Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.4-Linear-quadratic-binomial/70-1.2.1.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**391**]. This is test number [70].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (391)	0.00 (0)
Mathematica	100.00 (391)	0.00 (0)
Fricas	76.47 (299)	23.53 (92)
Maple	72.12 (282)	27.88 (109)
Giac	64.45 (252)	35.55 (139)
Reduce	63.43 (248)	36.57 (143)
Maxima	56.78 (222)	43.22 (169)
Mupad	54.22 (212)	45.78 (179)
Sympy	31.46 (123)	68.54 (268)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

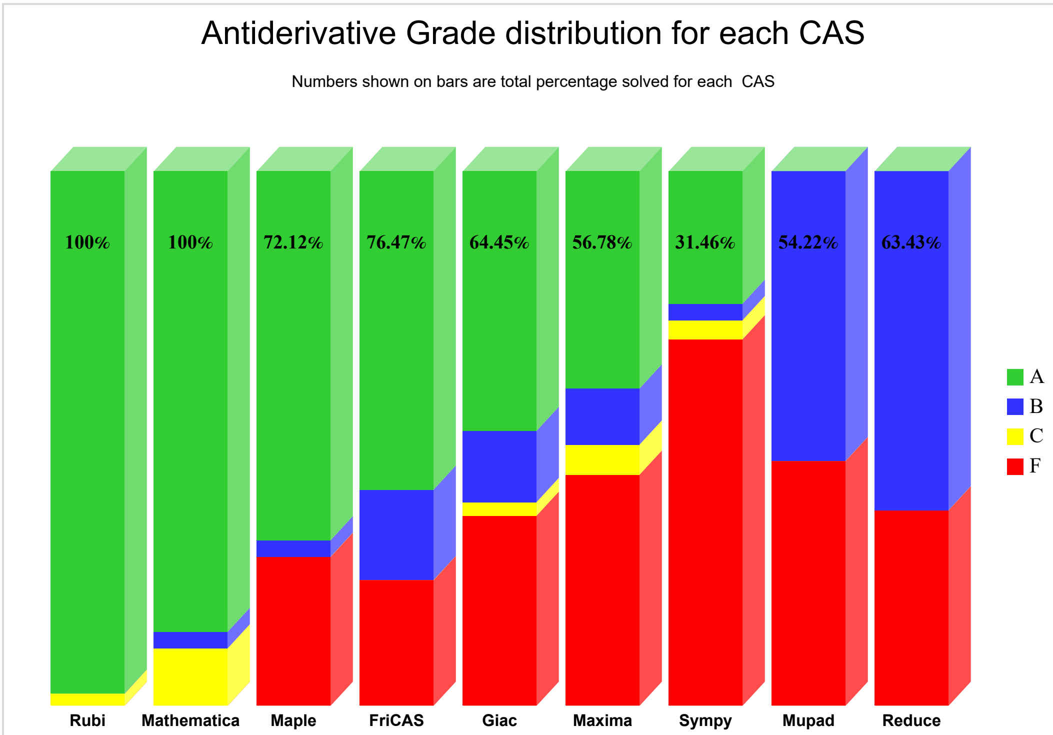
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

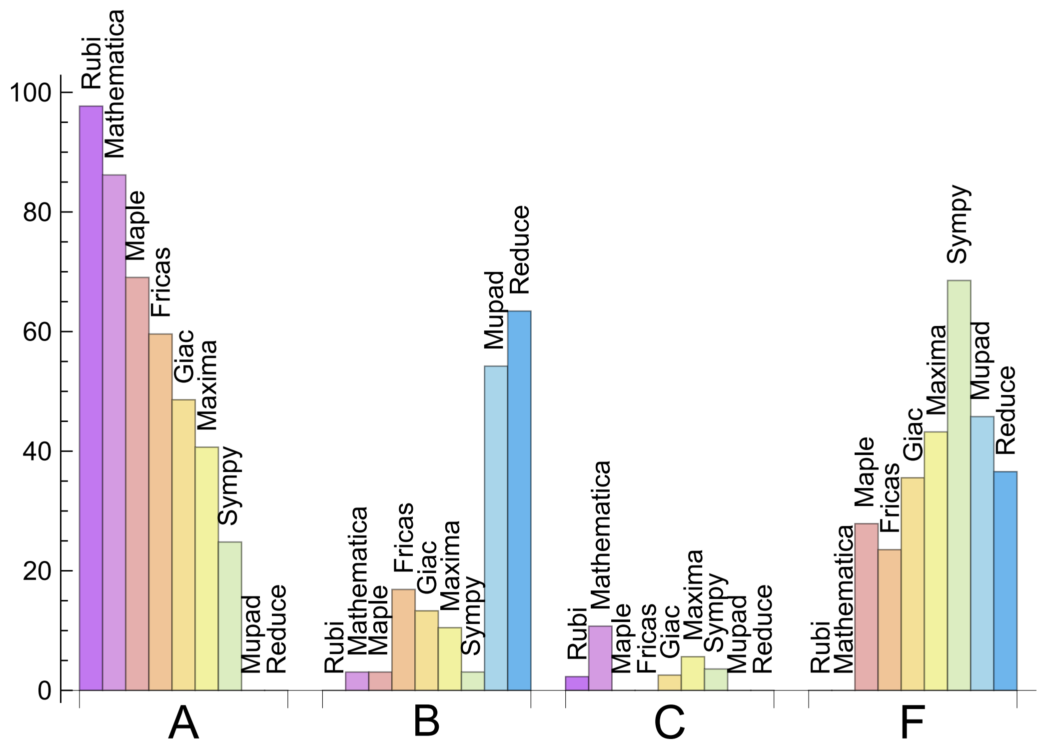
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.698	0.000	2.302	0.000
Mathematica	86.189	3.069	10.742	0.000
Maple	69.054	3.069	0.000	27.877
Fricas	59.591	16.880	0.000	23.529
Giac	48.593	13.299	2.558	35.550
Maxima	40.665	10.486	5.627	43.223
Sympy	24.808	3.069	3.581	68.542
Mupad	0.000	54.220	0.000	45.780
Reduce	0.000	63.427	0.000	36.573

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	92	100.00	0.00	0.00
Maple	109	100.00	0.00	0.00
Giac	139	91.37	0.00	8.63
Reduce	143	100.00	0.00	0.00
Maxima	169	100.00	0.00	0.00
Mupad	179	0.00	100.00	0.00
Sympy	268	93.28	6.72	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.11
Giac	0.13
Reduce	0.24
Maple	0.31
Rubi	0.36
Sympy	0.98
Mathematica	1.24
Mupad	5.56

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	77.47	1.04	65.50	0.96
Mathematica	80.00	0.95	72.00	0.95
Maple	83.59	1.00	55.00	0.90
Giac	107.39	1.30	76.50	0.93
Reduce	112.36	1.36	67.50	1.10
Maxima	115.92	1.47	66.50	1.05
Rubi	119.59	1.05	81.00	1.00
Fricas	133.43	1.44	94.00	1.29
Sympy	168.31	2.06	76.00	1.12

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

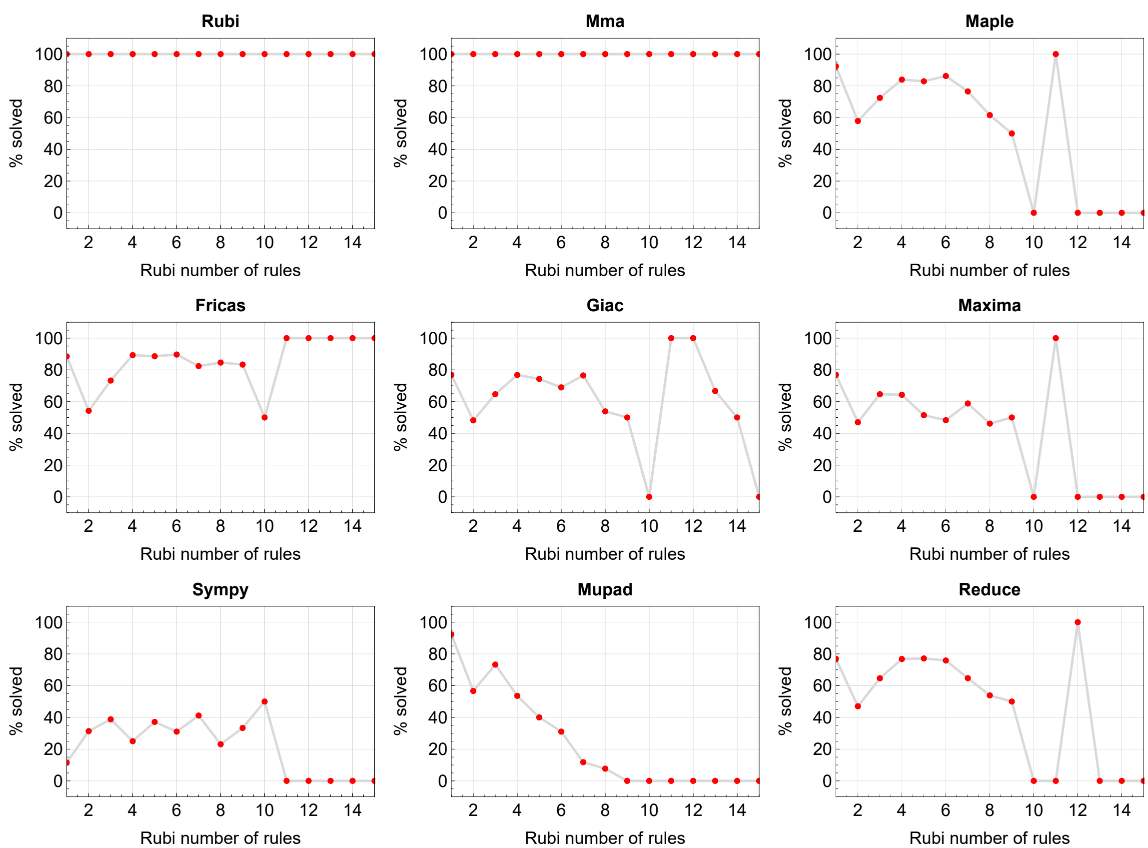


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

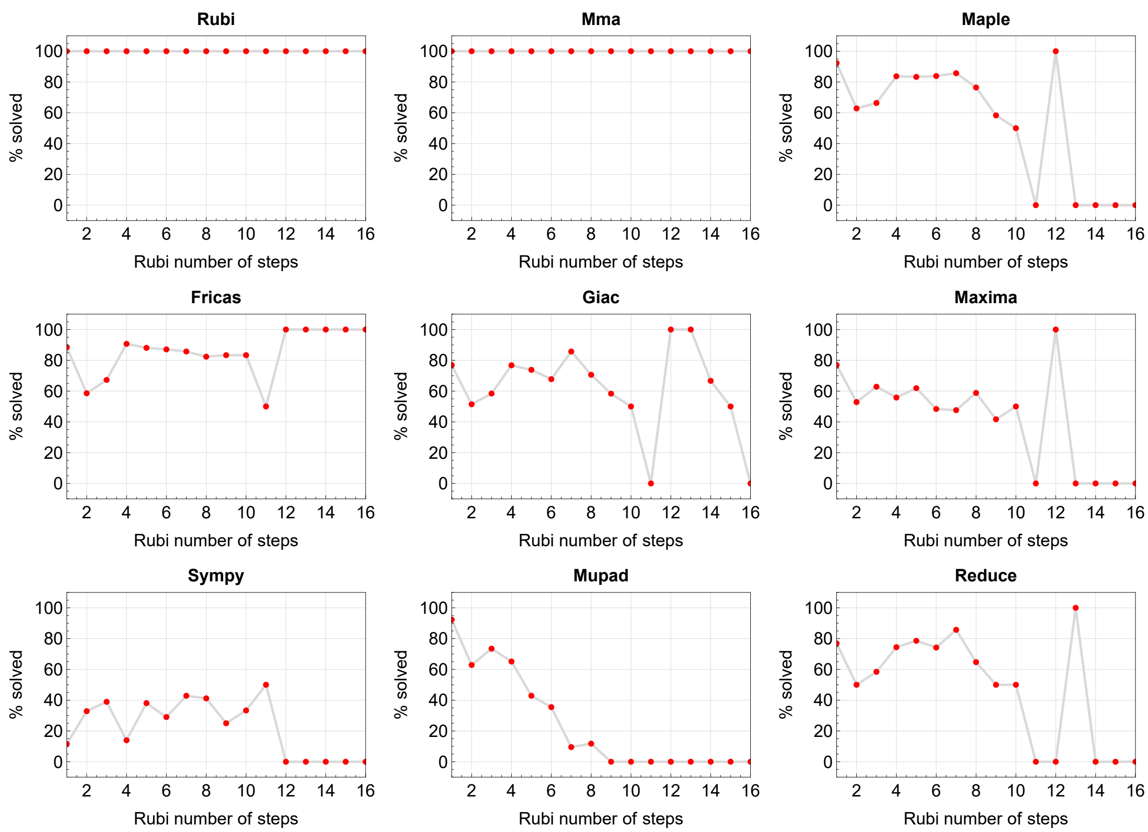


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

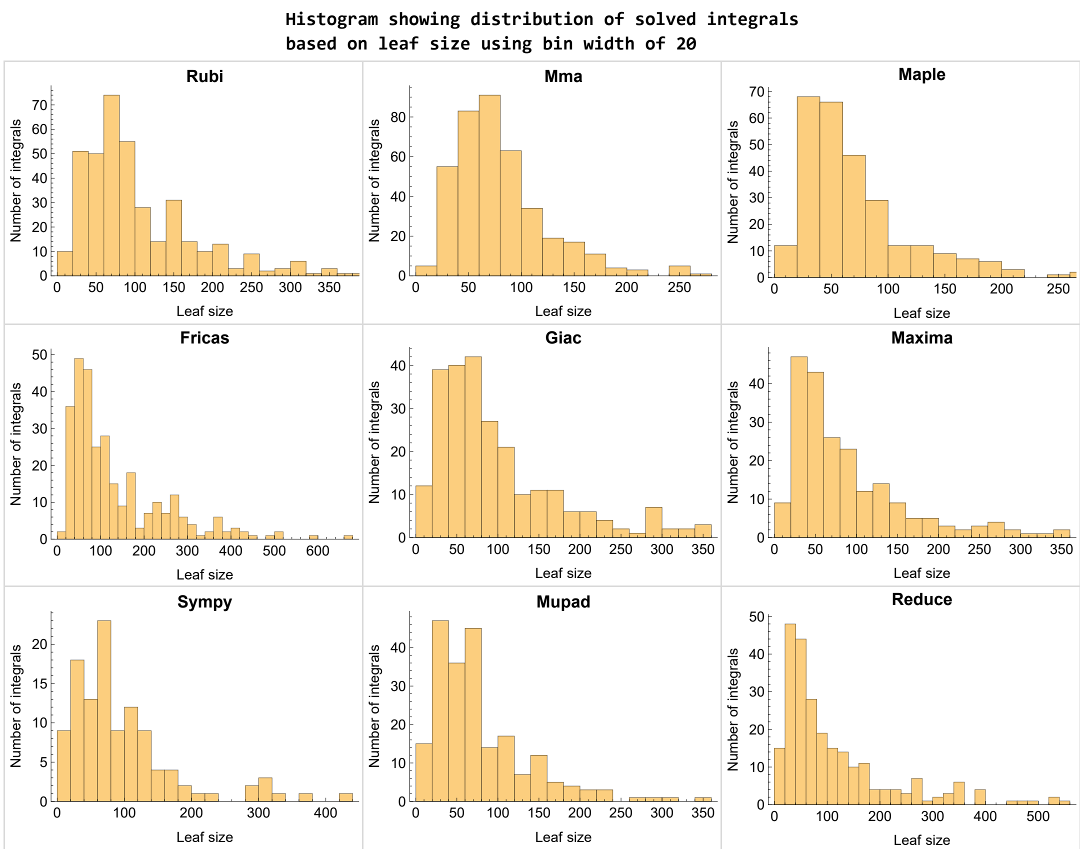


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

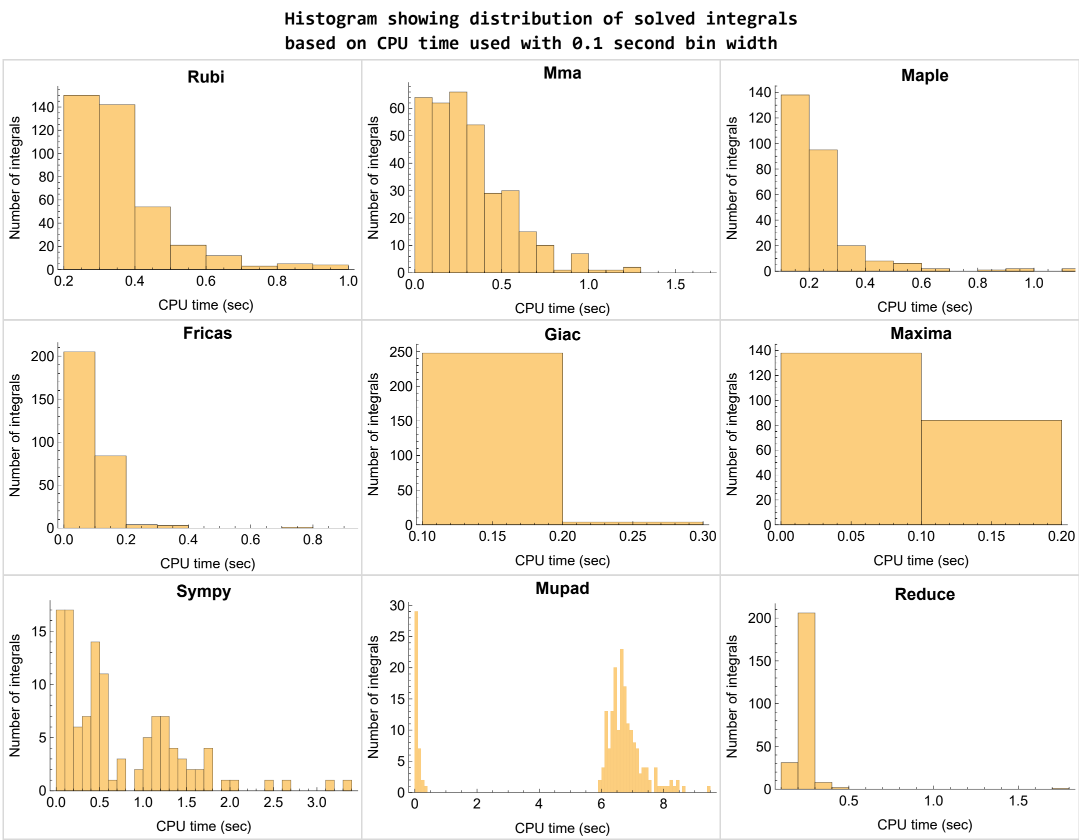


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

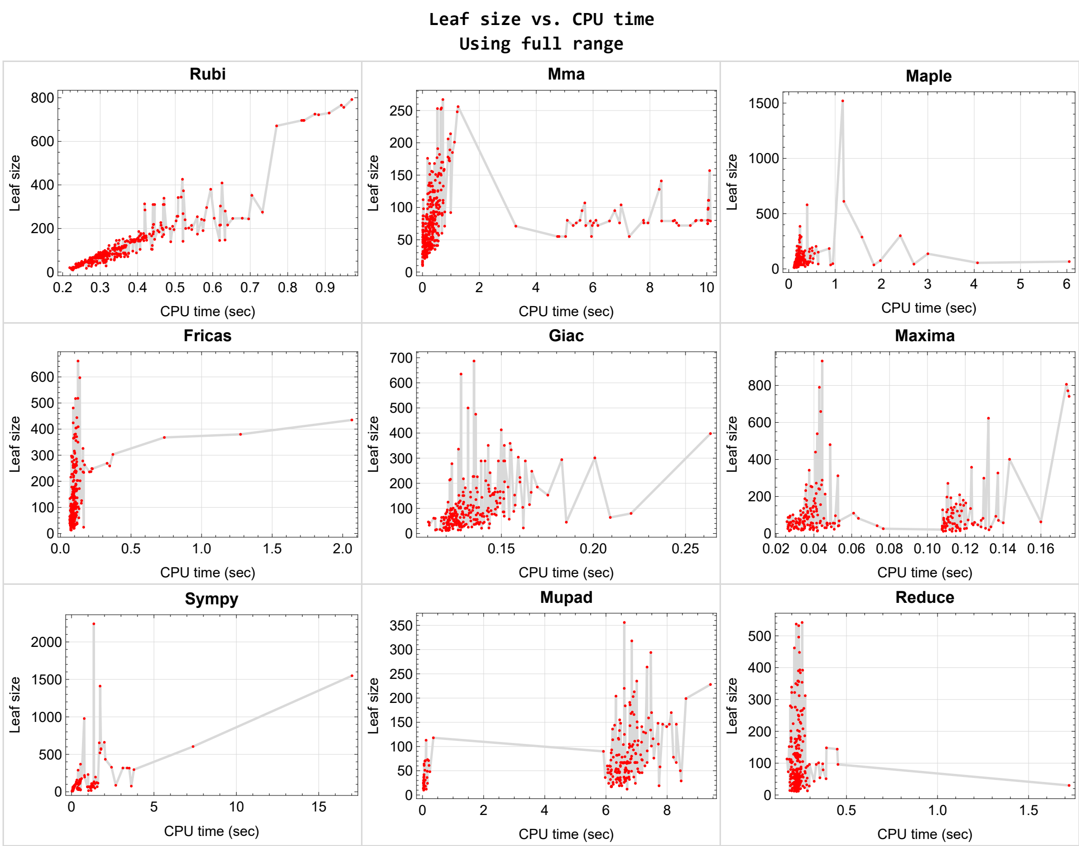


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {249, 250, 251, 252, 253, 256, 257, 258, 259, 260, 304, 305, 306, 311, 312, 313, 314, 327}

Mathematica {342, 362, 363, 366, 385}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

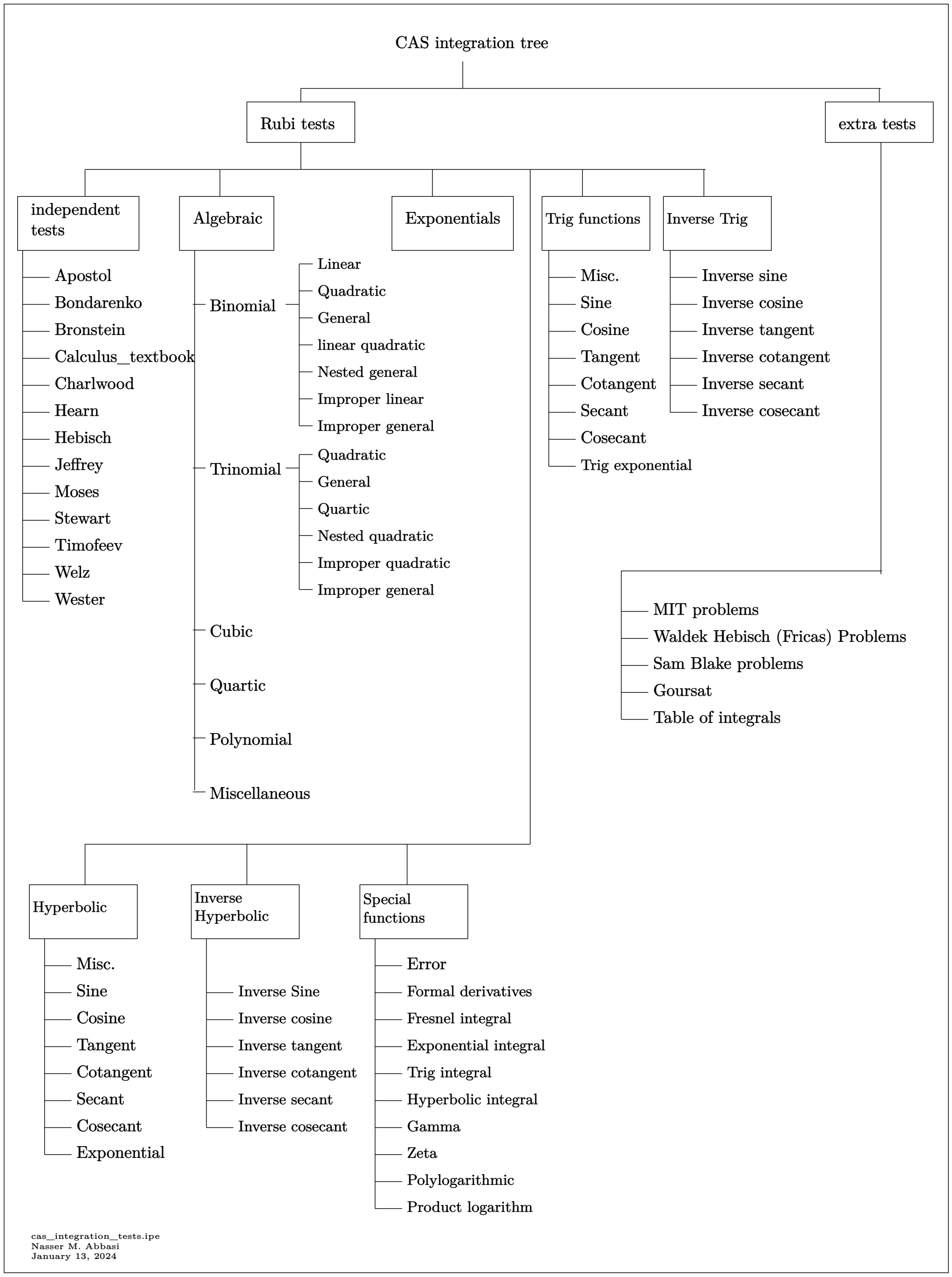
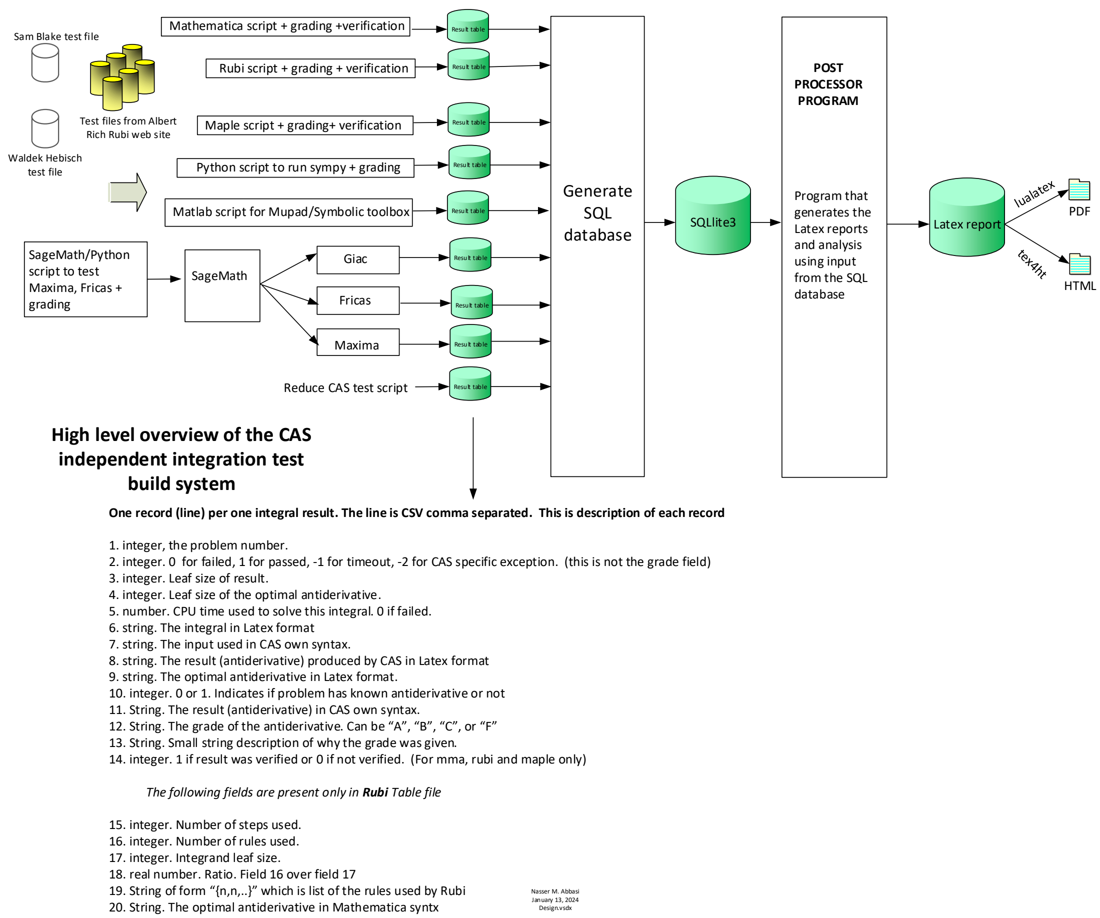


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391 }

B grade { }

C grade { 247, 248, 254, 255, 261, 262, 268, 269, 328 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 386, 387, 388, 391 }

B grade { 37, 78, 86, 163, 164, 227, 319, 326, 340, 341, 389, 390 }

C grade { 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 302, 303, 327, 328, 342, 362, 363, 366, 379, 380, 381, 385 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 240, 272, 273, 274, 275, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 298, 307, 308, 309, 310, 315, 316, 317, 318, 336, 337, 338, 348, 349, 350, 360, 361, 373, 374, 375, 379, 380, 381, 382 }

B grade { 44, 55, 70, 109, 129, 138, 149, 227, 239, 241, 372, 389 }

C grade { }

F normal fail { 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 276, 277, 278, 279, 280, 281, 282, 283, 288, 289, 290, 299, 300, 301, 302, 303, 304, 305, 306, 311, 312, 313, 314, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 356, 357, 358, 359, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 376, 377, 378, 383, 384, 385, 386, 387, 388, 390, 391 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17, 19, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 81, 82, 83, 84, 85, 87, 89, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 131, 136, 137, 138, 140, 141, 142, 146, 147, 148, 149, 152, 153, 154, 155, 159, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 221, 222, 223, 224, 225, 226, 230, 231, 232, 233, 237, 238, 240, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 304, 305, 306, 307, 308, 309, 310, 311, 312, 315, 316, 317, 318, 338, 360, 361, 375, 379, 380, 381, 382 }

B grade { 9, 10, 18, 20, 28, 29, 30, 45, 56, 57, 71, 72, 73, 74, 78, 79, 80, 86, 88, 96, 104, 124, 130, 132, 133, 134, 135, 139, 143, 144, 145, 150, 151, 156, 157, 158, 160, 161, 162, 163, 164, 199, 209, 210, 211, 212, 217, 218, 219, 220, 227, 228, 229, 234, 235, 236, 239, 241, 284, 313, 314, 336, 337, 372, 373, 374 }

C grade { }

F normal fail { 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 302, 303, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 376, 377, 378, 383, 384, 385, 386, 387, 388, 389, 390, 391 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 49, 50, 51, 53, 54, 55, 60, 61, 62, 66, 67, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 97, 99, 100, 101, 102, 103, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 127, 128, 131, 132, 133, 134, 141, 142, 143, 144, 145, 146, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 176, 182, 183, 184, 185, 189, 190, 191, 192, 196, 197, 198, 199, 238, 240, 272, 273, 274, 275, 295, 296, 297, 298, 360, 361, 372, 373, 374, 375 }

B grade { 45, 46, 47, 48, 56, 57, 58, 59, 68, 69, 70, 71, 72, 73, 74, 80, 88, 96, 98, 104, 106, 118, 125, 126, 129, 130, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 336, 337, 338 }

C grade { 52, 63, 64, 65, 206, 207, 208, 209, 213, 214, 215, 216, 217, 223, 224, 225, 226, 230, 231, 232, 233, 234 }

F normal fail { 169, 170, 171, 177, 178, 179, 180, 181, 186, 187, 188, 193, 194, 195, 200, 201, 202, 203, 204, 205, 210, 211, 212, 218, 219, 220, 221, 222, 227, 228, 229, 235, 236, 237, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 37, 39, 40, 41, 42, 43, 49, 50, 51, 52, 54, 55, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 86, 91, 92, 93, 94, 99, 100, 101, 102, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 120, 121, 122, 123, 127, 128, 129, 136, 137, 138, 147, 148, 149, 159, 163, 164, 167, 168, 169, 170, 171, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 305, 307, 309, 312, 313, 314, 316, 318, 360, 361 }

B grade { 34, 36, 38, 45, 46, 48, 53, 57, 58, 59, 64, 71, 72, 73, 74, 80, 81, 88, 89, 95, 96, 97, 98, 103, 104, 105, 106, 117, 118, 130, 131, 139, 140, 141, 150, 151, 152, 153, 154, 165, 166, 172, 173, 174, 175, 176, 238, 239, 336, 337, 338, 375 }

C grade { 47, 56, 82, 90, 116, 125, 134, 145, 161, 205 }

F normal fail { 124, 126, 132, 133, 135, 142, 143, 144, 146, 155, 156, 157, 158, 160, 162, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 295, 296, 297, 298, 299, 302, 303, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391 }

F(-1) timedout fail { }

F(-2) exception fail { 44, 79, 87, 300, 301, 304, 306, 308, 310, 311, 315, 317 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 45, 46, 47, 48, 51, 56, 57, 58, 59, 62, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 96, 97, 98, 101, 104, 105, 106, 108, 114, 115, 116, 117, 118, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 176, 182, 183, 184, 185, 189, 190, 191, 192, 196, 197, 198, 199, 206, 207, 208, 209, 213, 214, 215, 216, 217, 223, 224, 225, 226, 230, 231, 232, 233, 234, 238, 240, 244, 245, 251, 252, 258, 259, 265, 266, 270, 272, 273, 274, 275, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 298, 302, 307, 308, 309, 310, 315, 316, 317, 318, 321, 322, 327, 329, 330, 331, 332, 333, 334, 336, 337, 338, 349, 350, 355, 356, 360, 361, 372, 373, 374, 375, 379, 380, 381, 382 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 49, 50, 52, 53, 54, 55, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 91, 92, 94, 95, 99, 100, 102, 103, 107, 109, 110, 111, 112, 113, 119, 120, 121, 127, 128, 129, 136, 137, 138, 147, 148, 149, 169, 170, 171, 177, 178, 179, 180, 181, 186, 187, 188, 193, 194, 195, 200, 201, 202, 203, 204, 205, 210, 211, 212, 218, 219, 220, 221, 222, 227, 228, 229, 235, 236, 237, 239, 241, 242, 243, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 260, 261, 262, 263, 264, 267, 268, 269, 271, 276, 277, 278, 279, 280, 281, 282, 283, 288, 289, 290, 299, 300, 301, 303, 304, 305, 306, 311, 312, 313, 314, 319, 320, 323, 324, 325, 326, 328, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 357, 358, 359, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 376, 377, 378, 383, 384, 385, 386, 387, 388, 389, 390, 391 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 37, 39, 40, 41, 42, 49, 50, 51, 52, 60, 61, 62, 63, 64, 65, 75, 76, 77, 78, 83, 84, 85, 86, 91, 92, 93, 99, 100, 101, 108, 109, 110, 111, 112, 163, 164, 240, 242, 243, 244, 245, 249, 250, 251, 252, 256, 257, 258, 259, 263, 264, 265, 266, 270, 302, 321, 327, 329, 330, 331, 332, 333, 334, 349, 354, 355, 360, 361, 389 }

B grade { 25, 26, 31, 33, 35, 113, 114, 336, 337, 338, 374, 375 }

C grade { 123, 132, 143, 158, 159, 238, 319, 320, 322, 348, 350, 351, 356, 357 }

F normal fail { 32, 34, 36, 38, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 79, 80, 81, 82, 87, 88, 89, 90, 94, 95, 96, 97, 98, 102, 103, 104, 105, 106, 107, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 224, 225, 226, 227, 228, 229, 233, 234, 235, 236, 237, 239, 241, 246, 247, 248, 253, 254, 255, 260, 261, 262, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 311, 312, 313, 314, 315, 316, 317, 323, 324, 325, 326, 328, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 352, 353, 358, 359, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 390, 391 }

F(-1) timedout fail { 71, 72, 73, 74, 206, 213, 220, 221, 222, 223, 230, 231, 232, 286, 287, 309, 310, 318 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 272, 273, 274, 275, 284, 285, 286, 287, 298, 307, 308, 314, 316, 317, 318, 336, 337, 338, 360, 361 }

C grade { }

F normal fail { 47, 57, 70, 82, 90, 147, 148, 149, 150, 151, 152, 153, 154, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 276, 277, 278, 279, 280, 281, 282, 283, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 309, 310, 311, 312, 313, 315, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	60	60	65	65	72	64	60
N.S.	1	1.00	0.78	0.72	0.72	0.78	0.78	0.87	0.77	0.72
time (sec)	N/A	0.338	0.006	0.146	0.028	0.092	0.084	0.123	0.259	6.113

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	54	49	49	54	53	61	53	49
N.S.	1	1.00	0.82	0.74	0.74	0.82	0.80	0.92	0.80	0.74
time (sec)	N/A	0.309	0.006	0.154	0.035	0.066	0.080	0.113	0.208	6.162

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	38	38	42	37	49	42	38
N.S.	1	1.00	0.80	0.78	0.78	0.86	0.76	1.00	0.86	0.78
time (sec)	N/A	0.290	0.006	0.125	0.028	0.073	0.068	0.127	0.250	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	27	31	26	38	31	27
N.S.	1	1.00	1.00	0.96	0.96	1.11	0.93	1.36	1.11	0.96
time (sec)	N/A	0.270	0.005	0.125	0.029	0.070	0.059	0.123	0.242	0.051

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	18	20	14	19	19	18
N.S.	1	1.00	1.00	1.06	1.06	1.18	0.82	1.12	1.12	1.06
time (sec)	N/A	0.263	0.003	0.123	0.033	0.073	0.057	0.124	0.202	6.178

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	8	14	12	13
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.67	1.17	1.00	1.08
time (sec)	N/A	0.224	0.001	0.111	0.042	0.077	0.024	0.114	0.236	0.031

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	47	44	47	49	39	48	58	31
N.S.	1	1.00	1.34	1.26	1.34	1.40	1.11	1.37	1.66	0.89
time (sec)	N/A	0.279	0.012	0.164	0.037	0.067	0.120	0.121	0.195	0.090

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	54	67	89	58	51	112	51
N.S.	1	1.00	1.12	1.04	1.29	1.71	1.12	0.98	2.15	0.98
time (sec)	N/A	0.307	0.014	0.174	0.035	0.071	0.155	0.124	0.188	0.078

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	71	63	90	134	83	70	168	71
N.S.	1	1.00	1.03	0.91	1.30	1.94	1.20	1.01	2.43	1.03
time (sec)	N/A	0.345	0.018	0.179	0.035	0.068	0.201	0.123	0.261	0.094

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	74	112	178	107	81	225	93
N.S.	1	1.00	0.95	0.86	1.30	2.07	1.24	0.94	2.62	1.08
time (sec)	N/A	0.366	0.019	0.186	0.037	0.080	0.249	0.125	0.250	6.187

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	65	66	88	65	78	86	64
N.S.	1	1.00	1.01	0.93	0.94	1.26	0.93	1.11	1.23	0.91
time (sec)	N/A	0.365	0.019	0.136	0.029	0.071	0.121	0.123	0.278	0.052

