Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.8/65-1.1.3.8-b

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**113**]. This is test number [65].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (113)	0.00 (0)
Mathematica	100.00 (113)	0.00 (0)
Sympy	92.04 (104)	7.96 (9)
Maple	81.42 (92)	18.58 (21)
Fricas	61.06 (69)	38.94 (44)
Mupad	18.58 (21)	81.42 (92)
Giac	15.93 (18)	84.07 (95)
Maxima	15.93 (18)	84.07 (95)
Reduce	15.93 (18)	84.07 (95)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

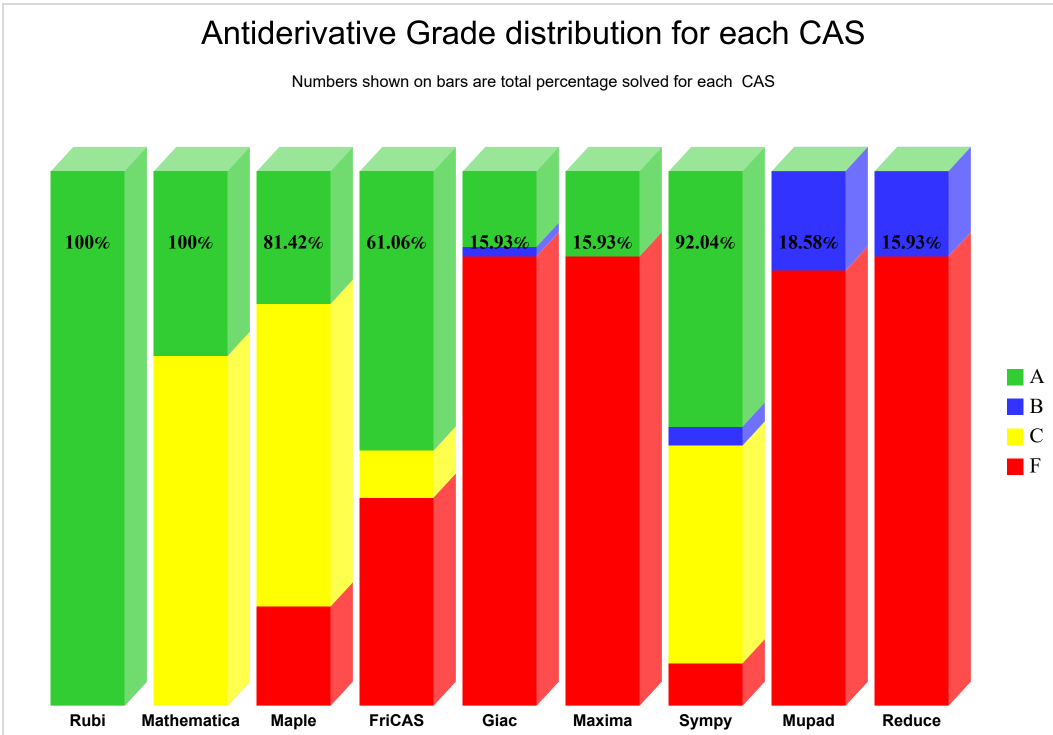
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

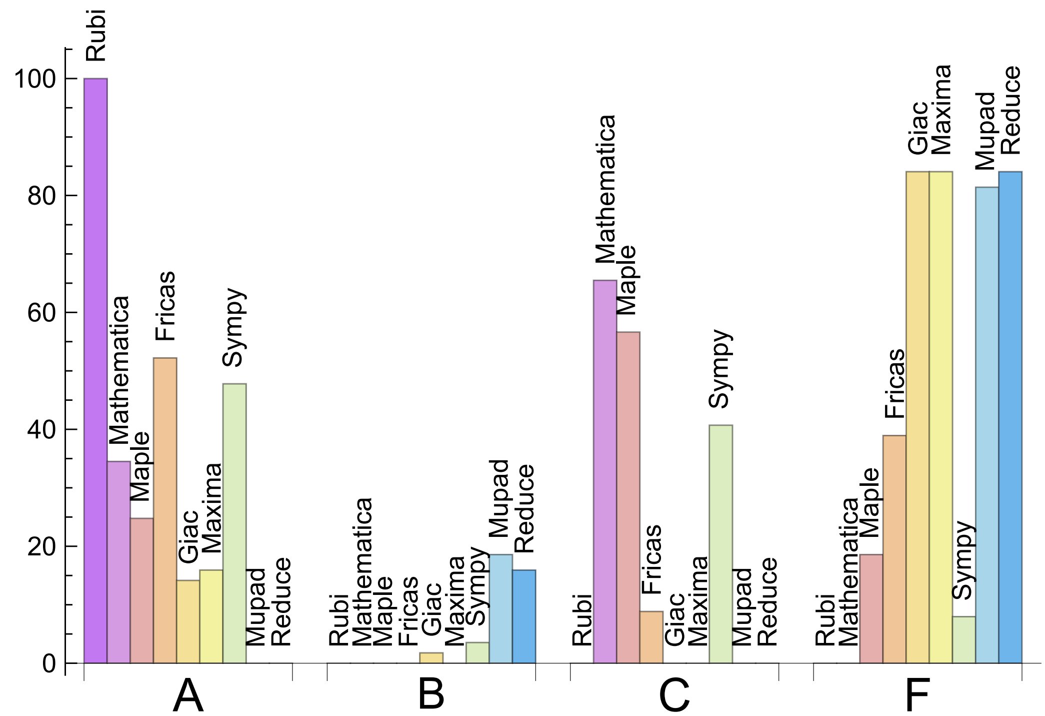
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Fricas	52.212	0.000	8.850	38.938
Sympy	47.788	3.540	40.708	7.965
Mathematica	34.513	0.000	65.487	0.000
Maple	24.779	0.000	56.637	18.584
Maxima	15.929	0.000	0.000	84.071
Giac	14.159	1.770	0.000	84.071
Mupad	0.000	18.584	0.000	81.416
Reduce	0.000	15.929	0.000	84.071

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	9	0.00	100.00	0.00
Maple	21	100.00	0.00	0.00
Fricas	44	100.00	0.00	0.00
Mupad	92	0.00	100.00	0.00
Giac	95	100.00	0.00	0.00
Maxima	95	100.00	0.00	0.00
Reduce	95	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.13
Reduce	0.18
Rubi	0.90
Fricas	1.12
Maple	1.97
Mupad	4.14
Sympy	6.36
Mathematica	8.07

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	163.27	0.67	149.00	0.59
Sympy	211.64	0.79	149.50	0.80
Maxima	226.61	1.08	204.50	1.10
Maple	232.16	0.81	244.50	0.78
Giac	236.89	1.16	281.00	1.06
Rubi	277.53	1.04	298.00	1.01
Reduce	530.44	2.10	228.50	1.57
Mupad	532.43	2.60	201.00	1.03
Fricas	24549.16	112.40	181.00	0.62

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

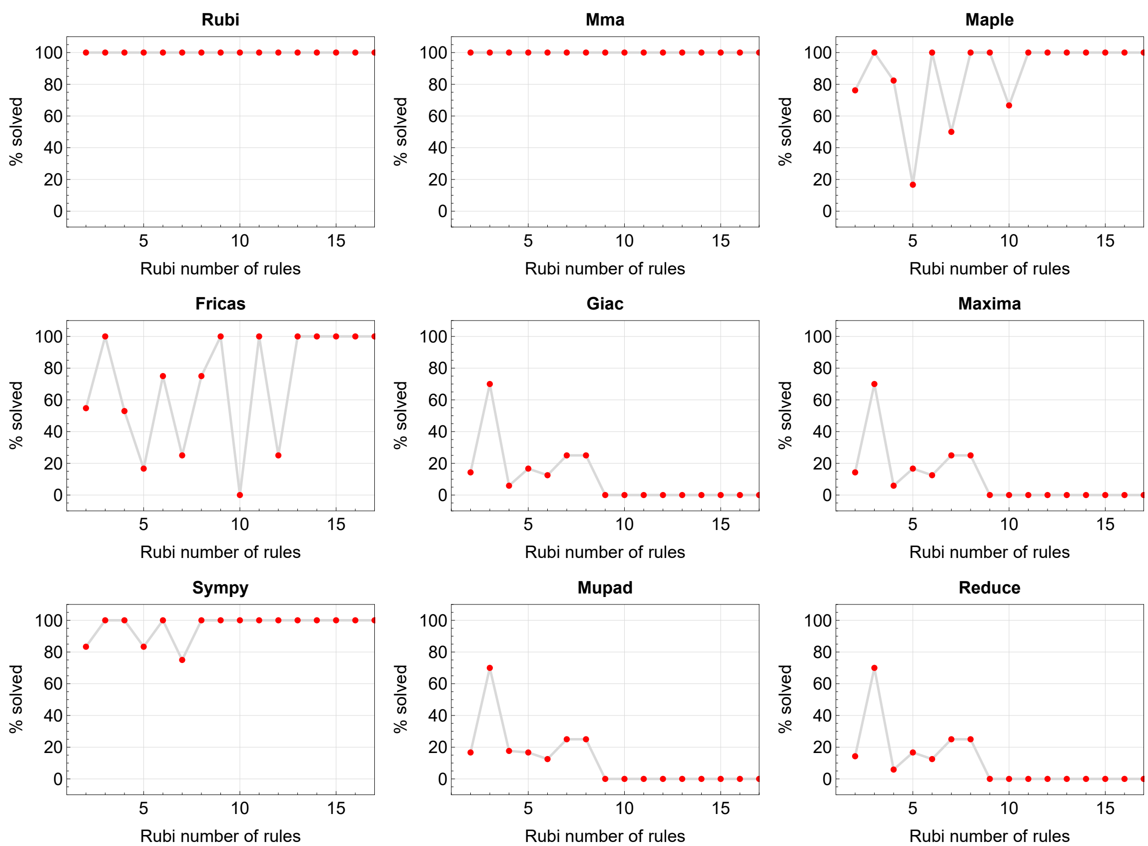


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

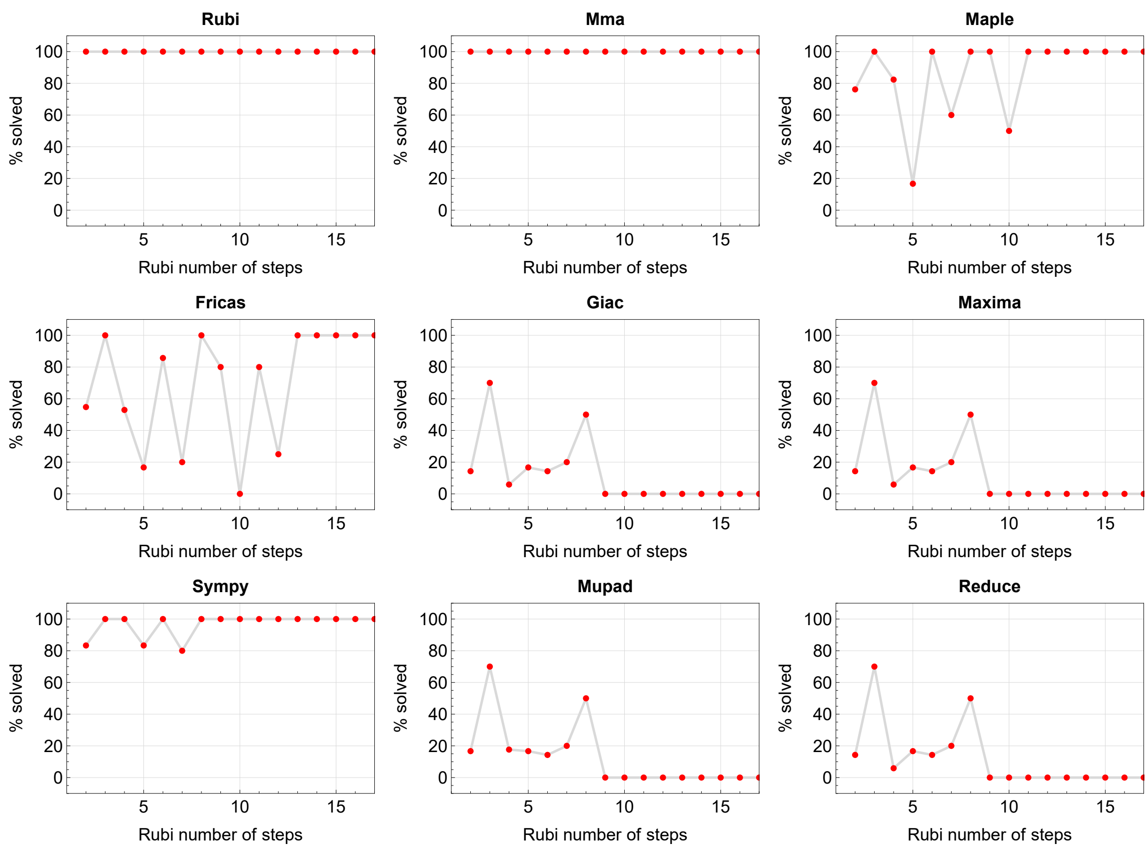


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

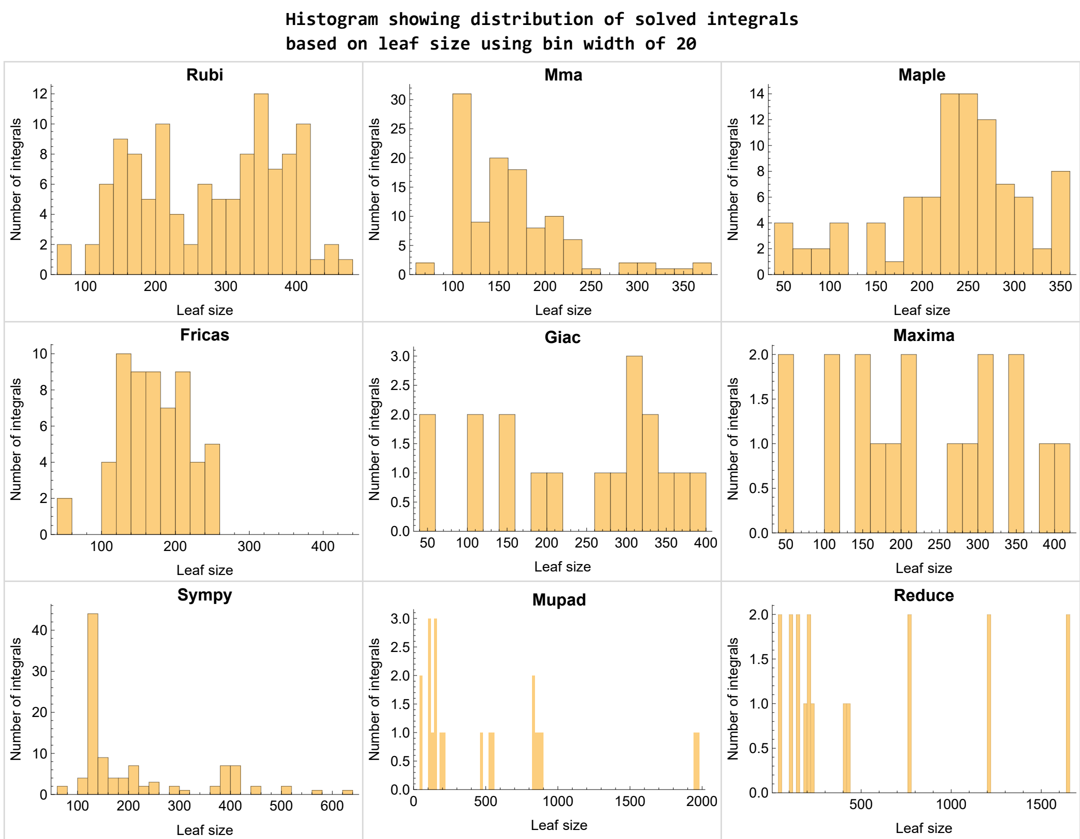


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

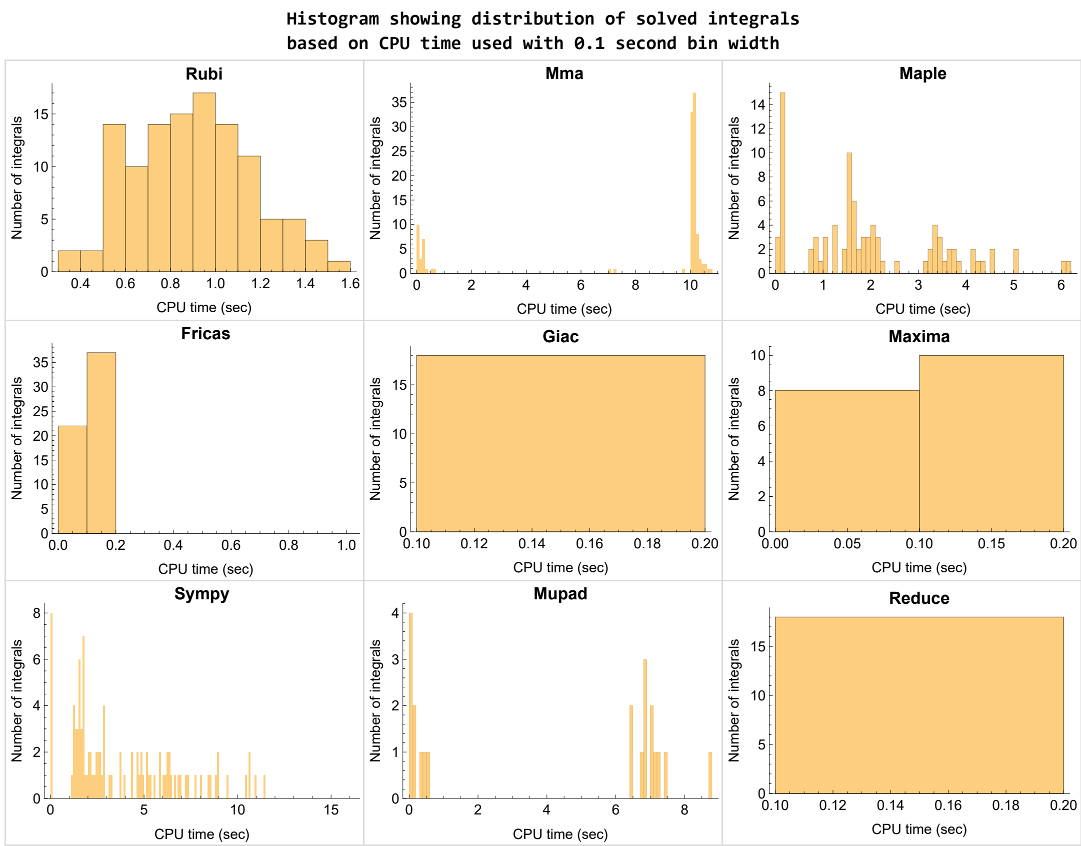


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

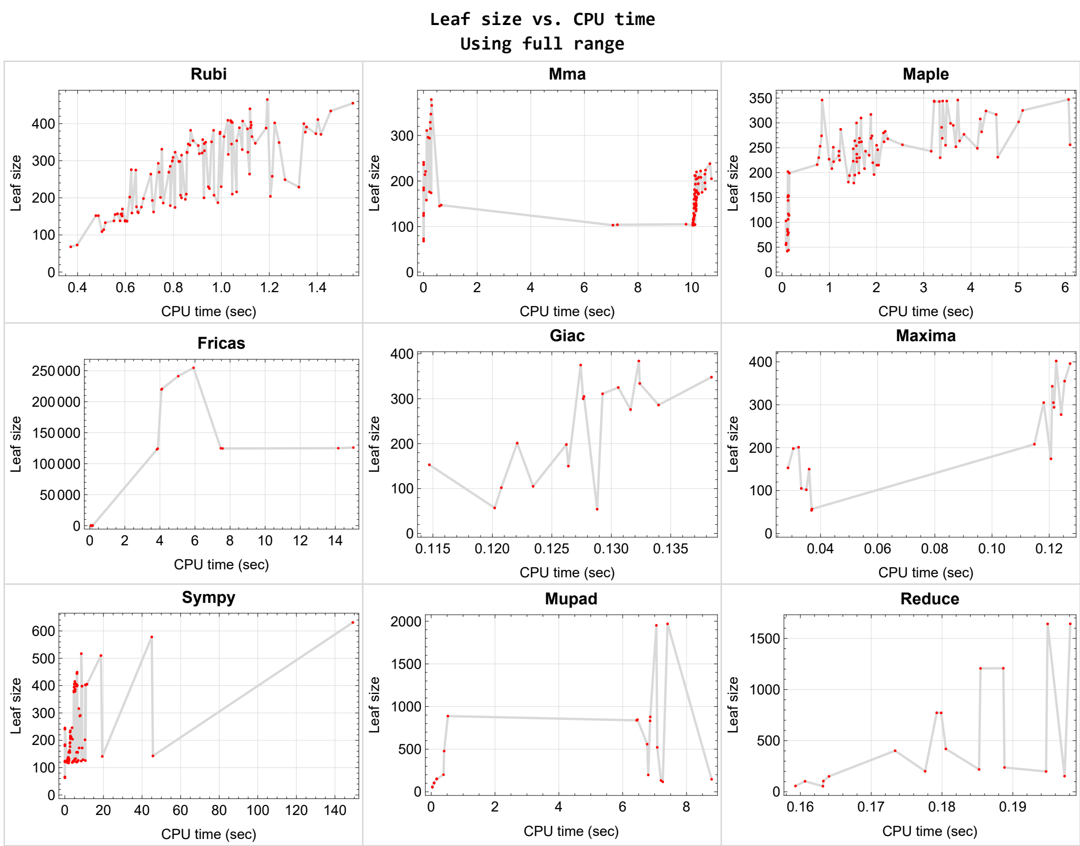


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

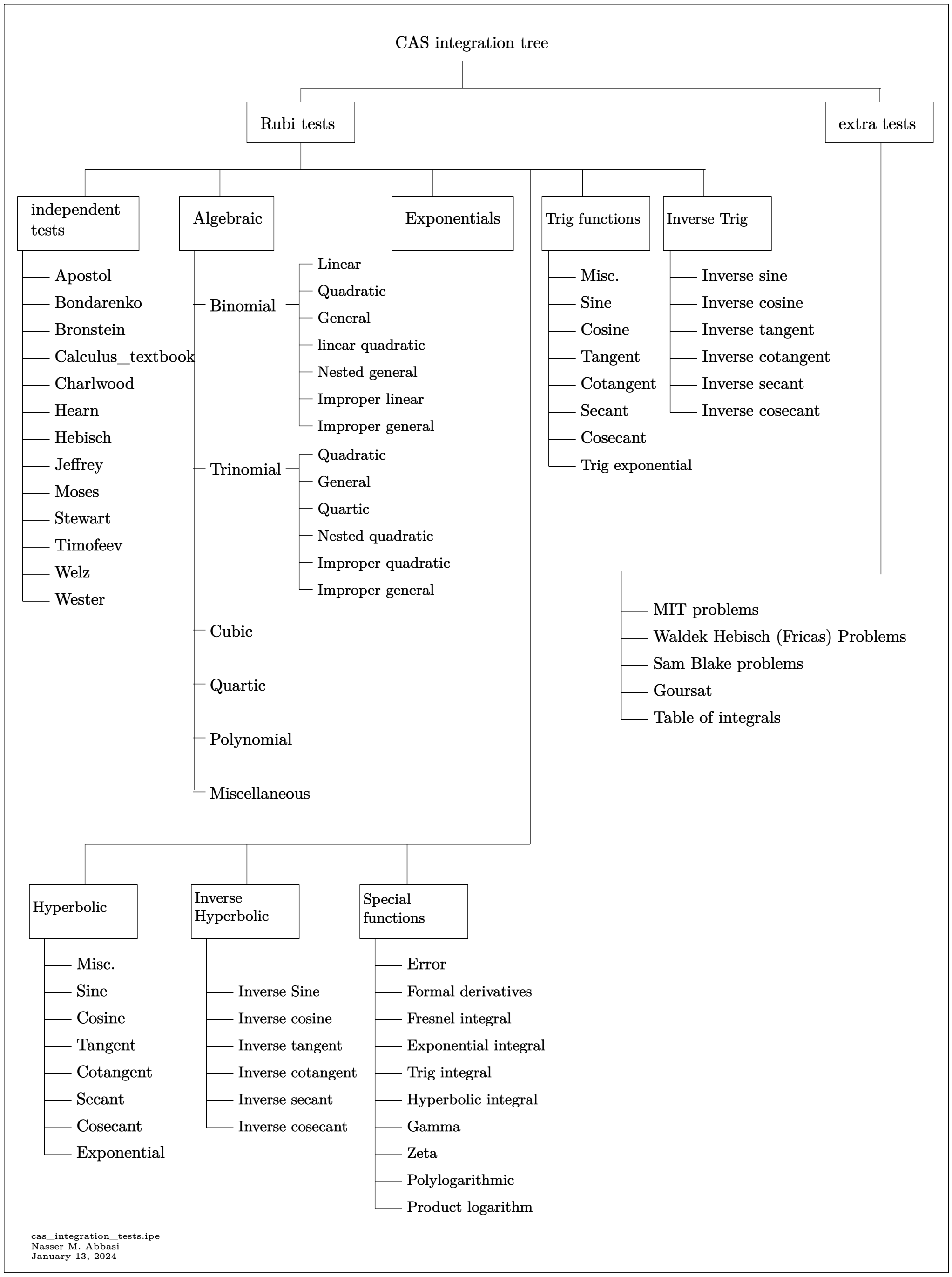
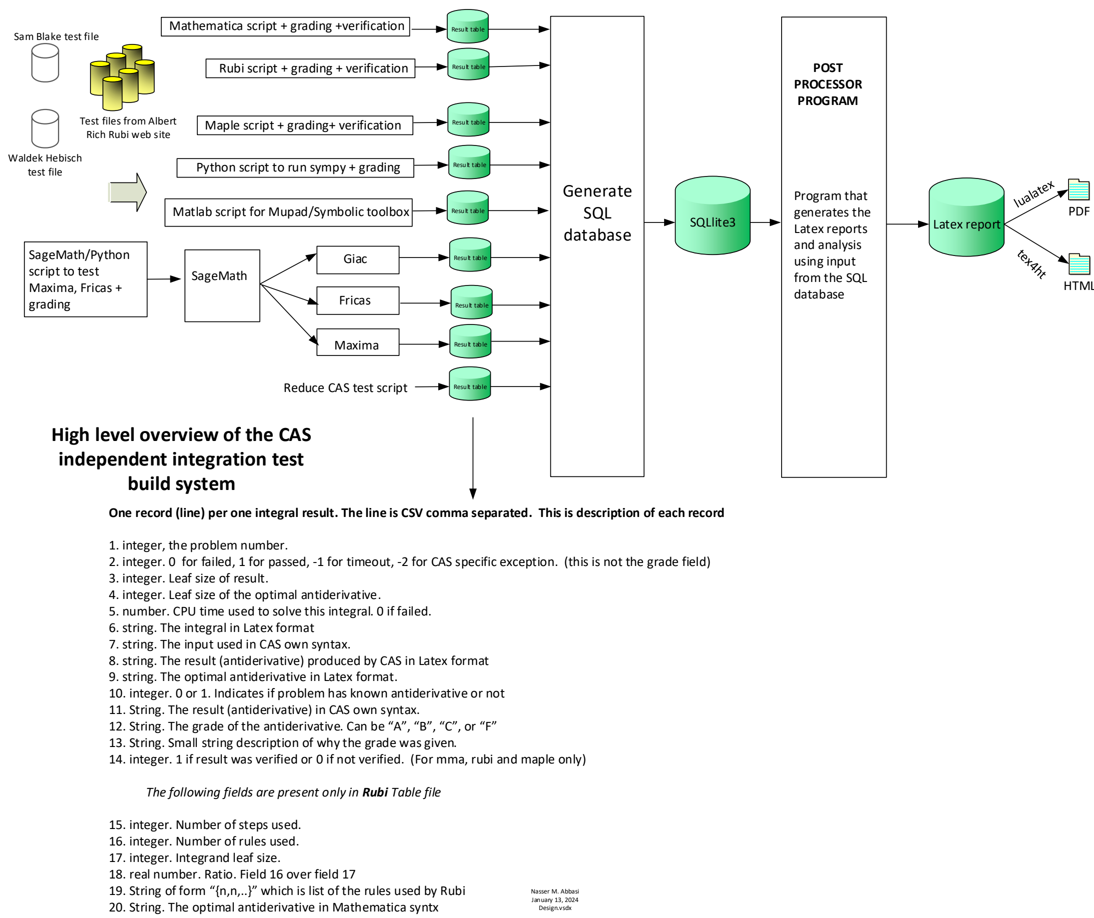


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade { }

C grade { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38 }

B grade { }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92 }

F normal fail { 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 21, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 51, 52, 53, 54, 55, 56, 57, 68, 69, 70, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92 }

B grade { }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 }

F normal fail { 22, 23, 24, 30, 39, 45, 46, 47, 48, 49, 50, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 }

B grade { }

C grade { }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18 }

B grade { 9, 10 }

C grade { }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 79, 91, 92 }

C grade { }

F normal fail { }

F(-1) timedout fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 54, 55, 56, 57, 58, 59, 60, 61, 72, 73, 74, 75, 76, 83, 84, 85, 86, 87, 88, 89, 112, 113 }

B grade { 13, 14, 15, 17 }

C grade { 39, 40, 45, 46, 47, 48, 49, 50, 51, 52, 53, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 77, 78, 79, 80, 81, 82, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108 }

F normal fail { }

F(-1) timedout fail { 9, 10, 11, 12, 16, 18, 109, 110, 111 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 }

C grade { }

F normal fail { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	68	55	54	54	63	54	55	54
N.S.	1	1.01	1.01	0.82	0.81	0.81	0.94	0.81	0.82	0.81
time (sec)	N/A	0.372	0.005	0.081	0.037	0.060	0.018	0.129	0.163	0.041

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	57	57	66	57	57	57
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.90	0.78	0.78	0.78
time (sec)	N/A	0.398	0.004	0.088	0.037	0.063	0.018	0.120	0.159	0.035

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	103	102	102	121	102	103	102
N.S.	1	1.00	1.14	0.94	0.94	0.94	1.11	0.94	0.94	0.94
time (sec)	N/A	0.502	0.003	0.082	0.035	0.071	0.022	0.121	0.161	0.094

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	129	106	105	105	124	105	105	105
N.S.	1	1.00	1.13	0.93	0.92	0.92	1.09	0.92	0.92	0.92
time (sec)	N/A	0.509	0.007	0.128	0.033	0.085	0.021	0.123	0.163	0.089

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	180	151	150	150	180	150	151	150
N.S.	1	1.00	1.19	1.00	0.99	0.99	1.19	0.99	1.00	0.99
time (sec)	N/A	0.586	0.004	0.121	0.036	0.099	0.025	0.126	0.164	0.179

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	185	154	153	153	184	153	153	153
N.S.	1	1.00	1.19	0.99	0.98	0.98	1.18	0.98	0.98	0.98
time (sec)	N/A	0.581	0.006	0.129	0.029	0.061	0.025	0.115	0.197	0.169

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	236	199	198	198	241	198	199	198
N.S.	1	1.00	1.22	1.03	1.03	1.03	1.25	1.03	1.03	1.03
time (sec)	N/A	0.712	0.005	0.153	0.030	0.064	0.028	0.126	0.195	6.800

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	241	202	201	201	245	201	201	201
N.S.	1	1.00	1.22	1.02	1.02	1.02	1.24	1.02	1.02	1.02
time (sec)	N/A	0.675	0.005	0.125	0.032	0.067	0.027	0.122	0.178	0.387

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	214	44	174	241149	0	276	219	1970
N.S.	1	1.01	1.62	0.33	1.32	1826.89	0.00	2.09	1.66	14.92
time (sec)	N/A	0.516	0.053	0.135	0.121	5.043	0.000	0.132	0.185	7.404

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	162	221	79	208	220680	0	325	238	846
N.S.	1	1.01	1.37	0.49	1.29	1370.68	0.00	2.02	1.48	5.25
time (sec)	N/A	0.651	0.077	0.135	0.115	4.117	0.000	0.131	0.189	6.462

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	293	296	42	277	254687	0	286	402	1952
N.S.	1	1.30	1.31	0.19	1.23	1126.93	0.00	1.27	1.78	8.64
time (sec)	N/A	0.737	0.169	0.109	0.124	5.919	0.000	0.134	0.173	7.055

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	321	311	75	305	219615	0	305	420	838
N.S.	1	1.26	1.22	0.30	1.20	864.63	0.00	1.20	1.65	3.30
time (sec)	N/A	0.920	0.121	0.118	0.121	4.086	0.000	0.128	0.181	6.437

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	323	315	86	305	124301	517	311	772	478
N.S.	1	1.26	1.23	0.33	1.19	483.66	2.01	1.21	3.00	1.86
time (sec)	N/A	0.807	0.260	0.114	0.118	3.879	8.525	0.129	0.180	0.407

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	315	294	82	294	122993	510	300	773	559
N.S.	1	1.22	1.14	0.32	1.14	476.72	1.98	1.16	3.00	2.17
time (sec)	N/A	0.834	0.224	0.116	0.122	3.827	18.705	0.128	0.179	6.767

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	365	347	117	355	124838	578	348	1207	832
N.S.	1	1.26	1.20	0.40	1.22	430.48	1.99	1.20	4.16	2.87
time (sec)	N/A	1.011	0.275	0.131	0.125	7.472	45.036	0.138	0.185	6.858

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	351	329	114	343	124542	0	334	1208	521
N.S.	1	1.14	1.07	0.37	1.11	404.36	0.00	1.08	3.92	1.69
time (sec)	N/A	0.959	0.253	0.147	0.121	7.575	0.000	0.132	0.189	7.082

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	404	379	153	402	125011	631	384	1642	879
N.S.	1	1.26	1.18	0.48	1.25	389.44	1.97	1.20	5.12	2.74
time (sec)	N/A	1.123	0.292	0.137	0.122	14.173	149.277	0.132	0.195	6.863

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	396	366	144	396	125996	0	375	1643	888
N.S.	1	1.08	1.00	0.39	1.08	344.25	0.00	1.02	4.49	2.43
time (sec)	N/A	1.125	0.306	0.127	0.127	15.032	0.000	0.127	0.198	0.524

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	264	140	291	0	167	131	0	215	0
N.S.	1	1.05	0.56	1.16	0.00	0.67	0.52	0.00	0.86	0.00
time (sec)	N/A	1.118	10.134	3.379	0.000	0.138	1.676	0.000	0.214	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	230	124	220	0	162	131	0	175	0
N.S.	1	1.02	0.55	0.98	0.00	0.72	0.58	0.00	0.78	0.00
time (sec)	N/A	0.946	10.094	1.921	0.000	0.101	1.591	0.000	0.218	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	201	115	229	0	128	129	0	154	0
N.S.	1	1.03	0.59	1.17	0.00	0.66	0.66	0.00	0.79	0.00
time (sec)	N/A	0.746	10.066	1.585	0.000	0.097	1.495	0.000	0.217	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	210	103	195	0	0	124	0	163	0
N.S.	1	1.03	0.50	0.96	0.00	0.00	0.61	0.00	0.80	0.00
time (sec)	N/A	0.854	7.062	1.540	0.000	0.000	1.755	0.000	0.214	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	200	104	208	0	0	119	0	172	0
N.S.	1	1.03	0.53	1.07	0.00	0.00	0.61	0.00	0.88	0.00
time (sec)	N/A	0.834	7.231	1.746	0.000	0.000	1.795	0.000	0.216	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	207	104	215	0	0	117	0	173	0
N.S.	1	1.02	0.51	1.06	0.00	0.00	0.58	0.00	0.86	0.00
time (sec)	N/A	0.969	10.051	2.008	0.000	0.000	1.937	0.000	0.216	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	216	104	252	0	123	129	0	173	0
N.S.	1	0.99	0.47	1.15	0.00	0.56	0.59	0.00	0.79	0.00
time (sec)	N/A	1.064	10.045	3.681	0.000	0.106	2.018	0.000	0.255	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	258	104	249	0	159	133	0	173	0
N.S.	1	1.00	0.40	0.97	0.00	0.62	0.52	0.00	0.67	0.00
time (sec)	N/A	1.212	10.125	4.136	0.000	0.086	2.184	0.000	0.334	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	225	141	230	0	127	126	0	154	0
N.S.	1	1.07	0.67	1.09	0.00	0.60	0.60	0.00	0.73	0.00
time (sec)	N/A	0.950	10.105	3.349	0.000	0.080	1.781	0.000	0.231	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	196	125	196	0	123	126	0	118	0
N.S.	1	1.04	0.66	1.04	0.00	0.65	0.67	0.00	0.63	0.00
time (sec)	N/A	0.850	10.110	1.948	0.000	0.078	1.670	0.000	0.208	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	159	113	179	0	106	124	0	100	0
N.S.	1	1.04	0.74	1.17	0.00	0.69	0.81	0.00	0.65	0.00
time (sec)	N/A	0.627	10.062	1.519	0.000	0.081	1.254	0.000	0.200	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	162	103	181	0	0	126	0	80	0
N.S.	1	1.03	0.66	1.15	0.00	0.00	0.80	0.00	0.51	0.00
time (sec)	N/A	0.717	10.070	1.404	0.000	0.000	1.284	0.000	0.192	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	179	104	199	0	102	129	0	81	0
N.S.	1	1.01	0.59	1.12	0.00	0.58	0.73	0.00	0.46	0.00
time (sec)	N/A	0.787	10.052	1.629	0.000	0.083	1.364	0.000	0.197	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	230	104	215	0	122	129	0	85	0
N.S.	1	1.04	0.47	0.97	0.00	0.55	0.58	0.00	0.38	0.00
time (sec)	N/A	0.999	10.059	2.043	0.000	0.079	1.762	0.000	0.201	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	249	138	256	0	218	126	0	349	0
N.S.	1	1.12	0.62	1.15	0.00	0.98	0.57	0.00	1.57	0.00
time (sec)	N/A	1.266	10.170	6.101	0.000	0.096	10.646	0.000	0.265	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	200	129	231	0	187	126	0	325	0
N.S.	1	1.05	0.68	1.22	0.00	0.98	0.66	0.00	1.71	0.00
time (sec)	N/A	0.928	10.074	4.568	0.000	0.082	6.813	0.000	0.247	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	186	119	223	0	181	126	0	288	0
N.S.	1	1.03	0.66	1.24	0.00	1.01	0.70	0.00	1.60	0.00
time (sec)	N/A	0.757	10.076	1.506	0.000	0.112	4.866	0.000	0.253	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	117	222	0	144	124	0	265	0
N.S.	1	1.05	0.70	1.33	0.00	0.86	0.74	0.00	1.59	0.00
time (sec)	N/A	0.646	10.081	1.092	0.000	0.076	4.374	0.000	0.238	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	210	142	243	0	173	126	0	110	0
N.S.	1	1.07	0.72	1.23	0.00	0.88	0.64	0.00	0.56	0.00
time (sec)	N/A	1.046	10.105	3.159	0.000	0.098	6.965	0.000	0.289	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	229	140	255	0	184	129	0	111	0
N.S.	1	1.06	0.65	1.18	0.00	0.85	0.59	0.00	0.51	0.00
time (sec)	N/A	1.323	10.086	3.463	0.000	0.119	9.436	0.000	0.219	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	264	104	194	0	0	121	0	74	0
N.S.	1	0.99	0.39	0.73	0.00	0.00	0.45	0.00	0.28	0.00
time (sec)	N/A	0.706	10.060	1.428	0.000	0.000	1.212	0.000	0.241	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	317	141	256	0	173	121	0	107	0
N.S.	1	1.01	0.45	0.82	0.00	0.55	0.39	0.00	0.34	0.00
time (sec)	N/A	1.029	10.106	2.549	0.000	0.092	6.231	0.000	0.255	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	407	215	269	0	205	212	0	239	0
N.S.	1	1.01	0.53	0.67	0.00	0.51	0.52	0.00	0.59	0.00
time (sec)	N/A	1.089	10.492	1.882	0.000	0.119	3.250	0.000	0.228	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	382	182	262	0	193	212	0	223	0
N.S.	1	1.00	0.48	0.69	0.00	0.51	0.56	0.00	0.59	0.00
time (sec)	N/A	0.968	10.510	1.681	0.000	0.137	3.195	0.000	0.220	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	354	211	254	0	170	158	0	190	0
N.S.	1	0.99	0.59	0.71	0.00	0.48	0.44	0.00	0.53	0.00
time (sec)	N/A	0.882	10.139	1.573	0.000	0.110	2.108	0.000	0.213	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	171	243	0	163	156	0	174	0
N.S.	1	1.00	0.52	0.73	0.00	0.49	0.47	0.00	0.53	0.00
time (sec)	N/A	0.752	10.101	1.206	0.000	0.110	2.028	0.000	0.217	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	208	274	0	0	204	0	197	0
N.S.	1	1.00	0.60	0.79	0.00	0.00	0.59	0.00	0.57	0.00
time (sec)	N/A	0.862	10.345	0.835	0.000	0.000	3.955	0.000	0.219	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	208	274	0	0	206	0	207	0
N.S.	1	1.00	0.61	0.80	0.00	0.00	0.60	0.00	0.61	0.00
time (sec)	N/A	0.904	10.263	1.904	0.000	0.000	2.846	0.000	0.219	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	204	269	0	0	230	0	206	0
N.S.	1	1.00	0.60	0.79	0.00	0.00	0.67	0.00	0.60	0.00
time (sec)	N/A	0.869	10.162	1.640	0.000	0.000	2.702	0.000	0.209	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	357	205	264	0	0	235	0	208	0
N.S.	1	1.04	0.60	0.77	0.00	0.00	0.69	0.00	0.61	0.00
time (sec)	N/A	0.922	10.202	1.540	0.000	0.000	2.836	0.000	0.212	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	330	175	261	0	0	211	0	224	0
N.S.	1	0.96	0.51	0.76	0.00	0.00	0.62	0.00	0.65	0.00
time (sec)	N/A	1.087	10.157	1.634	0.000	0.000	2.854	0.000	0.220	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	354	179	280	0	0	216	0	225	0
N.S.	1	0.96	0.49	0.76	0.00	0.00	0.59	0.00	0.61	0.00
time (sec)	N/A	1.064	10.181	2.148	0.000	0.000	2.883	0.000	0.235	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	345	145	262	0	166	189	0	189	0
N.S.	1	0.98	0.41	0.74	0.00	0.47	0.54	0.00	0.54	0.00
time (sec)	N/A	1.042	10.168	2.160	0.000	0.144	2.583	0.000	0.264	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	365	145	269	0	173	192	0	189	0
N.S.	1	0.97	0.38	0.71	0.00	0.46	0.51	0.00	0.50	0.00
time (sec)	N/A	1.128	10.167	3.393	0.000	0.128	2.673	0.000	0.279	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	388	146	277	0	196	246	0	218	0
N.S.	1	0.97	0.36	0.69	0.00	0.49	0.61	0.00	0.54	0.00
time (sec)	N/A	1.187	10.152	3.852	0.000	0.121	3.733	0.000	78.823	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	465	238	317	0	255	398	0	319	0
N.S.	1	1.02	0.52	0.69	0.00	0.56	0.87	0.00	0.70	0.00
time (sec)	N/A	1.192	10.679	1.881	0.000	0.126	8.911	0.000	0.397	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	440	205	310	0	247	398	0	303	0
N.S.	1	1.01	0.47	0.71	0.00	0.57	0.91	0.00	0.69	0.00
time (sec)	N/A	1.119	10.738	1.672	0.000	0.116	8.886	0.000	0.326	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	409	196	300	0	224	396	0	272	0
N.S.	1	0.99	0.48	0.73	0.00	0.54	0.96	0.00	0.66	0.00
time (sec)	N/A	1.027	10.495	1.577	0.000	0.126	4.954	0.000	0.261	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	175	287	0	214	394	0	256	0
N.S.	1	1.00	0.46	0.75	0.00	0.56	1.03	0.00	0.67	0.00
time (sec)	N/A	0.872	10.387	1.241	0.000	0.116	4.797	0.000	0.237	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	224	346	0	0	405	0	279	0
N.S.	1	1.00	0.56	0.86	0.00	0.00	1.00	0.00	0.69	0.00
time (sec)	N/A	1.046	10.515	0.846	0.000	0.000	11.461	0.000	0.214	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	222	346	0	0	406	0	289	0
N.S.	1	1.00	0.55	0.86	0.00	0.00	1.00	0.00	0.72	0.00
time (sec)	N/A	1.042	10.332	3.722	0.000	0.000	5.836	0.000	0.216	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	194	344	0	0	377	0	278	0
N.S.	1	1.00	0.48	0.85	0.00	0.00	0.93	0.00	0.68	0.00
time (sec)	N/A	1.042	10.246	3.488	0.000	0.000	4.646	0.000	0.212	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	194	343	0	0	381	0	282	0
N.S.	1	1.00	0.48	0.84	0.00	0.00	0.93	0.00	0.69	0.00
time (sec)	N/A	1.039	10.243	3.333	0.000	0.000	4.689	0.000	0.221	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	389	163	344	0	0	379	0	282	0
N.S.	1	0.97	0.40	0.85	0.00	0.00	0.94	0.00	0.70	0.00
time (sec)	N/A	1.075	10.187	3.402	0.000	0.000	5.270	0.000	0.222	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	386	165	344	0	0	386	0	282	0
N.S.	1	0.95	0.41	0.85	0.00	0.00	0.95	0.00	0.70	0.00
time (sec)	N/A	1.109	10.173	3.221	0.000	0.000	5.390	0.000	0.232	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	389	163	343	0	0	406	0	282	0
N.S.	1	0.94	0.40	0.83	0.00	0.00	0.99	0.00	0.68	0.00
time (sec)	N/A	1.122	10.164	3.224	0.000	0.000	4.898	0.000	0.309	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	402	164	299	0	0	415	0	282	0
N.S.	1	0.93	0.38	0.70	0.00	0.00	0.97	0.00	0.66	0.00
time (sec)	N/A	1.223	10.191	3.567	0.000	0.000	5.174	0.000	0.326	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	377	174	295	0	0	444	0	303	0
N.S.	1	0.87	0.40	0.68	0.00	0.00	1.03	0.00	0.70	0.00
time (sec)	N/A	1.350	10.228	3.627	0.000	0.000	6.224	0.000	74.071	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	400	174	324	0	0	449	0	303	0
N.S.	1	0.88	0.38	0.71	0.00	0.00	0.99	0.00	0.67	0.00
time (sec)	N/A	1.344	10.232	4.319	0.000	0.000	6.346	0.000	73.034	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	391	171	308	0	217	398	0	30	0
N.S.	1	0.91	0.40	0.72	0.00	0.51	0.93	0.00	0.07	0.00
time (sec)	N/A	1.355	10.246	4.198	0.000	0.154	6.165	0.000	200.025	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	411	172	317	0	227	401	0	30	0
N.S.	1	0.90	0.38	0.70	0.00	0.50	0.88	0.00	0.07	0.00
time (sec)	N/A	1.403	10.259	4.536	0.000	0.109	6.448	0.000	200.022	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	434	149	325	0	250	403	0	30	0
N.S.	1	0.90	0.31	0.68	0.00	0.52	0.84	0.00	0.06	0.00
time (sec)	N/A	1.456	10.171	5.096	0.000	0.110	10.606	0.000	200.022	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	455	151	347	0	258	403	0	30	0
N.S.	1	0.90	0.30	0.69	0.00	0.51	0.80	0.00	0.06	0.00
time (sec)	N/A	1.549	10.158	6.069	0.000	0.125	10.924	0.000	200.027	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	373	212	246	0	163	177	0	174	0
N.S.	1	1.01	0.57	0.66	0.00	0.44	0.48	0.00	0.47	0.00
time (sec)	N/A	0.994	10.116	2.026	0.000	0.113	2.673	0.000	0.193	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	212	235	0	156	156	0	159	0
N.S.	1	1.00	0.61	0.67	0.00	0.45	0.45	0.00	0.45	0.00
time (sec)	N/A	0.934	10.123	1.844	0.000	0.165	2.589	0.000	0.194	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	193	230	0	147	156	0	766	0
N.S.	1	1.00	0.60	0.71	0.00	0.46	0.48	0.00	2.38	0.00
time (sec)	N/A	0.858	10.139	1.641	0.000	0.151	2.471	0.000	0.210	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	160	222	0	133	129	0	312	0
N.S.	1	1.00	0.54	0.74	0.00	0.44	0.43	0.00	1.04	0.00
time (sec)	N/A	0.795	10.088	1.582	0.000	0.121	1.821	0.000	0.231	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	276	150	208	0	135	128	0	282	0
N.S.	1	1.00	0.54	0.75	0.00	0.49	0.46	0.00	1.02	0.00
time (sec)	N/A	0.624	10.085	1.054	0.000	0.113	1.444	0.000	0.200	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	285	159	216	0	0	126	0	143	0
N.S.	1	1.00	0.56	0.76	0.00	0.00	0.44	0.00	0.50	0.00
time (sec)	N/A	0.787	10.165	0.743	0.000	0.000	2.219	0.000	0.201	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	309	157	234	0	0	128	0	144	0
N.S.	1	1.07	0.54	0.81	0.00	0.00	0.44	0.00	0.50	0.00
time (sec)	N/A	0.799	10.165	1.201	0.000	0.000	1.754	0.000	0.255	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	300	148	225	0	139	126	0	291	118
N.S.	1	1.07	0.53	0.80	0.00	0.50	0.45	0.00	1.04	0.42
time (sec)	N/A	0.796	10.111	1.218	0.000	0.111	1.638	0.000	0.193	7.254

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	323	149	237	0	136	131	0	299	0
N.S.	1	1.07	0.49	0.78	0.00	0.45	0.43	0.00	0.99	0.00
time (sec)	N/A	0.856	10.137	1.582	0.000	0.115	1.732	0.000	0.212	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	346	147	243	0	150	158	0	720	0
N.S.	1	1.06	0.45	0.75	0.00	0.46	0.48	0.00	2.21	0.00
time (sec)	N/A	0.929	10.112	1.767	0.000	0.145	2.335	0.000	0.238	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	377	134	255	0	159	163	0	720	0
N.S.	1	1.06	0.38	0.72	0.00	0.45	0.46	0.00	2.03	0.00
time (sec)	N/A	0.995	10.161	2.002	0.000	0.105	2.455	0.000	0.252	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	373	220	302	0	242	202	0	396	0
N.S.	1	0.97	0.57	0.79	0.00	0.63	0.53	0.00	1.03	0.00
time (sec)	N/A	1.394	10.180	5.010	0.000	0.145	10.426	0.000	0.243	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	347	176	282	0	211	172	0	359	0
N.S.	1	0.96	0.48	0.78	0.00	0.58	0.47	0.00	0.99	0.00
time (sec)	N/A	1.142	10.144	4.227	0.000	0.140	8.940	0.000	0.239	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	319	166	264	0	223	172	0	343	0
N.S.	1	0.94	0.49	0.78	0.00	0.66	0.51	0.00	1.01	0.00
time (sec)	N/A	0.908	10.125	3.760	0.000	0.117	7.369	0.000	0.221	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	298	181	248	0	215	156	0	328	0
N.S.	1	0.90	0.55	0.75	0.00	0.65	0.47	0.00	0.99	0.00
time (sec)	N/A	0.827	10.104	1.507	0.000	0.112	6.647	0.000	0.217	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	327	165	251	0	201	156	0	1423	0
N.S.	1	0.96	0.49	0.74	0.00	0.59	0.46	0.00	4.19	0.00
time (sec)	N/A	0.929	10.168	1.085	0.000	0.109	6.048	0.000	0.293	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	298	116	227	0	147	133	0	1041	0
N.S.	1	0.96	0.38	0.73	0.00	0.48	0.43	0.00	3.37	0.00
time (sec)	N/A	0.822	10.073	0.996	0.000	0.092	5.539	0.000	0.238	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	275	116	230	0	129	131	0	655	0
N.S.	1	0.97	0.41	0.81	0.00	0.45	0.46	0.00	2.31	0.00
time (sec)	N/A	0.643	10.057	0.776	0.000	0.083	5.140	0.000	0.197	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	323	125	253	0	191	289	0	924	0
N.S.	1	1.03	0.40	0.80	0.00	0.61	0.92	0.00	2.93	0.00
time (sec)	N/A	1.111	10.109	0.806	0.000	0.120	7.748	0.000	0.225	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	349	123	268	0	215	291	0	929	133
N.S.	1	1.10	0.39	0.85	0.00	0.68	0.92	0.00	2.94	0.42
time (sec)	N/A	1.241	10.100	2.237	0.000	0.115	8.040	0.000	0.254	7.199

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	371	140	283	0	231	316	0	1164	147
N.S.	1	1.08	0.41	0.83	0.00	0.68	0.92	0.00	3.40	0.43
time (sec)	N/A	1.416	10.101	2.184	0.000	0.117	7.251	0.000	0.261	8.786

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	157	105	0	0	0	126	0	183	0
N.S.	1	1.14	0.76	0.00	0.00	0.00	0.91	0.00	1.33	0.00
time (sec)	N/A	0.585	10.090	0.000	0.000	0.000	1.586	0.000	0.205	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	157	105	0	0	0	126	0	144	0
N.S.	1	1.14	0.76	0.00	0.00	0.00	0.91	0.00	1.04	0.00
time (sec)	N/A	0.566	10.045	0.000	0.000	0.000	1.533	0.000	0.200	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	152	103	0	0	0	124	0	122	0
N.S.	1	1.13	0.77	0.00	0.00	0.00	0.93	0.00	0.91	0.00
time (sec)	N/A	0.477	10.039	0.000	0.000	0.000	1.375	0.000	0.213	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	138	104	0	0	0	126	0	150	0
N.S.	1	1.05	0.79	0.00	0.00	0.00	0.96	0.00	1.15	0.00
time (sec)	N/A	0.552	10.050	0.000	0.000	0.000	1.571	0.000	0.225	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	137	105	0	0	0	129	0	170	0
N.S.	1	1.02	0.78	0.00	0.00	0.00	0.96	0.00	1.27	0.00
time (sec)	N/A	0.599	9.790	0.000	0.000	0.000	1.589	0.000	0.226	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	174	105	0	0	0	138	0	170	0
N.S.	1	1.27	0.77	0.00	0.00	0.00	1.01	0.00	1.24	0.00
time (sec)	N/A	0.807	10.045	0.000	0.000	0.000	1.756	0.000	0.229	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	157	105	0	0	0	121	0	52	0
N.S.	1	1.14	0.76	0.00	0.00	0.00	0.88	0.00	0.38	0.00
time (sec)	N/A	0.566	10.076	0.000	0.000	0.000	1.542	0.000	0.288	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	152	103	0	0	0	119	0	48	0
N.S.	1	1.13	0.77	0.00	0.00	0.00	0.89	0.00	0.36	0.00
time (sec)	N/A	0.487	10.050	0.000	0.000	0.000	1.186	0.000	0.202	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	138	104	0	0	0	121	0	48	0
N.S.	1	1.05	0.79	0.00	0.00	0.00	0.92	0.00	0.37	0.00
time (sec)	N/A	0.577	10.056	0.000	0.000	0.000	1.263	0.000	0.199	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	137	105	0	0	0	124	0	48	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.93	0.00	0.36	0.00
time (sec)	N/A	0.606	10.055	0.000	0.000	0.000	1.310	0.000	0.199	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	139	105	0	0	0	133	0	52	0
N.S.	1	1.01	0.76	0.00	0.00	0.00	0.96	0.00	0.38	0.00
time (sec)	N/A	0.598	10.049	0.000	0.000	0.000	1.473	0.000	0.186	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	207	108	0	0	0	121	0	109	0
N.S.	1	1.44	0.75	0.00	0.00	0.00	0.84	0.00	0.76	0.00
time (sec)	N/A	0.830	10.048	0.000	0.000	0.000	5.877	0.000	0.183	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	160	108	0	0	0	121	0	109	0
N.S.	1	1.11	0.75	0.00	0.00	0.00	0.84	0.00	0.76	0.00
time (sec)	N/A	0.653	10.041	0.000	0.000	0.000	4.343	0.000	0.195	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	155	106	0	0	0	119	0	105	0
N.S.	1	1.16	0.79	0.00	0.00	0.00	0.89	0.00	0.78	0.00
time (sec)	N/A	0.557	10.044	0.000	0.000	0.000	3.777	0.000	0.189	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	187	107	0	0	0	121	0	104	0
N.S.	1	1.40	0.80	0.00	0.00	0.00	0.90	0.00	0.78	0.00
time (sec)	N/A	0.986	10.051	0.000	0.000	0.000	6.381	0.000	0.187	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	204	108	0	0	0	124	0	103	0
N.S.	1	1.49	0.79	0.00	0.00	0.00	0.91	0.00	0.75	0.00
time (sec)	N/A	1.205	10.056	0.000	0.000	0.000	8.494	0.000	0.194	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	174	0	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.781	0.283	0.000	0.000	0.000	0.000	0.000	0.702	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	202	158	0	0	0	0	0	0	0
N.S.	1	1.09	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	0.109	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	269	176	0	0	0	0	0	0	0
N.S.	1	1.14	0.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	0.222	0.000	0.000	0.000	0.000	0.000	0.442	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	170	147	0	0	0	141	0	0	0
N.S.	1	1.19	1.03	0.00	0.00	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.587	0.659	0.000	0.000	0.000	19.400	0.000	0.177	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	145	0	0	0	143	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.667	0.598	0.000	0.000	0.000	45.651	0.000	0.197	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [26] had the largest ratio of [.6071429999999999988]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.01	23	0.087
2	A	2	2	1.00	26	0.077
3	A	3	3	1.00	25	0.120
4	A	3	3	1.00	28	0.107
5	A	3	3	1.00	25	0.120
6	A	3	3	1.00	28	0.107
7	A	3	3	1.00	25	0.120
8	A	3	3	1.00	28	0.107
9	A	2	2	1.01	26	0.077
10	A	2	2	1.01	29	0.069
11	A	2	2	1.30	25	0.080
12	A	2	2	1.26	28	0.071
13	A	4	4	1.26	25	0.160
14	A	3	3	1.22	28	0.107
15	A	6	6	1.26	25	0.240
16	A	5	5	1.14	28	0.179
17	A	8	8	1.26	25	0.320
18	A	7	7	1.08	28	0.250
19	A	14	14	1.05	28	0.500
20	A	12	12	1.02	28	0.429
21	A	11	11	1.03	25	0.440

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	12	12	1.03	28	0.429
23	A	12	12	1.03	28	0.429
24	A	12	12	1.02	28	0.429
25	A	15	15	0.99	28	0.536
26	A	17	17	1.00	28	0.607
27	A	13	13	1.07	28	0.464
28	A	11	11	1.04	28	0.393
29	A	9	9	1.04	25	0.360
30	A	10	10	1.03	28	0.357
31	A	13	13	1.01	28	0.464
32	A	16	16	1.04	28	0.571
33	A	14	14	1.12	28	0.500
34	A	11	11	1.05	28	0.393
35	A	9	9	1.03	28	0.321
36	A	8	8	1.05	25	0.320
37	A	11	11	1.07	28	0.393
38	A	14	14	1.06	28	0.500
39	A	7	7	0.99	27	0.259
40	A	9	9	1.01	27	0.333
41	A	2	2	1.01	30	0.067
42	A	2	2	1.00	30	0.067
43	A	2	2	0.99	28	0.071
44	A	2	2	1.00	27	0.074
45	A	2	2	1.00	30	0.067
46	A	2	2	1.00	30	0.067
47	A	2	2	1.00	30	0.067
48	A	2	2	1.04	30	0.067
49	A	9	8	0.96	30	0.267
50	A	4	4	0.96	30	0.133
51	A	4	4	0.98	30	0.133
52	A	4	4	0.97	30	0.133
53	A	4	4	0.97	30	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.02	30	0.067
55	A	2	2	1.01	30	0.067
56	A	2	2	0.99	28	0.071
57	A	2	2	1.00	27	0.074
58	A	2	2	1.00	30	0.067
59	A	2	2	1.00	30	0.067
60	A	2	2	1.00	30	0.067
61	A	2	2	1.00	30	0.067
62	A	4	4	0.97	30	0.133
63	A	4	4	0.95	30	0.133
64	A	4	4	0.94	30	0.133
65	A	4	4	0.93	30	0.133
66	A	11	10	0.87	30	0.333
67	A	6	6	0.88	30	0.200
68	A	6	6	0.91	30	0.200
69	A	6	6	0.90	30	0.200
70	A	6	6	0.90	30	0.200
71	A	6	6	0.90	30	0.200
72	A	2	2	1.01	30	0.067
73	A	2	2	1.00	30	0.067
74	A	2	2	1.00	30	0.067
75	A	2	2	1.00	28	0.071
76	A	2	2	1.00	27	0.074
77	A	7	6	1.00	30	0.200
78	A	2	2	1.07	30	0.067
79	A	2	2	1.07	30	0.067
80	A	2	2	1.07	30	0.067
81	A	2	2	1.06	30	0.067
82	A	2	2	1.06	30	0.067
83	A	3	3	0.97	30	0.100
84	A	3	3	0.96	30	0.100
85	A	4	4	0.94	30	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	0.90	30	0.100
87	A	4	4	0.96	30	0.133
88	A	4	4	0.96	28	0.143
89	A	6	6	0.97	27	0.222
90	A	9	8	1.03	30	0.267
91	A	4	4	1.10	30	0.133
92	A	4	4	1.08	30	0.133
93	A	2	2	1.14	27	0.074
94	A	2	2	1.14	27	0.074
95	A	2	2	1.13	24	0.083
96	A	5	5	1.05	27	0.185
97	A	5	5	1.02	27	0.185
98	A	7	7	1.27	27	0.259
99	A	2	2	1.14	27	0.074
100	A	2	2	1.13	24	0.083
101	A	5	5	1.05	27	0.185
102	A	5	5	1.03	27	0.185
103	A	5	5	1.01	27	0.185
104	A	4	4	1.44	27	0.148
105	A	4	4	1.11	27	0.148
106	A	4	4	1.16	24	0.167
107	A	7	7	1.40	27	0.259
108	A	10	10	1.49	27	0.370
109	A	2	2	1.00	30	0.067
110	A	2	2	1.09	27	0.074
111	A	2	2	1.14	32	0.062
112	A	2	2	1.19	25	0.080
113	A	2	2	1.00	28	0.071

CHAPTER 3

Listing of integrals

3.1 $\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{f(a+bx^4)^2}{8b}$$

output
```
a*c*x+1/2*a*d*x^2+1/3*a*e*x^3+1/5*b*c*x^5+1/6*b*d*x^6+1/7*b*e*x^7+1/8*f*(b
*x^4+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/
6 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(ac+adx+aex^2+afx^3+bcx^4+bdx^5+bex^6+bfx^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$acx+\frac{1}{2}adx^2+\frac{1}{3}aex^3+\frac{1}{4}afx^4+\frac{1}{5}bcx^5+\frac{1}{6}bdx^6+\frac{1}{7}bex^7+\frac{1}{8}bfx^8$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (a*e*x^3)/3 + (a*f*x^4)/4 + (b*c*x^5)/5 + (b*d*x^6)/
6 + (b*e*x^7)/7 + (b*f*x^8)/8
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	55
default	$\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	55
norman	$\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	55
risch	$\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	55
parallelrisch	$\frac{1}{8}bf\,x^{8}+\frac{1}{7}be\,x^{7}+\frac{1}{6}x^{6}bd+\frac{1}{5}x^{5}bc+\frac{1}{4}af\,x^{4}+\frac{1}{3}ae\,x^{3}+\frac{1}{2}ad\,x^{2}+acx$	55
orering	$\frac{x\left(105bf\,x^{7}+120be\,x^{6}+140x^{5}bd+168bc\,x^{4}+210af\,x^{3}+280ae\,x^{2}+420adx+840ac\right)}{840}$	56

input
```
int((b*x^4+a)*(f*x^3+e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*b*f*x^8+1/7*b*e*x^7+1/6*x^6*b*d+1/5*x^5*b*c+1/4*a*f*x^4+1/3*a*e*x^3+1/
2*a*d*x^2+a*c*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \left(c+dx+ex^{2}+fx^{3}\right)\left(a+bx^{4}\right)\,dx=\frac{1}{8}\,bfx^{8}+\frac{1}{7}\,bex^{7}+\frac{1}{6}\,bdx^{6}+\frac{1}{5}\,bcx^{5}+\frac{1}{4}\,afx^{4}+\frac{1}{3}\,aex^{3}+\frac{1}{2}\,adx^{2}+acx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*
a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx = acx+\frac{adx^2}{2}+\frac{aex^3}{3}+\frac{afx^4}{4}+\frac{bcx^5}{5}+\frac{bdx^6}{6}+\frac{bex^7}{7}+\frac{bfx^8}{8}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

output
```
a*c*x + a*d*x**2/2 + a*e*x**3/3 + a*f*x**4/4 + b*c*x**5/5 + b*d*x**6/6 + b
*e*x**7/7 + b*f*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int (c+dx+ex^2+fx^3)(a+bx^4)\,dx = \frac{1}{8}bfx^8+\frac{1}{7}bex^7+\frac{1}{6}bdx^6+\frac{1}{5}bcx^5+\frac{1}{4}afx^4+\frac{1}{3}aex^3+\frac{1}{2}adx^2+acx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*
a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int (c+dx+ex^2+fx^3)\,(a+bx^4)\;dx=\frac{1}{8}bfx^8+\frac{1}{7}bex^7+\frac{1}{6}bdx^6+\frac{1}{5}bcx^5+\frac{1}{4}afx^4+\frac{1}{3}aex^3+\frac{1}{2}adx^2+acx$$

input

```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

output

```
1/8*b*f*x^8 + 1/7*b*e*x^7 + 1/6*b*d*x^6 + 1/5*b*c*x^5 + 1/4*a*f*x^4 + 1/3*
a*e*x^3 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int (c+dx+ex^2+fx^3)\,(a+bx^4)\;dx=\frac{b\,f\,x^8}{8}+\frac{b\,e\,x^7}{7}+\frac{b\,d\,x^6}{6}+\frac{b\,c\,x^5}{5}+\frac{a\,f\,x^4}{4}+\frac{a\,e\,x^3}{3}+\frac{a\,d\,x^2}{2}+a\,c\,x$$

input

```
int((a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

output

```
a*c*x + (a*d*x^2)/2 + (b*c*x^5)/5 + (a*e*x^3)/3 + (b*d*x^6)/6 + (a*f*x^4)/
4 + (b*e*x^7)/7 + (b*f*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int (c + dx + ex^2 + fx^3) (a + bx^4) \, dx$$
$$= \frac{x(105bf\, x^7 + 120be\, x^6 + 140bd\, x^5 + 168bc\, x^4 + 210af\, x^3 + 280ae\, x^2 + 420adx + 840ac)}{840}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

output
```
(x*(840*a*c + 420*a*d*x + 280*a*e*x**2 + 210*a*f*x**3 + 168*b*c*x**4 + 140
*b*d*x**5 + 120*b*e*x**6 + 105*b*f*x**7))/840
```

3.2 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

output
```
1/4*a*c*x^4+1/5*a*d*x^5+1/6*a*e*x^6+1/7*a*f*x^7+1/8*b*c*x^8+1/9*b*d*x^9+1/
10*b*e*x^10+1/11*b*f*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx=\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d
*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2360, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^4) \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \text{2360}$$

$$\int \left(acx^3+adx^4+aex^5+afx^6+bcx^7+bdx^8+bex^9+bfx^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}acx^4+\frac{1}{5}adx^5+\frac{1}{6}aex^6+\frac{1}{7}afx^7+\frac{1}{8}bcx^8+\frac{1}{9}bdx^9+\frac{1}{10}bex^{10}+\frac{1}{11}bfx^{11}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4),x]
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (a*e*x^6)/6 + (a*f*x^7)/7 + (b*c*x^8)/8 + (b*d
*x^9)/9 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2360
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}x^{5}ad+\frac{1}{6}ae\,x^{6}+\frac{1}{7}x^{7}af+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
default	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}x^{5}ad+\frac{1}{6}ae\,x^{6}+\frac{1}{7}x^{7}af+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
norman	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}x^{5}ad+\frac{1}{6}ae\,x^{6}+\frac{1}{7}x^{7}af+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
risch	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}x^{5}ad+\frac{1}{6}ae\,x^{6}+\frac{1}{7}x^{7}af+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
parallelrisch	$\frac{1}{4}ac\,x^{4}+\frac{1}{5}x^{5}ad+\frac{1}{6}ae\,x^{6}+\frac{1}{7}x^{7}af+\frac{1}{8}bc\,x^{8}+\frac{1}{9}bd\,x^{9}+\frac{1}{10}be\,x^{10}+\frac{1}{11}bf\,x^{11}$	58
orering	$\frac{x^{4}\left(2520bf\,x^{7}+2772be\,x^{6}+3080x^{5}bd+3465bc\,x^{4}+3960af\,x^{3}+4620ae\,x^{2}+5544adx+6930ac\right)}{27720}$	58

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*c*x^4+1/5*x^5*a*d+1/6*a*e*x^6+1/7*x^7*a*f+1/8*b*c*x^8+1/9*b*d*x^9+1/
10*b*e*x^10+1/11*b*f*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx=\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^9+\frac{1}{8}\,bcx^8+\frac{1}{7}\,afx^7+\frac{1}{6}\,aex^6+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 +
1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx=\frac{acx^4}{4}+\frac{adx^5}{5}+\frac{aex^6}{6}+\frac{afx^7}{7}+\frac{bcx^8}{8}+\frac{bdx^9}{9}+\frac{bex^{10}}{10}+\frac{bfx^{11}}{11}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a),x)
```

output
```
a*c*x**4/4 + a*d*x**5/5 + a*e*x**6/6 + a*f*x**7/7 + b*c*x**8/8 + b*d*x**9/
9 + b*e*x**10/10 + b*f*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx=\frac{1}{11}\,bfx^{11}+\frac{1}{10}\,bex^{10}+\frac{1}{9}\,bdx^9+\frac{1}{8}\,bcx^8+\frac{1}{7}\,afx^7+\frac{1}{6}\,aex^6+\frac{1}{5}\,adx^5+\frac{1}{4}\,acx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 +
1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right) dx = \frac{1}{11}bfx^{11}+\frac{1}{10}bex^{10}+\frac{1}{9}bdx^9+\frac{1}{8}bcx^8 + \frac{1}{7}afx^7+\frac{1}{6}aex^6+\frac{1}{5}adx^5+\frac{1}{4}acx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x, algorithm="giac")
```

output
```
1/11*b*f*x^11 + 1/10*b*e*x^10 + 1/9*b*d*x^9 + 1/8*b*c*x^8 + 1/7*a*f*x^7 +
1/6*a*e*x^6 + 1/5*a*d*x^5 + 1/4*a*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right) dx = \frac{b\,f\,x^{11}}{11}+\frac{b\,e\,x^{10}}{10}+\frac{b\,d\,x^9}{9}+\frac{b\,c\,x^8}{8} + \frac{a\,f\,x^7}{7}+\frac{a\,e\,x^6}{6}+\frac{a\,d\,x^5}{5}+\frac{a\,c\,x^4}{4}$$

input
```
int(x^3*(a + b*x^4)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a*c*x^4)/4 + (a*d*x^5)/5 + (b*c*x^8)/8 + (a*e*x^6)/6 + (b*d*x^9)/9 + (a*f
*x^7)/7 + (b*e*x^10)/10 + (b*f*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)\,dx$$
$$=\frac{x^4(2520bf\,x^7+2772be\,x^6+3080bd\,x^5+3465bc\,x^4+3960af\,x^3+4620ae\,x^2+5544adx+6930ac)}{27720}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a),x)
```

output
```
(x**4*(6930*a*c + 5544*a*d*x + 4620*a*e*x**2 + 3960*a*f*x**3 + 3465*b*c*x*
*4 + 3080*b*d*x**5 + 2772*b*e*x**6 + 2520*b*f*x**7))/27720
```

3.3 $\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 109

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{f(a+bx^4)^3}{12b}$$

output
```
a^2*c*x+1/2*a^2*d*x^2+1/3*a^2*e*x^3+2/5*a*b*c*x^5+1/3*a*b*d*x^6+2/7*a*b*e*
x^7+1/9*b^2*c*x^9+1/10*b^2*d*x^10+1/11*b^2*e*x^11+1/12*f*(b*x^4+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^2\,dx = a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{1}{4}a^2fx^4+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{1}{4}abfx^8+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}+\frac{1}{12}b^2fx^{12}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (a^2*f*x^4)/4 + (2*a*b*c*x^5)/5
+ (a*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4 + (b^2*c*x^9)/9 + (b^2*d
*x^10)/10 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(ex^2+dx+c\right) \left(bx^4+a\right)^2 dx+\frac{f\left(a+bx^4\right)^3}{12b}$$

$$\Big\downarrow \text{2188}$$

$$\int \left(b^2ex^{10}+b^2dx^9+b^2cx^8+2abex^6+2abdx^5+2abcx^4+a^2ex^2+a^2dx+a^2c\right)dx+\frac{f\left(a+bx^4\right)^3}{12b}$$

$$\Big\downarrow \text{2009}$$

$$a^2cx+\frac{1}{2}a^2dx^2+\frac{1}{3}a^2ex^3+\frac{2}{5}abcx^5+\frac{1}{3}abdx^6+\frac{2}{7}abex^7+\frac{f\left(a+bx^4\right)^3}{12b}+\frac{1}{9}b^2cx^9+\frac{1}{10}b^2dx^{10}+\frac{1}{11}b^2ex^{11}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
a^2*c*x + (a^2*d*x^2)/2 + (a^2*e*x^3)/3 + (2*a*b*c*x^5)/5 + (a*b*d*x^6)/3
+ (2*a*b*e*x^7)/7 + (b^2*c*x^9)/9 + (b^2*d*x^10)/10 + (b^2*e*x^11)/11 + (f
*(a + b*x^4)^3)/(12*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^{}$
default	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^{}$
norman	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^{}$
risch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^{}$
parallelrisch	$\frac{1}{12}b^2 f\,x^{12}+\frac{1}{11}b^2 e\,x^{11}+\frac{1}{10}b^2 d\,x^{10}+\frac{1}{9}b^2 c\,x^9+\frac{1}{4}abf\,x^8+\frac{2}{7}abe\,x^7+\frac{1}{3}abd\,x^6+\frac{2}{5}abc\,x^5+\frac{1}{4}a^{}$
orering	$\frac{x(1155b^2 f\,x^{11}+1260e\,b^2 x^{10}+1386b^2 d\,x^9+1540b^2 c\,x^8+3465x^7 abf+3960abe\,x^6+4620x^5 adb+5544abc\,x^4+3465a^2 f\,x^3+4620}{13860}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*f*x^12+1/11*b^2*e*x^11+1/10*b^2*d*x^10+1/9*b^2*c*x^9+1/4*a*b*f*x^
8+2/7*a*b*e*x^7+1/3*a*b*d*x^6+2/5*a*b*c*x^5+1/4*a^2*f*x^4+1/3*a^2*e*x^3+1/
2*a^2*d*x^2+a^2*c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{1}{12}\,b^2fx^{12}+\frac{1}{11}\,b^2ex^{11}+\frac{1}{10}\,b^2dx^{10}+\frac{1}{9}\,b^2cx^9+\frac{1}{4}\,abfx^8+\frac{2}{7}\,abex^7+\frac{1}{3}\,abdx^6+\frac{2}{5}\,abcx^5+\frac{1}{4}\,a^2fx^4+\frac{1}{3}\,a^2ex^3+\frac{1}{2}\,a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = a^2cx+\frac{a^2dx^2}{2}+\frac{a^2ex^3}{3}+\frac{a^2fx^4}{4}+\frac{2abcx^5}{5}+\frac{abdx^6}{3}+\frac{2abex^7}{7}+\frac{abfx^8}{4}+\frac{b^2cx^9}{9}+\frac{b^2dx^{10}}{10}+\frac{b^2ex^{11}}{11}+\frac{b^2fx^{12}}{12}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

output
```
a**2*c*x + a**2*d*x**2/2 + a**2*e*x**3/3 + a**2*f*x**4/4 + 2*a*b*c*x**5/5
+ a*b*d*x**6/3 + 2*a*b*e*x**7/7 + a*b*f*x**8/4 + b**2*c*x**9/9 + b**2*d*x*
*10/10 + b**2*e*x**11/11 + b**2*f*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{12}b^2fx^{12}+\frac{1}{11}b^2ex^{11}+\frac{1}{10}b^2dx^{10}+\frac{1}{9}b^2cx^9+\frac{1}{4}abfx^8+\frac{2}{7}abex^7+\frac{1}{3}abdx^6+\frac{2}{5}abcx^5+\frac{1}{4}a^2fx^4+\frac{1}{3}a^2ex^3+\frac{1}{2}a^2dx^2+a^2cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/12*b^2*f*x^12 + 1/11*b^2*e*x^11 + 1/10*b^2*d*x^10 + 1/9*b^2*c*x^9 + 1/4*
a*b*f*x^8 + 2/7*a*b*e*x^7 + 1/3*a*b*d*x^6 + 2/5*a*b*c*x^5 + 1/4*a^2*f*x^4
+ 1/3*a^2*e*x^3 + 1/2*a^2*d*x^2 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx = \frac{f\,a^2\,x^4}{4}+\frac{e\,a^2\,x^3}{3}+\frac{d\,a^2\,x^2}{2}+c\,a^2\,x+\frac{f\,a\,b\,x^8}{4}+\frac{2\,e\,a\,b\,x^7}{7}+\frac{d\,a\,b\,x^6}{3}+\frac{2\,c\,a\,b\,x^5}{5}+\frac{f\,b^2\,x^{12}}{12}+\frac{e\,b^2\,x^{11}}{11}+\frac{d\,b^2\,x^{10}}{10}+\frac{c\,b^2\,x^9}{9}$$

input
```
int((a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*c*x^9)/9 + (a^2*e*x^3)/3 + (b^2*d*x^10)/10 + (a^2*f*x
^4)/4 + (b^2*e*x^11)/11 + (b^2*f*x^12)/12 + a^2*c*x + (2*a*b*c*x^5)/5 + (a
*b*d*x^6)/3 + (2*a*b*e*x^7)/7 + (a*b*f*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx$$

$$=\frac{x(1155b^2f\,x^{11}+1260b^2e\,x^{10}+1386b^2d\,x^9+1540b^2c\,x^8+3465abf\,x^7+3960abe\,x^6+4620abd\,x^5+554}{13860}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

output
```
(x*(13860*a**2*c + 6930*a**2*d*x + 4620*a**2*e*x**2 + 3465*a**2*f*x**3 + 5
544*a*b*c*x**4 + 4620*a*b*d*x**5 + 3960*a*b*e*x**6 + 3465*a*b*f*x**7 + 154
0*b**2*c*x**8 + 1386*b**2*d*x**9 + 1260*b**2*e*x**10 + 1155*b**2*f*x**11))
/13860
```

3.4 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}+\frac{c\left(a+bx^4\right)^3}{12b}$$

output
```
1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a^2*f*x^7+2/9*a*b*d*x^9+1/5*a*b*e*x^10+2/1
1*a*b*f*x^11+1/13*b^2*d*x^13+1/14*b^2*e*x^14+1/15*b^2*f*x^15+1/12*c*(b*x^4
+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{1}{4}a^2cx^4+\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{1}{4}abcx^8+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{12}b^2cx^{12}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
(a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (a*b*c*x^8
)/4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11 + (b^2*c*x^12)/
12 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^4\right)^2 \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^4+a\right)^2 \left(x^3\left(fx^3+ex^2+dx+c\right)-cx^3\right)dx+\frac{c\left(a+bx^4\right)^3}{12b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^2fx^{14}+b^2ex^{13}+b^2dx^{12}+2abfx^{10}+2abex^9+2abdx^8+a^2fx^6+a^2ex^5+a^2dx^4\right)dx+\frac{c\left(a+bx^4\right)^3}{12b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^2dx^5+\frac{1}{6}a^2ex^6+\frac{1}{7}a^2fx^7+\frac{c\left(a+bx^4\right)^3}{12b}+\frac{2}{9}abdx^9+\frac{1}{5}abex^{10}+\frac{2}{11}abfx^{11}+\frac{1}{13}b^2dx^{13}+\frac{1}{14}b^2ex^{14}+\frac{1}{15}b^2fx^{15}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^2,x]
```

output
```
(a^2*d*x^5)/5 + (a^2*e*x^6)/6 + (a^2*f*x^7)/7 + (2*a*b*d*x^9)/9 + (a*b*e*x
^10)/5 + (2*a*b*f*x^11)/11 + (b^2*d*x^13)/13 + (b^2*e*x^14)/14 + (b^2*f*x^
15)/15 + (c*(a + b*x^4)^3)/(12*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

method	result
gosper	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
default	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
norman	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
risch	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
parallelrisch	$\frac{1}{4}a^2c\,x^4+\frac{1}{5}a^2d\,x^5+\frac{1}{6}a^2e\,x^6+\frac{1}{7}a^2f\,x^7+\frac{1}{4}abc\,x^8+\frac{2}{9}abd\,x^9+\frac{1}{5}abe\,x^{10}+\frac{2}{11}abf\,x^{11}+\frac{1}{12}b^2c$
orering	$\frac{x^4(12012b^2f\,x^{11}+12870e\,b^2x^{10}+13860b^2d\,x^9+15015b^2c\,x^8+32760x^7abf+36036abe\,x^6+40040x^5adb+45045abc\,x^4+25740a}{180180}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*c*x^4+1/5*a^2*d*x^5+1/6*a^2*e*x^6+1/7*a^2*f*x^7+1/4*a*b*c*x^8+2/9*
a*b*d*x^9+1/5*a*b*e*x^10+2/11*a*b*f*x^11+1/12*b^2*c*x^12+1/13*b^2*d*x^13+1
/14*b^2*e*x^14+1/15*b^2*f*x^15
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2 dx = \frac{1}{15}b^2fx^{15}+\frac{1}{14}b^2ex^{14}+\frac{1}{13}b^2dx^{13}+\frac{1}{12}b^2cx^{12} + \frac{2}{11}abfx^{11}+\frac{1}{5}abex^{10}+\frac{2}{9}abdx^9+\frac{1}{4}abcx^8 + \frac{1}{7}a^2fx^7+\frac{1}{6}a^2ex^6+\frac{1}{5}a^2dx^5+\frac{1}{4}a^2cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/
11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f
*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2 dx = \frac{a^2cx^4}{4}+\frac{a^2dx^5}{5}+\frac{a^2ex^6}{6}+\frac{a^2fx^7}{7} + \frac{abcx^8}{4}+\frac{2abdx^9}{9}+\frac{abex^{10}}{5}+\frac{2abfx^{11}}{11} + \frac{b^2cx^{12}}{12}+\frac{b^2dx^{13}}{13}+\frac{b^2ex^{14}}{14}+\frac{b^2fx^{15}}{15}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**2,x)
```

output
```
a**2*c*x**4/4 + a**2*d*x**5/5 + a**2*e*x**6/6 + a**2*f*x**7/7 + a*b*c*x**8
/4 + 2*a*b*d*x**9/9 + a*b*e*x**10/5 + 2*a*b*f*x**11/11 + b**2*c*x**12/12 +
 b**2*d*x**13/13 + b**2*e*x**14/14 + b**2*f*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/
11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f
*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^2\,dx = \frac{1}{15}\,b^2fx^{15}+\frac{1}{14}\,b^2ex^{14}+\frac{1}{13}\,b^2dx^{13}+\frac{1}{12}\,b^2cx^{12}+\frac{2}{11}\,abfx^{11}+\frac{1}{5}\,abex^{10}+\frac{2}{9}\,abdx^9+\frac{1}{4}\,abcx^8+\frac{1}{7}\,a^2fx^7+\frac{1}{6}\,a^2ex^6+\frac{1}{5}\,a^2dx^5+\frac{1}{4}\,a^2cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/15*b^2*f*x^15 + 1/14*b^2*e*x^14 + 1/13*b^2*d*x^13 + 1/12*b^2*c*x^12 + 2/
11*a*b*f*x^11 + 1/5*a*b*e*x^10 + 2/9*a*b*d*x^9 + 1/4*a*b*c*x^8 + 1/7*a^2*f
*x^7 + 1/6*a^2*e*x^6 + 1/5*a^2*d*x^5 + 1/4*a^2*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx=\frac{f\,a^2\,x^7}{7}+\frac{e\,a^2\,x^6}{6}+\frac{d\,a^2\,x^5}{5}+\frac{c\,a^2\,x^4}{4}+\frac{2\,f\,a\,b\,x^{11}}{11}+\frac{e\,a\,b\,x^{10}}{5}+\frac{2\,d\,a\,b\,x^9}{9}+\frac{c\,a\,b\,x^8}{4}+\frac{f\,b^2\,x^{15}}{15}+\frac{e\,b^2\,x^{14}}{14}+\frac{d\,b^2\,x^{13}}{13}+\frac{c\,b^2\,x^{12}}{12}$$

input
```
int(x^3*(a + b*x^4)^2*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^2*c*x^4)/4 + (a^2*d*x^5)/5 + (b^2*c*x^12)/12 + (a^2*e*x^6)/6 + (b^2*d*x
^13)/13 + (a^2*f*x^7)/7 + (b^2*e*x^14)/14 + (b^2*f*x^15)/15 + (a*b*c*x^8)/
4 + (2*a*b*d*x^9)/9 + (a*b*e*x^10)/5 + (2*a*b*f*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^2\,dx$$

$$=\frac{x^4(12012b^2f\,x^{11}+12870b^2e\,x^{10}+13860b^2d\,x^9+15015b^2c\,x^8+32760abf\,x^7+36036abe\,x^6+40040abd}{180180}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^2,x)
```

output
```
(x**4*(45045*a**2*c + 36036*a**2*d*x + 30030*a**2*e*x**2 + 25740*a**2*f*x*
*3 + 45045*a*b*c*x**4 + 40040*a*b*d*x**5 + 36036*a*b*e*x**6 + 32760*a*b*f*
x**7 + 15015*b**2*c*x**8 + 13860*b**2*d*x**9 + 12870*b**2*e*x**10 + 12012*
b**2*f*x**11))/180180
```

3.5 $\int (c+dx+ex^2+fx^3) (a+bx^4)^3 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 151

$$\int (c+dx+ex^2+fx^3) (a+bx^4)^3 \, dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5 +\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9 +\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{13}b^3cx^{13} +\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{f(a+bx^4)^4}{16b}$$

output

```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+3/5*a^2*b*c*x^5+1/2*a^2*b*d*x^6+3/7*a^
2*b*e*x^7+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+3/11*a*b^2*e*x^11+1/13*b^3*c*x
^13+1/14*b^3*d*x^14+1/15*b^3*e*x^15+1/16*f*(b*x^4+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{1}{4}a^3fx^4+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{3}{8}a^2bfx^8+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{1}{4}ab^2fx^{12}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}+\frac{1}{16}b^3fx^{16}$$

input

```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output

```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (a^3*f*x^4)/4 + (3*a^2*b*c*x^5)/
5 + (a^2*b*d*x^6)/2 + (3*a^2*b*e*x^7)/7 + (3*a^2*b*f*x^8)/8 + (a*b^2*c*x^9
)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2*e*x^11)/11 + (a*b^2*f*x^12)/4 + (b^3*
c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^3 (c+dx+ex^2+fx^3)\,dx$$

$$\Big\downarrow \text{2017}$$

$$\int (ex^2+dx+c)(bx^4+a)^3dx+\frac{f(a+bx^4)^4}{16b}$$

$$\Big\downarrow \text{2188}$$

$$\int \left(b^3ex^{14}+b^3dx^{13}+b^3cx^{12}+3ab^2ex^{10}+3ab^2dx^9+3ab^2cx^8+3a^2bex^6+3a^2bdx^5+3a^2bcx^4+a^3ex^2+a^3dx\right.$$

$$\frac{f\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \; 2009$$

$$a^3cx+\frac{1}{2}a^3dx^2+\frac{1}{3}a^3ex^3+\frac{3}{5}a^2bcx^5+\frac{1}{2}a^2bdx^6+\frac{3}{7}a^2bex^7+\frac{1}{3}ab^2cx^9+\frac{3}{10}ab^2dx^{10}+\frac{3}{11}ab^2ex^{11}+\frac{f\left(a+bx^4\right)^4}{16b}+\frac{1}{13}b^3cx^{13}+\frac{1}{14}b^3dx^{14}+\frac{1}{15}b^3ex^{15}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
a^3*c*x + (a^3*d*x^2)/2 + (a^3*e*x^3)/3 + (3*a^2*b*c*x^5)/5 + (a^2*b*d*x^6
)/2 + (3*a^2*b*e*x^7)/7 + (a*b^2*c*x^9)/3 + (3*a*b^2*d*x^10)/10 + (3*a*b^2
*e*x^11)/11 + (b^3*c*x^13)/13 + (b^3*d*x^14)/14 + (b^3*e*x^15)/15 + (f*(a
+ b*x^4)^4)/(16*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

method	result
gosper	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c$
default	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c$
norman	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c$
risch	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c$
parallelrisch	$a^3cx + \frac{1}{2}a^3d\,x^2 + \frac{1}{3}a^3e\,x^3 + \frac{1}{4}f\,a^3x^4 + \frac{3}{5}a^2bc\,x^5 + \frac{1}{2}a^2bd\,x^6 + \frac{3}{7}a^2be\,x^7 + \frac{3}{8}f\,a^2b\,x^8 + \frac{1}{3}a\,b^2c$
orering	$\frac{x(15015f\,b^3x^{15}+16016b^3e\,x^{14}+17160b^3d\,x^{13}+18480b^3c\,x^{12}+60060f\,a\,b^2x^{11}+65520a\,b^2e\,x^{10}+72072x^9a\,b^2d+80080a\,b^2c\,x^8}{2402}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*c*x+1/2*a^3*d*x^2+1/3*a^3*e*x^3+1/4*f*a^3*x^4+3/5*a^2*b*c*x^5+1/2*a^2*
b*d*x^6+3/7*a^2*b*e*x^7+3/8*f*a^2*b*x^8+1/3*a*b^2*c*x^9+3/10*a*b^2*d*x^10+
3/11*a*b^2*e*x^11+1/4*f*a*b^2*x^12+1/13*b^3*c*x^13+1/14*b^3*d*x^14+1/15*b^
3*e*x^15+1/16*f*b^3*x^16
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{1}{16}\,b^3fx^{16} + \frac{1}{15}\,b^3ex^{15} + \frac{1}{14}\,b^3dx^{14} + \frac{1}{13}\,b^3cx^{13} + \frac{1}{4}\,ab^2fx^{12} + \frac{3}{11}\,ab^2ex^{11} + \frac{3}{10}\,ab^2dx^{10} + \frac{1}{3}\,ab^2cx^9 + \frac{3}{8}\,a^2bfx^8 + \frac{3}{7}\,a^2bex^7 + \frac{1}{2}\,a^2bdx^6 + \frac{3}{5}\,a^2bcx^5 + \frac{1}{4}\,a^3fx^4 + \frac{1}{3}\,a^3ex^3 + \frac{1}{2}\,a^3dx^2 + a^3cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = a^3cx+\frac{a^3dx^2}{2}+\frac{a^3ex^3}{3}+\frac{a^3fx^4}{4}+\frac{3a^2bcx^5}{5}+\frac{a^2bdx^6}{2}+\frac{3a^2bex^7}{7}+\frac{3a^2bfx^8}{8}+\frac{ab^2cx^9}{3}+\frac{3ab^2dx^{10}}{10}+\frac{3ab^2ex^{11}}{11}+\frac{ab^2fx^{12}}{4}+\frac{b^3cx^{13}}{13}+\frac{b^3dx^{14}}{14}+\frac{b^3ex^{15}}{15}+\frac{b^3fx^{16}}{16}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

output
```
a**3*c*x + a**3*d*x**2/2 + a**3*e*x**3/3 + a**3*f*x**4/4 + 3*a**2*b*c*x**5
/5 + a**2*b*d*x**6/2 + 3*a**2*b*e*x**7/7 + 3*a**2*b*f*x**8/8 + a*b**2*c*x*
*9/3 + 3*a*b**2*d*x**10/10 + 3*a*b**2*e*x**11/11 + a*b**2*f*x**12/4 + b**3
*c*x**13/13 + b**3*d*x**14/14 + b**3*e*x**15/15 + b**3*f*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = \frac{1}{16}b^3fx^{16}+\frac{1}{15}b^3ex^{15}+\frac{1}{14}b^3dx^{14}+\frac{1}{13}b^3cx^{13}+\frac{1}{4}ab^2fx^{12}+\frac{3}{11}ab^2ex^{11}+\frac{3}{10}ab^2dx^{10}+\frac{1}{3}ab^2cx^9+\frac{3}{8}a^2bfx^8+\frac{3}{7}a^2bex^7+\frac{1}{2}a^2bdx^6+\frac{3}{5}a^2bcx^5+\frac{1}{4}a^3fx^4+\frac{1}{3}a^3ex^3+\frac{1}{2}a^3dx^2+a^3cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^3 \, dx = \frac{1}{16} b^3 f x^{16} + \frac{1}{15} b^3 e x^{15} + \frac{1}{14} b^3 d x^{14} + \frac{1}{13} b^3 c x^{13} + \frac{1}{4} a b^2 f x^{12} + \frac{3}{11} a b^2 e x^{11} + \frac{3}{10} a b^2 d x^{10} + \frac{1}{3} a b^2 c x^9 + \frac{3}{8} a^2 b f x^8 + \frac{3}{7} a^2 b e x^7 + \frac{1}{2} a^2 b d x^6 + \frac{3}{5} a^2 b c x^5 + \frac{1}{4} a^3 f x^4 + \frac{1}{3} a^3 e x^3 + \frac{1}{2} a^3 d x^2 + a^3 c x$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/16*b^3*f*x^16 + 1/15*b^3*e*x^15 + 1/14*b^3*d*x^14 + 1/13*b^3*c*x^13 + 1/
4*a*b^2*f*x^12 + 3/11*a*b^2*e*x^11 + 3/10*a*b^2*d*x^10 + 1/3*a*b^2*c*x^9 +
 3/8*a^2*b*f*x^8 + 3/7*a^2*b*e*x^7 + 1/2*a^2*b*d*x^6 + 3/5*a^2*b*c*x^5 + 1
/4*a^3*f*x^4 + 1/3*a^3*e*x^3 + 1/2*a^3*d*x^2 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3 dx = \frac{f a^3 x^4}{4} + \frac{e a^3 x^3}{3} + \frac{d a^3 x^2}{2} + c a^3 x + \frac{3 f a^2 b x^8}{8} + \frac{3 e a^2 b x^7}{7} + \frac{d a^2 b x^6}{2} + \frac{3 c a^2 b x^5}{5} + \frac{f a b^2 x^{12}}{4} + \frac{3 e a b^2 x^{11}}{11} + \frac{3 d a b^2 x^{10}}{10} + \frac{c a b^2 x^9}{3} + \frac{f b^3 x^{16}}{16} + \frac{e b^3 x^{15}}{15} + \frac{d b^3 x^{14}}{14} + \frac{c b^3 x^{13}}{13}$$

input
```
int((a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^3*d*x^2)/2 + (b^3*c*x^13)/13 + (a^3*e*x^3)/3 + (b^3*d*x^14)/14 + (a^3*f
*x^4)/4 + (b^3*e*x^15)/15 + (b^3*f*x^16)/16 + a^3*c*x + (3*a^2*b*c*x^5)/5
+ (a*b^2*c*x^9)/3 + (a^2*b*d*x^6)/2 + (3*a*b^2*d*x^10)/10 + (3*a^2*b*e*x^7
)/7 + (3*a*b^2*e*x^11)/11 + (3*a^2*b*f*x^8)/8 + (a*b^2*f*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^3 dx = \frac{x(15015b^3f x^{15} + 16016b^3e x^{14} + 17160b^3d x^{13} + 18480b^3c x^{12} + 60060a b^2f x^{11} + 65520a b^2e x^{10} + 7207}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

output
```
(x*(240240*a**3*c + 120120*a**3*d*x + 80080*a**3*e*x**2 + 60060*a**3*f*x**
3 + 144144*a**2*b*c*x**4 + 120120*a**2*b*d*x**5 + 102960*a**2*b*e*x**6 + 9
0090*a**2*b*f*x**7 + 80080*a*b**2*c*x**8 + 72072*a*b**2*d*x**9 + 65520*a*b
**2*e*x**10 + 60060*a*b**2*f*x**11 + 18480*b**3*c*x**12 + 17160*b**3*d*x**
13 + 16016*b**3*e*x**14 + 15015*b**3*f*x**15))/240240
```

3.6 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx=\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}+\frac{c(a+bx^4)^4}{16b}$$

output
```
1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+1/3*a^2*b*d*x^9+3/10*a^2*b*e*x^1
0+3/11*a^2*b*f*x^11+3/13*a*b^2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1
/17*b^3*d*x^17+1/18*b^3*e*x^18+1/19*b^3*f*x^19+1/16*c*(b*x^4+a)^4/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{4}a^3cx^4+\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{3}{8}a^2bcx^8+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{1}{4}ab^2cx^{12}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{1}{16}b^3cx^{16}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
(a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (3*a^2*b*c
*x^8)/8 + (a^2*b*d*x^9)/3 + (3*a^2*b*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (a
*b^2*c*x^12)/4 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)/14 + (a*b^2*f*x^15
)/5 + (b^3*c*x^16)/16 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/1
9
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^4\right)^3(c+dx+ex^2+fx^3)\,dx$$

$$\Big\downarrow \ 2017$$

$$\int \left(bx^4+a\right)^3\left(x^3(fx^3+ex^2+dx+c)-cx^3\right)dx+\frac{c\left(a+bx^4\right)^4}{16b}$$

$$\Big\downarrow \text{2389}$$

$$\int \left(b^3fx^{18}+b^3ex^{17}+b^3dx^{16}+3ab^2fx^{14}+3ab^2ex^{13}+3ab^2dx^{12}+3a^2bfx^{10}+3a^2bex^9+3a^2bdx^8+a^3fx^6+a^3\right.$$

$$\frac{c(a+bx^4)^4}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^3dx^5+\frac{1}{6}a^3ex^6+\frac{1}{7}a^3fx^7+\frac{1}{3}a^2bdx^9+\frac{3}{10}a^2bex^{10}+\frac{3}{11}a^2bfx^{11}+\frac{3}{13}ab^2dx^{13}+\frac{3}{14}ab^2ex^{14}+\frac{1}{5}ab^2fx^{15}+\frac{c(a+bx^4)^4}{16b}+\frac{1}{17}b^3dx^{17}+\frac{1}{18}b^3ex^{18}+\frac{1}{19}b^3fx^{19}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^3,x]
```

output
```
(a^3*d*x^5)/5 + (a^3*e*x^6)/6 + (a^3*f*x^7)/7 + (a^2*b*d*x^9)/3 + (3*a^2*b
*e*x^10)/10 + (3*a^2*b*f*x^11)/11 + (3*a*b^2*d*x^13)/13 + (3*a*b^2*e*x^14)
/14 + (a*b^2*f*x^15)/5 + (b^3*d*x^17)/17 + (b^3*e*x^18)/18 + (b^3*f*x^19)/
19 + (c*(a + b*x^4)^4)/(16*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

method	result
gosper	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}x^8c\,a^2b+\frac{1}{3}x^9a^2bd+\frac{3}{10}x^{10}e\,a^2b+\frac{3}{11}x^{11}f\,a^2b+$
default	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}x^8c\,a^2b+\frac{1}{3}x^9a^2bd+\frac{3}{10}x^{10}e\,a^2b+\frac{3}{11}x^{11}f\,a^2b+$
norman	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}x^8c\,a^2b+\frac{1}{3}x^9a^2bd+\frac{3}{10}x^{10}e\,a^2b+\frac{3}{11}x^{11}f\,a^2b+$
risch	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}x^8c\,a^2b+\frac{1}{3}x^9a^2bd+\frac{3}{10}x^{10}e\,a^2b+\frac{3}{11}x^{11}f\,a^2b+$
parallelrisch	$\frac{1}{4}a^3c\,x^4+\frac{1}{5}a^3d\,x^5+\frac{1}{6}a^3e\,x^6+\frac{1}{7}a^3f\,x^7+\frac{3}{8}x^8c\,a^2b+\frac{1}{3}x^9a^2bd+\frac{3}{10}x^{10}e\,a^2b+\frac{3}{11}x^{11}f\,a^2b+$
orering	$x^4\left(12252240f\,b^3x^{15}+12932920b^3e\,x^{14}+13693680b^3d\,x^{13}+145495535b^3c\,x^{12}+46558512fa\,b^2x^{11}+49884120a\,b^2e\,x^{10}+53721\right.$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*c*x^4+1/5*a^3*d*x^5+1/6*a^3*e*x^6+1/7*a^3*f*x^7+3/8*x^8*c*a^2*b+1/
3*x^9*a^2*b*d+3/10*x^10*e*a^2*b+3/11*x^11*f*a^2*b+1/4*a*b^2*c*x^12+3/13*a*
b^2*d*x^13+3/14*a*b^2*e*x^14+1/5*a*b^2*f*x^15+1/16*b^3*c*x^16+1/17*b^3*d*x
^17+1/18*b^3*e*x^18+1/19*b^3*f*x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\begin{aligned}\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^3\,dx &= \frac{1}{19}\,b^3fx^{19}+\frac{1}{18}\,b^3ex^{18}+\frac{1}{17}\,b^3dx^{17}\\ &\quad+\frac{1}{16}\,b^3cx^{16}+\frac{1}{5}\,ab^2fx^{15}+\frac{3}{14}\,ab^2ex^{14}\\ &\quad+\frac{3}{13}\,ab^2dx^{13}+\frac{1}{4}\,ab^2cx^{12}+\frac{3}{11}\,a^2bfx^{11}\\ &\quad+\frac{3}{10}\,a^2bex^{10}+\frac{1}{3}\,a^2bdx^9+\frac{3}{8}\,a^2bcx^8\\ &\quad+\frac{1}{7}\,a^3fx^7+\frac{1}{6}\,a^3ex^6+\frac{1}{5}\,a^3dx^5+\frac{1}{4}\,a^3cx^4\end{aligned}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/
5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12
+ 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^
8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.18

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{a^3cx^4}{4}+\frac{a^3dx^5}{5}+\frac{a^3ex^6}{6}+\frac{a^3fx^7}{7}+\frac{3a^2bcx^8}{8}+\frac{a^2bdx^9}{3}+\frac{3a^2bex^{10}}{10}+\frac{3a^2bfx^{11}}{11}+\frac{ab^2cx^{12}}{4}+\frac{3ab^2dx^{13}}{13}+\frac{3ab^2ex^{14}}{14}+\frac{ab^2fx^{15}}{5}+\frac{b^3cx^{16}}{16}+\frac{b^3dx^{17}}{17}+\frac{b^3ex^{18}}{18}+\frac{b^3fx^{19}}{19}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**3,x)
```

output
```
a**3*c*x**4/4 + a**3*d*x**5/5 + a**3*e*x**6/6 + a**3*f*x**7/7 + 3*a**2*b*c
*x**8/8 + a**2*b*d*x**9/3 + 3*a**2*b*e*x**10/10 + 3*a**2*b*f*x**11/11 + a*
b**2*c*x**12/4 + 3*a*b**2*d*x**13/13 + 3*a*b**2*e*x**14/14 + a*b**2*f*x**1
5/5 + b**3*c*x**16/16 + b**3*d*x**17/17 + b**3*e*x**18/18 + b**3*f*x**19/1
9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx=\frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3ex^{18}+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2ex^{14}+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bex^{10}+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3ex^6+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/
5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12
+ 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^
8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^3\,dx = \frac{1}{19}b^3fx^{19}+\frac{1}{18}b^3ex^{18}+\frac{1}{17}b^3dx^{17}+\frac{1}{16}b^3cx^{16}+\frac{1}{5}ab^2fx^{15}+\frac{3}{14}ab^2ex^{14}+\frac{3}{13}ab^2dx^{13}+\frac{1}{4}ab^2cx^{12}+\frac{3}{11}a^2bfx^{11}+\frac{3}{10}a^2bex^{10}+\frac{1}{3}a^2bdx^9+\frac{3}{8}a^2bcx^8+\frac{1}{7}a^3fx^7+\frac{1}{6}a^3ex^6+\frac{1}{5}a^3dx^5+\frac{1}{4}a^3cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/19*b^3*f*x^19 + 1/18*b^3*e*x^18 + 1/17*b^3*d*x^17 + 1/16*b^3*c*x^16 + 1/
5*a*b^2*f*x^15 + 3/14*a*b^2*e*x^14 + 3/13*a*b^2*d*x^13 + 1/4*a*b^2*c*x^12
+ 3/11*a^2*b*f*x^11 + 3/10*a^2*b*e*x^10 + 1/3*a^2*b*d*x^9 + 3/8*a^2*b*c*x^
8 + 1/7*a^3*f*x^7 + 1/6*a^3*e*x^6 + 1/5*a^3*d*x^5 + 1/4*a^3*c*x^4
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = \frac{f\,a^3\,x^7}{7}+\frac{e\,a^3\,x^6}{6}+\frac{d\,a^3\,x^5}{5}+\frac{c\,a^3\,x^4}{4}+\frac{3\,f\,a^2\,b\,x^{11}}{11}+\frac{3\,e\,a^2\,b\,x^{10}}{10}+\frac{d\,a^2\,b\,x^9}{3}+\frac{3\,c\,a^2\,b\,x^8}{8}+\frac{f\,a\,b^2\,x^{15}}{5}+\frac{3\,e\,a\,b^2\,x^{14}}{14}+\frac{3\,d\,a\,b^2\,x^{13}}{13}+\frac{c\,a\,b^2\,x^{12}}{4}+\frac{f\,b^3\,x^{19}}{19}+\frac{e\,b^3\,x^{18}}{18}+\frac{d\,b^3\,x^{17}}{17}+\frac{c\,b^3\,x^{16}}{16}$$

input
```
int(x^3*(a + b*x^4)^3*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^3*c*x^4)/4 + (a^3*d*x^5)/5 + (b^3*c*x^16)/16 + (a^3*e*x^6)/6 + (b^3*d*x
^17)/17 + (a^3*f*x^7)/7 + (b^3*e*x^18)/18 + (b^3*f*x^19)/19 + (3*a^2*b*c*x
^8)/8 + (a*b^2*c*x^12)/4 + (a^2*b*d*x^9)/3 + (3*a*b^2*d*x^13)/13 + (3*a^2*
b*e*x^10)/10 + (3*a*b^2*e*x^14)/14 + (3*a^2*b*f*x^11)/11 + (a*b^2*f*x^15)/
5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^3\,dx = \frac{x^4(12252240b^3f\,x^{15}+12932920b^3e\,x^{14}+13693680b^3d\,x^{13}+145495535b^3c\,x^{12}+46558512a\,b^2f\,x^{11}+498}{}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^3,x)
```

output

```
(x**4*(58198140*a**3*c + 46558512*a**3*d*x + 38798760*a**3*e*x**2 + 332560
80*a**3*f*x**3 + 87297210*a**2*b*c*x**4 + 77597520*a**2*b*d*x**5 + 6983776
8*a**2*b*e*x**6 + 63488880*a**2*b*f*x**7 + 58198140*a*b**2*c*x**8 + 537213
60*a*b**2*d*x**9 + 49884120*a*b**2*e*x**10 + 46558512*a*b**2*f*x**11 + 145
49535*b**3*c*x**12 + 13693680*b**3*d*x**13 + 12932920*b**3*e*x**14 + 12252
240*b**3*f*x**15))/232792560
```

