Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.7/63-1.1.3.7-c

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**70**]. This is test number [63].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (70)	0.00 (0)
Mathematica	100.00 (70)	0.00 (0)
Mupad	85.71 (60)	14.29 (10)
Maple	81.43 (57)	18.57 (13)
Fricas	80.00 (56)	20.00 (14)
Giac	80.00 (56)	20.00 (14)
Sympy	80.00 (56)	20.00 (14)
Maxima	77.14 (54)	22.86 (16)
Reduce	77.14 (54)	22.86 (16)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

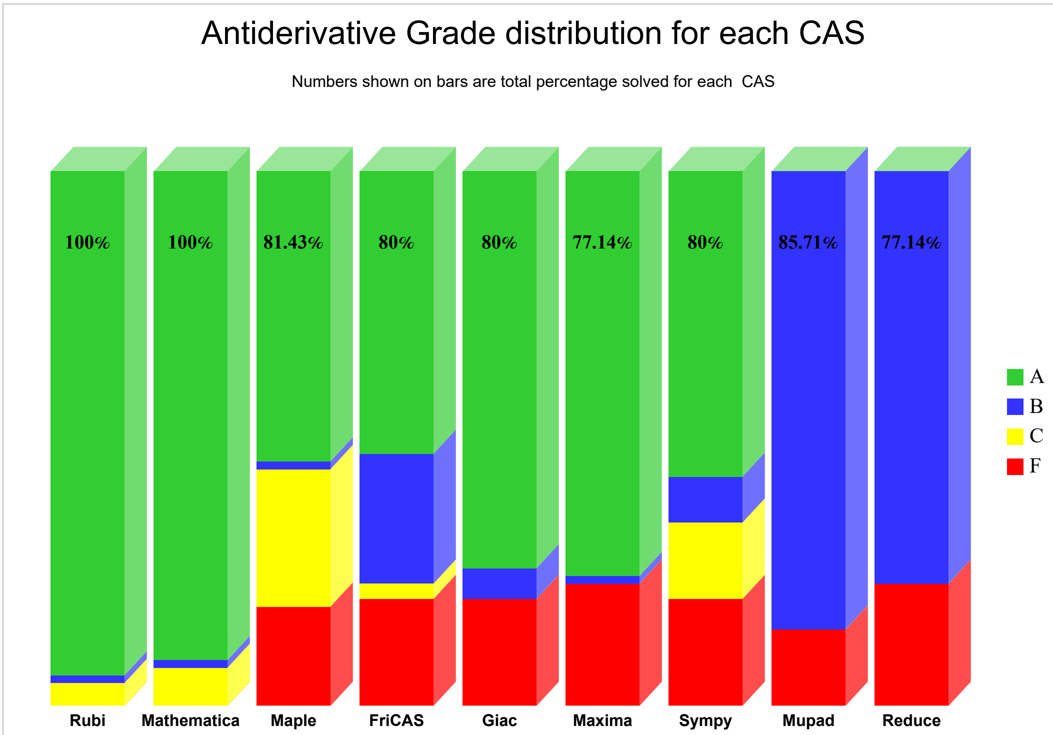
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

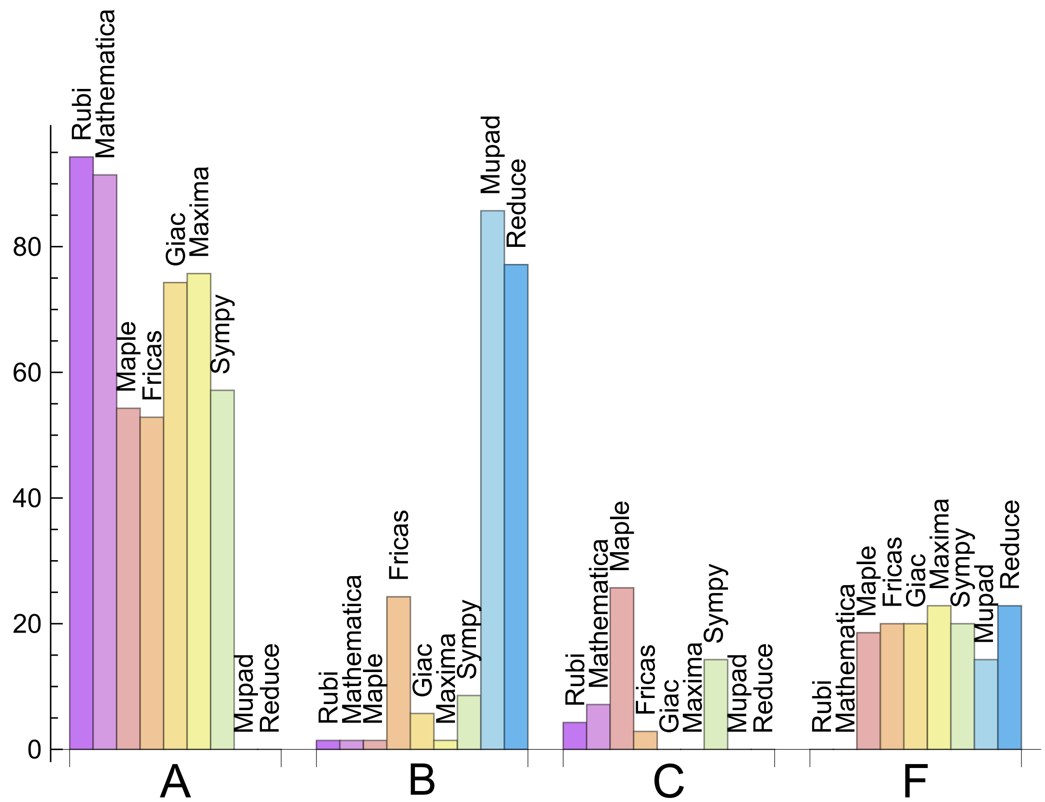
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.286	1.429	4.286	0.000
Mathematica	91.429	1.429	7.143	0.000
Maxima	75.714	1.429	0.000	22.857
Giac	74.286	5.714	0.000	20.000
Sympy	57.143	8.571	14.286	20.000
Maple	54.286	1.429	25.714	18.571
Fricas	52.857	24.286	2.857	20.000
Mupad	0.000	85.714	0.000	14.286
Reduce	0.000	77.143	0.000	22.857

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Mupad	10	0.00	100.00	0.00
Maple	13	100.00	0.00	0.00
Fricas	14	100.00	0.00	0.00
Giac	14	78.57	0.00	21.43
Sympy	14	0.00	92.86	7.14
Maxima	16	100.00	0.00	0.00
Reduce	16	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.13
Reduce	0.17
Maple	0.18
Mathematica	0.24
Rubi	0.61
Fricas	1.38
Mupad	4.50
Sympy	6.35

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	65.72	0.68	69.00	0.75
Mathematica	126.57	0.98	101.50	1.00
Maxima	145.74	0.97	84.00	0.92
Giac	155.14	1.09	94.00	0.92
Rubi	155.79	1.17	113.00	1.00
Sympy	219.27	2.52	105.00	1.00
Reduce	302.48	1.75	117.50	1.24
Mupad	855.48	3.41	88.50	0.95
Fricas	1982.77	7.77	109.00	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

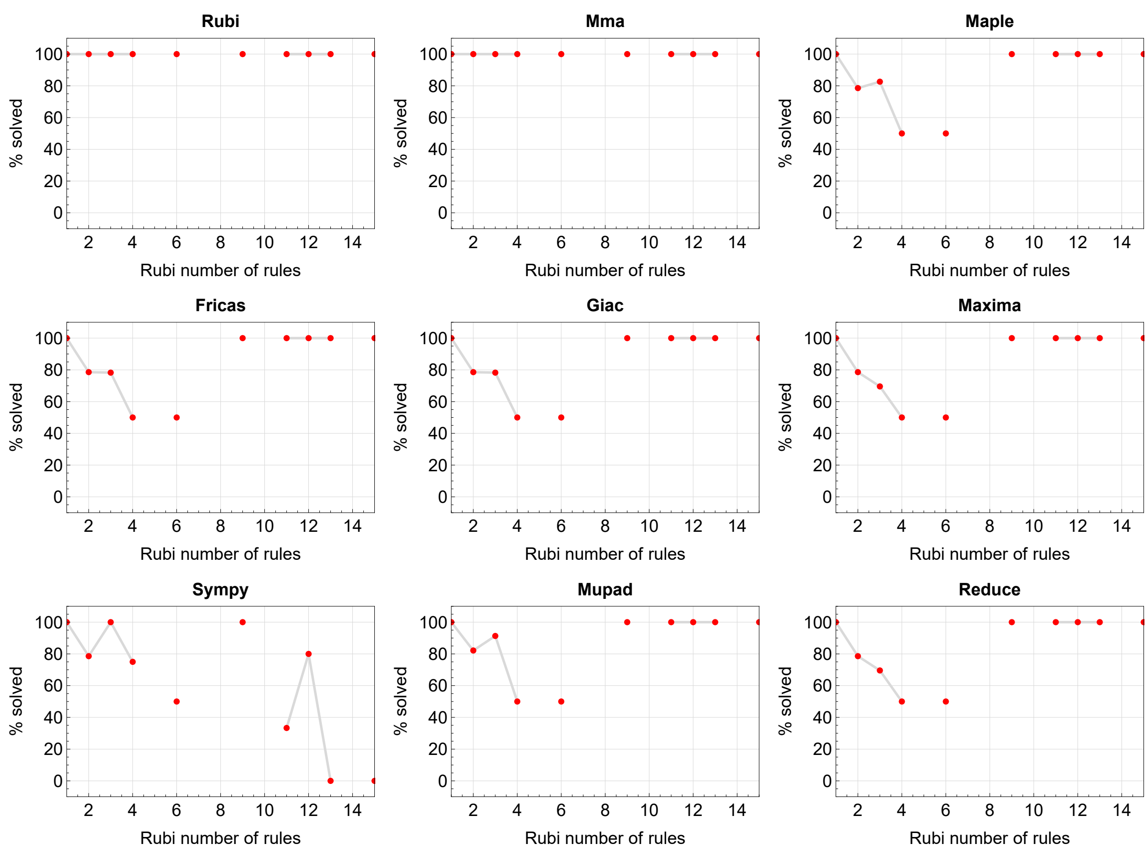


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

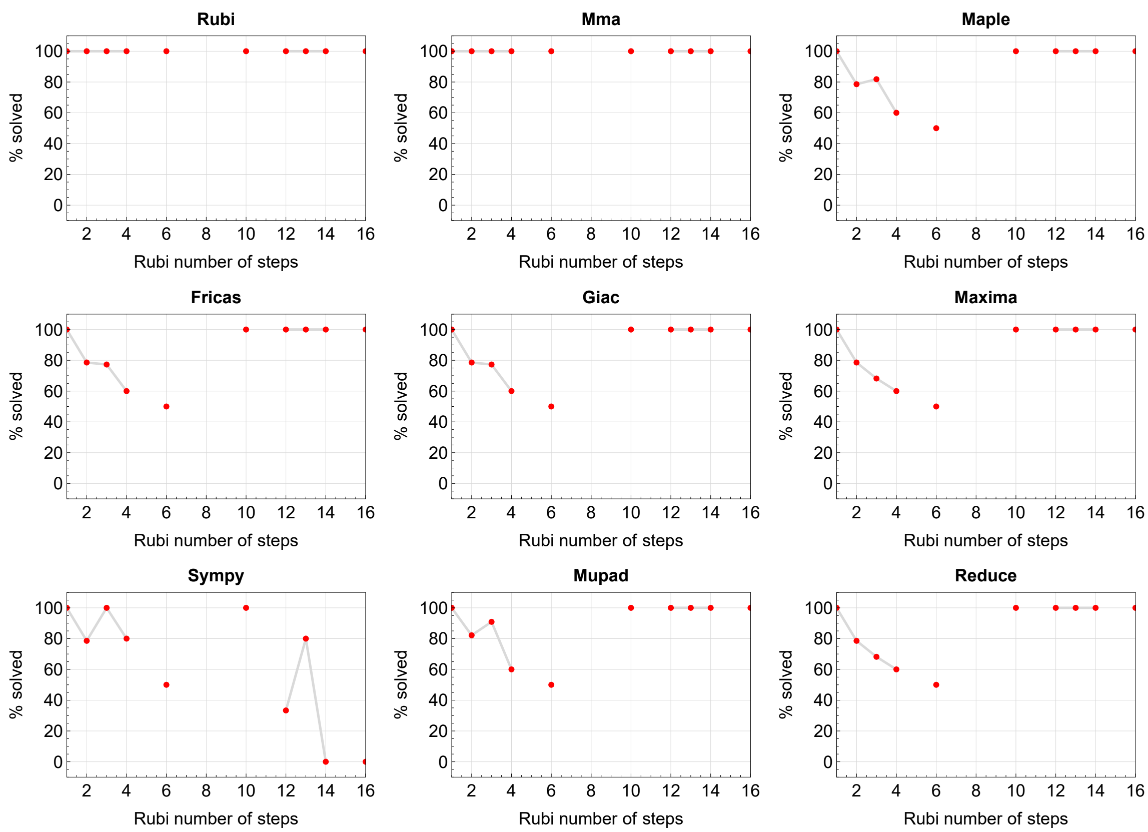


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

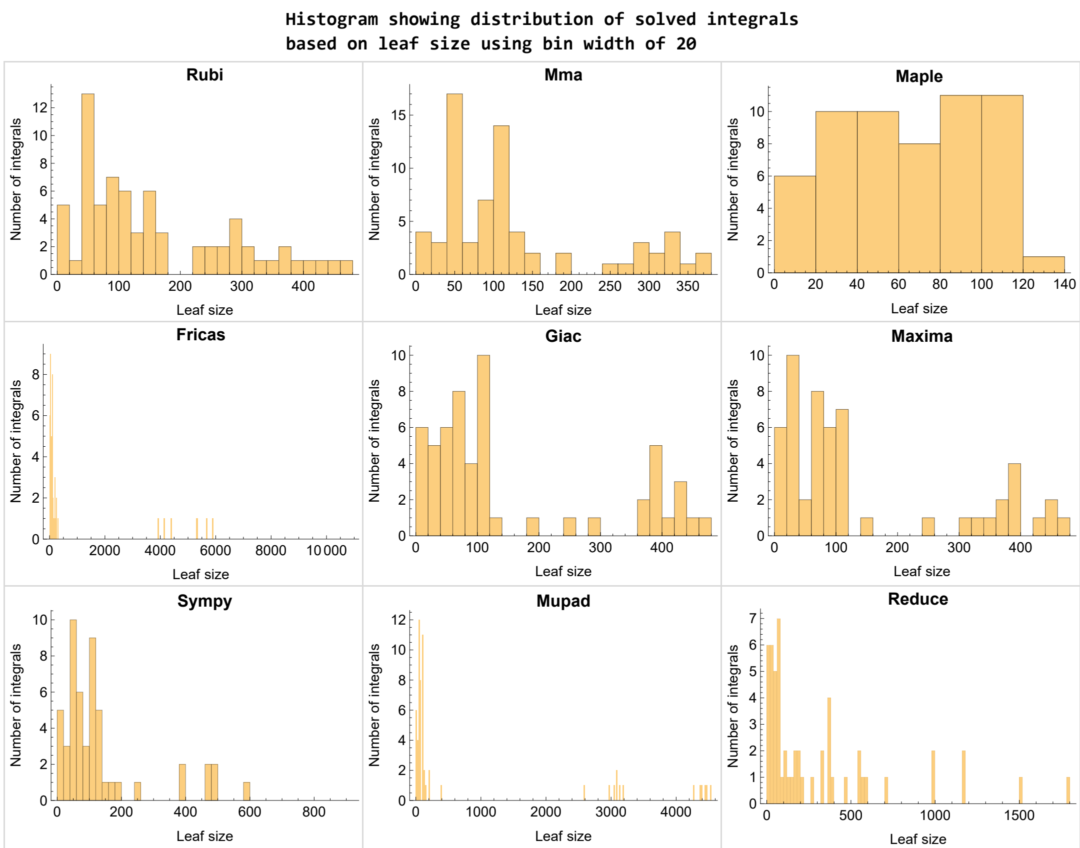


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

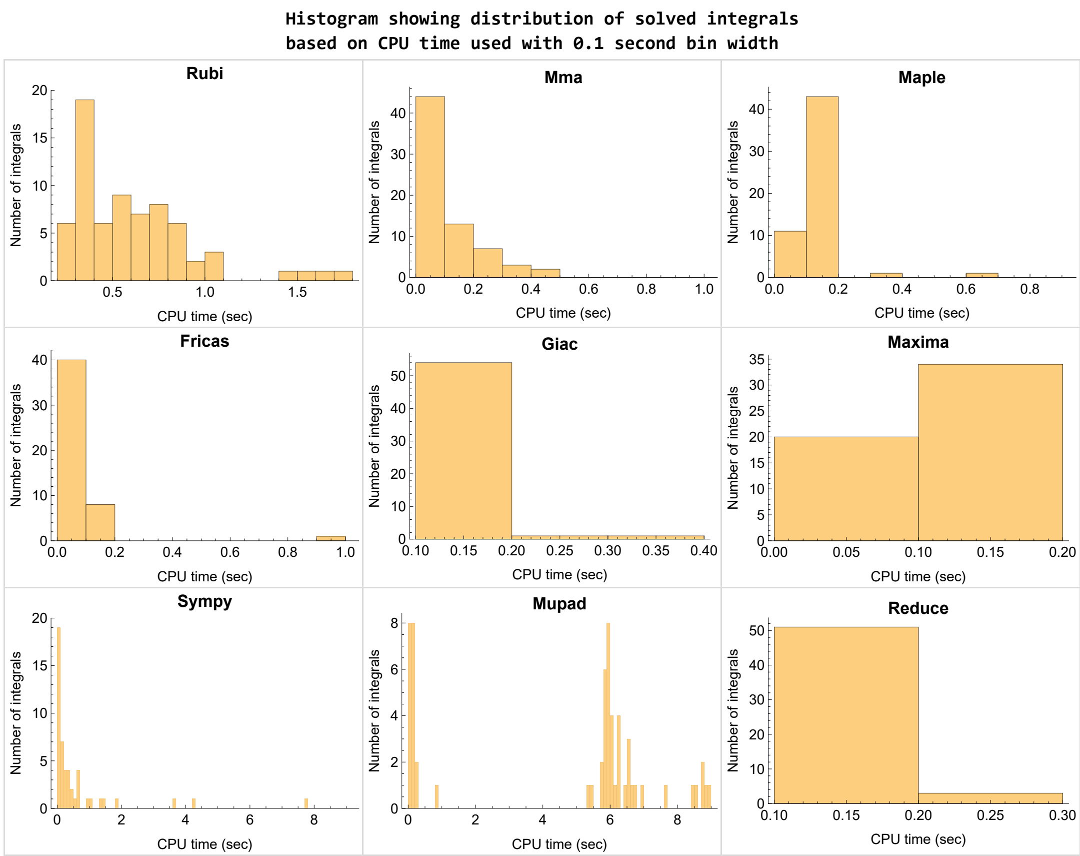


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

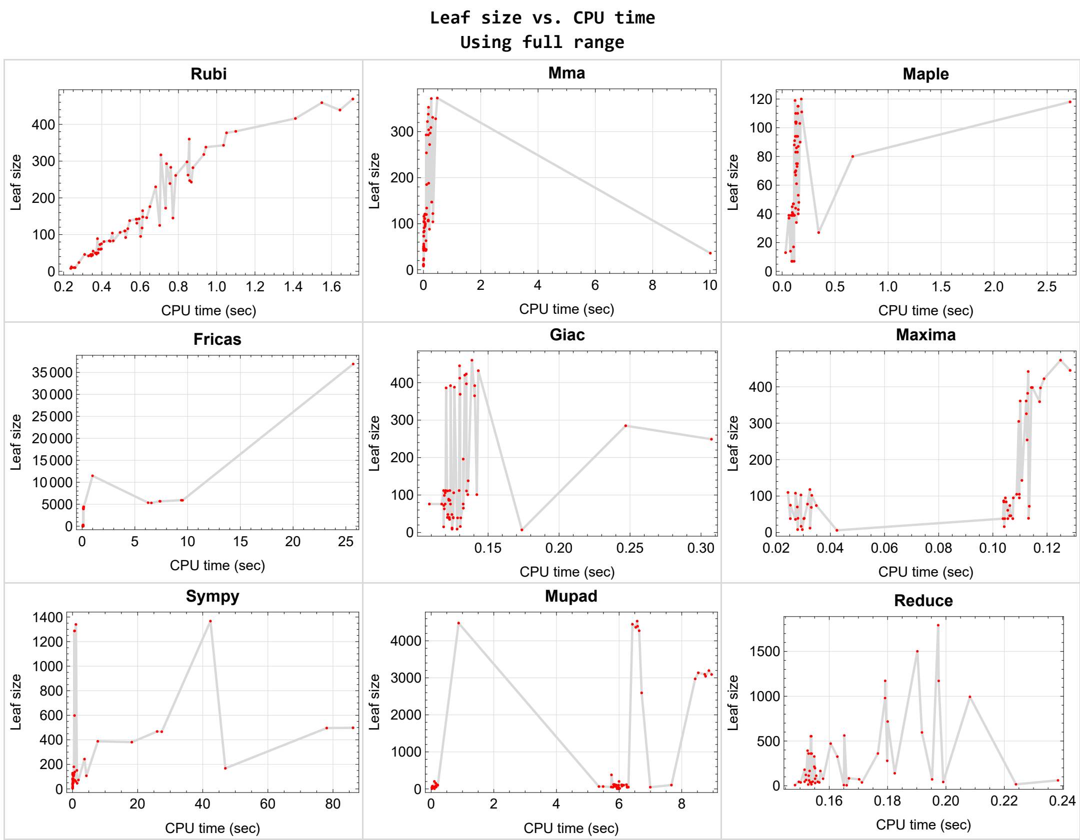


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

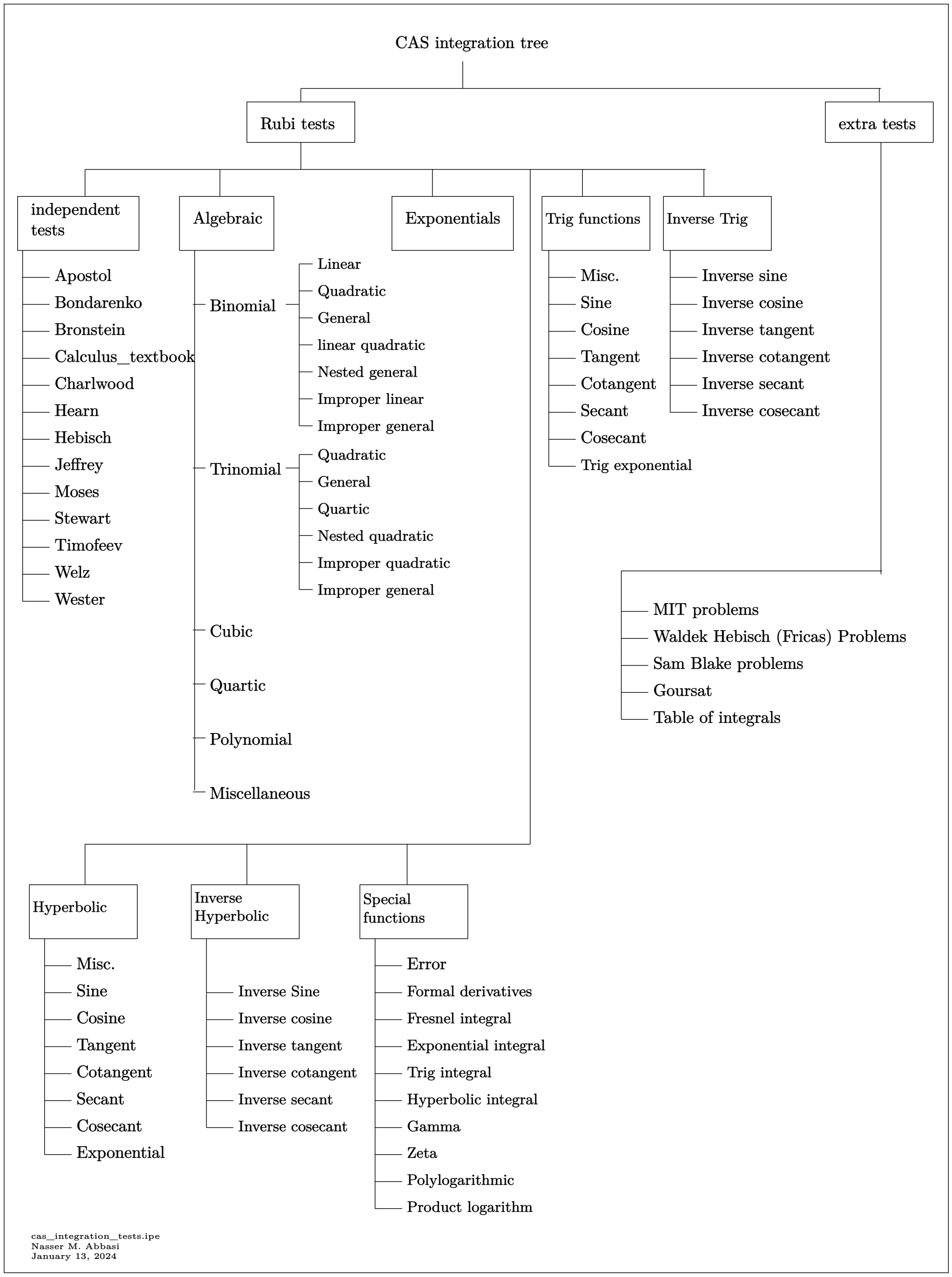
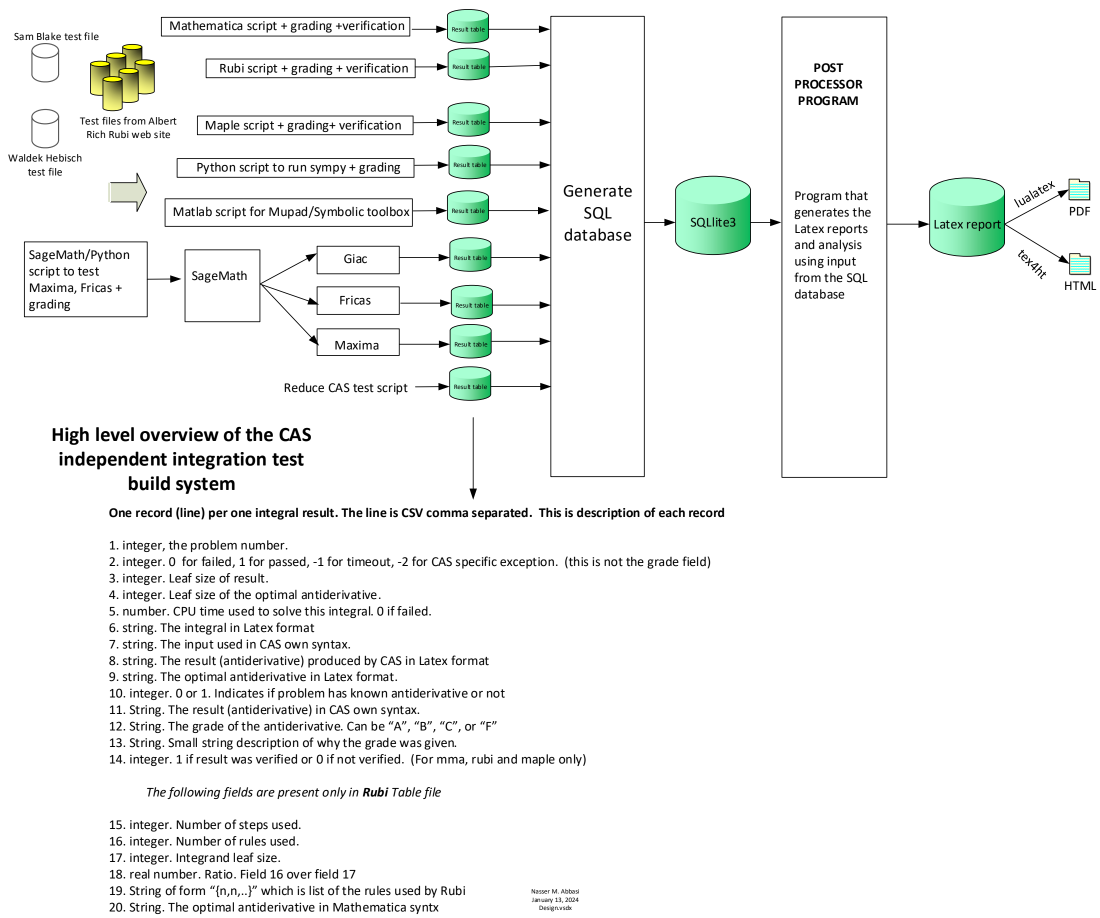


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade { 4 }

C grade { 50, 51, 52 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

B grade { 32 }

C grade { 1, 2, 42, 52, 53 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 6, 7, 8, 12, 13, 14, 18, 19, 20, 24, 25, 26, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57 }

B grade { 32 }

C grade { 1, 2, 4, 5, 9, 10, 11, 15, 16, 17, 21, 22, 23, 27, 28, 29, 51, 52 }

F normal fail { 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 6, 7, 8, 12, 13, 14, 18, 19, 20, 24, 25, 26, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 56, 57 }

B grade { 9, 10, 11, 15, 16, 17, 21, 22, 23, 27, 28, 29, 32, 44, 45, 54, 55 }

C grade { 4, 5 }

F normal fail { 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57 }

B grade { 32 }

C grade { }

F normal fail { 1, 2, 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57 }

B grade { 27, 32, 54, 55 }

C grade { }

F normal fail { 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67 }

F(-1) timedout fail { }

F(-2) exception fail { 68, 69, 70 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 67 }

C grade { }

F normal fail { }

F(-1) timedout fail { 53, 61, 62, 63, 64, 65, 66, 68, 69, 70 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 6, 7, 8, 12, 13, 14, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 57, 58 }

B grade { 1, 2, 32, 54, 55, 56 }

C grade { 51, 52, 53, 59, 60, 61, 64, 67, 68, 69 }

F normal fail { }

F(-1) timedout fail { 5, 9, 10, 11, 15, 16, 17, 23, 29, 62, 63, 65, 66 }

F(-2) exception fail { 70 }

Reduce

A grade { }

B grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57 }

C grade { }

F normal fail { 1, 2, 53, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	145	51	45	0	112	1287	101	20	64
N.S.	1	0.99	0.35	0.31	0.00	0.76	8.76	0.69	0.14	0.44
time (sec)	N/A	0.771	0.009	0.095	0.000	0.085	0.627	0.142	0.168	5.355

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	47	41	0	108	1287	101	16	65
N.S.	1	1.00	0.33	0.29	0.00	0.76	9.00	0.71	0.11	0.45
time (sec)	N/A	0.595	0.007	0.096	0.000	0.122	0.598	0.136	0.169	5.490

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.88	0.75	0.75
time (sec)	N/A	0.237	0.001	0.095	0.042	0.091	0.023	0.174	0.166	0.026

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	439	254	34	254	11478	151	249	279	380
N.S.	1	2.08	1.20	0.16	1.20	54.40	0.72	1.18	1.32	1.80
time (sec)	N/A	1.644	0.103	0.135	0.113	0.988	1.331	0.307	0.180	5.756

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	469	293	39	305	36914	0	285	361	2594
N.S.	1	1.95	1.22	0.16	1.27	153.17	0.00	1.18	1.50	10.76
time (sec)	N/A	1.712	0.094	0.113	0.110	25.698	0.000	0.247	0.177	6.719

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	110	108	108	128	112	84	104
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.10	0.97	0.72	0.90
time (sec)	N/A	0.536	0.015	0.147	0.027	0.073	0.030	0.119	0.167	7.670

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	75	74	74	83	76	61	74
N.S.	1	1.00	1.00	0.91	0.90	0.90	1.01	0.93	0.74	0.90
time (sec)	N/A	0.444	0.009	0.145	0.035	0.073	0.034	0.123	0.238	0.041

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	42	40	38	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.91	0.87	0.83	0.85
time (sec)	N/A	0.350	0.004	0.117	0.025	0.066	0.022	0.122	0.171	0.025

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	304	48	361	5351	0	365	140	3096
N.S.	1	1.00	0.89	0.14	1.05	15.60	0.00	1.06	0.41	9.03
time (sec)	N/A	1.036	0.182	0.158	0.112	6.255	0.000	0.140	0.182	8.734

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	416	331	87	398	5685	0	397	596	3137
N.S.	1	1.10	0.88	0.23	1.06	15.08	0.00	1.05	1.58	8.32
time (sec)	N/A	1.411	0.321	0.152	0.115	7.356	0.000	0.135	0.192	8.527

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	459	372	115	445	5912	0	432	993	3196
N.S.	1	1.12	0.91	0.28	1.09	14.45	0.00	1.06	2.43	7.81
time (sec)	N/A	1.549	0.275	0.152	0.129	9.401	0.000	0.143	0.208	8.866

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	110	110	110	128	112	73	106
N.S.	1	1.00	0.89	0.93	0.93	0.93	1.08	0.95	0.62	0.90
time (sec)	N/A	0.610	0.017	0.128	0.024	0.074	0.047	0.120	0.170	5.867

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	75	75	75	83	76	61	74
N.S.	1	1.00	0.99	0.90	0.90	0.90	1.00	0.92	0.73	0.89
time (sec)	N/A	0.460	0.008	0.135	0.025	0.065	0.039	0.109	0.153	0.040

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	40	39	39	42	40	31	40
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.89	0.85	0.66	0.85
time (sec)	N/A	0.372	0.006	0.103	0.030	0.066	0.032	0.121	0.155	0.028

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	297	53	382	5305	0	386	326	2973
N.S.	1	1.00	0.82	0.15	1.06	14.74	0.00	1.07	0.91	8.26
time (sec)	N/A	0.856	0.234	0.152	0.113	6.568	0.000	0.120	0.163	8.428

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	338	328	90	422	5673	0	423	718	3043
N.S.	1	0.86	0.83	0.23	1.07	14.44	0.00	1.08	1.83	7.74
time (sec)	N/A	0.944	0.426	0.170	0.119	7.405	0.000	0.135	0.180	8.762

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	381	373	120	473	5890	0	460	1171	3091
N.S.	1	0.88	0.87	0.28	1.10	13.67	0.00	1.07	2.72	7.17
time (sec)	N/A	1.100	0.484	0.181	0.125	9.525	0.000	0.139	0.198	8.953

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	103	102	102	117	112	113	102
N.S.	1	1.00	1.00	0.99	0.98	0.98	1.12	1.08	1.09	0.98
time (sec)	N/A	0.455	0.026	0.167	0.033	0.073	0.047	0.123	0.156	6.145

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	69	78	76	79	70
N.S.	1	1.00	1.00	0.96	0.95	0.95	1.07	1.04	1.08	0.96
time (sec)	N/A	0.390	0.011	0.128	0.033	0.065	0.038	0.120	0.155	5.864

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	36	36	39	40	45	38
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.93	0.95	1.07	0.90
time (sec)	N/A	0.330	0.006	0.065	0.027	0.068	0.035	0.123	0.156	0.047

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	262	272	67	326	3935	466	369	471	4366
N.S.	1	1.08	1.12	0.28	1.35	16.26	1.93	1.52	1.95	18.04
time (sec)	N/A	0.850	0.216	0.128	0.112	0.119	27.406	0.130	0.160	6.532

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	282	309	88	359	4144	496	392	980	4448
N.S.	1	1.08	1.18	0.34	1.38	15.88	1.90	1.50	3.75	17.04
time (sec)	N/A	0.876	0.260	0.115	0.117	0.119	78.070	0.124	0.179	6.425

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	318	322	115	398	4411	0	420	1502	4533
N.S.	1	1.07	1.09	0.39	1.34	14.90	0.00	1.42	5.07	15.31
time (sec)	N/A	0.933	0.139	0.149	0.114	0.133	0.000	0.134	0.190	6.576

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	104	103	103	103	117	112	113	104
N.S.	1	1.00	0.98	0.97	0.97	0.97	1.10	1.06	1.07	0.98
time (sec)	N/A	0.495	0.016	0.131	0.029	0.074	0.042	0.130	0.153	5.916

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	70	70	70	78	76	79	70
N.S.	1	1.00	0.97	0.93	0.93	0.93	1.04	1.01	1.05	0.93
time (sec)	N/A	0.397	0.009	0.130	0.028	0.067	0.028	0.117	0.158	5.839

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	38	38	39	40	45	38
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.89	0.91	1.02	0.86
time (sec)	N/A	0.351	0.005	0.064	0.030	0.068	0.034	0.125	0.150	0.048

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	239	293	69	361	3916	468	388	562	4273
N.S.	1	0.97	1.19	0.28	1.47	15.92	1.90	1.58	2.28	17.37
time (sec)	N/A	0.756	0.169	0.122	0.110	0.130	25.990	0.126	0.165	6.640

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	261	338	91	397	4136	498	412	1172	4394
N.S.	1	0.96	1.24	0.33	1.46	15.21	1.83	1.51	4.31	16.15
time (sec)	N/A	0.786	0.170	0.118	0.118	0.126	86.146	0.130	0.179	6.590

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	298	353	119	442	4398	0	445	1793	4479
N.S.	1	0.97	1.15	0.39	1.44	14.28	0.00	1.44	5.82	14.54
time (sec)	N/A	0.845	0.176	0.122	0.113	0.162	0.000	0.130	0.197	0.877

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	7	9	8	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.70	0.90	0.80	0.60
time (sec)	N/A	0.260	0.001	0.113	0.028	0.081	0.034	0.128	0.165	5.981

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	8	8	9	8	6
N.S.	1	1.00	1.00	0.70	0.80	0.80	0.80	0.90	0.80	0.60
time (sec)	N/A	0.255	0.001	0.091	0.030	0.069	0.034	0.125	0.148	5.934

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	14	17	17	15	15	17	6
N.S.	1	1.00	2.10	1.40	1.70	1.70	1.50	1.50	1.70	0.60
time (sec)	N/A	0.242	0.003	0.079	0.029	0.080	0.062	0.119	0.224	0.106

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	16	16	24	16	16	16
N.S.	1	1.00	1.00	0.71	0.67	0.67	1.00	0.67	0.67	0.67
time (sec)	N/A	0.280	0.007	0.108	0.104	0.080	0.062	0.130	0.154	5.990

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	46	39	38	49
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.92	0.78	0.76	0.98
time (sec)	N/A	0.372	0.012	0.096	0.104	0.092	0.100	0.129	0.154	0.158

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	39	38	38	46	39	38	48
N.S.	1	1.00	0.92	0.78	0.76	0.76	0.92	0.78	0.76	0.96
time (sec)	N/A	0.380	0.011	0.103	0.106	0.077	0.110	0.131	0.157	0.075

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	47	46	46	56	48	46	52
N.S.	1	1.00	0.93	0.78	0.77	0.77	0.93	0.80	0.77	0.87
time (sec)	N/A	0.383	0.010	0.105	0.107	0.086	0.133	0.124	0.152	0.124

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	46	46	56	48	46	52
N.S.	1	1.00	0.87	0.78	0.77	0.77	0.93	0.80	0.77	0.87
time (sec)	N/A	0.394	0.010	0.108	0.106	0.092	0.115	0.122	0.154	0.118

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	55	50	39	38	38	48	35	38	46
N.S.	1	1.10	1.00	0.78	0.76	0.76	0.96	0.70	0.76	0.92
time (sec)	N/A	0.354	0.005	0.085	0.107	0.082	0.104	0.123	0.153	5.924

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	38	38	48	39	38	46
N.S.	1	1.00	1.00	0.78	0.76	0.76	0.96	0.78	0.76	0.92
time (sec)	N/A	0.366	0.008	0.095	0.104	0.095	0.120	0.125	0.150	5.803

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	100	83	84	115	105	86	166	100
N.S.	1	1.00	0.91	0.75	0.76	1.05	0.95	0.78	1.51	0.91
time (sec)	N/A	0.521	0.067	0.128	0.105	0.083	0.295	0.122	0.153	0.211

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	83	84	115	105	86	166	100
N.S.	1	1.00	0.88	0.75	0.76	1.05	0.95	0.78	1.51	0.91
time (sec)	N/A	0.519	0.056	0.145	0.104	0.089	0.240	0.123	0.157	5.985

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	81	122	61	61	91	70	63	122	52
N.S.	1	1.11	1.67	0.84	0.84	1.25	0.96	0.86	1.67	0.71
time (sec)	N/A	0.411	0.327	0.136	0.105	0.077	0.127	0.119	0.152	5.748

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	73	74	126	82	76	214	77
N.S.	1	1.00	0.91	0.79	0.80	1.37	0.89	0.83	2.33	0.84
time (sec)	N/A	0.524	0.026	0.141	0.106	0.083	0.204	0.132	0.155	0.141

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	119	104	105	256	124	111	554	120
N.S.	1	1.00	0.80	0.70	0.71	1.73	0.84	0.75	3.74	0.81
time (sec)	N/A	0.613	0.040	0.133	0.109	0.092	0.404	0.135	0.154	5.822

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	121	104	105	257	124	111	554	121
N.S.	1	1.00	0.83	0.71	0.72	1.76	0.85	0.76	3.79	0.83
time (sec)	N/A	0.634	0.051	0.127	0.110	0.098	0.357	0.121	0.154	5.858

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	94	95	187	116	106	360	110
N.S.	1	1.00	0.78	0.66	0.67	1.32	0.82	0.75	2.54	0.77
time (sec)	N/A	0.582	0.041	0.135	0.110	0.084	0.289	0.126	0.154	6.092

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	94	95	187	116	106	360	111
N.S.	1	1.00	0.78	0.66	0.67	1.32	0.82	0.75	2.54	0.78
time (sec)	N/A	0.580	0.044	0.125	0.108	0.077	0.421	0.120	0.154	0.203

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	138	103	86	87	131	110	89	180	102
N.S.	1	1.22	0.91	0.76	0.77	1.16	0.97	0.79	1.59	0.90
time (sec)	N/A	0.545	0.036	0.135	0.104	0.095	0.317	0.122	0.152	0.190

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	111	94	95	187	119	99	360	111
N.S.	1	1.00	0.85	0.72	0.73	1.43	0.91	0.76	2.75	0.85
time (sec)	N/A	0.583	0.046	0.147	0.105	0.083	0.349	0.119	0.153	6.080

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	283	46	40	39	37	49	39	43	155
N.S.	1	5.78	0.94	0.82	0.80	0.76	1.00	0.80	0.88	3.16
time (sec)	N/A	0.760	0.036	0.152	0.113	0.086	0.181	0.122	0.199	0.145

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	293	91	44	72	70	71	72	72	204
N.S.	1	4.19	1.30	0.63	1.03	1.00	1.01	1.03	1.03	2.91
time (sec)	N/A	0.738	0.021	0.127	0.113	0.101	0.697	0.119	0.195	0.102

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	377	185	111	143	209	138	138	328	203
N.S.	1	2.50	1.23	0.74	0.95	1.38	0.91	0.91	2.17	1.34
time (sec)	N/A	1.051	0.109	0.185	0.111	0.100	0.684	0.136	0.155	5.997

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	36	27	0	0	61	0	44	0
N.S.	1	1.00	0.11	0.09	0.00	0.00	0.19	0.00	0.14	0.00
time (sec)	N/A	0.708	10.014	0.345	0.000	0.000	0.998	0.000	0.160	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	108	118	118	305	1340	392	393	115
N.S.	1	0.99	1.29	1.40	1.40	3.63	15.95	4.67	4.68	1.37
time (sec)	N/A	0.439	0.169	2.717	0.032	0.094	1.037	0.141	0.153	6.235

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	120	80	78	160	598	196	197	76
N.S.	1	1.00	1.97	1.31	1.28	2.62	9.80	3.21	3.23	1.25
time (sec)	N/A	0.397	0.103	0.666	0.032	0.083	0.616	0.132	0.155	6.094

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	43	42	43	39	56	180	65	66	38
N.S.	1	1.13	1.11	1.13	1.03	1.47	4.74	1.71	1.74	1.00
time (sec)	N/A	0.342	0.085	0.153	0.028	0.094	0.400	0.133	0.152	5.984

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	17	12	12	14	12
N.S.	1	1.00	1.00	1.08	1.00	1.42	1.00	1.00	1.17	1.00
time (sec)	N/A	0.241	0.001	0.033	0.032	0.083	0.024	0.125	0.153	6.019

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	73	0	33	43
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.74	0.00	0.79	1.02
time (sec)	N/A	0.345	0.069	0.000	0.000	0.000	1.818	0.000	0.155	6.256

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	388	0	84	49
N.S.	1	1.00	1.00	0.00	0.00	0.00	8.82	0.00	1.91	1.11
time (sec)	N/A	0.339	0.096	0.000	0.000	0.000	7.761	0.000	0.161	6.296

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	63	0	0	0	1367	0	176	59
N.S.	1	1.00	1.37	0.00	0.00	0.00	29.72	0.00	3.83	1.28
time (sec)	N/A	0.342	0.089	0.000	0.000	0.000	42.362	0.000	0.156	6.256

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	243	0	57	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	2.56	0.00	0.60	0.00
time (sec)	N/A	0.602	0.209	0.000	0.000	0.000	3.677	0.000	0.160	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	172	106	0	0	0	0	0	986	0
N.S.	1	1.20	0.74	0.00	0.00	0.00	0.00	0.00	6.90	0.00
time (sec)	N/A	0.733	0.175	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	243	106	0	0	0	0	0	0	0
N.S.	1	1.12	0.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.867	0.157	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	104	0	0	0	381	0	77	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	3.05	0.00	0.62	0.00
time (sec)	N/A	0.702	0.329	0.000	0.000	0.000	18.205	0.000	0.167	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	165	147	0	0	0	0	0	1368	0
N.S.	1	1.02	0.91	0.00	0.00	0.00	0.00	0.00	8.44	0.00
time (sec)	N/A	0.613	0.292	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	247	147	0	0	0	0	0	0	0
N.S.	1	1.01	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.859	0.285	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	46	0	96	47
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.00	0.00	2.09	1.02
time (sec)	N/A	0.310	0.008	0.000	0.000	0.000	1.417	0.000	0.155	6.995

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	0	0	107	0	790	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	1.20	0.00	8.88	0.00
time (sec)	N/A	0.377	0.053	0.000	0.000	0.000	4.253	0.000	0.156	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	176	134	0	0	0	168	0	0	0
N.S.	1	1.03	0.78	0.00	0.00	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.651	0.111	0.000	0.000	0.000	46.940	0.000	0.173	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	371	230	188	0	0	0	0	0	0	0
N.S.	1	0.62	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.681	0.183	0.000	0.000	0.000	0.000	0.000	0.209	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [48] had the largest ratio of [.705882000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.99	11	0.273
2	A	3	3	1.00	15	0.200
3	A	2	2	1.00	22	0.091
4	B	2	2	2.08	17	0.118
5	A	2	2	1.95	22	0.091
6	A	2	2	1.00	22	0.091
7	A	2	2	1.00	22	0.091
8	A	2	2	1.00	20	0.100
9	A	2	2	1.00	22	0.091
10	A	14	13	1.10	22	0.591
11	A	16	15	1.12	22	0.682
12	A	2	2	1.00	23	0.087
13	A	2	2	1.00	23	0.087
14	A	2	2	1.00	21	0.095
15	A	2	2	1.00	23	0.087
16	A	12	11	0.86	23	0.478
17	A	14	13	0.88	23	0.565
18	A	2	2	1.00	22	0.091
19	A	2	2	1.00	22	0.091
20	A	2	2	1.00	20	0.100
21	A	13	12	1.08	22	0.545

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	13	12	1.08	22	0.545
23	A	13	12	1.07	22	0.545
24	A	2	2	1.00	23	0.087
25	A	2	2	1.00	23	0.087
26	A	2	2	1.00	21	0.095
27	A	13	12	0.97	23	0.522
28	A	12	11	0.96	23	0.478
29	A	12	11	0.97	23	0.478
30	A	2	2	1.00	35	0.057
31	A	2	2	1.00	35	0.057
32	A	2	2	1.00	22	0.091
33	A	4	3	1.00	25	0.120
34	A	3	3	1.00	15	0.200
35	A	3	3	1.00	15	0.200
36	A	3	3	1.00	20	0.150
37	A	3	3	1.00	20	0.150
38	A	10	9	1.10	17	0.529
39	A	3	3	1.00	25	0.120
40	A	3	3	1.00	35	0.086
41	A	3	3	1.00	35	0.086
42	A	3	3	1.11	22	0.136
43	A	3	3	1.00	25	0.120
44	A	3	3	1.00	15	0.200
45	A	3	3	1.00	15	0.200
46	A	3	3	1.00	20	0.150
47	A	3	3	1.00	20	0.150
48	A	13	12	1.22	17	0.706
49	A	3	3	1.00	25	0.120
50	C	2	2	5.78	28	0.071
51	C	2	2	4.19	28	0.071
52	C	6	6	2.50	27	0.222
53	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	4	0.99	19	0.211
55	A	4	4	1.00	19	0.211
56	A	3	3	1.13	17	0.176
57	A	1	1	1.00	9	0.111
58	A	3	3	1.00	19	0.158
59	A	3	3	1.00	19	0.158
60	A	3	3	1.00	19	0.158
61	A	2	2	1.00	26	0.077
62	A	2	2	1.20	26	0.077
63	A	2	2	1.12	26	0.077
64	A	2	2	1.00	35	0.057
65	A	4	4	1.02	35	0.114
66	A	6	6	1.01	35	0.171
67	A	2	2	1.00	9	0.222
68	A	3	3	1.00	17	0.176
69	A	4	4	1.03	24	0.167
70	A	2	2	0.62	31	0.065

CHAPTER 3

Listing of integrals

3.1 $\int \frac{1+x}{1+x^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 147

$$\begin{aligned}\int \frac{1+x}{1+x^5}\,dx =& -\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)\\ &-\frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right)\right)\\ &-\frac{\log\left(2-x-\sqrt{5}x+2x^2\right)}{2\sqrt{5}}+\frac{\log\left(2-x+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}\end{aligned}$$

output
```
-1/5*(5-2*5^(1/2))^(1/2)*arctan((1-5^(1/2)-4*x)/(10+2*5^(1/2))^(1/2))-1/5*
(5+2*5^(1/2))^(1/2)*arctan(1/20*(50+10*5^(1/2))^(1/2)*(1+5^(1/2)-4*x))-1/1
0*ln(2-x-x*5^(1/2)+2*x^2)*5^(1/2)+1/10*ln(2-x+x*5^(1/2)+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1+x}{1+x^5} dx = \text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\&,\frac{\log(x-\#1)}{-1+2\#1-3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(1 + x)/(1 + x^5),x]
```

output
```
RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 + 
4*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2019, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^5+1} dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{x^4-x^3+x^2-x+1}dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{-2x+\sqrt{5}+1}{\sqrt{5}\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}-\frac{-2x-\sqrt{5}+1}{\sqrt{5}\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)+\frac{\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{2\sqrt{5}}$$

input
```
Int[(1 + x)/(1 + x^5),x]
```

output
```
-1/5*(Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] - 4*x)/Sqrt[2*(5 + Sqrt[5])]
]) - (Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]
])/5 + Log[2 - (1 - Sqrt[5])*x + 2*x^2]/(2*Sqrt[5]) - Log[2 - (1 + Sqrt[5]
)*x + 2*x^2]/(2*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.31

method	result
risch	$\sum_{\_R=\mathrm{RootOf}(\_Z^4-\_Z^3+\_Z^2-\_Z+1)}\frac{\ln(x-\_R)}{4\_R^3-3\_R^2+2\_R-1}$
default	$-\frac{\ln(2-x-\sqrt{5}\,x+2x^2)\sqrt{5}}{10}-\frac{2\left(-\frac{\sqrt{5}\,(-\sqrt{5}-1)}{2}-\sqrt{5}-5\right)\arctan\left(\frac{-1-\sqrt{5}+4x}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\ln(2-x+\sqrt{5}\,x+2x^2)\sqrt{5}}{10}+\frac{2\left(-\frac{\sqrt{5}\,(\sqrt{5}-1}{2}\right.}{}$
meijerg	$-\frac{x^2\ln\left(1+(x^5)^{\frac{1}{5}}\right)}{5(x^5)^{\frac{2}{5}}}-\frac{x^2\cos(\frac{2\pi}{5})\ln\left(1-2\cos(\frac{\pi}{5})(x^5)^{\frac{1}{5}}+(x^5)^{\frac{2}{5}}\right)}{5(x^5)^{\frac{2}{5}}}+\frac{2x^2\sin(\frac{2\pi}{5})\arctan\left(\frac{\sin(\frac{\pi}{5})(x^5)^{\frac{1}{5}}}{1-\cos(\frac{\pi}{5})(x^5)^{\frac{1}{5}}}\right)}{5(x^5)^{\frac{2}{5}}}+\frac{x^2\cos(\frac{\pi}{5})\ln}{}$

input
```
int((x+1)/(x^5+1),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3-3*_R^2+2*_R-1)*ln(x-_R),_R=RootOf(_Z^4-_Z^3+_Z^2-_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{1+x}{1+x^5}\,dx = -\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x+1)-5x\right)\sqrt{2\sqrt{5}+5}\right) + \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x+1)+5x\right)\sqrt{-2\sqrt{5}+5}\right) + \frac{1}{10}\sqrt{5}\log\left(2x^2+\sqrt{5}x-x+2\right) - \frac{1}{10}\sqrt{5}\log\left(2x^2-\sqrt{5}x-x+2\right)$$

input
```
integrate((1+x)/(x^5+1),x, algorithm="fricas")
```

output
```
-1/5*sqrt(2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x + 1) - 5*x)*sqrt(2*sqrt(5)
 + 5)) + 1/5*sqrt(-2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x + 1) + 5*x)*sqrt(
-2*sqrt(5) + 5)) + 1/10*sqrt(5)*log(2*x^2 + sqrt(5)*x - x + 2) - 1/10*sqrt
(5)*log(2*x^2 - sqrt(5)*x - x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. 2(122) = 244.

Time = 0.63 (sec) , antiderivative size = 1287, normalized size of antiderivative = 8.76

$$\int \frac{1+x}{1+x^5}\,dx = \text{Too large to display}$$

input
```
integrate((1+x)/(x**5+1),x)
```

output
```
sqrt(5)*log(x**2 + x*(-48/11 - 21*sqrt(5)/11 + 4*sqrt(10)*sqrt(sqrt(5) + 3
)/11 + 45*sqrt(2)*sqrt(sqrt(5) + 3)/22) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/
484 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10
 - sqrt(5)*log(x**2 + x*(-48/11 - 45*sqrt(2)*sqrt(3 - sqrt(5))/22 + 4*sqrt
(10)*sqrt(3 - sqrt(5))/11 + 21*sqrt(5)/11) - 2213*sqrt(5)/242 - 1381*sqrt(
10)*sqrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)
/10 + 2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*
sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt
(5)) + 15)) - 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*
sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*s
qrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 45*sqrt(2)*sqrt(3 - sqrt(5))/(-
8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqr
t(3 - sqrt(5)) + 15)) + 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*s
qrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqr
t(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) + 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*s
qrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5...
```

Maxima [F]

$$\int \frac{1+x}{1+x^5}\,dx = \int \frac{x+1}{x^5+1}\,dx$$

input
```
integrate((1+x)/(x^5+1),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(x^5 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\begin{aligned}\int \frac{1+x}{1+x^5}\,dx =& \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) \\ &+ \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) \\ &- \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) \\ &+ \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)\end{aligned}$$

input
```
integrate((1+x)/(x^5+1),x, algorithm="giac")
```

output
```
1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
+ 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)
) - 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2
 + 1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.44

$$\int \frac{1+x}{1+x^5}\,dx = \sum_{k=1}^{4} \ln\left(\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(-4\,x+\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)+15\,x-15\right)+1\right)\right)\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)$$

input
```
int((x + 1)/(x^5 + 1),x)
```

output
```
symsum(log(root(z^4 - z/25 + 1/125, z, k)*(root(z^4 - z/25 + 1/125, z, k)*
(25*root(z^4 - z/25 + 1/125, z, k) + 15*x - 15) - 4*x + 1))*root(z^4 - z/2
5 + 1/125, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{1+x}{1+x^5}\,dx = \int \frac{1}{x^4-x^3+x^2-x+1}dx$$

input
```
int((1+x)/(x^5+1),x)
```

output
```
int(1/(x**4 - x**3 + x**2 - x + 1),x)
```

3.2 $\int \frac{1-x}{1-x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 143

$$\int \frac{1-x}{1-x^5}\,dx = \frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{1-\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + \frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4x\right)\right) - \frac{\log\left(2+x-\sqrt{5}x+2x^2\right)}{2\sqrt{5}} + \frac{\log\left(2+x+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

output
```
1/5*(5-2*5^(1/2))^(1/2)*arctan((1-5^(1/2)+4*x)/(10+2*5^(1/2))^(1/2))+1/5*(
5+2*5^(1/2))^(1/2)*arctan(1/20*(50+10*5^(1/2))^(1/2)*(1+5^(1/2)+4*x))-1/10
*ln(2+x-x*5^(1/2)+2*x^2)*5^(1/2)+1/10*ln(2+x+x*5^(1/2)+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int \frac{1-x}{1-x^5}\,dx = \text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(1 - x)/(1 - x^5),x]
```

output
```
RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4
*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2019, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{1-x^5}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{x^4+x^3+x^2+x+1}dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{2x+\sqrt{5}+1}{\sqrt{5}\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}-\frac{2x-\sqrt{5}+1}{\sqrt{5}\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)-\frac{\log\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}{2\sqrt{5}}+\frac{\log\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}{2\sqrt{5}}$$

input
```
Int[(1 - x)/(1 - x^5),x]
```

output
```
(Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]])/5
+ (Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/
5 - Log[2 + (1 - Sqrt[5])*x + 2*x^2]/(2*Sqrt[5]) + Log[2 + (1 + Sqrt[5])*x
 + 2*x^2]/(2*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.29

method	result
risch	$\sum_{\_R=\mathrm{RootOf}(\_Z^4+\_Z^3+\_Z^2+\_Z+1)} \frac{\ln(x-\_R)}{4\_R^3+3\_R^2+2\_R+1}$
default	$\frac{\ln(2+x+\sqrt{5}\,x+2x^2)\sqrt{5}}{10}+\frac{2\left(-\frac{\sqrt{5}\,(\sqrt{5}+1)}{2}+5+\sqrt{5}\right)\arctan\left(\frac{1+\sqrt{5}+4x}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{\ln(2+x-\sqrt{5}\,x+2x^2)\sqrt{5}}{10}-\frac{2\left(-\frac{\sqrt{5}\,(-\sqrt{5}+1)}{2}+\sqrt{}\right.}{5\sqrt{}}$
meijerg	$-\frac{x\left(\ln\left(1-(x^5)^{\frac{1}{5}}\right)+\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}+(x^5)^{\frac{2}{5}}\right)-2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}}{1-\cos\left(\frac{2\pi}{5}\right)(x^5)^{\frac{1}{5}}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1+2\cos\left(\frac{\pi}{5}\right)\right.\right.}{5(x^5)^{\frac{1}{5}}}$

input
```
int((1-x)/(-x^5+1),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3+3*_R^2+2*_R+1)*ln(x-_R),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

$$\int \frac{1-x}{1-x^5}\,dx = -\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x-1)-5x\right)\sqrt{2\sqrt{5}+5}\right) + \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x-1)+5x\right)\sqrt{-2\sqrt{5}+5}\right) + \frac{1}{10}\sqrt{5}\log\left(2x^2+\sqrt{5}x+x+2\right) - \frac{1}{10}\sqrt{5}\log\left(2x^2-\sqrt{5}x+x+2\right)$$

input
```
integrate((1-x)/(-x^5+1),x, algorithm="fricas")
```

output
```
-1/5*sqrt(2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x - 1) - 5*x)*sqrt(2*sqrt(5)
 + 5)) + 1/5*sqrt(-2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x - 1) + 5*x)*sqrt(
-2*sqrt(5) + 5)) + 1/10*sqrt(5)*log(2*x^2 + sqrt(5)*x + x + 2) - 1/10*sqrt
(5)*log(2*x^2 - sqrt(5)*x + x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. 2(122) = 244.

Time = 0.60 (sec) , antiderivative size = 1287, normalized size of antiderivative = 9.00

$$\int \frac{1-x}{1-x^5}\,dx = \text{Too large to display}$$

input

```
integrate((1-x)/(-x**5+1),x)
```

output

```
sqrt(5)*log(x**2 + x*(-21*sqrt(5)/11 - 4*sqrt(10)*sqrt(3 - sqrt(5))/11 + 4
5*sqrt(2)*sqrt(3 - sqrt(5))/22 + 48/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)
*sqrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10
 - sqrt(5)*log(x**2 + x*(-45*sqrt(2)*sqrt(sqrt(5) + 3)/22 - 4*sqrt(10)*sqr
t(sqrt(5) + 3)/11 + 21*sqrt(5)/11 + 48/11) - 1381*sqrt(10)*sqrt(sqrt(5) +
3)/484 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)
/10 + 2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*
sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt
(5)) + 15)) - 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) +
15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 8*sqrt(10)*sqrt(3 - sqr
t(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sq
rt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt
(10)*sqrt(3 - sqrt(5)) + 15)) + 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*s
qrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqr
t(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(
5)) + 15)) + 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*s
qrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5...
```

Maxima [F]

$$\int \frac{1-x}{1-x^5}\,dx = \int \frac{x-1}{x^5-1}\,dx$$

input
```
integrate((1-x)/(-x^5+1),x, algorithm="maxima")
```

output
```
integrate((x - 1)/(x^5 - 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.71

$$\int \frac{1-x}{1-x^5}\,dx = \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) - \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

input
```
integrate((1-x)/(-x^5+1),x, algorithm="giac")
```

output
```
1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10))
+ 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10)
) + 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2
 - 1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.45

$$\int \frac{1-x}{1-x^5}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(4\,x+\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)+15\,x+15\right)+1\right)\right)\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)$$

input
```
int((x - 1)/(x^5 - 1),x)
```

output
```
symsum(log(-root(z^4 + z/25 + 1/125, z, k)*(4*x + root(z^4 + z/25 + 1/125,
 z, k)*(25*root(z^4 + z/25 + 1/125, z, k) + 15*x + 15) + 1))*root(z^4 + z/
25 + 1/125, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{1-x}{1-x^5}\,dx = \int \frac{1}{x^4+x^3+x^2+x+1}dx$$

input
```
int((1-x)/(-x^5+1),x)
```

output
```
int(1/(x**4 + x**3 + x**2 + x + 1),x)
```

3.3 $\int \frac{1+x+x^2+x^3+x^4}{1-x^5} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 8

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} \, dx = -\log(1-x)$$

output
```
-ln(1-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5} \, dx = -\log(1-x)$$

input
```
Integrate[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

output
```
-Log[1 - x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^3+x^2+x+1}{1-x^5} \, dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{ 16}$$

$$-\log(1-x)$$

input
```
Int[(1 + x + x^2 + x^3 + x^4)/(1 - x^5),x]
```

output
```
-Log[1 - x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$-\ln\left(x-1\right)$	7
norman	$-\ln\left(x-1\right)$	7
risch	$-\ln\left(x-1\right)$	7
parallelrisch	$-\ln\left(x-1\right)$	7
meijerg	Expression too large to display	542

input
```
int((x^4+x^3+x^2+x+1)/(-x^5+1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx=-\log\left(x-1\right)$$

input
```
integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="fricas")
```

output
```
-log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx=-\log\left(x-1\right)$$

input
```
integrate((x**4+x**3+x**2+x+1)/(-x**5+1),x)
```

output
```
-log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log{(x-1)}$$

input
```
integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="maxima")
```

output
```
-log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log{(|x-1|)}$$

input
```
integrate((x^4+x^3+x^2+x+1)/(-x^5+1),x, algorithm="giac")
```

output
```
-log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\ln{(x-1)}$$

input
```
int(-(x + x^2 + x^3 + x^4 + 1)/(x^5 - 1),x)
```

output
```
-log(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1+x+x^2+x^3+x^4}{1-x^5}\,dx = -\log(x-1)$$

input
```
int((x^4+x^3+x^2+x+1)/(-x^5+1),x)
```

output
```
- log(x - 1)
```

3.4 $\int \frac{A+Cx^4}{a+bx^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 211

$$\int \frac{A+Cx^4}{a+bx^6}\,dx=\frac{\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{5/6}}-\frac{\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}b^{5/6}}+\frac{\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}b^{5/6}}+\frac{\left(Ab^{2/3}-a^{2/3}C\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)}{2\sqrt{3}a^{5/6}b^{5/6}}$$

output
```
1/3*(A*b^(2/3)+a^(2/3)*C)*arctan(b^(1/6)*x/a^(1/6))/a^(5/6)/b^(5/6)+1/6*(A
*b^(2/3)+a^(2/3)*C)*arctan(-3^(1/2)+2*b^(1/6)*x/a^(1/6))/a^(5/6)/b^(5/6)+1
/6*(A*b^(2/3)+a^(2/3)*C)*arctan(3^(1/2)+2*b^(1/6)*x/a^(1/6))/a^(5/6)/b^(5/
6)+1/6*(A*b^(2/3)-a^(2/3)*C)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x/(a^(1/3)+b^
(1/3)*x^2))*3^(1/2)/a^(5/6)/b^(5/6)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.20

$$\int \frac{A+Cx^4}{a+bx^6}\,dx = \frac{4\sqrt[6]{a}\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)-2\sqrt[6]{a}\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)+2\sqrt[6]{a}\left(Ab^{2/3}+a^{2/3}\right.}{}$$

input
```
Integrate[(A + C*x^4)/(a + b*x^6),x]
```

output
```
(4*a^(1/6)*(A*b^(2/3) + a^(2/3)*C)*ArcTan[(b^(1/6)*x)/a^(1/6)] - 2*a^(1/6)
*(A*b^(2/3) + a^(2/3)*C)*ArcTan[Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] + 2*a^(1/
6)*(A*b^(2/3) + a^(2/3)*C)*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] + Sqrt[
3]*(-(a^(1/6)*A*b^(2/3)) + a^(5/6)*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6
)*x + b^(1/3)*x^2] - Sqrt[3]*(-(a^(1/6)*A*b^(2/3)) + a^(5/6)*C)*Log[a^(1/3
) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(12*a*b^(5/6))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 439 vs. 2(211) = 422.

Time = 1.64 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^4}{a+bx^6}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{A}{a+bx^6}+\frac{Cx^4}{a+bx^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A \arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}} - \frac{A \arctan\left(\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}} + \frac{A \arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}} - \frac{A \log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x + \sqrt[3]{a} + \sqrt[3]{b}x^2\right)}{4\sqrt{3}a^{5/6}\sqrt[6]{b}} + \frac{A \log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x + \sqrt[3]{a} + \sqrt[3]{b}x^2\right)}{4\sqrt{3}a^{5/6}\sqrt[6]{b}} + \frac{C \arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}} - \frac{C \arctan\left(\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}b^{5/6}} + \frac{C \arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}b^{5/6}} + \frac{C \log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x + \sqrt[3]{a} + \sqrt[3]{b}x^2\right)}{4\sqrt{3}\sqrt[6]{a}b^{5/6}} - \frac{C \log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x + \sqrt[3]{a} + \sqrt[3]{b}x^2\right)}{4\sqrt{3}\sqrt[6]{a}b^{5/6}}$$

input
```
Int[(A + C*x^4)/(a + b*x^6),x]
```

output
```
(A*ArcTan[(b^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*b^(1/6)) + (C*ArcTan[(b^(1/6)*x
)/a^(1/6)])/(3*a^(1/6)*b^(5/6)) - (A*ArcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x
)/a^(1/6)])/(6*a^(5/6)*b^(1/6)) - (C*ArcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x
)/a^(1/6)])/(6*a^(1/6)*b^(5/6)) + (A*ArcTan[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x
)/a^(1/6)])/(6*a^(5/6)*b^(1/6)) + (C*ArcTan[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x
)/a^(1/6)])/(6*a^(1/6)*b^(5/6)) - (A*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)
*x + b^(1/3)*x^2])/(4*Sqrt[3]*a^(5/6)*b^(1/6)) + (C*Log[a^(1/3) - Sqrt[3]*
a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*a^(1/6)*b^(5/6)) + (A*Log[a^(
1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*a^(5/6)*b^(1/6
)) - (C*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]
*a^(1/6)*b^(5/6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}b+a\right)}\frac{\left(C\_R^{4}+A\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6b}$
default	$-\frac{\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)C\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}}{12a}+\frac{\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)A\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{3}}{12a}+\frac{\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)C}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\arctan}{}$

input
```
int((C*x^4+A)/(b*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b*sum((C*_R^4+A)/_R^5*ln(x-_R),_R=RootOf(_Z^6*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 11478, normalized size of antiderivative = 54.40

$$\int \frac{A+Cx^4}{a+bx^6}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^4+A)/(b*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.72

$$\int \frac{A+Cx^4}{a+bx^6}\,dx$$
$$=\text{RootSum}\left(46656t^6a^5b^5+7776t^4ACa^4b^4+324t^2A^2C^2a^3b^3+A^6b^4+2A^3C^3a^2b^2+C^6a^4,\left(t\mapsto t\log\left(x\right.\right.\right.$$

input
```
integrate((C*x**4+A)/(b*x**6+a),x)
```

output
```
RootSum(46656*_t**6*a**5*b**5 + 7776*_t**4*A*C*a**4*b**4 + 324*_t**2*A**2*
C**2*a**3*b**3 + A**6*b**4 + 2*A**3*C**3*a**2*b**2 + C**6*a**4, Lambda(_t,
 _t*log(x + (7776*_t**5*C*a**5*b**4 + 1080*_t**3*A*C**2*a**4*b**3 - 6*_t*A
**5*a*b**4 + 30*_t*A**2*C**3*a**3*b**2)/(-A**6*b**4 + C**6*a**4))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{A+Cx^4}{a+bx^6}\,dx =& -\frac{\sqrt{3}\left(Ca^{\frac{2}{3}}-Ab^{\frac{2}{3}}\right)\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{5}{6}}b^{\frac{5}{6}}} \\ &+\frac{\sqrt{3}\left(Ca^{\frac{2}{3}}-Ab^{\frac{2}{3}}\right)\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{5}{6}}b^{\frac{5}{6}}} \\ &+\frac{\left(Ca^{\frac{2}{3}}+Ab^{\frac{2}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{3\,a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \\ &+\frac{\left(Cab^{\frac{1}{3}}+Aa^{\frac{1}{3}}b\right)\arctan\left(\frac{2\,b^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,ab\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \\ &+\frac{\left(Cab^{\frac{1}{3}}+Aa^{\frac{1}{3}}b\right)\arctan\left(\frac{2\,b^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,ab\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\end{aligned}$$

input
```
integrate((C*x^4+A)/(b*x^6+a),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*(C*a^(2/3) - A*b^(2/3))*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^
(1/6)*x + a^(1/3))/(a^(5/6)*b^(5/6)) + 1/12*sqrt(3)*(C*a^(2/3) - A*b^(2/3)
)*log(b^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3))/(a^(5/6)*b^(5/6))
 + 1/3*(C*a^(2/3) + A*b^(2/3))*arctan(b^(1/3)*x/sqrt(a^(1/3)*b^(1/3)))/(a^
(2/3)*b^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*(C*a*b^(1/3) + A*a^(1/3)*b)*arc
tan((2*b^(1/3)*x + sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(a*b*sq
rt(a^(1/3)*b^(1/3))) + 1/6*(C*a*b^(1/3) + A*a^(1/3)*b)*arctan((2*b^(1/3)*x
 - sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(a*b*sqrt(a^(1/3)*b^(1/
3)))
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.18

$$\int \frac{A+Cx^4}{a+bx^6}\,dx = \frac{\left(Ab^4+(ab^5)^{\frac{2}{3}}\,C\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6\,(ab^5)^{\frac{5}{6}}} + \frac{\left(Ab^4+(ab^5)^{\frac{2}{3}}\,C\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6\,(ab^5)^{\frac{5}{6}}} + \frac{\left(C\left(\frac{a}{b}\right)^{\frac{5}{6}}+A\left(\frac{a}{b}\right)^{\frac{1}{6}}\right)\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a} + \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ab^4-(ab^5)^{\frac{5}{6}}\,C\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,ab^5} - \frac{\sqrt{3}\left((ab^5)^{\frac{1}{6}}\,Ab^4-(ab^5)^{\frac{5}{6}}\,C\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,ab^5}$$

input
```
integrate((C*x^4+A)/(b*x^6+a),x, algorithm="giac")
```

output
```
1/6*(A*b^4 + (a*b^5)^(2/3)*C)*arctan((2*x + sqrt(3)*(a/b)^(1/6))/(a/b)^(1/
6))/(a*b^5)^(5/6) + 1/6*(A*b^4 + (a*b^5)^(2/3)*C)*arctan((2*x - sqrt(3)*(a
/b)^(1/6))/(a/b)^(1/6))/(a*b^5)^(5/6) + 1/3*(C*(a/b)^(5/6) + A*(a/b)^(1/6)
)*arctan(x/(a/b)^(1/6))/a + 1/12*sqrt(3)*((a*b^5)^(1/6)*A*b^4 - (a*b^5)^(5
/6)*C)*log(x^2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a*b^5) - 1/12*sqrt(
3)*((a*b^5)^(1/6)*A*b^4 - (a*b^5)^(5/6)*C)*log(x^2 - sqrt(3)*x*(a/b)^(1/6)
 + (a/b)^(1/3))/(a*b^5)
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.80

$$\int \frac{A+Cx^4}{a+bx^6}\,dx = \sum_{k=1}^{6} \ln\Big(-\mathrm{root}\big(46656\,a^5\,b^5\,z^6 + 7776\,A\,C\,a^4\,b^4\,z^4 + 324\,A^2\,C^2\,a^3\,b^3\,z^2 + 2\,A^3\,C^3\,a^2\,b^2 + C^6\,a^4 + A^6\,b^4, z, k\big)\,\Big(\mathrm{root}\big(46656\,a^5\,b^5\,z^6 + 7776\,A\,C\,a^4\,b^4\,z^4 + 324\,A^2\,C^2\,a^3\,b^3\,z^2 + 2\,A^3\,C^3\,a^2\,b^2 + C^6\,a^4 + A^6\,b^4, z$$

$$+ x\left(6\,A^4\,b^5 - 12\,A\,C^3\,a^2\,b^3\right)\Big) - A^4\,C\,b^4 - A\,C^4\,a^2\,b^2\Big)\,\mathrm{root}\big(46656\,a^5\,b^5\,z^6 + 7776\,A\,C\,a^4\,b^4\,z^4 + 324\,A^2\,C^2\,a^3\,b^3\,z^2 + 2\,A^3\,C^3\,a^2\,b^2 + C^6\,a^4 + A^6\,b^4, z, k\big)$$

input
```
int((A + C*x^4)/(a + b*x^6),x)
```

output
```
symsum(log(- root(46656*a^5*b^5*z^6 + 7776*A*C*a^4*b^4*z^4 + 324*A^2*C^2*a
^3*b^3*z^2 + 2*A^3*C^3*a^2*b^2 + C^6*a^4 + A^6*b^4, z, k)*(root(46656*a^5*
b^5*z^6 + 7776*A*C*a^4*b^4*z^4 + 324*A^2*C^2*a^3*b^3*z^2 + 2*A^3*C^3*a^2*b
^2 + C^6*a^4 + A^6*b^4, z, k)*(36*A^3*a*b^5 + 36*C^3*a^3*b^3 - 216*C^2*roo
t(46656*a^5*b^5*z^6 + 7776*A*C*a^4*b^4*z^4 + 324*A^2*C^2*a^3*b^3*z^2 + 2*A
^3*C^3*a^2*b^2 + C^6*a^4 + A^6*b^4, z, k)*a^3*b^4*x) + x*(6*A^4*b^5 - 12*A
*C^3*a^2*b^3)) - A^4*C*b^4 - A*C^4*a^2*b^2)*root(46656*a^5*b^5*z^6 + 7776*
A*C*a^4*b^4*z^4 + 324*A^2*C^2*a^3*b^3*z^2 + 2*A^3*C^3*a^2*b^2 + C^6*a^4 +
A^6*b^4, z, k), k, 1, 6)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

$$\int \frac{A + Cx^4}{a + bx^6} \, dx$$
$$= \frac{-2a^{\frac{1}{3}} atan\left(\frac{b^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3} - 2b^{\frac{1}{3}} x}{b^{\frac{1}{6}} a^{\frac{1}{6}}}\right) b - 2b^{\frac{1}{3}} atan\left(\frac{b^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3} - 2b^{\frac{1}{3}} x}{b^{\frac{1}{6}} a^{\frac{1}{6}}}\right) c + 2a^{\frac{1}{3}} atan\left(\frac{b^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3} + 2b^{\frac{1}{3}} x}{b^{\frac{1}{6}} a^{\frac{1}{6}}}\right) b + 2b^{\frac{1}{3}} atan\left(\frac{b^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3} + 2}{b^{\frac{1}{6}} a^{\frac{1}{6}}}\right.}{}$$

input
```
int((C*x^4+A)/(b*x^6+a),x)
```

output
```
(b**(1/6)*a**(1/6)*( - 2*a**(1/3)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*b**(
1/3)*x)/(b**(1/6)*a**(1/6)))*b - 2*b**(1/3)*atan((b**(1/6)*a**(1/6)*sqrt(3
) - 2*b**(1/3)*x)/(b**(1/6)*a**(1/6)))*c + 2*a**(1/3)*atan((b**(1/6)*a**(1
/6)*sqrt(3) + 2*b**(1/3)*x)/(b**(1/6)*a**(1/6)))*b + 2*b**(1/3)*atan((b**(
1/6)*a**(1/6)*sqrt(3) + 2*b**(1/3)*x)/(b**(1/6)*a**(1/6)))*c + 4*a**(1/3)*
atan((b**(1/3)*x)/(b**(1/6)*a**(1/6)))*b + 4*b**(1/3)*atan((b**(1/3)*x)/(b
**(1/6)*a**(1/6)))*c - a**(1/3)*sqrt(3)*log( - b**(1/6)*a**(1/6)*sqrt(3)*x
 + a**(1/3) + b**(1/3)*x**2)*b + a**(1/3)*sqrt(3)*log(b**(1/6)*a**(1/6)*sq
rt(3)*x + a**(1/3) + b**(1/3)*x**2)*b + b**(1/3)*sqrt(3)*log( - b**(1/6)*a
**(1/6)*sqrt(3)*x + a**(1/3) + b**(1/3)*x**2)*c - b**(1/3)*sqrt(3)*log(b**
(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + b**(1/3)*x**2)*c))/(12*b**(1/3)*a**(
1/3)*b)
```