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	54	55	76	49	66	75	53
N.S.	1	1.00	1.02	0.98	1.00	1.38	0.89	1.20	1.36	0.96
time (sec)	N/A	0.338	0.025	0.132	0.039	0.083	0.109	0.123	0.229	6.068

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	43	44	65	37	55	64	42
N.S.	1	1.00	1.02	0.98	1.00	1.48	0.84	1.25	1.45	0.95
time (sec)	N/A	0.313	0.016	0.130	0.038	0.066	0.100	0.119	0.194	0.050

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	32	33	51	26	34	50	31
N.S.	1	1.00	1.03	1.03	1.06	1.65	0.84	1.10	1.61	1.00
time (sec)	N/A	0.289	0.015	0.128	0.041	0.071	0.079	0.122	0.193	0.047

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	27	28	33	19	29	37	26
N.S.	1	1.00	0.88	1.04	1.08	1.27	0.73	1.12	1.42	1.00
time (sec)	N/A	0.277	0.008	0.123	0.035	0.076	0.075	0.123	0.206	0.045

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	14	14	10	14	13	12
N.S.	1	1.00	1.00	1.08	1.17	1.17	0.83	1.17	1.08	1.00
time (sec)	N/A	0.226	0.002	0.118	0.027	0.073	0.057	0.122	0.199	0.055

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	47	50	47	48	54	37	50	59	32
N.S.	1	1.31	1.39	1.31	1.33	1.50	1.03	1.39	1.64	0.89
time (sec)	N/A	0.268	0.010	0.133	0.029	0.070	0.107	0.117	0.251	6.147

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	87	71	90	136	85	79	168	70
N.S.	1	1.00	1.24	1.01	1.29	1.94	1.21	1.13	2.40	1.00
time (sec)	N/A	0.340	0.019	0.181	0.040	0.104	0.188	0.124	0.195	0.085

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	75	85	101	150	92	99	180	82
N.S.	1	1.00	0.86	0.98	1.16	1.72	1.06	1.14	2.07	0.94
time (sec)	N/A	0.376	0.042	0.181	0.036	0.083	0.227	0.123	0.196	6.348

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	112	96	135	227	133	101	276	115
N.S.	1	1.00	1.08	0.92	1.30	2.18	1.28	0.97	2.65	1.11
time (sec)	N/A	0.406	0.023	0.188	0.038	0.080	0.292	0.131	0.218	6.419

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	73	62	75	106	71	76	112	72
N.S.	1	1.00	1.03	0.87	1.06	1.49	1.00	1.07	1.58	1.01
time (sec)	N/A	0.374	0.026	0.136	0.033	0.085	0.172	0.126	0.186	6.162

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	62	51	64	95	60	65	101	61
N.S.	1	1.00	1.03	0.85	1.07	1.58	1.00	1.08	1.68	1.02
time (sec)	N/A	0.338	0.025	0.131	0.034	0.080	0.159	0.119	0.199	0.055

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	42	55	82	48	46	88	52
N.S.	1	1.00	0.84	0.86	1.12	1.67	0.98	0.94	1.80	1.06
time (sec)	N/A	0.316	0.038	0.129	0.031	0.092	0.123	0.127	0.198	6.180

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	36	49	60	41	40	70	46
N.S.	1	1.00	0.70	0.84	1.14	1.40	0.95	0.93	1.63	1.07
time (sec)	N/A	0.309	0.015	0.126	0.032	0.081	0.122	0.119	0.230	6.361

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	20	20	17	11	37	10
N.S.	1	1.00	1.00	1.10	2.00	2.00	1.70	1.10	3.70	1.00
time (sec)	N/A	0.227	0.005	0.123	0.028	0.098	0.092	0.118	0.236	0.039

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	24	24	24	14	23	24
N.S.	1	1.00	1.00	0.93	1.60	1.60	1.60	0.93	1.53	1.60
time (sec)	N/A	0.228	0.001	0.119	0.034	0.074	0.085	0.120	0.236	0.031

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	62	55	67	89	58	60	113	51
N.S.	1	1.00	1.15	1.02	1.24	1.65	1.07	1.11	2.09	0.94
time (sec)	N/A	0.316	0.012	0.148	0.029	0.091	0.153	0.125	0.174	6.317

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	65	69	90	134	87	79	168	70
N.S.	1	1.08	0.92	0.97	1.27	1.89	1.23	1.11	2.37	0.99
time (sec)	N/A	0.308	0.026	0.161	0.037	0.098	0.192	0.120	0.224	0.079

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	87	93	132	216	134	101	280	113
N.S.	1	1.00	0.83	0.89	1.26	2.06	1.28	0.96	2.67	1.08
time (sec)	N/A	0.437	0.031	0.188	0.041	0.074	0.296	0.127	0.192	0.118

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	98	104	156	266	158	125	339	136
N.S.	1	1.00	0.80	0.85	1.28	2.18	1.30	1.02	2.78	1.11
time (sec)	N/A	0.439	0.037	0.201	0.044	0.092	0.325	0.121	0.199	6.216

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	41	26	42	61	29	22	36
N.S.	1	1.00	1.57	1.46	0.93	1.50	2.18	1.04	0.79	1.29
time (sec)	N/A	0.248	0.103	0.167	0.113	0.078	0.365	0.129	0.204	5.976

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	44	53	26	42	0	29	23	36
N.S.	1	1.00	1.57	1.89	0.93	1.50	0.00	1.04	0.82	1.29
time (sec)	N/A	0.246	0.091	0.157	0.120	0.078	0.000	0.128	0.270	0.041

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	81	69	60	53	60	112	45	56	0
N.S.	1	1.33	1.13	0.98	0.87	0.98	1.84	0.74	0.92	0.00
time (sec)	N/A	0.324	0.231	0.202	0.124	0.093	0.510	0.141	0.242	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	84	69	60	64	60	0	135	56	0
N.S.	1	1.38	1.13	0.98	1.05	0.98	0.00	2.21	0.92	0.00
time (sec)	N/A	0.333	0.230	0.227	0.118	0.079	0.000	0.136	0.263	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	81	72	61	53	60	112	45	56	0
N.S.	1	1.31	1.16	0.98	0.85	0.97	1.81	0.73	0.90	0.00
time (sec)	N/A	0.309	0.209	0.187	0.113	0.098	0.492	0.137	0.229	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	82	72	61	63	60	0	121	56	0
N.S.	1	1.32	1.16	0.98	1.02	0.97	0.00	1.95	0.90	0.00
time (sec)	N/A	0.307	0.221	0.239	0.160	0.099	0.000	0.149	0.223	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	105	168	87	93	156	129	77	73	0
N.S.	1	1.31	2.10	1.09	1.16	1.95	1.61	0.96	0.91	0.00
time (sec)	N/A	0.352	0.250	0.227	0.035	0.094	0.492	0.146	0.205	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	109	151	87	100	156	0	164	73	0
N.S.	1	1.36	1.89	1.09	1.25	1.95	0.00	2.05	0.91	0.00
time (sec)	N/A	0.359	0.258	0.212	0.042	0.089	0.000	0.166	0.180	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	184	114	105	121	106	165	91	152	0
N.S.	1	1.23	0.76	0.70	0.81	0.71	1.10	0.61	1.01	0.00
time (sec)	N/A	0.457	0.481	0.232	0.116	0.098	0.520	0.142	0.200	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	148	111	94	96	95	150	81	128	0
N.S.	1	1.26	0.95	0.80	0.82	0.81	1.28	0.69	1.09	0.00
time (sec)	N/A	0.385	0.277	0.217	0.117	0.132	0.490	0.137	0.212	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	112	92	83	73	84	138	69	104	0
N.S.	1	1.30	1.07	0.97	0.85	0.98	1.60	0.80	1.21	0.00
time (sec)	N/A	0.337	0.299	0.187	0.122	0.079	0.462	0.140	0.204	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	78	89	72	53	73	114	56	80	75
N.S.	1	1.03	1.17	0.95	0.70	0.96	1.50	0.74	1.05	0.99
time (sec)	N/A	0.290	0.152	0.160	0.114	0.108	0.401	0.137	0.194	6.826

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	63	49	31	52	0	36	30	0
N.S.	1	1.00	1.37	1.07	0.67	1.13	0.00	0.78	0.65	0.00
time (sec)	N/A	0.265	0.112	0.205	0.110	0.089	0.000	0.135	0.241	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	63	130	40	66	0	0	59	0
N.S.	1	1.00	1.17	2.41	0.74	1.22	0.00	0.00	1.09	0.00
time (sec)	N/A	0.280	0.247	0.220	0.108	0.078	0.000	0.000	0.193	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	40	36	69	66	0	74	111	35
N.S.	1	1.00	1.21	1.09	2.09	2.00	0.00	2.24	3.36	1.06
time (sec)	N/A	0.238	0.247	0.256	0.033	0.075	0.000	0.136	0.196	6.686

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	123	104	0	165	68	47
N.S.	1	1.00	0.76	0.64	1.84	1.55	0.00	2.46	1.01	0.70
time (sec)	N/A	0.289	0.342	0.266	0.033	0.082	0.000	0.129	0.212	6.622

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	63	55	188	138	0	178	24	114
N.S.	1	1.08	0.63	0.55	1.88	1.38	0.00	1.78	0.24	1.14
time (sec)	N/A	0.338	0.447	0.326	0.035	0.078	0.000	0.143	200.038	6.221

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	149	74	66	264	171	0	289	322	143
N.S.	1	1.12	0.56	0.50	1.98	1.29	0.00	2.17	2.42	1.08
time (sec)	N/A	0.381	0.482	0.372	0.036	0.091	0.000	0.139	0.216	6.785

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	177	133	116	116	116	177	104	176	0
N.S.	1	1.26	0.94	0.82	0.82	0.82	1.26	0.74	1.25	0.00
time (sec)	N/A	0.416	0.353	0.216	0.110	0.103	0.611	0.138	0.218	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	141	114	105	93	105	163	92	152	0
N.S.	1	1.28	1.04	0.95	0.85	0.95	1.48	0.84	1.38	0.00
time (sec)	N/A	0.374	0.413	0.184	0.112	0.072	0.511	0.134	0.224	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	107	111	94	73	94	143	81	128	67
N.S.	1	1.07	1.11	0.94	0.73	0.94	1.43	0.81	1.28	0.67
time (sec)	N/A	0.321	0.226	0.165	0.111	0.089	0.477	0.132	0.214	8.299

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	78	89	72	88	72	134	56	80	0
N.S.	1	1.03	1.17	0.95	1.16	0.95	1.76	0.74	1.05	0.00
time (sec)	N/A	0.292	0.149	0.220	0.111	0.078	1.364	0.136	0.207	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	82	72	61	63	60	0	121	56	0
N.S.	1	1.32	1.16	0.98	1.02	0.97	0.00	1.95	0.90	0.00
time (sec)	N/A	0.318	0.237	0.238	0.111	0.076	0.000	0.148	0.208	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	94	79	83	0	77	123	0
N.S.	1	1.00	1.01	1.22	1.03	1.08	0.00	1.00	1.60	0.00
time (sec)	N/A	0.313	0.257	0.277	0.110	0.076	0.000	0.145	0.224	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	74	291	127	110	0	86	131	0
N.S.	1	1.00	0.89	3.51	1.53	1.33	0.00	1.04	1.58	0.00
time (sec)	N/A	0.323	0.323	0.268	0.113	0.099	0.000	0.143	0.216	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	179	96	0	160	69	37
N.S.	1	1.00	1.24	1.09	5.42	2.91	0.00	4.85	2.09	1.12
time (sec)	N/A	0.238	0.409	0.320	0.036	0.075	0.000	0.145	0.274	6.909

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	48	43	255	136	0	227	24	112
N.S.	1	1.00	0.72	0.64	3.81	2.03	0.00	3.39	0.36	1.67
time (sec)	N/A	0.281	0.422	0.385	0.038	0.089	0.000	0.135	200.032	7.055

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	60	55	342	170	0	289	322	141
N.S.	1	1.08	0.60	0.55	3.42	1.70	0.00	2.89	3.22	1.41
time (sec)	N/A	0.346	0.513	0.431	0.038	0.088	0.000	0.141	0.199	7.171

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	149	71	66	440	203	0	351	392	170
N.S.	1	1.12	0.53	0.50	3.31	1.53	0.00	2.64	2.95	1.28
time (sec)	N/A	0.391	0.567	0.460	0.041	0.118	0.000	0.143	0.259	7.491

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	235	177	160	165	160	231	150	272	0
N.S.	1	1.24	0.94	0.85	0.87	0.85	1.22	0.79	1.44	0.00
time (sec)	N/A	0.502	0.511	0.277	0.120	0.088	1.003	0.144	0.266	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	199	158	149	142	149	219	139	248	0
N.S.	1	1.26	1.00	0.94	0.90	0.94	1.39	0.88	1.57	0.00
time (sec)	N/A	0.466	0.757	0.214	0.114	0.094	0.786	0.151	0.238	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	165	155	138	122	138	197	129	224	67
N.S.	1	1.11	1.05	0.93	0.82	0.93	1.33	0.87	1.51	0.45
time (sec)	N/A	0.405	0.357	0.178	0.118	0.079	0.795	0.147	0.207	6.941

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	133	116	129	116	520	103	176	0
N.S.	1	1.10	1.07	0.94	1.04	0.94	4.19	0.83	1.42	0.00
time (sec)	N/A	0.380	0.282	0.237	0.111	0.106	1.723	0.137	0.196	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	142	114	105	139	105	328	248	152	0
N.S.	1	1.29	1.04	0.95	1.26	0.95	2.98	2.25	1.38	0.00
time (sec)	N/A	0.402	0.478	0.261	0.113	0.088	2.426	0.167	0.186	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	150	111	94	160	94	316	80	128	0
N.S.	1	1.15	0.85	0.72	1.23	0.72	2.43	0.62	0.98	0.00
time (sec)	N/A	0.445	0.318	0.303	0.115	0.093	3.502	0.140	0.210	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	144	92	83	156	83	0	69	104	0
N.S.	1	1.26	0.81	0.73	1.37	0.73	0.00	0.61	0.91	0.00
time (sec)	N/A	0.417	0.374	0.349	0.117	0.086	0.000	0.142	0.193	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	147	107	117	197	110	0	185	201	0
N.S.	1	1.12	0.82	0.89	1.50	0.84	0.00	1.41	1.53	0.00
time (sec)	N/A	0.633	0.502	0.467	0.110	0.089	0.000	0.170	0.195	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	141	101	151	271	144	0	155	275	0
N.S.	1	1.01	0.73	1.09	1.95	1.04	0.00	1.12	1.98	0.00
time (sec)	N/A	0.521	0.609	0.636	0.111	0.086	0.000	0.145	0.199	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	147	103	184	401	172	0	206	384	0
N.S.	1	1.04	0.73	1.30	2.84	1.22	0.00	1.46	2.72	0.00
time (sec)	N/A	0.408	0.483	0.872	0.143	0.089	0.000	0.152	0.247	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	98	611	623	200	0	210	24	0
N.S.	1	1.00	0.69	4.27	4.36	1.40	0.00	1.47	0.17	0.00
time (sec)	N/A	0.407	0.564	1.188	0.132	0.090	0.000	0.142	200.036	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	539	162	0	167	252	141
N.S.	1	1.00	1.24	1.09	16.33	4.91	0.00	5.06	7.64	4.27
time (sec)	N/A	0.245	0.534	1.836	0.042	0.093	0.000	0.149	0.257	7.986

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	48	43	659	202	0	351	390	170
N.S.	1	1.00	0.72	0.64	9.84	3.01	0.00	5.24	5.82	2.54
time (sec)	N/A	0.290	0.555	2.700	0.044	0.139	0.000	0.152	0.236	8.136

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	60	55	790	236	0	413	462	199
N.S.	1	1.08	0.60	0.55	7.90	2.36	0.00	4.13	4.62	1.99
time (sec)	N/A	0.347	0.683	4.072	0.043	0.206	0.000	0.150	0.214	8.616

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	149	71	66	932	269	0	475	532	228
N.S.	1	1.12	0.53	0.50	7.01	2.02	0.00	3.57	4.00	1.71
time (sec)	N/A	0.395	0.780	6.051	0.044	0.331	0.000	0.136	0.238	9.418

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	86	58	42	54	50	73	35	66	35
N.S.	1	1.37	0.92	0.67	0.86	0.79	1.16	0.56	1.05	0.56
time (sec)	N/A	0.314	0.151	0.245	0.110	0.081	0.151	0.124	0.200	6.682

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	61	53	37	40	45	34	30	53	30
N.S.	1	1.42	1.23	0.86	0.93	1.05	0.79	0.70	1.23	0.70
time (sec)	N/A	0.276	0.117	0.176	0.108	0.074	0.367	0.130	0.285	6.636

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	29	28	40	39	25	40	25
N.S.	1	1.00	1.21	0.76	0.74	1.05	1.03	0.66	1.05	0.66
time (sec)	N/A	0.245	0.083	0.164	0.108	0.073	0.088	0.128	0.213	0.031

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	32	18	12	28	15	12	12	12
N.S.	1	1.00	2.29	1.29	0.86	2.00	1.07	0.86	0.86	0.86
time (sec)	N/A	0.228	0.076	0.160	0.108	0.099	1.030	0.121	0.213	6.690

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	39	20	21	43	0	0	31	21
N.S.	1	1.00	1.70	0.87	0.91	1.87	0.00	0.00	1.35	0.91
time (sec)	N/A	0.239	0.120	0.196	0.112	0.088	0.000	0.000	0.194	0.045

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	20	38	34	0	41	64	19
N.S.	1	1.00	1.15	1.00	1.90	1.70	0.00	2.05	3.20	0.95
time (sec)	N/A	0.228	0.154	0.145	0.035	0.083	0.000	0.126	0.215	6.519

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	23	64	51	0	93	34	24
N.S.	1	1.00	0.68	0.56	1.56	1.24	0.00	2.27	0.83	0.59
time (sec)	N/A	0.256	0.140	0.151	0.026	0.079	0.000	0.133	0.231	0.043

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	35	30	95	68	0	118	17	31
N.S.	1	1.08	0.57	0.49	1.56	1.11	0.00	1.93	0.28	0.51
time (sec)	N/A	0.273	0.188	0.154	0.036	0.081	0.000	0.125	200.036	0.046

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	90	58	42	54	50	73	35	66	35
N.S.	1	1.38	0.89	0.65	0.83	0.77	1.12	0.54	1.02	0.54
time (sec)	N/A	0.301	0.124	0.208	0.109	0.070	0.194	0.130	0.218	6.608

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	63	53	37	40	45	34	30	53	30
N.S.	1	1.47	1.23	0.86	0.93	1.05	0.79	0.70	1.23	0.70
time (sec)	N/A	0.274	0.102	0.187	0.109	0.075	0.377	0.123	0.199	0.034

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	29	28	40	39	25	40	25
N.S.	1	1.00	1.21	0.76	0.74	1.05	1.03	0.66	1.05	0.66
time (sec)	N/A	0.236	0.083	0.164	0.131	0.073	0.086	0.132	0.189	0.034

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	34	19	14	30	17	14	14	14
N.S.	1	1.00	2.12	1.19	0.88	1.88	1.06	0.88	0.88	0.88
time (sec)	N/A	0.225	0.077	0.177	0.110	0.098	1.191	0.128	0.189	0.027

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	39	20	21	41	0	0	29	21
N.S.	1	1.00	1.56	0.80	0.84	1.64	0.00	0.00	1.16	0.84
time (sec)	N/A	0.240	0.132	0.187	0.108	0.081	0.000	0.000	0.192	0.043

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	25	20	38	33	0	41	65	19
N.S.	1	1.00	1.14	0.91	1.73	1.50	0.00	1.86	2.95	0.86
time (sec)	N/A	0.229	0.152	0.158	0.031	0.081	0.000	0.127	0.197	0.040

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	28	23	64	50	0	93	34	26
N.S.	1	1.00	0.62	0.51	1.42	1.11	0.00	2.07	0.76	0.58
time (sec)	N/A	0.260	0.138	0.159	0.034	0.076	0.000	0.125	0.201	0.043

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	35	30	95	67	0	151	19	31
N.S.	1	1.07	0.52	0.45	1.42	1.00	0.00	2.25	0.28	0.46
time (sec)	N/A	0.295	0.190	0.172	0.027	0.090	0.000	0.129	200.041	6.470

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	88	54	42	58	44	82	44	62	0
N.S.	1	1.31	0.81	0.63	0.87	0.66	1.22	0.66	0.93	0.00
time (sec)	N/A	0.307	0.122	0.197	0.032	0.070	0.173	0.127	0.235	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	65	49	37	46	39	46	39	51	0
N.S.	1	1.33	1.00	0.76	0.94	0.80	0.94	0.80	1.04	0.00
time (sec)	N/A	0.281	0.098	0.161	0.027	0.075	0.365	0.119	0.236	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	33	36	34	48	34	40	32
N.S.	1	1.00	0.95	0.75	0.82	0.77	1.09	0.77	0.91	0.73
time (sec)	N/A	0.253	0.075	0.145	0.030	0.092	0.079	0.126	0.231	6.638

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	20	0	21	18	0
N.S.	1	1.00	1.00	0.88	1.00	0.83	0.00	0.88	0.75	0.00
time (sec)	N/A	0.241	0.058	0.145	0.032	0.093	0.000	0.120	0.219	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	24	29	37	0	64	39	0
N.S.	1	1.00	1.21	0.83	1.00	1.28	0.00	2.21	1.34	0.00
time (sec)	N/A	0.247	0.083	0.164	0.031	0.079	0.000	0.130	0.226	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	18	34	31	0	33	40	17
N.S.	1	1.00	1.17	1.00	1.89	1.72	0.00	1.83	2.22	0.94
time (sec)	N/A	0.226	0.080	0.152	0.040	0.070	0.000	0.124	0.219	6.169

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	26	21	58	46	0	60	60	22
N.S.	1	1.00	0.70	0.57	1.57	1.24	0.00	1.62	1.62	0.59
time (sec)	N/A	0.247	0.109	0.161	0.027	0.089	0.000	0.123	0.227	6.240

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	33	28	87	65	0	151	82	59
N.S.	1	1.09	0.60	0.51	1.58	1.18	0.00	2.75	1.49	1.07
time (sec)	N/A	0.276	0.142	0.164	0.026	0.076	0.000	0.130	0.237	6.149

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	92	54	43	58	44	82	44	62	0
N.S.	1	1.33	0.78	0.62	0.84	0.64	1.19	0.64	0.90	0.00
time (sec)	N/A	0.311	0.125	0.187	0.027	0.092	0.151	0.121	0.228	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	67	49	38	46	39	46	39	51	0
N.S.	1	1.37	1.00	0.78	0.94	0.80	0.94	0.80	1.04	0.00
time (sec)	N/A	0.280	0.100	0.172	0.027	0.067	0.340	0.128	0.217	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	33	36	34	48	34	40	32
N.S.	1	1.00	0.95	0.75	0.82	0.77	1.09	0.77	0.91	0.73
time (sec)	N/A	0.249	0.080	0.152	0.026	0.073	0.091	0.124	0.229	6.411

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	30	23	26	22	0	23	20	0
N.S.	1	1.00	1.15	0.88	1.00	0.85	0.00	0.88	0.77	0.00
time (sec)	N/A	0.242	0.076	0.173	0.026	0.092	0.000	0.119	0.229	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	35	24	29	37	0	64	41	0
N.S.	1	1.00	1.13	0.77	0.94	1.19	0.00	2.06	1.32	0.00
time (sec)	N/A	0.242	0.085	0.184	0.038	0.118	0.000	0.133	0.234	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	18	34	31	0	33	40	17
N.S.	1	1.00	1.15	0.90	1.70	1.55	0.00	1.65	2.00	0.85
time (sec)	N/A	0.226	0.088	0.173	0.030	0.081	0.000	0.127	0.248	6.183

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	26	21	58	46	0	60	60	24
N.S.	1	1.00	0.63	0.51	1.41	1.12	0.00	1.46	1.46	0.59
time (sec)	N/A	0.243	0.110	0.183	0.034	0.083	0.000	0.122	0.240	6.488

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	66	33	28	87	65	0	118	82	59
N.S.	1	1.08	0.54	0.46	1.43	1.07	0.00	1.93	1.34	0.97
time (sec)	N/A	0.288	0.155	0.187	0.027	0.074	0.000	0.129	0.229	6.473

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	50	32	50	0	37	30	0
N.S.	1	1.00	1.36	1.06	0.68	1.06	0.00	0.79	0.64	0.00
time (sec)	N/A	0.290	0.133	0.221	0.116	0.076	0.000	0.131	0.220	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	97	105	86	84	216	124	87	96	91
N.S.	1	1.02	1.11	0.91	0.88	2.27	1.31	0.92	1.01	0.96
time (sec)	N/A	0.327	0.187	0.246	0.035	0.093	0.443	0.129	0.218	7.349

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	135	116	181	113	238	148	101	121	0
N.S.	1	1.29	1.10	1.72	1.08	2.27	1.41	0.96	1.15	0.00
time (sec)	N/A	0.380	0.272	0.220	0.034	0.112	0.506	0.143	0.236	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	175	127	197	144	260	160	113	146	0
N.S.	1	1.25	0.91	1.41	1.03	1.86	1.14	0.81	1.04	0.00
time (sec)	N/A	0.441	0.369	0.254	0.036	0.111	0.557	0.133	0.225	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	153	92	83	110	83	139	68	104	0
N.S.	1	1.22	0.74	0.66	0.88	0.66	1.11	0.54	0.83	0.00
time (sec)	N/A	0.413	0.395	0.251	0.114	0.088	0.539	0.141	0.245	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	117	83	72	85	72	126	56	80	0
N.S.	1	1.27	0.90	0.78	0.92	0.78	1.37	0.61	0.87	0.00
time (sec)	N/A	0.367	0.294	0.235	0.113	0.082	0.516	0.135	0.232	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	81	69	60	62	60	112	45	56	0
N.S.	1	1.33	1.13	0.98	1.02	0.98	1.84	0.74	0.92	0.00
time (sec)	N/A	0.309	0.239	0.198	0.109	0.130	0.481	0.139	0.220	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	50	41	50	94	37	29	54
N.S.	1	1.00	1.21	1.06	0.87	1.06	2.00	0.79	0.62	1.15
time (sec)	N/A	0.269	0.126	0.170	0.110	0.081	0.426	0.136	0.221	7.037

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	30	35	0	41	32	29
N.S.	1	1.00	1.00	0.94	0.97	1.13	0.00	1.32	1.03	0.94
time (sec)	N/A	0.243	0.136	0.206	0.108	0.088	0.000	0.136	0.227	6.456

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	67	41	37	74	70	0	74	64	36
N.S.	1	1.22	0.75	0.67	1.35	1.27	0.00	1.35	1.16	0.65
time (sec)	N/A	0.282	0.231	0.226	0.109	0.083	0.000	0.131	0.230	6.469

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	108	52	49	128	104	0	165	120	48
N.S.	1	1.23	0.59	0.56	1.45	1.18	0.00	1.88	1.36	0.55
time (sec)	N/A	0.333	0.330	0.263	0.108	0.094	0.000	0.137	0.244	6.446

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	149	63	60	193	137	0	227	91	117
N.S.	1	1.37	0.58	0.55	1.77	1.26	0.00	2.08	0.83	1.07
time (sec)	N/A	0.390	0.396	0.260	0.112	0.109	0.000	0.135	0.286	6.404

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	145	108	120	135	113	0	97	200	0
N.S.	1	1.12	0.84	0.93	1.05	0.88	0.00	0.75	1.55	0.00
time (sec)	N/A	0.619	0.347	0.334	0.123	0.114	0.000	0.147	0.241	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	104	90	108	110	101	0	86	164	0
N.S.	1	1.01	0.87	1.05	1.07	0.98	0.00	0.83	1.59	0.00
time (sec)	N/A	0.439	0.398	0.273	0.113	0.113	0.000	0.151	0.217	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	96	83	85	0	76	123	0
N.S.	1	1.00	1.07	1.30	1.12	1.15	0.00	1.03	1.66	0.00
time (sec)	N/A	0.325	0.253	0.258	0.109	0.090	0.000	0.139	0.219	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	63	94	59	72	0	56	57	72
N.S.	1	1.00	1.21	1.81	1.13	1.38	0.00	1.08	1.10	1.38
time (sec)	N/A	0.279	0.274	0.194	0.112	0.097	0.000	0.143	0.213	7.187

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	31	30	38	39	87	41	32	29
N.S.	1	1.00	1.11	1.07	1.36	1.39	3.11	1.46	1.14	1.04
time (sec)	N/A	0.243	0.147	0.176	0.035	0.093	2.683	0.137	0.239	6.643

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	46	65	102	0	0	82	56
N.S.	1	1.00	1.03	0.79	1.12	1.76	0.00	0.00	1.41	0.97
time (sec)	N/A	0.273	0.232	0.234	0.031	0.126	0.000	0.000	0.243	6.647

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	99	70	66	136	115	0	193	126	66
N.S.	1	1.25	0.89	0.84	1.72	1.46	0.00	2.44	1.59	0.84
time (sec)	N/A	0.313	0.353	0.258	0.035	0.117	0.000	0.143	0.239	6.553

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	140	82	77	232	171	0	0	174	133
N.S.	1	1.25	0.73	0.69	2.07	1.53	0.00	0.00	1.55	1.19
time (sec)	N/A	0.367	0.441	0.302	0.036	0.113	0.000	0.000	0.227	6.442

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	139	100	156	209	143	0	155	276	0
N.S.	1	1.01	0.73	1.14	1.53	1.04	0.00	1.13	2.01	0.00
time (sec)	N/A	0.488	0.496	0.441	0.117	0.099	0.000	0.146	0.220	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	145	180	128	0	142	237	0
N.S.	1	1.00	0.86	1.33	1.65	1.17	0.00	1.30	2.17	0.00
time (sec)	N/A	0.357	0.387	0.323	0.119	0.092	0.000	0.149	0.237	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	74	281	152	112	0	86	131	0
N.S.	1	1.00	0.91	3.47	1.88	1.38	0.00	1.06	1.62	0.00
time (sec)	N/A	0.315	0.378	0.246	0.119	0.076	0.000	0.151	0.231	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	39	36	80	63	0	74	88	35
N.S.	1	1.00	1.18	1.09	2.42	1.91	0.00	2.24	2.67	1.06
time (sec)	N/A	0.250	0.233	0.237	0.030	0.079	0.000	0.139	0.251	6.381

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	39	58	71	0	103	66	38
N.S.	1	1.00	0.79	0.74	1.09	1.34	0.00	1.94	1.25	0.72
time (sec)	N/A	0.260	0.231	0.203	0.033	0.079	0.000	0.135	0.242	6.225

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	58	53	60	102	296	0	81	54
N.S.	1	1.00	1.04	0.95	1.07	1.82	5.29	0.00	1.45	0.96
time (sec)	N/A	0.260	0.238	0.188	0.028	0.114	3.785	0.000	0.249	6.337

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	90	82	70	85	168	0	0	174	78
N.S.	1	1.10	1.00	0.85	1.04	2.05	0.00	0.00	2.12	0.95
time (sec)	N/A	0.296	0.316	0.251	0.037	0.117	0.000	0.000	0.246	6.240

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	131	93	88	156	203	0	242	186	139
N.S.	1	1.27	0.90	0.85	1.51	1.97	0.00	2.35	1.81	1.35
time (sec)	N/A	0.348	0.443	0.269	0.036	0.123	0.000	0.144	0.239	7.020

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	172	104	99	252	234	0	0	267	168
N.S.	1	1.26	0.76	0.73	1.85	1.72	0.00	0.00	1.96	1.24
time (sec)	N/A	0.408	0.572	0.317	0.039	0.165	0.000	0.000	0.229	6.818

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	174	111	204	358	190	0	223	393	0
N.S.	1	1.04	0.66	1.22	2.14	1.14	0.00	1.34	2.35	0.00
time (sec)	N/A	0.559	0.724	0.589	0.123	0.109	0.000	0.160	0.244	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	144	105	192	327	175	0	205	354	0
N.S.	1	1.04	0.76	1.38	2.35	1.26	0.00	1.47	2.55	0.00
time (sec)	N/A	0.407	0.478	0.503	0.137	0.093	0.000	0.160	0.256	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	86	580	299	159	0	180	229	0
N.S.	1	1.00	0.77	5.18	2.67	1.42	0.00	1.61	2.04	0.00
time (sec)	N/A	0.363	0.463	0.395	0.130	0.099	0.000	0.147	0.246	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	148	100	0	105	146	37
N.S.	1	1.00	1.24	1.09	4.48	3.03	0.00	3.18	4.42	1.12
time (sec)	N/A	0.239	0.357	0.316	0.040	0.096	0.000	0.148	0.239	6.590

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	44	123	104	0	165	179	49
N.S.	1	1.00	0.79	0.66	1.84	1.55	0.00	2.46	2.67	0.73
time (sec)	N/A	0.287	0.374	0.249	0.038	0.102	0.000	0.147	0.239	6.747

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	111	53	50	101	106	0	165	120	49
N.S.	1	1.29	0.62	0.58	1.17	1.23	0.00	1.92	1.40	0.57
time (sec)	N/A	0.355	0.346	0.247	0.033	0.106	0.000	0.148	0.222	6.406

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	70	65	78	116	0	0	128	66
N.S.	1	1.06	0.91	0.84	1.01	1.51	0.00	0.00	1.66	0.86
time (sec)	N/A	0.300	0.338	0.210	0.037	0.110	0.000	0.000	0.233	6.580

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	82	77	80	171	604	0	174	78
N.S.	1	1.10	1.02	0.96	1.00	2.14	7.55	0.00	2.18	0.98
time (sec)	N/A	0.316	0.293	0.202	0.032	0.116	7.373	0.000	0.245	6.525

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	122	104	92	105	236	0	0	264	155
N.S.	1	1.15	0.98	0.87	0.99	2.23	0.00	0.00	2.49	1.46
time (sec)	N/A	0.324	0.371	0.269	0.039	0.160	0.000	0.000	0.249	6.448

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	163	115	110	176	248	0	294	312	184
N.S.	1	1.28	0.91	0.87	1.39	1.95	0.00	2.31	2.46	1.45
time (sec)	N/A	0.379	0.505	0.293	0.040	0.226	0.000	0.183	0.240	6.629

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	204	126	121	272	259	0	0	356	213
N.S.	1	1.28	0.79	0.76	1.70	1.62	0.00	0.00	2.22	1.33
time (sec)	N/A	0.440	0.541	0.326	0.041	0.349	0.000	0.000	0.238	6.925

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	244	133	300	806	278	0	359	24	0
N.S.	1	1.07	0.59	1.32	3.55	1.22	0.00	1.58	0.11	0.00
time (sec)	N/A	0.696	0.732	2.406	0.174	0.158	0.000	0.155	200.033	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	214	127	288	771	263	0	333	24	0
N.S.	1	1.08	0.64	1.45	3.87	1.32	0.00	1.67	0.12	0.00
time (sec)	N/A	0.544	0.656	1.579	0.174	0.171	0.000	0.155	200.046	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	111	1520	741	247	0	304	24	0
N.S.	1	1.00	0.65	8.84	4.31	1.44	0.00	1.77	0.14	0.00
time (sec)	N/A	0.469	0.608	1.164	0.175	0.142	0.000	0.159	200.048	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	41	36	288	166	0	167	24	146
N.S.	1	1.00	1.24	1.09	8.73	5.03	0.00	5.06	0.73	4.42
time (sec)	N/A	0.246	0.496	0.903	0.044	0.111	0.000	0.143	200.035	8.301

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	49	44	263	170	0	289	24	146
N.S.	1	1.00	0.73	0.66	3.93	2.54	0.00	4.31	0.36	2.18
time (sec)	N/A	0.287	0.480	0.635	0.044	0.101	0.000	0.163	200.032	7.854

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	60	55	239	172	0	289	24	146
N.S.	1	1.08	0.60	0.55	2.39	1.72	0.00	2.89	0.24	1.46
time (sec)	N/A	0.335	0.514	0.514	0.043	0.134	0.000	0.152	200.031	8.074

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	147	75	66	213	171	0	289	24	148
N.S.	1	1.11	0.56	0.50	1.60	1.29	0.00	2.17	0.18	1.11
time (sec)	N/A	0.386	0.503	0.418	0.046	0.105	0.000	0.151	200.030	7.703

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	195	75	72	189	172	0	289	24	151
N.S.	1	1.39	0.54	0.51	1.35	1.23	0.00	2.06	0.17	1.08
time (sec)	N/A	0.468	0.497	0.331	0.040	0.100	0.000	0.148	200.032	7.018

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	185	93	88	163	183	0	0	125	166
N.S.	1	1.41	0.71	0.67	1.24	1.40	0.00	0.00	0.95	1.27
time (sec)	N/A	0.442	0.504	0.276	0.041	0.106	0.000	0.000	0.246	6.916

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	175	104	99	141	237	0	0	266	171
N.S.	1	1.31	0.78	0.74	1.05	1.77	0.00	0.00	1.99	1.28
time (sec)	N/A	0.432	0.492	0.244	0.039	0.216	0.000	0.000	0.257	6.860

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	146	115	110	118	249	0	0	312	186
N.S.	1	1.17	0.92	0.88	0.94	1.99	0.00	0.00	2.50	1.49
time (sec)	N/A	0.351	0.501	0.227	0.040	0.223	0.000	0.000	0.274	6.790

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	152	126	121	120	303	1549	0	358	191
N.S.	1	1.19	0.98	0.95	0.94	2.37	12.10	0.00	2.80	1.49
time (sec)	N/A	0.361	0.426	0.211	0.037	0.372	17.017	0.000	0.235	6.777

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	145	86	68	101	126	1411	91	114	101
N.S.	1	1.20	0.71	0.56	0.83	1.04	11.66	0.75	0.94	0.83
time (sec)	N/A	0.343	0.147	0.204	0.029	0.150	1.731	0.144	0.233	6.792

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	186	148	136	145	368	0	0	448	235
N.S.	1	1.21	0.96	0.88	0.94	2.39	0.00	0.00	2.91	1.53
time (sec)	N/A	0.403	0.534	0.295	0.041	0.736	0.000	0.000	0.242	7.007

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	227	159	154	216	380	0	398	496	264
N.S.	1	1.30	0.91	0.88	1.23	2.17	0.00	2.27	2.83	1.51
time (sec)	N/A	0.462	0.763	0.319	0.041	1.276	0.000	0.264	0.238	7.344

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	268	170	165	312	435	0	0	542	294
N.S.	1	1.29	0.82	0.79	1.50	2.09	0.00	0.00	2.61	1.41
time (sec)	N/A	0.521	0.713	0.351	0.053	2.065	0.000	0.000	0.257	7.468

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	34	15	14	30	10	14	13	14
N.S.	1	1.00	2.12	0.94	0.88	1.88	0.62	0.88	0.81	0.88
time (sec)	N/A	0.223	0.080	0.181	0.115	0.098	0.066	0.127	0.221	0.040

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	32	13	12	28	10	12	11	12
N.S.	1	1.00	2.29	0.93	0.86	2.00	0.71	0.86	0.79	0.86
time (sec)	N/A	0.229	0.067	0.167	0.113	0.073	0.069	0.120	0.227	0.124

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	166	75	65	82	78	0	278	55	103
N.S.	1	1.04	0.47	0.41	0.51	0.49	0.00	1.74	0.34	0.64
time (sec)	N/A	0.422	0.117	0.227	0.045	0.093	0.000	0.123	0.213	7.033