3.7 $\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{f(a+bx^4)^5}{20b}$$

output
```
a^4*c*x+1/2*a^4*d*x^2+1/3*a^4*e*x^3+4/5*a^3*b*c*x^5+2/3*a^3*b*d*x^6+4/7*a^
3*b*e*x^7+2/3*a^2*b^2*c*x^9+3/5*a^2*b^2*d*x^10+6/11*a^2*b^2*e*x^11+4/13*a*
b^3*c*x^13+2/7*a*b^3*d*x^14+4/15*a*b^3*e*x^15+1/17*b^4*c*x^17+1/18*b^4*d*x
^18+1/19*b^4*e*x^19+1/20*f*(b*x^4+a)^5/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.22

$$\int (c+dx+ex^2+fx^3) (a+bx^4)^4 \, dx = a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{1}{4}a^4fx^4 +\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{1}{2}a^3bfx^8 +\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11} +\frac{1}{2}a^2b^2fx^{12}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14} +\frac{4}{15}ab^3ex^{15}+\frac{1}{4}ab^3fx^{16}+\frac{1}{17}b^4cx^{17} +\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}+\frac{1}{20}b^4fx^{20}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (a^4*f*x^4)/4 + (4*a^3*b*c*x^5)/
5 + (2*a^3*b*d*x^6)/3 + (4*a^3*b*e*x^7)/7 + (a^3*b*f*x^8)/2 + (2*a^2*b^2*c
*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/
2 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3*e*x^15)/15 + (a*b^
3*f*x^16)/4 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (b^4*f
*x^20)/20
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2017, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^4 (c+dx+ex^2+fx^3) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(ex^2+dx+c\right)\left(bx^4+a\right)^4dx+\frac{f\left(a+bx^4\right)^5}{20b}$$

$\Big\downarrow$ 2188

$$\int \left(b^4ex^{18}+b^4dx^{17}+b^4cx^{16}+4ab^3ex^{14}+4ab^3dx^{13}+4ab^3cx^{12}+6a^2b^2ex^{10}+6a^2b^2dx^9+6a^2b^2cx^8+4a^3bex^6\right.$$
$$\frac{f\left(a+bx^4\right)^5}{20b}$$

$\Big\downarrow$ 2009

$$a^4cx+\frac{1}{2}a^4dx^2+\frac{1}{3}a^4ex^3+\frac{4}{5}a^3bcx^5+\frac{2}{3}a^3bdx^6+\frac{4}{7}a^3bex^7+\frac{2}{3}a^2b^2cx^9+\frac{3}{5}a^2b^2dx^{10}+\frac{6}{11}a^2b^2ex^{11}+\frac{4}{13}ab^3cx^{13}+\frac{2}{7}ab^3dx^{14}+\frac{4}{15}ab^3ex^{15}+\frac{f\left(a+bx^4\right)^5}{20b}+\frac{1}{17}b^4cx^{17}+\frac{1}{18}b^4dx^{18}+\frac{1}{19}b^4ex^{19}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
a^4*c*x + (a^4*d*x^2)/2 + (a^4*e*x^3)/3 + (4*a^3*b*c*x^5)/5 + (2*a^3*b*d*x
^6)/3 + (4*a^3*b*e*x^7)/7 + (2*a^2*b^2*c*x^9)/3 + (3*a^2*b^2*d*x^10)/5 + (
6*a^2*b^2*e*x^11)/11 + (4*a*b^3*c*x^13)/13 + (2*a*b^3*d*x^14)/7 + (4*a*b^3
*e*x^15)/15 + (b^4*c*x^17)/17 + (b^4*d*x^18)/18 + (b^4*e*x^19)/19 + (f*(a
+ b*x^4)^5)/(20*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

method	result
gosper	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}e\,a^4x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}c\,a^2b$
default	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}e\,a^4x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}c\,a^2b$
norman	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}e\,a^4x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}c\,a^2b$
risch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}e\,a^4x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}c\,a^2b$
parallelrisch	$a^4cx+\frac{1}{2}a^4d\,x^2+\frac{1}{3}e\,a^4x^3+\frac{1}{4}a^4f\,x^4+\frac{4}{5}a^3bc\,x^5+\frac{2}{3}a^3bd\,x^6+\frac{4}{7}a^3be\,x^7+\frac{1}{2}a^3bf\,x^8+\frac{2}{3}c\,a^2b$
orering	$\frac{x(2909907f\,b^4x^{19}+3063060b^4e\,x^{18}+3233230d\,b^4x^{17}+3423420b^4c\,x^{16}+14549535f\,a\,b^3x^{15}+15519504a\,b^3e\,x^{14}+16628040a\,b}{}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*c*x+1/2*a^4*d*x^2+1/3*e*a^4*x^3+1/4*a^4*f*x^4+4/5*a^3*b*c*x^5+2/3*a^3*
b*d*x^6+4/7*a^3*b*e*x^7+1/2*a^3*b*f*x^8+2/3*c*a^2*b^2*x^9+3/5*a^2*b^2*d*x^
10+6/11*a^2*b^2*e*x^11+1/2*f*a^2*b^2*x^12+4/13*a*b^3*c*x^13+2/7*a*b^3*d*x^
14+4/15*a*b^3*e*x^15+1/4*f*a*b^3*x^16+1/17*b^4*c*x^17+1/18*b^4*d*x^18+1/19
*b^4*e*x^19+1/20*f*b^4*x^20
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^4 \, dx = \frac{1}{20}\,b^4fx^{20}+\frac{1}{19}\,b^4ex^{19}+\frac{1}{18}\,b^4dx^{18}+\frac{1}{17}\,b^4cx^{17}+\frac{1}{4}\,ab^3fx^{16}+\frac{4}{15}\,ab^3ex^{15}+\frac{2}{7}\,ab^3dx^{14}+\frac{4}{13}\,ab^3cx^{13}+\frac{1}{2}\,a^2b^2fx^{12}+\frac{6}{11}\,a^2b^2ex^{11}+\frac{3}{5}\,a^2b^2dx^{10}+\frac{2}{3}\,a^2b^2cx^9+\frac{1}{2}\,a^3bfx^8+\frac{4}{7}\,a^3bex^7+\frac{2}{3}\,a^3bdx^6+\frac{4}{5}\,a^3bcx^5+\frac{1}{4}\,a^4fx^4+\frac{1}{3}\,a^4ex^3+\frac{1}{2}\,a^4dx^2+a^4cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/
4*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13
+ 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*
b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*
b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.25

$$\int \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^4 \, dx = a^4cx+\frac{a^4dx^2}{2}+\frac{a^4ex^3}{3}+\frac{a^4fx^4}{4}+\frac{4a^3bcx^5}{5}+\frac{2a^3bdx^6}{3}+\frac{4a^3bex^7}{7}+\frac{a^3bfx^8}{2}+\frac{2a^2b^2cx^9}{3}+\frac{3a^2b^2dx^{10}}{5}+\frac{6a^2b^2ex^{11}}{11}+\frac{a^2b^2fx^{12}}{2}+\frac{4ab^3cx^{13}}{13}+\frac{2ab^3dx^{14}}{7}+\frac{4ab^3ex^{15}}{15}+\frac{ab^3fx^{16}}{4}+\frac{b^4cx^{17}}{17}+\frac{b^4dx^{18}}{18}+\frac{b^4ex^{19}}{19}+\frac{b^4fx^{20}}{20}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

output
```
a**4*c*x + a**4*d*x**2/2 + a**4*e*x**3/3 + a**4*f*x**4/4 + 4*a**3*b*c*x**5
/5 + 2*a**3*b*d*x**6/3 + 4*a**3*b*e*x**7/7 + a**3*b*f*x**8/2 + 2*a**2*b**2
*c*x**9/3 + 3*a**2*b**2*d*x**10/5 + 6*a**2*b**2*e*x**11/11 + a**2*b**2*f*x
**12/2 + 4*a*b**3*c*x**13/13 + 2*a*b**3*d*x**14/7 + 4*a*b**3*e*x**15/15 +
a*b**3*f*x**16/4 + b**4*c*x**17/17 + b**4*d*x**18/18 + b**4*e*x**19/19 + b
**4*f*x**20/20
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^4\,dx = \frac{1}{20}b^4fx^{20} + \frac{1}{19}b^4ex^{19} + \frac{1}{18}b^4dx^{18} + \frac{1}{17}b^4cx^{17} + \frac{1}{4}ab^3fx^{16} + \frac{4}{15}ab^3ex^{15} + \frac{2}{7}ab^3dx^{14} + \frac{4}{13}ab^3cx^{13} + \frac{1}{2}a^2b^2fx^{12} + \frac{6}{11}a^2b^2ex^{11} + \frac{3}{5}a^2b^2dx^{10} + \frac{2}{3}a^2b^2cx^9 + \frac{1}{2}a^3bfx^8 + \frac{4}{7}a^3bex^7 + \frac{2}{3}a^3bdx^6 + \frac{4}{5}a^3bcx^5 + \frac{1}{4}a^4fx^4 + \frac{1}{3}a^4ex^3 + \frac{1}{2}a^4dx^2 + a^4cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/
4*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13
+ 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*
b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*
b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{1}{20}b^4fx^{20}+\frac{1}{19}b^4ex^{19}+\frac{1}{18}b^4dx^{18}+\frac{1}{17}b^4cx^{17}+\frac{1}{4}ab^3fx^{16}+\frac{4}{15}ab^3ex^{15}+\frac{2}{7}ab^3dx^{14}+\frac{4}{13}ab^3cx^{13}+\frac{1}{2}a^2b^2fx^{12}+\frac{6}{11}a^2b^2ex^{11}+\frac{3}{5}a^2b^2dx^{10}+\frac{2}{3}a^2b^2cx^9+\frac{1}{2}a^3bfx^8+\frac{4}{7}a^3bex^7+\frac{2}{3}a^3bdx^6+\frac{4}{5}a^3bcx^5+\frac{1}{4}a^4fx^4+\frac{1}{3}a^4ex^3+\frac{1}{2}a^4dx^2+a^4cx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/20*b^4*f*x^20 + 1/19*b^4*e*x^19 + 1/18*b^4*d*x^18 + 1/17*b^4*c*x^17 + 1/
4*a*b^3*f*x^16 + 4/15*a*b^3*e*x^15 + 2/7*a*b^3*d*x^14 + 4/13*a*b^3*c*x^13
+ 1/2*a^2*b^2*f*x^12 + 6/11*a^2*b^2*e*x^11 + 3/5*a^2*b^2*d*x^10 + 2/3*a^2*
b^2*c*x^9 + 1/2*a^3*b*f*x^8 + 4/7*a^3*b*e*x^7 + 2/3*a^3*b*d*x^6 + 4/5*a^3*
b*c*x^5 + 1/4*a^4*f*x^4 + 1/3*a^4*e*x^3 + 1/2*a^4*d*x^2 + a^4*c*x
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

$$\int (c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{f\,a^4\,x^4}{4}+\frac{e\,a^4\,x^3}{3}+\frac{d\,a^4\,x^2}{2}+c\,a^4\,x+\frac{f\,a^3\,b\,x^8}{2}+\frac{4\,e\,a^3\,b\,x^7}{7}+\frac{2\,d\,a^3\,b\,x^6}{3}+\frac{4\,c\,a^3\,b\,x^5}{5}+\frac{f\,a^2\,b^2\,x^{12}}{2}+\frac{6\,e\,a^2\,b^2\,x^{11}}{11}+\frac{3\,d\,a^2\,b^2\,x^{10}}{5}+\frac{2\,c\,a^2\,b^2\,x^9}{3}+\frac{f\,a\,b^3\,x^{16}}{4}+\frac{4\,e\,a\,b^3\,x^{15}}{15}+\frac{2\,d\,a\,b^3\,x^{14}}{7}+\frac{4\,c\,a\,b^3\,x^{13}}{13}+\frac{f\,b^4\,x^{20}}{20}+\frac{e\,b^4\,x^{19}}{19}+\frac{d\,b^4\,x^{18}}{18}+\frac{c\,b^4\,x^{17}}{17}$$

input
```
int((a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^4*d*x^2)/2 + (b^4*c*x^17)/17 + (a^4*e*x^3)/3 + (b^4*d*x^18)/18 + (a^4*f
*x^4)/4 + (b^4*e*x^19)/19 + (b^4*f*x^20)/20 + a^4*c*x + (2*a^2*b^2*c*x^9)/
3 + (3*a^2*b^2*d*x^10)/5 + (6*a^2*b^2*e*x^11)/11 + (a^2*b^2*f*x^12)/2 + (4
*a^3*b*c*x^5)/5 + (4*a*b^3*c*x^13)/13 + (2*a^3*b*d*x^6)/3 + (2*a*b^3*d*x^1
4)/7 + (4*a^3*b*e*x^7)/7 + (4*a*b^3*e*x^15)/15 + (a^3*b*f*x^8)/2 + (a*b^3*
f*x^16)/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^4 \, dx$$
$$= \frac{x(2909907b^4 f x^{19} + 3063060b^4 e x^{18} + 3233230b^4 d x^{17} + 3423420b^4 c x^{16} + 14549535a b^3 f x^{15} + 1551950\cdots}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

output
```
(x*(58198140*a**4*c + 29099070*a**4*d*x + 19399380*a**4*e*x**2 + 14549535*
a**4*f*x**3 + 46558512*a**3*b*c*x**4 + 38798760*a**3*b*d*x**5 + 33256080*a
**3*b*e*x**6 + 29099070*a**3*b*f*x**7 + 38798760*a**2*b**2*c*x**8 + 349188
84*a**2*b**2*d*x**9 + 31744440*a**2*b**2*e*x**10 + 29099070*a**2*b**2*f*x*
*11 + 17907120*a*b**3*c*x**12 + 16628040*a*b**3*d*x**13 + 15519504*a*b**3*
e*x**14 + 14549535*a*b**3*f*x**15 + 3423420*b**4*c*x**16 + 3233230*b**4*d*
x**17 + 3063060*b**4*e*x**18 + 2909907*b**4*f*x**19))/58198140
```

3.8 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^4\,dx$

Optimal result

Integrand size = 28, antiderivative size = 198

$$\begin{aligned}\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx &= \frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9\\ &\quad+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}\\ &\quad+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}\\ &\quad+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{1}{21}b^4dx^{21}\\ &\quad+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}+\frac{c(a+bx^4)^5}{20b}\end{aligned}$$

output
```
1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+4/9*a^3*b*d*x^9+2/5*a^3*b*e*x^10
+4/11*a^3*b*f*x^11+6/13*a^2*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^
15+4/17*a*b^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/21*b^4*d*x^21+1/
22*b^4*e*x^22+1/23*b^4*f*x^23+1/20*c*(b*x^4+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.22

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx=\frac{1}{4}a^4cx^4+\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{1}{2}a^3bcx^8+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{1}{2}a^2b^2cx^{12}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{1}{4}ab^3cx^{16}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{1}{20}b^4cx^{20}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
(a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (a^3*b*c*x
^8)/2 + (4*a^3*b*d*x^9)/9 + (2*a^3*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (a^
2*b^2*c*x^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^
2*f*x^15)/5 + (a*b^3*c*x^16)/4 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9
+ (4*a*b^3*f*x^19)/19 + (b^4*c*x^20)/20 + (b^4*d*x^21)/21 + (b^4*e*x^22)/2
2 + (b^4*f*x^23)/23
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^4\right)^4\left(c+dx+ex^2+fx^3\right)\,dx$$

$$\Big\downarrow \; 2017$$

$$\int \left(bx^4+a\right)^4\left(x^3\left(fx^3+ex^2+dx+c\right)-cx^3\right)dx+\frac{c\left(a+bx^4\right)^5}{20b}$$

$\big\downarrow$ 2389

$$\int \left(b^4fx^{22}+b^4ex^{21}+b^4dx^{20}+4ab^3fx^{18}+4ab^3ex^{17}+4ab^3dx^{16}+6a^2b^2fx^{14}+6a^2b^2ex^{13}+6a^2b^2dx^{12}+4a^3bf\right.$$

$$\frac{c\left(a+bx^4\right)^5}{20b}$$

$\big\downarrow$ 2009

$$\frac{1}{5}a^4dx^5+\frac{1}{6}a^4ex^6+\frac{1}{7}a^4fx^7+\frac{4}{9}a^3bdx^9+\frac{2}{5}a^3bex^{10}+\frac{4}{11}a^3bfx^{11}+\frac{6}{13}a^2b^2dx^{13}+\frac{3}{7}a^2b^2ex^{14}+\frac{2}{5}a^2b^2fx^{15}+\frac{4}{17}ab^3dx^{17}+\frac{2}{9}ab^3ex^{18}+\frac{4}{19}ab^3fx^{19}+\frac{c\left(a+bx^4\right)^5}{20b}+\frac{1}{21}b^4dx^{21}+\frac{1}{22}b^4ex^{22}+\frac{1}{23}b^4fx^{23}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^4,x]
```

output
```
(a^4*d*x^5)/5 + (a^4*e*x^6)/6 + (a^4*f*x^7)/7 + (4*a^3*b*d*x^9)/9 + (2*a^3
*b*e*x^10)/5 + (4*a^3*b*f*x^11)/11 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*
x^14)/7 + (2*a^2*b^2*f*x^15)/5 + (4*a*b^3*d*x^17)/17 + (2*a*b^3*e*x^18)/9
+ (4*a*b^3*f*x^19)/19 + (b^4*d*x^21)/21 + (b^4*e*x^22)/22 + (b^4*f*x^23)/2
3 + (c*(a + b*x^4)^5)/(20*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02

method	result
gosper	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+$
default	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+$
norman	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+$
risch	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+$
parallelrisch	$\frac{1}{4}c\,a^4x^4+\frac{1}{5}a^4d\,x^5+\frac{1}{6}a^4e\,x^6+\frac{1}{7}a^4f\,x^7+\frac{1}{2}c\,a^3b\,x^8+\frac{4}{9}a^3bd\,x^9+\frac{2}{5}a^3be\,x^{10}+\frac{4}{11}a^3bf\,x^{11}+$
orering	$x^4(58198140f\,b^4x^{19}+60843510b^4e\,x^{18}+63740820d\,b^4x^{17}+669278861b^4c\,x^{16}+281801520f\,a\,b^3x^{15}+297457160a\,b^3e\,x^{14}+314$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*c*a^4*x^4+1/5*a^4*d*x^5+1/6*a^4*e*x^6+1/7*a^4*f*x^7+1/2*c*a^3*b*x^8+4/
9*a^3*b*d*x^9+2/5*a^3*b*e*x^10+4/11*a^3*b*f*x^11+1/2*c*a^2*b^2*x^12+6/13*a
^2*b^2*d*x^13+3/7*a^2*b^2*e*x^14+2/5*a^2*b^2*f*x^15+1/4*a*b^3*c*x^16+4/17*
a*b^3*d*x^17+2/9*a*b^3*e*x^18+4/19*a*b^3*f*x^19+1/20*b^4*c*x^20+1/21*b^4*d
*x^21+1/22*b^4*e*x^22+1/23*b^4*f*x^23
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{1}{23}\,b^4fx^{23}+\frac{1}{22}\,b^4ex^{22}+\frac{1}{21}\,b^4dx^{21}+\frac{1}{20}\,b^4cx^{20}+\frac{4}{19}\,ab^3fx^{19}+\frac{2}{9}\,ab^3ex^{18}+\frac{4}{17}\,ab^3dx^{17}+\frac{1}{4}\,ab^3cx^{16}+\frac{2}{5}\,a^2b^2fx^{15}+\frac{3}{7}\,a^2b^2ex^{14}+\frac{6}{13}\,a^2b^2dx^{13}+\frac{1}{2}\,a^2b^2cx^{12}+\frac{4}{11}\,a^3bfx^{11}+\frac{2}{5}\,a^3bex^{10}+\frac{4}{9}\,a^3bdx^9+\frac{1}{2}\,a^3bcx^8+\frac{1}{7}\,a^4fx^7+\frac{1}{6}\,a^4ex^6+\frac{1}{5}\,a^4dx^5+\frac{1}{4}\,a^4cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/
19*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16
+ 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*
b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*
a^3*b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^
4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.24

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{a^4cx^4}{4}+\frac{a^4dx^5}{5}+\frac{a^4ex^6}{6}+\frac{a^4fx^7}{7}+\frac{a^3bcx^8}{2}+\frac{4a^3bdx^9}{9}+\frac{2a^3bex^{10}}{5}+\frac{4a^3bfx^{11}}{11}+\frac{a^2b^2cx^{12}}{2}+\frac{6a^2b^2dx^{13}}{13}+\frac{3a^2b^2ex^{14}}{7}+\frac{2a^2b^2fx^{15}}{5}+\frac{ab^3cx^{16}}{4}+\frac{4ab^3dx^{17}}{17}+\frac{2ab^3ex^{18}}{9}+\frac{4ab^3fx^{19}}{19}+\frac{b^4cx^{20}}{20}+\frac{b^4dx^{21}}{21}+\frac{b^4ex^{22}}{22}+\frac{b^4fx^{23}}{23}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**4,x)
```

output
```
a**4*c*x**4/4 + a**4*d*x**5/5 + a**4*e*x**6/6 + a**4*f*x**7/7 + a**3*b*c*x
**8/2 + 4*a**3*b*d*x**9/9 + 2*a**3*b*e*x**10/5 + 4*a**3*b*f*x**11/11 + a**
2*b**2*c*x**12/2 + 6*a**2*b**2*d*x**13/13 + 3*a**2*b**2*e*x**14/7 + 2*a**2
*b**2*f*x**15/5 + a*b**3*c*x**16/4 + 4*a*b**3*d*x**17/17 + 2*a*b**3*e*x**1
8/9 + 4*a*b**3*f*x**19/19 + b**4*c*x**20/20 + b**4*d*x**21/21 + b**4*e*x**
22/22 + b**4*f*x**23/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3 (c+dx+ex^2+fx^3) (a+bx^4)^4 \, dx = \frac{1}{23} b^4 f x^{23} + \frac{1}{22} b^4 e x^{22} + \frac{1}{21} b^4 d x^{21} + \frac{1}{20} b^4 c x^{20} + \frac{4}{19} a b^3 f x^{19} + \frac{2}{9} a b^3 e x^{18} + \frac{4}{17} a b^3 d x^{17} + \frac{1}{4} a b^3 c x^{16} + \frac{2}{5} a^2 b^2 f x^{15} + \frac{3}{7} a^2 b^2 e x^{14} + \frac{6}{13} a^2 b^2 d x^{13} + \frac{1}{2} a^2 b^2 c x^{12} + \frac{4}{11} a^3 b f x^{11} + \frac{2}{5} a^3 b e x^{10} + \frac{4}{9} a^3 b d x^9 + \frac{1}{2} a^3 b c x^8 + \frac{1}{7} a^4 f x^7 + \frac{1}{6} a^4 e x^6 + \frac{1}{5} a^4 d x^5 + \frac{1}{4} a^4 c x^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/
19*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16
+ 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*
b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*
a^3*b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^
4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{1}{23}\,b^4fx^{23}+\frac{1}{22}\,b^4ex^{22}+\frac{1}{21}\,b^4dx^{21}+\frac{1}{20}\,b^4cx^{20}+\frac{4}{19}\,ab^3fx^{19}+\frac{2}{9}\,ab^3ex^{18}+\frac{4}{17}\,ab^3dx^{17}+\frac{1}{4}\,ab^3cx^{16}+\frac{2}{5}\,a^2b^2fx^{15}+\frac{3}{7}\,a^2b^2ex^{14}+\frac{6}{13}\,a^2b^2dx^{13}+\frac{1}{2}\,a^2b^2cx^{12}+\frac{4}{11}\,a^3bfx^{11}+\frac{2}{5}\,a^3bex^{10}+\frac{4}{9}\,a^3bdx^9+\frac{1}{2}\,a^3bcx^8+\frac{1}{7}\,a^4fx^7+\frac{1}{6}\,a^4ex^6+\frac{1}{5}\,a^4dx^5+\frac{1}{4}\,a^4cx^4$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/23*b^4*f*x^23 + 1/22*b^4*e*x^22 + 1/21*b^4*d*x^21 + 1/20*b^4*c*x^20 + 4/
19*a*b^3*f*x^19 + 2/9*a*b^3*e*x^18 + 4/17*a*b^3*d*x^17 + 1/4*a*b^3*c*x^16
+ 2/5*a^2*b^2*f*x^15 + 3/7*a^2*b^2*e*x^14 + 6/13*a^2*b^2*d*x^13 + 1/2*a^2*
b^2*c*x^12 + 4/11*a^3*b*f*x^11 + 2/5*a^3*b*e*x^10 + 4/9*a^3*b*d*x^9 + 1/2*
a^3*b*c*x^8 + 1/7*a^4*f*x^7 + 1/6*a^4*e*x^6 + 1/5*a^4*d*x^5 + 1/4*a^4*c*x^
4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^4\,dx=\frac{f\,a^4\,x^7}{7}+\frac{e\,a^4\,x^6}{6}+\frac{d\,a^4\,x^5}{5}+\frac{c\,a^4\,x^4}{4}+\frac{4\,f\,a^3\,b\,x^{11}}{11}+\frac{2\,e\,a^3\,b\,x^{10}}{5}+\frac{4\,d\,a^3\,b\,x^9}{9}+\frac{c\,a^3\,b\,x^8}{2}+\frac{2\,f\,a^2\,b^2\,x^{15}}{5}+\frac{3\,e\,a^2\,b^2\,x^{14}}{7}+\frac{6\,d\,a^2\,b^2\,x^{13}}{13}+\frac{c\,a^2\,b^2\,x^{12}}{2}+\frac{4\,f\,a\,b^3\,x^{19}}{19}+\frac{2\,e\,a\,b^3\,x^{18}}{9}+\frac{4\,d\,a\,b^3\,x^{17}}{17}+\frac{c\,a\,b^3\,x^{16}}{4}+\frac{f\,b^4\,x^{23}}{23}+\frac{e\,b^4\,x^{22}}{22}+\frac{d\,b^4\,x^{21}}{21}+\frac{c\,b^4\,x^{20}}{20}$$

input
```
int(x^3*(a + b*x^4)^4*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
(a^4*c*x^4)/4 + (a^4*d*x^5)/5 + (b^4*c*x^20)/20 + (a^4*e*x^6)/6 + (b^4*d*x
^21)/21 + (a^4*f*x^7)/7 + (b^4*e*x^22)/22 + (b^4*f*x^23)/23 + (a^2*b^2*c*x
^12)/2 + (6*a^2*b^2*d*x^13)/13 + (3*a^2*b^2*e*x^14)/7 + (2*a^2*b^2*f*x^15)
/5 + (a^3*b*c*x^8)/2 + (a*b^3*c*x^16)/4 + (4*a^3*b*d*x^9)/9 + (4*a*b^3*d*x
^17)/17 + (2*a^3*b*e*x^10)/5 + (2*a*b^3*e*x^18)/9 + (4*a^3*b*f*x^11)/11 +
(4*a*b^3*f*x^19)/19
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^4\,dx = \frac{x^4(58198140b^4f\,x^{19}+60843510b^4e\,x^{18}+63740820b^4d\,x^{17}+669278 61b^4c\,x^{16}+281801520a\,b^3f\,x^{15}+29}{}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^4,x)
```

output
```
(x**4*(334639305*a**4*c + 267711444*a**4*d*x + 223092870*a**4*e*x**2 + 191
222460*a**4*f*x**3 + 669278610*a**3*b*c*x**4 + 594914320*a**3*b*d*x**5 + 5
35422888*a**3*b*e*x**6 + 486748080*a**3*b*f*x**7 + 669278610*a**2*b**2*c*x
**8 + 617795640*a**2*b**2*d*x**9 + 573667380*a**2*b**2*e*x**10 + 535422888
*a**2*b**2*f*x**11 + 334639305*a*b**3*c*x**12 + 314954640*a*b**3*d*x**13 +
 297457160*a*b**3*e*x**14 + 281801520*a*b**3*f*x**15 + 66927861*b**4*c*x**
16 + 63740820*b**4*d*x**17 + 60843510*b**4*e*x**18 + 58198140*b**4*f*x**19
))/1338557220
```

3.9 $\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 132

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx=\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}}+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

output

```
1/2*(c-a^(1/2)*e/b^(1/2))*arctan(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(1/4)+1/2*(b
^(1/2)*c+a^(1/2)*e)*arctanh(b^(1/4)*x/a^(1/4))/a^(3/4)/b^(3/4)+1/2*d*arcta
nh(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)-1/4*f*ln(-b*x^4+a)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.62

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \frac{\left(\sqrt[4]{a}\sqrt{b}c-a^{3/4}e\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2ab^{3/4}} - \frac{\left(\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d+a^{3/4}e\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}x\right)}{4ab^{3/4}} - \frac{\left(-\sqrt[4]{a}\sqrt{b}c+\sqrt{a}\sqrt[4]{b}d-a^{3/4}e\right)\log\left(\sqrt[4]{a}+\sqrt[4]{b}x\right)}{4ab^{3/4}} + \frac{d\log\left(\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{a}\sqrt{b}} - \frac{f\log\left(a-bx^4\right)}{4b}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

output
```
((a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a*b^(3/4)
) - ((a^(1/4)*Sqrt[b]*c + Sqrt[a]*b^(1/4)*d + a^(3/4)*e)*Log[a^(1/4) - b^(
1/4)*x])/(4*a*b^(3/4)) - ((-(a^(1/4)*Sqrt[b]*c) + Sqrt[a]*b^(1/4)*d - a^(3
/4)*e)*Log[a^(1/4) + b^(1/4)*x])/(4*a*b^(3/4)) + (d*Log[Sqrt[a] + Sqrt[b]*
x^2])/(4*Sqrt[a]*Sqrt[b]) - (f*Log[a - b*x^4])/(4*b)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$$

$$\Big\downarrow \; 2415$$

$$\int \left(\frac{c+ex^2}{a-bx^4}+\frac{x(d+fx^2)}{a-bx^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}c-\sqrt{a}e\right)}{2a^{3/4}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2a^{3/4}b^{3/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}-\frac{f\log\left(a-bx^4\right)}{4b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x]
```

output
```
((Sqrt[b]*c - Sqrt[a]*e)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
+ ((Sqrt[b]*c + Sqrt[a]*e)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4
)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - (f*Log[a - b
*x^4])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.33

method	result
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4-a\right)}\frac{\left(\_R^3f+\_R^2e+d\_R+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{e\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{f\ln(-bx^4+}{4b}$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*sum((_R^3*f+_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 241149, normalized size of antiderivative = 1826.89

$$\int\frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int\frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx=\text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.32

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx = \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{b}d+\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{4\sqrt{a}b} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
1/2*(sqrt(b)*c - sqrt(a)*e)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(
a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/4*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b
)*x^2 + sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*d + sqrt(a)*f)*log(sqrt(b)*x^2
 - sqrt(a))/(sqrt(a)*b) - 1/4*(sqrt(b)*c + sqrt(a)*e)*log((sqrt(b)*x - sqr
t(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(92) = 184$.

Time = 0.13 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.09

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4}\,dx$$
$$= -\frac{\sqrt{2}\left(b^2c-\sqrt{2}(-ab^3)^{\frac{1}{4}}bd+\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c+\sqrt{2}(-ab^3)^{\frac{1}{4}}bd-\sqrt{-ab}be\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^3\right)^{\frac{3}{4}}}$$
$$-\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}$$
$$+\frac{\sqrt{2}\left(b^2c-\sqrt{-ab}be\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^3\right)^{\frac{3}{4}}}-\frac{f\log\left(|bx^4-a|\right)}{4b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(b^2*c - sqrt(2)*(-a*b^3)^(1/4)*b*d + sqrt(-a*b)*b*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/
4*sqrt(2)*(b^2*c + sqrt(2)*(-a*b^3)^(1/4)*b*d - sqrt(-a*b)*b*e)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/8*s
qrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a
/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*e)*log(x^2 - sqrt(
2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4) - 1/4*f*log(abs(b*x^4 - a))
/b
```

Mupad [B] (verification not implemented)

Time = 7.40 (sec) , antiderivative size = 1970, normalized size of antiderivative = 14.92

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a - b*x^4),x)
```

output
```
symsum(log(b^2*c^2*e - b^2*c*d^2 - b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*ro
ot(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f
^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16
*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4
*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2 + 2*a
*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)^2*a*b^3*c
- 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*
z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*f^3*z + 4*a^2*b*d*e^2
*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e - 2*a^2*b*d^2*f^2
 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4 - b^3*c^4, z, k)*b^3*
c^2*x - b^2*c^2*f*x + 16*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3 - 64*a^2
*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*
z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 16*a^3*b*
f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^
2*e - 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a*b^2*d^4 + a^3*f^4 - a^2*b*e^4
- b^3*c^4, z, k)^2*a*b^3*d*x - 4*root(256*a^3*b^4*z^4 + 256*a^3*b^3*f*z^3
- 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 - 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z + 16*a*b^3*c^2*d*z + 1
6*a^3*b*f^3*z + 4*a^2*b*d*e^2*f - 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.66

$$\int \frac{c+dx+ex^2+fx^3}{a-bx^4} \, dx$$
$$= \frac{-2b^{\frac{1}{4}}a^{\frac{3}{4}} atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right) e+2b^{\frac{3}{4}}a^{\frac{1}{4}} atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right) c-b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right) e+b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}x\right) e-b^{\frac{3}{4}}a^{\frac{1}{4}}\log}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*e + 2*b**(3/
4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*c - b**(1/4)*a**(3/4)*lo
g(a**(1/4) - b**(1/4)*x)*e + b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)*x)*
e - b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*c + b**(3/4)*a**(1/4)*log
(a**(1/4) + b**(1/4)*x)*c - sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1/4)*x)*d -
 sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*d + sqrt(b)*sqrt(a)*log(sqrt(a
) + sqrt(b)*x**2)*d - log(a**(1/4) - b**(1/4)*x)*a*f - log(a**(1/4) + b**(
1/4)*x)*a*f - log(sqrt(a) + sqrt(b)*x**2)*a*f)/(4*a*b)
```

3.10 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 161

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = -\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}+\frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}f}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{5/4}}+\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2b^{7/4}}+\frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log(a-bx^4)}{4b}$$

output
```
-d*x/b-1/2*e*x^2/b-1/3*f*x^3/b+1/2*a^(1/4)*(d-a^(1/2)*f/b^(1/2))*arctan(b^
(1/4)*x/a^(1/4))/b^(5/4)+1/2*a^(1/4)*(b^(1/2)*d+a^(1/2)*f)*arctanh(b^(1/4)
*x/a^(1/4))/b^(7/4)+1/2*a^(1/2)*e*arctanh(b^(1/2)*x^2/a^(1/2))/b^(3/2)-1/4
*c*ln(-b*x^4+a)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.37

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$= \frac{-12b^{3/4}dx-6b^{3/4}ex^2-4b^{3/4}fx^3+6\left(\sqrt[4]{a}\sqrt{b}d-a^{3/4}f\right)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-3\left(\sqrt[4]{a}\sqrt{b}d+\sqrt{a}\sqrt[4]{b}e+a^{3/4}f\right.}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

output
```
(-12*b^(3/4)*d*x - 6*b^(3/4)*e*x^2 - 4*b^(3/4)*f*x^3 + 6*(a^(1/4)*Sqrt[b]*
d - a^(3/4)*f)*ArcTan[(b^(1/4)*x)/a^(1/4)] - 3*(a^(1/4)*Sqrt[b]*d + Sqrt[a
]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) - b^(1/4)*x] + 3*(a^(1/4)*Sqrt[b]*d -
 Sqrt[a]*b^(1/4)*e + a^(3/4)*f)*Log[a^(1/4) + b^(1/4)*x] + 3*Sqrt[a]*b^(1/
4)*e*Log[Sqrt[a] + Sqrt[b]*x^2] - 3*b^(3/4)*c*Log[a - b*x^4])/(12*b^(7/4))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$\Big\downarrow \text{2370}$$
$$\int \left(\frac{x^3(c+ex^2)}{a-bx^4}+\frac{x^4(d+fx^2)}{a-bx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{b}d-\sqrt{a}f\right)}{2b^{7/4}}+\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2b^{7/4}}+\frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}-\frac{c\log\left(a-bx^4\right)}{4b}-\frac{dx}{b}-\frac{ex^2}{2b}-\frac{fx^3}{3b}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x]
```

output
```
-((d*x)/b) - (e*x^2)/(2*b) - (f*x^3)/(3*b) + (a^(1/4)*(Sqrt[b]*d - Sqrt[a]
*f)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[
a]*f)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*b^(7/4)) + (Sqrt[a]*e*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a]])/(2*b^(3/2)) - (c*Log[a - b*x^4])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{f x^3}{3b}-\frac{e x^2}{2b}-\frac{dx}{b}+\frac{\sum_{\_R=\text{RootOf}\left(b\_Z^4-a\right)}\frac{\left(-\_R^3 bc-\_R^2 af-\_R ae-ad\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$-\frac{\frac{1}{3}f x^3+\frac{1}{2}e x^2+dx}{b}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4}+\frac{ae\ln\left(\frac{a+x^2\sqrt{ab}}{a-x^2\sqrt{ab}}\right)}{4\sqrt{ab}}-\frac{af\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-}{b}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*f*x^3/b-1/2*e*x^2/b-d*x/b+1/4/b^2*sum((-_R^3*b*c-_R^2*a*f-_R*a*e-a*d)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.12 (sec) , antiderivative size = 220680, normalized size of antiderivative = 1370.68

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(-b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.29

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx = -\frac{2\,fx^3+3\,ex^2+6\,dx}{6\,b} + \frac{\frac{2\left(a\sqrt{b}d-a^{\frac{3}{2}}f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\left(\sqrt{a}bc-a\sqrt{b}e\right)\log\left(\sqrt{b}x^2+\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(\sqrt{a}bc+a\sqrt{b}e\right)\log\left(\sqrt{b}x^2-\sqrt{a}\right)}{\sqrt{a}b} - \frac{\left(a\sqrt{b}d+a^{\frac{3}{2}}f\right)\log\left(\frac{\sqrt{b}x-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}x+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4\,b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="maxima")
```

output
```
-1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/4*(2*(a*sqrt(b)*d - a^(3/2)*f)*arct
an(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) - (sqrt(a)*b*c - a*sqrt(b)*e)*log(sqrt(b)*x^2 + sqrt(a))/(sqrt(a)*b) - (
sqrt(a)*b*c + a*sqrt(b)*e)*log(sqrt(b)*x^2 - sqrt(a))/(sqrt(a)*b) - (a*sqr
t(b)*d + a^(3/2)*f)*log((sqrt(b)*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + s
qrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.02

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$= -\frac{c\log(|bx^4-a|)}{4\,b}$$
$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-(-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{4\,b^4}$$
$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-ab}b^2e-(-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(-\frac{a}{b})^{\frac{1}{4}}\right)}{2\,(-\frac{a}{b})^{\frac{1}{4}}}\right)}{4\,b^4}$$
$$+\frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2+\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4}$$
$$-\frac{\sqrt{2}\left((-ab^3)^{\frac{1}{4}}\,b^2d-(-ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2-\sqrt{2}x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\,b^4}$$
$$-\frac{2\,b^2fx^3+3\,b^2ex^2+6\,b^2dx}{6\,b^3}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*c*log(abs(b*x^4 - a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-a
*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-
a/b)^(1/4))/(-a/b)^(1/4))/b^4 - 1/4*sqrt(2)*(sqrt(2)*sqrt(-a*b)*b^2*e - (-
a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
-a/b)^(1/4))/(-a/b)^(1/4))/b^4 + 1/8*sqrt(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b
^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/b^4 - 1/8*sqrt
(2)*((-a*b^3)^(1/4)*b^2*d - (-a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(-a/b)^(
1/4) + sqrt(-a/b))/b^4 - 1/6*(2*b^2*f*x^3 + 3*b^2*e*x^2 + 6*b^2*d*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 846, normalized size of antiderivative = 5.25

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(-\frac{a^4 f^3 - 2\,a^3\,b\,c\,e\,f - a^3\,b\,d^2\,f + a^3\,b\,d\,e^2 + a^2\,b^2\,c^2\,d}{b^2}\right.\right.$$
$$-\mathrm{root}\big(256\,b^7\,z^4 + 256\,b^6\,c\,z^3 - 64\,a\,b^4\,d\,f\,z^2 - 32\,a\,b^4\,e^2\,z^2 + 96\,b^5\,c^2\,z^2 - 32\,a\,b^3\,c\,d\,f\,z + 16\,a^2\,b^2\,e\,f^2\,z + 16\,a$$
$$\left. - \frac{x\,(a^3\,c\,f^2 - 2\,a^3\,d\,e\,f + a^3\,e^3 - b\,a^2\,c^2\,e + b\,a^2\,c\,d^2)}{b}\right)\,\mathrm{root}\big(256\,b^7\,z^4 + 256\,b^6\,c\,z^3$$
$$-64\,a\,b^4\,d\,f\,z^2 - 32\,a\,b^4\,e^2\,z^2 + 96\,b^5\,c^2\,z^2 - 32\,a\,b^3\,c\,d\,f\,z + 16\,a^2\,b^2\,e\,f^2\,z + 16\,a\,b^3\,d^2\,e\,z$$
$$-16\,a\,b^3\,c\,e^2\,z + 16\,b^4\,c^3\,z - 4\,a^2\,b\,d\,e^2\,f + 4\,a^2\,b\,c\,e\,f^2 - 4\,a\,b^2\,c^2\,d\,f + 4\,a\,b^2\,c\,d^2\,e$$
$$\left. +2\,a^2\,b\,d^2\,f^2 - 2\,a\,b^2\,c^2\,e^2 + a^2\,b\,e^4 + b^3\,c^4 - a\,b^2\,d^4 - a^3\,f^4, z, k\big)\right) - \frac{e\,x^2}{2\,b} - \frac{f\,x^3}{3\,b} - \frac{d\,x}{b}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a - b*x^4),x)
```

output
```
symsum(log(- (a^4*f^3 + a^2*b^2*c^2*d + a^3*b*d*e^2 - a^3*b*d^2*f - 2*a^3*
b*c*e*f)/b^2 - root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*
b^4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*
a*b^3*d^2*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*
b*c*e*f^2 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*
c^2*e^2 + a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k)*(root(256*b^7*z
^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2
- 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^
2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 - 4*a*b^2*c^2*d*f +
 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 + a^2*b*e^4 + b^3*c^4
 - a*b^2*d^4 - a^3*f^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) + (8*a^2*b^3
*c*d - 8*a^3*b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 + 4*a^2*b^2*d^2 - 8*a^2*b^2*c*
e))/b) - (x*(a^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f + a^2*b*c*d^2 - a^2*b*c^2*e
))/b)*root(256*b^7*z^4 + 256*b^6*c*z^3 - 64*a*b^4*d*f*z^2 - 32*a*b^4*e^2*z
^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z + 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2
*e*z - 16*a*b^3*c*e^2*z + 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2
 - 4*a*b^2*c^2*d*f + 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 - 2*a*b^2*c^2*e^2 +
 a^2*b*e^4 + b^3*c^4 - a*b^2*d^4 - a^3*f^4, z, k), k, 1, 4) - (e*x^2)/(2*b
) - (f*x^3)/(3*b) - (d*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.48

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a-bx^4}\,dx$$
$$= \frac{-6b^{\frac{1}{4}}a^{\frac{3}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)f+6b^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)d-3b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}-b^{\frac{1}{4}}x\right)f+3b^{\frac{1}{4}}a^{\frac{3}{4}}\log\left(a^{\frac{1}{4}}+b^{\frac{1}{4}}x\right)f-3b^{\frac{3}{4}}a}{}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(-b*x^4+a),x)
```

output
```
( - 6*b**(1/4)*a**(3/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*f + 6*b**(3/
4)*a**(1/4)*atan((sqrt(b)*x)/(b**(1/4)*a**(1/4)))*d - 3*b**(1/4)*a**(3/4)*
log(a**(1/4) - b**(1/4)*x)*f + 3*b**(1/4)*a**(3/4)*log(a**(1/4) + b**(1/4)
*x)*f - 3*b**(3/4)*a**(1/4)*log(a**(1/4) - b**(1/4)*x)*d + 3*b**(3/4)*a**(
1/4)*log(a**(1/4) + b**(1/4)*x)*d - 3*sqrt(b)*sqrt(a)*log(a**(1/4) - b**(1
/4)*x)*e - 3*sqrt(b)*sqrt(a)*log(a**(1/4) + b**(1/4)*x)*e + 3*sqrt(b)*sqrt
(a)*log(sqrt(a) + sqrt(b)*x**2)*e - 3*log(a**(1/4) - b**(1/4)*x)*b*c - 3*l
og(a**(1/4) + b**(1/4)*x)*b*c - 3*log(sqrt(a) + sqrt(b)*x**2)*b*c - 12*b*d
*x - 6*b*e*x**2 - 4*b*f*x**3)/(12*b**2)
```

3.11 $\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 226

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}} - \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}c-\sqrt{a}e\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{3/4}} + \frac{f\log\left(a+bx^4\right)}{4b}$$

output
```
1/2*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)+1/4*(b^(1/2)*c+a^(1/2)*e
)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4)+1/4*(b^(1/2
)*c+a^(1/2)*e)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4)
+1/4*(b^(1/2)*c-a^(1/2)*e)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1
/2)*x^2))*2^(1/2)/a^(3/4)/b^(3/4)+1/4*f*ln(b*x^4+a)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.31

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$$
$$= \frac{-2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{2}\sqrt{b}c+2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{2}\sqrt{b}c-2\sqrt[4]{a}\sqrt[4]{b}d+\sqrt{2}\sqrt{a}\right.}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

output
```
(-2*a^(1/4)*b^(1/4)*(Sqrt[2]*Sqrt[b]*c + 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqr
t[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(Sqrt[
2]*Sqrt[b]*c - 2*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*b^(1/4)*(a^(1/4)*Sqrt[b]*c - a^(3/4)*e)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(a^
(1/4)*Sqrt[b]*c - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] + 2*a*f*Log[a + b*x^4])/(8*a*b)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx$$
$$\Big\downarrow \text{ 2415}$$
$$\int \left(\frac{c+ex^2}{a+bx^4}+\frac{x(d+fx^2)}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{f\log\left(a+bx^4\right)}{4b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x]
```

output
```
(d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*c + Sqrt
[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)
) + ((Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c
 - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*S
qrt[2]*a^(3/4)*b^(3/4)) + (f*Log[a + b*x^4])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

method	result
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^3 f+\_R^2 e+d\_R+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-}{x^2+}\right.\right.}{}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*sum((_R^3*f+_R^2*e+_R*d+c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.92 (sec) , antiderivative size = 254687, normalized size of antiderivative = 1126.93

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx=\text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx + ex^2 + fx^3}{a + bx^4} \, dx = \text{Timed out}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.23

$$\begin{aligned}\int \frac{c + dx + ex^2 + fx^3}{a + bx^4} \, dx &= \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f + bc - \sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} \\
&+ \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f - bc + \sqrt{a}\sqrt{b}e\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{5}{4}}} \\
&+ \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{5}{4}}c + \sqrt{2}a^{\frac{3}{4}}b^{\frac{3}{4}}e - 2\sqrt{a}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}} \\
&+ \frac{\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{5}{4}}c + \sqrt{2}a^{\frac{3}{4}}b^{\frac{3}{4}}e + 2\sqrt{a}bd\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{b}}b^{\frac{5}{4}}}\end{aligned}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(2)*a^(3/4)*b^(1/4)*f + b*c - sqrt(a)*sqrt(b)*e)*log(sqrt
(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/8*sqr
t(2)*(sqrt(2)*a^(3/4)*b^(1/4)*f - b*c + sqrt(a)*sqrt(b)*e)*log(sqrt(b)*x^2
 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(5/4)) + 1/4*(sqrt(2)*a
^(1/4)*b^(5/4)*c + sqrt(2)*a^(3/4)*b^(3/4)*e - 2*sqrt(a)*b*d)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) + 1/4*(sqrt(2)*a^(1/4)*b^(5/4)*c + sqr
t(2)*a^(3/4)*b^(3/4)*e + 2*sqrt(a)*b*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x -
sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(b))*b^(5/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.27

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \frac{f\log\left(|bx^4+a|\right)}{4\,b} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2d-\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*f*log(abs(b*x^4 + a))/b - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*d - (a*b^
3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(
3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)
*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 1952, normalized size of antiderivative = 8.64

$$\int \frac{c+dx+ex^2+fx^3}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4),x)
```

output
```
symsum(log(b^2*c*d^2 - b^2*c^2*e + b^2*d^3*x - a*b*e^3 - a*b*c*f^2 - 16*ro
ot(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f
^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16
*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4
*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a
*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)^2*a*b^3*c
- 4*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2*b^3*c*e*z^2 + 96*a^3
*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*z - 16*a^2*b^2*d^2*f*
z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*f^3*z - 4*a^2*b*d*e^2
*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2
 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*b^3*
c^2*x + b^2*c^2*f*x + 16*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3 + 64*a^2
*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^2*c*e*f*
z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 16*a^3*b*
f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^
2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4
+ b^3*c^4, z, k)^2*a*b^3*d*x + 4*root(256*a^3*b^4*z^4 - 256*a^3*b^3*f*z^3
+ 64*a^2*b^3*c*e*z^2 + 96*a^3*b^2*f^2*z^2 + 32*a^2*b^3*d^2*z^2 - 32*a^2*b^
2*c*e*f*z - 16*a^2*b^2*d^2*f*z + 16*a^2*b^2*d*e^2*z - 16*a*b^3*c^2*d*z - 1
6*a^3*b*f^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.78

$$\int \frac{c + dx + ex^2 + fx^3}{a + bx^4} \, dx$$
$$= \frac{-2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) e - 2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) c - 4\sqrt{b}\,\sqrt{a}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) d + 2}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*e - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 4*
sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a
**(1/4)*sqrt(2)))*d + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*e + 2*b**(3/4)*a**(1/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4
)*sqrt(2)))*c - 4*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt
(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + b**(1/4)*a**(3/4)*sqrt(2)*log( - b
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*e - b**(1/4)*a**(3/4)
*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*e - b**
(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt
(b)*x**2)*c + b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x +
sqrt(a) + sqrt(b)*x**2)*c + 2*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
 + sqrt(b)*x**2)*a*f + 2*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(
b)*x**2)*a*f)/(8*a*b)
```

3.12 $\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 254

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx &= \frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}-\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}\\ &+\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}\\ &-\frac{\sqrt[4]{a}\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}}\\ &-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}b^{7/4}}\\ &+\frac{c\log\left(a+bx^4\right)}{4b}\end{aligned}$$

output
```
d*x/b+1/2*e*x^2/b+1/3*f*x^3/b-1/2*a^(1/2)*e*arctan(b^(1/2)*x^2/a^(1/2))/b^
(3/2)-1/4*a^(1/4)*(b^(1/2)*d+a^(1/2)*f)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4
))*2^(1/2)/b^(7/4)-1/4*a^(1/4)*(b^(1/2)*d+a^(1/2)*f)*arctan(1+2^(1/2)*b^(1
/4)*x/a^(1/4))*2^(1/2)/b^(7/4)-1/4*a^(1/4)*(b^(1/2)*d-a^(1/2)*f)*arctanh(2
^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/b^(7/4)+1/4*c*ln(b
*x^4+a)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.22

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$
$$= \frac{24b^{3/4}dx + 12b^{3/4}ex^2 + 8b^{3/4}fx^3 + 6\sqrt[4]{a}\left(\sqrt{2}\sqrt{b}d + 2\sqrt[4]{a}\sqrt[4]{b}e + \sqrt{2}\sqrt{a}f\right)\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) - 6\sqrt[4]{a}\left(\right.}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

output
```
(24*b^(3/4)*d*x + 12*b^(3/4)*e*x^2 + 8*b^(3/4)*f*x^3 + 6*a^(1/4)*(Sqrt[2]*
Sqrt[b]*d + 2*a^(1/4)*b^(1/4)*e + Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*x)/a^(1/4)] - 6*a^(1/4)*(Sqrt[2]*Sqrt[b]*d - 2*a^(1/4)*b^(1/4)*e +
Sqrt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*(-(
a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + 3*Sqrt[2]*(-(a^(1/4)*Sqrt[b]*d) + a^(3/4)*f)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 6*b^(3/4)*c*Log[a + b*x^4])/(24
*b^(7/4))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$
$$\Big\downarrow \text{ 2370}$$
$$\int \left(\frac{x^3(c+ex^2)}{a+bx^4} + \frac{x^4(d+fx^2)}{a+bx^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}b^{7/4}}-\frac{\sqrt{a}e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}}+\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}-\frac{\sqrt[4]{a}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{7/4}}+\frac{c\log\left(a+bx^4\right)}{4b}+\frac{dx}{b}+\frac{ex^2}{2b}+\frac{fx^3}{3b}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x]
```

output
```
(d*x)/b + (e*x^2)/(2*b) + (f*x^3)/(3*b) - (Sqrt[a]*e*ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]])/(2*b^(3/2)) + (a^(1/4)*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d + Sqrt[a
]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(7/4)) + (a^(1/
4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/(4*Sqrt[2]*b^(7/4)) - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(7/4)) + (c*Log
[a + b*x^4])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.30

method	result
risch	$\frac{f x^3}{3b}+\frac{e x^2}{2b}+\frac{dx}{b}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^3 bc-\_R^2 af-\_R ae-ad\right)\ln\left(x-\_R\right)}{\_R^3}}{4b^2}$
default	$\frac{\frac{1}{3}f x^3+\frac{1}{2}e x^2+dx}{b}+\frac{-\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\frac{ae\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{2\sqrt{ab}}-\frac{af\sqrt{}}{}}{b}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*f*x^3/b+1/2*e*x^2/b+d*x/b+1/4/b^2*sum((_R^3*b*c-_R^2*a*f-_R*a*e-a*d)/_
R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 219615, normalized size of antiderivative = 864.63

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.20

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx = \frac{2\,fx^3+3\,ex^2+6\,dx}{6\,b} + \frac{\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c-abd+a^{\frac{3}{2}}\sqrt{b}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}b^{\frac{5}{4}}c+abd-a^{\frac{3}{2}}\sqrt{b}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{5}{4}}} - \frac{2\left(\sqrt{2}a^{\frac{5}{4}}b^{\frac{5}{4}}\cdots\right.}{\cdots}}{8\,b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/6*(2*f*x^3 + 3*e*x^2 + 6*d*x)/b + 1/8*(sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*
c - a*b*d + a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*b^(5/4)*c + a*b*d
 - a^(3/2)*sqrt(b)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a
))/(a^(3/4)*b^(5/4)) - 2*(sqrt(2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3
/4)*f - 2*a^(3/2)*b*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)) - 2
*(sqrt(2)*a^(5/4)*b^(5/4)*d + sqrt(2)*a^(7/4)*b^(3/4)*f + 2*a^(3/2)*b*e)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(5/4)))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.20

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$
$$= \frac{c\log(|bx^4+a|)}{4\,b} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-(ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} + \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ab}b^2e-(ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,b^4} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,b^4} + \frac{2\,b^2fx^3+3\,b^2ex^2+6\,b^2dx}{6\,b^3}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*c*log(abs(b*x^4 + a))/b + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^
3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/b^4 + 1/4*sqrt(2)*(sqrt(2)*sqrt(a*b)*b^2*e - (a*b^3)^(
1/4)*b^2*d - (a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4
))/(a/b)^(1/4))/b^4 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 + 1/8*sqrt(2)*((a*b^3)^(1
/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
b^4 + 1/6*(2*b^2*f*x^3 + 3*b^2*e*x^2 + 6*b^2*d*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 838, normalized size of antiderivative = 3.30

$$
\begin{aligned}
&\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx \\
&= \left(\sum_{k=1}^{4} \ln\left(\frac{a^4\,f^3+2\,a^3\,b\,c\,e\,f+a^3\,b\,d^2\,f-a^3\,b\,d\,e^2+a^2\,b^2\,c^2\,d}{b^2}\right.\right. \\
&\quad +\mathrm{root}\big(256\,b^7\,z^4-256\,b^6\,c\,z^3+64\,a\,b^4\,d\,f\,z^2+32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z-16\,a^2\,b^2\,e\,f^2\,z+16\,a \\
&\qquad \left. -\frac{x\,(a^3\,c\,f^2-2\,a^3\,d\,e\,f+a^3\,e^3+b\,a^2\,c^2\,e-b\,a^2\,c\,d^2)}{b}\right)\mathrm{root}\big(256\,b^7\,z^4-256\,b^6\,c\,z^3 \\
&\quad +64\,a\,b^4\,d\,f\,z^2+32\,a\,b^4\,e^2\,z^2+96\,b^5\,c^2\,z^2-32\,a\,b^3\,c\,d\,f\,z-16\,a^2\,b^2\,e\,f^2\,z+16\,a\,b^3\,d^2\,e\,z \\
&\qquad -16\,a\,b^3\,c\,e^2\,z-16\,b^4\,c^3\,z-4\,a^2\,b\,d\,e^2\,f+4\,a^2\,b\,c\,e\,f^2+4\,a\,b^2\,c^2\,d\,f-4\,a\,b^2\,c\,d^2\,e \\
&\quad \left.+2\,a^2\,b\,d^2\,f^2+2\,a\,b^2\,c^2\,e^2+a^2\,b\,e^4+a\,b^2\,d^4+a^3\,f^4+b^3\,c^4,z,k\big)\right)+\frac{e\,x^2}{2\,b}+\frac{f\,x^3}{3\,b}+\frac{d\,x}{b}
\end{aligned}
$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4),x)
```

output
```
symsum(log((a^4*f^3 + a^2*b^2*c^2*d - a^3*b*d*e^2 + a^3*b*d^2*f + 2*a^3*b*
c*e*f)/b^2 + root(256*b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^
4*e^2*z^2 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*
b^3*d^2*e*z - 16*a*b^3*c*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*
c*e*f^2 + 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^
2*e^2 + a^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k)*(root(256*b^7*z^4
 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2 + 96*b^5*c^2*z^2 -
32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e*z - 16*a*b^3*c*e^2*
z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 + 4*a*b^2*c^2*d*f - 4
*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a^2*b*e^4 + a*b^2*d^4
 + a^3*f^4 + b^3*c^4, z, k)*(16*a^2*b^2*d - 16*a^2*b^2*e*x) - (8*a^2*b^3*c
*d + 8*a^3*b^2*e*f)/b^2 + (x*(4*a^3*b*f^2 - 4*a^2*b^2*d^2 + 8*a^2*b^2*c*e)
)/b) - (x*(a^3*e^3 + a^3*c*f^2 - 2*a^3*d*e*f - a^2*b*c*d^2 + a^2*b*c^2*e))
/b)*root(256*b^7*z^4 - 256*b^6*c*z^3 + 64*a*b^4*d*f*z^2 + 32*a*b^4*e^2*z^2
 + 96*b^5*c^2*z^2 - 32*a*b^3*c*d*f*z - 16*a^2*b^2*e*f^2*z + 16*a*b^3*d^2*e
*z - 16*a*b^3*c*e^2*z - 16*b^4*c^3*z - 4*a^2*b*d*e^2*f + 4*a^2*b*c*e*f^2 +
 4*a*b^2*c^2*d*f - 4*a*b^2*c*d^2*e + 2*a^2*b*d^2*f^2 + 2*a*b^2*c^2*e^2 + a
^2*b*e^4 + a*b^2*d^4 + a^3*f^4 + b^3*c^4, z, k), k, 1, 4) + (e*x^2)/(2*b)
+ (f*x^3)/(3*b) + (d*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.65

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{a+bx^4}\,dx$$
$$= \frac{6b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)f+6b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+12\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)e-6}{}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a),x)
```

output
```
(6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2)))*f + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + 12*sq
rt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**
(1/4)*sqrt(2)))*e - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*f - 6*b**(3/4)*a**(1/4)*
sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*
sqrt(2)))*d + 12*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*e - 3*b**(1/4)*a**(3/4)*sqrt(2)*log( -
b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*f + 3*b**(1/4)*a**(3
/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*f +
3*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(b)*x**2)*d - 3*b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(
2)*x + sqrt(a) + sqrt(b)*x**2)*d + 6*log( - b**(1/4)*a**(1/4)*sqrt(2)*x +
sqrt(a) + sqrt(b)*x**2)*b*c + 6*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(b)*x**2)*b*c + 24*b*d*x + 12*b*e*x**2 + 8*b*f*x**3)/(24*b**2)
```

3.13 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 257

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx &= -\frac{f}{4b(a+bx^4)}+\frac{x(c+dx+ex^2)}{4a(a+bx^4)}+\frac{d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}}\\ &\quad-\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}\\ &\quad+\frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}\\ &\quad+\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{3/4}}\end{aligned}$$

output

```
-1/4*f/b/(b*x^4+a)+1/4*x*(e*x^2+d*x+c)/a/(b*x^4+a)+1/4*d*arctan(b^(1/2)*x^
2/a^(1/2))/a^(3/2)/b^(1/2)+1/16*(3*b^(1/2)*c+a^(1/2)*e)*arctan(-1+2^(1/2)*
b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(3/4)+1/16*(3*b^(1/2)*c+a^(1/2)*e)*ar
ctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(3/4)+1/16*(3*b^(1/2)*
c-a^(1/2)*e)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1
/2)/a^(7/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.23

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^2} dx$$
$$= \frac{-\frac{8a(af - bx(c+x(d+ex)))}{a+bx^4} - 2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{b}c + 4\sqrt[4]{a}\sqrt[4]{b}d + \sqrt{2}\sqrt{a}e\right)\arctan\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt[4]{a}\sqrt[4]{b}\left(3\sqrt{2}\sqrt{\right.}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

output
```
((-8*a*(a*f - b*x*(c + x*(d + e*x))))/(a + b*x^4) - 2*a^(1/4)*b^(1/4)*(3*S
qrt[2]*Sqrt[b]*c + 4*a^(1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*b^(1/4)*(3*Sqrt[2]*Sqrt[b]*c - 4*a^(
1/4)*b^(1/4)*d + Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] + Sqrt[2]*b^(1/4)*(-3*a^(1/4)*Sqrt[b]*c + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(3*a^(1/4)*Sqrt[b]*c
 - a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*
a^2*b)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2393, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^2} dx$$

$$\Big\downarrow \text{ 2393}$$

$$-\frac{\int -\frac{ex^2+2dx+3c}{bx^4+a}dx}{4a} - \frac{af - bx(c + dx + ex^2)}{4ab(a + bx^4)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{ex^2+2dx+3c}{bx^4+a}dx}{4a}-\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}$$

$\Big\downarrow$ 2415

$$\frac{\int \left(\frac{2dx}{bx^4+a}+\frac{ex^2+3c}{bx^4+a}\right)dx}{4a}-\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e+3\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(3\sqrt{b}c-\right.}{}}{4a}$$
$$\frac{af-bx\left(c+dx+ex^2\right)}{4ab\left(a+bx^4\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x]
```

output
```
-1/4*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)) + ((d*ArcTan[(Sqrt[b]
*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((3*Sqrt[b]*c + Sqrt[a]*e)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c
+ Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b
^(3/4)) - ((3*Sqrt[b]*c - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((3*Sqrt[b]*c - Sqrt[a]*e
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/
4)*b^(3/4)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.33

method	result
risch	$\frac{\frac{e\,x^3}{4a}+\frac{d\,x^2}{4a}+\frac{cx}{4a}-\frac{f}{4b}}{b\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(\_R^2 e+2d\_R+3c\right)\ln\left(x-\_R\right)}{\_R^3}}{16ba}$
default	$c\left(\frac{x}{4a(b\,x^4+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+d\left(\frac{x^2}{4a(b\,x^4+a}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^3+1/4*d/a*x^2+1/4*c/a*x-1/4*f/b)/(b*x^4+a)+1/16/b/a*sum((_R^2*e
+2*_R*d+3*c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 124301, normalized size of antiderivative = 483.66

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(236) = 472.

Time = 8.53 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.01

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$
$$= \text{RootSum}\left(65536t^4a^7b^3 + t^2\cdot\left(3072a^4b^2ce + 2048a^4b^2d^2\right) + t\left(128a^3bde^2 - 1152a^2b^2c^2d\right) + a^2e^4 + 18abc\right.$$
$$+ \frac{-af + bcx + bdx^2 + bex^3}{4a^2b + 4ab^2x^4}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*b**3 + _t**2*(3072*a**4*b**2*c*e + 2048*a**4*b**2
*d**2) + _t*(128*a**3*b*d*e**2 - 1152*a**2*b**2*c**2*d) + a**2*e**4 + 18*a
*b*c**2*e**2 - 48*a*b*c*d**2*e + 16*a*b*d**4 + 81*b**2*c**4, Lambda(_t, _t
*log(x + (4096*_t**3*a**7*b**2*e**3 - 36864*_t**3*a**6*b**3*c**2*e + 98304
*_t**3*a**6*b**3*c*d**2 + 4608*_t**2*a**5*b**2*c*d*e**2 - 4096*_t**2*a**5*
b**2*d**3*e + 13824*_t**2*a**4*b**3*c**3*d + 144*_t*a**4*b*c*e**4 + 192*_t
*a**4*b*d**2*e**3 - 1728*_t*a**3*b**2*c**3*e**2 + 5184*_t*a**3*b**2*c**2*d
**2*e + 1536*_t*a**3*b**2*c*d**4 + 3888*_t*a**2*b**3*c**5 + 6*a**3*d*e**5
+ 120*a**2*b*c*d**3*e**2 - 64*a**2*b*d**5*e + 810*a*b**2*c**4*d*e - 1080*a
*b**2*c**3*d**3)/(a**3*e**6 - 9*a**2*b*c**2*e**4 + 96*a**2*b*c*d**2*e**3 -
 64*a**2*b*d**4*e**2 - 81*a*b**2*c**4*e**2 + 864*a*b**2*c**3*d**2*e - 576*
a*b**2*c**2*d**4 + 729*b**3*c**6)))) + (-a*f + b*c*x + b*d*x**2 + b*e*x**3
)/(4*a**2*b + 4*a*b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.19

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = \frac{bex^3+bdx^2+bcx-af}{4\,(ab^2x^4+a^2b)} + \frac{\frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(3\sqrt{b}c-\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e-4\sqrt{a}\sqrt{b}d\cdots\right.}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\cdots}}}{32\,a}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*f)/(a*b^2*x^4 + a^2*b) + 1/32*(sqrt(2)*
(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sq
rt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(3*sqrt(b)*c - sqrt(a)*e)*log(sqrt(b)*x
^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*b^(3/4)*c + sqrt(2)*a^(3/4)*b^(1/4)*e - 4*sqrt(a)*sqrt(b)*d)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)
*c + sqrt(2)*a^(3/4)*b^(1/4)*e + 4*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(
2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqr
t(sqrt(a)*sqrt(b))*b^(3/4)))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.21

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$
$$= \frac{bex^3+bdx^2+bcx-af}{4\,(bx^4+a)ab} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(2\,\sqrt{2}\sqrt{ab}b^2d+3\,(ab^3)^{\frac{1}{4}}\,b^2c+(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,b^2c-(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b*e*x^3 + b*d*x^2 + b*c*x - a*f)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(2*
sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt
(2)*(2*sqrt(2)*sqrt(a*b)*b^2*d + 3*(a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/
32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c -
(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.86

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\big(65536\,a^7\,b^3\,z^4+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a \right.\right.$$
$$-\frac{9\,b^2\,c^2\,e-12\,b^2\,c\,d^2+a\,b\,e^3}{64\,a^3}+\frac{x\,(2\,b^2\,d^3-3\,b^2\,c\,d\,e)}{16\,a^3}\bigg)\,\mathrm{root}\big(65536\,a^7\,b^3\,z^4$$
$$+3072\,a^4\,b^2\,c\,e\,z^2+2048\,a^4\,b^2\,d^2\,z^2-1152\,a^2\,b^2\,c^2\,d\,z+128\,a^3\,b\,d\,e^2\,z-48\,a\,b\,c\,d^2\,e$$
$$\left.+18\,a\,b\,c^2\,e^2+16\,a\,b\,d^4+81\,b^2\,c^4+a^2\,e^4,z,k\big)\right)+\frac{\frac{d\,x^2}{4\,a}-\frac{f}{4\,b}+\frac{e\,x^3}{4\,a}+\frac{c\,x}{4\,a}}{b\,x^4+a}$$

input

```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^2,x)
```

output

```
symsum(log((x*(2*b^2*d^3 - 3*b^2*c*d*e))/(16*a^3) - (9*b^2*c^2*e - 12*b^2*
c*d^2 + a*b*e^3)/(64*a^3) - root(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2
+ 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b
*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(root
(65536*a^7*b^3*z^4 + 3072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^
2*b^2*c^2*d*z + 128*a^3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a
*b*d^4 + 81*b^2*c^4 + a^2*e^4, z, k)*(12*b^3*c - 8*b^3*d*x) + (x*(36*a*b^3
*c^2 - 4*a^2*b^2*e^2))/(16*a^3) + (b^2*d*e)/a))*root(65536*a^7*b^3*z^4 + 3
072*a^4*b^2*c*e*z^2 + 2048*a^4*b^2*d^2*z^2 - 1152*a^2*b^2*c^2*d*z + 128*a^
3*b*d*e^2*z - 48*a*b*c*d^2*e + 18*a*b*c^2*e^2 + 16*a*b*d^4 + 81*b^2*c^4 +
a^2*e^4, z, k), k, 1, 4) + ((d*x^2)/(4*a) - f/(4*b) + (e*x^3)/(4*a) + (c*x
)/(4*a))/(a + b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.00

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*e - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*e*
x**4 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*c - 6*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b
*c*x**4 - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)
/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d*x**4 + 2*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*e + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*e*x**4 + 6*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a*c + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c*x**4 - 8
*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*a*d - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d*x**4 + b**(1/4)*a**(3/4)*
sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*e +
 b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)...
```

3.14 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = -\frac{c+ex^2}{4b(a+bx^4)} - \frac{x(d+fx^2)}{4b(a+bx^4)} + \frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{a}b^{3/2}} - \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} + \frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{7/4}} + \frac{\left(d-\frac{3\sqrt{a}f}{\sqrt{b}}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{3/4}b^{5/4}}$$

output
```
-1/4*(e*x^2+c)/b/(b*x^4+a)-1/4*x*(f*x^2+d)/b/(b*x^4+a)+1/4*e*arctan(b^(1/2
)*x^2/a^(1/2))/a^(1/2)/b^(3/2)+1/16*(b^(1/2)*d+3*a^(1/2)*f)*arctan(-1+2^(1
/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(7/4)+1/16*(b^(1/2)*d+3*a^(1/2)*f
)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(7/4)+1/16*(d-3*a^
(1/2)*f/b^(1/2))*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*
2^(1/2)/a^(3/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.14

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8b^{3/4}(c+x(d+x(e+fx)))}{a+bx^4}-\frac{2\left(\sqrt{2}\sqrt{b}d+4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\left(\sqrt{2}\sqrt{b}d-4\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1\cdots\right.}{a^{3/4}}\cdots}{32b^{7/4}}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

output
```
((-8*b^(3/4)*(c + x*(d + x*(e + f*x))))/(a + b*x^4) - (2*(Sqrt[2]*Sqrt[b]*
d + 4*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/a^(3/4) + (2*(Sqrt[2]*Sqrt[b]*d - 4*a^(1/4)*b^(1/4)*e + 3*Sq
rt[2]*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[
2]*(-(Sqrt[b]*d) + 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/a^(3/4) + (Sqrt[2]*(Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4))/(32*b^(7/4))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2363, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^2}dx$$

$$\Big\downarrow \ 2363$$

$$\frac{\int \frac{3fx^2+2ex+d}{bx^4+a}dx}{4b}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

$$\Big\downarrow \ 2415$$

$$\frac{\int \left(\frac{2ex}{bx^4+a}+\frac{3fx^2+d}{bx^4+a}\right)dx}{4b}-\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}d-\right.}{}}{4b}$$

$$\frac{c+dx+ex^2+fx^3}{4b\left(a+bx^4\right)}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x]
```

output
```
-1/4*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)) + ((e*ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((Sqrt[b]*d + 3*Sqrt[a]*f)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*d + 3*Sqrt
[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)
) - ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*d - 3*Sqrt[a]*f)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(
3/4)))/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.32

method	result
risch	$\frac{-\frac{f x^3}{4b}-\frac{e x^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4+a\right)}\frac{\left(3f\_R^2+2e\_R+d\right)\ln\left(x-\_R\right)}{\_R^3}}{16b^2}$
default	$\frac{-\frac{f x^3}{4b}-\frac{e x^2}{4b}-\frac{dx}{4b}-\frac{c}{4b}}{b x^4+a}+\frac{\frac{d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(x^2\sqrt{\frac{b}{a}}\right)}{\sqrt{ab}}+3f}{4b}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*f*x^3/b-1/4*e*x^2/b-1/4*d*x/b-1/4/b*c)/(b*x^4+a)+1/16/b^2*sum((3*_R^
2*f+2*_R*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.83 (sec) , antiderivative size = 122993, normalized size of antiderivative = 476.72

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(241) = 482$.

Time = 18.71 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.98

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$$
$$= \operatorname{RootSum}\left(65536t^4a^3b^7 + t^2\cdot\left(3072a^2b^4df + 2048a^2b^4e^2\right) + t\left(1152a^2b^2ef^2 - 128ab^3d^2e\right) + 81a^2f^4 + 18\right.$$
$$+ \frac{-c-dx-ex^2-fx^3}{4ab+4b^2x^4}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**3*b**7 + _t**2*(3072*a**2*b**4*d*f + 2048*a**2*b**4
*e**2) + _t*(1152*a**2*b**2*e*f**2 - 128*a*b**3*d**2*e) + 81*a**2*f**4 + 1
8*a*b*d**2*f**2 - 48*a*b*d*e**2*f + 16*a*b*e**4 + b**2*d**4, Lambda(_t, _t
*log(x + (110592*_t**3*a**4*b**5*f**3 - 12288*_t**3*a**3*b**6*d**2*f + 327
68*_t**3*a**3*b**6*d*e**2 + 13824*_t**2*a**3*b**4*d*e*f**2 - 12288*_t**2*a
**3*b**4*e**3*f + 512*_t**2*a**2*b**5*d**3*e + 3888*_t*a**3*b**2*d*f**4 +
5184*_t*a**3*b**2*e**2*f**3 - 576*_t*a**2*b**3*d**3*f**2 + 1728*_t*a**2*b*
*3*d**2*e**2*f + 512*_t*a**2*b**3*d*e**4 + 16*_t*a*b**4*d**5 + 1458*a**3*e
*f**5 + 360*a**2*b*d*e**3*f**2 - 192*a**2*b*e**5*f + 30*a*b**2*d**4*e*f -
40*a*b**2*d**3*e**3)/(729*a**3*f**6 - 81*a**2*b*d**2*f**4 + 864*a**2*b*d*e
**2*f**3 - 576*a**2*b*e**4*f**2 - 9*a*b**2*d**4*f**2 + 96*a*b**2*d**3*e**2
*f - 64*a*b**2*d**2*e**4 + b**3*d**6))))) + (-c - d*x - e*x**2 - f*x**3)/(4
*a*b + 4*b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.14

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = -\frac{fx^3+ex^2+dx+c}{4\,(b^2x^4+ab)}$$
$$+ \frac{\frac{\sqrt{2}\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac34}} - \frac{\sqrt{2}\left(\sqrt{b}d-3\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac34}} + \frac{2\left(\sqrt{2}a^{\frac14}b^{\frac34}d+3\sqrt{2}a^{\frac34}b^{\frac14}f-4\sqrt{a}\sqrt{\right.}}{a^{\frac34}\sqrt{\sqrt{}}}}{32\,b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(f*x^3 + e*x^2 + d*x + c)/(b^2*x^4 + a*b) + 1/32*(sqrt(2)*(sqrt(b)*d
- 3*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(
3/4)*b^(3/4)) - sqrt(2)*(sqrt(b)*d - 3*sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3
/4)*d + 3*sqrt(2)*a^(3/4)*b^(1/4)*f - 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4
)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2
)*a^(3/4)*b^(1/4)*f + 4*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
 - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(b))*b^(3/4)))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.16

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx$$

$$= -\frac{fx^3+ex^2+dx+c}{4\,(bx^4+a)b} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2e+(ab^3)^{\frac{1}{4}}b^2d+3\,(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ab}b^2e+(ab^3)^{\frac{1}{4}}b^2d+3\,(ab^3)^{\frac{3}{4}}f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^4} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-3\,(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2d-3\,(ab^3)^{\frac{3}{4}}f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^4}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)*b) + 1/16*sqrt(2)*(2*sqrt(2)*s
qrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/16*sqrt(2)*(2*sqrt
(2)*sqrt(a*b)*b^2*e + (a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/32*sqrt(2)*((
a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a*b^4) - 1/32*sqrt(2)*((a*b^3)^(1/4)*b^2*d - 3*(a*b^3)^(3/4)*f
)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.17

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{x\,(2\,e^3-3\,d\,e\,f)}{16\,b}-\frac{3\,b\,d^2\,f-4\,b\,d\,e^2+27\,a\,f^3}{64\,b^2}\right.\right.$$
$$-\mathrm{root}\big(65536\,a^3\,b^7\,z^4+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f+$$
$$+3072\,a^2\,b^4\,d\,f\,z^2+2048\,a^2\,b^4\,e^2\,z^2+1152\,a^2\,b^2\,e\,f^2\,z-128\,a\,b^3\,d^2\,e\,z-48\,a\,b\,d\,e^2\,f$$
$$\left.+18\,a\,b\,d^2\,f^2+16\,a\,b\,e^4+81\,a^2\,f^4+b^2\,d^4,z,k\big)\right)-\frac{\frac{c}{4\,b}+\frac{e\,x^2}{4\,b}+\frac{f\,x^3}{4\,b}+\frac{d\,x}{4\,b}}{b\,x^4+a}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^2,x)
```

output
```
symsum(log((x*(2*e^3 - 3*d*e*f))/(16*b) - (27*a*f^3 - 4*b*d*e^2 + 3*b*d^2*
f)/(64*b^2) - root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4
*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*
a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*(3*a*e*f + (b*d^2*x
)/4 - (9*a*f^2*x)/4 + 4*root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 20
48*a^2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e
^2*f + 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*d -
 8*root(65536*a^3*b^7*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 +
1152*a^2*b^2*e*f^2*z - 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2
 + 16*a*b*e^4 + 81*a^2*f^4 + b^2*d^4, z, k)*a*b^2*e*x))*root(65536*a^3*b^7
*z^4 + 3072*a^2*b^4*d*f*z^2 + 2048*a^2*b^4*e^2*z^2 + 1152*a^2*b^2*e*f^2*z
- 128*a*b^3*d^2*e*z - 48*a*b*d*e^2*f + 18*a*b*d^2*f^2 + 16*a*b*e^4 + 81*a^
2*f^4 + b^2*d^4, z, k), k, 1, 4) - (c/(4*b) + (e*x^2)/(4*b) + (f*x^3)/(4*b
) + (d*x)/(4*b))/(a + b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 773, normalized size of antiderivative = 3.00

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^2,x)
```

output
```
( - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*f - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*f*
x**4 - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d - 2*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b
*d*x**4 - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)
/(b**(1/4)*a**(1/4)*sqrt(2)))*a*e - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*e*x**4 + 6*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1
/4)*a**(1/4)*sqrt(2)))*a*f + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*f*x**4 + 2*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b*
*(1/4)*a**(1/4)*sqrt(2)))*a*d + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*d*x**4 - 8
*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*a*e - 8*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*e*x**4 + 3*b**(1/4)*a**(3/4
)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*f
 + 3*b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqr...
```

3.15 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 290

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx &= -\frac{f}{8b(a+bx^4)^2}+\frac{x(c+dx+ex^2)}{8a(a+bx^4)^2} \\ &\quad+\frac{x(7c+6dx+5ex^2)}{32a^2(a+bx^4)}+\frac{3d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}} \\ &\quad-\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} \\ &\quad+\frac{\left(21\sqrt{b}c+5\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}b^{3/4}} \\ &\quad+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{64\sqrt{2}a^{11/4}b^{3/4}}\end{aligned}$$

output
```
-1/8*f/b/(b*x^4+a)^2+1/8*x*(e*x^2+d*x+c)/a/(b*x^4+a)^2+1/32*x*(5*e*x^2+6*d
*x+7*c)/a^2/(b*x^4+a)+3/16*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(5/2)/b^(1/2)+1
/128*(21*b^(1/2)*c+5*a^(1/2)*e)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/
2)/a^(11/4)/b^(3/4)+1/128*(21*b^(1/2)*c+5*a^(1/2)*e)*arctan(1+2^(1/2)*b^(1
/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/b^(3/4)+1/128*(21*b^(1/2)*c-5*a^(1/2)*e)*a
rctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(11/4)/b
^(3/4)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.20

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \frac{\frac{8ax(7c+x(6d+5ex))}{a+bx^4} - \frac{32a^2(af-bx(c+x(d+ex)))}{b(a+bx^4)^2} - \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{b}c+24\sqrt[4]{a}\sqrt[4]{b}d+5\sqrt{2}\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}} + \frac{2\sqrt[4]{a}\left(21\sqrt{2}\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

output
```
((8*a*x*(7*c + x*(6*d + 5*e*x)))/(a + b*x^4) - (32*a^2*(a*f - b*x*(c + x*(
d + e*x))))/(b*(a + b*x^4)^2) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c + 24*a^(1
/4)*b^(1/4)*d + 5*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/b^(3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[b]*c - 24*a^(1/4)*b^(1/4)*d + 5*
Sqrt[2]*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqr
t[2]*(-21*a^(1/4)*Sqrt[b]*c + 5*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/b^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[b]*c - 5*a^(3
/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(2
56*a^3)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2393, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{5ex^2+6dx+7c}{(bx^4+a)^2}dx}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5ex^2+6dx+7c}{(bx^4+a)^2}dx}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}-\frac{\int -\frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{5ex^2+12dx+21c}{bx^4+a}dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\int \left(\frac{12dx}{bx^4+a}+\frac{5ex^2+21c}{bx^4+a}\right)dx}{4a}+\frac{x(7c+6dx+5ex^2)}{4a(a+bx^4)}}{8a}-\frac{af-bx(c+dx+ex^2)}{8ab(a+bx^4)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e+21\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(21\sqrt{b}c-5\sqrt{a}e\right)\log\left(\sqrt{\right.}}{4\sqrt{2}a}}{4a}}{8a}$$

$$\frac{af-bx\left(c+dx+ex^2\right)}{8ab\left(a+bx^4\right)^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x]
```

output
```
-1/8*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)^2) + ((x*(7*c + 6*d*x
+ 5*e*x^2))/(4*a*(a + b*x^4)) + ((6*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt
[a]*Sqrt[b]) - ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c + 5*Sqrt[a]*e)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((21*
Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + ((21*Sqrt[b]*c - 5*Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4
)))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{5be\,x^7}{32a^2}+\frac{3bd\,x^6}{16a^2}+\frac{7bc\,x^5}{32a^2}+\frac{9e\,x^3}{32a}+\frac{5d\,x^2}{16a}+\frac{11cx}{32a}-\frac{f}{8b}}{(b\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(5\_R^2e+12d\_R+21c\right)\ln\left(x-\_R\right)}{\_R^3}}{128b\,a^2}$
default	$c\left(\frac{x}{8a(b\,x^4+a)^2}+\frac{\frac{7x}{32a\left(b\,x^4+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+d\Bigg($

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*b*e/a^2*x^7+3/16*b*d/a^2*x^6+7/32*b/a^2*c*x^5+9/32*e/a*x^3+5/16*d/a*
x^2+11/32*c/a*x-1/8*f/b)/(b*x^4+a)^2+1/128/b/a^2*sum((5*_R^2*e+12*_R*d+21*
c)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.47 (sec) , antiderivative size = 124838, normalized size of antiderivative = 430.48

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(275) = 550.

Time = 45.04 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.99

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx$$
$$= \text{RootSum}\Big(268435456t^4a^{11}b^3 + t^2\cdot\left(6881280a^6b^2ce + 4718592a^6b^2d^2\right) + t\big(153600a^4bde^2 - 2709504a^3b^2$$
$$+ \frac{-4a^2f + 11abcx + 10abdx^2 + 9abex^3 + 7b^2cx^5 + 6b^2dx^6 + 5b^2ex^7}{32a^4b + 64a^3b^2x^4 + 32a^2b^3x^8}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*b**3 + _t**2*(6881280*a**6*b**2*c*e + 471859
2*a**6*b**2*d**2) + _t*(153600*a**4*b*d*e**2 - 2709504*a**3*b**2*c**2*d) +
 625*a**2*e**4 + 22050*a*b*c**2*e**2 - 60480*a*b*c*d**2*e + 20736*a*b*d**4
 + 194481*b**2*c**4, Lambda(_t, _t*log(x + (262144000*_t**3*a**10*b**2*e**
3 - 4624220160*_t**3*a**9*b**3*c**2*e + 12683575296*_t**3*a**9*b**3*c*d**2
 + 309657600*_t**2*a**7*b**2*c*d*e**2 - 283115520*_t**2*a**7*b**2*d**3*e +
 1820786688*_t**2*a**6*b**3*c**3*d + 5040000*_t*a**5*b*c*e**4 + 6912000*_t
*a**5*b*d**2*e**3 - 118540800*_t*a**4*b**2*c**3*e**2 + 365783040*_t*a**4*b
**2*c**2*d**2*e + 111476736*_t*a**4*b**2*c*d**4 + 522764928*_t*a**3*b**3*c
**5 + 112500*a**3*d*e**5 + 4536000*a**2*b*c*d**3*e**2 - 2488320*a**2*b*d**
5*e + 58344300*a*b**2*c**4*d*e - 80015040*a*b**2*c**3*d**3)/(15625*a**3*e*
*6 - 275625*a**2*b*c**2*e**4 + 3024000*a**2*b*c*d**2*e**3 - 2073600*a**2*b
*d**4*e**2 - 4862025*a*b**2*c**4*e**2 + 53343360*a*b**2*c**3*d**2*e - 3657
8304*a*b**2*c**2*d**4 + 85766121*b**3*c**6)))) + (-4*a**2*f + 11*a*b*c*x +
 10*a*b*d*x**2 + 9*a*b*e*x**3 + 7*b**2*c*x**5 + 6*b**2*d*x**6 + 5*b**2*e*x
**7)/(32*a**4*b + 64*a**3*b**2*x**4 + 32*a**2*b**3*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.22

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^3} \, dx$$

$$= \frac{5\,b^2ex^7 + 6\,b^2dx^6 + 7\,b^2cx^5 + 9\,abex^3 + 10\,abdx^2 + 11\,abcx - 4\,a^2f}{32\,(a^2b^3x^8 + 2\,a^3b^2x^4 + a^4b)}$$

$$+ \frac{\frac{\sqrt{2}\left(21\,\sqrt{b}c - 5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} - \frac{\sqrt{2}\left(21\,\sqrt{b}c - 5\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c + 5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}e - \right.}{a}}{256\,a^2}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*c*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2
 + 11*a*b*c*x - 4*a^2*f)/(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b) + 1/256*(sq
rt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(21*sqrt(b)*c - 5*sqrt(a)*e)*l
og(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) +
2*(21*sqrt(2)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e - 24*sqrt(a)
*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqr
t(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(21*sqrt(2
)*a^(1/4)*b^(3/4)*c + 5*sqrt(2)*a^(3/4)*b^(1/4)*e + 24*sqrt(a)*sqrt(b)*d)*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.20

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(12\sqrt{2}\sqrt{ab}b^2d+21\,(ab^3)^{\frac{1}{4}}\,b^2c+5\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^3b^3} + \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} - \frac{\sqrt{2}\left(21\,(ab^3)^{\frac{1}{4}}\,b^2c-5\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^3b^3} + \frac{5\,b^2ex^7+6\,b^2dx^6+7\,b^2cx^5+9\,abex^3+10\,abdx^2+11\,abcx-4\,a^2f}{32\,(bx^4+a)^2a^2b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b^2*c + 5*(a*
b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/
(a^3*b^3) + 1/128*sqrt(2)*(12*sqrt(2)*sqrt(a*b)*b^2*d + 21*(a*b^3)^(1/4)*b
^2*c + 5*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a^3*b^3) + 1/256*sqrt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^
(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/256*sq
rt(2)*(21*(a*b^3)^(1/4)*b^2*c - 5*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*(5*b^2*e*x^7 + 6*b^2*d*x^6 + 7*b^2*
c*x^5 + 9*a*b*e*x^3 + 10*a*b*d*x^2 + 11*a*b*c*x - 4*a^2*f)/((b*x^4 + a)^2*
a^2*b)
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.87

$$\int \frac{c + dx + ex^2 + fx^3}{(a + bx^4)^3} \, dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^3,x)
```

output
```
symsum(log(-(b*(125*a*e^3 - 3024*b*c*d^2 + 2205*b*c^2*e - 1728*b*d^3*x + 3
44064*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*
b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c
*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4,
 z, k)^2*a^5*b^2*c - 3200*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*
e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d
*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e
^4 + 194481*b^2*c^4, z, k)*a^3*b*e^2*x + 2520*b*c*d*e*x + 56448*root(26843
5456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 27
09504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a
*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^2*b^2*c
^2*x - 196608*root(268435456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718
592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 604
80*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4 + 625*a^2*e^4 + 194481*
b^2*c^4, z, k)^2*a^5*b^2*d*x + 15360*root(268435456*a^11*b^3*z^4 + 6881280
*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 2709504*a^3*b^2*c^2*d*z + 153
600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050*a*b*c^2*e^2 + 20736*a*b*d^4
+ 625*a^2*e^4 + 194481*b^2*c^4, z, k)*a^3*b*d*e))/(32768*a^6))*root(268435
456*a^11*b^3*z^4 + 6881280*a^6*b^2*c*e*z^2 + 4718592*a^6*b^2*d^2*z^2 - 270
9504*a^3*b^2*c^2*d*z + 153600*a^4*b*d*e^2*z - 60480*a*b*c*d^2*e + 22050...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1207, normalized size of antiderivative = 4.16

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

output

```
( - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*e - 20*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b*e*x**4 - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*e*x**8 - 42*b**(3/4)*a**
(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**
(1/4)*sqrt(2)))*a**2*c - 84*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*c*x**4 - 42*b*
*(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b**2*c*x**8 - 48*sqrt(b)*sqrt(a)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d - 96*s
qrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a*b*d*x**4 - 48*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*d*x**8 + 10*b**(1/
4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*a**2*e + 20*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)
*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*e*x**4 +
 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*e*x**8 + 42*b**(3/4)*a**(1/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt...
```

3.16 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 308

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = -\frac{c+ex^2}{8b(a+bx^4)^2}-\frac{x(d+fx^2)}{8b(a+bx^4)^2}+\frac{ex^2}{16ab(a+bx^4)} + \frac{x(d+3fx^2)}{32ab(a+bx^4)}+\frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16a^{3/2}b^{3/2}} - \frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d+\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}b^{7/4}} + \frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{64\sqrt{2}a^{7/4}b^{7/4}}$$

output
```
-1/8*(e*x^2+c)/b/(b*x^4+a)^2-1/8*x*(f*x^2+d)/b/(b*x^4+a)^2+1/16*e*x^2/a/b/
(b*x^4+a)+1/32*x*(3*f*x^2+d)/a/b/(b*x^4+a)+1/16*e*arctan(b^(1/2)*x^2/a^(1/
2))/a^(3/2)/b^(3/2)+3/128*(b^(1/2)*d+a^(1/2)*f)*arctan(-1+2^(1/2)*b^(1/4)*
x/a^(1/4))*2^(1/2)/a^(7/4)/b^(7/4)+3/128*(b^(1/2)*d+a^(1/2)*f)*arctan(1+2^
(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(7/4)+3/128*(b^(1/2)*d-a^(1/2)*
f)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4
)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.07

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$$

$$= \frac{\frac{8b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)} - \frac{32b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^2} - \frac{2\left(3\sqrt{2}\sqrt{b}d+8\sqrt[4]{a}\sqrt[4]{b}e+3\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{2\left(3\sqrt{2}\sqrt{b}d-8\sqrt[4]{a}\right.}{}}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

output
```
((8*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)) - (32*b^(3/4)*(c + x*
(d + x*(e + f*x))))/(a + b*x^4)^2 - (2*(3*Sqrt[2]*Sqrt[b]*d + 8*a^(1/4)*b^
(1/4)*e + 3*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^
(7/4) + (2*(3*Sqrt[2]*Sqrt[b]*d - 8*a^(1/4)*b^(1/4)*e + 3*Sqrt[2]*Sqrt[a]*
f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (3*Sqrt[2]*(-(Sqrt[b
]*d) + Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/
a^(7/4) + (3*Sqrt[2]*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(256*b^(7/4))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2363, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^3} \, dx$$

$$\Big\downarrow \text{2363}$$

$$\frac{\int \frac{3fx^2+2ex+d}{\left(bx^4+a\right)^2}dx}{8b}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x\left(d+2ex+3fx^2\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3fx^2+4ex+3d}{bx^4+a}dx}{4a}}{8b}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{3fx^2+4ex+3d}{bx^4+a}dx}{4a}+\frac{x\left(d+2ex+3fx^2\right)}{4a\left(a+bx^4\right)}}{8b}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\int \left(\frac{4ex}{bx^4+a}+\frac{3fx^2+3d}{bx^4+a}\right)dx}{4a}+\frac{x\left(d+2ex+3fx^2\right)}{4a\left(a+bx^4\right)}}{8b}-\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}f+\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{3\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{3/4}b^{3}}}{4a}}{8b}$$

$$\frac{c+dx+ex^2+fx^3}{8b\left(a+bx^4\right)^2}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x]
```

output
```
-1/8*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^2) + ((x*(d + 2*e*x + 3*f*x^
2))/(4*a*(a + b*x^4)) + ((2*e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt
[b]) - (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + (3*(Sqrt[b]*d + Sqrt[a]*f)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - (3*(Sqrt[b]*d - Sqr
t[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(3/4)) + (3*(Sqrt[b]*d - Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)))/(4*a))/(8*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.37

method	result
risch	$\frac{\frac{3 f x^{7}}{32 a}+\frac{e x^{6}}{16 a}+\frac{d x^{5}}{32 a}-\frac{f x^{3}}{32 b}-\frac{e x^{2}}{16 b}-\frac{3 d x}{32 b}-\frac{c}{8 b}}{\left(b x^{4}+a \right)^{2}}+\frac{\sum_{\_R =\operatorname{RootOf}\left(b \_Z^{4}+a \right)}\frac{\left(3 f \_R^{2}+4 e \_R +3 d \right) \ln \left(x -\_R \right)}{\_R^{3}}}{128 b^{2} a}$
default	$\frac{\frac{3 f x^{7}}{32 a}+\frac{e x^{6}}{16 a}+\frac{d x^{5}}{32 a}-\frac{f x^{3}}{32 b}-\frac{e x^{2}}{16 b}-\frac{3 d x}{32 b}-\frac{c}{8 b}}{\left(b x^{4}+a \right)^{2}}+\frac{\frac{3 d \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}+\cdots}{\cdots}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/32*f/a*x^7+1/16/a*e*x^6+1/32*d/a*x^5-1/32*f*x^3/b-1/16*e*x^2/b-3/32*d*x
/b-1/8/b*c)/(b*x^4+a)^2+1/128/b^2/a*sum((3*_R^2*f+4*_R*e+3*d)/_R^3*ln(x-_R
),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.58 (sec) , antiderivative size = 124542, normalized size of antiderivative = 404.36

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.11

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \frac{3\,bfx^7+2\,bex^6+bdx^5-afx^3-2\,aex^2-3\,adx-4\,ac}{32\,(ab^3x^8+2\,a^2b^2x^4+a^3b)}$$
$$+\frac{\frac{3\sqrt{2}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{3\sqrt{2}\left(\sqrt{b}d-\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+3\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-8\sqrt{a}\cdots\right.}{a^{\frac{3}{4}}\sqrt{\cdots}}}{256\,ab}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(3*b*f*x^7 + 2*b*e*x^6 + b*d*x^5 - a*f*x^3 - 2*a*e*x^2 - 3*a*d*x - 4*
a*c)/(a*b^3*x^8 + 2*a^2*b^2*x^4 + a^3*b) + 1/256*(3*sqrt(2)*(sqrt(b)*d - s
qrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*
b^(3/4)) - 3*sqrt(2)*(sqrt(b)*d - sqrt(a)*f)*log(sqrt(b)*x^2 - sqrt(2)*a^(
1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)
*d + 3*sqrt(2)*a^(3/4)*b^(1/4)*f - 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*s
qrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*b^(3/4)*d + 3*sqrt(2)
*a^(3/4)*b^(1/4)*f + 8*sqrt(a)*sqrt(b)*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x
- sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(b))*b^(3/4)))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.08

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \frac{3\,bfx^7+2\,bex^6+bdx^5-afx^3-2\,aex^2-3\,adx-4\,ac}{32\,(bx^4+a)^2ab} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ab}b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4} + \frac{\sqrt{2}\left(4\,\sqrt{2}\sqrt{ab}b^2e+3\,(ab^3)^{\frac{1}{4}}\,b^2d+3\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{128\,a^2b^4} + \frac{3\,\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4} - \frac{3\,\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^2d-(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{256\,a^2b^4}$$

input

```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x, algorithm="giac")
```

output

```
1/32*(3*b*f*x^7 + 2*b*e*x^6 + b*d*x^5 - a*f*x^3 - 2*a*e*x^2 - 3*a*d*x - 4*
a*c)/((b*x^4 + a)^2*a*b) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^2*e + 3*(a
*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 1/128*sqrt(2)*(4*sqrt(2)*sqrt(a*b)*b^
2*e + 3*(a*b^3)^(1/4)*b^2*d + 3*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/256*sqrt(2)*((a*b^3)^(1/4
)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
^2*b^4) - 3/256*sqrt(2)*((a*b^3)^(1/4)*b^2*d - (a*b^3)^(3/4)*f)*log(x^2 -
sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.69

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(268435456\,a^7\,b^7\,z^4+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2 \right.\right.\right.$$
$$-\frac{3\,(9\,b\,d^2\,f-16\,b\,d\,e^2+9\,a\,f^3)}{32768\,a^3\,b^2}+\frac{x\,(8\,e^3-9\,d\,e\,f)}{4096\,a^3\,b}\Bigg)\,\mathrm{root}\big(268435456\,a^7\,b^7\,z^4$$
$$+589824\,a^4\,b^4\,d\,f\,z^2+524288\,a^4\,b^4\,e^2\,z^2+18432\,a^3\,b^2\,e\,f^2\,z-18432\,a^2\,b^3\,d^2\,e\,z$$
$$\left.-576\,a\,b\,d\,e^2\,f+162\,a\,b\,d^2\,f^2+256\,a\,b\,e^4+81\,a^2\,f^4+81\,b^2\,d^4,z,k\big)\right)$$
$$-\frac{\frac{c}{8\,b}-\frac{d\,x^5}{32\,a}-\frac{e\,x^6}{16\,a}+\frac{e\,x^2}{16\,b}-\frac{3\,f\,x^7}{32\,a}+\frac{f\,x^3}{32\,b}+\frac{3\,d\,x}{32\,b}}{a^2+2\,a\,b\,x^4+b^2\,x^8}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^3,x)
```

output
```
symsum(log((x*(8*e^3 - 9*d*e*f))/(4096*a^3*b) - (3*(9*a*f^3 - 16*b*d*e^2 +
 9*b*d^2*f))/(32768*a^3*b^2) - root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4
*d*f*z^2 + 524288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*
d^2*e*z - 576*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 8
1*b^2*d^4, z, k)*(root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*z^2 + 52
4288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e*z - 576
*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2*d^4, z,
 k)*((3*b^2*d)/2 - 2*b^2*e*x) + (3*e*f)/(32*a) + (x*(144*a*b^2*d^2 - 144*a
^2*b*f^2))/(4096*a^3*b)))*root(268435456*a^7*b^7*z^4 + 589824*a^4*b^4*d*f*
z^2 + 524288*a^4*b^4*e^2*z^2 + 18432*a^3*b^2*e*f^2*z - 18432*a^2*b^3*d^2*e
*z - 576*a*b*d*e^2*f + 162*a*b*d^2*f^2 + 256*a*b*e^4 + 81*a^2*f^4 + 81*b^2
*d^4, z, k), k, 1, 4) - (c/(8*b) - (d*x^5)/(32*a) - (e*x^6)/(16*a) + (e*x^
2)/(16*b) - (3*f*x^7)/(32*a) + (f*x^3)/(32*b) + (3*d*x)/(32*b))/(a^2 + b^2
*x^8 + 2*a*b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1208, normalized size of antiderivative = 3.92

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^3,x)
```

output
```
( - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*f - 12*b**(1/4)*a**(3/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*
a*b*f*x**4 - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*f*x**8 - 6*b**(3/4)*a**(1/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*a**2*d - 12*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*d*x**4 - 6*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*b**2*d*x**8 - 16*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*e - 32*sqrt(
b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*b*e*x**4 - 16*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*e*x**8 + 6*b**(1/4)*a*
*(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**2*f + 12*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*f*x**4 + 6*b*
*(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b*
*(1/4)*a**(1/4)*sqrt(2)))*b**2*f*x**8 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a...
```

3.17 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4} dx$

Optimal result

Integrand size = 25, antiderivative size = 321

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4} dx = & -\frac{f}{12b(a+bx^4)^3} + \frac{x(c+dx+ex^2)}{12a(a+bx^4)^3} + \frac{x(11c+10dx+9ex^2)}{96a^2(a+bx^4)^2} \\ & + \frac{x(77c+60dx+45ex^2)}{384a^3(a+bx^4)} + \frac{5d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{7/2}\sqrt{b}} \\ & - \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\ & + \frac{\left(77\sqrt{b}c+15\sqrt{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{15/4}b^{3/4}} \\ & + \frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{256\sqrt{2}a^{15/4}b^{3/4}}\end{aligned}$$

output
```
-1/12*f/b/(b*x^4+a)^3+1/12*x*(e*x^2+d*x+c)/a/(b*x^4+a)^3+1/96*x*(9*e*x^2+1
0*d*x+11*c)/a^2/(b*x^4+a)^2+1/384*x*(45*e*x^2+60*d*x+77*c)/a^3/(b*x^4+a)+5
/32*d*arctan(b^(1/2)*x^2/a^(1/2))/a^(7/2)/b^(1/2)+1/512*(77*b^(1/2)*c+15*a
^(1/2)*e)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(15/4)/b^(3/4)+1/
512*(77*b^(1/2)*c+15*a^(1/2)*e)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2
)/a^(15/4)/b^(3/4)+1/512*(77*b^(1/2)*c-15*a^(1/2)*e)*arctanh(2^(1/2)*a^(1/
4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(15/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.18

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$

$$= \frac{\frac{8ax(77c+15x(4d+3ex))}{a+bx^4}+\frac{32a^2x(11c+x(10d+9ex))}{(a+bx^4)^2}-\frac{256a^3(af-bx(c+x(d+ex)))}{b(a+bx^4)^3}-\frac{6\sqrt[4]{a}\left(77\sqrt{2}\sqrt{b}c+80\sqrt[4]{a}\sqrt[4]{b}d+15\sqrt{2}\sqrt{a}e\right)\text{arcta}}{b^{3/4}}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

output
```
((8*a*x*(77*c + 15*x*(4*d + 3*e*x)))/(a + b*x^4) + (32*a^2*x*(11*c + x*(10
*d + 9*e*x)))/(a + b*x^4)^2 - (256*a^3*(a*f - b*x*(c + x*(d + e*x))))/(b*(
a + b*x^4)^3) - (6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c + 80*a^(1/4)*b^(1/4)*d +
15*Sqrt[2]*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (
6*a^(1/4)*(77*Sqrt[2]*Sqrt[b]*c - 80*a^(1/4)*b^(1/4)*d + 15*Sqrt[2]*Sqrt[a
]*e)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (3*Sqrt[2]*(-77*a^
(1/4)*Sqrt[b]*c + 15*a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/b^(3/4) + (3*Sqrt[2]*(77*a^(1/4)*Sqrt[b]*c - 15*a^(3/4)*e)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(3/4))/(3072*a^4)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2393, 25, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9ex^2+10dx+11c}{(bx^4+a)^3}dx}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}-\frac{\int -\frac{45ex^2+60dx+77c}{(bx^4+a)^2}dx}{8a}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{45ex^2+60dx+77c}{(bx^4+a)^2}dx}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x(77c+60dx+45ex^2)}{4a(a+bx^4)}-\frac{\int -\frac{3(15ex^2+40dx+77c)}{bx^4+a}dx}{4a}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab(a+bx^4)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{15ex^2+40dx+77c}{bx^4+a}dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\frac{3\int \left(\frac{40dx}{bx^4+a}+\frac{15ex^2+77c}{bx^4+a}\right)dx}{4a}+\frac{x\left(77c+60dx+45ex^2\right)}{4a\left(a+bx^4\right)}}{8a}+\frac{x(11c+10dx+9ex^2)}{8a(a+bx^4)^2}}{12a}-\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(15\sqrt{a}e+77\sqrt{b}c\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(77\sqrt{b}c-15\sqrt{a}e\right)}{}\right.}{4a}}{8a}}{12a}$$

$$\frac{af-bx(c+dx+ex^2)}{12ab\left(a+bx^4\right)^3}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x]
```

output
```
-1/12*(a*f - b*x*(c + d*x + e*x^2))/(a*b*(a + b*x^4)^3) + ((x*(11*c + 10*d
*x + 9*e*x^2))/(8*a*(a + b*x^4)^2) + ((x*(77*c + 60*d*x + 45*e*x^2))/(4*a*
(a + b*x^4)) + (3*((20*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
- ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((77*Sqrt[b]*c + 15*Sqrt[a]*e)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((77*Sqrt[b]*c -
 15*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*
Sqrt[2]*a^(3/4)*b^(3/4)) + ((77*Sqrt[b]*c - 15*Sqrt[a]*e)*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))))/(4*a
))/(8*a))/(12*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.48

method	result
risch	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}(b\_Z^4+a)}\left(15\_R^2e+40d\_\right.}{512b\,a^3}$
default	$\frac{\frac{15e\,b^2x^{11}}{128a^3}+\frac{5d\,b^2x^{10}}{32a^3}+\frac{77c\,b^2x^9}{384a^3}+\frac{21be\,x^7}{64a^2}+\frac{5bd\,x^6}{12a^2}+\frac{33bc\,x^5}{64a^2}+\frac{113e\,x^3}{384a}+\frac{11d\,x^2}{32a}+\frac{51cx}{128a}-\frac{f}{12b}}{(b\,x^4+a)^3}+\frac{77c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+\right.}{}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(15/128*e/a^3*b^2*x^11+5/32/a^3*d*b^2*x^10+77/384*c/a^3*b^2*x^9+21/64*b*e/
a^2*x^7+5/12*b*d/a^2*x^6+33/64*b/a^2*c*x^5+113/384*e/a*x^3+11/32*d/a*x^2+5
1/128*c/a*x-1/12*f/b)/(b*x^4+a)^3+1/512/b/a^3*sum((15*_R^2*e+40*_R*d+77*c)
/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.17 (sec) , antiderivative size = 125011, normalized size of antiderivative = 389.44

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(306) = 612$.