3.5 $\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 241

$$\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx = \frac{\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{5/6}}+\frac{B\arctan\left(\frac{\sqrt{b}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{b}} - \frac{\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}b^{5/6}} + \frac{\left(Ab^{2/3}+a^{2/3}C\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}b^{5/6}} + \frac{\left(Ab^{2/3}-a^{2/3}C\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x}{\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)}{2\sqrt{3}a^{5/6}b^{5/6}}$$

output
```
1/3*(A*b^(2/3)+a^(2/3)*C)*arctan(b^(1/6)*x/a^(1/6))/a^(5/6)/b^(5/6)+1/3*B*
arctan(b^(1/2)*x^3/a^(1/2))/a^(1/2)/b^(1/2)+1/6*(A*b^(2/3)+a^(2/3)*C)*arct
an(-3^(1/2)+2*b^(1/6)*x/a^(1/6))/a^(5/6)/b^(5/6)+1/6*(A*b^(2/3)+a^(2/3)*C)
*arctan(3^(1/2)+2*b^(1/6)*x/a^(1/6))/a^(5/6)/b^(5/6)+1/6*(A*b^(2/3)-a^(2/3
)*C)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*x/(a^(1/3)+b^(1/3)*x^2))*3^(1/2)/a^(5
/6)/b^(5/6)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx = \frac{4\sqrt[6]{a}\left(Ab^{2/3}-\sqrt[3]{a}\sqrt[3]{b}B+a^{2/3}C\right)\arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)-2\sqrt[6]{a}\left(Ab^{2/3}+2\sqrt[3]{a}\sqrt[3]{b}B+a^{2/3}C\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)\cdots}{\cdots}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^6),x]
```

output
```
(4*a^(1/6)*(A*b^(2/3) - a^(1/3)*b^(1/3)*B + a^(2/3)*C)*ArcTan[(b^(1/6)*x)/
a^(1/6)] - 2*a^(1/6)*(A*b^(2/3) + 2*a^(1/3)*b^(1/3)*B + a^(2/3)*C)*ArcTan[
Sqrt[3] - (2*b^(1/6)*x)/a^(1/6)] + 2*a^(1/6)*(A*b^(2/3) + 2*a^(1/3)*b^(1/3
)*B + a^(2/3)*C)*ArcTan[Sqrt[3] + (2*b^(1/6)*x)/a^(1/6)] + Sqrt[3]*(-(a^(1
/6)*A*b^(2/3)) + a^(5/6)*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1
/3)*x^2] - Sqrt[3]*(-(a^(1/6)*A*b^(2/3)) + a^(5/6)*C)*Log[a^(1/3) + Sqrt[3
]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(12*a*b^(5/6))
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{A}{a+bx^6}+\frac{Bx^2}{a+bx^6}+\frac{Cx^4}{a+bx^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{b}}-\frac{A\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}}+\frac{A\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6a^{5/6}\sqrt[6]{b}}-\frac{A\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{4\sqrt{3}a^{5/6}\sqrt[6]{b}}+\frac{A\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{4\sqrt{3}a^{5/6}\sqrt[6]{b}}+\frac{C\arctan\left(\frac{\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}b^{5/6}}-\frac{C\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}b^{5/6}}+\frac{C\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}+2\sqrt[6]{b}x}{\sqrt[6]{a}}\right)}{6\sqrt[6]{a}b^{5/6}}+\frac{B\arctan\left(\frac{\sqrt{b}x^3}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{b}}+\frac{C\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{4\sqrt{3}\sqrt[6]{a}b^{5/6}}-\frac{C\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{4\sqrt{3}\sqrt[6]{a}b^{5/6}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^6),x]
```

output
```
(A*ArcTan[(b^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*b^(1/6)) + (C*ArcTan[(b^(1/6)*x
)/a^(1/6)])/(3*a^(1/6)*b^(5/6)) + (B*ArcTan[(Sqrt[b]*x^3)/Sqrt[a]])/(3*Sqr
t[a]*Sqrt[b]) - (A*ArcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/6)])/(6*a^(
5/6)*b^(1/6)) - (C*ArcTan[(Sqrt[3]*a^(1/6) - 2*b^(1/6)*x)/a^(1/6)])/(6*a^(
1/6)*b^(5/6)) + (A*ArcTan[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1/6)])/(6*a^(
5/6)*b^(1/6)) + (C*ArcTan[(Sqrt[3]*a^(1/6) + 2*b^(1/6)*x)/a^(1/6)])/(6*a^(
1/6)*b^(5/6)) - (A*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])
/(4*Sqrt[3]*a^(5/6)*b^(1/6)) + (C*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x
+ b^(1/3)*x^2])/(4*Sqrt[3]*a^(1/6)*b^(5/6)) + (A*Log[a^(1/3) + Sqrt[3]*a^(
1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*a^(5/6)*b^(1/6)) - (C*Log[a^(1/3
) + Sqrt[3]*a^(1/6)*b^(1/6)*x + b^(1/3)*x^2])/(4*Sqrt[3]*a^(1/6)*b^(5/6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}b+a\right)}\frac{\left(C\_R^{4}+B\_R^{2}+A\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6b}$
default	$-\frac{\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)C\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}}{12a}+\frac{\ln\left(x^{2}+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)A\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{3}}{12a}+\frac{\arctan\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)C}{6b\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}\mathrm{arctan}}{}$

input
```
int((C*x^4+B*x^2+A)/(b*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b*sum((C*_R^4+B*_R^2+A)/_R^5*ln(x-_R),_R=RootOf(_Z^6*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.70 (sec) , antiderivative size = 36914, normalized size of antiderivative = 153.17

$$\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx = \text{Timed out}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**6+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx \\
&= -\frac{\sqrt{3}\left(Ca^{\frac{2}{3}}-Ab^{\frac{2}{3}}\right)\log\left(b^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{5}{6}}b^{\frac{5}{6}}} \\
&\quad+\frac{\sqrt{3}\left(Ca^{\frac{2}{3}}-Ab^{\frac{2}{3}}\right)\log\left(b^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{12\,a^{\frac{5}{6}}b^{\frac{5}{6}}} \\
&\quad+\frac{\left(Ca^{\frac{2}{3}}-Ba^{\frac{1}{3}}b^{\frac{1}{3}}+Ab^{\frac{2}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{3\,a^{\frac{2}{3}}b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \\
&\quad+\frac{\left(Cab^{\frac{1}{3}}-2\,Aa^{\frac{1}{3}}b+\left(2\,Ba^{\frac{2}{3}}+3\,Aa^{\frac{1}{3}}b^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\arctan\left(\frac{2\,b^{\frac{1}{3}}x+\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,ab\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} \\
&\quad+\frac{\left(Cab^{\frac{1}{3}}-2\,Aa^{\frac{1}{3}}b+\left(2\,Ba^{\frac{2}{3}}+3\,Aa^{\frac{1}{3}}b^{\frac{1}{3}}\right)b^{\frac{2}{3}}\right)\arctan\left(\frac{2\,b^{\frac{1}{3}}x-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{6\,ab\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}
\end{aligned}$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^6+a),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*(C*a^(2/3) - A*b^(2/3))*log(b^(1/3)*x^2 + sqrt(3)*a^(1/6)*b^
(1/6)*x + a^(1/3))/(a^(5/6)*b^(5/6)) + 1/12*sqrt(3)*(C*a^(2/3) - A*b^(2/3)
)*log(b^(1/3)*x^2 - sqrt(3)*a^(1/6)*b^(1/6)*x + a^(1/3))/(a^(5/6)*b^(5/6))
 + 1/3*(C*a^(2/3) - B*a^(1/3)*b^(1/3) + A*b^(2/3))*arctan(b^(1/3)*x/sqrt(a
^(1/3)*b^(1/3)))/(a^(2/3)*b^(2/3)*sqrt(a^(1/3)*b^(1/3))) + 1/6*(C*a*b^(1/3
) - 2*A*a^(1/3)*b + (2*B*a^(2/3) + 3*A*a^(1/3)*b^(1/3))*b^(2/3))*arctan((2
*b^(1/3)*x + sqrt(3)*a^(1/6)*b^(1/6))/sqrt(a^(1/3)*b^(1/3)))/(a*b*sqrt(a^(
1/3)*b^(1/3))) + 1/6*(C*a*b^(1/3) - 2*A*a^(1/3)*b + (2*B*a^(2/3) + 3*A*a^(
1/3)*b^(1/3))*b^(2/3))*arctan((2*b^(1/3)*x - sqrt(3)*a^(1/6)*b^(1/6))/sqrt
(a^(1/3)*b^(1/3)))/(a*b*sqrt(a^(1/3)*b^(1/3)))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.18

$$
\begin{aligned}
\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx &= \frac{\left(Ab^4+2\left(ab^5\right)^{\frac{1}{3}}Bb^2+\left(ab^5\right)^{\frac{2}{3}}C\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6\left(ab^5\right)^{\frac{5}{6}}} \\
&+ \frac{\left(Ab^4+2\left(ab^5\right)^{\frac{1}{3}}Bb^2+\left(ab^5\right)^{\frac{2}{3}}C\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{6\left(ab^5\right)^{\frac{5}{6}}} \\
&+ \frac{\left(C\left(\frac{a}{b}\right)^{\frac{5}{6}}-B\sqrt{\frac{a}{b}}+A\left(\frac{a}{b}\right)^{\frac{1}{6}}\right)\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,a} \\
&+ \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ab^4-\left(ab^5\right)^{\frac{5}{6}}C\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,ab^5} \\
&- \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ab^4-\left(ab^5\right)^{\frac{5}{6}}C\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{b}\right)^{\frac{1}{6}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{12\,ab^5}
\end{aligned}
$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^6+a),x, algorithm="giac")
```

output
```
1/6*(A*b^4 + 2*(a*b^5)^(1/3)*B*b^2 + (a*b^5)^(2/3)*C)*arctan((2*x + sqrt(3
)*(a/b)^(1/6))/(a/b)^(1/6))/(a*b^5)^(5/6) + 1/6*(A*b^4 + 2*(a*b^5)^(1/3)*B
*b^2 + (a*b^5)^(2/3)*C)*arctan((2*x - sqrt(3)*(a/b)^(1/6))/(a/b)^(1/6))/(a
*b^5)^(5/6) + 1/3*(C*(a/b)^(5/6) - B*sqrt(a/b) + A*(a/b)^(1/6))*arctan(x/(
a/b)^(1/6))/a + 1/12*sqrt(3)*((a*b^5)^(1/6)*A*b^4 - (a*b^5)^(5/6)*C)*log(x
^2 + sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/3))/(a*b^5) - 1/12*sqrt(3)*((a*b^5)^
(1/6)*A*b^4 - (a*b^5)^(5/6)*C)*log(x^2 - sqrt(3)*x*(a/b)^(1/6) + (a/b)^(1/
3))/(a*b^5)
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 2594, normalized size of antiderivative = 10.76

$$\int \frac{A+Bx^2+Cx^4}{a+bx^6}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^6),x)
```

output
```
symsum(log(A^3*B^2*b^4 - B^5*a*b^3 - A^4*C*b^4 + B^2*C^3*a^2*b^2 - 72*B^3*
root(46656*a^5*b^5*z^6 + 7776*A*C*a^4*b^4*z^4 + 3888*B^2*a^4*b^4*z^4 + 324
*A^2*C^2*a^3*b^3*z^2 + 216*B*C^3*a^4*b^2*z^2 + 216*A^3*B*a^2*b^4*z^2 + 108
*B^4*a^3*b^3*z^2 + 9*A^2*B^2*C^2*a^2*b^2 + 6*A^4*B*C*a*b^3 + 6*A*B*C^4*a^3
*b - 6*A*B^4*C*a^2*b^2 - 2*B^3*C^3*a^3*b - 2*A^3*B^3*a*b^3 + 2*A^3*C^3*a^2
*b^2 + B^6*a^2*b^2 + C^6*a^4 + A^6*b^4, z, k)^2*a^2*b^4 - 36*C^3*root(4665
6*a^5*b^5*z^6 + 7776*A*C*a^4*b^4*z^4 + 3888*B^2*a^4*b^4*z^4 + 324*A^2*C^2*
a^3*b^3*z^2 + 216*B*C^3*a^4*b^2*z^2 + 216*A^3*B*a^2*b^4*z^2 + 108*B^4*a^3*
b^3*z^2 + 9*A^2*B^2*C^2*a^2*b^2 + 6*A^4*B*C*a*b^3 + 6*A*B*C^4*a^3*b - 6*A*
B^4*C*a^2*b^2 - 2*B^3*C^3*a^3*b - 2*A^3*B^3*a*b^3 + 2*A^3*C^3*a^2*b^2 + B^
6*a^2*b^2 + C^6*a^4 + A^6*b^4, z, k)^2*a^3*b^3 - 6*A^4*root(46656*a^5*b^5*
z^6 + 7776*A*C*a^4*b^4*z^4 + 3888*B^2*a^4*b^4*z^4 + 324*A^2*C^2*a^3*b^3*z^
2 + 216*B*C^3*a^4*b^2*z^2 + 216*A^3*B*a^2*b^4*z^2 + 108*B^4*a^3*b^3*z^2 +
9*A^2*B^2*C^2*a^2*b^2 + 6*A^4*B*C*a*b^3 + 6*A*B*C^4*a^3*b - 6*A*B^4*C*a^2*
b^2 - 2*B^3*C^3*a^3*b - 2*A^3*B^3*a*b^3 + 2*A^3*C^3*a^2*b^2 + B^6*a^2*b^2
+ C^6*a^4 + A^6*b^4, z, k)*b^5*x - A*C^4*a^2*b^2 - 36*A^3*root(46656*a^5*b
^5*z^6 + 7776*A*C*a^4*b^4*z^4 + 3888*B^2*a^4*b^4*z^4 + 324*A^2*C^2*a^3*b^3
*z^2 + 216*B*C^3*a^4*b^2*z^2 + 216*A^3*B*a^2*b^4*z^2 + 108*B^4*a^3*b^3*z^2
 + 9*A^2*B^2*C^2*a^2*b^2 + 6*A^4*B*C*a*b^3 + 6*A*B*C^4*a^3*b - 6*A*B^4*C*a
^2*b^2 - 2*B^3*C^3*a^3*b - 2*A^3*B^3*a*b^3 + 2*A^3*C^3*a^2*b^2 + B^6*a^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.50

$$\int \frac{A + Bx^2 + Cx^4}{a + bx^6}\,dx$$

$$= \frac{-2b^{\frac{2}{3}}a^{\frac{2}{3}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2b^{\frac{1}{3}}x}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right) - 2a^{\frac{1}{3}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2b^{\frac{1}{3}}x}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)c - 4b^{\frac{4}{3}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2b^{\frac{1}{3}}x}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right) + 2b^{\frac{2}{3}}a^{\frac{2}{3}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}{b^{\frac{1}{6}}a}\right.}{}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^6+a),x)
```

output
```
(b**(1/6)*a**(1/6)*( - 2*b**(2/3)*a**(2/3)*atan((b**(1/6)*a**(1/6)*sqrt(3)
 - 2*b**(1/3)*x)/(b**(1/6)*a**(1/6))) - 2*a**(1/3)*atan((b**(1/6)*a**(1/6)
*sqrt(3) - 2*b**(1/3)*x)/(b**(1/6)*a**(1/6)))*c - 4*b**(1/3)*atan((b**(1/6
)*a**(1/6)*sqrt(3) - 2*b**(1/3)*x)/(b**(1/6)*a**(1/6)))*b + 2*b**(2/3)*a**
(2/3)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*b**(1/3)*x)/(b**(1/6)*a**(1/6)))
 + 2*a**(1/3)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*b**(1/3)*x)/(b**(1/6)*a*
*(1/6)))*c + 4*b**(1/3)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*b**(1/3)*x)/(b
**(1/6)*a**(1/6)))*b + 4*b**(2/3)*a**(2/3)*atan((b**(1/3)*x)/(b**(1/6)*a**
(1/6))) + 4*a**(1/3)*atan((b**(1/3)*x)/(b**(1/6)*a**(1/6)))*c - 4*b**(1/3)
*atan((b**(1/3)*x)/(b**(1/6)*a**(1/6)))*b - b**(2/3)*a**(2/3)*sqrt(3)*log(
 - b**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + b**(1/3)*x**2) + b**(2/3)*a**(
2/3)*sqrt(3)*log(b**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + b**(1/3)*x**2) +
 a**(1/3)*sqrt(3)*log( - b**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + b**(1/3)
*x**2)*c - a**(1/3)*sqrt(3)*log(b**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + b
**(1/3)*x**2)*c))/(12*a**(2/3)*b)
```

3.6 $\int (a+cx^6)^3 (A+Bx^3+Cx^6)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int (a+cx^6)^3 (A+Bx^3+Cx^6)\, dx = a^3Ax+\frac{1}{4}a^3Bx^4+\frac{1}{7}a^2(3Ac+aC)x^7+\frac{3}{10}a^2Bcx^{10}+\frac{3}{13}ac(Ac+aC)x^{13}+\frac{3}{16}aBc^2x^{16}+\frac{1}{19}c^2(Ac+3aC)x^{19}+\frac{1}{22}Bc^3x^{22}+\frac{1}{25}c^3Cx^{25}$$

output
```
a^3*A*x+1/4*a^3*B*x^4+1/7*a^2*(3*A*c+C*a)*x^7+3/10*a^2*B*c*x^10+3/13*a*c*(
A*c+C*a)*x^13+3/16*a*B*c^2*x^16+1/19*c^2*(A*c+3*C*a)*x^19+1/22*B*c^3*x^22+
1/25*c^3*C*x^25
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int (a+cx^6)^3 (A+Bx^3+Cx^6)\, dx = a^3Ax+\frac{1}{4}a^3Bx^4+\frac{1}{7}a^2(3Ac+aC)x^7+\frac{3}{10}a^2Bcx^{10}+\frac{3}{13}ac(Ac+aC)x^{13}+\frac{3}{16}aBc^2x^{16}+\frac{1}{19}c^2(Ac+3aC)x^{19}+\frac{1}{22}Bc^3x^{22}+\frac{1}{25}c^3Cx^{25}$$

input
```
Integrate[(a + c*x^6)^3*(A + B*x^3 + C*x^6),x]
```

output
```
a^3*A*x + (a^3*B*x^4)/4 + (a^2*(3*A*c + a*C)*x^7)/7 + (3*a^2*B*c*x^10)/10
+ (3*a*c*(A*c + a*C)*x^13)/13 + (3*a*B*c^2*x^16)/16 + (c^2*(A*c + 3*a*C)*x
^19)/19 + (B*c^3*x^22)/22 + (c^3*C*x^25)/25
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^6\right)^3 \left(A+Bx^3+Cx^6\right) \, dx$$

$$\Big\downarrow \; 2308$$

$$\int \left(a^3A+a^3Bx^3+a^2x^6(aC+3Ac)+3a^2Bcx^9+c^2x^{18}(3aC+Ac)+3acx^{12}(aC+Ac)+3aBc^2x^{15}+Bc^3x^{21}\right.$$

$$\Big\downarrow \; 2009$$

$$a^3Ax+\frac{1}{4}a^3Bx^4+\frac{1}{7}a^2x^7(aC+3Ac)+\frac{3}{10}a^2Bcx^{10}+\frac{1}{19}c^2x^{19}(3aC+Ac)+\frac{3}{13}acx^{13}(aC+Ac)+\frac{3}{16}aBc^2x^{16}+\frac{1}{22}Bc^3x^{22}+\frac{1}{25}c^3Cx^{25}$$

input
```
Int[(a + c*x^6)^3*(A + B*x^3 + C*x^6),x]
```

output
```
a^3*A*x + (a^3*B*x^4)/4 + (a^2*(3*A*c + a*C)*x^7)/7 + (3*a^2*B*c*x^10)/10
+ (3*a*c*(A*c + a*C)*x^13)/13 + (3*a*B*c^2*x^16)/16 + (c^2*(A*c + 3*a*C)*x
^19)/19 + (B*c^3*x^22)/22 + (c^3*C*x^25)/25
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95

method	result
norman	$\frac{c^3 C x^{25}}{25}+\left(\frac{1}{19} A c^3+\frac{3}{19} a c^2 C\right) x^{19}+\frac{B c^3 x^{22}}{22}+\frac{3 a B c^2 x^{16}}{16}+\left(\frac{3}{13} A a c^2+\frac{3}{13} a^2 c C\right) x^{13}+\frac{a^3 B x^4}{4}+$
default	$\frac{c^3 C x^{25}}{25}+\frac{B c^3 x^{22}}{22}+\frac{(A c^3+3 a c^2 C) x^{19}}{19}+\frac{3 a B c^2 x^{16}}{16}+\frac{(3 A a c^2+3 a^2 c C) x^{13}}{13}+\frac{3 a^2 B c x^{10}}{10}+\frac{(3 a^2 A c+a^3 C) x^7}{7}+$
gosper	$\frac{1}{25} c^3 C x^{25}+\frac{1}{19} x^{19} A c^3+\frac{3}{19} x^{19} a c^2 C+\frac{1}{22} B c^3 x^{22}+\frac{3}{16} a B c^2 x^{16}+\frac{3}{13} x^{13} A a c^2+\frac{3}{13} x^{13} a^2 c C$
risch	$\frac{1}{25} c^3 C x^{25}+\frac{1}{19} x^{19} A c^3+\frac{3}{19} x^{19} a c^2 C+\frac{1}{22} B c^3 x^{22}+\frac{3}{16} a B c^2 x^{16}+\frac{3}{13} x^{13} A a c^2+\frac{3}{13} x^{13} a^2 c C$
parallelrisch	$\frac{1}{25} c^3 C x^{25}+\frac{1}{19} x^{19} A c^3+\frac{3}{19} x^{19} a c^2 C+\frac{1}{22} B c^3 x^{22}+\frac{3}{16} a B c^2 x^{16}+\frac{3}{13} x^{13} A a c^2+\frac{3}{13} x^{13} a^2 c C$
orering	$\frac{x(304304 c^3 C x^{24}+345800 c^3 B x^{21}+400400 A c^3 x^{18}+1201200 C a c^2 x^{18}+1426425 a c^2 B x^{15}+1755600 A a c^2 x^{12}+1755600 C a^2}{7607600}$

input
```
int((c*x^6+a)^3*(C*x^6+B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/25*c^3*C*x^25+(1/19*A*c^3+3/19*a*c^2*C)*x^19+1/22*B*c^3*x^22+3/16*a*B*c^
2*x^16+(3/13*A*a*c^2+3/13*a^2*c*C)*x^13+1/4*a^3*B*x^4+(3/7*a^2*A*c+1/7*a^3
*C)*x^7+3/10*a^2*B*c*x^10+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \left(a+cx^6\right)^3 \left(A+Bx^3+Cx^6\right)\,dx = \frac{1}{25}\,Cc^3x^{25}+\frac{1}{22}\,Bc^3x^{22}+\frac{3}{16}\,Bac^2x^{16}+\frac{1}{19}\,\left(3\,Cac^2+Ac^3\right)x^{19}+\frac{3}{10}\,Ba^2cx^{10}+\frac{3}{13}\,\left(Ca^2c+Aac^2\right)x^{13}+\frac{1}{4}\,Ba^3x^4+\frac{1}{7}\,\left(Ca^3+3\,Aa^2c\right)x^7+Aa^3x$$

input
```
integrate((c*x^6+a)^3*(C*x^6+B*x^3+A),x, algorithm="fricas")
```

output
```
1/25*C*c^3*x^25 + 1/22*B*c^3*x^22 + 3/16*B*a*c^2*x^16 + 1/19*(3*C*a*c^2 +
A*c^3)*x^19 + 3/10*B*a^2*c*x^10 + 3/13*(C*a^2*c + A*a*c^2)*x^13 + 1/4*B*a^
3*x^4 + 1/7*(C*a^3 + 3*A*a^2*c)*x^7 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int \left(a+cx^6\right)^3 \left(A+Bx^3+Cx^6\right)\,dx = Aa^3x+\frac{Ba^3x^4}{4}+\frac{3Ba^2cx^{10}}{10}+\frac{3Bac^2x^{16}}{16}+\frac{Bc^3x^{22}}{22}+\frac{Cc^3x^{25}}{25}+x^{19}\left(\frac{Ac^3}{19}+\frac{3Cac^2}{19}\right)+x^{13}\cdot\left(\frac{3Aac^2}{13}+\frac{3Ca^2c}{13}\right)+x^7\cdot\left(\frac{3Aa^2c}{7}+\frac{Ca^3}{7}\right)$$

input
```
integrate((c*x**6+a)**3*(C*x**6+B*x**3+A),x)
```

output
```
A*a**3*x + B*a**3*x**4/4 + 3*B*a**2*c*x**10/10 + 3*B*a*c**2*x**16/16 + B*c
**3*x**22/22 + C*c**3*x**25/25 + x**19*(A*c**3/19 + 3*C*a*c**2/19) + x**13
*(3*A*a*c**2/13 + 3*C*a**2*c/13) + x**7*(3*A*a**2*c/7 + C*a**3/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int (a+cx^6)^3 (A+Bx^3+Cx^6)\,dx = \frac{1}{25}Cc^3x^{25} + \frac{1}{22}Bc^3x^{22} + \frac{3}{16}Bac^2x^{16} + \frac{1}{19}(3Cac^2+Ac^3)x^{19} + \frac{3}{10}Ba^2cx^{10} + \frac{3}{13}(Ca^2c+Aac^2)x^{13} + \frac{1}{4}Ba^3x^4 + \frac{1}{7}(Ca^3+3Aa^2c)x^7 + Aa^3x$$

input
```
integrate((c*x^6+a)^3*(C*x^6+B*x^3+A),x, algorithm="maxima")
```

output
```
1/25*C*c^3*x^25 + 1/22*B*c^3*x^22 + 3/16*B*a*c^2*x^16 + 1/19*(3*C*a*c^2 +
A*c^3)*x^19 + 3/10*B*a^2*c*x^10 + 3/13*(C*a^2*c + A*a*c^2)*x^13 + 1/4*B*a^
3*x^4 + 1/7*(C*a^3 + 3*A*a^2*c)*x^7 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int (a+cx^6)^3 (A+Bx^3+Cx^6)\,dx = \frac{1}{25}Cc^3x^{25} + \frac{1}{22}Bc^3x^{22} + \frac{3}{19}Cac^2x^{19} + \frac{1}{19}Ac^3x^{19} + \frac{3}{16}Bac^2x^{16} + \frac{3}{13}Ca^2cx^{13} + \frac{3}{13}Aac^2x^{13} + \frac{3}{10}Ba^2cx^{10} + \frac{1}{7}Ca^3x^7 + \frac{3}{7}Aa^2cx^7 + \frac{1}{4}Ba^3x^4 + Aa^3x$$

input
```
integrate((c*x^6+a)^3*(C*x^6+B*x^3+A),x, algorithm="giac")
```

output
```
1/25*C*c^3*x^25 + 1/22*B*c^3*x^22 + 3/19*C*a*c^2*x^19 + 1/19*A*c^3*x^19 +
3/16*B*a*c^2*x^16 + 3/13*C*a^2*c*x^13 + 3/13*A*a*c^2*x^13 + 3/10*B*a^2*c*x
^10 + 1/7*C*a^3*x^7 + 3/7*A*a^2*c*x^7 + 1/4*B*a^3*x^4 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 7.67 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (a+cx^6)^3 (A+Bx^3+Cx^6)\,dx = x^7\left(\frac{C\,a^3}{7}+\frac{3\,A\,c\,a^2}{7}\right)+x^{19}\left(\frac{A\,c^3}{19}+\frac{3\,C\,a\,c^2}{19}\right)+\frac{B\,a^3\,x^4}{4}+\frac{B\,c^3\,x^{22}}{22}+\frac{C\,c^3\,x^{25}}{25}+A\,a^3\,x+\frac{3\,a\,c\,x^{13}\,(A\,c+C\,a)}{13}+\frac{3\,B\,a^2\,c\,x^{10}}{10}+\frac{3\,B\,a\,c^2\,x^{16}}{16}$$

input
```
int((a + c*x^6)^3*(A + B*x^3 + C*x^6),x)
```

output
```
x^7*((C*a^3)/7 + (3*A*a^2*c)/7) + x^19*((A*c^3)/19 + (3*C*a*c^2)/19) + (B*
a^3*x^4)/4 + (B*c^3*x^22)/22 + (C*c^3*x^25)/25 + A*a^3*x + (3*a*c*x^13*(A*
c + C*a))/13 + (3*B*a^2*c*x^10)/10 + (3*B*a*c^2*x^16)/16
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int (a+cx^6)^3 (A+Bx^3+Cx^6)\,dx = \frac{x(304304c^4x^{24}+345800b\,c^3x^{21}+1601600a\,c^3x^{18}+1426425ab\,c^2x^{15}+3511200a^2c^2x^{12}+2282280a^2bc\,x}{7607600}$$

input
```
int((c*x^6+a)^3*(C*x^6+B*x^3+A),x)
```

output
```
(x*(7607600*a**4 + 1901900*a**3*b*x**3 + 4347200*a**3*c*x**6 + 2282280*a**
2*b*c*x**9 + 3511200*a**2*c**2*x**12 + 1426425*a*b*c**2*x**15 + 1601600*a*
c**3*x**18 + 345800*b*c**3*x**21 + 304304*c**4*x**24))/7607600
```

3.7 $\int (a+cx^6)^2 (A+Bx^3+Cx^6)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int (a+cx^6)^2 (A+Bx^3+Cx^6)\, dx = a^2Ax+\frac{1}{4}a^2Bx^4+\frac{1}{7}a(2Ac+aC)x^7+\frac{1}{5}aBcx^{10}+\frac{1}{13}c(Ac+2aC)x^{13}+\frac{1}{16}Bc^2x^{16}+\frac{1}{19}c^2Cx^{19}$$

output
```
a^2*A*x+1/4*a^2*B*x^4+1/7*a*(2*A*c+C*a)*x^7+1/5*a*B*c*x^10+1/13*c*(A*c+2*C
*a)*x^13+1/16*B*c^2*x^16+1/19*c^2*C*x^19
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+cx^6)^2 (A+Bx^3+Cx^6)\, dx = a^2Ax+\frac{1}{4}a^2Bx^4+\frac{1}{7}a(2Ac+aC)x^7+\frac{1}{5}aBcx^{10}+\frac{1}{13}c(Ac+2aC)x^{13}+\frac{1}{16}Bc^2x^{16}+\frac{1}{19}c^2Cx^{19}$$

input
```
Integrate[(a + c*x^6)^2*(A + B*x^3 + C*x^6),x]
```

output
```
a^2*A*x + (a^2*B*x^4)/4 + (a*(2*A*c + a*C)*x^7)/7 + (a*B*c*x^10)/5 + (c*(A
*c + 2*a*C)*x^13)/13 + (B*c^2*x^16)/16 + (c^2*C*x^19)/19
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^6\right)^2 \left(A+Bx^3+Cx^6\right) \, dx$$

$$\Big\downarrow \text{ 2308}$$

$$\int \left(a^2A+a^2Bx^3+cx^{12}(2aC+Ac)+ax^6(aC+2Ac)+2aBcx^9+Bc^2x^{15}+c^2Cx^{18}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2Ax+\frac{1}{4}a^2Bx^4+\frac{1}{13}cx^{13}(2aC+Ac)+\frac{1}{7}ax^7(aC+2Ac)+\frac{1}{5}aBcx^{10}+\frac{1}{16}Bc^2x^{16}+\frac{1}{19}c^2Cx^{19}$$

input
```
Int[(a + c*x^6)^2*(A + B*x^3 + C*x^6),x]
```

output

```
a^2*A*x + (a^2*B*x^4)/4 + (a*(2*A*c + a*C)*x^7)/7 + (a*B*c*x^10)/5 + (c*(A
*c + 2*a*C)*x^13)/13 + (B*c^2*x^16)/16 + (c^2*C*x^19)/19
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

method	result
default	$\frac{c^2 C x^{19}}{19}+\frac{B c^2 x^{16}}{16}+\frac{(A c^2+2acC)x^{13}}{13}+\frac{aBc x^{10}}{5}+\frac{(2Aac+a^2C)x^7}{7}+\frac{a^2 B x^4}{4}+a^2 Ax$
norman	$a^2Ax+\frac{a^2 B x^4}{4}+\left(\frac{2}{7}Aac+\frac{1}{7}a^2C\right)x^7+\frac{aBc x^{10}}{5}+\left(\frac{1}{13}A c^2+\frac{2}{13}acC\right)x^{13}+\frac{B c^2 x^{16}}{16}+\frac{c^2 C x^{19}}{19}$
gosper	$a^2Ax+\frac{1}{4}a^2B x^4+\frac{2}{7}x^7Aac+\frac{1}{7}x^7a^2C+\frac{1}{5}aBc x^{10}+\frac{1}{13}x^{13}A c^2+\frac{2}{13}x^{13}acC+\frac{1}{16}B c^2x^{16}+$
risch	$a^2Ax+\frac{1}{4}a^2B x^4+\frac{2}{7}x^7Aac+\frac{1}{7}x^7a^2C+\frac{1}{5}aBc x^{10}+\frac{1}{13}x^{13}A c^2+\frac{2}{13}x^{13}acC+\frac{1}{16}B c^2x^{16}+$
parallelrisch	$a^2Ax+\frac{1}{4}a^2B x^4+\frac{2}{7}x^7Aac+\frac{1}{7}x^7a^2C+\frac{1}{5}aBc x^{10}+\frac{1}{13}x^{13}A c^2+\frac{2}{13}x^{13}acC+\frac{1}{16}B c^2x^{16}+$
orering	$\frac{x(7280c^2C x^{18}+8645c^2B x^{15}+10640A c^2x^{12}+21280Cac x^{12}+27664x^9aBc+39520Aac x^6+19760C a^2x^6+34580a^2B x^3+13}{138320}$

input
```
int((c*x^6+a)^2*(C*x^6+B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/19*c^2*C*x^19+1/16*B*c^2*x^16+1/13*(A*c^2+2*C*a*c)*x^13+1/5*a*B*c*x^10+1
/7*(2*A*a*c+C*a^2)*x^7+1/4*a^2*B*x^4+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int (a+cx^6)^2 (A+Bx^3+Cx^6)\,dx = \frac{1}{19}Cc^2x^{19}+\frac{1}{16}Bc^2x^{16}+\frac{1}{13}(2Cac+Ac^2)x^{13}+\frac{1}{5}Bacx^{10}+\frac{1}{7}(Ca^2+2Aac)x^7+\frac{1}{4}Ba^2x^4+Aa^2x$$

input
```
integrate((c*x^6+a)^2*(C*x^6+B*x^3+A),x, algorithm="fricas")
```

output
```
1/19*C*c^2*x^19 + 1/16*B*c^2*x^16 + 1/13*(2*C*a*c + A*c^2)*x^13 + 1/5*B*a*
c*x^10 + 1/7*(C*a^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \left(a+cx^6\right)^2 \left(A+Bx^3+Cx^6\right)\,dx = Aa^2x+\frac{Ba^2x^4}{4}+\frac{Bacx^{10}}{5}+\frac{Bc^2x^{16}}{16}+\frac{Cc^2x^{19}}{19}+x^{13}\left(\frac{Ac^2}{13}+\frac{2Cac}{13}\right)+x^7\cdot\left(\frac{2Aac}{7}+\frac{Ca^2}{7}\right)$$

input
```
integrate((c*x**6+a)**2*(C*x**6+B*x**3+A),x)
```

output
```
A*a**2*x + B*a**2*x**4/4 + B*a*c*x**10/5 + B*c**2*x**16/16 + C*c**2*x**19/
19 + x**13*(A*c**2/13 + 2*C*a*c/13) + x**7*(2*A*a*c/7 + C*a**2/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int \left(a+cx^6\right)^2 \left(A+Bx^3+Cx^6\right)\,dx = \frac{1}{19}\,Cc^2x^{19}+\frac{1}{16}\,Bc^2x^{16}+\frac{1}{13}\,(2\,Cac+Ac^2)x^{13}+\frac{1}{5}\,Bacx^{10}+\frac{1}{7}\,(Ca^2+2\,Aac)x^7+\frac{1}{4}\,Ba^2x^4+Aa^2x$$

input
```
integrate((c*x^6+a)^2*(C*x^6+B*x^3+A),x, algorithm="maxima")
```

output
```
1/19*C*c^2*x^19 + 1/16*B*c^2*x^16 + 1/13*(2*C*a*c + A*c^2)*x^13 + 1/5*B*a*
c*x^10 + 1/7*(C*a^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int (a+cx^6)^2 (A+Bx^3+Cx^6) \, dx = \frac{1}{19} Cc^2x^{19} + \frac{1}{16} Bc^2x^{16} + \frac{2}{13} Cacx^{13} + \frac{1}{13} Ac^2x^{13} + \frac{1}{5} Bacx^{10} + \frac{1}{7} Ca^2x^7 + \frac{2}{7} Aacx^7 + \frac{1}{4} Ba^2x^4 + Aa^2x$$

input
```
integrate((c*x^6+a)^2*(C*x^6+B*x^3+A),x, algorithm="giac")
```

output
```
1/19*C*c^2*x^19 + 1/16*B*c^2*x^16 + 2/13*C*a*c*x^13 + 1/13*A*c^2*x^13 + 1/
5*B*a*c*x^10 + 1/7*C*a^2*x^7 + 2/7*A*a*c*x^7 + 1/4*B*a^2*x^4 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int (a+cx^6)^2 (A+Bx^3+Cx^6) \, dx = x^7 \left(\frac{C a^2}{7} + \frac{2 A c a}{7}\right) + x^{13} \left(\frac{A c^2}{13} + \frac{2 C a c}{13}\right) + \frac{B a^2 x^4}{4} + \frac{B c^2 x^{16}}{16} + \frac{C c^2 x^{19}}{19} + A a^2 x + \frac{B a c x^{10}}{5}$$

input
```
int((a + c*x^6)^2*(A + B*x^3 + C*x^6),x)
```

output
```
x^7*((C*a^2)/7 + (2*A*a*c)/7) + x^13*((A*c^2)/13 + (2*C*a*c)/13) + (B*a^2*
x^4)/4 + (B*c^2*x^16)/16 + (C*c^2*x^19)/19 + A*a^2*x + (B*a*c*x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int (a + cx^6)^2 (A + Bx^3 + Cx^6) \, dx$$
$$= \frac{x(7280c^3x^{18} + 8645b\,c^2x^{15} + 31920a\,c^2x^{12} + 27664abc\,x^9 + 59280a^2c\,x^6 + 34580a^2b\,x^3 + 138320a^3)}{138320}$$

input
```
int((c*x^6+a)^2*(C*x^6+B*x^3+A),x)
```

output
```
(x*(138320*a**3 + 34580*a**2*b*x**3 + 59280*a**2*c*x**6 + 27664*a*b*c*x**9
 + 31920*a*c**2*x**12 + 8645*b*c**2*x**15 + 7280*c**3*x**18))/138320
```

3.8 $\int (a+cx^6)(A+Bx^3+Cx^6)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 46

$$\int (a+cx^6)\left(A+Bx^3+Cx^6\right)\,dx = aAx+\frac{1}{4}aBx^4+\frac{1}{7}(Ac+aC)x^7+\frac{1}{10}Bcx^{10}+\frac{1}{13}cCx^{13}$$

output
```
a*A*x+1/4*a*B*x^4+1/7*(A*c+C*a)*x^7+1/10*B*c*x^10+1/13*c*C*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+cx^6)\left(A+Bx^3+Cx^6\right)\,dx = aAx+\frac{1}{4}aBx^4+\frac{1}{7}(Ac+aC)x^7+\frac{1}{10}Bcx^{10}+\frac{1}{13}cCx^{13}$$

input
```
Integrate[(a + c*x^6)*(A + B*x^3 + C*x^6),x]
```

output
```
a*A*x + (a*B*x^4)/4 + ((A*c + a*C)*x^7)/7 + (B*c*x^10)/10 + (c*C*x^13)/13
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^6\right) \left(A+Bx^3+Cx^6\right) \, dx$$

$$\Big\downarrow \text{ 2308}$$

$$\int \left(x^6(aC+Ac)+aA+aBx^3+Bcx^9+cCx^{12}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}x^7(aC+Ac)+aAx+\frac{1}{4}aBx^4+\frac{1}{10}Bcx^{10}+\frac{1}{13}cCx^{13}$$

input
```
Int[(a + c*x^6)*(A + B*x^3 + C*x^6),x]
```

output
```
a*A*x + (a*B*x^4)/4 + ((A*c + a*C)*x^7)/7 + (B*c*x^10)/10 + (c*C*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^{4}}{4}+\frac{\left(Ac+Ca\right)x^{7}}{7}+\frac{Bc\,x^{10}}{10}+\frac{cC\,x^{13}}{13}$	39
norman	$\frac{cC\,x^{13}}{13}+\frac{Bc\,x^{10}}{10}+\left(\frac{Ac}{7}+\frac{Ca}{7}\right)x^{7}+\frac{Ba\,x^{4}}{4}+aAx$	40
gosper	$\frac{1}{13}cC\,x^{13}+\frac{1}{10}Bc\,x^{10}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ca+\frac{1}{4}Ba\,x^{4}+aAx$	41
risch	$\frac{1}{13}cC\,x^{13}+\frac{1}{10}Bc\,x^{10}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ca+\frac{1}{4}Ba\,x^{4}+aAx$	41
parallelrisch	$\frac{1}{13}cC\,x^{13}+\frac{1}{10}Bc\,x^{10}+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ca+\frac{1}{4}Ba\,x^{4}+aAx$	41
orering	$\frac{x\left(140Cc\,x^{12}+182x^{9}Bc+260Ac\,x^{6}+260Ca\,x^{6}+455Ba\,x^{3}+1820Aa\right)}{1820}$	44

input
```
int((c*x^6+a)*(C*x^6+B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/4*B*a*x^4+1/7*(A*c+C*a)*x^7+1/10*B*c*x^10+1/13*c*C*x^13
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+cx^6\right)\left(A+Bx^3+Cx^6\right)\,dx=\frac{1}{13}\,Ccx^{13}+\frac{1}{10}\,Bcx^{10}+\frac{1}{7}\,(Ca+Ac)x^7+\frac{1}{4}\,Bax^4+Aax$$

input
```
integrate((c*x^6+a)*(C*x^6+B*x^3+A),x, algorithm="fricas")
```

output
```
1/13*C*c*x^13 + 1/10*B*c*x^10 + 1/7*(C*a + A*c)*x^7 + 1/4*B*a*x^4 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int (a+cx^6)(A+Bx^3+Cx^6)\,dx = Aax + \frac{Bax^4}{4} + \frac{Bcx^{10}}{10} + \frac{Ccx^{13}}{13} + x^7\left(\frac{Ac}{7} + \frac{Ca}{7}\right)$$

input
```
integrate((c*x**6+a)*(C*x**6+B*x**3+A),x)
```

output
```
A*a*x + B*a*x**4/4 + B*c*x**10/10 + C*c*x**13/13 + x**7*(A*c/7 + C*a/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+cx^6)(A+Bx^3+Cx^6)\,dx = \frac{1}{13}Ccx^{13} + \frac{1}{10}Bcx^{10} + \frac{1}{7}(Ca+Ac)x^7 + \frac{1}{4}Bax^4 + Aax$$

input
```
integrate((c*x^6+a)*(C*x^6+B*x^3+A),x, algorithm="maxima")
```

output
```
1/13*C*c*x^13 + 1/10*B*c*x^10 + 1/7*(C*a + A*c)*x^7 + 1/4*B*a*x^4 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int (a+cx^6)(A+Bx^3+Cx^6)\,dx = \frac{1}{13}Ccx^{13} + \frac{1}{10}Bcx^{10} + \frac{1}{7}Cax^7 + \frac{1}{7}Acx^7 + \frac{1}{4}Bax^4 + Aax$$

input
```
integrate((c*x^6+a)*(C*x^6+B*x^3+A),x, algorithm="giac")
```

output
```
1/13*C*c*x^13 + 1/10*B*c*x^10 + 1/7*C*a*x^7 + 1/7*A*c*x^7 + 1/4*B*a*x^4 +
A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int (a+cx^6) (A+Bx^3+Cx^6) \, dx = \frac{C\,c\,x^{13}}{13} + \frac{B\,c\,x^{10}}{10} + \left(\frac{A\,c}{7} + \frac{C\,a}{7}\right)\,x^7 + \frac{B\,a\,x^4}{4} + A\,a\,x$$

input
```
int((a + c*x^6)*(A + B*x^3 + C*x^6),x)
```

output
```
x^7*((A*c)/7 + (C*a)/7) + A*a*x + (B*a*x^4)/4 + (B*c*x^10)/10 + (C*c*x^13)
/13
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int (a+cx^6) (A+Bx^3+Cx^6) \, dx \\ = \frac{x(140c^2x^{12} + 182bc\,x^9 + 520ac\,x^6 + 455ab\,x^3 + 1820a^2)}{1820}$$

input
```
int((c*x^6+a)*(C*x^6+B*x^3+A),x)
```

output
```
(x*(1820*a**2 + 455*a*b*x**3 + 520*a*c*x**6 + 182*b*c*x**9 + 140*c**2*x**1
2))/1820
```

3.9 $\int \frac{A+Bx^3+Cx^6}{a+cx^6} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 343

$$\begin{aligned}\int \frac{A+Bx^3+Cx^6}{a+cx^6} \, dx &= \frac{Cx}{c}+\frac{(Ac-aC)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}c^{7/6}} \\ &\quad -\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}+Ac-aC\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{7/6}} \\ &\quad -\frac{\left(\sqrt{3}\sqrt{a}B\sqrt{c}-Ac+aC\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{7/6}} \\ &\quad -\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}} \\ &\quad +\frac{\left(\sqrt{a}B\sqrt{c}-\sqrt{3}Ac+\sqrt{3}aC\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{7/6}} \\ &\quad +\frac{\left(B\sqrt{c}+\frac{\sqrt{3}(Ac-aC)}{\sqrt{a}}\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12\sqrt[3]{a}c^{7/6}}\end{aligned}$$

output
```
C*x/c+1/3*(A*c-C*a)*arctan(c^(1/6)*x/a^(1/6))/a^(5/6)/c^(7/6)+1/6*(3^(1/2)
*a^(1/2)*B*c^(1/2)+A*c-C*a)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(5/6)/c
^(7/6)-1/6*(3^(1/2)*a^(1/2)*B*c^(1/2)-A*c+C*a)*arctan(3^(1/2)+2*c^(1/6)*x/
a^(1/6))/a^(5/6)/c^(7/6)-1/6*B*ln(a^(1/3)+c^(1/3)*x^2)/a^(1/3)/c^(2/3)+1/1
2*(a^(1/2)*B*c^(1/2)-3^(1/2)*A*c+3^(1/2)*a*C)*ln(a^(1/3)-3^(1/2)*a^(1/6)*c
^(1/6)*x+c^(1/3)*x^2)/a^(5/6)/c^(7/6)+1/12*(B*c^(1/2)+3^(1/2)*(A*c-C*a)/a^
(1/2))*ln(a^(1/3)+3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(1/3)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^3+Cx^6}{a+cx^6}\,dx = \frac{12a^{5/6}\sqrt[6]{c}Cx+4(Ac-aC)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)-2\left(\sqrt{3}\sqrt{a}B\sqrt{c}+Ac-aC\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)-2\left(\sqrt{3}\right.}{}$$

input
```
Integrate[(A + B*x^3 + C*x^6)/(a + c*x^6),x]
```

output
```
(12*a^(5/6)*c^(1/6)*C*x + 4*(A*c - a*C)*ArcTan[(c^(1/6)*x)/a^(1/6)] - 2*(S
qrt[3]*Sqrt[a]*B*Sqrt[c] + A*c - a*C)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/
6)] - 2*(Sqrt[3]*Sqrt[a]*B*Sqrt[c] - A*c + a*C)*ArcTan[Sqrt[3] + (2*c^(1/6
)*x)/a^(1/6)] - 2*Sqrt[a]*B*Sqrt[c]*Log[a^(1/3) + c^(1/3)*x^2] + (Sqrt[a]*
B*Sqrt[c] - Sqrt[3]*A*c + Sqrt[3]*a*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/
6)*x + c^(1/3)*x^2] + (Sqrt[a]*B*Sqrt[c] + Sqrt[3]*A*c - Sqrt[3]*a*C)*Log[
a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(7/6))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3+Cx^6}{a+cx^6}\,dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{-aC+Ac+Bcx^3}{c\,(a+cx^6)}+\frac{C}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)\left(\sqrt{3}\sqrt{a}B\sqrt{c}-aC+Ac\right)}{6a^{5/6}c^{7/6}}-\frac{\arctan\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+\sqrt{3}\right)\left(\sqrt{3}\sqrt{a}B\sqrt{c}+aC-Ac\right)}{6a^{5/6}c^{7/6}}+\frac{(Ac-aC)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}c^{7/6}}+\frac{\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)\left(\sqrt{a}B\sqrt{c}+\sqrt{3}aC-\sqrt{3}Ac\right)}{12a^{5/6}c^{7/6}}+\frac{\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)\left(\frac{\sqrt{3}(Ac-aC)}{\sqrt{a}}+B\sqrt{c}\right)}{12\sqrt[3]{a}c^{7/6}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{6\sqrt[3]{a}c^{2/3}}+\frac{Cx}{c}$$

input
```
Int[(A + B*x^3 + C*x^6)/(a + c*x^6),x]
```

output
```
(C*x)/c + ((A*c - a*C)*ArcTan[(c^(1/6)*x)/a^(1/6)])/(3*a^(5/6)*c^(7/6)) -
((Sqrt[3]*Sqrt[a]*B*Sqrt[c] + A*c - a*C)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^
(1/6)])/(6*a^(5/6)*c^(7/6)) - ((Sqrt[3]*Sqrt[a]*B*Sqrt[c] - A*c + a*C)*Arc
Tan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)])/(6*a^(5/6)*c^(7/6)) - (B*Log[a^(1/3)
 + c^(1/3)*x^2])/(6*a^(1/3)*c^(2/3)) + ((Sqrt[a]*B*Sqrt[c] - Sqrt[3]*A*c +
 Sqrt[3]*a*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*
a^(5/6)*c^(7/6)) + ((B*Sqrt[c] + (Sqrt[3]*(A*c - a*C))/Sqrt[a])*Log[a^(1/3
) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(1/3)*c^(7/6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.14

method	result
risch	$\frac{Cx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+a\right)}\frac{\left(B\_R^{3}c+Ac-Ca\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c^{2}}$
default	$\frac{Cx}{c}+ -\frac{c\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)A\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{12a}+\frac{c\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)B\left(\frac{a}{c}\right)^{\frac{2}{3}}}{12a}+\frac{\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)C\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{12}+\frac{c(}{}$

input
```
int((C*x^6+B*x^3+A)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
C*x/c+1/6/c^2*sum((B*_R^3*c+A*c-C*a)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5351 vs. 2(244) = 488.

Time = 6.25 (sec) , antiderivative size = 5351, normalized size of antiderivative = 15.60

$$\int \frac{A+Bx^3+Cx^6}{a+cx^6}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3+Cx^6}{a+cx^6}\,dx = \text{Timed out}$$

input
```
integrate((C*x**6+B*x**3+A)/(c*x**6+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3+Cx^6}{a+cx^6}\,dx = \frac{Cx}{c} - \frac{\frac{2Bc^{\frac{1}{3}}\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}} + \frac{4\left(Cac^{\frac{1}{3}}-Ac^{\frac{4}{3}}\right)\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}c^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} + \frac{\left(\sqrt{3}Ca^{\frac{7}{6}}\sqrt{c}-\left(\sqrt{3}Aa^{\frac{1}{6}}\sqrt{c}+Ba^{\frac{2}{3}}\right)c\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}} - \cdots}{\cdots}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a),x, algorithm="maxima")
```

output
```
C*x/c - 1/12*(2*B*c^(1/3)*log(c^(1/3)*x^2 + a^(1/3))/a^(1/3) + 4*(C*a*c^(1
/3) - A*c^(4/3))*arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c^(1/3)*
sqrt(a^(1/3)*c^(1/3))) + (sqrt(3)*C*a^(7/6)*sqrt(c) - (sqrt(3)*A*a^(1/6)*s
qrt(c) + B*a^(2/3))*c)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/
3))/(a*c^(2/3)) - (sqrt(3)*C*a^(7/6)*sqrt(c) - (sqrt(3)*A*a^(1/6)*sqrt(c)
- B*a^(2/3))*c)*log(c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*
c^(2/3)) + 2*(C*a^(4/3)*c^(2/3) + (sqrt(3)*B*a^(5/6)*c^(1/6) - A*a^(1/3)*c
^(2/3))*c)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(
1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) + 2*(C*a^(4/3)*c^(2/3) - (sqrt(3)
*B*a^(5/6)*c^(1/6) + A*a^(1/3)*c^(2/3))*c)*arctan((2*c^(1/3)*x - sqrt(3)*a
^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/
c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3+Cx^6}{a+cx^6}\,dx$$
$$= -\frac{B|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\,(ac^5)^{\frac{1}{3}}} + \frac{Cx}{c} - \frac{\left((ac^5)^{\frac{1}{6}}\,Ca-(ac^5)^{\frac{1}{6}}\,Ac\right)\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac^2}$$
$$-\frac{\left((ac^5)^{\frac{1}{6}}\,Cac^2-(ac^5)^{\frac{1}{6}}\,Ac^3+\sqrt{3}(ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}$$
$$-\frac{\left((ac^5)^{\frac{1}{6}}\,Cac^2-(ac^5)^{\frac{1}{6}}\,Ac^3-\sqrt{3}(ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}$$
$$-\frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,Cac^2-\sqrt{3}(ac^5)^{\frac{1}{6}}\,Ac^3-(ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$
$$+\frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,Cac^2-\sqrt{3}(ac^5)^{\frac{1}{6}}\,Ac^3+(ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a),x, algorithm="giac")
```

output
```
-1/6*B*abs(c)*log(x^2 + (a/c)^(1/3))/(a*c^5)^(1/3) + C*x/c - 1/3*((a*c^5)^
(1/6)*C*a - (a*c^5)^(1/6)*A*c)*arctan(x/(a/c)^(1/6))/(a*c^2) - 1/6*((a*c^5
)^(1/6)*C*a*c^2 - (a*c^5)^(1/6)*A*c^3 + sqrt(3)*(a*c^5)^(2/3)*B)*arctan((2
*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/6*((a*c^5)^(1/6)*C*a*c^
2 - (a*c^5)^(1/6)*A*c^3 - sqrt(3)*(a*c^5)^(2/3)*B)*arctan((2*x - sqrt(3)*(
a/c)^(1/6))/(a/c)^(1/6))/(a*c^4) - 1/12*(sqrt(3)*(a*c^5)^(1/6)*C*a*c^2 - s
qrt(3)*(a*c^5)^(1/6)*A*c^3 - (a*c^5)^(2/3)*B)*log(x^2 + sqrt(3)*x*(a/c)^(1
/6) + (a/c)^(1/3))/(a*c^4) + 1/12*(sqrt(3)*(a*c^5)^(1/6)*C*a*c^2 - sqrt(3)
*(a*c^5)^(1/6)*A*c^3 + (a*c^5)^(2/3)*B)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) +
(a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 3096, normalized size of antiderivative = 9.03

$$\int \frac{A + Bx^3 + Cx^6}{a + cx^6} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^3 + C*x^6)/(a + c*x^6),x)
```

output
```
log(B*x*(-a^5*c^7)^(1/2) + a^3*c^4*(-(A^3*c^3*(-a^5*c^7)^(1/2) - C^3*a^3*(
-a^5*c^7)^(1/2) + B^3*a^4*c^5 - 3*A^2*B*a^3*c^6 - 3*B*C^2*a^5*c^4 - 3*A*B^
2*a*c^2*(-a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(-a^5*c^7)^(1/2) - 3*A^2*C*a*c^2*
(-a^5*c^7)^(1/2) + 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*(-a^5*c^7)^(1/2))/(a^5*
c^7))^(1/3) + A*a^2*c^4*x - C*a^3*c^3*x)*(-(A^3*c^3*(-a^5*c^7)^(1/2) - C^3
*a^3*(-a^5*c^7)^(1/2) + B^3*a^4*c^5 - 3*A^2*B*a^3*c^6 - 3*B*C^2*a^5*c^4 -
3*A*B^2*a*c^2*(-a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(-a^5*c^7)^(1/2) - 3*A^2*C*
a*c^2*(-a^5*c^7)^(1/2) + 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*(-a^5*c^7)^(1/2))
/(216*a^5*c^7))^(1/3) + log(B*x*(-a^5*c^7)^(1/2) - a^3*c^4*((A^3*c^3*(-a^5
*c^7)^(1/2) - C^3*a^3*(-a^5*c^7)^(1/2) - B^3*a^4*c^5 + 3*A^2*B*a^3*c^6 + 3
*B*C^2*a^5*c^4 - 3*A*B^2*a*c^2*(-a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(-a^5*c^7)
^(1/2) - 3*A^2*C*a*c^2*(-a^5*c^7)^(1/2) - 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*
(-a^5*c^7)^(1/2))/(a^5*c^7))^(1/3) - A*a^2*c^4*x + C*a^3*c^3*x)*((A^3*c^3*
(-a^5*c^7)^(1/2) - C^3*a^3*(-a^5*c^7)^(1/2) - B^3*a^4*c^5 + 3*A^2*B*a^3*c^
6 + 3*B*C^2*a^5*c^4 - 3*A*B^2*a*c^2*(-a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(-a^5
*c^7)^(1/2) - 3*A^2*C*a*c^2*(-a^5*c^7)^(1/2) - 6*A*B*C*a^4*c^5 + 3*B^2*C*a
^2*c*(-a^5*c^7)^(1/2))/(216*a^5*c^7))^(1/3) + log(B*x*(-a^5*c^7)^(1/2) - (
a^3*c^4*(-(A^3*c^3*(-a^5*c^7)^(1/2) - C^3*a^3*(-a^5*c^7)^(1/2) + B^3*a^4*c
^5 - 3*A^2*B*a^3*c^6 - 3*B*C^2*a^5*c^4 - 3*A*B^2*a*c^2*(-a^5*c^7)^(1/2) +
3*A*C^2*a^2*c*(-a^5*c^7)^(1/2) - 3*A^2*C*a*c^2*(-a^5*c^7)^(1/2) + 6*A*B...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.41

$$\int \frac{A + Bx^3 + Cx^6}{a + cx^6} \, dx = \frac{-2\sqrt{3}\, atan\left(\frac{c^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3} - 2c^{\frac{1}{3}} x}{c^{\frac{1}{6}} a^{\frac{1}{6}}}\right) b - 2\sqrt{3}\, atan\left(\frac{c^{\frac{1}{6}} a^{\frac{1}{6}} \sqrt{3} + 2c^{\frac{1}{3}} x}{c^{\frac{1}{6}} a^{\frac{1}{6}}}\right) b + 12c^{\frac{2}{3}} a^{\frac{1}{3}} x - 2\log\left(a^{\frac{1}{3}} + c^{\frac{1}{3}} x^2\right) b + \log\left(-c^{\frac{1}{6}} a^{\frac{1}{6}}\right.}{12c^{\frac{2}{3}} a^{\frac{1}{3}}}$$

input
```
int((C*x^6+B*x^3+A)/(c*x^6+a),x)
```

output
```
( - 2*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a*
*(1/6)))*b - 2*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c*
*(1/6)*a**(1/6)))*b + 12*c**(2/3)*a**(1/3)*x - 2*log(a**(1/3) + c**(1/3)*x
**2)*b + log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*b
+ log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*b)/(12*c**(2
/3)*a**(1/3))
```

3.10 $\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 377

$$
\begin{aligned}
&\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx\\
&=\frac{x\left(A-\frac{aC}{c}+Bx^3\right)}{6a\,(a+cx^6)}+\frac{(5Ac+aC)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{11/6}c^{7/6}}\\
&\quad-\frac{\left(2\sqrt{3}\sqrt{a}B\sqrt{c}+5Ac+aC\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{11/6}c^{7/6}}\\
&\quad-\frac{\left(2\sqrt{3}\sqrt{a}B\sqrt{c}-5Ac-aC\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{11/6}c^{7/6}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{18a^{4/3}c^{2/3}}\\
&\quad+\frac{\left(2\sqrt{a}B\sqrt{c}-5\sqrt{3}Ac-\sqrt{3}aC\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{72a^{11/6}c^{7/6}}\\
&\quad+\frac{\left(2\sqrt{a}B\sqrt{c}+5\sqrt{3}Ac+\sqrt{3}aC\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{72a^{11/6}c^{7/6}}
\end{aligned}
$$

output
```
1/6*x*(A-a*C/c+B*x^3)/a/(c*x^6+a)+1/18*(5*A*c+C*a)*arctan(c^(1/6)*x/a^(1/6
))/a^(11/6)/c^(7/6)+1/36*(2*3^(1/2)*a^(1/2)*B*c^(1/2)+5*A*c+C*a)*arctan(-3
^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(11/6)/c^(7/6)-1/36*(2*3^(1/2)*a^(1/2)*B*c^(
1/2)-5*A*c-C*a)*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(11/6)/c^(7/6)-1/18*
B*ln(a^(1/3)+c^(1/3)*x^2)/a^(4/3)/c^(2/3)+1/72*(2*a^(1/2)*B*c^(1/2)-5*3^(1
/2)*A*c-3^(1/2)*a*C)*ln(a^(1/3)-3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(
11/6)/c^(7/6)+1/72*(2*a^(1/2)*B*c^(1/2)+5*3^(1/2)*A*c+3^(1/2)*a*C)*ln(a^(1
/3)+3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(11/6)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx$$
$$= \frac{\frac{12a^{5/6}\sqrt[6]{c}x(Ac-aC+Bcx^3)}{a+cx^6}+4(5Ac+aC)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)-2\left(2\sqrt{3}\sqrt{a}B\sqrt{c}+5Ac+aC\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{}}{\sqrt[6]{}}\right.}{}$$

input
```
Integrate[(A + B*x^3 + C*x^6)/(a + c*x^6)^2,x]
```

output
```
((12*a^(5/6)*c^(1/6)*x*(A*c - a*C + B*c*x^3))/(a + c*x^6) + 4*(5*A*c + a*C
)*ArcTan[(c^(1/6)*x)/a^(1/6)] - 2*(2*Sqrt[3]*Sqrt[a]*B*Sqrt[c] + 5*A*c + a
*C)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)] + 2*(-2*Sqrt[3]*Sqrt[a]*B*Sqrt
[c] + 5*A*c + a*C)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] - 4*Sqrt[a]*B*S
qrt[c]*Log[a^(1/3) + c^(1/3)*x^2] - (-2*Sqrt[a]*B*Sqrt[c] + 5*Sqrt[3]*A*c
+ Sqrt[3]*a*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + (2
*Sqrt[a]*B*Sqrt[c] + 5*Sqrt[3]*A*c + Sqrt[3]*a*C)*Log[a^(1/3) + Sqrt[3]*a^
(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(72*a^(11/6)*c^(7/6))
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {2397, 25, 1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}-\frac{\int -\frac{2Bcx^3+5Ac+aC}{cx^6+a}dx}{6ac}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2Bcx^3+5Ac+aC}{cx^6+a}dx}{6ac}+\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$$\Big\downarrow \text{ 1746}$$

$$\frac{\frac{\int \frac{\sqrt[3]{c}\left(5Ac-2\sqrt[3]{a}Bxc^{2/3}+aC\right)}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2B\sqrt{c}-\frac{\sqrt{3}(5Ac+aC)}{\sqrt{a}}\right)x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt{c}B+\frac{\sqrt{3}(5Ac\cdots}{\sqrt{a}\cdots}\right.\right.}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}}{6a^{2/3}\sqrt[3]{c}}}{6ac}$$
$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{5Ac-2\sqrt[3]{a}Bxc^{2/3}+aC}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2B\sqrt{c}-\frac{\sqrt{3}(5Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt{c}B+\frac{\sqrt{3}(5Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}}{6ac}+$$
$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$$\Big\downarrow \text{ 452}$$

$$\frac{\frac{(aC+5Ac)\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx-2\sqrt[3]{a}Bc^{2/3}\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2B\sqrt{c}-\frac{\sqrt{3}(5Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2\ldots\right.}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\ldots}}{6\ldots}}{6ac}$$
$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\downarrow$ 218

$$\frac{\frac{\frac{(aC+5Ac)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}\sqrt[6]{c}}-2\sqrt[3]{a}Bc^{2/3}\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2B\sqrt{c}-\frac{\sqrt{3}(5Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt{c}\ldots\right.}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{\ldots}}{\sqrt[6]{\ldots}}}}{6a}}{6ac}$$
$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\downarrow$ 240

$$\frac{\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2B\sqrt{c}-\frac{\sqrt{3}(5Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{2(5Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt{c}B+\frac{\sqrt{3}(5Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\frac{(aC+5Ac)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}\sqrt[6]{c}}-\sqrt[3]{a}B\sqrt[3]{\ldots}}{3a^{2/3}}}{6ac}$$
$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\downarrow$ 1142

$$\frac{\frac{\frac{a^{2/3}\left(2B\sqrt{c}-\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}+\frac{1}{2}\left(2\sqrt{3}\sqrt{a}B\sqrt{c}+aC+5Ac\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{a^{2/3}\left(\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}+2B\ldots\right.}{\ldots}}{\ldots}$$
$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\downarrow$ 25

$$\frac{\frac{\frac{1}{2}\left(2\sqrt{3}\sqrt{a}B\sqrt{c}+aC+5Ac\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{a^{2/3}\left(2B\sqrt{c}-\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}}{6a}+\frac{a^{2/3}\left(\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}+2B\sqrt{\cdots}\right.\cdots}{\cdots}}{\cdots}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{1}{2}\left(2\sqrt{3}\sqrt{a}B\sqrt{c}+aC+5Ac\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{1}{2}\sqrt[3]{a}\left(2B\sqrt{c}-\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\frac{1}{2}\sqrt[3]{a}\left(\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}+2B\sqrt{c}\right)\cdots}{\cdots}}{6a\cdots}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\big\downarrow$ 1082

$$\frac{\frac{\frac{\sqrt[6]{a}\left(2\sqrt{3}\sqrt{a}B\sqrt{c}+aC+5Ac\right)\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}-\frac{1}{2}\sqrt[3]{a}\left(2B\sqrt{c}-\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\frac{1}{2}\sqrt[3]{a}\left(\frac{\sqrt{3}(aC+5A\cdots}{\sqrt{a}}\right.\cdots}{\cdots}}{\cdots}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\big\downarrow$ 217

$$\frac{\frac{-\frac{1}{2}\sqrt[3]{a}\left(2B\sqrt{c}-\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(2\sqrt{3}\sqrt{a}B\sqrt{c}+aC+5Ac\right)}{\sqrt[6]{c}}}{6a}+\frac{\frac{1}{2}\sqrt[3]{a}\left(\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}+2B\sqrt{\cdots}\right.\cdots}{\cdots}}{6\cdots}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

$\big\downarrow$ 1103

$$\frac{\frac{\frac{(aC+5Ac)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}\sqrt[6]{c}}-\sqrt[3]{a}B\sqrt[3]{c}\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{3a^{2/3}}+\frac{\frac{a^{2/3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)\left(2B\sqrt{c}-\frac{\sqrt{3}(aC+5Ac)}{\sqrt{a}}\right)}{2\sqrt[6]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\right.\right.}{}}{6a}}{}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{6ac\left(a+cx^6\right)}$$

input
```
Int[(A + B*x^3 + C*x^6)/(a + c*x^6)^2,x]
```

output
```
(x*(A*c - a*C + B*c*x^3))/(6*a*c*(a + c*x^6)) + ((((5*A*c + a*C)*ArcTan[(c
^(1/6)*x)/a^(1/6)])/(a^(1/6)*c^(1/6)) - a^(1/3)*B*c^(1/3)*Log[a^(1/3) + c^
(1/3)*x^2])/(3*a^(2/3)) + (-((a^(1/6)*(2*Sqrt[3]*Sqrt[a]*B*Sqrt[c] + 5*A*c
 + a*C)*ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/6)) +
(a^(2/3)*(2*B*Sqrt[c] - (Sqrt[3]*(5*A*c + a*C))/Sqrt[a])*Log[a^(1/3) - Sqr
t[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a) + (-((a^(1/6)*(2
*Sqrt[3]*Sqrt[a]*B*Sqrt[c] - 5*A*c - a*C)*ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x
)/(Sqrt[3]*a^(1/6)))])/c^(1/6)) + (a^(2/3)*(2*B*Sqrt[c] + (Sqrt[3]*(5*A*c
+ a*C))/Sqrt[a])*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(
2*c^(1/6)))/(6*a))/(6*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\frac{B x^4}{6a}+\frac{(Ac-Ca)x}{6ac}}{c x^6+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^6+a)}\frac{\left(2B\_R^3+\frac{5Ac+Ca}{c}\right)\ln(x-\_R)}{\_R^5}}{36ac}$
default	$\frac{\frac{B x^4}{6a}+\frac{(Ac-Ca)x}{6ac}}{c x^6+a}+\frac{-\frac{5c\ln\left(-x^2+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)A\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{12a}+\frac{c\ln\left(-x^2+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)B\left(\frac{a}{c}\right)^{\frac{2}{3}}}{6a}-\frac{\ln\left(-x^2+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x-\left(\frac{a}{c}\right.\right.}{12}}{}$

input
```
int((C*x^6+B*x^3+A)/(c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/6*B/a*x^4+1/6*(A*c-C*a)/a/c*x)/(c*x^6+a)+1/36/a/c*sum((2*B*_R^3+1/c*(5*
A*c+C*a))/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5685 vs. $2(272) = 544$.

Time = 7.36 (sec) , antiderivative size = 5685, normalized size of antiderivative = 15.08

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**6+B*x**3+A)/(c*x**6+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx = \frac{Bcx^4-(Ca-Ac)x}{6\,(ac^2x^6+a^2c)} - \frac{\frac{4\,Bc^{\frac{1}{3}}\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}} - \frac{4\left(Cac^{\frac{1}{3}}+5\,Ac^{\frac{4}{3}}\right)\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}c^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}} - \frac{\left(\sqrt{3}Ca^{\frac{7}{6}}\sqrt{c}+\left(5\sqrt{3}Aa^{\frac{1}{6}}\sqrt{c}+2\,Ba^{\frac{2}{3}}\right)c\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{ac^{\frac{2}{3}}}}{\;}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a)^2,x, algorithm="maxima")
```

output
```
1/6*(B*c*x^4 - (C*a - A*c)*x)/(a*c^2*x^6 + a^2*c) - 1/72*(4*B*c^(1/3)*log(
c^(1/3)*x^2 + a^(1/3))/a^(1/3) - 4*(C*a*c^(1/3) + 5*A*c^(4/3))*arctan(c^(1
/3)*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c^(1/3)*sqrt(a^(1/3)*c^(1/3))) - (sq
rt(3)*C*a^(7/6)*sqrt(c) + (5*sqrt(3)*A*a^(1/6)*sqrt(c) + 2*B*a^(2/3))*c)*l
og(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) + (sqrt(
3)*C*a^(7/6)*sqrt(c) + (5*sqrt(3)*A*a^(1/6)*sqrt(c) - 2*B*a^(2/3))*c)*log(
c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3)) - 2*(C*a^(4
/3)*c^(2/3) - (2*sqrt(3)*B*a^(5/6)*c^(1/6) - 5*A*a^(1/3)*c^(2/3))*c)*arcta
n((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3
)*sqrt(a^(1/3)*c^(1/3))) - 2*(C*a^(4/3)*c^(2/3) + (2*sqrt(3)*B*a^(5/6)*c^(
1/6) + 5*A*a^(1/3)*c^(2/3))*c)*arctan((2*c^(1/3)*x - sqrt(3)*a^(1/6)*c^(1/
6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx$$
$$= -\frac{B|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{18\,(ac^5)^{\frac{1}{3}}\,a}+\frac{Bcx^4-Cax+Acx}{6\,(cx^6+a)ac}$$
$$+\frac{\left((ac^5)^{\frac{1}{6}}\,Ca+5\,(ac^5)^{\frac{1}{6}}\,Ac\right)\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{18\,a^2c^2}$$
$$+\frac{\left((ac^5)^{\frac{1}{6}}\,Cac^2+5\,(ac^5)^{\frac{1}{6}}\,Ac^3-2\,\sqrt{3}(ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^4}$$
$$+\frac{\left((ac^5)^{\frac{1}{6}}\,Cac^2+5\,(ac^5)^{\frac{1}{6}}\,Ac^3+2\,\sqrt{3}(ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^4}$$
$$+\frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,Cac^2+5\,\sqrt{3}(ac^5)^{\frac{1}{6}}\,Ac^3+2\,(ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^4}$$
$$-\frac{\left(\sqrt{3}(ac^5)^{\frac{1}{6}}\,Cac^2+5\,\sqrt{3}(ac^5)^{\frac{1}{6}}\,Ac^3-2\,(ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^4}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a)^2,x, algorithm="giac")
```

output
```
-1/18*B*abs(c)*log(x^2 + (a/c)^(1/3))/((a*c^5)^(1/3)*a) + 1/6*(B*c*x^4 - C
*a*x + A*c*x)/((c*x^6 + a)*a*c) + 1/18*((a*c^5)^(1/6)*C*a + 5*(a*c^5)^(1/6
)*A*c)*arctan(x/(a/c)^(1/6))/(a^2*c^2) + 1/36*((a*c^5)^(1/6)*C*a*c^2 + 5*(
a*c^5)^(1/6)*A*c^3 - 2*sqrt(3)*(a*c^5)^(2/3)*B)*arctan((2*x + sqrt(3)*(a/c
)^(1/6))/(a/c)^(1/6))/(a^2*c^4) + 1/36*((a*c^5)^(1/6)*C*a*c^2 + 5*(a*c^5)^
(1/6)*A*c^3 + 2*sqrt(3)*(a*c^5)^(2/3)*B)*arctan((2*x - sqrt(3)*(a/c)^(1/6)
)/(a/c)^(1/6))/(a^2*c^4) + 1/72*(sqrt(3)*(a*c^5)^(1/6)*C*a*c^2 + 5*sqrt(3)
*(a*c^5)^(1/6)*A*c^3 + 2*(a*c^5)^(2/3)*B)*log(x^2 + sqrt(3)*x*(a/c)^(1/6)
+ (a/c)^(1/3))/(a^2*c^4) - 1/72*(sqrt(3)*(a*c^5)^(1/6)*C*a*c^2 + 5*sqrt(3)
*(a*c^5)^(1/6)*A*c^3 - 2*(a*c^5)^(2/3)*B)*log(x^2 - sqrt(3)*x*(a/c)^(1/6)
+ (a/c)^(1/3))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 3137, normalized size of antiderivative = 8.32

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3 + C*x^6)/(a + c*x^6)^2,x)
```

output
```
((B*x^4)/(6*a) + (x*(A*c - C*a))/(6*a*c))/(a + c*x^6) + log(a^7*c^4*((125*
A^3*c^3*(-a^11*c^7)^(1/2) + C^3*a^3*(-a^11*c^7)^(1/2) - 8*B^3*a^7*c^5 + 15
0*A^2*B*a^6*c^6 + 6*B*C^2*a^8*c^4 - 60*A*B^2*a*c^2*(-a^11*c^7)^(1/2) + 15*
A*C^2*a^2*c*(-a^11*c^7)^(1/2) + 75*A^2*C*a*c^2*(-a^11*c^7)^(1/2) + 60*A*B*
C*a^7*c^5 - 12*B^2*C*a^2*c*(-a^11*c^7)^(1/2))/(a^11*c^7))^(1/3) - 2*B*x*(-
a^11*c^7)^(1/2) + 5*A*a^5*c^4*x + C*a^6*c^3*x)*((125*A^3*c^3*(-a^11*c^7)^(
1/2) + C^3*a^3*(-a^11*c^7)^(1/2) - 8*B^3*a^7*c^5 + 150*A^2*B*a^6*c^6 + 6*B
*C^2*a^8*c^4 - 60*A*B^2*a*c^2*(-a^11*c^7)^(1/2) + 15*A*C^2*a^2*c*(-a^11*c^
7)^(1/2) + 75*A^2*C*a*c^2*(-a^11*c^7)^(1/2) + 60*A*B*C*a^7*c^5 - 12*B^2*C*
a^2*c*(-a^11*c^7)^(1/2))/(46656*a^11*c^7))^(1/3) + log(2*B*x*(-a^11*c^7)^(
1/2) + a^7*c^4*(-(125*A^3*c^3*(-a^11*c^7)^(1/2) + C^3*a^3*(-a^11*c^7)^(1/2
) + 8*B^3*a^7*c^5 - 150*A^2*B*a^6*c^6 - 6*B*C^2*a^8*c^4 - 60*A*B^2*a*c^2*(
-a^11*c^7)^(1/2) + 15*A*C^2*a^2*c*(-a^11*c^7)^(1/2) + 75*A^2*C*a*c^2*(-a^1
1*c^7)^(1/2) - 60*A*B*C*a^7*c^5 - 12*B^2*C*a^2*c*(-a^11*c^7)^(1/2))/(a^11*
c^7))^(1/3) + 5*A*a^5*c^4*x + C*a^6*c^3*x)*(-(125*A^3*c^3*(-a^11*c^7)^(1/2
) + C^3*a^3*(-a^11*c^7)^(1/2) + 8*B^3*a^7*c^5 - 150*A^2*B*a^6*c^6 - 6*B*C^
2*a^8*c^4 - 60*A*B^2*a*c^2*(-a^11*c^7)^(1/2) + 15*A*C^2*a^2*c*(-a^11*c^7)^
(1/2) + 75*A^2*C*a*c^2*(-a^11*c^7)^(1/2) - 60*A*B*C*a^7*c^5 - 12*B^2*C*a^2
*c*(-a^11*c^7)^(1/2))/(46656*a^11*c^7))^(1/3) + log((3^(1/2)*a^7*c^4*((125
*A^3*c^3*(-a^11*c^7)^(1/2) + C^3*a^3*(-a^11*c^7)^(1/2) - 8*B^3*a^7*c^5 ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^6+B*x^3+A)/(c*x^6+a)^2,x)
```

output

```
( - 6*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)))*a - 6*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2
*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*x**6 - 2*sqrt(3)*atan((c**(1/6)*a**(1/
6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*b - 2*sqrt(3)*atan((c**(
1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*b*c*x**6 + 6*sq
rt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a*
*(1/6)))*a + 6*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3
)*x)/(c**(1/6)*a**(1/6)))*c*x**6 - 2*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(
3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*b - 2*sqrt(3)*atan((c**(1/6)*a**
(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*b*c*x**6 + 12*sqrt(c)*s
qrt(a)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a + 12*sqrt(c)*sqrt(a)*atan(
(c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c*x**6 - 3*sqrt(c)*sqrt(a)*sqrt(3)*log(
- c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a - 3*sqrt(c)*sq
rt(a)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**
2)*c*x**6 + 3*sqrt(c)*sqrt(a)*sqrt(3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a*
*(1/3) + c**(1/3)*x**2)*a + 3*sqrt(c)*sqrt(a)*sqrt(3)*log(c**(1/6)*a**(1/6
)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c*x**6 + 6*c**(2/3)*a**(1/3)*b*x**
4 - 2*log(a**(1/3) + c**(1/3)*x**2)*a*b - 2*log(a**(1/3) + c**(1/3)*x**2)*
b*c*x**6 + log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*
a*b + log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*b*...
```

3.11 $\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 409

$$\begin{aligned}
&\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx \\
&= \frac{x\left(A-\frac{aC}{c}+Bx^3\right)}{12a\left(a+cx^6\right)^2}+\frac{x\left(11A+\frac{aC}{c}+8Bx^3\right)}{72a^2\left(a+cx^6\right)}+\frac{5(11Ac+aC)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{216a^{17/6}c^{7/6}} \\
&-\frac{\left(16\sqrt{3}\sqrt{a}B\sqrt{c}+55Ac+5aC\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{432a^{17/6}c^{7/6}} \\
&-\frac{\left(16\sqrt{3}\sqrt{a}B\sqrt{c}-55Ac-5aC\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{432a^{17/6}c^{7/6}}-\frac{B\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{27a^{7/3}c^{2/3}} \\
&+\frac{\left(16B-\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}\sqrt{c}}\right)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{864a^{7/3}c^{2/3}} \\
&+\frac{\left(16\sqrt{a}B\sqrt{c}+55\sqrt{3}Ac+5\sqrt{3}aC\right)\log\left(\sqrt[3]{a}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{864a^{17/6}c^{7/6}}
\end{aligned}$$

output
```
1/12*x*(A-a*C/c+B*x^3)/a/(c*x^6+a)^2+1/72*x*(11*A+a*C/c+8*B*x^3)/a^2/(c*x^
6+a)+5/216*(11*A*c+C*a)*arctan(c^(1/6)*x/a^(1/6))/a^(17/6)/c^(7/6)+1/432*(
16*3^(1/2)*a^(1/2)*B*c^(1/2)+55*A*c+5*C*a)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(
1/6))/a^(17/6)/c^(7/6)-1/432*(16*3^(1/2)*a^(1/2)*B*c^(1/2)-55*A*c-5*C*a)*a
rctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(17/6)/c^(7/6)-1/27*B*ln(a^(1/3)+c^(1
/3)*x^2)/a^(7/3)/c^(2/3)+1/864*(16*B-5*3^(1/2)*(11*A*c+C*a)/a^(1/2)/c^(1/2
))*ln(a^(1/3)-3^(1/2)*a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(7/3)/c^(2/3)+1/864
*(16*a^(1/2)*B*c^(1/2)+55*3^(1/2)*A*c+5*3^(1/2)*a*C)*ln(a^(1/3)+3^(1/2)*a^
(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(17/6)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx = \frac{\frac{72a^{11/6}\sqrt[6]{c}x(Ac-aC+Bcx^3)}{(a+cx^6)^2}+\frac{12a^{5/6}\sqrt[6]{c}x(11Ac+aC+8Bcx^3)}{a+cx^6}+20(11Ac+aC)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)-2(16\sqrt{3}\sqrt{a}B\sqrt{c}}{}$$

input
```
Integrate[(A + B*x^3 + C*x^6)/(a + c*x^6)^3,x]
```

output
```
((72*a^(11/6)*c^(1/6)*x*(A*c - a*C + B*c*x^3))/(a + c*x^6)^2 + (12*a^(5/6)
*c^(1/6)*x*(11*A*c + a*C + 8*B*c*x^3))/(a + c*x^6) + 20*(11*A*c + a*C)*Arc
Tan[(c^(1/6)*x)/a^(1/6)] - 2*(16*Sqrt[3]*Sqrt[a]*B*Sqrt[c] + 55*A*c + 5*a*
C)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)] + 2*(-16*Sqrt[3]*Sqrt[a]*B*Sqrt
[c] + 55*A*c + 5*a*C)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] - 32*Sqrt[a]
*B*Sqrt[c]*Log[a^(1/3) + c^(1/3)*x^2] - (-16*Sqrt[a]*B*Sqrt[c] + 55*Sqrt[3
]*A*c + 5*Sqrt[3]*a*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x
^2] + (16*Sqrt[a]*B*Sqrt[c] + 55*Sqrt[3]*A*c + 5*Sqrt[3]*a*C)*Log[a^(1/3)
+ Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(864*a^(17/6)*c^(7/6))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {2397, 25, 1761, 25, 1746, 27, 452, 218, 240, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3+Cx^6}{\left(a+cx^6\right)^3}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}-\frac{\int -\frac{8Bcx^3+11Ac+aC}{\left(cx^6+a\right)^2}dx}{12ac}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{8Bcx^3+11Ac+aC}{\left(cx^6+a\right)^2}dx}{12ac}+\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$$\Big\downarrow \text{ 1761}$$

$$\frac{\frac{x\left(aC+11Ac+8Bcx^3\right)}{6a\left(a+cx^6\right)}-\frac{\int -\frac{16Bcx^3+5(11Ac+aC)}{cx^6+a}dx}{6a}}{12ac}+\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{16Bcx^3+5(11Ac+aC)}{cx^6+a}dx}{6a}+\frac{x\left(aC+11Ac+8Bcx^3\right)}{6a\left(a+cx^6\right)}}{12ac}+\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$$\Big\downarrow \text{ 1746}$$

$$\frac{\frac{\frac{\int \frac{\sqrt[3]{c}\left(5(11Ac+aC)-16\sqrt[3]{a}Bc^{2/3}x\right)}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16B\sqrt{c}-\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}}\right)x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{6a^{2/3}\sqrt[3]{c}}+\frac{\int \frac{\sqrt[3]{c}\left(10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16\sqrt{c}B+\cdots\right.\right.}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}\cdots\right.}}{6a^{2/3}\sqrt[3]{c}}\cdots}{6a}\cdots}{12ac}$$
$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{\int \frac{5(11Ac+aC)-16\sqrt[3]{a}Bc^{2/3}x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16B\sqrt{c}-\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16\sqrt{c}B+\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}}{6a}}{12ac}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 452

$$\frac{\frac{\frac{5(aC+11Ac)\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx-16\sqrt[3]{a}Bc^{2/3}\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16B\sqrt{c}-\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}}}{}}{6a}}{12ac}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{5(aC+11Ac)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}\sqrt[6]{c}}-16\sqrt[3]{a}Bc^{2/3}\int \frac{x}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16B\sqrt{c}-\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+}}{}}{6a}}{12ac}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 240

$$\frac{\frac{\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16B\sqrt{c}-\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\int \frac{10(11Ac+aC)+\sqrt[3]{a}\sqrt[6]{c}\left(16\sqrt{c}B+\frac{5\sqrt{3}(11Ac+aC)}{\sqrt{a}}\right)x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}+\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\frac{5(aC+11Ac)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}\sqrt[6]{c}}-8}{3a^{2/}}}{6a}}{12ac}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{a^{2/3}\left(16B\sqrt{c}-\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}\right)\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}+\frac{1}{2}\left(16\sqrt{3}\sqrt{a}B\sqrt{c}+5aC+55Ac\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{a^{2/3}\left(\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}+\cdots\right.}{}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{1}{2}\left(16\sqrt{3}\sqrt{a}B\sqrt{c}+5aC+55Ac\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{a^{2/3}\left(16B\sqrt{c}-\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}\right)\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{a}\left(\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}dx}{2\sqrt[6]{c}}}{6a}+\frac{a^{2/3}\left(\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}+16\cdots\right.}{}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}\left(16\sqrt{3}\sqrt{a}B\sqrt{c}+5aC+55Ac\right)\int \frac{1}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{1}{2}\sqrt[3]{a}\left(16B\sqrt{c}-\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\frac{1}{2}\sqrt[3]{a}\left(\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}+16B\sqrt{c}\right.\cdots}{6a\cdots}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{\sqrt[6]{a}\left(16\sqrt{3}\sqrt{a}B\sqrt{c}+5aC+55Ac\right)\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}-\frac{1}{2}\sqrt[3]{a}\left(16B\sqrt{c}-\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}\right)\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx}{6a}+\frac{\frac{1}{2}\sqrt[3]{a}\left(\frac{5\sqrt{3}(aC\cdots}{\sqrt{\cdots}}\right.}{}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{-\frac{1}{2}\sqrt[3]{a}\left(16B\sqrt{c}-\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}\right)\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\frac{\sqrt[3]{c}x^2}{\sqrt[3]{a}}-\frac{\sqrt{3}\sqrt[6]{c}x}{\sqrt[6]{a}}+1}dx-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)\left(16\sqrt{3}\sqrt{a}B\sqrt{c}+5aC+55Ac\right)}{\sqrt[6]{c}}}{6a}+\frac{\frac{1}{2}\sqrt[3]{a}\left(\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}+1\right.}{}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{\frac{5(aC+11Ac)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{a}\sqrt[6]{c}}-8\sqrt[3]{a}B\sqrt[3]{c}\log\left(\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{3a^{2/3}}+\frac{\frac{a^{2/3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)\left(16B\sqrt{c}-\frac{5\sqrt{3}(aC+11Ac)}{\sqrt{a}}\right)}{2\sqrt[6]{c}}-\frac{\sqrt[6]{a}\arctan\left(\sqrt{3}\right.}{}}{6a}$$

$$\frac{x\left(-aC+Ac+Bcx^3\right)}{12ac\left(a+cx^6\right)^2}$$

input
```
Int[(A + B*x^3 + C*x^6)/(a + c*x^6)^3,x]
```

output
```
(x*(A*c - a*C + B*c*x^3))/(12*a*c*(a + c*x^6)^2) + ((x*(11*A*c + a*C + 8*B
*c*x^3))/(6*a*(a + c*x^6)) + (((5*(11*A*c + a*C)*ArcTan[(c^(1/6)*x)/a^(1/6
)])/(a^(1/6)*c^(1/6)) - 8*a^(1/3)*B*c^(1/3)*Log[a^(1/3) + c^(1/3)*x^2])/(3
*a^(2/3)) + (-((a^(1/6)*(16*Sqrt[3]*Sqrt[a]*B*Sqrt[c] + 55*A*c + 5*a*C)*Ar
cTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))])/c^(1/6)) + (a^(2/3)*(
16*B*Sqrt[c] - (5*Sqrt[3]*(11*A*c + a*C))/Sqrt[a])*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a) + (-((a^(1/6)*(16*Sqrt
[3]*Sqrt[a]*B*Sqrt[c] - 55*A*c - 5*a*C)*ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x)/
(Sqrt[3]*a^(1/6)))])/c^(1/6)) + (a^(2/3)*(16*B*Sqrt[c] + (5*Sqrt[3]*(11*A*
c + a*C))/Sqrt[a])*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])
/(2*c^(1/6)))/(6*a))/(6*a))/(12*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1746
```
Int[((d_) + (e_.)*(x_)^3)/((a_) + (c_.)*(x_)^6), x_Symbol] :> With[{q = Rt[
c/a, 6]}, Simp[1/(3*a*q^2)   Int[(q^2*d - e*x)/(1 + q^2*x^2), x], x] + (Sim
p[1/(6*a*q^2)   Int[(2*q^2*d - (Sqrt[3]*q^3*d - e)*x)/(1 - Sqrt[3]*q*x + q^
2*x^2), x], x] + Simp[1/(6*a*q^2)   Int[(2*q^2*d + (Sqrt[3]*q^3*d + e)*x)/(
1 + Sqrt[3]*q*x + q^2*x^2), x], x])] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
 + a*e^2, 0] && PosQ[c/a]
```

rule 1761
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Simp[1
/(2*a*n*(p + 1))   Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
 ILtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.28

method	result
risch	$\frac{\frac{B c\,x^{10}}{9a^2}+\frac{(11Ac+Ca)x^7}{72a^2}+\frac{7B\,x^4}{36a}+\frac{(17Ac-5Ca)x}{72ac}}{(c\,x^6+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+a\right)}\frac{\left(16B\_R^3+\frac{55Ac+5Ca}{c}\right)\ln\left(x-\_R\right)}{\_R^5}}{432a^2c}$
default	$\frac{\frac{B c\,x^{10}}{9a^2}+\frac{(11Ac+Ca)x^7}{72a^2}+\frac{7B\,x^4}{36a}+\frac{(17Ac-5Ca)x}{72ac}}{(c\,x^6+a)^2}+\frac{-\frac{55c\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)A\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{12a}+\frac{4c\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)B\left(\frac{a}{c}\right)^{\frac{2}{3}}}{3a}}{}$

input
```
int((C*x^6+B*x^3+A)/(c*x^6+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9*B*c/a^2*x^10+1/72*(11*A*c+C*a)/a^2*x^7+7/36*B/a*x^4+1/72*(17*A*c-5*C*
a)/a/c*x)/(c*x^6+a)^2+1/432/a^2/c*sum((16*B*_R^3+5/c*(11*A*c+C*a))/_R^5*ln
(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5912 vs. $2(305) = 610$.