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	122	64	54	68	67	0	218	44	85
N.S.	1	1.03	0.54	0.45	0.57	0.56	0.00	1.83	0.37	0.71
time (sec)	N/A	0.369	0.106	0.194	0.053	0.083	0.000	0.129	0.224	6.976

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	53	43	54	56	0	86	33	69
N.S.	1	1.00	0.68	0.55	0.69	0.72	0.00	1.10	0.42	0.88
time (sec)	N/A	0.301	0.089	0.201	0.047	0.081	0.000	0.120	0.213	6.699

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	35	26	44	0	54	21	35
N.S.	1	1.00	1.05	0.92	0.68	1.16	0.00	1.42	0.55	0.92
time (sec)	N/A	0.249	0.074	0.194	0.077	0.084	0.000	0.121	0.230	6.688

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	98	89	0	233	0	64	46	0
N.S.	1	1.00	0.99	0.90	0.00	2.35	0.00	0.65	0.46	0.00
time (sec)	N/A	0.355	0.177	0.210	0.000	0.109	0.000	0.118	0.223	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	101	117	0	292	0	68	83	0
N.S.	1	1.00	1.03	1.19	0.00	2.98	0.00	0.69	0.85	0.00
time (sec)	N/A	0.339	0.245	0.208	0.000	0.082	0.000	0.124	0.251	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	113	176	0	356	0	101	101	0
N.S.	1	1.06	0.80	1.25	0.00	2.52	0.00	0.72	0.72	0.00
time (sec)	N/A	0.402	0.281	0.213	0.000	0.110	0.000	0.128	0.347	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	210	84	77	110	107	0	687	78	116
N.S.	1	1.04	0.42	0.38	0.55	0.53	0.00	3.42	0.39	0.58
time (sec)	N/A	0.493	0.297	0.194	0.061	0.073	0.000	0.135	0.214	7.542

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	166	73	66	96	95	0	500	67	128
N.S.	1	1.04	0.46	0.41	0.60	0.59	0.00	3.12	0.42	0.80
time (sec)	N/A	0.418	0.283	0.188	0.051	0.110	0.000	0.132	0.215	7.298

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	122	62	55	82	83	0	213	56	109
N.S.	1	1.03	0.52	0.46	0.69	0.70	0.00	1.79	0.47	0.92
time (sec)	N/A	0.369	0.252	0.185	0.064	0.076	0.000	0.122	0.249	7.163

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	51	44	55	71	0	216	44	60
N.S.	1	1.00	0.65	0.56	0.71	0.91	0.00	2.77	0.56	0.77
time (sec)	N/A	0.296	0.255	0.194	0.051	0.095	0.000	0.122	0.277	7.274

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	43	36	39	57	0	80	34	48
N.S.	1	1.00	1.13	0.95	1.03	1.50	0.00	2.11	0.89	1.26
time (sec)	N/A	0.250	0.242	0.190	0.039	0.095	0.000	0.122	0.229	7.056

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	139	110	112	0	267	0	94	64	0
N.S.	1	1.02	0.81	0.82	0.00	1.96	0.00	0.69	0.47	0.00
time (sec)	N/A	0.392	0.560	0.227	0.000	0.111	0.000	0.123	0.240	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	138	107	142	0	307	0	92	107	0
N.S.	1	1.04	0.80	1.07	0.00	2.31	0.00	0.69	0.80	0.00
time (sec)	N/A	0.391	0.725	0.244	0.000	0.101	0.000	0.122	0.231	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	109	176	0	367	0	101	144	0
N.S.	1	1.00	0.78	1.27	0.00	2.64	0.00	0.73	1.04	0.00
time (sec)	N/A	0.384	0.543	0.227	0.000	0.099	0.000	0.131	0.449	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	191	151	241	0	444	0	144	208	0
N.S.	1	1.07	0.85	1.35	0.00	2.49	0.00	0.81	1.17	0.00
time (sec)	N/A	0.453	0.565	0.235	0.000	0.116	0.000	0.139	0.261	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	240	153	303	0	518	0	181	148	0
N.S.	1	1.11	0.71	1.40	0.00	2.39	0.00	0.83	0.68	0.00
time (sec)	N/A	0.526	0.580	0.237	0.000	0.124	0.000	0.136	0.390	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	166	67	62	57	69	0	122	47	98
N.S.	1	1.04	0.42	0.39	0.36	0.43	0.00	0.76	0.29	0.61
time (sec)	N/A	0.437	0.124	0.204	0.043	0.072	0.000	0.127	0.226	6.761

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	122	56	51	58	58	0	85	36	79
N.S.	1	1.03	0.47	0.43	0.49	0.49	0.00	0.71	0.30	0.66
time (sec)	N/A	0.365	0.118	0.194	0.039	0.103	0.000	0.125	0.228	6.620

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	44	39	34	46	0	47	25	61
N.S.	1	1.00	0.56	0.50	0.44	0.59	0.00	0.60	0.32	0.78
time (sec)	N/A	0.293	0.109	0.205	0.039	0.088	0.000	0.125	0.215	6.472

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	32	29	39	0	23	17	32
N.S.	1	1.00	0.97	0.89	0.81	1.08	0.00	0.64	0.47	0.89
time (sec)	N/A	0.251	0.083	0.193	0.040	0.068	0.000	0.119	0.216	6.768

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	86	68	0	169	0	40	49	0
N.S.	1	1.00	1.32	1.05	0.00	2.60	0.00	0.62	0.75	0.00
time (sec)	N/A	0.275	0.118	0.200	0.000	0.110	0.000	0.127	0.241	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	122	123	0	297	0	77	117	0
N.S.	1	1.00	1.12	1.13	0.00	2.72	0.00	0.71	1.07	0.00
time (sec)	N/A	0.333	0.220	0.210	0.000	0.078	0.000	0.128	0.226	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	158	135	181	0	365	0	106	203	0
N.S.	1	1.05	0.90	1.21	0.00	2.43	0.00	0.71	1.35	0.00
time (sec)	N/A	0.412	0.236	0.215	0.000	0.093	0.000	0.128	0.231	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	162	66	66	45	77	0	111	49	104
N.S.	1	1.05	0.43	0.43	0.29	0.50	0.00	0.72	0.32	0.67
time (sec)	N/A	0.416	0.402	0.240	0.047	0.082	0.000	0.132	0.228	6.882

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	118	55	55	43	66	0	75	38	86
N.S.	1	1.04	0.48	0.48	0.38	0.58	0.00	0.66	0.33	0.75
time (sec)	N/A	0.358	0.358	0.221	0.053	0.095	0.000	0.129	0.215	6.969

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	43	44	23	55	0	46	27	66
N.S.	1	1.00	0.58	0.59	0.31	0.74	0.00	0.62	0.36	0.89
time (sec)	N/A	0.288	0.328	0.215	0.051	0.082	0.000	0.129	0.232	6.747

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	16	48	0	23	19	50
N.S.	1	1.00	0.97	1.00	0.44	1.33	0.00	0.64	0.53	1.39
time (sec)	N/A	0.235	0.303	0.196	0.042	0.100	0.000	0.122	0.244	6.658

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	124	91	0	281	0	72	80	0
N.S.	1	1.00	1.19	0.88	0.00	2.70	0.00	0.69	0.77	0.00
time (sec)	N/A	0.334	0.423	0.201	0.000	0.097	0.000	0.125	0.225	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	153	125	141	0	379	0	110	156	0
N.S.	1	1.33	1.09	1.23	0.00	3.30	0.00	0.96	1.36	0.00
time (sec)	N/A	0.399	0.538	0.211	0.000	0.100	0.000	0.126	0.238	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	202	138	202	0	409	0	125	251	0
N.S.	1	1.29	0.88	1.29	0.00	2.62	0.00	0.80	1.61	0.00
time (sec)	N/A	0.465	0.610	0.214	0.000	0.126	0.000	0.130	0.236	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	75	65	57	100	0	110	57	125
N.S.	1	1.04	0.49	0.43	0.38	0.66	0.00	0.72	0.38	0.82
time (sec)	N/A	0.423	0.401	0.246	0.047	0.083	0.000	0.135	0.230	6.603

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	65	55	56	90	0	81	46	104
N.S.	1	1.00	0.56	0.47	0.48	0.78	0.00	0.70	0.40	0.90
time (sec)	N/A	0.363	0.374	0.226	0.050	0.077	0.000	0.129	0.231	6.570

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	76	52	42	36	79	0	50	35	83
N.S.	1	0.99	0.68	0.55	0.47	1.03	0.00	0.65	0.45	1.08
time (sec)	N/A	0.309	0.354	0.211	0.049	0.087	0.000	0.133	0.228	6.428

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	36	31	71	0	37	27	32
N.S.	1	1.00	0.97	0.95	0.82	1.87	0.00	0.97	0.71	0.84
time (sec)	N/A	0.244	0.353	0.208	0.047	0.101	0.000	0.138	0.237	6.331

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	199	152	141	0	381	0	100	157	0
N.S.	1	1.30	0.99	0.92	0.00	2.49	0.00	0.65	1.03	0.00
time (sec)	N/A	0.486	0.599	0.217	0.000	0.107	0.000	0.130	0.233	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	205	152	163	0	369	0	130	186	0
N.S.	1	1.31	0.97	1.04	0.00	2.37	0.00	0.83	1.19	0.00
time (sec)	N/A	0.476	0.642	0.226	0.000	0.100	0.000	0.128	0.271	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	254	157	263	0	517	0	152	346	0
N.S.	1	1.57	0.97	1.62	0.00	3.19	0.00	0.94	2.14	0.00
time (sec)	N/A	0.559	0.748	0.227	0.000	0.107	0.000	0.127	0.231	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	303	172	279	0	597	0	182	358	0
N.S.	1	1.49	0.85	1.37	0.00	2.94	0.00	0.90	1.76	0.00
time (sec)	N/A	0.621	0.954	0.240	0.000	0.137	0.000	0.130	0.233	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	352	185	385	0	661	0	226	537	0
N.S.	1	1.44	0.76	1.58	0.00	2.71	0.00	0.93	2.20	0.00
time (sec)	N/A	0.704	1.053	0.242	0.000	0.125	0.000	0.137	0.224	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	31	32	39	0	24	0	33	14	0
N.S.	1	0.97	1.00	1.22	0.00	0.75	0.00	1.03	0.44	0.00
time (sec)	N/A	0.246	0.077	0.174	0.000	0.164	0.000	0.111	0.225	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	58	52	71	62	0	70	44	54
N.S.	1	1.00	0.67	0.60	0.82	0.71	0.00	0.80	0.51	0.62
time (sec)	N/A	0.343	0.224	0.181	0.138	0.075	0.000	0.123	0.243	6.334

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	44	60	54	0	53	36	73
N.S.	1	1.00	0.77	0.68	0.92	0.83	0.00	0.82	0.55	1.12
time (sec)	N/A	0.296	0.194	0.158	0.125	0.101	0.000	0.122	0.218	0.165

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	36	49	46	0	36	28	60
N.S.	1	1.00	0.98	0.84	1.14	1.07	0.00	0.84	0.65	1.40
time (sec)	N/A	0.282	0.174	0.151	0.120	0.088	0.000	0.124	0.219	6.045

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	33	30	25	37	0	16	19	29
N.S.	1	1.00	1.65	1.50	1.25	1.85	0.00	0.80	0.95	1.45
time (sec)	N/A	0.234	0.166	0.145	0.117	0.070	0.000	0.121	0.234	8.459

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	63	66	0	99	0	43	31	0
N.S.	1	1.00	1.37	1.43	0.00	2.15	0.00	0.93	0.67	0.00
time (sec)	N/A	0.277	0.252	0.175	0.000	0.109	0.000	0.125	0.238	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	86	0	116	0	52	56	0
N.S.	1	1.00	1.16	1.56	0.00	2.11	0.00	0.95	1.02	0.00
time (sec)	N/A	0.279	0.269	0.157	0.000	0.097	0.000	0.121	0.246	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	101	123	0	137	0	63	96	0
N.S.	1	1.00	1.17	1.43	0.00	1.59	0.00	0.73	1.12	0.00
time (sec)	N/A	0.310	0.317	0.171	0.000	0.118	0.000	0.119	0.334	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	67	60	93	78	0	94	60	87
N.S.	1	1.00	0.61	0.55	0.85	0.72	0.00	0.86	0.55	0.80
time (sec)	N/A	0.344	0.268	0.158	0.136	0.108	0.000	0.121	0.222	6.171

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	59	52	82	70	0	77	52	53
N.S.	1	1.00	0.68	0.60	0.94	0.80	0.00	0.89	0.60	0.61
time (sec)	N/A	0.320	0.242	0.177	0.129	0.113	0.000	0.124	0.233	6.403

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	44	71	62	0	60	44	71
N.S.	1	1.00	0.80	0.68	1.09	0.95	0.00	0.92	0.68	1.09
time (sec)	N/A	0.296	0.206	0.171	0.128	0.091	0.000	0.114	0.229	0.199

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	36	47	54	0	43	36	43
N.S.	1	1.00	0.96	0.80	1.04	1.20	0.00	0.96	0.80	0.96
time (sec)	N/A	0.282	0.222	0.174	0.127	0.081	0.000	0.120	0.246	6.168

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	35	30	36	45	0	23	28	36
N.S.	1	1.00	1.59	1.36	1.64	2.05	0.00	1.05	1.27	1.64
time (sec)	N/A	0.234	0.240	0.172	0.116	0.082	0.000	0.122	0.296	0.149

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	77	77	0	106	0	55	44	0
N.S.	1	0.97	1.17	1.17	0.00	1.61	0.00	0.83	0.67	0.00
time (sec)	N/A	0.306	0.315	0.188	0.000	0.092	0.000	0.124	0.252	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	99	0	122	0	64	72	0
N.S.	1	1.00	0.95	1.36	0.00	1.67	0.00	0.88	0.99	0.00
time (sec)	N/A	0.303	0.360	0.215	0.000	0.106	0.000	0.120	0.248	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	84	72	124	0	139	0	67	96	0
N.S.	1	0.98	0.84	1.44	0.00	1.62	0.00	0.78	1.12	0.00
time (sec)	N/A	0.308	0.362	0.178	0.000	0.078	0.000	0.126	0.453	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	80	165	0	163	0	84	140	0
N.S.	1	1.00	0.71	1.46	0.00	1.44	0.00	0.74	1.24	0.00
time (sec)	N/A	0.339	0.381	0.171	0.000	0.113	0.000	0.133	0.255	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	88	206	0	187	0	101	98	0
N.S.	1	1.00	0.61	1.43	0.00	1.30	0.00	0.70	0.68	0.00
time (sec)	N/A	0.371	0.692	0.179	0.000	0.110	0.000	0.124	0.369	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	53	47	45	54	0	63	36	74
N.S.	1	1.00	0.62	0.55	0.53	0.64	0.00	0.74	0.42	0.87
time (sec)	N/A	0.326	0.352	0.169	0.116	0.081	0.000	0.120	0.219	6.806

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	39	47	46	0	46	28	61
N.S.	1	1.00	0.69	0.60	0.72	0.71	0.00	0.71	0.43	0.94
time (sec)	N/A	0.295	0.279	0.164	0.124	0.084	0.000	0.110	0.215	0.215

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	30	28	37	0	27	20	49
N.S.	1	1.00	0.84	0.70	0.65	0.86	0.00	0.63	0.47	1.14
time (sec)	N/A	0.277	0.237	0.158	0.121	0.080	0.000	0.127	0.217	0.157

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	29	25	25	32	0	16	14	24
N.S.	1	1.00	1.45	1.25	1.25	1.60	0.00	0.80	0.70	1.20
time (sec)	N/A	0.219	0.194	0.144	0.117	0.079	0.000	0.117	0.229	0.163

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	30	71	50	0	64	0	35	30	0
N.S.	1	1.20	2.84	2.00	0.00	2.56	0.00	1.40	1.20	0.00
time (sec)	N/A	0.238	0.237	0.161	0.000	0.073	0.000	0.121	0.238	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	86	0	116	0	52	75	0
N.S.	1	1.00	1.30	1.51	0.00	2.04	0.00	0.91	1.32	0.00
time (sec)	N/A	0.278	0.302	0.165	0.000	0.148	0.000	0.126	0.226	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	124	0	139	0	67	131	0
N.S.	1	1.00	0.94	1.44	0.00	1.62	0.00	0.78	1.52	0.00
time (sec)	N/A	0.296	0.349	0.164	0.000	0.103	0.000	0.122	0.225	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	68	60	58	64	0	85	46	93
N.S.	1	1.00	0.61	0.54	0.52	0.58	0.00	0.77	0.41	0.84
time (sec)	N/A	0.337	0.393	0.201	0.140	0.109	0.000	0.125	0.232	6.899

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	59	51	36	55	0	64	38	80
N.S.	1	1.00	0.68	0.59	0.41	0.63	0.00	0.74	0.44	0.92
time (sec)	N/A	0.321	0.350	0.182	0.134	0.070	0.000	0.128	0.228	6.838

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	36	47	0	44	30	48
N.S.	1	1.00	0.76	0.64	0.54	0.70	0.00	0.66	0.45	0.72
time (sec)	N/A	0.300	0.329	0.198	0.128	0.081	0.000	0.125	0.235	0.241

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	35	20	39	0	31	22	52
N.S.	1	1.00	0.96	0.78	0.44	0.87	0.00	0.69	0.49	1.16
time (sec)	N/A	0.278	0.308	0.188	0.133	0.092	0.000	0.128	0.225	6.623

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	38	30	15	34	0	16	16	24
N.S.	1	1.00	1.73	1.36	0.68	1.55	0.00	0.73	0.73	1.09
time (sec)	N/A	0.229	0.276	0.161	0.117	0.077	0.000	0.115	0.221	6.088

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	83	60	0	106	0	52	55	0
N.S.	1	1.00	1.66	1.20	0.00	2.12	0.00	1.04	1.10	0.00
time (sec)	N/A	0.271	0.297	0.163	0.000	0.078	0.000	0.126	0.225	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	93	0	139	0	72	109	0
N.S.	1	1.00	1.10	1.18	0.00	1.76	0.00	0.91	1.38	0.00
time (sec)	N/A	0.305	0.331	0.169	0.000	0.084	0.000	0.132	0.234	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	87	135	0	147	0	89	175	0
N.S.	1	1.00	0.81	1.25	0.00	1.36	0.00	0.82	1.62	0.00
time (sec)	N/A	0.343	0.435	0.171	0.000	0.087	0.000	0.125	0.228	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	34	27	46	38	29	18
N.S.	1	1.00	1.00	0.83	1.48	1.17	2.00	1.65	1.26	0.78
time (sec)	N/A	0.236	0.036	0.208	0.116	0.079	0.473	0.122	0.231	0.097

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	40	0	45	0	37	29	0
N.S.	1	1.00	1.39	1.74	0.00	1.96	0.00	1.61	1.26	0.00
time (sec)	N/A	0.242	0.079	0.146	0.000	0.078	0.000	0.127	0.234	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	39	35	48	42	34	19
N.S.	1	1.00	1.00	0.85	1.44	1.30	1.78	1.56	1.26	0.70
time (sec)	N/A	0.237	0.031	0.163	0.115	0.077	1.160	0.127	0.227	6.437

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	50	0	63	0	43	34	0
N.S.	1	1.00	1.33	1.85	0.00	2.33	0.00	1.59	1.26	0.00
time (sec)	N/A	0.244	0.085	0.167	0.000	0.097	0.000	0.111	0.237	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	378	409	107	0	0	0	654	0	138	0
N.S.	1	1.08	0.28	0.00	0.00	0.00	1.73	0.00	0.37	0.00
time (sec)	N/A	0.625	5.710	0.000	0.000	0.000	1.683	0.000	0.317	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	347	373	95	0	0	0	110	0	113	0
N.S.	1	1.07	0.27	0.00	0.00	0.00	0.32	0.00	0.33	0.00
time (sec)	N/A	0.522	5.611	0.000	0.000	0.000	1.214	0.000	0.308	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	339	80	0	0	0	68	0	88	67
N.S.	1	1.01	0.24	0.00	0.00	0.00	0.20	0.00	0.26	0.20
time (sec)	N/A	0.470	5.541	0.000	0.000	0.000	1.014	0.000	0.290	6.340

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	55	0	0	0	31	0	39	46
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.12	0.15
time (sec)	N/A	0.418	5.034	0.000	0.000	0.000	0.457	0.000	0.265	6.105

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	76	0	0	0	0	0	24	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.471	5.410	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	64	79	0	0	0	0	0	135	0
N.S.	1	0.20	0.25	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.303	5.903	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	355	66	79	0	0	0	0	0	207	0
N.S.	1	0.19	0.22	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.287	8.818	0.000	0.000	0.000	0.000	0.000	0.304	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	707	792	104	0	0	0	197	0	138	0
N.S.	1	1.12	0.15	0.00	0.00	0.00	0.28	0.00	0.20	0.00
time (sec)	N/A	0.971	6.985	0.000	0.000	0.000	1.454	0.000	0.344	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	676	756	95	0	0	0	110	0	113	0
N.S.	1	1.12	0.14	0.00	0.00	0.00	0.16	0.00	0.17	0.00
time (sec)	N/A	0.949	6.764	0.000	0.000	0.000	1.270	0.000	0.287	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	666	722	80	0	0	0	68	0	88	67
N.S.	1	1.08	0.12	0.00	0.00	0.00	0.10	0.00	0.13	0.10
time (sec)	N/A	0.882	6.101	0.000	0.000	0.000	1.078	0.000	0.281	6.740

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	642	696	55	0	0	0	31	0	39	46
N.S.	1	1.08	0.09	0.00	0.00	0.00	0.05	0.00	0.06	0.07
time (sec)	N/A	0.843	5.944	0.000	0.000	0.000	0.474	0.000	0.259	6.441

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	637	696	76	0	0	0	0	0	24	0
N.S.	1	1.09	0.12	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.837	6.016	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	640	64	79	0	0	0	0	0	135	0
N.S.	1	0.10	0.12	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.295	6.578	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	684	66	79	0	0	0	0	0	207	0
N.S.	1	0.10	0.12	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.288	9.605	0.000	0.000	0.000	0.000	0.000	0.319	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	683	766	111	0	0	0	566	0	95	0
N.S.	1	1.12	0.16	0.00	0.00	0.00	0.83	0.00	0.14	0.00
time (sec)	N/A	0.943	10.054	0.000	0.000	0.000	1.790	0.000	0.254	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	652	730	97	0	0	0	107	0	67	0
N.S.	1	1.12	0.15	0.00	0.00	0.00	0.16	0.00	0.10	0.00
time (sec)	N/A	0.910	10.041	0.000	0.000	0.000	1.275	0.000	0.261	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	637	696	80	0	0	0	66	0	39	67
N.S.	1	1.09	0.13	0.00	0.00	0.00	0.10	0.00	0.06	0.11
time (sec)	N/A	0.841	9.624	0.000	0.000	0.000	0.959	0.000	0.298	6.721

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	613	671	55	0	0	0	29	0	16	46
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.05	0.00	0.03	0.08
time (sec)	N/A	0.770	4.801	0.000	0.000	0.000	0.416	0.000	0.265	6.737

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	652	725	72	0	0	0	0	0	38	0
N.S.	1	1.11	0.11	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.872	5.299	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	675	66	72	0	0	0	0	0	63	0
N.S.	1	0.10	0.11	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.305	5.973	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	708	66	72	0	0	0	0	0	88	0
N.S.	1	0.09	0.10	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.300	9.014	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	353	380	111	0	0	0	576	0	98	0
N.S.	1	1.08	0.31	0.00	0.00	0.00	1.63	0.00	0.28	0.00
time (sec)	N/A	0.595	10.068	0.000	0.000	0.000	1.786	0.000	0.282	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	344	99	0	0	0	107	0	67	0
N.S.	1	1.07	0.31	0.00	0.00	0.00	0.33	0.00	0.21	0.00
time (sec)	N/A	0.516	10.049	0.000	0.000	0.000	1.262	0.000	0.237	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	80	0	0	0	66	0	42	67
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.21	0.00	0.14	0.22
time (sec)	N/A	0.445	9.801	0.000	0.000	0.000	0.981	0.000	0.231	7.346

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	55	0	0	0	29	0	16	46
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.10	0.00	0.06	0.16
time (sec)	N/A	0.419	4.749	0.000	0.000	0.000	0.424	0.000	0.226	7.101

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	325	342	72	0	0	0	0	0	38	0
N.S.	1	1.05	0.22	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.508	5.743	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	346	66	72	0	0	0	0	0	63	0
N.S.	1	0.19	0.21	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.290	6.170	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	379	66	72	0	0	0	0	0	88	0
N.S.	1	0.17	0.19	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.297	9.421	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	80	0	0	0	66	0	42	67
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.21	0.00	0.14	0.22
time (sec)	N/A	0.441	5.094	0.000	0.000	0.000	1.146	0.000	0.240	7.016

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	76	0	0	0	0	0	24	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.468	0.010	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	69	63	67	75	0	0	54	100
N.S.	1	1.04	0.51	0.46	0.49	0.55	0.00	0.00	0.40	0.74
time (sec)	N/A	0.406	0.116	0.213	0.043	0.087	0.000	0.000	0.354	6.852

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	58	52	56	64	0	0	43	82
N.S.	1	1.03	0.57	0.51	0.55	0.63	0.00	0.00	0.43	0.81
time (sec)	N/A	0.337	0.115	0.206	0.041	0.073	0.000	0.000	0.323	6.371

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	41	45	53	0	0	32	66
N.S.	1	1.00	0.77	0.62	0.68	0.80	0.00	0.00	0.48	1.00
time (sec)	N/A	0.285	0.099	0.201	0.038	0.072	0.000	0.000	0.282	6.665

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	20	41	0	0	20	32
N.S.	1	1.00	1.00	1.03	0.62	1.28	0.00	0.00	0.62	1.00
time (sec)	N/A	0.247	0.079	0.196	0.034	0.093	0.000	0.000	0.248	6.619

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	195	253	0	0	253	0	0	39	0
N.S.	1	0.78	1.01	0.00	0.00	1.01	0.00	0.00	0.16	0.00
time (sec)	N/A	0.544	0.526	0.000	0.000	0.100	0.000	0.000	0.257	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	254	202	252	0	0	371	0	0	57	0
N.S.	1	0.80	0.99	0.00	0.00	1.46	0.00	0.00	0.22	0.00
time (sec)	N/A	0.536	0.647	0.000	0.000	0.115	0.000	0.000	0.268	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	242	254	0	0	424	0	0	75	0
N.S.	1	0.82	0.86	0.00	0.00	1.44	0.00	0.00	0.26	0.00
time (sec)	N/A	0.570	0.667	0.000	0.000	0.090	0.000	0.000	1.872	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	329	280	267	0	0	481	0	0	93	0
N.S.	1	0.85	0.81	0.00	0.00	1.46	0.00	0.00	0.28	0.00
time (sec)	N/A	0.633	0.718	0.000	0.000	0.091	0.000	0.000	0.307	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	303	296	201	0	0	286	0	301	86	0
N.S.	1	0.98	0.66	0.00	0.00	0.94	0.00	0.99	0.28	0.00
time (sec)	N/A	0.584	1.126	0.000	0.000	0.113	0.000	0.201	0.659	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	256	214	0	0	275	0	153	24	0
N.S.	1	0.96	0.80	0.00	0.00	1.03	0.00	0.57	0.09	0.00
time (sec)	N/A	0.507	0.987	0.000	0.000	0.116	0.000	0.157	0.265	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	218	178	0	0	261	0	115	39	0
N.S.	1	0.98	0.80	0.00	0.00	1.18	0.00	0.52	0.18	0.00
time (sec)	N/A	0.442	0.896	0.000	0.000	0.114	0.000	0.165	0.286	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	219	173	0	0	295	0	104	57	0
N.S.	1	0.98	0.77	0.00	0.00	1.32	0.00	0.46	0.25	0.00
time (sec)	N/A	0.443	0.936	0.000	0.000	0.086	0.000	0.162	0.310	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	36	0	66	0	22	30	73
N.S.	1	1.00	1.00	1.03	0.00	1.89	0.00	0.63	0.86	2.09
time (sec)	N/A	0.246	0.441	0.190	0.000	0.106	0.000	0.143	1.722	7.172

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	49	44	0	91	0	41	60	101
N.S.	1	1.00	0.69	0.62	0.00	1.28	0.00	0.58	0.85	1.42
time (sec)	N/A	0.309	0.515	0.195	0.000	0.089	0.000	0.140	0.238	7.487

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	114	60	55	0	113	0	57	82	130
N.S.	1	1.08	0.57	0.52	0.00	1.07	0.00	0.54	0.77	1.23
time (sec)	N/A	0.367	0.537	0.198	0.000	0.074	0.000	0.154	0.245	7.227

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	157	71	66	0	135	0	73	104	159
N.S.	1	1.11	0.50	0.47	0.00	0.96	0.00	0.52	0.74	1.13
time (sec)	N/A	0.419	3.283	0.213	0.000	0.099	0.000	0.147	0.245	7.393

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	256	182	0	0	275	0	153	54	0
N.S.	1	0.96	0.68	0.00	0.00	1.03	0.00	0.57	0.20	0.00
time (sec)	N/A	0.505	0.600	0.000	0.000	0.084	0.000	0.175	0.476	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	222	215	168	0	0	260	0	113	24	0
N.S.	1	0.97	0.76	0.00	0.00	1.17	0.00	0.51	0.11	0.00
time (sec)	N/A	0.462	0.423	0.000	0.000	0.090	0.000	0.157	0.306	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	177	138	0	0	204	0	88	24	0
N.S.	1	0.93	0.72	0.00	0.00	1.07	0.00	0.46	0.13	0.00
time (sec)	N/A	0.409	0.308	0.000	0.000	0.096	0.000	0.157	0.298	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	36	0	48	0	22	52	46
N.S.	1	1.00	1.00	1.03	0.00	1.37	0.00	0.63	1.49	1.31
time (sec)	N/A	0.253	0.205	0.190	0.000	0.095	0.000	0.162	0.343	7.179

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	43	44	0	69	0	45	84	76
N.S.	1	1.00	0.61	0.62	0.00	0.97	0.00	0.63	1.18	1.07
time (sec)	N/A	0.310	0.231	0.197	0.000	0.102	0.000	0.185	0.343	7.566

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	114	54	55	0	91	0	64	116	105
N.S.	1	1.08	0.51	0.52	0.00	0.86	0.00	0.60	1.09	0.99
time (sec)	N/A	0.350	0.358	0.192	0.000	0.092	0.000	0.209	0.429	7.714

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	157	65	66	0	113	0	80	148	134
N.S.	1	1.11	0.46	0.47	0.00	0.80	0.00	0.57	1.05	0.95
time (sec)	N/A	0.401	0.441	0.203	0.000	0.077	0.000	0.220	0.518	7.462

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	62	63	54	64	0	0	115	83
N.S.	1	1.04	0.46	0.46	0.40	0.47	0.00	0.00	0.85	0.61
time (sec)	N/A	0.394	0.376	0.207	0.049	0.080	0.000	0.000	0.358	7.134

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	104	50	51	42	52	0	0	80	67
N.S.	1	1.03	0.50	0.50	0.42	0.51	0.00	0.00	0.79	0.66
time (sec)	N/A	0.345	0.337	0.198	0.044	0.071	0.000	0.000	0.300	6.975

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	39	40	31	41	0	0	45	52
N.S.	1	1.00	0.59	0.61	0.47	0.62	0.00	0.00	0.68	0.79
time (sec)	N/A	0.290	0.293	0.191	0.046	0.116	0.000	0.000	0.296	6.674

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	20	34	0	0	13	26
N.S.	1	1.00	1.00	1.10	0.67	1.13	0.00	0.00	0.43	0.87
time (sec)	N/A	0.248	0.249	0.181	0.040	0.078	0.000	0.000	0.286	6.349

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	225	162	206	0	0	253	0	0	24	0
N.S.	1	0.72	0.92	0.00	0.00	1.12	0.00	0.00	0.11	0.00
time (sec)	N/A	0.465	0.901	0.000	0.000	0.090	0.000	0.000	0.289	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	200	248	0	0	350	0	0	52	0
N.S.	1	0.77	0.95	0.00	0.00	1.35	0.00	0.00	0.20	0.00
time (sec)	N/A	0.528	1.223	0.000	0.000	0.124	0.000	0.000	0.262	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	238	256	0	0	405	0	0	84	0
N.S.	1	0.80	0.86	0.00	0.00	1.36	0.00	0.00	0.28	0.00
time (sec)	N/A	0.577	1.252	0.000	0.000	0.113	0.000	0.000	0.262	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	65	0	40	67
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.86	0.00	0.53	0.88
time (sec)	N/A	0.318	10.032	0.000	0.000	0.000	1.160	0.000	0.229	6.960

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.314	6.932	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	246	150	0	0	241	0	0	88	0
N.S.	1	1.10	0.67	0.00	0.00	1.08	0.00	0.00	0.39	0.00
time (sec)	N/A	0.653	0.657	0.000	0.000	0.105	0.000	0.000	0.345	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	215	142	0	0	236	0	165	56	0
N.S.	1	1.17	0.77	0.00	0.00	1.28	0.00	0.90	0.30	0.00
time (sec)	N/A	0.619	0.450	0.000	0.000	0.099	0.000	0.133	0.347	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	216	151	0	0	263	0	0	22	0
N.S.	1	1.17	0.82	0.00	0.00	1.43	0.00	0.00	0.12	0.00
time (sec)	N/A	0.621	0.464	0.000	0.000	0.112	0.000	0.000	200.032	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	31	35	30	0	45	0	21	28	49
N.S.	1	0.89	1.00	0.86	0.00	1.29	0.00	0.60	0.80	1.40
time (sec)	N/A	0.254	0.225	0.167	0.000	0.077	0.000	0.139	0.261	6.837

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	63	47	35	0	61	0	0	51	37
N.S.	1	0.89	0.66	0.49	0.00	0.86	0.00	0.00	0.72	0.52
time (sec)	N/A	0.286	0.266	0.167	0.000	0.091	0.000	0.000	0.388	6.751

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	94	54	44	0	78	0	54	22	46
N.S.	1	0.89	0.51	0.42	0.00	0.74	0.00	0.51	0.21	0.43
time (sec)	N/A	0.308	0.305	0.171	0.000	0.079	0.000	0.136	200.044	6.060

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	125	62	52	0	94	0	0	22	118
N.S.	1	0.89	0.44	0.37	0.00	0.67	0.00	0.00	0.16	0.84
time (sec)	N/A	0.340	0.346	0.175	0.000	0.097	0.000	0.000	200.034	0.356

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	258	275	170	0	0	326	0	0	125	0
N.S.	1	1.07	0.66	0.00	0.00	1.26	0.00	0.00	0.48	0.00
time (sec)	N/A	0.733	0.673	0.000	0.000	0.160	0.000	0.000	0.264	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	247	161	0	0	316	0	185	88	0
N.S.	1	1.11	0.72	0.00	0.00	1.42	0.00	0.83	0.39	0.00
time (sec)	N/A	0.679	0.656	0.000	0.000	0.098	0.000	0.154	0.296	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	A	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	216	152	0	0	310	0	165	26	0
N.S.	1	1.17	0.83	0.00	0.00	1.68	0.00	0.90	0.14	0.00
time (sec)	N/A	0.640	0.578	0.000	0.000	0.101	0.000	0.151	0.255	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	A	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	190	111	0	0	261	0	145	105	0
N.S.	1	1.26	0.74	0.00	0.00	1.73	0.00	0.96	0.70	0.00
time (sec)	N/A	0.574	0.392	0.000	0.000	0.101	0.000	0.133	0.260	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	31	35	30	0	40	0	0	103	24
N.S.	1	0.89	1.00	0.86	0.00	1.14	0.00	0.00	2.94	0.69
time (sec)	N/A	0.244	0.279	0.168	0.000	0.086	0.000	0.000	0.335	6.962

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	63	40	35	0	53	0	39	63	65
N.S.	1	0.89	0.56	0.49	0.00	0.75	0.00	0.55	0.89	0.92
time (sec)	N/A	0.288	0.319	0.167	0.000	0.094	0.000	0.144	0.241	6.683

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	94	49	44	0	70	0	0	79	88
N.S.	1	0.89	0.46	0.42	0.00	0.66	0.00	0.00	0.75	0.83
time (sec)	N/A	0.317	0.430	0.173	0.000	0.079	0.000	0.000	0.372	6.706

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	125	57	52	0	86	0	69	96	111
N.S.	1	0.89	0.40	0.37	0.00	0.61	0.00	0.49	0.68	0.79
time (sec)	N/A	0.339	0.569	0.172	0.000	0.086	0.000	0.152	0.297	6.940

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	141	0	0	0	661	0	118	0
N.S.	1	0.99	2.10	0.00	0.00	0.00	9.87	0.00	1.76	0.00
time (sec)	N/A	0.298	8.401	0.000	0.000	0.000	1.991	0.000	0.456	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	64	128	0	0	0	110	0	107	0
N.S.	1	0.96	1.91	0.00	0.00	0.00	1.64	0.00	1.60	0.00
time (sec)	N/A	0.289	8.325	0.000	0.000	0.000	1.428	0.000	0.364	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	80	80	0	0	0	68	0	96	67
N.S.	1	1.19	1.19	0.00	0.00	0.00	1.01	0.00	1.43	1.00
time (sec)	N/A	0.308	7.773	0.000	0.000	0.000	1.177	0.000	0.331	6.722

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	31	0	16	46
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.56	0.00	0.29	0.84
time (sec)	N/A	0.270	7.273	0.000	0.000	0.000	0.515	0.000	0.234	6.435

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	76	0	0	0	0	0	67	0
N.S.	1	1.06	1.23	0.00	0.00	0.00	0.00	0.00	1.08	0.00
time (sec)	N/A	0.289	7.783	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	66	79	0	0	0	0	0	93	0
N.S.	1	1.02	1.22	0.00	0.00	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.291	8.412	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	79	0	0	0	0	0	170	0
N.S.	1	0.99	1.18	0.00	0.00	0.00	0.00	0.00	2.54	0.00
time (sec)	N/A	0.293	10.129	0.000	0.000	0.000	0.000	0.000	0.324	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	66	157	0	0	0	0	0	169	0
N.S.	1	0.99	2.34	0.00	0.00	0.00	0.00	0.00	2.52	0.00
time (sec)	N/A	0.290	10.099	0.000	0.000	0.000	0.000	0.000	0.498	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	323	426	75	0	0	0	65	0	93	67
N.S.	1	1.32	0.23	0.00	0.00	0.00	0.20	0.00	0.29	0.21
time (sec)	N/A	0.519	10.032	0.000	0.000	0.000	1.256	0.000	0.305	6.481

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	323	66	76	0	0	0	0	0	24	0
N.S.	1	0.20	0.24	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.290	7.932	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	80	80	0	0	0	68	0	67	67
N.S.	1	1.19	1.19	0.00	0.00	0.00	1.01	0.00	1.00	1.00
time (sec)	N/A	0.299	10.063	0.000	0.000	0.000	1.558	0.000	0.252	6.614

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	80	80	0	0	0	68	0	67	67
N.S.	1	1.19	1.19	0.00	0.00	0.00	1.01	0.00	1.00	1.00
time (sec)	N/A	0.315	9.823	0.000	0.000	0.000	1.338	0.000	0.238	6.615

