\left(x^2+\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$
$$- \frac{1}{48}\left(6^{\frac{3}{4}}a - 2\cdot 6^{\frac{1}{4}}c - 4d\right)\log\left(x^2-\sqrt{2}\left(\frac{2}{3}\right)^{\frac{1}{4}}x+\sqrt{\frac{2}{3}}\right)$$

input
```
integrate((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x, algorithm="giac")
```

output
```
1/24*(6^(3/4)*a - 2*sqrt(6)*b + 2*6^(1/4)*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4
)*(2*x + sqrt(2)*(2/3)^(1/4))) + 1/24*(6^(3/4)*a + 2*sqrt(6)*b + 2*6^(1/4)
*c)*arctan(3/4*sqrt(2)*(2/3)^(3/4)*(2*x - sqrt(2)*(2/3)^(1/4))) + 1/48*(6^
(3/4)*a - 2*6^(1/4)*c + 4*d)*log(x^2 + sqrt(2)*(2/3)^(1/4)*x + sqrt(2/3))
- 1/48*(6^(3/4)*a - 2*6^(1/4)*c - 4*d)*log(x^2 - sqrt(2)*(2/3)^(1/4)*x + s
qrt(2/3))
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 1168, normalized size of antiderivative = 8.40

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2 + d*x^3)/(3*x^4 + 2),x)
```

output

```
symsum(log(9*a*b^2 - 864*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/2
4 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*
z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 -
(a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 +
 b^4/9216 + a^4/6144, z, k)^2*a - 9*a^2*c - 6*a*d^2 + 9*b^3*x - 6*c^3 + 14
4*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c
*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 -
(b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d
^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216 + a^4/6144, z
, k)*a*d - 144*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z
^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (
d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^2*b*d)/2304 - (a*b^2*c)/
2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736 + c^4/13824 + b^4/9216
+ a^4/6144, z, k)*b*c + 12*b*c*d - 108*root(z^4 - (d*z^3)/3 + (a*c*z^2)/24
 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/288 + (b*c^2*z)
/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a*c*d^2)/3456 + (a^
2*b*d)/2304 - (a*b^2*c)/2304 + (b^2*d^2)/6912 + (a^2*c^2)/4608 + d^4/20736
 + c^4/13824 + b^4/9216 + a^4/6144, z, k)*a^2*x + 864*root(z^4 - (d*z^3)/3
 + (a*c*z^2)/24 + (d^2*z^2)/24 + (b^2*z^2)/48 - (a*c*d*z)/144 - (b^2*d*z)/
288 + (b*c^2*z)/288 - (a^2*b*z)/192 - (d^3*z)/432 - (b*c^2*d)/3456 + (a...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.23

$$\int \frac{a+bx+cx^2+dx^3}{2+3x^4}\,dx = -\frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} - \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}-2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)a}{24} + \frac{6^{\frac{1}{4}}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)c}{12} - \frac{\sqrt{6}\,atan\left(\frac{\left(\sqrt{2}\,6^{\frac{1}{4}}+2\sqrt{3}\,x\right)6^{\frac{3}{4}}}{6\sqrt{2}}\right)b}{12} - \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{\sqrt{6}\,6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)a}{48} + \frac{6^{\frac{1}{4}}\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} - \frac{6^{\frac{1}{4}}\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)c}{24} + \frac{\log\left(-\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12} + \frac{\log\left(\sqrt{2}\,6^{\frac{1}{4}}x+\sqrt{3}\,x^2+\sqrt{2}\right)d}{12}$$

input
```
int((d*x^3+c*x^2+b*x+a)/(3*x^4+2),x)
```

output
```
( - 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(
1/4)))*a - 4*6**(1/4)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1
/4)))*c - 4*sqrt(6)*atan((sqrt(2)*6**(1/4) - 2*sqrt(3)*x)/(sqrt(2)*6**(1/4
)))*b + 2*sqrt(6)*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*
6**(1/4)))*a + 4*6**(1/4)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6
**(1/4)))*c - 4*sqrt(6)*atan((sqrt(2)*6**(1/4) + 2*sqrt(3)*x)/(sqrt(2)*6**
(1/4)))*b - sqrt(6)*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sq
rt(2))*a + sqrt(6)*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2
))*a + 2*6**(1/4)*log( - sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c -
2*6**(1/4)*log(sqrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*c + 4*log( - s
qrt(2)*6**(1/4)*x + sqrt(3)*x**2 + sqrt(2))*d + 4*log(sqrt(2)*6**(1/4)*x +
 sqrt(3)*x**2 + sqrt(2))*d)/48
```

3.89 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

Optimal result . . . 732
Mathematica [A] (verified) . . . 732
Rubi [A] (verified) . . . 733
Maple [A] (verified) . . . 734
Fricas [A] (verification not implemented) . . . 734
Sympy [A] (verification not implemented) . . . 735
Maxima [A] (verification not implemented) . . . 735
Giac [A] (verification not implemented) . . . 735
Mupad [B] (verification not implemented) . . . 736
Reduce [B] (verification not implemented) . . . 736

Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

output
```
-ln(1-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{1-x^4} \, dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{1-x}dx$$
$$\Big\downarrow \text{16}$$
$$-\log(1-x)$$

input
```
Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln(x-1)$
norman	$-\ln(x-1)$
risch	$-\ln(x-1)$
parallelrisch	$-\ln(x-1)$
meijerg	$-\frac{\ln(-x^4+1)}{4}-\frac{x^3\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}(x^2)}{2}-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1-\right.\right.}{4}$

input
```
int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(x-1)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log{(x-1)}$$

input
```
integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

output
```
-log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(|x-1|\right)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\ln(x-1)$$

input
```
int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
```

output
```
-log(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(x-1)$$

input
```
int((x^3+x^2+x+1)/(-x^4+1),x)
```

output
```
- log(x - 1)
```

3.90 $\int \frac{1+x+x^2+x^3}{1+x^4} \, dx$

Optimal result . . . 737
Mathematica [A] (verified) . . . 737
Rubi [A] (verified) . . . 738
Maple [C] (verified) . . . 739
Fricas [A] (verification not implemented) . . . 740
Sympy [A] (verification not implemented) . . . 740
Maxima [A] (verification not implemented) . . . 741
Giac [A] (verification not implemented) . . . 741
Mupad [B] (verification not implemented) . . . 742
Reduce [B] (verification not implemented) . . . 742

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{1+x+x^2+x^3}{1+x^4} \, dx = \frac{\arctan(x^2)}{2} - \frac{\arctan(1-\sqrt{2}x)}{\sqrt{2}} + \frac{\arctan(1+\sqrt{2}x)}{\sqrt{2}} + \frac{1}{4}\log(1+x^4)$$

output
```
1/2*arctan(x^2)+1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(1+x*2^(1/2))*2
^(1/2)+1/4*ln(x^4+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1+x+x^2+x^3}{1+x^4} \, dx = \frac{1}{4}\left(-2\left(1+\sqrt{2}\right)\arctan\left(1-\sqrt{2}x\right) + 2\left(-1+\sqrt{2}\right)\arctan\left(1+\sqrt{2}x\right) + \log\left(1+x^4\right)\right)$$

input
```
Integrate[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

output
```
(-2*(1 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(-1 + Sqrt[2])*ArcTan[1 + Sqrt
[2]*x] + Log[1 + x^4])/4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{x^4+1} \, dx$$

$$\Big\downarrow \text{ 2415}$$

$$\int \left(\frac{x(x^2+1)}{x^4+1}+\frac{x^2+1}{x^4+1}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(x^2\right)}{2}-\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}+\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}+\frac{1}{4}\log\left(x^4+1\right)$$

input
```
Int[(1 + x + x^2 + x^3)/(1 + x^4),x]
```

output
```
ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/Sqrt[2] + ArcTan[1 + Sqrt[2]*x]/Sqrt
[2] + Log[1 + x^4]/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\left(\_R^3+\_R^2+\_R+1\right)\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{8}+\frac{\arctan\left(x^2\right)}{2}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\,\mathrm{arc}\right.}{8}$
meijerg	$\frac{\ln(x^4+1)}{4}+\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\mathrm{arc}}{}$

input
```
int((x^3+x^2+x+1)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\sqrt{2}x+1\right)+\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\sqrt{2}x-1\right) + \frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2) - 1)*arctan(sqrt(2)*x + 1) + 1/2*(sqrt(2) + 1)*arctan(sqrt(2)
*x - 1) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{\log\left(x^2-\sqrt{2}x+1\right)}{4}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4}+2 \cdot \left(\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x-1\right)+2\left(-\frac{1}{4}+\frac{\sqrt{2}}{4}\right)\operatorname{atan}\left(\sqrt{2}x+1\right)$$

input
```
integrate((x**3+x**2+x+1)/(x**4+1),x)
```

output
```
log(x**2 - sqrt(2)*x + 1)/4 + log(x**2 + sqrt(2)*x + 1)/4 + 2*(1/4 + sqrt(
2)/4)*atan(sqrt(2)*x - 1) + 2*(-1/4 + sqrt(2)/4)*atan(sqrt(2)*x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = -\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(
2)*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(
2)*x + 1) + 1/4*log(x^2 - sqrt(2)*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) + \frac{1}{4}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{4}\log\left(x^2-\sqrt{2}x+1\right)$$

input
```
integrate((x^3+x^2+x+1)/(x^4+1),x, algorithm="giac")
```

output
```
1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*(sqrt(2) + 1)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2))) + 1/4*log(x^2 + sqrt(2)*x + 1) + 1/4*l
og(x^2 - sqrt(2)*x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.94

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = \ln\left((16\,x-16)\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right)-8\,x\right)\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right)-\ln\left(8\,x+(16\,x-16)\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{-2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right)-\ln\left(8\,x+(16\,x-16)\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}-\frac{1}{4}\right)+\ln\left(8\,x-(16\,x-16)\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right)\right)\left(\frac{\sqrt{2\,\sqrt{2}-3}}{4}+\frac{1}{4}\right)$$

input
```
int((x + x^2 + x^3 + 1)/(x^4 + 1),x)
```

output
```
log((16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 + 1/4) - 8*x)*((- 2*2^(1/2) - 3
)^(1/2)/4 + 1/4) - log(8*x + (16*x - 16)*((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4)
)*((- 2*2^(1/2) - 3)^(1/2)/4 - 1/4) - log(8*x + (16*x - 16)*((2*2^(1/2) -
3)^(1/2)/4 - 1/4))*((2*2^(1/2) - 3)^(1/2)/4 - 1/4) + log(8*x - (16*x - 16)
*((2*2^(1/2) - 3)^(1/2)/4 + 1/4))*((2*2^(1/2) - 3)^(1/2)/4 + 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \frac{1+x+x^2+x^3}{1+x^4}\,dx = -\frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)}{2}-\frac{atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)}{2}+\frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)}{2}-\frac{atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)}{2}+\frac{\log\left(-\sqrt{2}\,x+x^2+1\right)}{4}+\frac{\log\left(\sqrt{2}\,x+x^2+1\right)}{4}$$

input
```
int((x^3+x^2+x+1)/(x^4+1),x)
```

output
```
( - 2*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2)) - 2*atan((sqrt(2) - 2*x)/sqrt(
2)) + 2*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2)) - 2*atan((sqrt(2) + 2*x)/sqr
t(2)) + log( - sqrt(2)*x + x**2 + 1) + log(sqrt(2)*x + x**2 + 1))/4
```

3.91 $\int \frac{1+x+x^2+x^3}{a-bx^4} \, dx$

Optimal result 744
Mathematica [A] (verified) 745
Rubi [A] (verified) 745
Maple [C] (verified) 747
Fricas [C] (verification not implemented) 747
Sympy [A] (verification not implemented) 748
Maxima [A] (verification not implemented) 748
Giac [B] (verification not implemented) 749
Mupad [B] (verification not implemented) 750
Reduce [B] (verification not implemented) 751

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{1+x+x^2+x^3}{a-bx^4} \, dx = -\frac{\left(\sqrt{a}-\sqrt{b}\right) \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right) \operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\log\left(a-bx^4\right)}{4b}$$

output
```
-1/2*(a^(1/2)-b^(1/2))*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(3/4)+1/2*(a^(1
/2)+b^(1/2))*arctanh(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(3/4)+1/2*arctanh(b^(1/2
)*x^2/a^(1/2))/a^(1/2)/b^(1/2)-1/4*ln(-b*x^4+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \frac{\left(-a^{3/4}+\sqrt[4]{a}\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}} - \frac{\left(a^{3/4}+\sqrt{a}\sqrt[4]{b}+\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}} - \frac{\left(-a^{3/4}+\sqrt{a}\sqrt[4]{b}-\sqrt[4]{a}\sqrt{b}\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}} + \frac{\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}} - \frac{\log\left(a-bx^4\right)}{4b}$$

input
```
Integrate[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

output
```
((-a^(3/4) + a^(1/4)*Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)) -
 ((a^(3/4) + Sqrt[a]*b^(1/4) + a^(1/4)*Sqrt[b])*Log[a^(1/4) - b^(1/4)*x])/
(4*a*b^(3/4)) - ((-a^(3/4) + Sqrt[a]*b^(1/4) - a^(1/4)*Sqrt[b])*Log[a^(1/4
) + b^(1/4)*x])/(4*a*b^(3/4)) + Log[Sqrt[a] + Sqrt[b]*x^2]/(4*Sqrt[a]*Sqrt
[b]) - Log[a - b*x^4]/(4*b)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{a-bx^4}\,dx$$

$$\Big\downarrow \; 2415$$

$$\int \left(\frac{x(x^2+1)}{a-bx^4}+\frac{x^2+1}{a-bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{\log\left(a-bx^4\right)}{4b}$$

input
```
Int[(1 + x + x^2 + x^3)/(a - b*x^4),x]
```

output
```
-1/2*((Sqrt[a] - Sqrt[b])*ArcTan[(b^(1/4)*x)/a^(1/4)])/(a^(3/4)*b^(3/4)) +
 ((Sqrt[a] + Sqrt[b])*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - Log[a - b*x^4]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.31

method	result	siz
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}-a\right)}\frac{\left(\_R^{3}+\_R^{2}+\_R+1\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$	38
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{\ln\left(\frac{a+x^{2}\sqrt{ab}}{a-x^{2}\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\ln\left(-b\,x^{4}+a\right)}{4b}$	15

input
```
int((x^3+x^2+x+1)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 91748, normalized size of antiderivative = 739.90

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.51

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx =$$
$$-\operatorname{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\cdot\left(96a^3b^2 - 96a^2b^3\right) + t\left(-16a^3b + 32a^2b^2 - 16ab^3\right) + a^3 - 3a^2\right.$$

input
```
integrate((x**3+x**2+x+1)/(-b*x**4+a),x)
```

output
```
-RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 -
 96*a**2*b**3) + _t*(-16*a**3*b + 32*a**2*b**2 - 16*a*b**3) + a**3 - 3*a**
2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**3 + 48*_t*
*2*a**3*b**2 + 16*_t**2*a**2*b**3 - 12*_t*a**3*b + 16*_t*a**2*b**2 - 4*_t*
a*b**3 + a**3 - 2*a**2*b + a*b**2)/(a**2*b - 2*a*b**2 + b**3))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = -\frac{\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*
sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(a) - sqrt(b))*log(sqrt(b)*x^2 +
 sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log(sqrt(b)*x^2 - sqrt(a))
/(sqrt(a)*b) - 1/4*(sqrt(a) + sqrt(b))*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(
b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.34

$$
\begin{aligned}
&\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx \\
&= -\frac{\log\left(|bx^4-a|\right)}{4\,b} \\
&\quad + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2 - \sqrt{2}\sqrt{-ab^3}b + (-ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} \\
&\quad + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2 + \sqrt{2}\sqrt{-ab^3}b + (-ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} \\
&\quad + \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2 - (-ab^3)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3} \\
&\quad - \frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2 - (-ab^3)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,ab^3}
\end{aligned}
$$

input
```
integrate((x^3+x^2+x+1)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*log(abs(b*x^4 - a))/b + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 - sqrt(2)*sqr
t(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/
4))/(-a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((-a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt
(-a*b^3)*b + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4
))/(-a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4
))*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(a*b^3) - 1/8*sqrt(2)*((
-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqr
t(-a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.52

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx = \sum_{k=1}^{4} \ln\Big(-\mathrm{root}\big(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2 - 96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3 +a^3,z,k\big)\,\big(\mathrm{root}\big(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2+16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2 \ldots - x\,\big(4\,a\,b^2-4\,b^3\big)\big)\Big)\,\mathrm{root}\big(256\,a^3\,b^4\,z^4+256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2-96\,a^2\,b^3\,z^2 +16\,a^3\,b\,z+16\,a\,b^3\,z-32\,a^2\,b^2\,z-3\,a^2\,b+3\,a\,b^2-b^3+a^3,z,k\big)$$

input
```
int((x + x^2 + x^3 + 1)/(a - b*x^4),x)
```

output
```
symsum(log(-root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a
^2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 -
b^3 + a^3, z, k)*(root(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2
- 96*a^2*b^3*z^2 + 16*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*
b^2 - b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) - x*(4*a*b^2 - 4*b^3)))*roo
t(256*a^3*b^4*z^4 + 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 - 96*a^2*b^3*z^2 + 16
*a^3*b*z + 16*a*b^3*z - 32*a^2*b^2*z - 3*a^2*b + 3*a*b^2 - b^3 + a^3, z, k
), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.67

$$\int \frac{1+x+x^2+x^3}{a-bx^4}\,dx$$
$$= \frac{-2b^{\frac{1}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)+2b^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)-b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)+b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}x\right)-b^{\frac{3}{4}}a^{\frac{1}{4}}\log\left(a^{\frac{1}{4}}-\right.}{}$$

input
```
int((x^3+x^2+x+1)/(-b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4))) + 2*b**(3/4)
*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4))) - b**(1/4)*a**(3/4)*log(a*
*(1/4) - b**(1/4)*x) + b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x) - b**(
3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x) + b**(3/4)*a**(1/4)*log(a**(1/4)
+ b**(1/4)*x) - sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x) - sqrt(b)*sqrt(
a)*log(a**(1/4) + b**(1/4)*x) + sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2
) - log(a**(1/4) - b**(1/4)*x)*a - log(a**(1/4) + b**(1/4)*x)*a - log(sqrt
(a) + sqrt(b)*x**2)*a)/(4*a*b)
```

3.92 $\int \frac{1+x+x^2+x^3}{a+bx^4} \, dx$

Optimal result 752
Mathematica [A] (verified) 753
Rubi [A] (verified) 753
Maple [C] (verified) 755
Fricas [C] (verification not implemented) 755
Sympy [A] (verification not implemented) 756
Maxima [B] (verification not implemented) 756
Giac [A] (verification not implemented) 757
Mupad [B] (verification not implemented) 758
Reduce [B] (verification not implemented) 759

Optimal result

Integrand size = 19, antiderivative size = 213

$$\int \frac{1+x+x^2+x^3}{a+bx^4} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\log(a+bx^4)}{4b}$$

output
```
1/2*arctan(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)+1/4*(a^(1/2)+b^(1/2))*arct
an(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4)+1/4*(a^(1/2)+b^(1
/2))*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4)-1/4*(a^(1
/2)-b^(1/2))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1
/2)/a^(3/4)/b^(3/4)+1/4*ln(b*x^4+a)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.33

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$$
$$= \frac{-2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}+2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\left(\sqrt{2}\sqrt{a}-2\sqrt[4]{a}\sqrt[4]{b}+\sqrt{2}\sqrt{b}\right)\sqrt[4]{b}\,\mathrm{arc}}{}$$

input
```
Integrate[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

output
```
(-2*a^(1/4)*(Sqrt[2]*Sqrt[a] + 2*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*(Sqrt[2]*Sqrt[a] - 2
*a^(1/4)*b^(1/4) + Sqrt[2]*Sqrt[b])*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)] + Sqrt[2]*(a^(3/4) - a^(1/4)*Sqrt[b])*b^(1/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(-a^(3/4) + a^(1/4)*Sqrt[b]
)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 2*a*Log
[a + b*x^4])/(8*a*b)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{a+bx^4}\,dx$$
$$\Big\downarrow \ 2415$$
$$\int \left(\frac{x(x^2+1)}{a+bx^4}+\frac{x^2+1}{a+bx^4}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(a+bx^4\right)}{4b}$$

input
```
Int[(1 + x + x^2 + x^3)/(a + b*x^4),x]
```

output
```
ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])*A
rcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sq
rt[a] + Sqrt[b])*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b])*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b])*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/
4)) + Log[a + b*x^4]/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.17

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(\_R^{3}+\_R^{2}+\_R+1\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b}$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)}{x^{2}+\left(\frac{a}{b}\right)}\right.\right.}{}$

input
```
int((x^3+x^2+x+1)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R^3+_R^2+_R+1)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 96349, normalized size of antiderivative = 452.34

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3b^4 - 256t^3a^3b^3 + t^2\cdot\left(96a^3b^2 + 96a^2b^3\right) + t\left(-16a^3b - 32a^2b^2 - 16ab^3\right) + a^3 + 3a^2b +\right.$$

input
```
integrate((x**3+x**2+x+1)/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*b**4 - 256*_t**3*a**3*b**3 + _t**2*(96*a**3*b**2 +
96*a**2*b**3) + _t*(-16*a**3*b - 32*a**2*b**2 - 16*a*b**3) + a**3 + 3*a**2
*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**3 - 48*_t**2
*a**3*b**2 + 16*_t**2*a**2*b**3 + 12*_t*a**3*b + 16*_t*a**2*b**2 + 4*_t*a*
b**3 - a**3 - 2*a**2*b - a*b**2)/(a**2*b + 2*a*b**2 + b**3))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(142) = 284.

Time = 0.11 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.39

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}} - \sqrt{a}\sqrt{b} + b\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}$$
$$+ \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}} + \sqrt{a}\sqrt{b} - b\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}}$$
$$+ \frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\,\sqrt{a}\right)b + \left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}} + 2\,a\right)\sqrt{b} - 2\,a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}}$$
$$+ \frac{\left(\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\,\sqrt{a}\right)b + \left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}} - 2\,a\right)\sqrt{b} + 2\,a\sqrt{b}\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}}$$

input
```
integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4) - sqrt(a)*sqrt(b) + b)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqrt(2)*(
sqrt(2)*a^(3/4)*b^(1/4) + sqrt(a)*sqrt(b) - b)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) + 2*a)*sqrt(b) - 2*a*sqrt(b)
)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*((sqrt(2)*a^(1/4)*
b^(1/4) + 2*sqrt(a))*b + (sqrt(2)*a^(3/4)*b^(1/4) - 2*a)*sqrt(b) + 2*a*sqr
t(b))*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt
(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.27

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \frac{\log\left(|bx^4+a|\right)}{4\,b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 - \sqrt{2}\sqrt{ab^3}b + (ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 + \sqrt{2}\sqrt{ab^3}b + (ab^3)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 - (ab^3)^{\frac{3}{4}}\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2 - (ab^3)^{\frac{3}{4}}\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((x^3+x^2+x+1)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 - sqrt(2)*sqrt(
a*b^3)*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2 + sqrt(2)*sqrt(a*b^3)
*b + (a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2 - (a*b^3)^(3/4))*log(x^2 +
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^
2 - (a*b^3)^(3/4))*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.43