Time = 149.28 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.97

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \text{RootSum}\Big(68719476736t^4a^{15}b^3 + t^2\cdot\left(1211105280a^8b^2ce + 838860800a^8b^2d^2\right) + t\big(18432000a^5bde^2 - 48$$
$$+ \frac{-32a^3f + 153a^2bcx + 132a^2bdx^2 + 113a^2bex^3 + 198ab^2cx^5 + 160ab^2dx^6 + 126ab^2ex^7 + 77b^3cx^9 + 60b}{384a^6b + 1152a^5b^2x^4 + 1152a^4b^3x^8 + 384a^3b^4x^{12}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
RootSum(68719476736*_t**4*a**15*b**3 + _t**2*(1211105280*a**8*b**2*c*e + 8
38860800*a**8*b**2*d**2) + _t*(18432000*a**5*b*d*e**2 - 485703680*a**4*b**
2*c**2*d) + 50625*a**2*e**4 + 2668050*a*b*c**2*e**2 - 7392000*a*b*c*d**2*e
 + 2560000*a*b*d**4 + 35153041*b**2*c**4, Lambda(_t, _t*log(x + (452984832
000*_t**3*a**13*b**2*e**3 - 11936653639680*_t**3*a**12*b**3*c**2*e + 33071
248179200*_t**3*a**12*b**3*c*d**2 + 544997376000*_t**2*a**9*b**2*c*d*e**2
- 503316480000*_t**2*a**9*b**2*d**3*e + 4787095470080*_t**2*a**8*b**3*c**3
*d + 5987520000*_t*a**6*b*c*e**4 + 8294400000*_t*a**6*b*d**2*e**3 - 210370
406400*_t*a**5*b**2*c**3*e**2 + 655699968000*_t*a**5*b**2*c**2*d**2*e + 20
1850880000*_t*a**5*b**2*c*d**4 + 1385873488384*_t*a**4*b**3*c**5 + 9112500
0*a**3*d*e**5 + 5544000000*a**2*b*c*d**3*e**2 - 3072000000*a**2*b*d**5*e +
 105459123000*a*b**2*c**4*d*e - 146090560000*a*b**2*c**3*d**3)/(11390625*a
**3*e**6 - 300155625*a**2*b*c**2*e**4 + 3326400000*a**2*b*c*d**2*e**3 - 23
04000000*a**2*b*d**4*e**2 - 7909434225*a*b**2*c**4*e**2 + 87654336000*a*b*
*2*c**3*d**2*e - 60712960000*a*b**2*c**2*d**4 + 208422380089*b**3*c**6)))
 + (-32*a**3*f + 153*a**2*b*c*x + 132*a**2*b*d*x**2 + 113*a**2*b*e*x**3 +
198*a*b**2*c*x**5 + 160*a*b**2*d*x**6 + 126*a*b**2*e*x**7 + 77*b**3*c*x**9
 + 60*b**3*d*x**10 + 45*b**3*e*x**11)/(384*a**6*b + 1152*a**5*b**2*x**4 +
1152*a**4*b**3*x**8 + 384*a**3*b**4*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.25

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9+126\,ab^2ex^7+160\,ab^2dx^6+198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+1\cdots}{384\,(a^3b^4x^{12}+3\,a^4b^3x^8+3\,a^5b^2x^4+a^6b)}$$
$$+\frac{\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(77\,\sqrt{b}c-15\,\sqrt{a}e\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(77\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}c+15\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}\cdots\right.}{\cdots}}{1024\,a^3\cdots}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(45*b^3*e*x^11 + 60*b^3*d*x^10 + 77*b^3*c*x^9 + 126*a*b^2*e*x^7 + 16
0*a*b^2*d*x^6 + 198*a*b^2*c*x^5 + 113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*
a^2*b*c*x - 32*a^3*f)/(a^3*b^4*x^12 + 3*a^4*b^3*x^8 + 3*a^5*b^2*x^4 + a^6*
b) + 1/1024*(sqrt(2)*(77*sqrt(b)*c - 15*sqrt(a)*e)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(77*sqrt(b)*c
- 15*sqrt(a)*e)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1
/4)*e - 80*sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^
(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/
4)) + 2*(77*sqrt(2)*a^(1/4)*b^(3/4)*c + 15*sqrt(2)*a^(3/4)*b^(1/4)*e + 80*
sqrt(a)*sqrt(b)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.20

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx$$
$$= \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+ \frac{\sqrt{2}\left(40\sqrt{2}\sqrt{ab}b^2d+77\,(ab^3)^{\frac{1}{4}}\,b^2c+15\,(ab^3)^{\frac{3}{4}}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^4b^3}$$
$$+ \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$- \frac{\sqrt{2}\left(77\,(ab^3)^{\frac{1}{4}}\,b^2c-15\,(ab^3)^{\frac{3}{4}}\,e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^4b^3}$$
$$+ \frac{45\,b^3ex^{11}+60\,b^3dx^{10}+77\,b^3cx^9+126\,ab^2ex^7+160\,ab^2dx^6+198\,ab^2cx^5+113\,a^2bex^3+132\,a^2bdx^2+}{384\,(bx^4+a)^3a^3b}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*b^2*c + 15*(a
*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^4*b^3) + 1/512*sqrt(2)*(40*sqrt(2)*sqrt(a*b)*b^2*d + 77*(a*b^3)^(1/4)*
b^2*c + 15*(a*b^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))
/(a/b)^(1/4))/(a^4*b^3) + 1/1024*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b
^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) - 1/10
24*sqrt(2)*(77*(a*b^3)^(1/4)*b^2*c - 15*(a*b^3)^(3/4)*e)*log(x^2 - sqrt(2)
*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^3) + 1/384*(45*b^3*e*x^11 + 60*b^3*d*x^
10 + 77*b^3*c*x^9 + 126*a*b^2*e*x^7 + 160*a*b^2*d*x^6 + 198*a*b^2*c*x^5 +
113*a^2*b*e*x^3 + 132*a^2*b*d*x^2 + 153*a^2*b*c*x - 32*a^3*f)/((b*x^4 + a)
^3*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 879, normalized size of antiderivative = 2.74

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^4,x)
```

output
```
symsum(log(-(b*(3375*a*e^3 - 123200*b*c*d^2 + 88935*b*c^2*e - 64000*b*d^3*
x + 20185088*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 +
838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e
^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153
041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*c - 115200*root(68719476736*a
^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485
703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 26
68050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z,
 k)*a^4*b*e^2*x + 92400*b*c*d*e*x + 3035648*root(68719476736*a^15*b^3*z^4
+ 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b
^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^
2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e^4, z, k)*a^3*b^2*
c^2*x - 10485760*root(68719476736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^
2 + 838860800*a^8*b^2*d^2*z^2 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b
*d*e^2*z - 7392000*a*b*c*d^2*e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 3
5153041*b^2*c^4 + 50625*a^2*e^4, z, k)^2*a^7*b^2*d*x + 614400*root(6871947
6736*a^15*b^3*z^4 + 1211105280*a^8*b^2*c*e*z^2 + 838860800*a^8*b^2*d^2*z^2
 - 485703680*a^4*b^2*c^2*d*z + 18432000*a^5*b*d*e^2*z - 7392000*a*b*c*d^2*
e + 2668050*a*b*c^2*e^2 + 2560000*a*b*d^4 + 35153041*b^2*c^4 + 50625*a^2*e
^4, z, k)*a^4*b*d*e))/(2097152*a^9))*root(68719476736*a^15*b^3*z^4 + 12...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1642, normalized size of antiderivative = 5.12

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

output
```
( - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*e - 270*b**(1/4)*a**(3/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))
)*a**2*b*e*x**4 - 270*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*e*x**8 - 90*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1
/4)*a**(1/4)*sqrt(2)))*b**3*e*x**12 - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3
*c - 1386*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*c*x**4 - 1386*b**(3/4)*a**(1/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/
4)*sqrt(2)))*a*b**2*c*x**8 - 462*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*c*x**12
- 480*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**3*d - 1440*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*d*x**4 - 14
40*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4
)*a**(1/4)*sqrt(2)))*a*b**2*d*x**8 - 480*sqrt(b)*sqrt(a)*atan((b**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*d*x**12 +
90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b...
```

3.18 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4} dx$

Optimal result

Integrand size = 28, antiderivative size = 366

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = &-\frac{c+ex^2}{12b\,(a+bx^4)^3}-\frac{x(d+fx^2)}{12b\,(a+bx^4)^3}+\frac{ex^2}{48ab\,(a+bx^4)^2}\\ &+\frac{x(d+3fx^2)}{96ab\,(a+bx^4)^2}+\frac{ex^2}{32a^2b\,(a+bx^4)}\\ &+\frac{x(7d+15fx^2)}{384a^2b\,(a+bx^4)}+\frac{e\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32a^{5/2}b^{3/2}}\\ &-\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}\\ &+\frac{\left(7\sqrt{b}d+5\sqrt{a}f\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}\\ &+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{256\sqrt{2}a^{11/4}b^{7/4}}\end{aligned}$$

output
```
-1/12*(e*x^2+c)/b/(b*x^4+a)^3-1/12*x*(f*x^2+d)/b/(b*x^4+a)^3+1/48*e*x^2/a/
b/(b*x^4+a)^2+1/96*x*(3*f*x^2+d)/a/b/(b*x^4+a)^2+1/32*e*x^2/a^2/b/(b*x^4+a
)+1/384*x*(15*f*x^2+7*d)/a^2/b/(b*x^4+a)+1/32*e*arctan(b^(1/2)*x^2/a^(1/2)
)/a^(5/2)/b^(3/2)+1/512*(7*b^(1/2)*d+5*a^(1/2)*f)*arctan(-1+2^(1/2)*b^(1/4
)*x/a^(1/4))*2^(1/2)/a^(11/4)/b^(7/4)+1/512*(7*b^(1/2)*d+5*a^(1/2)*f)*arct
an(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/b^(7/4)+1/512*(7*b^(1/2)*
d-5*a^(1/2)*f)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^
(1/2)/a^(11/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \frac{\frac{32b^{3/4}x(d+x(2e+3fx))}{a(a+bx^4)^2}+\frac{8b^{3/4}x(7d+3x(4e+5fx))}{a^2(a+bx^4)}-\frac{256b^{3/4}(c+x(d+x(e+fx)))}{(a+bx^4)^3}-\frac{6\left(7\sqrt{2}\sqrt{b}d+16\sqrt[4]{a}\sqrt[4]{b}e+5\sqrt{2}\sqrt{a}f\right)\arctan\left(1-\right.}{a^{11/4}}}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

output
```
((32*b^(3/4)*x*(d + x*(2*e + 3*f*x)))/(a*(a + b*x^4)^2) + (8*b^(3/4)*x*(7*
d + 3*x*(4*e + 5*f*x)))/(a^2*(a + b*x^4)) - (256*b^(3/4)*(c + x*(d + x*(e
+ f*x))))/(a + b*x^4)^3 - (6*(7*Sqrt[2]*Sqrt[b]*d + 16*a^(1/4)*b^(1/4)*e +
 5*Sqrt[2]*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) +
(6*(7*Sqrt[2]*Sqrt[b]*d - 16*a^(1/4)*b^(1/4)*e + 5*Sqrt[2]*Sqrt[a]*f)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(11/4) + (3*Sqrt[2]*(-7*Sqrt[b]*d +
 5*Sqrt[a]*f)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(1
1/4) + (3*Sqrt[2]*(7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*x + Sqrt[b]*x^2])/a^(11/4))/(3072*b^(7/4))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2363, 2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^4} \, dx$$

$$\Big\downarrow \text{2363}$$

$$\frac{\int \frac{3fx^2+2ex+d}{\left(bx^4+a\right)^3}dx}{12b}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x\left(d+2ex+3fx^2\right)}{8a\left(a+bx^4\right)^2}-\frac{\int -\frac{15fx^2+12ex+7d}{\left(bx^4+a\right)^2}dx}{8a}}{12b}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{15fx^2+12ex+7d}{\left(bx^4+a\right)^2}dx}{8a}+\frac{x\left(d+2ex+3fx^2\right)}{8a\left(a+bx^4\right)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{\frac{x\left(7d+12ex+15fx^2\right)}{4a\left(a+bx^4\right)}-\frac{\int -\frac{3\left(5fx^2+8ex+7d\right)}{bx^4+a}dx}{4a}}{8a}+\frac{x\left(d+2ex+3fx^2\right)}{8a\left(a+bx^4\right)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3\int \frac{5fx^2+8ex+7d}{bx^4+a}dx}{4a}+\frac{x\left(7d+12ex+15fx^2\right)}{4a\left(a+bx^4\right)}}{8a}+\frac{x\left(d+2ex+3fx^2\right)}{8a\left(a+bx^4\right)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{\frac{3\int\left(\frac{8ex}{bx^4+a}+\frac{5fx^2+7d}{bx^4+a}\right)dx}{4a}+\frac{x\left(7d+12ex+15fx^2\right)}{4a\left(a+bx^4\right)}}{8a}+\frac{x\left(d+2ex+3fx^2\right)}{8a(a+bx^4)^2}}{12b}-\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}f+7\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}f+7\sqrt{b}d\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(7\sqrt{b}d-5\sqrt{a}f\right)\log\left(\sqrt{\right.}}{4\sqrt{2}a}}{4a}}{8a}}{12b}$$

$$\frac{c+dx+ex^2+fx^3}{12b\left(a+bx^4\right)^3}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x]
```

output
```
-1/12*(c + d*x + e*x^2 + f*x^3)/(b*(a + b*x^4)^3) + ((x*(d + 2*e*x + 3*f*x
^2))/(8*a*(a + b*x^4)^2) + ((x*(7*d + 12*e*x + 15*f*x^2))/(4*a*(a + b*x^4)
) + (3*((4*e*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) - ((7*Sqrt[b
]*d + 5*Sqrt[a]*f)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*b^(3/4)) + ((7*Sqrt[b]*d + 5*Sqrt[a]*f)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^
(3/4)) + ((7*Sqrt[b]*d - 5*Sqrt[a]*f)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))))/(4*a))/(8*a))/(12*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.39

method	result
risch	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{eb\,x^{10}}{32a^2}+\frac{7db\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{\left(5f\_R^2+8e\_R+7d\right)\ln\left(x-\_\cdots\right.}{\_R^3}}{512b^2a^2}$
default	$\frac{\frac{5bf\,x^{11}}{128a^2}+\frac{eb\,x^{10}}{32a^2}+\frac{7db\,x^9}{384a^2}+\frac{7f\,x^7}{64a}+\frac{e\,x^6}{12a}+\frac{3d\,x^5}{64a}-\frac{5f\,x^3}{384b}-\frac{e\,x^2}{32b}-\frac{7dx}{128b}-\frac{c}{12b}}{(b\,x^4+a)^3}+\frac{\frac{7d\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\cdots\right.\right.}{8a}}{\cdots}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(5/128/a^2*b*f*x^11+1/32/a^2*e*b*x^10+7/384/a^2*d*b*x^9+7/64*f/a*x^7+1/12/
a*e*x^6+3/64*d/a*x^5-5/384*f*x^3/b-1/32*e*x^2/b-7/128*d*x/b-1/12/b*c)/(b*x
^4+a)^3+1/512/b^2/a^2*sum((5*_R^2*f+8*_R*e+7*d)/_R^3*ln(x-_R),_R=RootOf(_Z
^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.03 (sec) , antiderivative size = 125996, normalized size of antiderivative = 344.25

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.08

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$
$$= \frac{15\,b^2fx^{11}+12\,b^2ex^{10}+7\,b^2dx^9+42\,abfx^7+32\,abex^6+18\,abdx^5-5\,a^2fx^3-12\,a^2ex^2-21\,a^2dx-32\,\cdots}{384\,(a^2b^4x^{12}+3\,a^3b^3x^8+3\,a^4b^2x^4+a^5b)}$$
$$+\frac{\frac{\sqrt{2}\left(7\,\sqrt{b}d-5\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7\,\sqrt{b}d-5\,\sqrt{a}f\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{3}{4}}}+\frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}b^{\frac{3}{4}}d+5\,\sqrt{2}a^{\frac{3}{4}}b^{\frac{1}{4}}f-16\cdots\right.}{a^{\frac{3}{4}}\cdots}}{1024\,a^2b\cdots}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="maxima")
```

output
```
1/384*(15*b^2*f*x^11 + 12*b^2*e*x^10 + 7*b^2*d*x^9 + 42*a*b*f*x^7 + 32*a*b
*e*x^6 + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2*e*x^2 - 21*a^2*d*x - 32*a^2*c
)/(a^2*b^4*x^12 + 3*a^3*b^3*x^8 + 3*a^4*b^2*x^4 + a^5*b) + 1/1024*(sqrt(2)
*(7*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/(a^(3/4)*b^(3/4)) - sqrt(2)*(7*sqrt(b)*d - 5*sqrt(a)*f)*log(sqrt
(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) + 2*(7*sq
rt(2)*a^(1/4)*b^(3/4)*d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f - 16*sqrt(a)*sqrt(b)
*e)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a
)*sqrt(b)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)) + 2*(7*sqrt(2)*a^(1/4)
*b^(3/4)*d + 5*sqrt(2)*a^(3/4)*b^(1/4)*f + 16*sqrt(a)*sqrt(b)*e)*arctan(1/
2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(
a^(3/4)*sqrt(sqrt(a)*sqrt(b))*b^(3/4)))/(a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.02

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4}\,dx$$

$$= \frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\,(ab^3)^{\frac{1}{4}}\,b^2d+5\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}$$

$$+ \frac{\sqrt{2}\left(8\sqrt{2}\sqrt{ab}b^2e+7\,(ab^3)^{\frac{1}{4}}\,b^2d+5\,(ab^3)^{\frac{3}{4}}\,f\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{512\,a^3b^4}$$

$$+ \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{1}{4}}\,b^2d-5\,(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}$$

$$- \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{1}{4}}\,b^2d-5\,(ab^3)^{\frac{3}{4}}\,f\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{1024\,a^3b^4}$$

$$+ \frac{15\,b^2fx^{11}+12\,b^2ex^{10}+7\,b^2dx^9+42\,abfx^7+32\,abex^6+18\,abdx^5-5\,a^2fx^3-12\,a^2ex^2-21\,a^2dx-3}{384\,(bx^4+a)^3a^2b}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x, algorithm="giac")
```

output
```
1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d + 5*(a*b^
3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a
^3*b^4) + 1/512*sqrt(2)*(8*sqrt(2)*sqrt(a*b)*b^2*e + 7*(a*b^3)^(1/4)*b^2*d
 + 5*(a*b^3)^(3/4)*f)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a^3*b^4) + 1/1024*sqrt(2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4
)*f)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^4) - 1/1024*sqrt(
2)*(7*(a*b^3)^(1/4)*b^2*d - 5*(a*b^3)^(3/4)*f)*log(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a^3*b^4) + 1/384*(15*b^2*f*x^11 + 12*b^2*e*x^10 + 7*b^2
*d*x^9 + 42*a*b*f*x^7 + 32*a*b*e*x^6 + 18*a*b*d*x^5 - 5*a^2*f*x^3 - 12*a^2
*e*x^2 - 21*a^2*d*x - 32*a^2*c)/((b*x^4 + a)^3*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.43

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4} \, dx = \text{Too large to display}$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^4,x)
```

output
```
((3*d*x^5)/(64*a) - c/(12*b) + (e*x^6)/(12*a) - (e*x^2)/(32*b) + (7*f*x^7)
/(64*a) - (5*f*x^3)/(384*b) - (7*d*x)/(128*b) + (7*b*d*x^9)/(384*a^2) + (b
*e*x^10)/(32*a^2) + (5*b*f*x^11)/(128*a^2))/(a^3 + b^3*x^12 + 3*a^2*b*x^4
+ 3*a*b^2*x^8) + symsum(log(-(125*a*f^3 - 448*b*d*e^2 + 245*b*d^2*f - 512*
b*e^3*x + 1835008*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2
 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*
e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2
401*b^2*d^4, z, k)^2*a^5*b^4*d + 560*b*d*e*f*x + 25088*root(68719476736*a^
11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*
a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2
*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4, z, k)*a^2*b^3*d^2*x - 20
97152*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*
a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a
*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*a^2*f^4 + 2401*b^2*d^4,
 z, k)^2*a^5*b^4*e*x - 12800*root(68719476736*a^11*b^7*z^4 + 36700160*a^6*
b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4*b^2*e*f^2*z - 802816*a
^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2*f^2 + 4096*a*b*e^4 + 625*
a^2*f^4 + 2401*b^2*d^4, z, k)*a^3*b^2*f^2*x + 40960*root(68719476736*a^11*
b^7*z^4 + 36700160*a^6*b^4*d*f*z^2 + 33554432*a^6*b^4*e^2*z^2 + 409600*a^4
*b^2*e*f^2*z - 802816*a^3*b^3*d^2*e*z - 8960*a*b*d*e^2*f + 2450*a*b*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1643, normalized size of antiderivative = 4.49

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^4} \, dx = \text{Too large to display}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^4,x)
```

output

```
( - 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*f - 90*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*a**2*b*f*x**4 - 90*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*f*x**8 - 30*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4
)*a**(1/4)*sqrt(2)))*b**3*f*x**12 - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*d
- 126*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*d*x**4 - 126*b**(3/4)*a**(1/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sq
rt(2)))*a*b**2*d*x**8 - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*d*x**12 - 96*s
qrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a*
*(1/4)*sqrt(2)))*a**3*e - 288*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*e*x**4 - 288*sqrt(b
)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4
)*sqrt(2)))*a*b**2*e*x**8 - 96*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*e*x**12 + 30*b**(1/4
)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(...
```

3.19 $\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 251

$$\begin{aligned}\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx &= \frac{ax(5(11Ab+5aC)+77bBx^2)\sqrt{a-bx^4}}{1155b^2}-\frac{(11Ab+5aC)x(a-bx^4)^{3/2}}{77b^2}\\ &\quad-\frac{Bx^3(a-bx^4)^{3/2}}{9b}-\frac{Cx^5(a-bx^4)^{3/2}}{11b}+\frac{2a^{11/4}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{7/4}\sqrt{a-bx^4}}\\ &\quad+\frac{2a^{9/4}\left(55Ab-77\sqrt{a}\sqrt{b}B+25aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{1155b^{9/4}\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/1155*a*x*(77*B*b*x^2+55*A*b+25*C*a)*(-b*x^4+a)^(1/2)/b^2-1/77*(11*A*b+5*
C*a)*x*(-b*x^4+a)^(3/2)/b^2-1/9*B*x^3*(-b*x^4+a)^(3/2)/b-1/11*C*x^5*(-b*x^
4+a)^(3/2)/b+2/15*a^(11/4)*B*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4)
,I)/b^(7/4)/(-b*x^4+a)^(1/2)+2/1155*a^(9/4)*(55*A*b-77*a^(1/2)*b^(1/2)*B+2
5*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(9/4)/(-b*x^4+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.56

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{x\sqrt{a-bx^4}\left(-\left((a-bx^4)\sqrt{1-\frac{bx^4}{a}}(99Ab+45aC+77bBx^2+63bCx^4)\right)+9a(11Ab+5aC)\,\text{Hypergeo}}{693b^2\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x^4*Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - b*x^4]*(-((a - b*x^4)*Sqrt[1 - (b*x^4)/a]*(99*A*b + 45*a*C + 7
7*b*B*x^2 + 63*b*C*x^4)) + 9*a*(11*A*b + 5*a*C)*Hypergeometric2F1[-1/2, 1/
4, 5/4, (b*x^4)/a] + 77*a*b*B*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, (b*x^4
)/a]))/(693*b^2*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2375, 25, 1597, 1603, 27, 1603, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \; 2375$$

$$-\frac{\int -x^4\left(11bBx^2+11Ab+5aC\right)\sqrt{a-bx^4}dx}{11b}-\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$$\Big\downarrow \; 25$$

$$\frac{\int x^4\left(11bBx^2+11Ab+5aC\right)\sqrt{a-bx^4}dx}{11b}-\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{1597}$$

$$\frac{\frac{2}{63}a\int \frac{x^4\left(77bBx^2+9(11Ab+5aC)\right)}{\sqrt{a-bx^4}}dx+\frac{1}{63}x^5\sqrt{a-bx^4}\left(9(5aC+11Ab)+77bBx^2\right)}{11b}-\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{1603}$$

$$\frac{\frac{2}{63}a\left(\frac{\int \frac{3bx^2\left(15(11Ab+5aC)x^2+77aB\right)}{\sqrt{a-bx^4}}dx}{5b}-\frac{77}{5}Bx^3\sqrt{a-bx^4}\right)+\frac{1}{63}x^5\sqrt{a-bx^4}\left(9(5aC+11Ab)+77bBx^2\right)}{11b}-\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{63}a\left(\frac{3}{5}\int \frac{x^2\left(15(11Ab+5aC)x^2+77aB\right)}{\sqrt{a-bx^4}}dx-\frac{77}{5}Bx^3\sqrt{a-bx^4}\right)+\frac{1}{63}x^5\sqrt{a-bx^4}\left(9(5aC+11Ab)+77bBx^2\right)}{11b}-\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{1603}$$

$$\frac{\frac{2}{63}a\left(\frac{3}{5}\left(\frac{\int \frac{3a\left(77bBx^2+5(11Ab+5aC)\right)}{\sqrt{a-bx^4}}dx}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+11Ab)}{b}\right)-\frac{77}{5}Bx^3\sqrt{a-bx^4}\right)+\frac{1}{63}x^5\sqrt{a-bx^4}\left(9(5aC+11Ab)\cdots\right.}{11b}-\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{63}a\left(\frac{3}{5}\left(\frac{a\int \frac{77bBx^2+5(11Ab+5aC)}{\sqrt{a-bx^4}}dx}{b}-\frac{5x\sqrt{a-bx^4}(5aC+11Ab)}{b}\right)-\frac{77}{5}Bx^3\sqrt{a-bx^4}\right)+\frac{1}{63}x^5\sqrt{a-bx^4}\left(9(5aC+11Ab)+\cdots\right.}{11b}-\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\frac{2}{63}a\left(\frac{3}{5}\left(\frac{a\left(\left(-77\sqrt{a}\sqrt{b}B+25aC+55Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx+77\sqrt{a}\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx\right)}{b}-\frac{5x\sqrt{a-bx^4}(5aC+11Ab)}{b}\right)-\frac{77}{5}Bx^3\sqrt{a-bx^4}}{11b}$$

$$\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$\big\downarrow$ 27

$$\frac{\frac{2}{63}a\left(\frac{3}{5}\left(\frac{a\left(\left(-77\sqrt{a}\sqrt{b}B+25aC+55Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx+77\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)}{b}-\frac{5x\sqrt{a-bx^4}(5aC+11Ab)}{b}\right)-\frac{77}{5}Bx^3\sqrt{a-bx^4}\right)+}{11b}$$

$$\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$\big\downarrow$ 765

$$\frac{\frac{2}{63}a\left(\frac{3}{5}\left(\frac{a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\left(-77\sqrt{a}\sqrt{b}B+25aC+55Ab\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+77\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)}{b}-\frac{5x\sqrt{a-bx^4}(5aC+11Ab)}{b}\right)-\frac{77}{5}Bx^3\sqrt{a-bx^4}}{11b}$$

$$\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$\big\downarrow$ 762

$$\frac{\frac{2}{63}a\left(\frac{3}{5}\left(\frac{a\left(77\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-77\sqrt{a}\sqrt{b}B+25aC+55Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{b}-\frac{5x\sqrt{a-bx^4}(5aC+11Ab)}{b}\right)-}{11b}$$

$$\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$\big\downarrow$ 1390

$$\frac{2}{63}a\left(\frac{\frac{3}{5}\left(\frac{a\left(\frac{77\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-77\sqrt{a}\sqrt{b}B+25aC+55Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{b}-\frac{5x\sqrt{a-bx^4}(5aC+11Ab}{b}\right.}{11b}\right.$$

$$\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$\Big\downarrow$ 1389

$$\frac{2}{63}a\left(\frac{\frac{3}{5}\left(\frac{a\left(\frac{77\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-77\sqrt{a}\sqrt{b}B+25aC+55Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{b}-\frac{5x\sqrt{a-bx^4}(5aC+1}{b}\right.}{11b}\right.$$

$$\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

$\Big\downarrow$ 327

$$\frac{2}{63}a\left(\frac{\frac{3}{5}\left(\frac{a\left(\frac{77a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-77\sqrt{a}\sqrt{b}B+25aC+55Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{b}-\frac{5x\sqrt{a-}}{}\right.}{11b}\right.$$

$$\frac{Cx^5\left(a-bx^4\right)^{3/2}}{11b}$$

input
```
Int[x^4*Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
-1/11*(C*x^5*(a - b*x^4)^(3/2))/b + ((x^5*(9*(11*A*b + 5*a*C) + 77*b*B*x^2
)*Sqrt[a - b*x^4])/63 + (2*a*((-77*B*x^3*Sqrt[a - b*x^4])/5 + (3*((-5*(11*
A*b + 5*a*C)*x*Sqrt[a - b*x^4])/b + (a*((77*a^(3/4)*b^(1/4)*B*Sqrt[1 - (b*
x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + (a^(
1/4)*(55*A*b - 77*Sqrt[a]*Sqrt[b]*B + 25*a*C)*Sqrt[1 - (b*x^4)/a]*Elliptic
F[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])))/b))/5))/63
)/(11*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1597
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Simp[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && Gt
Q[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (
IntegerQ[p] || IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{C x^9 \sqrt{-b x^4+a}}{11}+\frac{B x^7\sqrt{-b x^4+a}}{9}-\frac{\left(-Ab+\frac{2Ca}{11}\right)x^5\sqrt{-b x^4+a}}{7b}-\frac{2Ba x^3\sqrt{-b x^4+a}}{45b}-\frac{\left(Aa+\frac{5\left(-Ab+\frac{2Ca}{11}\right)a}{7b}\right)x\sqrt{-b x^4+a}}{3b}$
risch	$-\frac{x\left(-315C x^8b^2-385B x^6b^2-495A b^2x^4+90Cab x^4+154Ba x^2b+330abA+150a^2C\right)\sqrt{-b x^4+a}}{3465b^2}+\frac{2a^2\left(\frac{55Ab\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}}{\sqrt{}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-}}\right.}{}$
default	$A\left(\frac{x^5\sqrt{-b x^4+a}}{7}-\frac{2ax\sqrt{-b x^4+a}}{21b}+\frac{2a^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b x^4+a}}\right)+B\left(\frac{x^7\sqrt{-b x^4+a}}{9}-\frac{2a x^3\sqrt{-}}{45}\right.$

input
```
int(x^4*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*C*x^9*(-b*x^4+a)^(1/2)+1/9*B*x^7*(-b*x^4+a)^(1/2)-1/7*(-A*b+2/11*C*a)
/b*x^5*(-b*x^4+a)^(1/2)-2/45*B*a/b*x^3*(-b*x^4+a)^(1/2)-1/3*(A*a+5/7*(-A*b
+2/11*C*a)/b*a)/b*x*(-b*x^4+a)^(1/2)+1/3*(A*a+5/7*(-A*b+2/11*C*a)/b*a)/b*a
/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^
(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-2/1
5*B*a^(5/2)/b^(3/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2
)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.67

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx =$$

$$-\frac{462\,Ba^2\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-6\left((77\,B+25\,C)a^2+55\,Aab\right)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)}{x}\right.}{}$$

input
```
integrate(x^4*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/3465*(462*B*a^2*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x)
, -1) - 6*((77*B + 25*C)*a^2 + 55*A*a*b)*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f
(arcsin((a/b)^(1/4)/x), -1) - (315*C*b^2*x^10 + 385*B*b^2*x^8 - 154*B*a*b*
x^4 - 45*(2*C*a*b - 11*A*b^2)*x^6 - 462*B*a^2 - 30*(5*C*a^2 + 11*A*a*b)*x^
2)*sqrt(-b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.52

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{A\sqrt{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{B\sqrt{a}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{C\sqrt{a}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(2*I*
pi)/a)/(4*gamma(9/4)) + B*sqrt(a)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4
,), b*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4)) + C*sqrt(a)*x**9*gamma(9/4
)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\sqrt{-bx^4+a}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\sqrt{-bx^4+a}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^4\,\sqrt{a-b\,x^4}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int(x^4*(a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int(x^4*(a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int x^4\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{-330\sqrt{-b\,x^4+a}\,a^2bx-150\sqrt{-b\,x^4+a}\,a^2cx+495\sqrt{-b\,x^4+a}\,a\,b^2x^5-154\sqrt{-b\,x^4+a}\,a\,b^2x^3-90\sqrt{-}}{}$$

input
```
int(x^4*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
( - 330*sqrt(a - b*x**4)*a**2*b*x - 150*sqrt(a - b*x**4)*a**2*c*x + 495*sq
rt(a - b*x**4)*a*b**2*x**5 - 154*sqrt(a - b*x**4)*a*b**2*x**3 - 90*sqrt(a
- b*x**4)*a*b*c*x**5 + 385*sqrt(a - b*x**4)*b**3*x**7 + 315*sqrt(a - b*x**
4)*b**2*c*x**9 + 330*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a**3*b + 150*int
(sqrt(a - b*x**4)/(a - b*x**4),x)*a**3*c + 462*int((sqrt(a - b*x**4)*x**2)
/(a - b*x**4),x)*a**2*b**2)/(3465*b**2)
```

3.20 $\int x^2\sqrt{a-bx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 225

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{x(5aB+7(3Ab+aC)x^2)\sqrt{a-bx^4}}{105b}-\frac{Bx(a-bx^4)^{3/2}}{7b} - \frac{Cx^3(a-bx^4)^{3/2}}{9b}+\frac{2a^{7/4}(3Ab+aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{7/4}\sqrt{a-bx^4}} - \frac{2a^{7/4}\left(21Ab-5\sqrt{a}\sqrt{b}B+7aC\right)\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{105b^{7/4}\sqrt{a-bx^4}}$$

output
```
1/105*x*(5*a*B+7*(3*A*b+C*a)*x^2)*(-b*x^4+a)^(1/2)/b-1/7*B*x*(-b*x^4+a)^(3
/2)/b-1/9*C*x^3*(-b*x^4+a)^(3/2)/b+2/15*a^(7/4)*(3*A*b+C*a)*(1-b*x^4/a)^(1
/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)-2/105*a^(7/4)*
(21*A*b-5*a^(1/2)*b^(1/2)*B+7*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a
^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.55

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{x\sqrt{a-bx^4}\left(\left(9B+7Cx^2\right)\left(-a+bx^4\right)\sqrt{1-\frac{bx^4}{a}}+9aB\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)+7(3Ab+a\right.}{63b\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x^2*Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - b*x^4]*((9*B + 7*C*x^2)*(-a + b*x^4)*Sqrt[1 - (b*x^4)/a] + 9*a
*B*Hypergeometric2F1[-1/2, 1/4, 5/4, (b*x^4)/a] + 7*(3*A*b + a*C)*x^2*Hype
rgeometric2F1[-1/2, 3/4, 7/4, (b*x^4)/a]))/(63*b*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2375, 27, 1597, 1603, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \; 2375$$

$$-\frac{\int -3x^2\left(3bBx^2+3Ab+aC\right)\sqrt{a-bx^4}dx}{9b}-\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int x^2\left(3bBx^2+3Ab+aC\right)\sqrt{a-bx^4}dx}{3b}-\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \ 1597$$

$$\frac{\frac{2}{35}a\int \frac{x^2\left(15bBx^2+7(3Ab+aC)\right)}{\sqrt{a-bx^4}}dx+\frac{1}{35}x^3\sqrt{a-bx^4}\left(7(aC+3Ab)+15bBx^2\right)}{3b}-\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \ 1603$$

$$\frac{\frac{2}{35}a\left(\frac{\int \frac{3b\left(7(3Ab+aC)x^2+5aB\right)}{\sqrt{a-bx^4}}dx}{3b}-5Bx\sqrt{a-bx^4}\right)+\frac{1}{35}x^3\sqrt{a-bx^4}\left(7(aC+3Ab)+15bBx^2\right)}{3b}-\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2}{35}a\left(\int \frac{7(3Ab+aC)x^2+5aB}{\sqrt{a-bx^4}}dx-5Bx\sqrt{a-bx^4}\right)+\frac{1}{35}x^3\sqrt{a-bx^4}\left(7(aC+3Ab)+15bBx^2\right)}{3b}-\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{2}{35}a\left(\sqrt{a}\left(5\sqrt{a}B-\frac{7(aC+3Ab)}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{7\sqrt{a}(aC+3Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-5Bx\sqrt{a-bx^4}\right)+\frac{1}{35}x^3\sqrt{a-bx^4}\left(7(aC\right.}{3b}$$

$$\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2}{35}a\left(\sqrt{a}\left(5\sqrt{a}B-\frac{7(aC+3Ab)}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{7(aC+3Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-5Bx\sqrt{a-bx^4}\right)+\frac{1}{35}x^3\sqrt{a-bx^4}\left(7(aC+3\right.}{3b}$$

$$\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{2}{35}a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{7(aC+3Ab)}{\sqrt{b}}\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{7(aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-5Bx\sqrt{a-bx^4}\right)+\frac{1}{35}x^3\sqrt{a-bx^4}(7(aC+}{3b}$$

$$\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{2}{35}a\left(\frac{7(aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{7(aC+3Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-5Bx\sqrt{a-bx^4}\right)+\frac{1}{35}x^3\sqrt{}}{3b}$$

$$\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{2}{35}a\left(\frac{7\sqrt{1-\frac{bx^4}{a}}(aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{7(aC+3Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-5Bx\sqrt{a-bx^4}\right)+}{3b}$$

$$\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{2}{35}a\left(\frac{7\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(aC+3Ab)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{7(aC+3Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-5Bx\sqrt{a-bx^4}\right)}{3b}$$

$$\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{2}{35}a\left(\frac{7a^{3/4}\sqrt{1-\frac{bx^4}{a}}(aC+3Ab)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{7(aC+3Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-5Bx\sqrt{\ }\right.}{3b}$$

$$\frac{Cx^3\left(a-bx^4\right)^{3/2}}{9b}$$

input
```
Int[x^2*Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
-1/9*(C*x^3*(a - b*x^4)^(3/2))/b + ((x^3*(7*(3*A*b + a*C) + 15*b*B*x^2)*Sq
rt[a - b*x^4])/35 + (2*a*(-5*B*x*Sqrt[a - b*x^4] + (7*a^(3/4)*(3*A*b + a*C
)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)
*Sqrt[a - b*x^4]) + (a^(3/4)*(5*Sqrt[a]*B - (7*(3*A*b + a*C))/Sqrt[b])*Sqr
t[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt
[a - b*x^4])))/35)/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1597
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Simp[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3)))   Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
 3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && Gt
Q[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (
IntegerQ[p] || IntegerQ[m])
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.98

method	result
risch	$\frac{x\left(35Cb\,x^{6}+45Bb\,x^{4}+63Ab\,x^{2}-14Ca\,x^{2}-30Ba\right)\sqrt{-b\,x^{4}+a}}{315b}+\frac{2a\left(-\frac{(21Ab+7Ca)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\text{E}\ldots\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}\right.}{1\ldots}$
elliptic	$\frac{C\,x^{7}\sqrt{-b\,x^{4}+a}}{9}+\frac{B\,x^{5}\sqrt{-b\,x^{4}+a}}{7}-\frac{\left(-Ab+\frac{2Ca}{9}\right)x^{3}\sqrt{-b\,x^{4}+a}}{5b}-\frac{2Bax\sqrt{-b\,x^{4}+a}}{21b}+\frac{2B\,a^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\text{Elliptic}\ldots}{21b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
default	$A\left(\frac{x^{3}\sqrt{-b\,x^{4}+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}\right)+B\left(\frac{x^{5}\sqrt{-b\,x^{4}+a}}{7}-\ldots\right.$

input
```
int(x^2*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(35*C*b*x^6+45*B*b*x^4+63*A*b*x^2-14*C*a*x^2-30*B*a)*(-b*x^4+a)^(1
/2)/b+2/105*a/b*(-(21*A*b+7*C*a)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1
/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1
/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/
2))^(1/2),I))+5*B*a/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2
)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^
(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx = -\frac{42\left(Ca^2+3\,Aab\right)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-6\left(7\,Ca^2+\left(21\,A+5\,B\right)ab\right)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsi$$

input
```
integrate(x^2*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/315*(42*(C*a^2 + 3*A*a*b)*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b
)^(1/4)/x), -1) - 6*(7*C*a^2 + (21*A + 5*B)*a*b)*sqrt(-b)*x*(a/b)^(3/4)*el
liptic_f(arcsin((a/b)^(1/4)/x), -1) - (35*C*b^2*x^8 + 45*B*b^2*x^6 - 30*B*
a*b*x^2 - 7*(2*C*a*b - 9*A*b^2)*x^4 - 42*C*a^2 - 126*A*a*b)*sqrt(-b*x^4 +
a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.58

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{A\sqrt{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{C\sqrt{a}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*
pi)/a)/(4*gamma(7/4)) + B*sqrt(a)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,
), b*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4)) + C*sqrt(a)*x**7*gamma(7/4)*
hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*gamma(11/4))
```

Maxima [F]

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int\left(Cx^4+Bx^2+A\right)\sqrt{-bx^4+a}x^2\,dx$$

input
```
integrate(x^2*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int\left(Cx^4+Bx^2+A\right)\sqrt{-bx^4+a}x^2\,dx$$

input
```
integrate(x^2*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^2\,\sqrt{a-b\,x^4}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int(x^2*(a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int(x^2*(a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int x^2\sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{63\sqrt{-b\,x^4+a}\,ab\,x^3-30\sqrt{-b\,x^4+a}\,abx-14\sqrt{-b\,x^4+a}\,ac\,x^3+45\sqrt{-b\,x^4+a}\,b^2x^5+35\sqrt{-b\,x^4+a}\,bc}{315b}$$

input
```
int(x^2*(-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
(63*sqrt(a - b*x**4)*a*b*x**3 - 30*sqrt(a - b*x**4)*a*b*x - 14*sqrt(a - b*
x**4)*a*c*x**3 + 45*sqrt(a - b*x**4)*b**2*x**5 + 35*sqrt(a - b*x**4)*b*c*x
**7 + 30*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a**2*b + 126*int((sqrt(a - b
*x**4)*x**2)/(a - b*x**4),x)*a**2*b + 42*int((sqrt(a - b*x**4)*x**2)/(a -
b*x**4),x)*a**2*c)/(315*b)
```

3.21 $\int \sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 195

$$\begin{aligned}\int \sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right) \, dx &= \frac{x(5(7Ab+aC)+21bBx^2)\sqrt{a-bx^4}}{105b}-\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}\\ &\quad+\frac{2a^{7/4}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{3/4}\sqrt{a-bx^4}}\\ &\quad+\frac{2a^{5/4}\left(35Ab-21\sqrt{a}\sqrt{b}B+5aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{105b^{5/4}\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/105*x*(21*B*b*x^2+35*A*b+5*C*a)*(-b*x^4+a)^(1/2)/b-1/7*C*x*(-b*x^4+a)^(3
/2)/b+2/5*a^(7/4)*B*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(3/
4)/(-b*x^4+a)^(1/2)+2/105*a^(5/4)*(35*A*b-21*a^(1/2)*b^(1/2)*B+5*C*a)*(1-b
*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(5/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.59

$$\int \sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{x\sqrt{a-bx^4}\left(-3C(a-bx^4)\sqrt{1-\frac{bx^4}{a}}+3(7Ab+aC)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)+7bBx^2\,\text{Hype}\right.}{21b\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
(x*Sqrt[a - b*x^4]*(-3*C*(a - b*x^4)*Sqrt[1 - (b*x^4)/a] + 3*(7*A*b + a*C)
*Hypergeometric2F1[-1/2, 1/4, 5/4, (b*x^4)/a] + 7*b*B*x^2*Hypergeometric2F
1[-1/2, 3/4, 7/4, (b*x^4)/a]))/(21*b*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.440$, Rules used = {2427, 25, 1491, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$\Big\downarrow \text{2427}$$
$$-\frac{\int -\left(\left(7bBx^2+7Ab+aC\right)\sqrt{a-bx^4}\right)dx}{7b}-\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \left(7bBx^2+7Ab+aC\right)\sqrt{a-bx^4}dx}{7b}-\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \ 1491$$

$$\frac{\frac{1}{15}\int \frac{2a\left(21bBx^2+5(7Ab+aC)\right)}{\sqrt{a-bx^4}}dx+\frac{1}{15}x\sqrt{a-bx^4}\left(5(aC+7Ab)+21bBx^2\right)}{7b}-\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2}{15}a\int \frac{21bBx^2+5(7Ab+aC)}{\sqrt{a-bx^4}}dx+\frac{1}{15}x\sqrt{a-bx^4}\left(5(aC+7Ab)+21bBx^2\right)}{7b}-\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{2}{15}a\left(\left(-21\sqrt{a}\sqrt{b}B+5aC+35Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+21\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx\right)+\frac{1}{15}x\sqrt{a-bx^4}\left(5(aC+7Ab)+21\right.}{7b}$$
$$\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2}{15}a\left(\left(-21\sqrt{a}\sqrt{b}B+5aC+35Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+21\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)+\frac{1}{15}x\sqrt{a-bx^4}\left(5(aC+7Ab)+21bBx\right.}{7b}$$
$$\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{2}{15}a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\left(-21\sqrt{a}\sqrt{b}B+5aC+35Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+21\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)+\frac{1}{15}x\sqrt{a-bx^4}\left(5(aC+7Ab)+21bBx^2\right.}{7b}$$
$$\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \ 762$$

$$\frac{\frac{2}{15}a\left(21\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-21\sqrt{a}\sqrt{b}B+5aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)+\frac{1}{15}x\sqrt{a-bx^4}\left(5(aC\right.}{7b}$$
$$\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$$\Big\downarrow \ 1390$$

$$\frac{\frac{2}{15}a\left(\frac{21\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-21\sqrt{a}\sqrt{b}B+5aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)+\frac{1}{15}x\sqrt{a-bx^4}(5}{7b}$$

$$\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$\Big\downarrow$ 1389

$$\frac{\frac{2}{15}a\left(\frac{21\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-21\sqrt{a}\sqrt{b}B+5aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)+\frac{1}{15}x\sqrt{a-bx}}{7b}$$

$$\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

$\Big\downarrow$ 327

$$\frac{\frac{2}{15}a\left(\frac{21a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-21\sqrt{a}\sqrt{b}B+5aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)+\frac{1}{15}}{7b}$$

$$\frac{Cx\left(a-bx^4\right)^{3/2}}{7b}$$

input
```
Int[Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4),x]
```

output
```
-1/7*(C*x*(a - b*x^4)^(3/2))/b + ((x*(5*(7*A*b + a*C) + 21*b*B*x^2)*Sqrt[a
 - b*x^4])/15 + (2*a*((21*a^(3/4)*b^(1/4)*B*Sqrt[1 - (b*x^4)/a]*EllipticE[
ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + (a^(1/4)*(35*A*b - 21*
Sqrt[a]*Sqrt[b]*B + 5*a*C)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x
)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])))/15)/(7*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.17

method	result
elliptic	$\frac{C x^5\sqrt{-b x^4+a}}{7}+\frac{B x^3\sqrt{-b x^4+a}}{5}-\frac{\left(-Ab+\frac{2Ca}{7}\right)x\sqrt{-b x^4+a}}{3b}+\frac{\left(Aa+\frac{a\left(-Ab+\frac{2Ca}{7}\right)}{3b}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}$
risch	$\frac{x\left(15Cb x^4+21Bb x^2+35Ab-10Ca\right)\sqrt{-b x^4+a}}{105b}+\frac{2a\left(\frac{35Ab\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}+\frac{5Ca\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{E}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4}}\right.}{}$
default	$A\left(\frac{x\sqrt{-b x^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}\right)+B\left(\frac{x^3\sqrt{-b x^4+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\right.}{5\sqrt{}}\right.$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*C*x^5*(-b*x^4+a)^(1/2)+1/5*B*x^3*(-b*x^4+a)^(1/2)-1/3*(-A*b+2/7*C*a)/b
*x*(-b*x^4+a)^(1/2)+(A*a+1/3*a/b*(-A*b+2/7*C*a))/(1/a^(1/2)*b^(1/2))^(1/2)
*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1
/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-2/5*B*a^(3/2)/(1/a^(1/2)*b^(1
/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b
*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(
x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.66

$$\int \sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx = -\frac{42\,Ba\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-2\left((21\,B+5\,C)a+35\,Ab\right)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-}{105\,bx}$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
-1/105*(42*B*a*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1
) - 2*((21*B + 5*C)*a + 35*A*b)*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(arcsin((
a/b)^(1/4)/x), -1) - (15*C*b*x^6 + 21*B*b*x^4 - 5*(2*C*a - 7*A*b)*x^2 - 42
*B*a)*sqrt(-b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

$$\int \sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{A\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{B\sqrt{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{C\sqrt{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A),x)
```

output
```
A*sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(2*I*pi)
/a)/(4*gamma(5/4)) + B*sqrt(a)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
b*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4)) + C*sqrt(a)*x**5*gamma(5/4)*hyp
er((-1/2, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \sqrt{a-bx^4}(A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)\sqrt{-bx^4+a}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \sqrt{a-bx^4}(A+Bx^2+Cx^4)\,dx = \int (Cx^4+Bx^2+A)\sqrt{-bx^4+a}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-bx^4}(A+Bx^2+Cx^4)\,dx = \int \sqrt{a-b\,x^4}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \sqrt{a-bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{35\sqrt{-b\,x^4+a}\,abx-10\sqrt{-b\,x^4+a}\,acx+21\sqrt{-b\,x^4+a}\,b^2x^3+15\sqrt{-b\,x^4+a}\,bc\,x^5+70\left(\int\frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)}{105b}$$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A),x)
```

output
```
(35*sqrt(a - b*x**4)*a*b*x - 10*sqrt(a - b*x**4)*a*c*x + 21*sqrt(a - b*x**
4)*b**2*x**3 + 15*sqrt(a - b*x**4)*b*c*x**5 + 70*int(sqrt(a - b*x**4)/(a -
 b*x**4),x)*a**2*b + 10*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a**2*c + 42*i
nt((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)*a*b**2)/(105*b)
```

3.22 $\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 204

$$\begin{aligned}\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2} dx &= \frac{x(5aB-3(5Ab-aC)x^2)\sqrt{a-bx^4}}{15a} - \frac{A(a-bx^4)^{3/2}}{ax} \\ &\quad - \frac{2a^{3/4}(5Ab-aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{3/4}\sqrt{a-bx^4}} \\ &\quad + \frac{2a^{3/4}\left(15Ab+5\sqrt{a}\sqrt{b}B-3aC\right)\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{15b^{3/4}\sqrt{a-bx^4}}\end{aligned}$$

output
```
1/15*x*(5*a*B-3*(5*A*b-C*a)*x^2)*(-b*x^4+a)^(1/2)/a-A*(-b*x^4+a)^(3/2)/a/x
-2/5*a^(3/4)*(5*A*b-C*a)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/
b^(3/4)/(-b*x^4+a)^(1/2)+2/15*a^(3/4)*(15*A*b+5*a^(1/2)*b^(1/2)*B-3*C*a)*(
1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(3/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$$
$$= \frac{\sqrt{a-bx^4}\left(-3A\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},\frac{bx^4}{a}\right)+3Bx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)+Cx\right.}{3x\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^2,x]
```

output
```
(Sqrt[a - b*x^4]*(-3*A*Hypergeometric2F1[-1/2, -1/4, 3/4, (b*x^4)/a] + 3*B
*x^2*Hypergeometric2F1[-1/2, 1/4, 5/4, (b*x^4)/a] + C*x^4*Hypergeometric2F
1[-1/2, 3/4, 7/4, (b*x^4)/a]))/(3*x*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2374, 9, 27, 1491, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$$
$$\Big\downarrow \; 2374$$
$$-\frac{\int -\frac{2(aBx-(5Ab-aC)x^3)\sqrt{a-bx^4}}{x}dx}{2a}-\frac{A(a-bx^4)^{3/2}}{ax}$$
$$\Big\downarrow \; 9$$
$$-\frac{\int -2(aB-(5Ab-aC)x^2)\sqrt{a-bx^4}dx}{2a}-\frac{A(a-bx^4)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \left(aB-(5Ab-aC)x^2\right)\sqrt{a-bx^4}dx}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 1491}$$

$$\frac{\frac{1}{15}\int \frac{2a\left(5aB-3(5Ab-aC)x^2\right)}{\sqrt{a-bx^4}}dx+\frac{1}{15}x\sqrt{a-bx^4}\left(5aB-3x^2(5Ab-aC)\right)}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2}{15}a\int \frac{5aB-3(5Ab-aC)x^2}{\sqrt{a-bx^4}}dx+\frac{1}{15}x\sqrt{a-bx^4}\left(5aB-3x^2(5Ab-aC)\right)}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 1513}$$

$$\frac{\frac{2}{15}a\left(\sqrt{a}\left(\frac{3(5Ab-aC)}{\sqrt{b}}+5\sqrt{a}B\right)\int \frac{1}{\sqrt{a-bx^4}}dx-\frac{3\sqrt{a}(5Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{15}x\sqrt{a-bx^4}\left(5aB-3x^2(5Ab-aC)\right)}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{2}{15}a\left(\sqrt{a}\left(\frac{3(5Ab-aC)}{\sqrt{b}}+5\sqrt{a}B\right)\int \frac{1}{\sqrt{a-bx^4}}dx-\frac{3(5Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{15}x\sqrt{a-bx^4}\left(5aB-3x^2(5Ab-aC)\right)}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\frac{2}{15}a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(5Ab-aC)}{\sqrt{b}}+5\sqrt{a}B\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-\frac{3(5Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{15}x\sqrt{a-bx^4}\left(5aB-3x^2(5Ab-aC)\right)}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\frac{2}{15}a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(5Ab-aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3(5Ab-aC)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{15}x\sqrt{a-bx^4}(5aB-3x}{a}$$

$$\frac{A(a-bx^4)^{3/2}}{ax}$$

$\big\downarrow$ 1390

$$\frac{\frac{2}{15}a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(5Ab-aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3\sqrt{1-\frac{bx^4}{a}}(5Ab-aC)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)+\frac{1}{15}x\sqrt{a-bx^4}(5a}{a}$$

$$\frac{A(a-bx^4)^{3/2}}{ax}$$

$\big\downarrow$ 1389

$$\frac{\frac{2}{15}a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(5Ab-aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(5Ab-aC)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)+\frac{1}{15}x\sqrt{a-bx^4}}{a}$$

$$\frac{A(a-bx^4)^{3/2}}{ax}$$

$\big\downarrow$ 327

$$\frac{\frac{2}{15}a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(5Ab-aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3a^{3/4}\sqrt{1-\frac{bx^4}{a}}(5Ab-aC)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{15}x\sqrt{}}{a}$$

$$\frac{A(a-bx^4)^{3/2}}{ax}$$

input
```
Int[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^2,x]
```

output
```
-((A*(a - b*x^4)^(3/2))/(a*x)) + ((x*(5*a*B - 3*(5*A*b - a*C)*x^2)*Sqrt[a
- b*x^4])/15 + (2*a*((-3*a^(3/4)*(5*A*b - a*C)*Sqrt[1 - (b*x^4)/a]*Ellipti
cE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) + (a^(3/4)*
(5*Sqrt[a]*B + (3*(5*A*b - a*C))/Sqrt[b])*Sqrt[1 - (b*x^4)/a]*EllipticF[Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])))/15)/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1491
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Simp[2*(p/((4*p + 1)*(4*p + 3)))   Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
 1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c
*d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-3C\,x^{4}-5B\,x^{2}+15A\right)}{15x}+\frac{2(15Ab-3Ca)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{15\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$
elliptic	$-\frac{A\sqrt{-b\,x^{4}+a}}{x}+\frac{C\,x^{3}\sqrt{-b\,x^{4}+a}}{5}+\frac{Bx\sqrt{-b\,x^{4}+a}}{3}+\frac{2Ba\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{\left(-2Ab+\frac{2Ca}{5}\right)\sqrt{a}}{}$
default	$B\left(\frac{x\sqrt{-b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right)+A\left(-\frac{\sqrt{-b\,x^{4}+a}}{x}+\frac{2\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}}{}\right.$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-b*x^4+a)^(1/2)*(-3*C*x^4-5*B*x^2+15*A)/x+2/15*(15*A*b-3*C*a)*a^(1/
2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/
a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(
1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))+2/3*B*a/(1/a^(1/2)*b^(1/
2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*
x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \frac{iA\sqrt{b}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{B\sqrt{a}x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{C\sqrt{a}x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/x**2,x)
```

output
```
I*A*sqrt(b)*x*gamma(1/4)*hyper((-1/2, -1/4), (3/4,), a/(b*x**4))/(4*gamma(
5/4)) + B*sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar
(2*I*pi)/a)/(4*gamma(5/4)) + C*sqrt(a)*x**3*gamma(3/4)*hyper((-1/2, 3/4),
(7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{\sqrt{a-b\,x^4}\,(C\,x^4+B\,x^2+A)}{x^2}\,dx$$

input
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^2,x)
```

output
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \frac{15\sqrt{-b\,x^4+a}\,ab-6\sqrt{-b\,x^4+a}\,ac+5\sqrt{-b\,x^4+a}\,b^2x^2+3\sqrt{-b\,x^4+a}\,bc\,x^4+30\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^6+a\,x^2}dx\right)a^2bx}{15bx}$$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^2,x)
```

output
```
(15*sqrt(a - b*x**4)*a*b - 6*sqrt(a - b*x**4)*a*c + 5*sqrt(a - b*x**4)*b**
2*x**2 + 3*sqrt(a - b*x**4)*b*c*x**4 + 30*int(sqrt(a - b*x**4)/(a*x**2 - b
*x**6),x)*a**2*b*x - 6*int(sqrt(a - b*x**4)/(a*x**2 - b*x**6),x)*a**2*c*x
 + 10*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a*b**2*x)/(15*b*x)
```

3.23 $\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 195

$$\begin{aligned}\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx &= -\frac{(3aB+(Ab-aC)x^2)\sqrt{a-bx^4}}{3ax}-\frac{A(a-bx^4)^{3/2}}{3ax^3}\\ &\quad-\frac{2a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}\\ &\quad-\frac{2\sqrt[4]{a}\left(Ab-3\sqrt{a}\sqrt{b}B-aC\right)\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{b}\sqrt{a-bx^4}}\end{aligned}$$

output
```
-1/3*(3*a*B+(A*b-C*a)*x^2)*(-b*x^4+a)^(1/2)/a/x-1/3*A*(-b*x^4+a)^(3/2)/a/x
^3-2*a^(3/4)*b^(1/4)*B*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/(-
b*x^4+a)^(1/2)-2/3*a^(1/4)*(A*b-3*a^(1/2)*b^(1/2)*B-C*a)*(1-b*x^4/a)^(1/2)
*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \frac{\sqrt{a-bx^4}\left(-A\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},\frac{bx^4}{a}\right)-3Bx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},\frac{bx^4}{a}\right)+3C\right.}{3x^3\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
(Sqrt[a - b*x^4]*(-(A*Hypergeometric2F1[-3/4, -1/2, 1/4, (b*x^4)/a]) - 3*B
*x^2*Hypergeometric2F1[-1/2, -1/4, 3/4, (b*x^4)/a] + 3*C*x^4*Hypergeometri
c2F1[-1/2, 1/4, 5/4, (b*x^4)/a]))/(3*x^3*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2374, 9, 27, 1595, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$$

$$\Big\downarrow \text{2374}$$

$$-\frac{\int -\frac{6(aBx-(Ab-aC)x^3)\sqrt{a-bx^4}}{x^3}dx}{6a}-\frac{A(a-bx^4)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{6(aB-(Ab-aC)x^2)\sqrt{a-bx^4}}{x^2}dx}{6a}-\frac{A(a-bx^4)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(aB-(Ab-aC)x^2\right)\sqrt{a-bx^4}}{x^2}dx}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 1595$$

$$\frac{-\frac{2}{3}\int \frac{a\left(3bBx^2+Ab-aC\right)}{\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC)+3aB\right)}{3x}}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{2}{3}a\int \frac{3bBx^2+Ab-aC}{\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC)+3aB\right)}{3x}}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 1513$$

$$\frac{-\frac{2}{3}a\left(\left(-3\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+3\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx\right)-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC)+3aB\right)}{3x}}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{2}{3}a\left(\left(-3\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC)+3aB\right)}{3x}}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 765$$

$$\frac{-\frac{2}{3}a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC)+3aB\right)}{3x}}{a}-\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 762$$

$$\frac{-\frac{2}{3}a\left(3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC)+3aB\right)}{3x}}{a}$$

$$\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 1390

$$\frac{-\frac{2}{3}a\left(\frac{3\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC)+3\right.}{3x}}{a}$$

$$\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 1389

$$\frac{-\frac{2}{3}a\left(\frac{3\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}\left(x^2(Ab-aC\right.}{3x}}{a}$$

$$\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 327

$$\frac{-\frac{2}{3}a\left(\frac{3a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\sqrt{a-bx}}{a}$$

$$\frac{A\left(a-bx^4\right)^{3/2}}{3ax^3}$$

input
```
Int[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
-1/3*(A*(a - b*x^4)^(3/2))/(a*x^3) + (-1/3*((3*a*B + (A*b - a*C)*x^2)*Sqrt
[a - b*x^4])/x - (2*a*((3*a^(3/4)*b^(1/4)*B*Sqrt[1 - (b*x^4)/a]*EllipticE[
ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + (a^(1/4)*(A*b - 3*Sqrt
[a]*Sqrt[b]*B - a*C)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])))/3)/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1595
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Simp[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
)   Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*
x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.07

method	result
elliptic	$-\frac{A\sqrt{-b\,x^4+a}}{3x^3}-\frac{B\sqrt{-b\,x^4+a}}{x}+\frac{Cx\sqrt{-b\,x^4+a}}{3}+\frac{\left(-\frac{2Ab}{3}+\frac{2Ca}{3}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{2B\sqrt{b}\,\sqrt{a}}{}$
risch	$-\frac{\sqrt{-b\,x^4+a}\,(-C\,x^4+3B\,x^2+A)}{3x^3}-\frac{2Ab\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{2Ca\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}(}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
default	$C\left(\frac{x\sqrt{-b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}\right)+A\left(-\frac{\sqrt{-b\,x^4+a}}{3x^3}-\frac{2b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{Elli}}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4}}\right.$

input

```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/3*A*(-b*x^4+a)^(1/2)/x^3-B*(-b*x^4+a)^(1/2)/x+1/3*C*x*(-b*x^4+a)^(1/2)+
(-2/3*A*b+2/3*C*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)
*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(
1/2))^(1/2),I)+2*B*b^(1/2)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^
2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF
(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^4}\,dx$$

input

```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output

```
integral((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \frac{iA\sqrt{b}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle|\frac{a}{bx^4}\right)}{4x\Gamma\left(\frac{3}{4}\right)} + \frac{iB\sqrt{b}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{C\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/x**4,x)
```

output
```
I*A*sqrt(b)*gamma(-1/4)*hyper((-1/2, 1/4), (5/4,), a/(b*x**4))/(4*x*gamma(
3/4)) + I*B*sqrt(b)*x*gamma(1/4)*hyper((-1/2, -1/4), (3/4,), a/(b*x**4))/(
4*gamma(5/4)) + C*sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*e
xp_polar(2*I*pi)/a)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{\sqrt{a-b\,x^4}\,(C\,x^4+B\,x^2+A)}{x^4}\,dx$$

input
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^4,x)
```

output
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$$
$$= \frac{-3\sqrt{-b\,x^4+a}\,ab+2\sqrt{-b\,x^4+a}\,ac+3\sqrt{-b\,x^4+a}\,b^2x^2+\sqrt{-b\,x^4+a}\,bc\,x^4-6\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^8+a\,x^4}dx\right)a^2b\,x^3}{3b\,x^3}$$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^4,x)
```

output
```
( - 3*sqrt(a - b*x**4)*a*b + 2*sqrt(a - b*x**4)*a*c + 3*sqrt(a - b*x**4)*b
**2*x**2 + sqrt(a - b*x**4)*b*c*x**4 - 6*int(sqrt(a - b*x**4)/(a*x**4 - b*
x**8),x)*a**2*b*x**3 + 6*int(sqrt(a - b*x**4)/(a*x**4 - b*x**8),x)*a**2*c*
x**3 + 6*int(sqrt(a - b*x**4)/(a*x**2 - b*x**6),x)*a*b**2*x**3)/(3*b*x**3)
```

3.24 $\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 202

$$
\begin{aligned}
\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx
&= -\frac{1}{15}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)\sqrt{a-bx^4}+\frac{2Ab\sqrt{a-bx^4}}{5ax} \\
&\quad+\frac{2\sqrt[4]{b}(Ab-5aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5\sqrt[4]{a}\sqrt{a-bx^4}} \\
&\quad-\frac{2\sqrt[4]{b}\left(3Ab+5\sqrt{a}\sqrt{b}B-15aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{15\sqrt[4]{a}\sqrt{a-bx^4}}
\end{aligned}
$$

output

```
-1/15*(3*A/x^5+5*B/x^3+15*C/x)*(-b*x^4+a)^(1/2)+2/5*A*b*(-b*x^4+a)^(1/2)/a
/x+2/5*b^(1/4)*(A*b-5*C*a)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I
)/a^(1/4)/(-b*x^4+a)^(1/2)-2/15*b^(1/4)*(3*A*b+5*a^(1/2)*b^(1/2)*B-15*C*a)
*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = -\frac{\sqrt{a-bx^4}\left(3A\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},\frac{bx^4}{a}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},\frac{bx^4}{a}\right)\right.}{15x^5\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
-1/15*(Sqrt[a - b*x^4]*(3*A*Hypergeometric2F1[-5/4, -1/2, -1/4, (b*x^4)/a]
 + 5*B*x^2*Hypergeometric2F1[-3/4, -1/2, 1/4, (b*x^4)/a] + 15*C*x^4*Hyperg
eometric2F1[-1/2, -1/4, 3/4, (b*x^4)/a]))/(x^5*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2364, 27, 2374, 9, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$$

$$\Big\downarrow \text{ 2364}$$

$$2b\int -\frac{15Cx^4+5Bx^2+3A}{15x^2\sqrt{a-bx^4}}dx-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2}{15}b\int \frac{15Cx^4+5Bx^2+3A}{x^2\sqrt{a-bx^4}}dx-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 2374$$

$$-\frac{2}{15}b\left(-\frac{\int -\frac{2(5aBx-3(Ab-5aC)x^3)}{x\sqrt{a-bx^4}}dx}{2a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 9$$

$$-\frac{2}{15}b\left(-\frac{\int -\frac{2(5aB-3(Ab-5aC)x^2)}{\sqrt{a-bx^4}}dx}{2a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{2}{15}b\left(\frac{\int \frac{5aB-3(Ab-5aC)x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 1513$$

$$-\frac{2}{15}b\left(\frac{\sqrt{a}\left(\frac{3(Ab-5aC)}{\sqrt{b}}+5\sqrt{a}B\right)\int \frac{1}{\sqrt{a-bx^4}}dx-\frac{3\sqrt{a}(Ab-5aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{2}{15}b\left(\frac{\sqrt{a}\left(\frac{3(Ab-5aC)}{\sqrt{b}}+5\sqrt{a}B\right)\int \frac{1}{\sqrt{a-bx^4}}dx-\frac{3(Ab-5aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 765$$

$$-\frac{2}{15}b\left(\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(Ab-5aC)}{\sqrt{b}}+5\sqrt{a}B\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-\frac{3(Ab-5aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 762$$

$$-\frac{2}{15}b\left(\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(Ab-5aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3(Ab-5aC)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-$$
$$\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$\Big\downarrow$ 1390

$$-\frac{2}{15}b\left(\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(Ab-5aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3\sqrt{1-\frac{bx^4}{a}}(Ab-5aC)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}}{a}-\frac{3A\sqrt{a-bx^4}}{ax}\right)-$$
$$\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$\Big\downarrow$ 1389

$$-\frac{2}{15}b\left(\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(Ab-5aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(Ab-5aC)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}}{a}-\frac{3A\sqrt{a-bx^4}}{ax}\right.$$
$$\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$\Big\downarrow$ 327

$$-\frac{2}{15}b\left(\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\frac{3(Ab-5aC)}{\sqrt{b}}+5\sqrt{a}B\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3a^{3/4}\sqrt{1-\frac{bx^4}{a}}(Ab-5aC)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{a}-3A\right.$$
$$\frac{1}{15}\sqrt{a-bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

input
```
Int[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
-1/15*(((3*A)/x^5 + (5*B)/x^3 + (15*C)/x)*Sqrt[a - b*x^4]) - (2*b*((-3*A*S
qrt[a - b*x^4])/(a*x) + ((-3*a^(3/4)*(A*b - 5*a*C)*Sqrt[1 - (b*x^4)/a]*Ell
ipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) + (a^(3
/4)*(5*Sqrt[a]*B + (3*(A*b - 5*a*C))/Sqrt[b])*Sqrt[1 - (b*x^4)/a]*Elliptic
F[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4]))/a))/15
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\sqrt{-b\,x^4+a}\,(-6Ab\,x^4+15Ca\,x^4+5Ba\,x^2+3Aa)}{15x^5a}+\frac{2b\left(-\frac{(3Ab-15Ca)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\cdots}\right.\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,\sqrt{b}}\right.}{15a}$
elliptic	$-\frac{A\sqrt{-b\,x^4+a}}{5x^5}-\frac{B\sqrt{-b\,x^4+a}}{3x^3}+\frac{(2Ab-5Ca)\sqrt{-b\,x^4+a}}{5ax}-\frac{2Bb\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\left(-Cb+\frac{b(2A\cdots}{}\right.}{}$
default	$A\left(-\frac{\sqrt{-b\,x^4+a}}{5x^5}+\frac{2b\sqrt{-b\,x^4+a}}{5ax}-\frac{2b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}\right)+B\left(-\cdots\right.$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-b*x^4+a)^(1/2)*(-6*A*b*x^4+15*C*a*x^4+5*B*a*x^2+3*A*a)/x^5/a+2/15*
b/a*(-(3*A*b-15*C*a)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1
/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(Ellipti
cF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
)-5*B*a/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)
*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2)
,I))
```

Fricas [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx=\int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^6, x)
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \frac{iA\sqrt{b}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} + \frac{iB\sqrt{b}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x\Gamma\left(\frac{3}{4}\right)} + \frac{iC\sqrt{b}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/x**6,x)
```

output
```
I*A*sqrt(b)*gamma(-3/4)*hyper((-1/2, 3/4), (7/4,), a/(b*x**4))/(4*x**3*gam
ma(1/4)) + I*B*sqrt(b)*gamma(-1/4)*hyper((-1/2, 1/4), (5/4,), a/(b*x**4))/
(4*x*gamma(3/4)) + I*C*sqrt(b)*x*gamma(1/4)*hyper((-1/2, -1/4), (3/4,), a/
(b*x**4))/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{\sqrt{a-b\,x^4}\,(C\,x^4+B\,x^2+A)}{x^6}\,dx$$

input
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^6,x)
```

output
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$$
$$= \frac{-\sqrt{-b\,x^4+a}\,ab+2\sqrt{-b\,x^4+a}\,ac-3\sqrt{-b\,x^4+a}\,b^2x^2+3\sqrt{-b\,x^4+a}\,bc\,x^4-2\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{10}+a\,x^6}dx\right)a^2b\,x^5}{3b\,x^5}$$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - sqrt(a - b*x**4)*a*b + 2*sqrt(a - b*x**4)*a*c - 3*sqrt(a - b*x**4)*b**
2*x**2 + 3*sqrt(a - b*x**4)*b*c*x**4 - 2*int(sqrt(a - b*x**4)/(a*x**6 - b*
x**10),x)*a**2*b*x**5 + 10*int(sqrt(a - b*x**4)/(a*x**6 - b*x**10),x)*a**2
*c*x**5 - 6*int(sqrt(a - b*x**4)/(a*x**4 - b*x**8),x)*a*b**2*x**5)/(3*b*x*
*5)
```

3.25 $\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 219

$$\begin{aligned}\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx &= -\frac{1}{105}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)\sqrt{a-bx^4}+\frac{2Ab\sqrt{a-bx^4}}{21ax^3} \\ &\quad +\frac{2bB\sqrt{a-bx^4}}{5ax}+\frac{2b^{5/4}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5\sqrt[4]{a}\sqrt{a-bx^4}} \\ &\quad -\frac{2b^{3/4}\left(5Ab+21\sqrt{a}\sqrt{b}B+35aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{105a^{3/4}\sqrt{a-bx^4}}\end{aligned}$$

output
```
-1/105*(15*A/x^7+21*B/x^5+35*C/x^3)*(-b*x^4+a)^(1/2)+2/21*A*b*(-b*x^4+a)^(
1/2)/a/x^3+2/5*b*B*(-b*x^4+a)^(1/2)/a/x+2/5*b^(5/4)*B*(1-b*x^4/a)^(1/2)*El
lipticE(b^(1/4)*x/a^(1/4),I)/a^(1/4)/(-b*x^4+a)^(1/2)-2/105*b^(3/4)*(5*A*b
+21*a^(1/2)*b^(1/2)*B+35*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4
),I)/a^(3/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = -\frac{\sqrt{a-bx^4}\left(15A\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},\frac{bx^4}{a}\right)+21Bx^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},\frac{bx^4}{a}\right.\right.}{105x^7\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^8,x]
```

output
```
-1/105*(Sqrt[a - b*x^4]*(15*A*Hypergeometric2F1[-7/4, -1/2, -3/4, (b*x^4)/
a] + 21*B*x^2*Hypergeometric2F1[-5/4, -1/2, -1/4, (b*x^4)/a] + 35*C*x^4*Hy
pergeometric2F1[-3/4, -1/2, 1/4, (b*x^4)/a]))/(x^7*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.536, Rules used = {2364, 27, 2374, 9, 27, 1605, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx$$

$$\Big\downarrow \text{2364}$$

$$2b\int -\frac{35Cx^4+21Bx^2+15A}{105x^4\sqrt{a-bx^4}}dx-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{105}b\int \frac{35Cx^4+21Bx^2+15A}{x^4\sqrt{a-bx^4}}dx-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \ 2374$$

$$-\frac{2}{105}b\left(-\frac{\int -\frac{6\left(5(Ab+7aC)x^3+21aBx\right)}{x^3\sqrt{a-bx^4}}dx}{6a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \ 9$$

$$-\frac{2}{105}b\left(-\frac{\int -\frac{6\left(5(Ab+7aC)x^2+21aB\right)}{x^2\sqrt{a-bx^4}}dx}{6a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{2}{105}b\left(\frac{\int \frac{5(Ab+7aC)x^2+21aB}{x^2\sqrt{a-bx^4}}dx}{a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \ 1605$$

$$-\frac{2}{105}b\left(\frac{-\frac{\int -\frac{a\left(5(Ab+7aC)-21bBx^2\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{21B\sqrt{a-bx^4}}{x}}{a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \ 25$$

$$-\frac{2}{105}b\left(\frac{\frac{\int \frac{a\left(5(Ab+7aC)-21bBx^2\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{21B\sqrt{a-bx^4}}{x}}{a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{2}{105}b\left(\frac{\int \frac{5(Ab+7aC)-21bBx^2}{\sqrt{a-bx^4}}dx-\frac{21B\sqrt{a-bx^4}}{x}}{a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$$\Big\downarrow \ 1513$$

$$-\frac{2}{105}b\left(\frac{\left(21\sqrt{a}\sqrt{b}B+35aC+5Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx-21\sqrt{a}\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx-\frac{21B\sqrt{a-bx^4}}{x}}{a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$\downarrow$ 27

$$-\frac{2}{105}b\left(\frac{\left(21\sqrt{a}\sqrt{b}B+35aC+5Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx-21\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx-\frac{21B\sqrt{a-bx^4}}{x}}{a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$\downarrow$ 765

$$-\frac{2}{105}b\left(\frac{\frac{\sqrt{1-\frac{bx^4}{a}}\left(21\sqrt{a}\sqrt{b}B+35aC+5Ab\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-21\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx-\frac{21B\sqrt{a-bx^4}}{x}}{a}-\frac{5A\sqrt{a-bx^4}}{ax^3}\right)-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$\downarrow$ 762

$$-\frac{2}{105}b\left(\frac{-21\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(21\sqrt{a}\sqrt{b}B+35aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{21B\sqrt{a-bx^4}}{x}}{a}-5A\sqrt{\ldots}\right.$$
$$-\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$\downarrow$ 1390

$$-\frac{2}{105}b\left(\frac{-\frac{21\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(21\sqrt{a}\sqrt{b}B+35aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{21B\sqrt{a-bx^4}}{x}}{a}-\right.$$

$$\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$\downarrow$ 1389

$$-\frac{2}{105}b\left(\frac{-\frac{21\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(21\sqrt{a}\sqrt{b}B+35aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{21B\sqrt{a-bx^4}}{x}}{a}\right.$$

$$\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

$\downarrow$ 327

$$-\frac{2}{105}b\left(\frac{-\frac{21a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(21\sqrt{a}\sqrt{b}B+35aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-2}{a}\right.$$

$$\frac{1}{105}\sqrt{a-bx^4}\left(\frac{15A}{x^7}+\frac{21B}{x^5}+\frac{35C}{x^3}\right)$$

input

```
Int[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^8,x]
```

output

```
-1/105*(((15*A)/x^7 + (21*B)/x^5 + (35*C)/x^3)*Sqrt[a - b*x^4]) - (2*b*((-
5*A*Sqrt[a - b*x^4])/(a*x^3) + ((-21*B*Sqrt[a - b*x^4])/x - (21*a^(3/4)*b^
(1/4)*B*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sq
rt[a - b*x^4] + (a^(1/4)*(5*A*b + 21*Sqrt[a]*Sqrt[b]*B + 35*a*C)*Sqrt[1 -
(b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b
*x^4]))/a))/105
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.15

method	result
elliptic	$-\frac{A\sqrt{-b\,x^4+a}}{7x^7}-\frac{B\sqrt{-b\,x^4+a}}{5x^5}+\frac{(2Ab-7Ca)\sqrt{-b\,x^4+a}}{21a\,x^3}+\frac{2bB\sqrt{-b\,x^4+a}}{5ax}+\frac{\left(-Cb-\frac{b(2Ab-7Ca)}{21a}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{E}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
risch	$-\frac{\sqrt{-b\,x^4+a}\,(-42Bb\,x^6-10Ab\,x^4+35Ca\,x^4+21Ba\,x^2+15Aa)}{105x^7a}-\frac{2b\left(\frac{5Ab\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{35Ca\sqrt{1-}}{}\right.}{}$
default	$A\left(-\frac{\sqrt{-b\,x^4+a}}{7x^7}+\frac{2b\sqrt{-b\,x^4+a}}{21a\,x^3}-\frac{2b^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}\right)+B\left(-\frac{\sqrt{-b\,x^4+a}}{5x^5}+\frac{2b\sqrt{-b\,x^4}}{5ax}\right.$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*A*(-b*x^4+a)^(1/2)/x^7-1/5*B*(-b*x^4+a)^(1/2)/x^5+1/21*(2*A*b-7*C*a)/
a*(-b*x^4+a)^(1/2)/x^3+2/5*b*B*(-b*x^4+a)^(1/2)/a/x+(-C*b-1/21*b*(2*A*b-7*
C*a)/a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)
*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2)
,I)-2/5*B*b^(3/2)/a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(
1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \frac{42\,B\sqrt{a}bx^7\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)-2\,(35\,Ca+(5\,A+21\,B)b)\sqrt{a}x^7\left(\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1}{105\,ax^7}$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
1/105*(42*B*sqrt(a)*b*x^7*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)^(1/4)), -1
) - 2*(35*C*a + (5*A + 21*B)*b)*sqrt(a)*x^7*(b/a)^(3/4)*elliptic_f(arcsin(
x*(b/a)^(1/4)), -1) + (42*B*b*x^6 - 5*(7*C*a - 2*A*b)*x^4 - 21*B*a*x^2 - 1
5*A*a)*sqrt(-b*x^4 + a))/(a*x^7)
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{iB\sqrt{b}\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} + \frac{iC\sqrt{b}\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/x**8,x)
```

output
```
A*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(2*I*pi
)/a)/(4*x**7*gamma(-3/4)) + I*B*sqrt(b)*gamma(-3/4)*hyper((-1/2, 3/4), (7/
4,), a/(b*x**4))/(4*x**3*gamma(1/4)) + I*C*sqrt(b)*gamma(-1/4)*hyper((-1/2
, 1/4), (5/4,), a/(b*x**4))/(4*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx = \int \frac{\sqrt{a-b\,x^4}\,\left(C\,x^4+B\,x^2+A\right)}{x^8}\,dx$$

input
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^8,x)
```

output
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^8}\,dx$$
$$= \frac{-3\sqrt{-b\,x^4+a}\,ab-6\sqrt{-b\,x^4+a}\,ac-5\sqrt{-b\,x^4+a}\,b^2x^2-15\sqrt{-b\,x^4+a}\,bc\,x^4-6\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{12}+a\,x^8}\,dx\right)a^2b}{15b\,x^7}$$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^8,x)
```

output
```
( - 3*sqrt(a - b*x**4)*a*b - 6*sqrt(a - b*x**4)*a*c - 5*sqrt(a - b*x**4)*b
**2*x**2 - 15*sqrt(a - b*x**4)*b*c*x**4 - 6*int(sqrt(a - b*x**4)/(a*x**8 -
 b*x**12),x)*a**2*b*x**7 - 42*int(sqrt(a - b*x**4)/(a*x**8 - b*x**12),x)*a
**2*c*x**7 - 10*int(sqrt(a - b*x**4)/(a*x**6 - b*x**10),x)*a*b**2*x**7)/(1
5*b*x**7)
```

3.26 $\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 257

$$\begin{aligned}
&\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx \\
&= -\frac{1}{315}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)\sqrt{a-bx^4}+\frac{2Ab\sqrt{a-bx^4}}{45ax^5}+\frac{2bB\sqrt{a-bx^4}}{21ax^3} \\
&\quad+\frac{2b(Ab+3aC)\sqrt{a-bx^4}}{15a^2x}+\frac{2b^{5/4}(Ab+3aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15a^{5/4}\sqrt{a-bx^4}} \\
&\quad-\frac{2b^{5/4}\left(7Ab+5\sqrt{a}\sqrt{b}B+21aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{105a^{5/4}\sqrt{a-bx^4}}
\end{aligned}$$

output

```
-1/315*(35*A/x^9+45*B/x^7+63*C/x^5)*(-b*x^4+a)^(1/2)+2/45*A*b*(-b*x^4+a)^(
1/2)/a/x^5+2/21*b*B*(-b*x^4+a)^(1/2)/a/x^3+2/15*b*(A*b+3*C*a)*(-b*x^4+a)^(
1/2)/a^2/x+2/15*b^(5/4)*(A*b+3*C*a)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/
a^(1/4),I)/a^(5/4)/(-b*x^4+a)^(1/2)-2/105*b^(5/4)*(7*A*b+5*a^(1/2)*b^(1/2)
*B+21*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(5/4)/(-b*x^
4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx = -\frac{\sqrt{a-bx^4}\left(35A\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{1}{2},-\frac{5}{4},\frac{bx^4}{a}\right)+45Bx^2\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},\frac{bx^4}{a}\right.\right.}{315x^9\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^10,x]
```

output
```
-1/315*(Sqrt[a - b*x^4]*(35*A*Hypergeometric2F1[-9/4, -1/2, -5/4, (b*x^4)/
a] + 45*B*x^2*Hypergeometric2F1[-7/4, -1/2, -3/4, (b*x^4)/a] + 63*C*x^4*Hy
pergeometric2F1[-5/4, -1/2, -1/4, (b*x^4)/a]))/(x^9*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.607, Rules used = {2364, 27, 2374, 9, 27, 1605, 27, 1605, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}dx$$

$$\Big\downarrow \; 2364$$

$$2b\int -\frac{63Cx^4+45Bx^2+35A}{315x^6\sqrt{a-bx^4}}dx-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{2}{315}b\int \frac{63Cx^4+45Bx^2+35A}{x^6\sqrt{a-bx^4}}dx-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \text{2374}$$

$$-\frac{2}{315}b\left(-\frac{\int -\frac{30\left(7(Ab+3aC)x^3+15aBx\right)}{x^5\sqrt{a-bx^4}}dx}{10a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \text{9}$$

$$-\frac{2}{315}b\left(-\frac{\int -\frac{30\left(7(Ab+3aC)x^2+15aB\right)}{x^4\sqrt{a-bx^4}}dx}{10a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{315}b\left(\frac{3\int \frac{7(Ab+3aC)x^2+15aB}{x^4\sqrt{a-bx^4}}dx}{a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \text{1605}$$

$$-\frac{2}{315}b\left(\frac{3\left(-\frac{\int -\frac{3a\left(5bBx^2+7(Ab+3aC)\right)}{x^2\sqrt{a-bx^4}}dx}{3a}-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{315}b\left(\frac{3\left(\int \frac{5bBx^2+7(Ab+3aC)}{x^2\sqrt{a-bx^4}}dx-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \text{1605}$$

$$-\frac{2}{315}b\left(\frac{3\left(-\frac{\int -\frac{b\left(5aB-7(Ab+3aC)x^2\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$\downarrow$ 25

$$-\frac{2}{315}b\left(\frac{3\left(\frac{\int \frac{b\left(5aB-7(Ab+3aC)x^2\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$\downarrow$ 27

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\int \frac{5aB-7(Ab+3aC)x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-\frac{7A\sqrt{a-bx^4}}{ax^5}\right)-\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$\downarrow$ 1513

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\left(5\sqrt{a}\sqrt{b}B+21aC+7Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{7\sqrt{a}(3aC+Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-\frac{7A\sqrt{}}{a}\right.$$

$$\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$\downarrow$ 27

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\left(5\sqrt{a}\sqrt{b}B+21aC+7Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{7(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-\frac{7A\sqrt{a-}}{ax^5}\right.$$

$$\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \; 765$$

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+21aC+7Ab\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{7(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}-\frac{5B\sqrt{a-bx^4}}{x^3}\right)}{a}-7\right.$$

$$\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$$\Big\downarrow \; 762$$

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+21aC+7Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{7(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}-5B\right.}{a}\right.$$

$$\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$\downarrow$ 1390

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+21aC+7Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{7\sqrt{1-\frac{bx^4}{a}}(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{7\sqrt{a-bx^4}(3aC+Ab)}{ax}\right.}{a}\right.$$

$$\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$\downarrow$ 1389

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+21aC+7Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{7\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(3aC+Ab)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{7\sqrt{a-bx^4}(3aC\cdots}{ax}\right)}{a}\right.$$

$$\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

$\downarrow$ 327

$$-\frac{2}{315}b\left(\frac{3\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+21aC+7Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{7a^{3/4}\sqrt{1-\frac{bx^4}{a}}(3aC+Ab)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{7\sqrt{a-\cdots}}{\cdots}\right)}{a}\right.$$

$$\frac{1}{315}\sqrt{a-bx^4}\left(\frac{35A}{x^9}+\frac{45B}{x^7}+\frac{63C}{x^5}\right)$$

input
```
Int[(Sqrt[a - b*x^4]*(A + B*x^2 + C*x^4))/x^10,x]
```

output
```
-1/315*(((35*A)/x^9 + (45*B)/x^7 + (63*C)/x^5)*Sqrt[a - b*x^4]) - (2*b*((-
7*A*Sqrt[a - b*x^4])/(a*x^5) + (3*((-5*B*Sqrt[a - b*x^4])/x^3 - (7*(A*b +
3*a*C)*Sqrt[a - b*x^4])/(a*x) + (b*((-7*a^(3/4)*(A*b + 3*a*C)*Sqrt[1 - (b*
x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^
4]) + (a^(3/4)*(7*A*b + 5*Sqrt[a]*Sqrt[b]*B + 21*a*C)*Sqrt[1 - (b*x^4)/a]*
EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/a)
)/a))/315
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,(-42A\,b^{2}x^{8}-126Cab\,x^{8}-30Bb\,x^{6}a-14Aab\,x^{4}+63C\,a^{2}x^{4}+45B\,a^{2}x^{2}+35a^{2}A)}{315x^{9}a^{2}}+\frac{2b^{2}\left(-\frac{(7Ab+21Ca)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\cdots}{\cdots}\right.}{\cdots}$ …
elliptic	$-\frac{A\sqrt{-b\,x^{4}+a}}{9x^{9}}-\frac{B\sqrt{-b\,x^{4}+a}}{7x^{7}}+\frac{(2Ab-9Ca)\sqrt{-b\,x^{4}+a}}{45a\,x^{5}}+\frac{2bB\sqrt{-b\,x^{4}+a}}{21a\,x^{3}}+\frac{2b(Ab+3Ca)\sqrt{-b\,x^{4}+a}}{15a^{2}x}-\frac{2B\,b^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\cdots}{21\cdots}$ …
default	$A\left(-\frac{\sqrt{-b\,x^{4}+a}}{9x^{9}}+\frac{2b\sqrt{-b\,x^{4}+a}}{45a\,x^{5}}+\frac{2b^{2}\sqrt{-b\,x^{4}+a}}{15a^{2}x}-\frac{2b^{\frac{5}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\cdots\right.\right.}{15a^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right.$ …

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(-b*x^4+a)^(1/2)*(-42*A*b^2*x^8-126*C*a*b*x^8-30*B*a*b*x^6-14*A*a*b
*x^4+63*C*a^2*x^4+45*B*a^2*x^2+35*A*a^2)/x^9/a^2+2/105*b^2/a^2*(-(7*A*b+21
*C*a)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b
^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)
*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))-5*B*a/(1/a^(1
/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx$$
$$=\frac{42\,(3\,Cab+Ab^2)\sqrt{a}x^9\left(\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-6\,((5\,B+21\,C)ab+7\,Ab^2)\sqrt{a}x^9\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(\cdots\right.\right.}{315\cdots}$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^10,x, algorithm="fricas")
```

output
```
1/315*(42*(3*C*a*b + A*b^2)*sqrt(a)*x^9*(b/a)^(3/4)*elliptic_e(arcsin(x*(b
/a)^(1/4)), -1) - 6*((5*B + 21*C)*a*b + 7*A*b^2)*sqrt(a)*x^9*(b/a)^(3/4)*e
lliptic_f(arcsin(x*(b/a)^(1/4)), -1) + (30*B*a*b*x^6 + 42*(3*C*a*b + A*b^2
)*x^8 - 45*B*a^2*x^2 - 7*(9*C*a^2 - 2*A*a*b)*x^4 - 35*A*a^2)*sqrt(-b*x^4 +
 a))/(a^2*x^9)
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx = \frac{iA\sqrt{b}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{iC\sqrt{b}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)*(C*x**4+B*x**2+A)/x**10,x)
```

output
```
I*A*sqrt(b)*gamma(-7/4)*hyper((-1/2, 7/4), (11/4,), a/(b*x**4))/(4*x**7*ga
mma(-3/4)) + B*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp
_polar(2*I*pi)/a)/(4*x**7*gamma(-3/4)) + I*C*sqrt(b)*gamma(-3/4)*hyper((-1
/2, 3/4), (7/4,), a/(b*x**4))/(4*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^{10}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^10,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^10, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx = \int \frac{(Cx^4+Bx^2+A)\sqrt{-bx^4+a}}{x^{10}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^10,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx = \int \frac{\sqrt{a-b\,x^4}\,(C\,x^4+B\,x^2+A)}{x^{10}}\,dx$$

input
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^10,x)
```

output
```
int(((a - b*x^4)^(1/2)*(A + B*x^2 + C*x^4))/x^10, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}(A+Bx^2+Cx^4)}{x^{10}}\,dx$$
$$= \frac{-15\sqrt{-b\,x^4+a}\,ab-10\sqrt{-b\,x^4+a}\,ac-21\sqrt{-b\,x^4+a}\,b^2x^2-35\sqrt{-b\,x^4+a}\,bc\,x^4-30\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{14}+a\,x^{10}}dx\right.}{105b\,x^9}$$

input
```
int((-b*x^4+a)^(1/2)*(C*x^4+B*x^2+A)/x^10,x)
```

output
```
( - 15*sqrt(a - b*x**4)*a*b - 10*sqrt(a - b*x**4)*a*c - 21*sqrt(a - b*x**4
)*b**2*x**2 - 35*sqrt(a - b*x**4)*b*c*x**4 - 30*int(sqrt(a - b*x**4)/(a*x*
*10 - b*x**14),x)*a**2*b*x**9 - 90*int(sqrt(a - b*x**4)/(a*x**10 - b*x**14
),x)*a**2*c*x**9 - 42*int(sqrt(a - b*x**4)/(a*x**8 - b*x**12),x)*a*b**2*x*
*9)/(105*b*x**9)
```

3.27 $\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 211

$$\begin{aligned}\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx &= -\frac{(7Ab+5aC)x\sqrt{a-bx^4}}{21b^2} - \frac{Bx^3\sqrt{a-bx^4}}{5b} - \frac{Cx^5\sqrt{a-bx^4}}{7b} \\ &\quad + \frac{3a^{7/4}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}} \\ &\quad + \frac{a^{5/4}\left(35Ab-63\sqrt{a}\sqrt{b}B+25aC\right)\sqrt{1-\frac{bx^4}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{105b^{9/4}\sqrt{a-bx^4}}\end{aligned}$$

output

```
-1/21*(7*A*b+5*C*a)*x*(-b*x^4+a)^(1/2)/b^2-1/5*B*x^3*(-b*x^4+a)^(1/2)/b-1/
7*C*x^5*(-b*x^4+a)^(1/2)/b+3/5*a^(7/4)*B*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/
4)*x/a^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)+1/105*a^(5/4)*(35*A*b-63*a^(1/2)*
b^(1/2)*B+25*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(9/4)
/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = \frac{x(-a+bx^4)(35Ab+25aC+21bBx^2+15bCx^4)+5a(7Ab+5aC)x\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\right.}{105b^2\sqrt{a-bx^4}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/Sqrt[a - b*x^4],x]
```

output
```
(x*(-a + b*x^4)*(35*A*b + 25*a*C + 21*b*B*x^2 + 15*b*C*x^4) + 5*a*(7*A*b +
 5*a*C)*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a]
+ 21*a*b*B*x^3*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (b*x^4
)/a])/(105*b^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {2375, 25, 1603, 27, 1603, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \text{2375}$$

$$-\frac{\int -\frac{x^4(7bBx^2+7Ab+5aC)}{\sqrt{a-bx^4}}dx}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^4(7bBx^2+7Ab+5aC)}{\sqrt{a-bx^4}}dx}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \ 1603$$

$$\frac{\frac{\int \frac{bx^2\left(5(7Ab+5aC)x^2+21aB\right)}{\sqrt{a-bx^4}}dx}{5b}-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}\int \frac{x^2(5(7Ab+5aC)x^2+21aB)}{\sqrt{a-bx^4}}dx-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \ 1603$$

$$\frac{\frac{1}{5}\left(\frac{\int \frac{a\left(63bBx^2+5(7Ab+5aC)\right)}{\sqrt{a-bx^4}}dx}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}\left(\frac{a\int \frac{63bBx^2+5(7Ab+5aC)}{\sqrt{a-bx^4}}dx}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{1}{5}\left(\frac{a\left(\left(-63\sqrt{a}\sqrt{b}B+25aC+35Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+63\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx\right)}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{5}\left(\frac{a\left(\left(-63\sqrt{a}\sqrt{b}B+25aC+35Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+63\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{1}{5}\left(\frac{a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\left(-63\sqrt{a}\sqrt{b}B+25aC+35Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+63\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$\Big\downarrow$ 762

$$\frac{\frac{1}{5}\left(\frac{a\left(63\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-63\sqrt{a}\sqrt{b}B+25aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$\Big\downarrow$ 1390

$$\frac{\frac{1}{5}\left(\frac{a\left(\frac{63\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-63\sqrt{a}\sqrt{b}B+25aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)-\frac{7}{5}Bx^3\sqrt{a-bx^4}}{7b}-\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$\Big\downarrow$ 1389

$$\frac{\frac{1}{5}\left(\frac{a\left(\frac{63\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-63\sqrt{a}\sqrt{b}B+25aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{3b}-\frac{5x\sqrt{a-bx^4}(5aC+7Ab)}{3b}\right)}{7b}$$

$$\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

$\downarrow$ 327

$$\frac{\frac{1}{5}\left(\frac{a\left(\frac{63a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-63\sqrt{a}\sqrt{b}B+25aC+35Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)}{3b}-\frac{5x\sqrt{a-bx^4}(5aC}{3b}\right)}{7b}$$

$$\frac{Cx^5\sqrt{a-bx^4}}{7b}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/Sqrt[a - b*x^4],x]
```

output
```
-1/7*(C*x^5*Sqrt[a - b*x^4])/b + ((-7*B*x^3*Sqrt[a - b*x^4])/5 + ((-5*(7*A
*b + 5*a*C)*x*Sqrt[a - b*x^4])/(3*b) + (a*((63*a^(3/4)*b^(1/4)*B*Sqrt[1 -
(b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + (
a^(1/4)*(35*A*b - 63*Sqrt[a]*Sqrt[b]*B + 25*a*C)*Sqrt[1 - (b*x^4)/a]*Ellip
ticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])))/(3*b))/
5)/(7*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.09

method	result
elliptic	$-\frac{C x^5\sqrt{-b x^4+a}}{7b}-\frac{B x^3\sqrt{-b x^4+a}}{5b}-\frac{\left(A+\frac{5aC}{7b}\right)x\sqrt{-b x^4+a}}{3b}+\frac{\left(A+\frac{5aC}{7b}\right)a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}-$
risch	$-\frac{x(15Cb x^4+21Bb x^2+35Ab+25Ca)\sqrt{-b x^4+a}}{105b^2}+\frac{a\left(\frac{35Ab\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}+\frac{25Ca\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b}}\right.}{}$
default	$A\left(-\frac{x\sqrt{-b x^4+a}}{3b}+\frac{a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}\right)+B\left(-\frac{x^3\sqrt{-b x^4+a}}{5b}-\frac{3a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}}{}\right.$

input
```
int(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*C*x^5*(-b*x^4+a)^(1/2)/b-1/5*B*x^3*(-b*x^4+a)^(1/2)/b-1/3*(A+5/7*a*C/
b)/b*x*(-b*x^4+a)^(1/2)+1/3*(A+5/7*a*C/b)/b*a/(1/a^(1/2)*b^(1/2))^(1/2)*(1
-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)
*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3/5*B*a^(3/2)/b^(3/2)/(1/a^(1/2)
*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2
)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(
1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.60

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = -\frac{63\,Ba\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-((63\,B+25\,C)a+35\,Ab)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-}{105\,b^2x}$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(63*B*a*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1
) - ((63*B + 25*C)*a + 35*A*b)*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(arcsin((a
/b)^(1/4)/x), -1) + (15*C*b*x^6 + 21*B*b*x^4 + 5*(5*C*a + 7*A*b)*x^2 + 63*
B*a)*sqrt(-b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.60

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}+\frac{Cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(-b*x**4+a)**(1/2),x)
```

output
```
A*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4
*sqrt(a)*gamma(9/4)) + B*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4
*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(11/4)) + C*x**9*gamma(9/4)*hyper((1
/2, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^4}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx=\int \frac{x^4\,(C\,x^4+B\,x^2+A)}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(a - b*x^4)^(1/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = \frac{-35\sqrt{-b\,x^4+a}\,abx-25\sqrt{-b\,x^4+a}\,acx-21\sqrt{-b\,x^4+a}\,b^2x^3-15\sqrt{-b\,x^4+a}\,bc\,x^5+35\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right.}{105b^2}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x)
```

output
```
( - 35*sqrt(a - b*x**4)*a*b*x - 25*sqrt(a - b*x**4)*a*c*x - 21*sqrt(a - b*
x**4)*b**2*x**3 - 15*sqrt(a - b*x**4)*b*c*x**5 + 35*int(sqrt(a - b*x**4)/(
a - b*x**4),x)*a**2*b + 25*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a**2*c + 6
3*int((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)*a*b**2)/(105*b**2)
```

3.28 $\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 188

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = -\frac{Bx\sqrt{a-bx^4}}{3b}-\frac{Cx^3\sqrt{a-bx^4}}{5b} + \frac{a^{3/4}(5Ab+3aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}} - \frac{a^{3/4}\left(15Ab-5\sqrt{a}\sqrt{b}B+9aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{15b^{7/4}\sqrt{a-bx^4}}$$

output

```
-1/3*B*x*(-b*x^4+a)^(1/2)/b-1/5*C*x^3*(-b*x^4+a)^(1/2)/b+1/5*a^(3/4)*(5*A*
b+3*C*a)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(7/4)/(-b*x^4+
a)^(1/2)-1/15*a^(3/4)*(15*A*b-5*a^(1/2)*b^(1/2)*B+9*C*a)*(1-b*x^4/a)^(1/2)
*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = \frac{x(5B+3Cx^2)(-a+bx^4)+5aBx\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)+(5Ab+3aC)x^3\sqrt{1-}}{15b\sqrt{a-bx^4}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/Sqrt[a - b*x^4],x]
```

output
```
(x*(5*B + 3*C*x^2)*(-a + b*x^4) + 5*a*B*x*Sqrt[1 - (b*x^4)/a]*Hypergeometr
ic2F1[1/4, 1/2, 5/4, (b*x^4)/a] + (5*A*b + 3*a*C)*x^3*Sqrt[1 - (b*x^4)/a]*
Hypergeometric2F1[1/2, 3/4, 7/4, (b*x^4)/a])/(15*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2375, 25, 1603, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \text{2375}$$

$$-\frac{\int -\frac{x^2(5bBx^2+5Ab+3aC)}{\sqrt{a-bx^4}}dx}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2(5bBx^2+5Ab+3aC)}{\sqrt{a-bx^4}}dx}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{1603}$$

$$\frac{\frac{\int \frac{b\left(3(5Ab+3aC)x^2+5aB\right)}{\sqrt{a-bx^4}}dx}{3b}-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\int \frac{3(5Ab+3aC)x^2+5aB}{\sqrt{a-bx^4}}dx-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\frac{1}{3}\left(\sqrt{a}\left(5\sqrt{a}B-\frac{3(3aC+5Ab)}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{3\sqrt{a}(3aC+5Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\left(\sqrt{a}\left(5\sqrt{a}B-\frac{3(3aC+5Ab)}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{3(3aC+5Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{1}{3}\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{3(3aC+5Ab)}{\sqrt{b}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{3(3aC+5Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{1}{3}\left(\frac{3(3aC+5Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{3(3aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{1-\frac{bx^4}{a}}(3aC+5Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{3(3aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\frac{1}{3}\left(\frac{3\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(3aC+5Ab)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{3(3aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{1}{3}\left(\frac{3a^{3/4}\sqrt{1-\frac{bx^4}{a}}(3aC+5Ab)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}B-\frac{3(3aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{5}{3}Bx\sqrt{a-bx^4}}{5b}-\frac{Cx^3\sqrt{a-bx^4}}{5b}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/Sqrt[a - b*x^4],x]
```

output
```
-1/5*(C*x^3*Sqrt[a - b*x^4])/b + ((-5*B*x*Sqrt[a - b*x^4])/3 + ((3*a^(3/4)
*(5*A*b + 3*a*C)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)]
, -1])/(b^(3/4)*Sqrt[a - b*x^4]) + (a^(3/4)*(5*Sqrt[a]*B - (3*(5*A*b + 3*a
*C))/Sqrt[b])*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(b^(1/4)*Sqrt[a - b*x^4]))/3)/(5*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1603
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
 1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ
[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[
m])
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{x(3Cx^2+5B)\sqrt{-bx^4+a}}{15b}+\frac{-\frac{(15Ab+9Ca)\sqrt{a}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}\sqrt{b}}+\frac{5Ba\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \cdots}{\cdots}}{15b}$
elliptic	$-\frac{Cx^3\sqrt{-bx^4+a}}{5b}-\frac{Bx\sqrt{-bx^4+a}}{3b}+\frac{Ba\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}-\frac{\left(A+\frac{3aC}{5b}\right)\sqrt{a}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\ \cdots}{\sqrt{\cdots}}$
default	$-\frac{A\sqrt{a}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}\sqrt{b}}+B\left(-\frac{x\sqrt{-bx^4+a}}{3b}+\frac{a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\cdots}{\cdots}}\ \cdots}{3b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\cdots}\right.$

input
```
int(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(3*C*x^2+5*B)*(-b*x^4+a)^(1/2)/b+1/15/b*(-(15*A*b+9*C*a)*a^(1/2)/(
1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1
/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2)
,I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))+5*B*a/(1/a^(1/2)*b^(1/2))^(1
/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)
^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.65