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	80	80	0	0	0	68	0	67	67
N.S.	1	1.19	1.19	0.00	0.00	0.00	1.01	0.00	1.00	1.00
time (sec)	N/A	0.298	8.868	0.000	0.000	0.000	1.153	0.000	0.236	6.519

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	80	80	0	0	0	66	0	109	67
N.S.	1	1.19	1.19	0.00	0.00	0.00	0.99	0.00	1.63	1.00
time (sec)	N/A	0.290	10.030	0.000	0.000	0.000	1.110	0.000	0.244	6.795

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	80	80	0	0	0	66	0	95	67
N.S.	1	1.19	1.19	0.00	0.00	0.00	0.99	0.00	1.42	1.00
time (sec)	N/A	0.295	10.030	0.000	0.000	0.000	1.082	0.000	0.262	6.702

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	75	0	0	0	65	0	114	67
N.S.	1	1.16	1.12	0.00	0.00	0.00	0.97	0.00	1.70	1.00
time (sec)	N/A	0.289	10.032	0.000	0.000	0.000	1.296	0.000	0.258	6.575

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	76	0	0	0	0	0	114	0
N.S.	1	1.06	1.23	0.00	0.00	0.00	0.00	0.00	1.84	0.00
time (sec)	N/A	0.280	8.939	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	71	181	480	404	2242	635	350	356
N.S.	1	1.00	0.74	1.89	5.00	4.21	23.35	6.61	3.65	3.71
time (sec)	N/A	0.387	0.182	0.454	0.049	0.109	1.353	0.128	0.222	6.601

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	97	242	221	978	336	183	204
N.S.	1	1.00	0.76	1.39	3.46	3.16	13.97	4.80	2.61	2.91
time (sec)	N/A	0.355	0.154	0.281	0.043	0.094	0.772	0.127	0.246	6.327

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	41	93	83	289	125	74	90
N.S.	1	1.00	0.88	0.98	2.21	1.98	6.88	2.98	1.76	2.14
time (sec)	N/A	0.300	0.087	0.188	0.039	0.085	0.397	0.127	0.272	5.918

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	62	0	0	0	0	0	28	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.68	0.00
time (sec)	N/A	0.272	0.213	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	105	0	0	0	0	0	38	0
N.S.	1	1.00	2.23	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.282	0.284	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	156	0	0	0	0	0	50	0
N.S.	1	1.00	3.18	0.00	0.00	0.00	0.00	0.00	1.02	0.00
time (sec)	N/A	0.286	0.321	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	191	0	0	0	0	0	59	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.353	0.537	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	86	0	0	0	0	0	23	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.348	0.326	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	84	0	0	0	0	0	25	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.345	0.371	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	51	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.353	0.431	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	92	0	0	0	0	0	74	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.354	0.591	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	91	0	0	0	0	0	99	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.366	0.688	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	57	86	75	0	0	116	0	653	0
N.S.	1	1.14	1.72	1.50	0.00	0.00	2.32	0.00	13.06	0.00
time (sec)	N/A	0.311	0.268	1.977	0.000	0.000	1.605	0.000	0.205	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	56	56	47	0	0	78	0	310	58
N.S.	1	1.12	1.12	0.94	0.00	0.00	1.56	0.00	6.20	1.16
time (sec)	N/A	0.277	0.145	0.951	0.000	0.000	1.329	0.000	0.202	7.767

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	0	0	24	0	138	19
N.S.	1	1.00	1.00	0.95	0.00	0.00	1.09	0.00	6.27	0.86
time (sec)	N/A	0.234	0.066	0.354	0.000	0.000	0.545	0.000	0.210	7.736

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	54	76	0	0	0	318	0	32	0
N.S.	1	1.32	1.85	0.00	0.00	0.00	7.76	0.00	0.78	0.00
time (sec)	N/A	0.310	0.252	0.000	0.000	0.000	3.373	0.000	0.236	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	57	76	0	0	0	0	0	143	0
N.S.	1	1.12	1.49	0.00	0.00	0.00	0.00	0.00	2.80	0.00
time (sec)	N/A	0.329	0.317	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	76	0	0	0	0	0	217	0
N.S.	1	1.08	1.43	0.00	0.00	0.00	0.00	0.00	4.09	0.00
time (sec)	N/A	0.316	0.357	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	134	0	0	0	124	0	646	0
N.S.	1	1.03	1.94	0.00	0.00	0.00	1.80	0.00	9.36	0.00
time (sec)	N/A	0.318	0.213	0.000	0.000	0.000	1.529	0.000	0.219	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	83	83	0	0	0	82	0	303	78
N.S.	1	1.20	1.20	0.00	0.00	0.00	1.19	0.00	4.39	1.13
time (sec)	N/A	0.313	0.156	0.000	0.000	0.000	1.201	0.000	0.203	8.207

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	53	53	0	0	0	29	0	131	50
N.S.	1	0.76	0.76	0.00	0.00	0.00	0.41	0.00	1.87	0.71
time (sec)	N/A	0.272	0.046	0.000	0.000	0.000	0.545	0.000	0.201	8.432

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	73	75	0	0	0	318	0	24	0
N.S.	1	1.26	1.29	0.00	0.00	0.00	5.48	0.00	0.41	0.00
time (sec)	N/A	0.309	0.209	0.000	0.000	0.000	3.115	0.000	0.211	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	73	75	0	0	0	0	0	136	0
N.S.	1	1.07	1.10	0.00	0.00	0.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.304	0.243	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	73	75	0	0	0	0	0	210	0
N.S.	1	1.01	1.04	0.00	0.00	0.00	0.00	0.00	2.92	0.00
time (sec)	N/A	0.311	0.276	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	26	36	104	26	37	26
N.S.	1	1.00	1.00	0.96	0.93	1.29	3.71	0.93	1.32	0.93
time (sec)	N/A	0.246	0.004	0.175	0.034	0.102	0.288	0.120	0.214	6.596

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	36	42	36	49	26	37	26
N.S.	1	1.00	1.00	1.29	1.50	1.29	1.75	0.93	1.32	0.93
time (sec)	N/A	0.316	0.001	0.412	0.073	0.096	1.422	0.134	0.210	6.233

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	176	0	0	0	0	0	635	0
N.S.	1	1.23	2.67	0.00	0.00	0.00	0.00	0.00	9.62	0.00
time (sec)	N/A	0.349	0.936	0.000	0.000	0.000	0.000	0.000	0.289	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	176	0	0	0	0	0	635	0
N.S.	1	1.23	2.67	0.00	0.00	0.00	0.00	0.00	9.62	0.00
time (sec)	N/A	0.332	0.179	0.000	0.000	0.000	0.000	0.000	0.306	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	79	90	0	0	0	0	0	78	0
N.S.	1	1.23	1.41	0.00	0.00	0.00	0.00	0.00	1.22	0.00
time (sec)	N/A	0.354	0.601	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	78	93	0	0	0	0	0	162	0
N.S.	1	1.26	1.50	0.00	0.00	0.00	0.00	0.00	2.61	0.00
time (sec)	N/A	0.351	0.784	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	189	0	0	0	0	0	628	0
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	7.39	0.00
time (sec)	N/A	0.376	0.963	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0	272	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	3.09	0.00
time (sec)	N/A	0.362	0.545	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0	71	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.397	0.697	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	0	0	0	0	0	155	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	1.76	0.00
time (sec)	N/A	0.366	0.993	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0	476	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	5.60	0.00
time (sec)	N/A	0.363	0.141	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	485	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	5.71	0.00
time (sec)	N/A	0.366	0.146	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	121	164	121	292	0	0	116	203
N.S.	1	1.00	1.51	2.05	1.51	3.65	0.00	0.00	1.45	2.54
time (sec)	N/A	0.354	0.181	0.506	0.041	0.087	0.000	0.000	0.217	6.887

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	85	97	79	178	0	0	84	144
N.S.	1	1.00	1.39	1.59	1.30	2.92	0.00	0.00	1.38	2.36
time (sec)	N/A	0.320	0.168	0.460	0.036	0.096	0.000	0.000	0.191	6.283

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	51	41	82	369	0	52	105
N.S.	1	1.00	1.25	1.28	1.02	2.05	9.22	0.00	1.30	2.62
time (sec)	N/A	0.294	0.122	0.331	0.053	0.118	0.544	0.000	0.196	6.764

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	35	36	22	34	76	70	24	35
N.S.	1	1.00	1.75	1.80	1.10	1.70	3.80	3.50	1.20	1.75
time (sec)	N/A	0.236	0.093	0.288	0.038	0.109	3.626	0.126	0.243	6.395

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.256	0.144	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	50	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.252	0.156	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	65	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.62	0.00
time (sec)	N/A	0.255	0.154	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	247	82	140	0	228	0	0	123	318
N.S.	1	1.24	0.41	0.70	0.00	1.15	0.00	0.00	0.62	1.60
time (sec)	N/A	0.604	0.178	0.556	0.000	0.088	0.000	0.000	0.192	6.845

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	195	82	93	0	154	0	0	103	220
N.S.	1	1.37	0.58	0.65	0.00	1.08	0.00	0.00	0.73	1.55
time (sec)	N/A	0.497	0.205	0.527	0.000	0.095	0.000	0.000	0.201	6.598

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	146	82	61	0	95	0	0	83	148
N.S.	1	1.62	0.91	0.68	0.00	1.06	0.00	0.00	0.92	1.64
time (sec)	N/A	0.424	0.161	0.389	0.000	0.093	0.000	0.000	0.192	6.479

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	46	45	0	50	0	0	63	48
N.S.	1	1.00	1.10	1.07	0.00	1.19	0.00	0.00	1.50	1.14
time (sec)	N/A	0.269	0.190	0.391	0.000	0.096	0.000	0.000	0.209	6.306

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0	43	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.369	0.156	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	0	0	28	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.350	0.114	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	138	0	0	0	0	0	62	0
N.S.	1	1.00	1.79	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.352	0.245	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	49	64	0	0	0	0	0	409	0
N.S.	1	1.11	1.45	0.00	0.00	0.00	0.00	0.00	9.30	0.00
time (sec)	N/A	0.285	0.154	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	67	0	0	0	0	0	27	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.267	0.263	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	49	49	0	0	0	0	0	393	0
N.S.	1	1.11	1.11	0.00	0.00	0.00	0.00	0.00	8.93	0.00
time (sec)	N/A	0.251	0.059	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	70	124	137	0	0	434	0	787	0
N.S.	1	1.56	2.76	3.04	0.00	0.00	9.64	0.00	17.49	0.00
time (sec)	N/A	0.301	0.367	3.001	0.000	0.000	2.050	0.000	0.207	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	50	124	0	0	0	0	0	787	0
N.S.	1	1.14	2.82	0.00	0.00	0.00	0.00	0.00	17.89	0.00
time (sec)	N/A	0.291	0.279	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	45	45	0	0	0	0	0	865	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	0.00	0.00	19.66	0.00
time (sec)	N/A	0.257	0.154	0.000	0.000	0.000	0.000	0.000	0.224	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [311] had the largest ratio of [.6250000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	22	0.136
2	A	3	3	1.00	22	0.136
3	A	3	3	1.00	22	0.136
4	A	3	3	1.00	22	0.136
5	A	3	3	1.00	22	0.136
6	A	2	2	1.00	20	0.100
7	A	3	3	1.00	22	0.136
8	A	3	3	1.00	22	0.136
9	A	3	3	1.00	22	0.136
10	A	3	3	1.00	22	0.136
11	A	3	3	1.00	22	0.136
12	A	3	3	1.00	22	0.136
13	A	3	3	1.00	22	0.136
14	A	3	3	1.00	22	0.136
15	A	3	3	1.00	22	0.136
16	A	2	2	1.00	22	0.091
17	A	2	2	1.31	20	0.100
18	A	3	3	1.00	22	0.136
19	A	3	3	1.00	22	0.136
20	A	3	3	1.00	22	0.136
21	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	22	0.136
23	A	3	3	1.00	22	0.136
24	A	3	3	1.00	22	0.136
25	A	2	2	1.00	22	0.091
26	A	2	2	1.00	22	0.091
27	A	3	3	1.00	22	0.136
28	A	3	3	1.08	20	0.150
29	A	3	3	1.00	22	0.136
30	A	3	3	1.00	22	0.136
31	A	2	2	1.00	20	0.100
32	A	2	2	1.00	23	0.087
33	A	5	4	1.33	24	0.167
34	A	5	4	1.38	25	0.160
35	A	5	4	1.31	25	0.160
36	A	5	4	1.32	24	0.167
37	A	5	4	1.31	27	0.148
38	A	6	5	1.36	31	0.161
39	A	8	7	1.23	24	0.292
40	A	7	6	1.26	24	0.250
41	A	6	5	1.30	24	0.208
42	A	5	4	1.03	22	0.182
43	A	4	3	1.00	24	0.125
44	A	5	4	1.00	24	0.167
45	A	1	1	1.00	24	0.042
46	A	2	2	1.00	24	0.083
47	A	3	3	1.08	24	0.125
48	A	4	4	1.12	24	0.167
49	A	8	7	1.26	24	0.292
50	A	7	6	1.28	24	0.250
51	A	6	5	1.07	22	0.227
52	A	5	4	1.03	24	0.167
53	A	5	4	1.32	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	1.00	24	0.167
55	A	6	5	1.00	24	0.208
56	A	1	1	1.00	24	0.042
57	A	2	2	1.00	24	0.083
58	A	3	3	1.08	24	0.125
59	A	4	4	1.12	24	0.167
60	A	10	9	1.24	24	0.375
61	A	9	8	1.26	24	0.333
62	A	8	7	1.11	22	0.318
63	A	7	6	1.10	24	0.250
64	A	8	7	1.29	24	0.292
65	A	8	7	1.15	24	0.292
66	A	8	7	1.26	24	0.292
67	A	9	8	1.12	24	0.333
68	A	10	9	1.01	24	0.375
69	A	7	6	1.04	24	0.250
70	A	8	7	1.00	24	0.292
71	A	1	1	1.00	24	0.042
72	A	2	2	1.00	24	0.083
73	A	3	3	1.08	24	0.125
74	A	4	4	1.12	24	0.167
75	A	5	5	1.37	17	0.294
76	A	4	4	1.42	17	0.235
77	A	3	3	1.00	15	0.200
78	A	2	2	1.00	17	0.118
79	A	3	3	1.00	17	0.176
80	A	1	1	1.00	17	0.059
81	A	2	2	1.00	17	0.118
82	A	3	3	1.08	17	0.176
83	A	5	5	1.38	19	0.263
84	A	4	4	1.47	19	0.211
85	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	19	0.105
87	A	3	3	1.00	19	0.158
88	A	1	1	1.00	19	0.053
89	A	2	2	1.00	19	0.105
90	A	3	3	1.07	19	0.158
91	A	7	6	1.31	15	0.400
92	A	6	5	1.33	15	0.333
93	A	5	4	1.00	13	0.308
94	A	4	3	1.00	15	0.200
95	A	5	4	1.00	15	0.267
96	A	1	1	1.00	15	0.067
97	A	2	2	1.00	15	0.133
98	A	3	3	1.09	15	0.200
99	A	7	6	1.33	17	0.353
100	A	6	5	1.37	17	0.294
101	A	5	4	1.00	15	0.267
102	A	4	3	1.00	17	0.176
103	A	5	4	1.00	17	0.235
104	A	1	1	1.00	17	0.059
105	A	2	2	1.00	17	0.118
106	A	3	3	1.08	17	0.176
107	A	4	3	1.00	25	0.120
108	A	5	4	1.02	25	0.160
109	A	6	5	1.29	27	0.185
110	A	7	6	1.25	27	0.222
111	A	7	6	1.22	24	0.250
112	A	6	5	1.27	24	0.208
113	A	5	4	1.33	24	0.167
114	A	4	3	1.00	22	0.136
115	A	1	1	1.00	24	0.042
116	A	2	2	1.22	24	0.083
117	A	3	3	1.23	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	4	1.37	24	0.167
119	A	9	8	1.12	24	0.333
120	A	8	7	1.01	24	0.292
121	A	5	4	1.00	24	0.167
122	A	4	3	1.00	24	0.125
123	A	1	1	1.00	22	0.045
124	A	2	2	1.00	24	0.083
125	A	3	3	1.25	24	0.125
126	A	4	4	1.25	24	0.167
127	A	9	8	1.01	24	0.333
128	A	6	5	1.00	24	0.208
129	A	5	4	1.00	24	0.167
130	A	1	1	1.00	24	0.042
131	A	2	2	1.00	24	0.083
132	A	2	2	1.00	22	0.091
133	A	3	3	1.10	24	0.125
134	A	4	4	1.27	24	0.167
135	A	5	5	1.26	24	0.208
136	A	10	9	1.04	24	0.375
137	A	7	6	1.04	24	0.250
138	A	6	5	1.00	24	0.208
139	A	1	1	1.00	24	0.042
140	A	2	2	1.00	24	0.083
141	A	4	4	1.29	24	0.167
142	A	3	3	1.06	24	0.125
143	A	3	3	1.10	22	0.136
144	A	4	4	1.15	24	0.167
145	A	5	5	1.28	24	0.208
146	A	6	6	1.28	24	0.250
147	A	12	11	1.07	24	0.458
148	A	9	8	1.08	24	0.333
149	A	8	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	1	1	1.00	24	0.042
151	A	2	2	1.00	24	0.083
152	A	3	3	1.08	24	0.125
153	A	4	4	1.11	24	0.167
154	A	6	6	1.39	24	0.250
155	A	6	6	1.41	24	0.250
156	A	6	6	1.31	24	0.250
157	A	5	5	1.17	24	0.208
158	A	5	5	1.19	22	0.227
159	A	5	5	1.20	16	0.312
160	A	6	6	1.21	24	0.250
161	A	7	7	1.30	24	0.292
162	A	8	8	1.29	24	0.333
163	A	2	2	1.00	15	0.133
164	A	2	2	1.00	17	0.118
165	A	4	4	1.04	29	0.138
166	A	3	3	1.03	29	0.103
167	A	2	2	1.00	29	0.069
168	A	1	1	1.00	29	0.034
169	A	4	3	1.00	29	0.103
170	A	4	3	1.00	29	0.103
171	A	5	4	1.06	29	0.138
172	A	5	5	1.04	29	0.172
173	A	4	4	1.04	29	0.138
174	A	3	3	1.03	29	0.103
175	A	2	2	1.00	29	0.069
176	A	1	1	1.00	29	0.034
177	A	5	4	1.02	29	0.138
178	A	5	4	1.04	29	0.138
179	A	5	4	1.00	29	0.138
180	A	6	5	1.07	29	0.172
181	A	7	6	1.11	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	4	1.04	29	0.138
183	A	3	3	1.03	29	0.103
184	A	2	2	1.00	29	0.069
185	A	1	1	1.00	29	0.034
186	A	3	2	1.00	29	0.069
187	A	4	3	1.00	29	0.103
188	A	5	4	1.05	29	0.138
189	A	4	4	1.05	29	0.138
190	A	3	3	1.04	29	0.103
191	A	2	2	1.00	29	0.069
192	A	1	1	1.00	29	0.034
193	A	4	3	1.00	29	0.103
194	A	5	4	1.33	29	0.138
195	A	6	5	1.29	29	0.172
196	A	4	4	1.04	29	0.138
197	A	3	3	1.00	29	0.103
198	A	2	2	0.99	29	0.069
199	A	1	1	1.00	29	0.034
200	A	6	5	1.30	29	0.172
201	A	6	5	1.31	29	0.172
202	A	7	6	1.57	29	0.207
203	A	8	7	1.49	29	0.241
204	A	9	8	1.44	29	0.276
205	A	3	2	0.97	19	0.105
206	A	3	3	1.00	24	0.125
207	A	3	3	1.00	24	0.125
208	A	3	3	1.00	24	0.125
209	A	2	2	1.00	24	0.083
210	A	6	5	1.00	24	0.208
211	A	6	5	1.00	24	0.208
212	A	7	6	1.00	24	0.250
213	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	3	3	1.00	24	0.125
215	A	3	3	1.00	24	0.125
216	A	3	3	1.00	24	0.125
217	A	2	2	1.00	24	0.083
218	A	7	6	0.97	24	0.250
219	A	7	6	1.00	24	0.250
220	A	7	6	0.98	24	0.250
221	A	8	7	1.00	24	0.292
222	A	9	8	1.00	24	0.333
223	A	3	3	1.00	24	0.125
224	A	3	3	1.00	24	0.125
225	A	3	3	1.00	24	0.125
226	A	2	2	1.00	24	0.083
227	A	4	3	1.20	24	0.125
228	A	6	5	1.00	24	0.208
229	A	7	6	1.00	24	0.250
230	A	3	3	1.00	24	0.125
231	A	3	3	1.00	24	0.125
232	A	3	3	1.00	24	0.125
233	A	3	3	1.00	24	0.125
234	A	2	2	1.00	24	0.083
235	A	6	5	1.00	24	0.208
236	A	7	6	1.00	24	0.250
237	A	8	7	1.00	24	0.292
238	A	3	2	1.00	15	0.133
239	A	4	3	1.00	19	0.158
240	A	3	2	1.00	18	0.111
241	A	4	3	1.00	24	0.125
242	A	9	8	1.08	24	0.333
243	A	7	6	1.07	24	0.250
244	A	5	4	1.01	22	0.182
245	A	4	3	1.00	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	5	4	1.00	24	0.167
247	C	3	2	0.20	24	0.083
248	C	3	2	0.19	24	0.083
249	A	11	10	1.12	24	0.417
250	A	9	8	1.12	24	0.333
251	A	7	6	1.08	22	0.273
252	A	6	5	1.08	16	0.312
253	A	7	6	1.09	24	0.250
254	C	3	2	0.10	24	0.083
255	C	3	2	0.10	24	0.083
256	A	10	9	1.12	24	0.375
257	A	8	7	1.12	24	0.292
258	A	6	5	1.09	22	0.227
259	A	5	4	1.09	16	0.250
260	A	8	7	1.11	24	0.292
261	C	3	2	0.10	24	0.083
262	C	3	2	0.09	24	0.083
263	A	8	7	1.08	24	0.292
264	A	6	5	1.07	24	0.208
265	A	4	3	1.00	22	0.136
266	A	3	2	1.00	16	0.125
267	A	6	5	1.05	24	0.208
268	C	3	2	0.19	24	0.083
269	C	3	2	0.17	24	0.083
270	A	4	3	1.00	23	0.130
271	A	5	4	1.00	24	0.167
272	A	4	4	1.04	26	0.154
273	A	3	3	1.03	26	0.115
274	A	2	2	1.00	26	0.077
275	A	1	1	1.00	26	0.038
276	A	9	8	0.78	26	0.308
277	A	9	8	0.80	26	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	10	9	0.82	26	0.346
279	A	11	10	0.85	26	0.385
280	A	6	6	0.98	26	0.231
281	A	5	5	0.96	26	0.192
282	A	4	4	0.98	26	0.154
283	A	4	4	0.98	26	0.154
284	A	1	1	1.00	26	0.038
285	A	2	2	1.00	26	0.077
286	A	3	3	1.08	26	0.115
287	A	4	4	1.11	26	0.154
288	A	5	5	0.96	26	0.192
289	A	4	4	0.97	26	0.154
290	A	3	3	0.93	26	0.115
291	A	1	1	1.00	26	0.038
292	A	2	2	1.00	26	0.077
293	A	3	3	1.08	26	0.115
294	A	4	4	1.11	26	0.154
295	A	4	4	1.04	26	0.154
296	A	3	3	1.03	26	0.115
297	A	2	2	1.00	26	0.077
298	A	1	1	1.00	26	0.038
299	A	8	7	0.72	26	0.269
300	A	9	8	0.77	26	0.308
301	A	10	9	0.80	26	0.346
302	A	3	3	1.00	23	0.130
303	A	4	4	1.00	24	0.167
304	A	15	14	1.10	24	0.583
305	A	14	13	1.17	24	0.542
306	A	14	13	1.17	24	0.542
307	A	2	2	0.89	24	0.083
308	A	4	4	0.89	24	0.167
309	A	5	5	0.89	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	6	6	0.89	24	0.250
311	A	16	15	1.07	24	0.625
312	A	15	14	1.11	24	0.583
313	A	14	13	1.17	24	0.542
314	A	13	12	1.26	24	0.500
315	A	2	2	0.89	24	0.083
316	A	4	4	0.89	24	0.167
317	A	5	5	0.89	24	0.208
318	A	6	6	0.89	24	0.250
319	A	2	2	0.99	24	0.083
320	A	2	2	0.96	24	0.083
321	A	3	3	1.19	22	0.136
322	A	2	2	1.00	16	0.125
323	A	2	2	1.06	24	0.083
324	A	2	2	1.02	24	0.083
325	A	2	2	0.99	24	0.083
326	A	2	2	0.99	25	0.080
327	A	5	4	1.32	23	0.174
328	C	2	2	0.20	24	0.083
329	A	3	3	1.19	22	0.136
330	A	3	3	1.19	22	0.136
331	A	3	3	1.19	22	0.136
332	A	3	3	1.19	22	0.136
333	A	3	3	1.19	22	0.136
334	A	3	3	1.16	23	0.130
335	A	2	2	1.06	24	0.083
336	A	3	3	1.00	25	0.120
337	A	3	3	1.00	25	0.120
338	A	3	3	1.00	23	0.130
339	A	2	2	1.00	25	0.080
340	A	2	2	1.00	25	0.080
341	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	3	3	1.00	24	0.125
343	A	3	3	1.00	24	0.125
344	A	3	3	1.00	24	0.125
345	A	3	3	1.00	24	0.125
346	A	3	3	1.00	24	0.125
347	A	3	3	1.00	24	0.125
348	A	2	2	1.14	23	0.087
349	A	2	2	1.12	21	0.095
350	A	1	1	1.00	15	0.067
351	A	2	2	1.32	23	0.087
352	A	2	2	1.12	23	0.087
353	A	2	2	1.08	23	0.087
354	A	2	2	1.03	22	0.091
355	A	3	3	1.20	20	0.150
356	A	2	2	0.76	14	0.143
357	A	2	2	1.26	22	0.091
358	A	2	2	1.07	22	0.091
359	A	2	2	1.01	22	0.091
360	A	2	2	1.00	17	0.118
361	A	1	1	1.00	38	0.026
362	A	3	3	1.23	25	0.120
363	A	3	3	1.23	25	0.120
364	A	3	3	1.23	25	0.120
365	A	3	3	1.26	25	0.120
366	A	3	3	1.00	24	0.125
367	A	3	3	1.00	24	0.125
368	A	3	3	1.00	24	0.125
369	A	3	3	1.00	24	0.125
370	A	3	3	1.00	22	0.136
371	A	3	3	1.00	23	0.130
372	A	3	3	1.00	24	0.125
373	A	3	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	3	3	1.00	24	0.125
375	A	2	2	1.00	22	0.091
376	A	2	2	1.00	24	0.083
377	A	2	2	1.00	24	0.083
378	A	2	2	1.00	24	0.083
379	A	6	6	1.24	26	0.231
380	A	5	5	1.37	26	0.192
381	A	4	4	1.62	26	0.154
382	A	1	1	1.00	26	0.038
383	A	3	3	1.00	26	0.115
384	A	3	3	1.00	24	0.125
385	A	3	3	1.00	26	0.115
386	A	2	2	1.11	20	0.100
387	A	2	2	1.00	25	0.080
388	A	1	1	1.11	18	0.056
389	A	2	2	1.56	21	0.095
390	A	2	2	1.14	22	0.091
391	A	1	1	1.02	18	0.056

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = -16a^4x-\frac{4a^3(a+bx)^2}{b}-\frac{4a^2(a+bx)^3}{3b}-\frac{a(a+bx)^4}{2b}-\frac{(a+bx)^5}{5b}-\frac{32a^5\log(a-bx)}{b}$$

output
```
-16*a^4*x-4*a^3*(b*x+a)^2/b-4/3*a^2*(b*x+a)^3/b-1/2*a*(b*x+a)^4/b-1/5*(b*x
+a)^5/b-32*a^5*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = -31a^4x-13a^3bx^2-\frac{16}{3}a^2b^2x^3-\frac{3}{2}ab^3x^4-\frac{b^4x^5}{5}-\frac{32a^5\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^6/(a^2 - b^2*x^2),x]
```

output
```
-31*a^4*x - 13*a^3*b*x^2 - (16*a^2*b^2*x^3)/3 - (3*a*b^3*x^4)/2 - (b^4*x^5
)/5 - (32*a^5*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{(a+bx)^5}{a-bx}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{32a^5}{a-bx}-16a^4-8a^3(a+bx)-4a^2(a+bx)^2-2a(a+bx)^3-(a+bx)^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{32a^5\log(a-bx)}{b}-16a^4x-\frac{4a^3(a+bx)^2}{b}-\frac{4a^2(a+bx)^3}{3b}-\frac{a(a+bx)^4}{2b}-\frac{(a+bx)^5}{5b}$$

input
```
Int[(a + b*x)^6/(a^2 - b^2*x^2),x]
```

output
```
-16*a^4*x - (4*a^3*(a + b*x)^2)/b - (4*a^2*(a + b*x)^3)/(3*b) - (a*(a + b*
x)^4)/(2*b) - (a + b*x)^5/(5*b) - (32*a^5*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{b^4x^5}{5}-\frac{3a\,b^3x^4}{2}-\frac{16a^2b^2x^3}{3}-13a^3b\,x^2-31a^4x-\frac{32a^5\ln(-bx+a)}{b}$	60
norman	$-\frac{b^4x^5}{5}-\frac{3a\,b^3x^4}{2}-\frac{16a^2b^2x^3}{3}-13a^3b\,x^2-31a^4x-\frac{32a^5\ln(-bx+a)}{b}$	60
risch	$-\frac{b^4x^5}{5}-\frac{3a\,b^3x^4}{2}-\frac{16a^2b^2x^3}{3}-13a^3b\,x^2-31a^4x-\frac{32a^5\ln(-bx+a)}{b}$	60
parallelrisch	$-\frac{6b^5x^5+45a\,b^4x^4+160a^2b^3x^3+390a^3b^2x^2+960a^5\ln(bx-a)+930a^4bx}{30b}$	66

input
```
int((b*x+a)^6/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*b^4*x^5-3/2*a*b^3*x^4-16/3*a^2*b^2*x^3-13*a^3*b*x^2-31*a^4*x-32*a^5*l
n(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = -\frac{6\,b^5x^5+45\,ab^4x^4+160\,a^2b^3x^3+390\,a^3b^2x^2+930\,a^4bx+960\,a^5\log(bx-a)}{30\,b}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/30*(6*b^5*x^5 + 45*a*b^4*x^4 + 160*a^2*b^3*x^3 + 390*a^3*b^2*x^2 + 930*
a^4*b*x + 960*a^5*log(b*x - a))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = -\frac{32a^5\log(-a+bx)}{b} - 31a^4x - 13a^3bx^2 - \frac{16a^2b^2x^3}{3} - \frac{3ab^3x^4}{2} - \frac{b^4x^5}{5}$$

input
```
integrate((b*x+a)**6/(-b**2*x**2+a**2),x)
```

output
```
-32*a**5*log(-a + b*x)/b - 31*a**4*x - 13*a**3*b*x**2 - 16*a**2*b**2*x**3/
3 - 3*a*b**3*x**4/2 - b**4*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = -\frac{1}{5}\,b^4x^5 - \frac{3}{2}\,ab^3x^4 - \frac{16}{3}\,a^2b^2x^3 - 13\,a^3bx^2 - 31\,a^4x - \frac{32\,a^5\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/5*b^4*x^5 - 3/2*a*b^3*x^4 - 16/3*a^2*b^2*x^3 - 13*a^3*b*x^2 - 31*a^4*x
- 32*a^5*log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = -\frac{32\,a^5\log\left(|bx-a|\right)}{b} - \frac{6\,b^9x^5+45\,ab^8x^4+160\,a^2b^7x^3+390\,a^3b^6x^2+930\,a^4b^5x}{30\,b^5}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-32*a^5*log(abs(b*x - a))/b - 1/30*(6*b^9*x^5 + 45*a*b^8*x^4 + 160*a^2*b^7
*x^3 + 390*a^3*b^6*x^2 + 930*a^4*b^5*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = -31\,a^4\,x-\frac{b^4\,x^5}{5}-13\,a^3\,b\,x^2-\frac{3\,a\,b^3\,x^4}{2}-\frac{32\,a^5\,\ln\left(b\,x-a\right)}{b}-\frac{16\,a^2\,b^2\,x^3}{3}$$

input
```
int((a + b*x)^6/(a^2 - b^2*x^2),x)
```

output
```
- 31*a^4*x - (b^4*x^5)/5 - 13*a^3*b*x^2 - (3*a*b^3*x^4)/2 - (32*a^5*log(b*
x - a))/b - (16*a^2*b^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^6}{a^2-b^2x^2}\,dx = \frac{-960\log(-bx+a)\,a^5-930a^4bx-390a^3b^2x^2-160a^2b^3x^3-45a\,b^4x^4-6b^5x^5}{30b}$$

input
```
int((b*x+a)^6/(-b^2*x^2+a^2),x)
```

output
```
( - 960*log(a - b*x)*a**5 - 930*a**4*b*x - 390*a**3*b**2*x**2 - 160*a**2*b
**3*x**3 - 45*a*b**4*x**4 - 6*b**5*x**5)/(30*b)
```

3.2 $\int \frac{(a+bx)^5}{a^2-b^2x^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{(a+bx)^5}{a^2-b^2x^2} dx = -8a^3x - \frac{2a^2(a+bx)^2}{b} - \frac{2a(a+bx)^3}{3b} - \frac{(a+bx)^4}{4b} - \frac{16a^4\log(a-bx)}{b}$$

output
```
-8*a^3*x-2*a^2*(b*x+a)^2/b-2/3*a*(b*x+a)^3/b-1/4*(b*x+a)^4/b-16*a^4*ln(-b*
x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^5}{a^2-b^2x^2} dx = -15a^3x - \frac{11}{2}a^2bx^2 - \frac{5}{3}ab^2x^3 - \frac{b^3x^4}{4} - \frac{16a^4\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^5/(a^2 - b^2*x^2),x]
```

output
```
-15*a^3*x - (11*a^2*b*x^2)/2 - (5*a*b^2*x^3)/3 - (b^3*x^4)/4 - (16*a^4*Log
[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx$$
$$\Big\downarrow \text{456}$$
$$\int \frac{(a+bx)^4}{a-bx}dx$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{16a^4}{a-bx}-8a^3-4a^2(a+bx)-2a(a+bx)^2-(a+bx)^3\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{16a^4\log(a-bx)}{b}-8a^3x-\frac{2a^2(a+bx)^2}{b}-\frac{2a(a+bx)^3}{3b}-\frac{(a+bx)^4}{4b}$$

input
```
Int[(a + b*x)^5/(a^2 - b^2*x^2),x]
```

output
```
-8*a^3*x - (2*a^2*(a + b*x)^2)/b - (2*a*(a + b*x)^3)/(3*b) - (a + b*x)^4/(
4*b) - (16*a^4*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{b^{3} x^{4}}{4}-\frac{5 a\, b^{2} x^{3}}{3}-\frac{11 a^{2} b\, x^{2}}{2}-15 a^{3} x -\frac{16 a^{4} \ln \left(-b x +a \right)}{b}$	49
norman	$-\frac{b^{3} x^{4}}{4}-\frac{5 a\, b^{2} x^{3}}{3}-\frac{11 a^{2} b\, x^{2}}{2}-15 a^{3} x -\frac{16 a^{4} \ln \left(-b x +a \right)}{b}$	49
risch	$-\frac{b^{3} x^{4}}{4}-\frac{5 a\, b^{2} x^{3}}{3}-\frac{11 a^{2} b\, x^{2}}{2}-15 a^{3} x -\frac{16 a^{4} \ln \left(-b x +a \right)}{b}$	49
parallelrisch	$-\frac{3 b^{4} x^{4}+20 a\, b^{3} x^{3}+66 a^{2} b^{2} x^{2}+192 a^{4} \ln \left(b x -a \right)+180 a^{3} b x}{12 b}$	55

input
```
int((b*x+a)^5/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3*x^4-5/3*a*b^2*x^3-11/2*a^2*b*x^2-15*a^3*x-16*a^4*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx=-\frac{3\,b^4x^4+20\,ab^3x^3+66\,a^2b^2x^2+180\,a^3bx+192\,a^4\log\left(bx-a\right)}{12\,b}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/12*(3*b^4*x^4 + 20*a*b^3*x^3 + 66*a^2*b^2*x^2 + 180*a^3*b*x + 192*a^4*l
og(b*x - a))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx = -\frac{16a^4\log(-a+bx)}{b} - 15a^3x - \frac{11a^2bx^2}{2} - \frac{5ab^2x^3}{3} - \frac{b^3x^4}{4}$$

input
```
integrate((b*x+a)**5/(-b**2*x**2+a**2),x)
```

output
```
-16*a**4*log(-a + b*x)/b - 15*a**3*x - 11*a**2*b*x**2/2 - 5*a*b**2*x**3/3
- b**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx = -\frac{1}{4}\,b^3x^4 - \frac{5}{3}\,ab^2x^3 - \frac{11}{2}\,a^2bx^2 - 15\,a^3x - \frac{16\,a^4\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/4*b^3*x^4 - 5/3*a*b^2*x^3 - 11/2*a^2*b*x^2 - 15*a^3*x - 16*a^4*log(b*x
- a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx = -\frac{16\,a^4\log(|bx-a|)}{b} - \frac{3\,b^7x^4 + 20\,ab^6x^3 + 66\,a^2b^5x^2 + 180\,a^3b^4x}{12\,b^4}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-16*a^4*log(abs(b*x - a))/b - 1/12*(3*b^7*x^4 + 20*a*b^6*x^3 + 66*a^2*b^5*
x^2 + 180*a^3*b^4*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx = -15\,a^3\,x-\frac{b^3\,x^4}{4}-\frac{11\,a^2\,b\,x^2}{2}-\frac{5\,a\,b^2\,x^3}{3}-\frac{16\,a^4\,\ln\left(b\,x-a\right)}{b}$$

input
```
int((a + b*x)^5/(a^2 - b^2*x^2),x)
```

output
```
- 15*a^3*x - (b^3*x^4)/4 - (11*a^2*b*x^2)/2 - (5*a*b^2*x^3)/3 - (16*a^4*lo
g(b*x - a))/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^5}{a^2-b^2x^2}\,dx = \frac{-192\log(-bx+a)\,a^4-180a^3bx-66a^2b^2x^2-20a\,b^3x^3-3b^4x^4}{12b}$$

input
```
int((b*x+a)^5/(-b^2*x^2+a^2),x)
```

output
```
( - 192*log(a - b*x)*a**4 - 180*a**3*b*x - 66*a**2*b**2*x**2 - 20*a*b**3*x
**3 - 3*b**4*x**4)/(12*b)
```