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+3\,a\,b^2+b^3+a^3,z,k\right)\,\left(\mathrm{root}\left(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,\ldots + x\,\left(4\,b^3+4\,a\,b^2\right)\right)\right)\,\mathrm{root}\left(256\,a^3\,b^4\,z^4-256\,a^3\,b^3\,z^3+96\,a^3\,b^2\,z^2+96\,a^2\,b^3\,z^2-16\,a^3\,b\,z-16\,a\,b^3\,z-32\,a^2\,b^2\,z+3\,a^2\,b+3\,a\,b^2+b^3+a^3,z,k\right)$$

input
```
int((x + x^2 + x^3 + 1)/(a + b*x^4),x)
```

output
```
symsum(log(-root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a
^2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 +
b^3 + a^3, z, k)*(root(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2
+ 96*a^2*b^3*z^2 - 16*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*
b^2 + b^3 + a^3, z, k)*(16*a*b^3 - 16*a*b^3*x) + x*(4*a*b^2 + 4*b^3)))*roo
t(256*a^3*b^4*z^4 - 256*a^3*b^3*z^3 + 96*a^3*b^2*z^2 + 96*a^2*b^3*z^2 - 16
*a^3*b*z - 16*a*b^3*z - 32*a^2*b^2*z + 3*a^2*b + 3*a*b^2 + b^3 + a^3, z, k
), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.83

$$\int \frac{1+x+x^2+x^3}{a+bx^4}\,dx = \frac{-2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-4\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2b^{\frac{1}{4}}a}{}$$

input
```
int((x^3+x^2+x+1)/(b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(
1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) - 4*sqrt
(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1
/4)*sqrt(2))) + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2
))) - 4*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b*
*(1/4)*a**(1/4)*sqrt(2))) + b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(
1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2) - b**(1/4)*a**(3/4)*sqrt(2)*log(b
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2) - b**(3/4)*a**(1/4)*s
qrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2) + b**(
3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*
x**2) + 2*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a +
 2*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a)/(8*a*b)
```

3.93 $\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx=-\frac{1}{4}(1-x)^4$$

output
```
-1/4*(1-x)^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx=-\frac{1}{4}(-1+x)^4$$

input
```
Integrate[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

output
```
-1/4*(-1 + x)^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2006, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^4)^3}{(x^3+x^2+x+1)^3}dx$$

$$\Big\downarrow \ 2006$$

$$\int (1-x)^3dx$$

$$\Big\downarrow \ 17$$

$$-\frac{1}{4}(1-x)^4$$

input
```
Int[(1 - x^4)^3/(1 + x + x^2 + x^3)^3,x]
```

output
```
-1/4*(1 - x)^4
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] && !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{(x-1)^{4}}{4}$	8
parallelrisch	$-\frac{1}{4}x^{4}+x^{3}-\frac{3}{2}x^{2}+x$	16
gosper	$-\frac{x(x^{3}-4x^{2}+6x-4)}{4}$	17
risch	$-\frac{1}{4}x^{4}+x^{3}-\frac{3}{2}x^{2}+x-\frac{1}{4}$	17
orering	$\frac{x(x^{3}-4x^{2}+6x-4)(-x^{4}+1)^{3}}{4(x-1)^{3}(x^{3}+x^{2}+x+1)^{3}}$	42
norman	$\frac{-2x^{5}-2x^{3}-x^{4}-\frac{7}{4}x^{2}-\frac{1}{2}x-\frac{1}{4}x^{8}+\frac{1}{2}x^{9}-\frac{1}{4}x^{10}-\frac{3}{4}}{(x^{3}+x^{2}+x+1)^{2}}$	53

input
```
int((-x^4+1)^3/(x^3+x^2+x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x-1)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx=-\frac{1}{4}\,x^4+x^3-\frac{3}{2}\,x^2+x$$

input
```
integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="fricas")
```

output
```
-1/4*x^4 + x^3 - 3/2*x^2 + x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{x^4}{4} + x^3 - \frac{3x^2}{2} + x$$

input
```
integrate((-x**4+1)**3/(x**3+x**2+x+1)**3,x)
```

output
```
-x**4/4 + x**3 - 3*x**2/2 + x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4 + x^3 - \frac{3}{2}\,x^2 + x$$

input
```
integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="maxima")
```

output
```
-1/4*x^4 + x^3 - 3/2*x^2 + x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{1}{4}\,x^4 + x^3 - \frac{3}{2}\,x^2 + x$$

input
```
integrate((-x^4+1)^3/(x^3+x^2+x+1)^3,x, algorithm="giac")
```

output
```
-1/4*x^4 + x^3 - 3/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = -\frac{x^4}{4}+x^3-\frac{3\,x^2}{2}+x$$

input
```
int(-(x^4 - 1)^3/(x + x^2 + x^3 + 1)^3,x)
```

output
```
x - (3*x^2)/2 + x^3 - x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{(1-x^4)^3}{(1+x+x^2+x^3)^3}\,dx = \frac{x(-x^3+4x^2-6x+4)}{4}$$

input
```
int((-x^4+1)^3/(x^3+x^2+x+1)^3,x)
```

output
```
(x*( - x**3 + 4*x**2 - 6*x + 4))/4
```

3.94 $\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx$

Optimal result 765
Mathematica [A] (verified) 765
Rubi [A] (verified) 766
Maple [A] (verified) 767
Fricas [A] (verification not implemented) 767
Sympy [A] (verification not implemented) 768
Maxima [A] (verification not implemented) 768
Giac [A] (verification not implemented) 768
Mupad [B] (verification not implemented) 769
Reduce [B] (verification not implemented) 769

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx = -\frac{1}{3}(1-x)^3$$

output
```
-1/3*(1-x)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2} dx = x - x^2 + \frac{x^3}{3}$$

input
```
Integrate[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

output
```
x - x^2 + x^3/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^4)^2}{(x^3+x^2+x+1)^2} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int (1-x)^2 dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{3}(1-x)^3$$

input
```
Int[(1 - x^4)^2/(1 + x + x^2 + x^3)^2,x]
```

output
```
-1/3*(1 - x)^3
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{(x-1)^3}{3}$	8
gosper	$\frac{x(x^2-3x+3)}{3}$	12
parallelrisch	$\frac{1}{3}x^3-x^2+x$	13
risch	$\frac{1}{3}x^3-x^2+x-\frac{1}{3}$	14
orering	$\frac{x(x^2-3x+3)(-x^4+1)^2}{3(x-1)^2(x^3+x^2+x+1)^2}$	37
norman	$\frac{-\frac{1}{3}x^2+\frac{2}{3}x+\frac{1}{3}x^4-\frac{2}{3}x^5+\frac{1}{3}x^6-\frac{1}{3}}{x^3+x^2+x+1}$	38

input
```
int((-x^4+1)^2/(x^3+x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx=\frac{1}{3}\,x^3-x^2+x$$

input
```
integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="fricas")
```

output
```
1/3*x^3 - x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{x^3}{3} - x^2 + x$$

input
```
integrate((-x**4+1)**2/(x**3+x**2+x+1)**2,x)
```

output
```
x**3/3 - x**2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3 - x^2 + x$$

input
```
integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="maxima")
```

output
```
1/3*x^3 - x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{1}{3}\,x^3 - x^2 + x$$

input
```
integrate((-x^4+1)^2/(x^3+x^2+x+1)^2,x, algorithm="giac")
```

output
```
1/3*x^3 - x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{x\,(x^2-3\,x+3)}{3}$$

input
```
int((x^4 - 1)^2/(x + x^2 + x^3 + 1)^2,x)
```

output
```
(x*(x^2 - 3*x + 3))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^4)^2}{(1+x+x^2+x^3)^2}\,dx = \frac{x(x^2-3x+3)}{3}$$

input
```
int((-x^4+1)^2/(x^3+x^2+x+1)^2,x)
```

output
```
(x*(x**2 - 3*x + 3))/3
```

3.95 $\int \frac{1-x^4}{1+x+x^2+x^3} \, dx$

Optimal result 770
Mathematica [A] (verified) 770
Rubi [A] (verified) 771
Maple [A] (verified) 772
Fricas [A] (verification not implemented) 772
Sympy [A] (verification not implemented) 773
Maxima [A] (verification not implemented) 773
Giac [A] (verification not implemented) 773
Mupad [B] (verification not implemented) 774
Reduce [B] (verification not implemented) 774

Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = -\frac{1}{2}(1-x)^2$$

output
```
-1/2*(1-x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1-x^4}{1+x+x^2+x^3} \, dx = x - \frac{x^2}{2}$$

input
```
Integrate[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

output
```
x - x^2/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^4}{x^3+x^2+x+1} \, dx$$
$$\Big\downarrow \; 2019$$
$$\int (1-x)dx$$
$$\Big\downarrow \; 17$$
$$-\frac{1}{2}(1-x)^2$$

input
```
Int[(1 - x^4)/(1 + x + x^2 + x^3),x]
```

output
```
-1/2*(1 - x)^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{x(x-2)}{2}$	7
default	$x-\frac{1}{2}x^{2}$	8
norman	$x-\frac{1}{2}x^{2}$	8
risch	$x-\frac{1}{2}x^{2}$	8
parallelrisch	$x-\frac{1}{2}x^{2}$	8
parts	$x-\frac{1}{2}x^{2}$	8
orering	$\frac{x(x-2)(-x^{4}+1)}{2(x-1)(x^{3}+x^{2}+x+1)}$	30

input
```
int((-x^4+1)/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(x-2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx=-\frac{1}{2}\,x^2+x$$

input
```
integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="fricas")
```

output
```
-1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = -\frac{x^2}{2}+x$$

input
```
integrate((-x**4+1)/(x**3+x**2+x+1),x)
```

output
```
-x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = -\frac{1}{2}\,x^2+x$$

input
```
integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="maxima")
```

output
```
-1/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = -\frac{1}{2}\,x^2+x$$

input
```
integrate((-x^4+1)/(x^3+x^2+x+1),x, algorithm="giac")
```

output
```
-1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = -\frac{x\,(x-2)}{2}$$

input
```
int(-(x^4 - 1)/(x + x^2 + x^3 + 1),x)
```

output
```
-(x*(x - 2))/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1-x^4}{1+x+x^2+x^3}\,dx = \frac{x(-x+2)}{2}$$

input
```
int((-x^4+1)/(x^3+x^2+x+1),x)
```

output
```
(x*( - x + 2))/2
```

3.96 $\int \frac{1+x+x^2+x^3}{1-x^4} \, dx$

Optimal result 775
Mathematica [A] (verified) 775
Rubi [A] (verified) 776
Maple [A] (verified) 777
Fricas [A] (verification not implemented) 777
Sympy [A] (verification not implemented) 778
Maxima [A] (verification not implemented) 778
Giac [A] (verification not implemented) 778
Mupad [B] (verification not implemented) 779
Reduce [B] (verification not implemented) 779

Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

output
```
-ln(1-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3}{1-x^4} \, dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2+x+1}{1-x^4} \, dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{1-x}dx$$
$$\Big\downarrow \text{16}$$
$$-\log(1-x)$$

input
```
Int[(1 + x + x^2 + x^3)/(1 - x^4),x]
```

output
```
-Log[1 - x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result
default	$-\ln\left(x-1\right)$
norman	$-\ln\left(x-1\right)$
risch	$-\ln\left(x-1\right)$
parallelrisch	$-\ln\left(x-1\right)$
meijerg	$-\frac{\ln\left(-x^{4}+1\right)}{4}-\frac{x^{3}\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{4\left(x^{4}\right)^{\frac{3}{4}}}+\frac{\operatorname{arctanh}\left(x^{2}\right)}{2}-\frac{x\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1-\right.\right.}{4}$

input
```
int((x^3+x^2+x+1)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx=-\log\left(x-1\right)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log{(x-1)}$$

input
```
integrate((x**3+x**2+x+1)/(-x**4+1),x)
```

output
```
-log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(x-1\right)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log\left(|x-1|\right)$$

input
```
integrate((x^3+x^2+x+1)/(-x^4+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\ln(x-1)$$

input
```
int(-(x + x^2 + x^3 + 1)/(x^4 - 1),x)
```

output
```
-log(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3}{1-x^4}\,dx = -\log(x-1)$$

input
```
int((x^3+x^2+x+1)/(-x^4+1),x)
```

output
```
 - log(x - 1)
```

3.97 $\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx$

Optimal result 780
Mathematica [A] (verified) 780
Rubi [A] (verified) 781
Maple [A] (verified) 782
Fricas [A] (verification not implemented) 782
Sympy [A] (verification not implemented) 783
Maxima [A] (verification not implemented) 783
Giac [A] (verification not implemented) 783
Mupad [B] (verification not implemented) 784
Reduce [B] (verification not implemented) 784

Optimal result

Integrand size = 21, antiderivative size = 7

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx = \frac{1}{1-x}$$

output
```
1/(1-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2} dx = -\frac{1}{-1+x}$$

input
```
Integrate[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

output
```
-(-1 + x)^(-1)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3+x^2+x+1)^2}{(1-x^4)^2} \, dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{(1-x)^2} dx$$
$$\Big\downarrow \text{17}$$
$$\frac{1}{1-x}$$

input
```
Int[(1 + x + x^2 + x^3)^2/(1 - x^4)^2,x]
```

output
```
(1 - x)^(-1)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result
gosper	$-\frac{1}{x-1}$
default	$-\frac{1}{x-1}$
risch	$-\frac{1}{x-1}$
parallelrisch	$-\frac{1}{x-1}$
norman	$\frac{-x^3-x^2-x-1}{x^4-1}$
orering	$\frac{(1-x)\left(x^3+x^2+x+1\right)^2}{\left(-x^4+1\right)^2}$
meijerg	$\frac{(-1)^{\frac{1}{4}}\left(-\frac{x^3(-1)^{\frac{3}{4}}}{-x^4+1}-\frac{3x^3(-1)^{\frac{3}{4}}\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{4\left(x^4\right)^{\frac{3}{4}}}\right)}{4}+\frac{i\left(-\frac{ix^2}{-x^4+1}+i\operatorname{arctanh}\left(x^2\right)\right)}{2}+ \frac{3(\cdots}{\cdots}$

input
```
int((x^3+x^2+x+1)^2/(-x^4+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\left(1+x+x^2+x^3\right)^2}{\left(1-x^4\right)^2}\,dx=-\frac{1}{x-1}$$

input
```
integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="fricas")
```

output
```
-1/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
integrate((x**3+x**2+x+1)**2/(-x**4+1)**2,x)
```

output
```
-1/(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="maxima")
```

output
```
-1/(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
integrate((x^3+x^2+x+1)^2/(-x^4+1)^2,x, algorithm="giac")
```

output
```
-1/(x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{1}{x-1}$$

input
```
int((x + x^2 + x^3 + 1)^2/(x^4 - 1)^2,x)
```

output
```
-1/(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{(1+x+x^2+x^3)^2}{(1-x^4)^2}\,dx = -\frac{x}{x-1}$$

input
```
int((x^3+x^2+x+1)^2/(-x^4+1)^2,x)
```

output
```
( - x)/(x - 1)
```

3.98 $\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx$

Optimal result 785
Mathematica [A] (verified) 785
Rubi [A] (verified) 786
Maple [A] (verified) 787
Fricas [A] (verification not implemented) 787
Sympy [A] (verification not implemented) 788
Maxima [A] (verification not implemented) 788
Giac [A] (verification not implemented) 788
Mupad [B] (verification not implemented) 789
Reduce [B] (verification not implemented) 789

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2(1-x)^2}$$

output
```
1/2/(1-x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2(-1+x)^2}$$

input
```
Integrate[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

output
```
1/(2*(-1 + x)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3+x^2+x+1)^3}{(1-x^4)^3}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(1-x)^3}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{2(1-x)^2}$$

input
```
Int[(1 + x + x^2 + x^3)^3/(1 - x^4)^3,x]
```

output
```
1/(2*(1 - x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result
gosper	$\frac{1}{2(x-1)^2}$
default	$\frac{1}{2(x-1)^2}$
risch	$\frac{1}{2(x-1)^2}$
parallelrisch	$\frac{1}{2(x-1)^2}$
orering	$\frac{\left(-\frac{x}{2}+\frac{1}{2}\right)\left(x^3+x^2+x+1\right)^3}{\left(-x^4+1\right)^3}$
norman	$\frac{x+x^5+\frac{3}{2}x^4+\frac{3}{2}x^2+2x^3+\frac{1}{2}x^6+\frac{1}{2}}{\left(x^4-1\right)^2}$
meijerg	$-\frac{(-1)^{\frac{3}{4}}\left(\frac{x(-1)^{\frac{1}{4}}\left(-7x^4+11\right)}{4\left(-x^4+1\right)^2}-\frac{21x(-1)^{\frac{1}{4}}\left(\ln\left(1-\left(x^4\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^4\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^4\right)^{\frac{1}{4}}\right)\right)}{16\left(x^4\right)^{\frac{1}{4}}}\right)}{8} - \frac{3(-1)^{\frac{1}{4}}\left(\frac{x^3(-1)^{\frac{3}{4}}\left(-15x\right.}{12\left(-x^4+1\right)}\right.}{}$

input
```
int((x^3+x^2+x+1)^3/(-x^4+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/(x-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\left(1+x+x^2+x^3\right)^3}{\left(1-x^4\right)^3}\,dx = \frac{1}{2\left(x^2-2\,x+1\right)}$$

input
```
integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="fricas")
```

output
```
1/2/(x^2 - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2x^2-4x+2}$$

input
```
integrate((x**3+x**2+x+1)**3/(-x**4+1)**3,x)
```

output
```
1/(2*x**2 - 4*x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x^2-2\,x+1)}$$

input
```
integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="maxima")
```

output
```
1/2/(x^2 - 2*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x-1)^2}$$

input
```
integrate((x^3+x^2+x+1)^3/(-x^4+1)^3,x, algorithm="giac")
```

output
```
1/2/(x - 1)^2
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2\,(x-1)^2}$$

input
```
int(-(x + x^2 + x^3 + 1)^3/(x^4 - 1)^3,x)
```

output
```
1/(2*(x - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{(1+x+x^2+x^3)^3}{(1-x^4)^3}\,dx = \frac{1}{2x^2-4x+2}$$

input
```
int((x^3+x^2+x+1)^3/(-x^4+1)^3,x)
```

output
```
1/(2*(x**2 - 2*x + 1))
```

3.99 $\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx$

Optimal result 790
Mathematica [A] (verified) 790
Rubi [A] (verified) 791
Maple [A] (verified) 792
Fricas [B] (verification not implemented) 792
Sympy [B] (verification not implemented) 793
Maxima [B] (verification not implemented) 793
Giac [A] (verification not implemented) 793
Mupad [B] (verification not implemented) 794
Reduce [B] (verification not implemented) 794

Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx = \frac{1}{3(1-x)^3}$$

output
```
1/3/(1-x)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4} dx = -\frac{1}{3(-1+x)^3}$$

input
```
Integrate[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

output
```
-1/3*1/(-1 + x)^3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^3+x^2+x+1)^4}{(1-x^4)^4} \, dx$$
$$\Big\downarrow \; 2019$$
$$\int \frac{1}{(1-x)^4} dx$$
$$\Big\downarrow \; 17$$
$$\frac{1}{3(1-x)^3}$$

input
```
Int[(1 + x + x^2 + x^3)^4/(1 - x^4)^4,x]
```

output
```
1/(3*(1 - x)^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{1}{3(x-1)^3}$	8
default	$-\frac{1}{3(x-1)^3}$	8
risch	$-\frac{1}{3(x-1)^3}$	8
parallelrisch	$-\frac{1}{3(x-1)^3}$	8
orering	$\frac{\left(-\frac{x}{3}+\frac{1}{3}\right)\left(x^3+x^2+x+1\right)^4}{\left(-x^4+1\right)^4}$	27
norman	$\frac{-4x^4-x^8-x-2x^2-\frac{10}{3}x^3-4x^5-\frac{10}{3}x^6-2x^7-\frac{1}{3}x^9-\frac{1}{3}}{\left(x^4-1\right)^3}$	54
meijerg	Expression too large to display	698

input
```
int((x^3+x^2+x+1)^4/(-x^4+1)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(x-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="fricas")
```

output
```
-1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3x^3-9x^2+9x-3}$$

input
```
integrate((x**3+x**2+x+1)**4/(-x**4+1)**4,x)
```

output
```
-1/(3*x**3 - 9*x**2 + 9*x - 3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="maxima")
```

output
```
-1/3/(x^3 - 3*x^2 + 3*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x-1)^3}$$

input
```
integrate((x^3+x^2+x+1)^4/(-x^4+1)^4,x, algorithm="giac")
```

output
```
-1/3/(x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3\,(x-1)^3}$$

input
```
int((x + x^2 + x^3 + 1)^4/(x^4 - 1)^4,x)
```

output
```
-1/(3*(x - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{(1+x+x^2+x^3)^4}{(1-x^4)^4}\,dx = -\frac{1}{3x^3-9x^2+9x-3}$$

input
```
int((x^3+x^2+x+1)^4/(-x^4+1)^4,x)
```

output
```
( - 1)/(3*(x**3 - 3*x**2 + 3*x - 1))
```

3.100 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$

Optimal result 795
Mathematica [A] (verified) 796
Rubi [A] (verified) 796
Maple [C] (verified) 798
Fricas [C] (verification not implemented) 798
Sympy [F(-1)] 799
Maxima [A] (verification not implemented) 799
Giac [B] (verification not implemented) 800
Mupad [B] (verification not implemented) 801
Reduce [B] (verification not implemented) 801

Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx = -\frac{gx}{b}+\frac{\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} +\frac{\left(bc+\sqrt{a}\sqrt{b}e+ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}} +\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

output
```
-g*x/b+1/2*(b*c-a^(1/2)*b^(1/2)*e+a*g)*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b
^(5/4)+1/2*(b*c+a^(1/2)*b^(1/2)*e+a*g)*arctanh(b^(1/4)*x/a^(1/4))/a^(3/4)/
b^(5/4)+1/2*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)-1/4*f*ln(-b*x^4
+a)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.68

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$
$$= \frac{-4a^{3/4}\sqrt[4]{b}gx+2\left(bc-\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\left(bc+\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e+ag\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

output
```
(-4*a^(3/4)*b^(1/4)*g*x + 2*(b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4
)*x)/a^(1/4)] - (b*c + a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e + a*g)*Log[a^
(1/4) - b^(1/4)*x] + b*c*Log[a^(1/4) + b^(1/4)*x] - a^(1/4)*b^(3/4)*d*Log[
a^(1/4) + b^(1/4)*x] + Sqrt[a]*Sqrt[b]*e*Log[a^(1/4) + b^(1/4)*x] + a*g*Lo
g[a^(1/4) + b^(1/4)*x] + a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2] - a^
(3/4)*b^(1/4)*f*Log[a - b*x^4])/(4*a^(3/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\frac{c+ex^2+gx^4}{a-bx^4}+\frac{x\left(d+fx^2\right)}{a-bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+bc\right)}{2a^{3/4}b^{5/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}-\frac{gx}{b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x]
```

output
```
-((g*x)/b) + ((b*c - Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])
/(2*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTanh[(b^(1/4)*x
)/a^(1/4)])/(2*a^(3/4)*b^(5/4)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sq
rt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(-\_R^3bf-\_R^2be-\_Rbd-ag-cb\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$-\frac{gx}{b}+\frac{\frac{(ag+cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln\left(-bx^4\right.}{4}}{b}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-g*x/b+1/4/b^2*sum((-_R^3*b*f-_R^2*b*e-_R*b*d-a*g-b*c)/_R^3*ln(x-_R),_R=Ro
otOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.73 (sec) , antiderivative size = 592528, normalized size of antiderivative = 4003.57

$$\int\frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx=\text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4} \, dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.36

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4} \, dx = -\frac{gx}{b} + \frac{\frac{2\left(b^{\frac{3}{2}}c - \sqrt{a}be + a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(b^{\frac{3}{2}}d - \sqrt{a}bf\right)\log\left(\sqrt{b}x^2 + \sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}d + \sqrt{a}bf\right)\log\left(\sqrt{b}x^2 - \sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(b^{\frac{3}{2}}c + \sqrt{a}be + a\sqrt{b}g\right)\,\mathrm{l}\cdots}{\sqrt{a}\sqrt{\sqrt{a}\cdots}}}{4b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
-g*x/b + 1/4*(2*(b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*arctan(sqrt(b)*x/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b^(3/2)*d
 - sqrt(a)*b*f)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (b^(3/2)*d + sqrt
(a)*b*f)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (b^(3/2)*c + sqrt(a)*b*e
 + a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(
sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(108) = 216$.