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = -\frac{3\,(3\,Ca+5\,Ab)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}\,E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1) - (9\,Ca+5\,(3\,A+B)b)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}\,F(\arcsin\left(\frac{\left(\frac{a}{b}\right)}{x}\right.}{15\,b^2x}$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(3*(3*C*a + 5*A*b)*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1
/4)/x), -1) - (9*C*a + 5*(3*A + B)*b)*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(ar
csin((a/b)^(1/4)/x), -1) + (3*C*b*x^4 + 5*B*b*x^2 + 9*C*a + 15*A*b)*sqrt(-
b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = \frac{Ax^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Cx^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(-b*x**4+a)**(1/2),x)
```

output
```
A*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4
*sqrt(a)*gamma(7/4)) + B*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*
exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + C*x**7*gamma(7/4)*hyper((1/2
, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx=\int \frac{x^2\,(C\,x^4+B\,x^2+A)}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(a - b*x^4)^(1/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt{a-bx^4}}\,dx$$
$$=\frac{-5\sqrt{-b\,x^4+a}\,bx-3\sqrt{-b\,x^4+a}\,c\,x^3+5\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)ab+15\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx\right)ab+9\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}\right.}{15b}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a - b*x**4)*b*x - 3*sqrt(a - b*x**4)*c*x**3 + 5*int(sqrt(a - b*
x**4)/(a - b*x**4),x)*a*b + 15*int((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)
*a*b + 9*int((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)*a*c)/(15*b)
```

3.29 $\int \frac{A+Bx^2+Cx^4}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 153

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-bx^4}}\,dx = -\frac{Cx\sqrt{a-bx^4}}{3b}+\frac{a^{3/4}B\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\left(3Ab-3\sqrt{a}\sqrt{b}B+aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3b^{5/4}\sqrt{a-bx^4}}$$

output
```
-1/3*C*x*(-b*x^4+a)^(1/2)/b+a^(3/4)*B*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*
x/a^(1/4),I)/b^(3/4)/(-b*x^4+a)^(1/2)+1/3*a^(1/4)*(3*A*b-3*a^(1/2)*b^(1/2)
*B+C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(5/4)/(-b*x^4+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-bx^4}}\,dx = \frac{-aCx+bCx^5+(3Ab+aC)x\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)+bBx^3\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeom}}{3b\sqrt{a-bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/Sqrt[a - b*x^4],x]
```

output
```
(-(a*C*x) + b*C*x^5 + (3*A*b + a*C)*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F
1[1/4, 1/2, 5/4, (b*x^4)/a] + b*B*x^3*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F
1[1/2, 3/4, 7/4, (b*x^4)/a])/(3*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2427, 25, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \; 2427$$

$$-\frac{\int -\frac{3bBx^2+3Ab+aC}{\sqrt{a-bx^4}}dx}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3bBx^2+3Ab+aC}{\sqrt{a-bx^4}}dx}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$$\Big\downarrow \; 1513$$

$$\frac{\left(-3\sqrt{a}\sqrt{b}B+aC+3Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+3\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$\Big\downarrow$ 27

$$\frac{\left(-3\sqrt{a}\sqrt{b}B+aC+3Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$\Big\downarrow$ 765

$$\frac{\frac{\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B+aC+3Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$\Big\downarrow$ 762

$$\frac{3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$\Big\downarrow$ 1390

$$\frac{\frac{3\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$\Big\downarrow$ 1389

$$\frac{\frac{3\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

$\Big\downarrow$ 327

$$\frac{\frac{3a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-3\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}}{3b}-\frac{Cx\sqrt{a-bx^4}}{3b}$$

input
```
Int[(A + B*x^2 + C*x^4)/Sqrt[a - b*x^4],x]
```

output
```
-1/3*(C*x*Sqrt[a - b*x^4])/b + ((3*a^(3/4)*b^(1/4)*B*Sqrt[1 - (b*x^4)/a]*E
llipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + (a^(1/4)*(3*A
*b - 3*Sqrt[a]*Sqrt[b]*B + a*C)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1
/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4]))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.17

method	result
elliptic	$-\frac{C x \sqrt{-b\,x^{4}+a}}{3 b}+\frac{\left(A+\frac{a C}{3 b}\right)\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{B\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},\right.\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$
default	$\frac{A\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{B\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$
risch	$-\frac{C x \sqrt{-b\,x^{4}+a}}{3 b}+\frac{\frac{C a\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{3 A b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-3 B\sqrt{b}\,\sqrt{a}\,\sqrt{1-}}{3 b}$

input
```
int((C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*C*x*(-b*x^4+a)^(1/2)/b+(A+1/3*a*C/b)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(
1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*Ell
ipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-B*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*
(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/
2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2
)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-bx^4}}\,dx = -\frac{3\,Ba\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-((3\,B+C)a+3\,Ab)\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+ \cdots}{3\,abx}$$

input
```
integrate((C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*B*a*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1) -
 ((3*B + C)*a + 3*A*b)*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4
)/x), -1) + sqrt(-b*x^4 + a)*(C*a*x^2 + 3*B*a))/(a*b*x)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2+Cx^4}{\sqrt{a-bx^4}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}+\frac{Cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/(-b*x**4+a)**(1/2),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp
_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + C*x**5*gamma(5/4)*hyper((1/2, 5
/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{a - bx^4}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-bx^4 + a}}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{a - bx^4}}\, dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-bx^4 + a}}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{a - bx^4}}\,dx = \int \frac{C\,x^4 + B\,x^2 + A}{\sqrt{a - b\,x^4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a - b*x^4)^(1/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{\sqrt{a - bx^4}}\,dx = \frac{-\sqrt{-b\,x^4 + a}\,cx + 3\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)ab + \left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)ac + 3\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx\right)b^2}{3b}$$

input
```
int((C*x^4+B*x^2+A)/(-b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - b*x**4)*c*x + 3*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a*b + in
t(sqrt(a - b*x**4)/(a - b*x**4),x)*a*c + 3*int((sqrt(a - b*x**4)*x**2)/(a
- b*x**4),x)*b**2)/(3*b)
```

3.30 $\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 157

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx &= -\frac{A\sqrt{a-bx^4}}{ax}-\frac{(Ab-aC)\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}b^{3/4}\sqrt{a-bx^4}}\\&\quad+\frac{\left(Ab+\sqrt{a}\sqrt{b}B-aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}b^{3/4}\sqrt{a-bx^4}}\end{aligned}$$

output
```
-A*(-b*x^4+a)^(1/2)/a/x-(A*b-C*a)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^
(1/4),I)/a^(1/4)/b^(3/4)/(-b*x^4+a)^(1/2)+(A*b+a^(1/2)*b^(1/2)*B-C*a)*(1-b
*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(1/4)/b^(3/4)/(-b*x^4+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx$$
$$=\frac{\sqrt{1-\frac{bx^4}{a}}\left(-3A\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{bx^4}{a}\right)+3Bx^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)+Cx^4\,\text{H}\right.}{3x\sqrt{a-bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[a - b*x^4]),x]
```

output
```
(Sqrt[1 - (b*x^4)/a]*(-3*A*Hypergeometric2F1[-1/4, 1/2, 3/4, (b*x^4)/a] +
3*B*x^2*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a] + C*x^4*Hypergeometric
2F1[1/2, 3/4, 7/4, (b*x^4)/a]))/(3*x*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2374, 9, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \; 2374$$

$$-\frac{\int -\frac{2\left(aBx-(Ab-aC)x^3\right)}{x\sqrt{a-bx^4}}dx}{2a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 9$$

$$-\frac{\int -\frac{2\left(aB-(Ab-aC)x^2\right)}{\sqrt{a-bx^4}}dx}{2a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{aB-(Ab-aC)x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 1513$$

$$\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}(Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{(Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{(Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{(Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 1390$$

$$\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{1-\frac{bx^4}{a}}(Ab-aC)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 1389$$

$$\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(Ab-aC)\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}(Ab-aC)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{a}-\frac{A\sqrt{a-bx^4}}{ax}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*Sqrt[a - b*x^4]),x]
```

output
```
-((A*Sqrt[a - b*x^4])/(a*x)) + (-((a^(3/4)*(A*b - a*C)*Sqrt[1 - (b*x^4)/a]
*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])) +
(a^(3/4)*(A*b + Sqrt[a]*Sqrt[b]*B - a*C)*Sqrt[1 - (b*x^4)/a]*EllipticF[Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]))/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

method	result
elliptic	$-\frac{A\sqrt{-b\,x^{4}+a}}{a x}+\frac{B\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{\left(C-\frac{Ab}{a}\right)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$
risch	$-\frac{A\sqrt{-b\,x^{4}+a}}{a x}-\frac{-\frac{(Ab-Ca)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}-\frac{Ba\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{Elliptic}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}}{a}$
default	$\frac{B\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+A\left(-\frac{\sqrt{-b\,x^{4}+a}}{a x}+\frac{\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{Elli}\right.}{\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right.$

input
```
int((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(-b*x^4+a)^(1/2)/a/x+B/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1
/2)*b^(1/2))^(1/2),I)-(C-A/a*b)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/
2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/
2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2
))^(1/2),I))
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-bx^4+a}\,x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-(C*x^4 + B*x^2 + A)*sqrt(-b*x^4 + a)/(b*x^6 - a*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{Bx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(-b*x**4+a)**(1/2),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*x*gamma(3/4)) + B*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_
polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + C*x**3*gamma(3/4)*hyper((1/2, 3/
4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{-bx^4+a}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-bx^4+ax^2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-b*x^4 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\sqrt{a-b\,x^4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a - b*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a - b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a-bx^4}}\,dx=\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^6+a\,x^2}dx\right)a+\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)b+\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a*x**2 - b*x**6),x)*a + int(sqrt(a - b*x**4)/(a - b*
x**4),x)*b + int((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)*c
```

3.31 $\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx = -\frac{A\sqrt{a-bx^4}}{3ax^3}-\frac{B\sqrt{a-bx^4}}{ax}-\frac{\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}+\frac{\left(Ab+3\sqrt{a}\sqrt{b}B+3aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}\sqrt[4]{b}\sqrt{a-bx^4}}$$

output
```
-1/3*A*(-b*x^4+a)^(1/2)/a/x^3-B*(-b*x^4+a)^(1/2)/a/x-b^(1/4)*B*(1-b*x^4/a)
^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/a^(1/4)/(-b*x^4+a)^(1/2)+1/3*(A*b+3*
a^(1/2)*b^(1/2)*B+3*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/
a^(3/4)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx$$
$$= \frac{\sqrt{1-\frac{bx^4}{a}}\left(-A\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\frac{bx^4}{a}\right)-3Bx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{bx^4}{a}\right)+3Cx^4\right.}{3x^3\sqrt{a-bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*Sqrt[a - b*x^4]),x]
```

output
```
(Sqrt[1 - (b*x^4)/a]*(-(A*Hypergeometric2F1[-3/4, 1/2, 1/4, (b*x^4)/a]) -
3*B*x^2*Hypergeometric2F1[-1/4, 1/2, 3/4, (b*x^4)/a] + 3*C*x^4*Hypergeomet
ric2F1[1/4, 1/2, 5/4, (b*x^4)/a]))/(3*x^3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {2374, 9, 27, 1605, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \; 2374$$

$$-\frac{\int -\frac{2\left((Ab+3aC)x^3+3aBx\right)}{x^3\sqrt{a-bx^4}}dx}{6a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \; 9$$

$$-\frac{\int -\frac{2\left((Ab+3aC)x^2+3aB\right)}{x^2\sqrt{a-bx^4}}dx}{6a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(Ab+3aC)x^2+3aB}{x^2\sqrt{a-bx^4}}dx}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 1605$$

$$\frac{-\frac{\int -\frac{a\left(-3bBx^2+Ab+3aC\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a\left(-3bBx^2+Ab+3aC\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-3bBx^2+Ab+3aC}{\sqrt{a-bx^4}}dx-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 1513$$

$$\frac{\left(3\sqrt{a}\sqrt{b}B+3aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx-3\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(3\sqrt{a}\sqrt{b}B+3aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx-3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{\sqrt{1-\frac{bx^4}{a}}\left(3\sqrt{a}\sqrt{b}B+3aC+Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 762$$

$$\frac{-3\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(3\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \ 1390$$

$$\frac{-\frac{3\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(3\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{1389}$$

$$\frac{-\frac{3\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(3\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{327}$$

$$\frac{-\frac{3a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(3\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{3B\sqrt{a-bx^4}}{x}}{3a}-\frac{A\sqrt{a-bx^4}}{3ax^3}$$

input

```
Int[(A + B*x^2 + C*x^4)/(x^4*Sqrt[a - b*x^4]),x]
```

output

```
-1/3*(A*Sqrt[a - b*x^4])/(a*x^3) + ((-3*B*Sqrt[a - b*x^4])/x - (3*a^(3/4)*
b^(1/4)*B*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/
Sqrt[a - b*x^4] + (a^(1/4)*(A*b + 3*Sqrt[a]*Sqrt[b]*B + 3*a*C)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x
^4]))/(3*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

method	result
elliptic	$-\frac{A\sqrt{-b\,x^{4}+a}}{3a\,x^{3}}-\frac{B\sqrt{-b\,x^{4}+a}}{ax}+\frac{\left(C+\frac{Ab}{3a}\right)\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{\sqrt{b}\,B\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,(\mathrm{Elli}}{\sqrt{a}\,\sqrt{}}$
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,(3B\,x^{2}+A)}{3a\,x^{3}}+\frac{\frac{Ab\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{3Ca\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{3B\sqrt{b}\,\sqrt{}}{}}{3a}$
default	$\frac{C\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+A\left(-\frac{\sqrt{-b\,x^{4}+a}}{3a\,x^{3}}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right)+B$

input
```
int((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(-b*x^4+a)^(1/2)/a/x^3-B*(-b*x^4+a)^(1/2)/a/x+(C+1/3*A/a*b)/(1/a^(1
/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+b^(1/2)/a^(
1/2)*B/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*
x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2)
,I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx = -\frac{3\,B\sqrt{a}bx^3\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1) - (3\,Ca+(A+3\,B)b)\sqrt{a}x^3\left(\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1) + }{3\,abx^3}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(3*B*sqrt(a)*b*x^3*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)^(1/4)), -1)
- (3*C*a + (A + 3*B)*b)*sqrt(a)*x^3*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^
(1/4)), -1) + sqrt(-b*x^4 + a)*(3*B*b*x^2 + A*b))/(a*b*x^3)
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx = \frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{B\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{Cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(-b*x**4+a)**(1/2),x)
```

output
```
A*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sq
rt(a)*x**3*gamma(1/4)) + B*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*e
xp_polar(2*I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) + C*x*gamma(1/4)*hyper((1/4,
1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-bx^4+a}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{-bx^4+a}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,\sqrt{a-b\,x^4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a - b*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a - b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt{a-bx^4}}\,dx = \left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^8+a\,x^4}dx\right)a+\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^6+a\,x^2}dx\right)b + \left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a*x**4 - b*x**8),x)*a + int(sqrt(a - b*x**4)/(a*x**2
 - b*x**6),x)*b + int(sqrt(a - b*x**4)/(a - b*x**4),x)*c
```

3.32 $\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 221

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-bx^4}}\,dx &= -\frac{A\sqrt{a-bx^4}}{5ax^5}-\frac{B\sqrt{a-bx^4}}{3ax^3}-\frac{(3Ab+5aC)\sqrt{a-bx^4}}{5a^2x} \\ &\quad -\frac{\sqrt[4]{b}(3Ab+5aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}} \\ &\quad +\frac{\sqrt[4]{b}\left(9Ab+5\sqrt{a}\sqrt{b}B+15aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{15a^{5/4}\sqrt{a-bx^4}}\end{aligned}$$

output

```
-1/5*A*(-b*x^4+a)^(1/2)/a/x^5-1/3*B*(-b*x^4+a)^(1/2)/a/x^3-1/5*(3*A*b+5*C*
a)*(-b*x^4+a)^(1/2)/a^2/x-1/5*b^(1/4)*(3*A*b+5*C*a)*(1-b*x^4/a)^(1/2)*Elli
pticE(b^(1/4)*x/a^(1/4),I)/a^(5/4)/(-b*x^4+a)^(1/2)+1/15*b^(1/4)*(9*A*b+5*
a^(1/2)*b^(1/2)*B+15*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)
/a^(5/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-bx^4}}\,dx =$$
$$-\frac{\sqrt{1-\frac{bx^4}{a}}\left(3A\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\frac{bx^4}{a}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\frac{bx^4}{a}\right)+15\right.}{15x^5\sqrt{a-bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*Sqrt[a - b*x^4]),x]
```

output
```
-1/15*(Sqrt[1 - (b*x^4)/a]*(3*A*Hypergeometric2F1[-5/4, 1/2, -1/4, (b*x^4)
/a] + 5*B*x^2*Hypergeometric2F1[-3/4, 1/2, 1/4, (b*x^4)/a] + 15*C*x^4*Hype
rgeometric2F1[-1/4, 1/2, 3/4, (b*x^4)/a]))/(x^5*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2374, 9, 27, 1605, 25, 27, 1605, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \text{2374}$$

$$-\frac{\int -\frac{2\left((3Ab+5aC)x^3+5aBx\right)}{x^5\sqrt{a-bx^4}}dx}{10a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{2\left((3Ab+5aC)x^2+5aB\right)}{x^4\sqrt{a-bx^4}}dx}{10a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(3Ab+5aC)x^2+5aB}{x^4\sqrt{a-bx^4}}dx}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 1605$$

$$\frac{-\frac{\int -\frac{a\left(5bBx^2+3(3Ab+5aC)\right)}{x^2\sqrt{a-bx^4}}dx}{3a}-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a\left(5bBx^2+3(3Ab+5aC)\right)}{x^2\sqrt{a-bx^4}}dx}{3a}-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}\int \frac{5bBx^2+3(3Ab+5aC)}{x^2\sqrt{a-bx^4}}dx-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 1605$$

$$\frac{\frac{1}{3}\left(-\frac{\int -\frac{b\left(5aB-3(3Ab+5aC)x^2\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{3}\left(\frac{\int \frac{b\left(5aB-3(3Ab+5aC)x^2\right)}{\sqrt{a-bx^4}}dx}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}\left(\frac{b\int \frac{5aB-3(3Ab+5aC)x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{1}{3}\left(\frac{b\left(\frac{\sqrt{a}\left(5\sqrt{a}\sqrt{b}B+15aC+9Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{3\sqrt{a}(5aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\left(\frac{b\left(\frac{\sqrt{a}\left(5\sqrt{a}\sqrt{b}B+15aC+9Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{3(5aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{765}$$

$$\frac{\frac{1}{3}\left(\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+15aC+9Ab\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{3(5aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-\frac{5B\sqrt{a-bx^4}}{3x^3}}{5a}-\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{762}$$

$$\frac{\frac{1}{3}\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+15aC+9Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{3(5aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-\frac{5B\sqrt{a-b}}{3x^3}}{5a}$$

$$\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$\downarrow$ 1390

$$\frac{\frac{1}{3}\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+15aC+9Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{3\sqrt{1-\frac{bx^4}{a}}(5aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)-5}{5a}$$

$$\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$\downarrow$ 1389

$$\frac{\frac{1}{3}\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+15aC+9Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{3\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(5aC+3Ab)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{3\sqrt{a-bx^4}(5aC+3Ab)}{ax}\right)}{5a}$$

$$\frac{A\sqrt{a-bx^4}}{5ax^5}$$

$\downarrow$ 327

$$\frac{\frac{1}{3}\left(\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(5\sqrt{a}\sqrt{b}B+15aC+9Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{3a^{3/4}\sqrt{1-\frac{bx^4}{a}}(5aC+3Ab)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{3\sqrt{a-bx^4}(5aC-}{ax}\right.}{5a}$$

$$\frac{A\sqrt{a-bx^4}}{5ax^5}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*Sqrt[a - b*x^4]),x]
```

output
```
-1/5*(A*Sqrt[a - b*x^4])/(a*x^5) + ((-5*B*Sqrt[a - b*x^4])/(3*x^3) + ((-3*
(3*A*b + 5*a*C)*Sqrt[a - b*x^4])/(a*x) + (b*((-3*a^(3/4)*(3*A*b + 5*a*C)*S
qrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sq
rt[a - b*x^4]) + (a^(3/4)*(9*A*b + 5*Sqrt[a]*Sqrt[b]*B + 15*a*C)*Sqrt[1 -
(b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b
*x^4])))/a)/3)/(5*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1605
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + S
imp[1/(a*f^2*(m + 1))   Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*
(m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
 IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{-b\,x^4+a}\,(9Ab\,x^4+15Ca\,x^4+5Ba\,x^2+3Aa)}{15a^2x^5}-\frac{b\left(-\frac{(9Ab+15Ca)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{}}{\sqrt{}}}\right.\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,\sqrt{b}}\right.}{15a^2}$
elliptic	$-\frac{A\sqrt{-b\,x^4+a}}{5a\,x^5}-\frac{B\sqrt{-b\,x^4+a}}{3a\,x^3}-\frac{(3Ab+5Ca)\sqrt{-b\,x^4+a}}{5a^2x}+\frac{bB\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}+\frac{(3Ab+5Ca)\sqrt{}}{}$
default	$A\left(-\frac{\sqrt{-b\,x^4+a}}{5a\,x^5}-\frac{3b\sqrt{-b\,x^4+a}}{5a^2x}+\frac{3b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}\right)+B\left(-\right.$

input
```
int((C*x^4+B*x^2+A)/x^6/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-b*x^4+a)^(1/2)*(9*A*b*x^4+15*C*a*x^4+5*B*a*x^2+3*A*a)/a^2/x^5-1/15
*b/a^2*(-(9*A*b+15*C*a)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a
^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(Elli
pticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2)
,I))-5*B*a/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1
/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1
/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.55

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a - bx^4}}\,dx = -\frac{3\,(5\,Ca + 3\,Ab)\sqrt{a}x^5\left(\frac{b}{a}\right)^{\frac{3}{4}} E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1) - (5\,(B + 3\,C)a + 9\,Ab)\sqrt{a}x^5\left(\frac{b}{a}\right)^{\frac{3}{4}} F(\arcsin\left(x\left(\frac{b}{a}\right.\right.}{15\,a^2x^5}$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(3*(5*C*a + 3*A*b)*sqrt(a)*x^5*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)
^(1/4)), -1) - (5*(B + 3*C)*a + 9*A*b)*sqrt(a)*x^5*(b/a)^(3/4)*elliptic_f(
arcsin(x*(b/a)^(1/4)), -1) + (3*(5*C*a + 3*A*b)*x^4 + 5*B*a*x^2 + 3*A*a)*s
qrt(-b*x^4 + a))/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.58

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a - bx^4}}\,dx = -\frac{iA\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\,\frac{a}{bx^4}\right)}{4\sqrt{b}x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{B\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4}, \frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\,\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{C\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4}, \frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\,\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(-b*x**4+a)**(1/2),x)
```

output
```
-I*A*gamma(-7/4)*hyper((1/2, 7/4), (11/4,), a/(b*x**4))/(4*sqrt(b)*x**7*ga
mma(-3/4)) + B*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(2*I
*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + C*gamma(-1/4)*hyper((-1/4, 1/2), (3/
4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a - bx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-bx^4 + a}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-b*x^4 + a)*x^6), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a - bx^4}}\,dx = \int \frac{Cx^4 + Bx^2 + A}{\sqrt{-bx^4 + a}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(-b*x^4 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{x^6\sqrt{a - bx^4}}\,dx = \int \frac{C\,x^4 + B\,x^2 + A}{x^6\,\sqrt{a - b\,x^4}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(a - b*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(a - b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt{a-bx^4}}\,dx = \left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{10}+a\,x^6}dx\right)a+\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^8+a\,x^4}dx\right)b + \left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^6+a\,x^2}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^6/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a*x**6 - b*x**10),x)*a + int(sqrt(a - b*x**4)/(a*x**
4 - b*x**8),x)*b + int(sqrt(a - b*x**4)/(a*x**2 - b*x**6),x)*c
```

3.33 $\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 223

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{x(aB+(Ab+aC)x^2)}{2b^2\sqrt{a-bx^4}}+\frac{Bx\sqrt{a-bx^4}}{3b^2} + \frac{Cx^3\sqrt{a-bx^4}}{5b^2}-\frac{3a^{3/4}(5Ab+7aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{10b^{11/4}\sqrt{a-bx^4}} + \frac{a^{3/4}\left(45Ab-25\sqrt{a}\sqrt{b}B+63aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{30b^{11/4}\sqrt{a-bx^4}}$$

output
```
1/2*x*(a*B+(A*b+C*a)*x^2)/b^2/(-b*x^4+a)^(1/2)+1/3*B*x*(-b*x^4+a)^(1/2)/b^
2+1/5*C*x^3*(-b*x^4+a)^(1/2)/b^2-3/10*a^(3/4)*(5*A*b+7*C*a)*(1-b*x^4/a)^(1
/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(11/4)/(-b*x^4+a)^(1/2)+1/30*a^(3/4)*
(45*A*b-25*a^(1/2)*b^(1/2)*B+63*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x
/a^(1/4),I)/b^(11/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.62

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{25aBx-30Abx^3-42aCx^3-10bBx^5-6bCx^7-25aBx\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeo}}{30}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x]
```

output
```
(25*a*B*x - 30*A*b*x^3 - 42*a*C*x^3 - 10*b*B*x^5 - 6*b*C*x^7 - 25*a*B*x*Sq
rt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a] + 6*(5*A*b +
 7*a*C)*x^3*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (b*x^4)/a
])/(30*b^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2367, 25, 2427, 25, 2427, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2367}$$

$$\frac{\int -\frac{2ab^2Cx^6+2ab^2Bx^4+3ab(Ab+aC)x^2+a^2bB}{\sqrt{a-bx^4}}dx}{2ab^3}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\int \frac{2ab^2Cx^6+2ab^2Bx^4+3ab(Ab+aC)x^2+a^2bB}{\sqrt{a-bx^4}}dx}{2ab^3}$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{-\frac{\int -\frac{10ab^3Bx^4+3ab^2(5Ab+7aC)x^2+5a^2b^2B}{\sqrt{a-bx^4}}dx}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$\downarrow$ 25

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\int \frac{10ab^3Bx^4+3ab^2(5Ab+7aC)x^2+5a^2b^2B}{\sqrt{a-bx^4}}dx}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$\downarrow$ 2427

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{-\frac{\int -\frac{ab^3\left(9(5Ab+7aC)x^2+25aB\right)}{\sqrt{a-bx^4}}dx}{3b}-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$\downarrow$ 25

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{\int \frac{ab^3\left(9(5Ab+7aC)x^2+25aB\right)}{\sqrt{a-bx^4}}dx}{3b}-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$\downarrow$ 27

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\int \frac{9(5Ab+7aC)x^2+25aB}{\sqrt{a-bx^4}}dx-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$\downarrow$ 1513

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\left(\sqrt{a}\left(25\sqrt{a}B-\frac{9(7aC+5Ab)}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{9\sqrt{a}(7aC+5Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$\downarrow$ 27

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\left(\sqrt{a}\left(25\sqrt{a}B-\frac{9(7aC+5Ab)}{\sqrt{b}}\right)\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{9(7aC+5Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$\downarrow$ 765

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(25\sqrt{a}B-\frac{9(7aC+5Ab)}{\sqrt{b}}\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{9(7aC+5Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx^3\sqrt{a-bx^4}}{2ab^3}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\left(\frac{9(7aC+5Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(25\sqrt{a}B-\frac{9(7aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx^3\sqrt{a-\cdots}}{2ab^3}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\left(\frac{9\sqrt{1-\frac{bx^4}{a}}(7aC+5Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(25\sqrt{a}B-\frac{9(7aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}abCx\cdots}{2ab^3}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\left(\frac{9\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(7aC+5Ab)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(25\sqrt{a}B-\frac{9(7aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}}{5b}-\frac{2}{5}ab\cdots}{2ab^3}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\frac{1}{3}ab^2\left(\frac{9a^{3/4}\sqrt{1-\frac{bx^4}{a}}(7aC+5Ab)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}+\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(25\sqrt{a}B-\frac{9(7aC+5Ab)}{\sqrt{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{10}{3}ab^2Bx\sqrt{a-bx^4}\cdots}{5b}\cdots}{2ab^3}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x]
```

output
```
(x*(a*B + (A*b + a*C)*x^2))/(2*b^2*Sqrt[a - b*x^4]) - ((-2*a*b*C*x^3*Sqrt[
a - b*x^4])/5 + ((-10*a*b^2*B*x*Sqrt[a - b*x^4])/3 + (a*b^2*((9*a^(3/4)*(5
*A*b + 7*a*C)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(b^(3/4)*Sqrt[a - b*x^4]) + (a^(3/4)*(25*Sqrt[a]*B - (9*(5*A*b + 7*a*C
))/Sqrt[b])*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(b^(1/4)*Sqrt[a - b*x^4])))/3)/(5*b))/(2*a*b^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 6.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{2b\left(\frac{(Ab+Ca)x^3}{4b^3}+\frac{Bax}{4b^3}\right)}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{C\,x^3\sqrt{-b\,x^4+a}}{5b^2}+\frac{Bx\sqrt{-b\,x^4+a}}{3b^2}-\frac{5Ba\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{\left(-\frac{3(Ab+}{2b^2}\right.}{}$
default	$A\left(\frac{x^3}{2b\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{3\sqrt{a}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}\right)+B\left(\frac{ax}{2b^2\sqrt{-\left(x^4-\frac{a}{b}\right)}}\right.$
risch	$\frac{x(3C\,x^2+5B)\sqrt{-b\,x^4+a}}{15b^2}-\frac{\frac{20Ba\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}-\frac{15A\sqrt{b}\sqrt{a}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}}{}$

input
```
int(x^6*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(1/4/b^3*(A*b+C*a)*x^3+1/4/b^3*B*a*x)/(-(x^4-a/b)*b)^(1/2)+1/5*C*x^3*(
-b*x^4+a)^(1/2)/b^2+1/3*B*x*(-b*x^4+a)^(1/2)/b^2-5/6*B*a/b^2/(1/a^(1/2)*b^
(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(
-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-(-3/2*(A*b+C*a)/b
^2-3/5*C/b^2*a)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^
(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*
(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{9\left((7\,Cab+5\,Ab^2)x^5-(7\,Ca^2+5\,Aab)x\right)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-}{}$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/30*(9*((7*C*a*b + 5*A*b^2)*x^5 - (7*C*a^2 + 5*A*a*b)*x)*sqrt(-b)*(a/b)^(
3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1) - ((63*C*a*b + 5*(9*A + 5*B)*b^
2)*x^5 - (63*C*a^2 + 5*(9*A + 5*B)*a*b)*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_f
(arcsin((a/b)^(1/4)/x), -1) + (6*C*b^2*x^8 + 10*B*b^2*x^6 - 25*B*a*b*x^2 +
 6*(7*C*a*b + 5*A*b^2)*x^4 - 63*C*a^2 - 45*A*a*b)*sqrt(-b*x^4 + a))/(b^4*x
^5 - a*b^3*x)
```

Sympy [A] (verification not implemented)

Time = 10.65 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.57

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{Ax^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{Bx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)} + \frac{Cx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(C*x**4+B*x**2+A)/(-b*x**4+a)**(3/2),x)
```

output
```
A*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(
4*a**(3/2)*gamma(11/4)) + B*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x
**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(13/4)) + C*x**11*gamma(11/4)*hy
per((3/2, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(15
/4))
```

Maxima [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^6}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^6}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^6/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^6\,(C\,x^4+B\,x^2+A)}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{-15\sqrt{-b\,x^4+a}\,ab\,x^3+25\sqrt{-b\,x^4+a}\,abx-21\sqrt{-b\,x^4+a}\,ac\,x^3-5\sqrt{-b\,x^4}}{}$$

input
```
int(x^6*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x)
```

output
```
( - 15*sqrt(a - b*x**4)*a*b*x**3 + 25*sqrt(a - b*x**4)*a*b*x - 21*sqrt(a -
 b*x**4)*a*c*x**3 - 5*sqrt(a - b*x**4)*b**2*x**5 - 3*sqrt(a - b*x**4)*b*c*
x**7 - 25*int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**3*b +
 25*int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2*x**4
 + 45*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**3*
b + 63*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**3
*c - 45*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**
2*b**2*x**4 - 63*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**
8),x)*a**2*b*c*x**4)/(15*b**2*(a - b*x**4))
```

3.34 $\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 190

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{x(Ab+aC+bBx^2)}{2b^2\sqrt{a-bx^4}} + \frac{Cx\sqrt{a-bx^4}}{3b^2} - \frac{3a^{3/4}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2b^{7/4}\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\left(3Ab-9\sqrt{a}\sqrt{b}B+5aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{6b^{9/4}\sqrt{a-bx^4}}$$

output
```
1/2*x*(B*b*x^2+A*b+C*a)/b^2/(-b*x^4+a)^(1/2)+1/3*C*x*(-b*x^4+a)^(1/2)/b^2-
3/2*a^(3/4)*B*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(7/4)/(-b
*x^4+a)^(1/2)-1/6*a^(1/4)*(3*A*b-9*a^(1/2)*b^(1/2)*B+5*C*a)*(1-b*x^4/a)^(1
/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(9/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{3Abx+5aCx-6bBx^3-2bCx^5-(3Ab+5aC)x\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2}}{6b^2\sqrt{a-b}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x]
```

output
```
(3*A*b*x + 5*a*C*x - 6*b*B*x^3 - 2*b*C*x^5 - (3*A*b + 5*a*C)*x*Sqrt[1 - (b
*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a] + 6*b*B*x^3*Sqrt[1 -
(b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (b*x^4)/a])/(6*b^2*Sqrt[a - b*
x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2367, 2427, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2367}$$

$$\frac{x(aC+Ab+bBx^2)}{2b^2\sqrt{a-bx^4}}-\frac{\int \frac{2abCx^4+3abBx^2+a(Ab+aC)}{\sqrt{a-bx^4}}dx}{2ab^2}$$

$$\Big\downarrow \text{2427}$$

$$\frac{x(aC+Ab+bBx^2)}{2b^2\sqrt{a-bx^4}}-\frac{-\frac{\int -\frac{ab(9bBx^2+3Ab+5aC)}{\sqrt{a-bx^4}}dx}{3b}-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \; 25$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{\int \frac{ab\left(9bBx^2+3Ab+5aC\right)}{\sqrt{a-bx^4}}dx}{3b}-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\int \frac{9bBx^2+3Ab+5aC}{\sqrt{a-bx^4}}dx-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \; 1513$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\left(\left(-9\sqrt{a}\sqrt{b}B+5aC+3Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+9\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx\right)-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\left(\left(-9\sqrt{a}\sqrt{b}B+5aC+3Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx+9\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \; 765$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\left(-9\sqrt{a}\sqrt{b}B+5aC+3Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+9\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \; 762$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\left(9\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-9\sqrt{a}\sqrt{b}B+5aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \; 1390$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\left(\frac{9\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-9\sqrt{a}\sqrt{b}B+5aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \text{1389}$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\left(\frac{9\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-9\sqrt{a}\sqrt{b}B+5aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{2}{3}aCx\sqrt{a-bx^4}}{2ab^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2b^2\sqrt{a-bx^4}}-\frac{\frac{1}{3}a\left(\frac{9a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(-9\sqrt{a}\sqrt{b}B+5aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}\right)-\frac{2}{3}aCx}{2ab^2}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x]
```

output
```
(x*(A*b + a*C + b*B*x^2))/(2*b^2*Sqrt[a - b*x^4]) - ((-2*a*C*x*Sqrt[a - b*
x^4])/3 + (a*((9*a^(3/4)*b^(1/4)*B*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b
^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^4] + (a^(1/4)*(3*A*b - 9*Sqrt[a]*Sqr
t[b]*B + 5*a*C)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(b^(1/4)*Sqrt[a - b*x^4])))/3)/(2*a*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{2b\left(\frac{B\,x^{3}}{4b^{2}}+\frac{(Ab+Ca)x}{4b^{3}}\right)}{\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{Cx\sqrt{-b\,x^{4}+a}}{3b^{2}}+\frac{\left(-\frac{Ab+Ca}{2b^{2}}-\frac{Ca}{3b^{2}}\right)\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{3B\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\cdots}{\cdots}$
default	$A\left(\frac{x}{2b\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right)+B\left(\frac{x^{3}}{2b\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{3\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\cdots}{\cdots}\right.$
risch	$\frac{Cx\sqrt{-b\,x^{4}+a}}{3b^{2}}-\frac{\frac{3Ab\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{4Ca\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{3B\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\cdots}{\cdots}}{\cdots}$

input
```
int(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(1/4*B*x^3/b^2+1/4/b^3*(A*b+C*a)*x)/(-(x^4-a/b)*b)^(1/2)+1/3*C*x*(-b*x
^4+a)^(1/2)/b^2+(-1/2*(A*b+C*a)/b^2-1/3*C/b^2*a)/(1/a^(1/2)*b^(1/2))^(1/2)
*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1
/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+3/2*B/b^(3/2)*a^(1/2)/(1/a^(1
/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(
x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{9\,(Babx^5-Ba^2x)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-(((9\,B+5\,C)ab+3\,A}{}$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(9*(B*a*b*x^5 - B*a^2*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^
(1/4)/x), -1) - (((9*B + 5*C)*a*b + 3*A*b^2)*x^5 - ((9*B + 5*C)*a^2 + 3*A*
a*b)*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) + (2*C*
a*b*x^6 + 6*B*a*b*x^4 - 9*B*a^2 - (5*C*a^2 + 3*A*a*b)*x^2)*sqrt(-b*x^4 + a
))/(a*b^3*x^5 - a^2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.66

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{Cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(-b*x**4+a)**(3/2),x)
```

output
```
A*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4
*a**(3/2)*gamma(9/4)) + B*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**
4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + C*x**9*gamma(9/4)*hyper(
(3/2, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx=\int \frac{x^4\,(C\,x^4+B\,x^2+A)}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx=\frac{3\sqrt{-b\,x^4+a}\,abx+5\sqrt{-b\,x^4+a}\,acx-3\sqrt{-b\,x^4+a}\,b^2x^3-\sqrt{-b\,x^4+a}\,bc\,x^5}{}$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x)
```

output
```
(3*sqrt(a - b*x**4)*a*b*x + 5*sqrt(a - b*x**4)*a*c*x - 3*sqrt(a - b*x**4)*
b**2*x**3 - sqrt(a - b*x**4)*b*c*x**5 - 3*int(sqrt(a - b*x**4)/(a**2 - 2*a
*b*x**4 + b**2*x**8),x)*a**3*b - 5*int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4
 + b**2*x**8),x)*a**3*c + 3*int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2
*x**8),x)*a**2*b**2*x**4 + 5*int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**
2*x**8),x)*a**2*b*c*x**4 + 9*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**
4 + b**2*x**8),x)*a**2*b**2 - 9*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*
x**4 + b**2*x**8),x)*a*b**3*x**4)/(3*b**2*(a - b*x**4))
```

3.35 $\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 180

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{x(aB+(Ab+aC)x^2)}{2ab\sqrt{a-bx^4}} - \frac{(Ab+3aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}b^{7/4}\sqrt{a-bx^4}} + \frac{\left(Ab-\sqrt{a}\sqrt{b}B+3aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{a}b^{7/4}\sqrt{a-bx^4}}$$

output

```
1/2*x*(a*B+(A*b+C*a)*x^2)/a/b/(-b*x^4+a)^(1/2)-1/2*(A*b+3*C*a)*(1-b*x^4/a)
^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/a^(1/4)/b^(7/4)/(-b*x^4+a)^(1/2)+1/2
*(A*b-a^(1/2)*b^(1/2)*B+3*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/
4),I)/a^(1/4)/b^(7/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{3ax(B-2Cx^2)-3aBx\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)+2(Ab}{6ab\sqrt{a-bx^4}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x]
```

output
```
(3*a*x*(B - 2*C*x^2) - 3*a*B*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4,
1/2, 5/4, (b*x^4)/a] + 2*(A*b + 3*a*C)*x^3*Sqrt[1 - (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 3/2, 7/4, (b*x^4)/a])/(6*a*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2367, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2367}$$

$$\frac{x(x^2(aC+Ab)+aB)}{2ab\sqrt{a-bx^4}}-\frac{\int \frac{b((Ab+3aC)x^2+aB)}{\sqrt{a-bx^4}}dx}{2ab^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(x^2(aC+Ab)+aB)}{2ab\sqrt{a-bx^4}}-\frac{\int \frac{(Ab+3aC)x^2+aB}{\sqrt{a-bx^4}}dx}{2ab}$$

$$\Big\downarrow \text{1513}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2ab\sqrt{a-bx^4}}-\frac{\frac{\sqrt{a}(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{b}B+3aC+Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{2ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2ab\sqrt{a-bx^4}}-\frac{\frac{(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{b}B+3aC+Ab\right)\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{2ab}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2ab\sqrt{a-bx^4}}-\frac{\frac{(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(-\sqrt{a}\sqrt{b}B+3aC+Ab\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}}{2ab}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2ab\sqrt{a-bx^4}}-\frac{\frac{(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2ab}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2ab\sqrt{a-bx^4}}-\frac{\frac{\sqrt{1-\frac{bx^4}{a}}(3aC+Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2ab}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{x\left(x^2(aC+Ab)+aB\right)}{2ab\sqrt{a-bx^4}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(3aC+Ab)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2ab}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{x(x^2(aC+Ab)+aB)}{2ab\sqrt{a-bx^4}}-\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}(3aC+Ab)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-\sqrt{a}\sqrt{b}B+3aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2ab}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x]
```

output
```
(x*(a*B + (A*b + a*C)*x^2))/(2*a*b*Sqrt[a - b*x^4]) - ((a^(3/4)*(A*b + 3*a
*C)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/
4)*Sqrt[a - b*x^4]) - (a^(3/4)*(A*b - Sqrt[a]*Sqrt[b]*B + 3*a*C)*Sqrt[1 -
(b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b
*x^4]))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.24

method	result
elliptic	$\frac{2b\left(\frac{(Ab+Ca)x^3}{4a\,b^2}+\frac{Bx}{4b^2}\right)}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{B\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-\frac{\left(-\frac{C}{b}-\frac{Ab+Ca}{2ab}\right)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{Ellipti}\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
default	$A\left(\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,\sqrt{b}}\right)+B\left(\frac{x}{2b\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\right.$

input
```
int(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(1/4/a*(A*b+C*a)/b^2*x^3+1/4*B*x/b^2)/(-(x^4-a/b)*b)^(1/2)-1/2*B/b/(1/
a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2
))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-(-1/b*C
-1/2/a*(A*b+C*a)/b)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/
2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(Elliptic
F(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{((3\,Cab+Ab^2)x^5-(3\,Ca^2+Aab)x)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-((3$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(((3*C*a*b + A*b^2)*x^5 - (3*C*a^2 + A*a*b)*x)*sqrt(-b)*(a/b)^(3/4)*el
liptic_e(arcsin((a/b)^(1/4)/x), -1) - ((3*C*a*b + (A + B)*b^2)*x^5 - (3*C*
a^2 + (A + B)*a*b)*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x
), -1) + (2*C*a*b*x^4 - B*a*b*x^2 - 3*C*a^2 - A*a*b)*sqrt(-b*x^4 + a))/(a*
b^3*x^5 - a^2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.70

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \frac{Ax^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(-b*x**4+a)**(3/2),x)
```

output
```
A*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4
*a**(3/2)*gamma(7/4)) + B*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4
*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + C*x**7*gamma(7/4)*hyper((3
/2, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^2}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx=\int \frac{x^2\,(C\,x^4+B\,x^2+A)}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a-bx^4)^{3/2}}\,dx=\frac{\sqrt{-b\,x^4+a}\,bx-\sqrt{-b\,x^4+a}\,c\,x^3-\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)a^2b+\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+}\right.}{}$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - b*x**4)*b*x - sqrt(a - b*x**4)*c*x**3 - int(sqrt(a - b*x**4)/(a*
*2 - 2*a*b*x**4 + b**2*x**8),x)*a**2*b + int(sqrt(a - b*x**4)/(a**2 - 2*a*
b*x**4 + b**2*x**8),x)*a*b**2*x**4 + int((sqrt(a - b*x**4)*x**2)/(a**2 - 2
*a*b*x**4 + b**2*x**8),x)*a**2*b + 3*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2
*a*b*x**4 + b**2*x**8),x)*a**2*c - int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a
*b*x**4 + b**2*x**8),x)*a*b**2*x**4 - 3*int((sqrt(a - b*x**4)*x**2)/(a**2
- 2*a*b*x**4 + b**2*x**8),x)*a*b*c*x**4)/(b*(a - b*x**4))
```

3.36 $\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx = \frac{x\left(A+\frac{aC}{b}+Bx^2\right)}{2a\sqrt{a-bx^4}} - \frac{B\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a-bx^4}} + \frac{\left(Ab+\sqrt{a}\sqrt{b}B-aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}b^{5/4}\sqrt{a-bx^4}}$$

output
```
1/2*x*(A+a*C/b+B*x^2)/a/(-b*x^4+a)^(1/2)-1/2*B*(1-b*x^4/a)^(1/2)*EllipticE
(b^(1/4)*x/a^(1/4),I)/a^(1/4)/b^(3/4)/(-b*x^4+a)^(1/2)+1/2*(A*b+a^(1/2)*b^
(1/2)*B-C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(3/4)/b^(5
/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx = \frac{3(Ab+aC)x+3(Ab-aC)x\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)+2bBx}{6ab\sqrt{a-bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a - b*x^4)^(3/2),x]
```

output
```
(3*(A*b + a*C)*x + 3*(A*b - a*C)*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, (b*x^4)/a] + 2*b*B*x^3*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1
[3/4, 3/2, 7/4, (b*x^4)/a])/(6*a*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2397, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2397}$$

$$\frac{\int \frac{-bBx^2+Ab-aC}{\sqrt{a-bx^4}}dx}{2ab}+\frac{x(aC+Ab+bBx^2)}{2ab\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{1513}$$

$$\frac{\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx-\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{2ab}+\frac{x(aC+Ab+bBx^2)}{2ab\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx-\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2ab\sqrt{a-bx^4}}$$

$\Big\downarrow$ 765

$$\frac{\frac{\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2ab\sqrt{a-bx^4}}$$

$\Big\downarrow$ 762

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2ab\sqrt{a-bx^4}}$$

$\Big\downarrow$ 1390

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2ab\sqrt{a-bx^4}}$$

$\Big\downarrow$ 1389

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2ab\sqrt{a-bx^4}}$$

$\Big\downarrow$ 327

$$\frac{\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B-aC+Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2ab\sqrt{a-bx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a - b*x^4)^(3/2),x]
```

output
```
(x*(A*b + a*C + b*B*x^2))/(2*a*b*Sqrt[a - b*x^4]) + (-((a^(3/4)*b^(1/4)*B*
Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b
*x^4]) + (a^(1/4)*(A*b + Sqrt[a]*Sqrt[b]*B - a*C)*Sqrt[1 - (b*x^4)/a]*Elli
pticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4]))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{2b\left(\frac{B x^3}{4ba}+\frac{(Ab+Ca)x}{4a b^2}\right)}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\left(-\frac{C}{b}+\frac{Ab+Ca}{2ab}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}+\frac{B\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(\cdots\right.\right.}{2\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-\cdots}}$
default	$A\left(\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}\right)+B\left(\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\left(\cdots\right.}{2\sqrt{\cdots}}\right.$

input
```
int((C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*b*(1/4/b/a*B*x^3+1/4/a*(A*b+C*a)/b^2*x)/(-(x^4-a/b)*b)^(1/2)+(-1/b*C+1/2
/a*(A*b+C*a)/b)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1
+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2
))^(1/2),I)+1/2*B/a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(
x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx = -\frac{(Bb^2x^4-Bab)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)+((Cab-(A+B)b^2)x^4-Ca^2+(A+B)ab)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}}{2\,(ab^3x^4-a^2b^2)}$$

input
```
integrate((C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((B*b^2*x^4 - B*a*b)*sqrt(a)*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)^(1
/4)), -1) + ((C*a*b - (A + B)*b^2)*x^4 - C*a^2 + (A + B)*a*b)*sqrt(a)*(b/a
)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) + (B*b^2*x^3 + (C*a*b + A*b^
2)*x)*sqrt(-b*x^4 + a))/(a*b^3*x^4 - a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/(-b*x**4+a)**(3/2),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(3/2)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*ex
p_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + C*x**5*gamma(5/4)*hyper((5/4,
 3/2), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a - b*x^4)^(3/2),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{(a-bx^4)^{3/2}}\,dx = \frac{\sqrt{-b\,x^4+a}\,cx+\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)a^2b-\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)a^2c-\left(\int \frac{\sqrt{-b}}{b^2x^8-2a}\right.}{}$$

input

```
int((C*x^4+B*x^2+A)/(-b*x^4+a)^(3/2),x)
```

output

```
(sqrt(a - b*x**4)*c*x + int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**
8),x)*a**2*b - int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**
2*c - int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a*b**2*x**4
+ int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a*b*c*x**4 + int
((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a*b**2 - int((
sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*b**3*x**4)/(b*(a
 - b*x**4))
```

3.37 $\int \frac{A+Bx^2+Cx^4}{x^2(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 197

$$\int \frac{A+Bx^2+Cx^4}{x^2(a-bx^4)^{3/2}}\,dx = \frac{x(aB+(Ab+aC)x^2)}{2a^2\sqrt{a-bx^4}} - \frac{A\sqrt{a-bx^4}}{a^2x} - \frac{(3Ab+aC)\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{5/4}b^{3/4}\sqrt{a-bx^4}} + \frac{\left(3Ab+\sqrt{a}\sqrt{b}B+aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{5/4}b^{3/4}\sqrt{a-bx^4}}$$

output
```
1/2*x*(a*B+(A*b+C*a)*x^2)/a^2/(-b*x^4+a)^(1/2)-A*(-b*x^4+a)^(1/2)/a^2/x-1/
2*(3*A*b+C*a)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/a^(5/4)/b^(
3/4)/(-b*x^4+a)^(1/2)+1/2*(3*A*b+a^(1/2)*b^(1/2)*B+C*a)*(1-b*x^4/a)^(1/2)*
EllipticF(b^(1/4)*x/a^(1/4),I)/a^(5/4)/b^(3/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a-bx^4\right)^{3/2}}\,dx = \frac{-6A\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},\frac{bx^4}{a}\right)+x^2\left(3B+3B\sqrt{1-\frac{bx^4}{a}}\,\text{Hyp}\right.}{6ax\sqrt{}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(a - b*x^4)^(3/2)),x]
```

output
```
(-6*A*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, (b*x^4)/a] + x
^2*(3*B + 3*B*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)
/a] + 2*C*x^2*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (b*x^4)
/a]))/(6*a*x*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2368, 2374, 9, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a-bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{\int \frac{-\frac{b(Ab+aC)x^4}{a}+bBx^2+2Ab}{x^2\sqrt{a-bx^4}}dx}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{2374}$$

$$\frac{-\frac{\int -\frac{2\left(abBx-b(3Ab+aC)x^3\right)}{x\sqrt{a-bx^4}}dx}{2a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 9$$

$$\frac{-\frac{\int -\frac{2b\left(aB-(3Ab+aC)x^2\right)}{\sqrt{a-bx^4}}dx}{2a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{aB-(3Ab+aC)x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 1513$$

$$\frac{\frac{b\left(\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B+aC+3Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}(aC+3Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\left(\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B+aC+3Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{(aC+3Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 765$$

$$\frac{\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B+aC+3Ab\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{(aC+3Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{(aC+3Ab)\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 1390$$

$$\frac{\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{1-\frac{bx^4}{a}}(aC+3Ab)\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 1389$$

$$\frac{\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}(aC+3Ab)\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(\sqrt{a}\sqrt{b}B+aC+3Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}(aC+3Ab)E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{2Ab\sqrt{a-bx^4}}{ax}}{2ab}+\frac{x\left(x^2(aC+Ab)+aB\right)}{2a^2\sqrt{a-bx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(a - b*x^4)^(3/2)),x]
```

output
```
(x*(a*B + (A*b + a*C)*x^2))/(2*a^2*Sqrt[a - b*x^4]) + ((-2*A*b*Sqrt[a - b*
x^4])/(a*x) + (b*(-((a^(3/4)*(3*A*b + a*C)*Sqrt[1 - (b*x^4)/a]*EllipticE[A
rcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])) + (a^(3/4)*(3*
A*b + Sqrt[a]*Sqrt[b]*B + a*C)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/a)/(2*a*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.23

method	result
elliptic	$-\frac{A\sqrt{-b\,x^{4}+a}}{a^{2}x}+\frac{2b\left(\frac{(Ab+Ca)x^{3}}{4b\,a^{2}}+\frac{Bx}{4ba}\right)}{\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{B\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-\frac{\left(-\frac{Ab}{a^{2}}-\frac{Ab+Ca}{2a^{2}}\right)\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\cdots}{}$
risch	$-\frac{A\sqrt{-b\,x^{4}+a}}{a^{2}x}-\frac{-\frac{A\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+a\left(\frac{2b\left(-\frac{(Ab+Ca)x^{3}}{4ba}-\frac{Bx}{4b}\right)}{\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{B\sqrt{1-\cdots}}{}\cdots\right.}{\cdots}$
default	$B\left(\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right)+A\left(-\frac{\sqrt{-b\,x^{4}+a}}{a^{2}x}+\frac{b\,x^{3}}{2a^{2}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{3\sqrt{b}\,\sqrt{\cdots}}{\cdots}\right.$

input
```
int((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(-b*x^4+a)^(1/2)/a^2/x+2*b*(1/4/b/a^2*(A*b+C*a)*x^3+1/4/b/a*B*x)/(-(x^4
-a/b)*b)^(1/2)+1/2*B/a/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(
1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)
*b^(1/2))^(1/2),I)-(-A*b/a^2-1/2/a^2*(A*b+C*a))*a^(1/2)/(1/a^(1/2)*b^(1/2)
)^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^
4+a)^(1/2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(
1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2+Cx^4}{x^2(a-bx^4)^{3/2}}\,dx =$$

$$-\frac{((Cab+3\,Ab^2)x^5-(Ca^2+3\,Aab)x)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-(((B+C)ab+3\,Ab^2)x^5-((B}{2\,(a^2b^2x^5}$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((C*a*b + 3*A*b^2)*x^5 - (C*a^2 + 3*A*a*b)*x)*sqrt(a)*(b/a)^(3/4)*el
liptic_e(arcsin(x*(b/a)^(1/4)), -1) - (((B + C)*a*b + 3*A*b^2)*x^5 - ((B +
 C)*a^2 + 3*A*a*b)*x)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4))
, -1) + (B*a*b*x^2 + (C*a*b + 3*A*b^2)*x^4 - 2*A*a*b)*sqrt(-b*x^4 + a))/(a
^2*b^2*x^5 - a^3*b*x)
```

Sympy [A] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a-bx^4\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)} + \frac{Bx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(-b*x**4+a)**(3/2),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(3/2)*x*gamma(3/4)) + B*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp
_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + C*x**3*gamma(3/4)*hyper((3/4,
3/2), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a-bx^4\right)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{\left(-bx^4+a\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-b*x^4 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2(a-bx^4)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(-bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2(a-bx^4)^{3/2}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{x^2\,(a-b\,x^4)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a - b*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a - b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2(a-bx^4)^{3/2}}\,dx = \left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)b + \left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^{10}-2ab\,x^6+a^2x^2}dx\right)a + \left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{b^2x^8-2ab\,x^4+a^2}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^2/(-b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*b + int(sqrt(a - b
*x**4)/(a**2*x**2 - 2*a*b*x**6 + b**2*x**10),x)*a + int((sqrt(a - b*x**4)*
x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*c
```

3.38 $\int \frac{A+Bx^2+Cx^4}{x^4(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 217

$$\int \frac{A+Bx^2+Cx^4}{x^4(a-bx^4)^{3/2}}\,dx = \frac{x\left(a\left(\frac{Ab}{a}+C\right)+bBx^2\right)}{2a^2\sqrt{a-bx^4}} - \frac{A\sqrt{a-bx^4}}{3a^2x^3} - \frac{B\sqrt{a-bx^4}}{a^2x} - \frac{3\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{5/4}\sqrt{a-bx^4}} + \frac{\left(5Ab+9\sqrt{a}\sqrt{b}B+3aC\right)\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{6a^{7/4}\sqrt[4]{b}\sqrt{a-bx^4}}$$

output
```
1/2*x*(a*(A*b/a+C)+B*b*x^2)/a^2/(-b*x^4+a)^(1/2)-1/3*A*(-b*x^4+a)^(1/2)/a^
2/x^3-B*(-b*x^4+a)^(1/2)/a^2/x-3/2*b^(1/4)*B*(1-b*x^4/a)^(1/2)*EllipticE(b
^(1/4)*x/a^(1/4),I)/a^(5/4)/(-b*x^4+a)^(1/2)+1/6*(5*A*b+9*a^(1/2)*b^(1/2)*
B+3*C*a)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(7/4)/b^(1/4)/
(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a-bx^4\right)^{3/2}}\,dx = \frac{-2A\sqrt{1-\frac{bx^4}{a}}\ \text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},\frac{bx^4}{a}\right)-6Bx^2\sqrt{1-\frac{bx^4}{a}}\ \text{Hypergeome}}{6ax^3\sqrt{}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(a - b*x^4)^(3/2)),x]
```

output
```
(-2*A*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, (b*x^4)/a] - 6
*B*x^2*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, (b*x^4)/a] +
3*C*x^4*(1 + Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/
a]))/(6*a*x^3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2368, 2374, 9, 27, 2374, 9, 27, 1513, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a-bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2368}$$

$$\frac{\int \frac{-\frac{b^2Bx^6}{a}+b\left(\frac{Ab}{a}+C\right)x^4+2bBx^2+2Ab}{x^4\sqrt{a-bx^4}}dx}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{2374}$$

$$\frac{-\frac{\int -\frac{2\left(-3b^2Bx^5+b(5Ab+3aC)x^3+6abBx\right)}{x^3\sqrt{a-bx^4}}dx}{6a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 9$$

$$\frac{-\frac{\int -\frac{2\left(-3b^2Bx^4+b(5Ab+3aC)x^2+6abB\right)}{x^2\sqrt{a-bx^4}}dx}{6a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{-3b^2Bx^4+b(5Ab+3aC)x^2+6abB}{x^2\sqrt{a-bx^4}}dx}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 2374$$

$$\frac{\frac{-\frac{\int -\frac{2\left(ab(5Ab+3aC)x-9ab^2Bx^3\right)}{x\sqrt{a-bx^4}}dx}{2a}-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 9$$

$$\frac{\frac{-\frac{\int -\frac{2ab\left(-9bBx^2+5Ab+3aC\right)}{\sqrt{a-bx^4}}dx}{2a}-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b\int \frac{-9bBx^2+5Ab+3aC}{\sqrt{a-bx^4}}dx-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 1513$$

$$\frac{\frac{b\left(\left(9\sqrt{a}\sqrt{b}B+3aC+5Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx-9\sqrt{a}\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx\right)-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b\left(\left(9\sqrt{a}\sqrt{b}B+3aC+5Ab\right)\int \frac{1}{\sqrt{a-bx^4}}dx-9\sqrt{b}B\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{\frac{b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\left(9\sqrt{a}\sqrt{b}B+3aC+5Ab\right)\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}-9\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$\Big\downarrow$ 762

$$\frac{\frac{b\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(9\sqrt{a}\sqrt{b}B+3aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-9\sqrt{b}B\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx\right)-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$\Big\downarrow$ 1390

$$\frac{\frac{b\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(9\sqrt{a}\sqrt{b}B+3aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{9\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}\right)-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$\Big\downarrow$ 1389

$$\frac{\frac{b\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(9\sqrt{a}\sqrt{b}B+3aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{9\sqrt{a}\sqrt{b}B\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{a-bx^4}}\right)-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx^4}}{3ax^3}}{2ab}+\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

$\Big\downarrow$ 327

$$\frac{\frac{b\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\left(9\sqrt{a}\sqrt{b}B+3aC+5Ab\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}-\frac{9a^{3/4}\sqrt[4]{b}B\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}}\right)-\frac{6bB\sqrt{a-bx^4}}{x}}{3a}-\frac{2Ab\sqrt{a-bx\ldots}}{3ax^3}}{2ab}$$

$$\frac{x\left(aC+Ab+bBx^2\right)}{2a^2\sqrt{a-bx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*(a - b*x^4)^(3/2)),x]
```

output
```
(x*(A*b + a*C + b*B*x^2))/(2*a^2*Sqrt[a - b*x^4]) + ((-2*A*b*Sqrt[a - b*x^
4])/(3*a*x^3) + ((-6*b*B*Sqrt[a - b*x^4])/x + b*((-9*a^(3/4)*b^(1/4)*B*Sqr
t[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/Sqrt[a - b*x^
4] + (a^(1/4)*(5*A*b + 9*Sqrt[a]*Sqrt[b]*B + 3*a*C)*Sqrt[1 - (b*x^4)/a]*El
lipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(1/4)*Sqrt[a - b*x^4])))/(3*a
))/(2*a*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.18

method	result
elliptic	$-\frac{A\sqrt{-b\,x^4+a}}{3a^2x^3}-\frac{B\sqrt{-b\,x^4+a}}{a^2x}+\frac{2b\left(\frac{B\,x^3}{4a^2}+\frac{(Ab+Ca)x}{4b\,a^2}\right)}{\sqrt{-(x^4-\frac{a}{b})b}}+\frac{\left(\frac{Ab}{3a^2}+\frac{Ab+Ca}{2a^2}\right)\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}+$
default	$C\left(\frac{x}{2a\sqrt{-(x^4-\frac{a}{b})b}}+\frac{\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}\right)+A\left(-\frac{\sqrt{-b\,x^4+a}}{3a^2x^3}+\frac{bx}{2a^2\sqrt{-(x^4-\frac{a}{b})b}}+\frac{5b\sqrt{1-}}{}\right.$
risch	$-\frac{\sqrt{-b\,x^4+a}\,(3B\,x^2+A)}{3a^2x^3}+\frac{\frac{Ab\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}+\frac{3B\sqrt{b}\sqrt{a}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{Ell}\right.}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-b\,x^4+a}}}{}$

input
```
int((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*(-b*x^4+a)^(1/2)/a^2/x^3-B*(-b*x^4+a)^(1/2)/a^2/x+2*b*(1/4*B/a^2*x^
3+1/4/b/a^2*(A*b+C*a)*x)/(-(x^4-a/b)*b)^(1/2)+(1/3*A*b/a^2+1/2/a^2*(A*b+C*
a))/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+
3/2*B*b^(1/2)/a^(3/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1
/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)
*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2+Cx^4}{x^4\,(a-bx^4)^{3/2}}\,dx = -\frac{9\,(Bb^2x^7-Babx^3)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)-\left(\left(3\,Cab+(5\,A+9\,B)b^2\right)x^7-\left(3\,Ca^2+(5\,A+9\,B\right.\right.}{6\,(a^2b^2x^7}$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(9*(B*b^2*x^7 - B*a*b*x^3)*sqrt(a)*(b/a)^(3/4)*elliptic_e(arcsin(x*(b
/a)^(1/4)), -1) - ((3*C*a*b + (5*A + 9*B)*b^2)*x^7 - (3*C*a^2 + (5*A + 9*B
)*a*b)*x^3)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) + (9
*B*b^2*x^6 - 6*B*a*b*x^2 + (3*C*a*b + 5*A*b^2)*x^4 - 2*A*a*b)*sqrt(-b*x^4
+ a))/(a^2*b^2*x^7 - a^3*b*x^3)
```

Sympy [A] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a-bx^4\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{B\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)} + \frac{Cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(-b*x**4+a)**(3/2),x)
```

output
```
A*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a*
*(3/2)*x**3*gamma(1/4)) + B*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*
exp_polar(2*I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + C*x*gamma(1/4)*hyper((1/4
, 3/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(3/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a-bx^4\right)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{\left(-bx^4+a\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-b*x^4 + a)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(a-bx^4)^{3/2}}\,dx=\int \frac{Cx^4+Bx^2+A}{(-bx^4+a)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((-b*x^4 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4(a-bx^4)^{3/2}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,(a-b\,x^4)^{3/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a - b*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a - b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4(a-bx^4)^{3/2}}\,dx=\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)c+\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^{12}-2ab\,x^8+a^2x^4}dx\right)a+\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^{10}-2ab\,x^6+a^2x^2}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/x^4/(-b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*c + int(sqrt(a - b
*x**4)/(a**2*x**4 - 2*a*b*x**8 + b**2*x**12),x)*a + int(sqrt(a - b*x**4)/(
a**2*x**2 - 2*a*b*x**6 + b**2*x**10),x)*b
```

3.39 $\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 266

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx = -\frac{A\sqrt{a+bx^4}}{ax}+\frac{(Ab+aC)x\sqrt{a+bx^4}}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(Ab+aC)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \frac{\left(Ab+\sqrt{a}\sqrt{b}B+aC\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
-A*(b*x^4+a)^(1/2)/a/x+(A*b+C*a)*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1
/2)*x^2)-(A*b+C*a)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b
^(3/4)/(b*x^4+a)^(1/2)+1/2*(A*b+a^(1/2)*b^(1/2)*B+C*a)*(a^(1/2)+b^(1/2)*x^
2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1
/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx = \frac{\sqrt{1+\frac{bx^4}{a}}\left(-3A\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)+3Bx^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)+Cx\right.}{3x\sqrt{a+bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
(Sqrt[1 + (b*x^4)/a]*(-3*A*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^4)/a)]
 + 3*B*x^2*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + C*x^4*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)]))/(3*x*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2374, 9, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2374}$$

$$-\frac{\int -\frac{2((Ab+aC)x^3+aBx)}{x\sqrt{bx^4+a}}dx}{2a}-\frac{A\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{2((Ab+aC)x^2+aB)}{\sqrt{bx^4+a}}dx}{2a}-\frac{A\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(Ab+aC)x^2+aB}{\sqrt{bx^4+a}}dx}{a}-\frac{A\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 1512$$

$$\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B+aC+Ab\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}(aC+Ab)\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{a}-\frac{A\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B+aC+Ab\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{(aC+Ab)\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{a}-\frac{A\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{b}B+aC+Ab\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{(aC+Ab)\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{a}-\frac{A\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{b}B+aC+Ab\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{(aC+Ab)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\text{arcta}\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}\right.}{\sqrt{b}}}{a}-\frac{A\sqrt{a+bx^4}}{ax}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
-((A*Sqrt[a + b*x^4])/(a*x)) + (-(((A*b + a*C)*(-((x*Sqrt[a + b*x^4])/(Sqr
t[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(1/4)*Sqrt[a + b*x^4])))/Sqrt[b]) + (a^(1/4)*(A*b + Sqrt[a]*Sqrt[b]*B +
a*C)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4]))
/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

method	result
elliptic	$-\frac{A\sqrt{b\,x^{4}+a}}{a x}+\frac{B\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i\left(\frac{A b}{a}+C\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$
risch	$-\frac{A\sqrt{b\,x^{4}+a}}{a x}+\frac{\frac{B a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i(A b+C a)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{Elliptic}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}}{a}$
default	$\frac{B\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+A\left(-\frac{\sqrt{b\,x^{4}+a}}{a x}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}, i\right)-\mathrm{E}\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(b*x^4+a)^(1/2)/a/x+B/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)+I*(A/a*b+C)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)
*b^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{\sqrt{bx^4+ax^2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*sqrt(b*x^4 + a)/(b*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.45

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{Bx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(b*x**4+a)**(1/2),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*x*gamma(3/4)) + B*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + C*x**3*gamma(3/4)*hyper((1/2, 3/4),
(7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bx^4+ax^2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{\sqrt{bx^4+ax^2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(sqrt(b*x^4 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(1/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt{a+bx^4}}\,dx = \left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^6+a\,x^2}dx\right)a+\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)b + \left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a + int(sqrt(a + b*x**4)/(a + b*
x**4),x)*b + int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*c
```

3.40 $\int \frac{A+Bx^2+Cx^4}{x^2(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 313

$$\int \frac{A+Bx^2+Cx^4}{x^2(a+bx^4)^{3/2}}\,dx = \frac{x(aB-(Ab-aC)x^2)}{2a^2\sqrt{a+bx^4}} - \frac{A\sqrt{a+bx^4}}{a^2x} + \frac{(3Ab-aC)x\sqrt{a+bx^4}}{2a^2\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(3Ab-aC)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^4}} + \frac{\left(3Ab+\sqrt{a}\sqrt{b}B-aC\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
1/2*x*(a*B-(A*b-C*a)*x^2)/a^2/(b*x^4+a)^(1/2)-A*(b*x^4+a)^(1/2)/a^2/x+1/2*
(3*A*b-C*a)*x*(b*x^4+a)^(1/2)/a^2/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)-1/2*(3*A*b
-C*a)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/b^(3/4)/(b*x^4
+a)^(1/2)+1/4*(3*A*b+a^(1/2)*b^(1/2)*B-C*a)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+
a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/
4)),1/2*2^(1/2))/a^(7/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.45

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{-6A\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^4}{a}\right)+x^2\left(3B+3B\sqrt{1+\frac{bx^4}{a}}\,\text{Hy}\right.}{6a}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
(-6*A*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)]
+ x^2*(3*B + 3*B*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b
*x^4)/a)] + 2*C*x^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -
((b*x^4)/a)]))/(6*a*x*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2368, 25, 2374, 9, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2368}$$

$$\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}-\frac{\int -\frac{\frac{b(Ab-aC)x^4}{a}+bBx^2+2Ab}{x^2\sqrt{bx^4+a}}dx}{2ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{b(Ab-aC)x^4}{a}+bBx^2+2Ab}{x^2\sqrt{bx^4+a}}dx}{2ab}+\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2374}$$

$$\frac{-\frac{\int -\frac{2\left(b(3Ab-aC)x^3+abBx\right)}{x\sqrt{bx^4+a}}dx}{2a}-\frac{2Ab\sqrt{a+bx^4}}{ax}}{2ab}+\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{9}$$

$$\frac{-\frac{\int -\frac{2b\left((3Ab-aC)x^2+aB\right)}{\sqrt{bx^4+a}}dx}{2a}-\frac{2Ab\sqrt{a+bx^4}}{ax}}{2ab}+\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\int \frac{(3Ab-aC)x^2+aB}{\sqrt{bx^4+a}}dx}{a}-\frac{2Ab\sqrt{a+bx^4}}{ax}}{2ab}+\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{\frac{b\left(\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B-aC+3Ab\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}(3Ab-aC)\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{2Ab\sqrt{a+bx^4}}{ax}}{2ab}+\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\left(\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}B-aC+3Ab\right)\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{(3Ab-aC)\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{2Ab\sqrt{a+bx^4}}{ax}}{2ab}+\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{b}B-aC+3Ab\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{(3Ab-aC)\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{2Ab\sqrt{a+bx^4}}{ax}}{2ab}+\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{x\left(aB-x^2(Ab-aC)\right)}{2a^2\sqrt{a+bx^4}}+\frac{\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{b}B-aC+3Ab\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{(3Ab-aC)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}\cdots\right.}{\sqrt{b}}\cdots\right.}{a}\cdots}{2ab}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
(x*(a*B - (A*b - a*C)*x^2))/(2*a^2*Sqrt[a + b*x^4]) + ((-2*A*b*Sqrt[a + b*
x^4])/(a*x) + (b*(-(((3*A*b - a*C)*(-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[
b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt
[a + b*x^4])))/Sqrt[b]) + (a^(1/4)*(3*A*b + Sqrt[a]*Sqrt[b]*B - a*C)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/a)/(2*a*
b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{2b\left(\frac{(Ab-Ca)x^3}{4a^2b}-\frac{Bx}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{A\sqrt{b\,x^4+a}}{a^2x}+\frac{B\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i\left(\frac{Ab-Ca}{2a^2}+\frac{Ab}{a^2}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}$
default	$B\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+A\left(-\frac{b\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{a^2x}+\frac{3i\sqrt{b}\,\sqrt{1-}}{}\right.$
risch	$-\frac{A\sqrt{b\,x^4+a}}{a^2x}+\frac{a^2B\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+b^2A\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{}\right.}{}$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*(A*b-C*a)/a^2/b*x^3-1/4/b/a*B*x)/((x^4+a/b)*b)^(1/2)-A*(b*x^4+a)
^(1/2)/a^2/x+1/2*B/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)+I*(1/2*(A*b-C*a)/a^2+A*b/a^2)*a^(1/2)/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/
(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{3/2}}\,dx=\frac{\left(\left(Cab-3\,Ab^2\right)x^5+\left(Ca^2-3\,Aab\right)x\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid-1\right)-(((B}{}$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(((C*a*b - 3*A*b^2)*x^5 + (C*a^2 - 3*A*a*b)*x)*sqrt(a)*(-b/a)^(3/4)*el
liptic_e(arcsin(x*(-b/a)^(1/4)), -1) - (((B + C)*a*b - 3*A*b^2)*x^5 + ((B
+ C)*a^2 - 3*A*a*b)*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/
4)), -1) + (B*a*b*x^2 + (C*a*b - 3*A*b^2)*x^4 - 2*A*a*b)*sqrt(b*x^4 + a))/
(a^2*b^2*x^5 + a^3*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^2+Cx^4}{x^2(a+bx^4)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)} + \frac{Bx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(b*x**4+a)**(3/2),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*x*gamma(3/4)) + B*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + C*x**3*gamma(3/4)*hyper((3/4, 3/2)
, (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2(a+bx^4)^{3/2}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2 (a + bx^4)^{3/2}} \, dx = \int \frac{Cx^4 + Bx^2 + A}{(bx^4 + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4}{x^2 (a + bx^4)^{3/2}} \, dx = \int \frac{C\,x^4 + B\,x^2 + A}{x^2\,(b\,x^4 + a)^{3/2}} \, dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(3/2)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^2 + Cx^4}{x^2 (a + bx^4)^{3/2}} \, dx = \left(\int \frac{\sqrt{b\,x^4 + a}}{b^2 x^8 + 2ab\,x^4 + a^2} dx \right) b + \left(\int \frac{\sqrt{b\,x^4 + a}}{b^2 x^{10} + 2ab\,x^6 + a^2 x^2} dx \right) a + \left(\int \frac{\sqrt{b\,x^4 + a}\,x^2}{b^2 x^8 + 2ab\,x^4 + a^2} dx \right) c$$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*b + int(sqrt(a + b
*x**4)/(a**2*x**2 + 2*a*b*x**6 + b**2*x**10),x)*a + int((sqrt(a + b*x**4)*
x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*c
```

3.41 $\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 404

$$\begin{aligned}
&\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx \\
&= \frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{1}{8}ex^6\sqrt{a+bx^4}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{ax(5d+7fx^2)\sqrt{a+bx^4}}{105b} \\
&\quad+\frac{c(a+bx^4)^{3/2}}{6b}+\frac{dx(a+bx^4)^{3/2}}{7b}+\frac{fx^3(a+bx^4)^{3/2}}{9b}-\frac{a^2e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}} \\
&\quad+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}} \\
&\quad-\frac{a^{7/4}\left(5\sqrt{b}d+7\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/16*a*e*x^2*(b*x^4+a)^(1/2)/b+1/8*e*x^6*(b*x^4+a)^(1/2)-2/15*a^2*f*x*(b*x
^4+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)-1/105*a*x*(7*f*x^2+5*d)*(b*x^4+a
)^(1/2)/b+1/6*c*(b*x^4+a)^(3/2)/b+1/7*d*x*(b*x^4+a)^(3/2)/b+1/9*f*x^3*(b*x
^4+a)^(3/2)/b-1/16*a^2*e*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)+2/15
*a^(9/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)
*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)
^(1/2)-1/105*a^(7/4)*(5*b^(1/2)*d+7*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x
^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^
(1/4)),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.49 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.53

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$= \frac{\sqrt{a+bx^4}\Big(168\sqrt{b}c(a+bx^4)+144\sqrt{b}dx(a+bx^4)+112\sqrt{b}fx^3(a+bx^4)+63e\Big(\sqrt{b}x^2(a+2bx^4)-\frac{a^{3/2}a}{}}{1008b^{3/2}}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(168*Sqrt[b]*c*(a + b*x^4) + 144*Sqrt[b]*d*x*(a + b*x^4)
+ 112*Sqrt[b]*f*x^3*(a + b*x^4) + 63*e*(Sqrt[b]*x^2*(a + 2*b*x^4) - (a^(3/
2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]) - (144*a*Sqrt[b]*d
*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] -
(112*a*Sqrt[b]*f*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt
[1 + (b*x^4)/a]))/(1008*b^(3/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^3\sqrt{a+bx^4}\left(c+ex^2\right)+x^4\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{7/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^4}}-\frac{a^2e\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^2fx\sqrt{a+bx^4}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\left(a+bx^4\right)^{3/2}}{6b}+\frac{1}{63}x^5\sqrt{a+bx^4}\left(9d+7fx^2\right)+\frac{2adx\sqrt{a+bx^4}}{21b}+\frac{ex^2\left(a+bx^4\right)^{3/2}}{8b}-\frac{aex^2\sqrt{a+bx^4}}{16b}+\frac{2afx^3\sqrt{a+bx^4}}{45b}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(2*a*d*x*Sqrt[a + b*x^4])/(21*b) - (a*e*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a
*f*x^3*Sqrt[a + b*x^4])/(45*b) - (2*a^2*f*x*Sqrt[a + b*x^4])/(15*b^(3/2)*(
Sqrt[a] + Sqrt[b]*x^2)) + (x^5*(9*d + 7*f*x^2)*Sqrt[a + b*x^4])/63 + (c*(a
 + b*x^4)^(3/2))/(6*b) + (e*x^2*(a + b*x^4)^(3/2))/(8*b) - (a^2*e*ArcTanh[
(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3/2)) + (2*a^(9/4)*f*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a + b*x^4]) - (a^(7/4)*(5*Sqrt
[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(7/4)
*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\left(560 b f \,x^{7}+630 b e \,x^{6}+720 x^{5} b d +840 b c \,x^{4}+224 a f \,x^{3}+315 a e \,x^{2}+480 a d x +840 a c \right) \sqrt{b \,x^{4}+a}}{5040 b}-\frac{a^{2} \left(\frac{80 d \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \text{Ellipt}}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right.}{}$
default	$\frac{c \left(b \,x^{4}+a \right)^{\frac{3}{2}}}{6 b}+d \left(\frac{x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{2 a x \sqrt{b \,x^{4}+a}}{21 b}-\frac{2 a^{2} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{21 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+e \left(\frac{x^{2} \left(b \,x^{4}+a \right)^{\frac{3}{2}}}{8 b}\right.$
elliptic	$\frac{f \,x^{7} \sqrt{b \,x^{4}+a}}{9}+\frac{e \,x^{6} \sqrt{b \,x^{4}+a}}{8}+\frac{d \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{c \,x^{4} \sqrt{b \,x^{4}+a}}{6}+\frac{2 a f \,x^{3} \sqrt{b \,x^{4}+a}}{45 b}+\frac{a e \,x^{2} \sqrt{b \,x^{4}+a}}{16 b}+\frac{2 a d x \sqrt{b \,x^{4}+a}}{21 b}+$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5040*(560*b*f*x^7+630*b*e*x^6+720*b*d*x^5+840*b*c*x^4+224*a*f*x^3+315*a*
e*x^2+480*a*d*x+840*a*c)/b*(b*x^4+a)^(1/2)-1/840*a^2/b*(80*d/(I/a^(1/2)*b^
(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/
2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+112*I*f*a^(1/2
)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+105/2*e*ln(b^(1/2)*x^2
+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.51

$$\int x^3\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx =$$

$$-\frac{1344\,a^2\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-315\,a^2\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+19}{}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/10080*(1344*a^2*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) - 315*a^2*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^
2 - a) + 192*(5*a*b*d - 7*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin(
(-a/b)^(1/4)/x), -1) - 2*(560*b^2*f*x^8 + 630*b^2*e*x^7 + 720*b^2*d*x^6 +
840*b^2*c*x^5 + 224*a*b*f*x^4 + 315*a*b*e*x^3 + 480*a*b*d*x^2 + 840*a*b*c*
x - 672*a^2*f)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.52

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{a^{\frac{3}{2}}ex^2}{16b\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}ex^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} - \frac{a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + c\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{bex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
a**(3/2)*e*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*d*x**5*gamma(5/4)*hype
r((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a
)*e*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*f*x**7*gamma(7/4)*hyper((-1/2,
7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) - a**2*e*asinh(sq
rt(b)*x**2/sqrt(a))/(16*b**(3/2)) + c*Piecewise((sqrt(a)*x**4/4, Eq(b, 0))
, ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*x**10/(8*sqrt(a)*sqrt(1 + b*x**
4/a))
```

Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^4 + a)^(3/2)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)*sqrt(b*x^4 +
 a), x)
```

Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x^3\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^3*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int x^3(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$= \frac{1680\sqrt{b\,x^4+a}\,abc+960\sqrt{b\,x^4+a}\,abdx+630\sqrt{b\,x^4+a}\,abe\,x^2+448\sqrt{b\,x^4+a}\,abf\,x^3+1680\sqrt{b\,x^4+a}}{}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

output
```
(1680*sqrt(a + b*x**4)*a*b*c + 960*sqrt(a + b*x**4)*a*b*d*x + 630*sqrt(a +
 b*x**4)*a*b*e*x**2 + 448*sqrt(a + b*x**4)*a*b*f*x**3 + 1680*sqrt(a + b*x*
*4)*b**2*c*x**4 + 1440*sqrt(a + b*x**4)*b**2*d*x**5 + 1260*sqrt(a + b*x**4
)*b**2*e*x**6 + 1120*sqrt(a + b*x**4)*b**2*f*x**7 + 315*sqrt(b)*log(sqrt(a
 + b*x**4) - sqrt(b)*x**2)*a**2*e - 315*sqrt(b)*log(sqrt(a + b*x**4) + sqr
t(b)*x**2)*a**2*e - 960*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**2*b*d - 13
44*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a**2*b*f)/(10080*b**2)
```

3.42 $\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 381

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\frac{afx^2\sqrt{a+bx^4}}{16b}+\frac{1}{8}fx^6\sqrt{a+bx^4}+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{x(5ae-21bcx^2)\sqrt{a+bx^4}}{105b}+\frac{d(a+bx^4)^{3/2}}{6b}+\frac{ex(a+bx^4)^{3/2}}{7b}-\frac{a^2f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{a^{5/4}\left(21\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/16*a*f*x^2*(b*x^4+a)^(1/2)/b+1/8*f*x^6*(b*x^4+a)^(1/2)+2/5*a*c*x*(b*x^4+
a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)-1/105*x*(-21*b*c*x^2+5*a*e)*(b*x^4+
a)^(1/2)/b+1/6*d*(b*x^4+a)^(3/2)/b+1/7*e*x*(b*x^4+a)^(3/2)/b-1/16*a^2*f*ar
ctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)-2/5*a^(5/4)*c*(a^(1/2)+b^(1/2)*
x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1
/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+1/105*a^(5/4)*(21*b^(
1/2)*c-5*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)
^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(5/4)
/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.51 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.48

$$\int x^2\left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{1}{336}\sqrt{a+bx^4}\left(\frac{56d(a+bx^4)}{b}+\frac{48ex(a+bx^4)}{b}+\frac{21fx^2(a+2bx^4)}{b}-\frac{21a^{3/2}f\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}\sqrt{1+\frac{bx^4}{a}}}-\frac{48aex\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{b\sqrt{1+\frac{bx^4}{a}}}+\frac{112cx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)$$

input
```
Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*((56*d*(a + b*x^4))/b + (48*e*x*(a + b*x^4))/b + (21*f*x^
2*(a + 2*b*x^4))/b - (21*a^(3/2)*f*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(3/2
)*Sqrt[1 + (b*x^4)/a]) - (48*a*e*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*
x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (112*c*x^3*Hypergeometric2F1[-1/2, 3/4
, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/336
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^2\sqrt{a+bx^4}(c+ex^2)+x^3\sqrt{a+bx^4}(d+fx^2)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{a^2f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16b^{3/2}}+\frac{1}{35}x^3\sqrt{a+bx^4}(7c+5ex^2)+\frac{2acx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\left(a+bx^4\right)^{3/2}}{6b}+\frac{2aex\sqrt{a+bx^4}}{21b}+\frac{fx^2\left(a+bx^4\right)^{3/2}}{8b}-\frac{afx^2\sqrt{a+bx^4}}{16b}$$

input
```
Int[x^2*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(2*a*e*x*Sqrt[a + b*x^4])/(21*b) - (a*f*x^2*Sqrt[a + b*x^4])/(16*b) + (2*a
*c*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*c + 5*
e*x^2)*Sqrt[a + b*x^4])/35 + (d*(a + b*x^4)^(3/2))/(6*b) + (f*x^2*(a + b*x
^4)^(3/2))/(8*b) - (a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*b^(3
/2)) - (2*a^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sq
rt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.69

method	result
risch	$\frac{\left(210 b f \,x^{6}+240 b e \,x^{5}+280 d b \,x^{4}+336 b c \,x^{3}+105 a f \,x^{2}+160 a e x +280 a d \right) \sqrt{b \,x^{4}+a}}{1680 b}-\frac{a \left(\frac{80 a e \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, \cdots\right.}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}} \cdots\right.}{\cdots}$
default	$c \left(\frac{x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{5 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}\, \sqrt{b}}\right)+\frac{d \left(b \,x^{4}+a \right)^{\frac{3}{2}}}{6 b}+e \left(\frac{x^{5} \cdots}{\cdots}\right.$
elliptic	$\frac{f \,x^{6} \sqrt{b \,x^{4}+a}}{8}+\frac{e \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{d \,x^{4} \sqrt{b \,x^{4}+a}}{6}+\frac{c \,x^{3} \sqrt{b \,x^{4}+a}}{5}+\frac{a f \,x^{2} \sqrt{b \,x^{4}+a}}{16 b}+\frac{2 a e x \sqrt{b \,x^{4}+a}}{21 b}+\frac{a d \sqrt{b \,x^{4}+a}}{6 b}-\frac{2 a^{2} e \cdots}{\cdots}$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1680*(210*b*f*x^6+240*b*e*x^5+280*b*d*x^4+336*b*c*x^3+105*a*f*x^2+160*a*
e*x+280*a*d)/b*(b*x^4+a)^(1/2)-1/840*a/b*(80*a*e/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+105/2*a*f*ln(b^(1/2)*x^2+(
b*x^4+a)^(1/2))/b^(1/2)-336*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.51

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$=\frac{1344\,ab^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+105\,a^2\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-64\,(21}{}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3360*(1344*a*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x),
 -1) + 105*a^2*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 -
a) - 64*(21*a*b*c + 5*a*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/
b)^(1/4)/x), -1) + 2*(210*b^2*f*x^7 + 240*b^2*e*x^6 + 280*b^2*d*x^5 + 336*
b^2*c*x^4 + 105*a*b*f*x^3 + 160*a*b*e*x^2 + 280*a*b*d*x + 672*a*b*c)*sqrt(
b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.56

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{a^{\frac{3}{2}}fx^2}{16b\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}fx^6}{16\sqrt{1+\frac{bx^4}{a}}} - \frac{a^2f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + d\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{bfx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
a**(3/2)*f*x**2/(16*b*sqrt(1 + b*x**4/a)) + sqrt(a)*c*x**3*gamma(3/4)*hype
r((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*
e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(9/4)) + 3*sqrt(a)*f*x**6/(16*sqrt(1 + b*x**4/a)) - a**2*f*asinh(sqrt
(b)*x**2/sqrt(a))/(16*b**(3/2)) + d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)),
((a + b*x**4)**(3/2)/(6*b), True)) + b*f*x**10/(8*sqrt(a)*sqrt(1 + b*x**4/
a))
```

Maxima [F]

$$\int x^2(c+dx+ex^2+fx^3)\,\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Giac [F]

$$\int x^2(c+dx+ex^2+fx^3)\,\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx+ex^2+fx^3)\,\sqrt{a+bx^4}\,dx=\int x^2\,\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^2*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int x^2(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$= \frac{560\sqrt{b\,x^4+a}\,abd+320\sqrt{b\,x^4+a}\,abex+210\sqrt{b\,x^4+a}\,abf\,x^2+672\sqrt{b\,x^4+a}\,b^2c\,x^3+560\sqrt{b\,x^4+a}\,b^2d}{}$$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

output
```
(560*sqrt(a + b*x**4)*a*b*d + 320*sqrt(a + b*x**4)*a*b*e*x + 210*sqrt(a +
b*x**4)*a*b*f*x**2 + 672*sqrt(a + b*x**4)*b**2*c*x**3 + 560*sqrt(a + b*x**
4)*b**2*d*x**4 + 480*sqrt(a + b*x**4)*b**2*e*x**5 + 420*sqrt(a + b*x**4)*b
**2*f*x**6 + 105*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*f - 105
*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a**2*f - 320*int(sqrt(a + b*
x**4)/(a + b*x**4),x)*a**2*b*e + 1344*int((sqrt(a + b*x**4)*x**2)/(a + b*x
**4),x)*a*b**2*c)/(3360*b**2)
```

3.43 $\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 356

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{x(5af-21bdx^2)\sqrt{a+bx^4}}{105b}+\frac{e(a+bx^4)^{3/2}}{6b}+\frac{fx(a+bx^4)^{3/2}}{7b}+\frac{a\text{carctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{a^{5/4}\left(21\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/4*c*x^2*(b*x^4+a)^(1/2)+2/5*a*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/
2)*x^2)-1/105*x*(-21*b*d*x^2+5*a*f)*(b*x^4+a)^(1/2)/b+1/6*e*(b*x^4+a)^(3/2
)/b+1/7*f*x*(b*x^4+a)^(3/2)/b+1/4*a*c*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))
/b^(1/2)-2/5*a^(5/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x
^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/
4)/(b*x^4+a)^(1/2)+1/105*a^(5/4)*(21*b^(1/2)*d-5*a^(1/2)*f)*(a^(1/2)+b^(1/
2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan
(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.59

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$=\frac{\sqrt{a+bx^4}\left(14ae\sqrt{1+\frac{bx^4}{a}}+12afx\sqrt{1+\frac{bx^4}{a}}+21bcx^2\sqrt{1+\frac{bx^4}{a}}+14bex^4\sqrt{1+\frac{bx^4}{a}}+12bfx^5\sqrt{1+\frac{bx^4}{a}}+\right.}{}$$

input
```
Integrate[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(14*a*e*Sqrt[1 + (b*x^4)/a] + 12*a*f*x*Sqrt[1 + (b*x^4)/a
] + 21*b*c*x^2*Sqrt[1 + (b*x^4)/a] + 14*b*e*x^4*Sqrt[1 + (b*x^4)/a] + 12*b
*f*x^5*Sqrt[1 + (b*x^4)/a] + 21*Sqrt[a]*Sqrt[b]*c*ArcSinh[(Sqrt[b]*x^2)/Sq
rt[a]] - 12*a*f*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 28*b*d
*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(84*b*Sqrt[1 + (b*x
^4)/a])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)\,dx$$
$$\Big\downarrow \text{2372}$$
$$\int \left(x\sqrt{a+bx^4}\left(c+ex^2\right)+x^2\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{5/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{ac\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{4}cx^2\sqrt{a+bx^4}+\frac{1}{35}x^3\sqrt{a+bx^4}(7d+5fx^2)+\frac{2adx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\left(a+bx^4\right)^{3/2}}{6b}+\frac{2afx\sqrt{a+bx^4}}{21b}$$

input

```
Int[x*(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output

```
(2*a*f*x*Sqrt[a + b*x^4])/(21*b) + (c*x^2*Sqrt[a + b*x^4])/4 + (2*a*d*x*Sq
rt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (x^3*(7*d + 5*f*x^2)*
Sqrt[a + b*x^4])/35 + (e*(a + b*x^4)^(3/2))/(6*b) + (a*c*ArcTanh[(Sqrt[b]*
x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (2*a^(5/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(5/4)*(21*Sqrt[b]*d - 5*
Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(5/4)*Sqrt[a +
b*x^4])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(60bf\,x^5+70be\,x^4+84bd\,x^3+105bc\,x^2+40af\,x+70ae)\sqrt{b\,x^4+a}}{420b} - \frac{a\left(\frac{20af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}} - \frac{105c\sqrt{b}\,\ln(\ldots}{\ldots}\right.}{\ldots}$
default	$c\left(\frac{x^2\sqrt{b\,x^4+a}}{4}+\frac{a\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{4\sqrt{b}}\right)+d\left(\frac{x^3\sqrt{b\,x^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{Ellip}\ldots\right.}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right.$
elliptic	$\frac{x^5f\sqrt{b\,x^4+a}}{7}+\frac{e\,x^4\sqrt{b\,x^4+a}}{6}+\frac{d\,x^3\sqrt{b\,x^4+a}}{5}+\frac{c\,x^2\sqrt{b\,x^4+a}}{4}+\frac{2afx\sqrt{b\,x^4+a}}{21b}+\frac{ae\sqrt{b\,x^4+a}}{6b}-\frac{2a^2f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\ldots}}{21b\sqrt{\ldots}}$

input
```
int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/420*(60*b*f*x^5+70*b*e*x^4+84*b*d*x^3+105*b*c*x^2+40*a*f*x+70*a*e)/b*(b*
x^4+a)^(1/2)-1/210*a/b*(20*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-105/2*c*b^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/
2))-84*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.48

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{336\,a\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+105\,a\sqrt{b}cx\log\left(-2\,bx^4-2\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-16\,(21\,a\ldots}{\ldots}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/840*(336*a*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -
1) + 105*a*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) -
 16*(21*a*d + 5*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)
/x), -1) + 2*(60*b*f*x^6 + 70*b*e*x^5 + 84*b*d*x^4 + 105*b*c*x^3 + 40*a*f*
x^2 + 70*a*e*x + 168*a*d)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.44

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{\sqrt{a}cx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{a}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{ac\,\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + e\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*c*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*d*x**3*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*f*x**5
*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(
9/4)) + a*c*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + e*Piecewise((sqrt(a)
*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Maxima [F]

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^
2))/sqrt(b) + 2*sqrt(b*x^4 + a)*a/((b - (b*x^4 + a)/x^4)*x^2))*c + integra
te(sqrt(b*x^4 + a)*(f*x^4 + e*x^3 + d*x^2), x)
```

Giac [F]

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}(fx^3+ex^2+dx+c)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int x\sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x*(a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int x(c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$=\frac{140\sqrt{b\,x^4+a}\,ae+80\sqrt{b\,x^4+a}\,afx+210\sqrt{b\,x^4+a}\,bc\,x^2+168\sqrt{b\,x^4+a}\,bd\,x^3+140\sqrt{b\,x^4+a}\,be\,x^4+}{}$$

input
```
int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

output
```
(140*sqrt(a + b*x**4)*a*e + 80*sqrt(a + b*x**4)*a*f*x + 210*sqrt(a + b*x**
4)*b*c*x**2 + 168*sqrt(a + b*x**4)*b*d*x**3 + 140*sqrt(a + b*x**4)*b*e*x**
4 + 120*sqrt(a + b*x**4)*b*f*x**5 - 105*sqrt(b)*log(sqrt(a + b*x**4) - sqr
t(b)*x**2)*a*c + 105*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*c - 80
*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**2*f + 336*int((sqrt(a + b*x**4)*x
**2)/(a + b*x**4),x)*a*b*d)/(840*b)
```

3.44 $\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 331

$$\begin{aligned}\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx &= \frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{15}x(5c+3ex^2)\sqrt{a+bx^4}+\frac{f(a+bx^4)^{3/2}}{6b}\\ &+\frac{ad\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}\\ &+\frac{a^{3/4}\left(5\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/4*d*x^2*(b*x^4+a)^(1/2)+2/5*a*e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/
2)*x^2)+1/15*x*(3*e*x^2+5*c)*(b*x^4+a)^(1/2)+1/6*f*(b*x^4+a)^(3/2)/b+1/4*a
*d*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/2)-2/5*a^(5/4)*e*(a^(1/2)+b^(
1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan
(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+1/15*a^(3/4)*(5*
b^(1/2)*c+3*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x
^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(3
/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.52

$$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx$$
$$=\frac{\sqrt{a+bx^4}\left(2af\sqrt{1+\frac{bx^4}{a}}+3bdx^2\sqrt{1+\frac{bx^4}{a}}+2bfx^4\sqrt{1+\frac{bx^4}{a}}+3\sqrt{a}\sqrt{b}d\,\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+12bcx\,\text{Hyperg}\right.}{12b\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(2*a*f*Sqrt[1 + (b*x^4)/a] + 3*b*d*x^2*Sqrt[1 + (b*x^4)/a
] + 2*b*f*x^4*Sqrt[1 + (b*x^4)/a] + 3*Sqrt[a]*Sqrt[b]*d*ArcSinh[(Sqrt[b]*x
^2)/Sqrt[a]] + 12*b*c*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)] +
4*b*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)]))/(12*b*Sqrt[1 +
 (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^4}(c+dx+ex^2+fx^3)\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(\sqrt{a+bx^4}(c+ex^2)+x\sqrt{a+bx^4}(d+fx^2)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}+\frac{ad\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{15}x\sqrt{a+bx^4}(5c+3ex^2)+\frac{1}{4}dx^2\sqrt{a+bx^4}+\frac{2aex\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\left(a+bx^4\right)^{3/2}}{6b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4],x]
```

output
```
(d*x^2*Sqrt[a + b*x^4])/4 + (2*a*e*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a]
+ Sqrt[b]*x^2)) + (x*(5*c + 3*e*x^2)*Sqrt[a + b*x^4])/15 + (f*(a + b*x^4)^
(3/2))/(6*b) + (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) -
(2*a^(5/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a +
b*x^4]) + (a^(3/4)*(5*Sqrt[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(10bf\,x^4+12be\,x^3+15bd\,x^2+20cbx+10af)\sqrt{b\,x^4+a}}{60b}+\frac{a\left(\frac{20c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{15d\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}\right.}{}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(\frac{x^2\sqrt{b\,x^4+a}}{4}+\frac{a\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{4\sqrt{b}}\right)+e\left(\right.$
elliptic	$\frac{f\,x^4\sqrt{b\,x^4+a}}{6}+\frac{x^3e\sqrt{b\,x^4+a}}{5}+\frac{d\,x^2\sqrt{b\,x^4+a}}{4}+\frac{cx\sqrt{b\,x^4+a}}{3}+\frac{af\sqrt{b\,x^4+a}}{6b}+\frac{2ac\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(10*b*f*x^4+12*b*e*x^3+15*b*d*x^2+20*b*c*x+10*a*f)/b*(b*x^4+a)^(1/2)+
1/30*a*(20*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2
))^(1/2),I)+15/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+12*I*e*a^(1/2)/
(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.49

$$\int \left(c+dx+ex^2+fx^3\right)\sqrt{a+bx^4}\,dx$$
$$=\frac{48\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+15\,a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+16\,(5\,bc-}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(48*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1
) + 15*a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 1
6*(5*b*c - 3*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) + 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 + 10*a*f*x +
24*a*e)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.47

$$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx = \frac{\sqrt{a}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{a}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + f\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*e*x**3*ga
mma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4
)) + a*d*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + f*Piecewise((sqrt(a)*x*
*4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True))
```

Maxima [F]

$$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Giac [F]

$$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{bx^4+a}\,(fx^3+ex^2+dx+c)\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx+ex^2+fx^3)\sqrt{a+bx^4}\,dx=\int \sqrt{b\,x^4+a}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int \left(c + dx + ex^2 + fx^3\right) \sqrt{a + bx^4} \, dx$$
$$= \frac{20\sqrt{b\,x^4 + a}\,af + 40\sqrt{b\,x^4 + a}\,bcx + 30\sqrt{b\,x^4 + a}\,bd\,x^2 + 24\sqrt{b\,x^4 + a}\,be\,x^3 + 20\sqrt{b\,x^4 + a}\,bf\,x^4 - 15\sqrt{b}}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2),x)
```

output
```
(20*sqrt(a + b*x**4)*a*f + 40*sqrt(a + b*x**4)*b*c*x + 30*sqrt(a + b*x**4)
*b*d*x**2 + 24*sqrt(a + b*x**4)*b*e*x**3 + 20*sqrt(a + b*x**4)*b*f*x**4 -
15*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*d + 15*sqrt(b)*log(sqrt(
a + b*x**4) + sqrt(b)*x**2)*a*d + 80*int(sqrt(a + b*x**4)/(a + b*x**4),x)*
a*b*c + 48*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a*b*e)/(120*b)
```

3.45 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 345

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx \\
&= \frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}(2c+ex^2)\sqrt{a+bx^4}+\frac{1}{15}x(5d+3fx^2)\sqrt{a+bx^4} \\
&+\frac{a\text{earctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{1}{2}\sqrt{a}c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}} \\
&+\frac{a^{3/4}\left(5\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
2/5*a*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)+1/4*(e*x^2+2*c)*(b
*x^4+a)^(1/2)+1/15*x*(3*f*x^2+5*d)*(b*x^4+a)^(1/2)+1/4*a*e*arctanh(b^(1/2)
*x^2/(b*x^4+a)^(1/2))/b^(1/2)-1/2*a^(1/2)*c*arctanh((b*x^4+a)^(1/2)/a^(1/2
))-2/5*a^(5/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*
x^4+a)^(1/2)+1/15*a^(3/4)*(5*b^(1/2)*d+3*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*
((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)
*x/a^(1/4)),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx$$

$$= \frac{3a^{3/2}e\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3\sqrt{b}\left((2c+ex^2)(a+bx^4)-2\sqrt{a}c\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)+12a\sqrt{}}{12\sqrt{}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

output
```
(3*a^(3/2)*e*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*Sqrt[b
]*((2*c + e*x^2)*(a + b*x^4) - 2*Sqrt[a]*c*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a
+ b*x^4]/Sqrt[a]]) + 12*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F
1[-1/2, 1/4, 5/4, -((b*x^4)/a)] + 4*a*Sqrt[b]*f*x^3*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[-1/2, 3/4, 7/4, -((b*x^4)/a)])/(12*Sqrt[b]*Sqrt[a + b*x^4]
)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x} \, dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{\sqrt{a+bx^4}\left(c+ex^2\right)}{x}+\sqrt{a+bx^4}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{2a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{ae\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}+\frac{1}{4}\sqrt{a+bx^4}\left(2c+ex^2\right)+\frac{1}{15}x\sqrt{a+bx^4}\left(5d+3fx^2\right)+\frac{2afx\sqrt{a+bx^4}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x,x]
```

output
```
(2*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*c + e*
x^2)*Sqrt[a + b*x^4])/4 + (x*(5*d + 3*f*x^2)*Sqrt[a + b*x^4])/15 + (a*e*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*c*ArcTanh[Sqr
t[a + b*x^4]/Sqrt[a]])/2 - (2*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (a^(3/4)*(5*Sqrt[b]*d + 3*Sqrt[a]*f)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

method	result
elliptic	$\frac{f\,x^{3}\sqrt{b\,x^{4}+a}}{5}+\frac{e\,x^{2}\sqrt{b\,x^{4}+a}}{4}+\frac{dx\sqrt{b\,x^{4}+a}}{3}+\frac{c\sqrt{b\,x^{4}+a}}{2}+\frac{2ad\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{ae\ln\left(2\sqrt{\right.}}{}$
default	$d\left(\frac{x\sqrt{b\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+e\left(\frac{x^{2}\sqrt{b\,x^{4}+a}}{4}+\frac{a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4\sqrt{b}}\right)+f\,($

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*f*x^3*(b*x^4+a)^(1/2)+1/4*e*x^2*(b*x^4+a)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2
)+1/2*c*(b*x^4+a)^(1/2)+2/3*a*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*a*e*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))
/b^(1/2)+2/5*I*a^(3/2)*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
-1/2*a^(1/2)*c*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = -\frac{\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{\sqrt{a}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ac}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + \frac{\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x,x)
```

output
```
-sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*d*x*gamma(1/4)*hyper(
(-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*e*
x**2*sqrt(1 + b*x**4/a)/4 + sqrt(a)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (
7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a*c/(2*sqrt(b)*x**2*sqrt
(a/(b*x**4) + 1)) + a*e*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*
c*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x}\,dx = \frac{60\sqrt{b\,x^4+a}\,bc+40\sqrt{b\,x^4+a}\,bdx+30\sqrt{b\,x^4+a}\,be\,x^2+24\sqrt{b\,x^4+a}\,bf\,x^3+30\sqrt{a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right.}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x,x)
```

output
```
(60*sqrt(a + b*x**4)*b*c + 40*sqrt(a + b*x**4)*b*d*x + 30*sqrt(a + b*x**4)
*b*e*x**2 + 24*sqrt(a + b*x**4)*b*f*x**3 + 30*sqrt(a)*log(sqrt(a + b*x**4)
 - sqrt(a))*b*c - 30*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*c - 15*sqrt
(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*e + 15*sqrt(b)*log(sqrt(a + b*x
**4) + sqrt(b)*x**2)*a*e + 80*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*b*d +
 48*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a*b*f)/(120*b)
```

3.46 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 341

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx \\
&= \frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2} - \frac{(3c-ex^2)\sqrt{a+bx^4}}{3x} + \frac{1}{4}(2d+fx^2)\sqrt{a+bx^4} \\
&\quad + \frac{af\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}} - \frac{1}{2}\sqrt{a}d\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad - \frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
2*b^(1/2)*c*x*(b*x^4+a)^(1/2)/(a^(1/2)+b^(1/2)*x^2)-1/3*(-e*x^2+3*c)*(b*x^
4+a)^(1/2)/x+1/4*(f*x^2+2*d)*(b*x^4+a)^(1/2)+1/4*a*f*arctanh(b^(1/2)*x^2/(
b*x^4+a)^(1/2))/b^(1/2)-1/2*a^(1/2)*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))-2*a
^(1/4)*b^(1/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(b*x^4+a)^(
1/2)+1/3*a^(1/4)*(3*b^(1/2)*c+a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/
(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4))
,1/2*2^(1/2))/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \frac{-4\sqrt{b}c\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)+x\left(\sqrt{a}f\sqrt{a+bx^4}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+\sqrt{b}\sqrt{1+\cdots}\right.}{4\sqrt{b}\cdots}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

output
```
(-4*Sqrt[b]*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^4)
/a)] + x*(Sqrt[a]*f*Sqrt[a + b*x^4]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[
b]*Sqrt[1 + (b*x^4)/a]*((2*d + f*x^2)*Sqrt[a + b*x^4] - 2*Sqrt[a]*d*ArcTan
h[Sqrt[a + b*x^4]/Sqrt[a]]) + 4*Sqrt[b]*e*x*Sqrt[a + b*x^4]*Hypergeometric
2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)]))/(4*Sqrt[b]*x*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^2}\,dx$$

$$\Big\downarrow \ 2372$$

$$\int \left(\frac{\sqrt{a+bx^4}(c+ex^2)}{x^2}+\frac{\sqrt{a+bx^4}(d+fx^2)}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}-\frac{1}{2}\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{af\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4\sqrt{b}}-\frac{\sqrt{a+bx^4}(3c-ex^2)}{3x}+\frac{2\sqrt{b}cx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}+\frac{1}{4}\sqrt{a+bx^4}(2d+fx^2)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^2,x]
```

output
```
(2*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((3*c - e*x^2)*S
qrt[a + b*x^4])/(3*x) + ((2*d + f*x^2)*Sqrt[a + b*x^4])/4 + (a*f*ArcTanh[(
Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*Sqrt[b]) - (Sqrt[a]*d*ArcTanh[Sqrt[a + b
*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.80