Time = 9.40 (sec) , antiderivative size = 5912, normalized size of antiderivative = 14.45

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x**6+B*x**3+A)/(c*x**6+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx=\frac{8\,Bc^2x^{10}+(Cac+11\,Ac^2)x^7+14\,Bacx^4-(5\,Ca^2-17\,Aac)x}{72\,(a^2c^3x^{12}+2\,a^3c^2x^6+a^4c)} - \frac{\frac{32\,Bc^{\frac{1}{3}}\log\left(c^{\frac{1}{3}}x^2+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{20\,\left(Cac^{\frac{1}{3}}+11\,Ac^{\frac{4}{3}}\right)\arctan\left(\frac{c^{\frac{1}{3}}x}{\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}c^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}-\frac{\left(5\,\sqrt{3}Ca^{\frac{7}{6}}\sqrt{c}+\left(55\,\sqrt{3}Aa^{\frac{1}{6}}\sqrt{c}+16\,Ba^{\frac{2}{3}}\right)c\right)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x\right.}{ac^{\frac{2}{3}}}}{}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a)^3,x, algorithm="maxima")
```

output
```
1/72*(8*B*c^2*x^10 + (C*a*c + 11*A*c^2)*x^7 + 14*B*a*c*x^4 - (5*C*a^2 - 17
*A*a*c)*x)/(a^2*c^3*x^12 + 2*a^3*c^2*x^6 + a^4*c) - 1/864*(32*B*c^(1/3)*lo
g(c^(1/3)*x^2 + a^(1/3))/a^(1/3) - 20*(C*a*c^(1/3) + 11*A*c^(4/3))*arctan(
c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c^(1/3)*sqrt(a^(1/3)*c^(1/3))) -
 (5*sqrt(3)*C*a^(7/6)*sqrt(c) + (55*sqrt(3)*A*a^(1/6)*sqrt(c) + 16*B*a^(2/
3))*c)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3))
 + (5*sqrt(3)*C*a^(7/6)*sqrt(c) + (55*sqrt(3)*A*a^(1/6)*sqrt(c) - 16*B*a^(2
/3))*c)*log(c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a*c^(2/3))
 - 2*(5*C*a^(4/3)*c^(2/3) - (16*sqrt(3)*B*a^(5/6)*c^(1/6) - 55*A*a^(1/3)*c
^(2/3))*c)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(
1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(1/3))) - 2*(5*C*a^(4/3)*c^(2/3) + (16*sq
rt(3)*B*a^(5/6)*c^(1/6) + 55*A*a^(1/3)*c^(2/3))*c)*arctan((2*c^(1/3)*x - s
qrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(2/3)*sqrt(a^(1/3)*c^(
1/3))))/(a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx$$
$$= -\frac{B|c|\log\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{27\,(ac^5)^{\frac{1}{3}}\,a^2} + \frac{5\left((ac^5)^{\frac{1}{6}}\,Ca+11\,(ac^5)^{\frac{1}{6}}\,Ac\right)\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{216\,a^3c^2}$$
$$+ \frac{8\,Bc^2x^{10}+Cacx^7+11\,Ac^2x^7+14\,Bacx^4-5\,Ca^2x+17\,Aacx}{72\,(cx^6+a)^2a^2c}$$
$$+ \frac{\left(5\,(ac^5)^{\frac{1}{6}}\,Cac^2+55\,(ac^5)^{\frac{1}{6}}\,Ac^3-16\,\sqrt{3}(ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^4}$$
$$+ \frac{\left(5\,(ac^5)^{\frac{1}{6}}\,Cac^2+55\,(ac^5)^{\frac{1}{6}}\,Ac^3+16\,\sqrt{3}(ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^4}$$
$$+ \frac{\left(5\,\sqrt{3}(ac^5)^{\frac{1}{6}}\,Cac^2+55\,\sqrt{3}(ac^5)^{\frac{1}{6}}\,Ac^3+16\,(ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^4}$$
$$- \frac{\left(5\,\sqrt{3}(ac^5)^{\frac{1}{6}}\,Cac^2+55\,\sqrt{3}(ac^5)^{\frac{1}{6}}\,Ac^3-16\,(ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^4}$$

input
```
integrate((C*x^6+B*x^3+A)/(c*x^6+a)^3,x, algorithm="giac")
```

output
```
-1/27*B*abs(c)*log(x^2 + (a/c)^(1/3))/((a*c^5)^(1/3)*a^2) + 5/216*((a*c^5)
^(1/6)*C*a + 11*(a*c^5)^(1/6)*A*c)*arctan(x/(a/c)^(1/6))/(a^3*c^2) + 1/72*
(8*B*c^2*x^10 + C*a*c*x^7 + 11*A*c^2*x^7 + 14*B*a*c*x^4 - 5*C*a^2*x + 17*A
*a*c*x)/((c*x^6 + a)^2*a^2*c) + 1/432*(5*(a*c^5)^(1/6)*C*a*c^2 + 55*(a*c^5
)^(1/6)*A*c^3 - 16*sqrt(3)*(a*c^5)^(2/3)*B)*arctan((2*x + sqrt(3)*(a/c)^(1
/6))/(a/c)^(1/6))/(a^3*c^4) + 1/432*(5*(a*c^5)^(1/6)*C*a*c^2 + 55*(a*c^5)^
(1/6)*A*c^3 + 16*sqrt(3)*(a*c^5)^(2/3)*B)*arctan((2*x - sqrt(3)*(a/c)^(1/6
))/(a/c)^(1/6))/(a^3*c^4) + 1/864*(5*sqrt(3)*(a*c^5)^(1/6)*C*a*c^2 + 55*sq
rt(3)*(a*c^5)^(1/6)*A*c^3 + 16*(a*c^5)^(2/3)*B)*log(x^2 + sqrt(3)*x*(a/c)^
(1/6) + (a/c)^(1/3))/(a^3*c^4) - 1/864*(5*sqrt(3)*(a*c^5)^(1/6)*C*a*c^2 +
55*sqrt(3)*(a*c^5)^(1/6)*A*c^3 - 16*(a*c^5)^(2/3)*B)*log(x^2 - sqrt(3)*x*(
a/c)^(1/6) + (a/c)^(1/3))/(a^3*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 3196, normalized size of antiderivative = 7.81

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3 + C*x^6)/(a + c*x^6)^3,x)
```

output
```
((7*B*x^4)/(36*a) + (x^7*(11*A*c + C*a))/(72*a^2) + (B*c*x^10)/(9*a^2) + (
x*(17*A*c - 5*C*a))/(72*a*c))/(a^2 + c^2*x^12 + 2*a*c*x^6) + log(a^11*c^4*
((166375*A^3*c^3*(-a^17*c^7)^(1/2) + 125*C^3*a^3*(-a^17*c^7)^(1/2) - 4096*
B^3*a^10*c^5 + 145200*A^2*B*a^9*c^6 + 1200*B*C^2*a^11*c^4 - 42240*A*B^2*a*
c^2*(-a^17*c^7)^(1/2) + 4125*A*C^2*a^2*c*(-a^17*c^7)^(1/2) + 45375*A^2*C*a
*c^2*(-a^17*c^7)^(1/2) + 26400*A*B*C*a^10*c^5 - 3840*B^2*C*a^2*c*(-a^17*c^
7)^(1/2))/(a^17*c^7))^(1/3) - 16*B*x*(-a^17*c^7)^(1/2) + 55*A*a^8*c^4*x +
5*C*a^9*c^3*x)*((166375*A^3*c^3*(-a^17*c^7)^(1/2) + 125*C^3*a^3*(-a^17*c^7
)^(1/2) - 4096*B^3*a^10*c^5 + 145200*A^2*B*a^9*c^6 + 1200*B*C^2*a^11*c^4 -
 42240*A*B^2*a*c^2*(-a^17*c^7)^(1/2) + 4125*A*C^2*a^2*c*(-a^17*c^7)^(1/2)
+ 45375*A^2*C*a*c^2*(-a^17*c^7)^(1/2) + 26400*A*B*C*a^10*c^5 - 3840*B^2*C*
a^2*c*(-a^17*c^7)^(1/2))/(80621568*a^17*c^7))^(1/3) + log(16*B*x*(-a^17*c^
7)^(1/2) + a^11*c^4*(-(166375*A^3*c^3*(-a^17*c^7)^(1/2) + 125*C^3*a^3*(-a^
17*c^7)^(1/2) + 4096*B^3*a^10*c^5 - 145200*A^2*B*a^9*c^6 - 1200*B*C^2*a^11
*c^4 - 42240*A*B^2*a*c^2*(-a^17*c^7)^(1/2) + 4125*A*C^2*a^2*c*(-a^17*c^7)^
(1/2) + 45375*A^2*C*a*c^2*(-a^17*c^7)^(1/2) - 26400*A*B*C*a^10*c^5 - 3840*
B^2*C*a^2*c*(-a^17*c^7)^(1/2))/(a^17*c^7))^(1/3) + 55*A*a^8*c^4*x + 5*C*a^
9*c^3*x)*(-(166375*A^3*c^3*(-a^17*c^7)^(1/2) + 125*C^3*a^3*(-a^17*c^7)^(1/
2) + 4096*B^3*a^10*c^5 - 145200*A^2*B*a^9*c^6 - 1200*B*C^2*a^11*c^4 - 4224
0*A*B^2*a*c^2*(-a^17*c^7)^(1/2) + 4125*A*C^2*a^2*c*(-a^17*c^7)^(1/2) + ...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 993, normalized size of antiderivative = 2.43

$$\int \frac{A+Bx^3+Cx^6}{(a+cx^6)^3}\,dx = \text{Too large to display}$$

input
```
int((C*x^6+B*x^3+A)/(c*x^6+a)^3,x)
```

output
```
( - 30*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c*
*(1/6)*a**(1/6)))*a**2 - 60*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3
) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*x**6 - 30*sqrt(c)*sqrt(a)*atan(
(c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c**2*x**12
 - 8*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**
(1/6)))*a**2*b - 16*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x
)/(c**(1/6)*a**(1/6)))*a*b*c*x**6 - 8*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt
(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*b*c**2*x**12 + 30*sqrt(c)*sqrt(a)
*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2
 + 60*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)))*a*c*x**6 + 30*sqrt(c)*sqrt(a)*atan((c**(1/6)*a**(1/6)*sqr
t(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c**2*x**12 - 8*sqrt(3)*atan((c**
(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*b - 16*sq
rt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))
*a*b*c*x**6 - 8*sqrt(3)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c
**(1/6)*a**(1/6)))*b*c**2*x**12 + 60*sqrt(c)*sqrt(a)*atan((c**(1/3)*x)/(c*
*(1/6)*a**(1/6)))*a**2 + 120*sqrt(c)*sqrt(a)*atan((c**(1/3)*x)/(c**(1/6)*a
**(1/6)))*a*c*x**6 + 60*sqrt(c)*sqrt(a)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/
6)))*c**2*x**12 - 15*sqrt(c)*sqrt(a)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt
(3)*x + a**(1/3) + c**(1/3)*x**2)*a**2 - 30*sqrt(c)*sqrt(a)*sqrt(3)*log...
```

3.12 $\int (a-cx^6)^3 (A+Bx^3+Cx^6) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int (a-cx^6)^3 (A+Bx^3+Cx^6) \, dx = a^3Ax+\frac{1}{4}a^3Bx^4-\frac{1}{7}a^2(3Ac-aC)x^7 -\frac{3}{10}a^2Bcx^{10}+\frac{3}{13}ac(Ac-aC)x^{13}+\frac{3}{16}aBc^2x^{16} -\frac{1}{19}c^2(Ac-3aC)x^{19}-\frac{1}{22}Bc^3x^{22}-\frac{1}{25}c^3Cx^{25}$$

output
```
a^3*A*x+1/4*a^3*B*x^4-1/7*a^2*(3*A*c-C*a)*x^7-3/10*a^2*B*c*x^10+3/13*a*c*(
A*c-C*a)*x^13+3/16*a*B*c^2*x^16-1/19*c^2*(A*c-3*C*a)*x^19-1/22*B*c^3*x^22-
1/25*c^3*C*x^25
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int (a-cx^6)^3 (A+Bx^3+Cx^6) \, dx = -\frac{3}{910}a^2cx^7(130A+91Bx^3+70Cx^6) +\frac{3ac^2x^{13}(304A+247Bx^3+208Cx^6)}{3952} -\frac{c^3x^{19}(550A+475Bx^3+418Cx^6)}{10450} +a^3\left(Ax+\frac{Bx^4}{4}+\frac{Cx^7}{7}\right)$$

input
```
Integrate[(a - c*x^6)^3*(A + B*x^3 + C*x^6),x]
```

output
```
(-3*a^2*c*x^7*(130*A + 91*B*x^3 + 70*C*x^6))/910 + (3*a*c^2*x^13*(304*A +
247*B*x^3 + 208*C*x^6))/3952 - (c^3*x^19*(550*A + 475*B*x^3 + 418*C*x^6))/
10450 + a^3*(A*x + (B*x^4)/4 + (C*x^7)/7)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^6\right)^3 \left(A+Bx^3+Cx^6\right) \, dx$$

$$\Big\downarrow \text{ 2308}$$

$$\int \left(a^3A+a^3Bx^3+a^2x^6(aC-3Ac)-3a^2Bcx^9-c^2x^{18}(Ac-3aC)-3acx^{12}(aC-Ac)+3aBc^2x^{15}-Bc^3x^{21}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$a^3Ax+\frac{1}{4}a^3Bx^4-\frac{1}{7}a^2x^7(3Ac-aC)-\frac{3}{10}a^2Bcx^{10}-\frac{1}{19}c^2x^{19}(Ac-3aC)+\frac{3}{13}acx^{13}(Ac-aC)+\frac{3}{16}aBc^2x^{16}-\frac{1}{22}Bc^3x^{22}-\frac{1}{25}c^3Cx^{25}$$

input
```
Int[(a - c*x^6)^3*(A + B*x^3 + C*x^6),x]
```

output
```
a^3*A*x + (a^3*B*x^4)/4 - (a^2*(3*A*c - a*C)*x^7)/7 - (3*a^2*B*c*x^10)/10
+ (3*a*c*(A*c - a*C)*x^13)/13 + (3*a*B*c^2*x^16)/16 - (c^2*(A*c - 3*a*C)*x
^19)/19 - (B*c^3*x^22)/22 - (c^3*C*x^25)/25
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

method	result
norman	$-\frac{c^3C\,x^{25}}{25}-\frac{B\,c^3x^{22}}{22}+\left(\frac{3}{13}Aa\,c^2-\frac{3}{13}a^2cC\right)x^{13}+\frac{3aB\,c^2x^{16}}{16}+\left(-\frac{1}{19}A\,c^3+\frac{3}{19}a\,c^2C\right)x^{19}-\frac{3a^2Bc}{10}$
default	$-\frac{c^3C\,x^{25}}{25}-\frac{B\,c^3x^{22}}{22}+\frac{(-A\,c^3+3a\,c^2C)x^{19}}{19}+\frac{3aB\,c^2x^{16}}{16}+\frac{(3Aa\,c^2-3a^2cC)x^{13}}{13}-\frac{3a^2Bc\,x^{10}}{10}+\frac{(-3a^2Ac+a^3C}{7}$
gosper	$-\frac{1}{25}c^3C\,x^{25}-\frac{1}{22}B\,c^3x^{22}+\frac{3}{13}x^{13}Aa\,c^2-\frac{3}{13}x^{13}a^2cC+\frac{3}{16}aB\,c^2x^{16}-\frac{1}{19}x^{19}A\,c^3+\frac{3}{19}x^{19}a\,c^2C$
risch	$-\frac{1}{25}c^3C\,x^{25}-\frac{1}{22}B\,c^3x^{22}+\frac{3}{13}x^{13}Aa\,c^2-\frac{3}{13}x^{13}a^2cC+\frac{3}{16}aB\,c^2x^{16}-\frac{1}{19}x^{19}A\,c^3+\frac{3}{19}x^{19}a\,c^2C$
parallelrisch	$-\frac{1}{25}c^3C\,x^{25}-\frac{1}{22}B\,c^3x^{22}+\frac{3}{13}x^{13}Aa\,c^2-\frac{3}{13}x^{13}a^2cC+\frac{3}{16}aB\,c^2x^{16}-\frac{1}{19}x^{19}A\,c^3+\frac{3}{19}x^{19}a\,c^2C$
orering	$\frac{x(-304304c^3C\,x^{24}-345800c^3B\,x^{21}-400400A\,c^3x^{18}+1201200Ca\,c^2x^{18}+1426425a\,c^2B\,x^{15}+1755600Aa\,c^2x^{12}-1755600C\,a}{7607600}$

input
```
int((-c*x^6+a)^3*(C*x^6+B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-1/25*c^3*C*x^25-1/22*B*c^3*x^22+(3/13*A*a*c^2-3/13*a^2*c*C)*x^13+3/16*a*B
*c^2*x^16+(-1/19*A*c^3+3/19*a*c^2*C)*x^19-3/10*a^2*B*c*x^10+1/4*a^3*B*x^4+
(-3/7*a^2*A*c+1/7*a^3*C)*x^7+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int \left(a-cx^6\right)^3 \left(A+Bx^3+Cx^6\right) \, dx = -\frac{1}{25}\, Cc^3x^{25} - \frac{1}{22}\, Bc^3x^{22} + \frac{3}{16}\, Bac^2x^{16} + \frac{1}{19}\, \left(3\, Cac^2 - Ac^3\right)x^{19} - \frac{3}{10}\, Ba^2cx^{10} - \frac{3}{13}\, \left(Ca^2c - Aac^2\right)x^{13} + \frac{1}{4}\, Ba^3x^4 + \frac{1}{7}\, \left(Ca^3 - 3\, Aa^2c\right)x^7 + Aa^3x$$

input
```
integrate((-c*x^6+a)^3*(C*x^6+B*x^3+A),x, algorithm="fricas")
```

output
```
-1/25*C*c^3*x^25 - 1/22*B*c^3*x^22 + 3/16*B*a*c^2*x^16 + 1/19*(3*C*a*c^2 -
 A*c^3)*x^19 - 3/10*B*a^2*c*x^10 - 3/13*(C*a^2*c - A*a*c^2)*x^13 + 1/4*B*a
^3*x^4 + 1/7*(C*a^3 - 3*A*a^2*c)*x^7 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \left(a-cx^6\right)^3 \left(A+Bx^3+Cx^6\right) \, dx = Aa^3x + \frac{Ba^3x^4}{4} - \frac{3Ba^2cx^{10}}{10} + \frac{3Bac^2x^{16}}{16} - \frac{Bc^3x^{22}}{22} - \frac{Cc^3x^{25}}{25} + x^{19}\left(-\frac{Ac^3}{19} + \frac{3Cac^2}{19}\right) + x^{13} \cdot \left(\frac{3Aac^2}{13} - \frac{3Ca^2c}{13}\right) + x^7\left(-\frac{3Aa^2c}{7} + \frac{Ca^3}{7}\right)$$

input
```
integrate((-c*x**6+a)**3*(C*x**6+B*x**3+A),x)
```

output
```
A*a**3*x + B*a**3*x**4/4 - 3*B*a**2*c*x**10/10 + 3*B*a*c**2*x**16/16 - B*c
**3*x**22/22 - C*c**3*x**25/25 + x**19*(-A*c**3/19 + 3*C*a*c**2/19) + x**1
3*(3*A*a*c**2/13 - 3*C*a**2*c/13) + x**7*(-3*A*a**2*c/7 + C*a**3/7)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int \left(a-cx^6\right)^3 \left(A+Bx^3+Cx^6\right)\,dx = -\frac{1}{25}\,Cc^3x^{25} - \frac{1}{22}\,Bc^3x^{22} + \frac{3}{16}\,Bac^2x^{16} + \frac{1}{19}\,(3\,Cac^2 - Ac^3)x^{19} - \frac{3}{10}\,Ba^2cx^{10} - \frac{3}{13}\,(Ca^2c - Aac^2)x^{13} + \frac{1}{4}\,Ba^3x^4 + \frac{1}{7}\,(Ca^3 - 3\,Aa^2c)x^7 + Aa^3x$$

input
```
integrate((-c*x^6+a)^3*(C*x^6+B*x^3+A),x, algorithm="maxima")
```

output
```
-1/25*C*c^3*x^25 - 1/22*B*c^3*x^22 + 3/16*B*a*c^2*x^16 + 1/19*(3*C*a*c^2 -
 A*c^3)*x^19 - 3/10*B*a^2*c*x^10 - 3/13*(C*a^2*c - A*a*c^2)*x^13 + 1/4*B*a
^3*x^4 + 1/7*(C*a^3 - 3*A*a^2*c)*x^7 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95

$$\int \left(a-cx^6\right)^3 \left(A+Bx^3+Cx^6\right)\,dx = -\frac{1}{25}\,Cc^3x^{25} - \frac{1}{22}\,Bc^3x^{22} + \frac{3}{19}\,Cac^2x^{19} - \frac{1}{19}\,Ac^3x^{19} + \frac{3}{16}\,Bac^2x^{16} - \frac{3}{13}\,Ca^2cx^{13} + \frac{3}{13}\,Aac^2x^{13} - \frac{3}{10}\,Ba^2cx^{10} + \frac{1}{7}\,Ca^3x^7 - \frac{3}{7}\,Aa^2cx^7 + \frac{1}{4}\,Ba^3x^4 + Aa^3x$$

input
```
integrate((-c*x^6+a)^3*(C*x^6+B*x^3+A),x, algorithm="giac")
```

output
```
-1/25*C*c^3*x^25 - 1/22*B*c^3*x^22 + 3/19*C*a*c^2*x^19 - 1/19*A*c^3*x^19 +
 3/16*B*a*c^2*x^16 - 3/13*C*a^2*c*x^13 + 3/13*A*a*c^2*x^13 - 3/10*B*a^2*c*
x^10 + 1/7*C*a^3*x^7 - 3/7*A*a^2*c*x^7 + 1/4*B*a^3*x^4 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\int (a-cx^6)^3 (A+Bx^3+Cx^6)\, dx = x^7 \left(\frac{C a^3}{7} - \frac{3 A a^2 c}{7}\right) - x^{19} \left(\frac{A c^3}{19} - \frac{3 C a c^2}{19}\right) + \frac{B a^3 x^4}{4} - \frac{B c^3 x^{22}}{22} - \frac{C c^3 x^{25}}{25} + A a^3 x + \frac{3 a c x^{13} (A c - C a)}{13} - \frac{3 B a^2 c x^{10}}{10} + \frac{3 B a c^2 x^{16}}{16}$$

input
```
int((a - c*x^6)^3*(A + B*x^3 + C*x^6),x)
```

output
```
x^7*((C*a^3)/7 - (3*A*a^2*c)/7) - x^19*((A*c^3)/19 - (3*C*a*c^2)/19) + (B*
a^3*x^4)/4 - (B*c^3*x^22)/22 - (C*c^3*x^25)/25 + A*a^3*x + (3*a*c*x^13*(A*
c - C*a))/13 - (3*B*a^2*c*x^10)/10 + (3*B*a*c^2*x^16)/16
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int (a-cx^6)^3 (A+Bx^3+Cx^6)\, dx = \frac{x(-23408c^4x^{24} - 26600b c^3x^{21} + 61600a c^3x^{18} + 109725ab c^2x^{15} - 175560a^2bc x^9 - 167200a^3c x^6 + 1463}{585200}$$

input
```
int((-c*x^6+a)^3*(C*x^6+B*x^3+A),x)
```

output
```
(x*(585200*a**4 + 146300*a**3*b*x**3 - 167200*a**3*c*x**6 - 175560*a**2*b*
c*x**9 + 109725*a*b*c**2*x**15 + 61600*a*c**3*x**18 - 26600*b*c**3*x**21 -
 23408*c**4*x**24))/585200
```

3.13 $\int (a-cx^6)^2 (A+Bx^3+Cx^6)\, dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int (a-cx^6)^2 (A+Bx^3+Cx^6)\, dx = a^2Ax+\frac{1}{4}a^2Bx^4-\frac{1}{7}a(2Ac-aC)x^7-\frac{1}{5}aBcx^{10}+\frac{1}{13}c(Ac-2aC)x^{13}+\frac{1}{16}Bc^2x^{16}+\frac{1}{19}c^2Cx^{19}$$

output
```
a^2*A*x+1/4*a^2*B*x^4-1/7*a*(2*A*c-C*a)*x^7-1/5*a*B*c*x^10+1/13*c*(A*c-2*C
*a)*x^13+1/16*B*c^2*x^16+1/19*c^2*C*x^19
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int (a-cx^6)^2 (A+Bx^3+Cx^6)\, dx = a^2Ax+\frac{1}{4}a^2Bx^4+\frac{1}{7}a(-2Ac+aC)x^7-\frac{1}{5}aBcx^{10}+\frac{1}{13}c(Ac-2aC)x^{13}+\frac{1}{16}Bc^2x^{16}+\frac{1}{19}c^2Cx^{19}$$

input
```
Integrate[(a - c*x^6)^2*(A + B*x^3 + C*x^6),x]
```

output
```
a^2*A*x + (a^2*B*x^4)/4 + (a*(-2*A*c + a*C)*x^7)/7 - (a*B*c*x^10)/5 + (c*(
A*c - 2*a*C)*x^13)/13 + (B*c^2*x^16)/16 + (c^2*C*x^19)/19
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^6\right)^2 \left(A+Bx^3+Cx^6\right) \, dx$$

$$\Big\downarrow \text{2308}$$

$$\int \left(a^2A+a^2Bx^3+cx^{12}(Ac-2aC)+ax^6(aC-2Ac)-2aBcx^9+Bc^2x^{15}+c^2Cx^{18}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{4}a^2Bx^4+\frac{1}{13}cx^{13}(Ac-2aC)-\frac{1}{7}ax^7(2Ac-aC)-\frac{1}{5}aBcx^{10}+\frac{1}{16}Bc^2x^{16}+\frac{1}{19}c^2Cx^{19}$$

input
```
Int[(a - c*x^6)^2*(A + B*x^3 + C*x^6),x]
```

output
```
a^2*A*x + (a^2*B*x^4)/4 - (a*(2*A*c - a*C)*x^7)/7 - (a*B*c*x^10)/5 + (c*(A
*c - 2*a*C)*x^13)/13 + (B*c^2*x^16)/16 + (c^2*C*x^19)/19
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^2 C x^{19}}{19}+\frac{B c^2 x^{16}}{16}+\frac{(A c^2-2acC)x^{13}}{13}-\frac{aBc x^{10}}{5}+\frac{(-2Aac+a^2C)x^7}{7}+\frac{a^2 B x^4}{4}+a^2Ax$
norman	$a^2Ax+\frac{a^2 B x^4}{4}+\left(-\frac{2}{7}Aac+\frac{1}{7}a^2C\right)x^7-\frac{aBc x^{10}}{5}+\left(\frac{1}{13}A c^2-\frac{2}{13}acC\right)x^{13}+\frac{B c^2 x^{16}}{16}+\frac{c^2 C x^{19}}{19}$
gosper	$a^2Ax+\frac{1}{4}a^2B x^4-\frac{2}{7}x^7Aac+\frac{1}{7}x^7a^2C-\frac{1}{5}aBc x^{10}+\frac{1}{13}x^{13}A c^2-\frac{2}{13}x^{13}acC+\frac{1}{16}B c^2x^{16}+$
risch	$a^2Ax+\frac{1}{4}a^2B x^4-\frac{2}{7}x^7Aac+\frac{1}{7}x^7a^2C-\frac{1}{5}aBc x^{10}+\frac{1}{13}x^{13}A c^2-\frac{2}{13}x^{13}acC+\frac{1}{16}B c^2x^{16}+$
parallelrisch	$a^2Ax+\frac{1}{4}a^2B x^4-\frac{2}{7}x^7Aac+\frac{1}{7}x^7a^2C-\frac{1}{5}aBc x^{10}+\frac{1}{13}x^{13}A c^2-\frac{2}{13}x^{13}acC+\frac{1}{16}B c^2x^{16}+$
orering	$\frac{x(7280c^2C x^{18}+8645c^2B x^{15}+10640A c^2x^{12}-21280Cac x^{12}-27664x^9aBc-39520Aac x^6+19760C a^2x^6+34580a^2B x^3+13}{138320}$

input
```
int((-c*x^6+a)^2*(C*x^6+B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
1/19*c^2*C*x^19+1/16*B*c^2*x^16+1/13*(A*c^2-2*C*a*c)*x^13-1/5*a*B*c*x^10+1
/7*(-2*A*a*c+C*a^2)*x^7+1/4*a^2*B*x^4+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \left(a-cx^6\right)^2\left(A+Bx^3+Cx^6\right)\,dx=\frac{1}{19}\,Cc^2x^{19}+\frac{1}{16}\,Bc^2x^{16}-\frac{1}{13}\,(2\,Cac-Ac^2)x^{13}-\frac{1}{5}\,Bacx^{10}+\frac{1}{7}\,(Ca^2-2\,Aac)x^7+\frac{1}{4}\,Ba^2x^4+Aa^2x$$

input
```
integrate((-c*x^6+a)^2*(C*x^6+B*x^3+A),x, algorithm="fricas")
```

output
```
1/19*C*c^2*x^19 + 1/16*B*c^2*x^16 - 1/13*(2*C*a*c - A*c^2)*x^13 - 1/5*B*a*
c*x^10 + 1/7*(C*a^2 - 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int (a-cx^6)^2 (A+Bx^3+Cx^6)\,dx = Aa^2x+\frac{Ba^2x^4}{4}-\frac{Bacx^{10}}{5}+\frac{Bc^2x^{16}}{16}+\frac{Cc^2x^{19}}{19}+x^{13}\left(\frac{Ac^2}{13}-\frac{2Cac}{13}\right)+x^7\left(-\frac{2Aac}{7}+\frac{Ca^2}{7}\right)$$

input
```
integrate((-c*x**6+a)**2*(C*x**6+B*x**3+A),x)
```

output
```
A*a**2*x + B*a**2*x**4/4 - B*a*c*x**10/5 + B*c**2*x**16/16 + C*c**2*x**19/
19 + x**13*(A*c**2/13 - 2*C*a*c/13) + x**7*(-2*A*a*c/7 + C*a**2/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int (a-cx^6)^2 (A+Bx^3+Cx^6)\,dx = \frac{1}{19}\,Cc^2x^{19}+\frac{1}{16}\,Bc^2x^{16}-\frac{1}{13}\,(2\,Cac-Ac^2)x^{13}-\frac{1}{5}\,Bacx^{10}+\frac{1}{7}\,(Ca^2-2\,Aac)x^7+\frac{1}{4}\,Ba^2x^4+Aa^2x$$

input
```
integrate((-c*x^6+a)^2*(C*x^6+B*x^3+A),x, algorithm="maxima")
```

output
```
1/19*C*c^2*x^19 + 1/16*B*c^2*x^16 - 1/13*(2*C*a*c - A*c^2)*x^13 - 1/5*B*a*
c*x^10 + 1/7*(C*a^2 - 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int (a-cx^6)^2 (A+Bx^3+Cx^6)\,dx = \frac{1}{19}Cc^2x^{19}+\frac{1}{16}Bc^2x^{16}-\frac{2}{13}Cacx^{13}+\frac{1}{13}Ac^2x^{13} -\frac{1}{5}Bacx^{10}+\frac{1}{7}Ca^2x^7-\frac{2}{7}Aacx^7+\frac{1}{4}Ba^2x^4+Aa^2x$$

input
```
integrate((-c*x^6+a)^2*(C*x^6+B*x^3+A),x, algorithm="giac")
```

output
```
1/19*C*c^2*x^19 + 1/16*B*c^2*x^16 - 2/13*C*a*c*x^13 + 1/13*A*c^2*x^13 - 1/
5*B*a*c*x^10 + 1/7*C*a^2*x^7 - 2/7*A*a*c*x^7 + 1/4*B*a^2*x^4 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int (a-cx^6)^2 (A+Bx^3+Cx^6)\,dx = x^7\left(\frac{C\,a^2}{7}-\frac{2\,A\,a\,c}{7}\right)+x^{13}\left(\frac{A\,c^2}{13}-\frac{2\,C\,a\,c}{13}\right) +\frac{B\,a^2\,x^4}{4}+\frac{B\,c^2\,x^{16}}{16}+\frac{C\,c^2\,x^{19}}{19}+A\,a^2\,x-\frac{B\,a\,c\,x^{10}}{5}$$

input
```
int((a - c*x^6)^2*(A + B*x^3 + C*x^6),x)
```

output
```
x^7*((C*a^2)/7 - (2*A*a*c)/7) + x^13*((A*c^2)/13 - (2*C*a*c)/13) + (B*a^2*
x^4)/4 + (B*c^2*x^16)/16 + (C*c^2*x^19)/19 + A*a^2*x - (B*a*c*x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \left(a-cx^6\right)^2 \left(A+Bx^3+Cx^6\right) dx$$
$$= \frac{x(7280c^3x^{18}+8645b\,c^2x^{15}-10640a\,c^2x^{12}-27664abc\,x^9-19760a^2c\,x^6+34580a^2b\,x^3+138320a^3)}{138320}$$

input
```
int((-c*x^6+a)^2*(C*x^6+B*x^3+A),x)
```

output
```
(x*(138320*a**3 + 34580*a**2*b*x**3 - 19760*a**2*c*x**6 - 27664*a*b*c*x**9
 - 10640*a*c**2*x**12 + 8645*b*c**2*x**15 + 7280*c**3*x**18))/138320
```

3.14 $\int (a - cx^6) (A + Bx^3 + Cx^6) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int (a - cx^6) (A + Bx^3 + Cx^6) \, dx = aAx + \frac{1}{4}aBx^4 - \frac{1}{7}(Ac - aC)x^7 - \frac{1}{10}Bcx^{10} - \frac{1}{13}cCx^{13}$$

output
```
a*A*x+1/4*a*B*x^4-1/7*(A*c-C*a)*x^7-1/10*B*c*x^10-1/13*c*C*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (a - cx^6) (A + Bx^3 + Cx^6) \, dx = aAx + \frac{1}{4}aBx^4 + \frac{1}{7}(-Ac + aC)x^7 - \frac{1}{10}Bcx^{10} - \frac{1}{13}cCx^{13}$$

input
```
Integrate[(a - c*x^6)*(A + B*x^3 + C*x^6),x]
```

output
```
a*A*x + (a*B*x^4)/4 + ((-(A*c) + a*C)*x^7)/7 - (B*c*x^10)/10 - (c*C*x^13)/
13
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^6)\left(A+Bx^3+Cx^6\right)dx$$

$$\Big\downarrow \text{ 2308}$$

$$\int \left(-x^6(Ac-aC)+aA+aBx^3-Bcx^9-cCx^{12}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{7}x^7(Ac-aC)+aAx+\frac{1}{4}aBx^4-\frac{1}{10}Bcx^{10}-\frac{1}{13}cCx^{13}$$

input
```
Int[(a - c*x^6)*(A + B*x^3 + C*x^6),x]
```

output
```
a*A*x + (a*B*x^4)/4 - ((A*c - a*C)*x^7)/7 - (B*c*x^10)/10 - (c*C*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{cC\,x^{13}}{13}-\frac{Bc\,x^{10}}{10}+\frac{(-Ac+Ca)x^{7}}{7}+\frac{Ba\,x^{4}}{4}+aAx$	40
norman	$-\frac{cC\,x^{13}}{13}-\frac{Bc\,x^{10}}{10}+\left(-\frac{Ac}{7}+\frac{Ca}{7}\right)x^{7}+\frac{Ba\,x^{4}}{4}+aAx$	40
gosper	$-\frac{1}{13}cC\,x^{13}-\frac{1}{10}Bc\,x^{10}-\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ca+\frac{1}{4}Ba\,x^{4}+aAx$	41
risch	$-\frac{1}{13}cC\,x^{13}-\frac{1}{10}Bc\,x^{10}-\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ca+\frac{1}{4}Ba\,x^{4}+aAx$	41
parallelrisch	$-\frac{1}{13}cC\,x^{13}-\frac{1}{10}Bc\,x^{10}-\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ca+\frac{1}{4}Ba\,x^{4}+aAx$	41
orering	$\frac{x\left(-140Cc\,x^{12}-182x^{9}Bc-260Ac\,x^{6}+260Ca\,x^{6}+455Ba\,x^{3}+1820Aa\right)}{1820}$	44

input
```
int((-c*x^6+a)*(C*x^6+B*x^3+A),x,method=_RETURNVERBOSE)
```

output
```
-1/13*c*C*x^13-1/10*B*c*x^10+1/7*(-A*c+C*a)*x^7+1/4*B*a*x^4+a*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int\left(a-cx^{6}\right)\left(A+Bx^{3}+Cx^{6}\right)\,dx=-\frac{1}{13}\,Ccx^{13}-\frac{1}{10}\,Bcx^{10}+\frac{1}{7}\,(Ca-Ac)x^{7}+\frac{1}{4}\,Bax^{4}+Aax$$

input
```
integrate((-c*x^6+a)*(C*x^6+B*x^3+A),x, algorithm="fricas")
```

output
```
-1/13*C*c*x^13 - 1/10*B*c*x^10 + 1/7*(C*a - A*c)*x^7 + 1/4*B*a*x^4 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(a-cx^6\right)\left(A+Bx^3+Cx^6\right)\,dx=Aax+\frac{Bax^4}{4}-\frac{Bcx^{10}}{10}-\frac{Ccx^{13}}{13}+x^7\left(-\frac{Ac}{7}+\frac{Ca}{7}\right)$$

input
```
integrate((-c*x**6+a)*(C*x**6+B*x**3+A),x)
```

output
```
A*a*x + B*a*x**4/4 - B*c*x**10/10 - C*c*x**13/13 + x**7*(-A*c/7 + C*a/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \left(a-cx^6\right)\left(A+Bx^3+Cx^6\right)\,dx=-\frac{1}{13}\,Ccx^{13}-\frac{1}{10}\,Bcx^{10}+\frac{1}{7}\,(Ca-Ac)x^7+\frac{1}{4}\,Bax^4+Aax$$

input
```
integrate((-c*x^6+a)*(C*x^6+B*x^3+A),x, algorithm="maxima")
```

output
```
-1/13*C*c*x^13 - 1/10*B*c*x^10 + 1/7*(C*a - A*c)*x^7 + 1/4*B*a*x^4 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \left(a-cx^6\right)\left(A+Bx^3+Cx^6\right)\,dx=-\frac{1}{13}\,Ccx^{13}-\frac{1}{10}\,Bcx^{10}+\frac{1}{7}\,Cax^7-\frac{1}{7}\,Acx^7+\frac{1}{4}\,Bax^4+Aax$$

input
```
integrate((-c*x^6+a)*(C*x^6+B*x^3+A),x, algorithm="giac")
```

output
```
-1/13*C*c*x^13 - 1/10*B*c*x^10 + 1/7*C*a*x^7 - 1/7*A*c*x^7 + 1/4*B*a*x^4 +
 A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int (a-cx^6) (A+Bx^3+Cx^6) \, dx = -\frac{C\,c\,x^{13}}{13} - \frac{B\,c\,x^{10}}{10} + \left(\frac{C\,a}{7} - \frac{A\,c}{7}\right) x^7 + \frac{B\,a\,x^4}{4} + A\,a\,x$$

input
```
int((a - c*x^6)*(A + B*x^3 + C*x^6),x)
```

output
```
A*a*x - x^7*((A*c)/7 - (C*a)/7) + (B*a*x^4)/4 - (B*c*x^10)/10 - (C*c*x^13)
/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int (a-cx^6) (A+Bx^3+Cx^6) \, dx = \frac{x(-20c^2x^{12} - 26bc\,x^9 + 65ab\,x^3 + 260a^2)}{260}$$

input
```
int((-c*x^6+a)*(C*x^6+B*x^3+A),x)
```

output
```
(x*(260*a**2 + 65*a*b*x**3 - 26*b*c*x**9 - 20*c**2*x**12))/260
```

3.15 $\int \frac{A+Bx^3+Cx^6}{a-cx^6} dx$

Optimal result

Integrand size = 23, antiderivative size = 360

$$
\begin{aligned}
\int \frac{A+Bx^3+Cx^6}{a-cx^6}\,dx = &-\frac{Cx}{c}+\frac{\left(\sqrt{a}B\sqrt{c}-Ac-aC\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{7/6}} \\
&+\frac{\left(\sqrt{a}B\sqrt{c}+Ac+aC\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{7/6}} \\
&-\frac{\left(\sqrt{a}B\sqrt{c}+Ac+aC\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{6a^{5/6}c^{7/6}} \\
&-\frac{\left(\sqrt{a}B\sqrt{c}-Ac-aC\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{6a^{5/6}c^{7/6}} \\
&+\frac{\left(\sqrt{a}B\sqrt{c}-Ac-aC\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{7/6}} \\
&+\frac{\left(\sqrt{a}B\sqrt{c}+Ac+aC\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{12a^{5/6}c^{7/6}}
\end{aligned}
$$

output
```
-C*x/c+1/6*(a^(1/2)*B*c^(1/2)-A*c-C*a)*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^
(1/2)/a^(1/6))*3^(1/2)/a^(5/6)/c^(7/6)+1/6*(a^(1/2)*B*c^(1/2)+A*c+C*a)*arc
tan(1/3*(a^(1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(5/6)/c^(7/6)-1/6
*(a^(1/2)*B*c^(1/2)+A*c+C*a)*ln(a^(1/6)-c^(1/6)*x)/a^(5/6)/c^(7/6)-1/6*(a^
(1/2)*B*c^(1/2)-A*c-C*a)*ln(a^(1/6)+c^(1/6)*x)/a^(5/6)/c^(7/6)+1/12*(a^(1/
2)*B*c^(1/2)-A*c-C*a)*ln(a^(1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(5/6)/c^
(7/6)+1/12*(a^(1/2)*B*c^(1/2)+A*c+C*a)*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3
)*x^2)/a^(5/6)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^3+Cx^6}{a-cx^6}\,dx$$

$$= \frac{-12a^{5/6}\sqrt[6]{c}Cx - 2\sqrt{3}\left(-\sqrt{a}B\sqrt{c}+Ac+aC\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt{a}B\sqrt{c}+Ac+aC\right)\arctan\left(\right.}{}$$

input
```
Integrate[(A + B*x^3 + C*x^6)/(a - c*x^6),x]
```

output
```
(-12*a^(5/6)*c^(1/6)*C*x - 2*Sqrt[3]*(-(Sqrt[a]*B*Sqrt[c]) + A*c + a*C)*Ar
cTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] + 2*Sqrt[3]*(Sqrt[a]*B*Sqrt[c] +
 A*c + a*C)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] - 2*(Sqrt[a]*B*Sqr
t[c] + A*c + a*C)*Log[a^(1/6) - c^(1/6)*x] + 2*(-(Sqrt[a]*B*Sqrt[c]) + A*c
 + a*C)*Log[a^(1/6) + c^(1/6)*x] - (-(Sqrt[a]*B*Sqrt[c]) + A*c + a*C)*Log[
a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + (Sqrt[a]*B*Sqrt[c] + A*c + a*
C)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(7/6))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3+Cx^6}{a-cx^6}dx$$

$$\Big\downarrow \text{2426}$$

$$\int \left(\frac{aC+Ac+Bcx^3}{c(a-cx^6)}-\frac{C}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)}{2\sqrt{3}a^{5/6}c^{7/6}}+\frac{\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)}{2\sqrt{3}a^{5/6}c^{7/6}}+\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)}{12a^{5/6}c^{7/6}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)}{12a^{5/6}c^{7/6}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)\left(\sqrt{a}B\sqrt{c}+aC+Ac\right)}{6a^{5/6}c^{7/6}}-\frac{\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)\left(\sqrt{a}B\sqrt{c}-aC-Ac\right)}{6a^{5/6}c^{7/6}}-\frac{Cx}{c}$$

input
```
Int[(A + B*x^3 + C*x^6)/(a - c*x^6),x]
```

output
```
-((C*x)/c) + ((Sqrt[a]*B*Sqrt[c] - A*c - a*C)*ArcTan[(a^(1/6) - 2*c^(1/6)*
x)/(Sqrt[3]*a^(1/6))])/(2*Sqrt[3]*a^(5/6)*c^(7/6)) + ((Sqrt[a]*B*Sqrt[c] +
 A*c + a*C)*ArcTan[(a^(1/6) + 2*c^(1/6)*x)/(Sqrt[3]*a^(1/6))])/(2*Sqrt[3]*
a^(5/6)*c^(7/6)) - ((Sqrt[a]*B*Sqrt[c] + A*c + a*C)*Log[a^(1/6) - c^(1/6)*
x])/(6*a^(5/6)*c^(7/6)) - ((Sqrt[a]*B*Sqrt[c] - A*c - a*C)*Log[a^(1/6) + c
^(1/6)*x])/(6*a^(5/6)*c^(7/6)) + ((Sqrt[a]*B*Sqrt[c] - A*c - a*C)*Log[a^(1
/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*a^(5/6)*c^(7/6)) + ((Sqrt[a]*B
*Sqrt[c] + A*c + a*C)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(12*
a^(5/6)*c^(7/6))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.15

method	result
risch	$-\frac{Cx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6-a\right)}\frac{\left(-B\_R^3c-Ac-Ca\right)\ln\left(x-\_R\right)}{\_R^5}}{6c^2}$
default	$-\frac{Cx}{c}+\frac{\frac{cB\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{cB\left(\frac{a}{c}\right)^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{6a}+\frac{cA\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{cA\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,\mathrm{arc}}{}}{}$

input
```
int((C*x^6+B*x^3+A)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-C*x/c+1/6/c^2*sum((-B*_R^3*c-A*c-C*a)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5305 vs. 2(257) = 514.

Time = 6.57 (sec) , antiderivative size = 5305, normalized size of antiderivative = 14.74

$$\int \frac{A+Bx^3+Cx^6}{a-cx^6}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3 + Cx^6}{a - cx^6}\, dx = \text{Timed out}$$

input
```
integrate((C*x**6+B*x**3+A)/(-c*x**6+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.06

$$\int \frac{A + Bx^3 + Cx^6}{a - cx^6}\, dx = -\frac{Cx}{c} + \frac{\frac{2\sqrt{3}(Ca\sqrt{c} + (B\sqrt{a} + A\sqrt{c})c)\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{2\sqrt{3}(Ca\sqrt{c} - (B\sqrt{a} - A\sqrt{c})c)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{(Ca\sqrt{c} + (B \cdots}{\cdots}}{\cdots}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-C*x/c + 1/12*(2*sqrt(3)*(C*a*sqrt(c) + (B*sqrt(a) + A*sqrt(c))*c)*arctan(
1/3*sqrt(3)*(2*x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt
(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 2*sqrt(3)*(C*a*sqrt(c) - (B*sqrt(a) - A*s
qrt(c))*c)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqr
t(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + (C*a*sqrt(c) + (B*sqrt(
a) + A*sqrt(c))*c)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))
^(2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - (C*a*sqrt(c) - (B*sqrt(a) -
A*sqrt(c))*c)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3
))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 2*(C*a*sqrt(c) - (B*sqrt(a) - A*s
qrt(c))*c)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(
2/3)) - 2*(C*a*sqrt(c) + (B*sqrt(a) + A*sqrt(c))*c)*log(x - (sqrt(a)/sqrt(
c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)))/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3+Cx^6}{a-cx^6}\,dx$$

$$= \frac{B|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6\left(-ac^5\right)^{\frac{1}{3}}} - \frac{Cx}{c} + \frac{\left(\left(-ac^5\right)^{\frac{1}{6}}Ca+\left(-ac^5\right)^{\frac{1}{6}}Ac\right)\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac^2}$$
$$+ \frac{\left(\left(-ac^5\right)^{\frac{1}{6}}Cac^2+\left(-ac^5\right)^{\frac{1}{6}}Ac^3-\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}$$
$$+ \frac{\left(\left(-ac^5\right)^{\frac{1}{6}}Cac^2+\left(-ac^5\right)^{\frac{1}{6}}Ac^3+\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^4}$$
$$+ \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Cac^2+\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Ac^3+\left(-ac^5\right)^{\frac{2}{3}}B\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$
$$- \frac{\left(\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Cac^2+\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Ac^3-\left(-ac^5\right)^{\frac{2}{3}}B\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^4}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a),x, algorithm="giac")
```

output
```
1/6*B*abs(c)*log(x^2 + (-a/c)^(1/3))/(-a*c^5)^(1/3) - C*x/c + 1/3*((-a*c^5
)^(1/6)*C*a + (-a*c^5)^(1/6)*A*c)*arctan(x/(-a/c)^(1/6))/(a*c^2) + 1/6*((-
a*c^5)^(1/6)*C*a*c^2 + (-a*c^5)^(1/6)*A*c^3 - sqrt(3)*(-a*c^5)^(2/3)*B)*ar
ctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/6*((-a*c^5)^(1
/6)*C*a*c^2 + (-a*c^5)^(1/6)*A*c^3 + sqrt(3)*(-a*c^5)^(2/3)*B)*arctan((2*x
 - sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^4) + 1/12*(sqrt(3)*(-a*c^5)^(1
/6)*C*a*c^2 + sqrt(3)*(-a*c^5)^(1/6)*A*c^3 + (-a*c^5)^(2/3)*B)*log(x^2 + s
qrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4) - 1/12*(sqrt(3)*(-a*c^5)^(1/
6)*C*a*c^2 + sqrt(3)*(-a*c^5)^(1/6)*A*c^3 - (-a*c^5)^(2/3)*B)*log(x^2 - sq
rt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 2973, normalized size of antiderivative = 8.26

$$\int \frac{A + Bx^3 + Cx^6}{a - cx^6} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^3 + C*x^6)/(a - c*x^6),x)
```

output
```
log(B*x*(a^5*c^7)^(1/2) + a^3*c^4*(-(A^3*c^3*(a^5*c^7)^(1/2) + C^3*a^3*(a^
5*c^7)^(1/2) + B^3*a^4*c^5 + 3*A^2*B*a^3*c^6 + 3*B*C^2*a^5*c^4 + 3*A*B^2*a
*c^2*(a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(a^5*c^7)^(1/2) + 3*A^2*C*a*c^2*(a^5*
c^7)^(1/2) + 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*(a^5*c^7)^(1/2))/(a^5*c^7))^(
1/3) + A*a^2*c^4*x + C*a^3*c^3*x)*(-(A^3*c^3*(a^5*c^7)^(1/2) + C^3*a^3*(a^
5*c^7)^(1/2) + B^3*a^4*c^5 + 3*A^2*B*a^3*c^6 + 3*B*C^2*a^5*c^4 + 3*A*B^2*a
*c^2*(a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(a^5*c^7)^(1/2) + 3*A^2*C*a*c^2*(a^5*
c^7)^(1/2) + 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*(a^5*c^7)^(1/2))/(216*a^5*c^7
))^(1/3) + log(a^3*c^4*((A^3*c^3*(a^5*c^7)^(1/2) + C^3*a^3*(a^5*c^7)^(1/2)
 - B^3*a^4*c^5 - 3*A^2*B*a^3*c^6 - 3*B*C^2*a^5*c^4 + 3*A*B^2*a*c^2*(a^5*c^
7)^(1/2) + 3*A*C^2*a^2*c*(a^5*c^7)^(1/2) + 3*A^2*C*a*c^2*(a^5*c^7)^(1/2) -
 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*(a^5*c^7)^(1/2))/(a^5*c^7))^(1/3) - B*x*(
a^5*c^7)^(1/2) + A*a^2*c^4*x + C*a^3*c^3*x)*((A^3*c^3*(a^5*c^7)^(1/2) + C^
3*a^3*(a^5*c^7)^(1/2) - B^3*a^4*c^5 - 3*A^2*B*a^3*c^6 - 3*B*C^2*a^5*c^4 +
3*A*B^2*a*c^2*(a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(a^5*c^7)^(1/2) + 3*A^2*C*a*
c^2*(a^5*c^7)^(1/2) - 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*(a^5*c^7)^(1/2))/(21
6*a^5*c^7))^(1/3) - log(B*x*(a^5*c^7)^(1/2) - (a^3*c^4*(-(A^3*c^3*(a^5*c^7
)^(1/2) + C^3*a^3*(a^5*c^7)^(1/2) + B^3*a^4*c^5 + 3*A^2*B*a^3*c^6 + 3*B*C^
2*a^5*c^4 + 3*A*B^2*a*c^2*(a^5*c^7)^(1/2) + 3*A*C^2*a^2*c*(a^5*c^7)^(1/2)
+ 3*A^2*C*a*c^2*(a^5*c^7)^(1/2) + 6*A*B*C*a^4*c^5 + 3*B^2*C*a^2*c*(a^5*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^3+Cx^6}{a-cx^6}\,dx = \frac{-4\sqrt{c}\sqrt{a}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}-2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)b+4\sqrt{c}\sqrt{a}\sqrt{3}\,atan\left(\frac{c^{\frac{1}{6}}a^{\frac{1}{6}}+2c^{\frac{1}{3}}x}{c^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}}\right)+2\sqrt{3}\,atan\left(\right.}{}$$

input
```
int((C*x^6+B*x^3+A)/(-c*x^6+a),x)
```

output

```
( - 4*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)*sqrt(3))) + 2*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*
x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b + 4*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/
6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3))) + 2*sqrt(3)*atan(
(c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b - 2*sqrt
(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2) + 4*sqr
t(c)*sqrt(a)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x) + 2*sqrt(c)*sqrt(a)*lo
g(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2) - 4*sqrt(c)*sqrt(a)*log(
c**(1/6)*a**(1/6) - c**(1/3)*x) - 12*c**(2/3)*a**(1/3)*x + log( - c**(1/6)
*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*b - 2*log( - c**(1/6)*a**(1/6) - c
**(1/3)*x)*b + log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*x**2)*b - 2*l
og(c**(1/6)*a**(1/6) - c**(1/3)*x)*b)/(12*c**(2/3)*a**(1/3))
```

3.16 $\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 393

$$\begin{aligned}\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx &= \frac{x\left(A+\frac{aC}{c}+Bx^3\right)}{6a\,(a-cx^6)} \\ &+ \frac{\left(2\sqrt{a}B\sqrt{c}-5Ac+aC\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{12\sqrt{3}a^{11/6}c^{7/6}} \\ &+ \frac{\left(2\sqrt{a}B\sqrt{c}+5Ac-aC\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{12\sqrt{3}a^{11/6}c^{7/6}} \\ &- \frac{\left(2\sqrt{a}B\sqrt{c}+5Ac-aC\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{36a^{11/6}c^{7/6}} \\ &- \frac{\left(2\sqrt{a}B\sqrt{c}-5Ac+aC\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{36a^{11/6}c^{7/6}} \\ &+ \frac{\left(2\sqrt{a}B\sqrt{c}-5Ac+aC\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{72a^{11/6}c^{7/6}} \\ &+ \frac{\left(2\sqrt{a}B\sqrt{c}+5Ac-aC\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{72a^{11/6}c^{7/6}}\end{aligned}$$

output
```
1/6*x*(A+a*C/c+B*x^3)/a/(-c*x^6+a)+1/36*(2*a^(1/2)*B*c^(1/2)-5*A*c+C*a)*ar
ctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(11/6)/c^(7/6)+1
/36*(2*a^(1/2)*B*c^(1/2)+5*A*c-C*a)*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)*3^(1/
2)/a^(1/6))*3^(1/2)/a^(11/6)/c^(7/6)-1/36*(2*a^(1/2)*B*c^(1/2)+5*A*c-C*a)*
ln(a^(1/6)-c^(1/6)*x)/a^(11/6)/c^(7/6)-1/36*(2*a^(1/2)*B*c^(1/2)-5*A*c+C*a
)*ln(a^(1/6)+c^(1/6)*x)/a^(11/6)/c^(7/6)+1/72*(2*a^(1/2)*B*c^(1/2)-5*A*c+C
*a)*ln(a^(1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(11/6)/c^(7/6)+1/72*(2*a^(
1/2)*B*c^(1/2)+5*A*c-C*a)*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(11/
6)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx$$

$$= \frac{\frac{12a^{5/6}\sqrt[6]{c}x(Ac+aC+Bcx^3)}{a-cx^6} + 2\sqrt{3}\left(2\sqrt{a}B\sqrt{c}-5Ac+aC\right)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right) - 2\sqrt{3}\left(-2\sqrt{a}B\sqrt{c}-5Ac+a\right.}{}$$

input
```
Integrate[(A + B*x^3 + C*x^6)/(a - c*x^6)^2,x]
```

output
```
((12*a^(5/6)*c^(1/6)*x*(A*c + a*C + B*c*x^3))/(a - c*x^6) + 2*Sqrt[3]*(2*S
qrt[a]*B*Sqrt[c] - 5*A*c + a*C)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]
] - 2*Sqrt[3]*(-2*Sqrt[a]*B*Sqrt[c] - 5*A*c + a*C)*ArcTan[(1 + (2*c^(1/6)*
x)/a^(1/6))/Sqrt[3]] + 2*(-2*Sqrt[a]*B*Sqrt[c] - 5*A*c + a*C)*Log[a^(1/6)
- c^(1/6)*x] - 2*(2*Sqrt[a]*B*Sqrt[c] - 5*A*c + a*C)*Log[a^(1/6) + c^(1/6)
*x] + (2*Sqrt[a]*B*Sqrt[c] - 5*A*c + a*C)*Log[a^(1/3) - a^(1/6)*c^(1/6)*x
+ c^(1/3)*x^2] - (-2*Sqrt[a]*B*Sqrt[c] - 5*A*c + a*C)*Log[a^(1/3) + a^(1/6
)*c^(1/6)*x + c^(1/3)*x^2])/(72*a^(11/6)*c^(7/6))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.86, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2397, 1747, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx$$

$$\Big\downarrow \; 2397$$

$$\frac{\int \frac{2Bcx^3+5Ac-aC}{a-cx^6}dx}{6ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$$\Big\downarrow \; 1747$$

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^3}dx-\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}+aC-5Ac\right)\int \frac{1}{\sqrt{a}\sqrt{c}x^3+a}dx}{6ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$$\Big\downarrow \; 750$$

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}dx}{3a^{2/3}}\right)-\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}+aC-5Ac\right)\left(\frac{\int \frac{\cdots}{\sqrt[3]{a}\cdots}}{\cdots}\right.}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}+aC-5Ac\right)\left(\frac{\int \frac{\sqrt[6]{a}\cdots}{\sqrt[3]{a}\sqrt[3]{\cdots}}}{\cdots}\right.}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}+aC-5Ac\right)\left(\frac{\int \frac{2}{\sqrt[3]{a}\sqrt[3]{c}}}{}\right.}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$\downarrow$ 1142

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}(2\sqrt{}}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}(2\sqrt{}}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}(2}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$\downarrow$ 1082

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}(2\sqrt{}}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$\Big\downarrow$ 217

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}+a\right.}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}-aC+5Ac\right)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(2\sqrt{a}B\sqrt{c}+aC\right.}{6ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{6ac\left(a-cx^6\right)}$$

input

```
Int[(A + B*x^3 + C*x^6)/(a - c*x^6)^2,x]
```

output
```
(x*(A*c + a*C + B*c*x^3))/(6*a*c*(a - c*x^6)) + (-1/2*((2*Sqrt[a]*B*Sqrt[c
] - 5*A*c + a*C)*(Log[a^(1/6) + c^(1/6)*x]/(3*a^(5/6)*c^(1/6)) + (-((Sqrt[
3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/(a^(1/3)*c^(1/6))) - Log[a
^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^(1/3)*c^(1/6)))/(3*Sqrt[a])
)) + ((2*Sqrt[a]*B*Sqrt[c] + 5*A*c - a*C)*(-1/3*Log[a^(1/6) - c^(1/6)*x]/(
a^(5/6)*c^(1/6)) + ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/
(a^(1/3)*c^(1/6)) + Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^(1
/3)*c^(1/6)))/(3*Sqrt[a])))/2)/(6*a*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1747
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Simp[(d + e*q)/2   Int[1/(a + c*q*x^n), x], x] + Simp[(d
- e*q)/2   Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\frac{B x^4}{6a}+\frac{(Ac+Ca)x}{6ac}}{-c x^6+a}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6-a\right)}\frac{\left(2B\_R^3+\frac{5Ac-Ca}{c}\right)\ln\left(x-\_R\right)}{\_R^5}}{36ac}$
default	$\frac{\frac{B x^4}{6a}+\frac{(Ac+Ca)x}{6ac}}{-c x^6+a}+\frac{\frac{cB\left(\frac{a}{c}\right)^{\frac{2}{3}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{6a}+\frac{5cA\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}-\frac{C\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12}+\frac{cB\left(\frac{a}{c}\right.}{}}{}$

input
```
int((C*x^6+B*x^3+A)/(-c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/6*B/a*x^4+1/6*(A*c+C*a)/a/c*x)/(-c*x^6+a)-1/36/a/c*sum((2*B*_R^3+1/c*(5
*A*c-C*a))/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5673 vs. $2(289) = 578$.

Time = 7.41 (sec) , antiderivative size = 5673, normalized size of antiderivative = 14.44

$$\int \frac{A + Bx^3 + Cx^6}{(a - cx^6)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3 + Cx^6}{(a - cx^6)^2} \, dx = \text{Timed out}$$

input
```
integrate((C*x**6+B*x**3+A)/(-c*x**6+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx = -\frac{Bcx^4+(Ca+Ac)x}{6\,(ac^2x^6-a^2c)} - \frac{\frac{2\sqrt{3}(Ca\sqrt{c}-(2B\sqrt{a}+5A\sqrt{c})c)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{2\sqrt{3}(Ca\sqrt{c}+(2B\sqrt{a}-5A\sqrt{c})c)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}c\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}} + \frac{(Ca}{}}{}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(B*c*x^4 + (C*a + A*c)*x)/(a*c^2*x^6 - a^2*c) - 1/72*(2*sqrt(3)*(C*a*
sqrt(c) - (2*B*sqrt(a) + 5*A*sqrt(c))*c)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a
)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2
/3)) + 2*sqrt(3)*(C*a*sqrt(c) + (2*B*sqrt(a) - 5*A*sqrt(c))*c)*arctan(1/3*
sqrt(3)*(2*x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*
c*(sqrt(a)/sqrt(c))^(2/3)) + (C*a*sqrt(c) - (2*B*sqrt(a) + 5*A*sqrt(c))*c)
*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c
*(sqrt(a)/sqrt(c))^(2/3)) - (C*a*sqrt(c) + (2*B*sqrt(a) - 5*A*sqrt(c))*c)*
log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*c*
(sqrt(a)/sqrt(c))^(2/3)) + 2*(C*a*sqrt(c) + (2*B*sqrt(a) - 5*A*sqrt(c))*c)
*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - 2*
(C*a*sqrt(c) - (2*B*sqrt(a) + 5*A*sqrt(c))*c)*log(x - (sqrt(a)/sqrt(c))^(1
/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx = \frac{B|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{18\,(-ac^5)^{\frac{1}{3}}\,a} - \frac{Bcx^4+Cax+Acx}{6\,(cx^6-a)ac} - \frac{\left((-ac^5)^{\frac{1}{6}}\,Ca-5\,(-ac^5)^{\frac{1}{6}}\,Ac\right)\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{18\,a^2c^2} - \frac{\left((-ac^5)^{\frac{1}{6}}\,Cac^2-5\,(-ac^5)^{\frac{1}{6}}\,Ac^3+2\,\sqrt{3}(-ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^4} - \frac{\left((-ac^5)^{\frac{1}{6}}\,Cac^2-5\,(-ac^5)^{\frac{1}{6}}\,Ac^3-2\,\sqrt{3}(-ac^5)^{\frac{2}{3}}\,B\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^4} - \frac{\left(\sqrt{3}(-ac^5)^{\frac{1}{6}}\,Cac^2-5\,\sqrt{3}(-ac^5)^{\frac{1}{6}}\,Ac^3-2\,(-ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^4} + \frac{\left(\sqrt{3}(-ac^5)^{\frac{1}{6}}\,Cac^2-5\,\sqrt{3}(-ac^5)^{\frac{1}{6}}\,Ac^3+2\,(-ac^5)^{\frac{2}{3}}\,B\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^4}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a)^2,x, algorithm="giac")
```

output
```
1/18*B*abs(c)*log(x^2 + (-a/c)^(1/3))/((-a*c^5)^(1/3)*a) - 1/6*(B*c*x^4 +
C*a*x + A*c*x)/((c*x^6 - a)*a*c) - 1/18*((-a*c^5)^(1/6)*C*a - 5*(-a*c^5)^(
1/6)*A*c)*arctan(x/(-a/c)^(1/6))/(a^2*c^2) - 1/36*((-a*c^5)^(1/6)*C*a*c^2
- 5*(-a*c^5)^(1/6)*A*c^3 + 2*sqrt(3)*(-a*c^5)^(2/3)*B)*arctan((2*x + sqrt(
3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^2*c^4) - 1/36*((-a*c^5)^(1/6)*C*a*c^2 -
5*(-a*c^5)^(1/6)*A*c^3 - 2*sqrt(3)*(-a*c^5)^(2/3)*B)*arctan((2*x - sqrt(3)
*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^2*c^4) - 1/72*(sqrt(3)*(-a*c^5)^(1/6)*C*a*
c^2 - 5*sqrt(3)*(-a*c^5)^(1/6)*A*c^3 - 2*(-a*c^5)^(2/3)*B)*log(x^2 + sqrt(
3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^2*c^4) + 1/72*(sqrt(3)*(-a*c^5)^(1/6)
*C*a*c^2 - 5*sqrt(3)*(-a*c^5)^(1/6)*A*c^3 + 2*(-a*c^5)^(2/3)*B)*log(x^2 -
sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 3043, normalized size of antiderivative = 7.74

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^3 + C*x^6)/(a - c*x^6)^2,x)
```

output
```
((B*x^4)/(6*a) + (x*(A*c + C*a))/(6*a*c))/(a - c*x^6) + log(2*B*x*(a^11*c^
7)^(1/2) - a^7*c^4*(-(C^3*a^3*(a^11*c^7)^(1/2) - 125*A^3*c^3*(a^11*c^7)^(1
/2) + 8*B^3*a^7*c^5 + 150*A^2*B*a^6*c^6 + 6*B*C^2*a^8*c^4 - 60*A*B^2*a*c^2
*(a^11*c^7)^(1/2) - 15*A*C^2*a^2*c*(a^11*c^7)^(1/2) + 75*A^2*C*a*c^2*(a^11
*c^7)^(1/2) - 60*A*B*C*a^7*c^5 + 12*B^2*C*a^2*c*(a^11*c^7)^(1/2))/(a^11*c^
7))^(1/3) - 5*A*a^5*c^4*x + C*a^6*c^3*x)*(-(C^3*a^3*(a^11*c^7)^(1/2) - 125
*A^3*c^3*(a^11*c^7)^(1/2) + 8*B^3*a^7*c^5 + 150*A^2*B*a^6*c^6 + 6*B*C^2*a^
8*c^4 - 60*A*B^2*a*c^2*(a^11*c^7)^(1/2) - 15*A*C^2*a^2*c*(a^11*c^7)^(1/2)
+ 75*A^2*C*a*c^2*(a^11*c^7)^(1/2) - 60*A*B*C*a^7*c^5 + 12*B^2*C*a^2*c*(a^1
1*c^7)^(1/2))/(46656*a^11*c^7))^(1/3) + log(2*B*x*(a^11*c^7)^(1/2) + a^7*c
^4*(-(125*A^3*c^3*(a^11*c^7)^(1/2) - C^3*a^3*(a^11*c^7)^(1/2) + 8*B^3*a^7*
c^5 + 150*A^2*B*a^6*c^6 + 6*B*C^2*a^8*c^4 + 60*A*B^2*a*c^2*(a^11*c^7)^(1/2
) + 15*A*C^2*a^2*c*(a^11*c^7)^(1/2) - 75*A^2*C*a*c^2*(a^11*c^7)^(1/2) - 60
*A*B*C*a^7*c^5 - 12*B^2*C*a^2*c*(a^11*c^7)^(1/2))/(a^11*c^7))^(1/3) + 5*A*
a^5*c^4*x - C*a^6*c^3*x)*(-(125*A^3*c^3*(a^11*c^7)^(1/2) - C^3*a^3*(a^11*c
^7)^(1/2) + 8*B^3*a^7*c^5 + 150*A^2*B*a^6*c^6 + 6*B*C^2*a^8*c^4 + 60*A*B^2
*a*c^2*(a^11*c^7)^(1/2) + 15*A*C^2*a^2*c*(a^11*c^7)^(1/2) - 75*A^2*C*a*c^2
*(a^11*c^7)^(1/2) - 60*A*B*C*a^7*c^5 - 12*B^2*C*a^2*c*(a^11*c^7)^(1/2))/(4
6656*a^11*c^7))^(1/3) + log(4*B*x*(a^11*c^7)^(1/2) + a^7*c^4*(-(C^3*a^3*(a
^11*c^7)^(1/2) - 125*A^3*c^3*(a^11*c^7)^(1/2) + 8*B^3*a^7*c^5 + 150*A^2...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^6+B*x^3+A)/(-c*x^6+a)^2,x)
```

output
```
( - 4*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)*sqrt(3)))*a + 4*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(
1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*c*x**6 + 2*sqrt(3)*atan(
(c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*b - 2*sq
rt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))
*b*c*x**6 + 4*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)
*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a - 4*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1
/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*c*x**6 + 2*sqrt(
3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*
b - 2*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*s
qrt(3)))*b*c*x**6 - 2*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(1/3
) + c**(1/3)*x**2)*a + 2*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(
1/3) + c**(1/3)*x**2)*c*x**6 + 4*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)
- c**(1/3)*x)*a - 4*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6) - c**(1/3)*x)
*c*x**6 + 2*sqrt(c)*sqrt(a)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*
x**2)*a - 2*sqrt(c)*sqrt(a)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)*
x**2)*c*x**6 - 4*sqrt(c)*sqrt(a)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a + 4
*sqrt(c)*sqrt(a)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*c*x**6 + 12*c**(2/3)*
a**(1/3)*a*x + 6*c**(2/3)*a**(1/3)*b*x**4 + log( - c**(1/6)*a**(1/6)*x + a
**(1/3) + c**(1/3)*x**2)*a*b - log( - c**(1/6)*a**(1/6)*x + a**(1/3) + ...
```

3.17 $\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 431

$$\begin{aligned}\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^3}\,dx &= \frac{x\left(A+\frac{aC}{c}+Bx^3\right)}{12a(a-cx^6)^2}+\frac{x\left(11A-\frac{aC}{c}+8Bx^3\right)}{72a^2(a-cx^6)} \\ &+ \frac{\left(16\sqrt{a}B\sqrt{c}-55Ac+5aC\right)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{144\sqrt{3}a^{17/6}c^{7/6}} \\ &+ \frac{\left(16\sqrt{a}B\sqrt{c}+55Ac-5aC\right)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{144\sqrt{3}a^{17/6}c^{7/6}} \\ &- \frac{\left(16\sqrt{a}B\sqrt{c}+55Ac-5aC\right)\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{432a^{17/6}c^{7/6}} \\ &- \frac{\left(16\sqrt{a}B\sqrt{c}-55Ac+5aC\right)\log\left(\sqrt[6]{a}+\sqrt[6]{c}x\right)}{432a^{17/6}c^{7/6}} \\ &+ \frac{\left(16\sqrt{a}B\sqrt{c}-55Ac+5aC\right)\log\left(\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{864a^{17/6}c^{7/6}} \\ &+ \frac{\left(16\sqrt{a}B\sqrt{c}+55Ac-5aC\right)\log\left(\sqrt[3]{a}+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{c}x^2\right)}{864a^{17/6}c^{7/6}}\end{aligned}$$

output
```
1/12*x*(A+a*C/c+B*x^3)/a/(-c*x^6+a)^2+1/72*x*(11*A-a*C/c+8*B*x^3)/a^2/(-c*
x^6+a)+1/432*(16*a^(1/2)*B*c^(1/2)-55*A*c+5*C*a)*arctan(1/3*(a^(1/6)-2*c^(
1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(17/6)/c^(7/6)+1/432*(16*a^(1/2)*B*c^(1
/2)+55*A*c-5*C*a)*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2
)/a^(17/6)/c^(7/6)-1/432*(16*a^(1/2)*B*c^(1/2)+55*A*c-5*C*a)*ln(a^(1/6)-c^
(1/6)*x)/a^(17/6)/c^(7/6)-1/432*(16*a^(1/2)*B*c^(1/2)-55*A*c+5*C*a)*ln(a^(
1/6)+c^(1/6)*x)/a^(17/6)/c^(7/6)+1/864*(16*a^(1/2)*B*c^(1/2)-55*A*c+5*C*a)
*ln(a^(1/3)-a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(17/6)/c^(7/6)+1/864*(16*a^(1
/2)*B*c^(1/2)+55*A*c-5*C*a)*ln(a^(1/3)+a^(1/6)*c^(1/6)*x+c^(1/3)*x^2)/a^(1
7/6)/c^(7/6)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^3}\,dx$$

$$= \frac{\frac{72a^{11/6}\sqrt[6]{c}x(Ac+aC+Bcx^3)}{(a-cx^6)^2} - \frac{12a^{5/6}\sqrt[6]{c}(-11Acx+aCx-8Bcx^4)}{a-cx^6} + 2\sqrt{3}(16\sqrt{a}B\sqrt{c}-55Ac+5aC)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}}{\sqrt[6]{c}}}{\sqrt{3}}\right.}{}$$

input
```
Integrate[(A + B*x^3 + C*x^6)/(a - c*x^6)^3,x]
```

output
```
((72*a^(11/6)*c^(1/6)*x*(A*c + a*C + B*c*x^3))/(a - c*x^6)^2 - (12*a^(5/6)
*c^(1/6)*(-11*A*c*x + a*C*x - 8*B*c*x^4))/(a - c*x^6) + 2*Sqrt[3]*(16*Sqrt
[a]*B*Sqrt[c] - 55*A*c + 5*a*C)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]
] - 2*Sqrt[3]*(-16*Sqrt[a]*B*Sqrt[c] - 55*A*c + 5*a*C)*ArcTan[(1 + (2*c^(1
/6)*x)/a^(1/6))/Sqrt[3]] + 2*(-16*Sqrt[a]*B*Sqrt[c] - 55*A*c + 5*a*C)*Log[
a^(1/6) - c^(1/6)*x] - 2*(16*Sqrt[a]*B*Sqrt[c] - 55*A*c + 5*a*C)*Log[a^(1/
6) + c^(1/6)*x] + (16*Sqrt[a]*B*Sqrt[c] - 55*A*c + 5*a*C)*Log[a^(1/3) - a^
(1/6)*c^(1/6)*x + c^(1/3)*x^2] - (-16*Sqrt[a]*B*Sqrt[c] - 55*A*c + 5*a*C)*
Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(864*a^(17/6)*c^(7/6))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.88, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {2397, 1761, 25, 1747, 750, 16, 27, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^3}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{\int \frac{8Bcx^3+11Ac-aC}{(a-cx^6)^2}dx}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$$\Big\downarrow \text{ 1761}$$

$$\frac{\frac{x\left(-aC+11Ac+8Bcx^3\right)}{6a(a-cx^6)}-\frac{\int -\frac{16Bcx^3+5(11Ac-aC)}{a-cx^6}dx}{6a}}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{16Bcx^3+5(11Ac-aC)}{a-cx^6}dx}{6a}+\frac{x\left(-aC+11Ac+8Bcx^3\right)}{6a(a-cx^6)}}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$$\Big\downarrow \text{ 1747}$$

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\int \frac{1}{a-\sqrt{a}\sqrt{c}x^3}dx-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5aC-55Ac\right)\int \frac{1}{\sqrt{a}\sqrt{c}x^3+a}dx}{6a}+\frac{x\left(-aC+11Ac+8Bcx^3\right)}{6a(a-cx^6)}}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[6]{a}\sqrt[6]{c}x}dx}{3a^{2/3}}\right)-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5aC-55Ac\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a \cdots}}{3a^{2/3}} \cdots\right)}{6a}}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$\downarrow$ 16

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(\sqrt[6]{c}x+2\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5aC-55Ac\right)\left(\frac{\int \frac{\sqrt[6]{a}\left(2\sqrt[6]{a}-\sqrt[6]{c}x\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}} \cdots}{3a^{2/3}} \cdots\right)}{6a}}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\left(\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5aC-55Ac\right)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{a}\sqrt[3]{c}x^2-\sqrt{a}\sqrt[6]{c}x+a^{2/3}} \cdots}{3\sqrt{a}} \cdots\right)}{6a}}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$\downarrow$ 1142

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5aC \cdots\right)}{6a}}{12ac}+\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{a}\sqrt[6]{c}\left(2\sqrt[6]{c}x+\sqrt[6]{a}\right)}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5aC\right.}{6a}}{12ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5a\right.}{6a}}{12ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}-5aC+55Ac\right)\left(\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx-\frac{3\int\frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}\left(16\sqrt{a}B\sqrt{c}+5aC\right.}{6a}}{12ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{1}{2}(16\sqrt{a}B\sqrt{c}-5aC+55Ac)\left(\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{a}\sqrt[3]{c}x^2+\sqrt{a}\sqrt[6]{c}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}(16\sqrt{a}B\sqrt{c}+5aC-55Ac)\left(\ldots\right.}{6a}}{12ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{1}{2}(16\sqrt{a}B\sqrt{c}-5aC+55Ac)\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[3]{a}\sqrt[6]{c}}}{3\sqrt{a}}-\frac{\log\left(\sqrt[6]{a}-\sqrt[6]{c}x\right)}{3a^{5/6}\sqrt[6]{c}}\right)-\frac{1}{2}(16\sqrt{a}B\sqrt{c}+5aC-55Ac)\left(\frac{\log\left(\ldots\right)}{3a\ldots}\right.}{6a}}{12ac}$$

$$\frac{x\left(aC+Ac+Bcx^3\right)}{12ac\left(a-cx^6\right)^2}$$

input

```
Int[(A + B*x^3 + C*x^6)/(a - c*x^6)^3,x]
```

output

```
(x*(A*c + a*C + B*c*x^3))/(12*a*c*(a - c*x^6)^2) + ((x*(11*A*c - a*C + 8*B
*c*x^3))/(6*a*(a - c*x^6)) + (-1/2*((16*Sqrt[a]*B*Sqrt[c] - 55*A*c + 5*a*C
)*(Log[a^(1/6) + c^(1/6)*x]/(3*a^(5/6)*c^(1/6)) + (-((Sqrt[3]*ArcTan[(1 -
(2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/(a^(1/3)*c^(1/6))) - Log[a^(1/3) - a^(1/6
)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^(1/3)*c^(1/6)))/(3*Sqrt[a]))) + ((16*Sqrt[
a]*B*Sqrt[c] + 55*A*c - 5*a*C)*(-1/3*Log[a^(1/6) - c^(1/6)*x]/(a^(5/6)*c^(
1/6)) + ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/(a^(1/3)*c^
(1/6)) + Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*a^(1/3)*c^(1/6)
))/(3*Sqrt[a])))/2)/(6*a))/(12*a*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1747
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> With[{
q = Rt[-a/c, 2]}, Simp[(d + e*q)/2   Int[1/(a + c*q*x^n), x], x] + Simp[(d
- e*q)/2   Int[1/(a - c*q*x^n), x], x]] /; FreeQ[{a, c, d, e, n}, x] && EqQ
[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && NegQ[a*c] && IntegerQ[n]
```

rule 1761
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Simp[1
/(2*a*n*(p + 1))   Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
 ILtQ[p, -1]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.28

method	result
risch	$\frac{-\frac{B c\,x^{10}}{9 a^{2}}-\frac{\left(11 A c -C a \right) x^{7}}{72 a^{2}}+\frac{7 B\,x^{4}}{36 a}+\frac{\left(17 A c +5 C a \right) x}{72 a c}}{\left(-c\,x^{6}+a \right)^{2}}-\frac{\sum_{\textit{\_R} =\operatorname{RootOf}\left(c\,\textit{\_Z}^{6}-a \right)}\frac{\left(16 B\,\textit{\_R}^{3}+\frac{55 A c -5 C a}{c}\right) \ln \left(x -\textit{\_R} \right)}{\textit{\_R}^{5}}}{432 a^{2} c}$
default	$\frac{-\frac{B c\,x^{10}}{9 a^{2}}-\frac{\left(11 A c -C a \right) x^{7}}{72 a^{2}}+\frac{7 B\,x^{4}}{36 a}+\frac{\left(17 A c +5 C a \right) x}{72 a c}}{\left(-c\,x^{6}+a \right)^{2}}+ \frac{\frac{4 c B \left(\frac{a}{c}\right)^{\frac{2}{3}} \ln \left(x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{6}} x +\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{3 a}+\frac{8 c B \left(\frac{a}{c}\right)^{\frac{2}{3}} \sqrt{3}\, \arctan \left(\frac{2 x \sqrt{3}}{3 \left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{3 a}+ 55 c A (}{}$

input
```
int((C*x^6+B*x^3+A)/(-c*x^6+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/9*B*c/a^2*x^10-1/72*(11*A*c-C*a)/a^2*x^7+7/36*B/a*x^4+1/72*(17*A*c+5*C
*a)/a/c*x)/(-c*x^6+a)^2-1/432/a^2/c*sum((16*B*_R^3+5/c*(11*A*c-C*a))/_R^5*
ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5890 vs. $2(329) = 658$.