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.02

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{a - bx^4}\,dx$$
$$= -\frac{\sqrt{2}\left(b^2c + abg - \sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd + \sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}$$
$$- \frac{\sqrt{2}\left(b^2c + abg + \sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd - \sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}$$
$$- \frac{\sqrt{2}(b^2c + abg - \sqrt{-ab}be)\log\left(x^2 + \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$
$$+ \frac{\sqrt{2}(b^2c + abg - \sqrt{-ab}be)\log\left(x^2 - \sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}} - \frac{gx}{b} - \frac{f\log\left(|bx^4 - a|\right)}{4\,b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c + a*b*g - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3
/4) - 1/4*sqrt(2)*(b^2*c + a*b*g + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)
*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^
3)^(3/4) - 1/8*sqrt(2)*(b^2*c + a*b*g - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*
x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + a*b*g -
 sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(
3/4) - g*x/b - 1/4*f*log(abs(b*x^4 - a))/b
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 5082, normalized size of antiderivative = 34.34

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4),x)
```

output
```
symsum(log(b^2*c^2*e - b^2*c*d^2 + a^2*e*g^2 - a^2*f^2*g - b^2*d^3*x - a*b
*e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3
 - 64*a^3*b^3*e*g*z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b
^4*d^2*z^2 - 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z
+ 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*
c^2*d*z + 16*a^3*b^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2
*b^2*d*e^2*f + 4*a^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4
*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b
^3*c^3*g - 6*a^2*b^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b
^3*c^2*e^2 + a^3*b*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k
)^2*a*b^3*c - 4*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*
z^2 - 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 - 32*a^2*b^4*d^2*z^2 - 32*a^
3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z + 16*a^3*b^2*d*g^2
*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z + 16*a*b^4*c^2*d*z + 16*a^3*b
^2*f^3*z + 8*a^2*b^2*c*d*f*g - 4*a^2*b^2*d^2*e*g + 4*a^2*b^2*d*e^2*f + 4*a
^2*b^2*c*e^2*g - 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4
*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g - 6*a^2*b
^2*c^2*g^2 - 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^3*b
*f^4 + a*b^3*d^4 - a^2*b^2*e^4 - a^4*g^4 - b^4*c^4, z, k)*b^3*c^2*x - b^2*
c^2*f*x - a^2*f*g^2*x - 16*root(256*a^3*b^5*z^4 + 256*a^3*b^4*f*z^3 - 6...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.95

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a-bx^4}\,dx$$

$$= \frac{-2b^{\frac{5}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)e+2b^{\frac{3}{4}}a^{\frac{5}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)g+2b^{\frac{7}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)c-b^{\frac{5}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)e+b^{\frac{5}{4}}a^{\frac{3}{4}}\log\left(\right.}{}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b*e + 2*b**(
3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*g + 2*b**(3/4)*a**(1
/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b*c - b**(1/4)*a**(3/4)*log(a**(
1/4) - b**(1/4)*x)*b*e + b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*b*e
- b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a*g - b**(3/4)*a**(1/4)*log
(a**(1/4) - b**(1/4)*x)*b*c + b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)
*a*g + b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*b*c - sqrt(b)*sqrt(a)*
log(a**(1/4) - b**(1/4)*x)*b*d - sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x
)*b*d + sqrt(b)*sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*b*d - log(a**(1/4) - b
**(1/4)*x)*a*b*f - log(a**(1/4) + b**(1/4)*x)*a*b*f - log(sqrt(a) + sqrt(b
)*x**2)*a*b*f - 4*a*b*g*x)/(4*a*b**2)
```

3.101 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$

Optimal result . . . 803
Mathematica [A] (verified) . . . 804
Rubi [A] (verified) . . . 804
Maple [C] (verified) . . . 806
Fricas [C] (verification not implemented) . . . 806
Sympy [F(-1)] . . . 807
Maxima [A] (verification not implemented) . . . 807
Giac [B] (verification not implemented) . . . 808
Mupad [B] (verification not implemented) . . . 809
Reduce [B] (verification not implemented) . . . 809

Optimal result

Integrand size = 31, antiderivative size = 172

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{4ab\,(a-bx^4)} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e-ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8a^{7/4}b^{5/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}$$

output
```
1/4*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)+1/8*(3*b*c-a^(1/2)*b^
(1/2)*e-a*g)*arctan(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(5/4)+1/8*(3*b*c+a^(1/2)*
b^(1/2)*e-a*g)*arctanh(b^(1/4)*x/a^(1/4))/a^(7/4)/b^(5/4)+1/4*d*arctanh(b^
(1/2)*x^2/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2} dx$$
$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{a-bx^4} - 2\left(-3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - \left(3bc+2\sqrt[4]{a}b^{3/4}d+\sqrt{a}\sqrt{b}e-\right.}{16a}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

output
```
((4*a^(3/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^4) - 2
*(-3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (3*b*c +
 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) - b^(1/4)*x] +
 (3*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[a]*Sqrt[b]*e - a*g)*Log[a^(1/4) + b^(
1/4)*x] + 2*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(16*a^(7/4)*b^(5
/4))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2397, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2} dx$$
$$\Big\downarrow \text{ 2397}$$
$$\frac{\int \frac{bex^2+2bdx+3bc-ag}{a-bx^4}dx}{4ab} + \frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$
$$\Big\downarrow \text{ 2415}$$

$$\frac{\int \left(\frac{2bdx}{a-bx^4}+\frac{bex^2+3bc-ag}{a-bx^4}\right)dx}{4ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-\sqrt{a}\sqrt{b}e-ag+3bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+3bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}}{4ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a-bx^4\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x]
```

output
```
(x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a - b*x^4)) + (((3*b*c
 - Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1/4
)) + ((3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a
^(3/4)*b^(1/4)) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a])/(4*a
*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

method	result
risch	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}+\frac{(ag+cb)x}{4ab}+\frac{f}{4b}}{-b x^4+a}-\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^2 e+2d\_R-\frac{ag-3cb}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e x^3}{4a}+\frac{d x^2}{4a}+\frac{(ag+cb)x}{4ab}+\frac{f}{4b}}{-b x^4+a}+\frac{\frac{(-ag+3cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{bd\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x\cdots}{x\cdots}\right)\cdots\right)}{4\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots}{4ba}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^3+1/4*d/a*x^2+1/4*(a*g+b*c)/a/b*x+1/4*f/b)/(-b*x^4+a)-1/16/b/a*
sum((_R^2*e+2*d*_R-1/b*(a*g-3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.13 (sec) , antiderivative size = 334837, normalized size of antiderivative = 1946.73

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a-bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = -\frac{bex^3+bdx^2+af+(bc+ag)x}{4\,(ab^2x^4-a^2b)} + \frac{\frac{2\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}} - \frac{2\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}} + \frac{2\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be-a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(3\,b^{\frac{3}{2}}c+\sqrt{a}be-a\sqrt{b}g\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{16\,ab}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(b*e*x^3 + b*d*x^2 + a*f + (b*c + a*g)*x)/(a*b^2*x^4 - a^2*b) + 1/16*
(2*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 2*sqrt(b)*d*log(sqrt(b)*
x^2 - sqrt(a))/sqrt(a) + 2*(3*b^(3/2)*c - sqrt(a)*b*e - a*sqrt(b)*g)*arcta
n(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b))
 - (3*b^(3/2)*c + sqrt(a)*b*e - a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)
*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(b))*sqrt(b)))/(a*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(133) = 266$.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx$$
$$= -\frac{\sqrt{2}\left(3\,b^2c-abg-2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}\left(3\,b^2c-abg+2\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{\sqrt{2}(3\,b^2c-abg-\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}(3\,b^2c-abg-\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{32\,(-ab^3)^{\frac{3}{4}}\,a} - \frac{bex^3+bdx^2+bcx+agx+af}{4\,(bx^4-a)ab}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*b^2*c - a*b*g - 2*sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)
*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/((-a*b
^3)^(3/4)*a) - 1/16*sqrt(2)*(3*b^2*c - a*b*g + 2*sqrt(2)*(-a*b^3)^(1/4)*b*
d - sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)
^(1/4))/((-a*b^3)^(3/4)*a) - 1/32*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*
e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) + 1/3
2*sqrt(2)*(3*b^2*c - a*b*g - sqrt(-a*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1
/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a) - 1/4*(b*e*x^3 + b*d*x^2 + b*c*x + a*
g*x + a*f)/((b*x^4 - a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 1393, normalized size of antiderivative = 8.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^2,x)
```

output
```
symsum(log(- (12*b^2*c*d^2 - 9*b^2*c^2*e - a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2
*g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^
2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 12
8*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^
2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a
^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*
a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2
*g^2*x - 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c
*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z
+ 128*a^3*b^3*d*e^2*z + 1152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2
*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^
2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4
*c^4 - a^2*b^2*e^4 - a^4*g^4, z, k)*a^3*b*g + a*b*e^2*x + 48*root(65536*a^
7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2
*z^2 - 768*a^3*b^3*c*d*g*z + 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z + 1
152*a^2*b^4*c^2*d*z + 16*a^2*b^2*d^2*e*g - 12*a^2*b^2*c*e^2*g - 48*a*b^3*c
*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 - 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e
^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 - 81*b^4*c^4 - a^2*b^2*e^4 - a^4*
g^4, z, k)*a^2*b^2*c - 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^
3*e*g*z^2 - 3072*a^4*b^4*c*e*z^2 - 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.31

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^2,x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*e + 2*b*
*(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**2*e*x**4 - 2*b**(
3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*g + 6*b**(3/4)*a*
*(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*c + 2*b**(3/4)*a**(1/4)*a
tan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b*g*x**4 - 6*b**(3/4)*a**(1/4)*atan
((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**2*c*x**4 - b**(1/4)*a**(3/4)*log(a**(
1/4) - b**(1/4)*x)*a*b*e + b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*b*
*2*e*x**4 + b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*b*e - b**(1/4)*
a**(3/4)*log(a**(1/4) + b**(1/4)*x)*b**2*e*x**4 + b**(3/4)*a**(1/4)*log(a*
*(1/4) - b**(1/4)*x)*a**2*g - 3*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*
x)*a*b*c - b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a*b*g*x**4 + 3*b**
(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*b**2*c*x**4 - b**(3/4)*a**(1/4)*
log(a**(1/4) + b**(1/4)*x)*a**2*g + 3*b**(3/4)*a**(1/4)*log(a**(1/4) + b**
(1/4)*x)*a*b*c + b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*a*b*g*x**4 -
 3*b**(3/4)*a**(1/4)*log(a**(1/4) + b**(1/4)*x)*b**2*c*x**4 - 2*sqrt(b)*sq
rt(a)*log(a**(1/4) - b**(1/4)*x)*a*b*d + 2*sqrt(b)*sqrt(a)*log(a**(1/4) -
b**(1/4)*x)*b**2*d*x**4 - 2*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*a*b
*d + 2*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*b**2*d*x**4 + 2*sqrt(b)*
sqrt(a)*log(sqrt(a) + sqrt(b)*x**2)*a*b*d - 2*sqrt(b)*sqrt(a)*log(sqrt(a)
+ sqrt(b)*x**2)*b**2*d*x**4 + 4*a**2*b*g*x + 4*a*b**2*c*x + 4*a*b**2*d*...
```

3.102 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$

Optimal result 811
Mathematica [A] (verified) 812
Rubi [A] (verified) 812
Maple [C] (verified) 815
Fricas [C] (verification not implemented) 815
Sympy [F(-1)] 816
Maxima [A] (verification not implemented) 816
Giac [A] (verification not implemented) 817
Mupad [B] (verification not implemented) 818
Reduce [B] (verification not implemented) 818

Optimal result

Integrand size = 31, antiderivative size = 236

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{8ab(a-bx^4)^2} + \frac{f}{8ab(a-bx^4)} + \frac{x(7bc-ag+6bdx+5bex^2)}{32a^2b(a-bx^4)} + \frac{\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{\left(21bc+5\sqrt{a}\sqrt{b}e-3ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64a^{11/4}b^{5/4}} + \frac{3d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}}$$

output
```
1/8*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^2+1/8*f/a/b/(-b*x^4+a
)+1/32*x*(5*b*e*x^2+6*b*d*x-a*g+7*b*c)/a^2/b/(-b*x^4+a)+1/64*(21*b*c-5*a^(
1/2)*b^(1/2)*e-3*a*g)*arctan(b^(1/4)*x/a^(1/4))/a^(11/4)/b^(5/4)+1/64*(21*
b*c+5*a^(1/2)*b^(1/2)*e-3*a*g)*arctanh(b^(1/4)*x/a^(1/4))/a^(11/4)/b^(5/4)
+3/16*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{b}(a^2(4f+3gx)-b^2x^5(7c+x(6d+5ex))+abx(11c+x(10d+9ex+gx^3)))}{(a-bx^4)^2}+2\left(21bc-5\sqrt{a}\sqrt{b}e-3ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\cdot$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

output
```
((4*a^(3/4)*b^(1/4)*(a^2*(4*f + 3*g*x) - b^2*x^5*(7*c + x*(6*d + 5*e*x)) +
 a*b*x*(11*c + x*(10*d + 9*e*x + g*x^3))))/(a - b*x^4)^2 + 2*(21*b*c - 5*S
qrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (21*b*c + 12*a^(1/
4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) - b^(1/4)*x] + (21
*b*c - 12*a^(1/4)*b^(3/4)*d + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*Log[a^(1/4) + b
^(1/4)*x] + 12*a^(1/4)*b^(3/4)*d*Log[Sqrt[a] + Sqrt[b]*x^2])/(128*a^(11/4)
*b^(5/4))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2397, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{4bfx^3+5bex^2+6bdx+7bc-ag}{(a-bx^4)^2}dx}{8ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{8ab(a-bx^4)^2}$$

$$\Big\downarrow \text{2393}$$

$$\frac{\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}-\frac{\int -\frac{5bex^2+12bdx+3(7bc-ag)}{a-bx^4}dx}{4a}}{8ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{5bex^2+12bdx+3(7bc-ag)}{a-bx^4}dx}{4a}+\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}}{8ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{\frac{\int \left(\frac{12bdx}{a-bx^4}+\frac{5bex^2+3(7bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}}{8ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e-3ag+21bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{6\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}}{4a}+\frac{x(-ag+7bc+6bdx+5bex^2)+4af}{4a(a-bx^4)}}{8ab}+\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a-bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x]
```

output
```
(x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a - b*x^4)^2) + ((4*a*
f + x*(7*b*c - a*g + 6*b*d*x + 5*b*e*x^2))/(4*a*(a - b*x^4)) + (((21*b*c -
 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(1
/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e - 3*a*g)*ArcTanh[(b^(1/4)*x)/a^(1/4)
])/(2*a^(3/4)*b^(1/4)) + (6*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt
[a])/(4*a))/(8*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.62

method	result
risch	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}+\frac{(ag-7cb)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{(3ag+11cb)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^4-a\right)}\frac{\left(5\_R^2e+12d\_R-\frac{3(ag-7cb)}{b}\right)\ln\left(x-\_\right.}{\_R^3}}{128b\,a^2}$
default	$\frac{-\frac{5be\,x^7}{32a^2}-\frac{3bd\,x^6}{16a^2}+\frac{(ag-7cb)x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{(3ag+11cb)x}{32ab}+\frac{f}{8b}}{(-b\,x^4+a)^2}+\frac{\frac{(-3ag+21cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{3bd\ln\left(\right.}{}}{32b\,a^2}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/32*b*e/a^2*x^7-3/16*b*d/a^2*x^6+1/32*(a*g-7*b*c)/a^2*x^5+9/32*e/a*x^3+
5/16*d/a*x^2+1/32*(3*a*g+11*b*c)/a/b*x+1/8*f/b)/(-b*x^4+a)^2-1/128/b/a^2*s
um((5*_R^2*e+12*d*_R-3/b*(a*g-7*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.07 (sec) , antiderivative size = 343626, normalized size of antiderivative = 1456.04

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.20

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx =$$
$$-\frac{5\,b^2ex^7+6\,b^2dx^6-9\,abex^3+(7\,b^2c-abg)x^5-10\,abdx^2-4\,a^2f-(11\,abc+3\,a^2g)x}{32\,(a^2b^3x^8-2\,a^3b^2x^4+a^4b)}$$
$$+\frac{\frac{12\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{12\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(21\,b^{\frac{3}{2}}c+5\,\sqrt{a}be-3\,a\sqrt{b}g\right)\log}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{128\,a^2b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 - 9*a*b*e*x^3 + (7*b^2*c - a*b*g)*x^5 - 1
0*a*b*d*x^2 - 4*a^2*f - (11*a*b*c + 3*a^2*g)*x)/(a^2*b^3*x^8 - 2*a^3*b^2*x
^4 + a^4*b) + 1/128*(12*sqrt(b)*d*log(sqrt(b)*x^2 + sqrt(a))/sqrt(a) - 12*
sqrt(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt(a) + 2*(21*b^(3/2)*c - 5*sqrt(a)
*b*e - 3*a*sqrt(b)*g)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) - (21*b^(3/2)*c + 5*sqrt(a)*b*e - 3*a*sqrt(b)*
g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(
b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.64

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^3}\,dx$$

$$= -\frac{\sqrt{2}\left(21\,b^2c-3\,abg-12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}\left(21\,b^2c-3\,abg+12\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-5\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{\sqrt{2}(21\,b^2c-3\,abg-5\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$+\frac{\sqrt{2}(21\,b^2c-3\,abg-5\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{256\,(-ab^3)^{\frac{3}{4}}\,a^2}$$
$$-\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5-abgx^5-9\,abex^3-10\,abdx^2-11\,abcx-3\,a^2gx-4\,a^2f}{32\,(bx^4-a)^2a^2b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/128*sqrt(2)*(21*b^2*c - 3*a*b*g - 12*sqrt(2)*(-a*b^3)^(1/4)*b*d + 5*sqr
t(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))
/((-a*b^3)^(3/4)*a^2) - 1/128*sqrt(2)*(21*b^2*c - 3*a*b*g + 12*sqrt(2)*(-a
*b^3)^(1/4)*b*d - 5*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/
b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^2) - 1/256*sqrt(2)*(21*b^2*c - 3
*a*b*g - 5*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/
((-a*b^3)^(3/4)*a^2) + 1/256*sqrt(2)*(21*b^2*c - 3*a*b*g - 5*sqrt(-a*b)*b*
e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^2) - 1
/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 - a*b*g*x^5 - 9*a*b*e*x^3 - 1
0*a*b*d*x^2 - 11*a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 - a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.25

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^3,x)
```

output
```
(f/(8*b) + (5*d*x^2)/(16*a) + (9*e*x^3)/(32*a) - (x^5*(7*b*c - a*g))/(32*a
^2) + (x*(11*b*c + 3*a*g))/(32*a*b) - (3*b*d*x^6)/(16*a^2) - (5*b*e*x^7)/(
32*a^2))/(a^2 + b^2*x^8 - 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 4718592*a^6*b^4
*d^2*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3
*d*e^2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*a^2*b^2*c
*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 - 238
14*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 625*a^2*b^2
*e^4 + 20736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*(root(26843545
6*a^11*b^5*z^4 + 983040*a^7*b^3*e*g*z^2 - 6881280*a^6*b^4*c*e*z^2 - 471859
2*a^6*b^4*d^2*z^2 - 774144*a^4*b^3*c*d*g*z + 55296*a^5*b^2*d*g^2*z + 15360
0*a^4*b^3*d*e^2*z + 2709504*a^3*b^4*c^2*d*z + 8640*a^2*b^2*d^2*e*g - 6300*
a^2*b^2*c*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*
g^3 - 23814*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 - 62
5*a^2*b^2*e^4 + 20736*a*b^3*d^4 - 81*a^4*g^4 - 194481*b^4*c^4, z, k)*((344
064*a^5*b^3*c - 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^
4*b*g^2 + 7056*a^2*b^3*c^2 + 400*a^3*b^2*e^2 - 2016*a^3*b^2*c*g))/(4096*a^
6) - (15*b^2*d*e)/(32*a^3)) - (3024*b^2*c*d^2 - 2205*b^2*c^2*e - 45*a^2*e*
g^2 + 125*a*b*e^3 - 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(216*b
^2*d^3 - 315*b^2*c*d*e + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.86

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a - bx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^3,x)
```

output
```
( - 10*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*e +
20*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*e*x**4 -
 10*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**3*e*x**8 -
6*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*g + 42*b**(
3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*c + 12*b**(3/4)
*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b*g*x**4 - 84*b**(3/4
)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*c*x**4 - 6*b**(3/4
)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**2*g*x**8 + 42*b**(3/
4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**3*c*x**8 - 5*b**(1/4)
*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a**2*b*e + 10*b**(1/4)*a**(3/4)*log(a
**(1/4) - b**(1/4)*x)*a*b**2*e*x**4 - 5*b**(1/4)*a**(3/4)*log(a**(1/4) - b
**(1/4)*x)*b**3*e*x**8 + 5*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*
*2*b*e - 10*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*a*b**2*e*x**4 + 5
*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*b**3*e*x**8 + 3*b**(3/4)*a**
(1/4)*log(a**(1/4) - b**(1/4)*x)*a**3*g - 21*b**(3/4)*a**(1/4)*log(a**(1/4
) - b**(1/4)*x)*a**2*b*c - 6*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*
a**2*b*g*x**4 + 42*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a*b**2*c*x
**4 + 3*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a*b**2*g*x**8 - 21*b*
*(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*b**3*c*x**8 - 3*b**(3/4)*a**(1/
4)*log(a**(1/4) + b**(1/4)*x)*a**3*g + 21*b**(3/4)*a**(1/4)*log(a**(1/4...
```

3.103 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$

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Optimal result

Integrand size = 31, antiderivative size = 281

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \frac{x(bc+ag+bdx+bex^2+bfx^3)}{12ab(a-bx^4)^3}+\frac{f}{12ab(a-bx^4)^2}+\frac{x(11bc-ag+10bdx+9bex^2)}{96a^2b(a-bx^4)^2}+\frac{x(7(11bc-ag)+60bdx+45bex^2)}{384a^3b(a-bx^4)}+\frac{\left(77bc-15\sqrt{a}\sqrt{b}e-7ag\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}}+\frac{\left(77bc+15\sqrt{a}\sqrt{b}e-7ag\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256a^{15/4}b^{5/4}}+\frac{5d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}}$$

output
```
1/12*x*(b*f*x^3+b*e*x^2+b*d*x+a*g+b*c)/a/b/(-b*x^4+a)^3+1/12*f/a/b/(-b*x^4
+a)^2+1/96*x*(9*b*e*x^2+10*b*d*x-a*g+11*b*c)/a^2/b/(-b*x^4+a)^2+1/384*x*(4
5*b*e*x^2+60*b*d*x-7*a*g+77*b*c)/a^3/b/(-b*x^4+a)+1/256*(77*b*c-15*a^(1/2)
*b^(1/2)*e-7*a*g)*arctan(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(5/4)+1/256*(77*b*c
+15*a^(1/2)*b^(1/2)*e-7*a*g)*arctanh(b^(1/4)*x/a^(1/4))/a^(15/4)/b^(5/4)+5
/32*d*arctanh(b^(1/2)*x^2/a^(1/2))/a^(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.11