3.3 $\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx = -4a^2x - \frac{a(a+bx)^2}{b} - \frac{(a+bx)^3}{3b} - \frac{8a^3\log(a-bx)}{b}$$

output
```
-4*a^2*x-a*(b*x+a)^2/b-1/3*(b*x+a)^3/b-8*a^3*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx = -7a^2x - 2abx^2 - \frac{b^2x^3}{3} - \frac{8a^3\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^4/(a^2 - b^2*x^2),x]
```

output
```
-7*a^2*x - 2*a*b*x^2 - (b^2*x^3)/3 - (8*a^3*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{(a+bx)^3}{a-bx}dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{8a^3}{a-bx}-4a^2-2a(a+bx)-(a+bx)^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{8a^3\log(a-bx)}{b}-4a^2x-\frac{a(a+bx)^2}{b}-\frac{(a+bx)^3}{3b}$$

input
```
Int[(a + b*x)^4/(a^2 - b^2*x^2),x]
```

output
```
-4*a^2*x - (a*(a + b*x)^2)/b - (a + b*x)^3/(3*b) - (8*a^3*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{b^2x^3}{3}-2ab\,x^2-7a^2x-\frac{8a^3\ln(-bx+a)}{b}$	38
norman	$-\frac{b^2x^3}{3}-2ab\,x^2-7a^2x-\frac{8a^3\ln(-bx+a)}{b}$	38
risch	$-\frac{b^2x^3}{3}-2ab\,x^2-7a^2x-\frac{8a^3\ln(-bx+a)}{b}$	38
parallelrisch	$-\frac{b^3x^3+6a\,b^2x^2+24a^3\ln(bx-a)+21a^2bx}{3b}$	43

input
```
int((b*x+a)^4/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*x^3-2*a*b*x^2-7*a^2*x-8*a^3*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx=-\frac{b^3x^3+6\,ab^2x^2+21\,a^2bx+24\,a^3\log(bx-a)}{3\,b}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/3*(b^3*x^3 + 6*a*b^2*x^2 + 21*a^2*b*x + 24*a^3*log(b*x - a))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx = -\frac{8a^3\log(-a+bx)}{b} - 7a^2x - 2abx^2 - \frac{b^2x^3}{3}$$

input
```
integrate((b*x+a)**4/(-b**2*x**2+a**2),x)
```

output
```
-8*a**3*log(-a + b*x)/b - 7*a**2*x - 2*a*b*x**2 - b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx = -\frac{1}{3}\,b^2x^3 - 2\,abx^2 - 7\,a^2x - \frac{8\,a^3\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/3*b^2*x^3 - 2*a*b*x^2 - 7*a^2*x - 8*a^3*log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx = -\frac{8\,a^3\log(|bx-a|)}{b} - \frac{b^5x^3+6\,ab^4x^2+21\,a^2b^3x}{3\,b^3}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-8*a^3*log(abs(b*x - a))/b - 1/3*(b^5*x^3 + 6*a*b^4*x^2 + 21*a^2*b^3*x)/b^
3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx = -7\,a^2\,x-\frac{b^2\,x^3}{3}-\frac{8\,a^3\,\ln\left(b\,x-a\right)}{b}-2\,a\,b\,x^2$$

input
```
int((a + b*x)^4/(a^2 - b^2*x^2),x)
```

output
```
- 7*a^2*x - (b^2*x^3)/3 - (8*a^3*log(b*x - a))/b - 2*a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)^4}{a^2-b^2x^2}\,dx = \frac{-24\log(-bx+a)\,a^3-21a^2bx-6a\,b^2x^2-b^3x^3}{3b}$$

input
```
int((b*x+a)^4/(-b^2*x^2+a^2),x)
```

output
```
( - 24*log(a - b*x)*a**3 - 21*a**2*b*x - 6*a*b**2*x**2 - b**3*x**3)/(3*b)
```

3.4 $\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx=-3ax-\frac{bx^2}{2}-\frac{4a^2\log(a-bx)}{b}$$

output
```
-3*a*x-1/2*b*x^2-4*a^2*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx=-3ax-\frac{bx^2}{2}-\frac{4a^2\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^3/(a^2 - b^2*x^2),x]
```

output
```
-3*a*x - (b*x^2)/2 - (4*a^2*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \ 456$$

$$\int \frac{(a+bx)^2}{a-bx}dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{4a^2}{a-bx}-3a-bx\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4a^2\log(a-bx)}{b}-3ax-\frac{bx^2}{2}$$

input
```
Int[(a + b*x)^3/(a^2 - b^2*x^2),x]
```

output
```
-3*a*x - (b*x^2)/2 - (4*a^2*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
default	$-3ax-\frac{b\,x^{2}}{2}-\frac{4a^{2}\ln\left(-bx+a\right)}{b}$	27
norman	$-3ax-\frac{b\,x^{2}}{2}-\frac{4a^{2}\ln\left(-bx+a\right)}{b}$	27
risch	$-3ax-\frac{b\,x^{2}}{2}-\frac{4a^{2}\ln\left(-bx+a\right)}{b}$	27
parallelrisch	$-\frac{b^{2}x^{2}+8a^{2}\ln\left(bx-a\right)+6abx}{2b}$	32

input
```
int((b*x+a)^3/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-3*a*x-1/2*b*x^2-4*a^2*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx=-\frac{b^2x^2+6\,abx+8\,a^2\log\left(bx-a\right)}{2\,b}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx = -\frac{4a^2\log(-a+bx)}{b} - 3ax - \frac{bx^2}{2}$$

input
```
integrate((b*x+a)**3/(-b**2*x**2+a**2),x)
```

output
```
-4*a**2*log(-a + b*x)/b - 3*a*x - b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx = -\frac{1}{2}\,bx^2 - 3\,ax - \frac{4\,a^2\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2*b*x^2 - 3*a*x - 4*a^2*log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx = -\frac{4\,a^2\log(|bx-a|)}{b} - \frac{b^3x^2+6\,ab^2x}{2\,b^2}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-4*a^2*log(abs(b*x - a))/b - 1/2*(b^3*x^2 + 6*a*b^2*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx = -3\,a\,x-\frac{b\,x^2}{2}-\frac{4\,a^2\,\ln\left(b\,x-a\right)}{b}$$

input
```
int((a + b*x)^3/(a^2 - b^2*x^2),x)
```

output
```
- 3*a*x - (b*x^2)/2 - (4*a^2*log(b*x - a))/b
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^3}{a^2-b^2x^2}\,dx = \frac{-8\log(-bx+a)\,a^2-6abx-b^2x^2}{2b}$$

input
```
int((b*x+a)^3/(-b^2*x^2+a^2),x)
```

output
```
( - 8*log(a - b*x)*a**2 - 6*a*b*x - b**2*x**2)/(2*b)
```

3.5 $\int \frac{(a+bx)^2}{a^2-b^2x^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 17

$$\int \frac{(a+bx)^2}{a^2-b^2x^2} \, dx = -x - \frac{2a\log(a-bx)}{b}$$

output
```
-x-2*a*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{a^2-b^2x^2} \, dx = -x - \frac{2a\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^2/(a^2 - b^2*x^2),x]
```

output
```
-x - (2*a*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{a^2-b^2x^2}dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{a+bx}{a-bx}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{2a}{a-bx}-1\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a\log(a-bx)}{b}-x$$

input
```
Int[(a + b*x)^2/(a^2 - b^2*x^2),x]
```

output
```
-x - (2*a*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
default	$-x-\frac{2a\ln(-bx+a)}{b}$	18
norman	$-x-\frac{2a\ln(-bx+a)}{b}$	18
risch	$-x-\frac{2a\ln(-bx+a)}{b}$	18
parallelrisch	$-\frac{2a\ln(bx-a)+bx}{b}$	21

input
```
int((b*x+a)^2/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-x-2*a*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^2}{a^2-b^2x^2}\,dx=-\frac{bx+2\,a\log\left(bx-a\right)}{b}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-(b*x + 2*a*log(b*x - a))/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2}{a^2-b^2x^2}\,dx = -\frac{2a\log(-a+bx)}{b} - x$$

input
```
integrate((b*x+a)**2/(-b**2*x**2+a**2),x)
```

output
```
-2*a*log(-a + b*x)/b - x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^2}{a^2-b^2x^2}\,dx = -x - \frac{2\,a\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-x - 2*a*log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^2}{a^2-b^2x^2}\,dx = -x - \frac{2\,a\log(|bx-a|)}{b}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-x - 2*a*log(abs(b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^2}{a^2-b^2x^2}\,dx = -x - \frac{2\,a\,\ln\left(b\,x-a\right)}{b}$$

input
```
int((a + b*x)^2/(a^2 - b^2*x^2),x)
```

output
```
- x - (2*a*log(b*x - a))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^2}{a^2-b^2x^2}\,dx = \frac{-2\log(-bx+a)\,a-bx}{b}$$

input
```
int((b*x+a)^2/(-b^2*x^2+a^2),x)
```

output
```
( - 2*log(a - b*x)*a - b*x)/b
```

3.6 $\int \frac{a+bx}{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 12

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx = -\frac{\log(a-bx)}{b}$$

output
```
-ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx = -\frac{\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)/(a^2 - b^2*x^2),x]
```

output
```
-(Log[a - b*x]/b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {451, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{a^2-b^2x^2} \, dx$$

$$\Big\downarrow \text{451}$$

$$\int \frac{1}{a-bx}dx$$

$$\Big\downarrow \text{16}$$

$$-\frac{\log(a-bx)}{b}$$

input
```
Int[(a + b*x)/(a^2 - b^2*x^2),x]
```

output
```
-(Log[a - b*x]/b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 451
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c^2/a   In
t[1/(c - d*x), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\ln(-bx+a)}{b}$	13
norman	$-\frac{\ln(-bx+a)}{b}$	13
risch	$-\frac{\ln(-bx+a)}{b}$	13
parallelrisch	$-\frac{\ln(bx-a)}{b}$	14

input
```
int((b*x+a)/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
-ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx = -\frac{\log\left(bx-a\right)}{b}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-log(b*x - a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx = -\frac{\log\left(-a+bx\right)}{b}$$

input
```
integrate((b*x+a)/(-b**2*x**2+a**2),x)
```

output
```
-log(-a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx=-\frac{\log(bx-a)}{b}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx=-\frac{\log(|bx-a|)}{b}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-log(abs(b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx=-\frac{\ln(b\,x-a)}{b}$$

input
```
int((a + b*x)/(a^2 - b^2*x^2),x)
```

output
```
-log(b*x - a)/b
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{a^2-b^2x^2}\,dx = -\frac{\log(-bx+a)}{b}$$

input
```
int((b*x+a)/(-b^2*x^2+a^2),x)
```

output
```
( - log(a - b*x))/b
```

3.7 $\int \frac{1}{(a+bx)(a^2-b^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)}\,dx = -\frac{1}{2ab(a+bx)}+\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^2b}$$

output
```
-1/2/a/b/(b*x+a)+1/2*arctanh(b*x/a)/a^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)}\,dx = \frac{-2a-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)}{4a^2b(a+bx)}$$

input
```
Integrate[1/((a + b*x)*(a^2 - b^2*x^2)),x]
```

output
```
(-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*(a + b*x
))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{1}{(a-bx)(a+bx)^2}dx$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(\frac{1}{2a(a^2-b^2x^2)}+\frac{1}{2a(a+bx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^2b}-\frac{1}{2ab(a+bx)}$$

input
```
Int[1/((a + b*x)*(a^2 - b^2*x^2)),x]
```

output
```
-1/2*1/(a*b*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

method	result	size
norman	$\frac{x}{2a^{2}(bx+a)}-\frac{\ln(-bx+a)}{4b\,a^{2}}+\frac{\ln(bx+a)}{4b\,a^{2}}$	44
default	$\frac{\ln(bx+a)}{4b\,a^{2}}-\frac{1}{2ab(bx+a)}-\frac{\ln(-bx+a)}{4b\,a^{2}}$	46
risch	$\frac{\ln(bx+a)}{4b\,a^{2}}-\frac{1}{2ab(bx+a)}-\frac{\ln(-bx+a)}{4b\,a^{2}}$	46
parallelrisch	$-\frac{\ln(bx-a)xb-\ln(bx+a)xb+a\ln(bx-a)-\ln(bx+a)a-2bx}{4a^{2}(bx+a)b}$	61

input
```
int(1/(b*x+a)/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*x/(b*x+a)-1/4/b/a^2*ln(-b*x+a)+1/4/b/a^2*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+bx)\,(a^2-b^2x^2)}\,dx=\frac{(bx+a)\log(bx+a)-(bx+a)\log(bx-a)-2\,a}{4\,(a^2b^2x+a^3b)}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*x + a
^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)}\,dx = -\frac{1}{2a^2b+2ab^2x} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4} - \frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2b}$$

input
```
integrate(1/(b*x+a)/(-b**2*x**2+a**2),x)
```

output
```
-1/(2*a**2*b + 2*a*b**2*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)}\,dx = -\frac{1}{2\left(ab^2x+a^2b\right)} + \frac{\log\left(bx+a\right)}{4\,a^2b} - \frac{\log\left(bx-a\right)}{4\,a^2b}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/2/(a*b^2*x + a^2*b) + 1/4*log(b*x + a)/(a^2*b) - 1/4*log(b*x - a)/(a^2*
b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)}\,dx = \frac{\log\left(|bx+a|\right)}{4\,a^2b} - \frac{\log\left(|bx-a|\right)}{4\,a^2b} - \frac{1}{2\,(bx+a)ab}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/4*log(abs(b*x + a))/(a^2*b) - 1/4*log(abs(b*x - a))/(a^2*b) - 1/2/((b*x
+ a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b} - \frac{1}{2\,a\,b\,(a+b\,x)}$$

input
```
int(1/((a^2 - b^2*x^2)*(a + b*x)),x)
```

output
```
atanh((b*x)/a)/(2*a^2*b) - 1/(2*a*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)}\,dx = \frac{-\log(-bx+a)\,a-\log(-bx+a)\,bx+\log(bx+a)\,a+\log(bx+a)\,bx+2bx}{4a^2b\,(bx+a)}$$

input
```
int(1/(b*x+a)/(-b^2*x^2+a^2),x)
```

output
```
( - log(a - b*x)*a - log(a - b*x)*b*x + log(a + b*x)*a + log(a + b*x)*b*x
+ 2*b*x)/(4*a**2*b*(a + b*x))
```

3.8 $\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{1}{(a+bx)^2\,(a^2-b^2x^2)}\,dx=-\frac{1}{4ab(a+bx)^2}-\frac{1}{4a^2b(a+bx)}+\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{4a^3b}$$

output
```
-1/4/a/b/(b*x+a)^2-1/4/a^2/b/(b*x+a)+1/4*arctanh(b*x/a)/a^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx)^2\,(a^2-b^2x^2)}\,dx = \frac{-2a(2a+bx)-(a+bx)^2\log(a-bx)+(a+bx)^2\log(a+bx)}{8a^3b(a+bx)^2}$$

input
```
Integrate[1/((a + b*x)^2*(a^2 - b^2*x^2)),x]
```

output
```
(-2*a*(2*a + b*x) - (a + b*x)^2*Log[a - b*x] + (a + b*x)^2*Log[a + b*x])/(
8*a^3*b*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}dx$$
$$\Big\downarrow \text{ 456}$$
$$\int \frac{1}{(a-bx)(a+bx)^3}dx$$
$$\Big\downarrow \text{ 54}$$
$$\int \left(\frac{1}{4a^2(a^2-b^2x^2)}+\frac{1}{4a^2(a+bx)^2}+\frac{1}{2a(a+bx)^3}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{4a^3b}-\frac{1}{4a^2b(a+bx)}-\frac{1}{4ab(a+bx)^2}$$

input
```
Int[1/((a + b*x)^2*(a^2 - b^2*x^2)),x]
```

output
```
-1/4*1/(a*b*(a + b*x)^2) - 1/(4*a^2*b*(a + b*x)) + ArcTanh[(b*x)/a]/(4*a^3
*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{-\frac{x}{4a^2}-\frac{1}{2ba}}{(bx+a)^2}-\frac{\ln(-bx+a)}{8b\,a^3}+\frac{\ln(bx+a)}{8b\,a^3}$	54
norman	$\frac{\frac{3x}{4a^2}+\frac{b\,x^2}{2a^3}}{(bx+a)^2}-\frac{\ln(-bx+a)}{8b\,a^3}+\frac{\ln(bx+a)}{8b\,a^3}$	55
default	$\frac{\ln(bx+a)}{8b\,a^3}-\frac{1}{4a^2b(bx+a)}-\frac{1}{4ab(bx+a)^2}-\frac{\ln(-bx+a)}{8b\,a^3}$	61
parallelrisch	$-\frac{\ln(bx-a)x^2b^2-\ln(bx+a)x^2b^2+2\ln(bx-a)xab-2\ln(bx+a)xab-4b^2x^2+a^2\ln(bx-a)-\ln(bx+a)a^2-6abx}{8a^3(bx+a)^2b}$	106

input
```
int(1/(b*x+a)^2/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a^2*x-1/2/b/a)/(b*x+a)^2-1/8/b/a^3*ln(-b*x+a)+1/8/b/a^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a+bx)^2\,(a^2-b^2x^2)}\,dx$$
$$= -\frac{2\,abx+4\,a^2-(b^2x^2+2\,abx+a^2)\log(bx+a)+(b^2x^2+2\,abx+a^2)\log(bx-a)}{8\,(a^3b^3x^2+2\,a^4b^2x+a^5b)}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/8*(2*a*b*x + 4*a^2 - (b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2
+ 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*x^2 + 2*a^4*b^2*x + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}\,dx = -\frac{2a+bx}{4a^4b+8a^3b^2x+4a^2b^3x^2} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8} - \frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3b}$$

input
```
integrate(1/(b*x+a)**2/(-b**2*x**2+a**2),x)
```

output
```
-(2*a + b*x)/(4*a**4*b + 8*a**3*b**2*x + 4*a**2*b**3*x**2) - (log(-a/b + x
)/8 - log(a/b + x)/8)/(a**3*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}\,dx = -\frac{bx+2\,a}{4\,(a^2b^3x^2+2\,a^3b^2x+a^4b)} + \frac{\log(bx+a)}{8\,a^3b} - \frac{\log(bx-a)}{8\,a^3b}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/4*(b*x + 2*a)/(a^2*b^3*x^2 + 2*a^3*b^2*x + a^4*b) + 1/8*log(b*x + a)/(a
^3*b) - 1/8*log(b*x - a)/(a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}\,dx = -\frac{\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}}{4\,a^2b^2} - \frac{\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{8\,a^3b}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
-1/4*(b/(b*x + a) + a*b/(b*x + a)^2)/(a^2*b^2) - 1/8*log(abs(-2*a/(b*x + a
) + 1))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^3\,b} - \frac{\frac{x}{4\,a^2}+\frac{1}{2\,a\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int(1/((a^2 - b^2*x^2)*(a + b*x)^2),x)
```

output
```
atanh((b*x)/a)/(4*a^3*b) - (x/(4*a^2) + 1/(2*a*b))/(a^2 + b^2*x^2 + 2*a*b*
x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)}\,dx = \frac{-\log(-bx+a)\,a^2-2\log(-bx+a)\,abx-\log(-bx+a)\,b^2x^2+\log(bx+a)\,a^2+2\log(bx+a)\,abx+\log(}{8a^3b\,(b^2x^2+2abx+a^2)}$$

input
```
int(1/(b*x+a)^2/(-b^2*x^2+a^2),x)
```

output
```
( - log(a - b*x)*a**2 - 2*log(a - b*x)*a*b*x - log(a - b*x)*b**2*x**2 + lo
g(a + b*x)*a**2 + 2*log(a + b*x)*a*b*x + log(a + b*x)*b**2*x**2 - 3*a**2 +
 b**2*x**2)/(8*a**3*b*(a**2 + 2*a*b*x + b**2*x**2))
```

3.9 $\int \frac{1}{(a+bx)^3(a^2-b^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)}\,dx = -\frac{1}{6ab(a+bx)^3}-\frac{1}{8a^2b(a+bx)^2}-\frac{1}{8a^3b(a+bx)}+\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{8a^4b}$$

output
```
-1/6/a/b/(b*x+a)^3-1/8/a^2/b/(b*x+a)^2-1/8/a^3/b/(b*x+a)+1/8*arctanh(b*x/a
)/a^4/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)}\,dx = \frac{-2a(10a^2+9abx+3b^2x^2)-3(a+bx)^3\log(a-bx)+3(a+bx)^3\log(a+bx)}{48a^4b(a+bx)^3}$$

input
```
Integrate[1/((a + b*x)^3*(a^2 - b^2*x^2)),x]
```

output
```
(-2*a*(10*a^2 + 9*a*b*x + 3*b^2*x^2) - 3*(a + b*x)^3*Log[a - b*x] + 3*(a +
 b*x)^3*Log[a + b*x])/(48*a^4*b*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)}dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{1}{(a-bx)(a+bx)^4}dx$$

$$\Big\downarrow \; 54$$

$$\int \left(\frac{1}{8a^3(a+bx)^2}+\frac{1}{4a^2(a+bx)^3}+\frac{1}{8a^3(a^2-b^2x^2)}+\frac{1}{2a(a+bx)^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{8a^4b}-\frac{1}{8a^3b(a+bx)}-\frac{1}{8a^2b(a+bx)^2}-\frac{1}{6ab(a+bx)^3}$$

input
```
Int[1/((a + b*x)^3*(a^2 - b^2*x^2)),x]
```

output
```
-1/6*1/(a*b*(a + b*x)^3) - 1/(8*a^2*b*(a + b*x)^2) - 1/(8*a^3*b*(a + b*x))
 + ArcTanh[(b*x)/a]/(8*a^4*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result
norman	$\frac{-\frac{5}{12ba}-\frac{3x}{8a^2}-\frac{b\,x^2}{8a^3}}{(bx+a)^3}-\frac{\ln(-bx+a)}{16a^4b}+\frac{\ln(bx+a)}{16a^4b}$
risch	$\frac{-\frac{5}{12ba}-\frac{3x}{8a^2}-\frac{b\,x^2}{8a^3}}{(bx+a)^3}-\frac{\ln(-bx+a)}{16a^4b}+\frac{\ln(bx+a)}{16a^4b}$
default	$\frac{\ln(bx+a)}{16a^4b}-\frac{1}{8a^3b(bx+a)}-\frac{1}{8a^2b(bx+a)^2}-\frac{1}{6ab(bx+a)^3}-\frac{\ln(-bx+a)}{16a^4b}$
parallelrisch	$-\frac{3\ln(bx-a)x^3b^5-3\ln(bx+a)x^3b^5+9\ln(bx-a)x^2a\,b^4-9\ln(bx+a)x^2a\,b^4+9\ln(bx-a)x\,a^2b^3-9\ln(bx+a)x\,a^2b^3+6x^2a\,b^4+3}{48a^4b^3(bx+a)^3}$

input
```
int(1/(b*x+a)^3/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
(-5/12/b/a-3/8/a^2*x-1/8*b/a^3*x^2)/(b*x+a)^3-1/16/a^4/b*ln(-b*x+a)+1/16/a
^4/b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(61) = 122$.

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)}\,dx = -\frac{6\,ab^2x^2 + 18\,a^2bx + 20\,a^3 - 3\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\log\left(bx+a\right) + 3\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx\right.}{48\left(a^4b^4x^3 + 3\,a^5b^3x^2 + 3\,a^6b^2x + a^7b\right)}$$

input
```
integrate(1/(b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/48*(6*a*b^2*x^2 + 18*a^2*b*x + 20*a^3 - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^
2*b*x + a^3)*log(b*x + a) + 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*lo
g(b*x - a))/(a^4*b^4*x^3 + 3*a^5*b^3*x^2 + 3*a^6*b^2*x + a^7*b)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)}\,dx = -\frac{10a^2 + 9abx + 3b^2x^2}{24a^6b + 72a^5b^2x + 72a^4b^3x^2 + 24a^3b^4x^3} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{16} - \frac{\log\left(\frac{a}{b}+x\right)}{16}}{a^4b}$$

input
```
integrate(1/(b*x+a)**3/(-b**2*x**2+a**2),x)
```

output
```
-(10*a**2 + 9*a*b*x + 3*b**2*x**2)/(24*a**6*b + 72*a**5*b**2*x + 72*a**4*b
**3*x**2 + 24*a**3*b**4*x**3) - (log(-a/b + x)/16 - log(a/b + x)/16)/(a**4
*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx)^3 (a^2-b^2x^2)} dx = -\frac{3 b^2 x^2 + 9 abx + 10 a^2}{24 (a^3 b^4 x^3 + 3 a^4 b^3 x^2 + 3 a^5 b^2 x + a^6 b)} + \frac{\log (bx+a)}{16 a^4 b} - \frac{\log (bx-a)}{16 a^4 b}$$

input
```
integrate(1/(b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/24*(3*b^2*x^2 + 9*a*b*x + 10*a^2)/(a^3*b^4*x^3 + 3*a^4*b^3*x^2 + 3*a^5*
b^2*x + a^6*b) + 1/16*log(b*x + a)/(a^4*b) - 1/16*log(b*x - a)/(a^4*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx)^3 (a^2-b^2x^2)} dx = \frac{\log (|bx+a|)}{16 a^4 b} - \frac{\log (|bx-a|)}{16 a^4 b} - \frac{3 ab^2 x^2 + 9 a^2 bx + 10 a^3}{24 (bx+a)^3 a^4 b}$$

input
```
integrate(1/(b*x+a)^3/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/16*log(abs(b*x + a))/(a^4*b) - 1/16*log(abs(b*x - a))/(a^4*b) - 1/24*(3*
a*b^2*x^2 + 9*a^2*b*x + 10*a^3)/((b*x + a)^3*a^4*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)^3 (a^2-b^2x^2)} dx = \frac{\mathrm{atanh}\left(\frac{b x}{a}\right)}{8 a^4 b} - \frac{\frac{3 x}{8 a^2} + \frac{5}{12 a b} + \frac{b x^2}{8 a^3}}{a^3 + 3 a^2 b x + 3 a b^2 x^2 + b^3 x^3}$$

input
```
int(1/((a^2 - b^2*x^2)*(a + b*x)^3),x)
```

output
```
atanh((b*x)/a)/(8*a^4*b) - ((3*x)/(8*a^2) + 5/(12*a*b) + (b*x^2)/(8*a^3))/
(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.43

$$\int \frac{1}{(a+bx)^3\,(a^2-b^2x^2)}\,dx$$
$$= \frac{-3\log(-bx+a)\,a^3-9\log(-bx+a)\,a^2bx-9\log(-bx+a)\,a\,b^2x^2-3\log(-bx+a)\,b^3x^3+3\log(bx+a}{48a^4b\,(b^3x^3+3a\,b^2x^2+}$$

input
```
int(1/(b*x+a)^3/(-b^2*x^2+a^2),x)
```

output
```
( - 3*log(a - b*x)*a**3 - 9*log(a - b*x)*a**2*b*x - 9*log(a - b*x)*a*b**2*
x**2 - 3*log(a - b*x)*b**3*x**3 + 3*log(a + b*x)*a**3 + 9*log(a + b*x)*a**
2*b*x + 9*log(a + b*x)*a*b**2*x**2 + 3*log(a + b*x)*b**3*x**3 - 18*a**3 -
12*a**2*b*x + 2*b**3*x**3)/(48*a**4*b*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
 b**3*x**3))
```

3.10 $\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx = -\frac{1}{8ab(a+bx)^4}-\frac{1}{12a^2b(a+bx)^3}-\frac{1}{16a^3b(a+bx)^2}-\frac{1}{16a^4b(a+bx)}+\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{16a^5b}$$

output
```
-1/8/a/b/(b*x+a)^4-1/12/a^2/b/(b*x+a)^3-1/16/a^3/b/(b*x+a)^2-1/16/a^4/b/(b
*x+a)+1/16*arctanh(b*x/a)/a^5/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx = \frac{-2a(16a^3+19a^2bx+12ab^2x^2+3b^3x^3)-3(a+bx)^4\log(a-bx)+3(a+bx)^4\log(a+bx)}{96a^5b(a+bx)^4}$$

input
```
Integrate[1/((a + b*x)^4*(a^2 - b^2*x^2)),x]
```

output
```
(-2*a*(16*a^3 + 19*a^2*b*x + 12*a*b^2*x^2 + 3*b^3*x^3) - 3*(a + b*x)^4*Log
[a - b*x] + 3*(a + b*x)^4*Log[a + b*x])/(96*a^5*b*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^4 (a^2-b^2x^2)} \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(a-bx)(a+bx)^5} dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{16a^4(a+bx)^2}+\frac{1}{8a^3(a+bx)^3}+\frac{1}{4a^2(a+bx)^4}+\frac{1}{16a^4(a^2-b^2x^2)}+\frac{1}{2a(a+bx)^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{16a^5b}-\frac{1}{16a^4b(a+bx)}-\frac{1}{16a^3b(a+bx)^2}-\frac{1}{12a^2b(a+bx)^3}-\frac{1}{8ab(a+bx)^4}$$

input
```
Int[1/((a + b*x)^4*(a^2 - b^2*x^2)),x]
```

output
```
-1/8*1/(a*b*(a + b*x)^4) - 1/(12*a^2*b*(a + b*x)^3) - 1/(16*a^3*b*(a + b*x
)^2) - 1/(16*a^4*b*(a + b*x)) + ArcTanh[(b*x)/a]/(16*a^5*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result
norman	$\frac{-\frac{1}{3ba}-\frac{b\,x^2}{4a^3}-\frac{b^2x^3}{16a^4}-\frac{19x}{48a^2}}{(bx+a)^4}-\frac{\ln(-bx+a)}{32a^5b}+\frac{\ln(bx+a)}{32a^5b}$
risch	$\frac{-\frac{1}{3ba}-\frac{b\,x^2}{4a^3}-\frac{b^2x^3}{16a^4}-\frac{19x}{48a^2}}{(bx+a)^4}-\frac{\ln(-bx+a)}{32a^5b}+\frac{\ln(bx+a)}{32a^5b}$
default	$\frac{\ln(bx+a)}{32a^5b}-\frac{1}{16a^4b(bx+a)}-\frac{1}{16a^3b(bx+a)^2}-\frac{1}{12a^2b(bx+a)^3}-\frac{1}{8ab(bx+a)^4}-\frac{\ln(-bx+a)}{32a^5b}$
parallelrisch	$-\frac{3\ln(bx-a)x^4b^7-3\ln(bx+a)x^4b^7+12\ln(bx-a)x^3a\,b^6-12\ln(bx+a)x^3a\,b^6+18\ln(bx-a)x^2a^2b^5-18\ln(bx+a)x^2a^2b^5+6x^3a}{96a^5b^4(bx+a)}$

input
```
int(1/(b*x+a)^4/(-b^2*x^2+a^2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3/b/a-1/4*b/a^3*x^2-1/16*b^2/a^4*x^3-19/48/a^2*x)/(b*x+a)^4-1/32/a^5/b
*ln(-b*x+a)+1/32/a^5/b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(76) = 152$.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a+bx)^4\,(a^2-b^2x^2)}\,dx = -\frac{6\,ab^3x^3 + 24\,a^2b^2x^2 + 38\,a^3bx + 32\,a^4 - 3\,(b^4x^4 + 4\,ab^3x^3 + 6\,a^2b^2x^2 + 4\,a^3bx + a^4)\log(bx+a) + 3\,(b}{96\,(a^5b^5x^4 + 4\,a^6b^4x^3 + 6\,a^7b^3x^2 + 4\,a^8b^2x + a^9b)}$$

input
```
integrate(1/(b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="fricas")
```

output
```
-1/96*(6*a*b^3*x^3 + 24*a^2*b^2*x^2 + 38*a^3*b*x + 32*a^4 - 3*(b^4*x^4 + 4
*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x + a) + 3*(b^4*x^4 +
4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*log(b*x - a))/(a^5*b^5*x^4
+ 4*a^6*b^4*x^3 + 6*a^7*b^3*x^2 + 4*a^8*b^2*x + a^9*b)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+bx)^4\,(a^2-b^2x^2)}\,dx = -\frac{16a^3 + 19a^2bx + 12ab^2x^2 + 3b^3x^3}{48a^8b + 192a^7b^2x + 288a^6b^3x^2 + 192a^5b^4x^3 + 48a^4b^5x^4} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{32} - \frac{\log\left(\frac{a}{b}+x\right)}{32}}{a^5b}$$

input
```
integrate(1/(b*x+a)**4/(-b**2*x**2+a**2),x)
```

output
```
-(16*a**3 + 19*a**2*b*x + 12*a*b**2*x**2 + 3*b**3*x**3)/(48*a**8*b + 192*a
**7*b**2*x + 288*a**6*b**3*x**2 + 192*a**5*b**4*x**3 + 48*a**4*b**5*x**4)
- (log(-a/b + x)/32 - log(a/b + x)/32)/(a**5*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx = -\frac{3\,b^3x^3+12\,ab^2x^2+19\,a^2bx+16\,a^3}{48\,(a^4b^5x^4+4\,a^5b^4x^3+6\,a^6b^3x^2+4\,a^7b^2x+a^8b)} + \frac{\log(bx+a)}{32\,a^5b} - \frac{\log(bx-a)}{32\,a^5b}$$

input
```
integrate(1/(b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="maxima")
```

output
```
-1/48*(3*b^3*x^3 + 12*a*b^2*x^2 + 19*a^2*b*x + 16*a^3)/(a^4*b^5*x^4 + 4*a^
5*b^4*x^3 + 6*a^6*b^3*x^2 + 4*a^7*b^2*x + a^8*b) + 1/32*log(b*x + a)/(a^5*
b) - 1/32*log(b*x - a)/(a^5*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx = \frac{\log(|bx+a|)}{32\,a^5b} - \frac{\log(|bx-a|)}{32\,a^5b} - \frac{3\,ab^3x^3+12\,a^2b^2x^2+19\,a^3bx+16\,a^4}{48\,(bx+a)^4a^5b}$$

input
```
integrate(1/(b*x+a)^4/(-b^2*x^2+a^2),x, algorithm="giac")
```

output
```
1/32*log(abs(b*x + a))/(a^5*b) - 1/32*log(abs(b*x - a))/(a^5*b) - 1/48*(3*
a*b^3*x^3 + 12*a^2*b^2*x^2 + 19*a^3*b*x + 16*a^4)/((b*x + a)^4*a^5*b)
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{bx}{a}\right)}{16\,a^5\,b} - \frac{\frac{19\,x}{48\,a^2} + \frac{1}{3\,a\,b} + \frac{b\,x^2}{4\,a^3} + \frac{b^2\,x^3}{16\,a^4}}{a^4 + 4\,a^3\,b\,x + 6\,a^2\,b^2\,x^2 + 4\,a\,b^3\,x^3 + b^4\,x^4}$$

input
```
int(1/((a^2 - b^2*x^2)*(a + b*x)^4),x)
```

output
```
atanh((b*x)/a)/(16*a^5*b) - ((19*x)/(48*a^2) + 1/(3*a*b) + (b*x^2)/(4*a^3)
 + (b^2*x^3)/(16*a^4))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^
3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+bx)^4(a^2-b^2x^2)}\,dx = \frac{-6\log(-bx+a)\,a^4 - 24\log(-bx+a)\,a^3bx - 36\log(-bx+a)\,a^2b^2x^2 - 24\log(-bx+a)\,a\,b^3x^3 - 6\log(-$$

input
```
int(1/(b*x+a)^4/(-b^2*x^2+a^2),x)
```

output
```
( - 6*log(a - b*x)*a**4 - 24*log(a - b*x)*a**3*b*x - 36*log(a - b*x)*a**2*
b**2*x**2 - 24*log(a - b*x)*a*b**3*x**3 - 6*log(a - b*x)*b**4*x**4 + 6*log
(a + b*x)*a**4 + 24*log(a + b*x)*a**3*b*x + 36*log(a + b*x)*a**2*b**2*x**2
 + 24*log(a + b*x)*a*b**3*x**3 + 6*log(a + b*x)*b**4*x**4 - 61*a**4 - 64*a
**3*b*x - 30*a**2*b**2*x**2 + 3*b**4*x**4)/(192*a**5*b*(a**4 + 4*a**3*b*x
+ 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.11 $\int \frac{(a+bx)^7}{\left(a^2-b^2x^2\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{(a+bx)^7}{\left(a^2-b^2x^2\right)^2} \, dx = 49a^3x+\frac{23}{2}a^2bx^2+\frac{7}{3}ab^2x^3+\frac{b^3x^4}{4}+\frac{32a^5}{b(a-bx)}+\frac{80a^4\log(a-bx)}{b}$$

output
```
49*a^3*x+23/2*a^2*b*x^2+7/3*a*b^2*x^3+1/4*b^3*x^4+32*a^5/b/(-b*x+a)+80*a^4
*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^7}{\left(a^2-b^2x^2\right)^2} \, dx = 49a^3x+\frac{23}{2}a^2bx^2+\frac{7}{3}ab^2x^3+\frac{b^3x^4}{4}-\frac{32a^5}{b(-a+bx)}+\frac{80a^4\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^7/(a^2 - b^2*x^2)^2,x]
```

output
```
49*a^3*x + (23*a^2*b*x^2)/2 + (7*a*b^2*x^3)/3 + (b^3*x^4)/4 - (32*a^5)/(b*
(-a + b*x)) + (80*a^4*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^2}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(a+bx)^5}{(a-bx)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{32a^5}{(a-bx)^2}-\frac{80a^4}{a-bx}+49a^3+23a^2bx+7ab^2x^2+b^3x^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{32a^5}{b(a-bx)}+\frac{80a^4\log(a-bx)}{b}+49a^3x+\frac{23}{2}a^2bx^2+\frac{7}{3}ab^2x^3+\frac{b^3x^4}{4}$$

input
```
Int[(a + b*x)^7/(a^2 - b^2*x^2)^2,x]
```

output
```
49*a^3*x + (23*a^2*b*x^2)/2 + (7*a*b^2*x^3)/3 + (b^3*x^4)/4 + (32*a^5)/(b*
(a - b*x)) + (80*a^4*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$49a^{3}x+\frac{23a^{2}b\,x^{2}}{2}+\frac{7a\,b^{2}x^{3}}{3}+\frac{b^{3}x^{4}}{4}+\frac{32a^{5}}{b\left(-bx+a\right)}+\frac{80a^{4}\ln\left(-bx+a\right)}{b}$	65
risch	$49a^{3}x+\frac{23a^{2}b\,x^{2}}{2}+\frac{7a\,b^{2}x^{3}}{3}+\frac{b^{3}x^{4}}{4}+\frac{32a^{5}}{b\left(-bx+a\right)}+\frac{80a^{4}\ln\left(-bx+a\right)}{b}$	65
norman	$\frac{81a^{5}x-\frac{b^{5}x^{6}}{4}-\frac{7a\,b^{4}x^{5}}{3}-\frac{45a^{2}b^{3}x^{4}}{4}+\frac{87a^{6}}{2b}-\frac{140a^{3}b^{2}x^{3}}{3}}{-b^{2}x^{2}+a^{2}}+\frac{80a^{4}\ln\left(-bx+a\right)}{b}$	86
parallelrisch	$\frac{3b^{5}x^{5}+25a\,b^{4}x^{4}+110a^{2}b^{3}x^{3}+960\ln\left(bx-a\right)x\,a^{4}b+450a^{3}b^{2}x^{2}-960a^{5}\ln\left(bx-a\right)-972a^{5}}{12\left(bx-a\right)b}$	88

input
```
int((b*x+a)^7/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
49*a^3*x+23/2*a^2*b*x^2+7/3*a*b^2*x^3+1/4*b^3*x^4+32*a^5/b/(-b*x+a)+80*a^4
*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26

$$\int\frac{(a+bx)^{7}}{(a^{2}-b^{2}x^{2})^{2}}\,dx$$
$$=\frac{3\,b^{5}x^{5}+25\,ab^{4}x^{4}+110\,a^{2}b^{3}x^{3}+450\,a^{3}b^{2}x^{2}-588\,a^{4}bx-384\,a^{5}+960\,(a^{4}bx-a^{5})\log{(bx-a)}}{12\,(b^{2}x-ab)}$$

input
```
integrate((b*x+a)^7/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^5*x^5 + 25*a*b^4*x^4 + 110*a^2*b^3*x^3 + 450*a^3*b^2*x^2 - 588*a
^4*b*x - 384*a^5 + 960*(a^4*b*x - a^5)*log(b*x - a))/(b^2*x - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^2}\,dx = -\frac{32a^5}{-ab+b^2x}+\frac{80a^4\log(-a+bx)}{b}+49a^3x+\frac{23a^2bx^2}{2}+\frac{7ab^2x^3}{3}+\frac{b^3x^4}{4}$$