Time = 9.52 (sec) , antiderivative size = 5890, normalized size of antiderivative = 13.67

$$\int \frac{A + Bx^3 + Cx^6}{(a - cx^6)^3} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^3 + Cx^6}{(a - cx^6)^3} \, dx = \text{Timed out}$$

input
```
integrate((C*x**6+B*x**3+A)/(-c*x**6+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^3+Cx^6}{(a-cx^6)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a)^3,x, algorithm="maxima")
```

output
```
-1/72*(8*B*c^2*x^10 - (C*a*c - 11*A*c^2)*x^7 - 14*B*a*c*x^4 - (5*C*a^2 + 1
7*A*a*c)*x)/(a^2*c^3*x^12 - 2*a^3*c^2*x^6 + a^4*c) - 1/864*(2*sqrt(3)*(5*C
*a*sqrt(c) - (16*B*sqrt(a) + 55*A*sqrt(c))*c)*arctan(1/3*sqrt(3)*(2*x + (s
qrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c
))^(2/3)) + 2*sqrt(3)*(5*C*a*sqrt(c) + (16*B*sqrt(a) - 55*A*sqrt(c))*c)*ar
ctan(1/3*sqrt(3)*(2*x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/
(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + (5*C*a*sqrt(c) - (16*B*sqrt(a) + 55*
A*sqrt(c))*c)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3
))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) - (5*C*a*sqrt(c) + (16*B*sqrt(a) -
55*A*sqrt(c))*c)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(
2/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)) + 2*(5*C*a*sqrt(c) + (16*B*sqrt(
a) - 55*A*sqrt(c))*c)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)
/sqrt(c))^(2/3)) - 2*(5*C*a*sqrt(c) - (16*B*sqrt(a) + 55*A*sqrt(c))*c)*log
(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*c*(sqrt(a)/sqrt(c))^(2/3)))/(a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^3+Cx^6}{\left(a-cx^6\right)^3}\,dx$$
$$= \frac{B|c|\log\left(x^2+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{27\left(-ac^5\right)^{\frac{1}{3}}a^2} - \frac{5\left(\left(-ac^5\right)^{\frac{1}{6}}Ca-11\left(-ac^5\right)^{\frac{1}{6}}Ac\right)\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{216\,a^3c^2}$$
$$- \frac{8\,Bc^2x^{10}-Cacx^7+11\,Ac^2x^7-14\,Bacx^4-5\,Ca^2x-17\,Aacx}{72\left(cx^6-a\right)^2a^2c}$$
$$- \frac{\left(5\left(-ac^5\right)^{\frac{1}{6}}Cac^2-55\left(-ac^5\right)^{\frac{1}{6}}Ac^3+16\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{2x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^4}$$
$$- \frac{\left(5\left(-ac^5\right)^{\frac{1}{6}}Cac^2-55\left(-ac^5\right)^{\frac{1}{6}}Ac^3-16\sqrt{3}\left(-ac^5\right)^{\frac{2}{3}}B\right)\arctan\left(\frac{2x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^4}$$
$$- \frac{\left(5\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Cac^2-55\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Ac^3-16\left(-ac^5\right)^{\frac{2}{3}}B\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^4}$$
$$+ \frac{\left(5\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Cac^2-55\sqrt{3}\left(-ac^5\right)^{\frac{1}{6}}Ac^3+16\left(-ac^5\right)^{\frac{2}{3}}B\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^4}$$

input
```
integrate((C*x^6+B*x^3+A)/(-c*x^6+a)^3,x, algorithm="giac")
```

output
```
1/27*B*abs(c)*log(x^2 + (-a/c)^(1/3))/((-a*c^5)^(1/3)*a^2) - 5/216*((-a*c^
5)^(1/6)*C*a - 11*(-a*c^5)^(1/6)*A*c)*arctan(x/(-a/c)^(1/6))/(a^3*c^2) - 1
/72*(8*B*c^2*x^10 - C*a*c*x^7 + 11*A*c^2*x^7 - 14*B*a*c*x^4 - 5*C*a^2*x -
17*A*a*c*x)/((c*x^6 - a)^2*a^2*c) - 1/432*(5*(-a*c^5)^(1/6)*C*a*c^2 - 55*(
-a*c^5)^(1/6)*A*c^3 + 16*sqrt(3)*(-a*c^5)^(2/3)*B)*arctan((2*x + sqrt(3)*(
-a/c)^(1/6))/(-a/c)^(1/6))/(a^3*c^4) - 1/432*(5*(-a*c^5)^(1/6)*C*a*c^2 - 5
5*(-a*c^5)^(1/6)*A*c^3 - 16*sqrt(3)*(-a*c^5)^(2/3)*B)*arctan((2*x - sqrt(3
)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^3*c^4) - 1/864*(5*sqrt(3)*(-a*c^5)^(1/6)*
C*a*c^2 - 55*sqrt(3)*(-a*c^5)^(1/6)*A*c^3 - 16*(-a*c^5)^(2/3)*B)*log(x^2 +
 sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^3*c^4) + 1/864*(5*sqrt(3)*(-a*c
^5)^(1/6)*C*a*c^2 - 55*sqrt(3)*(-a*c^5)^(1/6)*A*c^3 + 16*(-a*c^5)^(2/3)*B)
*log(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^3*c^4)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 3091, normalized size of antiderivative = 7.17

$$\int \frac{A + Bx^3 + Cx^6}{(a - cx^6)^3} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^3 + C*x^6)/(a - c*x^6)^3,x)
```

output
```
((7*B*x^4)/(36*a) - (x^7*(11*A*c - C*a))/(72*a^2) - (B*c*x^10)/(9*a^2) + (
x*(17*A*c + 5*C*a))/(72*a*c))/(a^2 + c^2*x^12 - 2*a*c*x^6) + log(16*B*x*(a
^17*c^7)^(1/2) + a^11*c^4*(-(166375*A^3*c^3*(a^17*c^7)^(1/2) - 125*C^3*a^3
*(a^17*c^7)^(1/2) + 4096*B^3*a^10*c^5 + 145200*A^2*B*a^9*c^6 + 1200*B*C^2*
a^11*c^4 + 42240*A*B^2*a*c^2*(a^17*c^7)^(1/2) + 4125*A*C^2*a^2*c*(a^17*c^7
)^(1/2) - 45375*A^2*C*a*c^2*(a^17*c^7)^(1/2) - 26400*A*B*C*a^10*c^5 - 3840
*B^2*C*a^2*c*(a^17*c^7)^(1/2))/(a^17*c^7))^(1/3) + 55*A*a^8*c^4*x - 5*C*a^
9*c^3*x)*(-(166375*A^3*c^3*(a^17*c^7)^(1/2) - 125*C^3*a^3*(a^17*c^7)^(1/2)
 + 4096*B^3*a^10*c^5 + 145200*A^2*B*a^9*c^6 + 1200*B*C^2*a^11*c^4 + 42240*
A*B^2*a*c^2*(a^17*c^7)^(1/2) + 4125*A*C^2*a^2*c*(a^17*c^7)^(1/2) - 45375*A
^2*C*a*c^2*(a^17*c^7)^(1/2) - 26400*A*B*C*a^10*c^5 - 3840*B^2*C*a^2*c*(a^1
7*c^7)^(1/2))/(80621568*a^17*c^7))^(1/3) + log(16*B*x*(a^17*c^7)^(1/2) - a
^11*c^4*(-(125*C^3*a^3*(a^17*c^7)^(1/2) - 166375*A^3*c^3*(a^17*c^7)^(1/2)
+ 4096*B^3*a^10*c^5 + 145200*A^2*B*a^9*c^6 + 1200*B*C^2*a^11*c^4 - 42240*A
*B^2*a*c^2*(a^17*c^7)^(1/2) - 4125*A*C^2*a^2*c*(a^17*c^7)^(1/2) + 45375*A^
2*C*a*c^2*(a^17*c^7)^(1/2) - 26400*A*B*C*a^10*c^5 + 3840*B^2*C*a^2*c*(a^17
*c^7)^(1/2))/(a^17*c^7))^(1/3) - 55*A*a^8*c^4*x + 5*C*a^9*c^3*x)*(-(125*C^
3*a^3*(a^17*c^7)^(1/2) - 166375*A^3*c^3*(a^17*c^7)^(1/2) + 4096*B^3*a^10*c
^5 + 145200*A^2*B*a^9*c^6 + 1200*B*C^2*a^11*c^4 - 42240*A*B^2*a*c^2*(a^17*
c^7)^(1/2) - 4125*A*C^2*a^2*c*(a^17*c^7)^(1/2) + 45375*A^2*C*a*c^2*(a^1...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.72

$$\int \frac{A + Bx^3 + Cx^6}{(a - cx^6)^3} \, dx = \text{Too large to display}$$

input
```
int((C*x^6+B*x^3+A)/(-c*x^6+a)^3,x)
```

output
```
( - 50*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c*
*(1/6)*a**(1/6)*sqrt(3)))*a**2 + 100*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6
)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*c*x**6 - 50*sqrt
(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(
1/6)*sqrt(3)))*c**2*x**12 + 16*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3
)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a**2*b - 32*sqrt(3)*atan((c**(1/6)*a**(1
/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*b*c*x**6 + 16*sqrt(3)*a
tan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b*c**2
*x**12 + 50*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x
)/(c**(1/6)*a**(1/6)*sqrt(3)))*a**2 - 100*sqrt(c)*sqrt(a)*sqrt(3)*atan((c*
*(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*c*x**6 + 50
*sqrt(c)*sqrt(a)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)
*a**(1/6)*sqrt(3)))*c**2*x**12 + 16*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c*
*(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a**2*b - 32*sqrt(3)*atan((c**(1/6)*
a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*b*c*x**6 + 16*sqrt
(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b
*c**2*x**12 - 25*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(1/3) + c
**(1/3)*x**2)*a**2 + 50*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6)*x + a**(1
/3) + c**(1/3)*x**2)*a*c*x**6 - 25*sqrt(c)*sqrt(a)*log( - c**(1/6)*a**(1/6
)*x + a**(1/3) + c**(1/3)*x**2)*c**2*x**12 + 50*sqrt(c)*sqrt(a)*log( - ...
```

3.18 $\int (a+cx^6)^3 (A+Bx^6+Cx^{12})\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\begin{aligned}\int (a+cx^6)^3 \left(A+Bx^6+Cx^{12}\right) dx &= a^3Ax+\frac{1}{7}a^2(aB+3Ac)x^7 \\ &+\frac{1}{13}a\left(3aBc+3Ac^2+a^2C\right)x^{13} \\ &+\frac{1}{19}c\left(3aBc+Ac^2+3a^2C\right)x^{19} \\ &+\frac{1}{25}c^2(Bc+3aC)x^{25}+\frac{1}{31}c^3Cx^{31}\end{aligned}$$

output
```
a^3*A*x+1/7*a^2*(3*A*c+B*a)*x^7+1/13*a*(3*A*c^2+3*B*a*c+C*a^2)*x^13+1/19*c
*(A*c^2+3*B*a*c+3*C*a^2)*x^19+1/25*c^2*(B*c+3*C*a)*x^25+1/31*c^3*C*x^31
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \left(a+cx^6\right)^3 \left(A+Bx^6+Cx^{12}\right)\,dx = a^3Ax+\frac{1}{7}a^2(aB+3Ac)x^7 +\frac{1}{13}a\left(3aBc+3Ac^2+a^2C\right)x^{13} +\frac{1}{19}c\left(3aBc+Ac^2+3a^2C\right)x^{19} +\frac{1}{25}c^2(Bc+3aC)x^{25}+\frac{1}{31}c^3Cx^{31}$$

input
```
Integrate[(a + c*x^6)^3*(A + B*x^6 + C*x^12),x]
```

output
```
a^3*A*x + (a^2*(a*B + 3*A*c)*x^7)/7 + (a*(3*a*B*c + 3*A*c^2 + a^2*C)*x^13)
/13 + (c*(3*a*B*c + A*c^2 + 3*a^2*C)*x^19)/19 + (c^2*(B*c + 3*a*C)*x^25)/2
5 + (c^3*C*x^31)/31
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^6\right)^3 \left(A+Bx^6+Cx^{12}\right)\,dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(a^3A+cx^{18}\left(3a^2C+3aBc+Ac^2\right)+ax^{12}\left(a^2C+3aBc+3Ac^2\right)+a^2x^6(aB+3Ac)+c^2x^{24}(3aC+Bc)+c^3C$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax+\frac{1}{19}cx^{19}\left(3a^2C+3aBc+Ac^2\right)+\frac{1}{13}ax^{13}\left(a^2C+3aBc+3Ac^2\right)+\frac{1}{7}a^2x^7(aB+3Ac)+\frac{1}{25}c^2x^{25}(3aC+Bc)+\frac{1}{31}c^3Cx^{31}$$

input
```
Int[(a + c*x^6)^3*(A + B*x^6 + C*x^12),x]
```

output
```
a^3*A*x + (a^2*(a*B + 3*A*c)*x^7)/7 + (a*(3*a*B*c + 3*A*c^2 + a^2*C)*x^13)
/13 + (c*(3*a*B*c + A*c^2 + 3*a^2*C)*x^19)/19 + (c^2*(B*c + 3*a*C)*x^25)/2
5 + (c^3*C*x^31)/31
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

method	result
default	$\frac{c^3 C x^{31}}{31}+\frac{\left(c^3 B+3 a c^2 C\right) x^{25}}{25}+\frac{\left(A c^3+3 a c^2 B+3 a^2 c C\right) x^{19}}{19}+\frac{\left(3 A a c^2+3 a^2 c B+a^3 C\right) x^{13}}{13}+\frac{\left(3 a^2 A c+B a^3\right) x^7}{7}+a^3$
norman	$\frac{c^3 C x^{31}}{31}+\left(\frac{1}{25} c^3 B+\frac{3}{25} a c^2 C\right) x^{25}+\left(\frac{1}{19} A c^3+\frac{3}{19} a c^2 B+\frac{3}{19} a^2 c C\right) x^{19}+\left(\frac{3}{13} A a c^2+\frac{3}{13} a^2 c B\right.$
gosper	$\frac{1}{31} c^3 C x^{31}+\frac{1}{25} x^{25} c^3 B+\frac{3}{25} x^{25} a c^2 C+\frac{1}{19} x^{19} A c^3+\frac{3}{19} x^{19} a c^2 B+\frac{3}{19} x^{19} a^2 c C+\frac{3}{13} x^{13} A a c^2$
risch	$\frac{1}{31} c^3 C x^{31}+\frac{1}{25} x^{25} c^3 B+\frac{3}{25} x^{25} a c^2 C+\frac{1}{19} x^{19} A c^3+\frac{3}{19} x^{19} a c^2 B+\frac{3}{19} x^{19} a^2 c C+\frac{3}{13} x^{13} A a c^2$
parallelrisch	$\frac{1}{31} c^3 C x^{31}+\frac{1}{25} x^{25} c^3 B+\frac{3}{25} x^{25} a c^2 C+\frac{1}{19} x^{19} A c^3+\frac{3}{19} x^{19} a c^2 B+\frac{3}{19} x^{19} a^2 c C+\frac{3}{13} x^{13} A a c^2$
orering	$\frac{x\left(43225 c^3 C x^{30}+53599 B c^3 x^{24}+160797 C a c^2 x^{24}+70525 A c^3 x^{18}+211575 B a c^2 x^{18}+211575 C a^2 c x^{18}+309225 A a c^2 x^{12}+30\right.}{1339975}$

input
```
int((c*x^6+a)^3*(C*x^12+B*x^6+A),x,method=_RETURNVERBOSE)
```

output
```
1/31*c^3*C*x^31+1/25*(B*c^3+3*C*a*c^2)*x^25+1/19*(A*c^3+3*B*a*c^2+3*C*a^2*
c)*x^19+1/13*(3*A*a*c^2+3*B*a^2*c+C*a^3)*x^13+1/7*(3*A*a^2*c+B*a^3)*x^7+a^
3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int (a+cx^6)^3 (A+Bx^6+Cx^{12})\,dx = \frac{1}{31}\,Cc^3x^{31} + \frac{1}{25}\,(3\,Cac^2+Bc^3)x^{25} + \frac{1}{19}\,(3\,Ca^2c+3\,Bac^2+Ac^3)x^{19} + \frac{1}{13}\,(Ca^3+3\,Ba^2c+3\,Aac^2)x^{13} + \frac{1}{7}\,(Ba^3+3\,Aa^2c)x^7 + Aa^3x$$

input
```
integrate((c*x^6+a)^3*(C*x^12+B*x^6+A),x, algorithm="fricas")
```

output
```
1/31*C*c^3*x^31 + 1/25*(3*C*a*c^2 + B*c^3)*x^25 + 1/19*(3*C*a^2*c + 3*B*a*
c^2 + A*c^3)*x^19 + 1/13*(C*a^3 + 3*B*a^2*c + 3*A*a*c^2)*x^13 + 1/7*(B*a^3
 + 3*A*a^2*c)*x^7 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int (a+cx^6)^3 (A+Bx^6+Cx^{12})\,dx = Aa^3x + \frac{Cc^3x^{31}}{31} + x^{25}\left(\frac{Bc^3}{25}+\frac{3Cac^2}{25}\right) + x^{19}\left(\frac{Ac^3}{19}+\frac{3Bac^2}{19}+\frac{3Ca^2c}{19}\right) + x^{13} \cdot \left(\frac{3Aac^2}{13}+\frac{3Ba^2c}{13}+\frac{Ca^3}{13}\right) + x^7 \cdot \left(\frac{3Aa^2c}{7}+\frac{Ba^3}{7}\right)$$

input
```
integrate((c*x**6+a)**3*(C*x**12+B*x**6+A),x)
```

output
```
A*a**3*x + C*c**3*x**31/31 + x**25*(B*c**3/25 + 3*C*a*c**2/25) + x**19*(A*
c**3/19 + 3*B*a*c**2/19 + 3*C*a**2*c/19) + x**13*(3*A*a*c**2/13 + 3*B*a**2
*c/13 + C*a**3/13) + x**7*(3*A*a**2*c/7 + B*a**3/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int (a+cx^6)^3 (A+Bx^6+Cx^{12})\,dx = \frac{1}{31}\,Cc^3x^{31} + \frac{1}{25}\,(3\,Cac^2+Bc^3)x^{25} + \frac{1}{19}\,(3\,Ca^2c+3\,Bac^2+Ac^3)x^{19} + \frac{1}{13}\,(Ca^3+3\,Ba^2c+3\,Aac^2)x^{13} + \frac{1}{7}\,(Ba^3+3\,Aa^2c)x^7 + Aa^3x$$

input
```
integrate((c*x^6+a)^3*(C*x^12+B*x^6+A),x, algorithm="maxima")
```

output
```
1/31*C*c^3*x^31 + 1/25*(3*C*a*c^2 + B*c^3)*x^25 + 1/19*(3*C*a^2*c + 3*B*a*
c^2 + A*c^3)*x^19 + 1/13*(C*a^3 + 3*B*a^2*c + 3*A*a*c^2)*x^13 + 1/7*(B*a^3
 + 3*A*a^2*c)*x^7 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int (a+cx^6)^3 (A+Bx^6+Cx^{12})\,dx = \frac{1}{31}\,Cc^3x^{31} + \frac{3}{25}\,Cac^2x^{25} + \frac{1}{25}\,Bc^3x^{25} + \frac{3}{19}\,Ca^2cx^{19} + \frac{3}{19}\,Bac^2x^{19} + \frac{1}{19}\,Ac^3x^{19} + \frac{1}{13}\,Ca^3x^{13} + \frac{3}{13}\,Ba^2cx^{13} + \frac{3}{13}\,Aac^2x^{13} + \frac{1}{7}\,Ba^3x^7 + \frac{3}{7}\,Aa^2cx^7 + Aa^3x$$

input
```
integrate((c*x^6+a)^3*(C*x^12+B*x^6+A),x, algorithm="giac")
```

output
```
1/31*C*c^3*x^31 + 3/25*C*a*c^2*x^25 + 1/25*B*c^3*x^25 + 3/19*C*a^2*c*x^19
+ 3/19*B*a*c^2*x^19 + 1/19*A*c^3*x^19 + 1/13*C*a^3*x^13 + 3/13*B*a^2*c*x^1
3 + 3/13*A*a*c^2*x^13 + 1/7*B*a^3*x^7 + 3/7*A*a^2*c*x^7 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \left(a+cx^6\right)^3 \left(A+Bx^6+Cx^{12}\right) \, dx = x^{13}\left(\frac{C\,a^3}{13}+\frac{3\,B\,a^2\,c}{13}+\frac{3\,A\,a\,c^2}{13}\right) + x^{19}\left(\frac{3\,C\,a^2\,c}{19}+\frac{3\,B\,a\,c^2}{19}+\frac{A\,c^3}{19}\right) + x^7\left(\frac{B\,a^3}{7}+\frac{3\,A\,c\,a^2}{7}\right) + x^{25}\left(\frac{B\,c^3}{25}+\frac{3\,C\,a\,c^2}{25}\right)+\frac{C\,c^3\,x^{31}}{31}+A\,a^3\,x$$

input
```
int((a + c*x^6)^3*(A + B*x^6 + C*x^12),x)
```

output
```
x^13*((C*a^3)/13 + (3*A*a*c^2)/13 + (3*B*a^2*c)/13) + x^19*((A*c^3)/19 + (
3*B*a*c^2)/19 + (3*C*a^2*c)/19) + x^7*((B*a^3)/7 + (3*A*a^2*c)/7) + x^25*(
(B*c^3)/25 + (3*C*a*c^2)/25) + (C*c^3*x^31)/31 + A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \left(a+cx^6\right)^3 \left(A+Bx^6+Cx^{12}\right) \, dx = \frac{x(43225c^4x^{30}+160797a\,c^3x^{24}+53599b\,c^3x^{24}+211575a^2c^2x^{18}+211575ab\,c^2x^{18}+70525a\,c^3x^{18}+1030}{1339975}$$

input
```
int((c*x^6+a)^3*(C*x^12+B*x^6+A),x)
```

output
```
(x*(1339975*a**4 + 191425*a**3*b*x**6 + 103075*a**3*c*x**12 + 574275*a**3*
c*x**6 + 309225*a**2*b*c*x**12 + 211575*a**2*c**2*x**18 + 309225*a**2*c**2
*x**12 + 211575*a*b*c**2*x**18 + 160797*a*c**3*x**24 + 70525*a*c**3*x**18
+ 53599*b*c**3*x**24 + 43225*c**4*x**30))/1339975
```

3.19 $\int (a+cx^6)^2 (A+Bx^6+Cx^{12})\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int (a+cx^6)^2 (A+Bx^6+Cx^{12})\,dx = a^2Ax+\frac{1}{7}a(aB+2Ac)x^7+\frac{1}{13}(2aBc+Ac^2+a^2C)x^{13}+\frac{1}{19}c(Bc+2aC)x^{19}+\frac{1}{25}c^2Cx^{25}$$

output
```
a^2*A*x+1/7*a*(2*A*c+B*a)*x^7+1/13*(A*c^2+2*B*a*c+C*a^2)*x^13+1/19*c*(B*c+
2*C*a)*x^19+1/25*c^2*C*x^25
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (a+cx^6)^2 (A+Bx^6+Cx^{12})\,dx = a^2Ax+\frac{1}{7}a(aB+2Ac)x^7+\frac{1}{13}(2aBc+Ac^2+a^2C)x^{13}+\frac{1}{19}c(Bc+2aC)x^{19}+\frac{1}{25}c^2Cx^{25}$$

input
```
Integrate[(a + c*x^6)^2*(A + B*x^6 + C*x^12),x]
```

output
```
a^2*A*x + (a*(a*B + 2*A*c)*x^7)/7 + ((2*a*B*c + A*c^2 + a^2*C)*x^13)/13 +
(c*(B*c + 2*a*C)*x^19)/19 + (c^2*C*x^25)/25
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^6\right)^2 \left(A+Bx^6+Cx^{12}\right) \, dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(x^{12}\left(a^2C+2aBc+Ac^2\right)+a^2A+ax^6(aB+2Ac)+cx^{18}(2aC+Bc)+c^2Cx^{24}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{13}x^{13}\left(a^2C+2aBc+Ac^2\right)+a^2Ax+\frac{1}{7}ax^7(aB+2Ac)+\frac{1}{19}cx^{19}(2aC+Bc)+\frac{1}{25}c^2Cx^{25}$$

input
```
Int[(a + c*x^6)^2*(A + B*x^6 + C*x^12),x]
```

output
```
a^2*A*x + (a*(a*B + 2*A*c)*x^7)/7 + ((2*a*B*c + A*c^2 + a^2*C)*x^13)/13 +
(c*(B*c + 2*a*C)*x^19)/19 + (c^2*C*x^25)/25
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
default	$\frac{c^2 C\,x^{25}}{25}+\frac{(c^2 B+2acC)x^{19}}{19}+\frac{(A\,c^2+2aBc+a^2 C)x^{13}}{13}+\frac{(2Aac+a^2 B)x^7}{7}+a^2 Ax$
norman	$\frac{c^2 C\,x^{25}}{25}+\left(\frac{1}{19}c^2 B+\frac{2}{19}acC\right)x^{19}+a^2 Ax+\left(\frac{2}{7}Aac+\frac{1}{7}a^2 B\right)x^7+\left(\frac{1}{13}A\,c^2+\frac{2}{13}aBc+\frac{1}{13}a^2 C\right)$
gosper	$\frac{1}{25}c^2 C\,x^{25}+\frac{1}{19}x^{19}c^2 B+\frac{2}{19}x^{19}acC+a^2 Ax+\frac{2}{7}x^7 Aac+\frac{1}{7}x^7 a^2 B+\frac{1}{13}x^{13}A\,c^2+\frac{2}{13}x^{13}aBc+$
risch	$\frac{1}{25}c^2 C\,x^{25}+\frac{1}{19}x^{19}c^2 B+\frac{2}{19}x^{19}acC+a^2 Ax+\frac{2}{7}x^7 Aac+\frac{1}{7}x^7 a^2 B+\frac{1}{13}x^{13}A\,c^2+\frac{2}{13}x^{13}aBc+$
parallelrisch	$\frac{1}{25}c^2 C\,x^{25}+\frac{1}{19}x^{19}c^2 B+\frac{2}{19}x^{19}acC+a^2 Ax+\frac{2}{7}x^7 Aac+\frac{1}{7}x^7 a^2 B+\frac{1}{13}x^{13}A\,c^2+\frac{2}{13}x^{13}aBc+$
orering	$\frac{x(1729c^2 C\,x^{24}+2275B\,c^2 x^{18}+4550Cac\,x^{18}+3325A\,c^2 x^{12}+6650Bac\,x^{12}+3325C\,a^2 x^{12}+12350Aac\,x^6+6175B\,a^2 x^6+43225}{43225}$

input
```
int((c*x^6+a)^2*(C*x^12+B*x^6+A),x,method=_RETURNVERBOSE)
```

output
```
1/25*c^2*C*x^25+1/19*(B*c^2+2*C*a*c)*x^19+1/13*(A*c^2+2*B*a*c+C*a^2)*x^13+
1/7*(2*A*a*c+B*a^2)*x^7+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \left(a+cx^6\right)^2\left(A+Bx^6+Cx^{12}\right)\,dx=\frac{1}{25}\,Cc^2x^{25}+\frac{1}{19}\,(2\,Cac+Bc^2)x^{19}+\frac{1}{13}\,(Ca^2+2\,Bac+Ac^2)x^{13}+\frac{1}{7}\,(Ba^2+2\,Aac)x^7+Aa^2x$$

input
```
integrate((c*x^6+a)^2*(C*x^12+B*x^6+A),x, algorithm="fricas")
```

output
```
1/25*C*c^2*x^25 + 1/19*(2*C*a*c + B*c^2)*x^19 + 1/13*(C*a^2 + 2*B*a*c + A*
c^2)*x^13 + 1/7*(B*a^2 + 2*A*a*c)*x^7 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int (a+cx^6)^2 (A+Bx^6+Cx^{12})\,dx = Aa^2x + \frac{Cc^2x^{25}}{25} + x^{19}\left(\frac{Bc^2}{19} + \frac{2Cac}{19}\right) + x^{13}\left(\frac{Ac^2}{13} + \frac{2Bac}{13} + \frac{Ca^2}{13}\right) + x^7\cdot\left(\frac{2Aac}{7} + \frac{Ba^2}{7}\right)$$

input
```
integrate((c*x**6+a)**2*(C*x**12+B*x**6+A),x)
```

output
```
A*a**2*x + C*c**2*x**25/25 + x**19*(B*c**2/19 + 2*C*a*c/19) + x**13*(A*c**
2/13 + 2*B*a*c/13 + C*a**2/13) + x**7*(2*A*a*c/7 + B*a**2/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int (a+cx^6)^2 (A+Bx^6+Cx^{12})\,dx = \frac{1}{25}\,Cc^2x^{25} + \frac{1}{19}\,(2\,Cac + Bc^2)x^{19} + \frac{1}{13}\,(Ca^2 + 2\,Bac + Ac^2)x^{13} + \frac{1}{7}\,(Ba^2 + 2\,Aac)x^7 + Aa^2x$$

input
```
integrate((c*x^6+a)^2*(C*x^12+B*x^6+A),x, algorithm="maxima")
```

output
```
1/25*C*c^2*x^25 + 1/19*(2*C*a*c + B*c^2)*x^19 + 1/13*(C*a^2 + 2*B*a*c + A*
c^2)*x^13 + 1/7*(B*a^2 + 2*A*a*c)*x^7 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \left(a+cx^6\right)^2 \left(A+Bx^6+Cx^{12}\right)\,dx = \frac{1}{25}Cc^2x^{25}+\frac{2}{19}Cacx^{19}+\frac{1}{19}Bc^2x^{19}+\frac{1}{13}Ca^2x^{13}+\frac{2}{13}Bacx^{13}+\frac{1}{13}Ac^2x^{13}+\frac{1}{7}Ba^2x^7+\frac{2}{7}Aacx^7+Aa^2x$$

input

```
integrate((c*x^6+a)^2*(C*x^12+B*x^6+A),x, algorithm="giac")
```

output

```
1/25*C*c^2*x^25 + 2/19*C*a*c*x^19 + 1/19*B*c^2*x^19 + 1/13*C*a^2*x^13 + 2/
13*B*a*c*x^13 + 1/13*A*c^2*x^13 + 1/7*B*a^2*x^7 + 2/7*A*a*c*x^7 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \left(a+cx^6\right)^2 \left(A+Bx^6+Cx^{12}\right)\,dx = x^7\left(\frac{B\,a^2}{7}+\frac{2\,A\,c\,a}{7}\right)+x^{19}\left(\frac{B\,c^2}{19}+\frac{2\,C\,a\,c}{19}\right)+x^{13}\left(\frac{C\,a^2}{13}+\frac{2\,B\,a\,c}{13}+\frac{A\,c^2}{13}\right)+\frac{C\,c^2\,x^{25}}{25}+A\,a^2\,x$$

input

```
int((a + c*x^6)^2*(A + B*x^6 + C*x^12),x)
```

output

```
x^7*((B*a^2)/7 + (2*A*a*c)/7) + x^19*((B*c^2)/19 + (2*C*a*c)/19) + x^13*((
A*c^2)/13 + (C*a^2)/13 + (2*B*a*c)/13) + (C*c^2*x^25)/25 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \left(a+cx^6\right)^2 \left(A+Bx^6+Cx^{12}\right)\,dx$$
$$= \frac{x(1729c^3x^{24}+4550a\,c^2x^{18}+2275b\,c^2x^{18}+3325a^2c\,x^{12}+6650abc\,x^{12}+3325a\,c^2x^{12}+6175a^2b\,x^6+123}{43225}$$

input
```
int((c*x^6+a)^2*(C*x^12+B*x^6+A),x)
```

output
```
(x*(43225*a**3 + 6175*a**2*b*x**6 + 3325*a**2*c*x**12 + 12350*a**2*c*x**6
+ 6650*a*b*c*x**12 + 4550*a*c**2*x**18 + 3325*a*c**2*x**12 + 2275*b*c**2*x
**18 + 1729*c**3*x**24))/43225
```

3.20 $\int \left(a+cx^6\right)\left(A+Bx^6+Cx^{12}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \left(a+cx^6\right)\left(A+Bx^6+Cx^{12}\right)\,dx=aAx+\frac{1}{7}(aB+Ac)x^7+\frac{1}{13}(Bc+aC)x^{13}+\frac{1}{19}cCx^{19}$$

output
```
a*A*x+1/7*(A*c+B*a)*x^7+1/13*(B*c+C*a)*x^13+1/19*c*C*x^19
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \left(a+cx^6\right)\left(A+Bx^6+Cx^{12}\right)\,dx=aAx+\frac{1}{7}(aB+Ac)x^7+\frac{1}{13}(Bc+aC)x^{13}+\frac{1}{19}cCx^{19}$$

input
```
Integrate[(a + c*x^6)*(A + B*x^6 + C*x^12),x]
```

output
```
a*A*x + ((a*B + A*c)*x^7)/7 + ((B*c + a*C)*x^13)/13 + (c*C*x^19)/19
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + cx^6) (A + Bx^6 + Cx^{12}) \, dx$$

$$\Big\downarrow \text{1737}$$

$$\int (x^6(aB + Ac) + aA + x^{12}(aC + Bc) + cCx^{18}) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(aB + Ac) + aAx + \frac{1}{13}x^{13}(aC + Bc) + \frac{1}{19}cCx^{19}$$

input
```
Int[(a + c*x^6)*(A + B*x^6 + C*x^12),x]
```

output
```
a*A*x + ((a*B + A*c)*x^7)/7 + ((B*c + a*C)*x^13)/13 + (c*C*x^19)/19
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
default	$aAx+\frac{(Ac+Ba)x^{7}}{7}+\frac{(Bc+Ca)x^{13}}{13}+\frac{cC\,x^{19}}{19}$	37
norman	$aAx+\left(\frac{Ac}{7}+\frac{Ba}{7}\right)x^{7}+\left(\frac{Bc}{13}+\frac{Ca}{13}\right)x^{13}+\frac{cC\,x^{19}}{19}$	39
gosper	$aAx+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ba+\frac{1}{13}x^{13}Bc+\frac{1}{13}x^{13}Ca+\frac{1}{19}cC\,x^{19}$	41
risch	$aAx+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ba+\frac{1}{13}x^{13}Bc+\frac{1}{13}x^{13}Ca+\frac{1}{19}cC\,x^{19}$	41
parallelrisch	$aAx+\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ba+\frac{1}{13}x^{13}Bc+\frac{1}{13}x^{13}Ca+\frac{1}{19}cC\,x^{19}$	41
orering	$\frac{x(91Cc\,x^{18}+133Bc\,x^{12}+133Ca\,x^{12}+247Ac\,x^{6}+247Ba\,x^{6}+1729Aa)}{1729}$	44

input
```
int((c*x^6+a)*(C*x^12+B*x^6+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/7*(A*c+B*a)*x^7+1/13*(B*c+C*a)*x^13+1/19*c*C*x^19
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int (a+cx^6)\left(A+Bx^6+Cx^{12}\right)\,dx=\frac{1}{19}\,Ccx^{19}+\frac{1}{13}\,(Ca+Bc)x^{13}+\frac{1}{7}\,(Ba+Ac)x^{7}+Aax$$

input
```
integrate((c*x^6+a)*(C*x^12+B*x^6+A),x, algorithm="fricas")
```

output
```
1/19*C*c*x^19 + 1/13*(C*a + B*c)*x^13 + 1/7*(B*a + A*c)*x^7 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \left(a+cx^6\right) \left(A+Bx^6+Cx^{12}\right) \, dx = Aax + \frac{Ccx^{19}}{19} + x^{13}\left(\frac{Bc}{13} + \frac{Ca}{13}\right) + x^7\left(\frac{Ac}{7} + \frac{Ba}{7}\right)$$

input
```
integrate((c*x**6+a)*(C*x**12+B*x**6+A),x)
```

output
```
A*a*x + C*c*x**19/19 + x**13*(B*c/13 + C*a/13) + x**7*(A*c/7 + B*a/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \left(a+cx^6\right) \left(A+Bx^6+Cx^{12}\right) \, dx = \frac{1}{19} \, Ccx^{19} + \frac{1}{13} \, (Ca + Bc)x^{13} + \frac{1}{7} \, (Ba + Ac)x^7 + Aax$$

input
```
integrate((c*x^6+a)*(C*x^12+B*x^6+A),x, algorithm="maxima")
```

output
```
1/19*C*c*x^19 + 1/13*(C*a + B*c)*x^13 + 1/7*(B*a + A*c)*x^7 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \left(a+cx^6\right) \left(A+Bx^6+Cx^{12}\right) \, dx = \frac{1}{19} \, Ccx^{19} + \frac{1}{13} \, Cax^{13} + \frac{1}{13} \, Bcx^{13} + \frac{1}{7} \, Bax^7 + \frac{1}{7} \, Acx^7 + Aax$$

input
```
integrate((c*x^6+a)*(C*x^12+B*x^6+A),x, algorithm="giac")
```

output
```
1/19*C*c*x^19 + 1/13*C*a*x^13 + 1/13*B*c*x^13 + 1/7*B*a*x^7 + 1/7*A*c*x^7
+ A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int (a+cx^6)\left(A+Bx^6+Cx^{12}\right)\,dx=\frac{C\,c\,x^{19}}{19}+\left(\frac{C\,a}{13}+\frac{B\,c}{13}\right)x^{13}+\left(\frac{B\,a}{7}+\frac{A\,c}{7}\right)x^7+A\,a\,x$$

input
```
int((a + c*x^6)*(A + B*x^6 + C*x^12),x)
```

output
```
x^7*((B*a)/7 + (A*c)/7) + x^13*((C*a)/13 + (B*c)/13) + A*a*x + (C*c*x^19)/
19
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int (a+cx^6)\left(A+Bx^6+Cx^{12}\right)\,dx = \frac{x(91c^2x^{18}+133ac\,x^{12}+133bc\,x^{12}+247ab\,x^6+247ac\,x^6+1729a^2)}{1729}$$

input
```
int((c*x^6+a)*(C*x^12+B*x^6+A),x)
```

output
```
(x*(1729*a**2 + 247*a*b*x**6 + 133*a*c*x**12 + 247*a*c*x**6 + 133*b*c*x**1
2 + 91*c**2*x**18))/1729
```

3.21 $\int \frac{A+Bx^6+Cx^{12}}{a+cx^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 242

$$\int \frac{A+Bx^6+Cx^{12}}{a+cx^6} dx = \frac{(Bc-aC)x}{c^2}+\frac{Cx^7}{7c}-\frac{(aBc-Ac^2-a^2C)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}c^{13/6}} + \frac{(aBc-Ac^2-a^2C)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{13/6}} - \frac{(aBc-Ac^2-a^2C)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{6a^{5/6}c^{13/6}} - \frac{(aBc-Ac^2-a^2C)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{2\sqrt{3}a^{5/6}c^{13/6}}$$

output
```
(B*c-C*a)*x/c^2+1/7*C*x^7/c-1/3*(-A*c^2+B*a*c-C*a^2)*arctan(c^(1/6)*x/a^(1
/6))/a^(5/6)/c^(13/6)-1/6*(-A*c^2+B*a*c-C*a^2)*arctan(-3^(1/2)+2*c^(1/6)*x
/a^(1/6))/a^(5/6)/c^(13/6)-1/6*(-A*c^2+B*a*c-C*a^2)*arctan(3^(1/2)+2*c^(1/
6)*x/a^(1/6))/a^(5/6)/c^(13/6)-1/6*(-A*c^2+B*a*c-C*a^2)*arctanh(3^(1/2)*a^
(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1/2)/a^(5/6)/c^(13/6)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^6+Cx^{12}}{a+cx^6}\,dx$$
$$= \frac{84a^{5/6}\sqrt[6]{c}(Bc-aC)x+12a^{5/6}c^{7/6}Cx^7+28(-aBc+Ac^2+a^2C)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)-14(-aBc+Ac^2+a}{}$$

input
```
Integrate[(A + B*x^6 + C*x^12)/(a + c*x^6),x]
```

output
```
(84*a^(5/6)*c^(1/6)*(B*c - a*C)*x + 12*a^(5/6)*c^(7/6)*C*x^7 + 28*(-(a*B*c
) + A*c^2 + a^2*C)*ArcTan[(c^(1/6)*x)/a^(1/6)] - 14*(-(a*B*c) + A*c^2 + a^
2*C)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)] + 14*(-(a*B*c) + A*c^2 + a^2*
C)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] - 7*Sqrt[3]*(-(a*B*c) + A*c^2 +
 a^2*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + 7*Sqrt[3]
*(-(a*B*c) + A*c^2 + a^2*C)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1
/3)*x^2])/(84*a^(5/6)*c^(13/6))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1741, 27, 913, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Cx^{12}}{a+cx^6}\,dx$$
$$\Big\downarrow \text{ 1741}$$
$$\frac{\int \frac{7((Bc-aC)x^6+Ac)}{cx^6+a}dx}{7c}+\frac{Cx^7}{7c}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(Bc-aC)x^6+Ac}{cx^6+a}dx}{c}+\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\int \frac{1}{cx^6+a}dx}{c}}{c}+\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 753}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{2\left(\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{2\left(\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}\right)}{c}}{c}+\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}\right)}{c}}{c}+\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}\right)}{c}}{c}+\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\sqrt{3}\int-\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\cdots}\cdots}{\cdots}\cdots\right.}{c}}{c}$$

$$\frac{Cx^7}{7c}$$

$\big\downarrow$ 25

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\sqrt{3}\int\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\cdots}\cdots}{\cdots}\cdots\right.}{c}}{c}$$

$$\frac{Cx^7}{7c}$$

$\big\downarrow$ 27

$$\frac{\frac{x(Bc-aC)}{c}-\frac{\left(a^2(-C)+aBc-Ac^2\right)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\cdots}\cdots}{\cdots}\cdots\right.}{c}}{c}$$

$$\frac{Cx^7}{7c}$$

$\big\downarrow$ 1082

$$\frac{\frac{x(Bc-aC)}{c}-\frac{(a^2(-C)+aBc-Ac^2)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{\ldots}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\ldots}\ldots\right.}{c}\ldots}{c}\ldots$$

$$\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{(a^2(-C)+aBc-Ac^2)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\ldots}{6a^{5/6}}\ldots\right.}{c}\ldots}{c}\ldots$$

$$\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{x(Bc-aC)}{c}-\frac{(a^2(-C)+aBc-Ac^2)\left(\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{c}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{c}}+\frac{\sqrt{3}\log\left(\ldots\right.}{\ldots}}{6a^{5/6}}\ldots\right.}{c}\ldots}{c}\ldots$$

$$\frac{Cx^7}{7c}$$

input
```
Int[(A + B*x^6 + C*x^12)/(a + c*x^6),x]
```

output
```
(C*x^7)/(7*c) + (((B*c - a*C)*x)/c - ((a*B*c - A*c^2 - a^2*C)*(ArcTan[(c^(
1/6)*x)/a^(1/6)]/(3*a^(5/6)*c^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x
)/(Sqrt[3]*a^(1/6)))]/c^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^
(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3]*(1 + (2
*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]/c^(1/6) + (Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a
^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(5/6))))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.28

method	result
risch	$\frac{C\,x^7}{7c}+\frac{Bx}{c}-\frac{Cax}{c^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^6+a\right)}\frac{\left(A\,c^2-aBc+a^2C\right)\ln\left(x-\_R\right)}{\_R^5}}{6c^3}$
default	$\frac{\frac{1}{7}C\,x^7c+Bcx-Cax}{c^2}+\frac{\left(-\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3a}+\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln \cdots}{\cdots}\right.}{c^2}$

input
```
int((C*x^12+B*x^6+A)/(c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*C*x^7/c+1/c*B*x-1/c^2*C*a*x+1/6/c^3*sum((A*c^2-B*a*c+C*a^2)/_R^5*ln(x-
_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3935 vs. 2(187) = 374.

Time = 0.12 (sec) , antiderivative size = 3935, normalized size of antiderivative = 16.26

$$\int \frac{A+Bx^6+Cx^{12}}{a+cx^6}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx^6+Cx^{12}}{a+cx^6}\,dx = \frac{Cx^7}{7c}+x\left(\frac{B}{c}-\frac{Ca}{c^2}\right)$$
$$+\text{RootSum}\left(46656t^6a^5c^{13}+A^6c^{12}-6A^5Bac^{11}+6A^5Ca^2c^{10}+15A^4B^2a^2c^{10}-30A^4BCa^3c^9+15A^4C^2\right.$$

input
```
integrate((C*x**12+B*x**6+A)/(c*x**6+a),x)
```

output
```
C*x**7/(7*c) + x*(B/c - C*a/c**2) + RootSum(46656*_t**6*a**5*c**13 + A**6*
c**12 - 6*A**5*B*a*c**11 + 6*A**5*C*a**2*c**10 + 15*A**4*B**2*a**2*c**10 -
 30*A**4*B*C*a**3*c**9 + 15*A**4*C**2*a**4*c**8 - 20*A**3*B**3*a**3*c**9 +
 60*A**3*B**2*C*a**4*c**8 - 60*A**3*B*C**2*a**5*c**7 + 20*A**3*C**3*a**6*c
**6 + 15*A**2*B**4*a**4*c**8 - 60*A**2*B**3*C*a**5*c**7 + 90*A**2*B**2*C**
2*a**6*c**6 - 60*A**2*B*C**3*a**7*c**5 + 15*A**2*C**4*a**8*c**4 - 6*A*B**5
*a**5*c**7 + 30*A*B**4*C*a**6*c**6 - 60*A*B**3*C**2*a**7*c**5 + 60*A*B**2*
C**3*a**8*c**4 - 30*A*B*C**4*a**9*c**3 + 6*A*C**5*a**10*c**2 + B**6*a**6*c
**6 - 6*B**5*C*a**7*c**5 + 15*B**4*C**2*a**8*c**4 - 20*B**3*C**3*a**9*c**3
 + 15*B**2*C**4*a**10*c**2 - 6*B*C**5*a**11*c + C**6*a**12, Lambda(_t, _t*
log(6*_t*a*c**2/(A*c**2 - B*a*c + C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^6+Cx^{12}}{a+cx^6}\,dx = \frac{Ccx^7-7\,(Ca-Bc)x}{7\,c^2} + \frac{\frac{\sqrt{3}(Ca^2-Bac+Ac^2)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}c^{\frac{1}{6}}} - \frac{\sqrt{3}(Ca^2-Bac+Ac^2)\log\left(c^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}c^{\frac{1}{6}}} + \frac{4\left(Ca^2c^{\frac{1}{3}}-Bac^{\frac{4}{3}}+Ac^{\frac{7}{3}}\right)\mathrm{arct}}{a^{\frac{2}{3}}c^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}c^{\frac{1}{3}}}}}{12}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a),x, algorithm="maxima")
```

output
```
1/7*(C*c*x^7 - 7*(C*a - B*c)*x)/c^2 + 1/12*(sqrt(3)*(C*a^2 - B*a*c + A*c^2
)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(5/6)*c^(1/6))
 - sqrt(3)*(C*a^2 - B*a*c + A*c^2)*log(c^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/
6)*x + a^(1/3))/(a^(5/6)*c^(1/6)) + 4*(C*a^2*c^(1/3) - B*a*c^(4/3) + A*c^(
7/3))*arctan(c^(1/3)*x/sqrt(a^(1/3)*c^(1/3)))/(a^(2/3)*c^(1/3)*sqrt(a^(1/3
)*c^(1/3))) + 2*(C*a^(7/3)*c^(1/3) - B*a^(4/3)*c^(4/3) + A*a^(1/3)*c^(7/3)
)*arctan((2*c^(1/3)*x + sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a
*c^(1/3)*sqrt(a^(1/3)*c^(1/3))) + 2*(C*a^(7/3)*c^(1/3) - B*a^(4/3)*c^(4/3)
 + A*a^(1/3)*c^(7/3))*arctan((2*c^(1/3)*x - sqrt(3)*a^(1/6)*c^(1/6))/sqrt(
a^(1/3)*c^(1/3)))/(a*c^(1/3)*sqrt(a^(1/3)*c^(1/3))))/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.52

$$\int \frac{A+Bx^6+Cx^{12}}{a+cx^6}\,dx$$
$$=\frac{\sqrt{3}\left((ac^5)^{\frac{1}{6}}Ca^2-(ac^5)^{\frac{1}{6}}Bac+(ac^5)^{\frac{1}{6}}Ac^2\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^3}$$
$$-\frac{\sqrt{3}\left((ac^5)^{\frac{1}{6}}Ca^2-(ac^5)^{\frac{1}{6}}Bac+(ac^5)^{\frac{1}{6}}Ac^2\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^3}$$
$$+\frac{\left((ac^5)^{\frac{1}{6}}Ca^2-(ac^5)^{\frac{1}{6}}Bac+(ac^5)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^3}$$
$$+\frac{\left((ac^5)^{\frac{1}{6}}Ca^2-(ac^5)^{\frac{1}{6}}Bac+(ac^5)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^3}$$
$$+\frac{\left((ac^5)^{\frac{1}{6}}Ca^2-(ac^5)^{\frac{1}{6}}Bac+(ac^5)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac^3}$$
$$+\frac{Cc^6x^7-7\,Cac^5x+7\,Bc^6x}{7\,c^7}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*((a*c^5)^(1/6)*C*a^2 - (a*c^5)^(1/6)*B*a*c + (a*c^5)^(1/6)*A*
c^2)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^3) - 1/12*sqrt(3)
*((a*c^5)^(1/6)*C*a^2 - (a*c^5)^(1/6)*B*a*c + (a*c^5)^(1/6)*A*c^2)*log(x^2
 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a*c^3) + 1/6*((a*c^5)^(1/6)*C*a^2
 - (a*c^5)^(1/6)*B*a*c + (a*c^5)^(1/6)*A*c^2)*arctan((2*x + sqrt(3)*(a/c)^
(1/6))/(a/c)^(1/6))/(a*c^3) + 1/6*((a*c^5)^(1/6)*C*a^2 - (a*c^5)^(1/6)*B*a
*c + (a*c^5)^(1/6)*A*c^2)*arctan((2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/
(a*c^3) + 1/3*((a*c^5)^(1/6)*C*a^2 - (a*c^5)^(1/6)*B*a*c + (a*c^5)^(1/6)*A
*c^2)*arctan(x/(a/c)^(1/6))/(a*c^3) + 1/7*(C*c^6*x^7 - 7*C*a*c^5*x + 7*B*c
^6*x)/c^7
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 4366, normalized size of antiderivative = 18.04

$$\int \frac{A + Bx^6 + Cx^{12}}{a + cx^6} dx = \text{Too large to display}$$

input
```
int((A + B*x^6 + C*x^12)/(a + c*x^6),x)
```

output
```
x*(B/c - (C*a)/c^2) + (C*x^7)/(7*c) + (atan(((((6*x*(A^4*c^8 + C^4*a^8 + B
^4*a^4*c^4 + 6*A^2*B^2*a^2*c^6 + 6*A^2*C^2*a^4*c^4 + 6*B^2*C^2*a^6*c^2 - 4
*A^3*B*a*c^7 - 4*B*C^3*a^7*c - 4*A*B^3*a^3*c^5 + 4*A*C^3*a^6*c^2 + 4*A^3*C
*a^2*c^6 - 4*B^3*C*a^5*c^3 - 12*A*B*C^2*a^5*c^3 + 12*A*B^2*C*a^4*c^4 - 12*
A^2*B*C*a^3*c^5))/c^3 - (6*(A*c^2 + C*a^2 - B*a*c)*(C^3*a^7 + A^3*a*c^6 -
B^3*a^4*c^3 - 3*B*C^2*a^6*c + 3*A*B^2*a^3*c^4 - 3*A^2*B*a^2*c^5 + 3*A*C^2*
a^5*c^2 + 3*A^2*C*a^3*c^4 + 3*B^2*C*a^5*c^2 - 6*A*B*C*a^4*c^3))/((-a)^(5/6
)*c^(19/6)))*(A*c^2 + C*a^2 - B*a*c)*1i)/(6*(-a)^(5/6)*c^(13/6)) + (((6*x*
(A^4*c^8 + C^4*a^8 + B^4*a^4*c^4 + 6*A^2*B^2*a^2*c^6 + 6*A^2*C^2*a^4*c^4 +
 6*B^2*C^2*a^6*c^2 - 4*A^3*B*a*c^7 - 4*B*C^3*a^7*c - 4*A*B^3*a^3*c^5 + 4*A
*C^3*a^6*c^2 + 4*A^3*C*a^2*c^6 - 4*B^3*C*a^5*c^3 - 12*A*B*C^2*a^5*c^3 + 12
*A*B^2*C*a^4*c^4 - 12*A^2*B*C*a^3*c^5))/c^3 + (6*(A*c^2 + C*a^2 - B*a*c)*(
C^3*a^7 + A^3*a*c^6 - B^3*a^4*c^3 - 3*B*C^2*a^6*c + 3*A*B^2*a^3*c^4 - 3*A^
2*B*a^2*c^5 + 3*A*C^2*a^5*c^2 + 3*A^2*C*a^3*c^4 + 3*B^2*C*a^5*c^2 - 6*A*B*
C*a^4*c^3))/((-a)^(5/6)*c^(19/6)))*(A*c^2 + C*a^2 - B*a*c)*1i)/(6*(-a)^(5/
6)*c^(13/6)))/((((6*x*(A^4*c^8 + C^4*a^8 + B^4*a^4*c^4 + 6*A^2*B^2*a^2*c^6
 + 6*A^2*C^2*a^4*c^4 + 6*B^2*C^2*a^6*c^2 - 4*A^3*B*a*c^7 - 4*B*C^3*a^7*c -
 4*A*B^3*a^3*c^5 + 4*A*C^3*a^6*c^2 + 4*A^3*C*a^2*c^6 - 4*B^3*C*a^5*c^3 - 1
2*A*B*C^2*a^5*c^3 + 12*A*B^2*C*a^4*c^4 - 12*A^2*B*C*a^3*c^5))/c^3 - (6*(A*
c^2 + C*a^2 - B*a*c)*(C^3*a^7 + A^3*a*c^6 - B^3*a^4*c^3 - 3*B*C^2*a^6*c...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.95

$$\int \frac{A + Bx^6 + Cx^{12}}{a + cx^6} dx = \text{Too large to display}$$

input
```
int((C*x^12+B*x^6+A)/(c*x^6+a),x)
```

output

```
( - 14*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(
c**(1/6)*a**(1/6)))*a + 14*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(
3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*b - 14*c**(1/6)*a**(1/6)*atan((c**
(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c + 14*c**(1/6
)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1
/6)))*a - 14*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3
)*x)/(c**(1/6)*a**(1/6)))*b + 14*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)
*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c + 28*c**(1/6)*a**(1/6)*ata
n((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a - 28*c**(1/6)*a**(1/6)*atan((c**(1/3
)*x)/(c**(1/6)*a**(1/6)))*b + 28*c**(1/6)*a**(1/6)*atan((c**(1/3)*x)/(c**(
1/6)*a**(1/6)))*c - 7*c**(1/6)*a**(1/6)*sqrt(3)*log( - c**(1/6)*a**(1/6)*s
qrt(3)*x + a**(1/3) + c**(1/3)*x**2)*a + 7*c**(1/6)*a**(1/6)*sqrt(3)*log(
- c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*b - 7*c**(1/6)*a
**(1/6)*sqrt(3)*log( - c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x
**2)*c + 7*c**(1/6)*a**(1/6)*sqrt(3)*log(c**(1/6)*a**(1/6)*sqrt(3)*x + a**
(1/3) + c**(1/3)*x**2)*a - 7*c**(1/6)*a**(1/6)*sqrt(3)*log(c**(1/6)*a**(1/
6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*b + 7*c**(1/6)*a**(1/6)*sqrt(3)*l
og(c**(1/6)*a**(1/6)*sqrt(3)*x + a**(1/3) + c**(1/3)*x**2)*c - 84*c**(1/3)
*a*x + 84*c**(1/3)*b*x + 12*c**(1/3)*c*x**7)/(84*c**(1/3)*c)
```

3.22 $\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 261

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^2}\,dx = \frac{Cx}{c^2} - \frac{(aBc-Ac^2-a^2C)\,x}{6ac^2\,(a+cx^6)} + \frac{(aBc+5Ac^2-7a^2C)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{11/6}c^{13/6}} - \frac{(aBc+5Ac^2-7a^2C)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{11/6}c^{13/6}} + \frac{(aBc+5Ac^2-7a^2C)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{36a^{11/6}c^{13/6}} + \frac{(aBc+5Ac^2-7a^2C)\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{12\sqrt{3}a^{11/6}c^{13/6}}$$

output
```
C*x/c^2-1/6*(-A*c^2+B*a*c-C*a^2)*x/a/c^2/(c*x^6+a)+1/18*(5*A*c^2+B*a*c-7*C
*a^2)*arctan(c^(1/6)*x/a^(1/6))/a^(11/6)/c^(13/6)+1/36*(5*A*c^2+B*a*c-7*C*
a^2)*arctan(-3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(11/6)/c^(13/6)+1/36*(5*A*c^2+
B*a*c-7*C*a^2)*arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(11/6)/c^(13/6)+1/36*
(5*A*c^2+B*a*c-7*C*a^2)*arctanh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)
*x^2))*3^(1/2)/a^(11/6)/c^(13/6)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^2}\,dx$$

$$= \frac{72\sqrt[6]{c}Cx + \frac{12\sqrt[6]{c}(-aBc+Ac^2+a^2C)x}{a(a+cx^6)} - \frac{4(-aBc-5Ac^2+7a^2C)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{11/6}} + \frac{2(-aBc-5Ac^2+7a^2C)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{a^{11/6}}}{}$$

input
```
Integrate[(A + B*x^6 + C*x^12)/(a + c*x^6)^2,x]
```

output
```
(72*c^(1/6)*C*x + (12*c^(1/6)*(-(a*B*c) + A*c^2 + a^2*C)*x)/(a*(a + c*x^6)
) - (4*(-(a*B*c) - 5*A*c^2 + 7*a^2*C)*ArcTan[(c^(1/6)*x)/a^(1/6)])/a^(11/6
) + (2*(-(a*B*c) - 5*A*c^2 + 7*a^2*C)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/
6)])/a^(11/6) - (2*(-(a*B*c) - 5*A*c^2 + 7*a^2*C)*ArcTan[Sqrt[3] + (2*c^(1
/6)*x)/a^(1/6)])/a^(11/6) + (Sqrt[3]*(-(a*B*c) - 5*A*c^2 + 7*a^2*C)*Log[a^
(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/a^(11/6) - (Sqrt[3]*(-(a
*B*c) - 5*A*c^2 + 7*a^2*C)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/
3)*x^2])/a^(11/6))/(72*c^(13/6))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {1739, 25, 913, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^2}\,dx$$

$$\Big\downarrow \; 1739$$

$$-\frac{\int -\frac{6acCx^6+5Ac^2+aBc-a^2C}{cx^6+a}dx}{6ac^2} - \frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{6acCx^6+5Ac^2+aBc-a^2C}{cx^6+a}dx}{6ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\Big\downarrow \ 913$$

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\int \frac{1}{cx^6+a}dx+6aCx}{6ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\Big\downarrow \ 753$$

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{2\left(\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{2\left(\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}\right)+6aC}{6ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}\right)+6aCx}{6ac^2}-$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}\right)+6aCx}{6ac^2}-$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\Big\downarrow \ 1142$$

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\sqrt{3}\int-\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}\ldots}\ldots}{\ldots}\right.\ldots}{6ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$\Big\downarrow$ 25

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\sqrt{3}\int\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x\ldots}\ldots}{\ldots}\right.\ldots}{6ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$\Big\downarrow$ 27

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{\ldots}}\ldots}{\ldots}\right.\ldots}{6ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$\Big\downarrow$ 1082

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}\ldots}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x\ldots}\ldots}{\ldots}\right.\ldots}{6ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\downarrow \text{217}$$

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+}{6a^{5/6}}\right.}{6ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

$$\downarrow \text{1103}$$

$$\frac{\left(-7a^2C+aBc+5Ac^2\right)\left(\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{c}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}+1\right)\right)}{\sqrt[6]{c}}+\frac{\sqrt{3}\log\left(\sqrt{3}\right.}{}}{6a^{5/6}}\right.}{6ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{6ac^2\left(a+cx^6\right)}$$

input

```
Int[(A + B*x^6 + C*x^12)/(a + c*x^6)^2,x]
```

output

```
-1/6*((a*B*c - A*c^2 - a^2*C)*x)/(a*c^2*(a + c*x^6)) + (6*a*C*x + (a*B*c +
 5*A*c^2 - 7*a^2*C)*(ArcTan[(c^(1/6)*x)/a^(1/6)]/(3*a^(5/6)*c^(1/6)) + (-(
ArcTan[Sqrt[3]*(1 - (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]/c^(1/6)) - (Sqrt[3]*
Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^
(5/6)) + (ArcTan[Sqrt[3]*(1 + (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]/c^(1/6) +
(Sqrt[3]*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6
)))/(6*a^(5/6))))/(6*a*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.34

method	result
risch	$\frac{Cx}{c^2}+\frac{(Ac^2-aBc+a^2C)x}{6ac^2(cx^6+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^6+a)}\frac{(5Ac^2+aBc-7a^2C)\ln(x-\_R)}{\_R^5}}{36c^3a}$
default	$\frac{Cx}{c^2}+\frac{\frac{(Ac^2-aBc+a^2C)x}{6a(cx^6+a)}+\frac{(5Ac^2+aBc-7a^2C)\left(-\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arctan\cdots}{3a}\cdots\right)}{6a}}{c^2}$

input
```
int((C*x^12+B*x^6+A)/(c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x/c^2+1/6*(A*c^2-B*a*c+C*a^2)/a*x/c^2/(c*x^6+a)+1/36/c^3/a*sum((5*A*c^2+
B*a*c-7*C*a^2)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4144 vs. 2(212) = 424.

Time = 0.12 (sec) , antiderivative size = 4144, normalized size of antiderivative = 15.88

$$\int \frac{A + Bx^6 + Cx^{12}}{(a + cx^6)^2}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 78.07 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.90

$$\int \frac{A + Bx^6 + Cx^{12}}{(a + cx^6)^2}\, dx = \frac{Cx}{c^2} + \frac{x(Ac^2 - Bac + Ca^2)}{6a^2c^2 + 6ac^3x^6} + \text{RootSum}\Big(2176782336t^6a^{11}c^{13} + 15625A^6c^{12} + 18750A^5Bac^{11} - 131250A^5Ca^2c^{10} + 9375A^4B^2a^2c^{10}$$

input
```
integrate((C*x**12+B*x**6+A)/(c*x**6+a)**2,x)
```

output
```
C*x/c**2 + x*(A*c**2 - B*a*c + C*a**2)/(6*a**2*c**2 + 6*a*c**3*x**6) + Roo
tSum(2176782336*_t**6*a**11*c**13 + 15625*A**6*c**12 + 18750*A**5*B*a*c**1
1 - 131250*A**5*C*a**2*c**10 + 9375*A**4*B**2*a**2*c**10 - 131250*A**4*B*C
*a**3*c**9 + 459375*A**4*C**2*a**4*c**8 + 2500*A**3*B**3*a**3*c**9 - 52500
*A**3*B**2*C*a**4*c**8 + 367500*A**3*B*C**2*a**5*c**7 - 857500*A**3*C**3*a
**6*c**6 + 375*A**2*B**4*a**4*c**8 - 10500*A**2*B**3*C*a**5*c**7 + 110250*
A**2*B**2*C**2*a**6*c**6 - 514500*A**2*B*C**3*a**7*c**5 + 900375*A**2*C**4
*a**8*c**4 + 30*A*B**5*a**5*c**7 - 1050*A*B**4*C*a**6*c**6 + 14700*A*B**3*
C**2*a**7*c**5 - 102900*A*B**2*C**3*a**8*c**4 + 360150*A*B*C**4*a**9*c**3
- 504210*A*C**5*a**10*c**2 + B**6*a**6*c**6 - 42*B**5*C*a**7*c**5 + 735*B*
*4*C**2*a**8*c**4 - 6860*B**3*C**3*a**9*c**3 + 36015*B**2*C**4*a**10*c**2
- 100842*B*C**5*a**11*c + 117649*C**6*a**12, Lambda(_t, _t*log(-36*_t*a**2
*c**2/(-5*A*c**2 - B*a*c + 7*C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.38

$$\int \frac{A + Bx^6 + Cx^{12}}{(a + cx^6)^2} \, dx = \frac{(Ca^2 - Bac + Ac^2)x}{6\,(ac^3x^6 + a^2c^2)} + \frac{Cx}{c^2} - \frac{\frac{\sqrt{3}(7\,Ca^2 - Bac - 5\,Ac^2)\log\left(c^{\frac{1}{3}}x^2 + \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}c^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ca^2 - Bac - 5\,Ac^2)\log\left(c^{\frac{1}{3}}x^2 - \sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x + a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}c^{\frac{1}{6}}} + \frac{4\left(7\,Ca^2c^{\frac{1}{3}} - Bac^{\frac{4}{3}} - 5\,A\right.}{a^{\frac{2}{3}}c^{\frac{1}{3}}\sqrt{}}}{}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a)^2,x, algorithm="maxima")
```

output
```
1/6*(C*a^2 - B*a*c + A*c^2)*x/(a*c^3*x^6 + a^2*c^2) + C*x/c^2 - 1/72*(sqrt
(3)*(7*C*a^2 - B*a*c - 5*A*c^2)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*
x + a^(1/3))/(a^(5/6)*c^(1/6)) - sqrt(3)*(7*C*a^2 - B*a*c - 5*A*c^2)*log(c
^(1/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(5/6)*c^(1/6)) + 4*(7
*C*a^2*c^(1/3) - B*a*c^(4/3) - 5*A*c^(7/3))*arctan(c^(1/3)*x/sqrt(a^(1/3)*
c^(1/3)))/(a^(2/3)*c^(1/3)*sqrt(a^(1/3)*c^(1/3))) + 2*(7*C*a^(7/3)*c^(1/3)
 - B*a^(4/3)*c^(4/3) - 5*A*a^(1/3)*c^(7/3))*arctan((2*c^(1/3)*x + sqrt(3)*
a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(1/3)*sqrt(a^(1/3)*c^(1/3)))
+ 2*(7*C*a^(7/3)*c^(1/3) - B*a^(4/3)*c^(4/3) - 5*A*a^(1/3)*c^(7/3))*arctan
((2*c^(1/3)*x - sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(1/3)
*sqrt(a^(1/3)*c^(1/3))))/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^2}\,dx$$
$$= \frac{Cx}{c^2} - \frac{\sqrt{3}\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2-(ac^5)^{\frac{1}{6}}\,Bac-5\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^3}$$
$$+ \frac{\sqrt{3}\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2-(ac^5)^{\frac{1}{6}}\,Bac-5\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^3}$$
$$+ \frac{Ca^2x-Bacx+Ac^2x}{6\,(cx^6+a)ac^2}$$
$$- \frac{\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2-(ac^5)^{\frac{1}{6}}\,Bac-5\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^3}$$
$$- \frac{\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2-(ac^5)^{\frac{1}{6}}\,Bac-5\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^3}$$
$$- \frac{\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2-(ac^5)^{\frac{1}{6}}\,Bac-5\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{18\,a^2c^3}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a)^2,x, algorithm="giac")
```

output
```
C*x/c^2 - 1/72*sqrt(3)*(7*(a*c^5)^(1/6)*C*a^2 - (a*c^5)^(1/6)*B*a*c - 5*(a
*c^5)^(1/6)*A*c^2)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^3
) + 1/72*sqrt(3)*(7*(a*c^5)^(1/6)*C*a^2 - (a*c^5)^(1/6)*B*a*c - 5*(a*c^5)^
(1/6)*A*c^2)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^2*c^3) + 1/
6*(C*a^2*x - B*a*c*x + A*c^2*x)/((c*x^6 + a)*a*c^2) - 1/36*(7*(a*c^5)^(1/6
)*C*a^2 - (a*c^5)^(1/6)*B*a*c - 5*(a*c^5)^(1/6)*A*c^2)*arctan((2*x + sqrt(
3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^2*c^3) - 1/36*(7*(a*c^5)^(1/6)*C*a^2 - (a*
c^5)^(1/6)*B*a*c - 5*(a*c^5)^(1/6)*A*c^2)*arctan((2*x - sqrt(3)*(a/c)^(1/6
))/(a/c)^(1/6))/(a^2*c^3) - 1/18*(7*(a*c^5)^(1/6)*C*a^2 - (a*c^5)^(1/6)*B*
a*c - 5*(a*c^5)^(1/6)*A*c^2)*arctan(x/(a/c)^(1/6))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 4448, normalized size of antiderivative = 17.04

$$\int \frac{A + Bx^6 + Cx^{12}}{(a + cx^6)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^6 + C*x^12)/(a + c*x^6)^2,x)
```

output
```
(C*x)/c^2 + (atan(((((x*(625*A^4*c^8 + 2401*C^4*a^8 + B^4*a^4*c^4 + 150*A^
2*B^2*a^2*c^6 + 7350*A^2*C^2*a^4*c^4 + 294*B^2*C^2*a^6*c^2 + 500*A^3*B*a*c
^7 - 1372*B*C^3*a^7*c + 20*A*B^3*a^3*c^5 - 6860*A*C^3*a^6*c^2 - 3500*A^3*C
*a^2*c^6 - 28*B^3*C*a^5*c^3 + 2940*A*B*C^2*a^5*c^3 - 420*A*B^2*C*a^4*c^4 -
 2100*A^2*B*C*a^3*c^5))/(216*a^4*c^3) - ((5*A*c^2 - 7*C*a^2 + B*a*c)*(125*
A^3*c^6 - 343*C^3*a^6 + B^3*a^3*c^3 + 75*A^2*B*a*c^5 + 147*B*C^2*a^5*c + 1
5*A*B^2*a^2*c^4 + 735*A*C^2*a^4*c^2 - 525*A^2*C*a^2*c^4 - 21*B^2*C*a^4*c^2
 - 210*A*B*C*a^3*c^3))/(216*(-a)^(23/6)*c^(19/6)))*(5*A*c^2 - 7*C*a^2 + B*
a*c)*1i)/(36*(-a)^(11/6)*c^(13/6)) + (((x*(625*A^4*c^8 + 2401*C^4*a^8 + B^
4*a^4*c^4 + 150*A^2*B^2*a^2*c^6 + 7350*A^2*C^2*a^4*c^4 + 294*B^2*C^2*a^6*c
^2 + 500*A^3*B*a*c^7 - 1372*B*C^3*a^7*c + 20*A*B^3*a^3*c^5 - 6860*A*C^3*a^
6*c^2 - 3500*A^3*C*a^2*c^6 - 28*B^3*C*a^5*c^3 + 2940*A*B*C^2*a^5*c^3 - 420
*A*B^2*C*a^4*c^4 - 2100*A^2*B*C*a^3*c^5))/(216*a^4*c^3) + ((5*A*c^2 - 7*C*
a^2 + B*a*c)*(125*A^3*c^6 - 343*C^3*a^6 + B^3*a^3*c^3 + 75*A^2*B*a*c^5 + 1
47*B*C^2*a^5*c + 15*A*B^2*a^2*c^4 + 735*A*C^2*a^4*c^2 - 525*A^2*C*a^2*c^4
- 21*B^2*C*a^4*c^2 - 210*A*B*C*a^3*c^3))/(216*(-a)^(23/6)*c^(19/6)))*(5*A*
c^2 - 7*C*a^2 + B*a*c)*1i)/(36*(-a)^(11/6)*c^(13/6)))/((((x*(625*A^4*c^8 +
 2401*C^4*a^8 + B^4*a^4*c^4 + 150*A^2*B^2*a^2*c^6 + 7350*A^2*C^2*a^4*c^4 +
 294*B^2*C^2*a^6*c^2 + 500*A^3*B*a*c^7 - 1372*B*C^3*a^7*c + 20*A*B^3*a^3*c
^5 - 6860*A*C^3*a^6*c^2 - 3500*A^3*C*a^2*c^6 - 28*B^3*C*a^5*c^3 + 2940*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 980, normalized size of antiderivative = 3.75

$$\int \frac{A + Bx^6 + Cx^{12}}{(a + cx^6)^2} \, dx = \text{Too large to display}$$

input
```
int((C*x^12+B*x^6+A)/(c*x^6+a)^2,x)
```

output
```
(14*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)))*a**2 - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3
) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*b + 14*c**(1/6)*a**(1/6)*atan((c*
*(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*x**6 - 10
*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/
6)*a**(1/6)))*a*c - 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) -
2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*b*c*x**6 - 10*c**(1/6)*a**(1/6)*atan((c
**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c**2*x**6 -
14*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(
1/6)*a**(1/6)))*a**2 + 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3)
 + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*b - 14*c**(1/6)*a**(1/6)*atan((c**
(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*x**6 + 10*
c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6
)*a**(1/6)))*a*c + 2*c**(1/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2
*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*b*c*x**6 + 10*c**(1/6)*a**(1/6)*atan((c*
*(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*c**2*x**6 - 2
8*c**(1/6)*a**(1/6)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2 + 4*c**(1/
6)*a**(1/6)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*b - 28*c**(1/6)*a**(1
/6)*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c*x**6 + 20*c**(1/6)*a**(1/6)
*atan((c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c + 4*c**(1/6)*a**(1/6)*atan(...
```

3.23 $\int \frac{A+Bx^6+Cx^{12}}{\left(a+cx^6\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 296

$$\int \frac{A+Bx^6+Cx^{12}}{\left(a+cx^6\right)^3}\,dx=-\frac{\left(aBc-Ac^2-a^2C\right)x}{12ac^2\left(a+cx^6\right)^2}+\frac{\left(aBc+11Ac^2-13a^2C\right)x}{72a^2c^2\left(a+cx^6\right)}+\frac{\left(5aBc+55Ac^2+7a^2C\right)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{216a^{17/6}c^{13/6}}-\frac{\left(5aBc+55Ac^2+7a^2C\right)\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{432a^{17/6}c^{13/6}}+\frac{\left(5aBc+55Ac^2+7a^2C\right)\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{432a^{17/6}c^{13/6}}+\frac{\left(5aBc+55Ac^2+7a^2C\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{144\sqrt{3}a^{17/6}c^{13/6}}$$

output
```
-1/12*(-A*c^2+B*a*c-C*a^2)*x/a/c^2/(c*x^6+a)^2+1/72*(11*A*c^2+B*a*c-13*C*a
^2)*x/a^2/c^2/(c*x^6+a)+1/216*(55*A*c^2+5*B*a*c+7*C*a^2)*arctan(c^(1/6)*x/
a^(1/6))/a^(17/6)/c^(13/6)+1/432*(55*A*c^2+5*B*a*c+7*C*a^2)*arctan(-3^(1/2
)+2*c^(1/6)*x/a^(1/6))/a^(17/6)/c^(13/6)+1/432*(55*A*c^2+5*B*a*c+7*C*a^2)*
arctan(3^(1/2)+2*c^(1/6)*x/a^(1/6))/a^(17/6)/c^(13/6)+1/432*(55*A*c^2+5*B*
a*c+7*C*a^2)*arctanh(3^(1/2)*a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))*3^(1
/2)/a^(17/6)/c^(13/6)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^3}\,dx$$

$$= \frac{\frac{72a^{11/6}\sqrt[6]{c}(-aBc+Ac^2+a^2C)x}{(a+cx^6)^2} - \frac{12a^{5/6}\sqrt[6]{c}(-aBc-11Ac^2+13a^2C)x}{a+cx^6} + 4(5aBc+55Ac^2+7a^2C)\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right) - 2}{}$$

input
```
Integrate[(A + B*x^6 + C*x^12)/(a + c*x^6)^3,x]
```

output
```
((72*a^(11/6)*c^(1/6)*(-(a*B*c) + A*c^2 + a^2*C)*x)/(a + c*x^6)^2 - (12*a^
(5/6)*c^(1/6)*(-(a*B*c) - 11*A*c^2 + 13*a^2*C)*x)/(a + c*x^6) + 4*(5*a*B*c
 + 55*A*c^2 + 7*a^2*C)*ArcTan[(c^(1/6)*x)/a^(1/6)] - 2*(5*a*B*c + 55*A*c^2
 + 7*a^2*C)*ArcTan[Sqrt[3] - (2*c^(1/6)*x)/a^(1/6)] + 2*(5*a*B*c + 55*A*c^
2 + 7*a^2*C)*ArcTan[Sqrt[3] + (2*c^(1/6)*x)/a^(1/6)] - Sqrt[3]*(5*a*B*c +
55*A*c^2 + 7*a^2*C)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]
 + Sqrt[3]*(5*a*B*c + 55*A*c^2 + 7*a^2*C)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*c^
(1/6)*x + c^(1/3)*x^2])/(864*a^(17/6)*c^(13/6))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {1739, 25, 910, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^3}dx$$

$$\Big\downarrow \ 1739$$

$$-\frac{\int -\frac{12acCx^6+11Ac^2+aBc-a^2C}{(cx^6+a)^2}dx}{12ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{12acCx^6+11Ac^2+aBc-a^2C}{(cx^6+a)^2}dx}{12ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\Big\downarrow \ 910$$

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\int \frac{1}{cx^6+a}dx}{6a}+\frac{x(-13a^2C+aBc+11Ac^2)}{6a(a+cx^6)}}{12ac^2}-\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\Big\downarrow \ 753$$

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{2\left(\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{2\left(\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}\right)}{6a}+\frac{x(-13a^2C+aBc+}{6a(a+cx^6)}}{12ac^2}$$
$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[3]{a}}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}\right)}{6a}+\frac{x\left(-13a^2C+aBc+11Ac^2\right)}{6a(a+cx^6)}}{12ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt{3}\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt{3}\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\arctan\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}\right)}{6a}+\frac{x\left(-13a^2C+aBc+11Ac^2\right)}{6a(a+cx^6)}}{12ac^2}-$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\sqrt{3}\int -\frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right)}{6a}\cdots}{12ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\sqrt{3}\int \frac{\sqrt[6]{c}\left(\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right)}{6a}\cdots}{12ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\downarrow \; 27$$

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right.}{6a}\cdots}{12ac^2}\cdots$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\downarrow \; 1082$$

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)^2-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{\sqrt{3}\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\cdots}{6a^{5/6}}\cdots\right.}{6a}\cdots}{12ac^2}\cdots$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\downarrow \; 217$$

$$\frac{\frac{(7a^2C+5aBc+55Ac^2)\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{2\sqrt[6]{c}x+\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\arctan\left(\sqrt{3}\left(\frac{2\cdots}{\sqrt{3}\cdots}\right.\right.}{\sqrt[6]{c}}\cdots}{6a^{5/6}}\cdots\right.}{6a}\cdots}{12ac^2}\cdots$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

$$\downarrow \; 1103$$

$$\frac{\frac{x\left(-13a^2C+aBc+11Ac^2\right)}{6a\left(a+cx^6\right)}+\frac{\left(7a^2C+5aBc+55Ac^2\right)\left(\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)\right)}{\sqrt[6]{c}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\cdots\right)\right)}{\sqrt[6]{c}}\cdots\right)}{6a}\cdots}{12ac^2}$$

$$\frac{x\left(a^2(-C)+aBc-Ac^2\right)}{12ac^2\left(a+cx^6\right)^2}$$

input
```
Int[(A + B*x^6 + C*x^12)/(a + c*x^6)^3,x]
```

output
```
-1/12*((a*B*c - A*c^2 - a^2*C)*x)/(a*c^2*(a + c*x^6)^2) + (((a*B*c + 11*A*
c^2 - 13*a^2*C)*x)/(6*a*(a + c*x^6)) + ((5*a*B*c + 55*A*c^2 + 7*a^2*C)*(Ar
cTan[(c^(1/6)*x)/a^(1/6)]/(3*a^(5/6)*c^(1/6)) + (-(ArcTan[Sqrt[3]*(1 - (2*
c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]/c^(1/6)) - (Sqrt[3]*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(5/6)) + (ArcTan[Sqrt[3
]*(1 + (2*c^(1/6)*x)/(Sqrt[3]*a^(1/6)))]/c^(1/6) + (Sqrt[3]*Log[a^(1/3) +
Sqrt[3]*a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(2*c^(1/6)))/(6*a^(5/6))))/(6*a)
)/(12*a*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\frac{\left(11A\,c^{2}+aBc-13a^{2}C\right)x^{7}}{72a^{2}c}+\frac{\left(17A\,c^{2}-5aBc-7a^{2}C\right)x}{72a\,c^{2}}}{\left(c\,x^{6}+a\right)^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}+a\right)}\frac{\left(55A\,c^{2}+5aBc+7a^{2}C\right)\ln\left(x-\_R\right)}{\_R^{5}}}{432a^{2}c^{3}}$
default	$\frac{\frac{\left(11A\,c^{2}+aBc-13a^{2}C\right)x^{7}}{72a^{2}c}+\frac{\left(17A\,c^{2}-5aBc-7a^{2}C\right)x}{72a\,c^{2}}}{\left(c\,x^{6}+a\right)^{2}}+\frac{\left(55A\,c^{2}+5aBc+7a^{2}C\right)\left(-\frac{\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^{2}-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\arcta}{}\right.}{}$

input
```
int((C*x^12+B*x^6+A)/(c*x^6+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/72*(11*A*c^2+B*a*c-13*C*a^2)/a^2/c*x^7+1/72*(17*A*c^2-5*B*a*c-7*C*a^2)/
a/c^2*x)/(c*x^6+a)^2+1/432/a^2/c^3*sum((55*A*c^2+5*B*a*c+7*C*a^2)/_R^5*ln(
x-_R),_R=RootOf(_Z^6*c+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4411 vs. 2(242) = 484.

Time = 0.13 (sec) , antiderivative size = 4411, normalized size of antiderivative = 14.90

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^6 + Cx^{12}}{(a + cx^6)^3}\, dx = \text{Timed out}$$

input
```
integrate((C*x**12+B*x**6+A)/(c*x**6+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.34

$$\int \frac{A + Bx^6 + Cx^{12}}{(a + cx^6)^3}\, dx$$
$$= -\frac{(13\,Ca^2c - Bac^2 - 11\,Ac^3)x^7 + (7\,Ca^3 + 5\,Ba^2c - 17\,Aac^2)x}{72\,(a^2c^4x^{12} + 2\,a^3c^3x^6 + a^4c^2)}$$
$$+ \frac{\frac{\sqrt{3}(7\,Ca^2+5\,Bac+55\,Ac^2)\log\left(c^{\frac{1}{3}}x^2+\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}c^{\frac{1}{6}}} - \frac{\sqrt{3}(7\,Ca^2+5\,Bac+55\,Ac^2)\log\left(c^{\frac{1}{3}}x^2-\sqrt{3}a^{\frac{1}{6}}c^{\frac{1}{6}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}c^{\frac{1}{6}}} + \frac{4\left(7\,Ca^2c^{\frac{1}{3}}+5\,Ba\right.}{}}{}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a)^3,x, algorithm="maxima")
```

output
```
-1/72*((13*C*a^2*c - B*a*c^2 - 11*A*c^3)*x^7 + (7*C*a^3 + 5*B*a^2*c - 17*A
*a*c^2)*x)/(a^2*c^4*x^12 + 2*a^3*c^3*x^6 + a^4*c^2) + 1/864*(sqrt(3)*(7*C*
a^2 + 5*B*a*c + 55*A*c^2)*log(c^(1/3)*x^2 + sqrt(3)*a^(1/6)*c^(1/6)*x + a^
(1/3))/(a^(5/6)*c^(1/6)) - sqrt(3)*(7*C*a^2 + 5*B*a*c + 55*A*c^2)*log(c^(1
/3)*x^2 - sqrt(3)*a^(1/6)*c^(1/6)*x + a^(1/3))/(a^(5/6)*c^(1/6)) + 4*(7*C*
a^2*c^(1/3) + 5*B*a*c^(4/3) + 55*A*c^(7/3))*arctan(c^(1/3)*x/sqrt(a^(1/3)*
c^(1/3)))/(a^(2/3)*c^(1/3)*sqrt(a^(1/3)*c^(1/3))) + 2*(7*C*a^(7/3)*c^(1/3)
 + 5*B*a^(4/3)*c^(4/3) + 55*A*a^(1/3)*c^(7/3))*arctan((2*c^(1/3)*x + sqrt(
3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c^(1/3)*sqrt(a^(1/3)*c^(1/3)
)) + 2*(7*C*a^(7/3)*c^(1/3) + 5*B*a^(4/3)*c^(4/3) + 55*A*a^(1/3)*c^(7/3))*
arctan((2*c^(1/3)*x - sqrt(3)*a^(1/6)*c^(1/6))/sqrt(a^(1/3)*c^(1/3)))/(a*c
^(1/3)*sqrt(a^(1/3)*c^(1/3))))/(a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^3}\,dx = \frac{\sqrt{3}\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2+5\,(ac^5)^{\frac{1}{6}}\,Bac+55\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\log\left(x^2+\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^3} - \frac{\sqrt{3}\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2+5\,(ac^5)^{\frac{1}{6}}\,Bac+55\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\log\left(x^2-\sqrt{3}x\left(\frac{a}{c}\right)^{\frac{1}{6}}+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^3} + \frac{\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2+5\,(ac^5)^{\frac{1}{6}}\,Bac+55\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^3} + \frac{\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2+5\,(ac^5)^{\frac{1}{6}}\,Bac+55\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(\frac{a}{c}\right)^{\frac{1}{6}}}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^3} + \frac{\left(7\,(ac^5)^{\frac{1}{6}}\,Ca^2+5\,(ac^5)^{\frac{1}{6}}\,Bac+55\,(ac^5)^{\frac{1}{6}}\,Ac^2\right)\arctan\left(\frac{x}{\left(\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{216\,a^3c^3} - \frac{13\,Ca^2cx^7-Bac^2x^7-11\,Ac^3x^7+7\,Ca^3x+5\,Ba^2cx-17\,Aac^2x}{72\,(cx^6+a)^2a^2c^2}$$

input
```
integrate((C*x^12+B*x^6+A)/(c*x^6+a)^3,x, algorithm="giac")
```

output
```
1/864*sqrt(3)*(7*(a*c^5)^(1/6)*C*a^2 + 5*(a*c^5)^(1/6)*B*a*c + 55*(a*c^5)^
(1/6)*A*c^2)*log(x^2 + sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^3*c^3) - 1/
864*sqrt(3)*(7*(a*c^5)^(1/6)*C*a^2 + 5*(a*c^5)^(1/6)*B*a*c + 55*(a*c^5)^(1
/6)*A*c^2)*log(x^2 - sqrt(3)*x*(a/c)^(1/6) + (a/c)^(1/3))/(a^3*c^3) + 1/43
2*(7*(a*c^5)^(1/6)*C*a^2 + 5*(a*c^5)^(1/6)*B*a*c + 55*(a*c^5)^(1/6)*A*c^2)
*arctan((2*x + sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^3*c^3) + 1/432*(7*(a*c
^5)^(1/6)*C*a^2 + 5*(a*c^5)^(1/6)*B*a*c + 55*(a*c^5)^(1/6)*A*c^2)*arctan((
2*x - sqrt(3)*(a/c)^(1/6))/(a/c)^(1/6))/(a^3*c^3) + 1/216*(7*(a*c^5)^(1/6)
*C*a^2 + 5*(a*c^5)^(1/6)*B*a*c + 55*(a*c^5)^(1/6)*A*c^2)*arctan(x/(a/c)^(1
/6))/(a^3*c^3) - 1/72*(13*C*a^2*c*x^7 - B*a*c^2*x^7 - 11*A*c^3*x^7 + 7*C*a
^3*x + 5*B*a^2*c*x - 17*A*a*c^2*x)/((c*x^6 + a)^2*a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 4533, normalized size of antiderivative = 15.31

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^6 + C*x^12)/(a + c*x^6)^3,x)
```

output
```
((x^7*(11*A*c^2 - 13*C*a^2 + B*a*c))/(72*a^2*c) - (x*(7*C*a^2 - 17*A*c^2 +
 5*B*a*c))/(72*a*c^2))/(a^2 + c^2*x^12 + 2*a*c*x^6) - (atan(((((x*(9150625
*A^4*c^8 + 2401*C^4*a^8 + 625*B^4*a^4*c^4 + 453750*A^2*B^2*a^2*c^6 + 88935
0*A^2*C^2*a^4*c^4 + 7350*B^2*C^2*a^6*c^2 + 3327500*A^3*B*a*c^7 + 6860*B*C^
3*a^7*c + 27500*A*B^3*a^3*c^5 + 75460*A*C^3*a^6*c^2 + 4658500*A^3*C*a^2*c^
6 + 3500*B^3*C*a^5*c^3 + 161700*A*B*C^2*a^5*c^3 + 115500*A*B^2*C*a^4*c^4 +
 1270500*A^2*B*C*a^3*c^5))/(4478976*a^8*c^3) - ((55*A*c^2 + 7*C*a^2 + 5*B*
a*c)*(166375*A^3*c^6 + 343*C^3*a^6 + 125*B^3*a^3*c^3 + 45375*A^2*B*a*c^5 +
 735*B*C^2*a^5*c + 4125*A*B^2*a^2*c^4 + 8085*A*C^2*a^4*c^2 + 63525*A^2*C*a
^2*c^4 + 525*B^2*C*a^4*c^2 + 11550*A*B*C*a^3*c^3))/(4478976*(-a)^(47/6)*c^
(19/6)))*(55*A*c^2 + 7*C*a^2 + 5*B*a*c)*1i)/(432*(-a)^(17/6)*c^(13/6)) + (
((x*(9150625*A^4*c^8 + 2401*C^4*a^8 + 625*B^4*a^4*c^4 + 453750*A^2*B^2*a^2
*c^6 + 889350*A^2*C^2*a^4*c^4 + 7350*B^2*C^2*a^6*c^2 + 3327500*A^3*B*a*c^7
 + 6860*B*C^3*a^7*c + 27500*A*B^3*a^3*c^5 + 75460*A*C^3*a^6*c^2 + 4658500*
A^3*C*a^2*c^6 + 3500*B^3*C*a^5*c^3 + 161700*A*B*C^2*a^5*c^3 + 115500*A*B^2
*C*a^4*c^4 + 1270500*A^2*B*C*a^3*c^5))/(4478976*a^8*c^3) + ((55*A*c^2 + 7*
C*a^2 + 5*B*a*c)*(166375*A^3*c^6 + 343*C^3*a^6 + 125*B^3*a^3*c^3 + 45375*A
^2*B*a*c^5 + 735*B*C^2*a^5*c + 4125*A*B^2*a^2*c^4 + 8085*A*C^2*a^4*c^2 + 6
3525*A^2*C*a^2*c^4 + 525*B^2*C*a^4*c^2 + 11550*A*B*C*a^3*c^3))/(4478976*(-
a)^(47/6)*c^(19/6)))*(55*A*c^2 + 7*C*a^2 + 5*B*a*c)*1i)/(432*(-a)^(17/6...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1502, normalized size of antiderivative = 5.07

$$\int \frac{A+Bx^6+Cx^{12}}{(a+cx^6)^3}\,dx = \text{Too large to display}$$

input
```
int((C*x^12+B*x^6+A)/(c*x^6+a)^3,x)
```

output
```
( - 14*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(
c**(1/6)*a**(1/6)))*a**3 - 10*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sq
rt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*b - 28*c**(5/6)*a**(1/6)*a
tan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*c
*x**6 - 110*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)
*x)/(c**(1/6)*a**(1/6)))*a**2*c - 20*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(
1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*b*c*x**6 - 14*c**(5/6)
*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/
6)))*a*c**2*x**12 - 220*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3)
- 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a*c**2*x**6 - 10*c**(5/6)*a**(1/6)*at
an((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*b*c**2*
x**12 - 110*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) - 2*c**(1/3)
*x)/(c**(1/6)*a**(1/6)))*c**3*x**12 + 14*c**(5/6)*a**(1/6)*atan((c**(1/6)*
a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**3 + 10*c**(5/6)*a
**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)
))*a**2*b + 28*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1
/3)*x)/(c**(1/6)*a**(1/6)))*a**2*c*x**6 + 110*c**(5/6)*a**(1/6)*atan((c**(
1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)))*a**2*c + 20*c**
(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqrt(3) + 2*c**(1/3)*x)/(c**(1/6)*a
**(1/6)))*a*b*c*x**6 + 14*c**(5/6)*a**(1/6)*atan((c**(1/6)*a**(1/6)*sqr...
```

3.24 $\int (a-cx^6)^3 (A+Bx^6+Cx^{12}) \, dx$

Optimal result

Integrand size = 23, antiderivative size = 106

$$\int (a-cx^6)^3 (A+Bx^6+Cx^{12}) \, dx = a^3Ax + \frac{1}{7}a^2(aB-3Ac)x^7 - \frac{1}{13}a(3aBc-3Ac^2-a^2C)\,x^{13} + \frac{1}{19}c(3aBc-Ac^2-3a^2C)\,x^{19} - \frac{1}{25}c^2(Bc-3aC)x^{25} - \frac{1}{31}c^3Cx^{31}$$

output
```
a^3*A*x+1/7*a^2*(-3*A*c+B*a)*x^7-1/13*a*(-3*A*c^2+3*B*a*c-C*a^2)*x^13+1/19
*c*(-A*c^2+3*B*a*c-3*C*a^2)*x^19-1/25*c^2*(B*c-3*C*a)*x^25-1/31*c^3*C*x^31
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int (a-cx^6)^3 (A+Bx^6+Cx^{12})\,dx = a^3Ax+\frac{1}{7}a^2(aB-3Ac)x^7 + \frac{1}{13}a\left(-3aBc+3Ac^2+a^2C\right)x^{13} - \frac{1}{19}c\left(-3aBc+Ac^2+3a^2C\right)x^{19} - \frac{1}{25}c^2(Bc-3aC)x^{25}-\frac{1}{31}c^3Cx^{31}$$

input
```
Integrate[(a - c*x^6)^3*(A + B*x^6 + C*x^12),x]
```

output
```
a^3*A*x + (a^2*(a*B - 3*A*c)*x^7)/7 + (a*(-3*a*B*c + 3*A*c^2 + a^2*C)*x^13
)/13 - (c*(-3*a*B*c + A*c^2 + 3*a^2*C)*x^19)/19 - (c^2*(B*c - 3*a*C)*x^25)
/25 - (c^3*C*x^31)/31
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^6)^3 (A+Bx^6+Cx^{12})\,dx$$

$$\Big\downarrow \ 1737$$

$$\int \left(a^3A-cx^{18}\left(3a^2C-3aBc+Ac^2\right)+ax^{12}\left(a^2C-3aBc+3Ac^2\right)+a^2x^6(aB-3Ac)-c^2x^{24}(Bc-3aC)-c^3C$$

$$\Big\downarrow \ 2009$$

$$a^3Ax + \frac{1}{19}cx^{19}(-3a^2C + 3aBc - Ac^2) - \frac{1}{13}ax^{13}(a^2(-C) + 3aBc - 3Ac^2) + \frac{1}{7}a^2x^7(aB - 3Ac) - \frac{1}{25}c^2x^{25}(Bc - 3aC) - \frac{1}{31}c^3Cx^{31}$$

input
```
Int[(a - c*x^6)^3*(A + B*x^6 + C*x^12),x]
```

output
```
a^3*A*x + (a^2*(a*B - 3*A*c)*x^7)/7 - (a*(3*a*B*c - 3*A*c^2 - a^2*C)*x^13)
/13 + (c*(3*a*B*c - A*c^2 - 3*a^2*C)*x^19)/19 - (c^2*(B*c - 3*a*C)*x^25)/2
5 - (c^3*C*x^31)/31
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

method	result
norman	$-\frac{c^3 C x^{31}}{31} + \left(-\frac{1}{25} c^3 B + \frac{3}{25} a c^2 C\right) x^{25} + \left(\frac{3}{13} A a c^2 - \frac{3}{13} a^2 c B + \frac{1}{13} a^3 C\right) x^{13} + \left(-\frac{1}{19} A c^3 + \frac{3}{19} a\right.$
default	$-\frac{c^3 C x^{31}}{31} + \frac{(-c^3 B + 3a c^2 C) x^{25}}{25} + \frac{(-A c^3 + 3a c^2 B - 3a^2 c C) x^{19}}{19} + \frac{(3Aa c^2 - 3a^2 c B + a^3 C) x^{13}}{13} + \frac{(-3a^2 A c + B a^3) x^7}{7}$
gosper	$-\frac{1}{31} c^3 C x^{31} - \frac{1}{25} x^{25} c^3 B + \frac{3}{25} x^{25} a c^2 C + \frac{3}{13} x^{13} A a c^2 - \frac{3}{13} x^{13} a^2 c B + \frac{1}{13} x^{13} a^3 C - \frac{1}{19} x^{19} A c^3$
risch	$-\frac{1}{31} c^3 C x^{31} - \frac{1}{25} x^{25} c^3 B + \frac{3}{25} x^{25} a c^2 C + \frac{3}{13} x^{13} A a c^2 - \frac{3}{13} x^{13} a^2 c B + \frac{1}{13} x^{13} a^3 C - \frac{1}{19} x^{19} A c^3$
parallelrisch	$-\frac{1}{31} c^3 C x^{31} - \frac{1}{25} x^{25} c^3 B + \frac{3}{25} x^{25} a c^2 C + \frac{3}{13} x^{13} A a c^2 - \frac{3}{13} x^{13} a^2 c B + \frac{1}{13} x^{13} a^3 C - \frac{1}{19} x^{19} A c^3$
orering	$\frac{x(-43225 c^3 C x^{30} - 53599 B c^3 x^{24} + 160797 C a c^2 x^{24} - 70525 A c^3 x^{18} + 211575 B a c^2 x^{18} - 211575 C a^2 c x^{18} + 309225 A a c^2 x^{12} -}{1339975}$

input
```
int((-c*x^6+a)^3*(C*x^12+B*x^6+A),x,method=_RETURNVERBOSE)
```

output
```
-1/31*c^3*C*x^31+(-1/25*c^3*B+3/25*a*c^2*C)*x^25+(3/13*A*a*c^2-3/13*a^2*c*
B+1/13*a^3*C)*x^13+(-1/19*A*c^3+3/19*a*c^2*B-3/19*a^2*c*C)*x^19+(-3/7*a^2*
A*c+1/7*B*a^3)*x^7+a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \left(a-cx^6\right)^3 \left(A+Bx^6+Cx^{12}\right) \, dx = -\frac{1}{31}\,Cc^3x^{31}+\frac{1}{25}\left(3\,Cac^2-Bc^3\right)x^{25} -\frac{1}{19}\left(3\,Ca^2c-3\,Bac^2+Ac^3\right)x^{19} +\frac{1}{13}\left(Ca^3-3\,Ba^2c+3\,Aac^2\right)x^{13} +\frac{1}{7}\left(Ba^3-3\,Aa^2c\right)x^7+Aa^3x$$

input
```
integrate((-c*x^6+a)^3*(C*x^12+B*x^6+A),x, algorithm="fricas")
```

output
```
-1/31*C*c^3*x^31 + 1/25*(3*C*a*c^2 - B*c^3)*x^25 - 1/19*(3*C*a^2*c - 3*B*a
*c^2 + A*c^3)*x^19 + 1/13*(C*a^3 - 3*B*a^2*c + 3*A*a*c^2)*x^13 + 1/7*(B*a^
3 - 3*A*a^2*c)*x^7 + A*a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

$$\int \left(a-cx^6\right)^3 \left(A+Bx^6+Cx^{12}\right) \, dx = Aa^3x-\frac{Cc^3x^{31}}{31}+x^{25}\left(-\frac{Bc^3}{25}+\frac{3Cac^2}{25}\right) +x^{19}\left(-\frac{Ac^3}{19}+\frac{3Bac^2}{19}-\frac{3Ca^2c}{19}\right) +x^{13}\cdot\left(\frac{3Aac^2}{13}-\frac{3Ba^2c}{13}+\frac{Ca^3}{13}\right) +x^7\left(-\frac{3Aa^2c}{7}+\frac{Ba^3}{7}\right)$$

input
```
integrate((-c*x**6+a)**3*(C*x**12+B*x**6+A),x)
```

output
```
A*a**3*x - C*c**3*x**31/31 + x**25*(-B*c**3/25 + 3*C*a*c**2/25) + x**19*(-
A*c**3/19 + 3*B*a*c**2/19 - 3*C*a**2*c/19) + x**13*(3*A*a*c**2/13 - 3*B*a*
*2*c/13 + C*a**3/13) + x**7*(-3*A*a**2*c/7 + B*a**3/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int (a-cx^6)^3 (A+Bx^6+Cx^{12}) \, dx = -\frac{1}{31} Cc^3x^{31} + \frac{1}{25} (3 Cac^2 - Bc^3)x^{25} - \frac{1}{19} (3 Ca^2c - 3 Bac^2 + Ac^3)x^{19} + \frac{1}{13} (Ca^3 - 3 Ba^2c + 3 Aac^2)x^{13} + \frac{1}{7} (Ba^3 - 3 Aa^2c)x^7 + Aa^3x$$

input
```
integrate((-c*x^6+a)^3*(C*x^12+B*x^6+A),x, algorithm="maxima")
```

output
```
-1/31*C*c^3*x^31 + 1/25*(3*C*a*c^2 - B*c^3)*x^25 - 1/19*(3*C*a^2*c - 3*B*a
*c^2 + A*c^3)*x^19 + 1/13*(C*a^3 - 3*B*a^2*c + 3*A*a*c^2)*x^13 + 1/7*(B*a^
3 - 3*A*a^2*c)*x^7 + A*a^3*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

$$\int (a-cx^6)^3 (A+Bx^6+Cx^{12}) \, dx = -\frac{1}{31} Cc^3x^{31} + \frac{3}{25} Cac^2x^{25} - \frac{1}{25} Bc^3x^{25} - \frac{3}{19} Ca^2cx^{19} + \frac{3}{19} Bac^2x^{19} - \frac{1}{19} Ac^3x^{19} + \frac{1}{13} Ca^3x^{13} - \frac{3}{13} Ba^2cx^{13} + \frac{3}{13} Aac^2x^{13} + \frac{1}{7} Ba^3x^7 - \frac{3}{7} Aa^2cx^7 + Aa^3x$$

input
```
integrate((-c*x^6+a)^3*(C*x^12+B*x^6+A),x, algorithm="giac")
```

output
```
-1/31*C*c^3*x^31 + 3/25*C*a*c^2*x^25 - 1/25*B*c^3*x^25 - 3/19*C*a^2*c*x^19
 + 3/19*B*a*c^2*x^19 - 1/19*A*c^3*x^19 + 1/13*C*a^3*x^13 - 3/13*B*a^2*c*x^
13 + 3/13*A*a*c^2*x^13 + 1/7*B*a^3*x^7 - 3/7*A*a^2*c*x^7 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int (a-cx^6)^3 (A+Bx^6+Cx^{12})\,dx = x^{13}\left(\frac{C\,a^3}{13}-\frac{3\,B\,a^2\,c}{13}+\frac{3\,A\,a\,c^2}{13}\right) - x^{19}\left(\frac{3\,C\,a^2\,c}{19}-\frac{3\,B\,a\,c^2}{19}+\frac{A\,c^3}{19}\right) + x^7\left(\frac{B\,a^3}{7}-\frac{3\,A\,a^2\,c}{7}\right) - x^{25}\left(\frac{B\,c^3}{25}-\frac{3\,C\,a\,c^2}{25}\right) - \frac{C\,c^3\,x^{31}}{31} + A\,a^3\,x$$

input
```
int((a - c*x^6)^3*(A + B*x^6 + C*x^12),x)
```

output
```
x^13*((C*a^3)/13 + (3*A*a*c^2)/13 - (3*B*a^2*c)/13) - x^19*((A*c^3)/19 - (
3*B*a*c^2)/19 + (3*C*a^2*c)/19) + x^7*((B*a^3)/7 - (3*A*a^2*c)/7) - x^25*(
(B*c^3)/25 - (3*C*a*c^2)/25) - (C*c^3*x^31)/31 + A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int (a-cx^6)^3 (A+Bx^6+Cx^{12})\,dx = \frac{x(-43225c^4x^{30}+160797a\,c^3x^{24}-53599b\,c^3x^{24}-211575a^2c^2x^{18}+211575ab\,c^2x^{18}-70525a\,c^3x^{18}+10}{1339975}$$

input
```
int((-c*x^6+a)^3*(C*x^12+B*x^6+A),x)
```

output