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$

$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}x(77bc-7ag+15bx(4d+3ex))}{a-bx^4}+\frac{16a^{7/4}\sqrt[4]{b}x(11bc-ag+bx(10d+9ex))}{(a-bx^4)^2}+\frac{128a^{11/4}\sqrt[4]{b}(a(f+gx)+bx(c+x(d+ex)))}{(a-bx^4)^3}+6\Big(77bc}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

output
```
((4*a^(3/4)*b^(1/4)*x*(77*b*c - 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a - b*x^4)
 + (16*a^(7/4)*b^(1/4)*x*(11*b*c - a*g + b*x*(10*d + 9*e*x)))/(a - b*x^4)^
2 + (128*a^(11/4)*b^(1/4)*(a*(f + g*x) + b*x*(c + x*(d + e*x))))/(a - b*x^
4)^3 + 6*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4
)] - 3*(77*b*c + 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*Log[
a^(1/4) - b^(1/4)*x] + 3*(77*b*c - 40*a^(1/4)*b^(3/4)*d + 15*Sqrt[a]*Sqrt[
b]*e - 7*a*g)*Log[a^(1/4) + b^(1/4)*x] + 120*a^(1/4)*b^(3/4)*d*Log[Sqrt[a]
 + Sqrt[b]*x^2])/(1536*a^(15/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2397, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$

$$\Big\downarrow \ \textbf{2397}$$

$$\frac{\int \frac{8bfx^3+9bex^2+10bdx+11bc-ag}{(a-bx^4)^3}dx}{12ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3}$$

$$\Big\downarrow \ \textbf{2393}$$

$$\frac{\frac{x(-ag+11bc+10bdx+9bex^2)+8af}{8a(a-bx^4)^2}-\frac{\int -\frac{45bex^2+60bdx+7(11bc-ag)}{(a-bx^4)^2}dx}{8a}}{12ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3}$$

$$\Big\downarrow \ \textbf{25}$$

$$\frac{\frac{\int \frac{45bex^2+60bdx+7(11bc-ag)}{(a-bx^4)^2}dx}{8a}+\frac{x(-ag+11bc+10bdx+9bex^2)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3}$$

$$\Big\downarrow \ \textbf{2394}$$

$$\frac{\frac{\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}-\frac{\int -\frac{3\left(15bex^2+40bdx+7(11bc-ag)\right)}{a-bx^4}dx}{4a}}{8a}+\frac{x(-ag+11bc+10bdx+9bex^2)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{\frac{3\int \frac{15bex^2+40bdx+7(11bc-ag)}{a-bx^4}dx}{4a}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(-ag+11bc+10bdx+9bex^2)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3}$$

$$\Big\downarrow \ \textbf{2415}$$

$$\frac{\frac{\frac{3\int \left(\frac{40bdx}{a-bx^4}+\frac{15bex^2+7(11bc-ag)}{a-bx^4}\right)dx}{4a}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}}{8a}+\frac{x(-ag+11bc+10bdx+9bex^2)+8af}{8a(a-bx^4)^2}}{12ab}+\frac{x(ag+bc+bdx+bex^2+bfx^3)}{12ab\,(a-bx^4)^3}$$

$$\Big\downarrow \ \textbf{2009}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(-15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e-7ag+77bc\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{20\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{4a}+\frac{x\left(7(11bc-ag)+60bdx+45bex^2\right)}{4a\left(a-bx^4\right)}}{8a}}{12ab}$$

$$\frac{x\left(ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a-bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x]
```

output
```
(x*(b*c + a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a - b*x^4)^3) + ((8*a
*f + x*(11*b*c - a*g + 10*b*d*x + 9*b*e*x^2))/(8*a*(a - b*x^4)^2) + ((x*(7
*(11*b*c - a*g) + 60*b*d*x + 45*b*e*x^2))/(4*a*(a - b*x^4)) + (3*(((77*b*c
 - 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b
^(1/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e - 7*a*g)*ArcTanh[(b^(1/4)*x)/a^(
1/4)])/(2*a^(3/4)*b^(1/4)) + (20*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])
/Sqrt[a]))/(4*a))/(8*a))/(12*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\frac{15 e b^2 x^{11}}{128 a^3}+\frac{5 d b^2 x^{10}}{32 a^3}-\frac{7(ag-11cb) b x^9}{384 a^3}-\frac{21 be x^7}{64 a^2}-\frac{5bd x^6}{12 a^2}+\frac{3(ag-11cb) x^5}{64 a^2}+\frac{113 e x^3}{384 a}+\frac{11 d x^2}{32 a}+\frac{(7ag+51cb) x}{128 ab}+\frac{f}{12 b}}{(-b x^4+a)^3}-\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4-}}{}$
default	$\frac{\frac{15 e b^2 x^{11}}{128 a^3}+\frac{5 d b^2 x^{10}}{32 a^3}-\frac{7(ag-11cb) b x^9}{384 a^3}-\frac{21 be x^7}{64 a^2}-\frac{5bd x^6}{12 a^2}+\frac{3(ag-11cb) x^5}{64 a^2}+\frac{113 e x^3}{384 a}+\frac{11 d x^2}{32 a}+\frac{(7ag+51cb) x}{128 ab}+\frac{f}{12 b}}{(-b x^4+a)^3}+\frac{\frac{(-7ag+77cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}(}{}}{}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10-7/384*(a*g-11*b*c)/a^3*b*x^9-21
/64*b*e/a^2*x^7-5/12*b*d/a^2*x^6+3/64/a^2*(a*g-11*b*c)*x^5+113/384*e/a*x^3
+11/32*d/a*x^2+1/128*(7*a*g+51*b*c)/a/b*x+1/12*f/b)/(-b*x^4+a)^3-1/512/b/a
^3*sum((15*_R^2*e+40*d*_R-7*(a*g-11*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b
-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 44.66 (sec) , antiderivative size = 343822, normalized size of antiderivative = 1223.57

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(-b*x**4+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = -\frac{45\,b^3ex^{11}+60\,b^3dx^{10}-126\,ab^2ex^7-160\,ab^2dx^6+7\,(11\,b^3c-ab^2g)x^9+113\,a^2bex^3+132\,a^2bdx^2-1\ldots}{384\,(a^3b^4x^{12}-3\,a^4b^3x^8+3\,a^5b^2x^4-a^6b)} + \frac{\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}}-\frac{40\,\sqrt{b}d\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}}+\frac{2\left(77\,b^{\frac{3}{2}}c-15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\left(77\,b^{\frac{3}{2}}c+15\,\sqrt{a}be-7\,a\sqrt{b}g\right)\mathrm{l}\ldots}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\ldots}}{512\,a^3b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="maxima")
```

output
```
-1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 - 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6
+ 7*(11*b^3*c - a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 - 18*(11*
a*b^2*c - a^2*b*g)*x^5 + 32*a^3*f + 3*(51*a^2*b*c + 7*a^3*g)*x)/(a^3*b^4*x
^12 - 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 - a^6*b) + 1/512*(40*sqrt(b)*d*log(sqr
t(b)*x^2 + sqrt(a))/sqrt(a) - 40*sqrt(b)*d*log(sqrt(b)*x^2 - sqrt(a))/sqrt
(a) + 2*(77*b^(3/2)*c - 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*arctan(sqrt(b)*x/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - (77*b^(3/2
)*c + 15*sqrt(a)*b*e - 7*a*sqrt(b)*g)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b
)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sq
rt(b)))/(a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.55

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx$$
$$= -\frac{\sqrt{2}\left(77\,b^2c-7\,abg-40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd+15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}\left(77\,b^2c-7\,abg+40\,\sqrt{2}(-ab^3)^{\frac{1}{4}}\,bd-15\,\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{512\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$+\frac{\sqrt{2}(77\,b^2c-7\,abg-15\,\sqrt{-ab}be)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{1024\,(-ab^3)^{\frac{3}{4}}\,a^3}$$
$$-\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9-7\,ab^2gx^9-126\,ab^2ex^7-160\,ab^2dx^6-198\,ab^2cx^5+18\,a^2bgx^5+11}{384\,(bx^4-a)^3a^3b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x, algorithm="giac")
```

output
```
-1/512*sqrt(2)*(77*b^2*c - 7*a*b*g - 40*sqrt(2)*(-a*b^3)^(1/4)*b*d + 15*sq
rt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4)
)/((-a*b^3)^(3/4)*a^3) - 1/512*sqrt(2)*(77*b^2*c - 7*a*b*g + 40*sqrt(2)*(-
a*b^3)^(1/4)*b*d - 15*sqrt(-a*b)*b*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/((-a*b^3)^(3/4)*a^3) - 1/1024*sqrt(2)*(77*b^2*c
 - 7*a*b*g - 15*sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/
b))/((-a*b^3)^(3/4)*a^3) + 1/1024*sqrt(2)*(77*b^2*c - 7*a*b*g - 15*sqrt(-a
*b)*b*e)*log(x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/((-a*b^3)^(3/4)*a^
3) - 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 - 7*a*b^2*g*x^9 -
 126*a*b^2*e*x^7 - 160*a*b^2*d*x^6 - 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 11
3*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x + 21*a^3*g*x + 32*a^3*f)/(
(b*x^4 - a)^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.76

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a - b*x^4)^4,x)
```

output
```
symsum(log(- root(68719476736*a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 +
110100480*a^9*b^3*e*g*z^2 - 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c
*d*g*z + 485703680*a^4*b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^
5*b^3*d*e^2*z + 672000*a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*
a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^
2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^
4 + 2560000*a*b^3*d^4 - 2401*a^4*g^4 - 35153041*b^4*c^4, z, k)*(root(68719
476736*a^15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z
^2 - 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z + 485703680*a^4*
b^4*c^2*d*z + 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z + 672000*
a^2*b^2*d^2*e*g - 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 1278292
4*a*b^3*c^3*g + 105644*a^3*b*c*g^3 - 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b
*e^2*g^2 + 2668050*a*b^3*c^2*e^2 - 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 -
 2401*a^4*g^4 - 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c - 1835008*a^8
*b^2*g)/(2097152*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3
*b^3*c^2 + 7200*a^4*b^2*e^2 - 34496*a^4*b^2*c*g))/(131072*a^9) - (75*b^2*d
*e)/(256*a^5)) - (123200*b^2*c*d^2 - 88935*b^2*c^2*e - 735*a^2*e*g^2 + 337
5*a*b*e^3 - 11200*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(4000*b^
2*d^3 - 5775*b^2*c*d*e + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^
15*b^5*z^4 - 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 - 8...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1255, normalized size of antiderivative = 4.47

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a-bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(-b*x^4+a)^4,x)
```

output

```
( - 90*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*b*e +
270*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b**2*e*x*
*4 - 270*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*b**3*e*
x**8 + 90*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*b**4*e*x
**12 - 42*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**4*g +
 462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*b*c + 12
6*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**3*b*g*x**4 -
1386*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b**2*c*x
**4 - 126*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a**2*b**
2*g*x**8 + 1386*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a*
b**3*c*x**8 + 42*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*a
*b**3*g*x**12 - 462*b**(3/4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4))
)*b**4*c*x**12 - 45*b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a**3*b*e
+ 135*b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a**2*b**2*e*x**4 - 135*
b**(1/4)*a**(3/4)*log(a**(1/4) - b**(1/4)*x)*a*b**3*e*x**8 + 45*b**(1/4)*a
**(3/4)*log(a**(1/4) - b**(1/4)*x)*b**4*e*x**12 + 45*b**(1/4)*a**(3/4)*log
(a**(1/4) + b**(1/4)*x)*a**3*b*e - 135*b**(1/4)*a**(3/4)*log(a**(1/4) + b*
*(1/4)*x)*a**2*b**2*e*x**4 + 135*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)
*x)*a*b**3*e*x**8 - 45*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*b**4*e
*x**12 + 21*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*a**4*g - 231*b...
```

3.104 $\int (a+bx^4)^3 (c+dx+ex^2+fx^3+gx^4) \, dx$

Optimal result . . . 830
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Reduce [B] (verification not implemented) . . . 836

Optimal result

Integrand size = 30, antiderivative size = 179

$$\begin{aligned}\int (a+bx^4)^3 (c+dx+ex^2+fx^3+gx^4) \, dx &= a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{5}a^2(3bc+ag)x^5+\frac{1}{2}a^2bdx^6 \\ &\quad+\frac{3}{7}a^2bex^7+\frac{1}{3}ab(bc+ag)x^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11} \\ &\quad+\frac{1}{13}b^2(bc+3ag)x^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{17}b^3gx^{17}+\frac{f(a+bx^4)^4}{16b}\end{aligned}$$

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/5*a^2*(a*g+3*b*c)*x^5+1/2*a^2*b*d*x^
6+3/7*a^2*b*e*x^7+1/3*a*b*(a*g+b*c)*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^1
1+1/13*b^2*(3*a*g+b*c)*x^13+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/17*b^3*g*x^1
7+1/16*f*(b*x^4+a)^4/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.85

$$\int (a+bx^4)^3 (c+dx+ex^2+fx^3+gx^4)\, dx$$
$$= \frac{ab^2x^9(2860c+3x(858d+780ex+715fx^2+660gx^3))}{8580} + \frac{b^3x^{13}(28560c+13x(2040d+7x(272e+255fx+240gx^2)))}{371280} + a^3\left(cx+\frac{1}{60}x^2(30d+x(20e+3x(5f+4gx)))\right) + \frac{1}{840}a^2bx^5(504c+5x(84d+x(72e+7x(9f+8gx))))$$

input
```
Integrate[(a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(a*b^2*x^9*(2860*c + 3*x*(858*d + 780*e*x + 715*f*x^2 + 660*g*x^3)))/8580
+ (b^3*x^13*(28560*c + 13*x*(2040*d + 7*x*(272*e + 255*f*x + 240*g*x^2))))
/371280 + a^3*(c*x + (x^2*(30*d + x*(20*e + 3*x*(5*f + 4*g*x))))/60) + (a^
2*b*x^5*(504*c + 5*x*(84*d + x*(72*e + 7*x*(9*f + 8*g*x)))))/840
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^3 (c+dx+ex^2+fx^3+gx^4)\, dx$$
$$\Big\downarrow \text{2017}$$
$$\int (bx^4+a)^3 (gx^4+ex^2+dx+c)\, dx + \frac{f(a+bx^4)^4}{16b}$$
$$\Big\downarrow \text{2389}$$

$$\int \left(b^3gx^{16}+b^3ex^{14}+b^3dx^{13}+b^2(bc+3ag)x^{12}+3ab^2ex^{10}+3ab^2dx^9+3ab(bc+ag)x^8+3a^2bex^6+3a^2bdx^5+\right.$$

$$\frac{f\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \ 2009$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{5}a^2x^5(ag+3bc)+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{13}b^2x^{13}(3ag+bc)+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{3}abx^9(ag+bc)+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{17}b^3gx^{17}$$

input
```
Int[(a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^2*(3*b*c + a*g)*x^5)/5 + (a^2
*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (a*b*(b*c + a*g)*x^9)/3 + (3*a*b^2*d*x^1
0)/10 + (3*a*b^2*e*x^11)/11 + (b^2*(b*c + 3*a*g)*x^13)/13 + (b^3*d*x^14)/1
4 + (b^3*e*x^15)/15 + (b^3*g*x^17)/17 + (f*(a + b*x^4)^4)/(16*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

method	result
norman	$a^{3}cx+\frac{a^{3}d\,x^{2}}{2}+\frac{a^{3}e\,x^{3}}{3}+\frac{f\,a^{3}x^{4}}{4}+\left(\frac{1}{5}a^{3}g+\frac{3}{5}c\,a^{2}b\right)x^{5}+\frac{a^{2}bd\,x^{6}}{2}+\frac{3a^{2}be\,x^{7}}{7}+\frac{3f\,a^{2}b\,x^{8}}{8}+\left(\frac{1}{3}a^{2}bg\right.$
default	$\frac{b^{3}g\,x^{17}}{17}+\frac{f\,b^{3}x^{16}}{16}+\frac{b^{3}e\,x^{15}}{15}+\frac{b^{3}d\,x^{14}}{14}+\frac{(3a\,b^{2}g+b^{3}c)x^{13}}{13}+\frac{f\,a\,b^{2}x^{12}}{4}+\frac{3a\,b^{2}e\,x^{11}}{11}+\frac{3a\,b^{2}d\,x^{10}}{10}+\frac{(3a^{2}bg+3c}{9}$
gosper	$a^{3}cx+\frac{1}{2}a^{3}d\,x^{2}+\frac{1}{3}a^{3}e\,x^{3}+\frac{1}{4}f\,a^{3}x^{4}+\frac{1}{5}x^{5}a^{3}g+\frac{3}{5}a^{2}bc\,x^{5}+\frac{1}{2}a^{2}bd\,x^{6}+\frac{3}{7}a^{2}be\,x^{7}+\frac{3}{8}f\,a^{2}b\,x$
risch	$a^{3}cx+\frac{1}{2}a^{3}d\,x^{2}+\frac{1}{3}a^{3}e\,x^{3}+\frac{1}{4}f\,a^{3}x^{4}+\frac{1}{5}x^{5}a^{3}g+\frac{3}{5}a^{2}bc\,x^{5}+\frac{1}{2}a^{2}bd\,x^{6}+\frac{3}{7}a^{2}be\,x^{7}+\frac{3}{8}f\,a^{2}b\,x$
parallelrisch	$a^{3}cx+\frac{1}{2}a^{3}d\,x^{2}+\frac{1}{3}a^{3}e\,x^{3}+\frac{1}{4}f\,a^{3}x^{4}+\frac{1}{5}x^{5}a^{3}g+\frac{3}{5}a^{2}bc\,x^{5}+\frac{1}{2}a^{2}bd\,x^{6}+\frac{3}{7}a^{2}be\,x^{7}+\frac{3}{8}f\,a^{2}b\,x$
orering	$x(240240b^{3}g\,x^{16}+255255f\,b^{3}x^{15}+272272e\,b^{3}x^{14}+291720b^{3}d\,x^{13}+942480a\,b^{2}g\,x^{12}+314160b^{3}c\,x^{12}+1021020f\,a\,b^{2}x^{11}+1113$

input
```
int((b*x^4+a)^3*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*f*a^3*x^4+(1/5*a^3*g+3/5*c*a^2*b)*
x^5+1/2*a^2*b*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+(1/3*a^2*b*g+1/3*c*a*b
^2)*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/4*f*a*b^2*x^12+(3/13*a*b^2*g
+1/13*b^3*c)*x^13+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*b^3*x^16+1/17*b^3
*g*x^17
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03

$$\int\left(a+bx^{4}\right)^{3}\left(c+dx+ex^{2}+fx^{3}+gx^{4}\right)\,dx$$
$$=\frac{1}{17}\,b^{3}gx^{17}+\frac{1}{16}\,b^{3}fx^{16}+\frac{1}{15}\,b^{3}ex^{15}+\frac{1}{14}\,b^{3}dx^{14}+\frac{1}{4}\,ab^{2}fx^{12}+\frac{3}{11}\,ab^{2}ex^{11}+\frac{3}{10}\,ab^{2}dx^{10}+\frac{1}{13}\,\left(b^{3}c+3\,ab^{2}g\right)x^{13}+\frac{3}{8}\,a^{2}bfx^{8}+\frac{3}{7}\,a^{2}bex^{7}+\frac{1}{2}\,a^{2}bdx^{6}+\frac{1}{3}\,\left(ab^{2}c+a^{2}bg\right)x^{9}+\frac{1}{4}\,a^{3}fx^{4}+\frac{1}{3}\,a^{3}ex^{3}+\frac{1}{2}\,a^{3}dx^{2}+\frac{1}{5}\,\left(3\,a^{2}bc+a^{3}g\right)x^{5}+a^{3}cx$$

input
```
integrate((b*x^4+a)^3*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/13*(b^3*c + 3*a
*b^2*g)*x^13 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 1/3*(
a*b^2*c + a^2*b*g)*x^9 + 1/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + 1
/5*(3*a^2*b*c + a^3*g)*x^5 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.21

$$\begin{aligned}\int (a+bx^4)^3 (c+dx+ex^2+fx^3+gx^4)\ dx &= a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{a^3fx^4}{4}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3a^2bfx^8}{8}\\&+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{ab^2fx^{12}}{4}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{b^3fx^{16}}{16}+\frac{b^3gx^{17}}{17}\\&+x^{13}\cdot\left(\frac{3ab^2g}{13}+\frac{b^3c}{13}\right)+x^9\left(\frac{a^2bg}{3}+\frac{ab^2c}{3}\right)+x^5\left(\frac{a^3g}{5}+\frac{3a^2bc}{5}\right)\end{aligned}$$

input
```
integrate((b*x**4+a)**3*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + a**2*b*d*x**6/2
 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + 3*a*b**2*d*x**10/10 + 3*a*b**2*
e*x**11/11 + a*b**2*f*x**12/4 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f
*x**16/16 + b**3*g*x**17/17 + x**13*(3*a*b**2*g/13 + b**3*c/13) + x**9*(a*
*2*b*g/3 + a*b**2*c/3) + x**5*(a**3*g/5 + 3*a**2*b*c/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03

$$\begin{aligned}\int (a+bx^4)^3 (c+dx+ex^2+fx^3+gx^4)\ dx &= \frac{1}{17}b^3gx^{17}+\frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3ex^{15}+\frac{1}{14}b^3dx^{14}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}\\&+\frac{3}{10}ab^2dx^{10}+\frac{1}{13}(b^3c+3ab^2g)x^{13}+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6\\&+\frac{1}{3}(ab^2c+a^2bg)x^9+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+\frac{1}{5}(3a^2bc+a^3g)x^5+a^3cx\end{aligned}$$

input
```
integrate((b*x^4+a)^3*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/13*(b^3*c + 3*a
*b^2*g)*x^13 + 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 1/3*(
a*b^2*c + a^2*b*g)*x^9 + 1/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + 1
/5*(3*a^2*b*c + a^3*g)*x^5 + a^3*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05

$$\int (a+bx^4)^3 (c+dx+ex^2+fx^3+gx^4)\,dx = \frac{1}{17}b^3gx^{17}+\frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3ex^{15}+\frac{1}{14}b^3dx^{14}+\frac{1}{13}b^3cx^{13}+\frac{3}{13}ab^2gx^{13}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{1}{3}a^2bgx^9+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{5}a^3gx^5+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+a^3cx$$

input
```
integrate((b*x^4+a)^3*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
1/17*b^3*g*x^17 + 1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/
13*b^3*c*x^13 + 3/13*a*b^2*g*x^13 + 1/4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 +
 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 + 1/3*a^2*b*g*x^9 + 3/8*a^2*b*f*x^8 +
 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1/5*a^3*g*x^5 + 1/4
*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \left(a+bx^4\right)^3 \left(c+dx+ex^2+fx^3+gx^4\right) dx$$
$$= x^5\left(\frac{g\,a^3}{5}+\frac{3\,b\,c\,a^2}{5}\right)+x^{13}\left(\frac{c\,b^3}{13}+\frac{3\,a\,g\,b^2}{13}\right)+\frac{a^3\,d\,x^2}{2}+\frac{a^3\,e\,x^3}{3}+\frac{b^3\,d\,x^{14}}{14}$$
$$+\frac{a^3\,f\,x^4}{4}+\frac{b^3\,e\,x^{15}}{15}+\frac{b^3\,f\,x^{16}}{16}+\frac{b^3\,g\,x^{17}}{17}+a^3\,c\,x+\frac{a\,b\,x^9\,(b\,c+a\,g)}{3}$$
$$+\frac{a^2\,b\,d\,x^6}{2}+\frac{3\,a\,b^2\,d\,x^{10}}{10}+\frac{3\,a^2\,b\,e\,x^7}{7}+\frac{3\,a\,b^2\,e\,x^{11}}{11}+\frac{3\,a^2\,b\,f\,x^8}{8}+\frac{a\,b^2\,f\,x^{12}}{4}$$

input
```
int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
x^5*((a^3*g)/5 + (3*a^2*b*c)/5) + x^13*((b^3*c)/13 + (3*a*b^2*g)/13) + (a^
3*d*x^2)/2 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f*x^4)/4 + (b^3*e*x^15
)/15 + (b^3*f*x^16)/16 + (b^3*g*x^17)/17 + a^3*c*x + (a*b*x^9*(b*c + a*g))
/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7)/7 + (3*a*b^2*
e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06

$$\int \left(a+bx^4\right)^3 \left(c+dx+ex^2+fx^3+gx^4\right) dx$$
$$= \frac{x(240240b^3g\,x^{16}+255255b^3f\,x^{15}+272272b^3e\,x^{14}+291720b^3d\,x^{13}+942480a\,b^2g\,x^{12}+314160b^3c\,x^{12}+$$