input
```
integrate((b*x+a)**7/(-b**2*x**2+a**2)**2,x)
```

output
```
-32*a**5/(-a*b + b**2*x) + 80*a**4*log(-a + b*x)/b + 49*a**3*x + 23*a**2*b
*x**2/2 + 7*a*b**2*x**3/3 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^2}\,dx = \frac{1}{4}\,b^3x^4+\frac{7}{3}\,ab^2x^3+\frac{23}{2}\,a^2bx^2-\frac{32\,a^5}{b^2x-ab}+49\,a^3x+\frac{80\,a^4\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^7/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + 7/3*a*b^2*x^3 + 23/2*a^2*b*x^2 - 32*a^5/(b^2*x - a*b) + 49*a
^3*x + 80*a^4*log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^2}\,dx = \frac{80\,a^4\log\left(|bx-a|\right)}{b} - \frac{32\,a^5}{(bx-a)b} + \frac{3\,b^{11}x^4+28\,ab^{10}x^3+138\,a^2b^9x^2+588\,a^3b^8x}{12\,b^8}$$

input
```
integrate((b*x+a)^7/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
80*a^4*log(abs(b*x - a))/b - 32*a^5/((b*x - a)*b) + 1/12*(3*b^11*x^4 + 28*
a*b^10*x^3 + 138*a^2*b^9*x^2 + 588*a^3*b^8*x)/b^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^2}\,dx = 49\,a^3\,x + \frac{b^3\,x^4}{4} + \frac{32\,a^5}{b\,(a-b\,x)} + \frac{80\,a^4\,\ln(a-b\,x)}{b} + \frac{23\,a^2\,b\,x^2}{2} + \frac{7\,a\,b^2\,x^3}{3}$$

input
```
int((a + b*x)^7/(a^2 - b^2*x^2)^2,x)
```

output
```
49*a^3*x + (b^3*x^4)/4 + (32*a^5)/(b*(a - b*x)) + (80*a^4*log(a - b*x))/b
+ (23*a^2*b*x^2)/2 + (7*a*b^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^2}\,dx$$
$$= \frac{960\log(-bx+a)\,a^5-960\log(-bx+a)\,a^4bx+972a^4bx-450a^3b^2x^2-110a^2b^3x^3-25a\,b^4x^4-3b^5x^5}{12b\,(-bx+a)}$$

input
```
int((b*x+a)^7/(-b^2*x^2+a^2)^2,x)
```

output
```
(960*log(a - b*x)*a**5 - 960*log(a - b*x)*a**4*b*x + 972*a**4*b*x - 450*a*
*3*b**2*x**2 - 110*a**2*b**3*x**3 - 25*a*b**4*x**4 - 3*b**5*x**5)/(12*b*(a
 - b*x))
```

3.12 $\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx = 17a^2x+3abx^2+\frac{b^2x^3}{3}+\frac{16a^4}{b(a-bx)}+\frac{32a^3\log(a-bx)}{b}$$

output
```
17*a^2*x+3*a*b*x^2+1/3*b^2*x^3+16*a^4/b/(-b*x+a)+32*a^3*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx = 17a^2x+3abx^2+\frac{b^2x^3}{3}-\frac{16a^4}{b(-a+bx)}+\frac{32a^3\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^6/(a^2 - b^2*x^2)^2,x]
```

output
```
17*a^2*x + 3*a*b*x^2 + (b^2*x^3)/3 - (16*a^4)/(b*(-a + b*x)) + (32*a^3*Log
[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2} \, dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{(a+bx)^4}{(a-bx)^2}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{16a^4}{(a-bx)^2}-\frac{32a^3}{a-bx}+17a^2+6abx+b^2x^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{16a^4}{b(a-bx)}+\frac{32a^3\log(a-bx)}{b}+17a^2x+3abx^2+\frac{b^2x^3}{3}$$

input
```
Int[(a + b*x)^6/(a^2 - b^2*x^2)^2,x]
```

output
```
17*a^2*x + 3*a*b*x^2 + (b^2*x^3)/3 + (16*a^4)/(b*(a - b*x)) + (32*a^3*Log[
a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

method	result	size
default	$17a^2x+3ab\,x^2+\frac{b^2x^3}{3}+\frac{16a^4}{b(-bx+a)}+\frac{32a^3\ln(-bx+a)}{b}$	54
risch	$17a^2x+3ab\,x^2+\frac{b^2x^3}{3}+\frac{16a^4}{b(-bx+a)}+\frac{32a^3\ln(-bx+a)}{b}$	54
norman	$\frac{\frac{19a^5}{b}+33a^4x-\frac{b^4x^5}{3}-3a\,b^3x^4-\frac{50a^2b^2x^3}{3}}{-b^2x^2+a^2}+\frac{32a^3\ln(-bx+a)}{b}$	75
parallelrisch	$\frac{b^4x^4+8a\,b^3x^3+96\ln(bx-a)x\,a^3b+42a^2b^2x^2-96a^4\ln(bx-a)-99a^4}{3(bx-a)b}$	76

input
```
int((b*x+a)^6/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
17*a^2*x+3*a*b*x^2+1/3*b^2*x^3+16*a^4/b/(-b*x+a)+32*a^3*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int\frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx=\frac{b^4x^4+8\,ab^3x^3+42\,a^2b^2x^2-51\,a^3bx-48\,a^4+96\,(a^3bx-a^4)\log(bx-a)}{3\,(b^2x-ab)}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^4 + 8*a*b^3*x^3 + 42*a^2*b^2*x^2 - 51*a^3*b*x - 48*a^4 + 96*(a^
3*b*x - a^4)*log(b*x - a))/(b^2*x - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx = -\frac{16a^4}{-ab+b^2x} + \frac{32a^3\log(-a+bx)}{b} + 17a^2x + 3abx^2 + \frac{b^2x^3}{3}$$

input
```
integrate((b*x+a)**6/(-b**2*x**2+a**2)**2,x)
```

output
```
-16*a**4/(-a*b + b**2*x) + 32*a**3*log(-a + b*x)/b + 17*a**2*x + 3*a*b*x**
2 + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx = \frac{1}{3}\,b^2x^3 + 3\,abx^2 - \frac{16\,a^4}{b^2x-ab} + 17\,a^2x + \frac{32\,a^3\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + 3*a*b*x^2 - 16*a^4/(b^2*x - a*b) + 17*a^2*x + 32*a^3*log(b*x
 - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx = \frac{32\,a^3\log\left(|bx-a|\right)}{b} - \frac{16\,a^4}{(bx-a)b} + \frac{b^8x^3+9\,ab^7x^2+51\,a^2b^6x}{3\,b^6}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
32*a^3*log(abs(b*x - a))/b - 16*a^4/((b*x - a)*b) + 1/3*(b^8*x^3 + 9*a*b^7
*x^2 + 51*a^2*b^6*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx = 17\,a^2\,x + \frac{b^2\,x^3}{3} + \frac{16\,a^4}{b\,(a-b\,x)} + \frac{32\,a^3\,\ln\left(a-b\,x\right)}{b} + 3\,a\,b\,x^2$$

input
```
int((a + b*x)^6/(a^2 - b^2*x^2)^2,x)
```

output
```
17*a^2*x + (b^2*x^3)/3 + (16*a^4)/(b*(a - b*x)) + (32*a^3*log(a - b*x))/b
+ 3*a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^2}\,dx = \frac{96\log(-bx+a)\,a^4-96\log(-bx+a)\,a^3bx+99a^3bx-42a^2b^2x^2-8a\,b^3x^3-b^4x^4}{3b\,(-bx+a)}$$

input
```
int((b*x+a)^6/(-b^2*x^2+a^2)^2,x)
```

output
```
(96*log(a - b*x)*a**4 - 96*log(a - b*x)*a**3*b*x + 99*a**3*b*x - 42*a**2*b
**2*x**2 - 8*a*b**3*x**3 - b**4*x**4)/(3*b*(a - b*x))
```

3.13 $\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2} dx = 5ax + \frac{bx^2}{2} + \frac{8a^3}{b(a-bx)} + \frac{12a^2\log(a-bx)}{b}$$

output
```
5*a*x+1/2*b*x^2+8*a^3/b/(-b*x+a)+12*a^2*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2} dx = 5ax + \frac{bx^2}{2} - \frac{8a^3}{b(-a+bx)} + \frac{12a^2\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^5/(a^2 - b^2*x^2)^2,x]
```

output
```
5*a*x + (b*x^2)/2 - (8*a^3)/(b*(-a + b*x)) + (12*a^2*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2} \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(a+bx)^3}{(a-bx)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{8a^3}{(a-bx)^2}-\frac{12a^2}{a-bx}+5a+bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8a^3}{b(a-bx)}+\frac{12a^2\log(a-bx)}{b}+5ax+\frac{bx^2}{2}$$

input
```
Int[(a + b*x)^5/(a^2 - b^2*x^2)^2,x]
```

output
```
5*a*x + (b*x^2)/2 + (8*a^3)/(b*(a - b*x)) + (12*a^2*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result	size
default	$5ax+\frac{b\,x^{2}}{2}+\frac{8a^{3}}{b(-bx+a)}+\frac{12a^{2}\ln(-bx+a)}{b}$	43
risch	$5ax+\frac{b\,x^{2}}{2}+\frac{8a^{3}}{b(-bx+a)}+\frac{12a^{2}\ln(-bx+a)}{b}$	43
norman	$\frac{13a^{3}x-\frac{b^{3}x^{4}}{2}-5a\,b^{2}x^{3}+\frac{17a^{4}}{2b}}{-b^{2}x^{2}+a^{2}}+\frac{12a^{2}\ln(-bx+a)}{b}$	64
parallelrisch	$\frac{b^{3}x^{3}+24\ln(bx-a)x\,a^{2}b+9a\,b^{2}x^{2}-24a^{3}\ln(bx-a)-26a^{3}}{2(bx-a)b}$	65

input
```
int((b*x+a)^5/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
5*a*x+1/2*b*x^2+8*a^3/b/(-b*x+a)+12*a^2*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2}\,dx=\frac{b^3x^3+9\,ab^2x^2-10\,a^2bx-16\,a^3+24\,(a^2bx-a^3)\log(bx-a)}{2\,(b^2x-ab)}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^3 + 9*a*b^2*x^2 - 10*a^2*b*x - 16*a^3 + 24*(a^2*b*x - a^3)*log(
b*x - a))/(b^2*x - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2}\,dx = -\frac{8a^3}{-ab+b^2x} + \frac{12a^2\log{(-a+bx)}}{b} + 5ax + \frac{bx^2}{2}$$

input
```
integrate((b*x+a)**5/(-b**2*x**2+a**2)**2,x)
```

output
```
-8*a**3/(-a*b + b**2*x) + 12*a**2*log(-a + b*x)/b + 5*a*x + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2}\,dx = \frac{1}{2}\,bx^2 - \frac{8\,a^3}{b^2x-ab} + 5\,ax + \frac{12\,a^2\log\left(bx-a\right)}{b}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*x^2 - 8*a^3/(b^2*x - a*b) + 5*a*x + 12*a^2*log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2}\,dx = \frac{12\,a^2\log\left(|bx-a|\right)}{b} - \frac{8\,a^3}{(bx-a)b} + \frac{b^5x^2+10\,ab^4x}{2\,b^4}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
12*a^2*log(abs(b*x - a))/b - 8*a^3/((b*x - a)*b) + 1/2*(b^5*x^2 + 10*a*b^4
*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2}\,dx = 5\,a\,x+\frac{b\,x^2}{2}+\frac{8\,a^3}{b\,(a-b\,x)}+\frac{12\,a^2\,\ln(a-b\,x)}{b}$$

input
```
int((a + b*x)^5/(a^2 - b^2*x^2)^2,x)
```

output
```
5*a*x + (b*x^2)/2 + (8*a^3)/(b*(a - b*x)) + (12*a^2*log(a - b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^2}\,dx = \frac{24\log(-bx+a)\,a^3-24\log(-bx+a)\,a^2bx+26a^2bx-9a\,b^2x^2-b^3x^3}{2b\,(-bx+a)}$$

input
```
int((b*x+a)^5/(-b^2*x^2+a^2)^2,x)
```

output
```
(24*log(a - b*x)*a**3 - 24*log(a - b*x)*a**2*b*x + 26*a**2*b*x - 9*a*b**2*
x**2 - b**3*x**3)/(2*b*(a - b*x))
```

3.14 $\int \frac{(a+bx)^4}{\left(a^2-b^2x^2\right)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2} dx = x + \frac{4a^2}{b(a-bx)} + \frac{4a\log(a-bx)}{b}$$

output
```
x+4*a^2/b/(-b*x+a)+4*a*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2} dx = x - \frac{4a^2}{b(-a+bx)} + \frac{4a\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^4/(a^2 - b^2*x^2)^2,x]
```

output
```
x - (4*a^2)/(b*(-a + b*x)) + (4*a*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(a+bx)^2}{(a-bx)^2}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{4a^2}{(a-bx)^2}-\frac{4a}{a-bx}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^2}{b(a-bx)}+\frac{4a\log(a-bx)}{b}+x$$

input
```
Int[(a + b*x)^4/(a^2 - b^2*x^2)^2,x]
```

output
```
x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
default	$x+\frac{4a^{2}}{b(-bx+a)}+\frac{4a\ln(-bx+a)}{b}$	32
risch	$x+\frac{4a^{2}}{b(-bx+a)}+\frac{4a\ln(-bx+a)}{b}$	32
norman	$\frac{\frac{4a^{3}}{b}+5a^{2}x-b^{2}x^{3}}{-b^{2}x^{2}+a^{2}}+\frac{4a\ln(-bx+a)}{b}$	53
parallelrisch	$\frac{4\ln(bx-a)xab+b^{2}x^{2}-4a^{2}\ln(bx-a)-5a^{2}}{(bx-a)b}$	53

input
```
int((b*x+a)^4/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x+4*a^2/b/(-b*x+a)+4*a*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2}\,dx=\frac{b^2x^2-abx-4\,a^2+4\,(abx-a^2)\log(bx-a)}{b^2x-ab}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
(b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*x - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2}\,dx = -\frac{4a^2}{-ab+b^2x} + \frac{4a\log(-a+bx)}{b} + x$$

input
```
integrate((b*x+a)**4/(-b**2*x**2+a**2)**2,x)
```

output
```
-4*a**2/(-a*b + b**2*x) + 4*a*log(-a + b*x)/b + x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2}\,dx = -\frac{4\,a^2}{b^2x-ab} + x + \frac{4\,a\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-4*a^2/(b^2*x - a*b) + x + 4*a*log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2}\,dx = x + \frac{4\,a\log(|bx-a|)}{b} - \frac{4\,a^2}{(bx-a)b}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
x + 4*a*log(abs(b*x - a))/b - 4*a^2/((b*x - a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2}\,dx = x+\frac{4\,a^2}{b\,(a-b\,x)}+\frac{4\,a\,\ln\left(a-b\,x\right)}{b}$$

input
```
int((a + b*x)^4/(a^2 - b^2*x^2)^2,x)
```

output
```
x + (4*a^2)/(b*(a - b*x)) + (4*a*log(a - b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^2}\,dx = \frac{4\log(-bx+a)\,a^2-4\log(-bx+a)\,abx+5abx-b^2x^2}{b\,(-bx+a)}$$

input
```
int((b*x+a)^4/(-b^2*x^2+a^2)^2,x)
```

output
```
(4*log(a - b*x)*a**2 - 4*log(a - b*x)*a*b*x + 5*a*b*x - b**2*x**2)/(b*(a -
 b*x))
```

3.15 $\int \frac{(a+bx)^3}{(a^2-b^2x^2)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 26

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^2} \, dx = \frac{2a}{b(a-bx)} + \frac{\log(a-bx)}{b}$$

output
```
2*a/b/(-b*x+a)+ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^2} \, dx = \frac{\frac{2a}{a-bx} + \log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^3/(a^2 - b^2*x^2)^2,x]
```

output
```
((2*a)/(a - b*x) + Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^2}\,dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{a+bx}{(a-bx)^2}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(\frac{2a}{(a-bx)^2}+\frac{1}{bx-a}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2a}{b(a-bx)}+\frac{\log(a-bx)}{b}$$

input
```
Int[(a + b*x)^3/(a^2 - b^2*x^2)^2,x]
```

output
```
(2*a)/(b*(a - b*x)) + Log[a - b*x]/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{2a}{b(-bx+a)}+\frac{\ln(-bx+a)}{b}$	27
risch	$\frac{2a}{b(-bx+a)}+\frac{\ln(-bx+a)}{b}$	27
parallelrisch	$\frac{\ln(bx-a)xb-a\ln(bx-a)-2a}{(bx-a)b}$	40
norman	$\frac{\frac{2a^2}{b}+2ax}{-b^2x^2+a^2}+\frac{\ln(-bx+a)}{b}$	41

input
```
int((b*x+a)^3/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
2*a/b/(-b*x+a)+ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^2}\,dx=\frac{(bx-a)\log(bx-a)-2\,a}{b^2x-ab}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
((b*x - a)*log(b*x - a) - 2*a)/(b^2*x - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx)^3}{\left(a^2-b^2x^2\right)^2}\,dx = -\frac{2a}{-ab+b^2x}+\frac{\log(-a+bx)}{b}$$

input
```
integrate((b*x+a)**3/(-b**2*x**2+a**2)**2,x)
```

output
```
-2*a/(-a*b + b**2*x) + log(-a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^3}{\left(a^2-b^2x^2\right)^2}\,dx = -\frac{2\,a}{b^2x-ab}+\frac{\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-2*a/(b^2*x - a*b) + log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^3}{\left(a^2-b^2x^2\right)^2}\,dx = \frac{\log(|bx-a|)}{b}-\frac{2\,a}{(bx-a)b}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
log(abs(b*x - a))/b - 2*a/((b*x - a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^2}\,dx = \frac{\ln(a-b\,x)}{b} + \frac{2\,a}{b\,(a-b\,x)}$$

input
```
int((a + b*x)^3/(a^2 - b^2*x^2)^2,x)
```

output
```
log(a - b*x)/b + (2*a)/(b*(a - b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^2}\,dx = \frac{\log(-bx+a)\,a - \log(-bx+a)\,bx + 2bx}{b\,(-bx+a)}$$

input
```
int((b*x+a)^3/(-b^2*x^2+a^2)^2,x)
```

output
```
(log(a - b*x)*a - log(a - b*x)*b*x + 2*b*x)/(b*(a - b*x))
```

3.16 $\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 12

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2} dx = \frac{1}{b(a-bx)}$$

output
```
1/b/(-b*x+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2} dx = \frac{1}{b(a-bx)}$$

input
```
Integrate[(a + b*x)^2/(a^2 - b^2*x^2)^2,x]
```

output
```
1/(b*(a - b*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2}\,dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{1}{(a-bx)^2}dx$$

$$\Big\downarrow \; 17$$

$$\frac{1}{b(a-bx)}$$

input
```
Int[(a + b*x)^2/(a^2 - b^2*x^2)^2,x]
```

output
```
1/(b*(a - b*x))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{1}{b(-bx+a)}$	13
default	$\frac{1}{b(-bx+a)}$	13
risch	$\frac{1}{b(-bx+a)}$	13
parallelrisch	$-\frac{1}{(bx-a)b}$	15
norman	$\frac{x+\frac{a}{b}}{-b^2x^2+a^2}$	23
orering	$\frac{(-bx+a)(bx+a)^2}{b(-b^2x^2+a^2)^2}$	32

input
```
int((b*x+a)^2/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b/(-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2}\,dx = -\frac{1}{b^2x-ab}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/(b^2*x - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2}\,dx = -\frac{1}{-ab+b^2x}$$

input
```
integrate((b*x+a)**2/(-b**2*x**2+a**2)**2,x)
```

output
```
-1/(-a*b + b**2*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2}\,dx = -\frac{1}{b^2x-ab}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/(b^2*x - a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2}\,dx = -\frac{1}{(bx-a)b}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/((b*x - a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2}\,dx = \frac{1}{b\,(a-b\,x)}$$

input
```
int((a + b*x)^2/(a^2 - b^2*x^2)^2,x)
```

output
```
1/(b*(a - b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^2}\,dx = \frac{x}{a\,(-bx+a)}$$

input
```
int((b*x+a)^2/(-b^2*x^2+a^2)^2,x)
```

output
```
x/(a*(a - b*x))
```

3.17 $\int \frac{a+bx}{\left(a^2-b^2x^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int \frac{a+bx}{\left(a^2-b^2x^2\right)^2}\,dx=\frac{1}{2ab(a-bx)}+\frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{2a^2b}$$

output
```
1/2/a/b/(-b*x+a)+1/2*arctanh(b*x/a)/a^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{a+bx}{\left(a^2-b^2x^2\right)^2}\,dx=\frac{2a+(-a+bx)\log(a-bx)+(a-bx)\log(a+bx)}{4a^2b(a-bx)}$$

input
```
Integrate[(a + b*x)/(a^2 - b^2*x^2)^2,x]
```

output
```
(2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*(a - b*x
))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {454, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(a^2-b^2x^2)^2} \, dx$$

$$\Big\downarrow \text{454}$$

$$\frac{\int \frac{1}{a^2-b^2x^2}dx}{2a}+\frac{a+bx}{2ab\left(a^2-b^2x^2\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^2b}+\frac{a+bx}{2ab\left(a^2-b^2x^2\right)}$$

input
```
Int[(a + b*x)/(a^2 - b^2*x^2)^2,x]
```

output
```
(a + b*x)/(2*a*b*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^2*b)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{\ln(bx+a)}{4b\,a^2}-\frac{\ln(-bx+a)}{4b\,a^2}+\frac{1}{2ab(-bx+a)}$	47
risch	$\frac{\ln(bx+a)}{4b\,a^2}-\frac{\ln(-bx+a)}{4b\,a^2}+\frac{1}{2ab(-bx+a)}$	47
norman	$\frac{\frac{x}{2a}+\frac{1}{2b}}{-b^2x^2+a^2}-\frac{\ln(-bx+a)}{4b\,a^2}+\frac{\ln(bx+a)}{4b\,a^2}$	58
parallelrisch	$-\frac{\ln(bx-a)xb-\ln(bx+a)xb-a\ln(bx-a)+\ln(bx+a)a+2a}{4a^2b(bx-a)}$	62

input
```
int((b*x+a)/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/b/a^2*ln(b*x+a)-1/4/b/a^2*ln(-b*x+a)+1/2/a/b/(-b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \frac{a+bx}{(a^2-b^2x^2)^2}\,dx = \frac{(bx-a)\log(bx+a)-(bx-a)\log(bx-a)-2\,a}{4\,(a^2b^2x-a^3b)}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*x - a
^3*b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{a+bx}{(a^2-b^2x^2)^2}\,dx = -\frac{1}{-2a^2b+2ab^2x}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2b}$$

input
```
integrate((b*x+a)/(-b**2*x**2+a**2)**2,x)
```

output
```
-1/(-2*a**2*b + 2*a*b**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4)/(a**2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{a+bx}{(a^2-b^2x^2)^2}\,dx = -\frac{1}{2\,(ab^2x-a^2b)}+\frac{\log(bx+a)}{4\,a^2b}-\frac{\log(bx-a)}{4\,a^2b}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/2/(a*b^2*x - a^2*b) + 1/4*log(b*x + a)/(a^2*b) - 1/4*log(b*x - a)/(a^2*
b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{a+bx}{(a^2-b^2x^2)^2}\,dx = \frac{\log(|bx+a|)}{4\,a^2b}-\frac{\log(|bx-a|)}{4\,a^2b}-\frac{1}{2\,(bx-a)ab}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
1/4*log(abs(b*x + a))/(a^2*b) - 1/4*log(abs(b*x - a))/(a^2*b) - 1/2/((b*x
- a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{a+bx}{(a^2-b^2x^2)^2}\,dx = \frac{1}{2\,a\,b\,(a-b\,x)} + \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b}$$

input
```
int((a + b*x)/(a^2 - b^2*x^2)^2,x)
```

output
```
1/(2*a*b*(a - b*x)) + atanh((b*x)/a)/(2*a^2*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{a+bx}{(a^2-b^2x^2)^2}\,dx = \frac{-\log(-bx+a)\,a+\log(-bx+a)\,bx+\log(bx+a)\,a-\log(bx+a)\,bx+2bx}{4a^2b\,(-bx+a)}$$

input
```
int((b*x+a)/(-b^2*x^2+a^2)^2,x)
```

output
```
( - log(a - b*x)*a + log(a - b*x)*b*x + log(a + b*x)*a - log(a + b*x)*b*x
+ 2*b*x)/(4*a**2*b*(a - b*x))
```

3.18 $\int \frac{1}{(a+bx)(a^2-b^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)^2}\,dx = \frac{1}{8a^3b(a-bx)} - \frac{1}{8a^2b(a+bx)^2} - \frac{1}{4a^3b(a+bx)} + \frac{3\text{arctanh}\left(\frac{bx}{a}\right)}{8a^4b}$$

output
```
1/8/a^3/b/(-b*x+a)-1/8/a^2/b/(b*x+a)^2-1/4/a^3/b/(b*x+a)+3/8*arctanh(b*x/a
)/a^4/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)^2}\,dx = -\frac{1}{8a^3b(-a+bx)} - \frac{1}{8a^2b(a+bx)^2} - \frac{1}{4a^3b(a+bx)} - \frac{3\log(a-bx)}{16a^4b} + \frac{3\log(a+bx)}{16a^4b}$$

input
```
Integrate[1/((a + b*x)*(a^2 - b^2*x^2)^2),x]
```

output
```
-1/8*1/(a^3*b*(-a + b*x)) - 1/(8*a^2*b*(a + b*x)^2) - 1/(4*a^3*b*(a + b*x)
) - (3*Log[a - b*x])/(16*a^4*b) + (3*Log[a + b*x])/(16*a^4*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^2}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(a-bx)^2(a+bx)^3}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{8a^3(a-bx)^2}+\frac{1}{4a^3(a+bx)^2}+\frac{1}{4a^2(a+bx)^3}+\frac{3}{8a^3\left(a^2-b^2x^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\text{arctanh}\left(\frac{bx}{a}\right)}{8a^4b}+\frac{1}{8a^3b(a-bx)}-\frac{1}{4a^3b(a+bx)}-\frac{1}{8a^2b(a+bx)^2}$$

input
```
Int[1/((a + b*x)*(a^2 - b^2*x^2)^2),x]
```

output
```
1/(8*a^3*b*(a - b*x)) - 1/(8*a^2*b*(a + b*x)^2) - 1/(4*a^3*b*(a + b*x)) +
(3*ArcTanh[(b*x)/a])/(8*a^4*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result
norman	$\frac{-\frac{1}{4ba}+\frac{3x}{8a^2}+\frac{3b\,x^2}{8a^3}}{(bx+a)^2(-bx+a)}-\frac{3\ln(-bx+a)}{16a^4b}+\frac{3\ln(bx+a)}{16a^4b}$
default	$\frac{3\ln(bx+a)}{16a^4b}-\frac{1}{4a^3b(bx+a)}-\frac{1}{8a^2b(bx+a)^2}-\frac{3\ln(-bx+a)}{16a^4b}+\frac{1}{8a^3b(-bx+a)}$
risch	$\frac{-\frac{1}{4ba}+\frac{3x}{8a^2}+\frac{3b\,x^2}{8a^3}}{(bx+a)(-b^2x^2+a^2)}-\frac{3\ln(-bx+a)}{16a^4b}+\frac{3\ln(bx+a)}{16a^4b}$
parallelrisch	$-\frac{3\ln(bx-a)x^3b^5-3\ln(bx+a)x^3b^5+3\ln(bx-a)x^2a\,b^4-3\ln(bx+a)x^2a\,b^4-3\ln(bx-a)x\,a^2b^3+3\ln(bx+a)x\,a^2b^3+6x^2a\,b^4-3}{16a^4b^3(bx+a)(b^2x^2-a^2)}$

input
```
int(1/(b*x+a)/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/b/a+3/8/a^2*x+3/8*b/a^3*x^2)/(b*x+a)^2/(-b*x+a)-3/16/a^4/b*ln(-b*x+a
)+3/16/a^4/b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^2}\,dx = -\frac{6\,ab^2x^2+6\,a^2bx-4\,a^3-3\left(b^3x^3+ab^2x^2-a^2bx-a^3\right)\log\left(bx+a\right)+3\left(b^3x^3+ab^2x^2-a^2bx-a^3\right)\log}{16\left(a^4b^4x^3+a^5b^3x^2-a^6b^2x-a^7b\right)}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/16*(6*a*b^2*x^2 + 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 + a*b^2*x^2 - a^2*b*x
- a^3)*log(b*x + a) + 3*(b^3*x^3 + a*b^2*x^2 - a^2*b*x - a^3)*log(b*x - a)
)/(a^4*b^4*x^3 + a^5*b^3*x^2 - a^6*b^2*x - a^7*b)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^2}\,dx = \frac{2a^2-3abx-3b^2x^2}{-8a^6b-8a^5b^2x+8a^4b^3x^2+8a^3b^4x^3} + \frac{-\frac{3\log\left(-\frac{a}{b}+x\right)}{16}+\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4b}$$

input
```
integrate(1/(b*x+a)/(-b**2*x**2+a**2)**2,x)
```

output
```
(2*a**2 - 3*a*b*x - 3*b**2*x**2)/(-8*a**6*b - 8*a**5*b**2*x + 8*a**4*b**3*
x**2 + 8*a**3*b**4*x**3) + (-3*log(-a/b + x)/16 + 3*log(a/b + x)/16)/(a**4
*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^2}\,dx = -\frac{3\,b^2x^2+3\,abx-2\,a^2}{8\,(a^3b^4x^3+a^4b^3x^2-a^5b^2x-a^6b)} + \frac{3\,\log\left(bx+a\right)}{16\,a^4b} - \frac{3\,\log\left(bx-a\right)}{16\,a^4b}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/8*(3*b^2*x^2 + 3*a*b*x - 2*a^2)/(a^3*b^4*x^3 + a^4*b^3*x^2 - a^5*b^2*x
- a^6*b) + 3/16*log(b*x + a)/(a^4*b) - 3/16*log(b*x - a)/(a^4*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^2}\,dx = \frac{3\,\log\left(|bx+a|\right)}{16\,a^4b} - \frac{3\,\log\left(|bx-a|\right)}{16\,a^4b} - \frac{3\,ab^2x^2+3\,a^2bx-2\,a^3}{8\,(bx+a)^2(bx-a)a^4b}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
3/16*log(abs(b*x + a))/(a^4*b) - 3/16*log(abs(b*x - a))/(a^4*b) - 1/8*(3*a
*b^2*x^2 + 3*a^2*b*x - 2*a^3)/((b*x + a)^2*(b*x - a)*a^4*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^2}\,dx = \frac{\frac{3\,x}{8\,a^2}-\frac{1}{4\,a\,b}+\frac{3\,b\,x^2}{8\,a^3}}{a^3+a^2\,b\,x-a\,b^2\,x^2-b^3\,x^3} + \frac{3\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{8\,a^4\,b}$$

input
```
int(1/((a^2 - b^2*x^2)^2*(a + b*x)),x)
```

output
```
((3*x)/(8*a^2) - 1/(4*a*b) + (3*b*x^2)/(8*a^3))/(a^3 - b^3*x^3 - a*b^2*x^2
 + a^2*b*x) + (3*atanh((b*x)/a))/(8*a^4*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.40

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)^2}\,dx$$
$$= \frac{-3\log(-bx+a)\,a^3-3\log(-bx+a)\,a^2bx+3\log(-bx+a)\,a\,b^2x^2+3\log(-bx+a)\,b^3x^3+3\log(bx+a}{16a^4b\,(-b^3x^3-a\,b^2x^2+}$$

input
```
int(1/(b*x+a)/(-b^2*x^2+a^2)^2,x)
```

output
```
( - 3*log(a - b*x)*a**3 - 3*log(a - b*x)*a**2*b*x + 3*log(a - b*x)*a*b**2*
x**2 + 3*log(a - b*x)*b**3*x**3 + 3*log(a + b*x)*a**3 + 3*log(a + b*x)*a**
2*b*x - 3*log(a + b*x)*a*b**2*x**2 - 3*log(a + b*x)*b**3*x**3 + 2*a**3 + 1
2*a**2*b*x - 6*b**3*x**3)/(16*a**4*b*(a**3 + a**2*b*x - a*b**2*x**2 - b**3
*x**3))
```

3.19 $\int \frac{1}{(a+bx)^2(a^2-b^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)^2}\,dx = \frac{1}{16a^4b(a-bx)} - \frac{1}{12a^2b(a+bx)^3} - \frac{1}{8a^3b(a+bx)^2} - \frac{3}{16a^4b(a+bx)} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{4a^5b}$$

output
```
1/16/a^4/b/(-b*x+a)-1/12/a^2/b/(b*x+a)^3-1/8/a^3/b/(b*x+a)^2-3/16/a^4/b/(b
*x+a)+1/4*arctanh(b*x/a)/a^5/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)^2}\,dx = \frac{\frac{2a(-4a^3+a^2bx+6ab^2x^2+3b^3x^3)}{(a-bx)(a+bx)^3} - 3\log(a-bx) + 3\log(a+bx)}{24a^5b}$$

input
```
Integrate[1/((a + b*x)^2*(a^2 - b^2*x^2)^2),x]
```

output
```
((2*a*(-4*a^3 + a^2*b*x + 6*a*b^2*x^2 + 3*b^3*x^3))/((a - b*x)*(a + b*x)^3
) - 3*Log[a - b*x] + 3*Log[a + b*x])/(24*a^5*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2 (a^2-b^2x^2)^2} \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(a-bx)^2(a+bx)^4}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{16a^4(a-bx)^2}+\frac{3}{16a^4(a+bx)^2}+\frac{1}{4a^3(a+bx)^3}+\frac{1}{4a^2(a+bx)^4}+\frac{1}{4a^4(a^2-b^2x^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{4a^5b}+\frac{1}{16a^4b(a-bx)}-\frac{3}{16a^4b(a+bx)}-\frac{1}{8a^3b(a+bx)^2}-\frac{1}{12a^2b(a+bx)^3}$$

input
```
Int[1/((a + b*x)^2*(a^2 - b^2*x^2)^2),x]
```

output
```
1/(16*a^4*b*(a - b*x)) - 1/(12*a^2*b*(a + b*x)^3) - 1/(8*a^3*b*(a + b*x)^2
) - 3/(16*a^4*b*(a + b*x)) + ArcTanh[(b*x)/a]/(4*a^5*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\frac{b\,x^2}{2a^3}+\frac{3x}{4a^2}-\frac{5b^2x^3}{12a^4}-\frac{b^3x^4}{3a^5}}{(bx+a)^3(-bx+a)}-\frac{\ln(-bx+a)}{8a^5b}+\frac{\ln(bx+a)}{8a^5b}$
risch	$\frac{\frac{b^2x^3}{4a^4}+\frac{b\,x^2}{2a^3}+\frac{x}{12a^2}-\frac{1}{3ba}}{(bx+a)^2(-b^2x^2+a^2)}-\frac{\ln(-bx+a)}{8a^5b}+\frac{\ln(bx+a)}{8a^5b}$
default	$\frac{\ln(bx+a)}{8a^5b}-\frac{3}{16a^4b(bx+a)}-\frac{1}{8a^3b(bx+a)^2}-\frac{1}{12a^2b(bx+a)^3}-\frac{\ln(-bx+a)}{8a^5b}+\frac{1}{16a^4b(-bx+a)}$
parallelrisch	$-\frac{3\ln(bx-a)x^4b^7-3\ln(bx+a)x^4b^7+6\ln(bx-a)x^3a\,b^6-6\ln(bx+a)x^3a\,b^6+6x^3a\,b^6-6\ln(bx-a)x\,a^3b^4+6\ln(bx+a)x\,a^3b^4+1}{24a^5b^4(bx+a)^2(b^2x^2-a^2)}$

input
```
int(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*b/a^3*x^2+3/4/a^2*x-5/12*b^2/a^4*x^3-1/3*b^3/a^5*x^4)/(b*x+a)^3/(-b*x
+a)-1/8/a^5/b*ln(-b*x+a)+1/8/a^5/b*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.72

$$\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^2}\,dx = -\frac{6\,ab^3x^3+12\,a^2b^2x^2+2\,a^3bx-8\,a^4-3\,(b^4x^4+2\,ab^3x^3-2\,a^3bx-a^4)\log(bx+a)+3\,(b^4x^4+2\,ab^3x^3}{24\,(a^5b^5x^4+2\,a^6b^4x^3-2\,a^8b^2x-a^9b)}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/24*(6*a*b^3*x^3 + 12*a^2*b^2*x^2 + 2*a^3*b*x - 8*a^4 - 3*(b^4*x^4 + 2*a
*b^3*x^3 - 2*a^3*b*x - a^4)*log(b*x + a) + 3*(b^4*x^4 + 2*a*b^3*x^3 - 2*a^
3*b*x - a^4)*log(b*x - a))/(a^5*b^5*x^4 + 2*a^6*b^4*x^3 - 2*a^8*b^2*x - a^
9*b)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^2}\,dx = \frac{4a^3-a^2bx-6ab^2x^2-3b^3x^3}{-12a^8b-24a^7b^2x+24a^5b^4x^3+12a^4b^5x^4} + \frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{8}+\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^5b}$$

input
```
integrate(1/(b*x+a)**2/(-b**2*x**2+a**2)**2,x)
```

output
```
(4*a**3 - a**2*b*x - 6*a*b**2*x**2 - 3*b**3*x**3)/(-12*a**8*b - 24*a**7*b*
*2*x + 24*a**5*b**4*x**3 + 12*a**4*b**5*x**4) + (-log(-a/b + x)/8 + log(a/
b + x)/8)/(a**5*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a+bx)^2\,(a^2-b^2x^2)^2}\,dx = -\frac{3\,b^3x^3+6\,ab^2x^2+a^2bx-4\,a^3}{12\,(a^4b^5x^4+2\,a^5b^4x^3-2\,a^7b^2x-a^8b)} + \frac{\log(bx+a)}{8\,a^5b} - \frac{\log(bx-a)}{8\,a^5b}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/12*(3*b^3*x^3 + 6*a*b^2*x^2 + a^2*b*x - 4*a^3)/(a^4*b^5*x^4 + 2*a^5*b^4
*x^3 - 2*a^7*b^2*x - a^8*b) + 1/8*log(b*x + a)/(a^5*b) - 1/8*log(b*x - a)/
(a^5*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a+bx)^2\,(a^2-b^2x^2)^2}\,dx = -\frac{\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{8\,a^5b} + \frac{1}{32\,a^5b\left(\frac{2\,a}{bx+a}-1\right)} - \frac{\frac{9\,a^2b^5}{bx+a}+\frac{6\,a^3b^5}{(bx+a)^2}+\frac{4\,a^4b^5}{(bx+a)^3}}{48\,a^6b^6}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
-1/8*log(abs(-2*a/(b*x + a) + 1))/(a^5*b) + 1/32/(a^5*b*(2*a/(b*x + a) - 1
)) - 1/48*(9*a^2*b^5/(b*x + a) + 6*a^3*b^5/(b*x + a)^2 + 4*a^4*b^5/(b*x +
a)^3)/(a^6*b^6)
```

Mupad [B] (verification not implemented)

Time = 6.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx)^2 (a^2-b^2x^2)^2}\,dx = \frac{\frac{x}{12\,a^2}-\frac{1}{3\,a\,b}+\frac{b\,x^2}{2\,a^3}+\frac{b^2\,x^3}{4\,a^4}}{a^4+2\,a^3\,b\,x-2\,a\,b^3\,x^3-b^4\,x^4}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^5\,b}$$

input
```
int(1/((a^2 - b^2*x^2)^2*(a + b*x)^2),x)
```

output
```
(x/(12*a^2) - 1/(3*a*b) + (b*x^2)/(2*a^3) + (b^2*x^3)/(4*a^4))/(a^4 - b^4*
x^4 - 2*a*b^3*x^3 + 2*a^3*b*x) + atanh((b*x)/a)/(4*a^5*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a+bx)^2 (a^2-b^2x^2)^2}\,dx$$
$$= \frac{-3\log(-bx+a)\,a^4-6\log(-bx+a)\,a^3bx+6\log(-bx+a)\,a\,b^3x^3+3\log(-bx+a)\,b^4x^4+3\log(bx+a}{24a^5b\,(-b^4x^4-2a\,b}$$

input
```
int(1/(b*x+a)^2/(-b^2*x^2+a^2)^2,x)
```

output
```
( - 3*log(a - b*x)*a**4 - 6*log(a - b*x)*a**3*b*x + 6*log(a - b*x)*a*b**3*
x**3 + 3*log(a - b*x)*b**4*x**4 + 3*log(a + b*x)*a**4 + 6*log(a + b*x)*a**
3*b*x - 6*log(a + b*x)*a*b**3*x**3 - 3*log(a + b*x)*b**4*x**4 - 5*a**4 + 8
*a**3*b*x + 12*a**2*b**2*x**2 - 3*b**4*x**4)/(24*a**5*b*(a**4 + 2*a**3*b*x
 - 2*a*b**3*x**3 - b**4*x**4))
```