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{x}+\frac{f\,x^2\sqrt{b\,x^4+a}}{4}+\frac{e x\sqrt{b\,x^4+a}}{3}+\frac{d\sqrt{b\,x^4+a}}{2}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+ af\ln\left(2\sqrt{\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{x}+\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sqrt{a}\,d\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}+\frac{af\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{4\sqrt{b}}$
default	$e\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+c\left(-\frac{\sqrt{b\,x^4+a}}{x}+ 2i\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*c*(b*x^4+a)^(1/2)+1/4*f*x^2*(b*x^4+a)^(1/2)+1/3*e*x*(b*x^4+a)^(1/2)+1
/2*d*(b*x^4+a)^(1/2)+2/3*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)+1/4*a*f*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))/b^
(1/2)+2*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(
I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*
a^(1/2)*d*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{ad}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + \frac{\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**2,x)
```

output
```
sqrt(a)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x*gamma(3/4)) - sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*e
*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamm
a(5/4)) + sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/4 + a*d/(2*sqrt(b)*x**2*sqrt(a
/(b*x**4) + 1)) + a*f*asinh(sqrt(b)*x**2/sqrt(a))/(4*sqrt(b)) + sqrt(b)*d*
x**2/(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^2}\,dx = \frac{24\sqrt{b\,x^4+a}\,bc+12\sqrt{b\,x^4+a}\,bdx+8\sqrt{b\,x^4+a}\,be\,x^2+6\sqrt{b\,x^4+a}\,bf\,x^3+6\sqrt{a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)bc}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^2,x)
```

output
```
(24*sqrt(a + b*x**4)*b*c + 12*sqrt(a + b*x**4)*b*d*x + 8*sqrt(a + b*x**4)*
b*e*x**2 + 6*sqrt(a + b*x**4)*b*f*x**3 + 6*sqrt(a)*log(sqrt(a + b*x**4) -
sqrt(a))*b*d*x - 6*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*d*x - 3*sqrt(
b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*f*x + 3*sqrt(b)*log(sqrt(a + b*x
**4) + sqrt(b)*x**2)*a*f*x + 48*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*
a*b*c*x + 16*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*b*e*x)/(24*b*x)
```

3.47 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 342

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx \\
&= \frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}-\frac{(c-ex^2)\sqrt{a+bx^4}}{2x^2}-\frac{(3d-fx^2)\sqrt{a+bx^4}}{3x} \\
&\quad+\frac{1}{2}\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&\quad-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad+\frac{\sqrt[4]{a}\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
2*b^(1/2)*d*x*(b*x^4+a)^(1/2)/(a^(1/2)+b^(1/2)*x^2)-1/2*(-e*x^2+c)*(b*x^4+
a)^(1/2)/x^2-1/3*(-f*x^2+3*d)*(b*x^4+a)^(1/2)/x+1/2*b^(1/2)*c*arctanh(b^(1
/2)*x^2/(b*x^4+a)^(1/2))-1/2*a^(1/2)*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))-2*
a^(1/4)*b^(1/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(b*x^4+a)^
(1/2)+1/3*a^(1/4)*(3*b^(1/2)*d+a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)
/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)
),1/2*2^(1/2))/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx$$
$$= \frac{-ac+aex^2-bcx^4+bex^6+\sqrt{a}\sqrt{b}cx^2\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)-\sqrt{a}ex^2\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-2}{2x^2}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

output
```
(-(a*c) + a*e*x^2 - b*c*x^4 + b*e*x^6 + Sqrt[a]*Sqrt[b]*c*x^2*Sqrt[1 + (b*
x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*e*x^2*Sqrt[a + b*x^4]*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)] + 2*a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hyperge
ometric2F1[-1/2, 1/4, 5/4, -((b*x^4)/a)])/(2*x^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^3}\,dx$$
$$\Big\downarrow \text{2372}$$
$$\int \left(\frac{\sqrt{a+bx^4}\left(c+ex^2\right)}{x^3}+\frac{\sqrt{a+bx^4}\left(d+fx^2\right)}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{1}{2}\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^4}\left(c-ex^2\right)}{2x^2}-\frac{\sqrt{a+bx^4}\left(3d-fx^2\right)}{3x}+\frac{2\sqrt{b}dx\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^3,x]
```

output
```
(2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - ((c - e*x^2)*Sqr
t[a + b*x^4])/(2*x^2) - ((3*d - f*x^2)*Sqrt[a + b*x^4])/(3*x) + (Sqrt[b]*c
*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 - (Sqrt[a]*e*ArcTanh[Sqrt[a + b
*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*(3*Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(2dx+c)}{2x^2}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sqrt{a}\,e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}+\frac{e\sqrt{b\,x^4+a}}{2}+\frac{c\sqrt{}}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{2x^2}-\frac{d\sqrt{b\,x^4+a}}{x}+\frac{fx\sqrt{b\,x^4+a}}{3}+\frac{e\sqrt{b\,x^4+a}}{2}+\frac{2af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{c\sqrt{b}\,\ln\left(2\sqrt{}\right.}{}$
default	$f\left(\frac{x\sqrt{b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+c\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\,x^2\sqrt{b\,x^4+a}}{2a}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,x^2\right.}{2}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^4+a)^(1/2)*(2*d*x+c)/x^2+2/3*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*a^(1/2)*e*ln((2*a+2*a^(1/2)*
(b*x^4+a)^(1/2))/x^2)+1/2*e*(b*x^4+a)^(1/2)+1/2*c*b^(1/2)*ln(b^(1/2)*x^2+(
b*x^4+a)^(1/2))+1/3*f*x*(b*x^4+a)^(1/2)+2*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*
(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx=-\frac{\sqrt{a}c}{2x^2\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}-\frac{\sqrt{a}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{\sqrt{a}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{ae}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}}+\frac{\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}+\frac{\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}}-\frac{bcx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**3,x)
```

output
```
-sqrt(a)*c/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*d*gamma(-1/4)*hyper((-1/2
, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - sqrt(a)*e*as
inh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*f*x*gamma(1/4)*hyper((-1/2, 1/4),
(5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a*e/(2*sqrt(b)*x**2*sqr
t(a/(b*x**4) + 1)) + sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)*e*x
**2/(2*sqrt(a/(b*x**4) + 1)) - b*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}}{x^3}\,dx = \int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Giac [F]

$$\int \frac{(c + dx + ex^2 + fx^3)\sqrt{a + bx^4}}{x^3}\,dx = \int \frac{\sqrt{bx^4 + a}(fx^3 + ex^2 + dx + c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^3}\,dx = \frac{-6\sqrt{b\,x^4+a}\,c+12\sqrt{b\,x^4+a}\,dx+6\sqrt{b\,x^4+a}\,e\,x^2+4\sqrt{b\,x^4+a}\,f\,x^3+3\sqrt{a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)e\,x^2 -}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^3,x)
```

output
```
( - 6*sqrt(a + b*x**4)*c + 12*sqrt(a + b*x**4)*d*x + 6*sqrt(a + b*x**4)*e*
x**2 + 4*sqrt(a + b*x**4)*f*x**3 + 3*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a
))*e*x**2 - 3*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*e*x**2 - 3*sqrt(b)*l
og(sqrt(a + b*x**4) - sqrt(b)*x**2)*c*x**2 + 3*sqrt(b)*log(sqrt(a + b*x**4
) + sqrt(b)*x**2)*c*x**2 + 24*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a*
d*x**2 + 8*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*f*x**2)/(12*x**2)
```

3.48 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 342

$$\begin{aligned}
&\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx \\
&= -\frac{2\sqrt{a}e\sqrt{a+bx^4}}{x\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{(c-3ex^2)\sqrt{a+bx^4}}{3x^3}-\frac{(d-fx^2)\sqrt{a+bx^4}}{2x^2} \\
&+\frac{1}{2}\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}f\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) \\
&-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&+\frac{\sqrt[4]{b}\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-2*a^(1/2)*e*(b*x^4+a)^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)-1/3*(-3*e*x^2+c)*(b*x
^4+a)^(1/2)/x^3-1/2*(-f*x^2+d)*(b*x^4+a)^(1/2)/x^2+1/2*b^(1/2)*d*arctanh(b
^(1/2)*x^2/(b*x^4+a)^(1/2))-1/2*a^(1/2)*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))
-2*a^(1/4)*b^(1/4)*e*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2
)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(b*x^4+
a)^(1/2)+1/3*b^(1/4)*(b^(1/2)*c+3*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4
+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1
/4)),1/2*2^(1/2))/a^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$$

$$= \frac{-2ac\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right)+3x\left(-ad+afx^2-bdx^4+bfx^6+\sqrt{a}\sqrt{b}dx^2\sqrt{\right.}{}}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

output
```
(-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^4)/a
)] + 3*x*(-(a*d) + a*f*x^2 - b*d*x^4 + b*f*x^6 + Sqrt[a]*Sqrt[b]*d*x^2*Sqr
t[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - Sqrt[a]*f*x^2*Sqrt[a + b
*x^4]*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]] - 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[-1/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^4}dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{\sqrt{a+bx^4}(c+ex^2)}{x^4}+\frac{\sqrt{a+bx^4}(d+fx^2)}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}}+\frac{1}{2}\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}\sqrt{a}f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^4}(c-3ex^2)}{3x^3}-\frac{\sqrt{a+bx^4}(d-fx^2)}{2x^2}-\frac{2e\sqrt{a+bx^4}}{x}+\frac{2\sqrt{b}ex\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^4,x]
```

output
```
(-2*e*Sqrt[a + b*x^4])/x + (2*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt
[b]*x^2) - ((c - 3*e*x^2)*Sqrt[a + b*x^4])/(3*x^3) - ((d - f*x^2)*Sqrt[a +
 b*x^4])/(2*x^2) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 -
(Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (2*a^(1/4)*b^(1/4)*e*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (b^(1/4)*(Sqrt[b]*c
 + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a
 + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6x^3}+\frac{2cb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sqrt{a}\,f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}+\frac{\sqrt{b}\,d\ln(\sqrt{}}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{3x^3}-\frac{d\sqrt{b\,x^4+a}}{2x^2}-\frac{e\sqrt{b\,x^4+a}}{x}+\frac{f\sqrt{b\,x^4+a}}{2}+\frac{2cb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\sqrt{b}\,d\ln\left(2\sqrt{b}\right.}{}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\,x^2\sqrt{b\,x^4+a}}{2a}+\frac{\sqrt{b}\,\ln\left(\sqrt{b}\,x^2\right.}{2}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^4+a)^(1/2)*(6*e*x^2+3*d*x+2*c)/x^3+2/3*c*b/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*a^(1/2)*f*ln((2*a+
2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+1/2*b^(1/2)*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/
2))+2*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*f*
(b*x^4+a)^(1/2)
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.69

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}d}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{a}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{af}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{bdx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**4,x)
```

output
```
sqrt(a)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - sqrt(a)*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*e*
gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - sqrt(a)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a*f/(2*sqrt(b)*x**2
*sqrt(a/(b*x**4) + 1)) + sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/2 + sqrt(b)
*f*x**2/(2*sqrt(a/(b*x**4) + 1)) - b*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^4}\,dx$$
$$= \frac{-4\sqrt{b\,x^4+a}\,c-2\sqrt{b\,x^4+a}\,dx+4\sqrt{b\,x^4+a}\,e\,x^2+2\sqrt{b\,x^4+a}\,f\,x^3+\sqrt{a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)f\,x^3-}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x**4)*c - 2*sqrt(a + b*x**4)*d*x + 4*sqrt(a + b*x**4)*e*x
**2 + 2*sqrt(a + b*x**4)*f*x**3 + sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*
f*x**3 - sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*f*x**3 - sqrt(b)*log(sqrt
(a + b*x**4) - sqrt(b)*x**2)*d*x**3 + sqrt(b)*log(sqrt(a + b*x**4) + sqrt(
b)*x**2)*d*x**3 - 8*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a*c*x**3 + 8
*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a*e*x**3)/(4*x**3)
```

3.49 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 343

$$\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx &= -\frac{2\sqrt{a}f\sqrt{a+bx^4}}{x\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{(c+2ex^2)\sqrt{a+bx^4}}{4x^4}-\frac{(d-3fx^2)\sqrt{a+bx^4}}{3x^3} \\
&\quad+\frac{1}{2}\sqrt{b}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{bc\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} \\
&\quad-\frac{2\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+bx^4}} \\
&\quad+\frac{\sqrt[4]{b}\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
-2*a^(1/2)*f*(b*x^4+a)^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)-1/4*(2*e*x^2+c)*(b*x^
4+a)^(1/2)/x^4-1/3*(-3*f*x^2+d)*(b*x^4+a)^(1/2)/x^3+1/2*b^(1/2)*e*arctanh(
b^(1/2)*x^2/(b*x^4+a)^(1/2))-1/4*b*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1
/2)-2*a^(1/4)*b^(1/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*
x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(b*x
^4+a)^(1/2)+1/3*b^(1/4)*(b^(1/2)*d+3*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*
x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a
^(1/4)),1/2*2^(1/2))/a^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = -\frac{\sqrt{1+\frac{bx^4}{a}}\left(3ac\sqrt{1+\frac{bx^4}{a}}+6aex^2\sqrt{1+\frac{bx^4}{a}}-6\sqrt{a}\sqrt{b}ex^4\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3bcx^4\operatorname{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)\right.}{12x^4\sqrt{a}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

output
```
-1/12*(Sqrt[1 + (b*x^4)/a]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 +
 (b*x^4)/a] - 6*Sqrt[a]*Sqrt[b]*e*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b
*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2F1[-3/4, -1/2
, 1/4, -((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*
x^4)/a)]))/(x^4*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2364, 27, 2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^5}dx$$

$$\Big\downarrow \text{2364}$$

$$-2b\int -\frac{12fx^3+6ex^2+4dx+3c}{12x\sqrt{bx^4+a}}dx-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}b\int \frac{12fx^3+6ex^2+4dx+3c}{x\sqrt{bx^4+a}}dx-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \text{2371}$$

$$\frac{1}{6}b\left(3c\int \frac{1}{x\sqrt{bx^4+a}}dx+\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{6}b\left(\frac{3}{4}c\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4+\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}b\left(\frac{3c\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2b}+\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}b\left(\int \frac{12fx^2+6ex+4d}{\sqrt{bx^4+a}}dx-\frac{3c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \text{2424}$$

$$\frac{1}{6}b\left(\int \left(\frac{6ex}{\sqrt{bx^4+a}}+\frac{12fx^2+4d}{\sqrt{bx^4+a}}\right)dx-\frac{3c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$\downarrow$ 2009

$$\frac{1}{6}b\left(\frac{2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{}{(}}}{b^3}\right.$$

$$\frac{1}{12}\sqrt{a+bx^4}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^5,x]
```

output
```
-1/12*(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*Sqrt[a + b*x^4]) + (
b*((12*f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (3*e*ArcTa
nh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/Sqrt[b] - (3*c*ArcTanh[Sqrt[a + b*x^4]/
Sqrt[a]])/(2*Sqrt[a]) - (12*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (2*(Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*b^(3/4)*Sqrt[a + b*x^4])))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(12 f\,x^3+6 e\,x^2+4 d x+3 c)}{12 x^4}+\frac{b\left(\frac{4 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2\sqrt{a}}+\frac{3e\ln\left(\sqrt{b}\,x^2+\cdots\right.}{\sqrt{b}\cdots}\cdots\right.}{6\cdots}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{4x^4}-\frac{d\sqrt{b\,x^4+a}}{3x^3}-\frac{e\sqrt{b\,x^4+a}}{2x^2}-\frac{f\sqrt{b\,x^4+a}}{x}+\frac{2bd\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{\sqrt{b}\,e\ln\left(2\sqrt{b}\cdots\right.}{\cdots}$
default	$c\left(-\frac{(b\,x^4+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{4\sqrt{a}}+\frac{b\sqrt{b\,x^4+a}}{4a}\right)+d\left(-\frac{\sqrt{b\,x^4+a}}{3x^3}+\frac{2b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\cdots}\right.}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\cdots\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^4+a)^(1/2)*(12*f*x^3+6*e*x^2+4*d*x+3*c)/x^4+1/6*b*(4*d/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*c/a^(
1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+3*e*ln(b^(1/2)*x^2+(b*x^4+a)^
(1/2))/b^(1/2)+12*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}e}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{\sqrt{b}e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} - \frac{bex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**5,x)
```

output
```
sqrt(a)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**3*gamma(1/4)) - sqrt(a)*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*f*
gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gam
ma(3/4)) - sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*e*asinh(sqrt(
b)*x**2/sqrt(a))/2 - b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) - b*e*x
**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/8*(b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sqrt(a
) - 2*sqrt(b*x^4 + a)/x^4)*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x + d)
/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^5}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^5}\,dx$$
$$= \frac{-2\sqrt{b\,x^4+a}\,ac-8\sqrt{b\,x^4+a}\,adx-4\sqrt{b\,x^4+a}\,ae\,x^2+8\sqrt{b\,x^4+a}\,af\,x^3+\sqrt{a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)bc}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^5,x)
```

output
```
( - 2*sqrt(a + b*x**4)*a*c - 8*sqrt(a + b*x**4)*a*d*x - 4*sqrt(a + b*x**4)
*a*e*x**2 + 8*sqrt(a + b*x**4)*a*f*x**3 + sqrt(a)*log(sqrt(a + b*x**4) - s
qrt(a))*b*c*x**4 - sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*c*x**4 - 2*sq
rt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*e*x**4 + 2*sqrt(b)*log(sqrt(a
 + b*x**4) + sqrt(b)*x**2)*a*e*x**4 - 16*int(sqrt(a + b*x**4)/(a*x**4 + b*
x**8),x)*a**2*d*x**4 + 16*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a**2*f
*x**4)/(8*a*x**4)
```

3.50 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 368

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx &= \frac{2e\sqrt{a+bx^4}}{3x^3}-\frac{2bc\sqrt{a+bx^4}}{5\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{(c+5ex^2)\sqrt{a+bx^4}}{5x^5}\\ &\quad-\frac{(d+2fx^2)\sqrt{a+bx^4}}{4x^4}+\frac{1}{2}\sqrt{b}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}\\ &\quad-\frac{2b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}\\ &\quad+\frac{b^{3/4}\left(3\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
2/3*e*(b*x^4+a)^(1/2)/x^3-2/5*b*c*(b*x^4+a)^(1/2)/a^(1/2)/x/(a^(1/2)+b^(1/
2)*x^2)-1/5*(5*e*x^2+c)*(b*x^4+a)^(1/2)/x^5-1/4*(2*f*x^2+d)*(b*x^4+a)^(1/2
)/x^4+1/2*b^(1/2)*f*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-1/4*b*d*arctanh((
b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-2/5*b^(5/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^
4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)+1/15*b^(3/4)*(3*b^(1/2)*c+5*a^(1
/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Inv
erseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.49

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = -\frac{\sqrt{a+bx^4}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5x\left(3ad\sqrt{1+\frac{bx^4}{a}}+6afx^2\sqrt{1+\frac{bx^4}{a}}-6\cdots\right.\right.}{60ax^5\cdots}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

output
```
-1/60*(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^
4)/a)] + 5*x*(3*a*d*Sqrt[1 + (b*x^4)/a] + 6*a*f*x^2*Sqrt[1 + (b*x^4)/a] -
6*Sqrt[a]*Sqrt[b]*f*x^4*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + 3*b*d*x^4*ArcTanh
[Sqrt[1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^
4)/a)])))/(a*x^5*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^6} \, dx$$

$$\Big\downarrow \text{ 2364}$$

$$-2b\int -\frac{30fx^3+20ex^2+15dx+12c}{60x^2\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{30}b\int \frac{30fx^3+20ex^2+15dx+12c}{x^2\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{30}b\int \left(\frac{20ex^2+12c}{x^2\sqrt{bx^4+a}}+\frac{30fx^2+15d}{x\sqrt{bx^4+a}}\right)dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{30}b\left(\frac{2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\ldots}}{\ldots}\right.$$
$$\frac{1}{60}\sqrt{a+bx^4}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^6,x]
```

output
```
-1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*Sqrt[a + b*x^4]
) + (b*((-12*c*Sqrt[a + b*x^4])/(a*x) + (12*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(
a*(Sqrt[a] + Sqrt[b]*x^2)) + (15*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])
/Sqrt[b] - (15*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (12*b^(1/
4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) +
(2*(3*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a
^(3/4)*b^(1/4)*Sqrt[a + b*x^4])))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(24bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60x^5a}+\frac{b\left(\frac{20ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+12ic\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}}{\cdots}}\cdots\right.}{\cdots}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5x^5}-\frac{d\sqrt{b\,x^4+a}}{4x^4}-\frac{e\sqrt{b\,x^4+a}}{3x^3}-\frac{f\sqrt{b\,x^4+a}}{2x^2}-\frac{2bc\sqrt{b\,x^4+a}}{5ax}+\frac{2be\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\cdots$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{5x^5}-\frac{2b\sqrt{b\,x^4+a}}{5ax}+\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\cdots\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(b*x^4+a)^(1/2)*(24*b*c*x^4+30*a*f*x^3+20*a*e*x^2+15*a*d*x+12*a*c)/x
^5/a+1/30*b/a*(20*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)+12*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2
)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I))-15/2*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+15*a*f*
ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,\sqrt{a+bx^4}}{x^6}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.88 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}f}{2x^2\sqrt{1+\frac{bx^4}{a}}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{\sqrt{b}f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} - \frac{bfx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**6,x)
```

output
```
sqrt(a)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + sqrt(a)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*f/(2*x**2*sqrt(1
 + b*x**4/a)) - sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) + sqrt(b)*f*asinh(
sqrt(b)*x**2/sqrt(a))/2 - b*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a)) -
b*f*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^6}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^6}\,dx$$
$$= \frac{-8\sqrt{b\,x^4+a}\,ac-6\sqrt{b\,x^4+a}\,adx-24\sqrt{b\,x^4+a}\,ae\,x^2-12\sqrt{b\,x^4+a}\,af\,x^3+3\sqrt{a}\,\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^6,x)
```

output
```
( - 8*sqrt(a + b*x**4)*a*c - 6*sqrt(a + b*x**4)*a*d*x - 24*sqrt(a + b*x**4
)*a*e*x**2 - 12*sqrt(a + b*x**4)*a*f*x**3 + 3*sqrt(a)*log(sqrt(a + b*x**4)
 - sqrt(a))*b*d*x**5 - 3*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*d*x**5
- 6*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*f*x**5 + 6*sqrt(b)*log(
sqrt(a + b*x**4) + sqrt(b)*x**2)*a*f*x**5 - 16*int(sqrt(a + b*x**4)/(a*x**
6 + b*x**10),x)*a**2*c*x**5 - 48*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)
*a**2*e*x**5)/(24*a*x**5)
```

3.51 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 352

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = -\frac{e\sqrt{a+bx^4}}{4x^4}+\frac{2f\sqrt{a+bx^4}}{3x^3}-\frac{2bd\sqrt{a+bx^4}}{5\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(d+5fx^2)\sqrt{a+bx^4}}{5x^5}-\frac{c(a+bx^4)^{3/2}}{6ax^6}-\frac{b e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{2b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} + \frac{b^{3/4}\left(3\sqrt{b}d+5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}$$

output
```
-1/4*e*(b*x^4+a)^(1/2)/x^4+2/3*f*(b*x^4+a)^(1/2)/x^3-2/5*b*d*(b*x^4+a)^(1/
2)/a^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)-1/5*(5*f*x^2+d)*(b*x^4+a)^(1/2)/x^5-1/6
*c*(b*x^4+a)^(3/2)/a/x^6-1/4*b*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-
2/5*b^(5/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1
/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4
+a)^(1/2)+1/15*b^(3/4)*(3*b^(1/2)*d+5*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b
*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/
a^(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.41

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(12adx\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5\left(\sqrt{1+\frac{bx^4}{a}}(2ac+3aex^2+2bcx^4)+3\right.\right.}{60ax^6\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

output
```
-1/60*(Sqrt[a + b*x^4]*(12*a*d*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*
x^4)/a)] + 5*(Sqrt[1 + (b*x^4)/a]*(2*a*c + 3*a*e*x^2 + 2*b*c*x^4) + 3*b*e*
x^6*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*f*x^3*Hypergeometric2F1[-3/4, -1/2,
 1/4, -((b*x^4)/a)])))/(a*x^6*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}\left(c+dx+ex^2+fx^3\right)}{x^7}dx$$

$$\Big\downarrow \text{ 2364}$$

$$-2b\int -\frac{20fx^3+15ex^2+12dx+10c}{60x^3\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{30}b\int \frac{20fx^3+15ex^2+12dx+10c}{x^3\sqrt{bx^4+a}}dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{30}b\int \left(\frac{15ex^2+10c}{x^3\sqrt{bx^4+a}}+\frac{20fx^2+12d}{x^2\sqrt{bx^4+a}}\right)dx-\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{30}b\left(\frac{2\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\ }}{}\right.$$
$$\frac{1}{60}\sqrt{a+bx^4}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^7,x]
```

output
```
-1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*Sqrt[a + b*x^4]
) + (b*((-5*c*Sqrt[a + b*x^4])/(a*x^2) - (12*d*Sqrt[a + b*x^4])/(a*x) + (1
2*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (15*e*ArcTanh
[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (12*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + (2*(3*Sqrt[b]*d + 5*Sqrt
[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*b^(1/4)*Sqrt[a +
b*x^4])))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(24x^5bd+10bc\,x^4+20af\,x^3+15ae\,x^2+12adx+10ac\right)}{60x^6a}+\frac{b\left(\frac{20af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{15\sqrt{a}\,e\ln\left(\ldots\right.}{\ldots}\right.}{\ldots}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{6x^6}-\frac{d\sqrt{b\,x^4+a}}{5x^5}-\frac{e\sqrt{b\,x^4+a}}{4x^4}-\frac{f\sqrt{b\,x^4+a}}{3x^3}-\frac{bc\sqrt{b\,x^4+a}}{6a\,x^2}-\frac{2db\sqrt{b\,x^4+a}}{5ax}+\frac{2fb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{Ellipti}\ldots}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$-\frac{c\left(b\,x^4+a\right)^{\frac{3}{2}}}{6a\,x^6}+d\left(-\frac{\sqrt{b\,x^4+a}}{5x^5}-\frac{2b\sqrt{b\,x^4+a}}{5ax}+\frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\ldots\right.\right.}{5\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/60*(b*x^4+a)^(1/2)*(24*b*d*x^5+10*b*c*x^4+20*a*f*x^3+15*a*e*x^2+12*a*d*
x+10*a*c)/x^6/a+1/30*b/a*(20*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-15/2*a^(1/2)*e*ln((2*a+2*a^(1/2)*(b*x^4+a)
^(1/2))/x^2)+12*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)
*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.47

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx =$$
$$-\frac{48\sqrt{a}bdx^6\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-15\sqrt{a}bex^6\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-16(3bd-5af$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/120*(48*sqrt(a)*b*d*x^6*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
 -1) - 15*sqrt(a)*b*e*x^6*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x
^4) - 16*(3*b*d - 5*a*f)*sqrt(a)*x^6*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/
a)^(1/4)), -1) + 2*(24*b*d*x^5 + 10*b*c*x^4 + 20*a*f*x^3 + 15*a*e*x^2 + 12
*a*d*x + 10*a*c)*sqrt(b*x^4 + a))/(a*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.58 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6a} - \frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**7,x)
```

output
```
sqrt(a)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**5*gamma(-1/4)) + sqrt(a)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(b)*c*sqrt(a/(b*x**4
) + 1)/(6*x**4) - sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c*sqr
t(a/(b*x**4) + 1)/(6*a) - b*e*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(b*x^4 + a)^(3/2)*c/(a*x^6) + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x
+ d)/x^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx=\int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^7}\,dx$$
$$=\frac{-4\sqrt{b\,x^4+a}\,ac-8\sqrt{b\,x^4+a}\,adx-6\sqrt{b\,x^4+a}\,ae\,x^2-24\sqrt{b\,x^4+a}\,af\,x^3-4\sqrt{b\,x^4+a}\,bc\,x^4+3\sqrt{a}\,\mathrm{log}}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^7,x)
```

output

```
( - 4*sqrt(a + b*x**4)*a*c - 8*sqrt(a + b*x**4)*a*d*x - 6*sqrt(a + b*x**4)
*a*e*x**2 - 24*sqrt(a + b*x**4)*a*f*x**3 - 4*sqrt(a + b*x**4)*b*c*x**4 + 3
*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*e*x**6 - 3*sqrt(a)*log(sqrt(a +
 b*x**4) + sqrt(a))*b*e*x**6 - 16*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),
x)*a**2*d*x**6 - 48*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a**2*f*x**6)
/(24*a*x**6)
```

3.52 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 377

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \frac{2e\sqrt{a+bx^4}}{15x^5} - \frac{f\sqrt{a+bx^4}}{4x^4} - \frac{2bc\sqrt{a+bx^4}}{21ax^3} - \frac{2be\sqrt{a+bx^4}}{5\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(3c+7ex^2)\sqrt{a+bx^4}}{21x^7} - \frac{d(a+bx^4)^{3/2}}{6ax^6} - \frac{bf\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{2b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}} - \frac{b^{5/4}\left(5\sqrt{b}c-21\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}$$

output
```
2/15*e*(b*x^4+a)^(1/2)/x^5-1/4*f*(b*x^4+a)^(1/2)/x^4-2/21*b*c*(b*x^4+a)^(1
/2)/a/x^3-2/5*b*e*(b*x^4+a)^(1/2)/a^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)-1/21*(7*
e*x^2+3*c)*(b*x^4+a)^(1/2)/x^7-1/6*d*(b*x^4+a)^(3/2)/a/x^6-1/4*b*f*arctanh
((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-2/5*b^(5/4)*e*(a^(1/2)+b^(1/2)*x^2)*((b*
x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)-1/105*b^(5/4)*(5*b^(1/2)*c-21*
a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)
*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/(b*x^4+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.38

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = -\frac{\sqrt{a+bx^4}\left(35x\left(\sqrt{1+\frac{bx^4}{a}}(2ad+3afx^2+2bdx^4)+3bfx^6\operatorname{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)\right)+60ac\,\text{Hypergeome}\right.}{420ax^7\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

output
```
-1/420*(Sqrt[a + b*x^4]*(35*x*(Sqrt[1 + (b*x^4)/a]*(2*a*d + 3*a*f*x^2 + 2*
b*d*x^4) + 3*b*f*x^6*ArcTanh[Sqrt[1 + (b*x^4)/a]]) + 60*a*c*Hypergeometric
2F1[-7/4, -1/2, -3/4, -((b*x^4)/a)] + 84*a*e*x^2*Hypergeometric2F1[-5/4, -
1/2, -1/4, -((b*x^4)/a)]))/(a*x^7*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^8}dx$$

$$\Big\downarrow \text{2364}$$

$$-2b\int -\frac{105fx^3+84ex^2+70dx+60c}{420x^4\sqrt{bx^4+a}}dx-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{210}b\int \frac{105fx^3+84ex^2+70dx+60c}{x^4\sqrt{bx^4+a}}dx-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{210}b\int \left(\frac{84ex^2+60c}{x^4\sqrt{bx^4+a}}+\frac{105fx^2+70d}{x^3\sqrt{bx^4+a}}\right)dx-\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{210}b\left(-\frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}c-21\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{84\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{\right.}}{}\right.$$
$$\frac{1}{420}\sqrt{a+bx^4}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^8,x]
```

output
```
-1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*Sqrt[a + b*x^
4]) + (b*((-20*c*Sqrt[a + b*x^4])/(a*x^3) - (35*d*Sqrt[a + b*x^4])/(a*x^2)
 - (84*e*Sqrt[a + b*x^4])/(a*x) + (84*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqr
t[a] + Sqrt[b]*x^2)) - (105*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]
) - (84*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt
[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
 + b*x^4]) - (2*b^(1/4)*(5*Sqrt[b]*c - 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(a^(5/4)*Sqrt[a + b*x^4])))/210
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(168 b e\,x^{6}+70 x^{5} b d+40 b c\,x^{4}+105 a f\,x^{3}+84 a e\,x^{2}+70 a d x+60 a c)}{420 x^{7} a}+\frac{b\left(-\frac{20 c b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{d\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{e\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{f\sqrt{b\,x^{4}+a}}{4x^{4}}-\frac{2bc\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{db\sqrt{b\,x^{4}+a}}{6a\,x^{2}}-\frac{2eb\sqrt{b\,x^{4}+a}}{5ax}-\frac{2b^{2}c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{21}$
default	$c\left(-\frac{\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{2b\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{2b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-\frac{d(b\,x^{4}+a)^{\frac{3}{2}}}{6a\,x^{6}}+e\left(-\frac{\sqrt{b\,x^{4}+a}}{5x^{5}}-\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/420*(b*x^4+a)^(1/2)*(168*b*e*x^6+70*b*d*x^5+40*b*c*x^4+105*a*f*x^3+84*a
*e*x^2+70*a*d*x+60*a*c)/x^7/a+1/210*b/a*(-20*c*b/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-105/2*a^(1/2)*f*ln((2*a+2*
a^(1/2)*(b*x^4+a)^(1/2))/x^2)+84*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx =$$

$$-\frac{336\sqrt{a}bex^7\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)-105\sqrt{a}bfx^7\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-16\,(5\,bc+21}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/840*(336*sqrt(a)*b*e*x^7*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4))
, -1) - 105*sqrt(a)*b*f*x^7*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)
/x^4) - 16*(5*b*c + 21*b*e)*sqrt(a)*x^7*(-b/a)^(3/4)*elliptic_f(arcsin(x*(
-b/a)^(1/4)), -1) + 2*(168*b*e*x^6 + 70*b*d*x^5 + 40*b*c*x^4 + 105*a*f*x^3
 + 84*a*e*x^2 + 70*a*d*x + 60*a*c)*sqrt(b*x^4 + a))/(a*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \frac{\sqrt{a}c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}e\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6a} - \frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**8,x)
```

output
```
sqrt(a)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + sqrt(a)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - sqrt(b)*d*sqrt(a/(b*x*
*4) + 1)/(6*x**4) - sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d*s
qrt(a/(b*x**4) + 1)/(6*a) - b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^8}\,dx$$
$$= \frac{-24\sqrt{b\,x^4+a}\,ac-20\sqrt{b\,x^4+a}\,adx-40\sqrt{b\,x^4+a}\,ae\,x^2-30\sqrt{b\,x^4+a}\,af\,x^3-20\sqrt{b\,x^4+a}\,bd\,x^5+15}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^8,x)
```

output
```
( - 24*sqrt(a + b*x**4)*a*c - 20*sqrt(a + b*x**4)*a*d*x - 40*sqrt(a + b*x*
*4)*a*e*x**2 - 30*sqrt(a + b*x**4)*a*f*x**3 - 20*sqrt(a + b*x**4)*b*d*x**5
 + 15*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*f*x**7 - 15*sqrt(a)*log(sq
rt(a + b*x**4) + sqrt(a))*b*f*x**7 - 48*int(sqrt(a + b*x**4)/(a*x**8 + b*x
**12),x)*a**2*c*x**7 - 80*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a**2*
e*x**7)/(120*a*x**7)
```

3.53 $\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 402

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx &= -\frac{c\sqrt{a+bx^4}}{8x^8}+\frac{2f\sqrt{a+bx^4}}{15x^5}-\frac{bc\sqrt{a+bx^4}}{16ax^4}\\
&-\frac{2bd\sqrt{a+bx^4}}{21ax^3}-\frac{2bf\sqrt{a+bx^4}}{5\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{(3d+7fx^2)\sqrt{a+bx^4}}{21x^7}-\frac{e(a+bx^4)^{3/2}}{6ax^6}\\
&+\frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}-\frac{2b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+bx^4}}\\
&-\frac{b^{5/4}\left(5\sqrt{b}d-21\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/8*c*(b*x^4+a)^(1/2)/x^8+2/15*f*(b*x^4+a)^(1/2)/x^5-1/16*b*c*(b*x^4+a)^(
1/2)/a/x^4-2/21*b*d*(b*x^4+a)^(1/2)/a/x^3-2/5*b*f*(b*x^4+a)^(1/2)/a^(1/2)/
x/(a^(1/2)+b^(1/2)*x^2)-1/21*(7*f*x^2+3*d)*(b*x^4+a)^(1/2)/x^7-1/6*e*(b*x^
4+a)^(3/2)/a/x^6+1/16*b^2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-2/5*b
^(5/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*E
llipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(
1/2)-1/105*b^(5/4)*(5*b^(1/2)*d-21*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^
4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(
1/4)),1/2*2^(1/2))/a^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.36

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(30a^3d\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7x\left(6a^3fx\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2}\right.\right.\right.}{210a^3x^7\sqrt{1+}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

output
```
-1/210*(Sqrt[a + b*x^4]*(30*a^3*d*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b
*x^4)/a)] + 7*x*(6*a^3*f*x*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^4)/a
)] + 5*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*(a^2*e + b^2*c*x^6*Hypergeometric2F
1[3/2, 3, 5/2, 1 + (b*x^4)/a]))))/(a^3*x^7*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^4}(c+dx+ex^2+fx^3)}{x^9}dx$$

$$\Big\downarrow \text{2364}$$

$$-2b\int -\frac{168fx^3+140ex^2+120dx+105c}{840x^5\sqrt{bx^4+a}}dx-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{420}b\int \frac{168fx^3+140ex^2+120dx+105c}{x^5\sqrt{bx^4+a}}dx-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{420}b\int \left(\frac{140ex^2+105c}{x^5\sqrt{bx^4+a}}+\frac{168fx^2+120d}{x^4\sqrt{bx^4+a}}\right)dx-\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{420}b\left(-\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{b}d-21\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{168\sqrt[4]{b}f\left(\sqrt{a}+\right.}{}\right.$$
$$\frac{1}{840}\sqrt{a+bx^4}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*Sqrt[a + b*x^4])/x^9,x]
```

output
```
-1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*Sqrt[a + b
*x^4]) + (b*((-105*c*Sqrt[a + b*x^4])/(4*a*x^4) - (40*d*Sqrt[a + b*x^4])/(
a*x^3) - (70*e*Sqrt[a + b*x^4])/(a*x^2) - (168*f*Sqrt[a + b*x^4])/(a*x) +
(168*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (105*b*c*A
rcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) - (168*b^(1/4)*f*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (4*b^(1/4)*(5*Sqr
t[b]*d - 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(5/4)*S
qrt[a + b*x^4])))/420
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(672 b f\,x^{7}+280 b e\,x^{6}+160 x^{5} b d+105 b c\,x^{4}+336 a f\,x^{3}+280 a e\,x^{2}+240 a d x+210 a c)}{1680 x^{8} a}-\frac{b^{2}\left(\frac{80 d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{Ell}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+}}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{d\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{e\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{f\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{bc\sqrt{b\,x^{4}+a}}{16a\,x^{4}}-\frac{2bd\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{eb\sqrt{b\,x^{4}+a}}{6a\,x^{2}}-\frac{2bf\sqrt{b\,x^{4}+a}}{5ax}-\frac{2d}{}$
default	$c\left(-\frac{(b\,x^{4}+a)^{\frac{3}{2}}}{8a\,x^{8}}+\frac{b(b\,x^{4}+a)^{\frac{3}{2}}}{16a^{2}x^{4}}+\frac{b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{16a^{\frac{3}{2}}}-\frac{b^{2}\sqrt{b\,x^{4}+a}}{16a^{2}}\right)+d\left(-\frac{\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{2b\sqrt{b\,x^{4}+a}}{21a\,x^{3}}-\frac{2b^{2}\sqrt{1}}{}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(b*x^4+a)^(1/2)*(672*b*f*x^7+280*b*e*x^6+160*b*d*x^5+105*b*c*x^4+3
36*a*f*x^3+280*a*e*x^2+240*a*d*x+210*a*c)/x^8/a-1/840/a*b^2*(80*d/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2
)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-105/2*c/a
^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-336*I*f*a^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.49

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx =$$

$$-\frac{1344\,a^{\frac{3}{2}}bfx^{8}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-105\sqrt{a}b^{2}cx^{8}\log\left(-\frac{bx^{4}+2\sqrt{bx^{4}+a}\sqrt{a}+2\,a}{x^{4}}\right)-64\,(5\,abd+}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/3360*(1344*a^(3/2)*b*f*x^8*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4
)), -1) - 105*sqrt(a)*b^2*c*x^8*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) +
2*a)/x^4) - 64*(5*a*b*d + 21*a*b*f)*sqrt(a)*x^8*(-b/a)^(3/4)*elliptic_f(ar
csin(x*(-b/a)^(1/4)), -1) + 2*(672*a*b*f*x^7 + 280*a*b*e*x^6 + 160*a*b*d*x
^5 + 105*a*b*c*x^4 + 336*a^2*f*x^3 + 280*a^2*e*x^2 + 240*a^2*d*x + 210*a^2
*c)*sqrt(b*x^4 + a))/(a^2*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \frac{\sqrt{a}d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}f\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} - \frac{ac}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}c}{16ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{6a} + \frac{b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(1/2)/x**9,x)
```

output
```
sqrt(a)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**7*gamma(-3/4)) + sqrt(a)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,
), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a*c/(8*sqrt(b)*x**10*s
qrt(a/(b*x**4) + 1)) - 3*sqrt(b)*c/(16*x**6*sqrt(a/(b*x**4) + 1)) - sqrt(b
)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c/(16*a*x**2*sqrt(a/(b*x**4)
+ 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(6*a) + b**2*c*asinh(sqrt(a)/(sqrt
(b)*x**2))/(16*a**(3/2))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/32*(b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^
(3/2) + 2*((b*x^4 + a)^(3/2)*b^2 + sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a)^2*a
 - 2*(b*x^4 + a)*a^2 + a^3))*c + integrate(sqrt(b*x^4 + a)*(f*x^2 + e*x +
d)/x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx=\int \frac{\sqrt{bx^4+a}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \int \frac{\sqrt{b\,x^4+a}\,(f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

input
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
```

output
```
int(((a + b*x^4)^(1/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\sqrt{a+bx^4}}{x^9}\,dx = \frac{-60\sqrt{b\,x^4+a}\,a^2c-96\sqrt{b\,x^4+a}\,a^2dx-80\sqrt{b\,x^4+a}\,a^2e\,x^2-160\sqrt{b\,x^4+a}\,a^2f\,x^3-30\sqrt{b\,x^4+a}\,abc\,x^4}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(1/2)/x^9,x)
```

output
```
( - 60*sqrt(a + b*x**4)*a**2*c - 96*sqrt(a + b*x**4)*a**2*d*x - 80*sqrt(a
+ b*x**4)*a**2*e*x**2 - 160*sqrt(a + b*x**4)*a**2*f*x**3 - 30*sqrt(a + b*x
**4)*a*b*c*x**4 - 80*sqrt(a + b*x**4)*a*b*e*x**6 - 15*sqrt(a)*log(sqrt(a +
 b*x**4) - sqrt(a))*b**2*c*x**8 + 15*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a
))*b**2*c*x**8 - 192*int(sqrt(a + b*x**4)/(a*x**8 + b*x**12),x)*a**3*d*x**
8 - 320*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a**3*f*x**8)/(480*a**2*
x**8)
```

3.54 $\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 458

$$\begin{aligned}\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx &= \frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{1}{16}aex^6\sqrt{a+bx^4}\\
&-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{2a^2x(65d+77fx^2)\sqrt{a+bx^4}}{5005b}+\frac{1}{12}ex^6\left(a+bx^4\right)^{3/2}\\
&-\frac{ax(39d+77fx^2)\left(a+bx^4\right)^{3/2}}{3003b}+\frac{c\left(a+bx^4\right)^{5/2}}{10b}+\frac{dx\left(a+bx^4\right)^{5/2}}{11b}+\frac{fx^3\left(a+bx^4\right)^{5/2}}{13b}\\
&-\frac{a^3e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}\\
&-\frac{2a^{11/4}\left(65\sqrt{b}d+77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/32*a^2*e*x^2*(b*x^4+a)^(1/2)/b+1/16*a*e*x^6*(b*x^4+a)^(1/2)-4/65*a^3*f*x
*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)-2/5005*a^2*x*(77*f*x^2+65*d
)*(b*x^4+a)^(1/2)/b+1/12*e*x^6*(b*x^4+a)^(3/2)-1/3003*a*x*(77*f*x^2+39*d)*
(b*x^4+a)^(3/2)/b+1/10*c*(b*x^4+a)^(5/2)/b+1/11*d*x*(b*x^4+a)^(5/2)/b+1/13
*f*x^3*(b*x^4+a)^(5/2)/b-1/32*a^3*e*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b
^(3/2)+4/65*a^(13/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x
^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/
4)/(b*x^4+a)^(1/2)-2/5005*a^(11/4)*(65*b^(1/2)*d+77*a^(1/2)*f)*(a^(1/2)+b^
(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arc
tan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.68 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.52

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{\sqrt{a+bx^4}\left(6864\sqrt{b}c(a+bx^4)^2+6240\sqrt{b}dx(a+bx^4)^2+5280\sqrt{b}fx^3(a+bx^4)^2+715e\left(\sqrt{b}x\right.\right.}{}$$

input
```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*(6864*Sqrt[b]*c*(a + b*x^4)^2 + 6240*Sqrt[b]*d*x*(a + b*x
^4)^2 + 5280*Sqrt[b]*f*x^3*(a + b*x^4)^2 + 715*e*(Sqrt[b]*x^2*(3*a^2 + 14*
a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[1 +
 (b*x^4)/a]) - (6240*a^2*Sqrt[b]*d*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((
b*x^4)/a)])/Sqrt[1 + (b*x^4)/a] - (5280*a^2*Sqrt[b]*f*x^3*Hypergeometric2F
1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/(68640*b^(3/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^4\right)^{3/2}\left(c+dx+ex^2+fx^3\right)dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^3\left(a+bx^4\right)^{3/2}\left(c+ex^2\right)+x^4\left(a+bx^4\right)^{3/2}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^{11/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{a}f+65\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005b^{7/4}\sqrt{a+bx^4}}+\frac{4a^{13/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^4}}-\frac{a^3e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^3fx\sqrt{a+bx^4}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2dx\sqrt{a+bx^4}}{77b}-\frac{a^2ex^2\sqrt{a+bx^4}}{32b}+\frac{4a^2fx^3\sqrt{a+bx^4}}{195b}+\frac{c\left(a+bx^4\right)^{5/2}}{10b}+\frac{1}{143}x^5\left(a+bx^4\right)^{3/2}\left(13d+11fx^2\right)+\frac{2ax^5\sqrt{a+bx^4}\left(117d+77fx^2\right)}{3003}+\frac{ex^2\left(a+bx^4\right)^{5/2}}{12b}-\frac{aex^2\left(a+bx^4\right)^{3/2}}{48b}$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(4*a^2*d*x*Sqrt[a + b*x^4])/(77*b) - (a^2*e*x^2*Sqrt[a + b*x^4])/(32*b) +
(4*a^2*f*x^3*Sqrt[a + b*x^4])/(195*b) - (4*a^3*f*x*Sqrt[a + b*x^4])/(65*b^
(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^5*(117*d + 77*f*x^2)*Sqrt[a + b*x^
4])/3003 - (a*e*x^2*(a + b*x^4)^(3/2))/(48*b) + (x^5*(13*d + 11*f*x^2)*(a
+ b*x^4)^(3/2))/143 + (c*(a + b*x^4)^(5/2))/(10*b) + (e*x^2*(a + b*x^4)^(5
/2))/(12*b) - (a^3*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(32*b^(3/2))
+ (4*a^(13/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[
a + b*x^4]) - (2*a^(11/4)*(65*Sqrt[b]*d + 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(5005*b^(7/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.69

method	result
risch	$\frac{(36960b^2fx^{11}+40040eb^2x^{10}+43680b^2dx^9+48048b^2cx^8+61600x^7abf+70070abex^6+81120x^5adb+96096abcx^4+9856a^2fx^3+15}{480480b}$
default	$\frac{c(bx^4+a)^{\frac{5}{2}}}{10b}+d\left(\frac{bx^9\sqrt{bx^4+a}}{11}+\frac{13ax^5\sqrt{bx^4+a}}{77}+\frac{4a^2x\sqrt{bx^4+a}}{77b}-\frac{4a^3\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{77b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}\right)$
elliptic	$\frac{fbx^{11}\sqrt{bx^4+a}}{13}+\frac{bex^{10}\sqrt{bx^4+a}}{12}+\frac{bdx^9\sqrt{bx^4+a}}{11}+\frac{cbx^8\sqrt{bx^4+a}}{10}+\frac{5afx^7\sqrt{bx^4+a}}{39}+\frac{7aex^6\sqrt{bx^4+a}}{48}+\frac{13adx^5\sqrt{bx^4}}{77}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/480480*(36960*b^2*f*x^11+40040*b^2*e*x^10+43680*b^2*d*x^9+48048*b^2*c*x^
8+61600*a*b*f*x^7+70070*a*b*e*x^6+81120*a*b*d*x^5+96096*a*b*c*x^4+9856*a^2
*f*x^3+15015*a^2*e*x^2+24960*a^2*d*x+48048*a^2*c)/b*(b*x^4+a)^(1/2)-1/8008
0*a^3/b*(4160*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)+5005/2*e*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+4928*I*f*a
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.56

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx =$$

$$-\frac{59136\,a^3\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-15015\,a^3\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+7}{}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/960960*(59136*a^3*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/
4)/x), -1) - 15015*a^3*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b
)*x^2 - a) + 768*(65*a^2*b*d - 77*a^3*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1) - 2*(36960*b^3*f*x^12 + 40040*b^3*e*x^11 + 43
680*b^3*d*x^10 + 48048*b^3*c*x^9 + 61600*a*b^2*f*x^8 + 70070*a*b^2*e*x^7 +
 81120*a*b^2*d*x^6 + 96096*a*b^2*c*x^5 + 9856*a^2*b*f*x^4 + 15015*a^2*b*e*
x^3 + 24960*a^2*b*d*x^2 + 48048*a^2*b*c*x - 29568*a^3*f)*sqrt(b*x^4 + a))/
(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.87

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^{\frac{5}{2}}ex^2}{32b\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{17a^{\frac{3}{2}}ex^6}{96\sqrt{1+\frac{bx^4}{a}}}+\frac{a^{\frac{3}{2}}fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{a}bdx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{11\sqrt{a}bex^{10}}{48\sqrt{1+\frac{bx^4}{a}}}+\frac{\sqrt{a}bfx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}-\frac{a^3e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+ac\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)+bc\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right)+\frac{b^2ex^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(5/2)*e*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*d*x**5*gamma(5/4)*hyp
er((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 17*a**(
3/2)*e*x**6/(96*sqrt(1 + b*x**4/a)) + a**(3/2)*f*x**7*gamma(7/4)*hyper((-1
/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*d
*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(13/4)) + 11*sqrt(a)*b*e*x**10/(48*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*
x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(15/4)) - a**3*e*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*c*Pi
ecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b
*c*Piecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(
30*b) + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**
2*e*x**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^4 + a)^(5/2)*c/b + integrate((b*f*x^10 + b*e*x^9 + b*d*x^8 + a*f
*x^6 + a*e*x^5 + a*d*x^4)*sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int x^3\,{\left(b\,x^4+a\right)}^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^3*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{96096\sqrt{b\,x^4+a}\,a^2bc+49920\sqrt{b\,x^4+a}\,a^2bdx+30030\sqrt{b\,x^4+a}\,a^2be\,x^2+19712\sqrt{b\,x^4+a}\,a}{}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

output
```
(96096*sqrt(a + b*x**4)*a**2*b*c + 49920*sqrt(a + b*x**4)*a**2*b*d*x + 300
30*sqrt(a + b*x**4)*a**2*b*e*x**2 + 19712*sqrt(a + b*x**4)*a**2*b*f*x**3 +
 192192*sqrt(a + b*x**4)*a*b**2*c*x**4 + 162240*sqrt(a + b*x**4)*a*b**2*d*
x**5 + 140140*sqrt(a + b*x**4)*a*b**2*e*x**6 + 123200*sqrt(a + b*x**4)*a*b
**2*f*x**7 + 96096*sqrt(a + b*x**4)*b**3*c*x**8 + 87360*sqrt(a + b*x**4)*b
**3*d*x**9 + 80080*sqrt(a + b*x**4)*b**3*e*x**10 + 73920*sqrt(a + b*x**4)*
b**3*f*x**11 + 15015*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**3*e -
 15015*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a**3*e - 49920*int(sqr
t(a + b*x**4)/(a + b*x**4),x)*a**3*b*d - 59136*int((sqrt(a + b*x**4)*x**2)
/(a + b*x**4),x)*a**3*b*f)/(960960*b**2)
```

3.55 $\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 437

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{1}{16}afx^6\sqrt{a+bx^4}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{2ax\left(15ae-77bcx^2\right)\sqrt{a+bx^4}}{1155b}+\frac{1}{12}fx^6\left(a+bx^4\right)^{3/2}-\frac{x\left(9ae-77bcx^2\right)\left(a+bx^4\right)^{3/2}}{693b}+\frac{d\left(a+bx^4\right)^{5/2}}{10b}+\frac{ex\left(a+bx^4\right)^{5/2}}{11b}-\frac{a^3f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{2a^{9/4}\left(77\sqrt{b}c-15\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/32*a^2*f*x^2*(b*x^4+a)^(1/2)/b+1/16*a*f*x^6*(b*x^4+a)^(1/2)+4/15*a^2*c*x
*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)-2/1155*a*x*(-77*b*c*x^2+15*
a*e)*(b*x^4+a)^(1/2)/b+1/12*f*x^6*(b*x^4+a)^(3/2)-1/693*x*(-77*b*c*x^2+9*a
*e)*(b*x^4+a)^(3/2)/b+1/10*d*(b*x^4+a)^(5/2)/b+1/11*e*x*(b*x^4+a)^(5/2)/b-
1/32*a^3*f*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)-4/15*a^(9/4)*c*(a^
(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin
(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+2/1155*
a^(9/4)*(77*b^(1/2)*c-15*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1
/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*
2^(1/2))/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.74 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.47

$$\int x^2(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}\,dx = \frac{\sqrt{a+bx^4}\left(\frac{528d(a+bx^4)^2}{b}+\frac{480ex(a+bx^4)^2}{b}+\frac{55f\left(\sqrt{b}x^2(3a^2+14abx^4+8b^2x^8)-\frac{3a^{5/2}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{1+\frac{bx^4}{a}}}\right)}{b^{3/2}}-480\cdots\right.}{5280}$$

input
```
Integrate[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*((528*d*(a + b*x^4)^2)/b + (480*e*x*(a + b*x^4)^2)/b + (5
5*f*(Sqrt[b]*x^2*(3*a^2 + 14*a*b*x^4 + 8*b^2*x^8) - (3*a^(5/2)*ArcSinh[(Sq
rt[b]*x^2)/Sqrt[a]])/Sqrt[1 + (b*x^4)/a]))/b^(3/2) - (480*a^2*e*x*Hypergeo
metric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (1760*a
*c*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a
]))/5280
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^4)^{3/2} (c+dx+ex^2+fx^3) \, dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x^2 (a+bx^4)^{3/2} (c+ex^2)+x^3 (a+bx^4)^{3/2} (d+fx^2)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}c-15\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{a^3f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{32b^{3/2}}+\frac{4a^2cx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2ex\sqrt{a+bx^4}}{77b}-\frac{a^2fx^2\sqrt{a+bx^4}}{32b}+\frac{2ax^3\sqrt{a+bx^4}(77c+45ex^2)}{1155}+\frac{1}{99}x^3(a+bx^4)^{3/2}(11c+9ex^2)+\frac{d(a+bx^4)^{5/2}}{10b}-\frac{afx^2(a+bx^4)^{3/2}}{48b}+\frac{fx^2(a+bx^4)^{5/2}}{12b}$$

input
```
Int[x^2*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(4*a^2*e*x*Sqrt[a + b*x^4])/(77*b) - (a^2*f*x^2*Sqrt[a + b*x^4])/(32*b) +
(4*a^2*c*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^
3*(77*c + 45*e*x^2)*Sqrt[a + b*x^4])/1155 - (a*f*x^2*(a + b*x^4)^(3/2))/(4
8*b) + (x^3*(11*c + 9*e*x^2)*(a + b*x^4)^(3/2))/99 + (d*(a + b*x^4)^(5/2))
/(10*b) + (f*x^2*(a + b*x^4)^(5/2))/(12*b) - (a^3*f*ArcTanh[(Sqrt[b]*x^2)/
Sqrt[a + b*x^4]])/(32*b^(3/2)) - (4*a^(9/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(9/4)*(77*Sqrt[b]*c - 15*
Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(1155*b^(5/4)*Sqrt[a +
 b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(9240b^2 f\,x^{10}+10080e\,b^2x^9+11088d\,b^2x^8+12320b^2c\,x^7+16170ab f\,x^6+18720abe\,x^5+22176abd\,x^4+27104abc\,x^3+3465a^2x^2f+5760}{110880b}$
default	$c\left(\frac{b\,x^7\sqrt{b\,x^4+a}}{9}+\frac{11a\,x^3\sqrt{b\,x^4+a}}{45}+\frac{4ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{15\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right)+\frac{d(}{}$
elliptic	$\frac{fb\,x^{10}\sqrt{b\,x^4+a}}{12}+\frac{be\,x^9\sqrt{b\,x^4+a}}{11}+\frac{bd\,x^8\sqrt{b\,x^4+a}}{10}+\frac{cb\,x^7\sqrt{b\,x^4+a}}{9}+\frac{7af\,x^6\sqrt{b\,x^4+a}}{48}+\frac{13ae\,x^5\sqrt{b\,x^4+a}}{77}+\frac{ad\,x^4\sqrt{b\,x^4+}}{5}$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/110880*(9240*b^2*f*x^10+10080*b^2*e*x^9+11088*b^2*d*x^8+12320*b^2*c*x^7+
16170*a*b*f*x^6+18720*a*b*e*x^5+22176*a*b*d*x^4+27104*a*b*c*x^3+3465*a^2*f
*x^2+5760*a^2*e*x+11088*a^2*d)/b*(b*x^4+a)^(1/2)-1/18480*a^2/b*(960*a*e/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+115
5/2*a*f*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)-4928*I*c*b^(1/2)*a^(1/2)/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-E
llipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.57

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{59136\,a^2b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+3465\,a^3\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x\right.}{}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/221760*(59136*a^2*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) + 3465*a^3*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*
x^2 - a) - 768*(77*a^2*b*c + 15*a^2*b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1) + 2*(9240*b^3*f*x^11 + 10080*b^3*e*x^10 + 110
88*b^3*d*x^9 + 12320*b^3*c*x^8 + 16170*a*b^2*f*x^7 + 18720*a*b^2*e*x^6 + 2
2176*a*b^2*d*x^5 + 27104*a*b^2*c*x^4 + 3465*a^2*b*f*x^3 + 5760*a^2*b*e*x^2
 + 11088*a^2*b*d*x + 29568*a^2*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.91

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{a^{\frac{5}{2}}fx^2}{32b\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{17a^{\frac{3}{2}}fx^6}{96\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bcx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{a}bex^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{11\sqrt{a}bfx^{10}}{48\sqrt{1+\frac{bx^4}{a}}} - \frac{a^3f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}} + ad\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + bd\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{b^2fx^{14}}{12\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(5/2)*f*x**2/(32*b*sqrt(1 + b*x**4/a)) + a**(3/2)*c*x**3*gamma(3/4)*hyp
er((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2
)*e*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(9/4)) + 17*a**(3/2)*f*x**6/(96*sqrt(1 + b*x**4/a)) + sqrt(a)*b*c*x
**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(11/4)) + sqrt(a)*b*e*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**
4*exp_polar(I*pi)/a)/(4*gamma(13/4)) + 11*sqrt(a)*b*f*x**10/(48*sqrt(1 + b
*x**4/a)) - a**3*f*asinh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) + a*d*Piecewi
se((sqrt(a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*d*Pi
ecewise((-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b)
 + x**8*sqrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*f*x
**14/(12*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Giac [F]

$$\int x^2(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}(fx^3+ex^2+dx+c)x^2\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int x^2\,\left(b\,x^4+a\right)^{3/2}\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^2*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int x^2\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{22176\sqrt{b\,x^4+a}\,a^2bd+11520\sqrt{b\,x^4+a}\,a^2bex+6930\sqrt{b\,x^4+a}\,a^2bf\,x^2+54208\sqrt{b\,x^4+a}\,a}{}$$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

output
```
(22176*sqrt(a + b*x**4)*a**2*b*d + 11520*sqrt(a + b*x**4)*a**2*b*e*x + 693
0*sqrt(a + b*x**4)*a**2*b*f*x**2 + 54208*sqrt(a + b*x**4)*a*b**2*c*x**3 +
44352*sqrt(a + b*x**4)*a*b**2*d*x**4 + 37440*sqrt(a + b*x**4)*a*b**2*e*x**
5 + 32340*sqrt(a + b*x**4)*a*b**2*f*x**6 + 24640*sqrt(a + b*x**4)*b**3*c*x
**7 + 22176*sqrt(a + b*x**4)*b**3*d*x**8 + 20160*sqrt(a + b*x**4)*b**3*e*x
**9 + 18480*sqrt(a + b*x**4)*b**3*f*x**10 + 3465*sqrt(b)*log(sqrt(a + b*x*
*4) - sqrt(b)*x**2)*a**3*f - 3465*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x
**2)*a**3*f - 11520*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**3*b*e + 59136*
int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a**2*b**2*c)/(221760*b**2)
```

3.56 $\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 412

$$\int x\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\frac{3}{16}acx^2\sqrt{a+bx^4}$$
$$+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{2ax(15af-77bdx^2)\sqrt{a+bx^4}}{1155b}$$
$$+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}-\frac{x(9af-77bdx^2)\left(a+bx^4\right)^{3/2}}{693b}+\frac{e(a+bx^4)^{5/2}}{10b}+\frac{fx(a+bx^4)^{5/2}}{11b}$$
$$+\frac{3a^2c\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}$$
$$+\frac{2a^{9/4}\left(77\sqrt{b}d-15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}$$

output
```
3/16*a*c*x^2*(b*x^4+a)^(1/2)+4/15*a^2*d*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)
+b^(1/2)*x^2)-2/1155*a*x*(-77*b*d*x^2+15*a*f)*(b*x^4+a)^(1/2)/b+1/8*c*x^2*
(b*x^4+a)^(3/2)-1/693*x*(-77*b*d*x^2+9*a*f)*(b*x^4+a)^(3/2)/b+1/10*e*(b*x^
4+a)^(5/2)/b+1/11*f*x*(b*x^4+a)^(5/2)/b+3/16*a^2*c*arctanh(b^(1/2)*x^2/(b*
x^4+a)^(1/2))/b^(1/2)-4/15*a^(9/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(
1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*
2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+2/1155*a^(9/4)*(77*b^(1/2)*d-15*a^(1/2)*f
)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJ
acobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.50 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.48

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{\sqrt{a+bx^4}\left(\frac{264e(a+bx^4)^2}{b}+\frac{240fx(a+bx^4)^2}{b}+165c\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(a+bx^4)}\right)-\cdots}{2640}$$

input
```
Integrate[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*((264*e*(a + b*x^4)^2)/b + (240*f*x*(a + b*x^4)^2)/b + 16
5*c*(5*a*x^2 + 2*b*x^6 + (3*a^(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x
^2)/Sqrt[a]])/(Sqrt[b]*(a + b*x^4))) - (240*a^2*f*x*Hypergeometric2F1[-3/2
, 1/4, 5/4, -((b*x^4)/a)])/(b*Sqrt[1 + (b*x^4)/a]) + (880*a*d*x^3*Hypergeo
metric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]))/2640
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^4\right)^{3/2}\left(c+dx+ex^2+fx^3\right)\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(x\left(a+bx^4\right)^{3/2}\left(c+ex^2\right)+x^2\left(a+bx^4\right)^{3/2}\left(d+fx^2\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(77\sqrt{b}d-15\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155b^{5/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{3a^2c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2dx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{4a^2fx\sqrt{a+bx^4}}{77b}+\frac{1}{8}cx^2\left(a+bx^4\right)^{3/2}+\frac{3}{16}acx^2\sqrt{a+bx^4}+\frac{1}{99}x^3\left(a+bx^4\right)^{3/2}\left(11d+9fx^2\right)+\frac{2ax^3\sqrt{a+bx^4}\left(77d+45fx^2\right)}{1155}+\frac{e\left(a+bx^4\right)^{5/2}}{10b}$$

input
```
Int[x*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(4*a^2*f*x*Sqrt[a + b*x^4])/(77*b) + (3*a*c*x^2*Sqrt[a + b*x^4])/16 + (4*a
^2*d*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x^3*(7
7*d + 45*f*x^2)*Sqrt[a + b*x^4])/1155 + (c*x^2*(a + b*x^4)^(3/2))/8 + (x^3
*(11*d + 9*f*x^2)*(a + b*x^4)^(3/2))/99 + (e*(a + b*x^4)^(5/2))/(10*b) + (
3*a^2*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b]) - (4*a^(9/4)*
d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) +
(2*a^(9/4)*(77*Sqrt[b]*d - 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(1155*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.73

method	result
risch	$\frac{\left(5040 f x^{9} b^{2}+5544 b^{2} e x^{8}+6160 b^{2} d x^{7}+6930 b^{2} c x^{6}+9360 a b f x^{5}+11088 a b e x^{4}+13552 a b d x^{3}+17325 a b c x^{2}+2880 f a^{2} x+5544 a^{2} e\right) \sqrt{\cdots}}{55440 b}$
default	$c \left(\frac{3 a^{2} \ln \left(\sqrt{b}\, x^{2}+\sqrt{b x^{4}+a}\right)}{16 \sqrt{b}}+\frac{b x^{6} \sqrt{b x^{4}+a}}{8}+\frac{5 a x^{2} \sqrt{b x^{4}+a}}{16}\right)+d \left(\frac{b x^{7} \sqrt{b x^{4}+a}}{9}+\frac{11 a x^{3} \sqrt{b x^{4}+a}}{45}+\frac{4 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\cdots}{\cdots}\right.$
elliptic	$\frac{f b x^{9} \sqrt{b x^{4}+a}}{11}+\frac{b e x^{8} \sqrt{b x^{4}+a}}{10}+\frac{b d x^{7} \sqrt{b x^{4}+a}}{9}+\frac{c b x^{6} \sqrt{b x^{4}+a}}{8}+\frac{13 a f x^{5} \sqrt{b x^{4}+a}}{77}+\frac{a e x^{4} \sqrt{b x^{4}+a}}{5}+\frac{11 a d x^{3} \sqrt{b x^{4}+\cdots}}{45}\cdots$

input
```
int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/55440*(5040*b^2*f*x^9+5544*b^2*e*x^8+6160*b^2*d*x^7+6930*b^2*c*x^6+9360*
a*b*f*x^5+11088*a*b*e*x^4+13552*a*b*d*x^3+17325*a*b*c*x^2+2880*a^2*f*x+554
4*a^2*e)/b*(b*x^4+a)^(1/2)-1/9240*a^2/b*(480*a*f/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3465/2*c*b^(1/2)*ln(b^(1/2
)*x^2+(b*x^4+a)^(1/2))-2464*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^
(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(
1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.54

$$\int x\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \frac{29568\,a^2\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+10395\,a^2\sqrt{b}cx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}\right.}{}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/110880*(29568*a^2*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) + 10395*a^2*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)
*x^2 - a) - 384*(77*a^2*d + 15*a^2*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(ar
csin((-a/b)^(1/4)/x), -1) + 2*(5040*b^2*f*x^10 + 5544*b^2*e*x^9 + 6160*b^2
*d*x^8 + 6930*b^2*c*x^7 + 9360*a*b*f*x^6 + 11088*a*b*e*x^5 + 13552*a*b*d*x
^4 + 17325*a*b*c*x^3 + 2880*a^2*f*x^2 + 5544*a^2*e*x + 14784*a^2*d)*sqrt(b
*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.96

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}cx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}cx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bcx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bdx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{\sqrt{a}bfx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)} + \frac{3a^2c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + ae\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + be\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{b^2cx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*c*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*c*x**2/(16*sqrt(1 + b*x**4
/a)) + a**(3/2)*d*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*gamma(7/4)) + a**(3/2)*f*x**5*gamma(5/4)*hyper((-1/2, 5/4)
, (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*c*x**6/(1
6*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11
/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + sqrt(a)*b*f*x**9*gamma(9
/4)*hyper((-1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
+ 3*a**2*c*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*e*Piecewise((sqrt(
a)*x**4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*e*Piecewise((
-a**2*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*s
qrt(a + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*c*x**10/(8*s
qrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/32*(3*a^2*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 +
a)/x^2))/sqrt(b) + 2*(3*sqrt(b*x^4 + a)*a^2*b/x^2 - 5*(b*x^4 + a)^(3/2)*a^
2/x^6)/(b^2 - 2*(b*x^4 + a)*b/x^4 + (b*x^4 + a)^2/x^8))*c + integrate((b*f
*x^8 + b*e*x^7 + b*d*x^6 + a*f*x^4 + a*e*x^3 + a*d*x^2)*sqrt(b*x^4 + a), x
)
```

Giac [F]

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int\left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)x\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\int x\left(b\,x^4+a\right)^{3/2}\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x*(a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int x(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}\,dx=\frac{11088\sqrt{b\,x^4+a}\,a^2e+5760\sqrt{b\,x^4+a}\,a^2fx+34650\sqrt{b\,x^4+a}\,abc\,x^2+27104\sqrt{b\,x^4+a}\,abd\,x}{}$$

input
```
int(x*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

output
```
(11088*sqrt(a + b*x**4)*a**2*e + 5760*sqrt(a + b*x**4)*a**2*f*x + 34650*sq
rt(a + b*x**4)*a*b*c*x**2 + 27104*sqrt(a + b*x**4)*a*b*d*x**3 + 22176*sqrt
(a + b*x**4)*a*b*e*x**4 + 18720*sqrt(a + b*x**4)*a*b*f*x**5 + 13860*sqrt(a
 + b*x**4)*b**2*c*x**6 + 12320*sqrt(a + b*x**4)*b**2*d*x**7 + 11088*sqrt(a
 + b*x**4)*b**2*e*x**8 + 10080*sqrt(a + b*x**4)*b**2*f*x**9 - 10395*sqrt(b
)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*c + 10395*sqrt(b)*log(sqrt(a +
 b*x**4) + sqrt(b)*x**2)*a**2*c - 5760*int(sqrt(a + b*x**4)/(a + b*x**4),x
)*a**3*f + 29568*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a**2*b*d)/(11
0880*b)
```

3.57 $\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 382

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx = \frac{3}{16}adx^2\sqrt{a+bx^4} + \frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{105}ax(15c+7ex^2)\sqrt{a+bx^4} + \frac{1}{8}dx^2(a+bx^4)^{3/2} + \frac{1}{63}x(9c+7ex^2)(a+bx^4)^{3/2} + \frac{f(a+bx^4)^{5/2}}{10b} + \frac{3a^2d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{7/4}\left(15\sqrt{b}c+7\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}$$

output
```
3/16*a*d*x^2*(b*x^4+a)^(1/2)+4/15*a^2*e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)
+b^(1/2)*x^2)+2/105*a*x*(7*e*x^2+15*c)*(b*x^4+a)^(1/2)+1/8*d*x^2*(b*x^4+a)
^(3/2)+1/63*x*(7*e*x^2+9*c)*(b*x^4+a)^(3/2)+1/10*f*(b*x^4+a)^(5/2)/b+3/16*
a^2*d*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/2)-4/15*a^(9/4)*e*(a^(1/2)
+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*ar
ctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7/4
)*(15*b^(1/2)*c+7*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(
1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2)
)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.46

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^{3/2}\,dx = \frac{1}{240}\sqrt{a+bx^4}\left(\frac{24f(a+bx^4)^2}{b}\right.$$
$$+15d\left(5ax^2+2bx^6+\frac{3a^{5/2}\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(a+bx^4)}\right)+\frac{240acx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{1+\frac{bx^4}{a}}}+\frac{80a\cdots}{\cdots}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(Sqrt[a + b*x^4]*((24*f*(a + b*x^4)^2)/b + 15*d*(5*a*x^2 + 2*b*x^6 + (3*a^
(5/2)*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(a + b*
x^4))) + (240*a*c*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[
1 + (b*x^4)/a] + (80*a*e*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a
)])/Sqrt[1 + (b*x^4)/a]))/240
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^{3/2}\left(c+dx+ex^2+fx^3\right)\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left((a+bx^4)^{3/2}(c+ex^2)+x(a+bx^4)^{3/2}(d+fx^2)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}e+15\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}}-\frac{4a^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}+\frac{3a^2d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}+\frac{4a^2ex\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{63}x(a+bx^4)^{3/2}(9c+7ex^2)+\frac{2}{105}ax\sqrt{a+bx^4}(15c+7ex^2)+\frac{1}{8}dx^2(a+bx^4)^{3/2}+\frac{3}{16}adx^2\sqrt{a+bx^4}+\frac{f(a+bx^4)^{5/2}}{10b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2),x]
```

output
```
(3*a*d*x^2*Sqrt[a + b*x^4])/16 + (4*a^2*e*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(
Sqrt[a] + Sqrt[b]*x^2)) + (2*a*x*(15*c + 7*e*x^2)*Sqrt[a + b*x^4])/105 + (
d*x^2*(a + b*x^4)^(3/2))/8 + (x*(9*c + 7*e*x^2)*(a + b*x^4)^(3/2))/63 + (f
*(a + b*x^4)^(5/2))/(10*b) + (3*a^2*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4
]])/(16*Sqrt[b]) - (4*a^(9/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7/4)*(15*Sqrt[b]*c + 7*Sqrt[a]*e)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(105*b^(3/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\left(504 b^{2} f \,x^{8}+560 b^{2} e \,x^{7}+630 b^{2} d \,x^{6}+720 x^{5} b^{2} c +1008 a b f \,x^{4}+1232 a b e \,x^{3}+1575 a b d \,x^{2}+2160 a b c x +504 f \,a^{2}\right) \sqrt{b \,x^{4}+a}}{5040 b}+\frac{a^{2} \left(\frac{480 c \sqrt{}}{}\right.}{}$
default	$c \left(\frac{b \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{3 a x \sqrt{b \,x^{4}+a}}{7}+\frac{4 a^{2} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+d \left(\frac{3 a^{2} \ln \left(\sqrt{b}\, x^{2}+\sqrt{b \,x^{4}+a}\right)}{16 \sqrt{b}}+\right.$
elliptic	$\frac{f b \,x^{8} \sqrt{b \,x^{4}+a}}{10}+\frac{b e \,x^{7} \sqrt{b \,x^{4}+a}}{9}+\frac{b d \,x^{6} \sqrt{b \,x^{4}+a}}{8}+\frac{c b \,x^{5} \sqrt{b \,x^{4}+a}}{7}+\frac{a f \,x^{4} \sqrt{b \,x^{4}+a}}{5}+\frac{11 a e \,x^{3} \sqrt{b \,x^{4}+a}}{45}+\frac{5 a d \,x^{2} \sqrt{b \,x^{4}+a}}{16}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5040*(504*b^2*f*x^8+560*b^2*e*x^7+630*b^2*d*x^6+720*b^2*c*x^5+1008*a*b*f
*x^4+1232*a*b*e*x^3+1575*a*b*d*x^2+2160*a*b*c*x+504*a^2*f)/b*(b*x^4+a)^(1/
2)+1/840*a^2*(480*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)+315/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+224*I*e
*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.56

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \frac{2688\,a^2\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+945\,a^2\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/10080*(2688*a^2*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/
x), -1) + 945*a^2*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2
 - a) + 384*(15*a*b*c - 7*a^2*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin(
(-a/b)^(1/4)/x), -1) + 2*(504*b^2*f*x^9 + 560*b^2*e*x^8 + 630*b^2*d*x^7 +
720*b^2*c*x^6 + 1008*a*b*f*x^5 + 1232*a*b*e*x^4 + 1575*a*b*d*x^3 + 2160*a*
b*c*x^2 + 504*a^2*f*x + 1344*a^2*e)*sqrt(b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.03

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}dx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}dx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bcx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bdx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{3a^2d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + af\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + bf\left(\begin{cases}-\frac{a^2\sqrt{a+bx^4}}{15b^2}+\frac{ax^4\sqrt{a+bx^4}}{30b}+\frac{x^8\sqrt{a+bx^4}}{10} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^8}{8} & \text{otherwise}\end{cases}\right) + \frac{b^2dx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*c*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(3/2)*d*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*d*x**2
/(16*sqrt(1 + b*x**4/a)) + a**(3/2)*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (
7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*c*x**5*gamma(5
/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) +
3*sqrt(a)*b*d*x**6/(16*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*x**7*gamma(7/4)*h
yper((-1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + 3*a
**2*d*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*f*Piecewise((sqrt(a)*x*
*4/4, Eq(b, 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b*f*Piecewise((-a**2
*sqrt(a + b*x**4)/(15*b**2) + a*x**4*sqrt(a + b*x**4)/(30*b) + x**8*sqrt(a
 + b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True)) + b**2*d*x**10/(8*sqrt(a
)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

Giac [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^{3/2}\,dx=\int \left(bx^4+a\right)^{\frac{3}{2}}\left(fx^3+ex^2+dx+c\right)\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}\,dx = \int (b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)\,dx$$

input
```
int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int (c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}\,dx = \frac{1008\sqrt{b\,x^4+a}\,a^2f+4320\sqrt{b\,x^4+a}\,abcx+3150\sqrt{b\,x^4+a}\,abd\,x^2+2464\sqrt{b\,x^4+a}\,abe\,x^3+}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2),x)
```

output
```
(1008*sqrt(a + b*x**4)*a**2*f + 4320*sqrt(a + b*x**4)*a*b*c*x + 3150*sqrt(
a + b*x**4)*a*b*d*x**2 + 2464*sqrt(a + b*x**4)*a*b*e*x**3 + 2016*sqrt(a +
b*x**4)*a*b*f*x**4 + 1440*sqrt(a + b*x**4)*b**2*c*x**5 + 1260*sqrt(a + b*x
**4)*b**2*d*x**6 + 1120*sqrt(a + b*x**4)*b**2*e*x**7 + 1008*sqrt(a + b*x**
4)*b**2*f*x**8 - 945*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*d +
 945*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a**2*d + 5760*int(sqrt(a
 + b*x**4)/(a + b*x**4),x)*a**2*b*c + 2688*int((sqrt(a + b*x**4)*x**2)/(a
+ b*x**4),x)*a**2*b*e)/(10080*b)
```

3.58 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 403

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a(8c+3ex^2)\sqrt{a+bx^4} + \frac{2}{105}ax(15d+7fx^2)\sqrt{a+bx^4} + \frac{1}{24}(4c+3ex^2)(a+bx^4)^{3/2} + \frac{1}{63}x(9d+7fx^2)(a+bx^4)^{3/2} + \frac{3a^2e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{1}{2}a^{3/2}c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} + \frac{2a^{7/4}(15\sqrt{b}\ldots}{\ldots}$$

output
```
4/15*a^2*f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)+1/16*a*(3*e*x^2
+8*c)*(b*x^4+a)^(1/2)+2/105*a*x*(7*f*x^2+15*d)*(b*x^4+a)^(1/2)+1/24*(3*e*x
^2+4*c)*(b*x^4+a)^(3/2)+1/63*x*(7*f*x^2+9*d)*(b*x^4+a)^(3/2)+3/16*a^2*e*ar
ctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/2)-1/2*a^(3/2)*c*arctanh((b*x^4+a)
^(1/2)/a^(1/2))-4/15*a^(9/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b
^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2
))/b^(3/4)/(b*x^4+a)^(1/2)+2/105*a^(7/4)*(15*b^(1/2)*d+7*a^(1/2)*f)*(a^(1/
2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(
2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.52 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x}\,dx = \frac{1}{16}e\sqrt{a+bx^4}\left(5ax^2 + 2bx^6 + \frac{3a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{1}{6}c\left(\sqrt{a+bx^4}(4a+bx^4)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right) + \frac{adx\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{b}{\ldots}\right.}{\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

output
```
(e*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/S
qrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (c*(Sqrt[a + b*x^4]*(4*a + b
*x^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 + (a*d*x*Sqrt[a + b
*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a)])/Sqrt[1 + (b*x^4)/a]
 + (a*f*x^3*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^4)/a)
])/(3*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x}\,dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{\left(a+bx^4\right)^{3/2} \left(c+ex^2\right)}{x} + \left(a+bx^4\right)^{3/2} \left(d+fx^2\right) \right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(7\sqrt{a}f+15\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105b^{3/4}\sqrt{a+bx^4}} - \frac{4a^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}} - \frac{1}{2}a^{3/2}c\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) + \frac{3a^2e\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} + \frac{4a^2fx\sqrt{a+bx^4}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{1}{16}a\sqrt{a+bx^4}\left(8c+3ex^2\right) + \frac{1}{24}\left(a+bx^4\right)^{3/2}\left(4c+3ex^2\right) + \frac{2}{105}ax\sqrt{a+bx^4}\left(15d+7fx^2\right) + \frac{1}{63}x\left(a+bx^4\right)^{3/2}\left(9d+7fx^2\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x,x]
```

output
```
(4*a^2*f*x*Sqrt[a + b*x^4])/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (a*(8*c
 + 3*e*x^2)*Sqrt[a + b*x^4])/16 + (2*a*x*(15*d + 7*f*x^2)*Sqrt[a + b*x^4])
/105 + ((4*c + 3*e*x^2)*(a + b*x^4)^(3/2))/24 + (x*(9*d + 7*f*x^2)*(a + b*
x^4)^(3/2))/63 + (3*a^2*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt
[b]) - (a^(3/2)*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (4*a^(9/4)*f*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a + b*x^4]) + (2*a^(7
/4)*(15*Sqrt[b]*d + 7*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(105*b^(3/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{f b x^7 \sqrt{b x^4+a}}{9}+\frac{b e x^6 \sqrt{b x^4+a}}{8}+\frac{b d x^5 \sqrt{b x^4+a}}{7}+\frac{c b x^4 \sqrt{b x^4+a}}{6}+\frac{11 a f x^3 \sqrt{b x^4+a}}{45}+\frac{5 a e x^2 \sqrt{b x^4+a}}{16}+\frac{3 a d x \sqrt{b x^4+a}}{7}$
default	$d\left(\frac{b x^5 \sqrt{b x^4+a}}{7}+\frac{3 a x \sqrt{b x^4+a}}{7}+\frac{4 a^2 \sqrt{1-\frac{i \sqrt{b}\, x^2}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^2}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b x^4+a}}\right)+e\left(\frac{3 a^2 \ln\left(\sqrt{b}\, x^2+\sqrt{b x^4+a}\right)}{16 \sqrt{b}}+\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/9*f*b*x^7*(b*x^4+a)^(1/2)+1/8*b*e*x^6*(b*x^4+a)^(1/2)+1/7*b*d*x^5*(b*x^4
+a)^(1/2)+1/6*c*b*x^4*(b*x^4+a)^(1/2)+11/45*a*f*x^3*(b*x^4+a)^(1/2)+5/16*a
*e*x^2*(b*x^4+a)^(1/2)+3/7*a*d*x*(b*x^4+a)^(1/2)+2/3*a*c*(b*x^4+a)^(1/2)+4
/7*a^2*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)+3/16*a^2*e*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))/b^(1/2)+4/15*I*f*a^
(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*c*arct
anh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x, x)
```

Sympy [A] (verification not implemented)

Time = 11.46 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x}\,dx &= -\frac{a^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} \\ &+ \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}ex^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1+\frac{bx^4}{a}}} \\ &+ \frac{a^{\frac{3}{2}}fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} \\ &+ \frac{3\sqrt{a}bex^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bfx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} \\ &+ \frac{a^2c}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a^2e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{a\sqrt{b}cx^2}{2\sqrt{\frac{a}{bx^4}+1}} \\ &+ bc\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{b^2ex^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}\end{aligned}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x,x)
```

output
```
-a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*d*x*gamma(1/4)*hype
r((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**(3/2)
*e*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*e*x**2/(16*sqrt(1 + b*x**4/a)) + a
**(3/2)*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(7/4)) + sqrt(a)*b*d*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,
), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*b*e*x**6/(16*sqrt(
1 + b*x**4/a)) + sqrt(a)*b*f*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b
*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + a**2*c/(2*sqrt(b)*x**2*sqrt(a/(
b*x**4) + 1)) + 3*a**2*e*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt
(b)*c*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*c*Piecewise((sqrt(a)*x**4/4, Eq(b,
 0)), ((a + b*x**4)**(3/2)/(6*b), True)) + b**2*e*x**10/(8*sqrt(a)*sqrt(1
+ b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x}\,dx = \frac{6720\sqrt{b\,x^4+a}\,abc+4320\sqrt{b\,x^4+a}\,abdx+3150\sqrt{b\,x^4+a}\,abe\,x}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x,x)
```

output
```
(6720*sqrt(a + b*x**4)*a*b*c + 4320*sqrt(a + b*x**4)*a*b*d*x + 3150*sqrt(a
 + b*x**4)*a*b*e*x**2 + 2464*sqrt(a + b*x**4)*a*b*f*x**3 + 1680*sqrt(a + b
*x**4)*b**2*c*x**4 + 1440*sqrt(a + b*x**4)*b**2*d*x**5 + 1260*sqrt(a + b*x
**4)*b**2*e*x**6 + 1120*sqrt(a + b*x**4)*b**2*f*x**7 + 2520*sqrt(a)*log(sq
rt(a + b*x**4) - sqrt(a))*a*b*c - 2520*sqrt(a)*log(sqrt(a + b*x**4) + sqrt
(a))*a*b*c - 945*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*e + 945
*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a**2*e + 5760*int(sqrt(a + b
*x**4)/(a + b*x**4),x)*a**2*b*d + 2688*int((sqrt(a + b*x**4)*x**2)/(a + b*
x**4),x)*a**2*b*f)/(10080*b)
```

3.59 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 404

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{2}{35}x\left(5ae+21bcx^2\right)\sqrt{a+bx^4} + \frac{1}{16}a\left(8d+3fx^2\right)\sqrt{a+bx^4} - \frac{(7c-ex^2)(a+bx^4)^{3/2}}{7x} + \frac{1}{24}\left(4d+3fx^2\right)\left(a+bx^4\right)^{3/2} + \frac{3a^2f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}} - \frac{1}{2}a^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}} + \frac{2a^{5/4}\left(2\right.}{}$$

output
```
12*a*b^(1/2)*c*x*(b*x^4+a)^(1/2)/(5*a^(1/2)+5*b^(1/2)*x^2)+2/35*x*(21*b*c*
x^2+5*a*e)*(b*x^4+a)^(1/2)+1/16*a*(3*f*x^2+8*d)*(b*x^4+a)^(1/2)-1/7*(-e*x^
2+7*c)*(b*x^4+a)^(3/2)/x+1/24*(3*f*x^2+4*d)*(b*x^4+a)^(3/2)+3/16*a^2*f*arc
tanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/2)-1/2*a^(3/2)*d*arctanh((b*x^4+a)^
(1/2)/a^(1/2))-12/5*a^(5/4)*b^(1/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(21*b^(1/2)*c+5*a^(1/2)*e)*(a^(1/2)
+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.55

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{1}{16}f\sqrt{a+bx^4}\left(5ax^2 + 2bx^6 + \frac{3a^{3/2}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{1}{6}d\left(\sqrt{a+bx^4}(4a+bx^4)-3a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right) - \frac{ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\right.}{x\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

output
```
(f*Sqrt[a + b*x^4]*(5*a*x^2 + 2*b*x^6 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*x^2)/S
qrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x^4)/a])))/16 + (d*(Sqrt[a + b*x^4]*(4*a + b
*x^4) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]))/6 - (a*c*Sqrt[a + b*x
^4]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)])/(x*Sqrt[1 + (b*x^4)/
a]) + (a*e*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^4)/a
)])/Sqrt[1 + (b*x^4)/a]
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^2}\,dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(\frac{(a+bx^4)^{3/2}(c+ex^2)}{x^2}+\frac{(a+bx^4)^{3/2}(d+fx^2)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+21\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3a^2f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{16\sqrt{b}}-\frac{(a+bx^4)^{3/2}(7c-ex^2)}{7x}+\frac{2}{35}x\sqrt{a+bx^4}(5ae+21bcx^2)+\frac{12a\sqrt{b}cx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{16}a\sqrt{a+bx^4}(8d+3fx^2)+\frac{1}{24}(a+bx^4)^{3/2}(4d+3fx^2)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^2,x]
```

output
```
(12*a*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + (2*x*(5*a
*e + 21*b*c*x^2)*Sqrt[a + b*x^4])/35 + (a*(8*d + 3*f*x^2)*Sqrt[a + b*x^4])
/16 - ((7*c - e*x^2)*(a + b*x^4)^(3/2))/(7*x) + ((4*d + 3*f*x^2)*(a + b*x^
4)^(3/2))/24 + (3*a^2*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(16*Sqrt[b
]) - (a^(3/2)*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*
c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4
)*(21*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(S
qrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3
5*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.72 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{fb\,x^6\sqrt{b\,x^4+a}}{8}+\frac{be\,x^5\sqrt{b\,x^4+a}}{7}+\frac{x^4d\sqrt{b\,x^4+a}\,b}{6}+\frac{cb\,x^3\sqrt{b\,x^4+a}}{5}+\frac{5af\,x^2\sqrt{b\,x^4+a}}{16}+\frac{3aex\sqrt{b\,x^4+a}}{7}+$
default	$e\left(\frac{b\,x^5\sqrt{b\,x^4+a}}{7}+\frac{3ax\sqrt{b\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+c\left(-\frac{a\sqrt{b\,x^4+a}}{x}+\frac{\sqrt{b\,x^4+a}\,b}{5}\right.$
risch	$-\frac{ac\sqrt{b\,x^4+a}}{x}+\frac{4a^2e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{a^{\frac{3}{2}}d\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}+\frac{cb\,x^3\sqrt{b\,x^4+a}}{5}+\frac{3i\sqrt{b}}{}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*c*(b*x^4+a)^(1/2)/x+1/8*f*b*x^6*(b*x^4+a)^(1/2)+1/7*b*e*x^5*(b*x^4+a)^(
1/2)+1/6*x^4*d*(b*x^4+a)^(1/2)*b+1/5*c*b*x^3*(b*x^4+a)^(1/2)+5/16*a*f*x^2*
(b*x^4+a)^(1/2)+3/7*a*e*x*(b*x^4+a)^(1/2)+2/3*a*d*(b*x^4+a)^(1/2)+4/7*a^2*
e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+3/16*f*a^2*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))/b^(1/2)+12/5*I*a^(3/2)*b^(
1/2)*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*d*arctanh(a^(
1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{a^{\frac{3}{2}}ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}fx^2\sqrt{1+\frac{bx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}fx^2}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bcx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}bfx^6}{16\sqrt{1+\frac{bx^4}{a}}} + \frac{a^2d}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a^2f\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} + \frac{a\sqrt{b}dx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bd\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right) + \frac{b^2fx^{10}}{8\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**2,x)
```

output
```
a**(3/2)*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x*gamma(3/4)) - a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2
)*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**(3/2)*f*x**2*sqrt(1 + b*x**4/a)/4 + a**(3/2)*f*x**2/(16*sq
rt(1 + b*x**4/a)) + sqrt(a)*b*c*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*b*e*x**5*gamma(5/4)*hy
per((-1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt
(a)*b*f*x**6/(16*sqrt(1 + b*x**4/a)) + a**2*d/(2*sqrt(b)*x**2*sqrt(a/(b*x*
*4) + 1)) + 3*a**2*f*asinh(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)) + a*sqrt(b)*
d*x**2/(2*sqrt(a/(b*x**4) + 1)) + b*d*Piecewise((sqrt(a)*x**4/4, Eq(b, 0))
, ((a + b*x**4)**(3/2)/(6*b), True)) + b**2*f*x**10/(8*sqrt(a)*sqrt(1 + b*
x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^2}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^2}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx=\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^2}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^2, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^2}\,dx=\frac{4704\sqrt{b\,x^4+a}\,abc+2240\sqrt{b\,x^4+a}\,abdx+1440\sqrt{b\,x^4+a}\,abe\,x}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^2,x)
```

output
```
(4704*sqrt(a + b*x**4)*a*b*c + 2240*sqrt(a + b*x**4)*a*b*d*x + 1440*sqrt(a
 + b*x**4)*a*b*e*x**2 + 1050*sqrt(a + b*x**4)*a*b*f*x**3 + 672*sqrt(a + b*
x**4)*b**2*c*x**4 + 560*sqrt(a + b*x**4)*b**2*d*x**5 + 480*sqrt(a + b*x**4
)*b**2*e*x**6 + 420*sqrt(a + b*x**4)*b**2*f*x**7 + 840*sqrt(a)*log(sqrt(a
+ b*x**4) - sqrt(a))*a*b*d*x - 840*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))
*a*b*d*x - 315*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*f*x + 315
*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a**2*f*x + 8064*int(sqrt(a +
 b*x**4)/(a*x**2 + b*x**6),x)*a**2*b*c*x + 1920*int(sqrt(a + b*x**4)/(a +
b*x**4),x)*a**2*b*e*x)/(3360*b*x)
```

3.60 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 406

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^3}\,dx = \frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{1}{4}\left(2ae+3bcx^2\right)\sqrt{a+bx^4}$$
$$+\frac{2}{35}x\left(5af+21bdx^2\right)\sqrt{a+bx^4}-\frac{(3c-ex^2)(a+bx^4)^{3/2}}{6x^2}-\frac{(7d-fx^2)(a+bx^4)^{3/2}}{7x}$$
$$+\frac{3}{4}a\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{1}{2}a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\Big(2}{5\sqrt{a+bx^4}}$$

output
```
12*a*b^(1/2)*d*x*(b*x^4+a)^(1/2)/(5*a^(1/2)+5*b^(1/2)*x^2)+1/4*(3*b*c*x^2+
2*a*e)*(b*x^4+a)^(1/2)+2/35*x*(21*b*d*x^2+5*a*f)*(b*x^4+a)^(1/2)-1/6*(-e*x
^2+3*c)*(b*x^4+a)^(3/2)/x^2-1/7*(-f*x^2+7*d)*(b*x^4+a)^(3/2)/x+3/4*a*b^(1/
2)*c*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-1/2*a^(3/2)*e*arctanh((b*x^4+a)^
(1/2)/a^(1/2))-12/5*a^(5/4)*b^(1/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))/(b*x^4+a)^(1/2)+2/35*a^(5/4)*(21*b^(1/2)*d+5*a^(1/2)*f)*(a^(1/2)
+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = \frac{-3ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-\frac{bx^4}{a}\right)+x\Big(ex$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

output
```
(-3*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] +
 x*(e*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*Arc
Tanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*d*Sqrt[a + b*x^4]*Hypergeometric2F1[-
3/2, -1/4, 3/4, -((b*x^4)/a)] + 6*a*f*x^2*Sqrt[a + b*x^4]*Hypergeometric2F
1[-3/2, 1/4, 5/4, -((b*x^4)/a)]))/(6*x^2*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}\,(c+dx+ex^2+fx^3)}{x^3}\,dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(\frac{(a+bx^4)^{3/2}\,(c+ex^2)}{x^3}+\frac{(a+bx^4)^{3/2}\,(d+fx^2)}{x^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+21\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}e\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/2}\left(3c-ex^2\right)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}\left(2ae+3bcx^2\right)-\frac{\left(a+bx^4\right)^{3/2}\left(7d-fx^2\right)}{7x}+\frac{2}{35}x\sqrt{a+bx^4}\left(5af+21bdx^2\right)+\frac{12a\sqrt{b}dx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^3,x]
```

output
```
(12*a*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) + ((2*a*e +
 3*b*c*x^2)*Sqrt[a + b*x^4])/4 + (2*x*(5*a*f + 21*b*d*x^2)*Sqrt[a + b*x^4]
)/35 - ((3*c - e*x^2)*(a + b*x^4)^(3/2))/(6*x^2) - ((7*d - f*x^2)*(a + b*x
^4)^(3/2))/(7*x) + (3*a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
4 - (a^(3/2)*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(1/4)*d
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a^(5/4)
*(21*Sqrt[b]*d + 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(35
*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.49 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.85

method	result
elliptic	$-\frac{ac\sqrt{b x^4+a}}{2x^2}-\frac{ad\sqrt{b x^4+a}}{x}+\frac{fb x^5\sqrt{b x^4+a}}{7}+\frac{be x^4\sqrt{b x^4+a}}{6}+\frac{x^3 d\sqrt{b x^4+a}\, b}{5}+\frac{cb x^2\sqrt{b x^4+a}}{4}+\frac{3afx\sqrt{b x^4+a}}{7}+2a$
default	$f\left(\frac{b x^5\sqrt{b x^4+a}}{7}+\frac{3ax\sqrt{b x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}\right)+c\left(-\frac{a\sqrt{b x^4+a}}{2x^2}+\frac{b x^2\sqrt{b x^4+}}{4}\right.$
risch	$-\frac{a\sqrt{b x^4+a}\,(2dx+c)}{2x^2}+\frac{4f a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}-\frac{a^{\frac{3}{2}}e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^4+a}}{x^2}\right)}{2}+\frac{cb x^2\sqrt{b x^4+a}}{4}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*c*(b*x^4+a)^(1/2)/x^2-a*d*(b*x^4+a)^(1/2)/x+1/7*f*b*x^5*(b*x^4+a)^(
1/2)+1/6*b*e*x^4*(b*x^4+a)^(1/2)+1/5*x^3*d*(b*x^4+a)^(1/2)*b+1/4*c*b*x^2*(
b*x^4+a)^(1/2)+3/7*a*f*x*(b*x^4+a)^(1/2)+2/3*a*e*(b*x^4+a)^(1/2)+4/7*f*a^2
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+
3/4*a*b^(1/2)*c*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))+12/5*I*d*a^(3/2)*b^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*a^(3/2)*e*arctanh(a^(1/2)
/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^3, x)
```

Sympy [A] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = -\frac{a^{\frac{3}{2}}c}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{a^{\frac{3}{2}}fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bcx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bcx^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a}bfx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{a^2e}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{a\sqrt{b}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} + be\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**3,x)
```

output
```
-a**(3/2)*c/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)*d*gamma(-1/4)*hyper((-1
/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**(3/2)*e
*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + a**(3/2)*f*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*c*x**2*sq
rt(1 + b*x**4/a)/4 - sqrt(a)*b*c*x**2/(2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*d
*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(7/4)) + sqrt(a)*b*f*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4
*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**2*e/(2*sqrt(b)*x**2*sqrt(a/(b*x**4
) + 1)) + 3*a*sqrt(b)*c*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*e*x**2/(
2*sqrt(a/(b*x**4) + 1)) + b*e*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a +
b*x**4)**(3/2)/(6*b), True))
```

Maxima [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^3} \, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^3} \, dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Giac [F]

$$\int \frac{(c + dx + ex^2 + fx^3)(a + bx^4)^{3/2}}{x^3} \, dx = \int \frac{(bx^4 + a)^{\frac{3}{2}}(fx^3 + ex^2 + dx + c)}{x^3} \, dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^3}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^3, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^3}\,dx = \frac{-420\sqrt{b\,x^4+a}\,ac+1176\sqrt{b\,x^4+a}\,adx+560\sqrt{b\,x^4+a}\,ae\,x^2+}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^3,x)
```

output
```
( - 420*sqrt(a + b*x**4)*a*c + 1176*sqrt(a + b*x**4)*a*d*x + 560*sqrt(a +
b*x**4)*a*e*x**2 + 360*sqrt(a + b*x**4)*a*f*x**3 + 210*sqrt(a + b*x**4)*b*
c*x**4 + 168*sqrt(a + b*x**4)*b*d*x**5 + 140*sqrt(a + b*x**4)*b*e*x**6 + 1
20*sqrt(a + b*x**4)*b*f*x**7 + 210*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))
*a*e*x**2 - 210*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*a*e*x**2 - 315*sqr
t(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*c*x**2 + 315*sqrt(b)*log(sqrt(
a + b*x**4) + sqrt(b)*x**2)*a*c*x**2 + 2016*int(sqrt(a + b*x**4)/(a*x**2 +
 b*x**6),x)*a**2*d*x**2 + 480*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**2*f*
x**2)/(840*x**2)
```

3.61 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 408

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(9ae-5bcx^2)\sqrt{a+bx^4}}{15x}$$
$$+\frac{1}{4}(2af+3bdx^2)\sqrt{a+bx^4} - \frac{(5c-3ex^2)(a+bx^4)^{3/2}}{15x^3} - \frac{(3d-fx^2)(a+bx^4)^{3/2}}{6x^2}$$
$$+\frac{3}{4}a\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{1}{2}a^{3/2}f\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\Big(2}{5\sqrt{a+bx^4}}$$

output
```
12*a*b^(1/2)*e*x*(b*x^4+a)^(1/2)/(5*a^(1/2)+5*b^(1/2)*x^2)-2/15*(-5*b*c*x^
2+9*a*e)*(b*x^4+a)^(1/2)/x+1/4*(3*b*d*x^2+2*a*f)*(b*x^4+a)^(1/2)-1/15*(-3*
e*x^2+5*c)*(b*x^4+a)^(3/2)/x^3-1/6*(-f*x^2+3*d)*(b*x^4+a)^(3/2)/x^2+3/4*a*
b^(1/2)*d*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-1/2*a^(3/2)*f*arctanh((b*x^
4+a)^(1/2)/a^(1/2))-12/5*a^(5/4)*b^(1/4)*e*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a
)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))/(b*x^4+a)^(1/2)+2/15*a^(3/4)*b^(1/4)*(5*b^(1/2)*c+9*a^(1/2)
*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Invers
eJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^4}\,dx = \frac{-2ac\sqrt{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)-3adx\sqrt{}}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

output
```
(-2*a*c*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] -
 3*a*d*x*Sqrt[a + b*x^4]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)]
+ x^2*(f*x*Sqrt[1 + (b*x^4)/a]*(Sqrt[a + b*x^4]*(4*a + b*x^4) - 3*a^(3/2)*
ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]) - 6*a*e*Sqrt[a + b*x^4]*Hypergeometric2F
1[-3/2, -1/4, 3/4, -((b*x^4)/a)]))/(6*x^3*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}\,(c+dx+ex^2+fx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{(a+bx^4)^{3/2}\,(c+ex^2)}{x^4}+\frac{(a+bx^4)^{3/2}\,(d+fx^2)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{3/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15\sqrt{a+bx^4}}-\frac{12a^{5/4}\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+bx^4}}-\frac{1}{2}a^{3/2}f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)+\frac{3}{4}a\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{2\sqrt{a+bx^4}(9ae-5bcx^2)}{15x}-\frac{\left(a+bx^4\right)^{3/2}\left(5c-3ex^2\right)}{15x^3}-\frac{\left(a+bx^4\right)^{3/2}\left(3d-fx^2\right)}{6x^2}+\frac{1}{4}\sqrt{a+bx^4}(2af+3bdx^2)+\frac{12a\sqrt{b}ex\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^4,x]
```

output
```
(12*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(5*(Sqrt[a] + Sqrt[b]*x^2)) - (2*(9*a*e
 - 5*b*c*x^2)*Sqrt[a + b*x^4])/(15*x) + ((2*a*f + 3*b*d*x^2)*Sqrt[a + b*x^
4])/4 - ((5*c - 3*e*x^2)*(a + b*x^4)^(3/2))/(15*x^3) - ((3*d - f*x^2)*(a +
 b*x^4)^(3/2))/(6*x^2) + (3*a*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x
^4]])/4 - (a^(3/2)*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (12*a^(5/4)*b^(
1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*Sqrt[a + b*x^4]) + (2*a
^(3/4)*b^(1/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(15*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{3x^3}-\frac{ad\sqrt{b\,x^4+a}}{2x^2}-\frac{ae\sqrt{b\,x^4+a}}{x}+\frac{fb\,x^4\sqrt{b\,x^4+a}}{6}+\frac{be\,x^3\sqrt{b\,x^4+a}}{5}+\frac{x^2d\sqrt{b\,x^4+a}\,b}{4}+\frac{cbx\sqrt{b\,x^4+a}}{3}+\frac{2af\sqrt{b}}{3}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{a\sqrt{b\,x^4+a}}{2x^2}+\frac{b\,x^2\sqrt{b\,x^4+a}}{4}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6x^3}+\frac{4abc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{cbx\sqrt{b\,x^4+a}}{3}+\frac{x^2d\sqrt{b\,x^4+a}\,b}{4}+\frac{3da}{}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a*c*(b*x^4+a)^(1/2)/x^3-1/2*a*d*(b*x^4+a)^(1/2)/x^2-a*e*(b*x^4+a)^(1/
2)/x+1/6*f*b*x^4*(b*x^4+a)^(1/2)+1/5*b*e*x^3*(b*x^4+a)^(1/2)+1/4*x^2*d*(b*
x^4+a)^(1/2)*b+1/3*c*b*x*(b*x^4+a)^(1/2)+2/3*a*f*(b*x^4+a)^(1/2)+4/3*a*b*c
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+
3/4*d*a*b^(1/2)*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*b^(1/2)
*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*f*a^(3/2)*arctanh(a^(1/2)
/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^4}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}d}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^{\frac{3}{2}}f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} + \frac{\sqrt{a}bcx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bdx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bdx^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{a^2f}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{a\sqrt{b}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} + bf\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for}\ b=0\\ \frac{(a+bx^4)^{\frac{3}{2}}}{6b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**4,x)
```

output
```
a**(3/2)*c*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**3*gamma(1/4)) - a**(3/2)*d/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)
*e*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*
gamma(3/4)) - a**(3/2)*f*asinh(sqrt(a)/(sqrt(b)*x**2))/2 + sqrt(a)*b*c*x*g
amma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/
4)) + sqrt(a)*b*d*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*d*x**2/(2*sqrt(1 +
 b*x**4/a)) + sqrt(a)*b*e*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**
4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + a**2*f/(2*sqrt(b)*x**2*sqrt(a/(b*x**
4) + 1)) + 3*a*sqrt(b)*d*asinh(sqrt(b)*x**2/sqrt(a))/4 + a*sqrt(b)*f*x**2/
(2*sqrt(a/(b*x**4) + 1)) + b*f*Piecewise((sqrt(a)*x**4/4, Eq(b, 0)), ((a +
 b*x**4)**(3/2)/(6*b), True))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx=\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^4}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^4, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^4}\,dx=\frac{-200\sqrt{b\,x^4+a}\,ac-60\sqrt{b\,x^4+a}\,adx+168\sqrt{b\,x^4+a}\,ae\,x^2+8}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^4,x)
```

output
```
( - 200*sqrt(a + b*x**4)*a*c - 60*sqrt(a + b*x**4)*a*d*x + 168*sqrt(a + b*
x**4)*a*e*x**2 + 80*sqrt(a + b*x**4)*a*f*x**3 + 40*sqrt(a + b*x**4)*b*c*x*
*4 + 30*sqrt(a + b*x**4)*b*d*x**5 + 24*sqrt(a + b*x**4)*b*e*x**6 + 20*sqrt
(a + b*x**4)*b*f*x**7 + 30*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*a*f*x**
3 - 30*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*a*f*x**3 - 45*sqrt(b)*log(s
qrt(a + b*x**4) - sqrt(b)*x**2)*a*d*x**3 + 45*sqrt(b)*log(sqrt(a + b*x**4)
 + sqrt(b)*x**2)*a*d*x**3 - 480*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*
a**2*c*x**3 + 288*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a**2*e*x**3)/(
120*x**3)
```

3.62 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 30, antiderivative size = 403