input
```
int((b*x^4+a)^3*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

output
```
(x*(4084080*a**3*c + 2042040*a**3*d*x + 1361360*a**3*e*x**2 + 1021020*a**3
*f*x**3 + 816816*a**3*g*x**4 + 2450448*a**2*b*c*x**4 + 2042040*a**2*b*d*x*
*5 + 1750320*a**2*b*e*x**6 + 1531530*a**2*b*f*x**7 + 1361360*a**2*b*g*x**8
 + 1361360*a*b**2*c*x**8 + 1225224*a*b**2*d*x**9 + 1113840*a*b**2*e*x**10
+ 1021020*a*b**2*f*x**11 + 942480*a*b**2*g*x**12 + 314160*b**3*c*x**12 + 2
91720*b**3*d*x**13 + 272272*b**3*e*x**14 + 255255*b**3*f*x**15 + 240240*b*
*3*g*x**16))/4084080
```

3.105 $\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4) \, dx$

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Optimal result

Integrand size = 30, antiderivative size = 131

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4) \, dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{5}a(2bc+ag)x^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7 + \frac{1}{9}b(bc+2ag)x^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{13}b^2gx^{13}+\frac{f(a+bx^4)^3}{12b}$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+1/5*a*(a*g+2*b*c)*x^5+1/3*a*b*d*x^6+2/
7*a*b*e*x^7+1/9*b*(2*a*g+b*c)*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/13*b^2
*g*x^13+1/12*f*(b*x^4+a)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4) \, dx = \frac{b^2x^9(2860c+3x(858d+780ex+715fx^2+660gx^3))}{25740} + a^2\left(cx+\frac{1}{60}x^2(30d+x(20e+3x(5f+4gx)))\right) + \frac{abx^5(504c+5x(84d+x(72e+7x(9f+8gx))))}{1260}$$

input
```
Integrate[(a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
(b^2*x^9*(2860*c + 3*x*(858*d + 780*e*x + 715*f*x^2 + 660*g*x^3)))/25740 +
 a^2*(c*x + (x^2*(30*d + x*(20*e + 3*x*(5*f + 4*g*x))))/60) + (a*b*x^5*(50
4*c + 5*x*(84*d + x*(72*e + 7*x*(9*f + 8*g*x)))))/1260
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4) \, dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int (bx^4+a)^2 (gx^4+ex^2+dx+c) \, dx + \frac{f(a+bx^4)^3}{12b}$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(b^2gx^{12}+b^2ex^{10}+b^2dx^9+b(bc+2ag)x^8+2abex^6+2abdx^5+a(2bc+ag)x^4+a^2ex^2+a^2dx+a^2c\right)dx+\frac{f\left(a+bx^4\right)^3}{12b}$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{9}bx^9(2ag+bc)+\frac{1}{5}ax^5(ag+2bc)+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{13}b^2gx^{13}$$

input
```
Int[(a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a*(2*b*c + a*g)*x^5)/5 + (a*b*d
*x^6)/3 + (2*a*b*e*x^7)/7 + (b*(b*c + 2*a*g)*x^9)/9 + (b^2*d*x^10)/10 + (b
^2*e*x^11)/11 + (b^2*g*x^13)/13 + (f*(a + b*x^4)^3)/(12*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2 g\,x^{13}}{13}+\frac{b^2 f\,x^{12}}{12}+\frac{b^2 e\,x^{11}}{11}+\frac{b^2 d\,x^{10}}{10}+\frac{(2abg+b^2c)x^9}{9}+\frac{abf\,x^8}{4}+\frac{2abe\,x^7}{7}+\frac{abd\,x^6}{3}+\frac{(a^2g+2abc)x^5}{5}+ \frac{a^2 f}{4}$
norman	$\frac{b^2 g\,x^{13}}{13}+\frac{b^2 f\,x^{12}}{12}+\frac{b^2 e\,x^{11}}{11}+\frac{b^2 d\,x^{10}}{10}+\left(\frac{2}{9}abg+\frac{1}{9}b^2c\right)x^9+\frac{abf\,x^8}{4}+\frac{2abe\,x^7}{7}+\frac{abd\,x^6}{3}+\left(\frac{1}{5}a^2g+\frac{2}{5}\right.$
gosper	$\frac{1}{13}b^2g\,x^{13}+\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{2}{9}x^9abg+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}$
risch	$\frac{1}{13}b^2g\,x^{13}+\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{2}{9}x^9abg+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}$
parallelrisch	$\frac{1}{13}b^2g\,x^{13}+\frac{1}{12}b^2f\,x^{12}+\frac{1}{11}b^2e\,x^{11}+\frac{1}{10}b^2d\,x^{10}+\frac{2}{9}x^9abg+\frac{1}{9}b^2c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}$
orering	$\frac{x(13860b^2g\,x^{12}+15015f\,b^2x^{11}+16380e\,b^2x^{10}+18018b^2d\,x^9+40040abg\,x^8+20020b^2c\,x^8+45045fab\,x^7+51480abe\,x^6+60060}{180180}$

input
```
int((b*x^4+a)^2*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*g*x^13+1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*(2*a*b
*g+b^2*c)*x^9+1/4*a*b*f*x^8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+1/5*(a^2*g+2*a*b*c
)*x^5+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/2*a^2*d*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{1}{13}\,b^2gx^{13}+\frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{9}\,\left(b^2c+2\,abg\right)x^9+\frac{1}{3}\,abdx^6+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{5}\,\left(2\,abc+a^2g\right)x^5+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((b*x^4+a)^2*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/
4*a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/9*(b^2*c + 2*a*b*g)*x^9 + 1/3*a*b*d*x^6 +
1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*c + a^2*g)*x^5 + 1/2*a^2*d*x^2
+ a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4)\, dx = a^2cx + \frac{a^2dx^2}{2} + \frac{a^2ex^3}{3} + \frac{a^2fx^4}{4} + \frac{abdx^6}{3} + \frac{2abex^7}{7} + \frac{abfx^8}{4} + \frac{b^2dx^{10}}{10} + \frac{b^2ex^{11}}{11} + \frac{b^2fx^{12}}{12} + \frac{b^2gx^{13}}{13} + x^9 \cdot \left(\frac{2abg}{9} + \frac{b^2c}{9}\right) + x^5\left(\frac{a^2g}{5} + \frac{2abc}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + a*b*d*x**6/3 +
2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*d*x**10/10 + b**2*e*x**11/11 + b**2*f
*x**12/12 + b**2*g*x**13/13 + x**9*(2*a*b*g/9 + b**2*c/9) + x**5*(a**2*g/5
 + 2*a*b*c/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4)\, dx = \frac{1}{13}b^2gx^{13} + \frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{9}(b^2c+2\,abg)x^9 + \frac{1}{3}abdx^6 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{5}(2\,abc+a^2g)x^5 + \frac{1}{2}a^2dx^2 + a^2cx$$

input
```
integrate((b*x^4+a)^2*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/
4*a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/9*(b^2*c + 2*a*b*g)*x^9 + 1/3*a*b*d*x^6 +
1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/5*(2*a*b*c + a^2*g)*x^5 + 1/2*a^2*d*x^2
+ a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4) \, dx = \frac{1}{13}b^2gx^{13} + \frac{1}{12}b^2fx^{12} + \frac{1}{11}b^2ex^{11} + \frac{1}{10}b^2dx^{10} + \frac{1}{9}b^2cx^9 + \frac{2}{9}abgx^9 + \frac{1}{4}abfx^8 + \frac{2}{7}abex^7 + \frac{1}{3}abdx^6 + \frac{2}{5}abcx^5 + \frac{1}{5}a^2gx^5 + \frac{1}{4}a^2fx^4 + \frac{1}{3}a^2ex^3 + \frac{1}{2}a^2dx^2 + a^2cx$$

input
```
integrate((b*x^4+a)^2*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
1/13*b^2*g*x^13 + 1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/
9*b^2*c*x^9 + 2/9*a*b*g*x^9 + 1/4*a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^
6 + 2/5*a*b*c*x^5 + 1/5*a^2*g*x^5 + 1/4*a^2*f*x^4 + 1/3*a^2*e*x^3 + 1/2*a^
2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int (a+bx^4)^2 (c+dx+ex^2+fx^3+gx^4) \, dx = x^5 \left(\frac{g\,a^2}{5} + \frac{2\,b\,c\,a}{5}\right) + x^9 \left(\frac{c\,b^2}{9} + \frac{2\,a\,g\,b}{9}\right) + \frac{a^2\,d\,x^2}{2} + \frac{a^2\,e\,x^3}{3} + \frac{b^2\,d\,x^{10}}{10} + \frac{a^2\,f\,x^4}{4} + \frac{b^2\,e\,x^{11}}{11} + \frac{b^2\,f\,x^{12}}{12} + \frac{b^2\,g\,x^{13}}{13} + a^2\,c\,x + \frac{a\,b\,d\,x^6}{3} + \frac{2\,a\,b\,e\,x^7}{7} + \frac{a\,b\,f\,x^8}{4}$$

input
```
int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
x^5*((a^2*g)/5 + (2*a*b*c)/5) + x^9*((b^2*c)/9 + (2*a*b*g)/9) + (a^2*d*x^2
)/2 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x^4)/4 + (b^2*e*x^11)/11 +
(b^2*f*x^12)/12 + (b^2*g*x^13)/13 + a^2*c*x + (a*b*d*x^6)/3 + (2*a*b*e*x^7
)/7 + (a*b*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \left(a+bx^4\right)^2 \left(c+dx+ex^2+fx^3+gx^4\right) dx$$
$$= \frac{x(13860b^2g\,x^{12}+15015b^2f\,x^{11}+16380b^2e\,x^{10}+18018b^2d\,x^9+40040abg\,x^8+20020b^2c\,x^8+45045abf\,x}{}$$

input
```
int((b*x^4+a)^2*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

output
```
(x*(180180*a**2*c + 90090*a**2*d*x + 60060*a**2*e*x**2 + 45045*a**2*f*x**3
 + 36036*a**2*g*x**4 + 72072*a*b*c*x**4 + 60060*a*b*d*x**5 + 51480*a*b*e*x
**6 + 45045*a*b*f*x**7 + 40040*a*b*g*x**8 + 20020*b**2*c*x**8 + 18018*b**2
*d*x**9 + 16380*b**2*e*x**10 + 15015*b**2*f*x**11 + 13860*b**2*g*x**12))/1
80180
```

3.106 $\int (a+bx^4) (c+dx+ex^2+fx^3+gx^4) \, dx$

Optimal result 845
Mathematica [A] (verified) 845
Rubi [A] (verified) 846
Maple [A] (verified) 847
Fricas [A] (verification not implemented) 847
Sympy [A] (verification not implemented) 848
Maxima [A] (verification not implemented) 848
Giac [A] (verification not implemented) 849
Mupad [B] (verification not implemented) 849
Reduce [B] (verification not implemented) 850

Optimal result

Integrand size = 28, antiderivative size = 82

$$\int (a+bx^4) (c+dx+ex^2+fx^3+gx^4) \, dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}(bc+ag)x^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9$$

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*(a*g+b*c)*x^5+1/6*b*d*x^6+1/
7*b*e*x^7+1/8*b*f*x^8+1/9*b*g*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int (a+bx^4) (c+dx+ex^2+fx^3+gx^4) \, dx = acx+b\left(\frac{cx^5}{5}+\frac{dx^6}{6}+\frac{ex^7}{7}+\frac{fx^8}{8}+\frac{gx^9}{9}\right)+\frac{1}{60}ax^2(30d+x(20e+3x(5f+4gx)))$$

input
```
Integrate[(a + b*x^4)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
a*c*x + b*((c*x^5)/5 + (d*x^6)/6 + (e*x^7)/7 + (f*x^8)/8 + (g*x^9)/9) + (a
*x^2*(30*d + x*(20*e + 3*x*(5*f + 4*g*x))))/60
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(c+dx+ex^2+fx^3+gx^4\right)dx$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(x^4(ag+bc)+ac+adx+aex^2+afx^3+bdx^5+bex^6+bfx^7+bgx^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(ag+bc)+acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8+\frac{1}{9}bgx^9$$

input
```
Int[(a + b*x^4)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + ((b*c + a*g)*x^5)/5 + (b
*d*x^6)/6 + (b*e*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result	si
default	$acx+\frac{ad\,x^{2}}{2}+\frac{ae\,x^{3}}{3}+\frac{af\,x^{4}}{4}+\frac{(ag+cb)x^{5}}{5}+\frac{x^{6}bd}{6}+\frac{be\,x^{7}}{7}+\frac{bf\,x^{8}}{8}+\frac{bg\,x^{9}}{9}$	67
norman	$\frac{bg\,x^{9}}{9}+\frac{bf\,x^{8}}{8}+\frac{be\,x^{7}}{7}+\frac{x^{6}bd}{6}+\left(\frac{ag}{5}+\frac{cb}{5}\right)x^{5}+\frac{af\,x^{4}}{4}+\frac{ae\,x^{3}}{3}+\frac{ad\,x^{2}}{2}+acx$	68
gosper	$\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}ag+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	69
risch	$\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}ag+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	69
parallelrisch	$\frac{1}{9}bg\,x^{9}+\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}ag+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	69
orering	$\frac{x\left(280bg\,x^{8}+315bf\,x^{7}+360be\,x^{6}+420x^{5}bd+504ag\,x^{4}+504bc\,x^{4}+630af\,x^{3}+840ae\,x^{2}+1260adx+2520ac\right)}{2520}$	70

input
```
int((b*x^4+a)*(g*x^4+f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/4*a*f*x^4+1/5*(a*g+b*c)*x^5+1/6*x^6*b*d+1/
7*b*e*x^7+1/8*b*f*x^8+1/9*b*g*x^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int (a+bx^4)\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{1}{9}\,bgx^{9}+\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,bex^{7}+\frac{1}{6}\,bdx^{6}+\frac{1}{4}\,afx^{4}+\frac{1}{5}\,(bc+ag)x^{5}+\frac{1}{3}\,aex^{3}+\frac{1}{2}\,adx^{2}+acx$$

input
```
integrate((b*x^4+a)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/4*a*f*x^4 + 1/5*
(b*c + a*g)*x^5 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int (a+bx^4)(c+dx+ex^2+fx^3+gx^4)\,dx = acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{afx^4}{4}+\frac{bdx^6}{6}+\frac{bex^7}{7}+\frac{bfx^8}{8}+\frac{bgx^9}{9}+x^5\left(\frac{ag}{5}+\frac{bc}{5}\right)$$

input
```
integrate((b*x**4+a)*(g*x**4+f*x**3+e*x**2+d*x+c),x)
```

output
```
a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*d*x**6/6 + b*e*x**7/7 + b
*f*x**8/8 + b*g*x**9/9 + x**5*(a*g/5 + b*c/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int (a+bx^4)(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{1}{9}\,bgx^9+\frac{1}{8}\,bfx^8+\frac{1}{7}\,bex^7+\frac{1}{6}\,bdx^6+\frac{1}{4}\,afx^4+\frac{1}{5}\,(bc+ag)x^5+\frac{1}{3}\,aex^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((b*x^4+a)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/4*a*f*x^4 + 1/5*
(b*c + a*g)*x^5 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int (a+bx^4)(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{1}{9}bgx^9+\frac{1}{8}bfx^8+\frac{1}{7}bex^7+\frac{1}{6}bdx^6+\frac{1}{5}bcx^5+\frac{1}{5}agx^5+\frac{1}{4}afx^4+\frac{1}{3}aex^3+\frac{1}{2}adx^2+acx$$

input
```
integrate((b*x^4+a)*(g*x^4+f*x^3+e*x^2+d*x+c),x, algorithm="giac")
```

output
```
1/9*b*g*x^9 + 1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/5*
a*g*x^5 + 1/4*a*f*x^4 + 1/3*a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int (a+bx^4)(c+dx+ex^2+fx^3+gx^4)\,dx = \frac{b\,g\,x^9}{9}+\frac{b\,f\,x^8}{8}+\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\left(\frac{b\,c}{5}+\frac{a\,g}{5}\right)x^5+\frac{a\,f\,x^4}{4}+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

input
```
int((a + b*x^4)*(c + d*x + e*x^2 + f*x^3 + g*x^4),x)
```

output
```
x^5*((b*c)/5 + (a*g)/5) + a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (b*d*x^6)/6
+ (a*f*x^4)/4 + (b*e*x^7)/7 + (b*f*x^8)/8 + (b*g*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)\left(c+dx+ex^2+fx^3+gx^4\right)\,dx$$
$$=\frac{x(280bg\,x^8+315bf\,x^7+360be\,x^6+420bd\,x^5+504ag\,x^4+504bc\,x^4+630af\,x^3+840ae\,x^2+1260adx\ -}{2520}$$

input
```
int((b*x^4+a)*(g*x^4+f*x^3+e*x^2+d*x+c),x)
```

output
```
(x*(2520*a*c + 1260*a*d*x + 840*a*e*x**2 + 630*a*f*x**3 + 504*a*g*x**4 + 5
04*b*c*x**4 + 420*b*d*x**5 + 360*b*e*x**6 + 315*b*f*x**7 + 280*b*g*x**8))/
2520
```

3.107 $\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4} dx$

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Optimal result

Integrand size = 30, antiderivative size = 247

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx &= \frac{gx}{b}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} \\ &\quad -\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} \\ &\quad +\frac{\left(bc+\sqrt{a}\sqrt{b}e-ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} \\ &\quad +\frac{\left(bc-\sqrt{a}\sqrt{b}e-ag\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} \\ &\quad +\frac{f\log\left(a+bx^4\right)}{4b}\end{aligned}$$

output
```
g*x/b+1/2*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)+1/4*(b*c+a^(1/2)*b
^(1/2)*e-a*g)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(5/4)
+1/4*(b*c+a^(1/2)*b^(1/2)*e-a*g)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/
2)/a^(3/4)/b^(5/4)+1/4*(b*c-a^(1/2)*b^(1/2)*e-a*g)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(5/4)+1/4*f*ln(b*x^4+a
)/b
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.26

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$= \frac{8a^{3/4}\sqrt[4]{b}gx-2\left(\sqrt{2}bc+2\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e-\sqrt{2}ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}bc-2\sqrt[4]{a}b^{3/4}d\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

output
```
(8*a^(3/4)*b^(1/4)*g*x - 2*(Sqrt[2]*b*c + 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sq
rt[a]*Sqrt[b]*e - Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2
*(Sqrt[2]*b*c - 2*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e - Sqrt[2]*
a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(b*c) + Sqrt[a]*S
qrt[b]*e + a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + S
qrt[2]*(b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2] + 2*a^(3/4)*b^(1/4)*f*Log[a + b*x^4])/(8*a^(3/4)*b^(5/
4))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\frac{c+ex^2+gx^4}{a+bx^4}+\frac{x\left(d+fx^2\right)}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e-ag+bc\right)}{2\sqrt{2}a^{3/4}b^{5/4}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e-ag+bc\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}+\frac{gx}{b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x]
```

output
```
(g*x)/b + (d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((b*c +
Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(5/4)) + ((b*c + Sqrt[a]*Sqrt[b]*e - a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - Sqrt[a]*Sqrt[b
]*e - a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt
[2]*a^(3/4)*b^(5/4)) + ((b*c - Sqrt[a]*Sqrt[b]*e - a*g)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + (f*Log
[a + b*x^4])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.23

method	result
risch	$\frac{gx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(\_R^{3}bf+\_R^{2}be+\_Rbd-ag+cb\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$
default	$\frac{gx}{b}+\frac{\frac{(-ag+cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{bd\arctan\left(x^{2}\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^{2}-}{x^{2}+}\right.\right.}{}}{b}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
g*x/b+1/4/b^2*sum((_R^3*b*f+_R^2*b*e+_R*b*d-a*g+b*c)/_R^3*ln(x-_R),_R=Root
Of(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 34.26 (sec) , antiderivative size = 622377, normalized size of antiderivative = 2519.74

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.33

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \frac{gx}{b} + \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f+b^2c-\sqrt{a}b^{\frac{3}{2}}e-abg\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}f-b^2c+\sqrt{a}b^{\frac{3}{2}}e+abg\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \cdots}{\cdots}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
g*x/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f + b^2*c - sqrt(a)*b^(3/2)*
e - a*b*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)
*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*f - b^2*c + sqrt(a)*b^(3/2)*e
 + a*b*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e - sqrt
(2)*a^(5/4)*b^(5/4)*g - 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
 sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqr
t(b))*b^(5/4)) + 2*(sqrt(2)*a^(1/4)*b^(9/4)*c + sqrt(2)*a^(3/4)*b^(7/4)*e
- sqrt(2)*a^(5/4)*b^(5/4)*g + 2*sqrt(a)*b^2*d)*arctan(1/2*sqrt(2)*(2*sqrt(
b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(
a)*sqrt(b))*b^(5/4)))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.37

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx$$
$$= \frac{gx}{b} + \frac{f\log\left(|bx^4+a|\right)}{4\,b}$$
$$- \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}$$
$$- \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d - \left(ab^3\right)^{\frac{1}{4}}b^2c + \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}$$
$$+ \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$
$$- \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c - \left(ab^3\right)^{\frac{1}{4}}abg - \left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
g*x/b + 1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d
 - (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*arctan(1/2
*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(s
qrt(2)*sqrt(a*b)*b^2*d - (a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^
3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)
*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 5042, normalized size of antiderivative = 20.41

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4),x)
```

output
```
symsum(log(b^2*c*d^2 - b^2*c^2*e - a^2*e*g^2 + a^2*f^2*g + b^2*d^3*x - a*b
*e^3 - a*b*c*f^2 - a*b*d^2*g - 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3
 - 64*a^3*b^3*e*g*z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b
^4*d^2*z^2 + 32*a^3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z
- 16*a^3*b^2*d*g^2*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*
c^2*d*z - 16*a^3*b^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2
*b^2*d*e^2*f - 4*a^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4
*a^3*b*d*f*g^2 + 4*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b
^3*c^3*g + 6*a^2*b^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b
^3*c^2*e^2 + a^2*b^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k
)^2*a*b^3*c - 4*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 64*a^3*b^3*e*g*
z^2 + 64*a^2*b^4*c*e*z^2 + 96*a^3*b^3*f^2*z^2 + 32*a^2*b^4*d^2*z^2 + 32*a^
3*b^2*e*f*g*z - 32*a^2*b^3*c*e*f*z + 32*a^2*b^3*c*d*g*z - 16*a^3*b^2*d*g^2
*z - 16*a^2*b^3*d^2*f*z + 16*a^2*b^3*d*e^2*z - 16*a*b^4*c^2*d*z - 16*a^3*b
^2*f^3*z - 8*a^2*b^2*c*d*f*g + 4*a^2*b^2*d^2*e*g - 4*a^2*b^2*d*e^2*f - 4*a
^2*b^2*c*e^2*g + 4*a^2*b^2*c*e*f^2 - 4*a^3*b*e*f^2*g + 4*a^3*b*d*f*g^2 + 4
*a*b^3*c^2*d*f - 4*a*b^3*c*d^2*e - 4*a^3*b*c*g^3 - 4*a*b^3*c^3*g + 6*a^2*b
^2*c^2*g^2 + 2*a^2*b^2*d^2*f^2 + 2*a^3*b*e^2*g^2 + 2*a*b^3*c^2*e^2 + a^2*b
^2*e^4 + a^3*b*f^4 + a*b^3*d^4 + a^4*g^4 + b^4*c^4, z, k)*b^3*c^2*x + b^2*
c^2*f*x + a^2*f*g^2*x + 16*root(256*a^3*b^5*z^4 - 256*a^3*b^4*f*z^3 - 6...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*e + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*g
 - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c - 4*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**
(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d + 2*b**(1/4)
*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)
*a**(1/4)*sqrt(2)))*b*e - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*g + 2*b**(3/4)*a
**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a
**(1/4)*sqrt(2)))*b*c - 4*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d + b**(1/4)*a**(3/4)*sqrt(2
)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*e - b**(1
/4)*a**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x
**2)*b*e + b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x +
sqrt(a) + sqrt(b)*x**2)*a*g - b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a*
*(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c - b**(3/4)*a**(1/4)*sqrt(2)
*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*g + b**(3/4)*
a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)
*b*c + 2*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a...
```