```
(x*(1339975*a**4 + 191425*a**3*b*x**6 + 103075*a**3*c*x**12 - 574275*a**3*
c*x**6 - 309225*a**2*b*c*x**12 - 211575*a**2*c**2*x**18 + 309225*a**2*c**2
*x**12 + 211575*a*b*c**2*x**18 + 160797*a*c**3*x**24 - 70525*a*c**3*x**18
- 53599*b*c**3*x**24 - 43225*c**4*x**30))/1339975
```

3.25 $\int (a-cx^6)^2 (A+Bx^6+Cx^{12})\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (a-cx^6)^2 (A+Bx^6+Cx^{12})\,dx = a^2Ax+\frac{1}{7}a(aB-2Ac)x^7-\frac{1}{13}(2aBc-Ac^2-a^2C)x^{13}+\frac{1}{19}c(Bc-2aC)x^{19}+\frac{1}{25}c^2Cx^{25}$$

output
```
a^2*A*x+1/7*a*(-2*A*c+B*a)*x^7-1/13*(-A*c^2+2*B*a*c-C*a^2)*x^13+1/19*c*(B*
c-2*C*a)*x^19+1/25*c^2*C*x^25
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (a-cx^6)^2 (A+Bx^6+Cx^{12})\,dx = a^2Ax+\frac{1}{7}a(aB-2Ac)x^7+\frac{1}{13}(-2aBc+Ac^2+a^2C)x^{13}+\frac{1}{19}c(Bc-2aC)x^{19}+\frac{1}{25}c^2Cx^{25}$$

input
```
Integrate[(a - c*x^6)^2*(A + B*x^6 + C*x^12),x]
```

output
```
a^2*A*x + (a*(a*B - 2*A*c)*x^7)/7 + ((-2*a*B*c + A*c^2 + a^2*C)*x^13)/13 +
 (c*(B*c - 2*a*C)*x^19)/19 + (c^2*C*x^25)/25
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-cx^6\right)^2 \left(A+Bx^6+Cx^{12}\right) dx$$

$$\Big\downarrow \text{1737}$$

$$\int \left(x^{12}\left(a^2C-2aBc+Ac^2\right)+a^2A+ax^6(aB-2Ac)+cx^{18}(Bc-2aC)+c^2Cx^{24}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{13}x^{13}\left(a^2(-C)+2aBc-Ac^2\right)+a^2Ax+\frac{1}{7}ax^7(aB-2Ac)+\frac{1}{19}cx^{19}(Bc-2aC)+\frac{1}{25}c^2Cx^{25}$$

input
```
Int[(a - c*x^6)^2*(A + B*x^6 + C*x^12),x]
```

output
```
a^2*A*x + (a*(a*B - 2*A*c)*x^7)/7 - ((2*a*B*c - A*c^2 - a^2*C)*x^13)/13 +
(c*(B*c - 2*a*C)*x^19)/19 + (c^2*C*x^25)/25
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

method	result
default	$\frac{c^2 C x^{25}}{25}+\frac{(c^2 B-2acC)x^{19}}{19}+\frac{(A c^2-2aBc+a^2 C)x^{13}}{13}+\frac{(-2Aac+a^2 B)x^7}{7}+a^2 A x$
norman	$\frac{c^2 C x^{25}}{25}+\left(\frac{1}{19}c^2 B-\frac{2}{19}acC\right)x^{19}+\left(\frac{1}{13}A c^2-\frac{2}{13}aBc+\frac{1}{13}a^2 C\right)x^{13}+a^2 A x+\left(-\frac{2}{7}Aac+\frac{1}{7}a^2\right.$
gosper	$\frac{1}{25}c^2 C x^{25}+\frac{1}{19}x^{19}c^2 B-\frac{2}{19}x^{19}acC+\frac{1}{13}x^{13}A c^2-\frac{2}{13}x^{13}aBc+\frac{1}{13}x^{13}a^2 C+a^2 A x-\frac{2}{7}x^7 Aac$
risch	$\frac{1}{25}c^2 C x^{25}+\frac{1}{19}x^{19}c^2 B-\frac{2}{19}x^{19}acC+\frac{1}{13}x^{13}A c^2-\frac{2}{13}x^{13}aBc+\frac{1}{13}x^{13}a^2 C+a^2 A x-\frac{2}{7}x^7 Aac$
parallelrisch	$\frac{1}{25}c^2 C x^{25}+\frac{1}{19}x^{19}c^2 B-\frac{2}{19}x^{19}acC+\frac{1}{13}x^{13}A c^2-\frac{2}{13}x^{13}aBc+\frac{1}{13}x^{13}a^2 C+a^2 A x-\frac{2}{7}x^7 Aac$
orering	$\frac{x(1729c^2 C x^{24}+2275B c^2 x^{18}-4550Cac x^{18}+3325A c^2 x^{12}-6650Bac x^{12}+3325C a^2 x^{12}-12350Aac x^6+6175B a^2 x^6+43225}{43225}$

input
```
int((-c*x^6+a)^2*(C*x^12+B*x^6+A),x,method=_RETURNVERBOSE)
```

output
```
1/25*c^2*C*x^25+1/19*(B*c^2-2*C*a*c)*x^19+1/13*(A*c^2-2*B*a*c+C*a^2)*x^13+
1/7*(-2*A*a*c+B*a^2)*x^7+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \left(a-cx^6\right)^2\left(A+Bx^6+Cx^{12}\right)\,dx=\frac{1}{25}\,Cc^2x^{25}-\frac{1}{19}\,\left(2\,Cac-Bc^2\right)x^{19}+\frac{1}{13}\,\left(Ca^2-2\,Bac+Ac^2\right)x^{13}+\frac{1}{7}\,\left(Ba^2-2\,Aac\right)x^7+Aa^2x$$

input
```
integrate((-c*x^6+a)^2*(C*x^12+B*x^6+A),x, algorithm="fricas")
```

output
```
1/25*C*c^2*x^25 - 1/19*(2*C*a*c - B*c^2)*x^19 + 1/13*(C*a^2 - 2*B*a*c + A*
c^2)*x^13 + 1/7*(B*a^2 - 2*A*a*c)*x^7 + A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int (a-cx^6)^2 (A+Bx^6+Cx^{12})\,dx = Aa^2x + \frac{Cc^2x^{25}}{25} + x^{19}\left(\frac{Bc^2}{19} - \frac{2Cac}{19}\right) + x^{13}\left(\frac{Ac^2}{13} - \frac{2Bac}{13} + \frac{Ca^2}{13}\right) + x^7\left(-\frac{2Aac}{7} + \frac{Ba^2}{7}\right)$$

input
```
integrate((-c*x**6+a)**2*(C*x**12+B*x**6+A),x)
```

output
```
A*a**2*x + C*c**2*x**25/25 + x**19*(B*c**2/19 - 2*C*a*c/19) + x**13*(A*c**
2/13 - 2*B*a*c/13 + C*a**2/13) + x**7*(-2*A*a*c/7 + B*a**2/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int (a-cx^6)^2 (A+Bx^6+Cx^{12})\,dx = \frac{1}{25}\,Cc^2x^{25} - \frac{1}{19}\,(2\,Cac - Bc^2)x^{19} + \frac{1}{13}\,(Ca^2 - 2\,Bac + Ac^2)x^{13} + \frac{1}{7}\,(Ba^2 - 2\,Aac)x^7 + Aa^2x$$

input
```
integrate((-c*x^6+a)^2*(C*x^12+B*x^6+A),x, algorithm="maxima")
```

output
```
1/25*C*c^2*x^25 - 1/19*(2*C*a*c - B*c^2)*x^19 + 1/13*(C*a^2 - 2*B*a*c + A*
c^2)*x^13 + 1/7*(B*a^2 - 2*A*a*c)*x^7 + A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int (a-cx^6)^2 (A+Bx^6+Cx^{12})\, dx = \frac{1}{25}\,Cc^2x^{25} - \frac{2}{19}\,Cacx^{19} + \frac{1}{19}\,Bc^2x^{19} + \frac{1}{13}\,Ca^2x^{13} - \frac{2}{13}\,Bacx^{13} + \frac{1}{13}\,Ac^2x^{13} + \frac{1}{7}\,Ba^2x^7 - \frac{2}{7}\,Aacx^7 + Aa^2x$$

input
```
integrate((-c*x^6+a)^2*(C*x^12+B*x^6+A),x, algorithm="giac")
```

output
```
1/25*C*c^2*x^25 - 2/19*C*a*c*x^19 + 1/19*B*c^2*x^19 + 1/13*C*a^2*x^13 - 2/
13*B*a*c*x^13 + 1/13*A*c^2*x^13 + 1/7*B*a^2*x^7 - 2/7*A*a*c*x^7 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int (a-cx^6)^2 (A+Bx^6+Cx^{12})\, dx = x^7\left(\frac{B\,a^2}{7}-\frac{2\,A\,a\,c}{7}\right)+x^{19}\left(\frac{B\,c^2}{19}-\frac{2\,C\,a\,c}{19}\right)+x^{13}\left(\frac{C\,a^2}{13}-\frac{2\,B\,a\,c}{13}+\frac{A\,c^2}{13}\right)+\frac{C\,c^2\,x^{25}}{25}+A\,a^2\,x$$

input
```
int((a - c*x^6)^2*(A + B*x^6 + C*x^12),x)
```

output
```
x^7*((B*a^2)/7 - (2*A*a*c)/7) + x^19*((B*c^2)/19 - (2*C*a*c)/19) + x^13*((
A*c^2)/13 + (C*a^2)/13 - (2*B*a*c)/13) + (C*c^2*x^25)/25 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \left(a - cx^6\right)^2 \left(A + Bx^6 + Cx^{12}\right)\, dx$$
$$= \frac{x(1729c^3x^{24} - 4550a\,c^2x^{18} + 2275b\,c^2x^{18} + 3325a^2c\,x^{12} - 6650abc\,x^{12} + 3325a\,c^2x^{12} + 6175a^2b\,x^6 - 123}{43225}$$

input
```
int((-c*x^6+a)^2*(C*x^12+B*x^6+A),x)
```

output
```
(x*(43225*a**3 + 6175*a**2*b*x**6 + 3325*a**2*c*x**12 - 12350*a**2*c*x**6
- 6650*a*b*c*x**12 - 4550*a*c**2*x**18 + 3325*a*c**2*x**12 + 2275*b*c**2*x
**18 + 1729*c**3*x**24))/43225
```

3.26 $\int (a - cx^6) (A + Bx^6 + Cx^{12}) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int (a - cx^6) (A + Bx^6 + Cx^{12}) \, dx = aAx + \frac{1}{7}(aB - Ac)x^7 - \frac{1}{13}(Bc - aC)x^{13} - \frac{1}{19}cCx^{19}$$

output
```
a*A*x+1/7*(-A*c+B*a)*x^7-1/13*(B*c-C*a)*x^13-1/19*c*C*x^19
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a - cx^6) (A + Bx^6 + Cx^{12}) \, dx = aAx + \frac{1}{7}(aB - Ac)x^7 + \frac{1}{13}(-Bc + aC)x^{13} - \frac{1}{19}cCx^{19}$$

input
```
Integrate[(a - c*x^6)*(A + B*x^6 + C*x^12),x]
```

output
```
a*A*x + ((a*B - A*c)*x^7)/7 + ((-(B*c) + a*C)*x^13)/13 - (c*C*x^19)/19
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-cx^6) (A+Bx^6+Cx^{12}) \, dx$$

$$\Big\downarrow \text{1737}$$

$$\int (x^6(aB-Ac)+aA-x^{12}(Bc-aC)-cCx^{18}) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(aB-Ac)+aAx-\frac{1}{13}x^{13}(Bc-aC)-\frac{1}{19}cCx^{19}$$

input
```
Int[(a - c*x^6)*(A + B*x^6 + C*x^12),x]
```

output
```
a*A*x + ((a*B - A*c)*x^7)/7 - ((B*c - a*C)*x^13)/13 - (c*C*x^19)/19
```

Defintions of rubi rules used

rule 1737
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{cC\,x^{19}}{19}+\frac{(-Bc+Ca)x^{13}}{13}+\frac{(-Ac+Ba)x^{7}}{7}+aAx$	39
norman	$aAx+\left(-\frac{Ac}{7}+\frac{Ba}{7}\right)x^{7}+\left(-\frac{Bc}{13}+\frac{Ca}{13}\right)x^{13}-\frac{cC\,x^{19}}{19}$	39
gosper	$aAx-\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ba-\frac{1}{13}x^{13}Bc+\frac{1}{13}x^{13}Ca-\frac{1}{19}cC\,x^{19}$	41
risch	$aAx-\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ba-\frac{1}{13}x^{13}Bc+\frac{1}{13}x^{13}Ca-\frac{1}{19}cC\,x^{19}$	41
parallelrisch	$aAx-\frac{1}{7}x^{7}Ac+\frac{1}{7}x^{7}Ba-\frac{1}{13}x^{13}Bc+\frac{1}{13}x^{13}Ca-\frac{1}{19}cC\,x^{19}$	41
orering	$\frac{x\left(-91Cc\,x^{18}-133Bc\,x^{12}+133Ca\,x^{12}-247Ac\,x^{6}+247Ba\,x^{6}+1729Aa\right)}{1729}$	44

input
```
int((-c*x^6+a)*(C*x^12+B*x^6+A),x,method=_RETURNVERBOSE)
```

output
```
-1/19*c*C*x^19+1/13*(-B*c+C*a)*x^13+1/7*(-A*c+B*a)*x^7+a*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \left(a-cx^6\right)\left(A+Bx^6+Cx^{12}\right)\,dx=-\frac{1}{19}\,Ccx^{19}+\frac{1}{13}\,(Ca-Bc)x^{13}+\frac{1}{7}\,(Ba-Ac)x^{7}+Aax$$

input
```
integrate((-c*x^6+a)*(C*x^12+B*x^6+A),x, algorithm="fricas")
```

output
```
-1/19*C*c*x^19 + 1/13*(C*a - B*c)*x^13 + 1/7*(B*a - A*c)*x^7 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \left(a-cx^6\right)\left(A+Bx^6+Cx^{12}\right)\,dx = Aax - \frac{Ccx^{19}}{19} + x^{13}\left(-\frac{Bc}{13}+\frac{Ca}{13}\right) + x^7\left(-\frac{Ac}{7}+\frac{Ba}{7}\right)$$

input
```
integrate((-c*x**6+a)*(C*x**12+B*x**6+A),x)
```

output
```
A*a*x - C*c*x**19/19 + x**13*(-B*c/13 + C*a/13) + x**7*(-A*c/7 + B*a/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \left(a-cx^6\right)\left(A+Bx^6+Cx^{12}\right)\,dx = -\frac{1}{19}\,Ccx^{19} + \frac{1}{13}\,(Ca-Bc)x^{13} + \frac{1}{7}\,(Ba-Ac)x^7 + Aax$$

input
```
integrate((-c*x^6+a)*(C*x^12+B*x^6+A),x, algorithm="maxima")
```

output
```
-1/19*C*c*x^19 + 1/13*(C*a - B*c)*x^13 + 1/7*(B*a - A*c)*x^7 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \left(a-cx^6\right)\left(A+Bx^6+Cx^{12}\right)\,dx = -\frac{1}{19}\,Ccx^{19} + \frac{1}{13}\,Cax^{13} - \frac{1}{13}\,Bcx^{13} + \frac{1}{7}\,Bax^7 - \frac{1}{7}\,Acx^7 + Aax$$

input
```
integrate((-c*x^6+a)*(C*x^12+B*x^6+A),x, algorithm="giac")
```

output
```
-1/19*C*c*x^19 + 1/13*C*a*x^13 - 1/13*B*c*x^13 + 1/7*B*a*x^7 - 1/7*A*c*x^7
 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (a-cx^6) (A+Bx^6+Cx^{12}) \, dx = -\frac{C\,c\,x^{19}}{19}+\left(\frac{C\,a}{13}-\frac{B\,c}{13}\right)\,x^{13}+\left(\frac{B\,a}{7}-\frac{A\,c}{7}\right)\,x^7+A\,a\,x$$

input
```
int((a - c*x^6)*(A + B*x^6 + C*x^12),x)
```

output
```
x^7*((B*a)/7 - (A*c)/7) + x^13*((C*a)/13 - (B*c)/13) + A*a*x - (C*c*x^19)/
19
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int (a-cx^6) (A+Bx^6+Cx^{12}) \, dx = \frac{x(-91c^2x^{18}+133ac\,x^{12}-133bc\,x^{12}+247ab\,x^6-247ac\,x^6+1729a^2)}{1729}$$

input
```
int((-c*x^6+a)*(C*x^12+B*x^6+A),x)
```

output
```
(x*(1729*a**2 + 247*a*b*x**6 + 133*a*c*x**12 - 247*a*c*x**6 - 133*b*c*x**1
2 - 91*c**2*x**18))/1729
```

3.27 $\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\begin{aligned}\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx =& -\frac{(Bc+aC)x}{c^2}-\frac{Cx^7}{7c} \\ &-\frac{(aBc+Ac^2+a^2C)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{13/6}} \\ &+\frac{(aBc+Ac^2+a^2C)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{2\sqrt{3}a^{5/6}c^{13/6}} \\ &+\frac{(aBc+Ac^2+a^2C)\operatorname{arctanh}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}c^{13/6}} \\ &+\frac{(aBc+Ac^2+a^2C)\operatorname{arctanh}\left(\frac{\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{6a^{5/6}c^{13/6}}\end{aligned}$$

output

```
-(B*c+C*a)*x/c^2-1/7*C*x^7/c-1/6*(A*c^2+B*a*c+C*a^2)*arctan(1/3*(a^(1/6)-2
*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(5/6)/c^(13/6)+1/6*(A*c^2+B*a*c+C*a
^2)*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(5/6)/c^(1
3/6)+1/3*(A*c^2+B*a*c+C*a^2)*arctanh(c^(1/6)*x/a^(1/6))/a^(5/6)/c^(13/6)+1
/6*(A*c^2+B*a*c+C*a^2)*arctanh(a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))/a^
(5/6)/c^(13/6)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx$$
$$= \frac{-84a^{5/6}\sqrt[6]{c}(Bc+aC)x-12a^{5/6}c^{7/6}Cx^7-14\sqrt{3}(aBc+Ac^2+a^2C)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)+14\sqrt{3}(aBc+\cdots}{\cdots}$$

input
```
Integrate[(A + B*x^6 + C*x^12)/(a - c*x^6),x]
```

output
```
(-84*a^(5/6)*c^(1/6)*(B*c + a*C)*x - 12*a^(5/6)*c^(7/6)*C*x^7 - 14*Sqrt[3]
*(a*B*c + A*c^2 + a^2*C)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] + 14*
Sqrt[3]*(a*B*c + A*c^2 + a^2*C)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]
] - 14*(a*B*c + A*c^2 + a^2*C)*Log[a^(1/6) - c^(1/6)*x] + 14*(a*B*c + A*c^
2 + a^2*C)*Log[a^(1/6) + c^(1/6)*x] - 7*(a*B*c + A*c^2 + a^2*C)*Log[a^(1/3
) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + 7*(a*B*c + A*c^2 + a^2*C)*Log[a^(1/
3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/(84*a^(5/6)*c^(13/6))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1741, 27, 913, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx$$
$$\Big\downarrow \text{ 1741}$$
$$-\frac{\int -\frac{7((Bc+aC)x^6+Ac)}{a-cx^6}dx}{7c}-\frac{Cx^7}{7c}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(Bc+aC)x^6+Ac}{a-cx^6}dx}{c}-\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\int \frac{1}{a-cx^6}dx}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 754}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{c}x^2}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{2\left(\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}+\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{2\left(\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}\right)}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{c}x^2}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}\right)}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}\right)}{c}-\frac{x(aC+Bc)}{c}}{c}-\frac{Cx^7}{7c}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int -\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}\cdots\right.}{\sqrt[3]{c}x^2+\sqrt[6]{a}\cdots}}{2\sqrt[6]{\cdots}}}{6a^{5/6}}\cdots\right.}{c}}{c}$$

$$\frac{Cx^7}{7c}$$

$\Big\downarrow$ 25

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int \frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int \frac{\sqrt[6]{c}\left(2\sqrt[6]{c}x+\cdots\right.}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{\cdots}}}{2\sqrt[6]{c}\cdots}}{6a^{5/6}}\cdots\right.}{c}}{c}$$

$$\frac{Cx^7}{7c}$$

$\Big\downarrow$ 27

$$\frac{\frac{(a^2C+aBc+Ac^2)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int \frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\int \frac{2\sqrt[6]{c}x\cdots}{\sqrt[3]{c}x^2+\sqrt[6]{a}\cdots}}{6a^{5/6}}\cdots\right.}{c}}{c}$$

$$\frac{Cx^7}{7c}$$

$\Big\downarrow$ 1082

$$\frac{(a^2C+aBc+Ac^2)\left(\frac{\frac{1}{2}\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1\right)^2 \cdots}}{\sqrt[6]{\cdots}}}{6a^{5/6}}\cdots\right.}{c}$$

$$c$$

$$\frac{Cx^7}{7c}$$

$\big\downarrow$ 217

$$\frac{(a^2C+aBc+Ac^2)\left(\frac{\frac{1}{2}\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\text{arctan}\cdots}{3a^{\cdots}}\right.}{c}$$

$$c$$

$$\frac{Cx^7}{7c}$$

$\big\downarrow$ 1103

$$\frac{(a^2C+aBc+Ac^2)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\text{arct}\cdots}{\cdots}\right.}{c}$$

$$c$$

$$\frac{Cx^7}{7c}$$

input
```
Int[(A + B*x^6 + C*x^12)/(a - c*x^6),x]
```

output
```
-1/7*(C*x^7)/c + (-(((B*c + a*C)*x)/c) + ((a*B*c + A*c^2 + a^2*C)*(ArcTanh
[(c^(1/6)*x)/a^(1/6)]/(3*a^(5/6)*c^(1/6)) + (-((Sqrt[3]*ArcTan[(1 - (2*c^(
1/6)*x)/a^(1/6))/Sqrt[3]])/c^(1/6)) - Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^
(1/3)*x^2]/(2*c^(1/6)))/(6*a^(5/6)) + ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x)/
a^(1/6))/Sqrt[3]])/c^(1/6) + Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2
]/(2*c^(1/6)))/(6*a^(5/6))))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.28

method	result
risch	$-\frac{C\,x^{7}}{7c}-\frac{Bx}{c}-\frac{Cax}{c^{2}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}-a\right)}\frac{\left(A\,c^{2}+aBc+a^{2}C\right)\ln\left(x-\_R\right)}{\_R^{5}}}{6c^{3}}$
default	$-\frac{\frac{1}{7}C\,x^{7}c+Bcx+Cax}{c^{2}}+\frac{\left(-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{6a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}x-x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}-\cdots\right.}{c^{2}}$

input
```
int((C*x^12+B*x^6+A)/(-c*x^6+a),x,method=_RETURNVERBOSE)
```

output
```
-1/7*C*x^7/c-1/c*B*x-1/c^2*C*a*x-1/6/c^3*sum((A*c^2+B*a*c+C*a^2)/_R^5*ln(x
-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3916 vs. 2(192) = 384.

Time = 0.13 (sec) , antiderivative size = 3916, normalized size of antiderivative = 15.92

$$\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 25.99 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx = -\frac{Cx^7}{7c} - x\left(\frac{B}{c}+\frac{Ca}{c^2}\right)$$
$$-\operatorname{RootSum}\left(46656t^6a^5c^{13} - A^6c^{12} - 6A^5Bac^{11} - 6A^5Ca^2c^{10} - 15A^4B^2a^2c^{10} - 30A^4BCa^3c^9 - 15A^4C\right.$$

input
```
integrate((C*x**12+B*x**6+A)/(-c*x**6+a),x)
```

output
```
-C*x**7/(7*c) - x*(B/c + C*a/c**2) - RootSum(46656*_t**6*a**5*c**13 - A**6
*c**12 - 6*A**5*B*a*c**11 - 6*A**5*C*a**2*c**10 - 15*A**4*B**2*a**2*c**10
- 30*A**4*B*C*a**3*c**9 - 15*A**4*C**2*a**4*c**8 - 20*A**3*B**3*a**3*c**9
- 60*A**3*B**2*C*a**4*c**8 - 60*A**3*B*C**2*a**5*c**7 - 20*A**3*C**3*a**6*
c**6 - 15*A**2*B**4*a**4*c**8 - 60*A**2*B**3*C*a**5*c**7 - 90*A**2*B**2*C*
*2*a**6*c**6 - 60*A**2*B*C**3*a**7*c**5 - 15*A**2*C**4*a**8*c**4 - 6*A*B**
5*a**5*c**7 - 30*A*B**4*C*a**6*c**6 - 60*A*B**3*C**2*a**7*c**5 - 60*A*B**2
*C**3*a**8*c**4 - 30*A*B*C**4*a**9*c**3 - 6*A*C**5*a**10*c**2 - B**6*a**6*
c**6 - 6*B**5*C*a**7*c**5 - 15*B**4*C**2*a**8*c**4 - 20*B**3*C**3*a**9*c**
3 - 15*B**2*C**4*a**10*c**2 - 6*B*C**5*a**11*c - C**6*a**12, Lambda(_t, _t
*log(-6*_t*a*c**2/(A*c**2 + B*a*c + C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx = -\frac{Ccx^7+7\,(Ca+Bc)x}{7\,c^2}$$
$$+\frac{\frac{2\sqrt{3}(Ca^2+Bac+Ac^2)\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}(Ca^2+Bac+Ac^2)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{(Ca^2+Bac+Ac^2)\log\left(x\right.}{\sqrt{a}\sqrt{c}(}}{}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a),x, algorithm="maxima")
```

output
```
-1/7*(C*c*x^7 + 7*(C*a + B*c)*x)/c^2 + 1/12*(2*sqrt(3)*(C*a^2 + B*a*c + A*
c^2)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^
(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)) + 2*sqrt(3)*(C*a^2 + B*a*
c + A*c^2)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqr
t(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)) + (C*a^2 + B*a*c +
A*c^2)*log(x^2 + x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqr
t(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)) - (C*a^2 + B*a*c + A*c^2)*log(x^2 -
x*(sqrt(a)/sqrt(c))^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*sqrt(c)*(sqr
t(a)/sqrt(c))^(2/3)) + 2*(C*a^2 + B*a*c + A*c^2)*log(x + (sqrt(a)/sqrt(c))
^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)) - 2*(C*a^2 + B*a*c + A*c
^2)*log(x - (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2
/3)))/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(192) = 384$.

Time = 0.13 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx = \frac{\sqrt{3}\left(\left(-ac^5\right)^{\frac{1}{6}}Ca^2+\left(-ac^5\right)^{\frac{1}{6}}Bac+\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^3}$$
$$-\frac{\sqrt{3}\left(\left(-ac^5\right)^{\frac{1}{6}}Ca^2+\left(-ac^5\right)^{\frac{1}{6}}Bac+\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12\,ac^3}$$
$$+\frac{\left(\left(-ac^5\right)^{\frac{1}{6}}Ca^2+\left(-ac^5\right)^{\frac{1}{6}}Bac+\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2\,x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^3}$$
$$+\frac{\left(\left(-ac^5\right)^{\frac{1}{6}}Ca^2+\left(-ac^5\right)^{\frac{1}{6}}Bac+\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2\,x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{6\,ac^3}$$
$$+\frac{\left(\left(-ac^5\right)^{\frac{1}{6}}Ca^2+\left(-ac^5\right)^{\frac{1}{6}}Bac+\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{3\,ac^3}$$
$$-\frac{Cc^6x^7+7\,Cac^5x+7\,Bc^6x}{7\,c^7}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*((-a*c^5)^(1/6)*C*a^2 + (-a*c^5)^(1/6)*B*a*c + (-a*c^5)^(1/6)
*A*c^2)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^3) - 1/12*sq
rt(3)*((-a*c^5)^(1/6)*C*a^2 + (-a*c^5)^(1/6)*B*a*c + (-a*c^5)^(1/6)*A*c^2)
*log(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a*c^3) + 1/6*((-a*c^5)^
(1/6)*C*a^2 + (-a*c^5)^(1/6)*B*a*c + (-a*c^5)^(1/6)*A*c^2)*arctan((2*x + s
qrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a*c^3) + 1/6*((-a*c^5)^(1/6)*C*a^2 + (
-a*c^5)^(1/6)*B*a*c + (-a*c^5)^(1/6)*A*c^2)*arctan((2*x - sqrt(3)*(-a/c)^(
1/6))/(-a/c)^(1/6))/(a*c^3) + 1/3*((-a*c^5)^(1/6)*C*a^2 + (-a*c^5)^(1/6)*B
*a*c + (-a*c^5)^(1/6)*A*c^2)*arctan(x/(-a/c)^(1/6))/(a*c^3) - 1/7*(C*c^6*x
^7 + 7*C*a*c^5*x + 7*B*c^6*x)/c^7
```

Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 4273, normalized size of antiderivative = 17.37

$$\int \frac{A+Bx^6+Cx^{12}}{a-cx^6}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^6 + C*x^12)/(a - c*x^6),x)
```

output
```
(atan(((((6*x*(A^4*c^8 + C^4*a^8 + B^4*a^4*c^4 + 6*A^2*B^2*a^2*c^6 + 6*A^2
*C^2*a^4*c^4 + 6*B^2*C^2*a^6*c^2 + 4*A^3*B*a*c^7 + 4*B*C^3*a^7*c + 4*A*B^3
*a^3*c^5 + 4*A*C^3*a^6*c^2 + 4*A^3*C*a^2*c^6 + 4*B^3*C*a^5*c^3 + 12*A*B*C^
2*a^5*c^3 + 12*A*B^2*C*a^4*c^4 + 12*A^2*B*C*a^3*c^5))/c^3 - (6*(A*c^2 + C*
a^2 + B*a*c)*(C^3*a^7 + A^3*a*c^6 + B^3*a^4*c^3 + 3*B*C^2*a^6*c + 3*A*B^2*
a^3*c^4 + 3*A^2*B*a^2*c^5 + 3*A*C^2*a^5*c^2 + 3*A^2*C*a^3*c^4 + 3*B^2*C*a^
5*c^2 + 6*A*B*C*a^4*c^3))/(a^(5/6)*c^(19/6)))*(A*c^2 + C*a^2 + B*a*c)*1i)/
(6*a^(5/6)*c^(13/6)) + (((6*x*(A^4*c^8 + C^4*a^8 + B^4*a^4*c^4 + 6*A^2*B^2
*a^2*c^6 + 6*A^2*C^2*a^4*c^4 + 6*B^2*C^2*a^6*c^2 + 4*A^3*B*a*c^7 + 4*B*C^3
*a^7*c + 4*A*B^3*a^3*c^5 + 4*A*C^3*a^6*c^2 + 4*A^3*C*a^2*c^6 + 4*B^3*C*a^5
*c^3 + 12*A*B*C^2*a^5*c^3 + 12*A*B^2*C*a^4*c^4 + 12*A^2*B*C*a^3*c^5))/c^3
+ (6*(A*c^2 + C*a^2 + B*a*c)*(C^3*a^7 + A^3*a*c^6 + B^3*a^4*c^3 + 3*B*C^2*
a^6*c + 3*A*B^2*a^3*c^4 + 3*A^2*B*a^2*c^5 + 3*A*C^2*a^5*c^2 + 3*A^2*C*a^3*
c^4 + 3*B^2*C*a^5*c^2 + 6*A*B*C*a^4*c^3))/(a^(5/6)*c^(19/6)))*(A*c^2 + C*a
^2 + B*a*c)*1i)/(6*a^(5/6)*c^(13/6)))/((((6*x*(A^4*c^8 + C^4*a^8 + B^4*a^4
*c^4 + 6*A^2*B^2*a^2*c^6 + 6*A^2*C^2*a^4*c^4 + 6*B^2*C^2*a^6*c^2 + 4*A^3*B
*a*c^7 + 4*B*C^3*a^7*c + 4*A*B^3*a^3*c^5 + 4*A*C^3*a^6*c^2 + 4*A^3*C*a^2*c
^6 + 4*B^3*C*a^5*c^3 + 12*A*B*C^2*a^5*c^3 + 12*A*B^2*C*a^4*c^4 + 12*A^2*B*
C*a^3*c^5))/c^3 - (6*(A*c^2 + C*a^2 + B*a*c)*(C^3*a^7 + A^3*a*c^6 + B^3*a^
4*c^3 + 3*B*C^2*a^6*c + 3*A*B^2*a^3*c^4 + 3*A^2*B*a^2*c^5 + 3*A*C^2*a^5...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.28

$$\int \frac{A + Bx^6 + Cx^{12}}{a - cx^6} \, dx = \text{Too large to display}$$

input
```
int((C*x^12+B*x^6+A)/(-c*x^6+a),x)
```

output

```
( - 14*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(
c**(1/6)*a**(1/6)*sqrt(3)))*a - 14*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6
)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b - 14*c**(1/6)*a*
*(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*
sqrt(3)))*c + 14*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**
(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a + 14*c**(1/6)*a**(1/6)*sqrt(3)*ata
n((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b + 14*c
**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)
*a**(1/6)*sqrt(3)))*c - 7*c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x + a
**(1/3) + c**(1/3)*x**2)*a - 7*c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*
x + a**(1/3) + c**(1/3)*x**2)*b - 7*c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(
1/6)*x + a**(1/3) + c**(1/3)*x**2)*c + 14*c**(1/6)*a**(1/6)*log( - c**(1/6
)*a**(1/6) - c**(1/3)*x)*a + 14*c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)
 - c**(1/3)*x)*b + 14*c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6) - c**(1/3
)*x)*c + 7*c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/3)
*x**2)*a + 7*c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(1/
3)*x**2)*b + 7*c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6)*x + a**(1/3) + c**(
1/3)*x**2)*c - 14*c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*a
- 14*c**(1/6)*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*b - 14*c**(1/6)
*a**(1/6)*log(c**(1/6)*a**(1/6) - c**(1/3)*x)*c - 84*c**(1/3)*a*x - 84*...
```

3.28 $\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 272

$$\begin{aligned}\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^2} dx &= \frac{Cx}{c^2} + \frac{(aBc+Ac^2+a^2C)\,x}{6ac^2(a-cx^6)} \\ &+ \frac{(aBc-5Ac^2+7a^2C)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{12\sqrt{3}a^{11/6}c^{13/6}} \\ &- \frac{(aBc-5Ac^2+7a^2C)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{12\sqrt{3}a^{11/6}c^{13/6}} \\ &- \frac{(aBc-5Ac^2+7a^2C)\,\text{arctanh}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{18a^{11/6}c^{13/6}} \\ &- \frac{(aBc-5Ac^2+7a^2C)\,\text{arctanh}\left(\frac{\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{36a^{11/6}c^{13/6}}\end{aligned}$$

output
```
C*x/c^2+1/6*(A*c^2+B*a*c+C*a^2)*x/a/c^2/(-c*x^6+a)+1/36*(-5*A*c^2+B*a*c+7*
C*a^2)*arctan(1/3*(a^(1/6)-2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(11/6)/
c^(13/6)-1/36*(-5*A*c^2+B*a*c+7*C*a^2)*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)*3^
(1/2)/a^(1/6))*3^(1/2)/a^(11/6)/c^(13/6)-1/18*(-5*A*c^2+B*a*c+7*C*a^2)*arc
tanh(c^(1/6)*x/a^(1/6))/a^(11/6)/c^(13/6)-1/36*(-5*A*c^2+B*a*c+7*C*a^2)*ar
ctanh(a^(1/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))/a^(11/6)/c^(13/6)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^6+Cx^{12}}{\left(a-cx^6\right)^2}\,dx$$

$$= \frac{72\sqrt[6]{c}Cx + \frac{12\sqrt[6]{c}(aBc+Ac^2+a^2C)x}{a(a-cx^6)} + \frac{2\sqrt{3}(aBc-5Ac^2+7a^2C)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{a^{11/6}} - \frac{2\sqrt{3}(aBc-5Ac^2+7a^2C)\arctan\left(\frac{1+\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{a^{11/6}}}{}$$

input
```
Integrate[(A + B*x^6 + C*x^12)/(a - c*x^6)^2,x]
```

output
```
(72*c^(1/6)*C*x + (12*c^(1/6)*(a*B*c + A*c^2 + a^2*C)*x)/(a*(a - c*x^6)) +
 (2*Sqrt[3]*(a*B*c - 5*A*c^2 + 7*a^2*C)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))
/Sqrt[3]])/a^(11/6) - (2*Sqrt[3]*(a*B*c - 5*A*c^2 + 7*a^2*C)*ArcTan[(1 + (
2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/a^(11/6) + (2*(a*B*c - 5*A*c^2 + 7*a^2*C)*
Log[a^(1/6) - c^(1/6)*x]])/a^(11/6) - (2*(a*B*c - 5*A*c^2 + 7*a^2*C)*Log[a^
(1/6) + c^(1/6)*x]])/a^(11/6) + ((a*B*c - 5*A*c^2 + 7*a^2*C)*Log[a^(1/3) -
a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]])/a^(11/6) - ((a*B*c - 5*A*c^2 + 7*a^2*C)*
Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]])/a^(11/6))/(72*c^(13/6))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1739, 913, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^2}dx$$

$$\Big\downarrow \text{1739}$$

$$\frac{x(a^2C+aBc+Ac^2)}{6ac^2(a-cx^6)}-\frac{\int \frac{6acCx^6-5Ac^2+aBc+a^2C}{a-cx^6}dx}{6ac^2}$$

$$\Big\downarrow \text{913}$$

$$\frac{x(a^2C+aBc+Ac^2)}{6ac^2(a-cx^6)}-\frac{(7a^2C+aBc-5Ac^2)\int \frac{1}{a-cx^6}dx-6aCx}{6ac^2}$$

$$\Big\downarrow \text{754}$$

$$\frac{x(a^2C+aBc+Ac^2)}{6ac^2(a-cx^6)}-\frac{(7a^2C+aBc-5Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{c}x^2}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{2\left(\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}+\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{2\left(\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}\right)-6aCx}{6ac^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(a^2C+aBc+Ac^2)}{6ac^2(a-cx^6)}-\frac{(7a^2C+aBc-5Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{c}x^2}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}\right)-6aCx}{6ac^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x\left(a^2C+aBc+Ac^2\right)}{6ac^2\left(a-cx^6\right)}-\left(7a^2C+aBc-5Ac^2\right)\left(\frac{\int\frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int\frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}\right)-6aCx}{6ac^2}$$

$\downarrow$ 1142

$$\frac{\frac{x\left(a^2C+aBc+Ac^2\right)}{6ac^2\left(a-cx^6\right)}-\left(7a^2C+aBc-5Ac^2\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int-\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right.}{6ac^2}$$

$\downarrow$ 25

$$\frac{\frac{x\left(a^2C+aBc+Ac^2\right)}{6ac^2\left(a-cx^6\right)}-\left(7a^2C+aBc-5Ac^2\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right.}{6ac^2}$$

$\downarrow$ 27

$$\frac{\frac{x\left(a^2C+aBc+Ac^2\right)}{6ac^2\left(a-cx^6\right)}-\left(7a^2C+aBc-5Ac^2\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right.}{6ac^2}$$

$\downarrow$ 1082

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{6ac^2\left(a-cx^6\right)}-\frac{\left(7a^2C+aBc-5Ac^2\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{3\int \cdots}{\cdots}}{6a^{5/6}}\cdots\right)}{6ac^2}$$

$\Big\downarrow$ 217

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{6ac^2\left(a-cx^6\right)}-\frac{\left(7a^2C+aBc-5Ac^2\right)\left(\frac{\frac{1}{2}\int \frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\int \frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\cdots}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}\cdots\right)}{6ac^2}$$

$\Big\downarrow$ 1103

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{6ac^2\left(a-cx^6\right)}-\frac{\left(7a^2C+aBc-5Ac^2\right)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}+\frac{\log\left(\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{\cdots}\right)}{2\sqrt[6]{c}}}{6a^{5/6}}\cdots\right)}{6ac^2}$$

input
```
Int[(A + B*x^6 + C*x^12)/(a - c*x^6)^2,x]
```

output
```
((a*B*c + A*c^2 + a^2*C)*x)/(6*a*c^2*(a - c*x^6)) - (-6*a*C*x + (a*B*c - 5
*A*c^2 + 7*a^2*C)*(ArcTanh[(c^(1/6)*x)/a^(1/6)]/(3*a^(5/6)*c^(1/6)) + (-((
Sqrt[3]*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/c^(1/6)) - Log[a^(1/3
) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(6*a^(5/6)) + ((Sqrt[3]*
ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]])/c^(1/6) + Log[a^(1/3) + a^(1/
6)*c^(1/6)*x + c^(1/3)*x^2]/(2*c^(1/6)))/(6*a^(5/6))))/(6*a*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1739
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.33

method	result
risch	$\frac{Cx}{c^2}+\frac{(A\,c^2+aBc+a^2C)x}{6a\,c^2(-c\,x^6+a)}-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^6-a\right)}\frac{\left(5A\,c^2-aBc-7a^2C\right)\ln\left(x-\_R\right)}{\_R^5}}{36c^3a}$
default	$\frac{Cx}{c^2}+\frac{\frac{\left(A\,c^2+aBc+a^2C\right)x}{6a\left(-c\,x^6+a\right)}+\frac{\left(5A\,c^2-aBc-7a^2C\right)\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^2+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{2x\sqrt{3}}{3\left(\frac{a}{c}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}}{3}\right)}{6a}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(\left(\frac{a}{c}\right)^{\frac{1}{6}}\cdots\right.}{12a}\cdots\right.}{6a}}{c^2}$

input
```
int((C*x^12+B*x^6+A)/(-c*x^6+a)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x/c^2+1/6*(A*c^2+B*a*c+C*a^2)*x/a/c^2/(-c*x^6+a)-1/36/c^3/a*sum((5*A*c^2
-B*a*c-7*C*a^2)/_R^5*ln(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4136 vs. $2(219) = 438$.

Time = 0.13 (sec) , antiderivative size = 4136, normalized size of antiderivative = 15.21

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 86.15 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^2}\,dx = \frac{Cx}{c^2} + \frac{x(-Ac^2-Bac-Ca^2)}{-6a^2c^2+6ac^3x^6}$$
$$+\text{RootSum}\left(2176782336t^6a^{11}c^{13} - 15625A^6c^{12} + 18750A^5Bac^{11} + 131250A^5Ca^2c^{10} - 9375A^4B^2a^2c^{10}\right.$$

input
```
integrate((C*x**12+B*x**6+A)/(-c*x**6+a)**2,x)
```

output
```
C*x/c**2 + x*(-A*c**2 - B*a*c - C*a**2)/(-6*a**2*c**2 + 6*a*c**3*x**6) + R
ootSum(2176782336*_t**6*a**11*c**13 - 15625*A**6*c**12 + 18750*A**5*B*a*c*
*11 + 131250*A**5*C*a**2*c**10 - 9375*A**4*B**2*a**2*c**10 - 131250*A**4*B
*C*a**3*c**9 - 459375*A**4*C**2*a**4*c**8 + 2500*A**3*B**3*a**3*c**9 + 525
00*A**3*B**2*C*a**4*c**8 + 367500*A**3*B*C**2*a**5*c**7 + 857500*A**3*C**3
*a**6*c**6 - 375*A**2*B**4*a**4*c**8 - 10500*A**2*B**3*C*a**5*c**7 - 11025
0*A**2*B**2*C**2*a**6*c**6 - 514500*A**2*B*C**3*a**7*c**5 - 900375*A**2*C*
*4*a**8*c**4 + 30*A*B**5*a**5*c**7 + 1050*A*B**4*C*a**6*c**6 + 14700*A*B**
3*C**2*a**7*c**5 + 102900*A*B**2*C**3*a**8*c**4 + 360150*A*B*C**4*a**9*c**
3 + 504210*A*C**5*a**10*c**2 - B**6*a**6*c**6 - 42*B**5*C*a**7*c**5 - 735*
B**4*C**2*a**8*c**4 - 6860*B**3*C**3*a**9*c**3 - 36015*B**2*C**4*a**10*c**
2 - 100842*B*C**5*a**11*c - 117649*C**6*a**12, Lambda(_t, _t*log(-36*_t*a*
*2*c**2/(-5*A*c**2 + B*a*c + 7*C*a**2) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^6+Cx^{12}}{\left(a-cx^6\right)^2}\,dx = -\frac{(Ca^2+Bac+Ac^2)x}{6\left(ac^3x^6-a^2c^2\right)}+\frac{Cx}{c^2} - \frac{\frac{2\sqrt{3}\left(7\,Ca^2+Bac-5\,Ac^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\left(7\,Ca^2+Bac-5\,Ac^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{(7\,Ca^2+Bac-5\,A}{}}{}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(C*a^2 + B*a*c + A*c^2)*x/(a*c^3*x^6 - a^2*c^2) + C*x/c^2 - 1/72*(2*s
qrt(3)*(7*C*a^2 + B*a*c - 5*A*c^2)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqrt
(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2
/3)) + 2*sqrt(3)*(7*C*a^2 + B*a*c - 5*A*c^2)*arctan(1/3*sqrt(3)*(2*x - (sq
rt(a)/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/s
qrt(c))^(2/3)) + (7*C*a^2 + B*a*c - 5*A*c^2)*log(x^2 + x*(sqrt(a)/sqrt(c))
^(1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)
) - (7*C*a^2 + B*a*c - 5*A*c^2)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sqr
t(a)/sqrt(c))^(2/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)) + 2*(7*C*a^
2 + B*a*c - 5*A*c^2)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)*(sq
rt(a)/sqrt(c))^(2/3)) - 2*(7*C*a^2 + B*a*c - 5*A*c^2)*log(x - (sqrt(a)/sqr
t(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)))/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^2}\,dx = \frac{Cx}{c^2} - \frac{\sqrt{3}\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2 + \left(-ac^5\right)^{\frac{1}{6}}Bac - 5\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^3} + \frac{\sqrt{3}\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2 + \left(-ac^5\right)^{\frac{1}{6}}Bac - 5\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{72\,a^2c^3} - \frac{Ca^2x+Bacx+Ac^2x}{6\,(cx^6-a)ac^2} - \frac{\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2 + \left(-ac^5\right)^{\frac{1}{6}}Bac - 5\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^3} - \frac{\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2 + \left(-ac^5\right)^{\frac{1}{6}}Bac - 5\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{36\,a^2c^3} - \frac{\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2 + \left(-ac^5\right)^{\frac{1}{6}}Bac - 5\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{18\,a^2c^3}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a)^2,x, algorithm="giac")
```

output
```
C*x/c^2 - 1/72*sqrt(3)*(7*(-a*c^5)^(1/6)*C*a^2 + (-a*c^5)^(1/6)*B*a*c - 5*
(-a*c^5)^(1/6)*A*c^2)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^
2*c^3) + 1/72*sqrt(3)*(7*(-a*c^5)^(1/6)*C*a^2 + (-a*c^5)^(1/6)*B*a*c - 5*(
-a*c^5)^(1/6)*A*c^2)*log(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^2
*c^3) - 1/6*(C*a^2*x + B*a*c*x + A*c^2*x)/((c*x^6 - a)*a*c^2) - 1/36*(7*(-
a*c^5)^(1/6)*C*a^2 + (-a*c^5)^(1/6)*B*a*c - 5*(-a*c^5)^(1/6)*A*c^2)*arctan
((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^2*c^3) - 1/36*(7*(-a*c^5)^(
1/6)*C*a^2 + (-a*c^5)^(1/6)*B*a*c - 5*(-a*c^5)^(1/6)*A*c^2)*arctan((2*x -
sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^2*c^3) - 1/18*(7*(-a*c^5)^(1/6)*C*a
^2 + (-a*c^5)^(1/6)*B*a*c - 5*(-a*c^5)^(1/6)*A*c^2)*arctan(x/(-a/c)^(1/6))
/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 4394, normalized size of antiderivative = 16.15

$$\int \frac{A + Bx^6 + Cx^{12}}{(a - cx^6)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^6 + C*x^12)/(a - c*x^6)^2,x)
```

output
```
(C*x)/c^2 + (x*(A*c^2 + C*a^2 + B*a*c))/(6*a*(a*c^2 - c^3*x^6)) - (atan(((
((x*(625*A^4*c^8 + 2401*C^4*a^8 + B^4*a^4*c^4 + 150*A^2*B^2*a^2*c^6 + 7350
*A^2*C^2*a^4*c^4 + 294*B^2*C^2*a^6*c^2 - 500*A^3*B*a*c^7 + 1372*B*C^3*a^7*
c - 20*A*B^3*a^3*c^5 - 6860*A*C^3*a^6*c^2 - 3500*A^3*C*a^2*c^6 + 28*B^3*C*
a^5*c^3 - 2940*A*B*C^2*a^5*c^3 - 420*A*B^2*C*a^4*c^4 + 2100*A^2*B*C*a^3*c^
5))/(216*a^4*c^3) - ((7*C*a^2 - 5*A*c^2 + B*a*c)*(343*C^3*a^6 - 125*A^3*c^
6 + B^3*a^3*c^3 + 75*A^2*B*a*c^5 + 147*B*C^2*a^5*c - 15*A*B^2*a^2*c^4 - 73
5*A*C^2*a^4*c^2 + 525*A^2*C*a^2*c^4 + 21*B^2*C*a^4*c^2 - 210*A*B*C*a^3*c^3
))/(216*a^(23/6)*c^(19/6)))*(7*C*a^2 - 5*A*c^2 + B*a*c)*1i)/(36*a^(11/6)*c
^(13/6)) + (((x*(625*A^4*c^8 + 2401*C^4*a^8 + B^4*a^4*c^4 + 150*A^2*B^2*a^
2*c^6 + 7350*A^2*C^2*a^4*c^4 + 294*B^2*C^2*a^6*c^2 - 500*A^3*B*a*c^7 + 137
2*B*C^3*a^7*c - 20*A*B^3*a^3*c^5 - 6860*A*C^3*a^6*c^2 - 3500*A^3*C*a^2*c^6
 + 28*B^3*C*a^5*c^3 - 2940*A*B*C^2*a^5*c^3 - 420*A*B^2*C*a^4*c^4 + 2100*A^
2*B*C*a^3*c^5))/(216*a^4*c^3) + ((7*C*a^2 - 5*A*c^2 + B*a*c)*(343*C^3*a^6
- 125*A^3*c^6 + B^3*a^3*c^3 + 75*A^2*B*a*c^5 + 147*B*C^2*a^5*c - 15*A*B^2*
a^2*c^4 - 735*A*C^2*a^4*c^2 + 525*A^2*C*a^2*c^4 + 21*B^2*C*a^4*c^2 - 210*A
*B*C*a^3*c^3))/(216*a^(23/6)*c^(19/6)))*(7*C*a^2 - 5*A*c^2 + B*a*c)*1i)/(3
6*a^(11/6)*c^(13/6)))/((((x*(625*A^4*c^8 + 2401*C^4*a^8 + B^4*a^4*c^4 + 15
0*A^2*B^2*a^2*c^6 + 7350*A^2*C^2*a^4*c^4 + 294*B^2*C^2*a^6*c^2 - 500*A^3*B
*a*c^7 + 1372*B*C^3*a^7*c - 20*A*B^3*a^3*c^5 - 6860*A*C^3*a^6*c^2 - 350...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1172, normalized size of antiderivative = 4.31

$$\int \frac{A + Bx^6 + Cx^{12}}{(a - cx^6)^2} \, dx = \text{Too large to display}$$

input
```
int((C*x^12+B*x^6+A)/(-c*x^6+a)^2,x)
```

output

```
(14*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**
(1/6)*a**(1/6)*sqrt(3)))*a**2 + 2*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)
*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*b - 14*c**(1/6)*a
**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)
*sqrt(3)))*a*c*x**6 - 10*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6)
 - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*c - 2*c**(1/6)*a**(1/6)*sq
rt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))
*b*c*x**6 + 10*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1
/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*c**2*x**6 - 14*c**(1/6)*a**(1/6)*sqrt(
3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*
*2 - 2*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(
c**(1/6)*a**(1/6)*sqrt(3)))*a*b + 14*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1
/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*c*x**6 + 10*c*
*(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*
a**(1/6)*sqrt(3)))*a*c + 2*c**(1/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/
6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b*c*x**6 - 10*c**(1/6)*a**
(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*s
qrt(3)))*c**2*x**6 + 7*c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x + a**(
1/3) + c**(1/3)*x**2)*a**2 + c**(1/6)*a**(1/6)*log( - c**(1/6)*a**(1/6)*x
+ a**(1/3) + c**(1/3)*x**2)*a*b - 7*c**(1/6)*a**(1/6)*log( - c**(1/6)*a...
```

3.29 $\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 308

$$\begin{aligned}\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^3}\,dx &= \frac{(aBc+Ac^2+a^2C)\,x}{12ac^2\,(a-cx^6)^2} - \frac{(aBc-11Ac^2+13a^2C)\,x}{72a^2c^2\,(a-cx^6)} \\ &\quad + \frac{(5aBc-55Ac^2-7a^2C)\arctan\left(\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{144\sqrt{3}a^{17/6}c^{13/6}} \\ &\quad - \frac{(5aBc-55Ac^2-7a^2C)\arctan\left(\frac{\sqrt[6]{a}+2\sqrt[6]{c}x}{\sqrt{3}\sqrt[6]{a}}\right)}{144\sqrt{3}a^{17/6}c^{13/6}} \\ &\quad - \frac{(5aBc-55Ac^2-7a^2C)\,\text{arctanh}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{216a^{17/6}c^{13/6}} \\ &\quad - \frac{(5aBc-55Ac^2-7a^2C)\,\text{arctanh}\left(\frac{\sqrt[6]{a}\sqrt[6]{c}x}{\sqrt[3]{a}+\sqrt[3]{c}x^2}\right)}{432a^{17/6}c^{13/6}}\end{aligned}$$

output
```
1/12*(A*c^2+B*a*c+C*a^2)*x/a/c^2/(-c*x^6+a)^2-1/72*(-11*A*c^2+B*a*c+13*C*a
^2)*x/a^2/c^2/(-c*x^6+a)+1/432*(-55*A*c^2+5*B*a*c-7*C*a^2)*arctan(1/3*(a^(
1/6)-2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(1/2)/a^(17/6)/c^(13/6)-1/432*(-55*A*
c^2+5*B*a*c-7*C*a^2)*arctan(1/3*(a^(1/6)+2*c^(1/6)*x)*3^(1/2)/a^(1/6))*3^(
1/2)/a^(17/6)/c^(13/6)-1/216*(-55*A*c^2+5*B*a*c-7*C*a^2)*arctanh(c^(1/6)*x
/a^(1/6))/a^(17/6)/c^(13/6)-1/432*(-55*A*c^2+5*B*a*c-7*C*a^2)*arctanh(a^(1
/6)*c^(1/6)*x/(a^(1/3)+c^(1/3)*x^2))/a^(17/6)/c^(13/6)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^3}\,dx$$

$$= \frac{\frac{72a^{11/6}\sqrt[6]{c}(aBc+Ac^2+a^2C)x}{(a-cx^6)^2}+\frac{12a^{5/6}\sqrt[6]{c}(aBc-11Ac^2+13a^2C)x}{-a+cx^6}-2\sqrt{3}(-5aBc+55Ac^2+7a^2C)\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right.}{}$$

input
```
Integrate[(A + B*x^6 + C*x^12)/(a - c*x^6)^3,x]
```

output
```
((72*a^(11/6)*c^(1/6)*(a*B*c + A*c^2 + a^2*C)*x)/(a - c*x^6)^2 + (12*a^(5/
6)*c^(1/6)*(a*B*c - 11*A*c^2 + 13*a^2*C)*x)/(-a + c*x^6) - 2*Sqrt[3]*(-5*a
*B*c + 55*A*c^2 + 7*a^2*C)*ArcTan[(1 - (2*c^(1/6)*x)/a^(1/6))/Sqrt[3]] + 2
*Sqrt[3]*(-5*a*B*c + 55*A*c^2 + 7*a^2*C)*ArcTan[(1 + (2*c^(1/6)*x)/a^(1/6)
)/Sqrt[3]] - 2*(-5*a*B*c + 55*A*c^2 + 7*a^2*C)*Log[a^(1/6) - c^(1/6)*x] +
2*(-5*a*B*c + 55*A*c^2 + 7*a^2*C)*Log[a^(1/6) + c^(1/6)*x] - (-5*a*B*c + 5
5*A*c^2 + 7*a^2*C)*Log[a^(1/3) - a^(1/6)*c^(1/6)*x + c^(1/3)*x^2] + (-5*a*
B*c + 55*A*c^2 + 7*a^2*C)*Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x^2])/
(864*a^(17/6)*c^(13/6))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1739, 910, 754, 27, 221, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^3}dx$$

$$\Big\downarrow \text{1739}$$

$$\frac{x(a^2C+aBc+Ac^2)}{12ac^2(a-cx^6)^2}-\frac{\int \frac{12acCx^6-11Ac^2+aBc+a^2C}{(a-cx^6)^2}dx}{12ac^2}$$

$$\Big\downarrow \text{910}$$

$$\frac{x(a^2C+aBc+Ac^2)}{12ac^2(a-cx^6)^2}-\frac{\frac{(-7a^2C+5aBc-55Ac^2)\int \frac{1}{a-cx^6}dx}{6a}+\frac{x(13a^2C+aBc-11Ac^2)}{6a(a-cx^6)}}{12ac^2}$$

$$\Big\downarrow \text{754}$$

$$\frac{x(a^2C+aBc+Ac^2)}{12ac^2(a-cx^6)^2}-\frac{\frac{(-7a^2C+5aBc-55Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{c}x^2}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{2\left(\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}+\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{2\left(\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}\right)}dx}{3a^{5/6}}\right)}{6a}+\frac{x(13a^2C+aBc-11Ac^2)}{6a(a-cx^6)}}{12ac^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(a^2C+aBc+Ac^2)}{12ac^2(a-cx^6)^2}-\frac{\frac{(-7a^2C+5aBc-55Ac^2)\left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{c}x^2}dx}{3a^{2/3}}+\frac{\int \frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int \frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}\right)}{6a}+\frac{x(13a^2C+aBc-11Ac^2)}{6a(a-cx^6)}}{12ac^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{12ac^2\left(a-cx^6\right)^2}-\frac{\frac{\left(-7a^2C+5aBc-55Ac^2\right)\left(\frac{\int\frac{2\sqrt[6]{a}-\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\int\frac{\sqrt[6]{c}x+2\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt[6]{c}}\right)}{6a}+\frac{x\left(13a^2C+aBc-11Ac^2\right)}{6a\left(a-cx^6\right)}}{12ac^2}$$

$\big\downarrow$ 1142

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{12ac^2\left(a-cx^6\right)^2}-\frac{\frac{\left(-7a^2C+5aBc-55Ac^2\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\int-\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right.}{6a}\cdots}{12ac^2}$$

$\big\downarrow$ 25

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{12ac^2\left(a-cx^6\right)^2}-\frac{\frac{\left(-7a^2C+5aBc-55Ac^2\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\int\frac{\sqrt[6]{c}\left(\sqrt[6]{a}-2\sqrt[6]{c}x\right)}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\cdots}{6a^{5/6}}\cdots\right.}{6a}\cdots}{12ac^2}$$

$\big\downarrow$ 27

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{12ac^2\left(a-cx^6\right)^2}-\frac{\frac{\left(-7a^2C+5aBc-55Ac^2\right)\left(\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx}{6a^{5/6}}+\frac{\frac{3}{2}\sqrt[6]{a}\int\frac{1}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{1}{2}\int\cdots}{6a^{5/6}}\cdots\right.}{6a}\cdots}{12ac^2}$$

$\big\downarrow$ 1082

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{12ac^2\left(a-cx^6\right)^2}-\frac{\frac{\left(-7a^2C+5aBc-55Ac^2\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{3\int\frac{\cdots}{-\left(\frac{2\sqrt[6]{c}\cdots}{\sqrt[6]{a}}\cdots\right)\cdots}\cdots}{\cdots}}{6a^{5/6}}\cdots\right)}{6a}\cdots}{12ac^2}$$

$$\Big\downarrow \; 217$$

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{12ac^2\left(a-cx^6\right)^2}-\frac{\frac{\left(-7a^2C+5aBc-55Ac^2\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[6]{a}-2\sqrt[6]{c}x}{\sqrt[3]{c}x^2-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{1}{2}\int\frac{2\sqrt[6]{c}x+\sqrt[6]{a}}{\sqrt[3]{c}x^2+\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}}{6a^{5/6}}+\cdots\right)}{6a}\cdots}{12ac^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{x\left(a^2C+aBc+Ac^2\right)}{12ac^2\left(a-cx^6\right)^2}-\frac{\frac{x\left(13a^2C+aBc-11Ac^2\right)}{6a\left(a-cx^6\right)}+\frac{\left(-7a^2C+5aBc-55Ac^2\right)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}}{\sqrt{3}}\right)}{\sqrt[6]{c}}-\frac{\log\left(-\sqrt[6]{a}\sqrt[6]{c}x+\sqrt[3]{a}+\sqrt[3]{c}x^2\right)}{2\sqrt[6]{c}}}{6a^{5/6}}+\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[6]{c}x}{\sqrt[6]{a}}+1}{\sqrt{3}}\right)}{\sqrt[6]{c}}+\frac{\log\cdots}{\cdots}}{6a^{5/6}\cdots}\cdots\right)}{6a}\cdots}{12ac^2}$$

input
```
Int[(A + B*x^6 + C*x^12)/(a - c*x^6)^3,x]
```

output
```
((a*B*c + A*c^2 + a^2*C)*x)/(12*a*c^2*(a - c*x^6)^2) - (((a*B*c - 11*A*c^2
 + 13*a^2*C)*x)/(6*a*(a - c*x^6)) + ((5*a*B*c - 55*A*c^2 - 7*a^2*C)*(ArcTa
nh[(c^(1/6)*x)/a^(1/6)]/(3*a^(5/6)*c^(1/6)) + (-((Sqrt[3]*ArcTan[(1 - (2*c
^(1/6)*x)/a^(1/6))/Sqrt[3]])/c^(1/6)) - Log[a^(1/3) - a^(1/6)*c^(1/6)*x +
c^(1/3)*x^2]/(2*c^(1/6)))/(6*a^(5/6)) + ((Sqrt[3]*ArcTan[(1 + (2*c^(1/6)*x
)/a^(1/6))/Sqrt[3]])/c^(1/6) + Log[a^(1/3) + a^(1/6)*c^(1/6)*x + c^(1/3)*x
^2]/(2*c^(1/6)))/(6*a^(5/6))))/(6*a))/(12*a*c^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 910

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1739

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Simp[1/(n*(q + 1)*d*e^2)   Int[(d + e*x^n)^(q + 1)*Si
mp[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.39

method	result
risch	$\frac{-\frac{\left(11A\,c^{2}-aBc-13a^{2}C\right)x^{7}}{72a^{2}c}+\frac{\left(17A\,c^{2}+5aBc-7a^{2}C\right)x}{72a\,c^{2}}}{\left(-c\,x^{6}+a\right)^{2}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{6}-a\right)}\frac{\left(55A\,c^{2}-5aBc+7a^{2}C\right)\ln\left(x-\_R\right)}{\_R^{5}}}{432a^{2}c^{3}}$
default	$\frac{-\frac{\left(11A\,c^{2}-aBc-13a^{2}C\right)x^{7}}{72a^{2}c}+\frac{\left(17A\,c^{2}+5aBc-7a^{2}C\right)x}{72a\,c^{2}}}{\left(-c\,x^{6}+a\right)^{2}}+\frac{\left(55A\,c^{2}-5aBc+7a^{2}C\right)\left(\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\ln\left(x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{6}}x+\left(\frac{a}{c}\right)^{\frac{1}{3}}\right)}{12a}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\cdots\right.}{6a}\cdots\right.}{\cdots}$

input
```
int((C*x^12+B*x^6+A)/(-c*x^6+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/72*(11*A*c^2-B*a*c-13*C*a^2)/a^2/c*x^7+1/72*(17*A*c^2+5*B*a*c-7*C*a^2)
/a/c^2*x)/(-c*x^6+a)^2-1/432/a^2/c^3*sum((55*A*c^2-5*B*a*c+7*C*a^2)/_R^5*l
n(x-_R),_R=RootOf(_Z^6*c-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4398 vs. 2(254) = 508.

Time = 0.16 (sec) , antiderivative size = 4398, normalized size of antiderivative = 14.28

$$\int \frac{A+Bx^6+Cx^{12}}{\left(a-cx^6\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^6+Cx^{12}}{\left(a-cx^6\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x**12+B*x**6+A)/(-c*x**6+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^6+Cx^{12}}{\left(a-cx^6\right)^3}\,dx=\frac{(13\,Ca^2c+Bac^2-11\,Ac^3)x^7-(7\,Ca^3-5\,Ba^2c-17\,Aac^2)x}{72\,(a^2c^4x^{12}-2\,a^3c^3x^6+a^4c^2)}+\frac{\frac{2\,\sqrt{3}(7\,Ca^2-5\,Bac+55\,Ac^2)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{2\,\sqrt{3}(7\,Ca^2-5\,Bac+55\,Ac^2)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}\right)}{3\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{1}{3}}}\right)}{\sqrt{a}\sqrt{c}\left(\frac{\sqrt{a}}{\sqrt{c}}\right)^{\frac{2}{3}}}+\frac{(7\,Ca^2-5\,B\cdots}{\cdots}}{\cdots}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a)^3,x, algorithm="maxima")
```

output
```
1/72*((13*C*a^2*c + B*a*c^2 - 11*A*c^3)*x^7 - (7*C*a^3 - 5*B*a^2*c - 17*A*
a*c^2)*x)/(a^2*c^4*x^12 - 2*a^3*c^3*x^6 + a^4*c^2) + 1/864*(2*sqrt(3)*(7*C
*a^2 - 5*B*a*c + 55*A*c^2)*arctan(1/3*sqrt(3)*(2*x + (sqrt(a)/sqrt(c))^(1/
3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)) + 2
*sqrt(3)*(7*C*a^2 - 5*B*a*c + 55*A*c^2)*arctan(1/3*sqrt(3)*(2*x - (sqrt(a)
/sqrt(c))^(1/3))/(sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c
))^(2/3)) + (7*C*a^2 - 5*B*a*c + 55*A*c^2)*log(x^2 + x*(sqrt(a)/sqrt(c))^(
1/3) + (sqrt(a)/sqrt(c))^(2/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3))
- (7*C*a^2 - 5*B*a*c + 55*A*c^2)*log(x^2 - x*(sqrt(a)/sqrt(c))^(1/3) + (sq
rt(a)/sqrt(c))^(2/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)) + 2*(7*C*a
^2 - 5*B*a*c + 55*A*c^2)*log(x + (sqrt(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)
*(sqrt(a)/sqrt(c))^(2/3)) - 2*(7*C*a^2 - 5*B*a*c + 55*A*c^2)*log(x - (sqrt
(a)/sqrt(c))^(1/3))/(sqrt(a)*sqrt(c)*(sqrt(a)/sqrt(c))^(2/3)))/(a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^3}\,dx$$
$$= \frac{\sqrt{3}\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2-5\left(-ac^5\right)^{\frac{1}{6}}Bac+55\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\log\left(x^2+\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^3}$$
$$-\frac{\sqrt{3}\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2-5\left(-ac^5\right)^{\frac{1}{6}}Bac+55\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\log\left(x^2-\sqrt{3}x\left(-\frac{a}{c}\right)^{\frac{1}{6}}+\left(-\frac{a}{c}\right)^{\frac{1}{3}}\right)}{864\,a^3c^3}$$
$$+\frac{\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2-5\left(-ac^5\right)^{\frac{1}{6}}Bac+55\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2x+\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^3}$$
$$+\frac{\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2-5\left(-ac^5\right)^{\frac{1}{6}}Bac+55\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{2x-\sqrt{3}\left(-\frac{a}{c}\right)^{\frac{1}{6}}}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{432\,a^3c^3}$$
$$+\frac{\left(7\left(-ac^5\right)^{\frac{1}{6}}Ca^2-5\left(-ac^5\right)^{\frac{1}{6}}Bac+55\left(-ac^5\right)^{\frac{1}{6}}Ac^2\right)\arctan\left(\frac{x}{\left(-\frac{a}{c}\right)^{\frac{1}{6}}}\right)}{216\,a^3c^3}$$
$$+\frac{13\,Ca^2cx^7+Bac^2x^7-11\,Ac^3x^7-7\,Ca^3x+5\,Ba^2cx+17\,Aac^2x}{72\left(cx^6-a\right)^2a^2c^2}$$

input
```
integrate((C*x^12+B*x^6+A)/(-c*x^6+a)^3,x, algorithm="giac")
```

output
```
1/864*sqrt(3)*(7*(-a*c^5)^(1/6)*C*a^2 - 5*(-a*c^5)^(1/6)*B*a*c + 55*(-a*c^
5)^(1/6)*A*c^2)*log(x^2 + sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^3*c^3)
 - 1/864*sqrt(3)*(7*(-a*c^5)^(1/6)*C*a^2 - 5*(-a*c^5)^(1/6)*B*a*c + 55*(-a
*c^5)^(1/6)*A*c^2)*log(x^2 - sqrt(3)*x*(-a/c)^(1/6) + (-a/c)^(1/3))/(a^3*c
^3) + 1/432*(7*(-a*c^5)^(1/6)*C*a^2 - 5*(-a*c^5)^(1/6)*B*a*c + 55*(-a*c^5)
^(1/6)*A*c^2)*arctan((2*x + sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^3*c^3)
+ 1/432*(7*(-a*c^5)^(1/6)*C*a^2 - 5*(-a*c^5)^(1/6)*B*a*c + 55*(-a*c^5)^(1/
6)*A*c^2)*arctan((2*x - sqrt(3)*(-a/c)^(1/6))/(-a/c)^(1/6))/(a^3*c^3) + 1/
216*(7*(-a*c^5)^(1/6)*C*a^2 - 5*(-a*c^5)^(1/6)*B*a*c + 55*(-a*c^5)^(1/6)*A
*c^2)*arctan(x/(-a/c)^(1/6))/(a^3*c^3) + 1/72*(13*C*a^2*c*x^7 + B*a*c^2*x^
7 - 11*A*c^3*x^7 - 7*C*a^3*x + 5*B*a^2*c*x + 17*A*a*c^2*x)/((c*x^6 - a)^2*
a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 4479, normalized size of antiderivative = 14.54

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^6 + C*x^12)/(a - c*x^6)^3,x)
```

output
```
((x^7*(13*C*a^2 - 11*A*c^2 + B*a*c))/(72*a^2*c) + (x*(17*A*c^2 - 7*C*a^2 +
 5*B*a*c))/(72*a*c^2))/(a^2 + c^2*x^12 - 2*a*c*x^6) + (atan(((((x*(9150625
*A^4*c^8 + 2401*C^4*a^8 + 625*B^4*a^4*c^4 + 453750*A^2*B^2*a^2*c^6 + 88935
0*A^2*C^2*a^4*c^4 + 7350*B^2*C^2*a^6*c^2 - 3327500*A^3*B*a*c^7 - 6860*B*C^
3*a^7*c - 27500*A*B^3*a^3*c^5 + 75460*A*C^3*a^6*c^2 + 4658500*A^3*C*a^2*c^
6 - 3500*B^3*C*a^5*c^3 - 161700*A*B*C^2*a^5*c^3 + 115500*A*B^2*C*a^4*c^4 -
 1270500*A^2*B*C*a^3*c^5))/(4478976*a^8*c^3) - ((55*A*c^2 + 7*C*a^2 - 5*B*
a*c)*(166375*A^3*c^6 + 343*C^3*a^6 - 125*B^3*a^3*c^3 - 45375*A^2*B*a*c^5 -
 735*B*C^2*a^5*c + 4125*A*B^2*a^2*c^4 + 8085*A*C^2*a^4*c^2 + 63525*A^2*C*a
^2*c^4 + 525*B^2*C*a^4*c^2 - 11550*A*B*C*a^3*c^3))/(4478976*a^(47/6)*c^(19
/6)))*(55*A*c^2 + 7*C*a^2 - 5*B*a*c)*1i)/(432*a^(17/6)*c^(13/6)) + (((x*(9
150625*A^4*c^8 + 2401*C^4*a^8 + 625*B^4*a^4*c^4 + 453750*A^2*B^2*a^2*c^6 +
 889350*A^2*C^2*a^4*c^4 + 7350*B^2*C^2*a^6*c^2 - 3327500*A^3*B*a*c^7 - 686
0*B*C^3*a^7*c - 27500*A*B^3*a^3*c^5 + 75460*A*C^3*a^6*c^2 + 4658500*A^3*C*
a^2*c^6 - 3500*B^3*C*a^5*c^3 - 161700*A*B*C^2*a^5*c^3 + 115500*A*B^2*C*a^4
*c^4 - 1270500*A^2*B*C*a^3*c^5))/(4478976*a^8*c^3) + ((55*A*c^2 + 7*C*a^2
- 5*B*a*c)*(166375*A^3*c^6 + 343*C^3*a^6 - 125*B^3*a^3*c^3 - 45375*A^2*B*a
*c^5 - 735*B*C^2*a^5*c + 4125*A*B^2*a^2*c^4 + 8085*A*C^2*a^4*c^2 + 63525*A
^2*C*a^2*c^4 + 525*B^2*C*a^4*c^2 - 11550*A*B*C*a^3*c^3))/(4478976*a^(47/6)
*c^(19/6)))*(55*A*c^2 + 7*C*a^2 - 5*B*a*c)*1i)/(432*a^(17/6)*c^(13/6)))...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1793, normalized size of antiderivative = 5.82

$$\int \frac{A+Bx^6+Cx^{12}}{(a-cx^6)^3}\,dx = \text{Too large to display}$$

input
```
int((C*x^12+B*x^6+A)/(-c*x^6+a)^3,x)
```

output
```
( - 14*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(
c**(1/6)*a**(1/6)*sqrt(3)))*a**3 + 10*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(
1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a**2*b + 28*c**
(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a
**(1/6)*sqrt(3)))*a**2*c*x**6 - 110*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/
6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a**2*c - 20*c**(5
/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**
(1/6)*sqrt(3)))*a*b*c*x**6 - 14*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a
**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a*c**2*x**12 + 220*c*
*(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*
a**(1/6)*sqrt(3)))*a*c**2*x**6 + 10*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/
6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*b*c**2*x**12 - 11
0*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) - 2*c**(1/3)*x)/(c**(1
/6)*a**(1/6)*sqrt(3)))*c**3*x**12 + 14*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**
(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a**3 - 10*c**(
5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a*
*(1/6)*sqrt(3)))*a**2*b - 28*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(
1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(1/6)*sqrt(3)))*a**2*c*x**6 + 110*c**(5/
6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**(1/6) + 2*c**(1/3)*x)/(c**(1/6)*a**(
1/6)*sqrt(3)))*a**2*c + 20*c**(5/6)*a**(1/6)*sqrt(3)*atan((c**(1/6)*a**...
```