3.20 $\int \frac{1}{(a+bx)^3(a^2-b^2x^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)^2}\,dx = \frac{1}{32a^5b(a-bx)} - \frac{1}{16a^2b(a+bx)^4} - \frac{1}{12a^3b(a+bx)^3} - \frac{3}{32a^4b(a+bx)^2} - \frac{1}{8a^5b(a+bx)} + \frac{5\text{arctanh}\left(\frac{bx}{a}\right)}{32a^6b}$$

output
```
1/32/a^5/b/(-b*x+a)-1/16/a^2/b/(b*x+a)^4-1/12/a^3/b/(b*x+a)^3-3/32/a^4/b/(
b*x+a)^2-1/8/a^5/b/(b*x+a)+5/32*arctanh(b*x/a)/a^6/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)^3(a^2-b^2x^2)^2}\,dx = \frac{-64a^5-30a^4bx+70a^3b^2x^2+90a^2b^3x^3+30ab^4x^4-15(a-bx)(a+bx)^4\log(a-bx)+15(a-bx)(a+}{192a^6b(a-bx)(a+bx)^4}$$

input
```
Integrate[1/((a + b*x)^3*(a^2 - b^2*x^2)^2),x]
```

output
```
(-64*a^5 - 30*a^4*b*x + 70*a^3*b^2*x^2 + 90*a^2*b^3*x^3 + 30*a*b^4*x^4 - 1
5*(a - b*x)*(a + b*x)^4*Log[a - b*x] + 15*(a - b*x)*(a + b*x)^4*Log[a + b*
x])/(192*a^6*b*(a - b*x)*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^3 (a^2-b^2x^2)^2} \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(a-bx)^2(a+bx)^5}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{32a^5(a-bx)^2}+\frac{1}{8a^5(a+bx)^2}+\frac{3}{16a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{4a^2(a+bx)^5}+\frac{5}{32a^5(a^2-b^2x^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\text{arctanh}\left(\frac{bx}{a}\right)}{32a^6b}+\frac{1}{32a^5b(a-bx)}-\frac{1}{8a^5b(a+bx)}-\frac{3}{32a^4b(a+bx)^2}-\frac{1}{12a^3b(a+bx)^3}-\frac{1}{16a^2b(a+bx)^4}$$

input
```
Int[1/((a + b*x)^3*(a^2 - b^2*x^2)^2),x]
```

output
```
1/(32*a^5*b*(a - b*x)) - 1/(16*a^2*b*(a + b*x)^4) - 1/(12*a^3*b*(a + b*x)^
3) - 3/(32*a^4*b*(a + b*x)^2) - 1/(8*a^5*b*(a + b*x)) + (5*ArcTanh[(b*x)/a
])/(32*a^6*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{27x}{32a^2}+\frac{33b\,x^2}{32a^3}-\frac{19b^2x^3}{96a^4}-\frac{27b^3x^4}{32a^5}-\frac{b^4x^5}{3a^6}}{(bx+a)^4(-bx+a)}-\frac{5\ln(-bx+a)}{64a^6b}+\frac{5\ln(bx+a)}{64a^6b}$
risch	$\frac{\frac{5b^3x^4}{32a^5}+\frac{15b^2x^3}{32a^4}+\frac{35b\,x^2}{96a^3}-\frac{5x}{32a^2}-\frac{1}{3ba}}{(bx+a)^3(-b^2x^2+a^2)}-\frac{5\ln(-bx+a)}{64a^6b}+\frac{5\ln(bx+a)}{64a^6b}$
default	$\frac{5\ln(bx+a)}{64a^6b}-\frac{1}{8a^5b(bx+a)}-\frac{3}{32a^4b(bx+a)^2}-\frac{1}{12a^3b(bx+a)^3}-\frac{1}{16a^2b(bx+a)^4}-\frac{5\ln(-bx+a)}{64a^6b}+\frac{1}{32a^5b(-bx+a)}$
parallelrisch	$-\frac{15\ln(bx-a)x^5b^5-15\ln(bx+a)x^5b^5+45\ln(bx-a)x^4a\,b^4-45\ln(bx+a)x^4a\,b^4-64b^5x^5+30\ln(bx-a)x^3a^2b^3-30\ln(bx+a)x^3}{}$

input
```
int(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(27/32/a^2*x+33/32*b/a^3*x^2-19/96*b^2/a^4*x^3-27/32*b^3/a^5*x^4-1/3*b^4/a
^6*x^5)/(b*x+a)^4/(-b*x+a)-5/64/a^6/b*ln(-b*x+a)+5/64/a^6/b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(93) = 186$.

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+bx)^3\,(a^2-b^2x^2)^2}\,dx = -\frac{30\,ab^4x^4+90\,a^2b^3x^3+70\,a^3b^2x^2-30\,a^4bx-64\,a^5-15\,(b^5x^5+3\,ab^4x^4+2\,a^2b^3x^3-2\,a^3b^2x^2-3\,a^4b}{192\,(a^6b^6x^5+3\,a^7b^5x^4+2\,a^8b^4x^3-2}$$

input
```
integrate(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="fricas")
```

output
```
-1/192*(30*a*b^4*x^4 + 90*a^2*b^3*x^3 + 70*a^3*b^2*x^2 - 30*a^4*b*x - 64*a
^5 - 15*(b^5*x^5 + 3*a*b^4*x^4 + 2*a^2*b^3*x^3 - 2*a^3*b^2*x^2 - 3*a^4*b*x
 - a^5)*log(b*x + a) + 15*(b^5*x^5 + 3*a*b^4*x^4 + 2*a^2*b^3*x^3 - 2*a^3*b
^2*x^2 - 3*a^4*b*x - a^5)*log(b*x - a))/(a^6*b^6*x^5 + 3*a^7*b^5*x^4 + 2*a
^8*b^4*x^3 - 2*a^9*b^3*x^2 - 3*a^10*b^2*x - a^11*b)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx)^3\,(a^2-b^2x^2)^2}\,dx = \frac{32a^4+15a^3bx-35a^2b^2x^2-45ab^3x^3-15b^4x^4}{-96a^{10}b-288a^9b^2x-192a^8b^3x^2+192a^7b^4x^3+288a^6b^5x^4+96a^5b^6x^5} + \frac{-\frac{5\log\left(-\frac{a}{b}+x\right)}{64}+\frac{5\log\left(\frac{a}{b}+x\right)}{64}}{a^6b}$$

input
```
integrate(1/(b*x+a)**3/(-b**2*x**2+a**2)**2,x)
```

output
```
(32*a**4 + 15*a**3*b*x - 35*a**2*b**2*x**2 - 45*a*b**3*x**3 - 15*b**4*x**4
)/(-96*a**10*b - 288*a**9*b**2*x - 192*a**8*b**3*x**2 + 192*a**7*b**4*x**3
 + 288*a**6*b**5*x**4 + 96*a**5*b**6*x**5) + (-5*log(-a/b + x)/64 + 5*log(
a/b + x)/64)/(a**6*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx)^3 (a^2-b^2x^2)^2}\,dx = -\frac{15\,b^4x^4+45\,ab^3x^3+35\,a^2b^2x^2-15\,a^3bx-32\,a^4}{96\,(a^5b^6x^5+3\,a^6b^5x^4+2\,a^7b^4x^3-2\,a^8b^3x^2-3\,a^9b^2x-a^{10}b)} + \frac{5\,\log(bx+a)}{64\,a^6b} - \frac{5\,\log(bx-a)}{64\,a^6b}$$

input
```
integrate(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="maxima")
```

output
```
-1/96*(15*b^4*x^4 + 45*a*b^3*x^3 + 35*a^2*b^2*x^2 - 15*a^3*b*x - 32*a^4)/(
a^5*b^6*x^5 + 3*a^6*b^5*x^4 + 2*a^7*b^4*x^3 - 2*a^8*b^3*x^2 - 3*a^9*b^2*x
- a^10*b) + 5/64*log(b*x + a)/(a^6*b) - 5/64*log(b*x - a)/(a^6*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx)^3 (a^2-b^2x^2)^2}\,dx = \frac{5\,\log(|bx+a|)}{64\,a^6b} - \frac{5\,\log(|bx-a|)}{64\,a^6b} - \frac{15\,ab^4x^4+45\,a^2b^3x^3+35\,a^3b^2x^2-15\,a^4bx-32\,a^5}{96\,(bx+a)^4(bx-a)a^6b}$$

input
```
integrate(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x, algorithm="giac")
```

output
```
5/64*log(abs(b*x + a))/(a^6*b) - 5/64*log(abs(b*x - a))/(a^6*b) - 1/96*(15
*a*b^4*x^4 + 45*a^2*b^3*x^3 + 35*a^3*b^2*x^2 - 15*a^4*b*x - 32*a^5)/((b*x
+ a)^4*(b*x - a)*a^6*b)
```

Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+bx)^3\,(a^2-b^2x^2)^2}\,dx = \frac{\frac{35\,b\,x^2}{96\,a^3}-\frac{1}{3\,a\,b}-\frac{5\,x}{32\,a^2}+\frac{15\,b^2\,x^3}{32\,a^4}+\frac{5\,b^3\,x^4}{32\,a^5}}{a^5+3\,a^4\,b\,x+2\,a^3\,b^2\,x^2-2\,a^2\,b^3\,x^3-3\,a\,b^4\,x^4-b^5\,x^5} + \frac{5\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{32\,a^6\,b}$$

input
```
int(1/((a^2 - b^2*x^2)^2*(a + b*x)^3),x)
```

output
```
((35*b*x^2)/(96*a^3) - 1/(3*a*b) - (5*x)/(32*a^2) + (15*b^2*x^3)/(32*a^4)
+ (5*b^3*x^4)/(32*a^5))/(a^5 - b^5*x^5 - 3*a*b^4*x^4 + 2*a^3*b^2*x^2 - 2*a
^2*b^3*x^3 + 3*a^4*b*x) + (5*atanh((b*x)/a))/(32*a^6*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.65

$$\int \frac{1}{(a+bx)^3\,(a^2-b^2x^2)^2}\,dx = \frac{-15\log(-bx+a)\,a^5-45\log(-bx+a)\,a^4bx-30\log(-bx+a)\,a^3b^2x^2+30\log(-bx+a)\,a^2b^3x^3+45\log}{}$$

input
```
int(1/(b*x+a)^3/(-b^2*x^2+a^2)^2,x)
```

output
```
( - 15*log(a - b*x)*a**5 - 45*log(a - b*x)*a**4*b*x - 30*log(a - b*x)*a**3
*b**2*x**2 + 30*log(a - b*x)*a**2*b**3*x**3 + 45*log(a - b*x)*a*b**4*x**4
+ 15*log(a - b*x)*b**5*x**5 + 15*log(a + b*x)*a**5 + 45*log(a + b*x)*a**4*
b*x + 30*log(a + b*x)*a**3*b**2*x**2 - 30*log(a + b*x)*a**2*b**3*x**3 - 45
*log(a + b*x)*a*b**4*x**4 - 15*log(a + b*x)*b**5*x**5 - 54*a**5 + 90*a**3*
b**2*x**2 + 70*a**2*b**3*x**3 - 10*b**5*x**5)/(192*a**6*b*(a**5 + 3*a**4*b
*x + 2*a**3*b**2*x**2 - 2*a**2*b**3*x**3 - 3*a*b**4*x**4 - b**5*x**5))
```

3.21 $\int \frac{(a+bx)^8}{\left(a^2-b^2x^2\right)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{(a+bx)^8}{\left(a^2-b^2x^2\right)^3} dx = -31a^2x-4abx^2-\frac{b^2x^3}{3}+\frac{16a^5}{b(a-bx)^2}-\frac{80a^4}{b(a-bx)}-\frac{80a^3\log(a-bx)}{b}$$

output
```
-31*a^2*x-4*a*b*x^2-1/3*b^2*x^3+16*a^5/b/(-b*x+a)^2-80*a^4/b/(-b*x+a)-80*a
^3*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^8}{\left(a^2-b^2x^2\right)^3} dx = -31a^2x-4abx^2-\frac{b^2x^3}{3}+\frac{16a^5}{b(-a+bx)^2} + \frac{80a^4}{b(-a+bx)}-\frac{80a^3\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^8/(a^2 - b^2*x^2)^3,x]
```

output
```
-31*a^2*x - 4*a*b*x^2 - (b^2*x^3)/3 + (16*a^5)/(b*(-a + b*x)^2) + (80*a^4)
/(b*(-a + b*x)) - (80*a^3*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^8}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(a+bx)^5}{(a-bx)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{32a^5}{(a-bx)^3}-\frac{80a^4}{(a-bx)^2}+\frac{80a^3}{a-bx}-31a^2-8abx-b^2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{16a^5}{b(a-bx)^2}-\frac{80a^4}{b(a-bx)}-\frac{80a^3\log(a-bx)}{b}-31a^2x-4abx^2-\frac{b^2x^3}{3}$$

input
```
Int[(a + b*x)^8/(a^2 - b^2*x^2)^3,x]
```

output
```
-31*a^2*x - 4*a*b*x^2 - (b^2*x^3)/3 + (16*a^5)/(b*(a - b*x)^2) - (80*a^4)/
(b*(a - b*x)) - (80*a^3*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{b^{2} x^{3}}{3}-4 a b\,x^{2}-31 a^{2} x+\frac{80 a^{4} x-\frac{64 a^{5}}{b}}{\left(-b x+a \right)^{2}}-\frac{80 a^{3} \ln \left(-b x+a \right)}{b}$	62
default	$-31 a^{2} x-4 a b\,x^{2}-\frac{b^{2} x^{3}}{3}+\frac{16 a^{5}}{b \left(-b x+a \right)^{2}}-\frac{80 a^{4}}{b \left(-b x+a \right)}-\frac{80 a^{3} \ln \left(-b x+a \right)}{b}$	70
norman	$\frac{72 b^{3} a^{3} x^{4}-79 a^{6} x-\frac{1}{3} b^{6} x^{7}-4 a\,b^{5} x^{6}-\frac{91}{3} a^{2} b^{4} x^{5}+\frac{425}{3} a^{4} b^{2} x^{3}-36 b\,a^{5} x^{2}}{\left(-b^{2} x^{2}+a^{2}\right)^{2}}-\frac{80 a^{3} \ln \left(-b x+a \right)}{b}$	98
parallelrisch	$-\frac{b^{6} x^{5}+10 a\,b^{5} x^{4}+240 \ln \left(b x-a \right) x^{2} a^{3} b^{3}+70 a^{2} b^{4} x^{3}-480 \ln \left(b x-a \right) x\,a^{4} b^{2}+240 \ln \left(b x-a \right) a^{5} b-495 x\,a^{4} b^{2}+366 a^{5} b}{3 b^{2} \left(b x-a \right)^{2}}$	108

input
```
int((b*x+a)^8/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*x^3-4*a*b*x^2-31*a^2*x+(80*a^4*x-64*a^5/b)/(-b*x+a)^2-80*a^3*ln(-
b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^8}{(a^2-b^2x^2)^3}\,dx = -\frac{b^5x^5+10\,ab^4x^4+70\,a^2b^3x^3-174\,a^3b^2x^2-147\,a^4bx+192\,a^5+240\,(a^3b^2x^2-2\,a^4bx+a^5)\log{(bx-a)}}{3\,(b^3x^2-2\,ab^2x+a^2b)}$$

input
```
integrate((b*x+a)^8/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/3*(b^5*x^5 + 10*a*b^4*x^4 + 70*a^2*b^3*x^3 - 174*a^3*b^2*x^2 - 147*a^4*
b*x + 192*a^5 + 240*(a^3*b^2*x^2 - 2*a^4*b*x + a^5)*log(b*x - a))/(b^3*x^2
 - 2*a*b^2*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^8}{(a^2-b^2x^2)^3}\,dx = -\frac{80a^3\log{(-a+bx)}}{b}-31a^2x-4abx^2-\frac{b^2x^3}{3}-\frac{64a^5-80a^4bx}{a^2b-2ab^2x+b^3x^2}$$

input
```
integrate((b*x+a)**8/(-b**2*x**2+a**2)**3,x)
```

output
```
-80*a**3*log(-a + b*x)/b - 31*a**2*x - 4*a*b*x**2 - b**2*x**3/3 - (64*a**5
 - 80*a**4*b*x)/(a**2*b - 2*a*b**2*x + b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^8}{(a^2-b^2x^2)^3}\,dx = -\frac{1}{3}\,b^2x^3-4\,abx^2-31\,a^2x-\frac{80\,a^3\log{(bx-a)}}{b}+\frac{16\,(5\,a^4bx-4\,a^5)}{b^3x^2-2\,ab^2x+a^2b}$$

input
```
integrate((b*x+a)^8/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/3*b^2*x^3 - 4*a*b*x^2 - 31*a^2*x - 80*a^3*log(b*x - a)/b + 16*(5*a^4*b*
x - 4*a^5)/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^8}{(a^2-b^2x^2)^3}\,dx = -\frac{80\,a^3\log\left(|bx-a|\right)}{b} + \frac{16\,(5\,a^4bx-4\,a^5)}{(bx-a)^2b} - \frac{b^{11}x^3+12\,ab^{10}x^2+93\,a^2b^9x}{3\,b^9}$$

input
```
integrate((b*x+a)^8/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-80*a^3*log(abs(b*x - a))/b + 16*(5*a^4*b*x - 4*a^5)/((b*x - a)^2*b) - 1/3
*(b^11*x^3 + 12*a*b^10*x^2 + 93*a^2*b^9*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^8}{(a^2-b^2x^2)^3}\,dx = \frac{80\,a^4\,x-\frac{64\,a^5}{b}}{a^2-2\,a\,b\,x+b^2\,x^2} - 31\,a^2\,x - \frac{b^2\,x^3}{3} - \frac{80\,a^3\,\ln\left(b\,x-a\right)}{b} - 4\,a\,b\,x^2$$

input
```
int((a + b*x)^8/(a^2 - b^2*x^2)^3,x)
```

output
```
(80*a^4*x - (64*a^5)/b)/(a^2 + b^2*x^2 - 2*a*b*x) - 31*a^2*x - (b^2*x^3)/3
 - (80*a^3*log(b*x - a))/b - 4*a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^8}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{-480\log(-bx+a)\,a^5+960\log(-bx+a)\,a^4bx-480\log(-bx+a)\,a^3b^2x^2-237a^5+495a^3b^2x^2-140a^2}{6b\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)^8/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 480*log(a - b*x)*a**5 + 960*log(a - b*x)*a**4*b*x - 480*log(a - b*x)*a
**3*b**2*x**2 - 237*a**5 + 495*a**3*b**2*x**2 - 140*a**2*b**3*x**3 - 20*a*
b**4*x**4 - 2*b**5*x**5)/(6*b*(a**2 - 2*a*b*x + b**2*x**2))
```

3.22 $\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx = -7ax-\frac{bx^2}{2}+\frac{8a^4}{b(a-bx)^2}-\frac{32a^3}{b(a-bx)}-\frac{24a^2\log(a-bx)}{b}$$

output
```
-7*a*x-1/2*b*x^2+8*a^4/b/(-b*x+a)^2-32*a^3/b/(-b*x+a)-24*a^2*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx = -7ax-\frac{bx^2}{2}+\frac{8a^4}{b(-a+bx)^2}+\frac{32a^3}{b(-a+bx)}-\frac{24a^2\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^7/(a^2 - b^2*x^2)^3,x]
```

output
```
-7*a*x - (b*x^2)/2 + (8*a^4)/(b*(-a + b*x)^2) + (32*a^3)/(b*(-a + b*x)) -
(24*a^2*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(a+bx)^4}{(a-bx)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{16a^4}{(a-bx)^3}-\frac{32a^3}{(a-bx)^2}+\frac{24a^2}{a-bx}-7a-bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8a^4}{b(a-bx)^2}-\frac{32a^3}{b(a-bx)}-\frac{24a^2\log(a-bx)}{b}-7ax-\frac{bx^2}{2}$$

input
```
Int[(a + b*x)^7/(a^2 - b^2*x^2)^3,x]
```

output
```
-7*a*x - (b*x^2)/2 + (8*a^4)/(b*(a - b*x)^2) - (32*a^3)/(b*(a - b*x)) - (2
4*a^2*Log[a - b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{b\,x^{2}}{2}-7ax+\frac{32a^{3}x-\frac{24a^{4}}{b}}{\left(-bx+a\right)^{2}}-\frac{24a^{2}\ln\left(-bx+a\right)}{b}$	51
default	$-7ax-\frac{b\,x^{2}}{2}+\frac{8a^{4}}{b\left(-bx+a\right)^{2}}-\frac{32a^{3}}{b\left(-bx+a\right)}-\frac{24a^{2}\ln\left(-bx+a\right)}{b}$	59
norman	$\frac{-23a^{5}x-\frac{b^{5}x^{6}}{2}-7a\,b^{4}x^{5}+46a^{3}b^{2}x^{3}-\frac{25a^{6}}{b}+\frac{83a^{4}b\,x^{2}}{2}}{\left(-b^{2}x^{2}+a^{2}\right)^{2}}-\frac{24a^{2}\ln\left(-bx+a\right)}{b}$	84
parallelrisch	$-\frac{b^{5}x^{4}+48\ln\left(bx-a\right)x^{2}a^{2}b^{3}+12a\,b^{4}x^{3}-96\ln\left(bx-a\right)x\,a^{3}b^{2}+48\ln\left(bx-a\right)a^{4}b-104x\,a^{3}b^{2}+75a^{4}b}{2b^{2}\left(bx-a\right)^{2}}$	97

input
```
int((b*x+a)^7/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*x^2-7*a*x+(32*a^3*x-24*a^4/b)/(-b*x+a)^2-24*a^2*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx = -\frac{b^4x^4+12\,ab^3x^3-27\,a^2b^2x^2-50\,a^3bx+48\,a^4+48\,(a^2b^2x^2-2\,a^3bx+a^4)\log(bx-a)}{2\,(b^3x^2-2\,ab^2x+a^2b)}$$

input
```
integrate((b*x+a)^7/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*x^4 + 12*a*b^3*x^3 - 27*a^2*b^2*x^2 - 50*a^3*b*x + 48*a^4 + 48*(
a^2*b^2*x^2 - 2*a^3*b*x + a^4)*log(b*x - a))/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx = -\frac{24a^2\log(-a+bx)}{b} - 7ax - \frac{bx^2}{2} - \frac{24a^4-32a^3bx}{a^2b-2ab^2x+b^3x^2}$$

input
```
integrate((b*x+a)**7/(-b**2*x**2+a**2)**3,x)
```

output
```
-24*a**2*log(-a + b*x)/b - 7*a*x - b*x**2/2 - (24*a**4 - 32*a**3*b*x)/(a**
2*b - 2*a*b**2*x + b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx = -\frac{1}{2}\,bx^2 - 7\,ax - \frac{24\,a^2\log(bx-a)}{b} + \frac{8\,(4\,a^3bx-3\,a^4)}{b^3x^2-2\,ab^2x+a^2b}$$

input
```
integrate((b*x+a)^7/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/2*b*x^2 - 7*a*x - 24*a^2*log(b*x - a)/b + 8*(4*a^3*b*x - 3*a^4)/(b^3*x^
2 - 2*a*b^2*x + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx = -\frac{24\,a^2\log\left(|bx-a|\right)}{b}+\frac{8\,(4\,a^3bx-3\,a^4)}{(bx-a)^2b}-\frac{b^7x^2+14\,ab^6x}{2\,b^6}$$

input
```
integrate((b*x+a)^7/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-24*a^2*log(abs(b*x - a))/b + 8*(4*a^3*b*x - 3*a^4)/((b*x - a)^2*b) - 1/2*
(b^7*x^2 + 14*a*b^6*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx = \frac{32\,a^3\,x-\frac{24\,a^4}{b}}{a^2-2\,a\,b\,x+b^2\,x^2}-7\,a\,x-\frac{b\,x^2}{2}-\frac{24\,a^2\,\ln\left(b\,x-a\right)}{b}$$

input
```
int((a + b*x)^7/(a^2 - b^2*x^2)^3,x)
```

output
```
(32*a^3*x - (24*a^4)/b)/(a^2 + b^2*x^2 - 2*a*b*x) - 7*a*x - (b*x^2)/2 - (2
4*a^2*log(b*x - a))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^7}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{-48\log(-bx+a)\,a^4+96\log(-bx+a)\,a^3bx-48\log(-bx+a)\,a^2b^2x^2-23a^4+52a^2b^2x^2-12a\,b^3x^3-}{2b\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)^7/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 48*log(a - b*x)*a**4 + 96*log(a - b*x)*a**3*b*x - 48*log(a - b*x)*a**2
*b**2*x**2 - 23*a**4 + 52*a**2*b**2*x**2 - 12*a*b**3*x**3 - b**4*x**4)/(2*
b*(a**2 - 2*a*b*x + b**2*x**2))
```

3.23 $\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx = -x+\frac{4a^3}{b(a-bx)^2}-\frac{12a^2}{b(a-bx)}-\frac{6a\log(a-bx)}{b}$$

output
```
-x+4*a^3/b/(-b*x+a)^2-12*a^2/b/(-b*x+a)-6*a*ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx = -x+\frac{4a^2(-2a+3bx)}{b(a-bx)^2}-\frac{6a\log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^6/(a^2 - b^2*x^2)^3,x]
```

output
```
-x + (4*a^2*(-2*a + 3*b*x))/(b*(a - b*x)^2) - (6*a*Log[a - b*x])/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(a+bx)^3}{(a-bx)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{8a^3}{(a-bx)^3}-\frac{12a^2}{(a-bx)^2}+\frac{6a}{a-bx}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^3}{b(a-bx)^2}-\frac{12a^2}{b(a-bx)}-\frac{6a\log(a-bx)}{b}-x$$

input
```
Int[(a + b*x)^6/(a^2 - b^2*x^2)^3,x]
```

output
```
-x + (4*a^3)/(b*(a - b*x)^2) - (12*a^2)/(b*(a - b*x)) - (6*a*Log[a - b*x])
/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
risch	$-x+\frac{12 a^{2} x-\frac{8 a^{3}}{b}}{\left(-b x+a\right)^{2}}-\frac{6 a \ln \left(-b x+a\right)}{b}$	42
default	$-x+\frac{4 a^{3}}{b \left(-b x+a\right)^{2}}-\frac{12 a^{2}}{b \left(-b x+a\right)}-\frac{6 a \ln \left(-b x+a\right)}{b}$	50
norman	$\frac{-5 a^{4} x-b^{4} x^{5}+14 a^{2} b^{2} x^{3}-\frac{8 a^{5}}{b}+16 a^{3} b \,x^{2}}{\left(-b^{2} x^{2}+a^{2}\right)^{2}}-\frac{6 a \ln \left(-b x+a\right)}{b}$	73
parallelrisch	$-\frac{6 \ln \left(b x-a\right) x^{2} a \,b^{3}+b^{4} x^{3}-12 \ln \left(b x-a\right) x \,a^{2} b^{2}+6 \ln \left(b x-a\right) a^{3} b-15 x \,a^{2} b^{2}+10 a^{3} b}{b^{2} \left(b x-a\right)^{2}}$	86

input
```
int((b*x+a)^6/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-x+(12*a^2*x-8*a^3/b)/(-b*x+a)^2-6*a*ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^6}{\left(a^2-b^2x^2\right)^3}\,dx = -\frac{b^3x^3-2\,ab^2x^2-11\,a^2bx+8\,a^3+6\,\left(ab^2x^2-2\,a^2bx+a^3\right)\log\left(bx-a\right)}{b^3x^2-2\,ab^2x+a^2b}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-(b^3*x^3 - 2*a*b^2*x^2 - 11*a^2*b*x + 8*a^3 + 6*(a*b^2*x^2 - 2*a^2*b*x +
a^3)*log(b*x - a))/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx = -\frac{6a\log(-a+bx)}{b} - x - \frac{8a^3-12a^2bx}{a^2b-2ab^2x+b^3x^2}$$

input
```
integrate((b*x+a)**6/(-b**2*x**2+a**2)**3,x)
```

output
```
-6*a*log(-a + b*x)/b - x - (8*a**3 - 12*a**2*b*x)/(a**2*b - 2*a*b**2*x + b
**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx = -x - \frac{6\,a\log(bx-a)}{b} + \frac{4\,(3\,a^2bx-2\,a^3)}{b^3x^2-2\,ab^2x+a^2b}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-x - 6*a*log(b*x - a)/b + 4*(3*a^2*b*x - 2*a^3)/(b^3*x^2 - 2*a*b^2*x + a^2
*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx = -x - \frac{6\,a\log\left(|bx-a|\right)}{b} + \frac{4\,(3\,a^2bx-2\,a^3)}{(bx-a)^2b}$$

input
```
integrate((b*x+a)^6/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-x - 6*a*log(abs(b*x - a))/b + 4*(3*a^2*b*x - 2*a^3)/((b*x - a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx = \frac{12\,a^2\,x-\frac{8\,a^3}{b}}{a^2-2\,a\,b\,x+b^2\,x^2} - x - \frac{6\,a\,\ln\left(b\,x-a\right)}{b}$$

input
```
int((a + b*x)^6/(a^2 - b^2*x^2)^3,x)
```

output
```
(12*a^2*x - (8*a^3)/b)/(a^2 + b^2*x^2 - 2*a*b*x) - x - (6*a*log(b*x - a))/
b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx)^6}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{-12\log(-bx+a)\,a^3+24\log(-bx+a)\,a^2bx-12\log(-bx+a)\,a\,b^2x^2-5a^3+15a\,b^2x^2-2b^3x^3}{2b\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)^6/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 12*log(a - b*x)*a**3 + 24*log(a - b*x)*a**2*b*x - 12*log(a - b*x)*a*b*
*2*x**2 - 5*a**3 + 15*a*b**2*x**2 - 2*b**3*x**3)/(2*b*(a**2 - 2*a*b*x + b*
*2*x**2))
```

3.24 $\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3} dx = \frac{2a^2}{b(a-bx)^2} - \frac{4a}{b(a-bx)} - \frac{\log(a-bx)}{b}$$

output
```
2*a^2/b/(-b*x+a)^2-4*a/b/(-b*x+a)-ln(-b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3} dx = -\frac{\frac{2a(a-2bx)}{(a-bx)^2} + \log(a-bx)}{b}$$

input
```
Integrate[(a + b*x)^5/(a^2 - b^2*x^2)^3,x]
```

output
```
-(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(a+bx)^2}{(a-bx)^3}dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{4a^2}{(a-bx)^3}-\frac{4a}{(a-bx)^2}+\frac{1}{a-bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2}{b(a-bx)^2}-\frac{4a}{b(a-bx)}-\frac{\log(a-bx)}{b}$$

input
```
Int[(a + b*x)^5/(a^2 - b^2*x^2)^3,x]
```

output
```
(2*a^2)/(b*(a - b*x)^2) - (4*a)/(b*(a - b*x)) - Log[a - b*x]/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{4ax-\frac{2a^2}{b}}{(-bx+a)^2}-\frac{\ln(-bx+a)}{b}$	36
default	$\frac{2a^2}{b(-bx+a)^2}-\frac{4a}{b(-bx+a)}-\frac{\ln(-bx+a)}{b}$	44
norman	$\frac{4a\,b^2x^3-\frac{2a^4}{b}+6a^2b\,x^2}{(-b^2x^2+a^2)^2}-\frac{\ln(-bx+a)}{b}$	56
parallelrisch	$-\frac{\ln(bx-a)x^2b^3-2\ln(bx-a)xa\,b^2+\ln(bx-a)a^2b-4xa\,b^2+2a^2b}{b^2(bx-a)^2}$	72

input
```
int((b*x+a)^5/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(4*a*x-2*a^2/b)/(-b*x+a)^2-ln(-b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3}\,dx = \frac{4\,abx-2\,a^2-(b^2x^2-2\,abx+a^2)\log(bx-a)}{b^3x^2-2\,ab^2x+a^2b}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
(4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*x^2 - 2*a*
b^2*x + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3}\,dx = -\frac{2a^2-4abx}{a^2b-2ab^2x+b^3x^2}-\frac{\log(-a+bx)}{b}$$

input
```
integrate((b*x+a)**5/(-b**2*x**2+a**2)**3,x)
```

output
```
-(2*a**2 - 4*a*b*x)/(a**2*b - 2*a*b**2*x + b**3*x**2) - log(-a + b*x)/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3}\,dx = \frac{2\,(2\,abx-a^2)}{b^3x^2-2\,ab^2x+a^2b}-\frac{\log(bx-a)}{b}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
2*(2*a*b*x - a^2)/(b^3*x^2 - 2*a*b^2*x + a^2*b) - log(b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3}\,dx = -\frac{\log(|bx-a|)}{b}+\frac{2\,(2\,abx-a^2)}{(bx-a)^2b}$$

input
```
integrate((b*x+a)^5/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-log(abs(b*x - a))/b + 2*(2*a*b*x - a^2)/((b*x - a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3}\,dx = \frac{4\,a\,x-\frac{2\,a^2}{b}}{a^2-2\,a\,b\,x+b^2\,x^2} - \frac{\ln\left(b\,x-a\right)}{b}$$

input
```
int((a + b*x)^5/(a^2 - b^2*x^2)^3,x)
```

output
```
(4*a*x - (2*a^2)/b)/(a^2 + b^2*x^2 - 2*a*b*x) - log(b*x - a)/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx)^5}{(a^2-b^2x^2)^3}\,dx = \frac{-\log(-bx+a)\,a^2+2\log(-bx+a)\,abx-\log(-bx+a)\,b^2x^2+2b^2x^2}{b\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)^5/(-b^2*x^2+a^2)^3,x)
```

output
```
( - log(a - b*x)*a**2 + 2*log(a - b*x)*a*b*x - log(a - b*x)*b**2*x**2 + 2*
b**2*x**2)/(b*(a**2 - 2*a*b*x + b**2*x**2))
```

3.25 $\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 10

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3} dx = \frac{x}{(a-bx)^2}$$

output
```
x/(-b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3} dx = \frac{x}{(a-bx)^2}$$

input
```
Integrate[(a + b*x)^4/(a^2 - b^2*x^2)^3,x]
```

output
```
x/(a - b*x)^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {456, 38}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{a+bx}{(a-bx)^3}dx$$

$$\Big\downarrow \text{38}$$

$$\frac{x}{(a-bx)^2}$$

input
```
Int[(a + b*x)^4/(a^2 - b^2*x^2)^3,x]
```

output
```
x/(a - b*x)^2
```

Defintions of rubi rules used

rule 38
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{x}{(-bx+a)^2}$	11
risch	$\frac{x}{(-bx+a)^2}$	11
parallelrisch	$\frac{x}{(bx-a)^2}$	12
default	$-\frac{1}{b(-bx+a)}+\frac{a}{b(-bx+a)^2}$	28
orering	$\frac{(-bx+a)x(bx+a)^3}{(-b^2x^2+a^2)^3}$	30
norman	$\frac{b^2x^3+2ab\,x^2+a^2x}{(-b^2x^2+a^2)^2}$	36

input
```
int((b*x+a)^4/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x/(-b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3}\,dx=\frac{x}{b^2x^2-2\,abx+a^2}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
x/(b^2*x^2 - 2*a*b*x + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{x}{a^2-2abx+b^2x^2}$$

input
```
integrate((b*x+a)**4/(-b**2*x**2+a**2)**3,x)
```

output
```
x/(a**2 - 2*a*b*x + b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{x}{b^2x^2-2\,abx+a^2}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
x/(b^2*x^2 - 2*a*b*x + a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{x}{(bx-a)^2}$$

input
```
integrate((b*x+a)^4/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
x/(b*x - a)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{x}{(a-b\,x)^2}$$

input
```
int((a + b*x)^4/(a^2 - b^2*x^2)^3,x)
```

output
```
x/(a - b*x)^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \frac{(a+bx)^4}{(a^2-b^2x^2)^3}\,dx = \frac{b^2x^2+a^2}{2ab\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)^4/(-b^2*x^2+a^2)^3,x)
```

output
```
(a**2 + b**2*x**2)/(2*a*b*(a**2 - 2*a*b*x + b**2*x**2))
```

3.26 $\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2b(a-bx)^2}$$

output
```
1/2/b/(-b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2b(a-bx)^2}$$

input
```
Integrate[(a + b*x)^3/(a^2 - b^2*x^2)^3,x]
```

output
```
1/(2*b*(a - b*x)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{1}{(a-bx)^3}dx$$

$$\Big\downarrow \; 17$$

$$\frac{1}{2b(a-bx)^2}$$

input
```
Int[(a + b*x)^3/(a^2 - b^2*x^2)^3,x]
```

output
```
1/(2*b*(a - b*x)^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{1}{2b(-bx+a)^2}$	14
default	$\frac{1}{2b(-bx+a)^2}$	14
risch	$\frac{1}{2b(-bx+a)^2}$	14
parallelrisch	$\frac{1}{2b(bx-a)^2}$	15
orering	$\frac{(-bx+a)(bx+a)^3}{2b(-b^2x^2+a^2)^3}$	33
norman	$\frac{ax+\frac{a^2}{2b}+\frac{b\,x^2}{2}}{(-b^2x^2+a^2)^2}$	34

input
```
int((b*x+a)^3/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(-b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2\,(b^3x^2-2\,ab^2x+a^2b)}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
1/2/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2a^2b-4ab^2x+2b^3x^2}$$