3.108 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2} dx$

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Optimal result

Integrand size = 30, antiderivative size = 269

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2} dx = \frac{x(bc-ag+bdx+bex^2+bfx^3)}{4ab(a+bx^4)} + \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}} - \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc+\sqrt{a}\sqrt{b}e+ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{\left(3bc-\sqrt{a}\sqrt{b}e+ag\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

output
```
1/4*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)+1/4*d*arctan(b^(1/2)*x
^2/a^(1/2))/a^(3/2)/b^(1/2)+1/16*(3*b*c+a^(1/2)*b^(1/2)*e+a*g)*arctan(-1+2
^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+1/16*(3*b*c+a^(1/2)*b^(1
/2)*e+a*g)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+1/1
6*(3*b*c-a^(1/2)*b^(1/2)*e+a*g)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)
+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.19

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{a+bx^4}-2\left(3\sqrt{2}bc+4\sqrt[4]{a}b^{3/4}d+\sqrt{2}\sqrt{a}\sqrt{b}e+\sqrt{2}ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

output
```
((-8*a^(3/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4) -
2*(3*Sqrt[2]*b*c + 4*a^(1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[
2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(3*Sqrt[2]*b*c - 4*a^(
1/4)*b^(3/4)*d + Sqrt[2]*Sqrt[a]*Sqrt[b]*e + Sqrt[2]*a*g)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-3*b*c + Sqrt[a]*Sqrt[b]*e - a*g)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c - Sqrt[
a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(32*a^(7/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2397, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{ 2397}$$
$$\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{4ab(a+bx^4)}-\frac{\int -\frac{bex^2+2bdx+3bc+ag}{bx^4+a}dx}{4ab}$$
$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{bex^2+2bdx+3bc+ag}{bx^4+a}dx}{4ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{\int \left(\frac{2bdx}{bx^4+a}+\frac{bex^2+3bc+ag}{bx^4+a}\right)dx}{4ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\sqrt{b}e+ag+3bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-\sqrt{a}\sqrt{b}e+ag+\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}}{4ab}$$

$$\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{4ab\left(a+bx^4\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x]
```

output
```
(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(4*a*b*(a + b*x^4)) + ((Sqrt[b
]*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - ((3*b*c + Sqrt[a]*Sqrt[b]*e +
 a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4))
 + ((3*b*c + Sqrt[a]*Sqrt[b]*e + a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(
1/4)) + ((3*b*c - Sqrt[a]*Sqrt[b]*e + a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}-\frac{(ag-cb)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^2 e+2d\_R+\frac{ag+3cb}{b}\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}-\frac{(ag-cb)x}{4ab}-\frac{f}{4b}}{b\,x^4+a}+\frac{\frac{(ag+3cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{bd\arctan\left(\right.}{\sqrt{a}}}{4ba}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^3+1/4*d/a*x^2-1/4*(a*g-b*c)/a/b*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*s
um((_R^2*e+2*d*_R+1/b*(a*g+3*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.43 (sec) , antiderivative size = 352423, normalized size of antiderivative = 1310.12

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.30

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2-af+(bc-ag)x}{4\,(ab^2x^4+a^2b)}$$

$$+\frac{\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,b^{\frac{3}{2}}c-\sqrt{a}be+a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{7}{4}}c+\sqrt{2}a\right.}{}}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 - a*f + (b*c - a*g)*x)/(a*b^2*x^4 + a^2*b) + 1/32*(
sqrt(2)*(3*b^(3/2)*c - sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*b^(3/2)*c -
sqrt(a)*b*e + a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + sqrt(2)*a^(3/
4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g - 4*sqrt(a)*b^(3/2)*d)*arctan(1/2
*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a
^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(7/4)*c + s
qrt(2)*a^(3/4)*b^(5/4)*e + sqrt(2)*a^(5/4)*b^(3/4)*g + 4*sqrt(a)*b^(3/2)*d
)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.34

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{bex^3+bdx^2+bcx-agx-af}{4\,(bx^4+a)ab}$$
$$+ \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3}$$
$$+ \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$
$$- \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{1}{4}}\,abg-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*g*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqr
t(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(1/4)*a*
b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(
1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g + (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^
(1/4)*b^2*c + (a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c +
(a*b^3)^(1/4)*a*b*g - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 1383, normalized size of antiderivative = 5.14

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^2,x)
```

output
```
symsum(log(- (9*b^2*c^2*e - 12*b^2*c*d^2 + a^2*e*g^2 + a*b*e^3 - 4*a*b*d^2
*g + 6*a*b*c*e*g)/(64*a^3) - (root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^
2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 12
8*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^
2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a
^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*
a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*b*(9*b^2*c^2*x + a^2
*g^2*x + 16*root(65536*a^7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c
*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z
+ 128*a^3*b^3*d*e^2*z - 1152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2
*b^2*c*e^2*g - 48*a*b^3*c*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^
2*b^2*c^2*g^2 + 2*a^3*b*e^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4
*c^4 + a^2*b^2*e^4 + a^4*g^4, z, k)*a^3*b*g - a*b*e^2*x + 48*root(65536*a^
7*b^5*z^4 + 1024*a^5*b^3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2
*z^2 - 768*a^3*b^3*c*d*g*z - 128*a^4*b^2*d*g^2*z + 128*a^3*b^3*d*e^2*z - 1
152*a^2*b^4*c^2*d*z - 16*a^2*b^2*d^2*e*g + 12*a^2*b^2*c*e^2*g - 48*a*b^3*c
*d^2*e + 108*a*b^3*c^3*g + 12*a^3*b*c*g^3 + 54*a^2*b^2*c^2*g^2 + 2*a^3*b*e
^2*g^2 + 18*a*b^3*c^2*e^2 + 16*a*b^3*d^4 + 81*b^4*c^4 + a^2*b^2*e^4 + a^4*
g^4, z, k)*a^2*b^2*c + 4*a*b*d*e - 32*root(65536*a^7*b^5*z^4 + 1024*a^5*b^
3*e*g*z^2 + 3072*a^4*b^4*c*e*z^2 + 2048*a^4*b^4*d^2*z^2 - 768*a^3*b^3*c...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1082, normalized size of antiderivative = 4.02

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

output

```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*e - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*
*2*e*x**4 - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*g - 6*b**(3/4)*a**(1/4)*sqr
t(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqr
t(2)))*a*b*c - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*g*x**4 - 6*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1
/4)*sqrt(2)))*b**2*c*x**4 - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d - 8*sqrt(b)*sqrt(a)*
atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))
)*b**2*d*x**4 + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*e + 2*b**(1/4)*a**(3/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*
sqrt(2)))*b**2*e*x**4 + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*g + 6*b**(3/4)*
a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*a*b*c + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*g*x**4 + 6...
```

3.109 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$

Optimal result . . . 868
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Optimal result

Integrand size = 30, antiderivative size = 334

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx &= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{8ab\,(a+bx^4)^2}-\frac{f}{8ab\,(a+bx^4)} \\ &+\frac{x(7bc+ag+6bdx+5bex^2)}{32a^2b\,(a+bx^4)}+\frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} \\ &-\frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\ &+\frac{\left(21bc+5\sqrt{a}\sqrt{b}e+3ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\ &+\frac{\left(21bc-5\sqrt{a}\sqrt{b}e+3ag\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{64\sqrt{2}a^{11/4}b^{5/4}}\end{aligned}$$

output
```
1/8*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^2-1/8*f/a/b/(b*x^4+a)+
1/32*x*(5*b*e*x^2+6*b*d*x+a*g+7*b*c)/a^2/b/(b*x^4+a)+3/16*d*arctan(b^(1/2)
*x^2/a^(1/2))/a^(5/2)/b^(1/2)+1/128*(21*b*c+5*a^(1/2)*b^(1/2)*e+3*a*g)*arc
tan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/b^(5/4)+1/128*(21*b*c+5
*a^(1/2)*b^(1/2)*e+3*a*g)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1
1/4)/b^(5/4)+1/128*(21*b*c-5*a^(1/2)*b^(1/2)*e+3*a*g)*arctanh(2^(1/2)*a^(1
/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(11/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.10

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8a^{3/4}\sqrt[4]{b}x(7bc+ag+bx(6d+5ex))}{a+bx^4} - \frac{32a^{7/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^2} - 2\left(21\sqrt{2}bc+24\sqrt[4]{a}b^{3/4}d+5\sqrt{2}\sqrt{a}\sqrt{b}e+3\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

output
```
((8*a^(3/4)*b^(1/4)*x*(7*b*c + a*g + b*x*(6*d + 5*e*x)))/(a + b*x^4) - (32
*a^(7/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^4)^2 - 2*
(21*Sqrt[2]*b*c + 24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*S
qrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(21*Sqrt[2]*b*c -
24*a^(1/4)*b^(3/4)*d + 5*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 3*Sqrt[2]*a*g)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-21*b*c + 5*Sqrt[a]*Sqrt[b]*e
 - 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]
*(21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(256*a^(11/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2397, 25, 2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}dx$$

$$\Big\downarrow \ 2397$$

$$\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}-\frac{\int -\frac{4bfx^3+5bex^2+6bdx+7bc+ag}{(bx^4+a)^2}dx}{8ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4bfx^3+5bex^2+6bdx+7bc+ag}{(bx^4+a)^2}dx}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 2393$$

$$\frac{-\frac{\int -\frac{5bex^2+12bdx+3(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{4a(a+bx^4)}}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{5bex^2+12bdx+3(7bc+ag)}{bx^4+a}dx}{4a}-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{4a(a+bx^4)}}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 2415$$

$$\frac{\frac{\int \left(\frac{12bdx}{bx^4+a}+\frac{5bex^2+3(7bc+ag)}{bx^4+a}\right)dx}{4a}-\frac{4af-x(ag+7bc+6bdx+5bex^2)}{4a(a+bx^4)}}{8ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-5\sqrt{a}\sqrt{b}e+3ag+21bc\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}+\cdots}{4a}}{8ab}$$

$$\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{8ab\left(a+bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x]
```

output
```
(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(8*a*b*(a + b*x^4)^2) + (-1/4*
(4*a*f - x*(7*b*c + a*g + 6*b*d*x + 5*b*e*x^2))/(a*(a + b*x^4)) + ((6*Sqrt
[b]*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - ((21*b*c + 5*Sqrt[a]*Sqrt[b
]*e + 3*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b
^(1/4)) + ((21*b*c + 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - ((21*b*c - 5*Sqrt[a]*Sqrt[b
]*e + 3*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(1/4)) + ((21*b*c - 5*Sqrt[a]*Sqrt[b]*e + 3*a*g)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4))
)/(4*a))/(8*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.43

method	result
risch	$\frac{\frac{5be\,x^{7}}{32a^{2}}+\frac{3bd\,x^{6}}{16a^{2}}+\frac{(ag+7cb)x^{5}}{32a^{2}}+\frac{9e\,x^{3}}{32a}+\frac{5d\,x^{2}}{16a}-\frac{(3ag-11cb)x}{32ab}-\frac{f}{8b}}{(b\,x^{4}+a)^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^{4}+a)}\frac{\left(5\_R^{2}e+12d\_R+\frac{3ag+21cb}{b}\right)\ln\left(x-\_R\right.}{\_R^{3}}}{128b\,a^{2}}$
default	$\frac{\frac{5be\,x^{7}}{32a^{2}}+\frac{3bd\,x^{6}}{16a^{2}}+\frac{(ag+7cb)x^{5}}{32a^{2}}+\frac{9e\,x^{3}}{32a}+\frac{5d\,x^{2}}{16a}-\frac{(3ag-11cb)x}{32ab}-\frac{f}{8b}}{(b\,x^{4}+a)^{2}}+\frac{\frac{(3ag+21cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{8a}}{}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+1/32*(a*g+7*b*c)/a^2*x^5+9/32*e/a*x^3+5
/16*d/a*x^2-1/32*(3*a*g-11*b*c)/a/b*x-1/8*f/b)/(b*x^4+a)^2+1/128/b/a^2*sum
((5*_R^2*e+12*d*_R+3/b*(a*g+7*b*c))/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 40.34 (sec) , antiderivative size = 358509, normalized size of antiderivative = 1073.38

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$
$$= \frac{5\,b^2ex^7+6\,b^2dx^6+9\,abex^3+(7\,b^2c+abg)x^5+10\,abdx^2-4\,a^2f+(11\,abc-3\,a^2g)x}{32\,(a^2b^3x^8+2\,a^3b^2x^4+a^4b)}$$
$$+ \frac{\frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,b^{\frac{3}{2}}c-5\,\sqrt{a}be+3\,a\sqrt{b}g\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\sqrt{2}a^{\frac{1}{4}}\right.}{}}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 9*a*b*e*x^3 + (7*b^2*c + a*b*g)*x^5 + 10
*a*b*d*x^2 - 4*a^2*f + (11*a*b*c - 3*a^2*g)*x)/(a^2*b^3*x^8 + 2*a^3*b^2*x^
4 + a^4*b) + 1/256*(sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)
*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4))
- sqrt(2)*(21*b^(3/2)*c - 5*sqrt(a)*b*e + 3*a*sqrt(b)*g)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(21*sqrt(2)*a^
(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2)*a^(5/4)*b^(3/4)*
g - 24*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4
)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4))
+ 2*(21*sqrt(2)*a^(1/4)*b^(7/4)*c + 5*sqrt(2)*a^(3/4)*b^(5/4)*e + 3*sqrt(2
)*a^(5/4)*b^(3/4)*g + 24*sqrt(a)*b^(3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*
x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*
sqrt(b))*b^(3/4)))/(a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.23

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx$$
$$= \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+\frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg+5\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3}$$
$$+\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$-\frac{\sqrt{2}\left(21\left(ab^3\right)^{\frac{1}{4}}b^2c+3\left(ab^3\right)^{\frac{1}{4}}abg-5\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3}$$
$$+\frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+abgx^5+9\,abex^3+10\,abdx^2+11\,abcx-3\,a^2gx-4\,a^2f}{32\left(bx^4+a\right)^2a^2b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*
b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^
2*d + 21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g + 5*(a*b^3)^(3/4)*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/
256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3
/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sqrt
(2)*(21*(a*b^3)^(1/4)*b^2*c + 3*(a*b^3)^(1/4)*a*b*g - 5*(a*b^3)^(3/4)*e)*l
og(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7
+ 6*b^2*d*x^6 + 7*b^2*c*x^5 + a*b*g*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2 + 11*
a*b*c*x - 3*a^2*g*x - 4*a^2*f)/((b*x^4 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.00

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^3,x)
```

output
```
((5*d*x^2)/(16*a) - f/(8*b) + (9*e*x^3)/(32*a) + (x^5*(7*b*c + a*g))/(32*a
^2) + (x*(11*b*c - 3*a*g))/(32*a*b) + (3*b*d*x^6)/(16*a^2) + (5*b*e*x^7)/(
32*a^2))/(a^2 + b^2*x^8 + 2*a*b*x^4) + symsum(log(- root(268435456*a^11*b^
5*z^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 4718592*a^6*b^4
*d^2*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 153600*a^4*b^3
*d*e^2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*a^2*b^2*c
*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*g^3 + 238
14*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 625*a^2*b^2
*e^4 + 20736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*(root(26843545
6*a^11*b^5*z^4 + 983040*a^7*b^3*e*g*z^2 + 6881280*a^6*b^4*c*e*z^2 + 471859
2*a^6*b^4*d^2*z^2 - 774144*a^4*b^3*c*d*g*z - 55296*a^5*b^2*d*g^2*z + 15360
0*a^4*b^3*d*e^2*z - 2709504*a^3*b^4*c^2*d*z - 8640*a^2*b^2*d^2*e*g + 6300*
a^2*b^2*c*e^2*g - 60480*a*b^3*c*d^2*e + 111132*a*b^3*c^3*g + 2268*a^3*b*c*
g^3 + 23814*a^2*b^2*c^2*g^2 + 450*a^3*b*e^2*g^2 + 22050*a*b^3*c^2*e^2 + 62
5*a^2*b^2*e^4 + 20736*a*b^3*d^4 + 81*a^4*g^4 + 194481*b^4*c^4, z, k)*((344
064*a^5*b^3*c + 49152*a^6*b^2*g)/(32768*a^6) - (6*b^3*d*x)/a) + (x*(144*a^
4*b*g^2 + 7056*a^2*b^3*c^2 - 400*a^3*b^2*e^2 + 2016*a^3*b^2*c*g))/(4096*a^
6) + (15*b^2*d*e)/(32*a^3)) - (2205*b^2*c^2*e - 3024*b^2*c*d^2 + 45*a^2*e*
g^2 + 125*a*b*e^3 - 432*a*b*d^2*g + 630*a*b*c*e*g)/(32768*a^6) - (x*(315*b
^2*c*d*e - 216*b^2*d^3 + 45*a*b*d*e*g))/(4096*a^6))*root(268435456*a^11...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1681, normalized size of antiderivative = 5.03

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

output

```
( - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*e - 20*b**(1/4)*a**(3/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)
))*a*b**2*e*x**4 - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*e*x**8 - 6*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)
*a**(1/4)*sqrt(2)))*a**3*g - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c - 12*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*g*x**4 - 84*b**(3/4)*a**(1/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b**2*c*x**4 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*g*x**8 - 42*b**(3/4)
*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)
*a**(1/4)*sqrt(2)))*b**3*c*x**8 - 48*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*d - 96*sqrt(
b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*b**2*d*x**4 - 48*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*d*x**8 + 10*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(...
```

3.110 $\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$

Optimal result . . . 878
Mathematica [A] (verified) . . . 879
Rubi [A] (verified) . . . 880
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Giac [A] (verification not implemented) . . . 885
Mupad [B] (verification not implemented) . . . 886
Reduce [B] (verification not implemented) . . . 887

Optimal result

Integrand size = 30, antiderivative size = 377

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx &= \frac{x(bc-ag+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3} - \frac{f}{12ab(a+bx^4)^2} \\ &+ \frac{x(11bc+ag+10bdx+9bex^2)}{96a^2b(a+bx^4)^2} \\ &+ \frac{x(7(11bc+ag)+60bdx+45bex^2)}{384a^3b(a+bx^4)} \\ &+ \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} \\ &- \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} \\ &+ \frac{\left(77bc+15\sqrt{a}\sqrt{b}e+7ag\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{5/4}} \\ &+ \frac{\left(77bc-15\sqrt{a}\sqrt{b}e+7ag\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{256\sqrt{2}a^{15/4}b^{5/4}}\end{aligned}$$

output
```
1/12*x*(b*f*x^3+b*e*x^2+b*d*x-a*g+b*c)/a/b/(b*x^4+a)^3-1/12*f/a/b/(b*x^4+a
)^2+1/96*x*(9*b*e*x^2+10*b*d*x+a*g+11*b*c)/a^2/b/(b*x^4+a)^2+1/384*x*(45*b
*e*x^2+60*b*d*x+7*a*g+77*b*c)/a^3/b/(b*x^4+a)+5/32*d*arctan(b^(1/2)*x^2/a^
(1/2))/a^(7/2)/b^(1/2)+1/512*(77*b*c+15*a^(1/2)*b^(1/2)*e+7*a*g)*arctan(-1
+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(15/4)/b^(5/4)+1/512*(77*b*c+15*a^(1
/2)*b^(1/2)*e+7*a*g)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(15/4)/
b^(5/4)+1/512*(77*b*c-15*a^(1/2)*b^(1/2)*e+7*a*g)*arctanh(2^(1/2)*a^(1/4)*
b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(15/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.09

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$
$$= \frac{\frac{8a^{3/4}\sqrt[4]{b}x(77bc+7ag+15bx(4d+3ex))}{a+bx^4}+\frac{32a^{7/4}\sqrt[4]{b}x(11bc+ag+bx(10d+9ex))}{(a+bx^4)^2}-\frac{256a^{11/4}\sqrt[4]{b}(a(f+gx)-bx(c+x(d+ex)))}{(a+bx^4)^3}-6\Big(77\sqrt{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

output
```
((8*a^(3/4)*b^(1/4)*x*(77*b*c + 7*a*g + 15*b*x*(4*d + 3*e*x)))/(a + b*x^4)
 + (32*a^(7/4)*b^(1/4)*x*(11*b*c + a*g + b*x*(10*d + 9*e*x)))/(a + b*x^4)^
2 - (256*a^(11/4)*b^(1/4)*(a*(f + g*x) - b*x*(c + x*(d + e*x))))/(a + b*x^
4)^3 - 6*(77*Sqrt[2]*b*c + 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[
b]*e + 7*Sqrt[2]*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*(77*Sqrt
[2]*b*c - 80*a^(1/4)*b^(3/4)*d + 15*Sqrt[2]*Sqrt[a]*Sqrt[b]*e + 7*Sqrt[2]*
a*g)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(77*b*c - 15*Sqrt
[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2] + 3*Sqrt[2]*(77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(3072*a^(15/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2397, 25, 2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}-\frac{\int -\frac{8bfx^3+9bex^2+10bdx+11bc+ag}{(bx^4+a)^3}dx}{12ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8bfx^3+9bex^2+10bdx+11bc+ag}{(bx^4+a)^3}dx}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2393}$$

$$\frac{-\frac{\int -\frac{45bex^2+60bdx+7(11bc+ag)}{(bx^4+a)^2}dx}{8a}-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{8a(a+bx^4)^2}}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45bex^2+60bdx+7(11bc+ag)}{(bx^4+a)^2}dx}{8a}-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{8a(a+bx^4)^2}}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3\left(15bex^2+40bdx+7(11bc+ag)\right)}{bx^4+a}dx}{4a}}{8a}-\frac{8af-x(ag+11bc+10bdx+9bex^2)}{8a(a+bx^4)^2}}{12ab}+\frac{x(-ag+bc+bdx+bex^2+bfx^3)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{15bex^2+40bdx+7(11bc+ag)}{bx^4+a}dx}{4a}+\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{8a\left(a+bx^4\right)^2}}{12ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{40bdx}{bx^4+a}+\frac{15bex^2+7(11bc+ag)}{bx^4+a}\right)dx}{4a}+\frac{x\left(7(ag+11bc)+60bdx+45bex^2\right)}{4a\left(a+bx^4\right)}}{8a}-\frac{8af-x\left(ag+11bc+10bdx+9bex^2\right)}{8a\left(a+bx^4\right)^2}}{12ab}+\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}\sqrt{b}e+7ag+77bc\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)\left(-15\sqrt{a}\sqrt{b}e+7ag+77bc\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}}{4a}}{8a}}{1}$$

$$\frac{x\left(-ag+bc+bdx+bex^2+bfx^3\right)}{12ab\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x]
```

output
```
(x*(b*c - a*g + b*d*x + b*e*x^2 + b*f*x^3))/(12*a*b*(a + b*x^4)^3) + (-1/8
*(8*a*f - x*(11*b*c + a*g + 10*b*d*x + 9*b*e*x^2))/(a*(a + b*x^4)^2) + ((x
*(7*(11*b*c + a*g) + 60*b*d*x + 45*b*e*x^2))/(4*a*(a + b*x^4)) + (3*((20*S
qrt[b]*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - ((77*b*c + 15*Sqrt[a]*Sq
rt[b]*e + 7*a*g)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/
4)*b^(1/4)) + ((77*b*c + 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*ArcTan[1 + (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - ((77*b*c - 15*Sqrt[a]*
Sqrt[b]*e + 7*a*g)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])
/(4*Sqrt[2]*a^(3/4)*b^(1/4)) + ((77*b*c - 15*Sqrt[a]*Sqrt[b]*e + 7*a*g)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b
^(1/4))))/(4*a))/(8*a))/(12*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{7(ag+11cb)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{3(ag+11cb)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}-\frac{(7ag-51cb)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+}}{}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{7(ag+11cb)b\,x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{3(ag+11cb)x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}-\frac{(7ag-51cb)x}{128ab}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\frac{(7ag+77cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}}{}}{}$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+7/384*(a*g+11*b*c)/a^3*b*x^9+21
/64*b*e/a^2*x^7+5/12*b*d/a^2*x^6+3/64/a^2*(a*g+11*b*c)*x^5+113/384*e/a*x^3
+11/32*d/a*x^2-1/128*(7*a*g-51*b*c)/a/b*x-1/12*f/b)/(b*x^4+a)^3+1/512/b/a^
3*sum((15*_R^2*e+40*d*_R+7*(a*g+11*b*c)/b)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+
a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 72.05 (sec) , antiderivative size = 358702, normalized size of antiderivative = 951.46