3.30 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 10

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} \, dx = \frac{1}{2}\log(3+2x)$$

output
```
1/2*ln(3+2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6} \, dx = \frac{1}{2}\log(3+2x)$$

input
```
Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6
),x]
```

output
```
Log[3 + 2*x]/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-32x^5+48x^4-72x^3+108x^2-162x+243}{729-64x^6}\,dx$$

$$\Big\downarrow \; 2019$$

$$\int \frac{1}{2x+3}dx$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\log(2x+3)$$

input
```
Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6),x]
```

output
```
Log[3 + 2*x]/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{\ln\left(x+\frac{3}{2}\right)}{2}$
default	$\frac{\ln(2x+3)}{2}$
norman	$\frac{\ln(2x+3)}{2}$
risch	$\frac{\ln(2x+3)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right.}{2}\right.}{12\left(x^{6}\right)^{\frac{1}{6}}}$

input
```
int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x,method=_RETU
RNVERBOSE)
```

output
```
1/2*ln(x+3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\,\log\left(2\,x+3\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algor
ithm="fricas")
```

output
```
1/2*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{\log(2x+3)}{2}$$

input
```
integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729),x)
```

output
```
log(2*x + 3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\log(2x+3)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algor
ithm="maxima")
```

output
```
1/2*log(2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx=\frac{1}{2}\log(|2x+3|)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x, algor
ithm="giac")
```

output
```
1/2*log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{2}$$

input
```
int((162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{729-64x^6}\,dx = \frac{\log(2x+3)}{2}$$

input
```
int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729),x)
```

output
```
log(2*x + 3)/2
```

3.31 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 10

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\log(3-2x)$$

output
```
-1/2*ln(3-2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\log(3-2x)$$

input
```
Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6
),x]
```

output
```
-1/2*Log[3 - 2*x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{32x^5+48x^4+72x^3+108x^2+162x+243}{729-64x^6}dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{3-2x}dx$$
$$\Big\downarrow \text{16}$$
$$-\frac{1}{2}\log(3-2x)$$

input
```
Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6),x]
```

output
```
-1/2*Log[3 - 2*x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result
parallelrisch	$-\frac{\ln(x-\frac{3}{2})}{2}$
default	$-\frac{\ln(2x-3)}{2}$
norman	$-\frac{\ln(2x-3)}{2}$
risch	$-\frac{\ln(2x-3)}{2}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}\right.}{12(x^6)^{\frac{1}{6}}}$

input
```
int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x,method=_RETUR
NVERBOSE)
```

output
```
-1/2*ln(x-3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx = -\frac{1}{2}\log\left(2\,x-3\right)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algori
thm="fricas")
```

output
```
-1/2*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{\log{(2x-3)}}{2}$$

input
```
integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729),x)
```

output
```
-log(2*x - 3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\,\log\left(2\,x-3\right)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algori
thm="maxima")
```

output
```
-1/2*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{1}{2}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x, algori
thm="giac")
```

output
```
-1/2*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{\ln\left(x-\frac{3}{2}\right)}{2}$$

input
```
int(-(162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729),x)
```

output
```
-log(x - 3/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{729-64x^6}\,dx=-\frac{\log(2x-3)}{2}$$

input
```
int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729),x)
```

output
```
( - log(2*x - 3))/2
```

3.32 $\int \frac{81+36x^2+16x^4}{729-64x^6} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 10

$$\int \frac{81+36x^2+16x^4}{729-64x^6} \, dx = \frac{1}{6}\text{arctanh}\left(\frac{2x}{3}\right)$$

output
```
1/6*arctanh(2/3*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{81+36x^2+16x^4}{729-64x^6} \, dx = -\frac{1}{12}\log(3-2x) + \frac{1}{12}\log(3+2x)$$

input
```
Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

output
```
-1/12*Log[3 - 2*x] + Log[3 + 2*x]/12
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2019, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{16x^4 + 36x^2 + 81}{729 - 64x^6} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{9 - 4x^2} dx$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}\text{arctanh}\left(\frac{2x}{3}\right)$$

input
```
Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6),x]
```

output
```
ArcTanh[(2*x)/3]/6
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{\ln(x-\frac{3}{2})}{12}+\frac{\ln(x+\frac{3}{2})}{12}$
default	$-\frac{\ln(2x-3)}{12}+\frac{\ln(2x+3)}{12}$
norman	$-\frac{\ln(2x-3)}{12}+\frac{\ln(2x+3)}{12}$
risch	$-\frac{\ln(2x-3)}{12}+\frac{\ln(2x+3)}{12}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}\right.}{36(x^6)^{\frac{1}{6}}}$

input
```
int((16*x^4+36*x^2+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/12*ln(x-3/2)+1/12*ln(x+3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx = \frac{1}{12}\log(2x+3)-\frac{1}{12}\log(2x-3)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{12}+\frac{\log\left(x+\frac{3}{2}\right)}{12}$$

input
```
integrate((16*x**4+36*x**2+81)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/12 + log(x + 3/2)/12
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx = \frac{1}{12}\log\left(2\,x+3\right)-\frac{1}{12}\log\left(2\,x-3\right)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/12*log(2*x + 3) - 1/12*log(2*x - 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx = \frac{1}{12}\log\left(\left|x+\frac{3}{2}\right|\right)-\frac{1}{12}\log\left(\left|x-\frac{3}{2}\right|\right)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/12*log(abs(x + 3/2)) - 1/12*log(abs(x - 3/2))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx=\frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{6}$$

input
```
int(-(36*x^2 + 16*x^4 + 81)/(64*x^6 - 729),x)
```

output
```
atanh((2*x)/3)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{81+36x^2+16x^4}{729-64x^6}\,dx=-\frac{\log(2x-3)}{12}+\frac{\log(2x+3)}{12}$$

input
```
int((16*x^4+36*x^2+81)/(-64*x^6+729),x)
```

output
```
( - log(2*x - 3) + log(2*x + 3))/12
```

3.33 $\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 24

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx=-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
-1/9*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx=\frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2019, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-16x^4-24x^3+54x+81}{729-64x^6}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{4x^2-6x+9}dx$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(8x-6)^2-108}d(8x-6)$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-6 + 8*x)/(6*Sqrt[3])]/(3*Sqrt[3])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{9}$
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{9}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\right.}{36(x^6)^{\frac{1}{6}}}$

input
```
int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx=\frac{1}{9}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6}\, dx = \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729),x)
```

output
```
sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6}\, dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x - 3)\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81 + 54x - 24x^3 - 16x^4}{729 - 64x^6}\, dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x - 3)\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3))
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx=\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(4\,x-3)}{9}\right)}{9}$$

input
```
int(-(54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729),x)
```

output
```
(3^(1/2)*atan((3^(1/2)*(4*x - 3))/9))/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{81+54x-24x^3-16x^4}{729-64x^6}\,dx=\frac{\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)}{9}$$

input
```
int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729),x)
```

output
```
(sqrt(3)*atan((4*x - 3)/(3*sqrt(3))))/9
```

3.34 $\int \frac{3-2x}{729-64x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{3-2x}{729-64x^6}\,dx=\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)$$

output
```
1/486*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)+1/486*ln(3+2*x)-1/972*ln(4*x^2-6
*x+9)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{3-2x}{729-64x^6}\,dx=\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{486}\log(3+2x)-\frac{1}{972}\log\left(9-6x+4x^2\right)$$

input
```
Integrate[(3 - 2*x)/(729 - 64*x^6),x]
```

output
```
ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6
*x + 4*x^2]/972
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-2x}{729-64x^6} \, dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{32x^5+48x^4+72x^3+108x^2+162x+243}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{3-4x}{486\left(4x^2-6x+9\right)}+\frac{1}{54\left(4x^2+6x+9\right)}+\frac{1}{243(2x+3)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{162\sqrt{3}}-\frac{1}{972}\log\left(4x^2-6x+9\right)+\frac{1}{486}\log(2x+3)$$

input
```
Int[(3 - 2*x)/(729 - 64*x^6),x]
```

output
```
ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(162*Sqrt[3]) + Log[3 + 2*x]/486 - Log[9 - 6
*x + 4*x^2]/972
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(4x^2-6x+9)}{972}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{486}+\frac{\ln(2x+3)}{486}$
risch	$\frac{\arctan\left(\frac{(3+4x)\sqrt{3}}{9}\right)\sqrt{3}}{486}+\frac{\ln(2x+3)}{486}-\frac{\ln(4x^2-6x+9)}{972}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\right.}{972\left(x^6\right)^{\frac{1}{6}}}$

input
```
int((3-2*x)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/972*ln(4*x^2-6*x+9)+1/486*3^(1/2)*arctan(1/18*(8*x+6)*3^(1/2))+1/486*ln
(2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right)+\frac{1}{486}\,\log\left(2\,x+3\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
 1/486*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\log\left(x+\frac{3}{2}\right)}{486} - \frac{\log\left(4x^2-6x+9\right)}{972} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{486}$$

input
```
integrate((3-2*x)/(-64*x**6+729),x)
```

output
```
log(x + 3/2)/486 - log(4*x**2 - 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
+ sqrt(3)/3)/486
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{486}\,\log\left(2\,x+3\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
 1/486*log(2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) - \frac{1}{972}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{486}\log\left(|2\,x+3|\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/972*log(4*x^2 - 6*x + 9) +
 1/486*log(abs(2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{486} - \frac{\ln\left(x^2-\frac{3\,x}{2}+\frac{9}{4}\right)}{972} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\,\left(\frac{x}{884736}+\frac{1}{884736}\right)} - \frac{\sqrt{3}\,x}{7962624\,\left(\frac{x}{884736}+\frac{1}{884736}\right)}\right)}{486}$$

input
```
int((2*x - 3)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/486 - log(x^2 - (3*x)/2 + 9/4)/972 - (3^(1/2)*atan(3^(1/2)/(1
327104*(x/884736 + 1/884736)) - (3^(1/2)*x)/(7962624*(x/884736 + 1/884736)
)))/486
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3-2x}{729-64x^6}\,dx = \frac{\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)}{486} - \frac{\log(4x^2-6x+9)}{972} + \frac{\log(2x+3)}{486}$$

input
```
int((3-2*x)/(-64*x^6+729),x)
```

output
```
(2*sqrt(3)*atan((4*x + 3)/(3*sqrt(3))) - log(4*x**2 - 6*x + 9) + 2*log(2*x
 + 3))/972
```

3.35 $\int \frac{3+2x}{729-64x^6} dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{3+2x}{729-64x^6} dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}} - \frac{1}{486}\log(3-2x) + \frac{1}{972}\log\left(9+6x+4x^2\right)$$

output
```
-1/486*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/486*ln(3-2*x)+1/972*ln(4*x^2+
6*x+9)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3+2x}{729-64x^6} dx = \frac{1}{972}\left(2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 2\log(3-2x) + \log\left(9+6x+4x^2\right)\right)$$

input
```
Integrate[(3 + 2*x)/(729 - 64*x^6),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 2*Log[3 - 2*x] + Log[9 + 6*x +
 4*x^2])/972
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{729-64x^6}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{-32x^5+48x^4-72x^3+108x^2-162x+243}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{4x+3}{486\left(4x^2+6x+9\right)}+\frac{1}{54\left(4x^2-6x+9\right)}-\frac{1}{243(2x-3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{162\sqrt{3}}+\frac{1}{972}\log\left(4x^2+6x+9\right)-\frac{1}{486}\log(3-2x)$$

input
```
Int[(3 + 2*x)/(729 - 64*x^6),x]
```

output
```
-1/162*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/486 + Log[9 +
6*x + 4*x^2]/972
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{486}-\frac{\ln(2x-3)}{486}+\frac{\ln(4x^2+6x+9)}{972}$
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{486}-\frac{\ln(2x-3)}{486}+\frac{\ln(4x^2+6x+9)}{972}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\right.}{972(x^6)^{\frac{1}{6}}}$

input
```
int((2*x+3)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
1/486*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/486*ln(2*x-3)+1/972*ln(4*x^2+
6*x+9)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{729-64x^6}\,dx=\frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right)-\frac{1}{486}\,\log\left(2\,x-3\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
 1/486*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{3+2x}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{486}+\frac{\log\left(4x^2+6x+9\right)}{972}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{486}$$

input
```
integrate((3+2*x)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/486 + log(4*x**2 + 6*x + 9)/972 + sqrt(3)*atan(4*sqrt(3)*x/9
 - sqrt(3)/3)/486
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{1}{486}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{972}\,\log\left(4\,x^2+6\,x+9\right)-\frac{1}{486}\,\log\left(2\,x-3\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
 1/486*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{1}{486}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{972}\log\left(4x^2+6x+9\right) - \frac{1}{486}\log\left(|2x-3|\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/486*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/972*log(4*x^2 + 6*x + 9) -
 1/486*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{\ln\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{972} - \frac{\ln\left(x-\frac{3}{2}\right)}{486} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{1327104\left(\frac{x}{884736}-\frac{1}{884736}\right)} + \frac{\sqrt{3}\,x}{7962624\left(\frac{x}{884736}-\frac{1}{884736}\right)}\right)}{486}$$

input
```
int(-(2*x + 3)/(64*x^6 - 729),x)
```

output
```
log((3*x)/2 + x^2 + 9/4)/972 - log(x - 3/2)/486 - (3^(1/2)*atan(3^(1/2)/(1
327104*(x/884736 - 1/884736)) + (3^(1/2)*x)/(7962624*(x/884736 - 1/884736)
)))/486
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{729-64x^6}\,dx = \frac{\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)}{486} + \frac{\log(4x^2+6x+9)}{972} - \frac{\log(2x-3)}{486}$$

input
```
int((3+2*x)/(-64*x^6+729),x)
```

output
```
(2*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) + log(4*x**2 + 6*x + 9) - 2*log(2*x
 - 3))/972
```

3.36 $\int \frac{9-6x+4x^2}{729-64x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx=\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(3+2x)-\frac{1}{324}\log\left(9+6x+4x^2\right)$$

output
```
1/162*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)-1/324*ln(3-2*x)+1/108*ln(3+2*x)-
1/324*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx=\frac{1}{324}\left(2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-\log(3-2x)+3\log(3+2x)-\log\left(9+6x+4x^2\right)\right)$$

input
```
Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
(2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - Log[3 - 2*x] + 3*Log[3 + 2*x] -
 Log[9 + 6*x + 4*x^2])/324
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-6x+9}{729-64x^6}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{-16x^4-24x^3+54x+81}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{3-2x}{81\left(4x^2+6x+9\right)}-\frac{1}{162(2x-3)}+\frac{1}{54(2x+3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{54\sqrt{3}}-\frac{1}{324}\log\left(4x^2+6x+9\right)-\frac{1}{324}\log(3-2x)+\frac{1}{108}\log(2x+3)$$

input
```
Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(54*Sqrt[3]) - Log[3 - 2*x]/324 + Log[3 + 2*
x]/108 - Log[9 + 6*x + 4*x^2]/324
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln(2x-3)}{324}-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{162}+\frac{\ln(2x+3)}{108}$
risch	$\frac{\ln(2x+3)}{108}-\frac{\ln(2x-3)}{324}-\frac{\ln(4x^2+6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{162}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\right.}{324(x^6)^{\frac{1}{6}}}$

input
```
int((4*x^2-6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/324*ln(2*x-3)-1/324*ln(4*x^2+6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x+6)*3
^(1/2))+1/108*ln(2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{324}\log\left(4x^2+6x+9\right) + \frac{1}{108}\log(2x+3) - \frac{1}{324}\log(2x-3)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
 1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{324} + \frac{\log\left(x+\frac{3}{2}\right)}{108} - \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{324} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{162}$$

input
```
integrate((4*x**2-6*x+9)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/324 + log(x + 3/2)/108 - log(x**2 + 3*x/2 + 9/4)/324 + sqrt(
3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/162
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{324}\log\left(4x^2+6x+9\right) + \frac{1}{108}\log\left(2x+3\right) - \frac{1}{324}\log\left(2x-3\right)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
 1/108*log(2*x + 3) - 1/324*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) - \frac{1}{324}\log\left(4x^2+6x+9\right) + \frac{1}{108}\log\left(|2x+3|\right) - \frac{1}{324}\log\left(|2x-3|\right)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) - 1/324*log(4*x^2 + 6*x + 9) +
 1/108*log(abs(2*x + 3)) - 1/324*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{108} - \frac{\ln\left(x-\frac{3}{2}\right)}{324} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

input
```
int(-(4*x^2 - 6*x + 9)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/108 - log(x - 3/2)/324 - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1
/2)*1i)/324 + 1/324) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/324 - 1
/324)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9-6x+4x^2}{729-64x^6}\,dx = \frac{\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)}{162} - \frac{\log(4x^2+6x+9)}{324} - \frac{\log(2x-3)}{324} + \frac{\log(2x+3)}{108}$$

input
```
int((4*x^2-6*x+9)/(-64*x^6+729),x)
```

output
```
(2*sqrt(3)*atan((4*x + 3)/(3*sqrt(3))) - log(4*x**2 + 6*x + 9) - log(2*x -
 3) + 3*log(2*x + 3))/324
```

3.37 $\int \frac{9+6x+4x^2}{729-64x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{108}\log(3-2x) + \frac{1}{324}\log(3+2x) + \frac{1}{324}\log\left(9-6x+4x^2\right)$$

output
```
-1/162*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/108*ln(3-2*x)+1/324*ln(3+2*x)
+1/324*ln(4*x^2-6*x+9)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{324}\left(2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 3\log(3-2x) + \log(3+2x) + \log\left(9-6x+4x^2\right)\right)$$

input
```
Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[3 + 2*x]
+ Log[9 - 6*x + 4*x^2])/324
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+6x+9}{729-64x^6}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{-16x^4+24x^3-54x+81}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{2x+3}{81(4x^2-6x+9)}-\frac{1}{54(2x-3)}+\frac{1}{162(2x+3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}+\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)$$

input
```
Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6),x]
```

output
```
-1/54*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/108 + Log[3 + 2
*x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result
default	$\frac{\ln(4x^2-6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{162}-\frac{\ln(2x-3)}{108}+\frac{\ln(2x+3)}{324}$
risch	$\frac{\ln(2x+3)}{324}+\frac{\ln(4x^2-6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{162}-\frac{\ln(2x-3)}{108}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\right.}{324(x^6)^{\frac{1}{6}}}$

input
```
int((4*x^2+6*x+9)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/108*ln(
2*x-3)+1/324*ln(2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{324}\log\left(4x^2-6x+9\right) + \frac{1}{324}\log\left(2x+3\right) - \frac{1}{108}\log\left(2x-3\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{108} + \frac{\log\left(x+\frac{3}{2}\right)}{324} + \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

input
```
integrate((4*x**2+6*x+9)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/324 + sqrt(
3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right) + \frac{1}{324}\log\left(2x+3\right)-\frac{1}{108}\log\left(2x-3\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{324}\log\left(4x^2-6x+9\right) + \frac{1}{324}\log\left(|2x+3|\right)-\frac{1}{108}\log\left(|2x-3|\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(abs(2*x + 3)) - 1/108*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{324} - \frac{\ln\left(x-\frac{3}{2}\right)}{108} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

input
```
int(-(6*x + 4*x^2 + 9)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/324 - log(x - 3/2)/108 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1
/2)*1i)/324 - 1/324) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/324 + 1
/324)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{9+6x+4x^2}{729-64x^6}\,dx = \frac{\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)}{162} + \frac{\log(4x^2-6x+9)}{324} - \frac{\log(2x-3)}{108} + \frac{\log(2x+3)}{324}$$

input
```
int((4*x^2+6*x+9)/(-64*x^6+729),x)
```

output
```
(2*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) + log(4*x**2 - 6*x + 9) - 3*log(2*x
 - 3) + log(2*x + 3))/324
```

3.38 $\int \frac{27-8x^3}{729-64x^6} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \frac{27-8x^3}{729-64x^6} \, dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)$$

output
```
-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/54*ln(3+2*x)-1/108*ln(4*x^2-6*
x+9)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{27-8x^3}{729-64x^6} \, dx = \frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}} + \frac{1}{54}\log(3+2x) - \frac{1}{108}\log\left(9-6x+4x^2\right)$$

input
```
Integrate[(27 - 8*x^3)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) + Log[3 + 2*x]/54 - Log[9 - 6*
x + 4*x^2]/108
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1386, 750, 16, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{27-8x^3}{729-64x^6}dx$$

$$\Big\downarrow \text{1386}$$

$$\int \frac{1}{8x^3+27}dx$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{27}\int \frac{1}{2x+3}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{27}\int \frac{3-x}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{1142}$$

$$\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx-\frac{1}{8}\int -\frac{2(3-4x)}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx+\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{1083}$$

$$\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx-\frac{9}{2}\int \frac{1}{-(8x-6)^2-108}d(8x-6)\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx+\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)\right)+\frac{1}{54}\log(2x+3)$$

$$\Big\downarrow \text{1103}$$

$$\frac{2}{27}\left(\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)-\frac{1}{8}\log\left(4x^2-6x+9\right)\right)+\frac{1}{54}\log(2x+3)$$

input
```
Int[(27 - 8*x^3)/(729 - 64*x^6),x]
```

output
```
Log[3 + 2*x]/54 + (2*((Sqrt[3]*ArcTan[(-6 + 8*x)/(6*Sqrt[3])])/4 - Log[9 -
 6*x + 4*x^2]/8))/27
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$-\frac{\ln\left(4x^{2}-6x+9\right)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}+\frac{\ln\left(2x+3\right)}{54}$
risch	$\frac{\ln\left(2x+3\right)}{54}-\frac{\ln\left(4x^{2}-6x+9\right)}{108}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{54}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\right.}{108\left(x^{6}\right)^{\frac{1}{6}}}$

input
```
int((-8*x^3+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
-1/108*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))+1/54*ln(2
*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{1}{108}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{54}\,\log\left(2\,x+3\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) +
1/54*log(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\log\left(x+\frac{3}{2}\right)}{54} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{108} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

input
```
integrate((-8*x**3+27)/(-64*x**6+729),x)
```

output
```
log(x + 3/2)/54 - log(x**2 - 3*x/2 + 9/4)/108 + sqrt(3)*atan(4*sqrt(3)*x/9
 - sqrt(3)/3)/54
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{1}{108}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{54}\,\log\left(2\,x+3\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(4*x^2 - 6*x + 9) +
1/54*log(2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) - \frac{1}{108}\log\left(x^2-\frac{3}{2}\,x+\frac{9}{4}\right) + \frac{1}{54}\log\left(\left|x+\frac{3}{2}\right|\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 1/108*log(x^2 - 3/2*x + 9/4)
+ 1/54*log(abs(x + 3/2))
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{54} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{108}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

input
```
int((8*x^3 - 27)/(64*x^6 - 729),x)
```

output
```
log(x + 3/2)/54 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/108)
 + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/108)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27-8x^3}{729-64x^6}\,dx = \frac{\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)}{54} - \frac{\log(4x^2-6x+9)}{108} + \frac{\log(2x+3)}{54}$$

input
```
int((-8*x^3+27)/(-64*x^6+729),x)
```

output
```
(2*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) - log(4*x**2 - 6*x + 9) + 2*log(2*x
 + 3))/108
```

3.39 $\int \frac{27+36x+24x^2+8x^3}{729-64x^6} dx$

Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6} dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}} - \frac{1}{18}\log(3-2x) + \frac{1}{36}\log\left(9-6x+4x^2\right)$$

output
```
-1/54*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/18*ln(3-2*x)+1/36*ln(4*x^2-6*x
+9)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6} dx = \frac{\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)}{18\sqrt{3}} - \frac{1}{18}\log(3-2x) + \frac{1}{36}\log\left(9-6x+4x^2\right)$$

input
```
Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

output
```
ArcTan[(-3 + 4*x)/(3*Sqrt[3])]/(18*Sqrt[3]) - Log[3 - 2*x]/18 + Log[9 - 6*
x + 4*x^2]/36
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^3+24x^2+36x+27}{729-64x^6}dx$$
$$\Big\downarrow \ 2019$$
$$\int \frac{1}{-8x^3+24x^2-36x+27}dx$$
$$\Big\downarrow \ 2462$$
$$\int \left(\frac{2x}{9(4x^2-6x+9)}-\frac{1}{9(2x-3)}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{18\sqrt{3}}+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log(3-2x)$$

input
```
Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6),x]
```

output
```
-1/18*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/18 + Log[9 - 6*
x + 4*x^2]/36
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
default	$\frac{\ln(4x^2-6x+9)}{36}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{54}-\frac{\ln(2x-3)}{18}$
risch	$-\frac{\ln(2x-3)}{18}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{54}+\frac{\ln(4x^2-6x+9)}{36}$
meijerg	$-\frac{x\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\right.}{108(x^6)^{\frac{1}{6}}}$

input
```
int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x,method=_RETURNVERBOSE)
```

output
```
1/36*ln(4*x^2-6*x+9)+1/54*3^(1/2)*arctan(1/18*(8*x-6)*3^(1/2))-1/18*ln(2*x
-3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx=\frac{1}{54}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{1}{36}\,\log\left(4\,x^2-6\,x+9\right)-\frac{1}{18}\,\log\left(2\,x-3\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="fricas")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1
/18*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{18}+\frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{36}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{54}$$

input
```
integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729),x)
```

output
```
-log(x - 3/2)/18 + log(x**2 - 3*x/2 + 9/4)/36 + sqrt(3)*atan(4*sqrt(3)*x/9
 - sqrt(3)/3)/54
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+\frac{1}{36}\log\left(4x^2-6x+9\right)-\frac{1}{18}\log\left(2x-3\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="maxima")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1
/18*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{36}\log\left(4x^2-6x+9\right) - \frac{1}{18}\log\left(|2x-3|\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x, algorithm="giac")
```

output
```
1/54*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/36*log(4*x^2 - 6*x + 9) - 1
/18*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = -\frac{\ln\left(x-\frac{3}{2}\right)}{18} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{108}\right)$$

input
```
int(-(36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729),x)
```

output
```
log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/108 + 1/36) - log(x - (3^(1/2)
*3i)/4 - 3/4)*((3^(1/2)*1i)/108 - 1/36) - log(x - 3/2)/18
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{729-64x^6}\,dx = \frac{\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)}{54} + \frac{\log(4x^2-6x+9)}{36} - \frac{\log(2x-3)}{18}$$

input
```
int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729),x)
```

output
```
(2*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) + 3*log(4*x**2 - 6*x + 9) - 6*log(2
*x - 3))/108
```

3.40 $\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 110

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx = -\frac{1}{2916(3+2x)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log(3-2x)}{17496}+\frac{5\log(3+2x)}{17496}-\frac{\log(9-6x+4x^2)}{17496}-\frac{\log(9+6x+4x^2)}{17496}$$

output
```
-1/2916/(3+2*x)-1/26244*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/8748*arctan(
1/9*(3+4*x)*3^(1/2))*3^(1/2)-1/17496*ln(3-2*x)+5/17496*ln(3+2*x)-1/17496*l
n(4*x^2-6*x+9)-1/17496*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{-\frac{18}{3+2x}+2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-3\log(3-2x)+15\log(3+2x)-3\log\left(9-6x+\right.}{52488}$$

input
```
Integrate[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6
)^2,x]
```

output
```
(-18/(3 + 2*x) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt[3]*ArcT
an[(3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + 15*Log[3 + 2*x] - 3*Log[9 - 6
*x + 4*x^2] - 3*Log[9 + 6*x + 4*x^2])/52488
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-32x^5+48x^4-72x^3+108x^2-162x+243}{(729-64x^6)^2}\,dx$$
$$\Big\downarrow \; 2019$$
$$\int \frac{1}{(2x+3)^2\left(-32x^5+48x^4-72x^3+108x^2-162x+243\right)}dx$$
$$\Big\downarrow \; 2462$$
$$\int \left(\frac{3-2x}{4374\left(4x^2-6x+9\right)}+\frac{3-2x}{4374\left(4x^2+6x+9\right)}-\frac{1}{8748(2x-3)}+\frac{5}{8748(2x+3)}+\frac{1}{1458(2x+3)^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{2916\sqrt{3}}-\frac{\log\left(4x^2-6x+9\right)}{17496}-\frac{\log\left(4x^2+6x+9\right)}{17496}-\frac{1}{2916(2x+3)}-\frac{\log(3-2x)}{17496}+\frac{5\log(2x+3)}{17496}$$

input
```
Int[(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5)/(729 - 64*x^6)^2,x]
```

output
```
-1/2916*1/(3 + 2*x) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) + ArcTa
n[(3 + 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) - Log[3 - 2*x]/17496 + (5*Log[3 +
2*x])/17496 - Log[9 - 6*x + 4*x^2]/17496 - Log[9 + 6*x + 4*x^2]/17496
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{1}{5832\left(x+\frac{3}{2}\right)}-\frac{\ln\left(16x^2+24x+36\right)}{17496}+\frac{\arctan\left(\frac{(3+4x)\sqrt{3}}{9}\right)\sqrt{3}}{8748}-\frac{\ln(2x-3)}{17496}+\frac{\sqrt{3}\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{26244}-\frac{\ln\left(4x^2-6x+9\right)}{17496}$
default	$-\frac{\ln\left(4x^2-6x+9\right)}{17496}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{26244}-\frac{\ln(2x-3)}{17496}-\frac{\ln\left(4x^2+6x+9\right)}{17496}+\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{8748}-\frac{1}{2916(2x+3)}+5$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right.}{3}\right.}{}\right.}{6\left(x^6\right)^{\frac{1}{6}}}\right.}{8748}$

input
```
int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x,method=_RE
TURNVERBOSE)
```

output
```
-1/5832/(x+3/2)-1/17496*ln(16*x^2+24*x+36)+1/8748*arctan(1/9*(3+4*x)*3^(1/
2))*3^(1/2)-1/17496*ln(2*x-3)+1/26244*3^(1/2)*arctan(2/9*(2*x-3/2)*3^(1/2)
)-1/17496*ln(4*x^2-6*x+9)+5/17496*ln(2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{6\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(2x+3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)-3(2x+3)\log\left(4x^2+6\right.}{52488(2x}$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, alg
orithm="fricas")
```

output
```
1/52488*(6*sqrt(3)*(2*x + 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(2*
x + 3)*arctan(1/9*sqrt(3)*(4*x - 3)) - 3*(2*x + 3)*log(4*x^2 + 6*x + 9) -
3*(2*x + 3)*log(4*x^2 - 6*x + 9) + 15*(2*x + 3)*log(2*x + 3) - 3*(2*x + 3)
*log(2*x - 3) - 18)/(2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{243 - 162x + 108x^2 - 72x^3 + 48x^4 - 32x^5}{(729 - 64x^6)^2}\,dx$$
$$= -\frac{\log\left(x - \frac{3}{2}\right)}{17496} + \frac{5\log\left(x + \frac{3}{2}\right)}{17496} - \frac{\log\left(x^2 - \frac{3x}{2} + \frac{9}{4}\right)}{17496} - \frac{\log\left(x^2 + \frac{3x}{2} + \frac{9}{4}\right)}{17496} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9} - \frac{\sqrt{3}}{3}\right)}{26244} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9} + \frac{\sqrt{3}}{3}\right)}{8748} - \frac{1}{5832x + 8748}$$

input
```
integrate((-32*x**5+48*x**4-72*x**3+108*x**2-162*x+243)/(-64*x**6+729)**2,
x)
```

output
```
-log(x - 3/2)/17496 + 5*log(x + 3/2)/17496 - log(x**2 - 3*x/2 + 9/4)/17496
 - log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)
/26244 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/8748 - 1/(5832*x + 8748)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$=\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)$$
$$-\frac{1}{2916\,(2\,x+3)}-\frac{1}{17496}\log\left(4\,x^2+6\,x+9\right)$$
$$-\frac{1}{17496}\log\left(4\,x^2-6\,x+9\right)+\frac{5}{17496}\log\left(2\,x+3\right)-\frac{1}{17496}\log\left(2\,x-3\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, alg
orithm="maxima")
```

output
```
1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/1
7496*log(4*x^2 - 6*x + 9) + 5/17496*log(2*x + 3) - 1/17496*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$=\frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+\frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)$$
$$-\frac{1}{2916\,(2\,x+3)}-\frac{1}{17496}\log\left(4\,x^2+6\,x+9\right)-\frac{1}{17496}\log\left(4\,x^2-6\,x+9\right)$$
$$+\frac{5}{17496}\log\left(|2\,x+3|\right)-\frac{1}{17496}\log\left(|2\,x-3|\right)$$

input
```
integrate((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x, alg
orithm="giac")
```

output
```
1/8748*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/26244*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x + 3) - 1/17496*log(4*x^2 + 6*x + 9) - 1/1
7496*log(4*x^2 - 6*x + 9) + 5/17496*log(abs(2*x + 3)) - 1/17496*log(abs(2*
x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$=\frac{5\,\ln\left(x+\frac{3}{2}\right)}{17496}-\frac{\ln\left(x-\frac{3}{2}\right)}{17496}-\frac{1}{5832\,\left(x+\frac{3}{2}\right)}-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right)$$
$$+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right)-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$
$$+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$

input
```
int(-(162*x - 108*x^2 + 72*x^3 - 48*x^4 + 32*x^5 - 243)/(64*x^6 - 729)^2,x
)
```

output
```
(5*log(x + 3/2))/17496 - log(x - 3/2)/17496 - 1/(5832*(x + 3/2)) - log(x -
 (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/17496 + 1/17496) + log(x + (3^(1/2)*3
i)/4 + 3/4)*((3^(1/2)*1i)/17496 - 1/17496) - log(x - (3^(1/2)*3i)/4 - 3/4)
*((3^(1/2)*1i)/52488 + 1/17496) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*
1i)/52488 - 1/17496)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.51

$$\int \frac{243-162x+108x^2-72x^3+48x^4-32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{4\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)x+6\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)+12\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)x+18\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)-6\log(4x^2-6x+9)}{}$$

input
```
int((-32*x^5+48*x^4-72*x^3+108*x^2-162*x+243)/(-64*x^6+729)^2,x)
```

output
```
(4*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x + 6*sqrt(3)*atan((4*x - 3)/(3*sqr
t(3))) + 12*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x + 18*sqrt(3)*atan((4*x +
 3)/(3*sqrt(3))) - 6*log(4*x**2 - 6*x + 9)*x - 9*log(4*x**2 - 6*x + 9) - 6
*log(4*x**2 + 6*x + 9)*x - 9*log(4*x**2 + 6*x + 9) - 6*log(2*x - 3)*x - 9*
log(2*x - 3) + 30*log(2*x + 3)*x + 45*log(2*x + 3) + 12*x)/(52488*(2*x + 3
))
```

3.41 $\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 110

$$\begin{aligned}\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx &= \frac{1}{2916(3-2x)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{8748\sqrt{3}}-\frac{5\log(3-2x)}{17496}\\ &\quad+\frac{\log(3+2x)}{17496}+\frac{\log(9-6x+4x^2)}{17496}+\frac{\log(9+6x+4x^2)}{17496}\end{aligned}$$

output
```
1/(8748-5832*x)-1/8748*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/26244*arctan(
1/9*(3+4*x)*3^(1/2))*3^(1/2)-5/17496*ln(3-2*x)+1/17496*ln(3+2*x)+1/17496*l
n(4*x^2-6*x+9)+1/17496*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{6\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)+3\left(\frac{6}{3-2x}-5\log(3-2x)+\log(3+2x)+\log\left(9-6x+4x^2\right)\right.}{52488}$$

input
```
Integrate[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6
)^2,x]
```

output
```
(6*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*
Sqrt[3])] + 3*(6/(3 - 2*x) - 5*Log[3 - 2*x] + Log[3 + 2*x] + Log[9 - 6*x +
 4*x^2] + Log[9 + 6*x + 4*x^2]))/52488
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{32x^5+48x^4+72x^3+108x^2+162x+243}{(729-64x^6)^2}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{(3-2x)^2\left(32x^5+48x^4+72x^3+108x^2+162x+243\right)}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{2x+3}{4374\left(4x^2-6x+9\right)}+\frac{2x+3}{4374\left(4x^2+6x+9\right)}-\frac{5}{8748(2x-3)}+\frac{1}{1458(2x-3)^2}+\frac{1}{8748(2x+3)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{2916\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{8748\sqrt{3}}+\frac{\log\left(4x^2-6x+9\right)}{17496}+\frac{\log\left(4x^2+6x+9\right)}{17496}+\frac{1}{2916(3-2x)}-\frac{5\log(3-2x)}{17496}+\frac{\log(2x+3)}{17496}$$

input
```
Int[(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5)/(729 - 64*x^6)^2,x]
```

output
```
1/(2916*(3 - 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(2916*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(8748*Sqrt[3]) - (5*Log[3 - 2*x])/17496 + Log[3 +
2*x]/17496 + Log[9 - 6*x + 4*x^2]/17496 + Log[9 + 6*x + 4*x^2]/17496
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{1}{5832\left(x-\frac{3}{2}\right)}+\frac{\arctan\left(\frac{(3+4x)\sqrt{3}}{9}\right)\sqrt{3}}{26244}+\frac{\ln(16x^2+24x+36)}{17496}+\frac{\ln(2x+3)}{17496}-\frac{5\ln(2x-3)}{17496}+\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}\arctan\left(\right.}{8}$
default	$\frac{\ln(4x^2-6x+9)}{17496}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{8748}-\frac{1}{2916(2x-3)}-\frac{5\ln(2x-3)}{17496}+\frac{\ln(4x^2+6x+9)}{17496}+\frac{\sqrt{3}\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{26244}+\frac{\ln(}{1}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{8748}$

input
```
int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x,method=_RET
URNVERBOSE)
```

output
```
-1/5832/(x-3/2)+1/26244*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)+1/17496*ln(16*
x^2+24*x+36)+1/17496*ln(2*x+3)-5/17496*ln(2*x-3)+1/17496*ln(4*x^2-6*x+9)+1
/8748*3^(1/2)*arctan(2/9*(2*x-3/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$=\frac{2\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+6\sqrt{3}(2x-3)\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right)+3(2x-3)\log\left(4x^2+6\right.}{52488\,(2x}$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algo
rithm="fricas")
```

output
```
1/52488*(2*sqrt(3)*(2*x - 3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 6*sqrt(3)*(2*
x - 3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(2*x - 3)*log(4*x^2 + 6*x + 9) +
3*(2*x - 3)*log(4*x^2 - 6*x + 9) + 3*(2*x - 3)*log(2*x + 3) - 15*(2*x - 3)
*log(2*x - 3) - 18)/(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{243 + 162x + 108x^2 + 72x^3 + 48x^4 + 32x^5}{(729 - 64x^6)^2}\,dx = -\frac{5\log\left(x-\frac{3}{2}\right)}{17496} + \frac{\log\left(x+\frac{3}{2}\right)}{17496} + \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{17496} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{17496} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{8748} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{26244} - \frac{1}{5832x-8748}$$

input
```
integrate((32*x**5+48*x**4+72*x**3+108*x**2+162*x+243)/(-64*x**6+729)**2,x
)
```

output
```
-5*log(x - 3/2)/17496 + log(x + 3/2)/17496 + log(x**2 - 3*x/2 + 9/4)/17496
 + log(x**2 + 3*x/2 + 9/4)/17496 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)
/8748 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/26244 - 1/(5832*x - 8748)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x-3)} + \frac{1}{17496}\log\left(4x^2+6x+9\right) + \frac{1}{17496}\log\left(4x^2-6x+9\right) + \frac{1}{17496}\log(2x+3) - \frac{5}{17496}\log(2x-3)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algo
rithm="maxima")
```

output
```
1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/1
7496*log(4*x^2 - 6*x + 9) + 1/17496*log(2*x + 3) - 5/17496*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.78

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx$$
$$= \frac{1}{26244}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{8748}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{1}{2916(2x-3)} + \frac{1}{17496}\log\left(4x^2+6x+9\right) + \frac{1}{17496}\log\left(4x^2-6x+9\right) + \frac{1}{17496}\log(|2x+3|) - \frac{5}{17496}\log(|2x-3|)$$

input
```
integrate((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x, algo
rithm="giac")
```

output
```
1/26244*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/8748*sqrt(3)*arctan(1/9*
sqrt(3)*(4*x - 3)) - 1/2916/(2*x - 3) + 1/17496*log(4*x^2 + 6*x + 9) + 1/1
7496*log(4*x^2 - 6*x + 9) + 1/17496*log(abs(2*x + 3)) - 5/17496*log(abs(2*
x - 3))
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{17496} - \frac{5\ln\left(x-\frac{3}{2}\right)}{17496} - \frac{1}{5832\left(x-\frac{3}{2}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{17496}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{17496}+\frac{\sqrt{3}\,1\mathrm{i}}{52488}\right)$$

input
```
int((162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5 + 243)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/17496 - (5*log(x - 3/2))/17496 - 1/(5832*(x - 3/2)) - log(x -
 (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/17496 - 1/17496) + log(x + (3^(1/2)*3
i)/4 - 3/4)*((3^(1/2)*1i)/17496 + 1/17496) - log(x - (3^(1/2)*3i)/4 + 3/4)
*((3^(1/2)*1i)/52488 - 1/17496) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*
1i)/52488 + 1/17496)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.51

$$\int \frac{243+162x+108x^2+72x^3+48x^4+32x^5}{(729-64x^6)^2}\,dx = \frac{12\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)x - 18\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right) + 4\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)x - 6\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right) + 6\log(4x^2-6x+9)}{}$$

input

```
int((32*x^5+48*x^4+72*x^3+108*x^2+162*x+243)/(-64*x^6+729)^2,x)
```

output

```
(12*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x - 18*sqrt(3)*atan((4*x - 3)/(3*s
qrt(3))) + 4*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x - 6*sqrt(3)*atan((4*x +
 3)/(3*sqrt(3))) + 6*log(4*x**2 - 6*x + 9)*x - 9*log(4*x**2 - 6*x + 9) + 6
*log(4*x**2 + 6*x + 9)*x - 9*log(4*x**2 + 6*x + 9) - 30*log(2*x - 3)*x + 4
5*log(2*x - 3) + 6*log(2*x + 3)*x - 9*log(2*x + 3) - 12*x)/(52488*(2*x - 3
))
```

3.42 $\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2}\,dx=\frac{x}{4374\left(9-4x^2\right)}-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2x}{3}\right)}{8748}$$

output
```
x/(-17496*x^2+39366)-1/39366*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/39366*a
rctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)+1/8748*arctanh(2/3*x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \frac{81+36x^2+16x^4}{\left(729-64x^6\right)^2}\,dx$$

$$=\frac{\frac{36x}{9-4x^2}+3\sqrt{3}\arctan\left(\frac{1}{3}\left(-i+\sqrt{3}\right)x\right)+4i\sqrt{3}\operatorname{arctanh}\left(\frac{1}{3}\left(1-i\sqrt{3}\right)x\right)+\left(-3+\frac{2}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}\right)\operatorname{arctanh}\left(\frac{1}{3}\left(\right.\right.}{157464}$$

input
```
Integrate[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
((36*x)/(9 - 4*x^2) + 3*Sqrt[3]*ArcTan[((-I + Sqrt[3])*x)/3] + (4*I)*Sqrt[
3]*ArcTanh[((1 - I*Sqrt[3])*x)/3] + (-3 + 2/Sqrt[(1 + I*Sqrt[3])/6])*ArcTa
nh[(x + I*Sqrt[3]*x)/3] - 9*Log[3 - 2*x] + 9*Log[3 + 2*x])/157464
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2019, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{16x^4+36x^2+81}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(9-4x^2)^2(16x^4+36x^2+81)}dx$$

$$\Big\downarrow \text{1484}$$

$$\int \left(\frac{1}{4374(4x^2-6x+9)}+\frac{1}{4374(4x^2+6x+9)}-\frac{1}{1458(4x^2-9)}+\frac{1}{8748(2x-3)^2}+\frac{1}{8748(2x+3)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{13122\sqrt{3}}+\frac{\text{arctanh}\left(\frac{2x}{3}\right)}{8748}+\frac{1}{17496(3-2x)}-\frac{1}{17496(2x+3)}$$

input
```
Int[(81 + 36*x^2 + 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
1/(17496*(3 - 2*x)) - 1/(17496*(3 + 2*x)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/
(13122*Sqrt[3]) + ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(13122*Sqrt[3]) + ArcTanh[
(2*x)/3]/8748
```

Defintions of rubi rules used

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{x}{17496\left(x^{2}-\frac{9}{4}\right)}+\frac{\ln(2x+3)}{17496}-\frac{\ln(2x-3)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{2x\sqrt{3}}{9}\right)}{39366}+\frac{\sqrt{3}\,\arctan\left(\frac{8\sqrt{3}\,x^{3}}{81}+\frac{4x\sqrt{3}}{9}\right)}{39366}$
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{39366}-\frac{1}{17496(2x-3)}-\frac{\ln(2x-3)}{17496}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)\sqrt{3}}{18}\right)}{39366}-\frac{1}{17496(2x+3)}+\frac{\ln(2x+3)}{17496}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^{6}\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^{6}\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}{3-\left(x^{6}\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^{6}\right.}{3}\right.}{\ldots}\right.}{6\left(x^{6}\right)^{\frac{1}{6}}}\right.}{26244}$

input
```
int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/17496*x/(x^2-9/4)+1/17496*ln(2*x+3)-1/17496*ln(2*x-3)+1/39366*3^(1/2)*a
rctan(2/9*x*3^(1/2))+1/39366*3^(1/2)*arctan(8/81*3^(1/2)*x^3+4/9*x*3^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{4\sqrt{3}(4x^2-9)\arctan\left(\frac{4}{81}\sqrt{3}(2x^3+9x)\right)+4\sqrt{3}(4x^2-9)\arctan\left(\frac{2}{9}\sqrt{3}x\right)+9(4x^2-9)\log(2x+3)-}{157464(4x^2-9)}$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/157464*(4*sqrt(3)*(4*x^2 - 9)*arctan(4/81*sqrt(3)*(2*x^3 + 9*x)) + 4*sqr
t(3)*(4*x^2 - 9)*arctan(2/9*sqrt(3)*x) + 9*(4*x^2 - 9)*log(2*x + 3) - 9*(4
*x^2 - 9)*log(2*x - 3) - 36*x)/(4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = -\frac{x}{17496x^2-39366} + \frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{9}\right)+2\,\mathrm{atan}\left(\frac{8\sqrt{3}x^3}{81}+\frac{4\sqrt{3}x}{9}\right)\right)}{78732} - \frac{\log\left(x-\frac{3}{2}\right)}{17496} + \frac{\log\left(x+\frac{3}{2}\right)}{17496}$$

input
```
integrate((16*x**4+36*x**2+81)/(-64*x**6+729)**2,x)
```

output
```
-x/(17496*x**2 - 39366) + sqrt(3)*(2*atan(2*sqrt(3)*x/9) + 2*atan(8*sqrt(3
)*x**3/81 + 4*sqrt(3)*x/9))/78732 - log(x - 3/2)/17496 + log(x + 3/2)/1749
6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(4x^2-9)} + \frac{1}{17496}\log(2x+3) - \frac{1}{17496}\log(2x-3)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9
*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(2*x + 3) - 1/1749
6*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{39366}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(4x^2-9)} + \frac{1}{17496}\log(|2x+3|) - \frac{1}{17496}\log(|2x-3|)$$

input
```
integrate((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/39366*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/39366*sqrt(3)*arctan(1/9
*sqrt(3)*(4*x - 3)) - 1/4374*x/(4*x^2 - 9) + 1/17496*log(abs(2*x + 3)) - 1
/17496*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{2x}{3}\right)}{8748} + \frac{\sqrt{3}\left(2\operatorname{atan}\left(\frac{8\sqrt{3}\,x^3}{81}+\frac{4\sqrt{3}\,x}{9}\right)+2\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{9}\right)\right)}{78732} - \frac{x}{17496\left(x^2-\frac{9}{4}\right)}$$

input
```
int((36*x^2 + 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

output
```
atanh((2*x)/3)/8748 + (3^(1/2)*(2*atan((4*3^(1/2)*x)/9 + (8*3^(1/2)*x^3)/8
1) + 2*atan((2*3^(1/2)*x)/9)))/78732 - x/(17496*(x^2 - 9/4))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \frac{81+36x^2+16x^4}{(729-64x^6)^2}\,dx = \frac{16\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)x^2-36\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)+16\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)x^2-36\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)-36\log(2x-3)}{629856x^2-1417176}$$

input
```
int((16*x^4+36*x^2+81)/(-64*x^6+729)^2,x)
```

output
```
(16*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**2 - 36*sqrt(3)*atan((4*x - 3)/(
3*sqrt(3))) + 16*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**2 - 36*sqrt(3)*ata
n((4*x + 3)/(3*sqrt(3))) - 36*log(2*x - 3)*x**2 + 81*log(2*x - 3) + 36*log
(2*x + 3)*x**2 - 81*log(2*x + 3) - 36*x)/(157464*(4*x**2 - 9))
```

3.43 $\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{x}{4374(9-6x+4x^2)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}} - \frac{\log(3-2x)}{26244} + \frac{\log(3+2x)}{78732} - \frac{\log(9-6x+4x^2)}{157464} + \frac{\log(9+6x+4x^2)}{52488}$$

output
```
x/(17496*x^2-26244*x+39366)-1/13122*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/
26244*ln(3-2*x)+1/78732*ln(3+2*x)-1/157464*ln(4*x^2-6*x+9)+1/52488*ln(4*x^
2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{36x}{9-6x+4x^2} + 12\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 6\log(3-2x) + 2\log(3+2x) - \log(9-6x+4x^2) + 3\log(9+6x+}{157464}$$

input
```
Integrate[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
((36*x)/(9 - 6*x + 4*x^2) + 12*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 6*
Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x
^2])/157464
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-16x^4-24x^3+54x+81}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{(4x^2-6x+9)^2(-16x^4-24x^3+54x+81)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{39-4x}{78732(4x^2-6x+9)}+\frac{4x+3}{26244(4x^2+6x+9)}+\frac{3-x}{729(4x^2-6x+9)^2}-\frac{1}{13122(2x-3)}+\frac{1}{39366(2x+3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{4374\sqrt{3}}+\frac{x}{4374(4x^2-6x+9)}-\frac{\log(4x^2-6x+9)}{157464}+\frac{\log(4x^2+6x+9)}{52488}-\frac{\log(3-2x)}{26244}+\frac{\log(2x+3)}{78732}$$

input
```
Int[(81 + 54*x - 24*x^3 - 16*x^4)/(729 - 64*x^6)^2,x]
```

output
```
x/(4374*(9 - 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(4374*Sqrt[3])
- Log[3 - 2*x]/26244 + Log[3 + 2*x]/78732 - Log[9 - 6*x + 4*x^2]/157464 +
Log[9 + 6*x + 4*x^2]/52488
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

method	result
default	$\frac{x}{17496x^2-26244x+39366}-\frac{\ln(4x^2-6x+9)}{157464}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}-\frac{\ln(2x-3)}{26244}+\frac{\ln(4x^2+6x+9)}{52488}+\frac{\ln(2x+3)}{78732}$
risch	$\frac{x}{17496x^2-26244x+39366}-\frac{\ln(2x-3)}{26244}+\frac{\ln(2x+3)}{78732}-\frac{\ln(64x^2-96x+144)}{157464}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{13122}+\frac{\ln(4x^2+6x+9)}{52488}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{26244}$

input
```
int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
1/17496*x/(x^2-3/2*x+9/4)-1/157464*ln(4*x^2-6*x+9)+1/13122*3^(1/2)*arctan(
1/18*(8*x-6)*3^(1/2))-1/26244*ln(2*x-3)+1/52488*ln(4*x^2+6*x+9)+1/78732*ln
(2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{12\sqrt{3}(4\,x^2-6\,x+9)\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+3\,(4\,x^2-6\,x+9)\log\left(4\,x^2+6\,x+9\right)-(4\,x^2-6\,x+9)}{157464\,(4\,x^2-6\,}$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/157464*(12*sqrt(3)*(4*x^2 - 6*x + 9)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(
4*x^2 - 6*x + 9)*log(4*x^2 + 6*x + 9) - (4*x^2 - 6*x + 9)*log(4*x^2 - 6*x
+ 9) + 2*(4*x^2 - 6*x + 9)*log(2*x + 3) - 6*(4*x^2 - 6*x + 9)*log(2*x - 3)
 + 36*x)/(4*x^2 - 6*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{x}{17496x^2-26244x+39366} - \frac{\log\left(x-\frac{3}{2}\right)}{26244} + \frac{\log\left(x+\frac{3}{2}\right)}{78732} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{157464} + \frac{\log\left(4x^2+6x+9\right)}{52488} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{13122}$$

input
```
integrate((-16*x**4-24*x**3+54*x+81)/(-64*x**6+729)**2,x)
```

output
```
x/(17496*x**2 - 26244*x + 39366) - log(x - 3/2)/26244 + log(x + 3/2)/78732
 - log(x**2 - 3*x/2 + 9/4)/157464 + log(4*x**2 + 6*x + 9)/52488 + sqrt(3)*
atan(4*sqrt(3)*x/9 - sqrt(3)/3)/13122
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.80

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{1}{13122}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(4\,x^2-6\,x+9)} + \frac{1}{52488}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{157464}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{78732}\,\log\left(2\,x+3\right) - \frac{1}{26244}\,\log\left(2\,x-3\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
 + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*
log(2*x + 3) - 1/26244*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{1}{13122}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(4\,x^2-6\,x+9)} + \frac{1}{52488}\,\log\left(4\,x^2+6\,x+9\right) - \frac{1}{157464}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{78732}\,\log\left(|2\,x+3|\right) - \frac{1}{26244}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/13122*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(4*x^2 - 6*x + 9)
 + 1/52488*log(4*x^2 + 6*x + 9) - 1/157464*log(4*x^2 - 6*x + 9) + 1/78732*
log(abs(2*x + 3)) - 1/26244*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{78732} - \frac{\ln\left(x-\frac{3}{2}\right)}{26244} + \frac{\ln\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{52488} + \frac{x}{17496\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{157464}+\frac{\sqrt{3}\,1\mathrm{i}}{26244}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{157464}+\frac{\sqrt{3}\,1\mathrm{i}}{26244}\right)$$

input
```
int((54*x - 24*x^3 - 16*x^4 + 81)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/78732 - log(x - 3/2)/26244 + log((3*x)/2 + x^2 + 9/4)/52488 +
 x/(17496*(x^2 - (3*x)/2 + 9/4)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)
*1i)/26244 + 1/157464) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/26244
 - 1/157464)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.33

$$\int \frac{81+54x-24x^3-16x^4}{(729-64x^6)^2}\,dx = \frac{48\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)x^2 - 72\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)x + 108\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right) - 4\log(4x^2-6x+9)\,x^2 + 6\log(4x^2}{}$$

input
```
int((-16*x^4-24*x^3+54*x+81)/(-64*x^6+729)^2,x)
```

output
```
(48*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**2 - 72*sqrt(3)*atan((4*x - 3)/(
3*sqrt(3)))*x + 108*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) - 4*log(4*x**2 - 6
*x + 9)*x**2 + 6*log(4*x**2 - 6*x + 9)*x - 9*log(4*x**2 - 6*x + 9) + 12*lo
g(4*x**2 + 6*x + 9)*x**2 - 18*log(4*x**2 + 6*x + 9)*x + 27*log(4*x**2 + 6*
x + 9) - 24*log(2*x - 3)*x**2 + 36*log(2*x - 3)*x - 54*log(2*x - 3) + 8*lo
g(2*x + 3)*x**2 - 12*log(2*x + 3)*x + 18*log(2*x + 3) + 24*x**2 + 54)/(157
464*(4*x**2 - 6*x + 9))
```

3.44 $\int \frac{3-2x}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = -\frac{1}{708588(3+2x)} + \frac{3-x}{708588(9-6x+4x^2)} + \frac{x}{236196(9+6x+4x^2)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\log(3-2x)}{4251528} + \frac{\log(3+2x)}{472392} - \frac{\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{8503056}$$

output
```
-1/708588/(3+2*x)+(3-x)/(2834352*x^2-4251528*x+6377292)+x/(944784*x^2+1417
176*x+2125764)-1/4251528*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/472392*arct
an(1/9*(3+4*x)*3^(1/2))*3^(1/2)-1/4251528*ln(3-2*x)+1/472392*ln(3+2*x)-1/9
44784*ln(4*x^2-6*x+9)+1/8503056*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{1944x}{243+162x+108x^2+72x^3+48x^4+32x^5} + 2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) + 18\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 2\log(3-2x) + 18\log(3+}{8503056}$$

input

```
Integrate[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

output

```
((1944*x)/(243 + 162*x + 108*x^2 + 72*x^3 + 48*x^4 + 32*x^5) + 2*Sqrt[3]*A
rcTan[(-3 + 4*x)/(3*Sqrt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] -
 2*Log[3 - 2*x] + 18*Log[3 + 2*x] - 9*Log[9 - 6*x + 4*x^2] + Log[9 + 6*x +
 4*x^2])/8503056
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx$$
$$\Big\downarrow \text{ 2019}$$
$$\int \frac{1}{(3-2x)(32x^5+48x^4+72x^3+108x^2+162x+243)^2}dx$$
$$\Big\downarrow \text{ 2462}$$
$$\int \left(\frac{7-6x}{708588(4x^2-6x+9)} + \frac{2x+33}{2125764(4x^2+6x+9)} - \frac{x}{39366(4x^2-6x+9)^2} + \frac{x+3}{39366(4x^2+6x+9)^2} - 212\right.$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{3-x}{708588\,(4x^2-6x+9)}+\frac{x}{236196\,(4x^2+6x+9)}-\frac{\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{8503056}-\frac{1}{708588(2x+3)}-\frac{\log(3-2x)}{4251528}+\frac{\log(2x+3)}{472392}$$

input
```
Int[(3 - 2*x)/(729 - 64*x^6)^2,x]
```

output
```
-1/708588*1/(3 + 2*x) + (3 - x)/(708588*(9 - 6*x + 4*x^2)) + x/(236196*(9
+ 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) + ArcTan
[(3 + 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) - Log[3 - 2*x]/4251528 + Log[3 +
2*x]/472392 - Log[9 - 6*x + 4*x^2]/944784 + Log[9 + 6*x + 4*x^2]/8503056
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882} - \frac{\ln(16x^2-24x+36)}{944784} + \frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{4251528} - \frac{\ln(2x-3)}{4251528} \cdots$
default	$-\frac{\frac{x}{4}-\frac{3}{4}}{708588\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)} - \frac{\ln(4x^2-6x+9)}{944784} + \frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{4251528} - \frac{\ln(2x-3)}{4251528} + \frac{x}{944784x^2+1417176x+2125764} + \frac{\ln(4x \cdots}{85 \cdots}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}} - \frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6 \cdots}{3 \cdots}\right)}{\cdots}\cdots\right)}{6(x^6)^{\frac{1}{6}}}\cdots\right)}{708588}$

input
```
int((3-2*x)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
1/139968*x/(x^5+3/2*x^4+9/4*x^3+27/8*x^2+81/16*x+243/32)-1/944784*ln(16*x^
2-24*x+36)+1/4251528*3^(1/2)*arctan(1/9*(4*x-3)*3^(1/2))-1/4251528*ln(2*x-
3)+1/8503056*ln(36*x^2+54*x+81)+1/472392*3^(1/2)*arctan(2/27*(6*x+9/2)*3^(
1/2))+1/472392*ln(2*x+3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(116) = 232$.

Time = 0.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx$$
$$= \frac{18\sqrt{3}(32x^5+48x^4+72x^3+108x^2+162x+243)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+2\sqrt{3}(32x^5+48x^4+72\cdots}{\cdots}$$

input
```
integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/8503056*(18*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 162*x + 243)*a
rctan(1/9*sqrt(3)*(4*x + 3)) + 2*sqrt(3)*(32*x^5 + 48*x^4 + 72*x^3 + 108*x
^2 + 162*x + 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + (32*x^5 + 48*x^4 + 72*x^
3 + 108*x^2 + 162*x + 243)*log(4*x^2 + 6*x + 9) - 9*(32*x^5 + 48*x^4 + 72*
x^3 + 108*x^2 + 162*x + 243)*log(4*x^2 - 6*x + 9) + 18*(32*x^5 + 48*x^4 +
72*x^3 + 108*x^2 + 162*x + 243)*log(2*x + 3) - 2*(32*x^5 + 48*x^4 + 72*x^3
 + 108*x^2 + 162*x + 243)*log(2*x - 3) + 1944*x)/(32*x^5 + 48*x^4 + 72*x^3
 + 108*x^2 + 162*x + 243)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{x}{139968x^5+209952x^4+314928x^3+472392x^2+708588x+1062882} - \frac{\log\left(x-\frac{3}{2}\right)}{4251528} + \frac{\log\left(x+\frac{3}{2}\right)}{472392} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{8503056} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{4251528} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

input
```
integrate((3-2*x)/(-64*x**6+729)**2,x)
```

output
```
x/(139968*x**5 + 209952*x**4 + 314928*x**3 + 472392*x**2 + 708588*x + 1062
882) - log(x - 3/2)/4251528 + log(x + 3/2)/472392 - log(x**2 - 3*x/2 + 9/4
)/944784 + log(x**2 + 3*x/2 + 9/4)/8503056 + sqrt(3)*atan(4*sqrt(3)*x/9 -
sqrt(3)/3)/4251528 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374\,(32x^5+48x^4+72x^3+108x^2+162x+243)} + \frac{1}{8503056}\log\left(4x^2+6x+9\right) - \frac{1}{944784}\log\left(4x^2-6x+9\right) + \frac{1}{472392}\log\left(2x+3\right) - \frac{1}{4251528}\log\left(2x-3\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(32*x^5 + 48*x^4 + 72*x^3 + 108*x^2 + 16
2*x + 243) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9
) + 1/472392*log(2*x + 3) - 1/4251528*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{x}{4374\,(4x^2+6x+9)(4x^2-6x+9)(2x+3)} + \frac{1}{8503056}\log\left(4x^2+6x+9\right) - \frac{1}{944784}\log\left(4x^2-6x+9\right) + \frac{1}{472392}\log\left(|2x+3|\right) - \frac{1}{4251528}\log\left(|2x-3|\right)$$

input
```
integrate((3-2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/4251528*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) + 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*
x + 3)) + 1/8503056*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) +
 1/472392*log(abs(2*x + 3)) - 1/4251528*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \frac{3-2x}{(729-64x^6)^2}\,dx &= \frac{\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{\ln\left(x-\frac{3}{2}\right)}{4251528} \\ &\quad - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) \\ &\quad - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) \\ &\quad + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) \\ &\quad + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) \\ &\quad + \frac{x}{139968\left(x^5+\frac{3x^4}{2}+\frac{9x^3}{4}+\frac{27x^2}{8}+\frac{81x}{16}+\frac{243}{32}\right)}\end{aligned}$$

input
```
int(-(2*x - 3)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/472392 - log(x - 3/2)/4251528 - log(x - (3^(1/2)*3i)/4 - 3/4)
*((3^(1/2)*1i)/8503056 + 1/944784) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/
2)*1i)/944784 - 1/8503056) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/8
503056 - 1/944784) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 +
1/8503056) + x/(139968*((81*x)/16 + (27*x^2)/8 + (9*x^3)/4 + (3*x^4)/2 + x
^5 + 243/32))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.74

$$\int \frac{3-2x}{(729-64x^6)^2}\,dx = \text{Too large to display}$$

input
```
int((3-2*x)/(-64*x^6+729)^2,x)
```

output
```
(64*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**5 + 96*sqrt(3)*atan((4*x - 3)/(
3*sqrt(3)))*x**4 + 144*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**3 + 216*sqrt
(3)*atan((4*x - 3)/(3*sqrt(3)))*x**2 + 324*sqrt(3)*atan((4*x - 3)/(3*sqrt(
3)))*x + 486*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) + 576*sqrt(3)*atan((4*x +
 3)/(3*sqrt(3)))*x**5 + 864*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**4 + 129
6*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**3 + 1944*sqrt(3)*atan((4*x + 3)/(
3*sqrt(3)))*x**2 + 2916*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x + 4374*sqrt(
3)*atan((4*x + 3)/(3*sqrt(3))) - 288*log(4*x**2 - 6*x + 9)*x**5 - 432*log(
4*x**2 - 6*x + 9)*x**4 - 648*log(4*x**2 - 6*x + 9)*x**3 - 972*log(4*x**2 -
 6*x + 9)*x**2 - 1458*log(4*x**2 - 6*x + 9)*x - 2187*log(4*x**2 - 6*x + 9)
 + 32*log(4*x**2 + 6*x + 9)*x**5 + 48*log(4*x**2 + 6*x + 9)*x**4 + 72*log(
4*x**2 + 6*x + 9)*x**3 + 108*log(4*x**2 + 6*x + 9)*x**2 + 162*log(4*x**2 +
 6*x + 9)*x + 243*log(4*x**2 + 6*x + 9) - 64*log(2*x - 3)*x**5 - 96*log(2*
x - 3)*x**4 - 144*log(2*x - 3)*x**3 - 216*log(2*x - 3)*x**2 - 324*log(2*x
- 3)*x - 486*log(2*x - 3) + 576*log(2*x + 3)*x**5 + 864*log(2*x + 3)*x**4
+ 1296*log(2*x + 3)*x**3 + 1944*log(2*x + 3)*x**2 + 2916*log(2*x + 3)*x +
4374*log(2*x + 3) + 1944*x)/(8503056*(32*x**5 + 48*x**4 + 72*x**3 + 108*x*
*2 + 162*x + 243))
```

3.45 $\int \frac{3+2x}{(729-64x^6)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\begin{aligned}\int \frac{3+2x}{(729-64x^6)^2} \, dx = & \frac{1}{708588(3-2x)} + \frac{x}{236196\,(9-6x+4x^2)} \\ & - \frac{3+x}{708588\,(9+6x+4x^2)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} \\ & + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{1417176\sqrt{3}} - \frac{\log(3-2x)}{472392} + \frac{\log(3+2x)}{4251528} \\ & - \frac{\log\left(9-6x+4x^2\right)}{8503056} + \frac{\log\left(9+6x+4x^2\right)}{944784}\end{aligned}$$

output

```
1/(2125764-1417176*x)+x/(944784*x^2-1417176*x+2125764)-(3+x)/(2834352*x^2+
4251528*x+6377292)-1/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/4251528*
arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)-1/472392*ln(3-2*x)+1/4251528*ln(3+2*x)
-1/8503056*ln(4*x^2-6*x+9)+1/944784*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{1944x}{243-162x+108x^2-72x^3+48x^4-32x^5}+18\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-18\log(3-2x)+2\log(3+}{8503056}$$

input
```
Integrate[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

output
```
((1944*x)/(243 - 162*x + 108*x^2 - 72*x^3 + 48*x^4 - 32*x^5) + 18*Sqrt[3]*
ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] -
 18*Log[3 - 2*x] + 2*Log[3 + 2*x] - Log[9 - 6*x + 4*x^2] + 9*Log[9 + 6*x +
 4*x^2])/8503056
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{(729-64x^6)^2}\,dx$$
$$\Big\downarrow \ 2019$$
$$\int \frac{1}{(2x+3)\left(-32x^5+48x^4-72x^3+108x^2-162x+243\right)^2}dx$$
$$\Big\downarrow \ 2462$$
$$\int \left(\frac{33-2x}{2125764\,(4x^2-6x+9)}+\frac{6x+7}{708588\,(4x^2+6x+9)}+\frac{3-x}{39366\,(4x^2-6x+9)^2}+\frac{x}{39366\,(4x^2+6x+9)^2}-\frac{}{236}\right.$$
$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{1417176\sqrt{3}}+\frac{x}{236196\left(4x^2-6x+9\right)}-\frac{x+3}{708588\left(4x^2+6x+9\right)}-\frac{\log\left(4x^2-6x+9\right)}{8503056}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{708588(3-2x)}-\frac{\log(3-2x)}{472392}+\frac{\log(2x+3)}{4251528}$$

input
```
Int[(3 + 2*x)/(729 - 64*x^6)^2,x]
```

output
```
1/(708588*(3 - 2*x)) + x/(236196*(9 - 6*x + 4*x^2)) - (3 + x)/(708588*(9 +
 6*x + 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(157464*Sqrt[3]) + ArcTan[(
3 + 4*x)/(3*Sqrt[3])]/(1417176*Sqrt[3]) - Log[3 - 2*x]/472392 + Log[3 + 2*
x]/4251528 - Log[9 - 6*x + 4*x^2]/8503056 + Log[9 + 6*x + 4*x^2]/944784
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{x}{139968\left(x^5-\frac{3}{2}x^4+\frac{9}{4}x^3-\frac{27}{8}x^2+\frac{81}{16}x-\frac{243}{32}\right)}-\frac{\ln(16x^2-24x+36)}{8503056}+\frac{\sqrt{3}\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{472392}-\frac{\ln(2x-3)}{472392}+\frac{\ln(2x+3)}{4251528}+\ln$
default	$\frac{x}{944784x^2-1417176x+2125764}-\frac{\ln(4x^2-6x+9)}{8503056}+\frac{\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}-\frac{1}{708588(2x-3)}-\frac{\ln(2x-3)}{472392}+\frac{-}{708588x^2+10}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6}{3}\right.}{}\right.}{6(x^6)^{\frac{1}{6}}}\right.}{708588}$

input
```
int((2*x+3)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/139968*x/(x^5-3/2*x^4+9/4*x^3-27/8*x^2+81/16*x-243/32)-1/8503056*ln(16*
x^2-24*x+36)+1/472392*3^(1/2)*arctan(1/9*(4*x-3)*3^(1/2))-1/472392*ln(2*x-
3)+1/4251528*ln(2*x+3)+1/944784*ln(4*x^2+6*x+9)+1/4251528*3^(1/2)*arctan(2
/9*(2*x+3/2)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(116) = 232$.

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.76

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx$$
$$= \frac{2\sqrt{3}(32x^5-48x^4+72x^3-108x^2+162x-243)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)+18\sqrt{3}(32x^5-48x^4+72}{}$$

input
```
integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/8503056*(2*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 162*x - 243)*ar
ctan(1/9*sqrt(3)*(4*x + 3)) + 18*sqrt(3)*(32*x^5 - 48*x^4 + 72*x^3 - 108*x
^2 + 162*x - 243)*arctan(1/9*sqrt(3)*(4*x - 3)) + 9*(32*x^5 - 48*x^4 + 72*
x^3 - 108*x^2 + 162*x - 243)*log(4*x^2 + 6*x + 9) - (32*x^5 - 48*x^4 + 72*
x^3 - 108*x^2 + 162*x - 243)*log(4*x^2 - 6*x + 9) + 2*(32*x^5 - 48*x^4 + 7
2*x^3 - 108*x^2 + 162*x - 243)*log(2*x + 3) - 18*(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)*log(2*x - 3) - 1944*x)/(32*x^5 - 48*x^4 + 72*x^3
 - 108*x^2 + 162*x - 243)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.85

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = -\frac{x}{139968x^5-209952x^4+314928x^3-472392x^2+708588x-1062882} - \frac{\log\left(x-\frac{3}{2}\right)}{472392} + \frac{\log\left(x+\frac{3}{2}\right)}{4251528} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{8503056} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{4251528}$$

input
```
integrate((3+2*x)/(-64*x**6+729)**2,x)
```

output
```
-x/(139968*x**5 - 209952*x**4 + 314928*x**3 - 472392*x**2 + 708588*x - 106
2882) - log(x - 3/2)/472392 + log(x + 3/2)/4251528 - log(x**2 - 3*x/2 + 9/
4)/8503056 + log(x**2 + 3*x/2 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 -
 sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/4251528
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{3+2x}{(729-64x^6)^2} \, dx = \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(32\,x^5-48\,x^4+72\,x^3-108\,x^2+162\,x-243)} + \frac{1}{944784}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{8503056}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{4251528}\log\left(2\,x+3\right) - \frac{1}{472392}\log\left(2\,x-3\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(32*x^5 - 48*x^4 + 72*x^3 - 108*x^2 + 16
2*x - 243) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9
) + 1/4251528*log(2*x + 3) - 1/472392*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int \frac{3+2x}{(729-64x^6)^2} \, dx = \frac{1}{4251528}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(4\,x^2+6\,x+9)(4\,x^2-6\,x+9)(2\,x-3)} + \frac{1}{944784}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{8503056}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{4251528}\log\left(|2\,x+3|\right) - \frac{1}{472392}\log\left(|2\,x-3|\right)$$

input
```
integrate((3+2*x)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/4251528*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/472392*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(4*x^2 - 6*x + 9)*(2*
x - 3)) + 1/944784*log(4*x^2 + 6*x + 9) - 1/8503056*log(4*x^2 - 6*x + 9) +
 1/4251528*log(abs(2*x + 3)) - 1/472392*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{4251528} - \frac{\ln\left(x-\frac{3}{2}\right)}{472392} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{8503056}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{8503056}\right) - \frac{x}{139968\left(x^5-\frac{3x^4}{2}+\frac{9x^3}{4}-\frac{27x^2}{8}+\frac{81x}{16}-\frac{243}{32}\right)}$$

input
```
int((2*x + 3)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/4251528 - log(x - 3/2)/472392 - log(x - (3^(1/2)*3i)/4 - 3/4)
*((3^(1/2)*1i)/944784 + 1/8503056) - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/
2)*1i)/8503056 - 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/9
44784 - 1/8503056) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/8503056 +
 1/944784) - x/(139968*((81*x)/16 - (27*x^2)/8 + (9*x^3)/4 - (3*x^4)/2 + x
^5 - 243/32))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.79

$$\int \frac{3+2x}{(729-64x^6)^2}\,dx = \text{Too large to display}$$

input
```
int((3+2*x)/(-64*x^6+729)^2,x)
```

output
```
(576*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**5 - 864*sqrt(3)*atan((4*x - 3)
/(3*sqrt(3)))*x**4 + 1296*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**3 - 1944*
sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**2 + 2916*sqrt(3)*atan((4*x - 3)/(3*
sqrt(3)))*x - 4374*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) + 64*sqrt(3)*atan((
4*x + 3)/(3*sqrt(3)))*x**5 - 96*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**4 +
 144*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**3 - 216*sqrt(3)*atan((4*x + 3)
/(3*sqrt(3)))*x**2 + 324*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x - 486*sqrt(
3)*atan((4*x + 3)/(3*sqrt(3))) - 32*log(4*x**2 - 6*x + 9)*x**5 + 48*log(4*
x**2 - 6*x + 9)*x**4 - 72*log(4*x**2 - 6*x + 9)*x**3 + 108*log(4*x**2 - 6*
x + 9)*x**2 - 162*log(4*x**2 - 6*x + 9)*x + 243*log(4*x**2 - 6*x + 9) + 28
8*log(4*x**2 + 6*x + 9)*x**5 - 432*log(4*x**2 + 6*x + 9)*x**4 + 648*log(4*
x**2 + 6*x + 9)*x**3 - 972*log(4*x**2 + 6*x + 9)*x**2 + 1458*log(4*x**2 +
6*x + 9)*x - 2187*log(4*x**2 + 6*x + 9) - 576*log(2*x - 3)*x**5 + 864*log(
2*x - 3)*x**4 - 1296*log(2*x - 3)*x**3 + 1944*log(2*x - 3)*x**2 - 2916*log
(2*x - 3)*x + 4374*log(2*x - 3) + 64*log(2*x + 3)*x**5 - 96*log(2*x + 3)*x
**4 + 144*log(2*x + 3)*x**3 - 216*log(2*x + 3)*x**2 + 324*log(2*x + 3)*x -
 486*log(2*x + 3) - 1944*x)/(8503056*(32*x**5 - 48*x**4 + 72*x**3 - 108*x*
*2 + 162*x - 243))
```

3.46 $\int \frac{9-6x+4x^2}{(729-64x^6)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2} \, dx = \frac{1}{472392(3-2x)} - \frac{1}{157464(3+2x)} + \frac{3+4x}{236196(9+6x+4x^2)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{354294} + \frac{\log(3+2x)}{118098} - \frac{\log(9-6x+4x^2)}{944784} - \frac{5\log(9+6x+4x^2)}{2834352}$$

output
```
1/(1417176-944784*x)-1/(472392+314928*x)+(3+4*x)/(944784*x^2+1417176*x+212
5764)-1/1417176*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/157464*arctan(1/9*(3
+4*x)*3^(1/2))*3^(1/2)-1/354294*ln(3-2*x)+1/118098*ln(3+2*x)-1/944784*ln(4
*x^2-6*x+9)-5/2834352*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{648x}{81+54x-24x^3-16x^4}+2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)+18\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-8\log(3-2x)+24\log(3+2x)-3\log}{2834352}$$

input
```
Integrate[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
((648*x)/(81 + 54*x - 24*x^3 - 16*x^4) + 2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] + 18*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 8*Log[3 - 2*x] + 24*L
og[3 + 2*x] - 3*Log[9 - 6*x + 4*x^2] - 5*Log[9 + 6*x + 4*x^2])/2834352
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2-6x+9}{(729-64x^6)^2}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{(4x^2-6x+9)(-16x^4-24x^3+54x+81)^2}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{21-10x}{708588(4x^2+6x+9)}+\frac{3-2x}{236196(4x^2-6x+9)}+\frac{1}{4374(4x^2+6x+9)^2}-\frac{1}{177147(2x-3)}+\frac{1}{59049(2x+3}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{472392\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{4x+3}{236196\,(4x^2+6x+9)}-\frac{\log\left(4x^2-6x+9\right)}{944784}-\frac{5\log\left(4x^2+6x+9\right)}{2834352}+\frac{1}{472392(3-2x)}-\frac{1}{157464(2x+3)}-\frac{\log(3-2x)}{354294}+\frac{\log(2x+3)}{118098}$$

input
```
Int[(9 - 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
1/(472392*(3 - 2*x)) - 1/(157464*(3 + 2*x)) + (3 + 4*x)/(236196*(9 + 6*x +
 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(472392*Sqrt[3]) + ArcTan[(3 + 4*
x)/(3*Sqrt[3])]/(52488*Sqrt[3]) - Log[3 - 2*x]/354294 + Log[3 + 2*x]/11809
8 - Log[9 - 6*x + 4*x^2]/944784 - (5*Log[9 + 6*x + 4*x^2])/2834352
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{x}{69984\left(x^{4}+\frac{3}{2}x^{3}-\frac{27}{8}x-\frac{81}{16}\right)}-\frac{\ln(2x-3)}{354294}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x-\frac{3}{2}\right)\sqrt{3}}{9}\right)}{1417176}-\frac{\ln(4x^{2}-6x+9)}{944784}+\frac{\ln(2x+3)}{118098}-\frac{5\ln(36x^{2}+54x+}{2834352}$
default	$-\frac{\ln(4x^{2}-6x+9)}{944784}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{1417176}-\frac{1}{472392(2x-3)}-\frac{\ln(2x-3)}{354294}-\frac{-3x-\frac{9}{4}}{708588\left(x^{2}+\frac{3}{2}x+\frac{9}{4}\right)}-\frac{5\ln(4x^{2}+6x+9)}{2834352}+$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^{6}}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^{6})^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^{6})^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^{6})^{\frac{1}{6}}}{3}+\frac{4(x^{6})^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(x^{6})^{\frac{1}{6}}}{3-(x^{6})^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^{6}}{3}\right.}{}\right.}{6(x^{6})^{\frac{1}{6}}}\right.}{236196}$

input
```
int((4*x^2-6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/69984*x/(x^4+3/2*x^3-27/8*x-81/16)-1/354294*ln(2*x-3)+1/1417176*3^(1/2)
*arctan(2/9*(2*x-3/2)*3^(1/2))-1/944784*ln(4*x^2-6*x+9)+1/118098*ln(2*x+3)
-5/2834352*ln(36*x^2+54*x+81)+1/157464*3^(1/2)*arctan(2/27*(6*x+9/2)*3^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx$$
$$=\frac{18\sqrt{3}(16\,x^4+24\,x^3-54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+2\sqrt{3}(16\,x^4+24\,x^3-54\,x-81)\arctan\left(\frac{1}{9}\sqrt{\right.}{}$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/2834352*(18*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*
x + 3)) + 2*sqrt(3)*(16*x^4 + 24*x^3 - 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
- 3)) - 5*(16*x^4 + 24*x^3 - 54*x - 81)*log(4*x^2 + 6*x + 9) - 3*(16*x^4 +
 24*x^3 - 54*x - 81)*log(4*x^2 - 6*x + 9) + 24*(16*x^4 + 24*x^3 - 54*x - 8
1)*log(2*x + 3) - 8*(16*x^4 + 24*x^3 - 54*x - 81)*log(2*x - 3) - 648*x)/(1
6*x^4 + 24*x^3 - 54*x - 81)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = -\frac{x}{69984x^4+104976x^3-236196x-354294} - \frac{\log\left(x-\frac{3}{2}\right)}{354294} + \frac{\log\left(x+\frac{3}{2}\right)}{118098} - \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} - \frac{5\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{2834352} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{1417176} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

input
```
integrate((4*x**2-6*x+9)/(-64*x**6+729)**2,x)
```

output
```
-x/(69984*x**4 + 104976*x**3 - 236196*x - 354294) - log(x - 3/2)/354294 +
log(x + 3/2)/118098 - log(x**2 - 3*x/2 + 9/4)/944784 - 5*log(x**2 + 3*x/2
+ 9/4)/2834352 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/1417176 + sqrt(3)
*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/157464
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(16\,x^4+24\,x^3-54\,x-81)} - \frac{5}{2834352}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{118098}\log\left(2\,x+3\right) - \frac{1}{354294}\log\left(2\,x-3\right)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 + 24*x^3 - 54*x - 81) - 5/283435
2*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/118098*log(2*x
+ 3) - 1/354294*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{1}{1417176}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(4\,x^2+6\,x+9)(2\,x+3)(2\,x-3)} - \frac{5}{2834352}\log\left(4\,x^2+6\,x+9\right) - \frac{1}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{1}{118098}\log\left(|2\,x+3|\right) - \frac{1}{354294}\log\left(|2\,x-3|\right)$$

input
```
integrate((4*x^2-6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/1417176*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 + 6*x + 9)*(2*x + 3)*(2*x - 3))
- 5/2834352*log(4*x^2 + 6*x + 9) - 1/944784*log(4*x^2 - 6*x + 9) + 1/11809
8*log(abs(2*x + 3)) - 1/354294*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx &= \frac{\ln\left(x+\frac{3}{2}\right)}{118098}-\frac{\ln\left(x-\frac{3}{2}\right)}{354294}\\ &\quad-\ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)\\ &\quad+\ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right)\\ &\quad-\ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)\\ &\quad+\ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)\\ &\quad+\frac{x}{69984\left(-x^4-\frac{3x^3}{2}+\frac{27x}{8}+\frac{81}{16}\right)}\end{aligned}$$

input
```
int((4*x^2 - 6*x + 9)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/118098 - log(x - 3/2)/354294 - log(x - (3^(1/2)*3i)/4 + 3/4)*
((3^(1/2)*1i)/314928 + 5/2834352) + log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2
)*1i)/314928 - 5/2834352) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/28
34352 + 1/944784) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/2834352 -
1/944784) + x/(69984*((27*x)/8 - (3*x^3)/2 - x^4 + 81/16))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.54

$$\int \frac{9-6x+4x^2}{(729-64x^6)^2}\,dx$$
$$= \frac{-648x+243\log(4x^2-6x+9)-1458\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)-1944\log(2x+3)+648\log(2x-3)+32\sqrt{3}\,at}{}$$

input
```
int((4*x^2-6*x+9)/(-64*x^6+729)^2,x)
```

output
```
(32*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**4 + 48*sqrt(3)*atan((4*x - 3)/(
3*sqrt(3)))*x**3 - 108*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x - 162*sqrt(3)
*atan((4*x - 3)/(3*sqrt(3))) + 288*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**
4 + 432*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**3 - 972*sqrt(3)*atan((4*x +
 3)/(3*sqrt(3)))*x - 1458*sqrt(3)*atan((4*x + 3)/(3*sqrt(3))) - 48*log(4*x
**2 - 6*x + 9)*x**4 - 72*log(4*x**2 - 6*x + 9)*x**3 + 162*log(4*x**2 - 6*x
 + 9)*x + 243*log(4*x**2 - 6*x + 9) - 80*log(4*x**2 + 6*x + 9)*x**4 - 120*
log(4*x**2 + 6*x + 9)*x**3 + 270*log(4*x**2 + 6*x + 9)*x + 405*log(4*x**2
+ 6*x + 9) - 128*log(2*x - 3)*x**4 - 192*log(2*x - 3)*x**3 + 432*log(2*x -
 3)*x + 648*log(2*x - 3) + 384*log(2*x + 3)*x**4 + 576*log(2*x + 3)*x**3 -
 1296*log(2*x + 3)*x - 1944*log(2*x + 3) - 648*x)/(2834352*(16*x**4 + 24*x
**3 - 54*x - 81))
```