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5} dx = \frac{12a\sqrt{b}fx\sqrt{a+bx^4}}{5\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(9af-5bdx^2)\sqrt{a+bx^4}}{15x} + \frac{3}{4}b(c+ex^2)\sqrt{a+bx^4} - \frac{(c+2ex^2)(a+bx^4)^{3/2}}{4x^4} - \frac{(5d-3fx^2)(a+bx^4)^{3/2}}{15x^3} + \frac{3}{4}a\sqrt{b}e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bc\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12a^{5/4}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(\right.}{5\sqrt{a+bx^4}}$$

output
```
12*a*b^(1/2)*f*x*(b*x^4+a)^(1/2)/(5*a^(1/2)+5*b^(1/2)*x^2)-2/15*(-5*b*d*x^
2+9*a*f)*(b*x^4+a)^(1/2)/x+3/4*b*(e*x^2+c)*(b*x^4+a)^(1/2)-1/4*(2*e*x^2+c)
*(b*x^4+a)^(3/2)/x^4-1/15*(-3*f*x^2+5*d)*(b*x^4+a)^(3/2)/x^3+3/4*a*b^(1/2)
*e*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-3/4*a^(1/2)*b*c*arctanh((b*x^4+a)^
(1/2)/a^(1/2))-12/5*a^(5/4)*b^(1/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))/(b*x^4+a)^(1/2)+2/15*a^(3/4)*b^(1/4)*(5*b^(1/2)*d+9*a^(1/2)*f)*(
a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.40

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{a+bx^4}\left(-10a^3d\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)+3x\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

output
```
(Sqrt[a + b*x^4]*(-10*a^3*d*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a
)] + 3*x*(-5*a^3*e*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] - 10*a
^3*f*x*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^4)/a)] + b*c*x^2*(a + b*x
^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))
/(30*a^2*x^3*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}\,(c+dx+ex^2+fx^3)}{x^5}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(12fx^3+6ex^2+4dx+3c)\sqrt{bx^4+a}}{12x}dx-\frac{1}{12}(a+bx^4)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}b\int \frac{(12fx^3+6ex^2+4dx+3c)\sqrt{bx^4+a}}{x}dx-\frac{1}{12}(a+bx^4)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\downarrow \text{2372}$$

$$\frac{1}{2}b\int\left(\frac{\sqrt{bx^4+a}(6ex^2+3c)}{x}+(12fx^2+4d)\sqrt{bx^4+a}\right)dx-\frac{1}{12}(a+bx^4)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}b\left(\frac{4a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^4}}-\frac{24a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right.}{}\right.$$
$$\frac{1}{12}(a+bx^4)^{3/2}\left(\frac{3c}{x^4}+\frac{4d}{x^3}+\frac{6e}{x^2}+\frac{12f}{x}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^5,x]
```

output
```
-1/12*(((3*c)/x^4 + (4*d)/x^3 + (6*e)/x^2 + (12*f)/x)*(a + b*x^4)^(3/2)) +
 (b*((24*a*f*x*Sqrt[a + b*x^4])/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (3*(
c + e*x^2)*Sqrt[a + b*x^4])/2 + (4*x*(5*d + 9*f*x^2)*Sqrt[a + b*x^4])/15 +
 (3*a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (3*Sqrt[a]*c
*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (24*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a + b*x^4]) + (4*a^(3/4)*(5*Sqrt[b]*d
+ 9*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a
 + b*x^4])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.40 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.85

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{4x^4}-\frac{ad\sqrt{b\,x^4+a}}{3x^3}-\frac{ae\sqrt{b\,x^4+a}}{2x^2}-\frac{af\sqrt{b\,x^4+a}}{x}+\frac{fb\,x^3\sqrt{b\,x^4+a}}{5}+\frac{be\,x^2\sqrt{b\,x^4+a}}{4}+\frac{xd\sqrt{b\,x^4+a}\,b}{3}+\frac{bc\sqrt{b\,x^4}}{2}$
default	$c\left(\frac{b\sqrt{b\,x^4+a}}{2}-\frac{a\sqrt{b\,x^4+a}}{4x^4}-\frac{3\sqrt{a}\,b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{4}\right)+d\left(-\frac{a\sqrt{b\,x^4+a}}{3x^3}+\frac{bx\sqrt{b\,x^4+a}}{3}+\frac{4ab\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}}{3\sqrt{}}\right.$
risch	$-\frac{a\sqrt{b\,x^4+a}\,(12f\,x^3+6e\,x^2+4dx+3c)}{12x^4}+\frac{4bda\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3b\sqrt{a}\,c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{4}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*c*(b*x^4+a)^(1/2)/x^4-1/3*a*d*(b*x^4+a)^(1/2)/x^3-1/2*a*e*(b*x^4+a)
^(1/2)/x^2-a*f*(b*x^4+a)^(1/2)/x+1/5*f*b*x^3*(b*x^4+a)^(1/2)+1/4*b*e*x^2*(
b*x^4+a)^(1/2)+1/3*x*d*(b*x^4+a)^(1/2)*b+1/2*b*c*(b*x^4+a)^(1/2)+4/3*b*d*a
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^
(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+
3/4*a*b^(1/2)*e*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))+12/5*I*a^(3/2)*b^(1/2)
*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*b*c*arctanh(a^(1/
2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.27 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}e}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bdx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bex^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bex^2}{2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bfx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}c}{2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}}cx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**5,x)
```

output
```
a**(3/2)*d*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**3*gamma(1/4)) - a**(3/2)*e/(2*x**2*sqrt(1 + b*x**4/a)) + a**(3/2)
*f*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*
gamma(3/4)) - 3*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b)*x**2))/4 + sqrt(a)*b*d*
x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma
(5/4)) + sqrt(a)*b*e*x**2*sqrt(1 + b*x**4/a)/4 - sqrt(a)*b*e*x**2/(2*sqrt(
1 + b*x**4/a)) + sqrt(a)*b*f*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/
(4*x**2) + a*sqrt(b)*c/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*e*asinh
(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*c*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
1/8*(3*sqrt(a)*b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a
))) + 4*sqrt(b*x^4 + a)*b - 2*sqrt(b*x^4 + a)*a/x^4)*c + integrate((b*f*x^
6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^4, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^5}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^5}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^5, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^5}\,dx = \frac{-30\sqrt{b\,x^4+a}\,ac-200\sqrt{b\,x^4+a}\,adx-60\sqrt{b\,x^4+a}\,ae\,x^2+16$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^5,x)
```

output
```
( - 30*sqrt(a + b*x**4)*a*c - 200*sqrt(a + b*x**4)*a*d*x - 60*sqrt(a + b*x
**4)*a*e*x**2 + 168*sqrt(a + b*x**4)*a*f*x**3 + 60*sqrt(a + b*x**4)*b*c*x*
*4 + 40*sqrt(a + b*x**4)*b*d*x**5 + 30*sqrt(a + b*x**4)*b*e*x**6 + 24*sqrt
(a + b*x**4)*b*f*x**7 + 45*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*c*x**
4 - 45*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*c*x**4 - 45*sqrt(b)*log(s
qrt(a + b*x**4) - sqrt(b)*x**2)*a*e*x**4 + 45*sqrt(b)*log(sqrt(a + b*x**4)
 + sqrt(b)*x**2)*a*e*x**4 - 480*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*
a**2*d*x**4 + 288*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a**2*f*x**4)/(
120*x**4)
```

3.63 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 405

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = -\frac{12\sqrt{a}bc\sqrt{a+bx^4}}{5x\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(5ae-9bcx^2)\sqrt{a+bx^4}}{15x^3} + \frac{3}{4}b(d+fx^2)\sqrt{a+bx^4} - \frac{(3c-5ex^2)(a+bx^4)^{3/2}}{15x^5} - \frac{(d+2fx^2)(a+bx^4)^{3/2}}{4x^4} + \frac{3}{4}a\sqrt{b}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bd\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\Big(}{5\sqrt{a+bx^4}}$$

output
```
-12/5*a^(1/2)*b*c*(b*x^4+a)^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)-2/15*(-9*b*c*x^2
+5*a*e)*(b*x^4+a)^(1/2)/x^3+3/4*b*(f*x^2+d)*(b*x^4+a)^(1/2)-1/15*(-5*e*x^2
+3*c)*(b*x^4+a)^(3/2)/x^5-1/4*(2*f*x^2+d)*(b*x^4+a)^(3/2)/x^4+3/4*a*b^(1/2
)*f*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-3/4*a^(1/2)*b*d*arctanh((b*x^4+a)
^(1/2)/a^(1/2))-12/5*a^(1/4)*b^(5/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a
^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/
2*2^(1/2))/(b*x^4+a)^(1/2)+2/15*a^(1/4)*b^(3/4)*(9*b^(1/2)*c+5*a^(1/2)*e)*
(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJac
obiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.41

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{\sqrt{a+bx^4}\left(-6a^3c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{bx^4}{a}\right)-1\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

output
```
(Sqrt[a + b*x^4]*(-6*a^3*c*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a
)] - 10*a^3*e*x^2*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] - 15*a^
3*f*x^3*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x^4)/a)] + 3*b*d*x^5*(a +
b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]
))/(30*a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^6}\,dx$$

$$\Big\downarrow \ 2364$$

$$-6b\int -\frac{(30fx^3+20ex^2+15dx+12c)\sqrt{bx^4+a}}{60x^2}dx - \frac{1}{60}(a+bx^4)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{10}b\int \frac{\left(30fx^3+20ex^2+15dx+12c\right)\sqrt{bx^4+a}}{x^2}dx-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{10}b\int \left(\frac{\sqrt{bx^4+a}\left(20ex^2+12c\right)}{x^2}+\frac{\left(30fx^2+15d\right)\sqrt{bx^4+a}}{x}\right)dx-\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}b\left(\frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{24\sqrt[4]{a}\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}\right.}{}\right.$$

$$\frac{1}{60}\left(a+bx^4\right)^{3/2}\left(\frac{12c}{x^5}+\frac{15d}{x^4}+\frac{20e}{x^3}+\frac{30f}{x^2}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^6,x]
```

output
```
-1/60*(((12*c)/x^5 + (15*d)/x^4 + (20*e)/x^3 + (30*f)/x^2)*(a + b*x^4)^(3/
2)) + (b*((24*Sqrt[b]*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - (4*(9
*c - 5*e*x^2)*Sqrt[a + b*x^4])/(3*x) + (15*(d + f*x^2)*Sqrt[a + b*x^4])/2
+ (15*a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (15*Sqrt[a
]*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (24*a^(1/4)*b^(1/4)*c*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (4*a^(1/4)*(9*Sqrt[b]*c +
 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a +
 b*x^4])))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.85

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{5x^5}-\frac{ad\sqrt{b\,x^4+a}}{4x^4}-\frac{ae\sqrt{b\,x^4+a}}{3x^3}-\frac{af\sqrt{b\,x^4+a}}{2x^2}-\frac{7cb\sqrt{b\,x^4+a}}{5x}+\frac{fb\,x^2\sqrt{b\,x^4+a}}{4}+\frac{bex\sqrt{b\,x^4+a}}{3}+\frac{d\sqrt{b\,x^4+a}}{2}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{5x^5}-\frac{7b\sqrt{b\,x^4+a}}{5x}+\frac{12ib^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d$
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(84bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac\right)}{60x^5}+\frac{4bea\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3b\sqrt{a}\,d\ln\left(2a\cdots\right)}{\cdots}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a*c*(b*x^4+a)^(1/2)/x^5-1/4*a*d*(b*x^4+a)^(1/2)/x^4-1/3*a*e*(b*x^4+a)
^(1/2)/x^3-1/2*a*f*(b*x^4+a)^(1/2)/x^2-7/5*c*b*(b*x^4+a)^(1/2)/x+1/4*f*b*x
^2*(b*x^4+a)^(1/2)+1/3*b*e*x*(b*x^4+a)^(1/2)+1/2*d*(b*x^4+a)^(1/2)*b+4/3*b
*e*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)+3/4*a*b^(1/2)*f*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))+12/5*I*b^(3/2)*c*a
^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*d*a^(1/2)*b*arctanh(a
^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^6}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.39 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^{\frac{3}{2}}f}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bc\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bd\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}bfx^2\sqrt{1+\frac{bx^4}{a}}}{4} - \frac{\sqrt{a}bfx^2}{2\sqrt{1+\frac{bx^4}{a}}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}d}{2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3a\sqrt{b}f\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^{\frac{3}{2}}dx^2}{2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**6,x)
```

output
```
a**(3/2)*c*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4
,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**(3/2)*f/(2*x**2*sqr
t(1 + b*x**4/a)) + sqrt(a)*b*c*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x
**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*d*asinh(sqrt(a)/(sqr
t(b)*x**2))/4 + sqrt(a)*b*e*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4
*exp_polar(I*pi)/a)/(4*gamma(5/4)) + sqrt(a)*b*f*x**2*sqrt(1 + b*x**4/a)/4
 - sqrt(a)*b*f*x**2/(2*sqrt(1 + b*x**4/a)) - a*sqrt(b)*d*sqrt(a/(b*x**4) +
 1)/(4*x**2) + a*sqrt(b)*d/(2*x**2*sqrt(a/(b*x**4) + 1)) + 3*a*sqrt(b)*f*a
sinh(sqrt(b)*x**2/sqrt(a))/4 + b**(3/2)*d*x**2/(2*sqrt(a/(b*x**4) + 1))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^6}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^6}\,dx=\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^6}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^6, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^6}\,dx = \frac{-24\sqrt{b\,x^4+a}\,ac-6\sqrt{b\,x^4+a}\,adx-40\sqrt{b\,x^4+a}\,ae\,x^2-12\sqrt{b}}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^6,x)
```

output
```
( - 24*sqrt(a + b*x**4)*a*c - 6*sqrt(a + b*x**4)*a*d*x - 40*sqrt(a + b*x**
4)*a*e*x**2 - 12*sqrt(a + b*x**4)*a*f*x**3 + 24*sqrt(a + b*x**4)*b*c*x**4
+ 12*sqrt(a + b*x**4)*b*d*x**5 + 8*sqrt(a + b*x**4)*b*e*x**6 + 6*sqrt(a +
b*x**4)*b*f*x**7 + 9*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*d*x**5 - 9*
sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*d*x**5 - 9*sqrt(b)*log(sqrt(a +
b*x**4) - sqrt(b)*x**2)*a*f*x**5 + 9*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b
)*x**2)*a*f*x**5 - 96*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a**2*c*x*
*5 - 96*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a**2*e*x**5)/(24*x**5)
```

3.64 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 412

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = -\frac{12\sqrt{a}bd\sqrt{a+bx^4}}{5x\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{2(5af-9bdx^2)\sqrt{a+bx^4}}{15x^3} - \frac{b(2c-3ex^2)\sqrt{a+bx^4}}{4x^2} - \frac{(2c+3ex^2)(a+bx^4)^{3/2}}{12x^6} - \frac{(3d-5fx^2)(a+bx^4)^{3/2}}{15x^5} + \frac{1}{2}b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}be\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\right.}{5\sqrt{a+bx^4}}$$

output

```
-12/5*a^(1/2)*b*d*(b*x^4+a)^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)-2/15*(-9*b*d*x^2
+5*a*f)*(b*x^4+a)^(1/2)/x^3-1/4*b*(-3*e*x^2+2*c)*(b*x^4+a)^(1/2)/x^2-1/12*
(3*e*x^2+2*c)*(b*x^4+a)^(3/2)/x^6-1/15*(-5*f*x^2+3*d)*(b*x^4+a)^(3/2)/x^5+
1/2*b^(3/2)*c*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-3/4*a^(1/2)*b*e*arctanh
((b*x^4+a)^(1/2)/a^(1/2))-12/5*a^(1/4)*b^(5/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b
*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))/(b*x^4+a)^(1/2)+2/15*a^(1/4)*b^(3/4)*(9*b^(1/2)*d+5*a
^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.40

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \frac{\sqrt{a+bx^4}\left(-5a^3c\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},-\frac{bx^4}{a}\right)-6\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

output
```
(Sqrt[a + b*x^4]*(-5*a^3*c*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a
)] - 6*a^3*d*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] - 10*a^3*
f*x^3*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^4)/a)] + 3*b*e*x^6*(a + b*
x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a]))
/(30*a^2*x^6*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^7}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(20fx^3+15ex^2+12dx+10c)\sqrt{bx^4+a}}{60x^3}dx-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}b\int \frac{(20fx^3+15ex^2+12dx+10c)\sqrt{bx^4+a}}{x^3}dx-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \ 2372$$

$$\frac{1}{10}b\int \left(\frac{\sqrt{bx^4+a}(15ex^2+10c)}{x^3}+\frac{(20fx^2+12d)\sqrt{bx^4+a}}{x^2}\right)dx-\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{10}b\left(\frac{4\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}f+9\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{24\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{\right.}}{}\right.$$

$$\frac{1}{60}(a+bx^4)^{3/2}\left(\frac{10c}{x^6}+\frac{12d}{x^5}+\frac{15e}{x^4}+\frac{20f}{x^3}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^7,x]
```

output
```
-1/60*(((10*c)/x^6 + (12*d)/x^5 + (15*e)/x^4 + (20*f)/x^3)*(a + b*x^4)^(3/
2)) + (b*((24*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - (5*(2
*c - 3*e*x^2)*Sqrt[a + b*x^4])/(2*x^2) - (4*(9*d - 5*f*x^2)*Sqrt[a + b*x^4
])/(3*x) + 5*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - (15*Sqrt[a
]*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2 - (24*a^(1/4)*b^(1/4)*d*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (4*a^(1/4)*(9*Sqrt[b]*d +
 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[a +
 b*x^4])))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.83

method	result
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{6x^6}-\frac{ad\sqrt{b\,x^4+a}}{5x^5}-\frac{ae\sqrt{b\,x^4+a}}{4x^4}-\frac{af\sqrt{b\,x^4+a}}{3x^3}-\frac{2cb\sqrt{b\,x^4+a}}{3x^2}-\frac{7bd\sqrt{b\,x^4+a}}{5x}+\frac{fbx\sqrt{b\,x^4+a}}{3}+\frac{be\sqrt{b\,x^4+a}}{2}$
default	$c\left(\frac{b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2}-\frac{a\sqrt{b\,x^4+a}}{6x^6}-\frac{2b\sqrt{b\,x^4+a}}{3x^2}\right)+d\left(-\frac{a\sqrt{b\,x^4+a}}{5x^5}-\frac{7b\sqrt{b\,x^4+a}}{5x}+\frac{12ib^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(84x^5bd+40bc\,x^4+20af\,x^3+15ae\,x^2+12adx+10ac)}{60x^6}+\frac{4bfa\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3b\sqrt{a}}{}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*c*(b*x^4+a)^(1/2)/x^6-1/5*a*d*(b*x^4+a)^(1/2)/x^5-1/4*a*e*(b*x^4+a)
^(1/2)/x^4-1/3*a*f*(b*x^4+a)^(1/2)/x^3-2/3*c*b*(b*x^4+a)^(1/2)/x^2-7/5*b*d
*(b*x^4+a)^(1/2)/x+1/3*f*b*x*(b*x^4+a)^(1/2)+1/2*b*e*(b*x^4+a)^(1/2)+4/3*b
*f*a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2)
,I)+1/2*b^(3/2)*c*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))+12/5*I*b^(3/2)*d*a^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-3/4*a^(1/2)*b*e*arctanh(a^(
1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^7, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bc}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bd\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{\sqrt{a}bfx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}e}{2x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}}ex^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^2cx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**7,x)
```

output
```
a**(3/2)*d*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**5*gamma(-1/4)) + a**(3/2)*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4
,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*c/(2*x**2*sq
rt(1 + b*x**4/a)) + sqrt(a)*b*d*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*
x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*sqrt(a)*b*e*asinh(sqrt(a)/(sq
rt(b)*x**2))/4 + sqrt(a)*b*f*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**
4*exp_polar(I*pi)/a)/(4*gamma(5/4)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(6*
x**4) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*e/(2*x**2*sq
rt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*c*asinh
(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*e*x**2/(2*sqrt(a/(b*x**4) + 1)) - b**2
*c*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/12*(3*b^(3/2)*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^
4 + a)/x^2)) + 6*sqrt(b*x^4 + a)*b/x^2 + 2*(b*x^4 + a)^(3/2)/x^6)*c + inte
grate((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a
)/x^6, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^7}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^7}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^7}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^7, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^7}\,dx = \frac{-4\sqrt{b\,x^4+a}\,ac-24\sqrt{b\,x^4+a}\,adx-6\sqrt{b\,x^4+a}\,ae\,x^2-40\sqrt{b\,}}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^7,x)
```

output
```
( - 4*sqrt(a + b*x**4)*a*c - 24*sqrt(a + b*x**4)*a*d*x - 6*sqrt(a + b*x**4
)*a*e*x**2 - 40*sqrt(a + b*x**4)*a*f*x**3 - 16*sqrt(a + b*x**4)*b*c*x**4 +
 24*sqrt(a + b*x**4)*b*d*x**5 + 12*sqrt(a + b*x**4)*b*e*x**6 + 8*sqrt(a +
b*x**4)*b*f*x**7 + 9*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*e*x**6 - 9*
sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*e*x**6 - 6*sqrt(b)*log(sqrt(a +
b*x**4) - sqrt(b)*x**2)*b*c*x**6 + 6*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b
)*x**2)*b*c*x**6 - 96*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a**2*d*x*
*6 - 96*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a**2*f*x**6)/(24*x**6)
```

3.65 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 430

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \frac{4bc\sqrt{a+bx^4}}{7x^3} - \frac{12\sqrt{a}be\sqrt{a+bx^4}}{5x\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

$$-\frac{6(7ae+5bcx^2)\sqrt{a+bx^4}}{35x^5} - \frac{b(2d-3fx^2)\sqrt{a+bx^4}}{4x^2}$$

$$-\frac{(c-7ex^2)(a+bx^4)^{3/2}}{7x^7} - \frac{(2d+3fx^2)(a+bx^4)^{3/2}}{12x^6}$$

$$+\frac{1}{2}b^{3/2}d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3}{4}\sqrt{a}bf\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right) - \frac{12\sqrt[4]{a}b^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\right.}{5\sqrt{a+bx^4}}$$

output
```
4/7*b*c*(b*x^4+a)^(1/2)/x^3-12/5*a^(1/2)*b*e*(b*x^4+a)^(1/2)/x/(a^(1/2)+b^
(1/2)*x^2)-6/35*(5*b*c*x^2+7*a*e)*(b*x^4+a)^(1/2)/x^5-1/4*b*(-3*f*x^2+2*d)
*(b*x^4+a)^(1/2)/x^2-1/7*(-7*e*x^2+c)*(b*x^4+a)^(3/2)/x^7-1/12*(3*f*x^2+2*
d)*(b*x^4+a)^(3/2)/x^6+1/2*b^(3/2)*d*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-
3/4*a^(1/2)*b*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))-12/5*a^(1/4)*b^(5/4)*e*(a
^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(b*x^4+a)^(1/2)+2/35*b^(5/4)*(
5*b^(1/2)*c+21*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2
)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a
^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.38

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{a+bx^4}\left(-30a^3c\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

output
```
(Sqrt[a + b*x^4]*(-30*a^3*c*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/
a)] + 7*x*(-5*a^3*d*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x^4)/a)] - 6*
a^3*e*x*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^4)/a)] + 3*b*f*x^6*(a +
 b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x^4)/a
])))/(210*a^2*x^7*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^8}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(105fx^3+84ex^2+70dx+60c)\sqrt{bx^4+a}}{420x^4}dx-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{70}b\int \frac{(105fx^3+84ex^2+70dx+60c)\sqrt{bx^4+a}}{x^4}dx-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{70}b\int \left(\frac{\sqrt{bx^4+a}(84ex^2+60c)}{x^4}+\frac{(105fx^2+70d)\sqrt{bx^4+a}}{x^3}\right)dx-\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{70}b\left(\frac{4\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}e+5\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+bx^4}}-\frac{168\sqrt[4]{a}\sqrt[4]{b}e\left(\sqrt{a}+\right.}{}\right.$$

$$\frac{1}{420}(a+bx^4)^{3/2}\left(\frac{60c}{x^7}+\frac{70d}{x^6}+\frac{84e}{x^5}+\frac{105f}{x^4}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^8,x]
```

output
```
-1/420*(((60*c)/x^7 + (70*d)/x^6 + (84*e)/x^5 + (105*f)/x^4)*(a + b*x^4)^(
3/2)) + (b*((-168*e*Sqrt[a + b*x^4])/x + (168*Sqrt[b]*e*x*Sqrt[a + b*x^4])
/(Sqrt[a] + Sqrt[b]*x^2) - (4*(5*c - 21*e*x^2)*Sqrt[a + b*x^4])/x^3 - (35*
(2*d - 3*f*x^2)*Sqrt[a + b*x^4])/(2*x^2) + 35*Sqrt[b]*d*ArcTanh[(Sqrt[b]*x
^2)/Sqrt[a + b*x^4]] - (105*Sqrt[a]*f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/2
- (168*a^(1/4)*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a +
 b*x^4] + (4*b^(1/4)*(5*Sqrt[b]*c + 21*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(a^(1/4)*Sqrt[a + b*x^4])))/70
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(588be\,x^6+280x^5bd+180bc\,x^4+105af\,x^3+84ae\,x^2+70adx+60ac)}{420x^7}+\frac{b\left(\frac{40cb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.}{}$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{7x^7}-\frac{ad\sqrt{b\,x^4+a}}{6x^6}-\frac{ae\sqrt{b\,x^4+a}}{5x^5}-\frac{af\sqrt{b\,x^4+a}}{4x^4}-\frac{3bc\sqrt{b\,x^4+a}}{7x^3}-\frac{2bd\sqrt{b\,x^4+a}}{3x^2}-\frac{7be\sqrt{b\,x^4+a}}{5x}+\frac{fb\sqrt{b\,x^4+a}}{2}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{7x^7}-\frac{3b\sqrt{b\,x^4+a}}{7x^3}+\frac{4b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(\frac{b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2}-\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/420*(b*x^4+a)^(1/2)*(588*b*e*x^6+280*b*d*x^5+180*b*c*x^4+105*a*f*x^3+84
*a*e*x^2+70*a*d*x+60*a*c)/x^7+1/70*b*(40*c*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-105/2*a^(1/2)*f*ln((2*a+2*a^(1
/2)*(b*x^4+a)^(1/2))/x^2)+35*b^(1/2)*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))+168
*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/
2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+35*f*(b*x^4
+a)^(1/2))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^8}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.17 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^8}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bd}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}be\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{3\sqrt{a}bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} + \frac{a\sqrt{b}f}{2x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{b^{\frac{3}{2}}fx^2}{2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^2dx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**8,x)
```

output
```
a**(3/2)*c*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*c*gamma(-3
/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/
4)) - sqrt(a)*b*d/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*e*gamma(-1/4)*hy
per((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - 3*s
qrt(a)*b*f*asinh(sqrt(a)/(sqrt(b)*x**2))/4 - a*sqrt(b)*d*sqrt(a/(b*x**4) +
 1)/(6*x**4) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) + a*sqrt(b)*f/(2*
x**2*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*
d*asinh(sqrt(b)*x**2/sqrt(a))/2 + b**(3/2)*f*x**2/(2*sqrt(a/(b*x**4) + 1))
 - b**2*d*x**2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^8}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^8}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^8, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^8}\,dx = \frac{24\sqrt{b\,x^4+a}\,ac-20\sqrt{b\,x^4+a}\,adx-120\sqrt{b\,x^4+a}\,ae\,x^2-30\sqrt{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^8,x)
```

output
```
(24*sqrt(a + b*x**4)*a*c - 20*sqrt(a + b*x**4)*a*d*x - 120*sqrt(a + b*x**4
)*a*e*x**2 - 30*sqrt(a + b*x**4)*a*f*x**3 - 120*sqrt(a + b*x**4)*b*c*x**4
- 80*sqrt(a + b*x**4)*b*d*x**5 + 120*sqrt(a + b*x**4)*b*e*x**6 + 60*sqrt(a
 + b*x**4)*b*f*x**7 + 45*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*f*x**7
- 45*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b*f*x**7 - 30*sqrt(b)*log(sqr
t(a + b*x**4) - sqrt(b)*x**2)*b*d*x**7 + 30*sqrt(b)*log(sqrt(a + b*x**4) +
 sqrt(b)*x**2)*b*d*x**7 + 288*int(sqrt(a + b*x**4)/(a*x**8 + b*x**12),x)*a
**2*c*x**7 - 480*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a**2*e*x**7)/(
120*x**7)
```

3.66 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 432

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \frac{4bd\sqrt{a+bx^4}}{7x^3} - \frac{12\sqrt{a}bf\sqrt{a+bx^4}}{5x\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$- \frac{6(7af+5bdx^2)\sqrt{a+bx^4}}{35x^5} - \frac{b(3c+8ex^2)\sqrt{a+bx^4}}{16x^4}$$
$$- \frac{(3c+4ex^2)(a+bx^4)^{3/2}}{24x^8} - \frac{(d-7fx^2)(a+bx^4)^{3/2}}{7x^7}$$
$$+\frac{1}{2}b^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right) - \frac{3b^2c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}} - \frac{12\sqrt[4]{a}b^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\right.}{5\sqrt{a+bx^4}}$$

output
```
4/7*b*d*(b*x^4+a)^(1/2)/x^3-12/5*a^(1/2)*b*f*(b*x^4+a)^(1/2)/x/(a^(1/2)+b^
(1/2)*x^2)-6/35*(5*b*d*x^2+7*a*f)*(b*x^4+a)^(1/2)/x^5-1/16*b*(8*e*x^2+3*c)
*(b*x^4+a)^(1/2)/x^4-1/24*(4*e*x^2+3*c)*(b*x^4+a)^(3/2)/x^8-1/7*(-7*f*x^2+
d)*(b*x^4+a)^(3/2)/x^7+1/2*b^(3/2)*e*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-
3/16*b^2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-12/5*a^(1/4)*b^(5/4)*f
*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE
(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(b*x^4+a)^(1/2)+2/35*b^(5/4
)*(5*b^(1/2)*d+21*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(
1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2)
)/a^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.40

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^9}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(240a^2dx\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)+7\left(15c\left(a(2a+5bx^4)\sqrt{1+\frac{bx^4}{a}}+3b^2x^8a\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

output
```
-1/1680*(Sqrt[a + b*x^4]*(240*a^2*d*x*Hypergeometric2F1[-7/4, -3/2, -3/4,
-((b*x^4)/a)] + 7*(15*c*(a*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 3*b^2*x^8
*ArcTanh[Sqrt[1 + (b*x^4)/a]]) + 40*a^2*e*x^2*Hypergeometric2F1[-3/2, -3/2
, -1/2, -((b*x^4)/a)] + 48*a^2*f*x^3*Hypergeometric2F1[-3/2, -5/4, -1/4, -
((b*x^4)/a)])))/(a*x^8*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2364, 27, 2364, 27, 2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^9}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(168fx^3+140ex^2+120dx+105c)\sqrt{bx^4+a}}{840x^5}dx-\frac{1}{840}(a+bx^4)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{140}b\int \frac{(168fx^3+140ex^2+120dx+105c)\sqrt{bx^4+a}}{x^5}dx-\frac{1}{840}(a+bx^4)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{2364}$$

$$\frac{1}{140}b\left(-2b\int -\frac{672fx^3+280ex^2+160dx+105c}{4x\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\right)-\frac{1}{840}(a+bx^4)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{140}b\left(\frac{1}{2}b\int \frac{672fx^3+280ex^2+160dx+105c}{x\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right)\right)-\frac{1}{840}(a+bx^4)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$$\Big\downarrow \text{2371}$$

$$\frac{1}{140}b\left(\frac{1}{2}b\left(105c\int\frac{1}{x\sqrt{bx^4+a}}dx+\int\frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{672f}{x}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$\downarrow$ 798

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\frac{105}{4}c\int\frac{1}{x^4\sqrt{bx^4+a}}dx^4+\int\frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{67}{}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$\downarrow$ 73

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\frac{105c\int\frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2b}+\int\frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{6}{}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$\downarrow$ 221

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\int\frac{672fx^2+280ex+160d}{\sqrt{bx^4+a}}dx-\frac{105c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{280e}{x^2}+\frac{67}{}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$\downarrow$ 2424

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\int\left(\frac{280ex}{\sqrt{bx^4+a}}+\frac{672fx^2+160d}{\sqrt{bx^4+a}}\right)dx-\frac{105c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{105c}{x^4}+\frac{160d}{x^3}+\frac{28}{x}\right.\right.$$
$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

$\downarrow$ 2009

$$\frac{1}{140}b\left(\frac{1}{2}b\left(\frac{16\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(21\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}-\frac{672\sqrt[4]{a}f\left(\sqrt{a}+\right.}{}\right.\right.$$

$$\frac{1}{840}\left(a+bx^4\right)^{3/2}\left(\frac{105c}{x^8}+\frac{120d}{x^7}+\frac{140e}{x^6}+\frac{168f}{x^5}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^9,x]
```

output
```
-1/840*(((105*c)/x^8 + (120*d)/x^7 + (140*e)/x^6 + (168*f)/x^5)*(a + b*x^4
)^(3/2)) + (b*(-1/4*(((105*c)/x^4 + (160*d)/x^3 + (280*e)/x^2 + (672*f)/x)
*Sqrt[a + b*x^4]) + (b*((672*f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt
[b]*x^2)) + (140*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/Sqrt[b] - (105*
c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (672*a^(1/4)*f*(Sqrt[a]
+ Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*Arc
Tan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (16*(5*Sqrt[b]
*d + 21*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*b^(3/
4)*Sqrt[a + b*x^4])))/2))/140
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(2352bf\,x^{7}+1120be\,x^{6}+720x^{5}bd+525bc\,x^{4}+336af\,x^{3}+280ae\,x^{2}+240adx+210ac)}{1680x^{8}}+\frac{b^{2}\left(\frac{160d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x}}\right.}{}$
elliptic	$-\frac{ac\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{ad\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{ae\sqrt{b\,x^{4}+a}}{6x^{6}}-\frac{af\sqrt{b\,x^{4}+a}}{5x^{5}}-\frac{5cb\sqrt{b\,x^{4}+a}}{16x^{4}}-\frac{3bd\sqrt{b\,x^{4}+a}}{7x^{3}}-\frac{2be\sqrt{b\,x^{4}+a}}{3x^{2}}-\frac{7fb\sqrt{b\,x^{4}+a}}{5x}$
default	$c\left(-\frac{a\sqrt{b\,x^{4}+a}}{8x^{8}}-\frac{5b\sqrt{b\,x^{4}+a}}{16x^{4}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}\right)+d\left(-\frac{a\sqrt{b\,x^{4}+a}}{7x^{7}}-\frac{3b\sqrt{b\,x^{4}+a}}{7x^{3}}+\frac{4b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+}}{7\sqrt{\frac{i}{}}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(b*x^4+a)^(1/2)*(2352*b*f*x^7+1120*b*e*x^6+720*b*d*x^5+525*b*c*x^4
+336*a*f*x^3+280*a*e*x^2+240*a*d*x+210*a*c)/x^8+1/280*b^2*(160*d/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-105/2*c/a^
(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+140*e*ln(b^(1/2)*x^2+(b*x^4+
a)^(1/2))/b^(1/2)+672*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*
(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^9}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^9, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}be}{2x^2\sqrt{1+\frac{bx^4}{a}}} + \frac{\sqrt{a}bf\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} - \frac{a^2c}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b}c}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}c}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}e\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} - \frac{b^2ex^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**9,x)
```

output
```
a**(3/2)*d*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**7*gamma(-3/4)) + a**(3/2)*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) + sqrt(a)*b*d*gamma(-3
/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/
4)) - sqrt(a)*b*e/(2*x**2*sqrt(1 + b*x**4/a)) + sqrt(a)*b*f*gamma(-1/4)*hy
per((-1/2, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) - a**
2*c/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*c/(16*x**6*sqrt(a
/(b*x**4) + 1)) - a*sqrt(b)*e*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2)*c*s
qrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*c/(16*x**2*sqrt(a/(b*x**4) + 1)) -
 b**(3/2)*e*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*e*asinh(sqrt(b)*x**2/sqrt(a)
)/2 - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*e*x**2/(2
*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
1/32*(3*b^2*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/s
qrt(a) - 2*(5*(b*x^4 + a)^(3/2)*b^2 - 3*sqrt(b*x^4 + a)*a*b^2)/((b*x^4 + a
)^2 - 2*(b*x^4 + a)*a + a^2))*c + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4 +
 a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^8, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^9}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^9}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^9}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^9, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^9}\,dx = \frac{-60\sqrt{b\,x^4+a}\,a^2c+96\sqrt{b\,x^4+a}\,a^2dx-80\sqrt{b\,x^4+a}\,a^2e\,x^2-4}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^9,x)
```

output
```
( - 60*sqrt(a + b*x**4)*a**2*c + 96*sqrt(a + b*x**4)*a**2*d*x - 80*sqrt(a
+ b*x**4)*a**2*e*x**2 - 480*sqrt(a + b*x**4)*a**2*f*x**3 - 150*sqrt(a + b*
x**4)*a*b*c*x**4 - 480*sqrt(a + b*x**4)*a*b*d*x**5 - 320*sqrt(a + b*x**4)*
a*b*e*x**6 + 480*sqrt(a + b*x**4)*a*b*f*x**7 + 45*sqrt(a)*log(sqrt(a + b*x
**4) - sqrt(a))*b**2*c*x**8 - 45*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*b
**2*c*x**8 - 120*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*b*e*x**8 +
 120*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*b*e*x**8 + 1152*int(sq
rt(a + b*x**4)/(a*x**8 + b*x**12),x)*a**3*d*x**8 - 1920*int(sqrt(a + b*x**
4)/(a*x**6 + b*x**10),x)*a**3*f*x**8)/(480*a*x**8)
```

3.67 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 454

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \frac{4bc\sqrt{a+bx^4}}{45x^5}$$
$$+\frac{4be\sqrt{a+bx^4}}{7x^3}-\frac{4b^2c\sqrt{a+bx^4}}{15\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2(27ae-7bcx^2)\sqrt{a+bx^4}}{63x^7}$$
$$-\frac{b(3d+8fx^2)\sqrt{a+bx^4}}{16x^4}-\frac{(c+9ex^2)(a+bx^4)^{3/2}}{9x^9}-\frac{(3d+4fx^2)(a+bx^4)^{3/2}}{24x^8}$$
$$+\frac{1}{2}b^{3/2}f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-\frac{3b^2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{4b^{9/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\cdots\right.\right.}{15a^{3/4}\sqrt{a+bx^4}}$$

output
```
4/45*b*c*(b*x^4+a)^(1/2)/x^5+4/7*b*e*(b*x^4+a)^(1/2)/x^3-4/15*b^2*c*(b*x^4
+a)^(1/2)/a^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)+2/63*(-7*b*c*x^2+27*a*e)*(b*x^4+
a)^(1/2)/x^7-1/16*b*(8*f*x^2+3*d)*(b*x^4+a)^(1/2)/x^4-1/9*(9*e*x^2+c)*(b*x
^4+a)^(3/2)/x^9-1/24*(4*f*x^2+3*d)*(b*x^4+a)^(3/2)/x^8+1/2*b^(3/2)*f*arcta
nh(b^(1/2)*x^2/(b*x^4+a)^(1/2))-3/16*b^2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2)
)/a^(1/2)-4/15*b^(9/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)
*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(
3/4)/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(7*b^(1/2)*c+15*a^(1/2)*e)*(a^(1/2)+b^(
1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arct
an(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.38

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(112a^2c\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{bx^4}{a}\right)+3x\left(48a^2ex\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2}\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

output
```
-1/1008*(Sqrt[a + b*x^4]*(112*a^2*c*Hypergeometric2F1[-9/4, -3/2, -5/4, -(
(b*x^4)/a)] + 3*x*(48*a^2*e*x*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4
)/a)] + 7*(3*a*d*(2*a + 5*b*x^4)*Sqrt[1 + (b*x^4)/a] + 9*b^2*d*x^8*ArcTanh
[Sqrt[1 + (b*x^4)/a]] + 8*a^2*f*x^2*Hypergeometric2F1[-3/2, -3/2, -1/2, -(
(b*x^4)/a)]))))/(a*x^9*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{10}}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(84fx^3+72ex^2+63dx+56c)\sqrt{bx^4+a}}{504x^6}dx-\frac{1}{504}(a+bx^4)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{84}b\int \frac{(84fx^3+72ex^2+63dx+56c)\sqrt{bx^4+a}}{x^6}dx-\frac{1}{504}(a+bx^4)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{2364}$$

$$\frac{1}{84}b\left(-2b\int -\frac{840fx^3+480ex^2+315dx+224c}{20x^2\sqrt{bx^4+a}}dx-\frac{1}{20}\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\right)-\frac{1}{504}(a+bx^4)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{84}b\left(\frac{1}{10}b\int \frac{840fx^3+480ex^2+315dx+224c}{x^2\sqrt{bx^4+a}}dx-\frac{1}{20}\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\right)-\frac{1}{504}(a+bx^4)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{84}b\left(\frac{1}{10}b\int\left(\frac{480ex^2+224c}{x^2\sqrt{bx^4+a}}+\frac{840fx^2+315d}{x\sqrt{bx^4+a}}\right)dx-\frac{1}{20}\sqrt{a+bx^4}\left(\frac{224c}{x^5}+\frac{315d}{x^4}+\frac{480e}{x^3}+\frac{840f}{x^2}\right)\right)-\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

$\big\downarrow$ 2009

$$\frac{1}{84}b\left(\frac{1}{10}b\left(\frac{16\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}e+7\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{224\sqrt[4]{b}c\left(\sqrt{a}+\right.}{}\right.\right.$$

$$\frac{1}{504}\left(a+bx^4\right)^{3/2}\left(\frac{56c}{x^9}+\frac{63d}{x^8}+\frac{72e}{x^7}+\frac{84f}{x^6}\right)$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^10,x]
```

output
```
-1/504*(((56*c)/x^9 + (63*d)/x^8 + (72*e)/x^7 + (84*f)/x^6)*(a + b*x^4)^(3
/2)) + (b*(-1/20*(((224*c)/x^5 + (315*d)/x^4 + (480*e)/x^3 + (840*f)/x^2)*
Sqrt[a + b*x^4]) + (b*((-224*c*Sqrt[a + b*x^4])/(a*x) + (224*Sqrt[b]*c*x*S
qrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (420*f*ArcTanh[(Sqrt[b]*x^2)
/Sqrt[a + b*x^4]])/Sqrt[b] - (315*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*S
qrt[a]) - (224*b^(1/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)
*Sqrt[a + b*x^4]) + (16*(7*Sqrt[b]*c + 15*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(a^(3/4)*b^(1/4)*Sqrt[a + b*x^4]))/10))/84
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.32 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(1344b^2c\,x^8+3360x^7abf+2160abe\,x^6+1575x^5adb+1232abc\,x^4+840a^2f\,x^3+720a^2e\,x^2+630a^2dx+560a^2c)}{5040x^9a}+\frac{b^2\left(\frac{480\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{9x^9}-\frac{ad\sqrt{b\,x^4+a}}{8x^8}-\frac{ae\sqrt{b\,x^4+a}}{7x^7}-\frac{af\sqrt{b\,x^4+a}}{6x^6}-\frac{11bc\sqrt{b\,x^4+a}}{45x^5}-\frac{5bd\sqrt{b\,x^4+a}}{16x^4}-\frac{3be\sqrt{b\,x^4+a}}{7x^3}-\frac{2fb\sqrt{b\,x^4+\cdots}}{3x^2}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{9x^9}-\frac{11b\sqrt{b\,x^4+a}}{45x^5}-\frac{4b^2\sqrt{b\,x^4+a}}{15ax}+\frac{4ib^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\cdots\right)\right.}{15\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/5040*(b*x^4+a)^(1/2)*(1344*b^2*c*x^8+3360*a*b*f*x^7+2160*a*b*e*x^6+1575
*a*b*d*x^5+1232*a*b*c*x^4+840*a^2*f*x^3+720*a^2*e*x^2+630*a^2*d*x+560*a^2*
c)/x^9/a+1/840/a*b^2*(480*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*b^(1/2))^(1/2),I)-315/2*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(
1/2))/x^2)+420*a*f*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+224*I*c*b^(1/2)
*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^
(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{10}}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
integral((b*f*x^7 + b*e*x^6 + b*d*x^5 + b*c*x^4 + a*f*x^3 + a*e*x^2 + a*d*
x + a*c)*sqrt(b*x^4 + a)/x^10, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.35 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{10}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{9}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{\sqrt{a}bf}{2x^2\sqrt{1+\frac{bx^4}{a}}} - \frac{a^2d}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{3a\sqrt{b}d}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{6x^4} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}d}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{6} + \frac{b^{\frac{3}{2}}f\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} - \frac{3b^2d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} - \frac{b^2fx^2}{2\sqrt{a}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**10,x)
```

output
```
a**(3/2)*c*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*c*gamma(-5
/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-
1/4)) + sqrt(a)*b*e*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**3*gamma(1/4)) - sqrt(a)*b*f/(2*x**2*sqrt(1 + b*x**4/a))
- a**2*d/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) + 1)) - 3*a*sqrt(b)*d/(16*x**6*s
qrt(a/(b*x**4) + 1)) - a*sqrt(b)*f*sqrt(a/(b*x**4) + 1)/(6*x**4) - b**(3/2
)*d*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*d/(16*x**2*sqrt(a/(b*x**4) +
1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/6 + b**(3/2)*f*asinh(sqrt(b)*x**2/sq
rt(a))/2 - 3*b**2*d*asinh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)) - b**2*f*x*
*2/(2*sqrt(a)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{10}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{10}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^10, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{10}}\,dx = \frac{-160\sqrt{b\,x^4+a}\,a^2c-420\sqrt{b\,x^4+a}\,a^2dx+672\sqrt{b\,x^4+a}\,a^2e\,x^2}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^10,x)
```

output
```
( - 160*sqrt(a + b*x**4)*a**2*c - 420*sqrt(a + b*x**4)*a**2*d*x + 672*sqrt
(a + b*x**4)*a**2*e*x**2 - 560*sqrt(a + b*x**4)*a**2*f*x**3 - 1120*sqrt(a
+ b*x**4)*a*b*c*x**4 - 1050*sqrt(a + b*x**4)*a*b*d*x**5 - 3360*sqrt(a + b*
x**4)*a*b*e*x**6 - 2240*sqrt(a + b*x**4)*a*b*f*x**7 + 315*sqrt(a)*log(sqrt
(a + b*x**4) - sqrt(a))*b**2*d*x**9 - 315*sqrt(a)*log(sqrt(a + b*x**4) + s
qrt(a))*b**2*d*x**9 - 840*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*b
*f*x**9 + 840*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*b*f*x**9 + 19
20*int(sqrt(a + b*x**4)/(a*x**10 + b*x**14),x)*a**3*c*x**9 + 8064*int(sqrt
(a + b*x**4)/(a*x**8 + b*x**12),x)*a**3*e*x**9)/(3360*a*x**9)
```

3.68 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 428

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = -\frac{ae\sqrt{a+bx^4}}{8x^8}+\frac{4bd\sqrt{a+bx^4}}{45x^5}-\frac{5be\sqrt{a+bx^4}}{16x^4}+\frac{4bf\sqrt{a+bx^4}}{7x^3}-\frac{4b^2d\sqrt{a+bx^4}}{15\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2(27af-7bdx^2)\sqrt{a+bx^4}}{63x^7}-\frac{(d+9fx^2)(a+bx^4)^{3/2}}{9x^9}-\frac{c(a+bx^4)^{5/2}}{10ax^{10}}-\frac{3b^2e\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{4b^{9/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}+\frac{2b^{7/4}\left(7\sqrt{b}d+15\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105a^{3/4}\sqrt{a+bx^4}}$$

output
```
-1/8*a*e*(b*x^4+a)^(1/2)/x^8+4/45*b*d*(b*x^4+a)^(1/2)/x^5-5/16*b*e*(b*x^4+
a)^(1/2)/x^4+4/7*b*f*(b*x^4+a)^(1/2)/x^3-4/15*b^2*d*(b*x^4+a)^(1/2)/a^(1/2
)/x/(a^(1/2)+b^(1/2)*x^2)+2/63*(-7*b*d*x^2+27*a*f)*(b*x^4+a)^(1/2)/x^7-1/9
*(9*f*x^2+d)*(b*x^4+a)^(3/2)/x^9-1/10*c*(b*x^4+a)^(5/2)/a/x^10-3/16*b^2*e*
arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-4/15*b^(9/4)*d*(a^(1/2)+b^(1/2)*x
^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)+2/105*b^(7/4)*(7*b^(1/
2)*d+15*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^
2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/
(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.40

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{11}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(63\sqrt{1+\frac{bx^4}{a}}\left(8b^2cx^8+2a^2(4c+5ex^2)+abx^4(16c+25ex^2)\right)+945b^2ex^{10}\text{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)\right.}{5040ax}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

output
```
-1/5040*(Sqrt[a + b*x^4]*(63*Sqrt[1 + (b*x^4)/a]*(8*b^2*c*x^8 + 2*a^2*(4*c
 + 5*e*x^2) + a*b*x^4*(16*c + 25*e*x^2)) + 945*b^2*e*x^10*ArcTanh[Sqrt[1 +
 (b*x^4)/a]] + 560*a^2*d*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a
)] + 720*a^2*f*x^3*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^4)/a)]))/(a*
x^10*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{11}}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(360fx^3+315ex^2+280dx+252c)\sqrt{bx^4+a}}{2520x^7}dx-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{420}b\int \frac{(360fx^3+315ex^2+280dx+252c)\sqrt{bx^4+a}}{x^7}dx-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{2364}$$

$$\frac{1}{420}b\left(-2b\int -\frac{480fx^3+315ex^2+224dx+168c}{4x^3\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{420}b\left(\frac{1}{2}b\int \frac{480fx^3+315ex^2+224dx+168c}{x^3\sqrt{bx^4+a}}dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{420}b\left(\frac{1}{2}b\int\left(\frac{315ex^2+168c}{x^3\sqrt{bx^4+a}}+\frac{480fx^2+224d}{x^2\sqrt{bx^4+a}}\right)dx-\frac{1}{4}\sqrt{a+bx^4}\left(\frac{168c}{x^6}+\frac{224d}{x^5}+\frac{315e}{x^4}+\frac{480f}{x^3}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

$\Big\downarrow$ 2009

$$\frac{1}{420}b\left(\frac{1}{2}b\left(\frac{16\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{a}f+7\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{224\sqrt[4]{b}d\left(\sqrt{a}+\right.}{}\right.\right.$$

$$\frac{(a+bx^4)^{3/2}\left(\frac{252c}{x^{10}}+\frac{280d}{x^9}+\frac{315e}{x^8}+\frac{360f}{x^7}\right)}{2520}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^11,x]
```

output
```
-1/2520*(((252*c)/x^10 + (280*d)/x^9 + (315*e)/x^8 + (360*f)/x^7)*(a + b*x
^4)^(3/2)) + (b*(-1/4*(((168*c)/x^6 + (224*d)/x^5 + (315*e)/x^4 + (480*f)/
x^3)*Sqrt[a + b*x^4]) + (b*((-84*c*Sqrt[a + b*x^4])/(a*x^2) - (224*d*Sqrt[
a + b*x^4])/(a*x) + (224*Sqrt[b]*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b
]*x^2)) - (315*e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (224*b^(1
/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) +
 (16*(7*Sqrt[b]*d + 15*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])))/2))/420
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(1344 b^{2} d\,x^{9}+504 b^{2} c\,x^{8}+2160 x^{7} a b f+1575 a b e\,x^{6}+1232 x^{5} a d b+1008 a b c\,x^{4}+720 a^{2} f\,x^{3}+630 a^{2} e\,x^{2}+560 a^{2} d x+504 a^{2} c\right)}{5040 x^{10} a}$
default	$-\frac{c\sqrt{b\,x^{4}+a}\,\left(b^{2} x^{8}+2 b\,x^{4} a+a^{2}\right)}{10 x^{10} a}+d\left(-\frac{a\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{11 b\sqrt{b\,x^{4}+a}}{45 x^{5}}-\frac{4 b^{2}\sqrt{b\,x^{4}+a}}{15 a x}+\frac{4 i b^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{Ellipt}\right.}{15\sqrt{a}\,\sqrt{}}\right.$
elliptic	$-\frac{a c\sqrt{b\,x^{4}+a}}{10 x^{10}}-\frac{a d\sqrt{b\,x^{4}+a}}{9 x^{9}}-\frac{a e\sqrt{b\,x^{4}+a}}{8 x^{8}}-\frac{a f\sqrt{b\,x^{4}+a}}{7 x^{7}}-\frac{c b\sqrt{b\,x^{4}+a}}{5 x^{6}}-\frac{11 b d\sqrt{b\,x^{4}+a}}{45 x^{5}}-\frac{5 b e\sqrt{b\,x^{4}+a}}{16 x^{4}}-\frac{3 b f\sqrt{b\,x^{4}+a}}{7 x^{3}}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/5040*(b*x^4+a)^(1/2)*(1344*b^2*d*x^9+504*b^2*c*x^8+2160*a*b*f*x^7+1575*
a*b*e*x^6+1232*a*b*d*x^5+1008*a*b*c*x^4+720*a^2*f*x^3+630*a^2*e*x^2+560*a^
2*d*x+504*a^2*c)/x^10/a+1/840/a*b^2*(480*a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-315/2*a^(1/2)*e*ln((2*a+2*a^(1
/2)*(b*x^4+a)^(1/2))/x^2)+224*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx =$$

$$-\frac{2688\sqrt{a}b^2dx^{10}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-945\sqrt{a}b^2ex^{10}\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-384\left(7b^2d\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/10080*(2688*sqrt(a)*b^2*d*x^10*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^
(1/4)), -1) - 945*sqrt(a)*b^2*e*x^10*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(
a) + 2*a)/x^4) - 384*(7*b^2*d - 15*a*b*f)*sqrt(a)*x^10*(-b/a)^(3/4)*ellipt
ic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(1344*b^2*d*x^9 + 504*b^2*c*x^8 + 216
0*a*b*f*x^7 + 1575*a*b*e*x^6 + 1232*a*b*d*x^5 + 1008*a*b*c*x^4 + 720*a^2*f
*x^3 + 630*a^2*e*x^2 + 560*a^2*d*x + 504*a^2*c)*sqrt(b*x^4 + a))/(a*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.17 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{11}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^9\Gamma\left(-\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} - \frac{a^2e}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{3a\sqrt{b}e}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}e}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}c\sqrt{\frac{a}{bx^4}+1}}{10a} - \frac{3b^2e\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

input

```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**11,x)
```

output

```
a**(3/2)*d*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**9*gamma(-5/4)) + a**(3/2)*f*gamma(-7/4)*hyper((-7/4, -1/2), (-3/
4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) + sqrt(a)*b*d*gamma(-5
/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**5*gamma(-
1/4)) + sqrt(a)*b*f*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*x**3*gamma(1/4)) - a**2*e/(8*sqrt(b)*x**10*sqrt(a/(b*x**4)
+ 1)) - a*sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*e/(16*x**
6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*c*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(3
/2)*e*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*e/(16*x**2*sqrt(a/(b*x**4)
+ 1)) - b**(5/2)*c*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*e*asinh(sqrt(a)/(s
qrt(b)*x**2))/(16*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{11}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/10*(b*x^4 + a)^(5/2)*c/(a*x^10) + integrate((b*f*x^6 + b*e*x^5 + b*d*x^
4 + a*f*x^2 + a*e*x + a*d)*sqrt(b*x^4 + a)/x^10, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{11}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{11}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{11}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^11, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{11}}\,dx=\int \frac{(f\,x^3+e\,x^2+dx+c)\,(b\,x^4+a)^{\frac{3}{2}}}{x^{11}}dx$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x)
```

output
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^11,x)
```

3.69 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 455

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = -\frac{af\sqrt{a+bx^4}}{8x^8}+\frac{12bc\sqrt{a+bx^4}}{385x^7}+\frac{4be\sqrt{a+bx^4}}{45x^5}-\frac{5bf\sqrt{a+bx^4}}{16x^4}-\frac{4b^2c\sqrt{a+bx^4}}{77ax^3}-\frac{4b^2e\sqrt{a+bx^4}}{15\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{2(55ae-27bcx^2)\sqrt{a+bx^4}}{495x^9}-\frac{(3c+11ex^2)(a+bx^4)^{3/2}}{33x^{11}}-\frac{d(a+bx^4)^{5/2}}{10ax^{10}}-\frac{3b^2f\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}-\frac{4b^{9/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}}-\frac{2b^{9/4}\left(15\sqrt{b}c-77\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/8*a*f*(b*x^4+a)^(1/2)/x^8+12/385*b*c*(b*x^4+a)^(1/2)/x^7+4/45*b*e*(b*x^
4+a)^(1/2)/x^5-5/16*b*f*(b*x^4+a)^(1/2)/x^4-4/77*b^2*c*(b*x^4+a)^(1/2)/a/x
^3-4/15*b^2*e*(b*x^4+a)^(1/2)/a^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)+2/495*(-27*b
*c*x^2+55*a*e)*(b*x^4+a)^(1/2)/x^9-1/33*(11*e*x^2+3*c)*(b*x^4+a)^(3/2)/x^1
1-1/10*d*(b*x^4+a)^(5/2)/a/x^10-3/16*b^2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2)
)/a^(1/2)-4/15*b^(9/4)*e*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)
*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(
3/4)/(b*x^4+a)^(1/2)-2/1155*b^(9/4)*(15*b^(1/2)*c-77*a^(1/2)*e)*(a^(1/2)+b
^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.38

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(720a^2c\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^4}{a}\right)+11x\left(9\sqrt{1+\frac{bx^4}{a}}(8b^2dx^8+2a^2(4d+5fx^2\right.\right.}{7920}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

output
```
-1/7920*(Sqrt[a + b*x^4]*(720*a^2*c*Hypergeometric2F1[-11/4, -3/2, -7/4, -
((b*x^4)/a)] + 11*x*(9*Sqrt[1 + (b*x^4)/a]*(8*b^2*d*x^8 + 2*a^2*(4*d + 5*f
*x^2) + a*b*x^4*(16*d + 25*f*x^2)) + 135*b^2*f*x^10*ArcTanh[Sqrt[1 + (b*x^
4)/a]] + 80*a^2*e*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4)/a)])))/(
a*x^11*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{12}}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(495fx^3+440ex^2+396dx+360c)\sqrt{bx^4+a}}{3960x^8}dx-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{660}b\int \frac{(495fx^3+440ex^2+396dx+360c)\sqrt{bx^4+a}}{x^8}dx-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{2364}$$

$$\frac{1}{660}b\left(-2b\int -\frac{3465fx^3+2464ex^2+1848dx+1440c}{28x^4\sqrt{bx^4+a}}dx-\frac{1}{28}\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{660}b\left(\frac{1}{14}b\int \frac{3465fx^3+2464ex^2+1848dx+1440c}{x^4\sqrt{bx^4+a}}dx-\frac{1}{28}\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$$\Big\downarrow \text{2372}$$

$$\frac{1}{660}b\left(\frac{1}{14}b\int\left(\frac{2464ex^2+1440c}{x^4\sqrt{bx^4+a}}+\frac{3465fx^2+1848d}{x^3\sqrt{bx^4+a}}\right)dx-\frac{1}{28}\sqrt{a+bx^4}\left(\frac{1440c}{x^7}+\frac{1848d}{x^6}+\frac{2464e}{x^5}+\frac{3465f}{x^4}\right)\right.$$
$$\frac{\left(a+bx^4\right)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

$\downarrow$ 2009

$$\frac{1}{660}b\left(\frac{1}{14}b\left(-\frac{16\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}c-77\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{2464\sqrt[4]{b}\cdots}{\cdots}\right.\right.$$
$$\frac{\left(a+bx^4\right)^{3/2}\left(\frac{360c}{x^{11}}+\frac{396d}{x^{10}}+\frac{440e}{x^9}+\frac{495f}{x^8}\right)}{3960}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^12,x]
```

output
```
-1/3960*(((360*c)/x^11 + (396*d)/x^10 + (440*e)/x^9 + (495*f)/x^8)*(a + b*
x^4)^(3/2)) + (b*(-1/28*(((1440*c)/x^7 + (1848*d)/x^6 + (2464*e)/x^5 + (34
65*f)/x^4)*Sqrt[a + b*x^4]) + (b*((-480*c*Sqrt[a + b*x^4])/(a*x^3) - (924*
d*Sqrt[a + b*x^4])/(a*x^2) - (2464*e*Sqrt[a + b*x^4])/(a*x) + (2464*Sqrt[b
]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (3465*f*ArcTanh[Sqrt[
a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (2464*b^(1/4)*e*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (16*b^(1/4)*(15*Sqrt[b]*c -
77*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*
x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(5/4)*Sqrt[a + b
*x^4])))/14))/660
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.54 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(14784 e\,b^2 x^{10}+5544 b^2 d\,x^9+2880 b^2 c\,x^8+17325 x^7 a b f+13552 a b e\,x^6+11088 x^5 a d b+9360 a b c\,x^4+6930 a^2 f\,x^3+6160 a^2 e\,x}{55440 x^{11} a}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{11x^{11}}-\frac{13b\sqrt{b\,x^4+a}}{77x^7}-\frac{4b^2\sqrt{b\,x^4+a}}{77a\,x^3}-\frac{4b^3\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{77a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)-\frac{d\sqrt{b\,x^4+a}\,(b^2x^8}{10x^{10}}$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{11x^{11}}-\frac{ad\sqrt{b\,x^4+a}}{10x^{10}}-\frac{ae\sqrt{b\,x^4+a}}{9x^9}-\frac{af\sqrt{b\,x^4+a}}{8x^8}-\frac{13bc\sqrt{b\,x^4+a}}{77x^7}-\frac{bd\sqrt{b\,x^4+a}}{5x^6}-\frac{11be\sqrt{b\,x^4+a}}{45x^5}-\frac{5bf\sqrt{b\,x^4+}}{16x^4}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/55440*(b*x^4+a)^(1/2)*(14784*b^2*e*x^10+5544*b^2*d*x^9+2880*b^2*c*x^8+1
7325*a*b*f*x^7+13552*a*b*e*x^6+11088*a*b*d*x^5+9360*a*b*c*x^4+6930*a^2*f*x
^3+6160*a^2*e*x^2+5544*a^2*d*x+5040*a^2*c)/x^11/a+1/9240/a*b^2*(-480*c*b/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-34
65/2*a^(1/2)*f*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+2464*I*b^(1/2)*e*a^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/
2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.50

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx =$$

$$-\frac{29568\sqrt{a}b^2ex^{11}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-10395\sqrt{a}b^2fx^{11}\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-384\,(15}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
-1/110880*(29568*sqrt(a)*b^2*e*x^11*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a
)^(1/4)), -1) - 10395*sqrt(a)*b^2*f*x^11*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*s
qrt(a) + 2*a)/x^4) - 384*(15*b^2*c + 77*b^2*e)*sqrt(a)*x^11*(-b/a)^(3/4)*e
lliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(14784*b^2*e*x^10 + 5544*b^2*d*x
^9 + 2880*b^2*c*x^8 + 17325*a*b*f*x^7 + 13552*a*b*e*x^6 + 11088*a*b*d*x^5
+ 9360*a*b*c*x^4 + 6930*a^2*f*x^3 + 6160*a^2*e*x^2 + 5544*a^2*d*x + 5040*a
^2*c)*sqrt(b*x^4 + a))/(a*x^11)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.45 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)} - \frac{a^2f}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{3a\sqrt{b}f}{16x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{4x^2} - \frac{b^{\frac{3}{2}}f}{16x^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}d\sqrt{\frac{a}{bx^4}+1}}{10a} - \frac{3b^2f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**12,x)
```

output
```
a**(3/2)*c*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*e*gamma(-9/4)*hyper((-9/4, -1/2), (
-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*c*gamma
(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamm
a(-3/4)) + sqrt(a)*b*e*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp
_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*f/(8*sqrt(b)*x**10*sqrt(a/(b*x
**4) + 1)) - a*sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(10*x**8) - 3*a*sqrt(b)*f/(1
6*x**6*sqrt(a/(b*x**4) + 1)) - b**(3/2)*d*sqrt(a/(b*x**4) + 1)/(5*x**4) -
b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(4*x**2) - b**(3/2)*f/(16*x**2*sqrt(a/(b*x
**4) + 1)) - b**(5/2)*d*sqrt(a/(b*x**4) + 1)/(10*a) - 3*b**2*f*asinh(sqrt(
a)/(sqrt(b)*x**2))/(16*sqrt(a))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{12}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{12}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{12}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{12}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^12, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{12}}\,dx=\int \frac{(f\,x^3+e\,x^2+dx+c)\left(b\,x^4+a\right)^{\frac{3}{2}}}{x^{12}}dx$$

input

```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x)
```

output

```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^12,x)
```

3.70 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 480

$$\begin{aligned}
\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx &= -\frac{ac\sqrt{a+bx^4}}{12x^{12}} - \frac{7bc\sqrt{a+bx^4}}{48x^8} \\
&+ \frac{12bd\sqrt{a+bx^4}}{385x^7} + \frac{4bf\sqrt{a+bx^4}}{45x^5} - \frac{b^2c\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2d\sqrt{a+bx^4}}{77ax^3} - \frac{4b^2f\sqrt{a+bx^4}}{15\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&+ \frac{2(55af-27bdx^2)\sqrt{a+bx^4}}{495x^9} - \frac{(3d+11fx^2)(a+bx^4)^{3/2}}{33x^{11}} - \frac{e(a+bx^4)^{5/2}}{10ax^{10}} \\
&+ \frac{b^3c\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} - \frac{4b^{9/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{a+bx^4}} \\
&- \frac{2b^{9/4}\left(15\sqrt{b}d-77\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{1155a^{5/4}\sqrt{a+bx^4}}
\end{aligned}$$

output

```
-1/12*a*c*(b*x^4+a)^(1/2)/x^12-7/48*b*c*(b*x^4+a)^(1/2)/x^8+12/385*b*d*(b*
x^4+a)^(1/2)/x^7+4/45*b*f*(b*x^4+a)^(1/2)/x^5-1/32*b^2*c*(b*x^4+a)^(1/2)/a
/x^4-4/77*b^2*d*(b*x^4+a)^(1/2)/a/x^3-4/15*b^2*f*(b*x^4+a)^(1/2)/a^(1/2)/x
/(a^(1/2)+b^(1/2)*x^2)+2/495*(-27*b*d*x^2+55*a*f)*(b*x^4+a)^(1/2)/x^9-1/33
*(11*f*x^2+3*d)*(b*x^4+a)^(3/2)/x^11-1/10*e*(b*x^4+a)^(5/2)/a/x^10+1/32*b^
3*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-4/15*b^(9/4)*f*(a^(1/2)+b^(1/
2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b
^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)-2/1155*b^(9/4)*(15
*b^(1/2)*d-77*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^
(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.31

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx =$$
$$-\frac{\sqrt{a+bx^4}\left(90a^5d\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^4}{a}\right)+11x\left(10a^5fx\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\right.\right.\right.}{990a^4x^{11}\sqrt{1+}}$$

input

```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

output

```
-1/990*(Sqrt[a + b*x^4]*(90*a^5*d*Hypergeometric2F1[-11/4, -3/2, -7/4, -((
b*x^4)/a)] + 11*x*(10*a^5*f*x*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^4
)/a)] + 9*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*e - b^3*c*x^10*Hypergeome
tric2F1[5/2, 4, 7/2, 1 + (b*x^4)/a]))))/(a^4*x^11*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{13}}dx$$

$$\Big\downarrow \text{ 2364}$$

$$-6b\int -\frac{(220fx^3+198ex^2+180dx+165c)\sqrt{bx^4+a}}{1980x^9}dx-\frac{(a+bx^4)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{330}b\int \frac{(220fx^3+198ex^2+180dx+165c)\sqrt{bx^4+a}}{x^9}dx-\frac{(a+bx^4)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 2364}$$

$$\frac{1}{330}b\left(-2b\int -\frac{2464fx^3+1848ex^2+1440dx+1155c}{56x^5\sqrt{bx^4+a}}dx-\frac{1}{56}\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{330}b\left(\frac{1}{28}b\int \frac{2464fx^3+1848ex^2+1440dx+1155c}{x^5\sqrt{bx^4+a}}dx-\frac{1}{56}\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\right)-\frac{(a+bx^4)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{ 2372}$$

$$\frac{1}{330}b\left(\frac{1}{28}b\int\left(\frac{1848ex^2+1155c}{x^5\sqrt{bx^4+a}}+\frac{2464fx^2+1440d}{x^4\sqrt{bx^4+a}}\right)dx-\frac{1}{56}\sqrt{a+bx^4}\left(\frac{1155c}{x^8}+\frac{1440d}{x^7}+\frac{1848e}{x^6}+\frac{2464f}{x^5}\right)\right.$$
$$\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{330}b\left(\frac{1}{28}b\left(-\frac{16\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(15\sqrt{b}d-77\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{a+bx^4}}-\frac{2464\sqrt[4]{b}\cdots}{\cdots}\right.\right.$$
$$\frac{\left(a+bx^4\right)^{3/2}\left(\frac{165c}{x^{12}}+\frac{180d}{x^{11}}+\frac{198e}{x^{10}}+\frac{220f}{x^9}\right)}{1980}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^13,x]
```

output
```
-1/1980*(((165*c)/x^12 + (180*d)/x^11 + (198*e)/x^10 + (220*f)/x^9)*(a + b
*x^4)^(3/2)) + (b*(-1/56*(((1155*c)/x^8 + (1440*d)/x^7 + (1848*e)/x^6 + (2
464*f)/x^5)*Sqrt[a + b*x^4]) + (b*((-1155*c*Sqrt[a + b*x^4])/(4*a*x^4) - (
480*d*Sqrt[a + b*x^4])/(a*x^3) - (924*e*Sqrt[a + b*x^4])/(a*x^2) - (2464*f
*Sqrt[a + b*x^4])/(a*x) + (2464*Sqrt[b]*f*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] +
 Sqrt[b]*x^2)) + (1155*b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) -
 (2464*b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[
b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a
+ b*x^4]) - (16*b^(1/4)*(15*Sqrt[b]*d - 77*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(a^(5/4)*Sqrt[a + b*x^4])))/28))/330
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(29568b^2 f\,x^{11}+11088e\,b^2x^{10}+5760b^2d\,x^9+3465b^2c\,x^8+27104x^7abf+22176abe\,x^6+18720x^5adb+16170abc\,x^4+12320a}{110880x^{12}a}$
default	$c\left(-\frac{a\sqrt{b\,x^4+a}}{12x^{12}}-\frac{7b\sqrt{b\,x^4+a}}{48x^8}-\frac{b^2\sqrt{b\,x^4+a}}{32a\,x^4}+\frac{b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{32a^{\frac{3}{2}}}\right)+d\left(-\frac{a\sqrt{b\,x^4+a}}{11x^{11}}-\frac{13b\sqrt{b\,x^4+a}}{77x^7}-\frac{4b^2\sqrt{}}{77}\right.$
elliptic	$-\frac{ac\sqrt{b\,x^4+a}}{12x^{12}}-\frac{ad\sqrt{b\,x^4+a}}{11x^{11}}-\frac{ae\sqrt{b\,x^4+a}}{10x^{10}}-\frac{af\sqrt{b\,x^4+a}}{9x^9}-\frac{7bc\sqrt{b\,x^4+a}}{48x^8}-\frac{13bd\sqrt{b\,x^4+a}}{77x^7}-\frac{be\sqrt{b\,x^4+a}}{5x^6}-\frac{11bf\sqrt{b\,x^4+}}{45x^5}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/110880*(b*x^4+a)^(1/2)*(29568*b^2*f*x^11+11088*b^2*e*x^10+5760*b^2*d*x^
9+3465*b^2*c*x^8+27104*a*b*f*x^7+22176*a*b*e*x^6+18720*a*b*d*x^5+16170*a*b
*c*x^4+12320*a^2*f*x^3+11088*a^2*e*x^2+10080*a^2*d*x+9240*a^2*c)/x^12/a-1/
18480/a*b^3*(960*d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-1155/2*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)
-4928*I*f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.52

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{13}}\,dx =$$

$$-\frac{59136\,a^{\frac{3}{2}}b^2fx^{12}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-3465\,\sqrt{a}b^3cx^{12}\log\left(-\frac{bx^4+2\,\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-768\,(15\,a}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="fricas")
```

output
```
-1/221760*(59136*a^(3/2)*b^2*f*x^12*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a
)^(1/4)), -1) - 3465*sqrt(a)*b^3*c*x^12*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sq
rt(a) + 2*a)/x^4) - 768*(15*a*b^2*d + 77*a*b^2*f)*sqrt(a)*x^12*(-b/a)^(3/4
)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*a*b^2*f*x^11 + 11088*a
*b^2*e*x^10 + 5760*a*b^2*d*x^9 + 3465*a*b^2*c*x^8 + 27104*a^2*b*f*x^7 + 22
176*a^2*b*e*x^6 + 18720*a^2*b*d*x^5 + 16170*a^2*b*c*x^4 + 12320*a^3*f*x^3
+ 11088*a^3*e*x^2 + 10080*a^3*d*x + 9240*a^3*c)*sqrt(b*x^4 + a))/(a^2*x^12
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.61 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx = \frac{a^{\frac{3}{2}}d\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{a^{\frac{3}{2}}f\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}bd\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)} + \frac{\sqrt{a}bf\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{5}\Gamma\left(-\frac{1}{4}\right)} - \frac{a^2c}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}} - \frac{11a\sqrt{b}c}{48x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{17b^{\frac{3}{2}}c}{96x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{5}{2}}c}{32ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}e\sqrt{\frac{a}{bx^4}+1}}{10a} + \frac{b^3c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**13,x)
```

output
```
a**(3/2)*d*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*x**11*gamma(-7/4)) + a**(3/2)*f*gamma(-9/4)*hyper((-9/4, -1/2), (
-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*gamma(-5/4)) + sqrt(a)*b*d*gamma
(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**7*gamm
a(-3/4)) + sqrt(a)*b*f*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**4*exp
_polar(I*pi)/a)/(4*x**5*gamma(-1/4)) - a**2*c/(12*sqrt(b)*x**14*sqrt(a/(b*
x**4) + 1)) - 11*a*sqrt(b)*c/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b)*e
*sqrt(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*c/(96*x**6*sqrt(a/(b*x**4) +
 1)) - b**(3/2)*e*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*c/(32*a*x**2*sq
rt(a/(b*x**4) + 1)) - b**(5/2)*e*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*c*asin
h(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{13}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="maxima")
```

output
```
-1/192*(3*b^3*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))
/a^(3/2) + 2*(3*(b*x^4 + a)^(5/2)*b^3 + 8*(b*x^4 + a)^(3/2)*a*b^3 - 3*sqrt
(b*x^4 + a)*a^2*b^3)/((b*x^4 + a)^3*a - 3*(b*x^4 + a)^2*a^2 + 3*(b*x^4 + a
)*a^3 - a^4))*c + integrate((b*f*x^6 + b*e*x^5 + b*d*x^4 + a*f*x^2 + a*e*x
 + a*d)*sqrt(b*x^4 + a)/x^12, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx=\int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{13}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^13, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{13}}\,dx=\int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{13}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^13, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{13}}\,dx=\int \frac{(f\,x^3+e\,x^2+dx+c)\,(b\,x^4+a)^{\frac{3}{2}}}{x^{13}}dx$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x)
```

output
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^13,x)
```

3.71 $\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}} dx$

Optimal result

Integrand size = 30, antiderivative size = 505

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}} dx = -\frac{ad\sqrt{a+bx^4}}{12x^{12}} + \frac{4bc\sqrt{a+bx^4}}{273x^9} - \frac{7bd\sqrt{a+bx^4}}{48x^8} + \frac{12be\sqrt{a+bx^4}}{385x^7} - \frac{4b^2c\sqrt{a+bx^4}}{195ax^5} - \frac{b^2d\sqrt{a+bx^4}}{32ax^4} - \frac{4b^2e\sqrt{a+bx^4}}{77ax^3} + \frac{4b^3c\sqrt{a+bx^4}}{65a^{3/2}x\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{6(91ae-55bcx^2)\sqrt{a+bx^4}}{5005x^{11}} - \frac{(5c+13ex^2)(a+bx^4)^{3/2}}{65x^{13}} - \frac{f(a+bx^4)^{5/2}}{10ax^{10}} + \frac{b^3d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{32a^{3/2}} + \frac{4b^{13/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65a^{7/4}\sqrt{a+bx^4}} - \frac{2b^{11/4}\left(77\sqrt{b}c+65\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5005a^{7/4}\sqrt{a+bx^4}}$$

output
```
-1/12*a*d*(b*x^4+a)^(1/2)/x^12+4/273*b*c*(b*x^4+a)^(1/2)/x^9-7/48*b*d*(b*x
^4+a)^(1/2)/x^8+12/385*b*e*(b*x^4+a)^(1/2)/x^7-4/195*b^2*c*(b*x^4+a)^(1/2)
/a/x^5-1/32*b^2*d*(b*x^4+a)^(1/2)/a/x^4-4/77*b^2*e*(b*x^4+a)^(1/2)/a/x^3+4
/65*b^3*c*(b*x^4+a)^(1/2)/a^(3/2)/x/(a^(1/2)+b^(1/2)*x^2)+6/5005*(-55*b*c*
x^2+91*a*e)*(b*x^4+a)^(1/2)/x^11-1/65*(13*e*x^2+5*c)*(b*x^4+a)^(3/2)/x^13-
1/10*f*(b*x^4+a)^(5/2)/a/x^10+1/32*b^3*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/
a^(3/2)+4/65*b^(13/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*
x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7
/4)/(b*x^4+a)^(1/2)-2/5005*b^(11/4)*(77*b^(1/2)*c+65*a^(1/2)*e)*(a^(1/2)+b
^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.30

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx =$$

$$-\frac{\sqrt{a+bx^4}\left(110a^5c\,\text{Hypergeometric2F1}\left(-\frac{13}{4},-\frac{3}{2},-\frac{9}{4},-\frac{bx^4}{a}\right)+13x^2\left(10a^5e\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\right.\right.\right.}{1430a^4x^{13}\sqrt{}}$$

input
```
Integrate[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

output
```
-1/1430*(Sqrt[a + b*x^4]*(110*a^5*c*Hypergeometric2F1[-13/4, -3/2, -9/4, -
((b*x^4)/a)] + 13*x^2*(10*a^5*e*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*
x^4)/a)] + 11*x*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*(a^3*f - b^3*d*x^10*Hype
rgeometric2F1[5/2, 4, 7/2, 1 + (b*x^4)/a]))))/(a^4*x^13*Sqrt[1 + (b*x^4)/a
])
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2364, 27, 2364, 27, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/2}(c+dx+ex^2+fx^3)}{x^{14}}dx$$

$$\Big\downarrow \text{2364}$$

$$-6b\int -\frac{(858fx^3+780ex^2+715dx+660c)\sqrt{bx^4+a}}{8580x^{10}}dx-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{(858fx^3+780ex^2+715dx+660c)\sqrt{bx^4+a}}{x^{10}}dx}{1430}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{2364}$$

$$\frac{b\left(-2b\int -\frac{24024fx^3+18720ex^2+15015dx+12320c}{168x^6\sqrt{bx^4+a}}dx-\frac{1}{168}\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\right)}{1430}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{1}{84}b\int \frac{24024fx^3+18720ex^2+15015dx+12320c}{x^6\sqrt{bx^4+a}}dx-\frac{1}{168}\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\right)}{1430}-\frac{(a+bx^4)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$$\Big\downarrow \text{2372}$$

$$\frac{b\left(\frac{1}{84}b\int\left(\frac{18720ex^2+12320c}{x^6\sqrt{bx^4+a}}+\frac{24024fx^2+15015d}{x^5\sqrt{bx^4+a}}\right)dx-\frac{1}{168}\sqrt{a+bx^4}\left(\frac{12320c}{x^9}+\frac{15015d}{x^8}+\frac{18720e}{x^7}+\frac{24024f}{x^6}\right)\right)}{1430}-\frac{\left(a+bx^4\right)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

$\big\downarrow$ 2009

$$\frac{b\left(\frac{1}{84}b\left(-\frac{48b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(65\sqrt{a}e+77\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{7/4}\sqrt{a+bx^4}}+\frac{7392b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}}{a^{7/4}\sqrt{a+bx^4}}\right.\right.}{}$$

$$\frac{\left(a+bx^4\right)^{3/2}\left(\frac{660c}{x^{13}}+\frac{715d}{x^{12}}+\frac{780e}{x^{11}}+\frac{858f}{x^{10}}\right)}{8580}$$

input
```
Int[((c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^(3/2))/x^14,x]
```

output
```
-1/8580*(((660*c)/x^13 + (715*d)/x^12 + (780*e)/x^11 + (858*f)/x^10)*(a +
b*x^4)^(3/2)) + (b*(-1/168*(((12320*c)/x^9 + (15015*d)/x^8 + (18720*e)/x^7
 + (24024*f)/x^6)*Sqrt[a + b*x^4]) + (b*((-2464*c*Sqrt[a + b*x^4])/(a*x^5)
 - (15015*d*Sqrt[a + b*x^4])/(4*a*x^4) - (6240*e*Sqrt[a + b*x^4])/(a*x^3)
- (12012*f*Sqrt[a + b*x^4])/(a*x^2) + (7392*b*c*Sqrt[a + b*x^4])/(a^2*x) -
 (7392*b^(3/2)*c*x*Sqrt[a + b*x^4])/(a^2*(Sqrt[a] + Sqrt[b]*x^2)) + (15015
*b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (7392*b^(5/4)*c*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a + b*x^4]) - (48*b^(3/4
)*(77*Sqrt[b]*c + 65*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
a^(7/4)*Sqrt[a + b*x^4]))/84))/1430
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m +
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{\sqrt{b x^4+a}\,(-29568 b^3 c x^{12}+48048 a b^2 f x^{11}+24960 a b^2 e x^{10}+15015 x^9 a b^2 d+9856 a b^2 c x^8+96096 a^2 b f x^7+81120 e a^2 b x^6+70070 a}{480480 x^{13} a^2}$
default	$c\left(-\frac{a\sqrt{b x^4+a}}{13 x^{13}}-\frac{5 b\sqrt{b x^4+a}}{39 x^9}-\frac{4 b^2\sqrt{b x^4+a}}{195 a x^5}+\frac{4 b^3\sqrt{b x^4+a}}{65 a^2 x}-\frac{4 i b^{\frac{7}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,(\mathrm{EllipticF}(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i)-\mathrm{Elli}}{65 a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right.$
elliptic	$-\frac{a c\sqrt{b x^4+a}}{13 x^{13}}-\frac{a d\sqrt{b x^4+a}}{12 x^{12}}-\frac{a e\sqrt{b x^4+a}}{11 x^{11}}-\frac{a f\sqrt{b x^4+a}}{10 x^{10}}-\frac{5 b c\sqrt{b x^4+a}}{39 x^9}-\frac{7 b d\sqrt{b x^4+a}}{48 x^8}-\frac{13 b e\sqrt{b x^4+a}}{77 x^7}-\frac{f b\sqrt{b x^4+a}}{5 x^6}$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/480480*(b*x^4+a)^(1/2)*(-29568*b^3*c*x^12+48048*a*b^2*f*x^11+24960*a*b^
2*e*x^10+15015*a*b^2*d*x^9+9856*a*b^2*c*x^8+96096*a^2*b*f*x^7+81120*a^2*b*
e*x^6+70070*a^2*b*d*x^5+61600*a^2*b*c*x^4+48048*a^3*f*x^3+43680*a^3*e*x^2+
40040*a^3*d*x+36960*a^3*c)/x^13/a^2-1/80080/a^2*b^3*(4160*a*e/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-5005/2*a^(1/2
)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+4928*I*c*b^(1/2)*a^(1/2)/(I/a^
(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellip
ticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \frac{59136\sqrt{a}b^3cx^{13}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)+15015\sqrt{a}b^3$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="fricas")
```

output
```
1/960960*(59136*sqrt(a)*b^3*c*x^13*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) + 15015*sqrt(a)*b^3*d*x^13*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sq
rt(a) + 2*a)/x^4) - 768*(77*b^3*c - 65*a*b^2*e)*sqrt(a)*x^13*(-b/a)^(3/4)*
elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + 2*(29568*b^3*c*x^12 - 48048*a*b^2
*f*x^11 - 24960*a*b^2*e*x^10 - 15015*a*b^2*d*x^9 - 9856*a*b^2*c*x^8 - 9609
6*a^2*b*f*x^7 - 81120*a^2*b*e*x^6 - 70070*a^2*b*d*x^5 - 61600*a^2*b*c*x^4
- 48048*a^3*f*x^3 - 43680*a^3*e*x^2 - 40040*a^3*d*x - 36960*a^3*c)*sqrt(b*
x^4 + a))/(a^2*x^13)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.92 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx+ex^2+fx^3)(a+bx^4)^{3/2}}{x^{14}}\,dx = \frac{a^{\frac{3}{2}}c\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4},-\frac{1}{2}\\-\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)} + \frac{a^{\frac{3}{2}}e\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{\sqrt{a}bc\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{9}\Gamma\left(-\frac{5}{4}\right)} + \frac{\sqrt{a}be\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{7}\Gamma\left(-\frac{3}{4}\right)} - \frac{a^2d}{12\sqrt{b}x^{14}\sqrt{\frac{a}{bx^4}+1}} - \frac{11a\sqrt{b}d}{48x^{10}\sqrt{\frac{a}{bx^4}+1}} - \frac{a\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{10x^8} - \frac{17b^{\frac{3}{2}}d}{96x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{3}{2}}f\sqrt{\frac{a}{bx^4}+1}}{5x^4} - \frac{b^{\frac{5}{2}}d}{32ax^2\sqrt{\frac{a}{bx^4}+1}} - \frac{b^{\frac{5}{2}}f\sqrt{\frac{a}{bx^4}+1}}{10a} + \frac{b^3d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{32a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**(3/2)/x**14,x)
```

output
```
a**(3/2)*c*gamma(-13/4)*hyper((-13/4, -1/2), (-9/4,), b*x**4*exp_polar(I*p
i)/a)/(4*x**13*gamma(-9/4)) + a**(3/2)*e*gamma(-11/4)*hyper((-11/4, -1/2),
 (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**11*gamma(-7/4)) + sqrt(a)*b*c*ga
mma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*x**9*g
amma(-5/4)) + sqrt(a)*b*e*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*x**7*gamma(-3/4)) - a**2*d/(12*sqrt(b)*x**14*sqrt(a/
(b*x**4) + 1)) - 11*a*sqrt(b)*d/(48*x**10*sqrt(a/(b*x**4) + 1)) - a*sqrt(b
)*f*sqrt(a/(b*x**4) + 1)/(10*x**8) - 17*b**(3/2)*d/(96*x**6*sqrt(a/(b*x**4
) + 1)) - b**(3/2)*f*sqrt(a/(b*x**4) + 1)/(5*x**4) - b**(5/2)*d/(32*a*x**2
*sqrt(a/(b*x**4) + 1)) - b**(5/2)*f*sqrt(a/(b*x**4) + 1)/(10*a) + b**3*d*a
sinh(sqrt(a)/(sqrt(b)*x**2))/(32*a**(3/2))
```

Maxima [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{14}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

Giac [F]

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{2}}(fx^3+ex^2+dx+c)}{x^{14}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/2)*(f*x^3 + e*x^2 + d*x + c)/x^14, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx+ex^2+fx^3)\,(a+bx^4)^{3/2}}{x^{14}}\,dx = \int \frac{(b\,x^4+a)^{3/2}\,(f\,x^3+e\,x^2+d\,x+c)}{x^{14}}\,dx$$

input
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14,x)
```

output
```
int(((a + b*x^4)^(3/2)*(c + d*x + e*x^2 + f*x^3))/x^14, x)
```

Reduce [F]

$$\int \frac{(c+dx+ex^2+fx^3)\left(a+bx^4\right)^{3/2}}{x^{14}}\,dx = \int \frac{(f\,x^3+e\,x^2+dx+c)\,(b\,x^4+a)^{\frac{3}{2}}}{x^{14}}dx$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x)
```

output
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^(3/2)/x^14,x)
```

3.72 $\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 370

$$\begin{aligned}\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx &= -\frac{af\sqrt{a+bx^4}}{2b^2}+\frac{cx\sqrt{a+bx^4}}{3b}+\frac{dx^2\sqrt{a+bx^4}}{4b}+\frac{ex^3\sqrt{a+bx^4}}{5b}\\
&\quad-\frac{3aex\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f(a+bx^4)^{3/2}}{6b^2}-\frac{ad\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}\\
&\quad+\frac{3a^{5/4}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}\\
&\quad-\frac{a^{3/4}\left(5\sqrt{b}c+9\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*a*f*(b*x^4+a)^(1/2)/b^2+1/3*c*x*(b*x^4+a)^(1/2)/b+1/4*d*x^2*(b*x^4+a)
^(1/2)/b+1/5*e*x^3*(b*x^4+a)^(1/2)/b-3/5*a*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^
(1/2)+b^(1/2)*x^2)+1/6*f*(b*x^4+a)^(3/2)/b^2-1/4*a*d*arctanh(b^(1/2)*x^2/(
b*x^4+a)^(1/2))/b^(3/2)+3/5*a^(5/4)*e*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)-1/30*a^(3/4)*(5*b^(1/2)*c+9*a^(1/2)*e)*(
a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJaco
biAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.57

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{-20a^2f+20abcx+15abdx^2+12abex^3-10abfx^4+20b^2cx^5+15b^2dx^6+12b^2ex^7+10b^2fx^8-15a\sqrt{b}}{}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(-20*a^2*f + 20*a*b*c*x + 15*a*b*d*x^2 + 12*a*b*e*x^3 - 10*a*b*f*x^4 + 20*
b^2*c*x^5 + 15*b^2*d*x^6 + 12*b^2*e*x^7 + 10*b^2*f*x^8 - 15*a*Sqrt[b]*d*Sq
rt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*b*c*x*Sqrt[1 +
 (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*b*e*x^3*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^
2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(c+d x+e x^2+f x^3\right)}{\sqrt{a+b x^4}} \, dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{x^4 \left(c+e x^2\right)}{\sqrt{a+b x^4}}+\frac{x^5 \left(d+f x^2\right)}{\sqrt{a+b x^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/4} \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \left(9 \sqrt{a} e+5 \sqrt{b} c\right) \text{EllipticF}\left(2 \arctan \left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30 b^{7/4} \sqrt{a+b x^4}}+\frac{3 a^{5/4} e \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} E\left(2 \arctan \left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5 b^{7/4} \sqrt{a+b x^4}}-\frac{a d \text{arctanh}\left(\frac{\sqrt{b} x^2}{\sqrt{a+b x^4}}\right)}{4 b^{3/2}}-\frac{3 a e x \sqrt{a+b x^4}}{5 b^{3/2} \left(\sqrt{a}+\sqrt{b} x^2\right)}-\frac{a f \sqrt{a+b x^4}}{3 b^2}+\frac{c x \sqrt{a+b x^4}}{3 b}+\frac{d x^2 \sqrt{a+b x^4}}{4 b}+\frac{e x^3 \sqrt{a+b x^4}}{5 b}+\frac{f x^4 \sqrt{a+b x^4}}{6 b}$$

input
```
Int[(x^4*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
-1/3*(a*f*Sqrt[a + b*x^4])/b^2 + (c*x*Sqrt[a + b*x^4])/(3*b) + (d*x^2*Sqrt
[a + b*x^4])/(4*b) + (e*x^3*Sqrt[a + b*x^4])/(5*b) + (f*x^4*Sqrt[a + b*x^4
])/(6*b) - (3*a*e*x*Sqrt[a + b*x^4])/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) -
 (a*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*e*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(
3/4)*(5*Sqrt[b]*c + 9*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/
(30*b^(7/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\left(-10bf\,x^{4}-12be\,x^{3}-15bd\,x^{2}-20cbx+20af\right)\sqrt{b\,x^{4}+a}}{60b^{2}}-\frac{a\left(\frac{10c\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{15d\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}\ldots}\right.}{2\sqrt{b}}\right.}{}$
default	$c\left(\frac{x\sqrt{b\,x^{4}+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+d\left(\frac{x^{2}\sqrt{b\,x^{4}+a}}{4b}-\frac{a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{3}{2}}}\right)+e\left(\ldots\right.$
elliptic	$\frac{f\,x^{4}\sqrt{b\,x^{4}+a}}{6b}+\frac{e\,x^{3}\sqrt{b\,x^{4}+a}}{5b}+\frac{d\,x^{2}\sqrt{b\,x^{4}+a}}{4b}+\frac{cx\sqrt{b\,x^{4}+a}}{3b}-\frac{af\sqrt{b\,x^{4}+a}}{3b^{2}}-\frac{ac\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\ldots}\right.}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(-10*b*f*x^4-12*b*e*x^3-15*b*d*x^2-20*b*c*x+20*a*f)/b^2*(b*x^4+a)^(1
/2)-1/30*a/b*(10*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)+15/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+18*I*e*a^
(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/
2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.44

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx =$$

$$-\frac{72\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-15\,a\sqrt{b}dx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+8\,(5\,bc-}{}$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(72*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -
1) - 15*a*sqrt(b)*d*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) +
8*(5*b*c - 9*a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) - 2*(10*b*f*x^5 + 12*b*e*x^4 + 15*b*d*x^3 + 20*b*c*x^2 - 20*a*f*x -
36*a*e)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.48

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}dx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + f\left(\begin{cases} -\frac{a\sqrt{a+bx^4}}{3b^2} + \frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*d*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + f*Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b
*x**4)/(6*b), Ne(b, 0)), (x**8/(8*sqrt(a)), True)) + c*x**5*gamma(5/4)*hyp
er((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) +
e*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^4\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{-40\sqrt{b\,x^4+a}\,af+40\sqrt{b\,x^4+a}\,bcx+30\sqrt{b\,x^4+a}\,bd\,x^2+24\sqrt{b\,x^4+a}\,be\,x^3+20\sqrt{b\,x^4+a}\,bf\,x^4+15\sqrt{}}{}$$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
( - 40*sqrt(a + b*x**4)*a*f + 40*sqrt(a + b*x**4)*b*c*x + 30*sqrt(a + b*x*
*4)*b*d*x**2 + 24*sqrt(a + b*x**4)*b*e*x**3 + 20*sqrt(a + b*x**4)*b*f*x**4
 + 15*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*d - 15*sqrt(b)*log(sq
rt(a + b*x**4) + sqrt(b)*x**2)*a*d - 40*int(sqrt(a + b*x**4)/(a + b*x**4),
x)*a*b*c - 72*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a*b*e)/(120*b**2
)
```

3.73 $\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 350

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx &= \frac{c\sqrt{a+bx^4}}{2b}+\frac{dx\sqrt{a+bx^4}}{3b}+\frac{ex^2\sqrt{a+bx^4}}{4b}+\frac{fx^3\sqrt{a+bx^4}}{5b}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\&\quad-\frac{ae\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}\\&\quad-\frac{a^{3/4}\left(5\sqrt{b}d+9\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*c*(b*x^4+a)^(1/2)/b+1/3*d*x*(b*x^4+a)^(1/2)/b+1/4*e*x^2*(b*x^4+a)^(1/2
)/b+1/5*f*x^3*(b*x^4+a)^(1/2)/b-3/5*a*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)
+b^(1/2)*x^2)-1/4*a*e*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)+3/5*a^(
5/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Ell
ipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/
2)-1/30*a^(3/4)*(5*b^(1/2)*d+9*a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)
/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)
),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.61

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{30\sqrt{b}c(a+bx^4)+20\sqrt{b}dx(a+bx^4)+15\sqrt{b}ex^2(a+bx^4)+12\sqrt{b}fx^3(a+bx^4)-15ae\sqrt{a+bx^4}\operatorname{arctanh}(}{}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(30*Sqrt[b]*c*(a + b*x^4) + 20*Sqrt[b]*d*x*(a + b*x^4) + 15*Sqrt[b]*e*x^2*
(a + b*x^4) + 12*Sqrt[b]*f*x^3*(a + b*x^4) - 15*a*e*Sqrt[a + b*x^4]*ArcTan
h[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 20*a*Sqrt[b]*d*x*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*a*Sqrt[b]*f*x^3*Sqrt[1 +
 (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(60*b^(3/2)*Sq
rt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \text{ 2372}$$
$$\int \left(\frac{x^3(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^4(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{a}f+5\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{7/4}\sqrt{a+bx^4}}+\frac{3a^{5/4}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}}-\frac{ae\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}-\frac{3afx\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\sqrt{a+bx^4}}{2b}+\frac{dx\sqrt{a+bx^4}}{3b}+\frac{ex^2\sqrt{a+bx^4}}{4b}+\frac{fx^3\sqrt{a+bx^4}}{5b}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(c*Sqrt[a + b*x^4])/(2*b) + (d*x*Sqrt[a + b*x^4])/(3*b) + (e*x^2*Sqrt[a +
b*x^4])/(4*b) + (f*x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*f*x*Sqrt[a + b*x^4])/
(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x^2)) - (a*e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/(4*b^(3/2)) + (3*a^(5/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(5*b^(7/4)*Sqrt[a + b*x^4]) - (a^(3/4)*(5*Sqrt[b]*d + 9*Sqrt[a]*f)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(30*b^(7/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.67

method	result
risch	$\frac{(12 f x^3+15 e x^2+20 d x+30 c)\sqrt{b x^4+a}}{60 b} - \frac{a\left(\frac{10 d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}} + \frac{18 i f\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{Ellip}\cdots\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{\cdots}}\right.}{30 b}$
default	$\frac{c\sqrt{b x^4+a}}{2b} + d\left(\frac{x\sqrt{b x^4+a}}{3b} - \frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right) + e\left(\frac{x^2\sqrt{b x^4+a}}{4b} - \frac{a\ln\left(\sqrt{b}\,x^2+\sqrt{b x^4+\cdots}\right.}{4b^{\frac{3}{2}}}\right.$
elliptic	$\frac{f x^3\sqrt{b x^4+a}}{5b} + \frac{e x^2\sqrt{b x^4+a}}{4b} + \frac{d x\sqrt{b x^4+a}}{3b} + \frac{c\sqrt{b x^4+a}}{2b} - \frac{a d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}} - \frac{a e\ln\left(2\sqrt{\cdots}\right.}{\cdots}$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(12*f*x^3+15*e*x^2+20*d*x+30*c)/b*(b*x^4+a)^(1/2)-1/30*a/b*(10*d/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+18*I*
f*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+15/2*e*ln(b^(1
/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.45

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx =$$
$$-\frac{72\,a\sqrt{b}fx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right) - 15\,a\sqrt{b}ex\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+8\,(5\,bd\cdots}{12\cdots}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(72*a*sqrt(b)*f*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -
1) - 15*a*sqrt(b)*e*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) +
8*(5*b*d - 9*a*f)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) - 2*(12*b*f*x^4 + 15*b*e*x^3 + 20*b*d*x^2 + 30*b*c*x - 36*a*f)*sqrt(
b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.45

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}ex^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + c\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*e*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*e*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + c*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)
/(2*b), True)) + d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((1/2, 7/4),
(11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^4 + a)*c/b + integrate((f*x^6 + e*x^5 + d*x^4)/sqrt(b*x^4 + a
), x)
```

Giac [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^3/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^3\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{60\sqrt{b\,x^4+a}\,bc+40\sqrt{b\,x^4+a}\,bdx+30\sqrt{b\,x^4+a}\,be\,x^2+24\sqrt{b\,x^4+a}\,bf\,x^3+15\sqrt{b}\log\Big(\sqrt{b\,x^4+a}-\sqrt{b}}{120b^2}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
(60*sqrt(a + b*x**4)*b*c + 40*sqrt(a + b*x**4)*b*d*x + 30*sqrt(a + b*x**4)
*b*e*x**2 + 24*sqrt(a + b*x**4)*b*f*x**3 + 15*sqrt(b)*log(sqrt(a + b*x**4)
 - sqrt(b)*x**2)*a*e - 15*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*e
 - 40*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*b*d - 72*int((sqrt(a + b*x**4
)*x**2)/(a + b*x**4),x)*a*b*f)/(120*b**2)
```

3.74 $\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx$

Optimal result

Integrand size = 30, antiderivative size = 322

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}} dx = \frac{d\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{3b} + \frac{fx^2\sqrt{a+bx^4}}{4b} + \frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{af\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}} - \frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

output

```
1/2*d*(b*x^4+a)^(1/2)/b+1/3*e*x*(b*x^4+a)^(1/2)/b+1/4*f*x^2*(b*x^4+a)^(1/2
)/b+c*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)-1/4*a*f*arctanh(b^(1
/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)-a^(1/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a
)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+1/6*a^(1/4)*(3*b^(1/2)*c-a^(1/2)*e)
*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJa
cobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{6\sqrt{b}d(a+bx^4)+4\sqrt{b}ex(a+bx^4)+3\sqrt{b}fx^2(a+bx^4)-3af\sqrt{a+bx^4}\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-4a\sqrt{b}ex\sqrt{1+}}{12b^{3/2}\sqrt{a+}}$$

input
```
Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(6*Sqrt[b]*d*(a + b*x^4) + 4*Sqrt[b]*e*x*(a + b*x^4) + 3*Sqrt[b]*f*x^2*(a
+ b*x^4) - 3*a*f*Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] -
4*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*
x^4)/a)] + 4*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4,
 7/4, -((b*x^4)/a)])/(12*b^(3/2)*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \text{2372}$$
$$\int \left(\frac{x^2(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^3(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}c-\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{af\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}+\frac{cx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\sqrt{a+bx^4}}{2b}+\frac{ex\sqrt{a+bx^4}}{3b}+\frac{fx^2\sqrt{a+bx^4}}{4b}$$

input
```
Int[(x^2*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(d*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(3*b) + (f*x^2*Sqrt[a +
b*x^4])/(4*b) + (c*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) -
(a*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(4*b^(3/2)) - (a^(1/4)*c*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*
(3*Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a
] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5
/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(3f\,x^2+4ex+6d)\sqrt{b\,x^4+a}}{12b}-\frac{\frac{2ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{3af\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}-\frac{6ic\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+i}}{}}{6b}$
default	$\frac{ic\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+\frac{d\sqrt{b\,x^4+a}}{2b}+e\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}\right.$
elliptic	$\frac{f\,x^2\sqrt{b\,x^4+a}}{4b}+\frac{ex\sqrt{b\,x^4+a}}{3b}+\frac{d\sqrt{b\,x^4+a}}{2b}-\frac{ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{af\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{4b^{\frac{3}{2}}}$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*f*x^2+4*e*x+6*d)/b*(b*x^4+a)^(1/2)-1/6/b*(2*a*e/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/2*a*f*ln(b^(1/2)*
x^2+(b*x^4+a)^(1/2))/b^(1/2)-6*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.46

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{24\,b^{\frac{3}{2}}cx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+3\,a\sqrt{b}fx\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)-8\,(3\,bc+be)\sqrt{}}{24\,b^2x}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(24*b^(3/2)*c*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
 3*a*sqrt(b)*f*x*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) - 8*(3*
b*c + b*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
 2*(3*b*f*x^3 + 4*b*e*x^2 + 6*b*d*x + 12*b*c)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.48

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}fx^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{af\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} + d\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*f*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*f*asinh(sqrt(b)*x**2/sqrt(a))/
(4*b**(3/2)) + d*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)
/(2*b), True)) + c*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((1/2, 5/4),
(9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^2\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
(3*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*f +
12*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*b*f*x**
4 - 3*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a**2*f
 - 12*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*b*f*
x**4 + 36*sqrt(b)*sqrt(a + b*x**4)*a*b*d*x**2 + 24*sqrt(b)*sqrt(a + b*x**4
)*a*b*e*x**3 + 18*sqrt(b)*sqrt(a + b*x**4)*a*b*f*x**4 + 48*sqrt(b)*sqrt(a
+ b*x**4)*b**2*d*x**6 + 32*sqrt(b)*sqrt(a + b*x**4)*b**2*e*x**7 + 24*sqrt(
b)*sqrt(a + b*x**4)*b**2*f*x**8 - 8*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/
(a + b*x**4),x)*a**2*b*e - 32*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a + b
*x**4),x)*a*b**2*e*x**4 + 24*sqrt(a + b*x**4)*int((sqrt(a + b*x**4)*x**2)/
(a + b*x**4),x)*a*b**2*c + 96*sqrt(a + b*x**4)*int((sqrt(a + b*x**4)*x**2)
/(a + b*x**4),x)*b**3*c*x**4 - 24*sqrt(b)*int(sqrt(a + b*x**4)/(a + b*x**4
),x)*a**2*b*e*x**2 - 32*sqrt(b)*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*b**
2*e*x**6 + 72*sqrt(b)*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a*b**2*c
*x**2 + 96*sqrt(b)*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*b**3*c*x**6
 + 9*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*b*f*x**2 + 12*log(sqrt(a +
b*x**4) - sqrt(b)*x**2)*a*b**2*f*x**6 - 9*log(sqrt(a + b*x**4) + sqrt(b)*x
**2)*a**2*b*f*x**2 - 12*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*b**2*f*x**6
 + 12*a**2*b*d + 8*a**2*b*e*x + 6*a**2*b*f*x**2 + 60*a*b**2*d*x**4 + 40*a*
b**2*e*x**5 + 30*a*b**2*f*x**6 + 48*b**3*d*x**8 + 32*b**3*e*x**9 + 24*b...
```

3.75 $\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 299

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{e\sqrt{a+bx^4}}{2b} + \frac{fx\sqrt{a+bx^4}}{3b} + \frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/2*e*(b*x^4+a)^(1/2)/b+1/3*f*x*(b*x^4+a)^(1/2)/b+d*x*(b*x^4+a)^(1/2)/b^(1
/2)/(a^(1/2)+b^(1/2)*x^2)+1/2*c*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/
2)-a^(1/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/
2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+
a)^(1/2)+1/6*a^(1/4)*(3*b^(1/2)*d-a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4
+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1
/4)),1/2*2^(1/2))/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.54

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = \frac{3ae+2afx+3bex^4+2bfx^5+3\sqrt{b}c\sqrt{a+bx^4}\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)-2afx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\ldots\right)}{6b\sqrt{a+bx^4}}$$

input
```
Integrate[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(3*a*e + 2*a*f*x + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[b]*c*Sqrt[a + b*x^4]*Arc
Tanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - 2*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hyp
ergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(6*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{x(c+ex^2)}{\sqrt{a+bx^4}}+\frac{x^2(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{dx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\sqrt{a+bx^4}}{2b}+\frac{fx\sqrt{a+bx^4}}{3b}$$

input
```
Int[(x*(c + d*x + e*x^2 + f*x^3))/Sqrt[a + b*x^4],x]
```

output
```
(e*Sqrt[a + b*x^4])/(2*b) + (f*x*Sqrt[a + b*x^4])/(3*b) + (d*x*Sqrt[a + b*
x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a
+ b*x^4]])/(2*Sqrt[b]) - (a^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(b^(3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*(3*Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a]
 + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(2fx+3e)\sqrt{b\,x^4+a}}{6b}-\frac{\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3c\sqrt{b}\,\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2}-\frac{3i\sqrt{b}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,(}{\sqrt{}}}{3b}$
default	$\frac{c\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}+f\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\right.$
elliptic	$\frac{fx\sqrt{b\,x^4+a}}{3b}+\frac{e\sqrt{b\,x^4+a}}{2b}-\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{c\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}}{\sqrt{}}}}{}$

input
```
int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*f*x+3*e)/b*(b*x^4+a)^(1/2)-1/3/b*(a*f/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*c*b^(1/2)*ln(b^(1/2)*x^2+(
b*x^4+a)^(1/2))-3*I*b^(1/2)*d*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.44