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^4} \, dx = \text{Timed out}$$

input
```
integrate((g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.25

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^4} \, dx$$
$$= \frac{45\, b^3 e x^{11} + 60\, b^3 d x^{10} + 126\, a b^2 e x^7 + 160\, a b^2 d x^6 + 7\, (11\, b^3 c + a b^2 g) x^9 + 113\, a^2 b e x^3 + 132\, a^2 b d x^2 + 18\, (}{384\, (a^3 b^4 x^{12} + 3\, a^4 b^3 x^8 + 3\, a^5 b^2 x^4 + a^6 b)}$$
$$+ \frac{\frac{\sqrt{2}\left(77\, b^{\frac{3}{2}} c - 15\, \sqrt{a} b e + 7\, a \sqrt{b} g\right) \log\left(\sqrt{b} x^2 + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(77\, b^{\frac{3}{2}} c - 15\, \sqrt{a} b e + 7\, a \sqrt{b} g\right) \log\left(\sqrt{b} x^2 - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}} x + \sqrt{a}\right)}{a^{\frac{3}{4}} b^{\frac{3}{4}}} + \frac{2\left(77\, \sqrt{2} a}{}}{}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 126*a*b^2*e*x^7 + 160*a*b^2*d*x^6 +
 7*(11*b^3*c + a*b^2*g)*x^9 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 18*(11*a
*b^2*c + a^2*b*g)*x^5 - 32*a^3*f + 3*(51*a^2*b*c - 7*a^3*g)*x)/(a^3*b^4*x^
12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*b) + 1/1024*(sqrt(2)*(77*b^(3/2)*
c - 15*sqrt(a)*b*e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1
/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*b^(3/2)*c - 15*sqrt(a)*b*
e + 7*a*sqrt(b)*g)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/
(a^(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 15*sqrt(2)*a^(3/4)*b
^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g - 80*sqrt(a)*b^(3/2)*d)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^
(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(7/4)*c + 1
5*sqrt(2)*a^(3/4)*b^(5/4)*e + 7*sqrt(2)*a^(5/4)*b^(3/4)*g + 80*sqrt(a)*b^(
3/2)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sq
rt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.22

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg+15\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$-\frac{\sqrt{2}\left(77\left(ab^3\right)^{\frac{1}{4}}b^2c+7\left(ab^3\right)^{\frac{1}{4}}abg-15\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$+\frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9+7\,ab^2gx^9+126\,ab^2ex^7+160\,ab^2dx^6+198\,ab^2cx^5+18\,a^2bgx^5+11}{384\left(bx^4+a\right)^3a^3b}$$

input
```
integrate((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*
b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b
^2*d + 77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g + 15*(a*b^3)^(3/4)*e
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^3) +
1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3
)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/1024
*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c + 7*(a*b^3)^(1/4)*a*b*g - 15*(a*b^3)^(3/4
)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^
3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 7*a*b^2*g*x^9 + 126*a*b^2*e*x^7
+ 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 18*a^2*b*g*x^5 + 113*a^2*b*e*x^3 + 1
32*a^2*b*d*x^2 + 153*a^2*b*c*x - 21*a^3*g*x - 32*a^3*f)/((b*x^4 + a)^3*a^3
*b)
```

Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 1053, normalized size of antiderivative = 2.79

$$\int \frac{c+dx+ex^2+fx^3+gx^4}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4)/(a + b*x^4)^4,x)
```

output
```
symsum(log(- root(68719476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 +
110100480*a^9*b^3*e*g*z^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c
*d*g*z - 485703680*a^4*b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^
5*b^3*d*e^2*z - 672000*a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*
a*b^3*c*d^2*e + 12782924*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^
2*c^2*g^2 + 22050*a^3*b*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^
4 + 2560000*a*b^3*d^4 + 2401*a^4*g^4 + 35153041*b^4*c^4, z, k)*(root(68719
476736*a^15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z
^2 + 838860800*a^8*b^4*d^2*z^2 - 88309760*a^5*b^3*c*d*g*z - 485703680*a^4*
b^4*c^2*d*z - 4014080*a^6*b^2*d*g^2*z + 18432000*a^5*b^3*d*e^2*z - 672000*
a^2*b^2*d^2*e*g + 485100*a^2*b^2*c*e^2*g - 7392000*a*b^3*c*d^2*e + 1278292
4*a*b^3*c^3*g + 105644*a^3*b*c*g^3 + 1743126*a^2*b^2*c^2*g^2 + 22050*a^3*b
*e^2*g^2 + 2668050*a*b^3*c^2*e^2 + 50625*a^2*b^2*e^4 + 2560000*a*b^3*d^4 +
 2401*a^4*g^4 + 35153041*b^4*c^4, z, k)*((20185088*a^7*b^3*c + 1835008*a^8
*b^2*g)/(2097152*a^9) - (5*b^3*d*x)/a^2) + (x*(1568*a^5*b*g^2 + 189728*a^3
*b^3*c^2 - 7200*a^4*b^2*e^2 + 34496*a^4*b^2*c*g))/(131072*a^9) + (75*b^2*d
*e)/(256*a^5)) - (88935*b^2*c^2*e - 123200*b^2*c*d^2 + 735*a^2*e*g^2 + 337
5*a*b*e^3 - 11200*a*b*d^2*g + 16170*a*b*c*e*g)/(2097152*a^9) - (x*(5775*b^
2*c*d*e - 4000*b^2*d^3 + 525*a*b*d*e*g))/(131072*a^9))*root(68719476736*a^
15*b^5*z^4 + 1211105280*a^8*b^4*c*e*z^2 + 110100480*a^9*b^3*e*g*z^2 + 8...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2279, normalized size of antiderivative = 6.05

$$\int \frac{c + dx + ex^2 + fx^3 + gx^4}{(a + bx^4)^4}\, dx = \text{Too large to display}$$

input
```
int((g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

output
```
( - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*e - 270*b**(1/4)*a**(3/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2
)))*a**2*b**2*e*x**4 - 270*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*e*x**8 - 90*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(
b**(1/4)*a**(1/4)*sqrt(2)))*b**4*e*x**12 - 42*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*
a**4*g - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*c - 126*b**(3/4)*a**(1/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*
sqrt(2)))*a**3*b*g*x**4 - 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c*x**
4 - 126*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt
(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*g*x**8 - 1386*b**(3/4)*a**(1
/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1
/4)*sqrt(2)))*a*b**3*c*x**8 - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*g*x**1
2 - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt
(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*c*x**12 - 480*sqrt(b)*sqrt(a)*...
```

3.111 $\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$

Optimal result 889
Mathematica [C] (verified) 889
Rubi [A] (verified) 890
Maple [C] (verified) 891
Fricas [A] (verification not implemented) 892
Sympy [C] (verification not implemented) 892
Maxima [F] 893
Giac [F] 893
Mupad [F(-1)] 893
Reduce [F] 894

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
1/2*d*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/2)+1/2*c*(a^(1/2)+b^(1/2)*
x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^
(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x)/Sqrt[a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a+bx^4}} \, dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\frac{c}{\sqrt{a+bx^4}}+\frac{dx}{\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}$	96
elliptic	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}$	99

input
```
int((d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+1/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{4\,b^{\frac{3}{2}}c\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+a\sqrt{b}d\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*b^(3/2)*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + a*s
qrt(b)*d*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a))/(a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \frac{d\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x)/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{-\sqrt{b}\,\log\left(\sqrt{b\,x^4+a}-\sqrt{b}\,x^2\right)d+\sqrt{b}\,\log\left(\sqrt{b\,x^4+a}+\sqrt{b}\,x^2\right)d+4\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)bc}{4b}$$

input
```
int((d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*d + sqrt(b)*log(sqrt(a +
b*x**4) + sqrt(b)*x**2)*d + 4*int(sqrt(a + b*x**4)/(a + b*x**4),x)*b*c)/(4
*b)
```

3.112 $\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$

Optimal result 895
Mathematica [C] (verified) 895
Rubi [A] (verified) 896
Maple [C] (verified) 897
Fricas [A] (verification not implemented) 898
Sympy [C] (verification not implemented) 898
Maxima [F] 899
Giac [F] 899
Mupad [F(-1)] 899
Reduce [F] 900

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}$$

output
```
1/2*d*arctan(b^(1/2)*x^2/(-b*x^4-a)^(1/2))/b^(1/2)+1/2*c*(a^(1/2)+b^(1/2)*
x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^
(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(1/4)/(-b*x^4-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{-a-bx^4}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[-a - b*x^4]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c}{\sqrt{-a-bx^4}}+\frac{dx}{\sqrt{-a-bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{-a-bx^4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-a-bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[-a - b*x^4]])/(2*Sqrt[b]) + (c*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[-a - b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{c\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4-a}}+\frac{d\arctan\left(\frac{\sqrt{b}\,x^2}{\sqrt{-b\,x^4-a}}\right)}{2\sqrt{b}}$	101
elliptic	$\frac{c\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4-a}}+\frac{d\ln\left(-\frac{2b\,x^2}{\sqrt{-b}}+2\sqrt{-b\,x^4-a}\right)}{2\sqrt{-b}}$	110

input
```
int((d*x+c)/(-b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(-I/a^(1/2)*b^(1/2))^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(-b*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*b^(1/2))^(1/2)
,I)+1/2*d*arctan(b^(1/2)*x^2/(-b*x^4-a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = -\frac{4\sqrt{-b}bc\left(-\frac{a}{b}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right) + a\sqrt{-b}d\log\left(-2\,bx^4+2\sqrt{-bx^4-a}\sqrt{-b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(-b)*b*c*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
 a*sqrt(-b)*d*log(-2*b*x^4 + 2*sqrt(-b*x^4 - a)*sqrt(-b)*x^2 - a))/(a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.52

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = -\frac{id\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(-b*x**4-a)**(1/2),x)
```

output
```
-I*d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - I*c*x*gamma(1/4)*hyper((1/4
, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{-b\,x^4-a}}\,dx$$

input
```
int((c + d*x)/(- a - b*x^4)^(1/2),x)
```

output
```
int((c + d*x)/(- a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{-a-bx^4}}\,dx = \frac{i\left(-2\sqrt{b}\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)c - \log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)d\right)}{2\sqrt{b}}$$

input
```
int((d*x+c)/(-b*x^4-a)^(1/2),x)
```

output
```
(i*( - 2*sqrt(b)*int(sqrt(a + b*x**4)/(a + b*x**4),x)*c - log((sqrt(a + b*
x**4) + sqrt(b)*x**2)/sqrt(a))*d))/(2*sqrt(b))
```

3.113 $\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$

Optimal result . . . 901
Mathematica [C] (verified) . . . 901
Rubi [A] (verified) . . . 902
Maple [A] (verified) . . . 903
Fricas [A] (verification not implemented) . . . 903
Sympy [A] (verification not implemented) . . . 904
Maxima [F] . . . 904
Giac [F] . . . 905
Mupad [F(-1)] . . . 905
Reduce [F] . . . 905

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

output

```
1/2*d*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(1/2)+a^(1/4)*c*(1-b*x^4/a)^(
1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx=\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{cx\sqrt{1-\frac{bx^4}{a}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{a-bx^4}}$$

input

```
Integrate[(c + d*x)/Sqrt[a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[a - b*x^4]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c}{\sqrt{a-bx^4}}+\frac{dx}{\sqrt{a-bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[a - b*x^4],x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[1
- (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a -
 b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{c\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{d\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2\sqrt{b}}$	90
elliptic	$\frac{c\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{d\ln\left(-\frac{2b\,x^{2}}{\sqrt{-b}}+2\sqrt{-b\,x^{4}+a}\right)}{2\sqrt{-b}}$	99

input
```
int((d*x+c)/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a
^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+1/
2*d*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \frac{4\sqrt{-b}bc\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-a\sqrt{-b}d\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*sqrt(-b)*b*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*
sqrt(-b)*d*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a))/(a*b)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = d\left(\begin{cases} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(-b*x**4+a)**(1/2),x)
```

output
```
d*Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1
), (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) + c*x*gamma(1/4)*hyper(
(1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \int \frac{dx+c}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate((d*x+c)/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \int \frac{c+d\,x}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int((c + d*x)/(a - b*x^4)^(1/2),x)
```

output
```
int((c + d*x)/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{a-bx^4}}\,dx = \frac{-\sqrt{b}\,atan\left(\frac{\sqrt{b}\,\sqrt{-b\,x^4+a}\,a-2\sqrt{b}\,\sqrt{-b\,x^4+a}\,b\,x^4}{-2b^2x^6+2ab\,x^2}\right)d+4\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)bc}{4b}$$

input
```
int((d*x+c)/(-b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(b)*atan((sqrt(b)*sqrt(a - b*x**4)*a - 2*sqrt(b)*sqrt(a - b*x**4)*
b*x**4)/(2*a*b*x**2 - 2*b**2*x**6))*d + 4*int(sqrt(a - b*x**4)/(a - b*x**4
),x)*b*c)/(4*b)
```

3.114 $\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx$

Optimal result 906
Mathematica [C] (verified) 906
Rubi [A] (verified) 907
Maple [A] (verified) 908
Fricas [A] (verification not implemented) 908
Sympy [A] (verification not implemented) 909
Maxima [F] 909
Giac [F] 910
Mupad [F(-1)] 910
Reduce [F] 910

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}} + \frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{-a+bx^4}}$$

output
```
1/2*d*arctanh(b^(1/2)*x^2/(b*x^4-a)^(1/2))/b^(1/2)+a^(1/4)*c*(1-b*x^4/a)^(
1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(1/4)/(b*x^4-a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{-a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1-\frac{bx^4}{a}}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{-a+bx^4}}$$

input
```
Integrate[(c + d*x)/Sqrt[-a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 - (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/Sqrt[-a + b*x^4]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{bx^4-a}}dx$$
$$\Big\downarrow \text{ 2424}$$
$$\int \left(\frac{c}{\sqrt{bx^4-a}}+\frac{dx}{\sqrt{bx^4-a}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sqrt[4]{a}c\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{bx^4-a}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{bx^4-a}}\right)}{2\sqrt{b}}$$

input
```
Int[(c + d*x)/Sqrt[-a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[-a + b*x^4]])/(2*Sqrt[b]) + (a^(1/4)*c*Sqrt[
1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[-
a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{c\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}-a}}+\frac{d\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}-a}\right)}{2\sqrt{b}}$	95
elliptic	$\frac{c\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}-a}}+\frac{d\ln\left(2\sqrt{b}\,x^{2}+2\sqrt{b\,x^{4}-a}\right)}{2\sqrt{b}}$	98

input
```
int((d*x+c)/(b*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(-1/a^(1/2)*b^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)*(1-b^(1/2)*x^2/
a^(1/2))^(1/2)/(b*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*b^(1/2))^(1/2),I)+1
/2*d*ln(b^(1/2)*x^2+(b*x^4-a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = -\frac{4\,b^{\frac{3}{2}}c\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1) - a\sqrt{b}d\log\left(2\,bx^4+2\,\sqrt{bx^4-a}\sqrt{b}x^2-a\right)}{4\,ab}$$

input
```
integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(4*b^(3/2)*c*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - a*sq
rt(b)*d*log(2*b*x^4 + 2*sqrt(b*x^4 - a)*sqrt(b)*x^2 - a))/(a*b)
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = d\left(\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}\right) - \frac{icx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)/(b*x**4-a)**(1/2),x)
```

output
```
d*Piecewise((acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1),
(-I*asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True)) - I*c*x*gamma(1/4)*hype
r((1/4, 1/2), (5/4,), b*x**4/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx=\int \frac{dx+c}{\sqrt{bx^4-a}}\,dx$$

input
```
integrate((d*x+c)/(b*x^4-a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/sqrt(b*x^4 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx=\int \frac{c+d\,x}{\sqrt{b\,x^4-a}}\,dx$$

input
```
int((c + d*x)/(b*x^4 - a)^(1/2),x)
```

output
```
int((c + d*x)/(b*x^4 - a)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{-a+bx^4}}\,dx = \frac{-\sqrt{b}\log\left(\sqrt{b\,x^4-a}-\sqrt{b}\,x^2\right)d+\sqrt{b}\log\left(\sqrt{b\,x^4-a}+\sqrt{b}\,x^2\right)d-4\left(\int \frac{\sqrt{b\,x^4-a}}{-b\,x^4+a}dx\right)bc}{4b}$$

input
```
int((d*x+c)/(b*x^4-a)^(1/2),x)
```

output
```
( - sqrt(b)*log(sqrt( - a + b*x**4) - sqrt(b)*x**2)*d + sqrt(b)*log(sqrt(
- a + b*x**4) + sqrt(b)*x**2)*d - 4*int(sqrt( - a + b*x**4)/(a - b*x**4),x
)*b*c)/(4*b)
```

3.115 $\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$

Optimal result 911
Mathematica [C] (verified) 912
Rubi [A] (verified) 912
Maple [C] (verified) 914
Fricas [A] (verification not implemented) 914
Sympy [C] (verification not implemented) 915
Maxima [F] 915
Giac [F] 916
Mupad [F(-1)] 916
Reduce [F] 916

Optimal result

Integrand size = 22, antiderivative size = 258

$$\begin{aligned}
&\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx \\
&= \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} \\
&\quad-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} \\
&\quad+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

output

```
e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)+1/2*d*arctanh(b^(1/2)*x^
2/(b*x^4+a)^(1/2))/b^(1/2)-a^(1/4)*e*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(
1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*
2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+1/2*(b^(1/2)*c+a^(1/2)*e)*(a^(1/2)+b^(1/2
)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(
b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

output
```
(d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) + (c*x*Sqrt[1 + (b*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] +
(e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])
/(3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \ 2424$$

$$\int \left(\frac{c+ex^2}{\sqrt{a+bx^4}}+\frac{dx}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2)/Sqrt[a + b*x^4],x]
```

output
```
(e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (d*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*c)/Sqrt[
a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2
]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4
])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
elliptic	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+1/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+I*e*a^(1/2)/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellipti
cE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.50

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{4\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+4\,(bc-ae)\sqrt{b}}{4\,abx}$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
 a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c
- a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 4*s
qrt(b*x^4 + a)*a*e)/(a*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.40

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gamma
(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma
(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \int \frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx=\int \frac{ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx=\int \frac{e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x + e*x^2)/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2}{\sqrt{a+bx^4}}\,dx = \frac{-\sqrt{b}\log\left(\sqrt{b\,x^4+a}-\sqrt{b}\,x^2\right)d+\sqrt{b}\log\left(\sqrt{b\,x^4+a}+\sqrt{b}\,x^2\right)d+4\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)bc+4\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx}{4b}$$

input
```
int((e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*d + sqrt(b)*log(sqrt(a +
b*x**4) + sqrt(b)*x**2)*d + 4*int(sqrt(a + b*x**4)/(a + b*x**4),x)*b*c + 4
*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*b*e)/(4*b)
```

3.116 $\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}} \, dx$

Optimal result 917
Mathematica [A] (verified) 917
Rubi [A] (verified) 918
Maple [A] (verified) 919
Fricas [A] (verification not implemented) 919
Sympy [C] (verification not implemented) 920
Maxima [A] (verification not implemented) 920
Giac [A] (verification not implemented) 921
Mupad [B] (verification not implemented) 921
Reduce [B] (verification not implemented) 921

Optimal result

Integrand size = 23, antiderivative size = 14

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}} \, dx = \frac{gx}{\sqrt{a+bx^4}}$$

output
```
g*x/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}} \, dx = \frac{gx}{\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^4]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {908}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag - bgx^4}{(a + bx^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{908}$$
$$\frac{gx}{\sqrt{a + bx^4}}$$

input
```
Int[(a*g - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(g*x)/Sqrt[a + b*x^4]
```

Defintions of rubi rules used

rule 908
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
default	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
trager	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
elliptic	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
pseudoelliptic	$\frac{gx}{\sqrt{b\,x^{4}+a}}$	13
orering	$\frac{x\left(-bg\,x^{4}+ag\right)}{\sqrt{b\,x^{4}+a}\,\left(-b\,x^{4}+a\right)}$	33

input
```
int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
g*x/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{gx}{\sqrt{bx^4+a}}$$

input
```
integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
g*x/sqrt(b*x^4 + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.71

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-b*g*x**4+a*g)/(b*x**4+a)**(3/2),x)
```

output
```
g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp
_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx}{\sqrt{bx^4+a}}$$

input
```
integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
g*x/sqrt(b*x^4 + a)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{gx}{\sqrt{bx^4+a}}$$

input
```
integrate((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
g*x/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{g\,x}{\sqrt{b\,x^4+a}}$$

input
```
int((a*g - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x)/(a + b*x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{ag-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{\sqrt{b\,x^4+a}\,gx}{b\,x^4+a}$$

input
```
int((-b*g*x^4+a*g)/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*g*x)/(a + b*x**4)
```

3.117 $\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result 922
Mathematica [A] (verified) 922
Rubi [A] (verified) 923
Maple [A] (verified) 923
Fricas [A] (verification not implemented) 924
Sympy [C] (verification not implemented) 924
Maxima [A] (verification not implemented) 925
Giac [A] (verification not implemented) 926
Mupad [B] (verification not implemented) 926
Reduce [B] (verification not implemented) 926

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

output
```
1/2*x*(2*a*g+e*x)/a/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(x*(2*a*g + e*x))/(2*a*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2395}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag - bgx^4 + ex}{(a + bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{2395}$$

$$\frac{2agx + ex^2}{2a\sqrt{a + bx^4}}$$

input
```
Int[(a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(2*a*g*x + e*x^2)/(2*a*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2395
```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{x(2ag+ex)}{2a\sqrt{b\,x^4+a}}$
trager	$\frac{x(2ag+ex)}{2a\sqrt{b\,x^4+a}}$
orering	$\frac{x(2ag+ex)}{2a\sqrt{b\,x^4+a}}$
elliptic	$\frac{e\,x^2}{2\sqrt{b\,x^4+a}\,a}+\frac{gx}{\sqrt{b\,x^4+a}}$
default	$ag\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+\frac{e\,x^2}{2\sqrt{b\,x^4+a}\,a}-bg\left(-\frac{x}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i}{}}}{}\right.$

input
```
int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(2*a*g+e*x)/a/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}(2\,agx+ex^2)}{2\,(abx^4+a^2)}$$

input
```
integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*a*g*x + e*x^2)/(a*b*x^4 + a^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.85

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((-b*g*x**4+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

output
```
g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp
_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + b*x*
*4/a))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{2\,agx+ex^2}{2\,\sqrt{bx^4+a}a}$$

input
```
integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a*g*x + e*x^2)/(sqrt(b*x^4 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\left(2\,g+\frac{ex}{a}\right)x}{2\,\sqrt{bx^4+a}}$$

input
```
integrate((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*g + e*x/a)*x/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{g\,x+\frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4+a}}$$

input
```
int((a*g + e*x - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{ag+ex-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{b\,x^4+a}\,x(2ag+ex)}{2a\,(b\,x^4+a)}$$

input
```
int((-b*g*x^4+a*g+e*x)/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*x*(2*a*g + e*x))/(2*a*(a + b*x**4))
```

3.118 $\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result 927
Mathematica [A] (verified) 927
Rubi [A] (verified) 928
Maple [A] (verified) 928
Fricas [A] (verification not implemented) 929
Sympy [A] (verification not implemented) 930
Maxima [A] (verification not implemented) 930
Giac [A] (verification not implemented) 931
Mupad [B] (verification not implemented) 931
Reduce [B] (verification not implemented) 931

Optimal result

Integrand size = 28, antiderivative size = 34

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=-\frac{f}{2b\sqrt{a+bx^4}}+\frac{gx}{\sqrt{a+bx^4}}$$

output
```
-1/2*f/b/(b*x^4+a)^(1/2)+g*x/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{ag+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{-f+2bgx}{2b\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(-f + 2*b*g*x)/(2*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2395}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag - bgx^4 + fx^3}{(a + bx^4)^{3/2}} \, dx$$
$$\Big\downarrow \; 2395$$
$$-\frac{f - 2bgx}{2b\sqrt{a + bx^4}}$$

input
```
Int[(a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(f - 2*b*g*x)/(b*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2395
```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result
gosper	$\frac{2bgx-f}{2\sqrt{b\,x^{4}+a}\,b}$
trager	$\frac{2bgx-f}{2\sqrt{b\,x^{4}+a}\,b}$
orering	$\frac{2bgx-f}{2\sqrt{b\,x^{4}+a}\,b}$
elliptic	$-\frac{f}{2b\sqrt{b\,x^{4}+a}}+\frac{gx}{\sqrt{b\,x^{4}+a}}$
default	$ag\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-\frac{f}{2b\sqrt{b\,x^{4}+a}}-bg\left(-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{}}{}}}{}\right.$

input
```
int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*b*g*x-f)/(b*x^4+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{ag+fx^3-bgx^4}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}(2\,bgx-f)}{2\left(b^2x^4+ab\right)}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*b*g*x - f)/(b^2*x^4 + a*b)
```