3.47 $\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{157464(3-2x)} - \frac{1}{472392(3+2x)} - \frac{3-4x}{236196\,(9-6x+4x^2)} - \frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{472392\sqrt{3}} - \frac{\log(3-2x)}{118098} + \frac{\log(3+2x)}{354294} + \frac{5\log\left(9-6x+4x^2\right)}{2834352} + \frac{\log\left(9+6x+4x^2\right)}{944784}$$

output
```
1/(472392-314928*x)-1/(1417176+944784*x)-(3-4*x)/(944784*x^2-1417176*x+212
5764)-1/157464*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/1417176*arctan(1/9*(3
+4*x)*3^(1/2))*3^(1/2)-1/118098*ln(3-2*x)+1/354294*ln(3+2*x)+5/2834352*ln(
4*x^2-6*x+9)+1/944784*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{648x}{81-54x+24x^3-16x^4} + 18\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) + 2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 24\log(3-2x) + 8\log(3+2x) + 5\log}{2834352}$$

input
```
Integrate[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
((648*x)/(81 - 54*x + 24*x^3 - 16*x^4) + 18*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*S
qrt[3])] + 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 24*Log[3 - 2*x] + 8*L
og[3 + 2*x] + 5*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/2834352
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+6x+9}{(729-64x^6)^2}\,dx$$
$$\Big\downarrow \; 2019$$
$$\int \frac{1}{(4x^2+6x+9)\left(-16x^4+24x^3-54x+81\right)^2}dx$$
$$\Big\downarrow \; 2462$$
$$\int \left(\frac{2x+3}{236196\,(4x^2+6x+9)} + \frac{10x+21}{708588\,(4x^2-6x+9)} + \frac{1}{4374\,(4x^2-6x+9)^2} - \frac{1}{59049(2x-3)} + \frac{1}{78732(2x-3)}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{52488\sqrt{3}}+\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{472392\sqrt{3}}-\frac{3-4x}{236196\left(4x^2-6x+9\right)}+\frac{5\log\left(4x^2-6x+9\right)}{2834352}+\frac{\log\left(4x^2+6x+9\right)}{944784}+\frac{1}{157464(3-2x)}-\frac{1}{472392(2x+3)}-\frac{\log(3-2x)}{118098}+\frac{\log(2x+3)}{354294}$$

input
```
Int[(9 + 6*x + 4*x^2)/(729 - 64*x^6)^2,x]
```

output
```
1/(157464*(3 - 2*x)) - 1/(472392*(3 + 2*x)) - (3 - 4*x)/(236196*(9 - 6*x +
 4*x^2)) - ArcTan[(3 - 4*x)/(3*Sqrt[3])]/(52488*Sqrt[3]) + ArcTan[(3 + 4*x
)/(3*Sqrt[3])]/(472392*Sqrt[3]) - Log[3 - 2*x]/118098 + Log[3 + 2*x]/35429
4 + (5*Log[9 - 6*x + 4*x^2])/2834352 + Log[9 + 6*x + 4*x^2]/944784
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{x}{69984\left(x^4-\frac{3}{2}x^3+\frac{27}{8}x-\frac{81}{16}\right)}+\frac{5\ln\left(16x^2-24x+36\right)}{2834352}+\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{157464}+\frac{\arctan\left(\frac{(3+4x)\sqrt{3}}{9}\right)\sqrt{3}}{1417176}+\frac{\ln\left(16x^2+24x+3\right.}{944784}$
default	$\frac{3x-\frac{9}{4}}{708588x^2-1062882x+1594323}+\frac{5\ln\left(4x^2-6x+9\right)}{2834352}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{157464}-\frac{1}{157464(2x-3)}-\frac{\ln(2x-3)}{118098}+\frac{\ln\left(4x^2+6x+\right.}{944784}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right.}{3}\right.}{}\right.}{6\left(x^6\right)^{\frac{1}{6}}}\right.}{236196}$

input
```
int((4*x^2+6*x+9)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/69984*x/(x^4-3/2*x^3+27/8*x-81/16)+5/2834352*ln(16*x^2-24*x+36)+1/15746
4*3^(1/2)*arctan(1/9*(4*x-3)*3^(1/2))+1/1417176*arctan(1/9*(3+4*x)*3^(1/2)
)*3^(1/2)+1/944784*ln(16*x^2+24*x+36)+1/354294*ln(2*x+3)-1/118098*ln(2*x-3
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{9+6x+4x^2}{\left(729-64x^6\right)^2}\,dx$$
$$=\frac{2\sqrt{3}(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+18\sqrt{3}(16\,x^4-24\,x^3+54\,x-81)\arctan\left(\frac{1}{9}\sqrt{\right.}{}$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/2834352*(2*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
 + 3)) + 18*sqrt(3)*(16*x^4 - 24*x^3 + 54*x - 81)*arctan(1/9*sqrt(3)*(4*x
- 3)) + 3*(16*x^4 - 24*x^3 + 54*x - 81)*log(4*x^2 + 6*x + 9) + 5*(16*x^4 -
 24*x^3 + 54*x - 81)*log(4*x^2 - 6*x + 9) + 8*(16*x^4 - 24*x^3 + 54*x - 81
)*log(2*x + 3) - 24*(16*x^4 - 24*x^3 + 54*x - 81)*log(2*x - 3) - 648*x)/(1
6*x^4 - 24*x^3 + 54*x - 81)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = -\frac{x}{69984x^4-104976x^3+236196x-354294} - \frac{\log\left(x-\frac{3}{2}\right)}{118098} + \frac{\log\left(x+\frac{3}{2}\right)}{354294} + \frac{5\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{2834352} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{157464} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{1417176}$$

input
```
integrate((4*x**2+6*x+9)/(-64*x**6+729)**2,x)
```

output
```
-x/(69984*x**4 - 104976*x**3 + 236196*x - 354294) - log(x - 3/2)/118098 +
log(x + 3/2)/354294 + 5*log(x**2 - 3*x/2 + 9/4)/2834352 + log(x**2 + 3*x/2
 + 9/4)/944784 + sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/157464 + sqrt(3)*
atan(4*sqrt(3)*x/9 + sqrt(3)/3)/1417176
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(16\,x^4-24\,x^3+54\,x-81)} + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) + \frac{5}{2834352}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{354294}\,\log\left(2\,x+3\right) - \frac{1}{118098}\,\log\left(2\,x-3\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(16*x^4 - 24*x^3 + 54*x - 81) + 1/944784
*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/354294*log(2*x
+ 3) - 1/118098*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{1}{1417176}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) - \frac{x}{4374\,(4\,x^2-6\,x+9)(2\,x+3)(2\,x-3)} + \frac{1}{944784}\,\log\left(4\,x^2+6\,x+9\right) + \frac{5}{2834352}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{354294}\,\log\left(|2\,x+3|\right) - \frac{1}{118098}\,\log\left(|2\,x-3|\right)$$

input
```
integrate((4*x^2+6*x+9)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/1417176*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 1/157464*sqrt(3)*arctan(
1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x + 3)*(2*x - 3))
+ 1/944784*log(4*x^2 + 6*x + 9) + 5/2834352*log(4*x^2 - 6*x + 9) + 1/35429
4*log(abs(2*x + 3)) - 1/118098*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{354294} - \frac{\ln\left(x-\frac{3}{2}\right)}{118098} - \frac{x}{69984\,\left(x^4-\frac{3\,x^3}{2}+\frac{27\,x}{8}-\frac{81}{16}\right)} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{5}{2834352}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{944784}+\frac{\sqrt{3}\,1\mathrm{i}}{2834352}\right)$$

input
```
int((6*x + 4*x^2 + 9)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/354294 - log(x - 3/2)/118098 - x/(69984*((27*x)/8 - (3*x^3)/2
 + x^4 - 81/16)) - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 - 5/
2834352) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/314928 + 5/2834352)
 - log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 - 1/944784) + log(x
 + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/2834352 + 1/944784)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.54

$$\int \frac{9+6x+4x^2}{(729-64x^6)^2}\,dx$$
$$= \frac{-648x-405\log(4x^2-6x+9)-162\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)-648\log(2x+3)+1944\log(2x-3)+288\sqrt{3}\,at}{}$$

input
```
int((4*x^2+6*x+9)/(-64*x^6+729)^2,x)
```

output
```
(288*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**4 - 432*sqrt(3)*atan((4*x - 3)
/(3*sqrt(3)))*x**3 + 972*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x - 1458*sqrt
(3)*atan((4*x - 3)/(3*sqrt(3))) + 32*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x
**4 - 48*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**3 + 108*sqrt(3)*atan((4*x
+ 3)/(3*sqrt(3)))*x - 162*sqrt(3)*atan((4*x + 3)/(3*sqrt(3))) + 80*log(4*x
**2 - 6*x + 9)*x**4 - 120*log(4*x**2 - 6*x + 9)*x**3 + 270*log(4*x**2 - 6*
x + 9)*x - 405*log(4*x**2 - 6*x + 9) + 48*log(4*x**2 + 6*x + 9)*x**4 - 72*
log(4*x**2 + 6*x + 9)*x**3 + 162*log(4*x**2 + 6*x + 9)*x - 243*log(4*x**2
+ 6*x + 9) - 384*log(2*x - 3)*x**4 + 576*log(2*x - 3)*x**3 - 1296*log(2*x
- 3)*x + 1944*log(2*x - 3) + 128*log(2*x + 3)*x**4 - 192*log(2*x + 3)*x**3
 + 432*log(2*x + 3)*x - 648*log(2*x + 3) - 648*x)/(2834352*(16*x**4 - 24*x
**3 + 54*x - 81))
```

3.48 $\int \frac{27-8x^3}{(729-64x^6)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{x}{4374(27+8x^3)} - \frac{7\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} + \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{52488\sqrt{3}} - \frac{\log(3-2x)}{157464} + \frac{7\log(3+2x)}{472392} - \frac{7\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{314928}$$

output
```
x/(34992*x^3+118098)-7/472392*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)+1/157464
*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)-1/157464*ln(3-2*x)+7/472392*ln(3+2*x)
-7/944784*ln(4*x^2-6*x+9)+1/314928*ln(4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{\frac{216x}{27+8x^3} + 14\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) + 6\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right) - 6\log(3-2x) + 14\log(3+2x) - 7\log(9-6x+}{944784}$$

input
```
Integrate[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

output
```
((216*x)/(27 + 8*x^3) + 14*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] + 6*Sqrt
[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 6*Log[3 - 2*x] + 14*Log[3 + 2*x] - 7*L
og[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {1388, 931, 27, 1020, 750, 16, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{27-8x^3}{(729-64x^6)^2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{1}{(27-8x^3)(8x^3+27)^2}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{x}{4374(8x^3+27)}-\frac{\int -\frac{8(135-16x^3)}{(27-8x^3)(8x^3+27)}dx}{34992}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{135-16x^3}{(27-8x^3)(8x^3+27)}dx}{4374}+\frac{x}{4374(8x^3+27)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{\frac{3}{2}\int \frac{1}{27-8x^3}dx+\frac{7}{2}\int \frac{1}{8x^3+27}dx}{4374}+\frac{x}{4374(8x^3+27)}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{7}{2}\left(\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{27}\int \frac{1}{2x+3}dx\right)+\frac{3}{2}\left(\frac{1}{27}\int \frac{2(x+3)}{4x^2+6x+9}dx+\frac{1}{27}\int \frac{1}{3-2x}dx\right)}{4374}+\frac{x}{4374(8x^3+27)}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{7}{2}\left(\frac{1}{27}\int \frac{2(3-x)}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{1}{27}\int \frac{2(x+3)}{4x^2+6x+9}dx-\frac{1}{54}\log(3-2x)\right)}{4374}+\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\int \frac{3-x}{4x^2-6x+9}dx+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\int \frac{x+3}{4x^2+6x+9}dx-\frac{1}{54}\log(3-2x)\right)}{4374}+\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx-\frac{1}{8}\int -\frac{2(3-4x)}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2+6x+9}dx+\frac{1}{8}\int \frac{2(4x+3)}{4x^2+6x+9}dx\right)-\frac{1}{54}\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2-6x+9}dx+\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{9}{4}\int \frac{1}{4x^2+6x+9}dx+\frac{1}{4}\int \frac{4x+3}{4x^2+6x+9}dx\right)-\frac{1}{54}\text{lo}\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx-\frac{9}{2}\int \frac{1}{-(8x-6)^2-108}d(8x-6)\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{4x+3}{4x^2+6x+9}dx-\frac{9}{2}\int \frac{1}{-(8x+6)^2-}\right.\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{3-4x}{4x^2-6x+9}dx+\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{1}{4}\int \frac{4x+3}{4x^2+6x+9}dx+\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)\right.\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{7}{2}\left(\frac{2}{27}\left(\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x-6}{6\sqrt{3}}\right)-\frac{1}{8}\log\left(4x^2-6x+9\right)\right)+\frac{1}{54}\log(2x+3)\right)+\frac{3}{2}\left(\frac{2}{27}\left(\frac{1}{4}\sqrt{3}\arctan\left(\frac{8x+6}{6\sqrt{3}}\right)+\frac{1}{8}\log\left(4x^2\right.\right.\right.}{4374}$$
$$\frac{x}{4374\left(8x^3+27\right)}$$

input
```
Int[(27 - 8*x^3)/(729 - 64*x^6)^2,x]
```

output
```
x/(4374*(27 + 8*x^3)) + ((7*(Log[3 + 2*x]/54 + (2*((Sqrt[3]*ArcTan[(-6 + 8
*x)/(6*Sqrt[3])])/4 - Log[9 - 6*x + 4*x^2]/8))/27))/2 + (3*(-1/54*Log[3 -
2*x] + (2*((Sqrt[3]*ArcTan[(6 + 8*x)/(6*Sqrt[3])])/4 + Log[9 + 6*x + 4*x^2
]/8))/27))/2)/4374
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

method	result
risch	$\frac{x}{34992x^3+118098}-\frac{\ln(2x-3)}{157464}+\frac{7\ln(2x+3)}{472392}+\frac{\ln(4x^2+6x+9)}{314928}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(2x+\frac{3}{2}\right)\sqrt{3}}{9}\right)}{157464}-\frac{7\ln(4x^2-6x+9)}{944784}+\frac{7\sqrt{3}\,\mathrm{a}}{}$
default	$-\frac{-\frac{3x}{4}-\frac{9}{8}}{118098\left(x^2-\frac{3}{2}x+\frac{9}{4}\right)}-\frac{7\ln(4x^2-6x+9)}{944784}+\frac{7\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}-\frac{\ln(2x-3)}{157464}+\frac{\ln(4x^2+6x+9)}{314928}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x+6)}{18}\right.}{157464}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2\left(x^6\right)^{\frac{1}{6}}}{3}+\frac{4\left(x^6\right)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}{3-\left(x^6\right)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2\left(x^6\right.}{3}\right.}{}\right.}{6\left(x^6\right)^{\frac{1}{6}}}\right.}{78732}$

input
```
int((-8*x^3+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
1/34992*x/(x^3+27/8)-1/157464*ln(2*x-3)+7/472392*ln(2*x+3)+1/314928*ln(4*x
^2+6*x+9)+1/157464*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))-7/944784*ln(4*x^2
-6*x+9)+7/472392*3^(1/2)*arctan(2/9*(2*x-3/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{27-8x^3}{\left(729-64x^6\right)^2}\,dx$$
$$=\frac{6\sqrt{3}(8\,x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right)+14\sqrt{3}(8\,x^3+27)\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right)+3\,(8\,x^3+27)\log\left(4\right.}{944}$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
1/944784*(6*sqrt(3)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x + 3)) + 14*sqrt(3
)*(8*x^3 + 27)*arctan(1/9*sqrt(3)*(4*x - 3)) + 3*(8*x^3 + 27)*log(4*x^2 +
6*x + 9) - 7*(8*x^3 + 27)*log(4*x^2 - 6*x + 9) + 14*(8*x^3 + 27)*log(2*x +
 3) - 6*(8*x^3 + 27)*log(2*x - 3) + 216*x)/(8*x^3 + 27)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{x}{34992x^3+118098} - \frac{\log\left(x-\frac{3}{2}\right)}{157464} + \frac{7\log\left(x+\frac{3}{2}\right)}{472392} - \frac{7\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928} + \frac{7\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{157464}$$

input
```
integrate((-8*x**3+27)/(-64*x**6+729)**2,x)
```

output
```
x/(34992*x**3 + 118098) - log(x - 3/2)/157464 + 7*log(x + 3/2)/472392 - 7*
log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x/2 + 9/4)/314928 + 7*sqrt(3
)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 + sqrt(3)*atan(4*sqrt(3)*x/9 + sq
rt(3)/3)/157464
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.77

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{1}{157464}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x+3)\right) + \frac{7}{472392}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(8\,x^3+27)} + \frac{1}{314928}\,\log\left(4\,x^2+6\,x+9\right) - \frac{7}{944784}\,\log\left(4\,x^2-6\,x+9\right) + \frac{7}{472392}\,\log\left(2\,x+3\right) - \frac{1}{157464}\,\log\left(2\,x-3\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x +
 9) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(2*x + 3) - 1/157464*log
(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{1}{157464}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x+3)\right) + \frac{7}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4\,x-3)\right) + \frac{x}{4374\,(8\,x^3+27)} + \frac{1}{314928}\log\left(4\,x^2+6\,x+9\right) - \frac{7}{944784}\log\left(4\,x^2-6\,x+9\right) + \frac{7}{472392}\log\left(|2\,x+3|\right) - \frac{1}{157464}\log\left(|2\,x-3|\right)$$

input
```
integrate((-8*x^3+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
1/157464*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 7/472392*sqrt(3)*arctan(1
/9*sqrt(3)*(4*x - 3)) + 1/4374*x/(8*x^3 + 27) + 1/314928*log(4*x^2 + 6*x +
 9) - 7/944784*log(4*x^2 - 6*x + 9) + 7/472392*log(abs(2*x + 3)) - 1/15746
4*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{7\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{\ln\left(x-\frac{3}{2}\right)}{157464} + \frac{x}{34992\left(x^3+\frac{27}{8}\right)} - \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) + \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{314928}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{7}{944784}+\frac{\sqrt{3}\,7\mathrm{i}}{944784}\right)$$

input
```
int(-(8*x^3 - 27)/(64*x^6 - 729)^2,x)
```

output
```
(7*log(x + 3/2))/472392 - log(x - 3/2)/157464 + x/(34992*(x^3 + 27/8)) - l
og(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 - 1/314928) + log(x + (3
^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/314928 + 1/314928) - log(x - (3^(1/2)*3i
)/4 - 3/4)*((3^(1/2)*7i)/944784 + 7/944784) + log(x + (3^(1/2)*3i)/4 - 3/4
)*((3^(1/2)*7i)/944784 - 7/944784)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.59

$$\int \frac{27-8x^3}{(729-64x^6)^2}\,dx = \frac{112\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)x^3+378\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)+48\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)x^3+162\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)-56\log(4x^2}{}$$

input
```
int((-8*x^3+27)/(-64*x^6+729)^2,x)
```

output
```
(112*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**3 + 378*sqrt(3)*atan((4*x - 3)
/(3*sqrt(3))) + 48*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**3 + 162*sqrt(3)*
atan((4*x + 3)/(3*sqrt(3))) - 56*log(4*x**2 - 6*x + 9)*x**3 - 189*log(4*x*
*2 - 6*x + 9) + 24*log(4*x**2 + 6*x + 9)*x**3 + 81*log(4*x**2 + 6*x + 9) -
 48*log(2*x - 3)*x**3 - 162*log(2*x - 3) + 112*log(2*x + 3)*x**3 + 378*log
(2*x + 3) + 216*x)/(944784*(8*x**3 + 27))
```

3.49 $\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2} dx = \frac{1}{26244(3-2x)} - \frac{3-2x}{26244(9-6x+4x^2)} - \frac{11\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)}{157464\sqrt{3}} - \frac{7\log(3-2x)}{157464} + \frac{\log(3+2x)}{472392} + \frac{17\log(9-6x+4x^2)}{944784} + \frac{\log(9+6x+4x^2)}{314928}$$

output
```
1/(78732-52488*x)-(3-2*x)/(104976*x^2-157464*x+236196)-11/472392*arctan(1/
9*(3-4*x)*3^(1/2))*3^(1/2)-1/472392*arctan(1/9*(3+4*x)*3^(1/2))*3^(1/2)-7/
157464*ln(3-2*x)+1/472392*ln(3+2*x)+17/944784*ln(4*x^2-6*x+9)+1/314928*ln(
4*x^2+6*x+9)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx$$
$$= \frac{\frac{216x}{27-36x+24x^2-8x^3}+22\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{3+4x}{3\sqrt{3}}\right)-42\log(3-2x)+2\log(3+2x)+17\log}{944784}$$

input
```
Integrate[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

output
```
((216*x)/(27 - 36*x + 24*x^2 - 8*x^3) + 22*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sq
rt[3])] - 2*Sqrt[3]*ArcTan[(3 + 4*x)/(3*Sqrt[3])] - 42*Log[3 - 2*x] + 2*Lo
g[3 + 2*x] + 17*Log[9 - 6*x + 4*x^2] + 3*Log[9 + 6*x + 4*x^2])/944784
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2019, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{8x^3+24x^2+36x+27}{(729-64x^6)^2}\,dx$$
$$\Big\downarrow \text{2019}$$
$$\int \frac{1}{(-8x^3+24x^2-36x+27)^2(8x^3+24x^2+36x+27)}dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{x}{39366(4x^2+6x+9)}+\frac{17x+3}{118098(4x^2-6x+9)}+\frac{2x+3}{4374(4x^2-6x+9)^2}-\frac{7}{78732(2x-3)}+\frac{1}{236196(2x+3)}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{11\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{\arctan\left(\frac{4x+3}{3\sqrt{3}}\right)}{157464\sqrt{3}}-\frac{3-2x}{26244\left(4x^2-6x+9\right)}+\frac{17\log\left(4x^2-6x+9\right)}{944784}+\frac{\log\left(4x^2+6x+9\right)}{314928}+\frac{1}{26244(3-2x)}-\frac{7\log(3-2x)}{157464}+\frac{\log(2x+3)}{472392}$$

input
```
Int[(27 + 36*x + 24*x^2 + 8*x^3)/(729 - 64*x^6)^2,x]
```

output
```
1/(26244*(3 - 2*x)) - (3 - 2*x)/(26244*(9 - 6*x + 4*x^2)) - (11*ArcTan[(3
- 4*x)/(3*Sqrt[3])])/(157464*Sqrt[3]) - ArcTan[(3 + 4*x)/(3*Sqrt[3])]/(157
464*Sqrt[3]) - (7*Log[3 - 2*x])/157464 + Log[3 + 2*x]/472392 + (17*Log[9 -
 6*x + 4*x^2])/944784 + Log[9 + 6*x + 4*x^2]/314928
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{x}{34992\left(x^3-3x^2+\frac{9}{2}x-\frac{27}{8}\right)}-\frac{7\ln(2x-3)}{157464}+\frac{17\ln(484x^2-726x+1089)}{944784}+\frac{11\sqrt{3}\arctan\left(\frac{2\left(22x-\frac{33}{2}\right)\sqrt{3}}{99}\right)}{472392}+\frac{\ln(2x+3)}{472392}-\cdots$
default	$\frac{\frac{9x}{4}-\frac{27}{8}}{118098x^2-177147x+\frac{531441}{2}}+\frac{17\ln(4x^2-6x+9)}{944784}+\frac{11\sqrt{3}\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{472392}-\frac{1}{26244(2x-3)}-\frac{7\ln(2x-3)}{157464}+\frac{\ln(4x^2+6x\cdots}{314928\cdots}$
meijerg	$-\frac{(-1)^{\frac{5}{6}}\left(\frac{4x(-1)^{\frac{1}{6}}}{6-\frac{128x^6}{243}}-\frac{5x(-1)^{\frac{1}{6}}\left(\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}\right)-\ln\left(1+\frac{2(x^6)^{\frac{1}{6}}}{3}\right)+\frac{\ln\left(1-\frac{2(x^6)^{\frac{1}{6}}}{3}+\frac{4(x^6)^{\frac{1}{3}}}{9}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(x^6)^{\frac{1}{6}}}{3-(x^6)^{\frac{1}{6}}}\right)-\frac{\ln\left(1+\frac{2(x^6\cdots}{3}\right.\cdots}{\cdots}\cdots\right)}{6(x^6)^{\frac{1}{6}}}\cdots\right)}{78732}$

input
```
int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/34992*x/(x^3-3*x^2+9/2*x-27/8)-7/157464*ln(2*x-3)+17/944784*ln(484*x^2-
726*x+1089)+11/472392*3^(1/2)*arctan(2/99*(22*x-33/2)*3^(1/2))+1/472392*ln
(2*x+3)-1/472392*3^(1/2)*arctan(2/9*(2*x+3/2)*3^(1/2))+1/314928*ln(4*x^2+6
*x+9)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{2\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right)-22\sqrt{3}(8x^3-24x^2+36x-27)\arctan\left(\frac{1}{9}\sqrt{\cdots}\right.\cdots}{\cdots}$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="fricas")
```

output
```
-1/944784*(2*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x
+ 3)) - 22*sqrt(3)*(8*x^3 - 24*x^2 + 36*x - 27)*arctan(1/9*sqrt(3)*(4*x -
3)) - 3*(8*x^3 - 24*x^2 + 36*x - 27)*log(4*x^2 + 6*x + 9) - 17*(8*x^3 - 24
*x^2 + 36*x - 27)*log(4*x^2 - 6*x + 9) - 2*(8*x^3 - 24*x^2 + 36*x - 27)*lo
g(2*x + 3) + 42*(8*x^3 - 24*x^2 + 36*x - 27)*log(2*x - 3) + 216*x)/(8*x^3
- 24*x^2 + 36*x - 27)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{x}{34992x^3-104976x^2+157464x-118098} - \frac{7\log\left(x-\frac{3}{2}\right)}{157464} + \frac{\log\left(x+\frac{3}{2}\right)}{472392} + \frac{17\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{944784} + \frac{\log\left(x^2+\frac{3x}{2}+\frac{9}{4}\right)}{314928} + \frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{472392} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{4\sqrt{3}x}{9}+\frac{\sqrt{3}}{3}\right)}{472392}$$

input
```
integrate((8*x**3+24*x**2+36*x+27)/(-64*x**6+729)**2,x)
```

output
```
-x/(34992*x**3 - 104976*x**2 + 157464*x - 118098) - 7*log(x - 3/2)/157464
+ log(x + 3/2)/472392 + 17*log(x**2 - 3*x/2 + 9/4)/944784 + log(x**2 + 3*x
/2 + 9/4)/314928 + 11*sqrt(3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/472392 - sqr
t(3)*atan(4*sqrt(3)*x/9 + sqrt(3)/3)/472392
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(8x^3-24x^2+36x-27)} + \frac{1}{314928}\log\left(4x^2+6x+9\right) + \frac{17}{944784}\log\left(4x^2-6x+9\right) + \frac{1}{472392}\log\left(2x+3\right) - \frac{7}{157464}\log\left(2x-3\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="maxima")
```

output
```
-1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan
(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/(8*x^3 - 24*x^2 + 36*x - 27) + 1/314928
*log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(2*x
+ 3) - 7/157464*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = -\frac{1}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x+3)\right) + \frac{11}{472392}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) - \frac{x}{4374\,(4x^2-6x+9)(2x-3)} + \frac{1}{314928}\log\left(4x^2+6x+9\right) + \frac{17}{944784}\log\left(4x^2-6x+9\right) + \frac{1}{472392}\log\left(|2x+3|\right) - \frac{7}{157464}\log\left(|2x-3|\right)$$

input
```
integrate((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x, algorithm="giac")
```

output
```
-1/472392*sqrt(3)*arctan(1/9*sqrt(3)*(4*x + 3)) + 11/472392*sqrt(3)*arctan
(1/9*sqrt(3)*(4*x - 3)) - 1/4374*x/((4*x^2 - 6*x + 9)*(2*x - 3)) + 1/31492
8*log(4*x^2 + 6*x + 9) + 17/944784*log(4*x^2 - 6*x + 9) + 1/472392*log(abs
(2*x + 3)) - 7/157464*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{472392} - \frac{7\ \ln\left(x-\frac{3}{2}\right)}{157464} - \frac{x}{34992\ \left(x^3-3\,x^2+\frac{9\,x}{2}-\frac{27}{8}\right)} + \ln\left(x+\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x+\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{314928}+\frac{\sqrt{3}\,1\mathrm{i}}{944784}\right) - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{17}{944784}+\frac{\sqrt{3}\,11\mathrm{i}}{944784}\right)$$

input
```
int((36*x + 24*x^2 + 8*x^3 + 27)/(64*x^6 - 729)^2,x)
```

output
```
log(x + 3/2)/472392 - (7*log(x - 3/2))/157464 - x/(34992*((9*x)/2 - 3*x^2
+ x^3 - 27/8)) + log(x - (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 + 1/31
4928) - log(x + (3^(1/2)*3i)/4 + 3/4)*((3^(1/2)*1i)/944784 - 1/314928) - l
og(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 - 17/944784) + log(x +
(3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*11i)/944784 + 17/944784)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.75

$$\int \frac{27+36x+24x^2+8x^3}{(729-64x^6)^2}\,dx$$
$$= \frac{-216x-528\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)x^2+48\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)x^2-459\log(4x^2-6x+9)+54\sqrt{3}\,atan\left(\frac{4x+3}{3\sqrt{3}}\right)-}{}$$

input
```
int((8*x^3+24*x^2+36*x+27)/(-64*x^6+729)^2,x)
```

output
```
(176*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x**3 - 528*sqrt(3)*atan((4*x - 3)
/(3*sqrt(3)))*x**2 + 792*sqrt(3)*atan((4*x - 3)/(3*sqrt(3)))*x - 594*sqrt(
3)*atan((4*x - 3)/(3*sqrt(3))) - 16*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x*
*3 + 48*sqrt(3)*atan((4*x + 3)/(3*sqrt(3)))*x**2 - 72*sqrt(3)*atan((4*x +
3)/(3*sqrt(3)))*x + 54*sqrt(3)*atan((4*x + 3)/(3*sqrt(3))) + 136*log(4*x**
2 - 6*x + 9)*x**3 - 408*log(4*x**2 - 6*x + 9)*x**2 + 612*log(4*x**2 - 6*x
+ 9)*x - 459*log(4*x**2 - 6*x + 9) + 24*log(4*x**2 + 6*x + 9)*x**3 - 72*lo
g(4*x**2 + 6*x + 9)*x**2 + 108*log(4*x**2 + 6*x + 9)*x - 81*log(4*x**2 + 6
*x + 9) - 336*log(2*x - 3)*x**3 + 1008*log(2*x - 3)*x**2 - 1512*log(2*x -
3)*x + 1134*log(2*x - 3) + 16*log(2*x + 3)*x**3 - 48*log(2*x + 3)*x**2 + 7
2*log(2*x + 3)*x - 54*log(2*x + 3) - 216*x)/(944784*(8*x**3 - 24*x**2 + 36
*x - 27))
```

3.50 $\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 49

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx = -\frac{1}{3}\left(15+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+\arctan(x)+\frac{1}{3}\left(15-8\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right)$$

output
```
1/3*(15+8*3^(1/2))*arctan(-3^(1/2)+2*x)+arctan(x)+1/3*(15-8*3^(1/2))*arcta
n(3^(1/2)+2*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx = \left(-5-\frac{8}{\sqrt{3}}\right)\arctan\left(\sqrt{3}-2x\right)+\arctan(x)+\frac{1}{3}\left(15-8\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right)$$

input
```
Integrate[(6 + 8*x + 9*x^2 + 8*x^3 + 6*x^4)/(1 + x^6),x]
```

output
```
(-5 - 8/Sqrt[3])*ArcTan[Sqrt[3] - 2*x] + ArcTan[x] + ((15 - 8*Sqrt[3])*Arc
Tan[Sqrt[3] + 2*x])/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 283, normalized size of antiderivative = 5.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6x^4+8x^3+9x^2+8x+6}{x^6+1}dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{(6x^3+8)x}{x^6+1}+\frac{8x^3+6}{x^6+1}+\frac{9x^2}{x^6+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\arctan(x^3)-\frac{1}{3}\left(3+4\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+2\arctan(x)+\frac{1}{3}\left(3-4\sqrt{3}\right)\arctan\left(2x+\sqrt{3}\right)-\frac{(3-4i)(-1)^{5/6}\arctan\left(\frac{1-2(-1)^{5/6}x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{(3+4i)(-1)^{5/6}\arctan\left(\frac{2(-1)^{5/6}x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{4}{3}\log(x^2+1)-\left(\frac{1}{2}+\frac{2i}{3}\right)(-1)^{5/6}\log\left(x^2-\sqrt[6]{-1}x+\sqrt[3]{-1}\right)+\left(\frac{1}{2}-\frac{2i}{3}\right)(-1)^{5/6}\log\left(x^2+\sqrt[6]{-1}x+\sqrt[3]{-1}\right)+\frac{1}{6}\left(4-3\sqrt{3}\right)\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{6}\left(4+3\sqrt{3}\right)\log\left(x^2+\sqrt{3}x+1\right)-\left(1-\frac{4i}{3}\right)(-1)^{5/6}\log\left(\sqrt[6]{-1}-x\right)+\left(1+\frac{4i}{3}\right)(-1)^{5/6}\log\left(x+\sqrt[6]{-1}\right)$$

input
```
Int[(6 + 8*x + 9*x^2 + 8*x^3 + 6*x^4)/(1 + x^6),x]
```

output
```
-1/3*((3 + 4*Sqrt[3])*ArcTan[Sqrt[3] - 2*x]) + 2*ArcTan[x] + 3*ArcTan[x^3]
 + ((3 - 4*Sqrt[3])*ArcTan[Sqrt[3] + 2*x])/3 - ((3 - 4*I)*(-1)^(5/6)*ArcTa
n[(1 - 2*(-1)^(5/6)*x)/Sqrt[3]])/Sqrt[3] + ((3 + 4*I)*(-1)^(5/6)*ArcTan[(1
 + 2*(-1)^(5/6)*x)/Sqrt[3]])/Sqrt[3] - (1 - (4*I)/3)*(-1)^(5/6)*Log[(-1)^(
1/6) - x] + (1 + (4*I)/3)*(-1)^(5/6)*Log[(-1)^(1/6) + x] - (4*Log[1 + x^2]
)/3 - (1/2 + (2*I)/3)*(-1)^(5/6)*Log[(-1)^(1/3) - (-1)^(1/6)*x + x^2] + (1
/2 - (2*I)/3)*(-1)^(5/6)*Log[(-1)^(1/3) + (-1)^(1/6)*x + x^2] + ((4 - 3*Sq
rt[3])*Log[1 - Sqrt[3]*x + x^2])/6 + ((4 + 3*Sqrt[3])*Log[1 + Sqrt[3]*x +
x^2])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result
default	$2\left(\frac{5}{2}-\frac{4\sqrt{3}}{3}\right)\arctan\left(\sqrt{3}+2x\right)-2\left(-\frac{5}{2}-\frac{4\sqrt{3}}{3}\right)\arctan\left(-\sqrt{3}+2x\right)+\arctan\left(x\right)$
risch	$\arctan\left(x\right)+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^{4}+834\_Z^{2}+121\right)}\_R\ln\left(24\_R^{3}+33\_R^{2}+2312\_R+1760x+1529\right)\right)}{2}$
meijerg	$\frac{x^{5}\sqrt{3}\,\ln\left(1-\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}+\left(x^{6}\right)^{\frac{1}{3}}\right)}{2\left(x^{6}\right)^{\frac{5}{6}}}+\frac{x^{5}\arctan\left(\frac{\left(x^{6}\right)^{\frac{1}{6}}}{2-\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}}\right)}{\left(x^{6}\right)^{\frac{5}{6}}}+\frac{2x^{5}\arctan\left(\left(x^{6}\right)^{\frac{1}{6}}\right)}{\left(x^{6}\right)^{\frac{5}{6}}}-\frac{x^{5}\sqrt{3}\,\ln\left(1+\sqrt{3}\left(x^{6}\right)^{\frac{1}{6}}+\left(x^{6}\right)^{\frac{1}{3}}\right)}{2\left(x^{6}\right)^{\frac{5}{6}}}+$

input
```
int((6*x^4+8*x^3+9*x^2+8*x+6)/(x^6+1),x,method=_RETURNVERBOSE)
```

output
```
2*(5/2-4/3*3^(1/2))*arctan(3^(1/2)+2*x)-2*(-5/2-4/3*3^(1/2))*arctan(-3^(1/
2)+2*x)+arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx = -\frac{1}{3}\left(8\sqrt{3}-15\right)\arctan\left(2\,x+\sqrt{3}\right) - \frac{1}{3}\left(8\sqrt{3}+15\right)\arctan\left(-2\,x+\sqrt{3}\right)+\arctan\left(x\right)$$

input
```
integrate((6*x^4+8*x^3+9*x^2+8*x+6)/(x^6+1),x, algorithm="fricas")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) - 1/3*(8*sqrt(3) + 15)*arctan(
-2*x + sqrt(3)) + arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx = \operatorname{atan}(x)+2\cdot\left(\frac{4\sqrt{3}}{3}+\frac{5}{2}\right)\operatorname{atan}\left(2x-\sqrt{3}\right) + 2\cdot\left(\frac{5}{2}-\frac{4\sqrt{3}}{3}\right)\operatorname{atan}\left(2x+\sqrt{3}\right)$$

input
```
integrate((6*x**4+8*x**3+9*x**2+8*x+6)/(x**6+1),x)
```

output
```
atan(x) + 2*(4*sqrt(3)/3 + 5/2)*atan(2*x - sqrt(3)) + 2*(5/2 - 4*sqrt(3)/3
)*atan(2*x + sqrt(3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx = -\frac{1}{3}\left(8\sqrt{3}-15\right)\arctan\left(2x+\sqrt{3}\right) + \frac{1}{3}\left(8\sqrt{3}+15\right)\arctan\left(2x-\sqrt{3}\right) + \arctan(x)$$

input
```
integrate((6*x^4+8*x^3+9*x^2+8*x+6)/(x^6+1),x, algorithm="maxima")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/3*(8*sqrt(3) + 15)*arctan(
2*x - sqrt(3)) + arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx = -\frac{1}{3}\left(8\sqrt{3}-15\right)\arctan\left(2x+\sqrt{3}\right) + \frac{1}{3}\left(8\sqrt{3}+15\right)\arctan\left(2x-\sqrt{3}\right) + \arctan(x)$$

input
```
integrate((6*x^4+8*x^3+9*x^2+8*x+6)/(x^6+1),x, algorithm="giac")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/3*(8*sqrt(3) + 15)*arctan(
2*x - sqrt(3)) + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.16

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx$$
$$= \operatorname{atan}(x) + 2\operatorname{atanh}\left(\frac{691200\,x\sqrt{-\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{5702400\,x+3294720\sqrt{3}\,x-3801600\sqrt{3}-6589440} + \frac{391680\sqrt{3}\,x\sqrt{-\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{5702400\,x+3294720\sqrt{3}\,x-3801600\sqrt{3}-6589440}\right)\sqrt{-\frac{20\sqrt{3}}{3}-\frac{139}{12}} + 2\operatorname{atanh}\left(\frac{691200\,x\sqrt{\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{5702400\,x-3294720\sqrt{3}\,x+3801600\sqrt{3}-6589440} - \frac{391680\sqrt{3}\,x\sqrt{\frac{20\sqrt{3}}{3}-\frac{139}{12}}}{5702400\,x-3294720\sqrt{3}\,x+3801600\sqrt{3}-6589440}\right)\sqrt{\frac{20\sqrt{3}}{3}-\frac{139}{12}}$$

input
```
int((8*x + 9*x^2 + 8*x^3 + 6*x^4 + 6)/(x^6 + 1),x)
```

output
```
atan(x) + 2*atanh((691200*x*(- (20*3^(1/2))/3 - 139/12)^(1/2))/(5702400*x
+ 3294720*3^(1/2)*x - 3801600*3^(1/2) - 6589440) + (391680*3^(1/2)*x*(- (2
0*3^(1/2))/3 - 139/12)^(1/2))/(5702400*x + 3294720*3^(1/2)*x - 3801600*3^(
1/2) - 6589440))*(- (20*3^(1/2))/3 - 139/12)^(1/2) + 2*atanh((691200*x*((2
0*3^(1/2))/3 - 139/12)^(1/2))/(5702400*x - 3294720*3^(1/2)*x + 3801600*3^(
1/2) - 6589440) - (391680*3^(1/2)*x*((20*3^(1/2))/3 - 139/12)^(1/2))/(5702
400*x - 3294720*3^(1/2)*x + 3801600*3^(1/2) - 6589440))*((20*3^(1/2))/3 -
139/12)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{6+8x+9x^2+8x^3+6x^4}{1+x^6}\,dx = -\frac{8\sqrt{3}\,atan\left(\sqrt{3}-2x\right)}{3} - 5atan\left(\sqrt{3}-2x\right) - \frac{8\sqrt{3}\,atan\left(\sqrt{3}+2x\right)}{3} + 5atan\left(\sqrt{3}+2x\right) + atan(x)$$

input
```
int((6*x^4+8*x^3+9*x^2+8*x+6)/(x^6+1),x)
```

output
```
( - 8*sqrt(3)*atan(sqrt(3) - 2*x) - 15*atan(sqrt(3) - 2*x) - 8*sqrt(3)*ata
n(sqrt(3) + 2*x) + 15*atan(sqrt(3) + 2*x) + 3*atan(x))/3
```

3.51 $\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6} dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6} dx = -\frac{1}{3}\left(15+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+\arctan(x)+\frac{1}{3}\left(15-8\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{\sqrt{3}}$$

output
```
1/3*(15+8*3^(1/2))*arctan(-3^(1/2)+2*x)+arctan(x)+1/3*(15-8*3^(1/2))*arcta
n(3^(1/2)+2*x)+1/3*arctanh(3^(1/2)*x/(x^2+1))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6} dx = \frac{1}{6}\left(-2\left(15+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+6\arctan(x)+2\left(15-8\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right)-\sqrt{3}\log\left(1-\sqrt{3}x+x^2\right)+\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)\right)$$

input
```
Integrate[(7 + 8*x + 9*x^2 + 8*x^3 + 5*x^4)/(1 + x^6),x]
```

output
```
(-2*(15 + 8*Sqrt[3])*ArcTan[Sqrt[3] - 2*x] + 6*ArcTan[x] + 2*(15 - 8*Sqrt[
3])*ArcTan[Sqrt[3] + 2*x] - Sqrt[3]*Log[1 - Sqrt[3]*x + x^2] + Sqrt[3]*Log
[1 + Sqrt[3]*x + x^2])/6
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 293, normalized size of antiderivative = 4.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4+8x^3+9x^2+8x+7}{x^6+1}dx$$

$$\Big\downarrow \text{2415}$$

$$\int \left(\frac{(5x^3+8)x}{x^6+1}+\frac{8x^3+7}{x^6+1}+\frac{9x^2}{x^6+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\arctan\left(x^3\right)-\frac{1}{6}\left(7+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+\frac{7\arctan(x)}{3}+\frac{1}{6}\left(7-8\sqrt{3}\right)\arctan\left(2x+\sqrt{3}\right)-\frac{\left(\frac{5}{2}-4i\right)(-1)^{5/6}\arctan\left(\frac{1-2(-1)^{5/6}x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\left(\frac{5}{2}+4i\right)(-1)^{5/6}\arctan\left(\frac{2(-1)^{5/6}x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{4}{3}\log\left(x^2+1\right)-\left(\frac{5}{12}+\frac{2i}{3}\right)(-1)^{5/6}\log\left(x^2-\sqrt[6]{-1}x+\sqrt[3]{-1}\right)+\left(\frac{5}{12}-\frac{2i}{3}\right)(-1)^{5/6}\log\left(x^2+\sqrt[6]{-1}x+\sqrt[3]{-1}\right)+\frac{1}{12}\left(8-7\sqrt{3}\right)\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{12}\left(8+7\sqrt{3}\right)\log\left(x^2+\sqrt{3}x+1\right)-\left(\frac{5}{6}-\frac{4i}{3}\right)(-1)^{5/6}\log\left(\sqrt[6]{-1}-x\right)+\left(\frac{5}{6}+\frac{4i}{3}\right)(-1)^{5/6}\log\left(x+\sqrt[6]{-1}\right)$$

input
```
Int[(7 + 8*x + 9*x^2 + 8*x^3 + 5*x^4)/(1 + x^6),x]
```

output
```
-1/6*((7 + 8*Sqrt[3])*ArcTan[Sqrt[3] - 2*x]) + (7*ArcTan[x])/3 + 3*ArcTan[
x^3] + ((7 - 8*Sqrt[3])*ArcTan[Sqrt[3] + 2*x])/6 - ((5/2 - 4*I)*(-1)^(5/6)
*ArcTan[(1 - 2*(-1)^(5/6)*x)/Sqrt[3]])/Sqrt[3] + ((5/2 + 4*I)*(-1)^(5/6)*A
rcTan[(1 + 2*(-1)^(5/6)*x)/Sqrt[3]])/Sqrt[3] - (5/6 - (4*I)/3)*(-1)^(5/6)*
Log[(-1)^(1/6) - x] + (5/6 + (4*I)/3)*(-1)^(5/6)*Log[(-1)^(1/6) + x] - (4*
Log[1 + x^2])/3 - (5/12 + (2*I)/3)*(-1)^(5/6)*Log[(-1)^(1/3) - (-1)^(1/6)*
x + x^2] + (5/12 - (2*I)/3)*(-1)^(5/6)*Log[(-1)^(1/3) + (-1)^(1/6)*x + x^2
] + ((8 - 7*Sqrt[3])*Log[1 - Sqrt[3]*x + x^2])/12 + ((8 + 7*Sqrt[3])*Log[1
 + Sqrt[3]*x + x^2])/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

method	result
risch	$\left(\sum_{\_R=\operatorname{RootOf}\left(9\_Z^4+207\_Z^2-120\_Z+25\right)} \_R\ln\left(1413\_R^3+1380\_R^2+34539\_R+13585x+1740\right)\right)$
default	$\frac{\sqrt{3}\,\ln\left(x^2+x\sqrt{3}+1\right)}{6}+\frac{2\left(\frac{15}{2}-4\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right)}{3}-\frac{\sqrt{3}\,\ln\left(x^2-x\sqrt{3}+1\right)}{6}-\frac{2\left(-\frac{15}{2}-4\sqrt{3}\right)\arctan\left(-\sqrt{3}+2x\right)}{3}+\text{arcta}$
meijerg	$\frac{5x^5\sqrt{3}\,\ln\left(1-\sqrt{3}\left(x^6\right)^{\frac{1}{6}}+\left(x^6\right)^{\frac{1}{3}}\right)}{12\left(x^6\right)^{\frac{5}{6}}}+\frac{5x^5\arctan\left(\frac{\left(x^6\right)^{\frac{1}{6}}}{2-\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}\right)}{6\left(x^6\right)^{\frac{5}{6}}}+\frac{5x^5\arctan\left(\left(x^6\right)^{\frac{1}{6}}\right)}{3\left(x^6\right)^{\frac{5}{6}}}-\frac{5x^5\sqrt{3}\,\ln\left(1+\sqrt{3}\left(x^6\right)^{\frac{1}{6}}+\left(x^6\right)^{\frac{1}{3}}\right)}{12\left(x^6\right)^{\frac{5}{6}}}$

input
```
int((5*x^4+8*x^3+9*x^2+8*x+7)/(x^6+1),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(1413*_R^3+1380*_R^2+34539*_R+13585*x+1740),_R=RootOf(9*_Z^4+207*
_Z^2-120*_Z+25))+arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6}\,dx = -\frac{1}{3}\left(8\sqrt{3}-15\right)\arctan\left(2\,x+\sqrt{3}\right) - \frac{1}{3}\left(8\sqrt{3}+15\right)\arctan\left(-2\,x+\sqrt{3}\right) + \frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \arctan\left(x\right)$$

input
```
integrate((5*x^4+8*x^3+9*x^2+8*x+7)/(x^6+1),x, algorithm="fricas")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) - 1/3*(8*sqrt(3) + 15)*arctan(
-2*x + sqrt(3)) + 1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/6*sqrt(3)*log(x
^2 - sqrt(3)*x + 1) + arctan(x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6}\,dx = -\frac{i\log\left(x-i\right)}{2} + \frac{i\log\left(x+i\right)}{2} + \operatorname{RootSum}\left(9t^4+207t^2-120t+25, \left(t\mapsto t\log\left(\frac{2893284t^5}{1146533245} + \frac{673488t^4}{17638973} + \frac{185798493t^3}{1146533245} + \frac{2169510}{229306}\right.\right.\right.$$

input
```
integrate((5*x**4+8*x**3+9*x**2+8*x+7)/(x**6+1),x)
```

output
```
-I*log(x - I)/2 + I*log(x + I)/2 + RootSum(9*_t**4 + 207*_t**2 - 120*_t +
25, Lambda(_t, _t*log(2893284*_t**5/1146533245 + 673488*_t**4/17638973 + 1
85798493*_t**3/1146533245 + 216951060*_t**2/229306649 + 2339335283*_t/1146
533245 + x + 53690556/229306649)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6}\,dx = -\frac{1}{3}\left(8\sqrt{3}-15\right)\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{3}\left(8\sqrt{3}+15\right)\arctan\left(2\,x-\sqrt{3}\right) + \frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \arctan\left(x\right)$$

input
```
integrate((5*x^4+8*x^3+9*x^2+8*x+7)/(x^6+1),x, algorithm="maxima")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/3*(8*sqrt(3) + 15)*arctan(
2*x - sqrt(3)) + 1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/6*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6}\,dx = -\frac{1}{3}\left(8\sqrt{3}-15\right)\arctan\left(2\,x+\sqrt{3}\right) + \frac{1}{3}\left(8\sqrt{3}+15\right)\arctan\left(2\,x-\sqrt{3}\right) + \frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right) - \frac{1}{6}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \arctan\left(x\right)$$

input
```
integrate((5*x^4+8*x^3+9*x^2+8*x+7)/(x^6+1),x, algorithm="giac")
```

output
```
-1/3*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/3*(8*sqrt(3) + 15)*arctan(
2*x - sqrt(3)) + 1/6*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/6*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6}\,dx = \left(\sum_{k=1}^{4}\ln\left(-15552\,\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)\right.\right.$$
$$-4320\,x-\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)\,x\,1368$$
$$+\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)^2\,x\,19872$$
$$-\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)^3\,x\,35640$$
$$+\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)^4\,x\,10368$$
$$+13608\,\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)^2$$
$$+6912\,\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)^3$$
$$-11664\,\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)^4$$
$$\left.\left.+360\right)\,\mathrm{root}\left(z^4+23\,z^2-\frac{40\,z}{3}+\frac{25}{9},z,k\right)\right)$$
$$-\frac{\ln\left(x-\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{\ln\left(x+1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int((8*x + 9*x^2 + 8*x^3 + 5*x^4 + 7)/(x^6 + 1),x)
```

output
```
(log(x + 1i)*1i)/2 - (log(x - 1i)*1i)/2 + symsum(log(19872*root(z^4 + 23*z
^2 - (40*z)/3 + 25/9, z, k)^2*x - 4320*x - 1368*root(z^4 + 23*z^2 - (40*z)
/3 + 25/9, z, k)*x - 15552*root(z^4 + 23*z^2 - (40*z)/3 + 25/9, z, k) - 35
640*root(z^4 + 23*z^2 - (40*z)/3 + 25/9, z, k)^3*x + 10368*root(z^4 + 23*z
^2 - (40*z)/3 + 25/9, z, k)^4*x + 13608*root(z^4 + 23*z^2 - (40*z)/3 + 25/
9, z, k)^2 + 6912*root(z^4 + 23*z^2 - (40*z)/3 + 25/9, z, k)^3 - 11664*roo
t(z^4 + 23*z^2 - (40*z)/3 + 25/9, z, k)^4 + 360)*root(z^4 + 23*z^2 - (40*z
)/3 + 25/9, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{7+8x+9x^2+8x^3+5x^4}{1+x^6}\,dx = -\frac{8\sqrt{3}\,atan(\sqrt{3}-2x)}{3} - 5atan\left(\sqrt{3}-2x\right) - \frac{8\sqrt{3}\,atan(\sqrt{3}+2x)}{3} + 5atan\left(\sqrt{3}+2x\right) + atan(x) - \frac{\sqrt{3}\log(-\sqrt{3}\,x+x^2+1)}{6} + \frac{\sqrt{3}\log(\sqrt{3}\,x+x^2+1)}{6}$$

input
```
int((5*x^4+8*x^3+9*x^2+8*x+7)/(x^6+1),x)
```

output
```
( - 16*sqrt(3)*atan(sqrt(3) - 2*x) - 30*atan(sqrt(3) - 2*x) - 16*sqrt(3)*a
tan(sqrt(3) + 2*x) + 30*atan(sqrt(3) + 2*x) + 6*atan(x) - sqrt(3)*log( - s
qrt(3)*x + x**2 + 1) + sqrt(3)*log(sqrt(3)*x + x**2 + 1))/6
```

3.52 $\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 151

$$\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx = \frac{x(1-x-x^2-x^3-x^4-x^5)}{12\,(1+x^6)^2} - \frac{x(13-2x-3x^2-4x^3-5x^4-6x^5)}{72\,(1+x^6)} - \frac{1}{216}\left(15+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+\frac{\arctan(x)}{72} + \frac{1}{216}\left(15-8\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}x}{1+x^2}\right)}{72\sqrt{3}}$$

output

```
1/12*x*(-x^5-x^4-x^3-x^2-x+1)/(x^6+1)^2-x*(-6*x^5-5*x^4-4*x^3-3*x^2-2*x+13
)/(72*x^6+72)+1/216*(15+8*3^(1/2))*arctan(-3^(1/2)+2*x)+1/72*arctan(x)+1/2
16*(15-8*3^(1/2))*arctan(3^(1/2)+2*x)+1/216*arctanh(3^(1/2)*x/(x^2+1))*3^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.23

$$\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx = \frac{1}{432}\left(-\frac{36(-1-x+x^2+x^3+x^4+x^5)}{(1+x^6)^2} + \frac{6(-12-13x+2x^2+3x^3+4x^4+5x^5)}{1+x^6} - 30\arctan\left(\sqrt{3}-2x\right)+6\arctan(x) + 30\arctan\left(\sqrt{3}+2x\right) + 8i\sqrt{3}\log\left(i+\sqrt{3}-2ix^2\right) - 8i\sqrt{3}\log\left(-i+\sqrt{3}+2ix^2\right) - \sqrt{3}\log\left(1-\sqrt{3}x+x^2\right) + \sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)\right)$$

input
```
Integrate[(x^7 + x^8 + x^9 + x^10 + x^11 + x^12)/(1 + x^6)^3,x]
```

output
```
((-36*(-1 - x + x^2 + x^3 + x^4 + x^5))/(1 + x^6)^2 + (6*(-12 - 13*x + 2*x
^2 + 3*x^3 + 4*x^4 + 5*x^5))/(1 + x^6) - 30*ArcTan[Sqrt[3] - 2*x] + 6*ArcT
an[x] + 30*ArcTan[Sqrt[3] + 2*x] + (8*I)*Sqrt[3]*Log[I + Sqrt[3] - (2*I)*x
^2] - (8*I)*Sqrt[3]*Log[-I + Sqrt[3] + (2*I)*x^2] - Sqrt[3]*Log[1 - Sqrt[3
]*x + x^2] + Sqrt[3]*Log[1 + Sqrt[3]*x + x^2])/432
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.50, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2390, 9, 2367, 2397, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}+x^{11}+x^{10}+x^9+x^8+x^7}{(x^6+1)^3}dx$$

$$\Big\downarrow \text{ 2390}$$

$$\int \frac{x(x^{11}+x^{10}+x^9+x^8+x^7+x^6)}{(x^6+1)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^7(x^5+x^4+x^3+x^2+x+1)}{(x^6+1)^3}dx$$

$$\Big\downarrow \text{ 2367}$$

$$\frac{x(-x^5-x^4-x^3-x^2-x+1)}{12(x^6+1)^2}-\frac{1}{12}\int \frac{-12x^6-6x^5-5x^4-4x^3-3x^2-2x+1}{(x^6+1)^2}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{1}{12}\left(\frac{1}{6}\int \frac{5x^4+8x^3+9x^2+8x+7}{x^6+1}dx-\frac{x(-6x^5-5x^4-4x^3-3x^2-2x+13)}{6(x^6+1)}\right)+\frac{x(-x^5-x^4-x^3-x^2-x+1)}{12(x^6+1)^2}$$

$$\Big\downarrow \text{ 2415}$$

$$\frac{1}{12}\left(\frac{1}{6}\int \left(\frac{9x^2}{x^6+1}+\frac{(5x^3+8)x}{x^6+1}+\frac{8x^3+7}{x^6+1}\right)dx-\frac{x(-6x^5-5x^4-4x^3-3x^2-2x+13)}{6(x^6+1)}\right)+\frac{x(-x^5-x^4-x^3-x^2-x+1)}{12(x^6+1)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x(-x^5-x^4-x^3-x^2-x+1)}{12(x^6+1)^2}+\frac{1}{12}\left(-\frac{x(-6x^5-5x^4-4x^3-3x^2-2x+13)}{6(x^6+1)}+\frac{1}{6}\left(3\arctan(x^3)-\frac{1}{6}\left(7+8\sqrt{3}\right)\arctan\left(\sqrt{3}-2x\right)+\frac{7\arctan(}{3}\right.\right.$$

input
```
Int[(x^7 + x^8 + x^9 + x^10 + x^11 + x^12)/(1 + x^6)^3,x]
```

output
```
(x*(1 - x - x^2 - x^3 - x^4 - x^5))/(12*(1 + x^6)^2) + (-1/6*(x*(13 - 2*x
- 3*x^2 - 4*x^3 - 5*x^4 - 6*x^5))/(1 + x^6) + (-1/6*((7 + 8*Sqrt[3])*ArcTa
n[Sqrt[3] - 2*x]) + (7*ArcTan[x])/3 + 3*ArcTan[x^3] + ((7 - 8*Sqrt[3])*Arc
Tan[Sqrt[3] + 2*x])/6 - ((5/2 - 4*I)*(-1)^(5/6)*ArcTan[(1 - 2*(-1)^(5/6)*x
)/Sqrt[3]])/Sqrt[3] + ((5/2 + 4*I)*(-1)^(5/6)*ArcTan[(1 + 2*(-1)^(5/6)*x)/
Sqrt[3]])/Sqrt[3] - (5/6 - (4*I)/3)*(-1)^(5/6)*Log[(-1)^(1/6) - x] + (5/6
+ (4*I)/3)*(-1)^(5/6)*Log[(-1)^(1/6) + x] - (4*Log[1 + x^2])/3 - (5/12 + (
2*I)/3)*(-1)^(5/6)*Log[(-1)^(1/3) - (-1)^(1/6)*x + x^2] + (5/12 - (2*I)/3)
*(-1)^(5/6)*Log[(-1)^(1/3) + (-1)^(1/6)*x + x^2] + ((8 - 7*Sqrt[3])*Log[1
- Sqrt[3]*x + x^2])/12 + ((8 + 7*Sqrt[3])*Log[1 + Sqrt[3]*x + x^2])/12)/6)
/12
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2390
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Int[x*PolynomialQuot
ient[Pq, x, x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x]
&& EqQ[Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\frac{5}{72}x^{11}+\frac{1}{18}x^{10}+\frac{1}{24}x^9+\frac{1}{36}x^8-\frac{13}{72}x^7-\frac{1}{6}x^6-\frac{1}{72}x^5-\frac{1}{36}x^4-\frac{1}{24}x^3-\frac{1}{18}x^2-\frac{7}{72}x-\frac{1}{12}}{(x^6+1)^2}+\frac{\arctan(x)}{72}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+207\_Z^2-\right.}\right.}{}$
default	$-\frac{-\frac{5}{2}x^7-2x^6+\frac{19}{4}x^5+4x^4-\frac{13}{4}x^3-\frac{5}{2}x^2+\frac{9}{4}x+2}{27(x^4-x^2+1)^2}+\frac{\sqrt{3}\,\ln\left(x^2+x\sqrt{3}+1\right)}{432}-\frac{\left(-\frac{15}{2}+4\sqrt{3}\right)\arctan\left(\sqrt{3}+2x\right)}{108}-\frac{\sqrt{3}\,\ln\left(x^2-x\sqrt{3}+\right.}{432}$
meijerg	$-\frac{x(169x^6+91)}{936(x^6+1)^2}+\frac{7x\left(-\frac{\sqrt{3}\,\ln\left(1-\sqrt{3}\left(x^6\right)^{\frac{1}{6}}+\left(x^6\right)^{\frac{1}{3}}\right)}{2\left(x^6\right)^{\frac{1}{6}}}+\frac{\arctan\left(\frac{\left(x^6\right)^{\frac{1}{6}}}{2-\sqrt{3}\left(x^6\right)^{\frac{1}{6}}}\right)}{\left(x^6\right)^{\frac{1}{6}}}+\frac{2\arctan\left(\left(x^6\right)^{\frac{1}{6}}\right)}{\left(x^6\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\,\ln\left(1+\sqrt{3}\left(x^6\right)^{\frac{1}{6}}+\left(x^6\right)^{\frac{1}{3}}\right)}{2\left(x^6\right)^{\frac{1}{6}}}\right.}{432}$

input
```
int((x^12+x^11+x^10+x^9+x^8+x^7)/(x^6+1)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/72*x^11+1/18*x^10+1/24*x^9+1/36*x^8-13/72*x^7-1/6*x^6-1/72*x^5-1/36*x^4
-1/24*x^3-1/18*x^2-7/72*x-1/12)/(x^6+1)^2+1/72*arctan(x)+1/72*sum(_R*ln(14
13*_R^3+1380*_R^2+34539*_R+13585*x+1740),_R=RootOf(9*_Z^4+207*_Z^2-120*_Z+
25))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.38

$$\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx$$
$$= \frac{30\,x^{11}+24\,x^{10}+18\,x^9+12\,x^8-78\,x^7-72\,x^6-6\,x^5-12\,x^4-18\,x^3+\sqrt{3}(x^{12}+2\,x^6+1)\log\left(x^2+\sqrt{\right.}}{}$$

input
```
integrate((x^12+x^11+x^10+x^9+x^8+x^7)/(x^6+1)^3,x, algorithm="fricas")
```

output
```
1/432*(30*x^11 + 24*x^10 + 18*x^9 + 12*x^8 - 78*x^7 - 72*x^6 - 6*x^5 - 12*
x^4 - 18*x^3 + sqrt(3)*(x^12 + 2*x^6 + 1)*log(x^2 + sqrt(3)*x + 1) - sqrt(
3)*(x^12 + 2*x^6 + 1)*log(x^2 - sqrt(3)*x + 1) - 24*x^2 + 2*(15*x^12 + 30*
x^6 - 8*sqrt(3)*(x^12 + 2*x^6 + 1) + 15)*arctan(2*x + sqrt(3)) + 6*(x^12 +
 2*x^6 + 1)*arctan(x) - 2*(15*x^12 + 30*x^6 + 8*sqrt(3)*(x^12 + 2*x^6 + 1)
 + 15)*arctan(-2*x + sqrt(3)) - 42*x - 36)/(x^12 + 2*x^6 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx = -\frac{i\log(x-i)}{144}+\frac{i\log(x+i)}{144}$$
$$+\operatorname{RootSum}\left(241864704t^4+1073088t^2-8640t+25,\left(t\mapsto t\log\left(\frac{5598266225983488t^5}{1146533245}+\frac{18099219529}{1763897}\right.\right.\right.$$
$$+\frac{5x^{11}+4x^{10}+3x^9+2x^8-13x^7-12x^6-x^5-2x^4-3x^3-4x^2-7x-6}{72x^{12}+144x^6+72}$$

input
```
integrate((x**12+x**11+x**10+x**9+x**8+x**7)/(x**6+1)**3,x)
```

output
```
-I*log(x - I)/144 + I*log(x + I)/144 + RootSum(241864704*_t**4 + 1073088*_
t**2 - 8640*_t + 25, Lambda(_t, _t*log(5598266225983488*_t**5/1146533245 +
 18099219529728*_t**4/17638973 + 69348915915264*_t**3/1146533245 + 1124674
295040*_t**2/229306649 + 168432140376*_t/1146533245 + x + 53690556/2293066
49))) + (5*x**11 + 4*x**10 + 3*x**9 + 2*x**8 - 13*x**7 - 12*x**6 - x**5 -
2*x**4 - 3*x**3 - 4*x**2 - 7*x - 6)/(72*x**12 + 144*x**6 + 72)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx &= -\frac{1}{216}\left(8\sqrt{3}-15\right)\arctan\left(2x+\sqrt{3}\right)+\frac{1}{216}\left(8\sqrt{3}+15\right)\arctan\left(2x-\sqrt{3}\right)\\ &\quad+\frac{1}{432}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{432}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)\\ &\quad+\frac{5x^{11}+4x^{10}+3x^9+2x^8-13x^7-12x^6-x^5-2x^4-3x^3-4x^2-7x-6}{72\left(x^{12}+2x^6+1\right)}\\ &\quad+\frac{1}{72}\arctan\left(x\right)\end{aligned}$$

input
```
integrate((x^12+x^11+x^10+x^9+x^8+x^7)/(x^6+1)^3,x, algorithm="maxima")
```

output
```
-1/216*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/216*(8*sqrt(3) + 15)*arc
tan(2*x - sqrt(3)) + 1/432*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/432*sqrt(3
)*log(x^2 - sqrt(3)*x + 1) + 1/72*(5*x^11 + 4*x^10 + 3*x^9 + 2*x^8 - 13*x^
7 - 12*x^6 - x^5 - 2*x^4 - 3*x^3 - 4*x^2 - 7*x - 6)/(x^12 + 2*x^6 + 1) + 1
/72*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx$$
$$= -\frac{1}{216}\left(8\sqrt{3}-15\right)\arctan\left(2x+\sqrt{3}\right)+\frac{1}{216}\left(8\sqrt{3}+15\right)\arctan\left(2x-\sqrt{3}\right) + \frac{1}{432}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{432}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right) + \frac{5x^{11}+4x^{10}+3x^9+2x^8-13x^7-12x^6-x^5-2x^4-3x^3-4x^2-7x-6}{72\,(x^6+1)^2} + \frac{1}{72}\arctan(x)$$

input
```
integrate((x^12+x^11+x^10+x^9+x^8+x^7)/(x^6+1)^3,x, algorithm="giac")
```

output
```
-1/216*(8*sqrt(3) - 15)*arctan(2*x + sqrt(3)) + 1/216*(8*sqrt(3) + 15)*arc
tan(2*x - sqrt(3)) + 1/432*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/432*sqrt(3
)*log(x^2 - sqrt(3)*x + 1) + 1/72*(5*x^11 + 4*x^10 + 3*x^9 + 2*x^8 - 13*x^
7 - 12*x^6 - x^5 - 2*x^4 - 3*x^3 - 4*x^2 - 7*x - 6)/(x^6 + 1)^2 + 1/72*arc
tan(x)
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.34

$$\int \frac{x^7+x^8+x^9+x^{10}+x^{11}+x^{12}}{(1+x^6)^3}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{5x}{2239488} - \mathrm{root}\left(z^4+\frac{23z^2}{5184}-\frac{5z}{139968}+\frac{25}{241864704},z,k\right)\left(\frac{19x}{373248} - \mathrm{root}\left(z^4+\frac{23z^2}{5184}-\frac{5z}{139968}+\frac{25}{241864704},z,k\right) + \frac{5}{26873856}\right)\mathrm{root}\left(z^4+\frac{23z^2}{5184}-\frac{5z}{139968}+\frac{25}{241864704},z,k\right)\right) - \frac{-\frac{5x^{11}}{72}-\frac{x^{10}}{18}-\frac{x^9}{24}-\frac{x^8}{36}+\frac{13x^7}{72}+\frac{x^6}{6}+\frac{x^5}{72}+\frac{x^4}{36}+\frac{x^3}{24}+\frac{x^2}{18}+\frac{7x}{72}+\frac{1}{12}}{x^{12}+2x^6+1} - \frac{\ln(x-\mathrm{i})\,1\mathrm{i}}{144}+\frac{\ln(x+1\mathrm{i})\,1\mathrm{i}}{144}$$

input
```
int((x^7 + x^8 + x^9 + x^10 + x^11 + x^12)/(x^6 + 1)^3,x)
```

output
```
(log(x + 1i)*1i)/144 - (log(x - 1i)*1i)/144 - ((7*x)/72 + x^2/18 + x^3/24
+ x^4/36 + x^5/72 + x^6/6 + (13*x^7)/72 - x^8/36 - x^9/24 - x^10/18 - (5*x
^11)/72 + 1/12)/(2*x^6 + x^12 + 1) + symsum(log(5/26873856 - root(z^4 + (2
3*z^2)/5184 - (5*z)/139968 + 25/241864704, z, k)*((19*x)/373248 - root(z^4
 + (23*z^2)/5184 - (5*z)/139968 + 25/241864704, z, k)*((23*x)/432 + root(z
^4 + (23*z^2)/5184 - (5*z)/139968 + 25/241864704, z, k)*(root(z^4 + (23*z^
2)/5184 - (5*z)/139968 + 25/241864704, z, k)*(144*x - 162) - (55*x)/8 + 4/
3) + 7/192) + 1/1728) - (5*x)/2239488)*root(z^4 + (23*z^2)/5184 - (5*z)/13
9968 + 25/241864704, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.17

$$\int \frac{x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12}}{(1 + x^6)^3}\,dx$$
$$= \frac{-42x - 24x^2 - 30atan(\sqrt{3} - 2x)\,x^{12} - 60atan(\sqrt{3} - 2x)\,x^6 + 30atan(\sqrt{3} + 2x)\,x^{12} + 60atan(\sqrt{3} + 2}{}$$

input
```
int((x^12+x^11+x^10+x^9+x^8+x^7)/(x^6+1)^3,x)
```

output
```
( - 16*sqrt(3)*atan(sqrt(3) - 2*x)*x**12 - 32*sqrt(3)*atan(sqrt(3) - 2*x)*
x**6 - 16*sqrt(3)*atan(sqrt(3) - 2*x) - 30*atan(sqrt(3) - 2*x)*x**12 - 60*
atan(sqrt(3) - 2*x)*x**6 - 30*atan(sqrt(3) - 2*x) - 16*sqrt(3)*atan(sqrt(3
) + 2*x)*x**12 - 32*sqrt(3)*atan(sqrt(3) + 2*x)*x**6 - 16*sqrt(3)*atan(sqr
t(3) + 2*x) + 30*atan(sqrt(3) + 2*x)*x**12 + 60*atan(sqrt(3) + 2*x)*x**6 +
 30*atan(sqrt(3) + 2*x) + 6*atan(x)*x**12 + 12*atan(x)*x**6 + 6*atan(x) -
sqrt(3)*log( - sqrt(3)*x + x**2 + 1)*x**12 - 2*sqrt(3)*log( - sqrt(3)*x +
x**2 + 1)*x**6 - sqrt(3)*log( - sqrt(3)*x + x**2 + 1) + sqrt(3)*log(sqrt(3
)*x + x**2 + 1)*x**12 + 2*sqrt(3)*log(sqrt(3)*x + x**2 + 1)*x**6 + sqrt(3)
*log(sqrt(3)*x + x**2 + 1) + 36*x**12 + 30*x**11 + 24*x**10 + 18*x**9 + 12
*x**8 - 78*x**7 - 6*x**5 - 12*x**4 - 18*x**3 - 24*x**2 - 42*x)/(432*(x**12
 + 2*x**6 + 1))
```

3.53 $\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 317

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx=\frac{\left(1+\sqrt{3}\right)x\sqrt{1-x^6}}{2\left(1-\left(1+\sqrt{3}\right)x^2\right)}-\frac{\sqrt[4]{3}x(1-x^2)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}E\left(\arccos\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt{-\frac{x^2(1-x^2)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}+\frac{\left(1+\sqrt{3}\right)x(1-x^2)\sqrt{\frac{1+x^2+x^4}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{-\frac{x^2(1-x^2)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}$$

output
```
(1+3^(1/2))*x*(-x^6+1)^(1/2)/(2-2*(1+3^(1/2))*x^2)-1/2*3^(1/4)*x*(-x^2+1)*
((x^4+x^2+1)/(1-(1+3^(1/2))*x^2)^2)^(1/2)*EllipticE((1-(1-(1-3^(1/2))*x^2)
^2/(1-(1+3^(1/2))*x^2)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/(-x^2*(-x^2+1)/(1
-(1+3^(1/2))*x^2)^2)^(1/2)/(-x^6+1)^(1/2)+1/12*(1+3^(1/2))*x*(-x^2+1)*((x^
4+x^2+1)/(1-(1+3^(1/2))*x^2)^2)^(1/2)*InverseJacobiAM(arccos((1-(1-3^(1/2)
)*x^2)/(1-(1+3^(1/2))*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/(-x^2*(-x^2+1
)/(1-(1+3^(1/2))*x^2)^2)^(1/2)/(-x^6+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.11

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},x^6\right) + \frac{1}{5}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},x^6\right)$$

input
```
Integrate[(1 + x^4)/Sqrt[1 - x^6],x]
```

output
```
x*Hypergeometric2F1[1/6, 1/2, 7/6, x^6] + (x^5*Hypergeometric2F1[1/2, 5/6,
 11/6, x^6])/5
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2421, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{\sqrt{1-x^6}}\,dx$$

$$\Big\downarrow \text{2421}$$

$$\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^6}}dx + \frac{1}{2}\int \frac{2x^4-\sqrt{3}+1}{\sqrt{1-x^6}}dx$$

$$\Big\downarrow \text{766}$$

$$\frac{1}{2}\int \frac{2x^4-\sqrt{3}+1}{\sqrt{1-x^6}}dx+\frac{\left(1+\sqrt{3}\right)x\left(1-x^2\right)\sqrt{\frac{x^4+x^2+1}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}$$

$$\Big\downarrow \; 2420$$

$$\frac{\left(1+\sqrt{3}\right)x\left(1-x^2\right)\sqrt{\frac{x^4+x^2+1}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}+\frac{1}{2}\left(\frac{\left(1+\sqrt{3}\right)x\sqrt{1-x^6}}{1-\left(1+\sqrt{3}\right)x^2}-\frac{\sqrt[4]{3}x\left(1-x^2\right)\sqrt{\frac{x^4+x^2+1}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}E\left(\arccos\left(\frac{1-\left(1-\sqrt{3}\right)x^2}{1-\left(1+\sqrt{3}\right)x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{-\frac{x^2\left(1-x^2\right)}{\left(1-\left(1+\sqrt{3}\right)x^2\right)^2}}\sqrt{1-x^6}}\right)$$

input
```
Int[(1 + x^4)/Sqrt[1 - x^6],x]
```

output
```
(((1 + Sqrt[3])*x*Sqrt[1 - x^6])/(1 - (1 + Sqrt[3])*x^2) - (3^(1/4)*x*(1 -
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 + Sqrt[3])*x^2)^2]*EllipticE[ArcCos[(1
- (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])/(Sqrt[-((
x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*x^2)^2)]*Sqrt[1 - x^6]))/2 + ((1 + Sqrt[
3])*x*(1 - x^2)*Sqrt[(1 + x^2 + x^4)/(1 - (1 + Sqrt[3])*x^2)^2]*EllipticF[
ArcCos[(1 - (1 - Sqrt[3])*x^2)/(1 - (1 + Sqrt[3])*x^2)], (2 + Sqrt[3])/4])
/(4*3^(1/4)*Sqrt[-((x^2*(1 - x^2))/(1 - (1 + Sqrt[3])*x^2)^2)]*Sqrt[1 - x^
6])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 2421
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{q =
 Rt[b/a, 3]}, Simp[(2*c*q^2 - (1 - Sqrt[3])*d)/(2*q^2)   Int[1/Sqrt[a + b*x
^6], x], x] + Simp[d/(2*q^2)   Int[(1 - Sqrt[3] + 2*q^2*x^4)/Sqrt[a + b*x^6
], x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[2*Rt[b/a, 3]^2*c - (1 - Sqrt[3]
)*d, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

method	result	size
meijerg	$\frac{x^5 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],x^6\right)}{5}+x \operatorname{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],x^6\right)$	27

input
```
int((x^4+1)/(-x^6+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([1/2,5/6],[11/6],x^6)+x*hypergeom([1/6,1/2],[7/6],x^6)
```

Fricas [F]

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx=\int \frac{x^4+1}{\sqrt{-x^6+1}}\,dx$$

input
```
integrate((x^4+1)/(-x^6+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^6 + 1)*(x^4 + 1)/(x^6 - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.19

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx = \frac{x^5\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|x^6e^{2i\pi}\right)}{6\Gamma\left(\frac{11}{6}\right)} + \frac{x\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|x^6e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((x**4+1)/(-x**6+1)**(1/2),x)
```

output
```
x**5*gamma(5/6)*hyper((1/2, 5/6), (11/6,), x**6*exp_polar(2*I*pi))/(6*gamm
a(11/6)) + x*gamma(1/6)*hyper((1/6, 1/2), (7/6,), x**6*exp_polar(2*I*pi))/
(6*gamma(7/6))
```

Maxima [F]

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx = \int \frac{x^4+1}{\sqrt{-x^6+1}}\,dx$$

input
```
integrate((x^4+1)/(-x^6+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)/sqrt(-x^6 + 1), x)
```

Giac [F]

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx = \int \frac{x^4+1}{\sqrt{-x^6+1}}\,dx$$

input
```
integrate((x^4+1)/(-x^6+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 1)/sqrt(-x^6 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx = \int \frac{x^4+1}{\sqrt{1-x^6}}\,dx$$

input
```
int((x^4 + 1)/(1 - x^6)^(1/2),x)
```

output
```
int((x^4 + 1)/(1 - x^6)^(1/2), x)
```

Reduce [F]

$$\int \frac{1+x^4}{\sqrt{1-x^6}}\,dx = -\left(\int \frac{\sqrt{-x^6+1}}{x^6-1}dx\right) - \left(\int \frac{\sqrt{-x^6+1}\,x^4}{x^6-1}dx\right)$$

input
```
int((x^4+1)/(-x^6+1)^(1/2),x)
```

output
```
- (int(sqrt( - x**6 + 1)/(x**6 - 1),x) + int((sqrt( - x**6 + 1)*x**4)/(x*
*6 - 1),x))
```

3.54 $\int \left(c+dx^{-1+n}\right) \left(a+bx^n\right)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \left(c+dx^{-1+n}\right) \left(a+bx^n\right)^3 \, dx = a^3cx+\frac{3a^2bcx^{1+n}}{1+n}+\frac{3ab^2cx^{1+2n}}{1+2n}+\frac{b^3cx^{1+3n}}{1+3n}+\frac{d(a+bx^n)^4}{4bn}$$

output
```
a^3*c*x+3*a^2*b*c*x^(1+n)/(1+n)+3*a*b^2*c*x^(1+2*n)/(1+2*n)+b^3*c*x^(1+3*n
)/(1+3*n)+1/4*d*(a+b*x^n)^4/b/n
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \left(c+dx^{-1+n}\right) \left(a+bx^n\right)^3 \, dx = \frac{x(c+dx^{-1+n}) \left(4a^3cx+\frac{12a^2bcx^{1+n}}{1+n}+\frac{12ab^2cx^{1+2n}}{1+2n}+\frac{4b^3cx^{1+3n}}{1+3n}+\frac{d(a+bx^n)^4}{bn}\right)}{4\left(cx+dx^n\right)}$$

input
```
Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

output
```
(x*(c + d*x^(-1 + n))*(4*a^3*c*x + (12*a^2*b*c*x^(1 + n))/(1 + n) + (12*a*
b^2*c*x^(1 + 2*n))/(1 + 2*n) + (4*b^3*c*x^(1 + 3*n))/(1 + 3*n) + (d*(a + b
*x^n)^4)/(b*n)))/(4*(c*x + d*x^n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2430, 775, 793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^3 \left(c+dx^{n-1}\right) dx$$

$$\Big\downarrow \text{ 2430}$$

$$c\int (bx^n+a)^3 dx + d\int x^{n-1}(bx^n+a)^3 dx$$

$$\Big\downarrow \text{ 775}$$

$$c\int \left(3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}+a^3\right) dx + d\int x^{n-1}(bx^n+a)^3 dx$$

$$\Big\downarrow \text{ 793}$$

$$c\int \left(3a^2bx^n+3ab^2x^{2n}+b^3x^{3n}+a^3\right) dx + \frac{d(a+bx^n)^4}{4bn}$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(a^3x+\frac{3a^2bx^{n+1}}{n+1}+\frac{3ab^2x^{2n+1}}{2n+1}+\frac{b^3x^{3n+1}}{3n+1}\right)+\frac{d(a+bx^n)^4}{4bn}$$

input
```
Int[(c + d*x^(-1 + n))*(a + b*x^n)^3,x]
```

output
```
(d*(a + b*x^n)^4)/(4*b*n) + c*(a^3*x + (3*a^2*b*x^(1 + n))/(1 + n) + (3*a*
b^2*x^(1 + 2*n))/(1 + 2*n) + (b^3*x^(1 + 3*n))/(1 + 3*n))
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
risch	$a^3cx+\frac{b^3d\,x^{4n}}{4n}+\frac{b^2(nbcx+3dna+ad)x^{3n}}{n(1+3n)}+\frac{3ab(2nbcx+2dna+ad)x^{2n}}{2n(1+2n)}+\frac{a^2(3nbcx+dna+ad)x^n}{n(1+n)}$
norman	$a^3cx+\frac{a^3d\,\mathrm{e}^{n\ln(x)}}{n}+\frac{b^3cx\,\mathrm{e}^{3n\ln(x)}}{1+3n}+\frac{da\,b^2\mathrm{e}^{3n\ln(x)}}{n}+\frac{b^3d\,\mathrm{e}^{4n\ln(x)}}{4n}+\frac{3a^2bd\,\mathrm{e}^{2n\ln(x)}}{2n}+\frac{3ca\,b^2x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{3c\,a^2}{}$
parallelrisch	$\frac{8x^{3n}b^3c\,n^3x+12x^{3n}b^3c\,n^2x+4x^{3n}b^3cnx+x\,x^{3n}x^{-1+n}b^3d+4a^3cxn+72x^na^2bc\,n^3x+60x^na^2bc\,n^2x+12x^na^2bcnx+24a^3c\,n^4}{}$
orering	Expression too large to display

input
```
int((c+d*x^(-1+n))*(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/4*b^3*d/n*(x^n)^4+b^2*(b*c*n*x+3*a*d*n+a*d)/n/(1+3*n)*(x^n)^3+3/
2*a*b*(2*b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a^2*(3*b*c*n*x+a*d*n+a*d)/
n/(1+n)*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(82) = 164$.

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.63

$$\int (c + dx^{-1+n}) (a + bx^n)^3 \, dx = \frac{4\,(6\,a^3cn^4 + 11\,a^3cn^3 + 6\,a^3cn^2 + a^3cn)x + (6\,b^3dn^3 + 11\,b^3dn^2 + 6\,b^3dn + b^3d)x^{4\,n} + 4\,(6\,ab^2dn^3 + 11\,a}{}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/4*(4*(6*a^3*c*n^4 + 11*a^3*c*n^3 + 6*a^3*c*n^2 + a^3*c*n)*x + (6*b^3*d*n
^3 + 11*b^3*d*n^2 + 6*b^3*d*n + b^3*d)*x^(4*n) + 4*(6*a*b^2*d*n^3 + 11*a*b
^2*d*n^2 + 6*a*b^2*d*n + a*b^2*d + (2*b^3*c*n^3 + 3*b^3*c*n^2 + b^3*c*n)*x
)*x^(3*n) + 6*(6*a^2*b*d*n^3 + 11*a^2*b*d*n^2 + 6*a^2*b*d*n + a^2*b*d + 2*
(3*a*b^2*c*n^3 + 4*a*b^2*c*n^2 + a*b^2*c*n)*x)*x^(2*n) + 4*(6*a^3*d*n^3 +
11*a^3*d*n^2 + 6*a^3*d*n + a^3*d + 3*(6*a^2*b*c*n^3 + 5*a^2*b*c*n^2 + a^2*
b*c*n)*x)*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. $2(75) = 150$.

Time = 1.04 (sec) , antiderivative size = 1340, normalized size of antiderivative = 15.95

$$\int (c + dx^{-1+n}) (a + bx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((c+d*x**(-1+n))*(a+b*x**n)**3,x)
```

output
```
Piecewise((a**3*c*x - a**3*d/x + 3*a**2*b*c*log(x) - 3*a**2*b*d/(2*x**2) -
 3*a*b**2*c/x - a*b**2*d/x**3 - b**3*c/(2*x**2) - b**3*d/(4*x**4), Eq(n, -
1)), (a**3*c*x - 2*a**3*d/sqrt(x) + 6*a**2*b*c*sqrt(x) - 3*a**2*b*d/x + 3*
a*b**2*c*log(x) - 2*a*b**2*d/x**(3/2) - 2*b**3*c/sqrt(x) - b**3*d/(2*x**2)
, Eq(n, -1/2)), (a**3*c*x - 3*a**3*d/x**(1/3) + 9*a**2*b*c*x**(2/3)/2 - 9*
a**2*b*d/(2*x**(2/3)) + 9*a*b**2*c*x**(1/3) - 3*a*b**2*d/x + b**3*c*log(x)
 - 3*b**3*d/(4*x**(4/3)), Eq(n, -1/3)), ((a + b)**3*(c*x + d*log(x)), Eq(n
, 0)), (24*a**3*c*n**4*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**3*c*n
**3*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*c*n**2*x/(24*n**4 + 44
*n**3 + 24*n**2 + 4*n) + 4*a**3*c*n*x/(24*n**4 + 44*n**3 + 24*n**2 + 4*n)
+ 24*a**3*d*n**3*x*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 44*a**
3*d*n**2*x*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 24*a**3*d*n*x*
x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 4*a**3*d*x*x**(n - 1)/(24
*n**4 + 44*n**3 + 24*n**2 + 4*n) + 72*a**2*b*c*n**3*x*x**n/(24*n**4 + 44*n
**3 + 24*n**2 + 4*n) + 60*a**2*b*c*n**2*x*x**n/(24*n**4 + 44*n**3 + 24*n**
2 + 4*n) + 12*a**2*b*c*n*x*x**n/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a
**2*b*d*n**3*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 66*a*
*2*b*d*n**2*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a**
2*b*d*n*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 6*a**2*b*d
*x*x**n*x**(n - 1)/(24*n**4 + 44*n**3 + 24*n**2 + 4*n) + 36*a*b**2*c*n*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^3\,dx = a^3cx+\frac{b^3dx^{4\,n}}{4\,n}+\frac{ab^2dx^{3\,n}}{n}+\frac{3\,a^2bdx^{2\,n}}{2\,n}+\frac{b^3cx^{3\,n+1}}{3\,n+1}+\frac{3\,ab^2cx^{2\,n+1}}{2\,n+1}+\frac{3\,a^2bcx^{n+1}}{n+1}+\frac{a^3dx^n}{n}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
a^3*c*x + 1/4*b^3*d*x^(4*n)/n + a*b^2*d*x^(3*n)/n + 3/2*a^2*b*d*x^(2*n)/n
+ b^3*c*x^(3*n + 1)/(3*n + 1) + 3*a*b^2*c*x^(2*n + 1)/(2*n + 1) + 3*a^2*b*
c*x^(n + 1)/(n + 1) + a^3*d*x^n/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(82) = 164$.

Time = 0.14 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.67

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx$$
$$= \frac{24\,a^3cn^4x+8\,b^3cn^3xx^{3\,n}+36\,ab^2cn^3xx^{2\,n}+72\,a^2bcn^3xx^n+44\,a^3cn^3x+6\,b^3dn^3x^{4\,n}+24\,ab^2dn^3x^{3\,n}+1}{}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^3,x, algorithm="giac")
```

output
```
1/4*(24*a^3*c*n^4*x + 8*b^3*c*n^3*x*x^(3*n) + 36*a*b^2*c*n^3*x*x^(2*n) + 7
2*a^2*b*c*n^3*x*x^n + 44*a^3*c*n^3*x + 6*b^3*d*n^3*x^(4*n) + 24*a*b^2*d*n^
3*x^(3*n) + 12*b^3*c*n^2*x*x^(3*n) + 36*a^2*b*d*n^3*x^(2*n) + 48*a*b^2*c*n
^2*x*x^(2*n) + 24*a^3*d*n^3*x^n + 60*a^2*b*c*n^2*x*x^n + 24*a^3*c*n^2*x +
11*b^3*d*n^2*x^(4*n) + 44*a*b^2*d*n^2*x^(3*n) + 4*b^3*c*n*x*x^(3*n) + 66*a
^2*b*d*n^2*x^(2*n) + 12*a*b^2*c*n*x*x^(2*n) + 44*a^3*d*n^2*x^n + 12*a^2*b*
c*n*x*x^n + 4*a^3*c*n*x + 6*b^3*d*n*x^(4*n) + 24*a*b^2*d*n*x^(3*n) + 36*a^
2*b*d*n*x^(2*n) + 24*a^3*d*n*x^n + b^3*d*x^(4*n) + 4*a*b^2*d*x^(3*n) + 6*a
^2*b*d*x^(2*n) + 4*a^3*d*x^n)/(6*n^4 + 11*n^3 + 6*n^2 + n)
```

Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.37

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^3\,dx = a^3\,c\,x+\frac{a^3\,d\,x^n}{n}+\frac{b^3\,d\,x^{4\,n}}{4\,n}+\frac{b^3\,c\,x\,x^{3\,n}}{3\,n+1}+\frac{3\,a^2\,b\,d\,x^{2\,n}}{2\,n}+\frac{a\,b^2\,d\,x^{3\,n}}{n}+\frac{3\,a\,b^2\,c\,x\,x^{2\,n}}{2\,n+1}+\frac{3\,a^2\,b\,c\,x\,x^n}{n+1}$$

input
```
int((c + d*x^(n - 1))*(a + b*x^n)^3,x)
```

output
```
a^3*c*x + (a^3*d*x^n)/n + (b^3*d*x^(4*n))/(4*n) + (b^3*c*x*x^(3*n))/(3*n +
 1) + (3*a^2*b*d*x^(2*n))/(2*n) + (a*b^2*d*x^(3*n))/n + (3*a*b^2*c*x*x^(2*
n))/(2*n + 1) + (3*a^2*b*c*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 393, normalized size of antiderivative = 4.68

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^3\,dx$$
$$=\frac{36x^{2n}ab^2cn^3x+48x^{2n}ab^2cn^2x+12x^{2n}ab^2cnx+72x^na^2bcn^3x+60x^na^2bcn^2x+12x^na^2bcnx+24x^{3n}a}{}$$

input
```
int((c+d*x^(-1+n))*(a+b*x^n)^3,x)
```

output
```
(6*x**(4*n)*b**3*d*n**3 + 11*x**(4*n)*b**3*d*n**2 + 6*x**(4*n)*b**3*d*n +
x**(4*n)*b**3*d + 24*x**(3*n)*a*b**2*d*n**3 + 44*x**(3*n)*a*b**2*d*n**2 +
24*x**(3*n)*a*b**2*d*n + 4*x**(3*n)*a*b**2*d + 8*x**(3*n)*b**3*c*n**3*x +
12*x**(3*n)*b**3*c*n**2*x + 4*x**(3*n)*b**3*c*n*x + 36*x**(2*n)*a**2*b*d*n
**3 + 66*x**(2*n)*a**2*b*d*n**2 + 36*x**(2*n)*a**2*b*d*n + 6*x**(2*n)*a**2
*b*d + 36*x**(2*n)*a*b**2*c*n**3*x + 48*x**(2*n)*a*b**2*c*n**2*x + 12*x**(
2*n)*a*b**2*c*n*x + 24*x**n*a**3*d*n**3 + 44*x**n*a**3*d*n**2 + 24*x**n*a*
*3*d*n + 4*x**n*a**3*d + 72*x**n*a**2*b*c*n**3*x + 60*x**n*a**2*b*c*n**2*x
 + 12*x**n*a**2*b*c*n*x + 24*a**3*c*n**4*x + 44*a**3*c*n**3*x + 24*a**3*c*
n**2*x + 4*a**3*c*n*x)/(4*n*(6*n**3 + 11*n**2 + 6*n + 1))
```

3.55 $\int (c+dx^{-1+n}) (a+bx^n)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int (c+dx^{-1+n}) (a+bx^n)^2 \, dx = a^2cx + \frac{2abcx^{1+n}}{1+n} + \frac{b^2cx^{1+2n}}{1+2n} + \frac{d(a+bx^n)^3}{3bn}$$

output
```
a^2*c*x+2*a*b*c*x^(1+n)/(1+n)+b^2*c*x^(1+2*n)/(1+2*n)+1/3*d*(a+b*x^n)^3/b/
n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.97

$$\int (c+dx^{-1+n}) (a+bx^n)^2 \, dx$$
$$= \frac{a^3d(1+3n+2n^2) + 3a^2b(1+3n+2n^2)(cnx+dx^n) + 3ab^2(1+2n)x^n(2cnx+d(1+n)x^n) + b^3(1+n}{3bn(1+n)(1+2n)}$$

input
```
Integrate[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

output
```
(a^3*d*(1 + 3*n + 2*n^2) + 3*a^2*b*(1 + 3*n + 2*n^2)*(c*n*x + d*x^n) + 3*a
*b^2*(1 + 2*n)*x^n*(2*c*n*x + d*(1 + n)*x^n) + b^3*(1 + n)*x^(2*n)*(3*c*n*
x + d*(1 + 2*n)*x^n))/(3*b*n*(1 + n)*(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2430, 775, 793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 \left(c+dx^{n-1}\right) dx$$

$$\Big\downarrow \text{2430}$$

$$c\int (bx^n+a)^2dx+d\int x^{n-1}(bx^n+a)^2dx$$

$$\Big\downarrow \text{775}$$

$$c\int \left(2abx^n+b^2x^{2n}+a^2\right)dx+d\int x^{n-1}(bx^n+a)^2dx$$

$$\Big\downarrow \text{793}$$

$$c\int \left(2abx^n+b^2x^{2n}+a^2\right)dx+\frac{d(a+bx^n)^3}{3bn}$$

$$\Big\downarrow \text{2009}$$

$$c\left(a^2x+\frac{2abx^{n+1}}{n+1}+\frac{b^2x^{2n+1}}{2n+1}\right)+\frac{d(a+bx^n)^3}{3bn}$$

input
```
Int[(c + d*x^(-1 + n))*(a + b*x^n)^2,x]
```

output
```
(d*(a + b*x^n)^3)/(3*b*n) + c*(a^2*x + (2*a*b*x^(1 + n))/(1 + n) + (b^2*x^
(1 + 2*n))/(1 + 2*n))
```

Defintions of rubi rules used

rule 775
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

method	result
risch	$a^{2} c x +\frac{b^{2} d \,x^{3 n}}{3 n}+\frac{b \left(n b c x +2 d n a +a d \right) x^{2 n}}{n \left(1+2 n \right)}+\frac{a \left(2 n b c x +d n a +a d \right) x^{n}}{n \left(1+n \right)}$
norman	$a^{2} c x +\frac{a^{2} d \,{\mathrm e}^{n \ln \left(x \right)}}{n}+\frac{b^{2} c x \,{\mathrm e}^{2 n \ln \left(x \right)}}{1+2 n}+\frac{d a b \,{\mathrm e}^{2 n \ln \left(x \right)}}{n}+\frac{b^{2} d \,{\mathrm e}^{3 n \ln \left(x \right)}}{3 n}+\frac{2 a b c x \,{\mathrm e}^{n \ln \left(x \right)}}{1+n}$
parallelrisch	$\frac{2 x \,x^{2 n} x^{-1+n} b^{2} d \,n^{2}+3 x \,x^{2 n} x^{-1+n} b^{2} d n +3 x^{2 n} b^{2} c \,n^{2} x +6 x \,x^{n} x^{-1+n} a b d \,n^{2}+x \,x^{2 n} x^{-1+n} b^{2} d +3 x^{2 n} b^{2} c n x +9 x \,x^{n} x^{-1+n} a b d}{3 n (1+}$
orering	Expression too large to display

input
```
int((c+d*x^(-1+n))*(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x+1/3*b^2*d/n*(x^n)^3+b*(b*c*n*x+2*a*d*n+a*d)/n/(1+2*n)*(x^n)^2+a*(2
*b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(59) = 118$.

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.62

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx$$
$$= \frac{3\,(2\,a^2cn^3+3\,a^2cn^2+a^2cn)x+(2\,b^2dn^2+3\,b^2dn+b^2d)x^{3\,n}+3\,(2\,abdn^2+3\,abdn+abd+(b^2cn^2+b^2c}{3\,(2\,n^3+3\,n^2+n)}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(2*a^2*c*n^3 + 3*a^2*c*n^2 + a^2*c*n)*x + (2*b^2*d*n^2 + 3*b^2*d*n
+ b^2*d)*x^(3*n) + 3*(2*a*b*d*n^2 + 3*a*b*d*n + a*b*d + (b^2*c*n^2 + b^2*c
*n)*x)*x^(2*n) + 3*(2*a^2*d*n^2 + 3*a^2*d*n + a^2*d + 2*(2*a*b*c*n^2 + a*b
*c*n)*x)*x^n)/(2*n^3 + 3*n^2 + n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(53) = 106$.