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{12\sqrt{b}dx\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-4\sqrt{b}(3d+f)x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+3\sqrt{b}cx\log}{12\,bx}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(12*sqrt(b)*d*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) -
 4*sqrt(b)*(3*d + f)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1)
 + 3*sqrt(b)*c*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*sqr
t(b*x^4 + a)*(2*f*x^2 + 3*e*x + 6*d))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.43

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx = e\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } b=0\\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
e*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True))
+ c*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + d*x**3*gamma(3/4)*hyper((1/2
, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + f*x**5*
gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*
gamma(9/4))
```

Maxima [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx=\int \frac{(fx^3+ex^2+dx+c)x}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*c*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2
))/sqrt(b) + integrate((f*x^4 + e*x^3 + d*x^2)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx=\int \frac{(fx^3+ex^2+dx+c)x}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx=\int \frac{x\,(f\,x^3+e\,x^2+d\,x+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2),x)
```

output
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\sqrt{b\,x^4+a}-\sqrt{b}\,x^2\right)c+3\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\sqrt{b\,x^4+a}+\sqrt{b}\,x^2\right)c+6\sqrt{b}\,\sqrt{b\,x^4+a}}{}$$

input
```
int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*c + 3*
sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*c + 6*sqrt(b
)*sqrt(a + b*x**4)*e*x**2 + 4*sqrt(b)*sqrt(a + b*x**4)*f*x**3 - 4*sqrt(a +
 b*x**4)*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*f + 12*sqrt(a + b*x**4)*in
t((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*b*d - 4*sqrt(b)*int(sqrt(a + b*x
**4)/(a + b*x**4),x)*a*f*x**2 + 12*sqrt(b)*int((sqrt(a + b*x**4)*x**2)/(a
+ b*x**4),x)*b*d*x**2 - 3*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*b*c*x**2 +
3*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*b*c*x**2 + 6*a*e + 4*a*f*x + 6*b*e*
x**4 + 4*b*f*x**5)/(12*b*(sqrt(a + b*x**4) + sqrt(b)*x**2))
```

3.76 $\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 277

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}} \, dx = \frac{f\sqrt{a+bx^4}}{2b} + \frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}$$

output
```
1/2*f*(b*x^4+a)^(1/2)/b+e*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)+
1/2*d*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/2)-a^(1/4)*e*(a^(1/2)+b^(1
/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(
b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+1/2*(b^(1/2)*c+a^
(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*I
nverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(3/4)/(b
*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.54

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \frac{f\sqrt{a+bx^4}}{2b}+\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} + \frac{cx\sqrt{1+\frac{bx^4}{a}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{ex^3\sqrt{1+\frac{bx^4}{a}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

output
```
(f*Sqrt[a + b*x^4])/(2*b) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*
Sqrt[b]) + (c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*
x^4)/a)])/Sqrt[a + b*x^4] + (e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1
/2, 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 2424$$

$$\int \left(\frac{c+ex^2}{\sqrt{a+bx^4}}+\frac{x(d+fx^2)}{\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}c}{\sqrt{a}}+e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{ex\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\sqrt{a+bx^4}}{2b}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/Sqrt[a + b*x^4],x]
```

output
```
(f*Sqrt[a + b*x^4])/(2*b) + (e*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt
[b]*x^2)) + (d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (a^(1
/4)*e*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a + b*x^4]) +
 (a^(1/4)*((Sqrt[b]*c)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.75

method	result
default	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
risch	$\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
elliptic	$\frac{f\sqrt{b\,x^4+a}}{2b}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\right.}{\sqrt{}}$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+1/2*d*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))/b^(1/2)+I*e*a^(1/2)/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellipti
cE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*f*(b*x^4+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.49

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{4\,a\sqrt{b}ex\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+a\sqrt{b}dx\log\left(-2\,bx^4-2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+4\,(bc-ae)\sqrt{b}}{4\,abx}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*a*sqrt(b)*e*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) +
 a*sqrt(b)*d*x*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 4*(b*c
- a*e)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 2*s
qrt(b*x^4 + a)*(a*f*x + 2*a*e))/(a*b*x)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = f\left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for } b=0 \\ \frac{\sqrt{a+bx^4}}{2b} & \text{otherwise} \end{cases}\right) + \frac{d\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
f*Piecewise((x**4/(4*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**4)/(2*b), True))
+ d*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*x*gamma(1/4)*hyper((1/4, 1
/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e*x**3*gam
ma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gam
ma(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx^4}}\,dx = \frac{-\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\sqrt{b\,x^4+a}-\sqrt{b}\,x^2\right)d+\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\sqrt{b\,x^4+a}+\sqrt{b}\,x^2\right)d+2\sqrt{b}\,\sqrt{b\,x^4+a}\,f}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*d + sqrt
(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*d + 2*sqrt(b)*sq
rt(a + b*x**4)*f*x**2 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a + b*x**
4),x)*b*c + 4*sqrt(a + b*x**4)*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)
*b*e + 4*sqrt(b)*int(sqrt(a + b*x**4)/(a + b*x**4),x)*b*c*x**2 + 4*sqrt(b)
*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*b*e*x**2 - log(sqrt(a + b*x**
4) - sqrt(b)*x**2)*b*d*x**2 + log(sqrt(a + b*x**4) + sqrt(b)*x**2)*b*d*x**
2 + 2*a*f + 2*b*f*x**4)/(4*b*(sqrt(a + b*x**4) + sqrt(b)*x**2))
```

3.77 $\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 286

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx &= \frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\ &\quad-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}\\ &\quad+\frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
f*x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)+1/2*e*arctanh(b^(1/2)*x^
2/(b*x^4+a)^(1/2))/b^(1/2)-1/2*c*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-
a^(1/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*
EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^
(1/2)+1/2*(b^(1/2)*d+a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+
b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1
/2))/a^(1/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.56

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \frac{e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{dx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}} + \frac{fx^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

output
```
(e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) + (d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4] + (f*x^3*Sqrt[1 + (b*x^4)
/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \; 2371$$

$$c\int \frac{1}{x\sqrt{bx^4+a}}dx+\int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx$$

$$\big\downarrow 798$$

$$\frac{1}{4}c\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4+\int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx$$

$$\big\downarrow 73$$

$$\frac{c\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2b}+\int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx$$

$$\big\downarrow 221$$

$$\int \frac{fx^2+ex+d}{\sqrt{bx^4+a}}dx-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

$$\big\downarrow 2424$$

$$\int \left(\frac{ex}{\sqrt{bx^4+a}}+\frac{fx^2+d}{\sqrt{bx^4+a}}\right)dx-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

$$\big\downarrow 2009$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}+f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}+\frac{fx\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x*Sqrt[a + b*x^4]),x]
```

output
```
(f*x*Sqrt[a + b*x^4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt
[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a
]])/(2*Sqrt[a]) - (a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(
3/4)*Sqrt[a + b*x^4]) + (a^(1/4)*((Sqrt[b]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.76

method	result
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{e\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{i f\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\cdots}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\cdots}$
default	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{e\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{i f\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{\cdots}}{\sqrt{\cdots}}}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}\cdots}$

input
```
int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b
^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
+1/2*e*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))/b^(1/2)+I*f*a^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*c/a^(1/2)*arctanh(a^(1/2)/(b*x^
4+a)^(1/2))
```

Fricas [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}\,x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^5 + a*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \frac{e\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} - \frac{c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(1/2),x)
```

output
```
e*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) - c*asinh(sqrt(a)/(sqrt(b)*x**2)
)/(2*sqrt(a)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(
I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + f*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4
,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}\,x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx^4}}\,dx$$
$$= \frac{\sqrt{a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)bc-\sqrt{a}\log\left(\sqrt{b\,x^4+a}+\sqrt{a}\right)bc-\sqrt{b}\log\left(\sqrt{b\,x^4+a}-\sqrt{b}\,x^2\right)ae+\sqrt{b}\log\Big(\sqrt{}}{4ab}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*c - sqrt(a)*log(sqrt(a + b*x**4
) + sqrt(a))*b*c - sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*e + sqrt
(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*e + 4*int(sqrt(a + b*x**4)/(a +
 b*x**4),x)*a*b*d + 4*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a*b*f)/(
4*a*b)
```

3.78 $\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 289

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\ &\quad-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}\\ &\quad+\frac{\left(\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}\end{aligned}$$

output

```
-c*(b*x^4+a)^(1/2)/a^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)+1/2*f*arctanh(b^(1/2)*x
^2/(b*x^4+a)^(1/2))/b^(1/2)-1/2*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
-b^(1/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)
*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)
^(1/2)+1/2*(b^(1/2)*c+a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)
+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(
1/2))/a^(3/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.54

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}} - \frac{d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{c\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}} + \frac{ex\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
(f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) - (d*ArcTanh[Sqrt[a
 + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[-1/4, 1/2, 3/4, -((b*x^4)/a)])/(x*Sqrt[a + b*x^4]) + (e*x*Sqrt[1 + (b*x^4
)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2372}$$

$$\int \left(\frac{c+ex^2}{x^2\sqrt{a+bx^4}}+\frac{d+fx^2}{x\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{f\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}-\frac{c\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}cx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
-((c*Sqrt[a + b*x^4])/(a*x)) + (Sqrt[b]*c*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] +
 Sqrt[b]*x^2)) + (f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]) -
(d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*c*(Sqrt[a] + S
qrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*c + Sqr
t[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2
]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a
 + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.81

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{a x}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2\sqrt{b}}+\frac{i\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}$
default	$\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+c\left(-\frac{\sqrt{b\,x^4+a}}{a x}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{Ell}\right.}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{a x}+\frac{\frac{a e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{i c\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}}{a}$

input
```
int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^4+a)^(1/2)/a/x+e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^
(1/2)*b^(1/2))^(1/2),I)+1/2*f*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))/b^(1/2)+
I*b^(1/2)/a^(1/2)*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*d/a^(1/2
)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^2}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(f*x^3 + e*x^2 + d*x + c)/(b*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.44

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \frac{f\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} + \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(1/2),x)
```

output
```
f*asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)) + c*gamma(-1/4)*hyper((-1/4, 1/2
), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - d*asinh(sq
rt(a)/(sqrt(b)*x**2))/(2*sqrt(a)) + e*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,
), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^2}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^2\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx^4}}\,dx$$
$$=\frac{\sqrt{a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)bd-\sqrt{a}\log\left(\sqrt{b\,x^4+a}+\sqrt{a}\right)bd-\sqrt{b}\log\left(\sqrt{b\,x^4+a}-\sqrt{b}\,x^2\right)af+\sqrt{b}\log\Big(\sqrt{}}{4ab}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b*d - sqrt(a)*log(sqrt(a + b*x**4
) + sqrt(a))*b*d - sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*f + sqrt
(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*f + 4*int(sqrt(a + b*x**4)/(a*x
**2 + b*x**6),x)*a*b*c + 4*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*b*e)/(4*
a*b)
```

3.79 $\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 280

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\ &\quad-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}\\ &\quad+\frac{\left(\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*c*(b*x^4+a)^(1/2)/a/x^2-d*(b*x^4+a)^(1/2)/a^(1/2)/x/(a^(1/2)+b^(1/2)*
x^2)-1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)-b^(1/4)*d*(a^(1/2)+b^(
1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan
(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)+1/2*(b^(1/2)*d+a
^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/b^(1/4)/(
b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.53

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{d\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}} + \frac{fx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*(c*Sqrt[a + b*x^4])/(a*x^2) - (e*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2
*Sqrt[a]) - (d*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*
x^4)/a)])/(x*Sqrt[a + b*x^4]) + (f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[1/4, 1/2, 5/4, -((b*x^4)/a)])/Sqrt[a + b*x^4]
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \ 2372$$

$$\int \left(\frac{c+ex^2}{x^3\sqrt{a+bx^4}}+\frac{d+fx^2}{x^2\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{e\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{c\sqrt{a+bx^4}}{2ax^2}-\frac{d\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}dx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^3*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*(c*Sqrt[a + b*x^4])/(a*x^2) - (d*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*d*
x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (e*ArcTanh[Sqrt[a + b*x^4
]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(a^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + Sqrt[a]*f)*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.80

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{d\sqrt{b\,x^4+a}}{ax}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{id\sqrt{b}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\right.\right.}{\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x}}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(2dx+c)}{2a\,x^2}+\frac{\frac{af\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{\sqrt{a}\,e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}+\frac{i\sqrt{b}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}}{a}$
default	$\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{c\sqrt{b\,x^4+a}}{2a\,x^2}+d\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(\right.\right.}{\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{}}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(b*x^4+a)^(1/2)/a/x^2-d/a*(b*x^4+a)^(1/2)/x+f/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+I/a^(1/2)*d*b^(1/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2*e/a^(1/2)*arctanh(a^(1/2)/(b*x
^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.50

$$\int\frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx=$$
$$-\frac{4\sqrt{a}bdx^2\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid-1\right)-\sqrt{a}bex^2\log\left(-\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-4\,(bd-af)\sqrt{a}x^2(-}{4\,abx^2}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(a)*b*d*x^2*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1
) - sqrt(a)*b*e*x^2*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) -
4*(b*d - a*f)*sqrt(a)*x^2*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
-1) + 2*sqrt(b*x^4 + a)*(2*b*d*x + b*c))/(a*b*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.45

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} + \frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(2*a) + d*gamma(-1/4)*hyper((-1/4, 1/2), (
3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) - e*asinh(sqrt(a
)/(sqrt(b)*x**2))/(2*sqrt(a)) + f*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b
*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx=\frac{f\,x\,\sqrt{\frac{b\,x^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\sqrt{b\,x^4+a}}-\frac{c\,\sqrt{b\,x^4+a}}{2\,a\,x^2}-\frac{d\,\sqrt{\frac{a}{b\,x^4}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{b\,x^4+a}}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(1/2)),x)
```

output
```
(f*x*((b*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^4)/a))/(a + b*
x^4)^(1/2) - (c*(a + b*x^4)^(1/2))/(2*a*x^2) - (d*(a/(b*x^4) + 1)^(1/2)*hy
pergeom([1/2, 3/4], 7/4, -a/(b*x^4)))/(3*x*(a + b*x^4)^(1/2)) - (e*atanh((
a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx^4}}\,dx$$
$$= \frac{\sqrt{a}\,\sqrt{b\,x^4+a}\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)e\,x^2-\sqrt{a}\,\sqrt{b\,x^4+a}\log\left(\sqrt{b\,x^4+a}+\sqrt{a}\right)e\,x^2-4\sqrt{b}\,\sqrt{b\,x^4+a}\,c\,x^2}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*e*x**2 - sqrt(a)
*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*e*x**2 - 4*sqrt(b)*sqrt(
a + b*x**4)*c*x**2 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*x**2 + b*x
**6),x)*a*d*x**2 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a + b*x**4),x)
*a*f*x**2 + sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*e*x**4 - sqrt(
b)*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*e*x**4 + 4*sqrt(b)*int(sqrt(a +
 b*x**4)/(a*x**2 + b*x**6),x)*a*d*x**4 + 4*sqrt(b)*int(sqrt(a + b*x**4)/(a
 + b*x**4),x)*a*f*x**4 - 2*a*c - 4*b*c*x**4)/(4*a*x**2*(sqrt(a + b*x**4) +
 sqrt(b)*x**2))
```

3.80 $\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 303

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{f\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}\\&\quad-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}\\&\quad-\frac{\sqrt[4]{b}\left(\sqrt{b}c-3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/3*c*(b*x^4+a)^(1/2)/a/x^3-1/2*d*(b*x^4+a)^(1/2)/a/x^2-e*(b*x^4+a)^(1/2)
/a^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)-1/2*f*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^
(1/2)-b^(1/4)*e*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^
(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x
^4+a)^(1/2)-1/6*b^(1/4)*(b^(1/2)*c-3*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*
x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a
^(1/4)),1/2*2^(1/2))/a^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.49

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$$
$$=\frac{-2ac\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right)-3x\left(ad+bdx^4+\sqrt{a}fx^2\sqrt{a+bx^4}\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right.\right.}{6ax^3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
(-2*a*c*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)
] - 3*x*(a*d + b*d*x^4 + Sqrt[a]*f*x^2*Sqrt[a + b*x^4]*ArcTanh[Sqrt[a + b*
x^4]/Sqrt[a]] + 2*a*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -((b*x^4)/a)]))/(6*a*x^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \text{2372}$$
$$\int \left(\frac{c+ex^2}{x^4\sqrt{a+bx^4}}+\frac{d+fx^2}{x^3\sqrt{a+bx^4}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}c-3\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}-\frac{f\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{c\sqrt{a+bx^4}}{3ax^3}-\frac{d\sqrt{a+bx^4}}{2ax^2}-\frac{e\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}ex\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
-1/3*(c*Sqrt[a + b*x^4])/(a*x^3) - (d*Sqrt[a + b*x^4])/(2*a*x^2) - (e*Sqrt
[a + b*x^4])/(a*x) + (Sqrt[b]*e*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x
^2)) - (f*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(2*Sqrt[a]) - (b^(1/4)*e*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(
Sqrt[b]*c - 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/
4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(6e\,x^2+3dx+2c)}{6a\,x^3}+\frac{-\frac{cb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{3\sqrt{a}\,f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^4+a}}{x^2}\right)}{2}+\frac{3i\sqrt{b}\,e\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}}{3a}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{d\sqrt{b\,x^4+a}}{2a\,x^2}-\frac{e\sqrt{b\,x^4+a}}{ax}-\frac{bc\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{ie\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-\frac{d\sqrt{b\,x^4+a}}{2a\,x^2}+e\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^4+a)^(1/2)*(6*e*x^2+3*d*x+2*c)/a/x^3+1/3/a*(-c*b/(I/a^(1/2)*b^(1
/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*a^(1/2)*f*ln
((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)+3*I*b^(1/2)*e*a^(1/2)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.45

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx =$$
$$-\frac{12\sqrt{a}ex^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-4\sqrt{a}(c+3e)x^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-3\sqrt{}}{12\,ax^3}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(12*sqrt(a)*e*x^3*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1
) - 4*sqrt(a)*(c + 3*e)*x^3*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4))
, -1) - 3*sqrt(a)*f*x^3*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4
) + 2*sqrt(b*x^4 + a)*(6*e*x^2 + 3*d*x + 2*c))/(a*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} - \frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**4/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(2*a) + c*gamma(-3/4)*hyper((-3/4, 1/2), (
1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + e*gamma(-1/
4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma
(3/4)) - f*asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^4}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+a}x^4}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^4\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^4*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx^4}}\,dx$$
$$=\frac{\sqrt{a}\,\sqrt{b\,x^4+a}\,\log\left(\sqrt{b\,x^4+a}-\sqrt{a}\right)f\,x^2-\sqrt{a}\,\sqrt{b\,x^4+a}\,\log\left(\sqrt{b\,x^4+a}+\sqrt{a}\right)f\,x^2-4\sqrt{b}\,\sqrt{b\,x^4+a}\,d\,x^2}{}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x^4/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*f*x**2 - sqrt(a)
*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*f*x**2 - 4*sqrt(b)*sqrt(
a + b*x**4)*d*x**2 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*x**4 + b*x
**8),x)*a*c*x**2 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*x**2 + b*x**
6),x)*a*e*x**2 + sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*f*x**4 -
sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*f*x**4 + 4*sqrt(b)*int(sqr
t(a + b*x**4)/(a*x**4 + b*x**8),x)*a*c*x**4 + 4*sqrt(b)*int(sqrt(a + b*x**
4)/(a*x**2 + b*x**6),x)*a*e*x**4 - 2*a*d - 4*b*d*x**4)/(4*a*x**2*(sqrt(a +
 b*x**4) + sqrt(b)*x**2))
```

3.81 $\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 326

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{\sqrt{a}x\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} - \frac{\sqrt[4]{b}\left(\sqrt{b}d-3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/4*c*(b*x^4+a)^(1/2)/a/x^4-1/3*d*(b*x^4+a)^(1/2)/a/x^3-1/2*e*(b*x^4+a)^(
1/2)/a/x^2-f*(b*x^4+a)^(1/2)/a^(1/2)/x/(a^(1/2)+b^(1/2)*x^2)+1/4*b*c*arcta
nh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-b^(1/4)*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^
4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/
4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)-1/6*b^(1/4)*(b^(1/2)*d-3*a^(1/2)
*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Invers
eJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.45

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}\left(3ac\sqrt{1+\frac{bx^4}{a}}+6aex^2\sqrt{1+\frac{bx^4}{a}}-3bcx^4\operatorname{arctanh}\left(\sqrt{1+\frac{bx^4}{a}}\right)+4adx\,\text{Hypergeometric2F1}\right.}{12a^2x^4\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

output
```
-1/12*(Sqrt[a + b*x^4]*(3*a*c*Sqrt[1 + (b*x^4)/a] + 6*a*e*x^2*Sqrt[1 + (b*
x^4)/a] - 3*b*c*x^4*ArcTanh[Sqrt[1 + (b*x^4)/a]] + 4*a*d*x*Hypergeometric2
F1[-3/4, 1/2, 1/4, -((b*x^4)/a)] + 12*a*f*x^3*Hypergeometric2F1[-1/4, 1/2,
 3/4, -((b*x^4)/a)]))/(a^2*x^4*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{c+ex^2}{x^5\sqrt{a+bx^4}}+\frac{d+fx^2}{x^4\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-3\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}}+\frac{bc\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{c\sqrt{a+bx^4}}{4ax^4}-\frac{d\sqrt{a+bx^4}}{3ax^3}-\frac{e\sqrt{a+bx^4}}{2ax^2}-\frac{f\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}fx\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^5*Sqrt[a + b*x^4]),x]
```

output
```
-1/4*(c*Sqrt[a + b*x^4])/(a*x^4) - (d*Sqrt[a + b*x^4])/(3*a*x^3) - (e*Sqrt
[a + b*x^4])/(2*a*x^2) - (f*Sqrt[a + b*x^4])/(a*x) + (Sqrt[b]*f*x*Sqrt[a +
 b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) + (b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a
]])/(4*a^(3/2)) - (b^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(
3/4)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*d - 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(12 f\,x^{3}+6 e\,x^{2}+4 d x+3 c)}{12 a\,x^{4}}-\frac{b\left(\frac{2 d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{3 c\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}-\frac{6 i f\sqrt{a}\,\sqrt{1-i}}{}\right.}{6 a}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{4 a\,x^{4}}-\frac{d\sqrt{b\,x^{4}+a}}{3 a\,x^{3}}-\frac{e\sqrt{b\,x^{4}+a}}{2 a\,x^{2}}-\frac{f\sqrt{b\,x^{4}+a}}{a x}-\frac{d b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{i\sqrt{b}\,f\sqrt{1-\frac{i\sqrt{b}}{\sqrt{}}}}{}$
default	$c\left(-\frac{\sqrt{b\,x^{4}+a}}{4 a\,x^{4}}+\frac{b\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{4 a^{\frac{3}{2}}}\right)+d\left(-\frac{\sqrt{b\,x^{4}+a}}{3 a\,x^{3}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3 a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)-\frac{e\sqrt{}}{}$

input
```
int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^4+a)^(1/2)*(12*f*x^3+6*e*x^2+4*d*x+3*c)/a/x^4-1/6*b/a*(2*d/(I/a
^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)
*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/2*c
/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2))/x^2)-6*I*f*a^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx =$$

$$-\frac{24\,a^{\frac{3}{2}}fx^4\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-3\,\sqrt{a}bcx^4\log\left(-\frac{bx^4+2\,\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)-8\,(ad+3\,af)\sqrt{a}x}{24\,a^2x^4}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(24*a^(3/2)*f*x^4*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1
) - 3*sqrt(a)*b*c*x^4*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4)
- 8*(a*d + 3*a*f)*sqrt(a)*x^4*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4
)), -1) + 2*(12*a*f*x^3 + 6*a*e*x^2 + 4*a*d*x + 3*a*c)*sqrt(b*x^4 + a))/(a
^2*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.48

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}c\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{f\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**5/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*c*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**4) + 1
)/(2*a) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*sqrt(a)*x**3*gamma(1/4)) + f*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,),
b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4)) + b*c*asinh(sqrt(a)/(sq
rt(b)*x**2))/(4*a**(3/2))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^5}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*c*(2*sqrt(b*x^4 + a)*b/((b*x^4 + a)*a - a^2) + b*log((sqrt(b*x^4 + a)
 - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2)) + integrate((f*x^2 + e*x
 + d)/(sqrt(b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^5}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x^5\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^5*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x^5/(b*x^4+a)^(1/2),x)
```

output
```
( - 6*sqrt(b)*sqrt(a)*sqrt(a + b*x**4)*a*c*x**2 - 16*sqrt(b)*sqrt(a)*sqrt(
a + b*x**4)*a*e*x**4 - 8*sqrt(b)*sqrt(a)*sqrt(a + b*x**4)*b*c*x**6 - 32*sq
rt(b)*sqrt(a)*sqrt(a + b*x**4)*b*e*x**8 + 8*sqrt(a)*sqrt(a + b*x**4)*int(s
qrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a**2*d*x**4 + 32*sqrt(a)*sqrt(a + b*x
**4)*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a*b*d*x**8 + 8*sqrt(a)*sqrt
(a + b*x**4)*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a**2*f*x**4 + 32*sq
rt(a)*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a*b*f*x**
8 - sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*c*x**4 - 4*sqrt(a
 + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*c*x**8 + sqrt(a + b*x**4)*
log(sqrt(a + b*x**4) + sqrt(a))*a*b*c*x**4 + 4*sqrt(a + b*x**4)*log(sqrt(a
 + b*x**4) + sqrt(a))*b**2*c*x**8 + 24*sqrt(b)*sqrt(a)*int(sqrt(a + b*x**4
)/(a*x**4 + b*x**8),x)*a**2*d*x**6 + 32*sqrt(b)*sqrt(a)*int(sqrt(a + b*x**
4)/(a*x**4 + b*x**8),x)*a*b*d*x**10 + 24*sqrt(b)*sqrt(a)*int(sqrt(a + b*x*
*4)/(a*x**2 + b*x**6),x)*a**2*f*x**6 + 32*sqrt(b)*sqrt(a)*int(sqrt(a + b*x
**4)/(a*x**2 + b*x**6),x)*a*b*f*x**10 - 2*sqrt(a)*a**2*c - 4*sqrt(a)*a**2*
e*x**2 - 10*sqrt(a)*a*b*c*x**4 - 32*sqrt(a)*a*b*e*x**6 - 8*sqrt(a)*b**2*c*
x**8 - 32*sqrt(a)*b**2*e*x**10 - 3*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(a))
*a*b*c*x**6 - 4*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*c*x**10 + 3*s
qrt(b)*log(sqrt(a + b*x**4) + sqrt(a))*a*b*c*x**6 + 4*sqrt(b)*log(sqrt(a +
 b*x**4) + sqrt(a))*b**2*c*x**10)/(8*sqrt(a)*a*x**4*(sqrt(a + b*x**4)*a...
```

3.82 $\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 354

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}+\frac{3bc\sqrt{a+bx^4}}{5a^{3/2}x\left(\sqrt{a}+\sqrt{b}x^2\right)}\\&\quad+\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}\\&\quad-\frac{b^{3/4}\left(9\sqrt{b}c+5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/5*c*(b*x^4+a)^(1/2)/a/x^5-1/4*d*(b*x^4+a)^(1/2)/a/x^4-1/3*e*(b*x^4+a)^(
1/2)/a/x^3-1/2*f*(b*x^4+a)^(1/2)/a/x^2+3/5*b*c*(b*x^4+a)^(1/2)/a^(3/2)/x/(
a^(1/2)+b^(1/2)*x^2)+1/4*b*d*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)+3/5*
b^(5/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*
EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^
(1/2)-1/30*b^(3/4)*(9*b^(1/2)*c+5*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4
+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1
/4)),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.38

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}\left(12ac\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)+5x\left(3a(d+2fx^2)\sqrt{1+\frac{bx^4}{a}}-3bdx^4\text{arctan}\right.\right.}{60a^2x^5\sqrt{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
-1/60*(Sqrt[a + b*x^4]*(12*a*c*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^4
)/a)] + 5*x*(3*a*(d + 2*f*x^2)*Sqrt[1 + (b*x^4)/a] - 3*b*d*x^4*ArcTanh[Sqr
t[1 + (b*x^4)/a]] + 4*a*e*x*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)
])))/(a^2*x^5*Sqrt[1 + (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(\frac{c+ex^2}{x^6\sqrt{a+bx^4}}+\frac{d+fx^2}{x^5\sqrt{a+bx^4}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(5\sqrt{a}e+9\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30a^{7/4}\sqrt{a+bx^4}}+\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}+\frac{bd\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{3b^{3/2}cx\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{3bc\sqrt{a+bx^4}}{5a^2x}-\frac{c\sqrt{a+bx^4}}{5ax^5}-\frac{d\sqrt{a+bx^4}}{4ax^4}-\frac{e\sqrt{a+bx^4}}{3ax^3}-\frac{f\sqrt{a+bx^4}}{2ax^2}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
-1/5*(c*Sqrt[a + b*x^4])/(a*x^5) - (d*Sqrt[a + b*x^4])/(4*a*x^4) - (e*Sqrt
[a + b*x^4])/(3*a*x^3) - (f*Sqrt[a + b*x^4])/(2*a*x^2) + (3*b*c*Sqrt[a + b
*x^4])/(5*a^2*x) - (3*b^(3/2)*c*x*Sqrt[a + b*x^4])/(5*a^2*(Sqrt[a] + Sqrt[
b]*x^2)) + (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(3/2)) + (3*b^(5/4)
*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a + b*x^4]) -
(b^(3/4)*(9*Sqrt[b]*c + 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(30*a^(7/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(-36bc\,x^4+30af\,x^3+20ae\,x^2+15adx+12ac)}{60a^2x^5}-\frac{b\left(\frac{10ae\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{15\sqrt{a}\,d\ln\left(\frac{2a+2\sqrt{}}{}\right.}{2}\right.}{}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5a\,x^5}-\frac{d\sqrt{b\,x^4+a}}{4a\,x^4}-\frac{e\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{f\sqrt{b\,x^4+a}}{2a\,x^2}+\frac{3bc\sqrt{b\,x^4+a}}{5a^2x}-\frac{eb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{5a\,x^5}+\frac{3b\sqrt{b\,x^4+a}}{5a^2x}-\frac{3ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60*(b*x^4+a)^(1/2)*(-36*b*c*x^4+30*a*f*x^3+20*a*e*x^2+15*a*d*x+12*a*c)/
a^2/x^5-1/30*b/a^2*(10*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)-15/2*a^(1/2)*d*ln((2*a+2*a^(1/2)*(b*x^4+a)^(1/2)
)/x^2)+18*I*c*b^(1/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.45

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx$$
$$=\frac{72\sqrt{a}bcx^5\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)+15\sqrt{a}bdx^5\log\left(-\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-8\left(9bc-5ae\right)\sqrt{a}}{120a^2x}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(72*sqrt(a)*b*c*x^5*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)),
-1) + 15*sqrt(a)*b*d*x^5*log(-(b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^
4) - 8*(9*b*c - 5*a*e)*sqrt(a)*x^5*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)
^(1/4)), -1) + 2*(36*b*c*x^4 - 30*a*f*x^3 - 20*a*e*x^2 - 15*a*d*x - 12*a*c
)*sqrt(b*x^4 + a))/(a^2*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}d\sqrt{\frac{a}{bx^4}+1}}{4ax^2} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^4}+1}}{2a} + \frac{c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{e\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**6/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*d*sqrt(a/(b*x**4) + 1)/(4*a*x**2) - sqrt(b)*f*sqrt(a/(b*x**4) + 1
)/(2*a) + c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)
/a)/(4*sqrt(a)*x**5*gamma(-1/4)) + e*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4)) + b*d*asinh(sqrt(a
)/(sqrt(b)*x**2))/(4*a**(3/2))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{fx^3+ex^2+dx+c}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{f\,x^3+e\,x^2+d\,x+c}{x^6\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x^6*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^6\sqrt{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x^6/(b*x^4+a)^(1/2),x)
```

output
```
( - 6*sqrt(b)*sqrt(a)*sqrt(a + b*x**4)*a*d*x**2 - 16*sqrt(b)*sqrt(a)*sqrt(
a + b*x**4)*a*f*x**4 - 8*sqrt(b)*sqrt(a)*sqrt(a + b*x**4)*b*d*x**6 - 32*sq
rt(b)*sqrt(a)*sqrt(a + b*x**4)*b*f*x**8 + 8*sqrt(a)*sqrt(a + b*x**4)*int(s
qrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a**2*c*x**4 + 32*sqrt(a)*sqrt(a + b*
x**4)*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a*b*c*x**8 + 8*sqrt(a)*sq
rt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a**2*e*x**4 + 32*
sqrt(a)*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a*b*e*x
**8 - sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*d*x**4 - 4*sqrt
(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*d*x**8 + sqrt(a + b*x**4
)*log(sqrt(a + b*x**4) + sqrt(a))*a*b*d*x**4 + 4*sqrt(a + b*x**4)*log(sqrt
(a + b*x**4) + sqrt(a))*b**2*d*x**8 + 24*sqrt(b)*sqrt(a)*int(sqrt(a + b*x*
*4)/(a*x**6 + b*x**10),x)*a**2*c*x**6 + 32*sqrt(b)*sqrt(a)*int(sqrt(a + b*
x**4)/(a*x**6 + b*x**10),x)*a*b*c*x**10 + 24*sqrt(b)*sqrt(a)*int(sqrt(a +
b*x**4)/(a*x**4 + b*x**8),x)*a**2*e*x**6 + 32*sqrt(b)*sqrt(a)*int(sqrt(a +
 b*x**4)/(a*x**4 + b*x**8),x)*a*b*e*x**10 - 2*sqrt(a)*a**2*d - 4*sqrt(a)*a
**2*f*x**2 - 10*sqrt(a)*a*b*d*x**4 - 32*sqrt(a)*a*b*f*x**6 - 8*sqrt(a)*b**
2*d*x**8 - 32*sqrt(a)*b**2*f*x**10 - 3*sqrt(b)*log(sqrt(a + b*x**4) - sqrt
(a))*a*b*d*x**6 - 4*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*d*x**10 +
 3*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(a))*a*b*d*x**6 + 4*sqrt(b)*log(sqrt
(a + b*x**4) + sqrt(a))*b**2*d*x**10)/(8*sqrt(a)*a*x**4*(sqrt(a + b*x**...
```

3.83 $\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 383

$$\begin{aligned}\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= \frac{x(ae-bcx^2)}{2b^2\sqrt{a+bx^4}}+\frac{a(d+fx^2)}{2b^2\sqrt{a+bx^4}}\\
&+\frac{d\sqrt{a+bx^4}}{2b^2}+\frac{ex\sqrt{a+bx^4}}{3b^2}+\frac{fx^2\sqrt{a+bx^4}}{4b^2}+\frac{3cx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\
&-\frac{3af\,\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}-\frac{3\sqrt[4]{a}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}}\\
&+\frac{\sqrt[4]{a}\left(9\sqrt{b}c-5\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*x*(-b*c*x^2+a*e)/b^2/(b*x^4+a)^(1/2)+1/2*a*(f*x^2+d)/b^2/(b*x^4+a)^(1/
2)+1/2*d*(b*x^4+a)^(1/2)/b^2+1/3*e*x*(b*x^4+a)^(1/2)/b^2+1/4*f*x^2*(b*x^4+
a)^(1/2)/b^2+3/2*c*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)-3/4*a*f
*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(5/2)-3/2*a^(1/4)*c*(a^(1/2)+b^(1/
2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b
^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)+1/12*a^(1/4)*(9*b^
(1/2)*c-5*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2
)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(9/4
)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.57

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{12a\sqrt{b}d+10a\sqrt{b}ex+9a\sqrt{b}fx^2+12b^{3/2}cx^3+6b^{3/2}dx^4+4b^{3/2}ex^5+3b^{3/}}{}$$

input
```
Integrate[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(12*a*Sqrt[b]*d + 10*a*Sqrt[b]*e*x + 9*a*Sqrt[b]*f*x^2 + 12*b^(3/2)*c*x^3
+ 6*b^(3/2)*d*x^4 + 4*b^(3/2)*e*x^5 + 3*b^(3/2)*f*x^6 - 9*a^(3/2)*f*Sqrt[1
 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 10*a*Sqrt[b]*e*x*Sqrt[1 + (
b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 12*b^(3/2)*c*x^
3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(12*
b^(5/2)*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2367, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx+ex^2+fx^3\right)}{\left(a+bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2367$$

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}}-\frac{\int \frac{-2ab^2fx^5-2ab^2ex^4-4ab^2dx^3-3ab^2cx^2+2a^2bfx+a^2be}{\sqrt{bx^4+a}}dx}{2ab^3}$$

$$\Big\downarrow \; 2424$$

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}}-\frac{\int \left(\frac{-2ab^2ex^4-3ab^2cx^2+a^2be}{\sqrt{bx^4+a}}+\frac{x\left(-2ab^2fx^4-4ab^2dx^2+2a^2bf\right)}{\sqrt{bx^4+a}}\right)dx}{2ab^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2b^2\sqrt{a+bx^4}}-$$

$$\frac{-\frac{a^{5/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}c-5\sqrt{a}e\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt{a+bx^4}}+\frac{3a^{5/4}b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arcta\right.}{\sqrt{a+bx^4}}}{2a}$$

input
```
Int[(x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*b^2*Sqrt[a + b*x^4]) - (-2*a*b*d*
Sqrt[a + b*x^4] - (2*a*b*e*x*Sqrt[a + b*x^4])/3 - (a*b*f*x^2*Sqrt[a + b*x^
4])/2 - (3*a*b^(3/2)*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (3*a^2
*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/2 + (3*a^(5/4)*b^(5/4)*
c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] - (a^(5/4)*b^(3
/4)*(9*Sqrt[b]*c - 5*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
6*Sqrt[a + b*x^4]))/(2*a*b^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.01 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.79

method	result
elliptic	$-\frac{2b\left(\frac{c\,x^{3}}{4b^{2}}-\frac{af\,x^{2}}{4b^{3}}-\frac{aex}{4b^{3}}-\frac{da}{4b^{3}}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{f\,x^{2}\sqrt{b\,x^{4}+a}}{4b^{2}}+\frac{ex\sqrt{b\,x^{4}+a}}{3b^{2}}+\frac{d\sqrt{b\,x^{4}+a}}{2b^{2}}-\frac{5ae\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.}{6b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
default	$c\left(-\frac{x^{3}}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+\frac{d\left(b\,x^{4}+2a\right)}{2\sqrt{b\,x^{4}+a}\,b^{2}}+e$
risch	$\frac{\left(3f\,x^{2}+4ex+6d\right)\sqrt{b\,x^{4}+a}}{12b^{2}}+\frac{aex}{2b^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{5ae\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{af\,x^{2}}{2b^{2}\sqrt{b\,x^{4}+a}}-\frac{c\,x}{2b\sqrt{(x}}$

input
```
int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2*b*(1/4*c/b^2*x^3-1/4/b^3*a*f*x^2-1/4/b^3*a*e*x-1/4*d/b^3*a)/((x^4+a/b)*
b)^(1/2)+1/4*f*x^2*(b*x^4+a)^(1/2)/b^2+1/3*e*x*(b*x^4+a)^(1/2)/b^2+1/2*d*(
b*x^4+a)^(1/2)/b^2-5/6/b^2*a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)-3/4*a*f/b^(5/2)*ln(2*b^(1/2)*x^2+2*(b*x^4+a)
^(1/2))+3/2*I/b^(3/2)*c*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.63

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{36\,(b^2cx^5+abcx)\sqrt{b}(-\frac{a}{b})^{\frac{3}{4}}\,E(\arcsin\left(\frac{(-\frac{a}{b})^{\frac{1}{4}}}{x}\right)\,|-1)-4\,((9\,b^2c+5\,b^2e)x^5}{}$$

input

```
integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output

```
1/24*(36*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/
b)^(1/4)/x), -1) - 4*((9*b^2*c + 5*b^2*e)*x^5 + (9*a*b*c + 5*a*b*e)*x)*sqr
t(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 9*(a*b*f*x^5 +
a^2*f*x)*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(3*
b^2*f*x^7 + 4*b^2*e*x^6 + 6*b^2*d*x^5 + 12*b^2*c*x^4 + 9*a*b*f*x^3 + 10*a*
b*e*x^2 + 12*a*b*d*x + 18*a*b*c)*sqrt(b*x^4 + a))/(b^4*x^5 + a*b^3*x)
```

Sympy [A] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.53

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}}+\frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + f\left(\frac{3\sqrt{a}x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^6}{4\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{ex^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**6*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
d*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b
, 0)), (x**8/(8*a**(3/2)), True)) + f*(3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x
**4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b**(5/2)) + x**6/(4*sqrt(a)*b
*sqrt(1 + b*x**4/a))) + c*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**
4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + e*x**9*gamma(9/4)*hyper((3
/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^6\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^6*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{24\sqrt{b\,x^4+a}\,abd+40\sqrt{b\,x^4+a}\,abex+18\sqrt{b\,x^4+a}\,abf\,x^2+24\sqrt{b\,x^4+a}}{}$$

input
```
int(x^6*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

output
```
(24*sqrt(a + b*x**4)*a*b*d + 40*sqrt(a + b*x**4)*a*b*e*x + 18*sqrt(a + b*x
**4)*a*b*f*x**2 + 24*sqrt(a + b*x**4)*b**2*c*x**3 + 12*sqrt(a + b*x**4)*b*
*2*d*x**4 + 8*sqrt(a + b*x**4)*b**2*e*x**5 + 6*sqrt(a + b*x**4)*b**2*f*x**
6 + 9*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*f + 9*sqrt(b)*log(
sqrt(a + b*x**4) - sqrt(b)*x**2)*a*b*f*x**4 - 9*sqrt(b)*log(sqrt(a + b*x**
4) + sqrt(b)*x**2)*a**2*f - 9*sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)
*a*b*f*x**4 - 40*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a
**3*b*e - 40*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*
b**2*e*x**4 - 72*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**
8),x)*a**2*b**2*c - 72*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b*
*2*x**8),x)*a*b**3*c*x**4)/(24*b**3*(a + b*x**4))
```

3.84 $\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 363

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{ae-bcx^2}{2b^2\sqrt{a+bx^4}}+\frac{x(af-bdx^2)}{2b^2\sqrt{a+bx^4}} + \frac{e\sqrt{a+bx^4}}{2b^2}+\frac{fx\sqrt{a+bx^4}}{3b^2}+\frac{3dx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} - \frac{3\sqrt[4]{a}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(9\sqrt{b}d-5\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}$$

output
```
1/2*(-b*c*x^2+a*e)/b^2/(b*x^4+a)^(1/2)+1/2*x*(-b*d*x^2+a*f)/b^2/(b*x^4+a)^
(1/2)+1/2*e*(b*x^4+a)^(1/2)/b^2+1/3*f*x*(b*x^4+a)^(1/2)/b^2+3/2*d*x*(b*x^4
+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)+1/2*c*arctanh(b^(1/2)*x^2/(b*x^4+a
)^(1/2))/b^(3/2)-3/2*a^(1/4)*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b
^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2
))/b^(7/4)/(b*x^4+a)^(1/2)+1/12*a^(1/4)*(9*b^(1/2)*d-5*a^(1/2)*f)*(a^(1/2)
+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.48

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6ae+5afx-3bcx^2+6bdx^3+3bex^4+2bfx^5+3\sqrt{a}\sqrt{b}c\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinl}}{}$$

input
```
Integrate[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(6*a*e + 5*a*f*x - 3*b*c*x^2 + 6*b*d*x^3 + 3*b*e*x^4 + 2*b*f*x^5 + 3*Sqrt[
a]*Sqrt[b]*c*Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] - 5*a*f*x*
Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 6*b*d
*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(
6*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2367, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (c + dx + ex^2 + fx^3)}{(a + bx^4)^{3/2}} \, dx$$

$\Big\downarrow$ 2367

$$\frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \frac{-2abfx^4 - 4abex^3 - 3abdx^2 - 2abcx + a^2 f}{\sqrt{bx^4 + a}} dx}{2ab^2}$$

$\Big\downarrow$ 2424

$$\frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} - \frac{\int \left(\frac{x(-4abex^2 - 2abc)}{\sqrt{bx^4 + a}} + \frac{-2abfx^4 - 3abdx^2 + a^2 f}{\sqrt{bx^4 + a}}\right) dx}{2ab^2}$$

$\Big\downarrow$ 2009

$$\frac{x(af - bcx - bdx^2 - bex^3)}{2b^2\sqrt{a + bx^4}} - \frac{-\frac{a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(9\sqrt{b}d-5\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{b}\sqrt{a+bx^4}} + \frac{3a^{5/4}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{a+bx^4}}}{2ab^2}$$

input
```
Int[(x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*b^2*Sqrt[a + b*x^4]) - (-2*a*e*Sq
rt[a + b*x^4] - (2*a*f*x*Sqrt[a + b*x^4])/3 - (3*a*Sqrt[b]*d*x*Sqrt[a + b*
x^4])/(Sqrt[a] + Sqrt[b]*x^2) - a*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
 b*x^4]] + (3*a^(5/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/S
qrt[a + b*x^4] - (a^(5/4)*(9*Sqrt[b]*d - 5*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*b^(1/4)*Sqrt[a + b*x^4]))/(2*a*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.23 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.78

method	result
elliptic	$-\frac{2b\left(\frac{d x^3}{4b^2}+\frac{c x^2}{4b^2}-\frac{a f x}{4b^3}-\frac{a e}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f x\sqrt{b x^4+a}}{3b^2}+\frac{e\sqrt{b x^4+a}}{2b^2}-\frac{5 f a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{c\ln\left(2\sqrt{b}\right.}{}$
default	$c\left(-\frac{x^2}{2b\sqrt{b x^4+a}}+\frac{\ln\left(\sqrt{b}\,x^2+\sqrt{b x^4+a}\right)}{2b^{\frac{3}{2}}}\right)+d\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right.$
risch	$\frac{(2fx+3e)\sqrt{b x^4+a}}{6b^2}+\frac{f a x}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{5 f a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}-\frac{c x^2}{2b\sqrt{b x^4+a}}+\frac{c\ln\left(\sqrt{b}\,x^2+\sqrt{b}\right.}{2b^{\frac{3}{2}}}$

input
```
int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*d/b^2*x^3+1/4/b^2*c*x^2-1/4/b^3*a*f*x-1/4*a*e/b^3)/((x^4+a/b)*b)
^(1/2)+1/3*f*x*(b*x^4+a)^(1/2)/b^2+1/2*e*(b*x^4+a)^(1/2)/b^2-5/6/b^2*f*a/(
I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1
/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/
2/b^(3/2)*c*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))+3/2*I*d/b^(3/2)*a^(1/2)/(I
/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-El
lipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.58

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{18\,(bdx^5+adx)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-2\,((9\,bd+5\,bf)x^5+$$

input
```
integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(18*(b*d*x^5 + a*d*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(
1/4)/x), -1) - 2*((9*b*d + 5*b*f)*x^5 + (9*a*d + 5*a*f)*x)*sqrt(b)*(-a/b)^
(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + 3*(b*c*x^5 + a*c*x)*sqrt(b)
*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*b*f*x^6 + 3*b*e*
x^5 + 6*b*d*x^4 - 3*b*c*x^3 + 5*a*f*x^2 + 6*a*e*x + 9*a*d)*sqrt(b*x^4 + a)
)/(b^3*x^5 + a*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.47

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = c\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + e\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{fx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**5*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b
*x**4/a))) + e*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b
*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + d*x**7*gamma(7/4)*hyper((
3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4)) + f
*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^5}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*c*(2*x^2/(sqrt(b*x^4 + a)*b) + log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(
sqrt(b) + sqrt(b*x^4 + a)/x^2))/b^(3/2)) + integrate((f*x^8 + e*x^7 + d*x^
6)/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\int \frac{(fx^3+ex^2+dx+c)x^5}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^5/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^5\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^5*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^5(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx=\frac{12\sqrt{b\,x^4+a}\,ae+20\sqrt{b\,x^4+a}\,afx-6\sqrt{b\,x^4+a}\,bc\,x^2+12\sqrt{b\,x^4+a}\,bd\,x^{3}}{}$$

input
```
int(x^5*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

output
```
(12*sqrt(a + b*x**4)*a*e + 20*sqrt(a + b*x**4)*a*f*x - 6*sqrt(a + b*x**4)*
b*c*x**2 + 12*sqrt(a + b*x**4)*b*d*x**3 + 6*sqrt(a + b*x**4)*b*e*x**4 + 4*
sqrt(a + b*x**4)*b*f*x**5 - 3*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)
*a*c - 3*sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*b*c*x**4 + 3*sqrt(b)
*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*c + 3*sqrt(b)*log(sqrt(a + b*x**4)
 + sqrt(b)*x**2)*b*c*x**4 - 20*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b
**2*x**8),x)*a**3*f - 20*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x*
*8),x)*a**2*b*f*x**4 - 36*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 +
 b**2*x**8),x)*a**2*b*d - 36*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**
4 + b**2*x**8),x)*a*b**2*d*x**4)/(12*b**2*(a + b*x**4))
```

3.85 $\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 338

$$\begin{aligned}\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= \frac{af-bdx^2}{2b^2\sqrt{a+bx^4}}-\frac{x(c+ex^2)}{2b\sqrt{a+bx^4}} \\ &+ \frac{f\sqrt{a+bx^4}}{2b^2}+\frac{3ex\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} \\ &- \frac{3\sqrt[4]{a}e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} \\ &+ \frac{\left(\sqrt{b}c+3\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*(-b*d*x^2+a*f)/b^2/(b*x^4+a)^(1/2)-1/2*x*(e*x^2+c)/b/(b*x^4+a)^(1/2)+1
/2*f*(b*x^4+a)^(1/2)/b^2+3/2*e*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*
x^2)+1/2*d*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)-3/2*a^(1/4)*e*(a^(
1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(
2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)+1/4*(b^(
1/2)*c+3*a^(1/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)
^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)
/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.49

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{2af-bcx-bdx^2+2bex^3+bfx^4+\sqrt{a}\sqrt{b}d\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+bc}{}$$

input
```
Integrate[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(2*a*f - b*c*x - b*d*x^2 + 2*b*e*x^3 + b*f*x^4 + Sqrt[a]*Sqrt[b]*d*Sqrt[1
+ (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + b*c*x*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*b*e*x^3*Sqrt[1 + (b*x^4)/
a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2367, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2367}$$

$$-\frac{\int -\frac{4abfx^3+3abex^2+2abdx+abc}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4abfx^3+3abex^2+2abdx+abc}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 2424}$$

$$\frac{\int \left(\frac{3abex^2+abc}{\sqrt{bx^4+a}}+\frac{x(4abfx^2+2abd)}{\sqrt{bx^4+a}}\right)dx}{2ab^2}-\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^{3/4}\sqrt[4]{b}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}}(3\sqrt{a}e+\sqrt{b}c)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{a+bx^4}}-\frac{3a^{5/4}\sqrt[4]{b}e(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{a+bx^4}}}{2ab^2}$$

$$\frac{x(c+dx+ex^2+fx^3)}{2b\sqrt{a+bx^4}}$$

input
```
Int[(x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(x*(c + d*x + e*x^2 + f*x^3))/(b*Sqrt[a + b*x^4]) + (2*a*f*Sqrt[a + b
*x^4] + (3*a*Sqrt[b]*e*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + a*Sqrt
[b]*d*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] - (3*a^(5/4)*b^(1/4)*e*(Sqrt[
a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*
ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(3/4)*b^(1/4)*(Sqr
t[b]*c + 3*Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*Sqrt[a +
 b*x^4]))/(2*a*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.76 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.78

method	result
elliptic	$-\frac{2b\left(\frac{e\,x^3}{4b^2}+\frac{d\,x^2}{4b^2}+\frac{cx}{4b^2}-\frac{af}{4b^3}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{f\sqrt{b\,x^4+a}}{2b^2}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{d\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}+$
default	$c\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}\right)+$
risch	$\frac{f\sqrt{b\,x^4+a}}{2b^2}+\frac{cb\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+bd\left(-\frac{x^2}{2b\sqrt{b\,x^4+a}}+\frac{\ln\left(\sqrt{b}\,x^2+\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}\right)+be\left(-\right.}{b}$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*e/b^2*x^3+1/4*d/b^2*x^2+1/4/b^2*c*x-1/4/b^3*a*f)/((x^4+a/b)*b)^(
1/2)+1/2*f*(b*x^4+a)^(1/2)/b^2+1/2/b*c/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/2*d/b^(3/2)*ln(2*b^(1/2)*x^2+2*(b*
x^4+a)^(1/2))+3/2*I*e/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/
2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(Ell
ipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2
),I))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.66

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6\,(abex^5+a^2ex)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+2\,((b^2c-3\,abe)x^5+\cdots}{\cdots}$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(6*(a*b*e*x^5 + a^2*e*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)
^(1/4)/x), -1) + 2*((b^2*c - 3*a*b*e)*x^5 + (a*b*c - 3*a^2*e)*x)*sqrt(b)*(
-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*d*x^5 + a^2*d*x)
*sqrt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^5
+ 2*a*b*e*x^4 - a*b*d*x^3 - a*b*c*x^2 + 2*a^2*f*x + 3*a^2*e)*sqrt(b*x^4 +
a))/(a*b^3*x^5 + a^2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.51

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + f\left(\begin{cases}\frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0\\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{ex^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**4*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
d*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b
*x**4/a))) + f*Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b
*x**4)), Ne(b, 0)), (x**8/(8*a**(3/2)), True)) + c*x**5*gamma(5/4)*hyper((
5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + e*x
**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^4\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^4*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{4\sqrt{b\,x^4+a}\,af-4\sqrt{b\,x^4+a}\,bcx-2\sqrt{b\,x^4+a}\,bd\,x^2+4\sqrt{b\,x^4+a}\,be\,x^3+}{}$$

input
```
int(x^4*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

output
```
(4*sqrt(a + b*x**4)*a*f - 4*sqrt(a + b*x**4)*b*c*x - 2*sqrt(a + b*x**4)*b*
d*x**2 + 4*sqrt(a + b*x**4)*b*e*x**3 + 2*sqrt(a + b*x**4)*b*f*x**4 - sqrt(
b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*d - sqrt(b)*log(sqrt(a + b*x**4)
 - sqrt(b)*x**2)*b*d*x**4 + sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a
*d + sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*b*d*x**4 + 4*int(sqrt(a
+ b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b*c + 4*int(sqrt(a + b*x
**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**2*c*x**4 - 12*int((sqrt(a + b
*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b*e - 12*int((sqrt(a
+ b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**2*e*x**4)/(4*b**2*
(a + b*x**4))
```

3.86 $\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 331

$$\begin{aligned}\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx &= -\frac{c}{2b\sqrt{a+bx^4}}-\frac{ex^2}{2b\sqrt{a+bx^4}} \\ &-\frac{x(d+fx^2)}{2b\sqrt{a+bx^4}}+\frac{3fx\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}} \\ &-\frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} \\ &+\frac{\left(\sqrt{b}d+3\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*c/b/(b*x^4+a)^(1/2)-1/2*e*x^2/b/(b*x^4+a)^(1/2)-1/2*x*(f*x^2+d)/b/(b*
x^4+a)^(1/2)+3/2*f*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)+1/2*e*a
rctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)-3/2*a^(1/4)*f*(a^(1/2)+b^(1/2)
*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(
1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)+1/4*(b^(1/2)*d+3*a^(
1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*In
verseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(7/4)/(b*
x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.55

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b}c-\sqrt{b}dx-\sqrt{b}ex^2+2\sqrt{b}fx^3+\sqrt{a}e\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+\sqrt{b}d\ldots}{\ldots}$$

input
```
Integrate[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(-(Sqrt[b]*c) - Sqrt[b]*d*x - Sqrt[b]*e*x^2 + 2*Sqrt[b]*f*x^3 + Sqrt[a]*e*
Sqrt[1 + (b*x^4)/a]*ArcSinh[(Sqrt[b]*x^2)/Sqrt[a]] + Sqrt[b]*d*x*Sqrt[1 +
(b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] - 2*Sqrt[b]*f*x^
3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(2*b
^(3/2)*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2363, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2363$$

$$\frac{\int \frac{3fx^2+2ex+d}{\sqrt{bx^4+a}}dx}{2b}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 2424$$

$$\frac{\int \left(\frac{2ex}{\sqrt{bx^4+a}}+\frac{3fx^2+d}{\sqrt{bx^4+a}}\right)dx}{2b}-\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(3\sqrt{a}f+\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a+bx^4}}-\frac{3\sqrt[4]{a}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}}}{2b}$$

$$\frac{c+dx+ex^2+fx^3}{2b\sqrt{a+bx^4}}$$

input
```
Int[(x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(c + d*x + e*x^2 + f*x^3)/(b*Sqrt[a + b*x^4]) + ((3*f*x*Sqrt[a + b*x^
4])/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x^2)) + (e*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a +
b*x^4]])/Sqrt[b] - (3*a^(1/4)*f*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(3/4)*Sqrt[a + b*x^4]) + ((Sqrt[b]*d + 3*Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(3/4)*Sqrt[a + b*x^4]))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2363
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Pq*((
a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[1/(b*n*(p + 1))   Int[D[Pq, x]
*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, m, n}, x] && PolyQ[Pq, x] && E
qQ[m - n + 1, 0] && LtQ[p, -1]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.75

method	result
elliptic	$-\frac{2 b \left(\frac{f\,x^{3}}{4 b^{2}}+\frac{e\,x^{2}}{4 b^{2}}+\frac{d x}{4 b^{2}}+\frac{c}{4 b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{d \sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{2 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+\frac{e \ln \left(2 \sqrt{b}\,x^{2}+2 \sqrt{b\,x^{4}+a}\right)}{2 b^{\frac{3}{2}}}+3 i f \sqrt{a}\,\sqrt{1-\ldots}$
default	$-\frac{c}{2 b \sqrt{b\,x^{4}+a}}+d \left(-\frac{x}{2 b \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\sqrt{1-\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{2 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+e \left(-\frac{x^{2}}{2 b \sqrt{b\,x^{4}+a}}+\ln \left(\sqrt{b}\,x^{2}\ldots\right.\right.$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*f*x^3/b^2+1/4*e/b^2*x^2+1/4*d/b^2*x+1/4*c/b^2)/((x^4+a/b)*b)^(1/
2)+1/2*d/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)+1/2*e/b^(3/2)*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))+3/2*I*f/b^(3/2
)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))
^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.65

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{6\left(abfx^5+a^2fx\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+2\left(\left(b^2d-3\,abf\right)x^5\right.}{}$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(6*(a*b*f*x^5 + a^2*f*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)
^(1/4)/x), -1) + 2*((b^2*d - 3*a*b*f)*x^5 + (a*b*d - 3*a^2*f)*x)*sqrt(b)*(
-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (a*b*e*x^5 + a^2*e*x)
*sqrt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(2*a*b*f*x^
4 - a*b*e*x^3 - a*b*d*x^2 - a*b*c*x + 3*a^2*f)*sqrt(b*x^4 + a))/(a*b^3*x^5
 + a^2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.47

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = c\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + e\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{fx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + e*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt
(1 + b*x**4/a))) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4)) + f*x**7*gamma(7/4)*hyper((3/2, 7/4
), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*c/(sqrt(b*x^4 + a)*b) + integrate((f*x^6 + e*x^5 + d*x^4)/(b*x^4 + a)
^(3/2), x)
```

Giac [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^3}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^3/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^3\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^3*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^4+a}\,bc-4\sqrt{b\,x^4+a}\,bdx-2\sqrt{b\,x^4+a}\,be\,x^2+4\sqrt{b\,x^4+a}\,bf\,x^3\,-}{}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**4)*b*c - 4*sqrt(a + b*x**4)*b*d*x - 2*sqrt(a + b*x**4)
*b*e*x**2 + 4*sqrt(a + b*x**4)*b*f*x**3 - sqrt(b)*log(sqrt(a + b*x**4) - s
qrt(b)*x**2)*a*e - sqrt(b)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*b*e*x**4 +
 sqrt(b)*log(sqrt(a + b*x**4) + sqrt(b)*x**2)*a*e + sqrt(b)*log(sqrt(a + b
*x**4) + sqrt(b)*x**2)*b*e*x**4 + 4*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**
4 + b**2*x**8),x)*a**2*b*d + 4*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b
**2*x**8),x)*a*b**2*d*x**4 - 12*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*
x**4 + b**2*x**8),x)*a**2*b*f - 12*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a
*b*x**4 + b**2*x**8),x)*a*b**2*f*x**4)/(4*b**2*(a + b*x**4))
```

3.87 $\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 340

$$\begin{aligned}\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = &-\frac{d}{2b\sqrt{a+bx^4}}-\frac{fx^2}{2b\sqrt{a+bx^4}}\\
&-\frac{x(ae-bcx^2)}{2ab\sqrt{a+bx^4}}-\frac{cx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}\\
&+\frac{c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}\\
&-\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/2*d/b/(b*x^4+a)^(1/2)-1/2*f*x^2/b/(b*x^4+a)^(1/2)-1/2*x*(-b*c*x^2+a*e)/
a/b/(b*x^4+a)^(1/2)-1/2*c*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1/2)*x^2
)+1/2*f*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2)+1/2*c*(a^(1/2)+b^(1/2
)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^
(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)-1/4*(c-a^(1
/2)*e/b^(1/2))*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(
1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/b^(3
/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.49

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-3a\sqrt{b}(d+x(e+fx))+3a^{3/2}f\sqrt{1+\frac{bx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)+3a\sqrt{b}ex\sqrt{1+$$

input
```
Integrate[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(-3*a*Sqrt[b]*(d + x*(e + f*x)) + 3*a^(3/2)*f*Sqrt[1 + (b*x^4)/a]*ArcSinh[
(Sqrt[b]*x^2)/Sqrt[a]] + 3*a*Sqrt[b]*e*x*Sqrt[1 + (b*x^4)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b^(3/2)*c*x^3*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b^(3/2)*Sqrt[a + b*x^4]
)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2367, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$$

$\Big\downarrow$ 2367

$$-\frac{\int -\frac{-2b^2dx^3-b^2cx^2+2abfx+abe}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{-2b^2dx^3-b^2cx^2+2abfx+abe}{\sqrt{bx^4+a}}dx}{2ab^2}-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2424

$$\frac{\int \left(\frac{abe-b^2cx^2}{\sqrt{bx^4+a}}+\frac{x(2abf-2b^2dx^2)}{\sqrt{bx^4+a}}\right)dx}{2ab^2}-\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\sqrt[4]{a}b^{3/4}(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}}(\sqrt{b}c-\sqrt{a}e)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}b^{5/4}c(\sqrt{a}+\sqrt{b}x^2)\sqrt{\frac{a+bx^4}{(\sqrt{a}+\sqrt{b}x^2)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt{a+bx^4}}}{2ab^2}$$

$$\frac{x(ae+afx-bcx^2-bdx^3)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(a*b*Sqrt[a + b*x^4]) + (-(b*d*
Sqrt[a + b*x^4]) - (b^(3/2)*c*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) +
 a*Sqrt[b]*f*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]] + (a^(1/4)*b^(5/4)*c*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] - (a^(1/4)*b^(3/4)
*(Sqrt[b]*c - Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*Sqrt[
a + b*x^4]))/(2*a*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.74

method	result
elliptic	$-\frac{2b\left(-\frac{c\,x^3}{4ab}+\frac{f\,x^2}{4b^2}+\frac{ex}{4b^2}+\frac{d}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+\frac{f\ln\left(2\sqrt{b}\,x^2+2\sqrt{b\,x^4+a}\right)}{2b^{\frac{3}{2}}}-\frac{ic\sqrt{1-\frac{i\sqrt{b}}{\sqrt{}}}}{}$
default	$c\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right)-\frac{d}{2b\sqrt{b\,x^4+a}}+e\left(-\frac{}{2b}\right.$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4*c/a/b*x^3+1/4*f*x^2/b^2+1/4*e*x/b^2+1/4*d/b^2)/((x^4+a/b)*b)^(1
/2)+1/2*e/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)+1/2*f/b^(3/2)*ln(2*b^(1/2)*x^2+2*(b*x^4+a)^(1/2))-1/2*I*c/a^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx =$$

$$-\frac{2\left(b^2cx^5+abcx\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-2\left(\left(b^2c+b^2e\right)x^5+\left(abc+abe\right)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\mathrm{arcsi}}{4}$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*(b^2*c*x^5 + a*b*c*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b
)^(1/4)/x), -1) - 2*((b^2*c + b^2*e)*x^5 + (a*b*c + a*b*e)*x)*sqrt(b)*(-a/
b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - (a*b*f*x^5 + a^2*f*x)*sq
rt(b)*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a) + 2*(a*b*f*x^3 + a
*b*e*x^2 + a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a*b^3*x^5 + a^2*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.46

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = d\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}\right) + \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**2*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
d*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + f*(asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt
(1 + b*x**4/a))) + c*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + e*x**5*gamma(5/4)*hyper((5/4, 3/2
), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^2\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^2*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a**2*f
*x**2 - 4*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)*a*
b*f*x**6 - 2*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(b)*x**2)
*b**2*f*x**10 + 2*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(b)*
x**2)*a**2*f*x**2 + 4*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt
(b)*x**2)*a*b*f*x**6 + 2*sqrt(b)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + s
qrt(b)*x**2)*b**2*f*x**10 - 2*sqrt(b)*sqrt(a + b*x**4)*a**2*d - 4*sqrt(b)*
sqrt(a + b*x**4)*a**2*e*x - 6*sqrt(b)*sqrt(a + b*x**4)*a**2*f*x**2 - 6*sqr
t(b)*sqrt(a + b*x**4)*a*b*d*x**4 - 12*sqrt(b)*sqrt(a + b*x**4)*a*b*e*x**5
- 14*sqrt(b)*sqrt(a + b*x**4)*a*b*f*x**6 - 4*sqrt(b)*sqrt(a + b*x**4)*b**2
*d*x**8 - 8*sqrt(b)*sqrt(a + b*x**4)*b**2*e*x**9 - 8*sqrt(b)*sqrt(a + b*x*
*4)*b**2*f*x**10 + 8*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x
**4 + b**2*x**8),x)*a**3*b*e*x**2 + 16*sqrt(a + b*x**4)*int(sqrt(a + b*x**
4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2*e*x**6 + 8*sqrt(a + b*x**4
)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**3*e*x**10 +
 8*sqrt(a + b*x**4)*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*
x**8),x)*a**2*b**2*c*x**2 + 16*sqrt(a + b*x**4)*int((sqrt(a + b*x**4)*x**2
)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**3*c*x**6 + 8*sqrt(a + b*x**4)*in
t((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*b**4*c*x**10
+ 4*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**...
```

3.88 $\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 309

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = -\frac{e}{2b\sqrt{a+bx^4}}+\frac{cx^2}{2a\sqrt{a+bx^4}}-\frac{x(af-bdx^2)}{2ab\sqrt{a+bx^4}} -\frac{dx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} -\frac{\left(d-\frac{\sqrt{a}f}{\sqrt{b}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
-1/2*e/b/(b*x^4+a)^(1/2)+1/2*c*x^2/a/(b*x^4+a)^(1/2)-1/2*x*(-b*d*x^2+a*f)/
a/b/(b*x^4+a)^(1/2)-1/2*d*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1/2)*x^2
)+1/2*d*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*El
lipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x
^4+a)^(1/2)-1/4*(d-a^(1/2)*f/b^(1/2))*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/
2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.38

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{-3ae-3afx+3bcx^2+3afx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right.}{6ab\sqrt{a+bx^4}}$$

input
```
Integrate[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
(-3*a*e - 3*a*f*x + 3*b*c*x^2 + 3*a*f*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*d*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2367, 25, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 2367$$

$$-\frac{\int -\frac{-2bex^3-bdx^2+af}{\sqrt{bx^4+a}}dx}{2ab}-\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-2bex^3-bdx^2+af}{\sqrt{bx^4+a}}dx}{2ab}-\frac{x(af-bcx-bdx^2-bex^3)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 2424$$

$$\frac{\int \left(\frac{af-bdx^2}{\sqrt{bx^4+a}}-\frac{2bex^3}{\sqrt{bx^4+a}}\right)dx}{2ab}-\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}$$

$\big\downarrow$ 2009

$$\frac{-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{b}d-\sqrt{a}f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{\sqrt{a+bx^4}}}{2ab}$$

$$\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(a*b*Sqrt[a + b*x^4]) + (-(e*Sq
rt[a + b*x^4]) - (Sqrt[b]*d*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) + (
a^(1/4)*b^(1/4)*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt
[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4]
 - (a^(1/4)*(Sqrt[b]*d - Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(1/4)*Sqrt[a + b*x^4]))/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.73

method	result
elliptic	$-\frac{2b\left(-\frac{d x^{3}}{4ba}-\frac{c x^{2}}{4ab}+\frac{f x}{4b^{2}}+\frac{e}{4b^{2}}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{i d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x}}$
default	$\frac{c\,x^{2}}{2a\sqrt{b\,x^{4}+a}}+d\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}\right)+f\left(-\frac{}{2}\right.$

input
```
int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4*d/b/a*x^3-1/4*c/a/b*x^2+1/4*f*x/b^2+1/4*e/b^2)/((x^4+a/b)*b)^(1
/2)+1/2*f/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/2*I*d/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \frac{(b^2dx^4+abd)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right) - ((b^2d+abf)x^4+abd}{2(}$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((b^2*d*x^4 + a*b*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(
1/4)), -1) - ((b^2*d + a*b*f)*x^4 + a*b*d + a^2*f)*sqrt(a)*(-b/a)^(3/4)*el
liptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*d*x^3 + b^2*c*x^2 - a*b*f*x -
a*b*e)*sqrt(b*x^4 + a))/(a*b^3*x^4 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.43

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = e\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{fx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x*(f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
e*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + c*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + d*x**3*gamma(3/4)*hyper((3/
4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + f*x**
5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*c*x^2/(sqrt(b*x^4 + a)*a) + integrate((f*x^4 + e*x^3 + d*x^2)/(b*x^4 +
 a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(fx^3+ex^2+dx+c)x}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*x/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x\,(f\,x^3+e\,x^2+d\,x+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2),x)
```

output
```
int((x*(c + d*x + e*x^2 + f*x^3))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(c+dx+ex^2+fx^3)}{(a+bx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x*(f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

output

```
( - sqrt(b)*sqrt(a + b*x**4)*a**3*e - 2*sqrt(b)*sqrt(a + b*x**4)*a**3*f*x
+ 3*sqrt(b)*sqrt(a + b*x**4)*a**2*b*c*x**2 - 3*sqrt(b)*sqrt(a + b*x**4)*a*
*2*b*e*x**4 - 6*sqrt(b)*sqrt(a + b*x**4)*a**2*b*f*x**5 + 7*sqrt(b)*sqrt(a
+ b*x**4)*a*b**2*c*x**6 - 2*sqrt(b)*sqrt(a + b*x**4)*a*b**2*e*x**8 - 4*sqr
t(b)*sqrt(a + b*x**4)*a*b**2*f*x**9 + 4*sqrt(b)*sqrt(a + b*x**4)*b**3*c*x*
*10 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x*
*8),x)*a**4*b*f*x**2 + 8*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a
*b*x**4 + b**2*x**8),x)*a**3*b**2*f*x**6 + 4*sqrt(a + b*x**4)*int(sqrt(a +
 b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b**3*f*x**10 + 4*sqrt(a +
 b*x**4)*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*
*3*b**2*d*x**2 + 8*sqrt(a + b*x**4)*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*
a*b*x**4 + b**2*x**8),x)*a**2*b**3*d*x**6 + 4*sqrt(a + b*x**4)*int((sqrt(a
 + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**4*d*x**10 + 2*sqr
t(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**5*f + 8*sq
rt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**4*b*f*x**
4 + 10*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**
3*b**2*f*x**8 + 4*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x
**8),x)*a**2*b**3*f*x**12 + 2*sqrt(b)*int((sqrt(a + b*x**4)*x**2)/(a**2 +
2*a*b*x**4 + b**2*x**8),x)*a**4*b*d + 8*sqrt(b)*int((sqrt(a + b*x**4)*x**2
)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b**2*d*x**4 + 10*sqrt(b)*int(...
```

3.89 $\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 284

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx=-\frac{f}{2b\sqrt{a+bx^4}}+\frac{x(c+dx+ex^2)}{2a\sqrt{a+bx^4}}-\frac{ex\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{e\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}+\frac{\left(\sqrt{b}c-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
-1/2*f/b/(b*x^4+a)^(1/2)+1/2*x*(e*x^2+d*x+c)/a/(b*x^4+a)^(1/2)-1/2*e*x*(b*
x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)+1/2*e*(a^(1/2)+b^(1/2)*x^2)*(
(b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/
a^(1/4))),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(b^(1/2)*c-a^(1
/2)*e)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Inv
erseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/b^(3/4)/(b*x
^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.41

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \frac{-3af+3bcx+3bdx^2+3bcx\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)+}{6ab\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

output
```
(-3*a*f + 3*b*c*x + 3*b*d*x^2 + 3*b*c*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*b*e*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(6*a*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2393, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2393}$$

$$-\frac{\int -\frac{c-ex^2}{\sqrt{bx^4+a}}dx}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{c-ex^2}{\sqrt{bx^4+a}}dx}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 1512}$$

$$\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{bx^4+a}}dx+\frac{\sqrt{a}e\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\int \frac{1}{\sqrt{bx^4+a}}dx+\frac{e\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{e\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}+\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(c-\frac{\sqrt{a}e}{\sqrt{b}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{e\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+\cdots}}{\sqrt{a}+\sqrt{\cdots}}\right.}{\sqrt{b}}}{2a}-\frac{af-bx(c+dx+ex^2)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*(a*f - b*x*(c + d*x + e*x^2))/(a*b*Sqrt[a + b*x^4]) + ((e*(-((x*Sqrt[
a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4])))/Sqrt[b] + ((c - (Sqrt[a]*e)/Sqr
t[b])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a +
 b*x^4]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.81

method	result
elliptic	$-\frac{2b\left(-\frac{e\,x^3}{4ba}-\frac{d\,x^2}{4ab}-\frac{cx}{4ab}+\frac{f}{4b^2}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-\frac{ie\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x}}$
default	$c\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{d\,x^2}{2\sqrt{b\,x^4+a}\,a}+e\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4*e/b/a*x^3-1/4*d/a/b*x^2-1/4*c/a/b*x+1/4*f/b^2)/((x^4+a/b)*b)^(1
/2)+1/2*c/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/2*I/a^(1/2)*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.45

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx=\frac{(bex^4+ae)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\left((bc+be)x^4+ac+ae\right)\sqrt{a}}{2\left(ab^2x^4+\right.}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((b*e*x^4 + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)
), -1) - ((b*c + b*e)*x^4 + a*c + a*e)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arc
sin(x*(-b/a)^(1/4)), -1) + (b*e*x^3 + b*d*x^2 + b*c*x - a*f)*sqrt(b*x^4 +
a))/(a*b^2*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.46

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = f\left(\begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} \frac{1}{4},\frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}} + \frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix} \frac{3}{4},\frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/(b*x**4+a)**(3/2),x)
```

output
```
f*Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), Tru
e)) + c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*a**(3/2)*gamma(5/4)) + d*x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a)) + e*x**3*g
amma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*
gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{(a+bx^4)^{3/2}}\,dx=\frac{-\sqrt{b}\,\sqrt{b\,x^4+a}\,a^2f+3\sqrt{b}\,\sqrt{b\,x^4+a}\,abd\,x^2-2\sqrt{b}\,\sqrt{b\,x^4+a}\,abf\,x^4+4\sqrt{b}\,\sqrt{b}$$

input
```
int((f*x^3+e*x^2+d*x+c)/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(b)*sqrt(a + b*x**4)*a**2*f + 3*sqrt(b)*sqrt(a + b*x**4)*a*b*d*x**
2 - 2*sqrt(b)*sqrt(a + b*x**4)*a*b*f*x**4 + 4*sqrt(b)*sqrt(a + b*x**4)*b**
2*d*x**6 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b*
*2*x**8),x)*a**2*b**2*c*x**2 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*
*2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**3*c*x**6 + 4*sqrt(a + b*x**4)*int((sq
rt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2*e*x**2 +
 4*sqrt(a + b*x**4)*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*
x**8),x)*a*b**3*e*x**6 + 2*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4
 + b**2*x**8),x)*a**3*b*c + 6*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x
**4 + b**2*x**8),x)*a**2*b**2*c*x**4 + 4*sqrt(b)*int(sqrt(a + b*x**4)/(a**
2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**3*c*x**8 + 2*sqrt(b)*int((sqrt(a + b*x
**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b*e + 6*sqrt(b)*int((sq
rt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2*e*x**4 +
 4*sqrt(b)*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*
a*b**3*e*x**8 + a**2*b*d - 2*a**2*b*f*x**2 + 5*a*b**2*d*x**4 - 2*a*b**2*f*
x**6 + 4*b**3*d*x**8)/(2*a*b*(2*sqrt(a + b*x**4)*a*b*x**2 + 2*sqrt(a + b*x
**4)*b**2*x**6 + sqrt(b)*a**2 + 3*sqrt(b)*a*b*x**4 + 2*sqrt(b)*b**2*x**8))
```

3.90 $\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 315

$$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx = \frac{c+ex^2}{2a\sqrt{a+bx^4}}+\frac{x(d+fx^2)}{2a\sqrt{a+bx^4}}-\frac{fx\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}d-\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
1/2*(e*x^2+c)/a/(b*x^4+a)^(1/2)+1/2*x*(f*x^2+d)/a/(b*x^4+a)^(1/2)-1/2*f*x*
(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)-1/2*c*arctanh((b*x^4+a)^(1
/2)/a^(1/2))/a^(3/2)+1/2*f*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/
2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a
^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/4*(b^(1/2)*d-a^(1/2)*f)*(a^(1/2)+b^(1/2)*
x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^
(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.40

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{3c\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)+x\left(3d+3ex+3d\sqrt{1+\frac{bx^4}{a}}\,\text{Hype}\right.}{6a}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

output
```
(3*c*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] + x*(3*d + 3*e*x + 3*d
*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)] + 2*f*
x^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(
6*a*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2368, 25, 2371, 798, 73, 221, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{\int -\frac{\frac{2b^2cx^4}{a}-bfx^3+bdx+2bc}{x\sqrt{bx^4+a}}dx}{2ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{2b^2cx^4}{a}-bfx^3+bdx+2bc}{x\sqrt{bx^4+a}}dx}{2ab}+\frac{x\left(ad+aex+afx^2-bcx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2371

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx+2bc\int \frac{1}{x\sqrt{bx^4+a}}dx}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 798

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx+\frac{1}{2}bc\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 73

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx+c\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 221

$$\frac{\int \frac{\frac{2b^2cx^3}{a}-bfx^2+bd}{\sqrt{bx^4+a}}dx-\frac{bc\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2424

$$\frac{\int \left(\frac{2b^2cx^3}{a\sqrt{bx^4+a}}+\frac{bd-bfx^2}{\sqrt{bx^4+a}}\right)dx-\frac{bc\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2ab}+\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}$$

$\Big\downarrow$ 2009

$$\frac{x(ad+aex+afx^2-bcx^3)}{2a^2\sqrt{a+bx^4}}+$$
$$\frac{\frac{\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{b}d}{\sqrt{a}}-f\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt{a+bx^4}}+\frac{\sqrt[4]{a}\sqrt[4]{b}f\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{\sqrt{a+bx^4}}}{2ab}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x]
```

output
```
(x*(a*d + a*e*x + a*f*x^2 - b*c*x^3))/(2*a^2*Sqrt[a + b*x^4]) + ((b*c*Sqrt
[a + b*x^4])/a - (Sqrt[b]*f*x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2) - (
b*c*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/Sqrt[a] + (a^(1/4)*b^(1/4)*f*(Sqrt[a
] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*A
rcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/Sqrt[a + b*x^4] + (a^(1/4)*b^(1/4)*((Sqr
t[b]*d)/Sqrt[a] - f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + S
qrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*Sqrt[a +
b*x^4]))/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.80

method	result
elliptic	$-\frac{2b\left(-\frac{f\,x^{3}}{4ba}-\frac{x^{2}e}{4ab}-\frac{dx}{4ab}-\frac{c}{4ab}\right)}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}-\frac{if\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}$
default	$d\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+\frac{e\,x^{2}}{2\sqrt{b\,x^{4}+a}\,a}+f\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}}{}\right.$

input
```
int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-1/4/b/a*f*x^3-1/4/a/b*x^2*e-1/4*d/a/b*x-1/4*c/a/b)/((x^4+a/b)*b)^(1
/2)+1/2*d/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-1/2*I*f/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I
))-1/2*c/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.61

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{2\left(abfx^4+a^2f\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right) - 2\left(\left(abd+abf\right)x^4+a^2\right.}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*(a*b*f*x^4 + a^2*f)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) - 2*((a*b*d + a*b*f)*x^4 + a^2*d + a^2*f)*sqrt(a)*(-b/a)^(3/4
)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (b^2*c*x^4 + a*b*c)*sqrt(a)*log
(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(a*b*f*x^3 + a*b*e*x^
2 + a*b*d*x + a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^4 + a^3*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.75 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx = c\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}+\frac{fx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x/(b*x**4+a)**(3/2),x)
```

output
```
c*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b
*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a)
+ 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9
/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a*
*(9/2) + 4*a**(7/2)*b*x**4)) + d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(
1 + b*x**4/a)) + f*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{\left(bx^4+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{f\,x^3+e\,x^2+d\,x+c}{x\,\left(b\,x^4+a\right)^{3/2}}\,dx$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)),x)
```

output
```
int((c + d*x + e*x^2 + f*x^3)/(x*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x\left(a+bx^4\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x/(b*x^4+a)^(3/2),x)
```

output
```
(2*sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*c*x**2 + 2
*sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*c*x**6 - 2*
sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*a*b*c*x**2 - 2*sq
rt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*b**2*c*x**6 + 2*sqr
t(b)*sqrt(a + b*x**4)*a**2*c + 6*sqrt(b)*sqrt(a + b*x**4)*a**2*e*x**2 + 4*
sqrt(b)*sqrt(a + b*x**4)*a*b*c*x**4 + 8*sqrt(b)*sqrt(a + b*x**4)*a*b*e*x**
6 + 8*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8
),x)*a**3*b*d*x**2 + 8*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b
*x**4 + b**2*x**8),x)*a**2*b**2*d*x**6 + 8*sqrt(a + b*x**4)*int((sqrt(a +
b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b*f*x**2 + 8*sqrt(a
+ b*x**4)*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a
**2*b**2*f*x**6 + sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*a**2*c +
 3*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*c*x**4 + 2*sqrt(b)*
sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*c*x**8 - sqrt(b)*sqrt(a)*log(
sqrt(a + b*x**4) + sqrt(a))*a**2*c - 3*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4
) + sqrt(a))*a*b*c*x**4 - 2*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a)
)*b**2*c*x**8 + 4*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x
**8),x)*a**4*d + 12*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2
*x**8),x)*a**3*b*d*x**4 + 8*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**
4 + b**2*x**8),x)*a**2*b**2*d*x**8 + 4*sqrt(b)*int((sqrt(a + b*x**4)*x*...
```

3.91 $\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \frac{c+ex^2}{2ax\sqrt{a+bx^4}}+\frac{d+fx^2}{2a\sqrt{a+bx^4}}-\frac{3c\sqrt{a+bx^4}}{2a^{3/2}x\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{d\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} - \frac{3\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{\left(3\sqrt{b}c+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
1/2*(e*x^2+c)/a/x/(b*x^4+a)^(1/2)+1/2*(f*x^2+d)/a/(b*x^4+a)^(1/2)-3/2*c*(b
*x^4+a)^(1/2)/a^(3/2)/x/(a^(1/2)+b^(1/2)*x^2)-1/2*d*arctanh((b*x^4+a)^(1/2
)/a^(1/2))/a^(3/2)-3/2*b^(1/4)*c*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)
+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1
/2))/a^(7/4)/(b*x^4+a)^(1/2)+1/4*(3*b^(1/2)*c+a^(1/2)*e)*(a^(1/2)+b^(1/2)*
x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^
(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(7/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.39

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)-2c\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}}{2ax}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
(d*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^4)/a] - 2*c*Sqrt[1 + (b*x^4)
/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)] + x^2*(e + f*x + e*Sqr
t[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(2*a*x*S
qrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2368, 25, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2368$$

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{\int -\frac{\frac{2b^2dx^5}{a}+\frac{b^2cx^4}{a}+bex^2+2bdx+2bc}{x^2\sqrt{bx^4+a}}dx}{2ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{2b^2dx^5}{a}+\frac{b^2cx^4}{a}+bex^2+2bdx+2bc}{x^2\sqrt{bx^4+a}}dx}{2ab}+\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2372}$$

$$\frac{\int \left(\frac{\frac{b^2cx^4}{a}+bex^2+2bc}{x^2\sqrt{bx^4+a}}+\frac{\frac{2b^2dx^4}{a}+2bd}{x\sqrt{bx^4+a}}\right)dx}{2ab}+\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}e+3\sqrt{b}c\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+bx^4}}-\frac{3b^{5/4}c\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{a^{3/4}\sqrt{a+bx^4}}}{2ab}$$

$$\frac{x\left(ae+afx-bcx^2-bdx^3\right)}{2a^2\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
(x*(a*e + a*f*x - b*c*x^2 - b*d*x^3))/(2*a^2*Sqrt[a + b*x^4]) + ((b*d*Sqrt
[a + b*x^4])/a - (2*b*c*Sqrt[a + b*x^4])/(a*x) + (3*b^(3/2)*c*x*Sqrt[a + b
*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*d*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]
)/Sqrt[a] - (3*b^(5/4)*c*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)
*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*c + Sqrt[a]*e)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[a + b*x^4]))/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.85

method	result
elliptic	$-\frac{2b\left(\frac{c x^3}{4a^2}-\frac{x^2 f}{4ab}-\frac{xe}{4ab}-\frac{d}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b x^4+a}}{a^2 x}+\frac{e\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}+\frac{3i\sqrt{b}\,c\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\cdots}{\sqrt{\cdots}}}\cdots}{\cdots}$
default	$e\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right)+c\left(-\frac{b x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b x^4+a}}{a^2 x}+\frac{3i\sqrt{b}\,\sqrt{1-\cdots}}{\cdots}\cdots\right.$
risch	$-\frac{c\sqrt{b x^4+a}}{a^2 x}+\frac{a^2 e\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b x^4+a}}\right)+a^2 d\left(\frac{1}{2a\sqrt{b x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^4+a}}{x^2}\right)}{2a^{\frac{3}{2}}}\right)+b\cdots}{a^2\cdots}$

input
```
int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*c/a^2*x^3-1/4/a/b*x^2*f-1/4/a/b*x*e-1/4*d/b/a)/((x^4+a/b)*b)^(1/
2)-1/a^2*c*(b*x^4+a)^(1/2)/x+1/2/a*e/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/2*I*b^(1/2)/a^(3/2)*c/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2
)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I
/a^(1/2)*b^(1/2))^(1/2),I))-1/2*d/a^(3/2)*arctanh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.68

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx =$$

$$-\frac{6\left(b^2cx^5+abcx\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-2\left(\left(3\,b^2c-abe\right)x^5+\left(3\,abc-a^2e\right)x\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(6*(b^2*c*x^5 + a*b*c*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b
/a)^(1/4)), -1) - 2*((3*b^2*c - a*b*e)*x^5 + (3*a*b*c - a^2*e)*x)*sqrt(a)*
(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*d*x^5 + a*b*d*x
)*sqrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3*b^2*c
*x^4 - a*b*f*x^3 - a*b*e*x^2 - a*b*d*x + 2*a*b*c)*sqrt(b*x^4 + a))/(a^2*b^
2*x^5 + a^3*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.04 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = d\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right)+\frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{ex\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{fx^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**2/(b*x**4+a)**(3/2),x)
```

output
```
d*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b
*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a)
+ 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9
/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a*
*(9/2) + 4*a**(7/2)*b*x**4)) + c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + e*x*gamma(1/4)*hyper((
1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4)) + f*x
**2/(2*a**(3/2)*sqrt(1 + b*x**4/a))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx = \int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.42

$$\int \frac{c+dx+ex^2+fx^3}{x^2(a+bx^4)^{3/2}}\,dx=\frac{d}{2\,a\,\sqrt{b\,x^4+a}}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}+\frac{f\,x^2}{2\,a\,\sqrt{b\,x^4+a}}-\frac{c\,\left(\frac{a}{b\,x^4}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}}+\frac{e\,x\,\left(\frac{b\,x^4}{a}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^2*(a + b*x^4)^(3/2)),x)
```

output
```
d/(2*a*(a + b*x^4)^(1/2)) - (d*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2
)) + (f*x^2)/(2*a*(a + b*x^4)^(1/2)) - (c*(a/(b*x^4) + 1)^(3/2)*hypergeom(
[3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2)) + (e*x*((b*x^4)/a +
 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b*x^4)^(3/2)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x^2/(b*x^4+a)^(3/2),x)
```

output
```
(2*sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*d*x**2 + 2
*sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*d*x**6 - 2*
sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*a*b*d*x**2 - 2*sq
rt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*b**2*d*x**6 + 2*sqr
t(b)*sqrt(a + b*x**4)*a**2*d + 6*sqrt(b)*sqrt(a + b*x**4)*a**2*f*x**2 + 4*
sqrt(b)*sqrt(a + b*x**4)*a*b*d*x**4 + 8*sqrt(b)*sqrt(a + b*x**4)*a*b*f*x**
6 + 8*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8
),x)*a**3*b*e*x**2 + 8*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b
*x**4 + b**2*x**8),x)*a**2*b**2*e*x**6 + 8*sqrt(a + b*x**4)*int(sqrt(a + b
*x**4)/(a**2*x**2 + 2*a*b*x**6 + b**2*x**10),x)*a**3*b*c*x**2 + 8*sqrt(a +
 b*x**4)*int(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 + b**2*x**10),x)*a**
2*b**2*c*x**6 + sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*a**2*d + 3
*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*d*x**4 + 2*sqrt(b)*sq
rt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*d*x**8 - sqrt(b)*sqrt(a)*log(sq
rt(a + b*x**4) + sqrt(a))*a**2*d - 3*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4)
+ sqrt(a))*a*b*d*x**4 - 2*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*
b**2*d*x**8 + 4*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**
8),x)*a**4*e + 12*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x
**8),x)*a**3*b*e*x**4 + 8*sqrt(b)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4
+ b**2*x**8),x)*a**2*b**2*e*x**8 + 4*sqrt(b)*int(sqrt(a + b*x**4)/(a**2...
```

3.92 $\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 342

$$\begin{aligned}\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx &= \frac{ae-bcx^2}{2a^2\sqrt{a+bx^4}}+\frac{d+fx^2}{2ax\sqrt{a+bx^4}} \\ &- \frac{c\sqrt{a+bx^4}}{2a^2x^2}-\frac{3d\sqrt{a+bx^4}}{2a^{3/2}x\left(\sqrt{a}+\sqrt{b}x^2\right)}-\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}} \\ &- \frac{3\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} \\ &+ \frac{\left(3\sqrt{b}d+\sqrt{a}f\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt[4]{b}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2*(-b*c*x^2+a*e)/a^2/(b*x^4+a)^(1/2)+1/2*(f*x^2+d)/a/x/(b*x^4+a)^(1/2)-1
/2*c*(b*x^4+a)^(1/2)/a^2/x^2-3/2*d*(b*x^4+a)^(1/2)/a^(3/2)/x/(a^(1/2)+b^(1
/2)*x^2)-1/2*e*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)-3/2*b^(1/4)*d*(a^(
1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(
2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)+1/4*(3*b
^(1/2)*d+a^(1/2)*f)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)
^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(7/4)
/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.41

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx = \frac{-ac+afx^3-2bcx^4+aex^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{bx^4}{a}\right)-2adx\sqrt{}}{}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

output
```
(-(a*c) + a*f*x^3 - 2*b*c*x^4 + a*e*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ (b*x^4)/a] - 2*a*d*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/
4, -((b*x^4)/a)] + a*f*x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((b*x^4)/a)])/(2*a^2*x^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2368, 25, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx+ex^2+fx^3}{x^3\left(a+bx^4\right)^{3/2}}\,dx$$

$\big\downarrow$ 2368

$$\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}-\frac{\int -\frac{\frac{2b^2ex^6}{a}+\frac{b^2dx^5}{a}+bfx^3+2bex^2+2bdx+2bc}{x^3\sqrt{bx^4+a}}dx}{2ab}$$

$\big\downarrow$ 25

$$\frac{\int \frac{\frac{2b^2ex^6}{a}+\frac{b^2dx^5}{a}+bfx^3+2bex^2+2bdx+2bc}{x^3\sqrt{bx^4+a}}dx}{2ab}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}$$

$\big\downarrow$ 2372

$$\frac{\int \left(\frac{\frac{b^2dx^4}{a}+bfx^2+2bd}{x^2\sqrt{bx^4+a}}+\frac{\frac{2b^2ex^6}{a}+2bex^2+2bc}{x^3\sqrt{bx^4+a}}\right)dx}{2ab}+\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}$$

$\big\downarrow$ 2009

$$\frac{\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}f+3\sqrt{b}d\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+bx^4}}-\frac{3b^{5/4}d\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right|}{a^{3/4}\sqrt{a+bx^4}}}{2ab}$$

$$\frac{x\left(af-bcx-bdx^2-bex^3\right)}{2a^2\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x]
```

output
```
(x*(a*f - b*c*x - b*d*x^2 - b*e*x^3))/(2*a^2*Sqrt[a + b*x^4]) + ((b*e*Sqrt
[a + b*x^4])/a - (b*c*Sqrt[a + b*x^4])/(a*x^2) - (2*b*d*Sqrt[a + b*x^4])/(
a*x) + (3*b^(3/2)*d*x*Sqrt[a + b*x^4])/(a*(Sqrt[a] + Sqrt[b]*x^2)) - (b*e*
ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/Sqrt[a] - (3*b^(5/4)*d*(Sqrt[a] + Sqrt[b
]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(
1/4)*x)/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a + b*x^4]) + (b^(3/4)*(3*Sqrt[b]*d
+ Sqrt[a]*f)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[a +
b*x^4]))/(2*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.83

method	result
elliptic	$-\frac{2b\left(\frac{d\,x^3}{4a^2}+\frac{c\,x^2}{4a^2}-\frac{xf}{4ab}-\frac{e}{4ba}\right)}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{2a^2x^2}-\frac{d\sqrt{b\,x^4+a}}{a^2x}+\frac{f\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+\frac{3id\sqrt{b}\sqrt{1-\frac{i}{}}}{}$
default	$f\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)-\frac{c(2b\,x^4+a)}{2x^2\sqrt{b\,x^4+a}\,a^2}+d\left(-\frac{b\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x}}{a^2}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(2dx+c)}{2a^2x^2}+\frac{f\,a^2\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+a^2e\left(\frac{1}{2a\sqrt{b\,x^4+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{b\,x^4+}}{x^2}\right.}{2a^{\frac{3}{2}}}\right.}{}$

input
```
int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(1/4*d/a^2*x^3+1/4*c/a^2*x^2-1/4/a/b*x*f-1/4*e/b/a)/((x^4+a/b)*b)^(1/
2)-1/2/a^2*c*(b*x^4+a)^(1/2)/x^2-d/a^2*(b*x^4+a)^(1/2)/x+1/2*f/a/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+3/2*I/a^(3
/2)*d*b^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))-1/2/a^(3/2)*e*arct
anh(a^(1/2)/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.68

$$\int \frac{c+dx+ex^2+fx^3}{x^3\,(a+bx^4)^{3/2}}\,dx =$$

$$-\frac{6\,(b^2dx^6+abdx^2)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)-2\left((3\,b^2d-abf)x^6+(3\,abd-a^2f)x^2\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{}}}{}$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(6*(b^2*d*x^6 + a*b*d*x^2)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(
-b/a)^(1/4)), -1) - 2*((3*b^2*d - a*b*f)*x^6 + (3*a*b*d - a^2*f)*x^2)*sqrt
(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (b^2*e*x^6 + a*b
*e*x^2)*sqrt(a)*log(-(b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) + 2*(3
*b^2*d*x^5 + 2*b^2*c*x^4 - a*b*f*x^3 - a*b*e*x^2 + 2*a*b*d*x + a*b*c)*sqrt
(b*x^4 + a))/(a^2*b^2*x^6 + a^3*b*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.92

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx = c\left(-\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}}-\frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}\right) + e\left(\frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}\right) + \frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{fx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((f*x**3+e*x**2+d*x+c)/x**3/(b*x**4+a)**(3/2),x)
```

output
```
c*(-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x*
*4) + 1))) + e*(2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)
 + a**3*log(b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1
 + b*x**4/a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**
4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/
a) + 1)/(4*a**(9/2) + 4*a**(7/2)*b*x**4)) + d*gamma(-1/4)*hyper((-1/4, 3/2
), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4)) + f*x*gamma
(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamm
a(5/4))
```

Maxima [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3 (a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3 (a+bx^4)^{3/2}}\,dx=\int \frac{fx^3+ex^2+dx+c}{(bx^4+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)/((b*x^4 + a)^(3/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.43

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx = \frac{e}{2\,a\,\sqrt{b\,x^4+a}} - \frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{2\,c\,(b\,x^4+a)-a\,c}{2\,a^2\,x^2\,\sqrt{b\,x^4+a}} - \frac{d\,\left(\frac{a}{b\,x^4}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}} + \frac{f\,x\,\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

input
```
int((c + d*x + e*x^2 + f*x^3)/(x^3*(a + b*x^4)^(3/2)),x)
```

output
```
e/(2*a*(a + b*x^4)^(1/2)) - (e*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(2*a^(3/2
)) - (2*c*(a + b*x^4) - a*c)/(2*a^2*x^2*(a + b*x^4)^(1/2)) - (d*(a/(b*x^4)
 + 1)^(3/2)*hypergeom([3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a + b*x^4)^(3/2
)) + (f*x*((b*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a
 + b*x^4)^(3/2)
```

Reduce [F]

$$\int \frac{c+dx+ex^2+fx^3}{x^3(a+bx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)/x^3/(b*x^4+a)^(3/2),x)
```

output

```
(sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*a**2*e*x**2 + 5*
sqrt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*e*x**6 + 4*sq
rt(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*e*x**10 - sqrt
(a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*a**2*e*x**2 - 5*sqrt(
a)*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*a*b*e*x**6 - 4*sqrt(a)
*sqrt(a + b*x**4)*log(sqrt(a + b*x**4) + sqrt(a))*b**2*e*x**10 - 10*sqrt(b
)*sqrt(a + b*x**4)*a**2*c*x**2 + 6*sqrt(b)*sqrt(a + b*x**4)*a**2*e*x**4 -
40*sqrt(b)*sqrt(a + b*x**4)*a*b*c*x**6 + 8*sqrt(b)*sqrt(a + b*x**4)*a*b*e*
x**8 - 32*sqrt(b)*sqrt(a + b*x**4)*b**2*c*x**10 + 4*sqrt(a + b*x**4)*int(s
qrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**4*f*x**2 + 20*sqrt(a
 + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b*
f*x**6 + 16*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**
2*x**8),x)*a**2*b**2*f*x**10 + 4*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a*
*2*x**2 + 2*a*b*x**6 + b**2*x**10),x)*a**4*d*x**2 + 20*sqrt(a + b*x**4)*in
t(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 + b**2*x**10),x)*a**3*b*d*x**6
+ 16*sqrt(a + b*x**4)*int(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 + b**2*
x**10),x)*a**2*b**2*d*x**10 + 3*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqr
t(a))*a**2*e*x**4 + 7*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*a*b*
e*x**8 + 4*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) - sqrt(a))*b**2*e*x**12 -
3*sqrt(b)*sqrt(a)*log(sqrt(a + b*x**4) + sqrt(a))*a**2*e*x**4 - 7*sqrt(...
```

3.93 $\int x^4 \sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 138

$$\int x^4 \sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)\,dx = \frac{3Cx^5(a+bx^4)^{4/3}}{31b} + \frac{(31Ab-15aC)x^5\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{155b\sqrt[3]{1+\frac{bx^4}{a}}} + \frac{Bx^7\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{4},\frac{11}{4},-\frac{bx^4}{a}\right)}{7\sqrt[3]{1+\frac{bx^4}{a}}}$$

output
```
3/31*C*x^5*(b*x^4+a)^(4/3)/b+1/155*(31*A*b-15*C*a)*x^5*(b*x^4+a)^(1/3)*hyp
ergeom([-1/3, 5/4],[9/4],-b*x^4/a)/b/(1+b*x^4/a)^(1/3)+1/7*B*x^7*(b*x^4+a)
^(1/3)*hypergeom([-1/3, 7/4],[11/4],-b*x^4/a)/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int x^4 \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{\sqrt[3]{a+bx^4}\left(63Ax^5\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)+45Bx^7\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{4},\frac{11}{4},-\frac{bx^4}{a}\right)\right.}{315\sqrt[3]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[x^4*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
((a + b*x^4)^(1/3)*(63*A*x^5*Hypergeometric2F1[-1/3, 5/4, 9/4, -((b*x^4)/a
)] + 45*B*x^7*Hypergeometric2F1[-1/3, 7/4, 11/4, -((b*x^4)/a)] + 35*C*x^9*
Hypergeometric2F1[-1/3, 9/4, 13/4, -((b*x^4)/a)]))/(315*(1 + (b*x^4)/a)^(1
/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \ 2432$$

$$\int \left(Ax^4\sqrt[3]{a+bx^4}+Bx^6\sqrt[3]{a+bx^4}+Cx^8\sqrt[3]{a+bx^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{Ax^5\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{5\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{Bx^7\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{4},\frac{11}{4},-\frac{bx^4}{a}\right)}{7\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{Cx^9\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{9}{4},\frac{13}{4},-\frac{bx^4}{a}\right)}{9\sqrt[3]{\frac{bx^4}{a}+1}}$$

input
```
Int[x^4*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
(A*x^5*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/3, 5/4, 9/4, -((b*x^4)/a)])/
(5*(1 + (b*x^4)/a)^(1/3)) + (B*x^7*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/
3, 7/4, 11/4, -((b*x^4)/a)])/(7*(1 + (b*x^4)/a)^(1/3)) + (C*x^9*(a + b*x^4
)^(1/3)*Hypergeometric2F1[-1/3, 9/4, 13/4, -((b*x^4)/a)])/(9*(1 + (b*x^4)/
a)^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int x^4\left(b\,x^4+a\right)^{\frac{1}{3}}\left(C\,x^4+B\,x^2+A\right)dx$$

input
```
int(x^4*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
int(x^4*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int x^4 \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^8 + B*x^6 + A*x^4)*(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int x^4 \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\frac{A\sqrt[3]{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{B\sqrt[3]{a}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{C\sqrt[3]{a}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(b*x**4+a)**(1/3)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(1/3)*x**5*gamma(5/4)*hyper((-1/3, 5/4), (9/4,), b*x**4*exp_polar(I*p
i)/a)/(4*gamma(9/4)) + B*a**(1/3)*x**7*gamma(7/4)*hyper((-1/3, 7/4), (11/4
,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4)) + C*a**(1/3)*x**9*gamma(9/4)
*hyper((-1/3, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int x^4\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^4\,{\left(b\,x^4+a\right)}^{1/3}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int(x^4*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x)
```

output
```
int(x^4*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int x^4 \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right) dx$$

$$= \frac{\frac{12(b\,x^4+a)^{\frac{1}{3}}a^2bx}{133} - \frac{180(b\,x^4+a)^{\frac{1}{3}}a^2cx}{4123} + \frac{3(b\,x^4+a)^{\frac{1}{3}}a\,b^2x^5}{19} + \frac{12(b\,x^4+a)^{\frac{1}{3}}a\,b^2x^3}{325} + \frac{12(b\,x^4+a)^{\frac{1}{3}}abc\,x^5}{589} + \frac{3(b\,x^4+a)^{\frac{1}{3}}b^3x^7}{25} + \frac{3(b}{}}{b^2}$$

input
```
int(x^4*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
(3*(40300*(a + b*x**4)**(1/3)*a**2*b*x - 19500*(a + b*x**4)**(1/3)*a**2*c*
x + 70525*(a + b*x**4)**(1/3)*a*b**2*x**5 + 16492*(a + b*x**4)**(1/3)*a*b*
*2*x**3 + 9100*(a + b*x**4)**(1/3)*a*b*c*x**5 + 53599*(a + b*x**4)**(1/3)*
b**3*x**7 + 43225*(a + b*x**4)**(1/3)*b**2*c*x**9 - 40300*int((a + b*x**4)
**(1/3)/(a + b*x**4),x)*a**3*b + 19500*int((a + b*x**4)**(1/3)/(a + b*x**4
),x)*a**3*c - 49476*int(((a + b*x**4)**(1/3)*x**2)/(a + b*x**4),x)*a**2*b*
*2))/(1339975*b**2)
```

3.94 $\int x^2\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 138

$$\begin{aligned}\int x^2\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)\,dx &= \frac{3Cx^3(a+bx^4)^{4/3}}{25b}+\frac{(25Ab-9aC)x^3\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{75b\sqrt[3]{1+\frac{bx^4}{a}}}\\&\quad+\frac{Bx^5\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{5\sqrt[3]{1+\frac{bx^4}{a}}}\end{aligned}$$

output
```
3/25*C*x^3*(b*x^4+a)^(4/3)/b+1/75*(25*A*b-9*C*a)*x^3*(b*x^4+a)^(1/3)*hyper
geom([-1/3, 3/4],[7/4],-b*x^4/a)/b/(1+b*x^4/a)^(1/3)+1/5*B*x^5*(b*x^4+a)^(
1/3)*hypergeom([-1/3, 5/4],[9/4],-b*x^4/a)/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int x^2\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx = \frac{\sqrt[3]{a+bx^4}\left(35Ax^3\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)+21Bx^5\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)\right.}{105\sqrt[3]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[x^2*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
((a + b*x^4)^(1/3)*(35*A*x^3*Hypergeometric2F1[-1/3, 3/4, 7/4, -((b*x^4)/a
)] + 21*B*x^5*Hypergeometric2F1[-1/3, 5/4, 9/4, -((b*x^4)/a)] + 15*C*x^7*H
ypergeometric2F1[-1/3, 7/4, 11/4, -((b*x^4)/a)]))/(105*(1 + (b*x^4)/a)^(1/
3))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(Ax^2\sqrt[3]{a+bx^4}+Bx^4\sqrt[3]{a+bx^4}+Cx^6\sqrt[3]{a+bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Ax^3\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{Bx^5\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{5\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{Cx^7\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{7}{4},\frac{11}{4},-\frac{bx^4}{a}\right)}{7\sqrt[3]{\frac{bx^4}{a}+1}}$$

input
```
Int[x^2*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
(A*x^3*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/3, 3/4, 7/4, -((b*x^4)/a)])/
(3*(1 + (b*x^4)/a)^(1/3)) + (B*x^5*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/
3, 5/4, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^(1/3)) + (C*x^7*(a + b*x^4)
^(1/3)*Hypergeometric2F1[-1/3, 7/4, 11/4, -((b*x^4)/a)])/(7*(1 + (b*x^4)/a
)^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int x^2\left(b\,x^4+a\right)^{\frac{1}{3}}\left(C\,x^4+B\,x^2+A\right)dx$$

input
```
int(x^2*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
int(x^2*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int x^2 \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}x^2\,dx$$

input
```
integrate(x^2*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^6 + B*x^4 + A*x^2)*(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int x^2 \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\frac{A\sqrt[3]{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{B\sqrt[3]{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{C\sqrt[3]{a}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(b*x**4+a)**(1/3)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(1/3)*x**3*gamma(3/4)*hyper((-1/3, 3/4), (7/4,), b*x**4*exp_polar(I*p
i)/a)/(4*gamma(7/4)) + B*a**(1/3)*x**5*gamma(5/4)*hyper((-1/3, 5/4), (9/4,
), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + C*a**(1/3)*x**7*gamma(7/4)*h
yper((-1/3, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(11/4))
```