Sympy [A] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.21

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\, dx = f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-b*g*x**4+f*x**3+a*g)/(b*x**4+a)**(3/2),x)
```

output
```
f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\, dx = \frac{2\, bgx - f}{2\, \sqrt{bx^4 + a}b}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*g*x - f)/(sqrt(b*x^4 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{2\,gx - \frac{f}{b}}{2\,\sqrt{bx^4 + a}}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*g*x - f/b)/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{g\,x - \frac{f}{2\,b}}{\sqrt{b\,x^4 + a}}$$

input
```
int((a*g + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x - f/(2*b))/(a + b*x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{ag + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{\sqrt{b\,x^4 + a}\,(2bgx - f)}{2b\,(b\,x^4 + a)}$$

input
```
int((-b*g*x^4+f*x^3+a*g)/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*(2*b*g*x - f))/(2*b*(a + b*x**4))
```

3.119 $\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result 932
Mathematica [A] (verified) 932
Rubi [A] (verified) 933
Maple [A] (verified) 933
Fricas [A] (verification not implemented) 934
Sympy [A] (verification not implemented) 935
Maxima [A] (verification not implemented) 935
Giac [A] (verification not implemented) 936
Mupad [B] (verification not implemented) 936
Reduce [B] (verification not implemented) 936

Optimal result

Integrand size = 31, antiderivative size = 47

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = -\frac{f}{2b\sqrt{a+bx^4}}+\frac{x(2ag+ex)}{2a\sqrt{a+bx^4}}$$

output
```
-1/2*f/b/(b*x^4+a)^(1/2)+1/2*x*(2*a*g+e*x)/a/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx = \frac{-af+2abgx+bex^2}{2ab\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(-(a*f) + 2*a*b*g*x + b*e*x^2)/(2*a*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2395}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag - bgx^4 + ex + fx^3}{(a + bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{2395}$$

$$-\frac{-2abgx + af - bex^2}{2ab\sqrt{a + bx^4}}$$

input
```
Int[(a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(a*f - 2*a*b*g*x - b*e*x^2)/(a*b*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2395
```
Int[(P4_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P4, x,
0], e = Coeff[P4, x, 1], f = Coeff[P4, x, 3], g = Coeff[P4, x, 4]}, Simp[-(
a*f + 2*a*g*x - b*e*x^2)/(2*a*b*Sqrt[a + b*x^4]), x] /; EqQ[b*d + a*g, 0]]
/; FreeQ[{a, b}, x] && PolyQ[P4, x, 4] && EqQ[Coeff[P4, x, 2], 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{2abgx+be\,x^{2}-af}{2\sqrt{b\,x^{4}+a}\,ab}$
trager	$\frac{2abgx+be\,x^{2}-af}{2\sqrt{b\,x^{4}+a}\,ab}$
orering	$\frac{2abgx+be\,x^{2}-af}{2\sqrt{b\,x^{4}+a}\,ab}$
elliptic	$-\frac{-be\,x^{2}+af}{2\sqrt{b\,x^{4}+a}\,ab}+\frac{gx}{\sqrt{b\,x^{4}+a}}$
default	$ag\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+\frac{e\,x^{2}}{2\sqrt{b\,x^{4}+a}\,a}-\frac{f}{2b\sqrt{b\,x^{4}+a}}-bg\left(-\frac{x}{2b\sqrt{\left(x^{4}+\right.}}\right.$

input
```
int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a*b*g*x+b*e*x^2-a*f)/(b*x^4+a)^(1/2)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{ag+ex+fx^3-bgx^4}{(a+bx^4)^{3/2}}\,dx=\frac{\sqrt{bx^4+a}(2\,abgx+bex^2-af)}{2\,(ab^2x^4+a^2b)}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.83

$$\int \frac{ag + ex + fx^3 - bgx^4}{(a + bx^4)^{3/2}} \, dx = f \left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1 + \frac{bx^4}{a}}}$$

input
```
integrate((-b*g*x**4+f*x**3+a*g+e*x)/(b*x**4+a)**(3/2),x)
```

output
```
f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*sqrt(a)*gamma(5/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x**2/(2*a**(3/2)*sqrt(1
+ b*x**4/a))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{ag + ex + fx^3 - bgx^4}{(a + bx^4)^{3/2}} \, dx = \frac{\sqrt{bx^4 + a}(2\, abgx + bex^2 - af)}{2\,(ab^2x^4 + a^2b)}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^4 + a)*(2*a*b*g*x + b*e*x^2 - a*f)/(a*b^2*x^4 + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

$$\int \frac{ag + ex + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{\left(2\,g + \frac{ex}{a}\right)x - \frac{f}{b}}{2\,\sqrt{bx^4 + a}}$$

input
```
integrate((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*((2*g + e*x/a)*x - f/b)/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{ag + ex + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{g\,x - \frac{f}{2\,b} + \frac{e\,x^2}{2\,a}}{\sqrt{b\,x^4 + a}}$$

input
```
int((a*g + e*x + f*x^3 - b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(g*x - f/(2*b) + (e*x^2)/(2*a))/(a + b*x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{ag + ex + fx^3 - bgx^4}{(a + bx^4)^{3/2}}\,dx = \frac{\sqrt{b\,x^4 + a}\,(2abgx + be\,x^2 - af)}{2ab\,(b\,x^4 + a)}$$

input
```
int((-b*g*x^4+f*x^3+a*g+e*x)/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*(2*a*b*g*x - a*f + b*e*x**2))/(2*a*b*(a + b*x**4))
```

3.120 $\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$

Optimal result 937
Mathematica [C] (verified) 938
Rubi [A] (verified) 939
Maple [C] (verified) 940
Fricas [A] (verification not implemented) 941
Sympy [A] (verification not implemented) 942
Maxima [F] 942
Giac [F] 943
Mupad [F(-1)] 943
Reduce [F] 944

Optimal result

Integrand size = 42, antiderivative size = 388

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx &= \frac{f\sqrt{a+bx^4}}{2b} \\ &+ \frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b} \\ &+ \frac{(5be-3ai)x\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{(2bd-ah)\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} \\ &- \frac{\sqrt[4]{a}(5be-3ai)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} \\ &+ \frac{\left(5\sqrt{b}(3bc-ag)+3\sqrt{a}(5be-3ai)\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*f*(b*x^4+a)^(1/2)/b+1/3*g*x*(b*x^4+a)^(1/2)/b+1/4*h*x^2*(b*x^4+a)^(1/2
)/b+1/5*i*x^3*(b*x^4+a)^(1/2)/b+1/5*(-3*a*i+5*b*e)*x*(b*x^4+a)^(1/2)/b^(3/
2)/(a^(1/2)+b^(1/2)*x^2)+1/4*(-a*h+2*b*d)*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1
/2))/b^(3/2)-1/5*a^(1/4)*(-3*a*i+5*b*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(
a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1
/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)+1/30*(5*b^(1/2)*(-a*g+3*b*c)+3*a^(1/2)
*(-3*a*i+5*b*e))*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^
(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.72

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{30a\sqrt{b}f+20a\sqrt{b}gx+15a\sqrt{b}hx^2+12a\sqrt{b}ix^3+30b^{3/2}fx^4+20b^{3/2}gx^5+15b^{3/2}hx^6+12b^{3/2}ix^7+30bd}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4
],x]
```

output
```
(30*a*Sqrt[b]*f + 20*a*Sqrt[b]*g*x + 15*a*Sqrt[b]*h*x^2 + 12*a*Sqrt[b]*i*x
^3 + 30*b^(3/2)*f*x^4 + 20*b^(3/2)*g*x^5 + 15*b^(3/2)*h*x^6 + 12*b^(3/2)*i
*x^7 + 30*b*d*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 15*
a*h*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*Sqrt[b]*(-
3*b*c + a*g)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x
^4)/a)] + 4*Sqrt[b]*(5*b*e - 3*a*i)*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{c+ex^2+gx^4+ix^6}{\sqrt{a+bx^4}}+\frac{x\left(d+fx^2+hx^4\right)}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(\frac{5\sqrt{b}(3bc-ag)}{\sqrt{a}}-9ai+15be\right)}{30b^{7/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(5be-3ai)E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)(2bd-ah)}{4b^{3/2}}+\frac{x\sqrt{a+bx^4}(5be-3ai)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}+\frac{gx\sqrt{a+bx^4}}{3b}+\frac{hx^2\sqrt{a+bx^4}}{4b}+\frac{ix^3\sqrt{a+bx^4}}{5b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/Sqrt[a + b*x^4],x]
```

output
```
(f*Sqrt[a + b*x^4])/(2*b) + (g*x*Sqrt[a + b*x^4])/(3*b) + (h*x^2*Sqrt[a +
b*x^4])/(4*b) + (i*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*e - 3*a*i)*x*Sqrt[a
+ b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*b*d - a*h)*ArcTanh[(Sq
rt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*(5*b*e - 3*a*i)*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) + (a^(1/4)*
(15*b*e + (5*Sqrt[b]*(3*b*c - a*g))/Sqrt[a] - 9*a*i)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.74

method	result
elliptic	$\frac{i x^3 \sqrt{b x^4+a}}{5b}+\frac{h x^2 \sqrt{b x^4+a}}{4b}+\frac{g x \sqrt{b x^4+a}}{3b}+\frac{f \sqrt{b x^4+a}}{2b}+\frac{\left(c-\frac{a g}{3b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{(d-\cdots}{\cdots}$
risch	$\frac{\left(12x^3 i+15h x^2+20g x+30f\right)\sqrt{b x^4+a}}{60b}-\frac{\frac{(15ah-30bd)\ln\left(\sqrt{b}\,x^2+\sqrt{b x^4+a}\right)}{2\sqrt{b}}+\frac{i(18ai-30be)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\cdots}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}\,\sqrt{b}}}{\cdots}$
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{d\ln\left(\sqrt{b}\,x^2+\sqrt{b x^4+a}\right)}{2\sqrt{b}}+\frac{i e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\cdots}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}\,\sqrt{b}}$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/5*i*x^3*(b*x^4+a)^(1/2)/b+1/4*h*x^2*(b*x^4+a)^(1/2)/b+1/3*g*x*(b*x^4+a)^
(1/2)/b+1/2*f*(b*x^4+a)^(1/2)/b+(c-1/3*a/b*g)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*(d-1/2*a/b*h)*ln(2*b^(1/2
)*x^2+2*(b*x^4+a)^(1/2))/b^(1/2)+I*(e-3/5*a/b*i)*a^(1/2)/(I/a^(1/2)*b^(1/2
))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE
(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.53

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{24\,(5\,abe-3\,a^2i)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+8\,(15\,b^2c-15\,abe-5\,abg+9\,a^2i)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(}{}$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorit
hm="fricas")
```

output
```
1/120*(24*(5*a*b*e - 3*a^2*i)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a
/b)^(1/4)/x), -1) + 8*(15*b^2*c - 15*a*b*e - 5*a*b*g + 9*a^2*i)*sqrt(b)*x*
(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - 15*(2*a*b*d - a^2*h)
*sqrt(b)*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(12*a*b*i
*x^4 + 15*a*b*h*x^3 + 20*a*b*g*x^2 + 30*a*b*f*x + 60*a*b*e - 36*a^2*i)*sqr
t(b*x^4 + a))/(a*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.67

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}hx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ah\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{d\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{gx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{ix^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((i*x**6+h*x**5+g*x**4+f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*h*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*h*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)
/(2*b), True)) + d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4
)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4
)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)
/(4*sqrt(a)*gamma(7/4)) + g*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + i*x**7*gamma(7/4)*hyper((1/
2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx = \int \frac{ix^6+hx^5+gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a
), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx = \int \frac{ix^6+hx^5+gx^4+fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((i*x^6 + h*x^5 + g*x^4 + f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx = \int \frac{i\,x^6+h\,x^5+g\,x^4+f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3 + g*x^4 + h*x^5 + i*x^6)/(a + b*x^4)^(1/2), x
)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3+gx^4+hx^5+ix^6}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{60\sqrt{b\,x^4+a}\,bf+40\sqrt{b\,x^4+a}\,bgx+30\sqrt{b\,x^4+a}\,bh\,x^2+24\sqrt{b\,x^4+a}\,bi\,x^3+15\sqrt{b}\log\left(\sqrt{b\,x^4+a}-\sqrt{b}\right.}{}$$

input
```
int((i*x^6+h*x^5+g*x^4+f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
(60*sqrt(a + b*x**4)*b*f + 40*sqrt(a + b*x**4)*b*g*x + 30*sqrt(a + b*x**4)
*b*h*x**2 + 24*sqrt(a + b*x**4)*b*i*x**3 + 15*sqrt(b)*log(sqrt(a + b*x**4)
 - sqrt(b)*x**2)*a*h - 30*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*b*d
 - 15*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*h + 30*sqrt(b)*log(sq
rt(a + b*x**4) + sqrt(b)*x**2)*b*d - 40*int(sqrt(a + b*x**4)/(a + b*x**4),
x)*a*b*g + 120*int(sqrt(a + b*x**4)/(a + b*x**4),x)*b**2*c - 72*int((sqrt(
a + b*x**4)*x**2)/(a + b*x**4),x)*a*b*i + 120*int((sqrt(a + b*x**4)*x**2)/
(a + b*x**4),x)*b**2*e)/(120*b**2)
```

3.121 $\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx$

Optimal result . . . 945
Mathematica [A] (verified) . . . 945
Rubi [A] (verified) . . . 946
Maple [A] (verified) . . . 947
Fricas [A] (verification not implemented) . . . 947
Sympy [C] (verification not implemented) . . . 948
Maxima [A] (verification not implemented) . . . 948
Giac [A] (verification not implemented) . . . 949
Mupad [B] (verification not implemented) . . . 949
Reduce [B] (verification not implemented) . . . 949

Optimal result

Integrand size = 37, antiderivative size = 21

$$\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx=\frac{x(g-hx^2)}{\sqrt{a+bx^4}}$$

output
```
x*(-h*x^2+g)/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx=\frac{x(g-hx^2)}{\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g - 3*a*h*x^2 - b*g*x^4 - b*h*x^6)/(a + b*x^4)^(3/2),x]
```

output
```
(x*(g - h*x^2))/Sqrt[a + b*x^4]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = $\{2396\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag - 3ahx^2 - bgx^4 - bhx^6}{(a + bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \; 2396$$

$$\frac{gx - hx^3}{\sqrt{a + bx^4}}$$

input
```
Int[(a*g - 3*a*h*x^2 - b*g*x^4 - b*h*x^6)/(a + b*x^4)^(3/2),x]
```

output
```
(g*x - h*x^3)/Sqrt[a + b*x^4]
```

Defintions of rubi rules used

rule 2396
```
Int[(P6_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P6, x,
0], e = Coeff[P6, x, 2], f = Coeff[P6, x, 3], g = Coeff[P6, x, 4], h = Coef
f[P6, x, 6]}, Simp[-(a*f - 2*b*d*x - 2*a*h*x^3)/(2*a*b*Sqrt[a + b*x^4]), x]
 /; EqQ[b*e - 3*a*h, 0] && EqQ[b*d + a*g, 0]] /; FreeQ[{a, b}, x] && PolyQ[
P6, x, 6] && EqQ[Coeff[P6, x, 1], 0] && EqQ[Coeff[P6, x, 5], 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result
gosper	$\frac{x\left(-h\,x^{2}+g\right)}{\sqrt{b\,x^{4}+a}}$
trager	$\frac{x\left(-h\,x^{2}+g\right)}{\sqrt{b\,x^{4}+a}}$
orering	$\frac{x\left(-h\,x^{2}+g\right)}{\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{2b\left(\frac{h\,x^{3}}{2b}-\frac{gx}{2b}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}$
default	$ag\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-3ah\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x}{\sqrt{a}}}}{}\right.$

input
```
int((-b*h*x^6-b*g*x^4-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
x*(-h*x^2+g)/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{ag-3ahx^{2}-bgx^{4}-bhx^{6}}{\left(a+bx^{4}\right)^{3/2}}\,dx=-\frac{hx^{3}-gx}{\sqrt{bx^{4}+a}}$$

input
```
integrate((-b*h*x^6-b*g*x^4-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x, algorithm="f
ricas")
```

output
```
-(h*x^3 - g*x)/sqrt(b*x^4 + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.99 (sec) , antiderivative size = 165, normalized size of antiderivative = 7.86

$$\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx = \frac{gx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{3hx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} - \frac{bhx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((-b*h*x**6-b*g*x**4-3*a*h*x**2+a*g)/(b*x**4+a)**(3/2),x)
```

output
```
g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) - 3*h*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) - b*g*x**5*gamma(5/4)*hyper((5/4, 3
/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) - b*h*x**7*
gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx = -\frac{hx^3-gx}{\sqrt{bx^4+a}}$$

input
```
integrate((-b*h*x^6-b*g*x^4-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x, algorithm="m
axima")
```

output
```
-(h*x^3 - g*x)/sqrt(b*x^4 + a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx = -\frac{(hx^2-g)x}{\sqrt{bx^4+a}}$$

input
```
integrate((-b*h*x^6-b*g*x^4-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x, algorithm="g
iac")
```

output
```
-(h*x^2 - g)*x/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx = \frac{x\,(g-h\,x^2)}{\sqrt{b\,x^4+a}}$$

input
```
int(-(b*h*x^6 - a*g + 3*a*h*x^2 + b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(x*(g - h*x^2))/(a + b*x^4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{ag-3ahx^2-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{b\,x^4+a}\,x(-h\,x^2+g)}{b\,x^4+a}$$

input
```
int((-b*h*x^6-b*g*x^4-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*x*(g - h*x**2))/(a + b*x**4)
```

3.122 $\int \frac{ag-3ahx^2-bfx^3-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx$

Optimal result 951
Mathematica [A] (verified) 951
Rubi [A] (verified) 952
Maple [A] (verified) 953
Fricas [A] (verification not implemented) 953
Sympy [A] (verification not implemented) 954
Maxima [A] (verification not implemented) 954
Giac [A] (verification not implemented) 955
Mupad [B] (verification not implemented) 955
Reduce [B] (verification not implemented) 955

Optimal result

Integrand size = 44, antiderivative size = 38

$$\int \frac{ag-3ahx^2-bfx^3-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx = \frac{f}{2\sqrt{a+bx^4}} + \frac{x(g-hx^2)}{\sqrt{a+bx^4}}$$

output
```
1/2*f/(b*x^4+a)^(1/2)+x*(-h*x^2+g)/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{ag-3ahx^2-bfx^3-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx = \frac{f+2gx-2hx^3}{2\sqrt{a+bx^4}}$$

input
```
Integrate[(a*g - 3*a*h*x^2 - b*f*x^3 - b*g*x^4 - b*h*x^6)/(a + b*x^4)^(3/2
),x]
```

output
```
(f + 2*g*x - 2*h*x^3)/(2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2396}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ag-3ahx^2-bfx^3-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2396$$

$$\frac{f+2gx-2hx^3}{2\sqrt{a+bx^4}}$$

input
```
Int[(a*g - 3*a*h*x^2 - b*f*x^3 - b*g*x^4 - b*h*x^6)/(a + b*x^4)^(3/2),x]
```

output
```
(f + 2*g*x - 2*h*x^3)/(2*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2396
```
Int[(P6_)/((a_) + (b_.)*(x_)^4)^(3/2), x_Symbol] :> With[{d = Coeff[P6, x,
0], e = Coeff[P6, x, 2], f = Coeff[P6, x, 3], g = Coeff[P6, x, 4], h = Coef
f[P6, x, 6]}, Simp[-(a*f - 2*b*d*x - 2*a*h*x^3)/(2*a*b*Sqrt[a + b*x^4]), x]
 /; EqQ[b*e - 3*a*h, 0] && EqQ[b*d + a*g, 0]] /; FreeQ[{a, b}, x] && PolyQ[
P6, x, 6] && EqQ[Coeff[P6, x, 1], 0] && EqQ[Coeff[P6, x, 5], 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result
gosper	$\frac{-2h\,x^{3}+2gx+f}{2\sqrt{b\,x^{4}+a}}$
trager	$\frac{-2h\,x^{3}+2gx+f}{2\sqrt{b\,x^{4}+a}}$
orering	$\frac{-2h\,x^{3}+2gx+f}{2\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{2b\left(\frac{h\,x^{3}}{2b}-\frac{gx}{2b}-\frac{f}{4b}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}$
default	$ag\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-3ah\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x}{\sqrt{a}}}}{}\right.$

input
```
int((-b*h*x^6-b*g*x^4-b*f*x^3-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2*(-2*h*x^3+2*g*x+f)/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{ag-3ahx^2-bfx^3-bgx^4-bhx^6}{(a+bx^4)^{3/2}}\,dx=-\frac{2\,hx^3-2\,gx-f}{2\,\sqrt{bx^4+a}}$$

input
```
integrate((-b*h*x^6-b*g*x^4-b*f*x^3-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x, algo
rithm="fricas")
```

output
```
-1/2*(2*h*x^3 - 2*g*x - f)/sqrt(b*x^4 + a)
```

Sympy [A] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 196, normalized size of antiderivative = 5.16

$$\int \frac{ag - 3ahx^2 - bfx^3 - bgx^4 - bhx^6}{(a + bx^4)^{3/2}}\, dx = -bf\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{gx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} - \frac{3hx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} - \frac{bgx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} - \frac{bhx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((-b*h*x**6-b*g*x**4-b*f*x**3-3*a*h*x**2+a*g)/(b*x**4+a)**(3/2),x
)
```

output
```
-b*f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)),
True)) + g*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(5/4)) - 3*h*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) - b*g*x**5*gamma(5/4)*hype
r((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) -
b*h*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(
4*a**(3/2)*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{ag - 3ahx^2 - bfx^3 - bgx^4 - bhx^6}{(a + bx^4)^{3/2}}\, dx = -\frac{2\,hx^3 - 2\,gx - f}{2\,\sqrt{bx^4 + a}}$$

input
```
integrate((-b*h*x^6-b*g*x^4-b*f*x^3-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x, algo
rithm="maxima")
```

output
```
-1/2*(2*h*x^3 - 2*g*x - f)/sqrt(b*x^4 + a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{ag - 3ahx^2 - bfx^3 - bgx^4 - bhx^6}{(a + bx^4)^{3/2}}\,dx = -\frac{2\,(hx^2 - g)x - f}{2\,\sqrt{bx^4 + a}}$$

input
```
integrate((-b*h*x^6-b*g*x^4-b*f*x^3-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x, algo
rithm="giac")
```

output
```
-1/2*(2*(h*x^2 - g)*x - f)/sqrt(b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{ag - 3ahx^2 - bfx^3 - bgx^4 - bhx^6}{(a + bx^4)^{3/2}}\,dx = \frac{-2\,h\,x^3 + 2\,g\,x + f}{2\,\sqrt{b\,x^4 + a}}$$

input
```
int(-(b*h*x^6 - a*g + b*f*x^3 + 3*a*h*x^2 + b*g*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
(f + 2*g*x - 2*h*x^3)/(2*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{ag - 3ahx^2 - bfx^3 - bgx^4 - bhx^6}{(a + bx^4)^{3/2}}\,dx = \frac{\sqrt{b\,x^4 + a}\,(-2h\,x^3 + 2gx + f)}{2b\,x^4 + 2a}$$

input
```
int((-b*h*x^6-b*g*x^4-b*f*x^3-3*a*h*x^2+a*g)/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*(f + 2*g*x - 2*h*x**3))/(2*(a + b*x**4))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 957
4.2 Links to plain text integration problems used in this report for each CAS . 975

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file