Time = 0.62 (sec) , antiderivative size = 598, normalized size of antiderivative = 9.80

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)^2\,dx$$
$$= \begin{cases} a^2cx - \frac{a^2d}{x} + 2abc\log(x) - \frac{abd}{x^2} - \frac{b^2c}{x} - \frac{b^2d}{3x^3} \\ a^2cx - \frac{2a^2d}{\sqrt{x}} + 4abc\sqrt{x} - \frac{2abd}{x} + b^2c\log(x) - \frac{2b^2d}{3x^{\frac{3}{2}}} \\ (a+b)^2\,(cx+d\log(x)) \\ \frac{6a^2cn^3x}{6n^3+9n^2+3n} + \frac{9a^2cn^2x}{6n^3+9n^2+3n} + \frac{3a^2cnx}{6n^3+9n^2+3n} + \frac{6a^2dn^2xx^{n-1}}{6n^3+9n^2+3n} + \frac{9a^2dnxx^{n-1}}{6n^3+9n^2+3n} + \frac{3a^2dxx^{n-1}}{6n^3+9n^2+3n} + \frac{12abcn^2xx^n}{6n^3+9n^2+3n} + \frac{6abcnxx}{6n^3+9n^2+} \end{cases}$$

input
```
integrate((c+d*x**(-1+n))*(a+b*x**n)**2,x)
```

output
```
Piecewise((a**2*c*x - a**2*d/x + 2*a*b*c*log(x) - a*b*d/x**2 - b**2*c/x -
b**2*d/(3*x**3), Eq(n, -1)), (a**2*c*x - 2*a**2*d/sqrt(x) + 4*a*b*c*sqrt(x
) - 2*a*b*d/x + b**2*c*log(x) - 2*b**2*d/(3*x**(3/2)), Eq(n, -1/2)), ((a +
 b)**2*(c*x + d*log(x)), Eq(n, 0)), (6*a**2*c*n**3*x/(6*n**3 + 9*n**2 + 3*
n) + 9*a**2*c*n**2*x/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*c*n*x/(6*n**3 + 9*n*
*2 + 3*n) + 6*a**2*d*n**2*x*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 9*a**2*d*
n*x*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*a**2*d*x*x**(n - 1)/(6*n**3 + 9
*n**2 + 3*n) + 12*a*b*c*n**2*x*x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*c*n*x*
x**n/(6*n**3 + 9*n**2 + 3*n) + 6*a*b*d*n**2*x*x**n*x**(n - 1)/(6*n**3 + 9*
n**2 + 3*n) + 9*a*b*d*n*x*x**n*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*a*b*
d*x*x**n*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*c*n**2*x*x**(2*n)/(6*
n**3 + 9*n**2 + 3*n) + 3*b**2*c*n*x*x**(2*n)/(6*n**3 + 9*n**2 + 3*n) + 2*b
**2*d*n**2*x*x**(2*n)*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + 3*b**2*d*n*x*x*
*(2*n)*x**(n - 1)/(6*n**3 + 9*n**2 + 3*n) + b**2*d*x*x**(2*n)*x**(n - 1)/(
6*n**3 + 9*n**2 + 3*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx = a^2cx + \frac{b^2dx^{3n}}{3n} + \frac{abdx^{2n}}{n} + \frac{b^2cx^{2n+1}}{2n+1} + \frac{2abcx^{n+1}}{n+1} + \frac{a^2dx^n}{n}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
a^2*c*x + 1/3*b^2*d*x^(3*n)/n + a*b*d*x^(2*n)/n + b^2*c*x^(2*n + 1)/(2*n +
 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^n/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(59) = 118$.

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.21

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx$$
$$= \frac{6a^2cn^3x + 3b^2cn^2xx^{2n} + 12abcn^2xx^n + 9a^2cn^2x + 2b^2dn^2x^{3n} + 6abdn^2x^{2n} + 3b^2cnxx^{2n} + 6a^2dn^2x^n}{3(2n^3 + 3n^2 -}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n)^2,x, algorithm="giac")
```

output
```
1/3*(6*a^2*c*n^3*x + 3*b^2*c*n^2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 9*a^2*c*
n^2*x + 2*b^2*d*n^2*x^(3*n) + 6*a*b*d*n^2*x^(2*n) + 3*b^2*c*n*x*x^(2*n) +
6*a^2*d*n^2*x^n + 6*a*b*c*n*x*x^n + 3*a^2*c*n*x + 3*b^2*d*n*x^(3*n) + 9*a*
b*d*n*x^(2*n) + 9*a^2*d*n*x^n + b^2*d*x^(3*n) + 3*a*b*d*x^(2*n) + 3*a^2*d*
x^n)/(2*n^3 + 3*n^2 + n)
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx = a^2\,c\,x+\frac{a^2\,d\,x^n}{n}+\frac{b^2\,d\,x^{3\,n}}{3\,n}+\frac{b^2\,c\,x\,x^{2\,n}}{2\,n+1}+\frac{a\,b\,d\,x^{2\,n}}{n}+\frac{2\,a\,b\,c\,x\,x^n}{n+1}$$

input
```
int((c + d*x^(n - 1))*(a + b*x^n)^2,x)
```

output
```
a^2*c*x + (a^2*d*x^n)/n + (b^2*d*x^(3*n))/(3*n) + (b^2*c*x*x^(2*n))/(2*n +
 1) + (a*b*d*x^(2*n))/n + (2*a*b*c*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.23

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)^2\,dx$$
$$=\frac{2x^{3n}b^2d\,n^2+3x^{3n}b^2dn+x^{3n}b^2d+6x^{2n}abd\,n^2+9x^{2n}abdn+3x^{2n}abd+3x^{2n}b^2c\,n^2x+3x^{2n}b^2cnx+6x^n a}{3n\left(2n^2+3n+1\right.}$$

input
```
int((c+d*x^(-1+n))*(a+b*x^n)^2,x)
```

output
```
(2*x**(3*n)*b**2*d*n**2 + 3*x**(3*n)*b**2*d*n + x**(3*n)*b**2*d + 6*x**(2*
n)*a*b*d*n**2 + 9*x**(2*n)*a*b*d*n + 3*x**(2*n)*a*b*d + 3*x**(2*n)*b**2*c*
n**2*x + 3*x**(2*n)*b**2*c*n*x + 6*x**n*a**2*d*n**2 + 9*x**n*a**2*d*n + 3*
x**n*a**2*d + 12*x**n*a*b*c*n**2*x + 6*x**n*a*b*c*n*x + 6*a**2*c*n**3*x +
9*a**2*c*n**2*x + 3*a**2*c*n*x)/(3*n*(2*n**2 + 3*n + 1))
```

3.56 $\int \left(c+dx^{-1+n}\right) (a+bx^n) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \left(c+dx^{-1+n}\right) (a+bx^n) \, dx = acx+\frac{bcx^{1+n}}{1+n}+\frac{d(a+bx^n)^2}{2bn}$$

output
```
a*c*x+b*c*x^(1+n)/(1+n)+1/2*d*(a+b*x^n)^2/b/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \left(c+dx^{-1+n}\right) (a+bx^n) \, dx = \frac{2a(cnx+dx^n)+bx^n\left(\frac{2cnx}{1+n}+dx^n\right)}{2n}$$

input
```
Integrate[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

output
```
(2*a*(c*n*x + d*x^n) + b*x^n*((2*c*n*x)/(1 + n) + d*x^n))/(2*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2430, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)\left(c+dx^{n-1}\right)\,dx$$

$$\Big\downarrow \text{2430}$$

$$c\int (bx^n+a)\,dx+d\int x^{n-1}(bx^n+a)\,dx$$

$$\Big\downarrow \text{802}$$

$$c\int (bx^n+a)\,dx+d\int \left(ax^{n-1}+bx^{2n-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\left(ax+\frac{bx^{n+1}}{n+1}\right)+d\left(\frac{ax^n}{n}+\frac{bx^{2n}}{2n}\right)$$

input
```
Int[(c + d*x^(-1 + n))*(a + b*x^n),x]
```

output
```
d*((a*x^n)/n + (b*x^(2*n))/(2*n)) + c*(a*x + (b*x^(1 + n))/(1 + n))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

method	result
risch	$acx+\frac{bd\,x^{2n}}{2n}+\frac{(nbcx+dna+ad)x^{n}}{n(1+n)}$
norman	$acx+\frac{ad\,\mathrm{e}^{n\ln(x)}}{n}+\frac{cbx\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{bd\,\mathrm{e}^{2n\ln(x)}}{2n}$
parallelrisch	$\frac{x\,x^{n}x^{-1+n}bdn+x\,x^{n}x^{-1+n}bd+2x^{n}bcnx+2x\,x^{-1+n}adn+2acx\,n^{2}+2x\,x^{-1+n}ad+2acxn}{2n(1+n)}$
orering	$x\left(c+d\,x^{-1+n}\right)\left(a+b\,x^{n}\right)-\frac{(3bc\,n^{2}x+3ad\,n^{2}-nbcx+dna-2ad)x^{2}\left(\frac{d\,x^{-1+n}(-1+n)(a+b\,x^{n})}{x}+\frac{\left(c+d\,x^{-1+n}\right)b\,x^{n}n}{x}\right.}{2n^{2}(1+n)(cbx+ad)}$

input
```
int((c+d*x^(-1+n))*(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+1/2*b*d/n*(x^n)^2+(b*c*n*x+a*d*n+a*d)/n/(1+n)*x^n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int\left(c+dx^{-1+n}\right)(a+bx^{n})\,dx=\frac{2\,(acn^{2}+acn)x+(bdn+bd)x^{2\,n}+2\,(bcnx+adn+ad)x^{n}}{2\,(n^{2}+n)}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*(a*c*n^2 + a*c*n)*x + (b*d*n + b*d)*x^(2*n) + 2*(b*c*n*x + a*d*n +
a*d)*x^n)/(n^2 + n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(31) = 62$.

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.74

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = \begin{cases} acx - \frac{ad}{x} + bc\log(x) - \frac{bd}{2x^2} & \text{for } n=-1 \\ (a+b)(cx+d\log(x)) & \text{for } n=0 \\ \frac{2acn^2x}{2n^2+2n} + \frac{2acnx}{2n^2+2n} + \frac{2adnxx^{n-1}}{2n^2+2n} + \frac{2adxx^{n-1}}{2n^2+2n} + \frac{2bcnxx^n}{2n^2+2n} + \frac{bdnxx^nx^{n-1}}{2n^2+2n} + \frac{bdxx^nx^{n-1}}{2n^2+2n} & \text{otherwise} \end{cases}$$

input
```
integrate((c+d*x**(-1+n))*(a+b*x**n),x)
```

output
```
Piecewise((a*c*x - a*d/x + b*c*log(x) - b*d/(2*x**2), Eq(n, -1)), ((a + b)
*(c*x + d*log(x)), Eq(n, 0)), (2*a*c*n**2*x/(2*n**2 + 2*n) + 2*a*c*n*x/(2*
n**2 + 2*n) + 2*a*d*n*x*x**(n - 1)/(2*n**2 + 2*n) + 2*a*d*x*x**(n - 1)/(2*
n**2 + 2*n) + 2*b*c*n*x*x**n/(2*n**2 + 2*n) + b*d*n*x*x**n*x**(n - 1)/(2*n
**2 + 2*n) + b*d*x*x**n*x**(n - 1)/(2*n**2 + 2*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \left(c+dx^{-1+n}\right)(a+bx^n)\,dx = acx + \frac{bdx^{2n}}{2n} + \frac{bcx^{n+1}}{n+1} + \frac{adx^n}{n}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="maxima")
```

output
```
a*c*x + 1/2*b*d*x^(2*n)/n + b*c*x^(n + 1)/(n + 1) + a*d*x^n/n
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)\,dx = \frac{2\,acn^2x+2\,bcnxx^n+2\,acnx+bdnx^{2\,n}+2\,adnx^n+bdx^{2\,n}+2\,adx^n}{2\,(n^2+n)}$$

input
```
integrate((c+d*x^(-1+n))*(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*(2*a*c*n^2*x + 2*b*c*n*x*x^n + 2*a*c*n*x + b*d*n*x^(2*n) + 2*a*d*n*x^n
 + b*d*x^(2*n) + 2*a*d*x^n)/(n^2 + n)
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)\,dx = a\,c\,x+\frac{a\,d\,x^n}{n}+\frac{b\,d\,x^{2\,n}}{2\,n}+\frac{b\,c\,x\,x^n}{n+1}$$

input
```
int((c + d*x^(n - 1))*(a + b*x^n),x)
```

output
```
a*c*x + (a*d*x^n)/n + (b*d*x^(2*n))/(2*n) + (b*c*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \left(c+dx^{-1+n}\right)\left(a+bx^n\right)\,dx = \frac{x^{2n}bdn+x^{2n}bd+2x^nadn+2x^nad+2x^nbcnx+2ac\,n^2x+2acnx}{2n\,(n+1)}$$

input
```
int((c+d*x^(-1+n))*(a+b*x^n),x)
```

output
```
(x**(2*n)*b*d*n + x**(2*n)*b*d + 2*x**n*a*d*n + 2*x**n*a*d + 2*x**n*b*c*n*
x + 2*a*c*n**2*x + 2*a*c*n*x)/(2*n*(n + 1))
```

3.57 $\int (c+dx^{-1+n}) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (c+dx^{-1+n}) \, dx = cx + \frac{dx^n}{n}$$

output
```
c*x+d*x^n/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (c+dx^{-1+n}) \, dx = cx + \frac{dx^n}{n}$$

input
```
Integrate[c + d*x^(-1 + n),x]
```

output
```
c*x + (d*x^n)/n
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^{n-1}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$cx+\frac{dx^n}{n}$$

input
```
Int[c + d*x^(-1 + n),x]
```

output
```
c*x + (d*x^n)/n
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$cx + \frac{d\,x^n}{n}$	13
parts	$cx + \frac{d\,x^n}{n}$	13
risch	$cx + \frac{dx\,x^{-1+n}}{n}$	16
parallelrisch	$cx + \frac{dx\,x^{-1+n}}{n}$	16
norman	$cx + \frac{dx\,\mathrm{e}^{(-1+n)\ln(x)}}{n}$	18
orering	$x(c + d\,x^{-1+n}) - \frac{xd\,x^{-1+n}(-1+n)}{n}$	28

input
```
int(c+d*x^(-1+n),x,method=_RETURNVERBOSE)
```

output
```
c*x+d*x^n/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \left(c + dx^{-1+n}\right)\,dx = \frac{cnx + dxx^{n-1}}{n}$$

input
```
integrate(c+d*x^(-1+n),x, algorithm="fricas")
```

output
```
(c*n*x + d*x*x^(n - 1))/n
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right)\,dx=cx+d\left(\begin{cases}\frac{x^n}{n} & \text{for } n\neq 0\\ \log\left(x\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(c+d*x**(-1+n),x)
```

output
```
c*x + d*Piecewise((x**n/n, Ne(n, 0)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right)\,dx=cx+\frac{dx^n}{n}$$

input
```
integrate(c+d*x^(-1+n),x, algorithm="maxima")
```

output
```
c*x + d*x^n/n
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right)\,dx=cx+\frac{dx^n}{n}$$

input
```
integrate(c+d*x^(-1+n),x, algorithm="giac")
```

output
```
c*x + d*x^n/n
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(c+dx^{-1+n}\right)\,dx = c\,x+\frac{d\,x^n}{n}$$

input
```
int(c + d*x^(n - 1),x)
```

output
```
c*x + (d*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \left(c+dx^{-1+n}\right)\,dx = \frac{x^n d+cnx}{n}$$

input
```
int(c+d*x^(-1+n),x)
```

output
```
(x**n*d + c*n*x)/n
```

3.58 $\int \frac{c+dx^{-1+n}}{a+bx^n} dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

output
```
c*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a+d*ln(a+b*x^n)/b/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

input
```
Integrate[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

output
```
(c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a
+ b*x^n])/(b*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2430, 778, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n-1}}{a+bx^n}dx$$

$$\Big\downarrow \text{2430}$$

$$c\int \frac{1}{bx^n+a}dx+d\int \frac{x^{n-1}}{bx^n+a}dx$$

$$\Big\downarrow \text{778}$$

$$d\int \frac{x^{n-1}}{bx^n+a}dx+\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}$$

$$\Big\downarrow \text{792}$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d\log\left(a+bx^n\right)}{bn}$$

input
```
Int[(c + d*x^(-1 + n))/(a + b*x^n),x]
```

output
```
(c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a + (d*Log[a
+ b*x^n])/(b*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Maple [F]

$$\int \frac{c + d\,x^{-1+n}}{a + b\,x^{n}}dx$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n),x)
```

output
```
int((c+d*x^(-1+n))/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{c + dx^{-1+n}}{a + bx^{n}}\,dx = \int \frac{dx^{n-1} + c}{bx^{n} + a}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + d\left(\begin{cases}\frac{x^n}{an} & \text{for } b=0\\ \tilde{\infty}x^n & \text{for } n=0\\ \frac{\log(an+bnx^n)}{bn} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((c+d*x**(-1+n))/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) + d*Piecewise((x**n/(a*n), Eq(b, 0)), (zoo*x*
*n, Eq(n, 0)), (log(a*n + b*n*x**n)/(b*n), True))
```

Maxima [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="maxima")
```

output
```
d*log(x)/b + integrate((b*c*x - a*d)/(b^2*x*x^n + a*b*x), x)
```

Giac [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx = \int \frac{dx^{n-1}+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^(n - 1) + c)/(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx=\frac{c\,x\,{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a}+\frac{d\,\ln\left(a+b\,x^n\right)}{b\,n}$$

input
```
int((c + d*x^(n - 1))/(a + b*x^n),x)
```

output
```
(c*x*hypergeom([1, 1/n], 1/n + 1, -(b*x^n)/a))/a + (d*log(a + b*x^n))/(b*n
)
```

Reduce [F]

$$\int \frac{c+dx^{-1+n}}{a+bx^n}\,dx=\frac{\left(\int \frac{1}{x^nb+a}dx\right)bcn+\log(x^nb+a)\,d}{bn}$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n),x)
```

output
```
(int(1/(x**n*b + a),x)*b*c*n + log(x**n*b + a)*d)/(b*n)
```

3.59 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} dx = -\frac{d}{bn(a+bx^n)} + \frac{cx \operatorname{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)}{a^2}$$

output
```
-d/b/n/(a+b*x^n)+c*x*hypergeom([2, 1/n],[1+1/n],-b*x^n/a)/a^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} dx = -\frac{d}{abn+b^2nx^n} + \frac{cx \operatorname{Hypergeometric2F1}\left(2, \frac{1}{n}, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)}{a^2}$$

input
```
Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

output
```
-(d/(a*b*n + b^2*n*x^n)) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -
((b*x^n)/a)])/a^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2430, 778, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n-1}}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{2430}$$

$$c\int \frac{1}{(bx^n+a)^2}dx+d\int \frac{x^{n-1}}{(bx^n+a)^2}dx$$

$$\Big\downarrow \text{778}$$

$$d\int \frac{x^{n-1}}{(bx^n+a)^2}dx+\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}$$

$$\Big\downarrow \text{793}$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}-\frac{d}{bn\,(a+bx^n)}$$

input
```
Int[(c + d*x^(-1 + n))/(a + b*x^n)^2,x]
```

output
```
-(d/(b*n*(a + b*x^n))) + (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((
b*x^n)/a)])/a^2
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx=\int \frac{dx^{n-1}+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x^(n - 1) + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.76 (sec) , antiderivative size = 388, normalized size of antiderivative = 8.82

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2}\,dx = c\left(\frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}nx\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{aa^{\frac{1}{n}}a^{-2-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bnxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)} - \frac{a^{\frac{1}{n}}a^{-2-\frac{1}{n}}bxx^n\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^3\Gamma\left(1+\frac{1}{n}\right)+bn^3x^n\Gamma\left(1+\frac{1}{n}\right)}\right) + d\left(\begin{cases}\tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0\\ -\frac{xx^{-2n}x^{n-1}}{b^2n} & \text{for } a=0\\ \frac{\tilde{\infty}xx^{n-1}}{n} & \text{for } b=-ax^{-n}\\ \frac{\log(x)}{(a+b)^2} & \text{for } n=0\\ \frac{xx^{n-1}}{a^2n+abnx^n} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((c+d*x**(-1+n))/(a+b*x**n)**2,x)
```

output
```
c*(a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)
*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1
/n)*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamm
a(1 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a
, 1, 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))
+ a**(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**
(1/n)*a**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + d*Piecewi
se((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (-x*x**(n - 1)/(b**2*n*x*
*(2*n)), Eq(a, 0)), (zoo*x*x**(n - 1)/n, Eq(b, -a/x**n)), (log(x)/(a + b)*
*2, Eq(n, 0)), (x*x**(n - 1)/(a**2*n + a*b*n*x**n), True))
```

Maxima [F]

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^2}\,dx = \int \frac{dx^{n-1} + c}{(bx^n + a)^2}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
c*(n - 1)*integrate(1/(a*b*n*x^n + a^2*n), x) + (b*c*x - a*d)/(a*b^2*n*x^n
 + a^2*b*n)
```

Giac [F]

$$\int \frac{c + dx^{-1+n}}{(a + bx^n)^2}\,dx = \int \frac{dx^{n-1} + c}{(bx^n + a)^2}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^(n - 1) + c)/(b*x^n + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} dx = \frac{c\,x\,{}_2\mathrm{F}_1\left(2,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^2} - \frac{a\,d}{b\,(a^2\,n+a\,b\,n\,x^n)}$$

input
```
int((c + d*x^(n - 1))/(a + b*x^n)^2,x)
```

output
```
(c*x*hypergeom([2, 1/n], 1/n + 1, -(b*x^n)/a))/a^2 - (a*d)/(b*(a^2*n + a*b
*n*x^n))
```

Reduce [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^2} dx = \frac{x^n\left(\int \frac{1}{x^{2n}b^2+2x^n ab+a^2}dx\right) abcn + x^n d + \left(\int \frac{1}{x^{2n}b^2+2x^n ab+a^2}dx\right) a^2cn}{an\,(x^n b+a)}$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n)^2,x)
```

output
```
(x**n*int(1/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)*a*b*c*n + x**n*d + int(
1/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)*a**2*c*n)/(a*n*(x**n*b + a))
```

3.60 $\int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx = -\frac{d}{2bn(a+bx^n)^2} + \frac{cx \,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

output
```
-1/2*d/b/n/(a+b*x^n)^2+c*x*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3} dx = \frac{-a^3d + 2bcnx(a+bx^n)^2 \,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^3bn(a+bx^n)^2}$$

input
```
Integrate[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

output
```
(-(a^3*d) + 2*b*c*n*x*(a + b*x^n)^2*Hypergeometric2F1[3, n^(-1), 1 + n^(-1
), -((b*x^n)/a)])/(2*a^3*b*n*(a + b*x^n)^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2430, 778, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n-1}}{(a+bx^n)^3}dx$$

$$\Big\downarrow \; 2430$$

$$c\int \frac{1}{(bx^n+a)^3}dx+d\int \frac{x^{n-1}}{(bx^n+a)^3}dx$$

$$\Big\downarrow \; 778$$

$$d\int \frac{x^{n-1}}{(bx^n+a)^3}dx+\frac{cx\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

$$\Big\downarrow \; 793$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}-\frac{d}{2bn\,(a+bx^n)^2}$$

input
```
Int[(c + d*x^(-1 + n))/(a + b*x^n)^3,x]
```

output
```
-1/2*d/(b*n*(a + b*x^n)^2) + (c*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/a^3
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2430
```
Int[((A_) + (B_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Simp[A   Int[(a + b*x^n)^p, x], x] + Simp[B   Int[x^m*(a + b*x^n)^p, x], x]
 /; FreeQ[{a, b, A, B, m, n, p}, x] && EqQ[m - n + 1, 0]
```

Maple [F]

$$\int \frac{c+d\,x^{-1+n}}{\left(a+b\,x^{n}\right)^{3}}dx$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx=\int \frac{dx^{n-1}+c}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d*x^(n - 1) + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n +
a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.36 (sec) , antiderivative size = 1367, normalized size of antiderivative = 29.72

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c+d*x**(-1+n))/(a+b*x**n)**3,x)
```

output
```
c*(2*a**2*a**(1/n)*a**(-3 - 1/n)*n**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1
+ 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 3*a**2*a**(1/n)*a**(-3 - 1
/n)*n**2*x*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(
1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) - 3*a**2*a**(1/n)*a**(-3 -
 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n*
*4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*
gamma(1 + 1/n)) - a**2*a**(1/n)*a**(-3 - 1/n)*n*x*gamma(1/n)/(2*a**2*n**4*
gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gam
ma(1 + 1/n)) + a**2*a**(1/n)*a**(-3 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gam
ma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 4*a*a**(1/n)*a**(-3 -
 1/n)*b*n**2*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/
(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4
*x**(2*n)*gamma(1 + 1/n)) + 2*a*a**(1/n)*a**(-3 - 1/n)*b*n**2*x*x**n*gamma
(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**
2*n**4*x**(2*n)*gamma(1 + 1/n)) - 6*a*a**(1/n)*a**(-3 - 1/n)*b*n*x*x**n*le
rchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 +
 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/
n)) - a*a**(1/n)*a**(-3 - 1/n)*b*n*x*x**n*gamma(1/n)/(2*a**2*n**4*gamma...
```

Maxima [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx=\int \frac{dx^{n-1}+c}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(2*n^2 - 3*n + 1)*c*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b^2
*c*(2*n - 1)*x*x^n + a*b*c*(3*n - 1)*x - a^2*d*n)/(a^2*b^3*n^2*x^(2*n) + 2
*a^3*b^2*n^2*x^n + a^4*b*n^2)
```

Giac [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx=\int \frac{dx^{n-1}+c}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^(-1+n))/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x^(n - 1) + c)/(b*x^n + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx=\frac{c\,x\,{}_2\mathrm{F}_1\left(3,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{a^3}-\frac{d}{2\,b\,\left(a^2\,n+b^2\,n\,x^{2\,n}+2\,a\,b\,n\,x^n\right)}$$

input
```
int((c + d*x^(n - 1))/(a + b*x^n)^3,x)
```

output
```
(c*x*hypergeom([3, 1/n], 1/n + 1, -(b*x^n)/a))/a^3 - d/(2*b*(a^2*n + b^2*n
*x^(2*n) + 2*a*b*n*x^n))
```

Reduce [F]

$$\int \frac{c+dx^{-1+n}}{(a+bx^n)^3}\,dx$$
$$= \frac{2x^{2n}\left(\int \frac{1}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2b+a^3}dx\right)b^3cn+4x^n\left(\int \frac{1}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2b+a^3}dx\right)a\,b^2cn+2\left(\int \frac{1}{x^{3n}b^3+3x^{2n}a\,b^2+3x^n\right.}{2bn\left(x^{2n}b^2+2x^n ab+a^2\right)}$$

input
```
int((c+d*x^(-1+n))/(a+b*x^n)^3,x)
```

output
```
(2*x**(2*n)*int(1/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**
3),x)*b**3*c*n + 4*x**n*int(1/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*
a**2*b + a**3),x)*a*b**2*c*n + 2*int(1/(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2
+ 3*x**n*a**2*b + a**3),x)*a**2*b*c*n - d)/(2*b*n*(x**(2*n)*b**2 + 2*x**n*
a*b + a**2))
```

3.61 $\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 95

$$\begin{aligned}\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx &= \frac{ex}{b} \\ &+ \frac{2dx^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a(2+n)} \\ &+ \frac{(bc-ae)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}\end{aligned}$$

output
```
e*x/b+2*d*x^(1+1/2*n)*hypergeom([1, 1/2+1/n],[3/2+1/n],-b*x^n/a)/a/(2+n)+(
-a*e+b*c)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx = \frac{x\left(2bdx^{n/2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-\frac{bx^n}{a}\right)-(2+n)\left(-ae+(-bc+\cdots\right.\right.}{ab(2+n)}$$

input
```
Integrate[(c + d*x^(n/2) + e*x^n)/(a + b*x^n),x]
```

output
```
(x*(2*b*d*x^(n/2)*Hypergeometric2F1[1, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x^
n)/a)] - (2 + n)*(-(a*e) + (-(b*c) + a*e)*Hypergeometric2F1[1, n^(-1), 1 +
 n^(-1), -((b*x^n)/a)])))/(a*b*(2 + n))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(\frac{-ae+bc+bdx^{n/2}}{b\,(a+bx^n)}+\frac{e}{b}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x(bc-ae)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}+\frac{2dx^{\frac{n+2}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a(n+2)}+\frac{ex}{b}$$

input
```
Int[(c + d*x^(n/2) + e*x^n)/(a + b*x^n),x]
```

output
```
(e*x)/b + (2*d*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2
, -((b*x^n)/a)])/(a*(2 + n)) + ((b*c - a*e)*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((b*x^n)/a)])/(a*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{c + d\,x^{\frac{n}{2}} + e\,x^{n}}{a + b\,x^{n}}dx$$

input
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n),x)
```

output
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{c + dx^{n/2} + ex^n}{a + bx^n}\,dx = \int \frac{dx^{\frac{1}{2}\,n} + ex^n + c}{bx^n + a}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(1/2*n) + e*x^n + c)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.56

$$\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}dx^{\frac{n}{2}+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{n}\right)}{2n\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}dx^{\frac{n}{2}+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)} - \frac{a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}ex\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**(1/2*n)+e*x**n)/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) + a**(-3/2 - 1/n)*a**(1/2 + 1/n)*d*x**(n/2 +
1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/2 + 1/n)*gamma(1/2 + 1/n)/(2*n*
gamma(3/2 + 1/n)) + a**(-3/2 - 1/n)*a**(1/2 + 1/n)*d*x**(n/2 + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, 1/2 + 1/n)*gamma(1/2 + 1/n)/(n**2*gamma(3/2
+ 1/n)) - a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*e*x*lerchphi(a*exp_polar(I*p
i)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1
/n))
```

Maxima [F]

$$\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx = \int \frac{dx^{\frac{1}{2}n}+ex^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
e*x/b + integrate((b*d*x^(1/2*n) + b*c - a*e)/(b^2*x^n + a*b), x)
```

Giac [F]

$$\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx=\int \frac{dx^{\frac{1}{2}n}+ex^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^(1/2*n) + e*x^n + c)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx=\int \frac{c+e\,x^n+d\,x^{n/2}}{a+b\,x^n}\,dx$$

input
```
int((c + e*x^n + d*x^(n/2))/(a + b*x^n),x)
```

output
```
int((c + e*x^n + d*x^(n/2))/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{c+dx^{n/2}+ex^n}{a+bx^n}\,dx=\frac{\left(\int \frac{x^{\frac{n}{2}}}{x^nb+a}dx\right)bd-\left(\int \frac{1}{x^nb+a}dx\right)ae+\left(\int \frac{1}{x^nb+a}dx\right)bc+ex}{b}$$

input
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n),x)
```

output
```
(int(x**(n/2)/(x**n*b + a),x)*b*d - int(1/(x**n*b + a),x)*a*e + int(1/(x**
n*b + a),x)*b*c + e*x)/b
```

3.62 $\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 143

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2} \, dx = \frac{x\left(bc-ae+bdx^{n/2}\right)}{abn\,(a+bx^n)} - \frac{d(2-n)x^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a^2n(2+n)} + \frac{(ae-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}$$

output
```
x*(b*c-a*e+b*d*x^(1/2*n))/a/b/n/(a+b*x^n)-d*(2-n)*x^(1+1/2*n)*hypergeom([1
, 1/2+1/n],[3/2+1/n],-b*x^n/a)/a^2/n/(2+n)+(a*e-b*c*(1-n))*x*hypergeom([1,
 1/n],[1+1/n],-b*x^n/a)/a^2/b/n
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2} \, dx = \frac{x\left(ae(2+n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+2bdx^{n/2}\,\text{Hypergeometric2F}\right.}{a^2b}$$

input
```
Integrate[(c + d*x^(n/2) + e*x^n)/(a + b*x^n)^2,x]
```

output
```
(x*(a*e*(2 + n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)] + 2
*b*d*x^(n/2)*Hypergeometric2F1[2, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x^n)/a)
] + (b*c - a*e)*(2 + n)*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)
/a)]))/(a^2*b*(2 + n))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{-ae+bc+bdx^{n/2}}{b(a+bx^n)^2}+\frac{e}{b(a+bx^n)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(1-n)x(bc-ae)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}-\frac{d(2-n)x^{\frac{n+2}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a^2n(n+2)}+\frac{x\left(-ae+bc+bdx^{n/2}\right)}{abn(a+bx^n)}+\frac{ex\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}$$

input
```
Int[(c + d*x^(n/2) + e*x^n)/(a + b*x^n)^2,x]
```

output
```
(x*(b*c - a*e + b*d*x^(n/2)))/(a*b*n*(a + b*x^n)) - (d*(2 - n)*x^((2 + n)/
2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2, -((b*x^n)/a)])/(a^2*n*(2
 + n)) + (e*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b
) - ((b*c - a*e)*(1 - n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x
^n)/a)])/(a^2*b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^{n}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{c+dx^{n/2}+ex^{n}}{(a+bx^{n})^{2}}\,dx=\int \frac{dx^{\frac{1}{2}\,n}+ex^{n}+c}{(bx^{n}+a)^{2}}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2}\,dx = \text{Timed out}$$

input
```
integrate((c+d*x**(1/2*n)+e*x**n)/(a+b*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2}\,dx = \int \frac{dx^{\frac{1}{2}\,n}+ex^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(b*d*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n) + integrate(1/2*
(b*d*(n - 2)*x^(1/2*n) + 2*b*c*(n - 1) + 2*a*e)/(a*b^2*n*x^n + a^2*b*n), x
)
```

Giac [F]

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2}\,dx = \int \frac{dx^{\frac{1}{2}\,n}+ex^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2}\,dx=\int \frac{c+e\,x^n+d\,x^{n/2}}{(a+b\,x^n)^2}\,dx$$

input
```
int((c + e*x^n + d*x^(n/2))/(a + b*x^n)^2,x)
```

output
```
int((c + e*x^n + d*x^(n/2))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^2}\,dx=\text{Too large to display}$$

input
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^2,x)
```

output
```
(x**n*int(x**(n/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**
n*a*b + a**2*n - a**2),x)*b**2*d*n**2 - 2*x**n*int(x**(n/2)/(x**(2*n)*b**2
*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*b**2*d*
n + x**n*int(x**(n/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*
x**n*a*b + a**2*n - a**2),x)*b**2*d + x**n*int(1/(x**(2*n)*b**2*n - x**(2*
n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*a*b*e*n - x**n*int
(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n -
 a**2),x)*a*b*e + x**n*int(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b
*n - 2*x**n*a*b + a**2*n - a**2),x)*b**2*c*n**2 - 2*x**n*int(1/(x**(2*n)*b
**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*b**2
*c*n + x**n*int(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n
*a*b + a**2*n - a**2),x)*b**2*c + int(x**(n/2)/(x**(2*n)*b**2*n - x**(2*n)
*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*a*b*d*n**2 - 2*int(x
**(n/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**
2*n - a**2),x)*a*b*d*n + int(x**(n/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2
*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*a*b*d + int(1/(x**(2*n)*b**2*
n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*a**2*e*n
 - int(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a*
*2*n - a**2),x)*a**2*e + int(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a
*b*n - 2*x**n*a*b + a**2*n - a**2),x)*a*b*c*n**2 - 2*int(1/(x**(2*n)*b*...
```

3.63 $\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 216

$$\begin{aligned}\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^3} dx &= \frac{x\left(bc-ae+bdx^{n/2}\right)}{2abn\left(a+bx^n\right)^2} \\ &+ \frac{x\left(2(ae-bc(1-2n))-bd(2-3n)x^{n/2}\right)}{4a^2bn^2\left(a+bx^n\right)} \\ &+ \frac{d(2-3n)(2-n)x^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{4a^3n^2(2+n)} \\ &- \frac{(ae-bc(1-2n))(1-n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^3bn^2}\end{aligned}$$

output
```
1/2*x*(b*c-a*e+b*d*x^(1/2*n))/a/b/n/(a+b*x^n)^2+1/4*x*(2*a*e-2*b*c*(1-2*n)
-b*d*(2-3*n)*x^(1/2*n))/a^2/b/n^2/(a+b*x^n)+1/4*d*(2-3*n)*(2-n)*x^(1+1/2*n
)*hypergeom([1, 1/2+1/n],[3/2+1/n],-b*x^n/a)/a^3/n^2/(2+n)-1/2*(a*e-b*c*(1
-2*n))*(1-n)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^3/b/n^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.49

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^3}\,dx = \frac{x\left(ae(2+n)\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+2bdx^{n/2}\,\text{Hypergeometric2F}\right.}{a^3b}$$

input
```
Integrate[(c + d*x^(n/2) + e*x^n)/(a + b*x^n)^3,x]
```

output
```
(x*(a*e*(2 + n)*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)] + 2
*b*d*x^(n/2)*Hypergeometric2F1[3, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x^n)/a)
] + (b*c - a*e)*(2 + n)*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)
/a)]))/(a^3*b*(2 + n))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{-ae+bc+bdx^{n/2}}{b\,(a+bx^n)^3}+\frac{e}{b\,(a+bx^n)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(1-2n)(1-n)x(bc-ae)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^3bn^2}+\frac{d(2-3n)(2-n)x^{\frac{n+2}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{4a^3n^2(n+2)}-\frac{x\left(2(1-2n)(bc-ae)+bd(2-3n)x^{n/2}\right)}{4a^2bn^2\left(a+bx^n\right)}+\frac{ex\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b}+\frac{x\left(-ae+bc+bdx^{n/2}\right)}{2abn\left(a+bx^n\right)^2}$$

input
```
Int[(c + d*x^(n/2) + e*x^n)/(a + b*x^n)^3,x]
```

output
```
(x*(b*c - a*e + b*d*x^(n/2)))/(2*a*b*n*(a + b*x^n)^2) - (x*(2*(b*c - a*e)*
(1 - 2*n) + b*d*(2 - 3*n)*x^(n/2)))/(4*a^2*b*n^2*(a + b*x^n)) + (d*(2 - 3*
n)*(2 - n)*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2, -(
(b*x^n)/a)])/(4*a^3*n^2*(2 + n)) + ((b*c - a*e)*(1 - 2*n)*(1 - n)*x*Hyperg
eometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(2*a^3*b*n^2) + (e*x*Hyp
ergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^{n}}{\left(a+b\,x^{n}\right)^{3}}dx$$

input
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^3}\,dx=\int \frac{dx^{\frac{1}{2}n}+ex^n+c}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d*x^(1/2*n) + e*x^n + c)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*
b*x^n + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^3}\,dx=\text{Timed out}$$

input
```
integrate((c+d*x**(1/2*n)+e*x**n)/(a+b*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^{n/2}+ex^n}{(a+bx^n)^3}\,dx=\int \frac{dx^{\frac{1}{2}n}+ex^n+c}{(bx^n+a)^3}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/4*(b^2*d*(3*n - 2)*x*x^(3/2*n) + a*b*d*(5*n - 2)*x*x^(1/2*n) + 2*(b^2*c*
(2*n - 1) + a*b*e)*x*x^n + 2*(a*b*c*(3*n - 1) - a^2*e*(n - 1))*x)/(a^2*b^3
*n^2*x^(2*n) + 2*a^3*b^2*n^2*x^n + a^4*b*n^2) + integrate(1/8*((3*n^2 - 8*
n + 4)*b*d*x^(1/2*n) + 4*(2*n^2 - 3*n + 1)*b*c + 4*a*e*(n - 1))/(a^2*b^2*n
^2*x^n + a^3*b*n^2), x)
```

Giac [F]

$$\int \frac{c + dx^{n/2} + ex^n}{(a + bx^n)^3} \, dx = \int \frac{dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^3} \, dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^{n/2} + ex^n}{(a + bx^n)^3} \, dx = \int \frac{c + e\,x^n + d\,x^{n/2}}{(a + b\,x^n)^3} \, dx$$

input
```
int((c + e*x^n + d*x^(n/2))/(a + b*x^n)^3,x)
```

output
```
int((c + e*x^n + d*x^(n/2))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{c + dx^{n/2} + ex^n}{(a + bx^n)^3} \, dx = \text{too large to display}$$

input
```
int((c+d*x^(1/2*n)+e*x^n)/(a+b*x^n)^3,x)
```

output

```
(4*x**(2*n)*int(x**(n/2)/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a
*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n -
 a**3),x)*b**3*d*n**2 - 4*x**(2*n)*int(x**(n/2)/(2*x**(3*n)*b**3*n - x**(3
*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x
**n*a**2*b + 2*a**3*n - a**3),x)*b**3*d*n + x**(2*n)*int(x**(n/2)/(2*x**(3
*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x
**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*b**3*d + 2*x**(2*n)*int
(1/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a
*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*a*b**2*e*n -
 x**(2*n)*int(1/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n -
 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)
*a*b**2*e + 4*x**(2*n)*int(1/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*
n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3
*n - a**3),x)*b**3*c*n**2 - 4*x**(2*n)*int(1/(2*x**(3*n)*b**3*n - x**(3*n)
*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n
*a**2*b + 2*a**3*n - a**3),x)*b**3*c*n + x**(2*n)*int(1/(2*x**(3*n)*b**3*n
 - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b
*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*b**3*c + 8*x**n*int(x**(n/2)/(2*x
**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 +
 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*a*b**2*d*n**2 - ...
```

3.64 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n} \, dx = \frac{ex}{b}+\frac{2fx^{\frac{2+n}{2}}}{b(2+n)} + \frac{2(bd-af)x^{\frac{2+n}{2}} \text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{ab(2+n)} + \frac{(bc-ae)x \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}$$

output
```
e*x/b+2*f*x^(1+1/2*n)/b/(2+n)+2*(-a*f+b*d)*x^(1+1/2*n)*hypergeom([1, 1/2+1
/n],[3/2+1/n],-b*x^n/a)/a/b/(2+n)+(-a*e+b*c)*x*hypergeom([1, 1/n],[1+1/n],
-b*x^n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n} \, dx = \frac{x\left(ae(2+n)+2afx^{n/2}+2(bd-af)x^{n/2} \text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1}{n},\right.\right.}{ab(2}$$

input
```
Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n),x]
```

output
```
(x*(a*e*(2 + n) + 2*a*f*x^(n/2) + 2*(b*d - a*f)*x^(n/2)*Hypergeometric2F1[
1, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x^n)/a)] + (b*c - a*e)*(2 + n)*Hyperge
ometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(a*b*(2 + n))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{a + bx^n} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x^{n/2}(bd - af) - ae + bc}{b\,(a + bx^n)} + \frac{e}{b} + \frac{fx^{n/2}}{b} \right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(bc - ae)\,\text{Hypergeometric2F1}\left(1, \frac{1}{n}, 1 + \frac{1}{n}, -\frac{bx^n}{a}\right)}{ab} + \frac{2x^{\frac{n+2}{2}}(bd - af)\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 + \frac{2}{n}\right), \frac{1}{2}\left(3 + \frac{2}{n}\right), -\frac{bx^n}{a}\right)}{ab(n+2)} + \frac{ex}{b} + \frac{2fx^{\frac{n+2}{2}}}{b(n+2)}$$

input
```
Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n),x]
```

output
```
(e*x)/b + (2*f*x^((2 + n)/2))/(b*(2 + n)) + (2*(b*d - a*f)*x^((2 + n)/2)*H
ypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2, -((b*x^n)/a)])/(a*b*(2 + n))
 + ((b*c - a*e)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/
(a*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^{n}+f\,x^{\frac{3n}{2}}}{a+b\,x^{n}}dx$$

input
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n),x)
```

output
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n}\,dx=\int \frac{fx^{\frac{3}{2}\,n}+dx^{\frac{1}{2}\,n}+ex^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n),x, algorithm="fricas
")
```

output
```
integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.20 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.05

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n}\,dx = \frac{a^{\frac{1}{n}}a^{-1-\frac{1}{n}}cx\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{3a^{-\frac{5}{2}-\frac{1}{n}}a^{\frac{3}{2}+\frac{1}{n}}fx^{\frac{3n}{2}+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)}{2n\Gamma\left(\frac{5}{2}+\frac{1}{n}\right)} + \frac{a^{-\frac{5}{2}-\frac{1}{n}}a^{\frac{3}{2}+\frac{1}{n}}fx^{\frac{3n}{2}+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{3}{2}+\frac{1}{n}\right)\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{5}{2}+\frac{1}{n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}dx^{\frac{n}{2}+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{n}\right)}{2n\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)} + \frac{a^{-\frac{3}{2}-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}dx^{\frac{n}{2}+1}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{2}+\frac{1}{n}\right)\Gamma\left(\frac{1}{2}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)} - \frac{a^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}ex\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n),x)
```

output
```
a**(1/n)*a**(-1 - 1/n)*c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) + 3*a**(-5/2 - 1/n)*a**(3/2 + 1/n)*f*x**(3*n/
2 + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3/2 + 1/n)*gamma(3/2 + 1/n)/(
2*n*gamma(5/2 + 1/n)) + a**(-5/2 - 1/n)*a**(3/2 + 1/n)*f*x**(3*n/2 + 1)*le
rchphi(b*x**n*exp_polar(I*pi)/a, 1, 3/2 + 1/n)*gamma(3/2 + 1/n)/(n**2*gamm
a(5/2 + 1/n)) + a**(-3/2 - 1/n)*a**(1/2 + 1/n)*d*x**(n/2 + 1)*lerchphi(b*x
**n*exp_polar(I*pi)/a, 1, 1/2 + 1/n)*gamma(1/2 + 1/n)/(2*n*gamma(3/2 + 1/n
)) + a**(-3/2 - 1/n)*a**(1/2 + 1/n)*d*x**(n/2 + 1)*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 1/2 + 1/n)*gamma(1/2 + 1/n)/(n**2*gamma(3/2 + 1/n)) - a**(1
 + 1/n)*b**(1/n)*b**(-1 - 1/n)*e*x*lerchphi(a*exp_polar(I*pi)/(b*x**n), 1,
 exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n}\,dx=\int \frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n),x, algorithm="maxima
")
```

output
```
(e*(n + 2)*x + 2*f*x*x^(1/2*n))/(b*(n + 2)) - integrate(-(b*c - a*e + (b*d
 - a*f)*x^(1/2*n))/(b^2*x^n + a*b), x)
```

Giac [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n}\,dx=\int \frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{bx^n+a}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n}\,dx=\int \frac{c+e\,x^n+d\,x^{n/2}+f\,x^{\frac{3\,n}{2}}}{a+b\,x^n}\,dx$$

input
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n),x)
```

output
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n), x)
```

Reduce [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{a+bx^n}\,dx=\frac{\left(\int \frac{x^{\frac{3n}{2}}}{x^n b+a}dx\right)bf+\left(\int \frac{x^{\frac{n}{2}}}{x^n b+a}dx\right)bd-\left(\int \frac{1}{x^n b+a}dx\right)ae+\left(\int \frac{1}{x^n b+a}dx\right)bc+}{b}$$

input
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n),x)
```

output
```
(int(x**((3*n)/2)/(x**n*b + a),x)*b*f + int(x**(n/2)/(x**n*b + a),x)*b*d -
 int(1/(x**n*b + a),x)*a*e + int(1/(x**n*b + a),x)*b*c + e*x)/b
```

3.65 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 162

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} \, dx = \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{abn\,(a+bx^n)} - \frac{(bd(2-n)-af(2+n))x^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a^2bn(2+n)} + \frac{(ae-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}$$

output

```
x*(b*c-a*e+(-a*f+b*d)*x^(1/2*n))/a/b/n/(a+b*x^n)-(b*d*(2-n)-a*f*(2+n))*x^(
1+1/2*n)*hypergeom([1, 1/2+1/n],[3/2+1/n],-b*x^n/a)/a^2/b/n/(2+n)+(a*e-b*c
*(1-n))*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a^2/b/n
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2} \, dx = \frac{x\left(\frac{2afx^{n/2}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-\frac{bx^n}{a}\right)}{2+n} + ae\,\text{Hypergeometric2F1}\left(1,\right.\right.}{}$$

input
```
Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

output
```
(x*((2*a*f*x^(n/2)*Hypergeometric2F1[1, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x
^n)/a)])/(2 + n) + a*e*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/
a)] + (2*(b*d - a*f)*x^(n/2)*Hypergeometric2F1[2, 1/2 + n^(-1), 3/2 + n^(-
1), -((b*x^n)/a)])/(2 + n) + (b*c - a*e)*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((b*x^n)/a)]))/(a^2*b)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {2431, 1748, 778, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{2431}$$

$$\frac{\int \frac{2(ae-bc(1-n))-(bd(2-n)-af(n+2))x^{n/2}}{bx^n+a}dx}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{1748}$$

$$\frac{2(ae-bc(1-n))\int \frac{1}{bx^n+a}dx-(bd(2-n)-af(n+2))\int \frac{x^{n/2}}{bx^n+a}dx}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{778}$$

$$\frac{\frac{2x(ae-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}-(bd(2-n)-af(n+2))\int \frac{x^{n/2}}{bx^n+a}dx}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\frac{2x(ae-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}-\frac{2x^{\frac{n+2}{2}}(bd(2-n)-af(n+2))\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{a(n+2)}}{2abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{abn\,(a+bx^n)}$$

input

```
Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^2,x]
```

output

```
(x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(a*b*n*(a + b*x^n)) + ((-2*(b*d*(2 -
 n) - a*f*(2 + n))*x^((2 + n)/2)*Hypergeometric2F1[1, (1 + 2/n)/2, (3 + 2/
n)/2, -((b*x^n)/a)])/(a*(2 + n)) + (2*(a*e - b*c*(1 - n))*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/a)/(2*a*b*n)
```

Defintions of rubi rules used

rule 778

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1748

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Simp[d
   Int[1/(a + c*x^(2*n)), x], x] + Simp[e   Int[x^n/(a + c*x^(2*n)), x], x]
 /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (
PosQ[a*c] ||  !IntegerQ[n])
```

rule 2431
```
Int[(P3_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{A = Coeff[P3, x
^(n/2), 0], B = Coeff[P3, x^(n/2), 1], C = Coeff[P3, x^(n/2), 2], D = Coeff
[P3, x^(n/2), 3]}, Simp[-(x*(b*A - a*C + (b*B - a*D)*x^(n/2))*(a + b*x^n)^(
p + 1))/(a*b*n*(p + 1)), x] - Simp[1/(2*a*b*n*(p + 1))   Int[(a + b*x^n)^(p
 + 1)*Simp[2*a*C - 2*b*A*(n*(p + 1) + 1) + (a*D*(n + 2) - b*B*(n*(2*p + 3)
+ 2))*x^(n/2), x], x], x]] /; FreeQ[{a, b, n}, x] && PolyQ[P3, x^(n/2), 3]
&& ILtQ[p, -1]
```

Maple [F]

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^{n}+f\,x^{\frac{3n}{2}}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

output
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx=\int \frac{fx^{\frac{3}{2}\,n}+dx^{\frac{1}{2}\,n}+ex^{n}+c}{\left(bx^{n}+a\right)^{2}}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="fric
as")
```

output
```
integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n
+ a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \text{Timed out}$$

input
```
integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \int \frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="maxi
ma")
```

output
```
((b*d - a*f)*x*x^(1/2*n) + (b*c - a*e)*x)/(a*b^2*n*x^n + a^2*b*n) + integr
ate(1/2*(2*b*c*(n - 1) + 2*a*e + (a*f*(n + 2) + b*d*(n - 2))*x^(1/2*n))/(a
*b^2*n*x^n + a^2*b*n), x)
```

Giac [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^2}\,dx = \int \frac{fx^{\frac{3}{2}n}+dx^{\frac{1}{2}n}+ex^n+c}{(bx^n+a)^2}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x, algorithm="giac
")
```

output
```
integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^2}\,dx = \int \frac{c + e\,x^n + d\,x^{n/2} + f\,x^{\frac{3\,n}{2}}}{(a + b\,x^n)^2}\,dx$$

input
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2,x)
```

output
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^2}\,dx = \text{Too large to display}$$

input
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^2,x)
```

output

```
(x**n*int(x**((3*n)/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2
*x**n*a*b + a**2*n - a**2),x)*b**2*f*n**2 - 2*x**n*int(x**((3*n)/2)/(x**(2
*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)
*b**2*f*n + x**n*int(x**((3*n)/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**
n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*b**2*f + x**n*int(x**(n/2)/(x**(2
*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)
*b**2*d*n**2 - 2*x**n*int(x**(n/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x*
*n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*b**2*d*n + x**n*int(x**(n/2)/(x*
*(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2)
,x)*b**2*d + x**n*int(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n -
2*x**n*a*b + a**2*n - a**2),x)*a*b*e*n - x**n*int(1/(x**(2*n)*b**2*n - x**
(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*a*b*e + x**n*in
t(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n
- a**2),x)*b**2*c*n**2 - 2*x**n*int(1/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2
*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*b**2*c*n + x**n*int(1/(x**(2*
n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*
b**2*c + int(x**((3*n)/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n
- 2*x**n*a*b + a**2*n - a**2),x)*a*b*f*n**2 - 2*int(x**((3*n)/2)/(x**(2*n)
*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*n - 2*x**n*a*b + a**2*n - a**2),x)*a*
b*f*n + int(x**((3*n)/2)/(x**(2*n)*b**2*n - x**(2*n)*b**2 + 2*x**n*a*b*...
```

3.66 $\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 244

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^3}\,dx = \frac{x\left(bc-ae+(bd-af)x^{n/2}\right)}{2abn\,(a+bx^n)^2}$$
$$+\frac{x\left(2(ae-bc(1-2n))-(bd(2-3n)-af(2+n))x^{n/2}\right)}{4a^2bn^2\,(a+bx^n)}$$
$$+\frac{(2-n)(bd(2-3n)-af(2+n))x^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-\frac{bx^n}{a}\right)}{4a^3bn^2(2+n)}$$
$$-\frac{(ae-bc(1-2n))(1-n)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^3bn^2}$$

output
```
1/2*x*(b*c-a*e+(-a*f+b*d)*x^(1/2*n))/a/b/n/(a+b*x^n)^2+1/4*x*(2*a*e-2*b*c*
(1-2*n)-(b*d*(2-3*n)-a*f*(2+n))*x^(1/2*n))/a^2/b/n^2/(a+b*x^n)+1/4*(2-n)*(
b*d*(2-3*n)-a*f*(2+n))*x^(1+1/2*n)*hypergeom([1, 1/2+1/n],[3/2+1/n],-b*x^n
/a)/a^3/b/n^2/(2+n)-1/2*(a*e-b*c*(1-2*n))*(1-n)*x*hypergeom([1, 1/n],[1+1/
n],-b*x^n/a)/a^3/b/n^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.60

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^3}\,dx = \frac{x\left(\frac{2afx^{n/2}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-\frac{bx^n}{a}\right)}{2+n}+ae\,\text{Hypergeometric2F1}\left(2,\right.\right.}{}$$

input
```
Integrate[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^3,x]
```

output
```
(x*((2*a*f*x^(n/2)*Hypergeometric2F1[2, 1/2 + n^(-1), 3/2 + n^(-1), -((b*x
^n)/a)])/(2 + n) + a*e*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/
a)] + (2*(b*d - a*f)*x^(n/2)*Hypergeometric2F1[3, 1/2 + n^(-1), 3/2 + n^(-
1), -((b*x^n)/a)])/(2 + n) + (b*c - a*e)*Hypergeometric2F1[3, n^(-1), 1 +
n^(-1), -((b*x^n)/a)]))/(a^3*b)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2431, 1761, 27, 1748, 778, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \; 2431$$

$$\frac{\int \frac{2(ae-bc(1-2n))-(bd(2-3n)-af(n+2))x^{n/2}}{(bx^n+a)^2}dx}{4abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{2abn\,(a+bx^n)^2}$$

$$\Big\downarrow \; 1761$$

$$\frac{\frac{x(2(ae-bc(1-2n))-x^{n/2}(bd(2-3n)-af(n+2)))}{an(a+bx^n)}-\frac{\int \frac{4(ae-bc(1-2n))(1-n)-(2-n)(bd(2-3n)-af(n+2))x^{n/2}}{2(bx^n+a)}dx}{an}}{4abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{2abn\left(a+bx^n\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x(2(ae-bc(1-2n))-x^{n/2}(bd(2-3n)-af(n+2)))}{an(a+bx^n)}-\frac{\int \frac{4(ae-bc(1-2n))(1-n)-(2-n)(bd(2-3n)-af(n+2))x^{n/2}}{bx^n+a}dx}{2an}}{4abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{2abn\left(a+bx^n\right)^2}$$

$$\Big\downarrow \; 1748$$

$$\frac{\frac{x(2(ae-bc(1-2n))-x^{n/2}(bd(2-3n)-af(n+2)))}{an(a+bx^n)}-\frac{4(1-n)(ae-bc(1-2n))\int \frac{1}{bx^n+a}dx-(2-n)(bd(2-3n)-af(n+2))\int \frac{x^{n/2}}{bx^n+a}dx}{2an}}{4abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{2abn\left(a+bx^n\right)^2}$$

$$\Big\downarrow \; 778$$

$$\frac{\frac{x(2(ae-bc(1-2n))-x^{n/2}(bd(2-3n)-af(n+2)))}{an(a+bx^n)}-\frac{\frac{4(1-n)x(ae-bc(1-2n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}-(2-n)(bd(2-3n)-af(n+2))\int}{2an}}{4abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{2abn\left(a+bx^n\right)^2}$$

$$\Big\downarrow \; 888$$

$$\frac{\frac{x(2(ae-bc(1-2n))-x^{n/2}(bd(2-3n)-af(n+2)))}{an(a+bx^n)}-\frac{\frac{4(1-n)x(ae-bc(1-2n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a}-\frac{2(2-n)x^{\frac{n+2}{2}}(bd(2-3n)-af(n+2)}{}}{2an}}{4abn}+\frac{x\left(x^{n/2}(bd-af)-ae+bc\right)}{2abn\left(a+bx^n\right)^2}$$

input
```
Int[(c + d*x^(n/2) + e*x^n + f*x^((3*n)/2))/(a + b*x^n)^3,x]
```

output
```
(x*(b*c - a*e + (b*d - a*f)*x^(n/2)))/(2*a*b*n*(a + b*x^n)^2) + ((x*(2*(a*
e - b*c*(1 - 2*n)) - (b*d*(2 - 3*n) - a*f*(2 + n))*x^(n/2)))/(a*n*(a + b*x
^n)) - ((-2*(2 - n)*(b*d*(2 - 3*n) - a*f*(2 + n))*x^((2 + n)/2)*Hypergeome
tric2F1[1, (1 + 2/n)/2, (3 + 2/n)/2, -((b*x^n)/a)])/(a*(2 + n)) + (4*(a*e
- b*c*(1 - 2*n))*(1 - n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x
^n)/a)])/a)/(2*a*n))/(4*a*b*n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1748
```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (c_.)*(x_)^(n2_)), x_Symbol] :> Simp[d
   Int[1/(a + c*x^(2*n)), x], x] + Simp[e   Int[x^n/(a + c*x^(2*n)), x], x]
 /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && (
PosQ[a*c] ||  !IntegerQ[n])
```

rule 1761
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> S
imp[(-x)*(d + e*x^n)*((a + c*x^(2*n))^(p + 1)/(2*a*n*(p + 1))), x] + Simp[1
/(2*a*n*(p + 1))   Int[(d*(2*n*p + 2*n + 1) + e*(2*n*p + 3*n + 1)*x^n)*(a +
 c*x^(2*n))^(p + 1), x], x] /; FreeQ[{a, c, d, e, n}, x] && EqQ[n2, 2*n] &&
 ILtQ[p, -1]
```

rule 2431
```
Int[(P3_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{A = Coeff[P3, x
^(n/2), 0], B = Coeff[P3, x^(n/2), 1], C = Coeff[P3, x^(n/2), 2], D = Coeff
[P3, x^(n/2), 3]}, Simp[-(x*(b*A - a*C + (b*B - a*D)*x^(n/2))*(a + b*x^n)^(
p + 1))/(a*b*n*(p + 1)), x] - Simp[1/(2*a*b*n*(p + 1))   Int[(a + b*x^n)^(p
 + 1)*Simp[2*a*C - 2*b*A*(n*(p + 1) + 1) + (a*D*(n + 2) - b*B*(n*(2*p + 3)
+ 2))*x^(n/2), x], x], x]] /; FreeQ[{a, b, n}, x] && PolyQ[P3, x^(n/2), 3]
&& ILtQ[p, -1]
```

Maple [F]

$$\int \frac{c+d\,x^{\frac{n}{2}}+e\,x^{n}+f\,x^{\frac{3n}{2}}}{\left(a+b\,x^{n}\right)^{3}}dx$$

input
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^3,x)
```

output
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{c+dx^{n/2}+ex^n+fx^{3n/2}}{(a+bx^n)^3}\,dx=\int \frac{fx^{\frac{3}{2}\,n}+dx^{\frac{1}{2}\,n}+ex^{n}+c}{\left(bx^{n}+a\right)^{3}}\,dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^3,x, algorithm="fric
as")
```

output
```
integral((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b^3*x^(3*n) + 3*a*b^2*x^
(2*n) + 3*a^2*b*x^n + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^3} dx = \text{Timed out}$$

input
```
integrate((c+d*x**(1/2*n)+e*x**n+f*x**(3/2*n))/(a+b*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^3} dx = \int \frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^3} dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^3,x, algorithm="maxi
ma")
```

output
```
1/4*((b^2*d*(3*n - 2) + a*b*f*(n + 2))*x*x^(3/2*n) + (a*b*d*(5*n - 2) - a^
2*f*(n - 2))*x*x^(1/2*n) + 2*(b^2*c*(2*n - 1) + a*b*e)*x*x^n + 2*(a*b*c*(3
*n - 1) - a^2*e*(n - 1))*x)/(a^2*b^3*n^2*x^(2*n) + 2*a^3*b^2*n^2*x^n + a^4
*b*n^2) + integrate(1/8*(4*(2*n^2 - 3*n + 1)*b*c + 4*a*e*(n - 1) + ((3*n^2
 - 8*n + 4)*b*d + (n^2 - 4)*a*f)*x^(1/2*n))/(a^2*b^2*n^2*x^n + a^3*b*n^2),
 x)
```

Giac [F]

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^3} dx = \int \frac{fx^{\frac{3}{2}n} + dx^{\frac{1}{2}n} + ex^n + c}{(bx^n + a)^3} dx$$

input
```
integrate((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^3,x, algorithm="giac
")
```

output
```
integrate((f*x^(3/2*n) + d*x^(1/2*n) + e*x^n + c)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^3} \, dx = \int \frac{c + e\,x^n + d\,x^{n/2} + f\,x^{\frac{3\,n}{2}}}{(a + b\,x^n)^3} \, dx$$

input
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^3,x)
```

output
```
int((c + e*x^n + d*x^(n/2) + f*x^((3*n)/2))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{c + dx^{n/2} + ex^n + fx^{3n/2}}{(a + bx^n)^3} \, dx = \text{too large to display}$$

input
```
int((c+d*x^(1/2*n)+e*x^n+f*x^(3/2*n))/(a+b*x^n)^3,x)
```

output

```
(4*x**(2*n)*int(x**((3*n)/2)/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*
n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3
*n - a**3),x)*b**3*f*n**2 - 4*x**(2*n)*int(x**((3*n)/2)/(2*x**(3*n)*b**3*n
 - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b
*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*b**3*f*n + x**(2*n)*int(x**((3*n)
/2)/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*
a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*b**3*f + 4*
x**(2*n)*int(x**(n/2)/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b*
*2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a*
*3),x)*b**3*d*n**2 - 4*x**(2*n)*int(x**(n/2)/(2*x**(3*n)*b**3*n - x**(3*n)
*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n
*a**2*b + 2*a**3*n - a**3),x)*b**3*d*n + x**(2*n)*int(x**(n/2)/(2*x**(3*n)
*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n
*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*b**3*d + 2*x**(2*n)*int(1/
(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*x**(2*n)*a*b*
*2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*a*b**2*e*n - x*
*(2*n)*int(1/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*a*b**2*n - 3*
x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3*n - a**3),x)*a*
b**2*e + 4*x**(2*n)*int(1/(2*x**(3*n)*b**3*n - x**(3*n)*b**3 + 6*x**(2*n)*
a*b**2*n - 3*x**(2*n)*a*b**2 + 6*x**n*a**2*b*n - 3*x**n*a**2*b + 2*a**3...
```

3.67 $\int (a+bx^n)^p\,dx$

Optimal result

Integrand size = 9, antiderivative size = 46

$$\int (a+bx^n)^p\,dx = x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

output
```
x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p\,dx = x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Integrate[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p\,dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int\left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

input
```
Int[(a + b*x^n)^p,x]
```

output
```
(x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (a+b\,x^n)^p\,dx$$

input
```
int((a+b*x^n)^p,x)
```

output
```
int((a+b*x^n)^p,x)
```

Fricas [F]

$$\int (a+bx^n)^p\,dx = \int (bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^p\,dx = \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p,x)
```

output
```
a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p, x)
```

Giac [F]

$$\int (a+bx^n)^p \, dx = \int (bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 7.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int (a+bx^n)^p \, dx = \frac{x\,(a+b\,x^n)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,x^n}{a}\right)}{\left(\frac{b\,x^n}{a}+1\right)^p}$$

input
```
int((a + b*x^n)^p,x)
```

output
```
(x*(a + b*x^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*x^n)/a))/((b*x^n)/a + 1
)^p
```

Reduce [F]

$$\int (a+bx^n)^p \, dx$$
$$= \frac{(x^n b+a)^p x + \left(\int \frac{(x^n b+a)^p}{x^n bnp+x^n b+anp+a}dx\right) a\, n^2 p^2 + \left(\int \frac{(x^n b+a)^p}{x^n bnp+x^n b+anp+a}dx\right) anp}{np+1}$$

input
```
int((a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*x + int((x**n*b + a)**p/(x**n*b*n*p + x**n*b + a*n*p + a)
,x)*a*n**2*p**2 + int((x**n*b + a)**p/(x**n*b*n*p + x**n*b + a*n*p + a),x)
*a*n*p)/(n*p + 1)
```

3.68 $\int (a+bx^n)^p (A+Bx^n) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int (a+bx^n)^p (A+Bx^n) \, dx = \frac{Bx(a+bx^n)^{1+p}}{b(1+n+np)} + \left(A - \frac{aB}{b+bn+bnp}\right) x(a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n}, -p, 1+\frac{1}{n}, -\frac{bx^n}{a}\right)$$

output
```
B*x*(a+b*x^n)^(p+1)/b/(n*p+n+1)+(A-a*B/(b*n*p+b*n+b))*x*(a+b*x^n)^p*hyperg
eom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int (a+bx^n)^p (A+Bx^n) \, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(B(a+bx^n)\left(1+\frac{bx^n}{a}\right)^p + (-aB+Ab(1+n+np)) \text{Hypergeometric2F1}\left(\frac{1}{n}, -p\right.\right.}{b(1+n+np)}$$

input
```
Integrate[(a + b*x^n)^p*(A + B*x^n),x]
```

output
```
(x*(a + b*x^n)^p*(B*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*B) + A*b*(1 + n +
 n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n
 + n*p)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx^n)(a + bx^n)^p \, dx$$

$$\Big\downarrow \text{913}$$

$$\left(A - \frac{aB}{bnp + bn + b}\right) \int (bx^n + a)^p \, dx + \frac{Bx(a + bx^n)^{p+1}}{b(np + n + 1)}$$

$$\Big\downarrow \text{779}$$

$$(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(A - \frac{aB}{bnp + bn + b}\right) \int \left(\frac{bx^n}{a} + 1\right)^p dx + \frac{Bx(a + bx^n)^{p+1}}{b(np + n + 1)}$$

$$\Big\downarrow \text{778}$$

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(A - \frac{aB}{bnp + bn + b}\right) \text{Hypergeometric2F1}\left(\frac{1}{n}, -p, 1 + \frac{1}{n}, -\frac{bx^n}{a}\right) + \frac{Bx(a + bx^n)^{p+1}}{b(np + n + 1)}$$

input
```
Int[(a + b*x^n)^p*(A + B*x^n),x]
```

output
```
(B*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((A - (a*B)/(b + b*n + b*n*p
))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]
)/(1 + (b*x^n)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{p} (A+B\,x^{n})\,dx$$

input
```
int((a+b*x^n)^p*(A+B*x^n),x)
```

output
```
int((a+b*x^n)^p*(A+B*x^n),x)
```

Fricas [F]

$$\int (a+bx^n)^p (A+Bx^n)\ dx = \int (Bx^n+A)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int (a+bx^n)^p\,(A+Bx^n)\,dx = \frac{Aa^{\frac{1}{n}}a^{p-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{Ba^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(A+B*x**n),x)
```

output
```
A*a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + B*a**(1 + 1/n)*a**(p - 1 - 1/n)*x**
(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p\,(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (A+Bx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (A+Bx^n) \, dx = \int (A+B\,x^n)\,(a+b\,x^n)^p \, dx$$

input
```
int((A + B*x^n)*(a + b*x^n)^p,x)
```

output
```
int((A + B*x^n)*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int (a+bx^n)^p (A+Bx^n) \, dx$$
$$= \frac{x^n(x^n b+a)^p bnpx + x^n(x^n b+a)^p bx + 2(x^n b+a)^p anpx + (x^n b+a)^p anx + (x^n b+a)^p ax + \left(\int \frac{}{x^n b n^2 p^2 \cdots}\right.}{}$$

input
```
int((a+b*x^n)^p*(A+B*x^n),x)
```

output

```
(x**n*(x**n*b + a)**p*b*n*p*x + x**n*(x**n*b + a)**p*b*x + 2*(x**n*b + a)*
*p*a*n*p*x + (x**n*b + a)**p*a*n*x + (x**n*b + a)**p*a*x + int((x**n*b + a
)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b
+ a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n**4*p**4 + 2*int((x
**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n
+ x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n**4*p**3 +
 int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x*
*n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n**4
*p**2 + 2*int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b
*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*
a**2*n**3*p**3 + 3*int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p +
 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n
 + a),x)*a**2*n**3*p**2 + int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n
**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*
p + a*n + a),x)*a**2*n**3*p + int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n
*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*
a*n*p + a*n + a),x)*a**2*n**2*p**2 + int((x**n*b + a)**p/(x**n*b*n**2*p**2
 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2
*p + 2*a*n*p + a*n + a),x)*a**2*n**2*p)/(n**2*p**2 + n**2*p + 2*n*p + n +
1)
```

3.69 $\int (a+bx^n)^p (A+Bx^n+Cx^{2n})\, dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int (a+bx^n)^p (A+Bx^n+Cx^{2n})\, dx = -\frac{(aC(1+n)-b(B+Bn(2+p)))x(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{Cx^{1+n}(a+bx^n)^{1+p}}{b(1+n(2+p))} + \left(A+\frac{a(aC(1+n)-b(B+Bn(2+p)))}{b^2(1+n+np)(1+n(2+p))}\right)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

output
```
-(a*C*(1+n)-b*(B+B*n*(2+p)))*x*(a+b*x^n)^(p+1)/b^2/(n*p+n+1)/(1+n*(2+p))+C
*x^(1+n)*(a+b*x^n)^(p+1)/b/(1+n*(2+p))+(A+a*(a*C*(1+n)-b*(B+B*n*(2+p)))/b^
2/(n*p+n+1)/(1+n*(2+p)))*x*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],-b*x^n/
a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}\right) \, dx$$
$$= \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(B(1+2n)x^n \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)+(1+n)\left(Cx^{2n} \text{Hy}\right.\right.}{(1+n)(1}$$

input
```
Integrate[(a + b*x^n)^p*(A + B*x^n + C*x^(2*n)),x]
```

output
```
(x*(a + b*x^n)^p*(B*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^
(-1), -((b*x^n)/a)] + (1 + n)*(C*x^(2*n)*Hypergeometric2F1[2 + n^(-1), -p,
 3 + n^(-1), -((b*x^n)/a)] + A*(1 + 2*n)*Hypergeometric2F1[n^(-1), -p, 1 +
 n^(-1), -((b*x^n)/a)])))/((1 + n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1741, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}\right) \, dx$$

$$\Big\downarrow \text{ 1741}$$

$$\frac{\int (bx^n+a)^p (Ab(n(p+2)+1)-(aC(n+1)-b(n(p+2)B+B))x^n) \, dx}{b(n(p+2)+1)} + \frac{Cx^{n+1}(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

$$\Big\downarrow \text{ 913}$$

$$\frac{\left(\frac{a(aC(n+1)-b(Bn(p+2)+B))}{b(np+n+1)}+Ab(n(p+2)+1)\right)\int (bx^n+a)^p\,dx-\frac{x(a+bx^n)^{p+1}(aC(n+1)-b(Bn(p+2)+B))}{b(np+n+1)}}{b(n(p+2)+1)}+\frac{Cx^{n+1}(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

$\downarrow$ 779

$$\frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{a(aC(n+1)-b(Bn(p+2)+B))}{b(np+n+1)}+Ab(n(p+2)+1)\right)\int\left(\frac{bx^n}{a}+1\right)^p dx-\frac{x(a+bx^n)^{p+1}(aC(n+1)-b(B}{b(np+n+1)}}{b(n(p+2)+1)}$$
$$\frac{Cx^{n+1}(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

$\downarrow$ 778

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)\left(\frac{a(aC(n+1)-b(Bn(p+2)+B))}{b(np+n+1)}+Ab(n(p+2)+1)\right.}{b(n(p+2)+1)}$$
$$\frac{Cx^{n+1}(a+bx^n)^{p+1}}{b(n(p+2)+1)}$$

input
```
Int[(a + b*x^n)^p*(A + B*x^n + C*x^(2*n)),x]
```

output
```
(C*x^(1 + n)*(a + b*x^n)^(1 + p))/(b*(1 + n*(2 + p))) + (-(((a*C*(1 + n) -
 b*(B + B*n*(2 + p)))*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p))) + ((A*b*(1
 + n*(2 + p)) + (a*(a*C*(1 + n) - b*(B + B*n*(2 + p))))/(b*(1 + n + n*p)))
*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p)/(b*(1 + n*(2 + p)))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1741
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Simp[1/(e*(n*(q + 2) + 1))   Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1)
- (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
 n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0]
```

Maple [F]

$$\int (a+b\,x^n)^p\left(A+B\,x^n+C\,x^{2n}\right)dx$$

input
```
int((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)),x)
```

output
```
int((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)),x)
```

Fricas [F]

$$\int (a+bx^n)^p\left(A+Bx^n+Cx^{2n}\right)\,dx=\int (Cx^{2\,n}+Bx^n+A)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)),x, algorithm="fricas")
```

output
```
integral((C*x^(2*n) + B*x^n + A)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 46.94 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.98

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}\right) dx$$
$$= \frac{Aa^{\frac{1}{n}}a^{p-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{Ba^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{Ca^{2+\frac{1}{n}}a^{p-2-\frac{1}{n}}x^{2n+1}\Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 2+\frac{1}{n} \\ 3+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(A+B*x**n+C*x**(2*n)),x)
```

output
```
A*a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + B*a**(1 + 1/n)*a**(p - 1 - 1/n)*x**
(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(2 + 1/n)) + C*a**(2 + 1/n)*a**(p - 2 - 1/n)*x**(2*n + 1)*
gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar(I*pi)/a)/
(n*gamma(3 + 1/n))
```

Maxima [F]

$$\int (a + bx^n)^p \left(A + Bx^n + Cx^{2n}\right) dx = \int (Cx^{2n} + Bx^n + A)(bx^n + a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((C*x^(2*n) + B*x^n + A)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a + bx^n)^p \left(A + Bx^n + Cx^{2n}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,1,0,0]%%%}+%%%{-3,[1,0,4,3,1,2,3,1,0,0]%
%%}+%%%{-
```

Mupad [F(-1)]

Timed out.

$$\int (a + bx^n)^p \left(A + Bx^n + Cx^{2n}\right) dx = \int (a + b\,x^n)^p \left(A + C\,x^{2\,n} + B\,x^n\right) dx$$

input
```
int((a + b*x^n)^p*(A + C*x^(2*n) + B*x^n),x)
```

output
```
int((a + b*x^n)^p*(A + C*x^(2*n) + B*x^n), x)
```

Reduce [F]

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}\right) dx = \text{too large to display}$$

input

```
int((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)),x)
```

output

```
(x**(2*n)*(x**n*b + a)**p*b**2*c*n**2*p**2*x + x**(2*n)*(x**n*b + a)**p*b*
*2*c*n**2*p*x + 2*x**(2*n)*(x**n*b + a)**p*b**2*c*n*p*x + x**(2*n)*(x**n*b
 + a)**p*b**2*c*n*x + x**(2*n)*(x**n*b + a)**p*b**2*c*x + x**n*(x**n*b + a
)**p*a*b*c*n**2*p**2*x + x**n*(x**n*b + a)**p*a*b*c*n*p*x + x**n*(x**n*b +
 a)**p*b**3*n**2*p**2*x + 2*x**n*(x**n*b + a)**p*b**3*n**2*p*x + 2*x**n*(x
**n*b + a)**p*b**3*n*p*x + 2*x**n*(x**n*b + a)**p*b**3*n*x + x**n*(x**n*b
+ a)**p*b**3*x - (x**n*b + a)**p*a**2*c*n**2*p*x - (x**n*b + a)**p*a**2*c*
n*p*x + 2*(x**n*b + a)**p*a*b**2*n**2*p**2*x + 5*(x**n*b + a)**p*a*b**2*n*
*2*p*x + 2*(x**n*b + a)**p*a*b**2*n**2*x + 3*(x**n*b + a)**p*a*b**2*n*p*x
+ 3*(x**n*b + a)**p*a*b**2*n*x + (x**n*b + a)**p*a*b**2*x + int((x**n*b +
a)**p/(x**n*b*n**3*p**3 + 3*x**n*b*n**3*p**2 + 2*x**n*b*n**3*p + 3*x**n*b*
n**2*p**2 + 6*x**n*b*n**2*p + 2*x**n*b*n**2 + 3*x**n*b*n*p + 3*x**n*b*n +
x**n*b + a*n**3*p**3 + 3*a*n**3*p**2 + 2*a*n**3*p + 3*a*n**2*p**2 + 6*a*n*
*2*p + 2*a*n**2 + 3*a*n*p + 3*a*n + a),x)*a**3*c*n**5*p**4 + 3*int((x**n*b
 + a)**p/(x**n*b*n**3*p**3 + 3*x**n*b*n**3*p**2 + 2*x**n*b*n**3*p + 3*x**n
*b*n**2*p**2 + 6*x**n*b*n**2*p + 2*x**n*b*n**2 + 3*x**n*b*n*p + 3*x**n*b*n
 + x**n*b + a*n**3*p**3 + 3*a*n**3*p**2 + 2*a*n**3*p + 3*a*n**2*p**2 + 6*a
*n**2*p + 2*a*n**2 + 3*a*n*p + 3*a*n + a),x)*a**3*c*n**5*p**3 + 2*int((x**
n*b + a)**p/(x**n*b*n**3*p**3 + 3*x**n*b*n**3*p**2 + 2*x**n*b*n**3*p + 3*x
**n*b*n**2*p**2 + 6*x**n*b*n**2*p + 2*x**n*b*n**2 + 3*x**n*b*n*p + 3*x*...
```

3.70 $\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right) dx$

Optimal result

Integrand size = 31, antiderivative size = 371

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right) dx$$
$$= \frac{(a^2D(1+3n+2n^2)-abC(1+n)(1+n(3+p))+b^2B(1+n(5+2p)+n^2(6+5p+p^2)))\,x(a+bx^n)^{1+}}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$
$$-\frac{(aD(1+2n)-b(C+Cn(3+p)))x^{1+n}(a+bx^n)^{1+p}}{b^2(1+n(2+p))(1+n(3+p))}+\frac{Dx^{1+2n}(a+bx^n)^{1+p}}{b(1+n(3+p))}$$
$$+\frac{(Ab^3(1+3n(2+p)+n^2(11+12p+3p^2)+n^3(6+11p+6p^2+p^3))-a(a^2D(1+3n+2n^2)-abC(1}{b^3(1+n}$$

output
```
(a^2*D*(2*n^2+3*n+1)-a*b*C*(1+n)*(1+n*(3+p))+b^2*B*(1+n*(5+2*p)+n^2*(p^2+5
*p+6)))*x*(a+b*x^n)^(p+1)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p))-(a*D*(1+2*
n)-b*(C+C*n*(3+p)))*x^(1+n)*(a+b*x^n)^(p+1)/b^2/(1+n*(2+p))/(1+n*(3+p))+D*
x^(1+2*n)*(a+b*x^n)^(p+1)/b/(1+n*(3+p))+(A*b^3*(1+3*n*(2+p)+n^2*(3*p^2+12*
p+11)+n^3*(p^3+6*p^2+11*p+6))-a*(a^2*D*(2*n^2+3*n+1)-a*b*C*(1+n)*(1+n*(3+p
))+b^2*B*(1+n*(5+2*p)+n^2*(p^2+5*p+6))))*x*(a+b*x^n)^p*hypergeom([-p, 1/n]
,[1+1/n],-b*x^n/a)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.51

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right) dx$$
$$= \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(B(1+5n+6n^2)\,x^n \text{Hypergeometric2F1}\left(1+\frac{1}{n},-p,2+\frac{1}{n},-\frac{bx^n}{a}\right)+(1+n)\,(C\right.}{}$$

input
```
Integrate[(a + b*x^n)^p*(A + B*x^n + C*x^(2*n) + D*x^(3*n)),x]
```

output
```
(x*(a + b*x^n)^p*(B*(1 + 5*n + 6*n^2)*x^n*Hypergeometric2F1[1 + n^(-1), -p
, 2 + n^(-1), -((b*x^n)/a)] + (1 + n)*(C*(1 + 3*n)*x^(2*n)*Hypergeometric2
F1[2 + n^(-1), -p, 3 + n^(-1), -((b*x^n)/a)] + (1 + 2*n)*(D*x^(3*n)*Hyperg
eometric2F1[3 + n^(-1), -p, 4 + n^(-1), -((b*x^n)/a)] + A*(1 + 3*n)*Hyperg
eometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))))/((1 + n)*(1 + 2*n)*(
1 + 3*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right) dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(A(a+bx^n)^p+Bx^n(a+bx^n)^p+Cx^{2n}(a+bx^n)^p+Dx^{3n}(a+bx^n)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$Ax(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n}, -p, 1+\frac{1}{n}, -\frac{bx^n}{a}\right) + \frac{Bx^{n+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(1+\frac{1}{n}, -p, 2+\frac{1}{n}, -\frac{bx^n}{a}\right)}{n+1} + \frac{Cx^{2n+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(2+\frac{1}{n}, -p, 3+\frac{1}{n}, -\frac{bx^n}{a}\right)}{2n+1} + \frac{Dx^{3n+1}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(3+\frac{1}{n}, -p, 4+\frac{1}{n}, -\frac{bx^n}{a}\right)}{3n+1}$$

input
```
Int[(a + b*x^n)^p*(A + B*x^n + C*x^(2*n) + D*x^(3*n)),x]
```

output
```
(B*x^(1 + n)*(a + b*x^n)^p*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1), -
((b*x^n)/a)])/((1 + n)*(1 + (b*x^n)/a)^p) + (C*x^(1 + 2*n)*(a + b*x^n)^p*H
ypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1), -((b*x^n)/a)])/((1 + 2*n)*(1
+ (b*x^n)/a)^p) + (D*x^(1 + 3*n)*(a + b*x^n)^p*Hypergeometric2F1[3 + n^(-1
), -p, 4 + n^(-1), -((b*x^n)/a)])/((1 + 3*n)*(1 + (b*x^n)/a)^p) + (A*x*(a
+ b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(1 + (
b*x^n)/a)^p
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int (a+b\,x^n)^p \left(A+B\,x^n+C\,x^{2n}+Dx^{3n}\right) dx$$

input
```
int((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)+D*x^(3*n)),x)
```

output
```
int((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)+D*x^(3*n)),x)
```

Fricas [F]

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right) dx = \int \left(Dx^{3\,n}+Cx^{2\,n}+Bx^n+A\right)(bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)+D*x^(3*n)),x, algorithm="fricas")
```

output
```
integral((D*x^(3*n) + C*x^(2*n) + B*x^n + A)*(b*x^n + a)^p, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**n)**p*(A+B*x**n+C*x**(2*n)+D*x**(3*n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right) dx = \int \left(Dx^{3\,n}+Cx^{2\,n}+Bx^n+A\right)(bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)+D*x^(3*n)),x, algorithm="maxima")
```

output
```
integrate((D*x^(3*n) + C*x^(2*n) + B*x^n + A)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)+D*x^(3*n)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,2,4,4,1,0,0,0]%%%}+%%%{4,[2,0,6,4,2,3,4,1,0,0,0
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right)\,dx = \int (a+b\,x^n)^p\,\left(A+C\,x^{2\,n}+x^{3\,n}\,D+B\,x^n\right)\,dx$$

input
```
int((a + b*x^n)^p*(A + C*x^(2*n) + x^(3*n)*D + B*x^n),x)
```

output
```
int((a + b*x^n)^p*(A + C*x^(2*n) + x^(3*n)*D + B*x^n), x)
```

Reduce [F]

$$\int (a+bx^n)^p \left(A+Bx^n+Cx^{2n}+Dx^{3n}\right)\,dx = \text{too large to display}$$

input
```
int((a+b*x^n)^p*(A+B*x^n+C*x^(2*n)+D*x^(3*n)),x)
```

output
```
(x**(3*n)*(x**n*b + a)**p*b**3*d*n**3*p**3*x + 3*x**(3*n)*(x**n*b + a)**p*
b**3*d*n**3*p**2*x + 2*x**(3*n)*(x**n*b + a)**p*b**3*d*n**3*p*x + 3*x**(3*
n)*(x**n*b + a)**p*b**3*d*n**2*p**2*x + 6*x**(3*n)*(x**n*b + a)**p*b**3*d*
n**2*p*x + 2*x**(3*n)*(x**n*b + a)**p*b**3*d*n**2*x + 3*x**(3*n)*(x**n*b +
 a)**p*b**3*d*n*p*x + 3*x**(3*n)*(x**n*b + a)**p*b**3*d*n*x + x**(3*n)*(x*
*n*b + a)**p*b**3*d*x + x**(2*n)*(x**n*b + a)**p*a*b**2*d*n**3*p**3*x + x*
*(2*n)*(x**n*b + a)**p*a*b**2*d*n**3*p**2*x + 2*x**(2*n)*(x**n*b + a)**p*a
*b**2*d*n**2*p**2*x + x**(2*n)*(x**n*b + a)**p*a*b**2*d*n**2*p*x + x**(2*n
)*(x**n*b + a)**p*a*b**2*d*n*p*x + x**(2*n)*(x**n*b + a)**p*b**3*c*n**3*p*
*3*x + 4*x**(2*n)*(x**n*b + a)**p*b**3*c*n**3*p**2*x + 3*x**(2*n)*(x**n*b
+ a)**p*b**3*c*n**3*p*x + 3*x**(2*n)*(x**n*b + a)**p*b**3*c*n**2*p**2*x +
8*x**(2*n)*(x**n*b + a)**p*b**3*c*n**2*p*x + 3*x**(2*n)*(x**n*b + a)**p*b*
*3*c*n**2*x + 3*x**(2*n)*(x**n*b + a)**p*b**3*c*n*p*x + 4*x**(2*n)*(x**n*b
 + a)**p*b**3*c*n*x + x**(2*n)*(x**n*b + a)**p*b**3*c*x - 2*x**n*(x**n*b +
 a)**p*a**2*b*d*n**3*p**2*x - x**n*(x**n*b + a)**p*a**2*b*d*n**2*p**2*x -
2*x**n*(x**n*b + a)**p*a**2*b*d*n**2*p*x - x**n*(x**n*b + a)**p*a**2*b*d*n
*p*x + x**n*(x**n*b + a)**p*a*b**2*c*n**3*p**3*x + 3*x**n*(x**n*b + a)**p*
a*b**2*c*n**3*p**2*x + 2*x**n*(x**n*b + a)**p*a*b**2*c*n**2*p**2*x + 3*x**
n*(x**n*b + a)**p*a*b**2*c*n**2*p*x + x**n*(x**n*b + a)**p*a*b**2*c*n*p*x
+ x**n*(x**n*b + a)**p*b**4*n**3*p**3*x + 5*x**n*(x**n*b + a)**p*b**4*n...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file