Maxima [F]

$$\int x^2\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int\left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}x^2\,dx$$

input
```
integrate(x^2*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int\left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}x^2\,dx$$

input
```
integrate(x^2*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int x^2\,\left(b\,x^4+a\right)^{1/3}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int(x^2*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x)
```

output
```
int(x^2*(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int x^2\sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{9975(b\,x^4+a)^{\frac{1}{3}}\,ab\,x^3+3900(b\,x^4+a)^{\frac{1}{3}}\,abx+1596(b\,x^4+a)^{\frac{1}{3}}\,ac\,x^3+6825(b\,x^4+a)^{\frac{1}{3}}\,b^2x^5+5187(b\,x^4}{43225b}$$

input
```
int(x^2*(b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
(9975*(a + b*x**4)**(1/3)*a*b*x**3 + 3900*(a + b*x**4)**(1/3)*a*b*x + 1596
*(a + b*x**4)**(1/3)*a*c*x**3 + 6825*(a + b*x**4)**(1/3)*b**2*x**5 + 5187*
(a + b*x**4)**(1/3)*b*c*x**7 - 3900*int((a + b*x**4)**(1/3)/(a + b*x**4),x
)*a**2*b + 13300*int(((a + b*x**4)**(1/3)*x**2)/(a + b*x**4),x)*a**2*b - 4
788*int(((a + b*x**4)**(1/3)*x**2)/(a + b*x**4),x)*a**2*c)/(43225*b)
```

3.95 $\int \sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)\,dx = \frac{3Cx(a+bx^4)^{4/3}}{19b} + \frac{(19Ab-3aC)x\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{19b\sqrt[3]{1+\frac{bx^4}{a}}} + \frac{Bx^3\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{1+\frac{bx^4}{a}}}$$

output

```
3/19*C*x*(b*x^4+a)^(4/3)/b+1/19*(19*A*b-3*C*a)*x*(b*x^4+a)^(1/3)*hypergeom
([-1/3, 1/4],[5/4],-b*x^4/a)/b/(1+b*x^4/a)^(1/3)+1/3*B*x^3*(b*x^4+a)^(1/3)
*hypergeom([-1/3, 3/4],[7/4],-b*x^4/a)/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$=\frac{\sqrt[3]{a+bx^4}\left(15Ax\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)+5Bx^3\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)+\right.}{15\sqrt[3]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
((a + b*x^4)^(1/3)*(15*A*x*Hypergeometric2F1[-1/3, 1/4, 5/4, -((b*x^4)/a)]
 + 5*B*x^3*Hypergeometric2F1[-1/3, 3/4, 7/4, -((b*x^4)/a)] + 3*C*x^5*Hyper
geometric2F1[-1/3, 5/4, 9/4, -((b*x^4)/a)]))/(15*(1 + (b*x^4)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(A\sqrt[3]{a+bx^4}+Bx^2\sqrt[3]{a+bx^4}+Cx^4\sqrt[3]{a+bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Ax\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{Bx^3\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{Cx^5\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{5\sqrt[3]{\frac{bx^4}{a}+1}}$$

input
```
Int[(a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x]
```

output
```
(A*x*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/3, 1/4, 5/4, -((b*x^4)/a)])/(1
 + (b*x^4)/a)^(1/3) + (B*x^3*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/3, 3/4
, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^(1/3)) + (C*x^5*(a + b*x^4)^(1/3)
*Hypergeometric2F1[-1/3, 5/4, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^(1/3)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{1}{3}} \left(C\,x^{4}+B\,x^{2}+A\right)dx$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\frac{A\sqrt[3]{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{B\sqrt[3]{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{C\sqrt[3]{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/3)*(C*x**4+B*x**2+A),x)
```

output
```
A*a**(1/3)*x*gamma(1/4)*hyper((-1/3, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + B*a**(1/3)*x**3*gamma(3/4)*hyper((-1/3, 3/4), (7/4,),
b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + C*a**(1/3)*x**5*gamma(5/4)*hype
r((-1/3, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx=\int {\left(b\,x^4+a\right)}^{1/3}\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4),x)
```

output
```
int((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^4}\left(A+Bx^2+Cx^4\right)\,dx$$
$$= \frac{741(b\,x^4+a)^{\frac{1}{3}}\,abx+156(b\,x^4+a)^{\frac{1}{3}}\,acx+399(b\,x^4+a)^{\frac{1}{3}}\,b^2x^3+273(b\,x^4+a)^{\frac{1}{3}}\,bc\,x^5+988\left(\int \frac{1}{(b\,x^4+a)^{\frac{2}{3}}}d\right.}{1729b}$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A),x)
```

output
```
(741*(a + b*x**4)**(1/3)*a*b*x + 156*(a + b*x**4)**(1/3)*a*c*x + 399*(a +
b*x**4)**(1/3)*b**2*x**3 + 273*(a + b*x**4)**(1/3)*b*c*x**5 + 988*int((a +
 b*x**4)**(1/3)/(a + b*x**4),x)*a**2*b - 156*int((a + b*x**4)**(1/3)/(a +
b*x**4),x)*a**2*c + 532*int(((a + b*x**4)**(1/3)*x**2)/(a + b*x**4),x)*a*b
**2)/(1729*b)
```

3.96 $\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 131

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = -\frac{A(a+bx^4)^{4/3}}{ax} + \frac{Bx\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{1+\frac{bx^4}{a}}} + \frac{(13Ab+3aC)x^3\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{9a\sqrt[3]{1+\frac{bx^4}{a}}}$$

output
```
-A*(b*x^4+a)^(4/3)/a/x+B*x*(b*x^4+a)^(1/3)*hypergeom([-1/3, 1/4],[5/4],-b*
x^4/a)/(1+b*x^4/a)^(1/3)+1/9*(13*A*b+3*C*a)*x^3*(b*x^4+a)^(1/3)*hypergeom(
[-1/3, 3/4],[7/4],-b*x^4/a)/a/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$$
$$= \frac{\sqrt[3]{a+bx^4}\left(-3A\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)+3Bx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)+\right.}{3x\sqrt[3]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^2,x]
```

output
```
((a + b*x^4)^(1/3)*(-3*A*Hypergeometric2F1[-1/3, -1/4, 3/4, -((b*x^4)/a)]
+ 3*B*x^2*Hypergeometric2F1[-1/3, 1/4, 5/4, -((b*x^4)/a)] + C*x^4*Hypergeo
metric2F1[-1/3, 3/4, 7/4, -((b*x^4)/a)]))/(3*x*(1 + (b*x^4)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2374, 9, 27, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$$
$$\Big\downarrow \text{ 2374}$$
$$-\frac{\int -\frac{2((13Ab+3aC)x^3+3aBx)\sqrt[3]{bx^4+a}}{3x}dx}{2a}-\frac{A(a+bx^4)^{4/3}}{ax}$$
$$\Big\downarrow \text{ 9}$$
$$-\frac{\int -\frac{2}{3}\left((13Ab+3aC)x^2+3aB\right)\sqrt[3]{bx^4+a}dx}{2a}-\frac{A(a+bx^4)^{4/3}}{ax}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \left((13Ab+3aC)x^2+3aB\right)\sqrt[3]{bx^4+a}dx}{3a}-\frac{A\left(a+bx^4\right)^{4/3}}{ax}$$

$\Big\downarrow$ 1516

$$\frac{\int \left((13Ab+3aC)\sqrt[3]{bx^4+a}x^2+3aB\sqrt[3]{bx^4+a}\right)dx}{3a}-\frac{A\left(a+bx^4\right)^{4/3}}{ax}$$

$\Big\downarrow$ 2009

$$\frac{\frac{x^3\sqrt[3]{a+bx^4}(3aC+13Ab)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{\frac{bx^4}{a}+1}}+\frac{3aBx\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{\frac{bx^4}{a}+1}}}{3a}-\frac{A\left(a+bx^4\right)^{4/3}}{ax}$$

input
```
Int[((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^2,x]
```

output
```
-((A*(a + b*x^4)^(4/3))/(a*x)) + ((3*a*B*x*(a + b*x^4)^(1/3)*Hypergeometri
c2F1[-1/3, 1/4, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/3) + ((13*A*b + 3*a
*C)*x^3*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/3, 3/4, 7/4, -((b*x^4)/a)])
/(3*(1 + (b*x^4)/a)^(1/3)))/(3*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{1}{3}}\,(C\,x^{4}+B\,x^{2}+A)}{x^{2}}dx$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^2,x)
```

output
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx=\int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \frac{A\sqrt[3]{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} + \frac{B\sqrt[3]{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{C\sqrt[3]{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/3)*(C*x**4+B*x**2+A)/x**2,x)
```

output
```
A*a**(1/3)*gamma(-1/4)*hyper((-1/3, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x*gamma(3/4)) + B*a**(1/3)*x*gamma(1/4)*hyper((-1/3, 1/4), (5/4,), b
*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4)) + C*a**(1/3)*x**3*gamma(3/4)*hyper
((-1/3, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx = \int \frac{(b\,x^4+a)^{1/3}\,(C\,x^4+B\,x^2+A)}{x^2}\,dx$$

input
```
int(((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^2,x)
```

output
```
int(((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^2}\,dx$$
$$= \frac{273(b\,x^4+a)^{\frac{1}{3}}\,ab+84(b\,x^4+a)^{\frac{1}{3}}\,ac+39(b\,x^4+a)^{\frac{1}{3}}\,b^2x^2+21(b\,x^4+a)^{\frac{1}{3}}\,bc\,x^4+364\left(\int \frac{(b\,x^4+a)^{\frac{1}{3}}}{b\,x^6+a\,x^2}dx\right)a^2}{91bx}$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^2,x)
```

output

```
(273*(a + b*x**4)**(1/3)*a*b + 84*(a + b*x**4)**(1/3)*a*c + 39*(a + b*x**4
)**(1/3)*b**2*x**2 + 21*(a + b*x**4)**(1/3)*b*c*x**4 + 364*int((a + b*x**4
)**(1/3)/(a*x**2 + b*x**6),x)*a**2*b*x + 84*int((a + b*x**4)**(1/3)/(a*x**
2 + b*x**6),x)*a**2*c*x + 52*int((a + b*x**4)**(1/3)/(a + b*x**4),x)*a*b**
2*x)/(91*b*x)
```

3.97 $\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = -\frac{A(a+bx^4)^{4/3}}{3ax^3} - \frac{B\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt[3]{1+\frac{bx^4}{a}}} + \frac{(7Ab+9aC)x\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{9a\sqrt[3]{1+\frac{bx^4}{a}}}$$

output
```
-1/3*A*(b*x^4+a)^(4/3)/a/x^3-B*(b*x^4+a)^(1/3)*hypergeom([-1/3, -1/4],[3/4
],-b*x^4/a)/x/(1+b*x^4/a)^(1/3)+1/9*(7*A*b+9*C*a)*x*(b*x^4+a)^(1/3)*hyperg
eom([-1/3, 1/4],[5/4],-b*x^4/a)/a/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 9.79 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \frac{\sqrt[3]{a+bx^4}\left(-A\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{3},\frac{1}{4},-\frac{bx^4}{a}\right)-3Bx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)\right.}{3x^3\sqrt[3]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
((a + b*x^4)^(1/3)*(-(A*Hypergeometric2F1[-3/4, -1/3, 1/4, -((b*x^4)/a)])
- 3*B*x^2*Hypergeometric2F1[-1/3, -1/4, 3/4, -((b*x^4)/a)] + 3*C*x^4*Hyper
geometric2F1[-1/3, 1/4, 5/4, -((b*x^4)/a)]))/(3*x^3*(1 + (b*x^4)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2374, 9, 27, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$$

$$\Big\downarrow \text{2374}$$

$$-\frac{\int -\frac{2((7Ab+9aC)x^3+9aBx)\sqrt[3]{bx^4+a}}{3x^3}dx}{6a}-\frac{A(a+bx^4)^{4/3}}{3ax^3}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{2((7Ab+9aC)x^2+9aB)\sqrt[3]{bx^4+a}}{3x^2}dx}{6a}-\frac{A(a+bx^4)^{4/3}}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{((7Ab+9aC)x^2+9aB)\sqrt[3]{bx^4+a}}{x^2}dx}{9a}-\frac{A(a+bx^4)^{4/3}}{3ax^3}$$

$$\Big\downarrow \; 1675$$

$$\frac{\int \left(\frac{9a\sqrt[3]{bx^4+a}B}{x^2}+7Ab\left(\frac{9aC}{7Ab}+1\right)\sqrt[3]{bx^4+a}\right)dx}{9a}-\frac{A(a+bx^4)^{4/3}}{3ax^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{x\sqrt[3]{a+bx^4}(9aC+7Ab)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{\frac{bx^4}{a}+1}}-\frac{9aB\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt[3]{\frac{bx^4}{a}+1}}}{9a}-\frac{A(a+bx^4)^{4/3}}{3ax^3}$$

input
```
Int[((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^4,x]
```

output
```
-1/3*(A*(a + b*x^4)^(4/3))/(a*x^3) + ((-9*a*B*(a + b*x^4)^(1/3)*Hypergeome
tric2F1[-1/3, -1/4, 3/4, -((b*x^4)/a)])/(x*(1 + (b*x^4)/a)^(1/3)) + ((7*A*
b + 9*a*C)*x*(a + b*x^4)^(1/3)*Hypergeometric2F1[-1/3, 1/4, 5/4, -((b*x^4)
/a)])/(1 + (b*x^4)/a)^(1/3))/(9*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{3}}\,(C\,x^4+B\,x^2+A)}{x^4}dx$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^4,x)
```

output
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \frac{A\sqrt[3]{a}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{3}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} + \frac{B\sqrt[3]{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)} + \frac{C\sqrt[3]{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/3)*(C*x**4+B*x**2+A)/x**4,x)
```

output
```
A*a**(1/3)*gamma(-3/4)*hyper((-3/4, -1/3), (1/4,), b*x**4*exp_polar(I*pi)/
a)/(4*x**3*gamma(1/4)) + B*a**(1/3)*gamma(-1/4)*hyper((-1/3, -1/4), (3/4,)
, b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4)) + C*a**(1/3)*x*gamma(1/4)*hyp
er((-1/3, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx = \int \frac{(b\,x^4+a)^{1/3}\,(C\,x^4+B\,x^2+A)}{x^4}\,dx$$

input
```
int(((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^4,x)
```

output
```
int(((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^4}\,dx$$
$$= \frac{-21(b\,x^4+a)^{\frac{1}{3}}\,ab-12(b\,x^4+a)^{\frac{1}{3}}\,ac+105(b\,x^4+a)^{\frac{1}{3}}\,b^2x^2+15(b\,x^4+a)^{\frac{1}{3}}\,bc\,x^4-28\left(\int \frac{(b\,x^4+a)^{\frac{1}{3}}}{b\,x^8+a\,x^4}dx\right) a}{35b\,x^3}$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^4,x)
```

output
```
( - 21*(a + b*x**4)**(1/3)*a*b - 12*(a + b*x**4)**(1/3)*a*c + 105*(a + b*x
**4)**(1/3)*b**2*x**2 + 15*(a + b*x**4)**(1/3)*b*c*x**4 - 28*int((a + b*x*
*4)**(1/3)/(a*x**4 + b*x**8),x)*a**2*b*x**3 - 36*int((a + b*x**4)**(1/3)/(
a*x**4 + b*x**8),x)*a**2*c*x**3 + 140*int((a + b*x**4)**(1/3)/(a*x**2 + b*
x**6),x)*a*b**2*x**3)/(35*b*x**3)
```

3.98 $\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = -\frac{A(a+bx^4)^{4/3}}{5ax^5}-\frac{B\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{3},\frac{1}{4},-\frac{bx^4}{a}\right)}{3x^3\sqrt[3]{1+\frac{bx^4}{a}}}-\frac{(Ab+15aC)\sqrt[3]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{15ax\sqrt[3]{1+\frac{bx^4}{a}}}$$

output
```
-1/5*A*(b*x^4+a)^(4/3)/a/x^5-1/3*B*(b*x^4+a)^(1/3)*hypergeom([-3/4, -1/3],
[1/4],-b*x^4/a)/x^3/(1+b*x^4/a)^(1/3)-1/15*(A*b+15*C*a)*(b*x^4+a)^(1/3)*hy
pergeom([-1/3, -1/4],[3/4],-b*x^4/a)/a/x/(1+b*x^4/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = -\frac{\sqrt[3]{a+bx^4}\Big(3A\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{3},-\frac{1}{4},-\frac{bx^4}{a}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{3},\frac{1}{4},-\frac{bx}{a}\right.}{15x^5\sqrt[3]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
-1/15*((a + b*x^4)^(1/3)*(3*A*Hypergeometric2F1[-5/4, -1/3, -1/4, -((b*x^4
)/a)] + 5*B*x^2*Hypergeometric2F1[-3/4, -1/3, 1/4, -((b*x^4)/a)] + 15*C*x^
4*Hypergeometric2F1[-1/3, -1/4, 3/4, -((b*x^4)/a)]))/(x^5*(1 + (b*x^4)/a)^
(1/3))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2364, 27, 2374, 9, 27, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx$$

$$\Big\downarrow \text{2364}$$

$$-\frac{4}{3}b\int -\frac{15Cx^4+5Bx^2+3A}{15x^2\,(bx^4+a)^{2/3}}dx-\frac{1}{15}\sqrt[3]{a+bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{45}b\int \frac{15Cx^4+5Bx^2+3A}{x^2\,(bx^4+a)^{2/3}}dx-\frac{1}{15}\sqrt[3]{a+bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 2374$$

$$\frac{4}{45}b\left(-\frac{\int -\frac{2((Ab+15aC)x^3+5aBx)}{x(bx^4+a)^{2/3}}dx}{2a}-\frac{3A\sqrt[3]{a+bx^4}}{ax}\right)-\frac{1}{15}\sqrt[3]{a+bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 9$$

$$\frac{4}{45}b\left(-\frac{\int -\frac{2((Ab+15aC)x^2+5aB)}{(bx^4+a)^{2/3}}dx}{2a}-\frac{3A\sqrt[3]{a+bx^4}}{ax}\right)-\frac{1}{15}\sqrt[3]{a+bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{4}{45}b\left(\frac{\int \frac{(Ab+15aC)x^2+5aB}{(bx^4+a)^{2/3}}dx}{a}-\frac{3A\sqrt[3]{a+bx^4}}{ax}\right)-\frac{1}{15}\sqrt[3]{a+bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 1516$$

$$\frac{4}{45}b\left(\frac{\int \left(\frac{(Ab+15aC)x^2}{(bx^4+a)^{2/3}}+\frac{5aB}{(bx^4+a)^{2/3}}\right)dx}{a}-\frac{3A\sqrt[3]{a+bx^4}}{ax}\right)-\frac{1}{15}\sqrt[3]{a+bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{4}{45}b\left(\frac{\frac{x^3\left(\frac{bx^4}{a}+1\right)^{2/3}(15aC+Ab)\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3(a+bx^4)^{2/3}}+\frac{5aBx\left(\frac{bx^4}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{(a+bx^4)^{2/3}}}{a}-\frac{3A\sqrt[3]{a}}{ax}\right.$$

$$\frac{1}{15}\sqrt[3]{a+bx^4}\left(\frac{3A}{x^5}+\frac{5B}{x^3}+\frac{15C}{x}\right)$$

input
```
Int[((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^6,x]
```

output
```
-1/15*(((3*A)/x^5 + (5*B)/x^3 + (15*C)/x)*(a + b*x^4)^(1/3)) + (4*b*((-3*A
*(a + b*x^4)^(1/3))/(a*x) + ((5*a*B*x*(1 + (b*x^4)/a)^(2/3)*Hypergeometric
2F1[1/4, 2/3, 5/4, -((b*x^4)/a)])/(a + b*x^4)^(2/3) + ((A*b + 15*a*C)*x^3*
(1 + (b*x^4)/a)^(2/3)*Hypergeometric2F1[2/3, 3/4, 7/4, -((b*x^4)/a)])/(3*(
a + b*x^4)^(2/3)))/a))/45
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, 
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, 
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] && 
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] && 
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2364
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{u
 = IntHide[x^m*Pq, x]}, Simp[u*(a + b*x^n)^p, x] - Simp[b*n*p   Int[x^(m + 
n)*(a + b*x^n)^(p - 1)*ExpandToSum[u/x^(m + 1), x], x], x]] /; FreeQ[{a, b}
, x] && PolyQ[Pq, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m + Expon[Pq, x] + 1
, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && 
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{1}{3}} \left(C\,x^{4}+B\,x^{2}+A\right)}{x^{6}}dx$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^6,x)
```

output
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \frac{A\sqrt[3]{a}\Gamma\left(-\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},-\frac{1}{3}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{B\sqrt[3]{a}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{3}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)} + \frac{C\sqrt[3]{a}\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/3)*(C*x**4+B*x**2+A)/x**6,x)
```

output
```
A*a**(1/3)*gamma(-5/4)*hyper((-5/4, -1/3), (-1/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**5*gamma(-1/4)) + B*a**(1/3)*gamma(-3/4)*hyper((-3/4, -1/3), (1/4
,), b*x**4*exp_polar(I*pi)/a)/(4*x**3*gamma(1/4)) + C*a**(1/3)*gamma(-1/4)
*hyper((-1/3, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx=\int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx=\int \frac{(Cx^4+Bx^2+A)(bx^4+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(1/3)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \int \frac{{\left(b\,x^4+a\right)}^{1/3}\,\left(C\,x^4+B\,x^2+A\right)}{x^6} \,dx$$

input
```
int(((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^6,x)
```

output
```
int(((a + b*x^4)^(1/3)*(A + B*x^2 + C*x^4))/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^4}(A+Bx^2+Cx^4)}{x^6}\,dx = \frac{-15(b\,x^4+a)^{\frac{1}{3}}\,ab-60(b\,x^4+a)^{\frac{1}{3}}\,ac-33(b\,x^4+a)^{\frac{1}{3}}\,b^2x^2+165(b\,x^4+a)^{\frac{1}{3}}\,bc\,x^4-20\left(\int \frac{(b\,x^4+a)^{\frac{1}{3}}}{b\,x^{10}+a\,x^6}dx\right) a}{55b\,x^5}$$

input
```
int((b*x^4+a)^(1/3)*(C*x^4+B*x^2+A)/x^6,x)
```

output
```
( - 15*(a + b*x**4)**(1/3)*a*b - 60*(a + b*x**4)**(1/3)*a*c - 33*(a + b*x*
*4)**(1/3)*b**2*x**2 + 165*(a + b*x**4)**(1/3)*b*c*x**4 - 20*int((a + b*x*
*4)**(1/3)/(a*x**6 + b*x**10),x)*a**2*b*x**5 - 300*int((a + b*x**4)**(1/3)
/(a*x**6 + b*x**10),x)*a**2*c*x**5 - 44*int((a + b*x**4)**(1/3)/(a*x**4 +
b*x**8),x)*a*b**2*x**5)/(55*b*x**5)
```

3.99 $\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 138

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx = \frac{3Cx^3(a+bx^4)^{2/3}}{17b} + \frac{(17Ab-9aC)x^3\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{51b\sqrt[3]{a+bx^4}} + \frac{Bx^5\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{5\sqrt[3]{a+bx^4}}$$

output
```
3/17*C*x^3*(b*x^4+a)^(2/3)/b+1/51*(17*A*b-9*C*a)*x^3*(1+b*x^4/a)^(1/3)*hyp
ergeom([1/3, 3/4],[7/4],-b*x^4/a)/b/(b*x^4+a)^(1/3)+1/5*B*x^5*(1+b*x^4/a)^
(1/3)*hypergeom([1/3, 5/4],[9/4],-b*x^4/a)/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx = \frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(35Ax^3\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)+21Bx^5\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)+1\right.}{105\sqrt[3]{a+bx^4}}$$

input

```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(1/3),x]
```

output

```
((1 + (b*x^4)/a)^(1/3)*(35*A*x^3*Hypergeometric2F1[1/3, 3/4, 7/4, -((b*x^4
)/a)] + 21*B*x^5*Hypergeometric2F1[1/3, 5/4, 9/4, -((b*x^4)/a)] + 15*C*x^7
*Hypergeometric2F1[1/3, 7/4, 11/4, -((b*x^4)/a)]))/(105*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4\right)}{\sqrt[3]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{Ax^2}{\sqrt[3]{a+bx^4}}+\frac{Bx^4}{\sqrt[3]{a+bx^4}}+\frac{Cx^6}{\sqrt[3]{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Ax^3\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{a+bx^4}}+\frac{Bx^5\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{5\sqrt[3]{a+bx^4}}+\frac{Cx^7\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{4},\frac{11}{4},-\frac{bx^4}{a}\right)}{7\sqrt[3]{a+bx^4}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(1/3),x]
```

output
```
(A*x^3*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, -((b*x^4)/a)
])/(3*(a + b*x^4)^(1/3)) + (B*x^5*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[
1/3, 5/4, 9/4, -((b*x^4)/a)])/(5*(a + b*x^4)^(1/3)) + (C*x^7*(1 + (b*x^4)/
a)^(1/3)*Hypergeometric2F1[1/3, 7/4, 11/4, -((b*x^4)/a)])/(7*(a + b*x^4)^(
1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{x^2(C\,x^4+B\,x^2+A)}{(b\,x^4+a)^{\frac{1}{3}}}dx$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x)
```

output
```
int(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^2}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((C*x^6 + B*x^4 + A*x^2)/(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx=\frac{Ax^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{7}{4}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{9}{4}\right)}+\frac{Cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(b*x**4+a)**(1/3),x)
```

output
```
A*x**3*gamma(3/4)*hyper((1/3, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(1/3)*gamma(7/4)) + B*x**5*gamma(5/4)*hyper((1/3, 5/4), (9/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*a**(1/3)*gamma(9/4)) + C*x**7*gamma(7/4)*hyper((1/3,
7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/3)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^2}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(b*x^4 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^2}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(b*x^4 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{x^2\,(C\,x^4+B\,x^2+A)}{(b\,x^4+a)^{1/3}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(1/3),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{\sqrt[3]{a+bx^4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)c + \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)a$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x)
```

output
```
int(x**6/(a + b*x**4)**(1/3),x)*c + int(x**4/(a + b*x**4)**(1/3),x)*b + in
t(x**2/(a + b*x**4)**(1/3),x)*a
```

3.100 $\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx = \frac{3Cx(a+bx^4)^{2/3}}{11b} + \frac{(11Ab-3aC)x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{11b\sqrt[3]{a+bx^4}} + \frac{Bx^3\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{a+bx^4}}$$

output
```
3/11*C*x*(b*x^4+a)^(2/3)/b+1/11*(11*A*b-3*C*a)*x*(1+b*x^4/a)^(1/3)*hyperge
om([1/4, 1/3],[5/4],-b*x^4/a)/b/(b*x^4+a)^(1/3)+1/3*B*x^3*(1+b*x^4/a)^(1/3
)*hypergeom([1/3, 3/4],[7/4],-b*x^4/a)/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx = \frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(15Ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)+5Bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)+3C\right.}{15\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^4)^(1/3),x]
```

output
```
((1 + (b*x^4)/a)^(1/3)*(15*A*x*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/
a)] + 5*B*x^3*Hypergeometric2F1[1/3, 3/4, 7/4, -((b*x^4)/a)] + 3*C*x^5*Hyp
ergeometric2F1[1/3, 5/4, 9/4, -((b*x^4)/a)]))/(15*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{A}{\sqrt[3]{a+bx^4}}+\frac{Bx^2}{\sqrt[3]{a+bx^4}}+\frac{Cx^4}{\sqrt[3]{a+bx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Ax\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}+\frac{Bx^3\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{a+bx^4}}+\frac{Cx^5\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{4},\frac{9}{4},-\frac{bx^4}{a}\right)}{5\sqrt[3]{a+bx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^4)^(1/3),x]
```

output
```
(A*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])
/(a + b*x^4)^(1/3) + (B*x^3*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/3, 3
/4, 7/4, -((b*x^4)/a)])/(3*(a + b*x^4)^(1/3)) + (C*x^5*(1 + (b*x^4)/a)^(1/
3)*Hypergeometric2F1[1/3, 5/4, 9/4, -((b*x^4)/a)])/(5*(a + b*x^4)^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{C\,x^4+B\,x^2+A}{(b\,x^4+a)^{\frac{1}{3}}}dx$$

input
```
int((C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x)
```

output
```
int((C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)/(b*x^4 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx=\frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{3}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{7}{4}\right)}+\frac{Cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**4+a)**(1/3),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 1/3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/3)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((1/3, 3/4), (7/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(1/3)*gamma(7/4)) + C*x**5*gamma(5/4)*hyper((1/3, 5/4
), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/3)*gamma(9/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^4 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^4 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{{(b\,x^4+a)}^{1/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^4)^(1/3),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^4)^(1/3), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{\sqrt[3]{a+bx^4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)c+\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)b + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)a$$

input

```
int((C*x^4+B*x^2+A)/(b*x^4+a)^(1/3),x)
```

output

```
int(x**4/(a + b*x**4)**(1/3),x)*c + int(x**2/(a + b*x**4)**(1/3),x)*b + in
t(1/(a + b*x**4)**(1/3),x)*a
```

3.101 $\int \frac{A+Bx^2+Cx^4}{x^2 \sqrt[3]{a+bx^4}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 131

$$\int \frac{A+Bx^2+Cx^4}{x^2 \sqrt[3]{a+bx^4}} \, dx = -\frac{A(a+bx^4)^{2/3}}{ax} + \frac{Bx\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}} + \frac{(5Ab+3aC)x^3\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{9a\sqrt[3]{a+bx^4}}$$

output

```
-A*(b*x^4+a)^(2/3)/a/x+B*x*(1+b*x^4/a)^(1/3)*hypergeom([1/4, 1/3],[5/4],-b
*x^4/a)/(b*x^4+a)^(1/3)+1/9*(5*A*b+3*C*a)*x^3*(1+b*x^4/a)^(1/3)*hypergeom(
[1/3, 3/4],[7/4],-b*x^4/a)/a/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx = \frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(-3A\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{3}{4},-\frac{bx^4}{a}\right)+3Bx^2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)+C\right.}{3x\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(1/3)),x]
```

output
```
((1 + (b*x^4)/a)^(1/3)*(-3*A*Hypergeometric2F1[-1/4, 1/3, 3/4, -((b*x^4)/a
)] + 3*B*x^2*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)] + C*x^4*Hyperg
eometric2F1[1/3, 3/4, 7/4, -((b*x^4)/a)]))/(3*x*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2374, 9, 27, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{2374}$$

$$-\frac{\int -\frac{2\left((5Ab+3aC)x^3+3aBx\right)}{3x\sqrt[3]{bx^4+a}}dx}{2a}-\frac{A\left(a+bx^4\right)^{2/3}}{ax}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{2\left((5Ab+3aC)x^2+3aB\right)}{3\sqrt[3]{bx^4+a}}dx}{2a}-\frac{A\left(a+bx^4\right)^{2/3}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(5Ab+3aC)x^2+3aB}{\sqrt[3]{bx^4+a}}dx}{3a} - \frac{A(a+bx^4)^{2/3}}{ax}$$

$$\Big\downarrow \text{1516}$$

$$\frac{\int \left(\frac{(5Ab+3aC)x^2}{\sqrt[3]{bx^4+a}} + \frac{3aB}{\sqrt[3]{bx^4+a}}\right) dx}{3a} - \frac{A(a+bx^4)^{2/3}}{ax}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{x^3\sqrt[3]{\frac{bx^4}{a}+1}(3aC+5Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{a+bx^4}} + \frac{3aBx\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}}{3a} - \frac{A(a+bx^4)^{2/3}}{ax}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(1/3)),x]
```

output
```
-((A*(a + b*x^4)^(2/3))/(a*x)) + ((3*a*B*x*(1 + (b*x^4)/a)^(1/3)*Hypergeom
etric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(a + b*x^4)^(1/3) + ((5*A*b + 3*a*C
)*x^3*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, -((b*x^4)/a)]
)/(3*(a + b*x^4)^(1/3)))/(3*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{2} \left(b\,x^{4}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/3),x)
```

output
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^{4}+Bx^{2}+A}{{\left(bx^{4}+a\right)}^{\frac{1}{3}}x^{2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(2/3)/(b*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{3}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{Bx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Cx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(b*x**4+a)**(1/3),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 1/3), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/3)*x*gamma(3/4)) + B*x*gamma(1/4)*hyper((1/4, 1/3), (5/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(1/3)*gamma(5/4)) + C*x**3*gamma(3/4)*hyper((1/3, 3/4)
, (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/3)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(1/3)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(1/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,{(b\,x^4+a)}^{1/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(1/3)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(1/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\sqrt[3]{a+bx^4}}\,dx=\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)c+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)b+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,x^2}dx\right)a$$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(1/3),x)
```

output
```
int(x**2/(a + b*x**4)**(1/3),x)*c + int(1/(a + b*x**4)**(1/3),x)*b + int(1
/((a + b*x**4)**(1/3)*x**2),x)*a
```

3.102 $\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 133

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx = -\frac{A(a+bx^4)^{2/3}}{3ax^3} - \frac{B\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt[3]{a+bx^4}} - \frac{(Ab-9aC)x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{9a\sqrt[3]{a+bx^4}}$$

output
```
-1/3*A*(b*x^4+a)^(2/3)/a/x^3-B*(1+b*x^4/a)^(1/3)*hypergeom([-1/4, 1/3],[3/
4],-b*x^4/a)/x/(b*x^4+a)^(1/3)-1/9*(A*b-9*C*a)*x*(1+b*x^4/a)^(1/3)*hyperge
om([1/4, 1/3],[5/4],-b*x^4/a)/a/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx$$
$$= \frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(-A\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{1}{4},-\frac{bx^4}{a}\right)-3Bx^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{3}{4},-\frac{bx^4}{a}\right)+3\right.}{3x^3\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(1/3)),x]
```

output
```
((1 + (b*x^4)/a)^(1/3)*(-(A*Hypergeometric2F1[-3/4, 1/3, 1/4, -((b*x^4)/a)
]) - 3*B*x^2*Hypergeometric2F1[-1/4, 1/3, 3/4, -((b*x^4)/a)] + 3*C*x^4*Hyp
ergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)]))/(3*x^3*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2374, 9, 27, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx$$
$$\Big\downarrow \text{ 2374}$$
$$-\frac{\int -\frac{2\left(9aBx-(Ab-9aC)x^3\right)}{3x^3\sqrt[3]{bx^4+a}}dx}{6a}-\frac{A\left(a+bx^4\right)^{2/3}}{3ax^3}$$
$$\Big\downarrow \text{ 9}$$
$$-\frac{\int -\frac{2\left(9aB-(Ab-9aC)x^2\right)}{3x^2\sqrt[3]{bx^4+a}}dx}{6a}-\frac{A\left(a+bx^4\right)^{2/3}}{3ax^3}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{9aB-(Ab-9aC)x^2}{x^2\sqrt[3]{bx^4+a}}dx}{9a}-\frac{A(a+bx^4)^{2/3}}{3ax^3}$$

$$\Big\downarrow \text{1675}$$

$$\frac{\int \left(\frac{9aB}{x^2\sqrt[3]{bx^4+a}}-\frac{Ab\left(1-\frac{9aC}{Ab}\right)}{\sqrt[3]{bx^4+a}}\right)dx}{9a}-\frac{A(a+bx^4)^{2/3}}{3ax^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{x\sqrt[3]{\frac{bx^4}{a}+1}(Ab-9aC)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}-\frac{9aB\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt[3]{a+bx^4}}}{9a}-\frac{A(a+bx^4)^{2/3}}{3ax^3}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(1/3)),x]
```

output
```
-1/3*(A*(a + b*x^4)^(2/3))/(a*x^3) + ((-9*a*B*(1 + (b*x^4)/a)^(1/3)*Hyperg
eometric2F1[-1/4, 1/3, 3/4, -((b*x^4)/a)])/(x*(a + b*x^4)^(1/3)) - ((A*b -
 9*a*C)*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)
/a)])/(a + b*x^4)^(1/3))/(9*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{4}\left(b\,x^{4}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(1/3),x)
```

output
```
int((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(2/3)/(b*x^8 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx = \frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{3}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{B\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{3}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{Cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(b*x**4+a)**(1/3),x)
```

output
```
A*gamma(-3/4)*hyper((-3/4, 1/3), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/3)*x**3*gamma(1/4)) + B*gamma(-1/4)*hyper((-1/4, 1/3), (3/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(1/3)*x*gamma(3/4)) + C*x*gamma(1/4)*hyper((1/4, 1/
3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/3)*gamma(5/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(1/3)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(1/3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^4\,(b\,x^4+a)^{1/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(1/3)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(1/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\sqrt[3]{a+bx^4}}\,dx=\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}}dx\right)c+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,x^4}dx\right)a+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,x^2}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(1/3),x)
```

output
```
int(1/(a + b*x**4)**(1/3),x)*c + int(1/((a + b*x**4)**(1/3)*x**4),x)*a + i
nt(1/((a + b*x**4)**(1/3)*x**2),x)*b
```

3.103 $\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 138

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx = -\frac{A(a+bx^4)^{2/3}}{5ax^5} - \frac{B\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{1}{4},-\frac{bx^4}{a}\right)}{3x^3\sqrt[3]{a+bx^4}} + \frac{(7Ab-15aC)\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{3}{4},-\frac{bx^4}{a}\right)}{15ax\sqrt[3]{a+bx^4}}$$

output
```
-1/5*A*(b*x^4+a)^(2/3)/a/x^5-1/3*B*(1+b*x^4/a)^(1/3)*hypergeom([-3/4, 1/3]
,[1/4],-b*x^4/a)/x^3/(b*x^4+a)^(1/3)+1/15*(7*A*b-15*C*a)*(1+b*x^4/a)^(1/3)
*hypergeom([-1/4, 1/3],[3/4],-b*x^4/a)/a/x/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx = -\frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(3A\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{3},-\frac{1}{4},-\frac{bx^4}{a}\right)+5Bx^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{1}{4},-\frac{bx^4}{a}\right)\right.}{15x^5\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^6*(a + b*x^4)^(1/3)),x]
```

output
```
-1/15*((1 + (b*x^4)/a)^(1/3)*(3*A*Hypergeometric2F1[-5/4, 1/3, -1/4, -((b*
x^4)/a)] + 5*B*x^2*Hypergeometric2F1[-3/4, 1/3, 1/4, -((b*x^4)/a)] + 15*C*
x^4*Hypergeometric2F1[-1/4, 1/3, 3/4, -((b*x^4)/a)]))/(x^5*(a + b*x^4)^(1/
3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2374, 9, 27, 1675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}dx$$

$$\Big\downarrow \text{2374}$$

$$-\frac{\int -\frac{2(15aBx-(7Ab-15aC)x^3)}{3x^5\sqrt[3]{bx^4+a}}dx}{10a}-\frac{A(a+bx^4)^{2/3}}{5ax^5}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{2(15aB-(7Ab-15aC)x^2)}{3x^4\sqrt[3]{bx^4+a}}dx}{10a}-\frac{A(a+bx^4)^{2/3}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{15aB-(7Ab-15aC)x^2}{x^4\sqrt[3]{bx^4+a}}dx}{15a}-\frac{A\left(a+bx^4\right)^{2/3}}{5ax^5}$$

$$\Big\downarrow \text{1675}$$

$$\frac{\int \left(\frac{15aB}{x^4\sqrt[3]{bx^4+a}}+\frac{15aC-7Ab}{x^2\sqrt[3]{bx^4+a}}\right)dx}{15a}-\frac{A\left(a+bx^4\right)^{2/3}}{5ax^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt[3]{\frac{bx^4}{a}+1}(7Ab-15aC)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{3},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt[3]{a+bx^4}}-\frac{5aB\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{3},\frac{1}{4},-\frac{bx^4}{a}\right)}{x^3\sqrt[3]{a+bx^4}}}{15a}-\frac{A\left(a+bx^4\right)^{2/3}}{5ax^5}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^6*(a + b*x^4)^(1/3)),x]
```

output
```
-1/5*(A*(a + b*x^4)^(2/3))/(a*x^5) + ((-5*a*B*(1 + (b*x^4)/a)^(1/3)*Hyperg
eometric2F1[-3/4, 1/3, 1/4, -((b*x^4)/a)])/(x^3*(a + b*x^4)^(1/3)) + ((7*A
*b - 15*a*C)*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[-1/4, 1/3, 3/4, -((b*
x^4)/a)])/(x*(a + b*x^4)^(1/3)))/(15*a)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1675
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{6}\left(b\,x^{4}+a\right)^{\frac{1}{3}}}dx$$

input
```
int((C*x^4+B*x^2+A)/x^6/(b*x^4+a)^(1/3),x)
```

output
```
int((C*x^4+B*x^2+A)/x^6/(b*x^4+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^{4}+Bx^{2}+A}{\left(bx^{4}+a\right)^{\frac{1}{3}}x^{6}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(b*x^4+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(2/3)/(b*x^10 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx = \frac{A\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{3}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{B\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{3}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{C\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{3}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[3]{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**6/(b*x**4+a)**(1/3),x)
```

output
```
A*gamma(-5/4)*hyper((-5/4, 1/3), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(1/3)*x**5*gamma(-1/4)) + B*gamma(-3/4)*hyper((-3/4, 1/3), (1/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(1/3)*x**3*gamma(1/4)) + C*gamma(-1/4)*hyper((-1/
4, 1/3), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/3)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(b*x^4+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(1/3)*x^6), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx=\int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{1}{3}}x^6}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^6/(b*x^4+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(1/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^6\,{(b\,x^4+a)}^{1/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^6*(a + b*x^4)^(1/3)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^6*(a + b*x^4)^(1/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^6\sqrt[3]{a+bx^4}}\,dx=\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,x^6}dx\right)a+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,x^4}dx\right)b+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,x^2}dx\right)c$$

input
```
int((C*x^4+B*x^2+A)/x^6/(b*x^4+a)^(1/3),x)
```

output
```
int(1/((a + b*x**4)**(1/3)*x**6),x)*a + int(1/((a + b*x**4)**(1/3)*x**4),x
)*b + int(1/((a + b*x**4)**(1/3)*x**2),x)*c
```

3.104 $\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \frac{3Cx^5}{11b\sqrt[3]{a+bx^4}} + \frac{(11Ab-15aC)x^5\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{4}{3},\frac{9}{4},-\frac{bx^4}{a}\right)}{55ab\sqrt[3]{a+bx^4}} + \frac{Bx^7\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{7}{4},\frac{11}{4},-\frac{bx^4}{a}\right)}{7a\sqrt[3]{a+bx^4}}$$

output
```
3/11*C*x^5/b/(b*x^4+a)^(1/3)+1/55*(11*A*b-15*C*a)*x^5*(1+b*x^4/a)^(1/3)*hy
pergeom([5/4, 4/3],[9/4],-b*x^4/a)/a/b/(b*x^4+a)^(1/3)+1/7*B*x^7*(1+b*x^4/
a)^(1/3)*hypergeom([4/3, 7/4],[11/4],-b*x^4/a)/a/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(63Ax^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{4}{3},\frac{9}{4},-\frac{bx^4}{a}\right)+45Bx^7\,\text{Hypergeome}\right.}{315a\sqrt[3]{a+}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3),x]
```

output
```
((1 + (b*x^4)/a)^(1/3)*(63*A*x^5*Hypergeometric2F1[5/4, 4/3, 9/4, -((b*x^4
)/a)] + 45*B*x^7*Hypergeometric2F1[4/3, 7/4, 11/4, -((b*x^4)/a)] + 35*C*x^
9*Hypergeometric2F1[4/3, 9/4, 13/4, -((b*x^4)/a)]))/(315*a*(a + b*x^4)^(1/
3))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2367, 27, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{3\int -\frac{4abCx^4+9abBx^2+3a(Ab-aC)}{3\sqrt[3]{bx^4+a}}dx}{4ab^2}-\frac{3x(-aC+Ab+bBx^2)}{4b^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4abCx^4+9abBx^2+3a(Ab-aC)}{\sqrt[3]{bx^4+a}}dx}{4ab^2}-\frac{3x(-aC+Ab+bBx^2)}{4b^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{2432}$$

$$\frac{\int \left(\frac{4abCx^4}{\sqrt[3]{bx^4+a}}+\frac{9abBx^2}{\sqrt[3]{bx^4+a}}-\frac{3a(aC-Ab)}{\sqrt[3]{bx^4+a}}\right)dx}{4ab^2}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4b^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3ax\sqrt[3]{\frac{bx^4}{a}+1}(Ab-aC)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}+\frac{3abBx^3\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}+\frac{4abCx^5\sqrt[3]{\frac{bx}{a}}}{}}{4ab^2}$$

$$\frac{3x\left(-aC+Ab+bBx^2\right)}{4b^2\sqrt[3]{a+bx^4}}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3),x]
```

output
```
(-3*x*(A*b - a*C + b*B*x^2))/(4*b^2*(a + b*x^4)^(1/3)) + ((3*a*(A*b - a*C)
*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(
a + b*x^4)^(1/3) + (3*a*b*B*x^3*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/
3, 3/4, 7/4, -((b*x^4)/a)])/(a + b*x^4)^(1/3) + (4*a*b*C*x^5*(1 + (b*x^4)/
a)^(1/3)*Hypergeometric2F1[1/3, 5/4, 9/4, -((b*x^4)/a)])/(5*(a + b*x^4)^(1
/3)))/(4*a*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{x^{4} \left(C \,x^{4}+B \,x^{2}+A \right)}{\left(b \,x^{4}+a \right)^{\frac{4}{3}}}d x$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

output
```
int(x^4*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^4}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((C*x^8 + B*x^6 + A*x^4)*(b*x^4 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^4 +
a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.88 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \frac{Ax^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{4}{3}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{9}{4}\right)} + \frac{Bx^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{11}{4}\right)} + \frac{Cx^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(C*x**4+B*x**2+A)/(b*x**4+a)**(4/3),x)
```

output
```
A*x**5*gamma(5/4)*hyper((5/4, 4/3), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(4/3)*gamma(9/4)) + B*x**7*gamma(7/4)*hyper((4/3, 7/4), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(4/3)*gamma(11/4)) + C*x**9*gamma(9/4)*hyper((4/3
, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(4/3)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^4}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(b*x^4 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx=\int \frac{(Cx^4+Bx^2+A)x^4}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^4/(b*x^4 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx=\int \frac{x^4\,(C\,x^4+B\,x^2+A)}{(b\,x^4+a)^{4/3}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx=\left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)c$$
$$+\left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)b$$
$$+\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)a$$

input
```
int(x^4*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

output
```
int(x**8/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c + int(x
**6/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*b + int(x**4/(
(a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*a
```

3.105 $\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 144

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \frac{3Cx^3}{5b\sqrt[3]{a+bx^4}} + \frac{(5Ab-9aC)x^3\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{4}{3},\frac{7}{4},-\frac{bx^4}{a}\right)}{15ab\sqrt[3]{a+bx^4}} + \frac{Bx^5\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{4}{3},\frac{9}{4},-\frac{bx^4}{a}\right)}{5a\sqrt[3]{a+bx^4}}$$

output
```
3/5*C*x^3/b/(b*x^4+a)^(1/3)+1/15*(5*A*b-9*C*a)*x^3*(1+b*x^4/a)^(1/3)*hyper
geom([3/4, 4/3],[7/4],-b*x^4/a)/a/b/(b*x^4+a)^(1/3)+1/5*B*x^5*(1+b*x^4/a)^
(1/3)*hypergeom([5/4, 4/3],[9/4],-b*x^4/a)/a/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(35Ax^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{4}{3},\frac{7}{4},-\frac{bx^4}{a}\right)+21Bx^5\,\text{Hypergeome}\right.}{105a\sqrt[3]{a+}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3),x]
```

output
```
((1 + (b*x^4)/a)^(1/3)*(35*A*x^3*Hypergeometric2F1[3/4, 4/3, 7/4, -((b*x^4
)/a)] + 21*B*x^5*Hypergeometric2F1[5/4, 4/3, 9/4, -((b*x^4)/a)] + 15*C*x^7
*Hypergeometric2F1[4/3, 7/4, 11/4, -((b*x^4)/a)]))/(105*a*(a + b*x^4)^(1/3
))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2367, 27, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx$$

$$\Big\downarrow \text{2367}$$

$$-\frac{3\int -\frac{b(3aB-(5Ab-9aC)x^2)}{3\sqrt[3]{bx^4+a}}dx}{4ab^2}-\frac{3x(aB-x^2(Ab-aC))}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3aB-(5Ab-9aC)x^2}{\sqrt[3]{bx^4+a}}dx}{4ab}-\frac{3x(aB-x^2(Ab-aC))}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{1516}$$

$$\frac{\int \left(\frac{3aB}{\sqrt[3]{bx^4+a}}-\frac{(5Ab-9aC)x^2}{\sqrt[3]{bx^4+a}}\right)dx}{4ab}-\frac{3x(aB-x^2(Ab-aC))}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3aBx\sqrt[3]{\frac{bx^4}{a}+1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}-\frac{x^3\sqrt[3]{\frac{bx^4}{a}+1}(5Ab-9aC)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{a+bx^4}}}{4ab}-\frac{3x(aB-x^2(Ab-aC))}{4ab\sqrt[3]{a+bx^4}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3),x]
```

output
```
(-3*x*(a*B - (A*b - a*C)*x^2))/(4*a*b*(a + b*x^4)^(1/3)) + ((3*a*B*x*(1 +
(b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(a + b*x^
4)^(1/3) - ((5*A*b - 9*a*C)*x^3*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/
3, 3/4, 7/4, -((b*x^4)/a)])/(3*(a + b*x^4)^(1/3)))/(4*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2367
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
 m + Expon[Pq, x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)
*x^m*Pq, a + b*x^n, x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x]}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floo
r[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   I
nt[(a + b*x^n)^(p + 1)*ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x],
 x], x], x]] /; GeQ[q, n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0
] && LtQ[p, -1] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x^{2} \left(C \,x^{4}+B \,x^{2}+A \right)}{\left(b \,x^{4}+a \right)^{\frac{4}{3}}}d x$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

output
```
int(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((C*x^6 + B*x^4 + A*x^2)*(b*x^4 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^4 +
a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.34 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.84

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \frac{Ax^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{4}{3}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{4}{3}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{9}{4}\right)} + \frac{Cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(C*x**4+B*x**2+A)/(b*x**4+a)**(4/3),x)
```

output
```
A*x**3*gamma(3/4)*hyper((3/4, 4/3), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(4/3)*gamma(7/4)) + B*x**5*gamma(5/4)*hyper((5/4, 4/3), (9/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*a**(4/3)*gamma(9/4)) + C*x**7*gamma(7/4)*hyper((4/3,
7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(4/3)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(b*x^4 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \int \frac{(Cx^4+Bx^2+A)x^2}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*x^2/(b*x^4 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \int \frac{x^2\,(C\,x^4+B\,x^2+A)}{(b\,x^4+a)^{4/3}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4))/(a + b*x^4)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^2+Cx^4)}{(a+bx^4)^{4/3}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)c + \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)b + \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)a$$

input
```
int(x^2*(C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

output
```
int(x**6/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c + int(x
**4/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*b + int(x**2/(
(a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*a
```

3.106 $\int \frac{A+Bx^2+Cx^4}{\left(a+bx^4\right)^{4/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^4\right)^{4/3}}\,dx = -\frac{3Cx}{b\sqrt[3]{a+bx^4}} + \frac{(Ab+3aC)x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{4}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{ab\sqrt[3]{a+bx^4}} + \frac{Bx^3\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{4}{3},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a\sqrt[3]{a+bx^4}}$$

output
```
-3*C*x/b/(b*x^4+a)^(1/3)+(A*b+3*C*a)*x*(1+b*x^4/a)^(1/3)*hypergeom([1/4, 4
/3],[5/4],-b*x^4/a)/a/b/(b*x^4+a)^(1/3)+1/3*B*x^3*(1+b*x^4/a)^(1/3)*hyperg
eom([3/4, 4/3],[7/4],-b*x^4/a)/a/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^4\right)^{4/3}}\,dx=\frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(15Ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{4}{3},\frac{5}{4},-\frac{bx^4}{a}\right)+5Bx^3\,\text{Hypergeometric2F}\right.}{15a\sqrt[3]{a+bx^4}}$$

input

```
Integrate[(A + B*x^2 + C*x^4)/(a + b*x^4)^(4/3),x]
```

output

```
((1 + (b*x^4)/a)^(1/3)*(15*A*x*Hypergeometric2F1[1/4, 4/3, 5/4, -((b*x^4)/
a)] + 5*B*x^3*Hypergeometric2F1[3/4, 4/3, 7/4, -((b*x^4)/a)] + 3*C*x^5*Hyp
ergeometric2F1[5/4, 4/3, 9/4, -((b*x^4)/a)]))/(15*a*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2397, 27, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{\left(a+bx^4\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 2397}$$

$$\frac{3x\left(-aC+Ab+bBx^2\right)}{4ab\sqrt[3]{a+bx^4}}-\frac{3\int -\frac{-5bBx^2+Ab+3aC}{3\sqrt[3]{bx^4+a}}dx}{4ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-5bBx^2+Ab+3aC}{\sqrt[3]{bx^4+a}}dx}{4ab}+\frac{3x\left(-aC+Ab+bBx^2\right)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{ 1516}$$

$$\frac{\int \left(\frac{Ab\left(\frac{3aC}{Ab}+1\right)}{\sqrt[3]{bx^4+a}}-\frac{5bBx^2}{\sqrt[3]{bx^4+a}}\right)dx}{4ab}+\frac{3x(-aC+Ab+bBx^2)}{4ab\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{x\sqrt[3]{\frac{bx^4}{a}+1}(3aC+Ab)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}-\frac{5bBx^3\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{a+bx^4}}}{4ab}+\frac{3x(-aC+Ab+bBx^2)}{4ab\sqrt[3]{a+bx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(a + b*x^4)^(4/3),x]
```

output
```
(3*x*(A*b - a*C + b*B*x^2))/(4*a*b*(a + b*x^4)^(1/3)) + (((A*b + 3*a*C)*x*
(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])/(a +
 b*x^4)^(1/3) - (5*b*B*x^3*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/3, 3/
4, 7/4, -((b*x^4)/a)])/(3*(a + b*x^4)^(1/3)))/(4*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{\left(b\,x^{4}+a\right)^{\frac{4}{3}}}dx$$

input
```
int((C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

output
```
int((C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^4)^{4/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^4 + a^2)
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^4)^{4/3}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{4}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{4}{3}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{7}{4}\right)} + \frac{Cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{4}{3}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/(b*x**4+a)**(4/3),x)
```

output
```
A*x*gamma(1/4)*hyper((1/4, 4/3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
4/3)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((3/4, 4/3), (7/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(4/3)*gamma(7/4)) + C*x**5*gamma(5/4)*hyper((5/4, 4/3
), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(4/3)*gamma(9/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^4)^{4/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^4 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^4)^{4/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/(b*x^4 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{(a+bx^4)^{4/3}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{(b\,x^4+a)^{4/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(a + b*x^4)^(4/3),x)
```

output
```
int((A + B*x^2 + C*x^4)/(a + b*x^4)^(4/3), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{(a+bx^4)^{4/3}}\,dx &= \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)c \\ &+ \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)b \\ &+ \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)a\end{aligned}$$

input
```
int((C*x^4+B*x^2+A)/(b*x^4+a)^(4/3),x)
```

output
```
int(x**4/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c + int(x
**2/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*b + int(1/((a
+ b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*a
```

3.107 $\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 134

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx=-\frac{A}{ax\sqrt[3]{a+bx^4}}+\frac{Bx\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{4}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{a\sqrt[3]{a+bx^4}}-\frac{(7Ab-3aC)x^3\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{4}{3},\frac{7}{4},-\frac{bx^4}{a}\right)}{9a^2\sqrt[3]{a+bx^4}}$$

output
```
-A/a/x/(b*x^4+a)^(1/3)+B*x*(1+b*x^4/a)^(1/3)*hypergeom([1/4, 4/3],[5/4],-b
*x^4/a)/a/(b*x^4+a)^(1/3)-1/9*(7*A*b-3*C*a)*x^3*(1+b*x^4/a)^(1/3)*hypergeo
m([3/4, 4/3],[7/4],-b*x^4/a)/a^2/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx=\frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(-3A\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{4}{3},\frac{3}{4},-\frac{bx^4}{a}\right)+3Bx^2\,\text{Hypergeometric2}\right.}{3ax\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(4/3)),x]
```

output
```
((1 + (b*x^4)/a)^(1/3)*(-3*A*Hypergeometric2F1[-1/4, 4/3, 3/4, -((b*x^4)/a
)] + 3*B*x^2*Hypergeometric2F1[1/4, 4/3, 5/4, -((b*x^4)/a)] + C*x^4*Hyperg
eometric2F1[3/4, 4/3, 7/4, -((b*x^4)/a)]))/(3*a*x*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2368, 27, 2374, 9, 27, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx$$

$$\Big\downarrow \ 2368$$

$$\frac{3x\left(aB-x^2(Ab-aC)\right)}{4a^2\sqrt[3]{a+bx^4}}-\frac{3\int -\frac{\frac{5b(Ab-aC)x^4}{a}+bBx^2+4Ab}{3x^2\sqrt[3]{bx^4+a}}dx}{4ab}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\frac{5b(Ab-aC)x^4}{a}+bBx^2+4Ab}{x^2\sqrt[3]{bx^4+a}}dx}{4ab}+\frac{3x\left(aB-x^2(Ab-aC)\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \ 2374$$

$$\frac{-\frac{\int -\frac{2\left(5b(7Ab-3aC)x^3+3abBx\right)}{3x\sqrt[3]{bx^4+a}}dx}{2a}-\frac{4Ab(a+bx^4)^{2/3}}{ax}}{4ab}+\frac{3x\left(aB-x^2(Ab-aC)\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \textbf{9}$$

$$\frac{-\frac{\int -\frac{2b\left(5(7Ab-3aC)x^2+3aB\right)}{3\sqrt[3]{bx^4+a}}dx}{2a}-\frac{4Ab(a+bx^4)^{2/3}}{ax}}{4ab}+\frac{3x\left(aB-x^2(Ab-aC)\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{b\int \frac{5(7Ab-3aC)x^2+3aB}{\sqrt[3]{bx^4+a}}dx}{3a}-\frac{4Ab(a+bx^4)^{2/3}}{ax}}{4ab}+\frac{3x\left(aB-x^2(Ab-aC)\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \textbf{1516}$$

$$\frac{\frac{b\int \left(\frac{5(7Ab-3aC)x^2}{\sqrt[3]{bx^4+a}}+\frac{3aB}{\sqrt[3]{bx^4+a}}\right)dx}{3a}-\frac{4Ab(a+bx^4)^{2/3}}{ax}}{4ab}+\frac{3x\left(aB-x^2(Ab-aC)\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{3x\left(aB-x^2(Ab-aC)\right)}{4a^2\sqrt[3]{a+bx^4}}+\frac{\frac{b\left(\frac{5x^3\sqrt[3]{\frac{bx^4}{a}+1}(7Ab-3aC)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[3]{a+bx^4}}+\frac{3aBx\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}\right)}{3a}-\frac{4Ab(a+bx^4)^{2/3}}{ax}}{4ab}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(4/3)),x]
```

output
```
(3*x*(a*B - (A*b - a*C)*x^2))/(4*a^2*(a + b*x^4)^(1/3)) + ((-4*A*b*(a + b*
x^4)^(2/3))/(a*x) + (b*((3*a*B*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1
/4, 1/3, 5/4, -((b*x^4)/a)])/(a + b*x^4)^(1/3) + (5*(7*A*b - 3*a*C)*x^3*(1
 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, -((b*x^4)/a)])/(3*(a
+ b*x^4)^(1/3))))/(3*a))/(4*a*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{2} \left(b\,x^{4}+a \right)^{\frac{4}{3}}}d x$$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(4/3),x)
```

output
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx=\int \frac{Cx^{4}+Bx^{2}+A}{{\left(bx^{4}+a\right)}^{\frac{4}{3}}x^{2}}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(2/3)/(b^2*x^10 + 2*a*b*x^6 + a^2
*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx=\frac{A\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{4}{3}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}x\Gamma\left(\frac{3}{4}\right)}+\frac{Bx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{4}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{5}{4}\right)}+\frac{Cx^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{4}{3}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**2/(b*x**4+a)**(4/3),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 4/3), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
4/3)*x*gamma(3/4)) + B*x*gamma(1/4)*hyper((1/4, 4/3), (5/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*a**(4/3)*gamma(5/4)) + C*x**3*gamma(3/4)*hyper((3/4, 4/3)
, (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(4/3)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(bx^4+a\right)^{\frac{4}{3}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(4/3)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2\left(a+bx^4\right)^{4/3}}\,dx=\int \frac{Cx^4+Bx^2+A}{\left(bx^4+a\right)^{\frac{4}{3}}x^2}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(4/3)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^2(a+bx^4)^{4/3}}\,dx=\int \frac{C\,x^4+B\,x^2+A}{x^2\,(b\,x^4+a)^{4/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(4/3)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^2*(a + b*x^4)^(4/3)), x)
```

Reduce [F]

$$\int \frac{A+Bx^2+Cx^4}{x^2(a+bx^4)^{4/3}}\,dx=\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)c+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a\,x^2+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^6}dx\right)a+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)b$$

input
```
int((C*x^4+B*x^2+A)/x^2/(b*x^4+a)^(4/3),x)
```

output
```
int(x**2/((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c + int(1
/((a + b*x**4)**(1/3)*a*x**2 + (a + b*x**4)**(1/3)*b*x**6),x)*a + int(1/((
a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*b
```

3.108 $\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^4\right)^{4/3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^4\right)^{4/3}}\,dx = -\frac{A}{3ax^3\sqrt[3]{a+bx^4}} - \frac{B\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{4}{3},\frac{3}{4},-\frac{bx^4}{a}\right)}{ax\sqrt[3]{a+bx^4}} - \frac{(13Ab-9aC)x\sqrt[3]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{4}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{9a^2\sqrt[3]{a+bx^4}}$$

output
```
-1/3*A/a/x^3/(b*x^4+a)^(1/3)-B*(1+b*x^4/a)^(1/3)*hypergeom([-1/4, 4/3],[3/
4],-b*x^4/a)/a/x/(b*x^4+a)^(1/3)-1/9*(13*A*b-9*C*a)*x*(1+b*x^4/a)^(1/3)*hy
pergeom([1/4, 4/3],[5/4],-b*x^4/a)/a^2/(b*x^4+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^4\right)^{4/3}}\,dx = \frac{\sqrt[3]{1+\frac{bx^4}{a}}\left(-A\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{4}{3},\frac{1}{4},-\frac{bx^4}{a}\right)-3Bx^2\,\text{Hypergeometric2F}\right.}{3ax^3\sqrt[3]{a+bx^4}}$$

input
```
Integrate[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(4/3)),x]
```

output
```
((1 + (b*x^4)/a)^(1/3)*(-(A*Hypergeometric2F1[-3/4, 4/3, 1/4, -((b*x^4)/a)
]) - 3*B*x^2*Hypergeometric2F1[-1/4, 4/3, 3/4, -((b*x^4)/a)] + 3*C*x^4*Hyp
ergeometric2F1[1/4, 4/3, 5/4, -((b*x^4)/a)]))/(3*a*x^3*(a + b*x^4)^(1/3))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2368, 27, 2374, 9, 27, 2374, 9, 27, 1516, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^4\right)^{4/3}}\,dx$$

$$\Big\downarrow \; 2368$$

$$-\frac{3\int -\frac{\frac{5b^2Bx^6}{a}-b\left(\frac{Ab}{a}-C\right)x^4+4bBx^2+4Ab}{3x^4\sqrt[3]{bx^4+a}}dx}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{5b^2Bx^6}{a}-b\left(\frac{Ab}{a}-C\right)x^4+4bBx^2+4Ab}{x^4\sqrt[3]{bx^4+a}}dx}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 2374$$

$$\frac{-\frac{\int -\frac{2\left(45b^2Bx^5-b(13Ab-9aC)x^3+36abBx\right)}{3x^3\sqrt[3]{bx^4+a}}dx}{6a}-\frac{4Ab\left(a+bx^4\right)^{2/3}}{3ax^3}}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 9$$

$$\frac{-\frac{\int -\frac{2\left(45b^2Bx^4-b(13Ab-9aC)x^2+36abB\right)}{3x^2\sqrt[3]{bx^4+a}}dx}{6a}-\frac{4Ab\left(a+bx^4\right)^{2/3}}{3ax^3}}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{45b^2Bx^4-b(13Ab-9aC)x^2+36abB}{x^2\sqrt[3]{bx^4+a}}dx}{9a}-\frac{4Ab\left(a+bx^4\right)^{2/3}}{3ax^3}}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 2374$$

$$\frac{\frac{-\frac{\int \frac{2\left(ab(13Ab-9aC)x-105ab^2Bx^3\right)}{x\sqrt[3]{bx^4+a}}dx}{2a}-\frac{36bB\left(a+bx^4\right)^{2/3}}{x}}{9a}-\frac{4Ab\left(a+bx^4\right)^{2/3}}{3ax^3}}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{-\frac{\int \frac{2ab\left(-105bBx^2+13Ab-9aC\right)}{\sqrt[3]{bx^4+a}}dx}{2a}-\frac{36bB\left(a+bx^4\right)^{2/3}}{x}}{9a}-\frac{4Ab\left(a+bx^4\right)^{2/3}}{3ax^3}}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-b\int \frac{-105bBx^2+13Ab-9aC}{\sqrt[3]{bx^4+a}}dx-\frac{36bB\left(a+bx^4\right)^{2/3}}{x}}{9a}-\frac{4Ab\left(a+bx^4\right)^{2/3}}{3ax^3}}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 1516$$

$$\frac{\frac{-b\int \left(\frac{13Ab\left(1-\frac{9aC}{13Ab}\right)}{\sqrt[3]{bx^4+a}}-\frac{105bBx^2}{\sqrt[3]{bx^4+a}}\right)dx-\frac{36bB\left(a+bx^4\right)^{2/3}}{x}}{9a}-\frac{4Ab\left(a+bx^4\right)^{2/3}}{3ax^3}}{4ab}-\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{-b\left(\frac{x\sqrt[3]{\frac{bx^4}{a}+1}(13Ab-9aC)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}-\frac{35bBx^3\sqrt[3]{\frac{bx^4}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{\sqrt[3]{a+bx^4}}\right)-\frac{36bB\left(a+bx^4\right)^{2/3}}{x}}{9a}-}{4ab}$$

$$\frac{3x\left(-aC+Ab+bBx^2\right)}{4a^2\sqrt[3]{a+bx^4}}$$

input
```
Int[(A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(4/3)),x]
```

output
```
(-3*x*(A*b - a*C + b*B*x^2))/(4*a^2*(a + b*x^4)^(1/3)) + ((-4*A*b*(a + b*x
^4)^(2/3))/(3*a*x^3) + ((-36*b*B*(a + b*x^4)^(2/3))/x - b*(((13*A*b - 9*a*
C)*x*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1/4, 1/3, 5/4, -((b*x^4)/a)])
/(a + b*x^4)^(1/3) - (35*b*B*x^3*(1 + (b*x^4)/a)^(1/3)*Hypergeometric2F1[1
/3, 3/4, 7/4, -((b*x^4)/a)])/(a + b*x^4)^(1/3)))/(9*a))/(4*a*b)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1516
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2368
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Simp[(-x)*R*((a + b*x^n)^(p + 1)/(a^2*n*(p + 1)*b^(
Floor[(q - 1)/n] + 1))), x] + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))
   Int[x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p +
 1) + i + 1)/a)*Coeff[R, x, i]*x^(i - m), {i, 0, n - 1}], x], x], x]]] /; F
reeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2374
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wit
h[{Pq0 = Coeff[Pq, x, 0]}, Simp[Pq0*(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c
*(m + 1))), x] + Simp[1/(2*a*c*(m + 1))   Int[(c*x)^(m + 1)*ExpandToSum[2*a
*(m + 1)*((Pq - Pq0)/x) - 2*b*Pq0*(m + n*(p + 1) + 1)*x^(n - 1), x]*(a + b*
x^n)^p, x], x] /; NeQ[Pq0, 0]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] &&
 IGtQ[n, 0] && LtQ[m, -1] && LeQ[n - 1, Expon[Pq, x]]
```

Maple [F]

$$\int \frac{C\,x^{4}+B\,x^{2}+A}{x^{4} \left(b\,x^{4}+a\right)^{\frac{4}{3}}}d x$$

input
```
int((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(4/3),x)
```

output
```
int((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4 \left(a+bx^4\right)^{4/3}}\, dx = \int \frac{Cx^{4} + Bx^{2} + A}{{\left(bx^{4} + a\right)}^{\frac{4}{3}} x^{4}}\, dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^(2/3)/(b^2*x^12 + 2*a*b*x^8 + a^2
*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.49 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^4\right)^{4/3}}\,dx = \frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{4}{3}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{B\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{4}{3}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}x\Gamma\left(\frac{3}{4}\right)} + \frac{Cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{4}{3}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{4}{3}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((C*x**4+B*x**2+A)/x**4/(b*x**4+a)**(4/3),x)
```

output
```
A*gamma(-3/4)*hyper((-3/4, 4/3), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
4/3)*x**3*gamma(1/4)) + B*gamma(-1/4)*hyper((-1/4, 4/3), (3/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(4/3)*x*gamma(3/4)) + C*x*gamma(1/4)*hyper((1/4, 4/
3), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(4/3)*gamma(5/4))
```

Maxima [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4\left(a+bx^4\right)^{4/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{\left(bx^4+a\right)^{\frac{4}{3}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(4/3)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^2+Cx^4}{x^4 (a+bx^4)^{4/3}}\,dx = \int \frac{Cx^4+Bx^2+A}{(bx^4+a)^{\frac{4}{3}}x^4}\,dx$$

input
```
integrate((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)/((b*x^4 + a)^(4/3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4}{x^4 (a+bx^4)^{4/3}}\,dx = \int \frac{C\,x^4+B\,x^2+A}{x^4\,(b\,x^4+a)^{4/3}}\,dx$$

input
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(4/3)),x)
```

output
```
int((A + B*x^2 + C*x^4)/(x^4*(a + b*x^4)^(4/3)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4}{x^4 (a+bx^4)^{4/3}}\,dx &= \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a\,x^4+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^8}dx\right)a\\ &+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a\,x^2+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^6}dx\right)b\\ &+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{3}}\,a+(b\,x^4+a)^{\frac{1}{3}}\,b\,x^4}dx\right)c\end{aligned}$$

input
```
int((C*x^4+B*x^2+A)/x^4/(b*x^4+a)^(4/3),x)
```

output
```
int(1/((a + b*x**4)**(1/3)*a*x**4 + (a + b*x**4)**(1/3)*b*x**8),x)*a + int
(1/((a + b*x**4)**(1/3)*a*x**2 + (a + b*x**4)**(1/3)*b*x**6),x)*b + int(1/
((a + b*x**4)**(1/3)*a + (a + b*x**4)**(1/3)*b*x**4),x)*c
```

3.109 $\int (gx)^m (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx$

Optimal result

Integrand size = 30, antiderivative size = 269

$$\begin{aligned}
\int (gx)^m (c+dx+ex^2+fx^3) (a+bx^4)^p \, dx &= \frac{c(gx)^{1+m}(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{g(1+m)} \\
&+ \frac{d(gx)^{2+m}(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{2+m}{4},-p,\frac{6+m}{4},-\frac{bx^4}{a}\right)}{g^2(2+m)} \\
&+ \frac{e(gx)^{3+m}(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{g^3(3+m)} \\
&+ \frac{f(gx)^{4+m}(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{4+m}{4},-p,\frac{8+m}{4},-\frac{bx^4}{a}\right)}{g^4(4+m)}
\end{aligned}$$

output
```
c*(g*x)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/
g/(1+m)/((1+b*x^4/a)^p)+d*(g*x)^(2+m)*(b*x^4+a)^p*hypergeom([-p, 1/2+1/4*m
],[3/2+1/4*m],-b*x^4/a)/g^2/(2+m)/((1+b*x^4/a)^p)+e*(g*x)^(3+m)*(b*x^4+a)^
p*hypergeom([-p, 3/4+1/4*m],[7/4+1/4*m],-b*x^4/a)/g^3/(3+m)/((1+b*x^4/a)^p
)+f*(g*x)^(4+m)*(b*x^4+a)^p*hypergeom([-p, 1+1/4*m],[2+1/4*m],-b*x^4/a)/g^
4/(4+m)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.65

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \, dx$$
$$= x(gx)^m \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(\frac{c \,\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{1+m} + x\left(\frac{d \,\text{Hypergeometric2F1}\left(\frac{2+m}{4},-p,\frac{6+m}{4},-\frac{bx^4}{a}\right)}{2+m} + x\left(\frac{e \,\text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{3+m} + \frac{fx \,\text{Hypergeometric2F1}\left(\frac{4+m}{4},-p,\frac{8+m}{4},-\frac{bx^4}{a}\right)}{4+m}\right)\right)\right)$$

input
```
Integrate[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(x*(g*x)^m*(a + b*x^4)^p*((c*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -
((b*x^4)/a)])/(1 + m) + x*((d*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4,
-((b*x^4)/a)])/(2 + m) + x*((e*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4,
 -((b*x^4)/a)])/(3 + m) + (f*x*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4,
 -((b*x^4)/a)])/(4 + m)))))/(1 + (b*x^4)/a)^p
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m \left(a+bx^4\right)^p \left(c+dx+ex^2+fx^3\right) \, dx$$

$$\Big\downarrow \; 2372$$

$$\int \left((c+ex^2)(gx)^m (a+bx^4)^p + \frac{(d+fx^2)(gx)^{m+1}(a+bx^4)^p}{g} \right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(gx)^{m+1}(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{g(m+1)} + \frac{d(gx)^{m+2}(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+2}{4},-p,\frac{m+6}{4},-\frac{bx^4}{a}\right)}{g^2(m+2)} + \frac{e(gx)^{m+3}(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+3}{4},-p,\frac{m+7}{4},-\frac{bx^4}{a}\right)}{g^3(m+3)} + \frac{f(gx)^{m+4}(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+4}{4},-p,\frac{m+8}{4},-\frac{bx^4}{a}\right)}{g^4(m+4)}$$

input
```
Int[(g*x)^m*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(c*(g*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
 -((b*x^4)/a)])/(g*(1 + m)*(1 + (b*x^4)/a)^p) + (d*(g*x)^(2 + m)*(a + b*x^
4)^p*Hypergeometric2F1[(2 + m)/4, -p, (6 + m)/4, -((b*x^4)/a)])/(g^2*(2 +
m)*(1 + (b*x^4)/a)^p) + (e*(g*x)^(3 + m)*(a + b*x^4)^p*Hypergeometric2F1[(
3 + m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(g^3*(3 + m)*(1 + (b*x^4)/a)^p) +
(f*(g*x)^(4 + m)*(a + b*x^4)^p*Hypergeometric2F1[(4 + m)/4, -p, (8 + m)/4,
 -((b*x^4)/a)])/(g^4*(4 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int (gx)^m \left(f\,x^3+e\,x^2+dx+c\right) \left(b\,x^4+a\right)^p dx$$

input
```
int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p\, dx = \int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m\, dx$$

input
```
integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p\, dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \,dx = \int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m \,dx$$

input
```
integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \,dx = \int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p (gx)^m \,dx$$

input
```
integrate((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m \left(c+dx+ex^2+fx^3\right) \left(a+bx^4\right)^p \,dx = \int (g\,x)^m \left(b\,x^4+a\right)^p \left(f\,x^3+e\,x^2+d\,x+c\right) \,dx$$

input
```
int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((g*x)^m*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int (gx)^m \left(c + dx + ex^2 + fx^3\right) \left(a + bx^4\right)^p \, dx = \text{too large to display}$$

input
```
int((g*x)^m*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
(g**m*(4*x**m*(a + b*x**4)**p*a*f*m**3*p + 48*x**m*(a + b*x**4)**p*a*f*m**
2*p**2 + 24*x**m*(a + b*x**4)**p*a*f*m**2*p + 192*x**m*(a + b*x**4)**p*a*f
*m*p**3 + 192*x**m*(a + b*x**4)**p*a*f*m*p**2 + 44*x**m*(a + b*x**4)**p*a*
f*m*p + 256*x**m*(a + b*x**4)**p*a*f*p**4 + 384*x**m*(a + b*x**4)**p*a*f*p
**3 + 176*x**m*(a + b*x**4)**p*a*f*p**2 + 24*x**m*(a + b*x**4)**p*a*f*p +
x**m*(a + b*x**4)**p*b*c*m**4*x + 16*x**m*(a + b*x**4)**p*b*c*m**3*p*x + 9
*x**m*(a + b*x**4)**p*b*c*m**3*x + 96*x**m*(a + b*x**4)**p*b*c*m**2*p**2*x
 + 108*x**m*(a + b*x**4)**p*b*c*m**2*p*x + 26*x**m*(a + b*x**4)**p*b*c*m**
2*x + 256*x**m*(a + b*x**4)**p*b*c*m*p**3*x + 432*x**m*(a + b*x**4)**p*b*c
*m*p**2*x + 208*x**m*(a + b*x**4)**p*b*c*m*p*x + 24*x**m*(a + b*x**4)**p*b
*c*m*x + 256*x**m*(a + b*x**4)**p*b*c*p**4*x + 576*x**m*(a + b*x**4)**p*b*
c*p**3*x + 416*x**m*(a + b*x**4)**p*b*c*p**2*x + 96*x**m*(a + b*x**4)**p*b
*c*p*x + x**m*(a + b*x**4)**p*b*d*m**4*x**2 + 16*x**m*(a + b*x**4)**p*b*d*
m**3*p*x**2 + 8*x**m*(a + b*x**4)**p*b*d*m**3*x**2 + 96*x**m*(a + b*x**4)*
*p*b*d*m**2*p**2*x**2 + 96*x**m*(a + b*x**4)**p*b*d*m**2*p*x**2 + 19*x**m*
(a + b*x**4)**p*b*d*m**2*x**2 + 256*x**m*(a + b*x**4)**p*b*d*m*p**3*x**2 +
 384*x**m*(a + b*x**4)**p*b*d*m*p**2*x**2 + 152*x**m*(a + b*x**4)**p*b*d*m
*p*x**2 + 12*x**m*(a + b*x**4)**p*b*d*m*x**2 + 256*x**m*(a + b*x**4)**p*b*
d*p**4*x**2 + 512*x**m*(a + b*x**4)**p*b*d*p**3*x**2 + 304*x**m*(a + b*x**
4)**p*b*d*p**2*x**2 + 48*x**m*(a + b*x**4)**p*b*d*p*x**2 + x**m*(a + b*...
```

3.110 $\int (cx)^m (a+bx^4)^p (A+Bx^2+Cx^4) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 186

$$\int (cx)^m (a+bx^4)^p (A+Bx^2+Cx^4) \, dx = \frac{C(cx)^{1+m} (a+bx^4)^{1+p}}{bc(5+m+4p)} + \frac{\left(\frac{A}{1+m}-\frac{aC}{b(5+m+4p)}\right)(cx)^{1+m} (a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{c} + \frac{B(cx)^{3+m} (a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{c^3(3+m)}$$

output

```
C*(c*x)^(1+m)*(b*x^4+a)^(p+1)/b/c/(5+m+4*p)+(A/(1+m)-a*C/b/(5+m+4*p))*(c*x
)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/c/((1+
b*x^4/a)^p)+B*(c*x)^(3+m)*(b*x^4+a)^p*hypergeom([-p, 3/4+1/4*m],[7/4+1/4*m
],-b*x^4/a)/c^3/(3+m)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.85

$$\int (cx)^m (a+bx^4)^p (A+Bx^2+Cx^4)\, dx$$
$$= \frac{x(cx)^m (a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(A(15+8m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)+(1+m\right.}{(1+}$$

input
```
Integrate[(c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(x*(c*x)^m*(a + b*x^4)^p*(A*(15 + 8*m + m^2)*Hypergeometric2F1[(1 + m)/4,
-p, (5 + m)/4, -((b*x^4)/a)] + (1 + m)*x^2*(B*(5 + m)*Hypergeometric2F1[(3
 + m)/4, -p, (7 + m)/4, -((b*x^4)/a)] + C*(3 + m)*x^2*Hypergeometric2F1[(5
 + m)/4, -p, (9 + m)/4, -((b*x^4)/a)])))/((1 + m)*(3 + m)*(5 + m)*(1 + (b*
x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (a+bx^4)^p (A+Bx^2+Cx^4)\, dx$$

$$\Big\downarrow \text{2383}$$

$$\int \left(A(cx)^m (a+bx^4)^p+\frac{B(cx)^{m+2}(a+bx^4)^p}{c^2}+\frac{C(cx)^{m+4}(a+bx^4)^p}{c^4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A(cx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{c(m+1)}+\frac{B(cx)^{m+3}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+3}{4},-p,\frac{m+7}{4},-\frac{bx^4}{a}\right)}{c^3(m+3)}+\frac{C(cx)^{m+5}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+5}{4},-p,\frac{m+9}{4},-\frac{bx^4}{a}\right)}{c^5(m+5)}$$

input
```
Int[(c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4),x]
```

output
```
(A*(c*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
 -((b*x^4)/a)])/(c*(1 + m)*(1 + (b*x^4)/a)^p) + (B*(c*x)^(3 + m)*(a + b*x^
4)^p*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(c^3*(3 +
m)*(1 + (b*x^4)/a)^p) + (C*(c*x)^(5 + m)*(a + b*x^4)^p*Hypergeometric2F1[(
5 + m)/4, -p, (9 + m)/4, -((b*x^4)/a)])/(c^5*(5 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Maple [F]

$$\int (cx)^m \left(b\,x^4+a\right)^p \left(C\,x^4+B\,x^2+A\right)dx$$

input
```
int((c*x)^m*(b*x^4+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
int((c*x)^m*(b*x^4+a)^p*(C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^4+a)^p*(C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((C*x^4 + B*x^2 + A)*(b*x^4 + a)^p*(c*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right) \, dx = \text{Timed out}$$

input
```
integrate((c*x)**m*(b*x**4+a)**p*(C*x**4+B*x**2+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right) \, dx = \int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^4+a)^p*(C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int \left(Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^4+a)^p*(C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((C*x^4 + B*x^2 + A)*(b*x^4 + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\int (c\,x)^m\,\left(b\,x^4+a\right)^p\,\left(C\,x^4+B\,x^2+A\right)\,dx$$

input
```
int((c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4),x)
```

output
```
int((c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4), x)
```

Reduce [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4\right)\,dx=\text{too large to display}$$

input
```
int((c*x)^m*(b*x^4+a)^p*(C*x^4+B*x^2+A),x)
```

output
```
(c**m*(x**m*(a + b*x**4)**p*a*b*m**2*x + 8*x**m*(a + b*x**4)**p*a*b*m*p*x
+ 8*x**m*(a + b*x**4)**p*a*b*m*x + 16*x**m*(a + b*x**4)**p*a*b*p**2*x + 32
*x**m*(a + b*x**4)**p*a*b*p*x + 15*x**m*(a + b*x**4)**p*a*b*x + 4*x**m*(a
+ b*x**4)**p*a*c*m*p*x + 16*x**m*(a + b*x**4)**p*a*c*p**2*x + 12*x**m*(a +
 b*x**4)**p*a*c*p*x + x**m*(a + b*x**4)**p*b**2*m**2*x**3 + 8*x**m*(a + b*
x**4)**p*b**2*m*p*x**3 + 6*x**m*(a + b*x**4)**p*b**2*m*x**3 + 16*x**m*(a +
 b*x**4)**p*b**2*p**2*x**3 + 24*x**m*(a + b*x**4)**p*b**2*p*x**3 + 5*x**m*
(a + b*x**4)**p*b**2*x**3 + x**m*(a + b*x**4)**p*b*c*m**2*x**5 + 8*x**m*(a
 + b*x**4)**p*b*c*m*p*x**5 + 4*x**m*(a + b*x**4)**p*b*c*m*x**5 + 16*x**m*(
a + b*x**4)**p*b*c*p**2*x**5 + 16*x**m*(a + b*x**4)**p*b*c*p*x**5 + 3*x**m
*(a + b*x**4)**p*b*c*x**5 + 4*int((x**m*(a + b*x**4)**p*x**2)/(a*m**3 + 12
*a*m**2*p + 9*a*m**2 + 48*a*m*p**2 + 72*a*m*p + 23*a*m + 64*a*p**3 + 144*a
*p**2 + 92*a*p + 15*a + b*m**3*x**4 + 12*b*m**2*p*x**4 + 9*b*m**2*x**4 + 4
8*b*m*p**2*x**4 + 72*b*m*p*x**4 + 23*b*m*x**4 + 64*b*p**3*x**4 + 144*b*p**
2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a*b**2*m**5*p + 80*int((x**m*(a + b*x
**4)**p*x**2)/(a*m**3 + 12*a*m**2*p + 9*a*m**2 + 48*a*m*p**2 + 72*a*m*p +
23*a*m + 64*a*p**3 + 144*a*p**2 + 92*a*p + 15*a + b*m**3*x**4 + 12*b*m**2*
p*x**4 + 9*b*m**2*x**4 + 48*b*m*p**2*x**4 + 72*b*m*p*x**4 + 23*b*m*x**4 +
64*b*p**3*x**4 + 144*b*p**2*x**4 + 92*b*p*x**4 + 15*b*x**4),x)*a*b**2*m**4
*p**2 + 60*int((x**m*(a + b*x**4)**p*x**2)/(a*m**3 + 12*a*m**2*p + 9*a*...
```

3.111 $\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 237

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \frac{C(cx)^{1+m}\left(a+bx^4\right)^{1+p}}{bc(5+m+4p)}+\frac{D(cx)^{3+m}\left(a+bx^4\right)^{1+p}}{bc^3(7+m+4p)} + \frac{\left(\frac{A}{1+m}-\frac{aC}{b(5+m+4p)}\right)(cx)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{c} + \frac{\left(\frac{B}{3+m}-\frac{aD}{b(7+m+4p)}\right)(cx)^{3+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{c^3}$$

output
```
C*(c*x)^(1+m)*(b*x^4+a)^(p+1)/b/c/(5+m+4*p)+D*(c*x)^(3+m)*(b*x^4+a)^(p+1)/
b/c^3/(7+m+4*p)+(A/(1+m)-a*C/b/(5+m+4*p))*(c*x)^(1+m)*(b*x^4+a)^p*hypergeo
m([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/c/((1+b*x^4/a)^p)+(B/(3+m)-a*D/b/(
7+m+4*p))*(c*x)^(3+m)*(b*x^4+a)^p*hypergeom([-p, 3/4+1/4*m],[7/4+1/4*m],-b
*x^4/a)/c^3/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.74

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$
$$= x(cx)^m \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{1+m} + \frac{Bx^2\,\text{Hypergeometric2F1}\left(\frac{3+m}{4},-p,\frac{7+m}{4},-\frac{bx^4}{a}\right)}{3+m} + \frac{Cx^4\,\text{Hypergeometric2F1}\left(\frac{5+m}{4},-p,\frac{9+m}{4},-\frac{bx^4}{a}\right)}{5+m} + \frac{Dx^6\,\text{Hypergeometric2F1}\left(\frac{7+m}{4},-p,\frac{11+m}{4},-\frac{bx^4}{a}\right)}{7+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(x*(c*x)^m*(a + b*x^4)^p*((A*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -
((b*x^4)/a)])/(1 + m) + (B*x^2*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4,
 -((b*x^4)/a)])/(3 + m) + (C*x^4*Hypergeometric2F1[(5 + m)/4, -p, (9 + m)/
4, -((b*x^4)/a)])/(5 + m) + (D*x^6*Hypergeometric2F1[(7 + m)/4, -p, (11 +
m)/4, -((b*x^4)/a)])/(7 + m)))/(1 + (b*x^4)/a)^p
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2383, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$

$$\Big\downarrow \text{2383}$$

$$\int \left(A(cx)^m\left(a+bx^4\right)^p+\frac{B(cx)^{m+2}\left(a+bx^4\right)^p}{c^2}+\frac{D(cx)^{m+6}\left(a+bx^4\right)^p}{c^6}+\frac{C(cx)^{m+4}\left(a+bx^4\right)^p}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{A(cx)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{c(m+1)}+\frac{B(cx)^{m+3}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+3}{4},-p,\frac{m+7}{4},-\frac{bx^4}{a}\right)}{c^3(m+3)}+\frac{D(cx)^{m+7}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+7}{4},-p,\frac{m+11}{4},-\frac{bx^4}{a}\right)}{c^7(m+7)}+\frac{C(cx)^{m+5}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+5}{4},-p,\frac{m+9}{4},-\frac{bx^4}{a}\right)}{c^5(m+5)}$$

input
```
Int[(c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4 + D*x^6),x]
```

output
```
(A*(c*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
 -((b*x^4)/a)])/(c*(1 + m)*(1 + (b*x^4)/a)^p) + (B*(c*x)^(3 + m)*(a + b*x^
4)^p*Hypergeometric2F1[(3 + m)/4, -p, (7 + m)/4, -((b*x^4)/a)])/(c^3*(3 +
m)*(1 + (b*x^4)/a)^p) + (C*(c*x)^(5 + m)*(a + b*x^4)^p*Hypergeometric2F1[(
5 + m)/4, -p, (9 + m)/4, -((b*x^4)/a)])/(c^5*(5 + m)*(1 + (b*x^4)/a)^p) +
(D*(c*x)^(7 + m)*(a + b*x^4)^p*Hypergeometric2F1[(7 + m)/4, -p, (11 + m)/4
, -((b*x^4)/a)])/(c^7*(7 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2383
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IGtQ[m, 0]
```

Maple [F]

$$\int (cx)^m \left(b\,x^4+a\right)^p \left(Dx^6+C\,x^4+B\,x^2+A\right)dx$$

input
```
int((c*x)^m*(b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
int((c*x)^m*(b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

Fricas [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx$$
$$=\int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="fricas")
```

output
```
integral((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + a)^p*(c*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right)\,dx=\text{Timed out}$$

input
```
integrate((c*x)**m*(b*x**4+a)**p*(D*x**6+C*x**4+B*x**2+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$
$$= \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$
$$= \int \left(Dx^6+Cx^4+Bx^2+A\right)\left(bx^4+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x, algorithm="giac")
```

output
```
integrate((D*x^6 + C*x^4 + B*x^2 + A)*(b*x^4 + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx$$
$$= \int (c\,x)^m \left(b\,x^4+a\right)^p \left(A+B\,x^2+C\,x^4+x^6\,D\right) dx$$

input
```
int((c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4 + x^6*D),x)
```

output
```
int((c*x)^m*(a + b*x^4)^p*(A + B*x^2 + C*x^4 + x^6*D), x)
```

Reduce [F]

$$\int (cx)^m \left(a+bx^4\right)^p \left(A+Bx^2+Cx^4+Dx^6\right) dx = \text{too large to display}$$

input
```
int((c*x)^m*(b*x^4+a)^p*(D*x^6+C*x^4+B*x^2+A),x)
```

output
```
(c**m*(x**m*(a + b*x**4)**p*a*b*m**3*x + 12*x**m*(a + b*x**4)**p*a*b*m**2*
p*x + 15*x**m*(a + b*x**4)**p*a*b*m**2*x + 48*x**m*(a + b*x**4)**p*a*b*m*p
**2*x + 120*x**m*(a + b*x**4)**p*a*b*m*p*x + 71*x**m*(a + b*x**4)**p*a*b*m
*x + 64*x**m*(a + b*x**4)**p*a*b*p**3*x + 240*x**m*(a + b*x**4)**p*a*b*p**
2*x + 284*x**m*(a + b*x**4)**p*a*b*p*x + 105*x**m*(a + b*x**4)**p*a*b*x +
4*x**m*(a + b*x**4)**p*a*c*m**2*p*x + 32*x**m*(a + b*x**4)**p*a*c*m*p**2*x
 + 40*x**m*(a + b*x**4)**p*a*c*m*p*x + 64*x**m*(a + b*x**4)**p*a*c*p**3*x
+ 160*x**m*(a + b*x**4)**p*a*c*p**2*x + 84*x**m*(a + b*x**4)**p*a*c*p*x +
4*x**m*(a + b*x**4)**p*a*d*m**2*p*x**3 + 32*x**m*(a + b*x**4)**p*a*d*m*p**
2*x**3 + 24*x**m*(a + b*x**4)**p*a*d*m*p*x**3 + 64*x**m*(a + b*x**4)**p*a*
d*p**3*x**3 + 96*x**m*(a + b*x**4)**p*a*d*p**2*x**3 + 20*x**m*(a + b*x**4)
**p*a*d*p*x**3 + x**m*(a + b*x**4)**p*b**2*m**3*x**3 + 12*x**m*(a + b*x**4
)**p*b**2*m**2*p*x**3 + 13*x**m*(a + b*x**4)**p*b**2*m**2*x**3 + 48*x**m*(
a + b*x**4)**p*b**2*m*p**2*x**3 + 104*x**m*(a + b*x**4)**p*b**2*m*p*x**3 +
 47*x**m*(a + b*x**4)**p*b**2*m*x**3 + 64*x**m*(a + b*x**4)**p*b**2*p**3*x
**3 + 208*x**m*(a + b*x**4)**p*b**2*p**2*x**3 + 188*x**m*(a + b*x**4)**p*b
**2*p*x**3 + 35*x**m*(a + b*x**4)**p*b**2*x**3 + x**m*(a + b*x**4)**p*b*c*
m**3*x**5 + 12*x**m*(a + b*x**4)**p*b*c*m**2*p*x**5 + 11*x**m*(a + b*x**4)
**p*b*c*m**2*x**5 + 48*x**m*(a + b*x**4)**p*b*c*m*p**2*x**5 + 88*x**m*(a +
 b*x**4)**p*b*c*m*p*x**5 + 31*x**m*(a + b*x**4)**p*b*c*m*x**5 + 64*x**m...
```

3.112 $\int (c+dx+ex^2+fx^3)(a+bx^4)^p \, dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (c+dx+ex^2+fx^3)(a+bx^4)^p \, dx = \frac{f(a+bx^4)^{1+p}}{4b(1+p)}+\frac{cx(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{5}{4}+p,\frac{5}{4},-\frac{bx^4}{a}\right)}{a}+\frac{dx^2(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2}+p,\frac{3}{2},-\frac{bx^4}{a}\right)}{2a}+\frac{ex^3(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(1,\frac{7}{4}+p,\frac{7}{4},-\frac{bx^4}{a}\right)}{3a}$$

output
```
1/4*f*(b*x^4+a)^(p+1)/b/(p+1)+c*x*(b*x^4+a)^(p+1)*hypergeom([1, 5/4+p],[5/
4],-b*x^4/a)/a+1/2*d*x^2*(b*x^4+a)^(p+1)*hypergeom([1, 3/2+p],[3/2],-b*x^4
/a)/a+1/3*e*x^3*(b*x^4+a)^(p+1)*hypergeom([1, 7/4+p],[7/4],-b*x^4/a)/a
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int (c+dx+ex^2+fx^3)(a+bx^4)^p\,dx\\&=\frac{1}{12}(a+bx^4)^p\left(\frac{3f(a+bx^4)}{b(1+p)}\right.\\&\quad+12cx\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)\\&\quad+6dx^2\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)\\&\quad\left.+4ex^3\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right)\end{aligned}$$

input
```
Integrate[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
((a + b*x^4)^p*((3*f*(a + b*x^4))/(b*(1 + p)) + (12*c*x*Hypergeometric2F1[
1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (6*d*x^2*Hypergeometric2F
1[1/2, -p, 3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (4*e*x^3*Hypergeometric
2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p))/12
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^p(c+dx+ex^2+fx^3)\,dx$$

$$\Big\downarrow \; 2424$$

$$\int \left((c+ex^2)(a+bx^4)^p+x(d+fx^2)(a+bx^4)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$cx(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{2}dx^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+\frac{f(a+bx^4)^{p+1}}{4b(p+1)}$$

input
```
Int[(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(f*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c*x*(a + b*x^4)^p*Hypergeometric2
F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (d*x^2*(a + b*x^4)^p*H
ypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)])/(2*(1 + (b*x^4)/a)^p) + (e*x
^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b
*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int \left(f\,x^{3}+e\,x^{2}+dx+c\right)\left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 19.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx = \frac{a^p cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^p dx^2{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2}+\frac{a^p ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+f\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^4)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(a+bx^4\right) & \text{otherwise}\end{cases}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

output
```
a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I*pi)/
a)/2 + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*
pi)/a)/(4*gamma(7/4)) + f*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewise(((
a + b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4*b),
True))
```

Maxima [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int \left(b\,x^4+a\right)^p\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int((a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int (c + dx + ex^2 + fx^3) (a + bx^4)^p \, dx = \text{too large to display}$$

input
```
int((f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
(32*(a + b*x**4)**p*a*f*p**3 + 48*(a + b*x**4)**p*a*f*p**2 + 22*(a + b*x**
4)**p*a*f*p + 3*(a + b*x**4)**p*a*f + 32*(a + b*x**4)**p*b*c*p**3*x + 72*(
a + b*x**4)**p*b*c*p**2*x + 52*(a + b*x**4)**p*b*c*p*x + 12*(a + b*x**4)**
p*b*c*x + 32*(a + b*x**4)**p*b*d*p**3*x**2 + 64*(a + b*x**4)**p*b*d*p**2*x
**2 + 38*(a + b*x**4)**p*b*d*p*x**2 + 6*(a + b*x**4)**p*b*d*x**2 + 32*(a +
 b*x**4)**p*b*e*p**3*x**3 + 56*(a + b*x**4)**p*b*e*p**2*x**3 + 28*(a + b*x
**4)**p*b*e*p*x**3 + 4*(a + b*x**4)**p*b*e*x**3 + 32*(a + b*x**4)**p*b*f*p
**3*x**4 + 48*(a + b*x**4)**p*b*f*p**2*x**4 + 22*(a + b*x**4)**p*b*f*p*x**
4 + 3*(a + b*x**4)**p*b*f*x**4 + 4096*int((a + b*x**4)**p/(32*a*p**3 + 48*
a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*
b*x**4),x)*a*b*c*p**7 + 15360*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 +
 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),
x)*a*b*c*p**6 + 23296*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*p
+ 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*b*c
*p**5 + 18240*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*a +
32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*b*c*p**4 +
7744*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*a + 32*b*p**3
*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*b*c*p**3 + 1680*int(
(a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 4
8*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*b*c*p**2 + 144*int((a + b*...
```

3.113 $\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\begin{aligned}\int x^3(c+dx+ex^2+fx^3)(a+bx^4)^p\,dx &= \frac{c(a+bx^4)^{1+p}}{4b(1+p)}+\frac{1}{5}dx^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)\\ &\quad+\frac{1}{6}ex^6(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^4}{a}\right)\\ &\quad+\frac{1}{7}fx^7(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)\end{aligned}$$

output
```
1/4*c*(b*x^4+a)^(p+1)/b/(p+1)+1/5*d*x^5*(b*x^4+a)^p*hypergeom([5/4, -p],[9
/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/6*e*x^6*(b*x^4+a)^p*hypergeom([3/2, -p],[5
/2],-b*x^4/a)/((1+b*x^4/a)^p)+1/7*f*x^7*(b*x^4+a)^p*hypergeom([7/4, -p],[1
1/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx$$
$$=\frac{(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(105c(a+bx^4)\left(1+\frac{bx^4}{a}\right)^p+84bd(1+p)x^5\,\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right.\right.}{42}$$

input

```
Integrate[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output

```
((a + b*x^4)^p*(105*c*(a + b*x^4)*(1 + (b*x^4)/a)^p + 84*b*d*(1 + p)*x^5*H
ypergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 70*b*e*(1 + p)*x^6*Hypergeo
metric2F1[3/2, -p, 5/2, -((b*x^4)/a)] + 60*b*f*(1 + p)*x^7*Hypergeometric2
F1[7/4, -p, 11/4, -((b*x^4)/a)]))/(420*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^4\right)^p\left(c+dx+ex^2+fx^3\right)\,dx$$

$$\Big\downarrow \; 2372$$

$$\int \left(x^3\left(c+ex^2\right)\left(a+bx^4\right)^p+x^4\left(d+fx^2\right)\left(a+bx^4\right)^p\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{c(a+bx^4)^{p+1}}{4b(p+1)}+\frac{1}{5}dx^5(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^4}{a}\right)+\frac{1}{6}ex^6(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^4}{a}\right)+\frac{1}{7}fx^7(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[x^3*(c + d*x + e*x^2 + f*x^3)*(a + b*x^4)^p,x]
```

output
```
(c*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (d*x^5*(a + b*x^4)^p*Hypergeometri
c2F1[5/4, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p) + (e*x^6*(a + b*x^
4)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^4)/a)])/(6*(1 + (b*x^4)/a)^p)
+ (f*x^7*(a + b*x^4)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^4)/a)])/(7*
(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int x^3(f\,x^3+e\,x^2+dx+c)\left(b\,x^4+a\right)^p dx$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx=\int\left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((f*x^6 + e*x^5 + d*x^4 + c*x^3)*(b*x^4 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 45.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.82

$$\int x^3\left(c+dx+ex^2+fx^3\right)\left(a+bx^4\right)^p\,dx$$
$$=\frac{a^p d x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{a^p e x^6{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6}$$
$$+\frac{a^p f x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},-p\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}+c\left(\begin{cases}\frac{a^px^4}{4} & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^4)^{p+1}}{p+1} & \text{for } p\neq-1\\ \log\left(a+bx^4\right) & \text{otherwise}\end{cases}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**3*(f*x**3+e*x**2+d*x+c)*(b*x**4+a)**p,x)
```

output
```
a**p*d*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(9/4)) + a**p*e*x**6*hyper((3/2, -p), (5/2,), b*x**4*exp_polar(I*p
i)/a)/6 + a**p*f*x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_pola
r(I*pi)/a)/(4*gamma(11/4)) + c*Piecewise((a**p*x**4/4, Eq(b, 0)), (Piecewi
se(((a + b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**4), True))/(4
*b), True))
```

Maxima [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
1/4*(b*x^4 + a)^(p + 1)*c/(b*(p + 1)) + integrate((f*x^6 + e*x^5 + d*x^4)*
(b*x^4 + a)^p, x)
```

Giac [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int \left(fx^3+ex^2+dx+c\right)\left(bx^4+a\right)^p x^3\,dx$$

input
```
integrate(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((f*x^3 + e*x^2 + d*x + c)*(b*x^4 + a)^p*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\int x^3\,\left(b\,x^4+a\right)^p\,\left(f\,x^3+e\,x^2+d\,x+c\right)\,dx$$

input
```
int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3),x)
```

output
```
int(x^3*(a + b*x^4)^p*(c + d*x + e*x^2 + f*x^3), x)
```

Reduce [F]

$$\int x^3(c+dx+ex^2+fx^3)\left(a+bx^4\right)^p\,dx=\text{too large to display}$$

input
```
int(x^3*(f*x^3+e*x^2+d*x+c)*(b*x^4+a)^p,x)
```

output
```
(1024*(a + b*x**4)**p*a*c*p**6 + 6144*(a + b*x**4)**p*a*c*p**5 + 14464*(a
+ b*x**4)**p*a*c*p**4 + 16896*(a + b*x**4)**p*a*c*p**3 + 10180*(a + b*x**4
)**p*a*c*p**2 + 2952*(a + b*x**4)**p*a*c*p + 315*(a + b*x**4)**p*a*c + 102
4*(a + b*x**4)**p*a*d*p**6*x + 5632*(a + b*x**4)**p*a*d*p**5*x + 11840*(a
+ b*x**4)**p*a*d*p**4*x + 11840*(a + b*x**4)**p*a*d*p**3*x + 5616*(a + b*x
**4)**p*a*d*p**2*x + 1008*(a + b*x**4)**p*a*d*p*x + 1024*(a + b*x**4)**p*a
*e*p**6*x**2 + 5120*(a + b*x**4)**p*a*e*p**5*x**2 + 9600*(a + b*x**4)**p*a
*e*p**4*x**2 + 8320*(a + b*x**4)**p*a*e*p**3*x**2 + 3236*(a + b*x**4)**p*a
*e*p**2*x**2 + 420*(a + b*x**4)**p*a*e*p*x**2 + 1024*(a + b*x**4)**p*a*f*p
**6*x**3 + 4608*(a + b*x**4)**p*a*f*p**5*x**3 + 7744*(a + b*x**4)**p*a*f*p
**4*x**3 + 5952*(a + b*x**4)**p*a*f*p**3*x**3 + 2032*(a + b*x**4)**p*a*f*p
**2*x**3 + 240*(a + b*x**4)**p*a*f*p*x**3 + 1024*(a + b*x**4)**p*b*c*p**6*
x**4 + 6144*(a + b*x**4)**p*b*c*p**5*x**4 + 14464*(a + b*x**4)**p*b*c*p**4
*x**4 + 16896*(a + b*x**4)**p*b*c*p**3*x**4 + 10180*(a + b*x**4)**p*b*c*p*
*2*x**4 + 2952*(a + b*x**4)**p*b*c*p*x**4 + 315*(a + b*x**4)**p*b*c*x**4 +
 1024*(a + b*x**4)**p*b*d*p**6*x**5 + 5888*(a + b*x**4)**p*b*d*p**5*x**5 +
 13248*(a + b*x**4)**p*b*d*p**4*x**5 + 14800*(a + b*x**4)**p*b*d*p**3*x**5
 + 8576*(a + b*x**4)**p*b*d*p**2*x**5 + 2412*(a + b*x**4)**p*b*d*p*x**5 +
252*(a + b*x**4)**p*b*d*x**5 + 1024*(a + b*x**4)**p*b*e*p**6*x**6 + 5632*(
a + b*x**4)**p*b*e*p**5*x**6 + 12160*(a + b*x**4)**p*b*e*p**4*x**6 + 13...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file