input
```
integrate((b*x+a)**3/(-b**2*x**2+a**2)**3,x)
```

output
```
1/(2*a**2*b - 4*a*b**2*x + 2*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2\,(b^3x^2-2\,ab^2x+a^2b)}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
1/2/(b^3*x^2 - 2*a*b^2*x + a^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2\,(bx-a)^2b}$$

input
```
integrate((b*x+a)^3/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/2/((b*x - a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2\,a^2\,b-4\,a\,b^2\,x+2\,b^3\,x^2}$$

input
```
int((a + b*x)^3/(a^2 - b^2*x^2)^3,x)
```

output
```
1/(2*a^2*b + 2*b^3*x^2 - 4*a*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)^3}{(a^2-b^2x^2)^3}\,dx = \frac{1}{2b\,(b^2x^2-2abx+a^2)}$$

input
```
int((b*x+a)^3/(-b^2*x^2+a^2)^3,x)
```

output
```
1/(2*b*(a**2 - 2*a*b*x + b**2*x**2))
```

3.27 $\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3} \, dx = \frac{1}{4ab(a-bx)^2} + \frac{1}{4a^2b(a-bx)} + \frac{\operatorname{arctanh}\left(\frac{bx}{a}\right)}{4a^3b}$$

output
```
1/4/a/b/(-b*x+a)^2+1/4/a^2/b/(-b*x+a)+1/4*arctanh(b*x/a)/a^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3} \, dx = \frac{2a(2a-bx) - (a-bx)^2 \log(a-bx) + (a-bx)^2 \log(a+bx)}{8a^3b(a-bx)^2}$$

input
```
Integrate[(a + b*x)^2/(a^2 - b^2*x^2)^3,x]
```

output
```
(2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8
*a^3*b*(a - b*x)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(a-bx)^3(a+bx)}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{4a^2\left(a^2-b^2x^2\right)}+\frac{1}{4a^2(a-bx)^2}+\frac{1}{2a(a-bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{4a^3b}+\frac{1}{4a^2b(a-bx)}+\frac{1}{4ab(a-bx)^2}$$

input
```
Int[(a + b*x)^2/(a^2 - b^2*x^2)^3,x]
```

output
```
1/(4*a*b*(a - b*x)^2) + 1/(4*a^2*b*(a - b*x)) + ArcTanh[(b*x)/a]/(4*a^3*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{-\frac{x}{4a^2}+\frac{1}{2ba}}{(-bx+a)^2}-\frac{\ln(-bx+a)}{8b\,a^3}+\frac{\ln(bx+a)}{8b\,a^3}$	55
default	$\frac{\ln(bx+a)}{8b\,a^3}-\frac{\ln(-bx+a)}{8b\,a^3}+\frac{1}{4a^2b(-bx+a)}+\frac{1}{4ab(-bx+a)^2}$	63
norman	$\frac{\frac{3x}{4}-\frac{x^3b^2}{4a^2}+\frac{a}{2b}}{(-b^2x^2+a^2)^2}-\frac{\ln(-bx+a)}{8b\,a^3}+\frac{\ln(bx+a)}{8b\,a^3}$	67
parallelrisch	$-\frac{\ln(bx-a)x^2b^2-\ln(bx+a)x^2b^2-2\ln(bx-a)xab+2\ln(bx+a)xab+4b^2x^2+a^2\ln(bx-a)-\ln(bx+a)a^2-6abx}{8a^3(bx-a)^2b}$	108

input
```
int((b*x+a)^2/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a^2*x+1/2/b/a)/(-b*x+a)^2-1/8/b/a^3*ln(-b*x+a)+1/8/b/a^3*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3}\,dx = -\frac{2\,abx-4\,a^2-(b^2x^2-2\,abx+a^2)\log(bx+a)+(b^2x^2-2\,abx+a^2)\log(bx-a)}{8\,(a^3b^3x^2-2\,a^4b^2x+a^5b)}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2
- 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*x^2 - 2*a^4*b^2*x + a^5*b)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3}\,dx = -\frac{-2a+bx}{4a^4b-8a^3b^2x+4a^2b^3x^2} - \frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8} - \frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3b}$$

input
```
integrate((b*x+a)**2/(-b**2*x**2+a**2)**3,x)
```

output
```
-(-2*a + b*x)/(4*a**4*b - 8*a**3*b**2*x + 4*a**2*b**3*x**2) - (log(-a/b +
x)/8 - log(a/b + x)/8)/(a**3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx)^2}{(a^2-b^2x^2)^3}\,dx = -\frac{bx-2\,a}{4\,(a^2b^3x^2-2\,a^3b^2x+a^4b)} + \frac{\log\left(bx+a\right)}{8\,a^3b} - \frac{\log\left(bx-a\right)}{8\,a^3b}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(b*x - 2*a)/(a^2*b^3*x^2 - 2*a^3*b^2*x + a^4*b) + 1/8*log(b*x + a)/(a
^3*b) - 1/8*log(b*x - a)/(a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)^2}{\left(a^2-b^2x^2\right)^3}\,dx = \frac{\log\left(|bx+a|\right)}{8\,a^3b} - \frac{\log\left(|bx-a|\right)}{8\,a^3b} - \frac{abx-2\,a^2}{4\,(bx-a)^2a^3b}$$

input
```
integrate((b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
1/8*log(abs(b*x + a))/(a^3*b) - 1/8*log(abs(b*x - a))/(a^3*b) - 1/4*(a*b*x
 - 2*a^2)/((b*x - a)^2*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx)^2}{\left(a^2-b^2x^2\right)^3}\,dx = \frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^3\,b} - \frac{\frac{x}{4\,a^2}-\frac{1}{2\,a\,b}}{a^2-2\,a\,b\,x+b^2\,x^2}$$

input
```
int((a + b*x)^2/(a^2 - b^2*x^2)^3,x)
```

output
```
atanh((b*x)/a)/(4*a^3*b) - (x/(4*a^2) - 1/(2*a*b))/(a^2 + b^2*x^2 - 2*a*b*
x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.09

$$\int \frac{(a+bx)^2}{\left(a^2-b^2x^2\right)^3}\,dx$$
$$= \frac{-\log(-bx+a)\,a^2+2\log(-bx+a)\,abx-\log(-bx+a)\,b^2x^2+\log(bx+a)\,a^2-2\log(bx+a)\,abx+\log(}{8a^3b\left(b^2x^2-2abx+a^2\right)}$$

input
```
int((b*x+a)^2/(-b^2*x^2+a^2)^3,x)
```

output
```
( - log(a - b*x)*a**2 + 2*log(a - b*x)*a*b*x - log(a - b*x)*b**2*x**2 + lo
g(a + b*x)*a**2 - 2*log(a + b*x)*a*b*x + log(a + b*x)*b**2*x**2 + 3*a**2 -
 b**2*x**2)/(8*a**3*b*(a**2 - 2*a*b*x + b**2*x**2))
```

3.28 $\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx=\frac{1}{8a^2b(a-bx)^2}+\frac{1}{4a^3b(a-bx)}-\frac{1}{8a^3b(a+bx)}+\frac{3\text{arctanh}\left(\frac{bx}{a}\right)}{8a^4b}$$

output
```
1/8/a^2/b/(-b*x+a)^2+1/4/a^3/b/(-b*x+a)-1/8/a^3/b/(b*x+a)+3/8*arctanh(b*x/
a)/a^4/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx=\frac{\frac{2a(2a^2+3abx-3b^2x^2)}{(a-bx)^2(a+bx)}-3\log(a-bx)+3\log(a+bx)}{16a^4b}$$

input
```
Integrate[(a + b*x)/(a^2 - b^2*x^2)^3,x]
```

output
```
((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b
*x] + 3*Log[a + b*x])/(16*a^4*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {454, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx$$

$$\Big\downarrow \text{454}$$

$$\frac{3\int \frac{1}{(a^2-b^2x^2)^2}dx}{4a}+\frac{a+bx}{4ab\,(a^2-b^2x^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{\int \frac{1}{a^2-b^2x^2}dx}{2a^2}+\frac{x}{2a^2(a^2-b^2x^2)}\right)}{4a}+\frac{a+bx}{4ab\,(a^2-b^2x^2)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{a+bx}{4ab\,(a^2-b^2x^2)^2}+\frac{3\left(\frac{\text{arctanh}\left(\frac{bx}{a}\right)}{2a^3b}+\frac{x}{2a^2(a^2-b^2x^2)}\right)}{4a}$$

input
```
Int[(a + b*x)/(a^2 - b^2*x^2)^3,x]
```

output
```
(a + b*x)/(4*a*b*(a^2 - b^2*x^2)^2) + (3*(x/(2*a^2*(a^2 - b^2*x^2)) + ArcT
anh[(b*x)/a]/(2*a^3*b)))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{5x}{8a}+\frac{1}{4b}-\frac{3b^2x^3}{8a^3}}{\left(-b^2x^2+a^2\right)^2}-\frac{3\ln\left(-bx+a\right)}{16a^4b}+\frac{3\ln\left(bx+a\right)}{16a^4b}$
default	$\frac{3\ln\left(bx+a\right)}{16a^4b}-\frac{1}{8a^3b\left(bx+a\right)}-\frac{3\ln\left(-bx+a\right)}{16a^4b}+\frac{1}{4a^3b\left(-bx+a\right)}+\frac{1}{8a^2b\left(-bx+a\right)^2}$
risch	$\frac{-\frac{3b\,x^2}{8a^3}+\frac{3x}{8a^2}+\frac{1}{4ba}}{\left(-bx+a\right)\left(-b^2x^2+a^2\right)}-\frac{3\ln\left(-bx+a\right)}{16a^4b}+\frac{3\ln\left(bx+a\right)}{16a^4b}$
parallelrisch	$-\frac{3\ln\left(bx-a\right)x^3b^5-3\ln\left(bx+a\right)x^3b^5-3\ln\left(bx-a\right)x^2a\,b^4+3\ln\left(bx+a\right)x^2a\,b^4-3\ln\left(bx-a\right)x\,a^2b^3+3\ln\left(bx+a\right)x\,a^2b^3+6x^2a\,b^4+3}{16a^4b^3\left(bx-a\right)\left(b^2x^2-a^2\right)}$

input
```
int((b*x+a)/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/8/a*x+1/4/b-3/8*b^2/a^3*x^3)/(-b^2*x^2+a^2)^2-3/16/a^4/b*ln(-b*x+a)+3/1
6/a^4/b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx = -\frac{6\,ab^2x^2-6\,a^2bx-4\,a^3-3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log(bx+a)+3\,(b^3x^3-ab^2x^2-a^2bx+a^3)\log}{16\,(a^4b^4x^3-a^5b^3x^2-a^6b^2x+a^7b)}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x
+ a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a)
)/(a^4*b^4*x^3 - a^5*b^3*x^2 - a^6*b^2*x + a^7*b)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx = -\frac{-2a^2-3abx+3b^2x^2}{8a^6b-8a^5b^2x-8a^4b^3x^2+8a^3b^4x^3} - \frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16} - \frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4b}$$

input
```
integrate((b*x+a)/(-b**2*x**2+a**2)**3,x)
```

output
```
-(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b - 8*a**5*b**2*x - 8*a**4*b**3
*x**2 + 8*a**3*b**4*x**3) - (3*log(-a/b + x)/16 - 3*log(a/b + x)/16)/(a**4
*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx = -\frac{3\,b^2x^2-3\,abx-2\,a^2}{8\,(a^3b^4x^3-a^4b^3x^2-a^5b^2x+a^6b)}+\frac{3\,\log(bx+a)}{16\,a^4b}-\frac{3\,\log(bx-a)}{16\,a^4b}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*x^3 - a^4*b^3*x^2 - a^5*b^2*x
+ a^6*b) + 3/16*log(b*x + a)/(a^4*b) - 3/16*log(b*x - a)/(a^4*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx = \frac{3\,\log(|bx+a|)}{16\,a^4b}-\frac{3\,\log(|bx-a|)}{16\,a^4b}-\frac{3\,ab^2x^2-3\,a^2bx-2\,a^3}{8\,(bx+a)(bx-a)^2a^4b}$$

input
```
integrate((b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
3/16*log(abs(b*x + a))/(a^4*b) - 3/16*log(abs(b*x - a))/(a^4*b) - 1/8*(3*a
*b^2*x^2 - 3*a^2*b*x - 2*a^3)/((b*x + a)*(b*x - a)^2*a^4*b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx = \frac{\frac{3\,x}{8\,a^2}+\frac{1}{4\,a\,b}-\frac{3\,b\,x^2}{8\,a^3}}{a^3-a^2\,b\,x-a\,b^2\,x^2+b^3\,x^3}+\frac{3\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{8\,a^4\,b}$$

input
```
int((a + b*x)/(a^2 - b^2*x^2)^3,x)
```

output
```
((3*x)/(8*a^2) + 1/(4*a*b) - (3*b*x^2)/(8*a^3))/(a^3 + b^3*x^3 - a*b^2*x^2
 - a^2*b*x) + (3*atanh((b*x)/a))/(8*a^4*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.37

$$\int \frac{a+bx}{(a^2-b^2x^2)^3}\,dx$$
$$= \frac{-3\log(-bx+a)\,a^3+3\log(-bx+a)\,a^2bx+3\log(-bx+a)\,a\,b^2x^2-3\log(-bx+a)\,b^3x^3+3\log(bx+a}{16a^4b\,(b^3x^3-a\,b^2x^2-}$$

input
```
int((b*x+a)/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 3*log(a - b*x)*a**3 + 3*log(a - b*x)*a**2*b*x + 3*log(a - b*x)*a*b**2*
x**2 - 3*log(a - b*x)*b**3*x**3 + 3*log(a + b*x)*a**3 - 3*log(a + b*x)*a**
2*b*x - 3*log(a + b*x)*a*b**2*x**2 + 3*log(a + b*x)*b**3*x**3 - 2*a**3 + 1
2*a**2*b*x - 6*b**3*x**3)/(16*a**4*b*(a**3 - a**2*b*x - a*b**2*x**2 + b**3
*x**3))
```

3.29 $\int \frac{1}{(a+bx)(a^2-b^2x^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)^3} dx = \frac{1}{32a^4b(a-bx)^2}+\frac{1}{8a^5b(a-bx)}-\frac{1}{24a^3b(a+bx)^3} -\frac{3}{32a^4b(a+bx)^2}-\frac{3}{16a^5b(a+bx)}+\frac{5\text{arctanh}\left(\frac{bx}{a}\right)}{16a^6b}$$

output
```
1/32/a^4/b/(-b*x+a)^2+1/8/a^5/b/(-b*x+a)-1/24/a^3/b/(b*x+a)^3-3/32/a^4/b/(
b*x+a)^2-3/16/a^5/b/(b*x+a)+5/16*arctanh(b*x/a)/a^6/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)^3} dx = \frac{-\frac{2a(8a^4-25a^3bx-25a^2b^2x^2+15ab^3x^3+15b^4x^4)}{(a-bx)^2(a+bx)^3}-15\log(a-bx)+15\log(a+bx)}{96a^6b}$$

input
```
Integrate[1/((a + b*x)*(a^2 - b^2*x^2)^3),x]
```

output
```
((-2*a*(8*a^4 - 25*a^3*b*x - 25*a^2*b^2*x^2 + 15*a*b^3*x^3 + 15*b^4*x^4))/
((a - b*x)^2*(a + b*x)^3) - 15*Log[a - b*x] + 15*Log[a + b*x])/(96*a^6*b)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(a^2-b^2x^2)^3}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(a-bx)^3(a+bx)^4}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{8a^5(a-bx)^2}+\frac{3}{16a^5(a+bx)^2}+\frac{1}{16a^4(a-bx)^3}+\frac{3}{16a^4(a+bx)^3}+\frac{1}{8a^3(a+bx)^4}+\frac{5}{16a^5(a^2-b^2x^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\text{arctanh}\left(\frac{bx}{a}\right)}{16a^6b}+\frac{1}{8a^5b(a-bx)}-\frac{3}{16a^5b(a+bx)}+\frac{1}{32a^4b(a-bx)^2}-\frac{3}{32a^4b(a+bx)^2}-\frac{1}{24a^3b(a+bx)^3}$$

input
```
Int[1/((a + b*x)*(a^2 - b^2*x^2)^3),x]
```

output
```
1/(32*a^4*b*(a - b*x)^2) + 1/(8*a^5*b*(a - b*x)) - 1/(24*a^3*b*(a + b*x)^3
) - 3/(32*a^4*b*(a + b*x)^2) - 3/(16*a^5*b*(a + b*x)) + (5*ArcTanh[(b*x)/a
])/(16*a^6*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result
norman	$\frac{\frac{7}{48ba}-\frac{15b^2x^3}{16a^4}+\frac{5b^4x^5}{16a^6}+\frac{5x}{6a^2}-\frac{5b\,x^2}{48a^3}}{(bx+a)^3(-bx+a)^2}-\frac{5\ln(-bx+a)}{32a^6b}+\frac{5\ln(bx+a)}{32a^6b}$
risch	$\frac{-\frac{5b^3x^4}{16a^5}-\frac{5b^2x^3}{16a^4}+\frac{25b\,x^2}{48a^3}+\frac{25x}{48a^2}-\frac{1}{6ba}}{(bx+a)(-b^2x^2+a^2)^2}-\frac{5\ln(-bx+a)}{32a^6b}+\frac{5\ln(bx+a)}{32a^6b}$
default	$\frac{5\ln(bx+a)}{32a^6b}-\frac{3}{16a^5b(bx+a)}-\frac{3}{32a^4b(bx+a)^2}-\frac{1}{24a^3b(bx+a)^3}-\frac{5\ln(-bx+a)}{32a^6b}+\frac{1}{8a^5b(-bx+a)}+\frac{1}{32a^4b(-bx+a)^2}$
parallelrisch	$-\frac{15\ln(bx-a)x^5b^8-15\ln(bx+a)x^5b^8+15\ln(bx-a)x^4a\,b^7-15\ln(bx+a)x^4a\,b^7-30x^5b^8-30\ln(bx-a)x^3a^2b^6+30\ln(bx+a)x^3}{}$

input
```
int(1/(b*x+a)/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(7/48/b/a-15/16*b^2/a^4*x^3+5/16*b^4/a^6*x^5+5/6/a^2*x-5/48*b/a^3*x^2)/(b*
x+a)^3/(-b*x+a)^2-5/32/a^6/b*ln(-b*x+a)+5/32/a^6/b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(95) = 190$.

Time = 0.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.06

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx = -\frac{30\,ab^4x^4+30\,a^2b^3x^3-50\,a^3b^2x^2-50\,a^4bx+16\,a^5-15\,(b^5x^5+ab^4x^4-2\,a^2b^3x^3-2\,a^3b^2x^2+a^4bx+}{96\,(a^6b^6x^5+a^7b^5x^4-2\,a^8b^4x^3-2\,a^9b}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/96*(30*a*b^4*x^4 + 30*a^2*b^3*x^3 - 50*a^3*b^2*x^2 - 50*a^4*b*x + 16*a^
5 - 15*(b^5*x^5 + a*b^4*x^4 - 2*a^2*b^3*x^3 - 2*a^3*b^2*x^2 + a^4*b*x + a^
5)*log(b*x + a) + 15*(b^5*x^5 + a*b^4*x^4 - 2*a^2*b^3*x^3 - 2*a^3*b^2*x^2
+ a^4*b*x + a^5)*log(b*x - a))/(a^6*b^6*x^5 + a^7*b^5*x^4 - 2*a^8*b^4*x^3
- 2*a^9*b^3*x^2 + a^10*b^2*x + a^11*b)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx &= -\frac{8a^4-25a^3bx-25a^2b^2x^2+15ab^3x^3+15b^4x^4}{48a^{10}b+48a^9b^2x-96a^8b^3x^2-96a^7b^4x^3+48a^6b^5x^4+48a^5b^6x^5} \\ &\quad -\frac{\frac{5\log\left(-\frac{a}{b}+x\right)}{32}-\frac{5\log\left(\frac{a}{b}+x\right)}{32}}{a^6b}\end{aligned}$$

input
```
integrate(1/(b*x+a)/(-b**2*x**2+a**2)**3,x)
```

output
```
-(8*a**4 - 25*a**3*b*x - 25*a**2*b**2*x**2 + 15*a*b**3*x**3 + 15*b**4*x**4
)/(48*a**10*b + 48*a**9*b**2*x - 96*a**8*b**3*x**2 - 96*a**7*b**4*x**3 + 4
8*a**6*b**5*x**4 + 48*a**5*b**6*x**5) - (5*log(-a/b + x)/32 - 5*log(a/b +
x)/32)/(a**6*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx = -\frac{15\,b^4x^4+15\,ab^3x^3-25\,a^2b^2x^2-25\,a^3bx+8\,a^4}{48\,(a^5b^6x^5+a^6b^5x^4-2\,a^7b^4x^3-2\,a^8b^3x^2+a^9b^2x+a^{10}b)} + \frac{5\,\log\left(bx+a\right)}{32\,a^6b} - \frac{5\,\log\left(bx-a\right)}{32\,a^6b}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/48*(15*b^4*x^4 + 15*a*b^3*x^3 - 25*a^2*b^2*x^2 - 25*a^3*b*x + 8*a^4)/(a
^5*b^6*x^5 + a^6*b^5*x^4 - 2*a^7*b^4*x^3 - 2*a^8*b^3*x^2 + a^9*b^2*x + a^1
0*b) + 5/32*log(b*x + a)/(a^6*b) - 5/32*log(b*x - a)/(a^6*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx = \frac{5\,\log\left(|bx+a|\right)}{32\,a^6b} - \frac{5\,\log\left(|bx-a|\right)}{32\,a^6b} - \frac{15\,ab^4x^4+15\,a^2b^3x^3-25\,a^3b^2x^2-25\,a^4bx+8\,a^5}{48\,(bx+a)^3(bx-a)^2a^6b}$$

input
```
integrate(1/(b*x+a)/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
5/32*log(abs(b*x + a))/(a^6*b) - 5/32*log(abs(b*x - a))/(a^6*b) - 1/48*(15
*a*b^4*x^4 + 15*a^2*b^3*x^3 - 25*a^3*b^2*x^2 - 25*a^4*b*x + 8*a^5)/((b*x +
 a)^3*(b*x - a)^2*a^6*b)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx=\frac{5\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{16\,a^6\,b}-\frac{\frac{1}{6\,a\,b}-\frac{25\,x}{48\,a^2}-\frac{25\,b\,x^2}{48\,a^3}+\frac{5\,b^2\,x^3}{16\,a^4}+\frac{5\,b^3\,x^4}{16\,a^5}}{a^5+a^4\,b\,x-2\,a^3\,b^2\,x^2-2\,a^2\,b^3\,x^3+a\,b^4\,x^4+b^5\,x^5}$$

input
```
int(1/((a^2 - b^2*x^2)^3*(a + b*x)),x)
```

output
```
(5*atanh((b*x)/a))/(16*a^6*b) - (1/(6*a*b) - (25*x)/(48*a^2) - (25*b*x^2)/
(48*a^3) + (5*b^2*x^3)/(16*a^4) + (5*b^3*x^4)/(16*a^5))/(a^5 + b^5*x^5 + a
*b^4*x^4 - 2*a^3*b^2*x^2 - 2*a^2*b^3*x^3 + a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+bx)\left(a^2-b^2x^2\right)^3}\,dx=\frac{-15\log(-bx+a)\,a^5-15\log(-bx+a)\,a^4bx+30\log(-bx+a)\,a^3b^2x^2+30\log(-bx+a)\,a^2b^3x^3-15\log}{}$$

input
```
int(1/(b*x+a)/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 15*log(a - b*x)*a**5 - 15*log(a - b*x)*a**4*b*x + 30*log(a - b*x)*a**3
*b**2*x**2 + 30*log(a - b*x)*a**2*b**3*x**3 - 15*log(a - b*x)*a*b**4*x**4
- 15*log(a - b*x)*b**5*x**5 + 15*log(a + b*x)*a**5 + 15*log(a + b*x)*a**4*
b*x - 30*log(a + b*x)*a**3*b**2*x**2 - 30*log(a + b*x)*a**2*b**3*x**3 + 15
*log(a + b*x)*a*b**4*x**4 + 15*log(a + b*x)*b**5*x**5 + 14*a**5 + 80*a**4*
b*x - 10*a**3*b**2*x**2 - 90*a**2*b**3*x**3 + 30*b**5*x**5)/(96*a**6*b*(a*
*5 + a**4*b*x - 2*a**3*b**2*x**2 - 2*a**2*b**3*x**3 + a*b**4*x**4 + b**5*x
**5))
```

3.30 $\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^3}\,dx = \frac{1}{64a^5b(a-bx)^2}+\frac{5}{64a^6b(a-bx)}-\frac{1}{32a^3b(a+bx)^4}-\frac{1}{16a^4b(a+bx)^3}-\frac{3}{32a^5b(a+bx)^2}-\frac{5}{32a^6b(a+bx)}+\frac{15\text{arctanh}\left(\frac{bx}{a}\right)}{64a^7b}$$

output
```
1/64/a^5/b/(-b*x+a)^2+5/64/a^6/b/(-b*x+a)-1/32/a^3/b/(b*x+a)^4-1/16/a^4/b/
(b*x+a)^3-3/32/a^5/b/(b*x+a)^2-5/32/a^6/b/(b*x+a)+15/64*arctanh(b*x/a)/a^7
/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^3}\,dx = \frac{\frac{2a\left(-16a^5+17a^4bx+50a^3b^2x^2+10a^2b^3x^3-30ab^4x^4-15b^5x^5\right)}{(a-bx)^2(a+bx)^4}-15\log(a-bx)+15\log(a+bx)}{128a^7b}$$

input
```
Integrate[1/((a + b*x)^2*(a^2 - b^2*x^2)^3),x]
```

output
```
((2*a*(-16*a^5 + 17*a^4*b*x + 50*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 30*a*b^4*x
^4 - 15*b^5*x^5))/((a - b*x)^2*(a + b*x)^4) - 15*Log[a - b*x] + 15*Log[a +
 b*x])/(128*a^7*b)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2\left(a^2-b^2x^2\right)^3}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(a-bx)^3(a+bx)^5}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{5}{64a^6(a-bx)^2}+\frac{5}{32a^6(a+bx)^2}+\frac{1}{32a^5(a-bx)^3}+\frac{3}{16a^5(a+bx)^3}+\frac{3}{16a^4(a+bx)^4}+\frac{1}{8a^3(a+bx)^5}+\frac{}{64a^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{15\text{arctanh}\left(\frac{bx}{a}\right)}{64a^7b}+\frac{5}{64a^6b(a-bx)}-\frac{5}{32a^6b(a+bx)}+\frac{1}{64a^5b(a-bx)^2}-\frac{3}{32a^5b(a+bx)^2}-\frac{1}{16a^4b(a+bx)^3}-\frac{1}{32a^3b(a+bx)^4}$$

input
```
Int[1/((a + b*x)^2*(a^2 - b^2*x^2)^3),x]
```

output
```
1/(64*a^5*b*(a - b*x)^2) + 5/(64*a^6*b*(a - b*x)) - 1/(32*a^3*b*(a + b*x)^
4) - 1/(16*a^4*b*(a + b*x)^3) - 3/(32*a^5*b*(a + b*x)^2) - 5/(32*a^6*b*(a
+ b*x)) + (15*ArcTanh[(b*x)/a])/(64*a^7*b)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.85

method	result
norman	$\frac{-\frac{1}{4ba}+\frac{25b\,x^2}{32a^3}-\frac{15b^3x^4}{32a^5}-\frac{15b^4x^5}{64a^6}+\frac{5b^2x^3}{32a^4}+\frac{17x}{64a^2}}{(bx+a)^4(-bx+a)^2}-\frac{15\ln(-bx+a)}{128a^7b}+\frac{15\ln(bx+a)}{128a^7b}$
risch	$\frac{-\frac{1}{4ba}+\frac{25b\,x^2}{32a^3}-\frac{15b^3x^4}{32a^5}-\frac{15b^4x^5}{64a^6}+\frac{5b^2x^3}{32a^4}+\frac{17x}{64a^2}}{(bx+a)^2(-b^2x^2+a^2)^2}-\frac{15\ln(-bx+a)}{128a^7b}+\frac{15\ln(bx+a)}{128a^7b}$
default	$\frac{15\ln(bx+a)}{128a^7b}-\frac{5}{32a^6b(bx+a)}-\frac{3}{32a^5b(bx+a)^2}-\frac{1}{16a^4b(bx+a)^3}-\frac{1}{32a^3b(bx+a)^4}-\frac{15\ln(-bx+a)}{128a^7b}+\frac{5}{64a^6b(-bx+a}$
parallelrisch	$-\frac{-60\ln(bx-a)x^3a^3b^8-30\ln(bx+a)x^5a\,b^{10}+30x^5a\,b^{10}+32a^6b^5-15\ln(bx-a)x^4a^2b^9+15\ln(bx+a)x^4a^2b^9+30\ln(bx-a)x^5a}{}$

input
```
int(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/b/a+25/32*b/a^3*x^2-15/32*b^3/a^5*x^4-15/64*b^4/a^6*x^5+5/32*b^2/a^4
*x^3+17/64/a^2*x)/(b*x+a)^4/(-b*x+a)^2-15/128/a^7/b*ln(-b*x+a)+15/128/a^7/
b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(110) = 220$.

Time = 0.09 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.18

$$\int \frac{1}{(a+bx)^2\,(a^2-b^2x^2)^3}\,dx = -\frac{30\,ab^5x^5+60\,a^2b^4x^4-20\,a^3b^3x^3-100\,a^4b^2x^2-34\,a^5bx+32\,a^6-15\,(b^6x^6+2\,ab^5x^5-a^2b^4x^4-4\,a^3}{128\,(a^7b^7x^6+2\,a^8b^6x^5-a^9}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="fricas")
```

output
```
-1/128*(30*a*b^5*x^5 + 60*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 100*a^4*b^2*x^2 -
 34*a^5*b*x + 32*a^6 - 15*(b^6*x^6 + 2*a*b^5*x^5 - a^2*b^4*x^4 - 4*a^3*b^3
*x^3 - a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a) + 15*(b^6*x^6 + 2*a*b^5
*x^5 - a^2*b^4*x^4 - 4*a^3*b^3*x^3 - a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*
x - a))/(a^7*b^7*x^6 + 2*a^8*b^6*x^5 - a^9*b^5*x^4 - 4*a^10*b^4*x^3 - a^11
*b^3*x^2 + 2*a^12*b^2*x + a^13*b)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx)^2\,(a^2-b^2x^2)^3}\,dx = -\frac{16a^5-17a^4bx-50a^3b^2x^2-10a^2b^3x^3+30ab^4x^4+15b^5x^5}{64a^{12}b+128a^{11}b^2x-64a^{10}b^3x^2-256a^9b^4x^3-64a^8b^5x^4+128a^7b^6x^5+64a^6b^7x^6} - \frac{\frac{15\log\left(-\frac{a}{b}+x\right)}{128}-\frac{15\log\left(\frac{a}{b}+x\right)}{128}}{a^7b}$$

input
```
integrate(1/(b*x+a)**2/(-b**2*x**2+a**2)**3,x)
```

output
```
-(16*a**5 - 17*a**4*b*x - 50*a**3*b**2*x**2 - 10*a**2*b**3*x**3 + 30*a*b**
4*x**4 + 15*b**5*x**5)/(64*a**12*b + 128*a**11*b**2*x - 64*a**10*b**3*x**2
 - 256*a**9*b**4*x**3 - 64*a**8*b**5*x**4 + 128*a**7*b**6*x**5 + 64*a**6*b
**7*x**6) - (15*log(-a/b + x)/128 - 15*log(a/b + x)/128)/(a**7*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)^3}\,dx = -\frac{15\,b^5x^5+30\,ab^4x^4-10\,a^2b^3x^3-50\,a^3b^2x^2-17\,a^4bx+16\,a^5}{64\,(a^6b^7x^6+2\,a^7b^6x^5-a^8b^5x^4-4\,a^9b^4x^3-a^{10}b^3x^2+2\,a^{11}b^2x+a^{12}b)} + \frac{15\,\log(bx+a)}{128\,a^7b} - \frac{15\,\log(bx-a)}{128\,a^7b}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="maxima")
```

output
```
-1/64*(15*b^5*x^5 + 30*a*b^4*x^4 - 10*a^2*b^3*x^3 - 50*a^3*b^2*x^2 - 17*a^
4*b*x + 16*a^5)/(a^6*b^7*x^6 + 2*a^7*b^6*x^5 - a^8*b^5*x^4 - 4*a^9*b^4*x^3
 - a^10*b^3*x^2 + 2*a^11*b^2*x + a^12*b) + 15/128*log(b*x + a)/(a^7*b) - 1
5/128*log(b*x - a)/(a^7*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+bx)^2(a^2-b^2x^2)^3}\,dx = -\frac{15\,\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{128\,a^7b} + \frac{\frac{24\,a}{bx+a}-11}{256\,a^7b\left(\frac{2\,a}{bx+a}-1\right)^2} - \frac{\frac{5\,a^6b^{11}}{bx+a}+\frac{3\,a^7b^{11}}{(bx+a)^2}+\frac{2\,a^8b^{11}}{(bx+a)^3}+\frac{a^9b^{11}}{(bx+a)^4}}{32\,a^{12}b^{12}}$$

input
```
integrate(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x, algorithm="giac")
```

output
```
-15/128*log(abs(-2*a/(b*x + a) + 1))/(a^7*b) + 1/256*(24*a/(b*x + a) - 11)
/(a^7*b*(2*a/(b*x + a) - 1)^2) - 1/32*(5*a^6*b^11/(b*x + a) + 3*a^7*b^11/(
b*x + a)^2 + 2*a^8*b^11/(b*x + a)^3 + a^9*b^11/(b*x + a)^4)/(a^12*b^12)
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+bx)^2 (a^2-b^2x^2)^3} \, dx = \frac{\frac{17\,x}{64\,a^2}-\frac{1}{4\,a\,b}+\frac{25\,b\,x^2}{32\,a^3}+\frac{5\,b^2\,x^3}{32\,a^4}-\frac{15\,b^3\,x^4}{32\,a^5}-\frac{15\,b^4\,x^5}{64\,a^6}}{a^6+2\,a^5\,b\,x-a^4\,b^2\,x^2-4\,a^3\,b^3\,x^3-a^2\,b^4\,x^4+2\,a\,b^5\,x^5+b^6\,x^6}+\frac{15\,\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{64\,a^7\,b}$$

input
```
int(1/((a^2 - b^2*x^2)^3*(a + b*x)^2),x)
```

output
```
((17*x)/(64*a^2) - 1/(4*a*b) + (25*b*x^2)/(32*a^3) + (5*b^2*x^3)/(32*a^4)
- (15*b^3*x^4)/(32*a^5) - (15*b^4*x^5)/(64*a^6))/(a^6 + b^6*x^6 + 2*a*b^5*
x^5 - a^4*b^2*x^2 - 4*a^3*b^3*x^3 - a^2*b^4*x^4 + 2*a^5*b*x) + (15*atanh((
b*x)/a))/(64*a^7*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.78

$$\int \frac{1}{(a+bx)^2 (a^2-b^2x^2)^3} \, dx = \frac{-15\log(-bx+a)\,a^6-30\log(-bx+a)\,a^5bx+15\log(-bx+a)\,a^4b^2x^2+60\log(-bx+a)\,a^3b^3x^3+15\log}{}$$

input
```
int(1/(b*x+a)^2/(-b^2*x^2+a^2)^3,x)
```

output
```
( - 15*log(a - b*x)*a**6 - 30*log(a - b*x)*a**5*b*x + 15*log(a - b*x)*a**4
*b**2*x**2 + 60*log(a - b*x)*a**3*b**3*x**3 + 15*log(a - b*x)*a**2*b**4*x*
*4 - 30*log(a - b*x)*a*b**5*x**5 - 15*log(a - b*x)*b**6*x**6 + 15*log(a +
b*x)*a**6 + 30*log(a + b*x)*a**5*b*x - 15*log(a + b*x)*a**4*b**2*x**2 - 60
*log(a + b*x)*a**3*b**3*x**3 - 15*log(a + b*x)*a**2*b**4*x**4 + 30*log(a +
 b*x)*a*b**5*x**5 + 15*log(a + b*x)*b**6*x**6 - 17*a**6 + 64*a**5*b*x + 85
*a**4*b**2*x**2 - 40*a**3*b**3*x**3 - 75*a**2*b**4*x**4 + 15*b**6*x**6)/(1
28*a**7*b*(a**6 + 2*a**5*b*x - a**4*b**2*x**2 - 4*a**3*b**3*x**3 - a**2*b*
*4*x**4 + 2*a*b**5*x**5 + b**6*x**6))
```

3.31 $\int \frac{1+ax}{\sqrt{1-a^2x^2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 28

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}} \, dx = -\frac{\sqrt{1-a^2x^2}}{a} + \frac{\arcsin(ax)}{a}$$

output
```
-(-a^2*x^2+1)^(1/2)/a+arcsin(a*x)/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}} \, dx = -\frac{\sqrt{1-a^2x^2} - 2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a}$$

input
```
Integrate[(1 + a*x)/Sqrt[1 - a^2*x^2],x]
```

output
```
-((Sqrt[1 - a^2*x^2] - 2*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2])])/a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax+1}{\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \text{455}$$

$$\int \frac{1}{\sqrt{1-a^2x^2}}dx - \frac{\sqrt{1-a^2x^2}}{a}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(ax)}{a} - \frac{\sqrt{1-a^2x^2}}{a}$$

input
```
Int[(1 + a*x)/Sqrt[1 - a^2*x^2],x]
```

output
```
-(Sqrt[1 - a^2*x^2]/a) + ArcSin[a*x]/a
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

method	result	size
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-a^2x^2+1}}{2a\sqrt{\pi}}+\frac{\arcsin(ax)}{a}$	41
default	$\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}-\frac{\sqrt{-a^2x^2+1}}{a}$	45
risch	$\frac{a^2x^2-1}{a\sqrt{-a^2x^2+1}}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}$	53

input
```
int((a*x+1)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/Pi^(1/2)*(-2*Pi^(1/2)+2*Pi^(1/2)*(-a^2*x^2+1)^(1/2))+arcsin(a*x)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx=-\frac{\sqrt{-a^2x^2+1}+2\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{a}$$

input
```
integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-a^2*x^2 + 1) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx = \begin{cases} \frac{\log\left(-2a^2x+2\sqrt{-a^2}\sqrt{-a^2x^2+1}\right)}{\sqrt{-a^2}} - \frac{\sqrt{-a^2x^2+1}}{a} & \text{for } a^2 \neq 0 \\ \frac{ax^2}{2} + x & \text{otherwise} \end{cases}$$

input
```
integrate((a*x+1)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Piecewise((log(-2*a**2*x + 2*sqrt(-a**2)*sqrt(-a**2*x**2 + 1))/sqrt(-a**2)
 - sqrt(-a**2*x**2 + 1)/a, Ne(a**2, 0)), (a*x**2/2 + x, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx = \frac{\arcsin(ax)}{a} - \frac{\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(a*x)/a - sqrt(-a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx = \frac{\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate((a*x+1)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
arcsin(a*x)*sgn(a)/abs(a) - sqrt(-a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a}$$

input
```
int((a*x + 1)/(1 - a^2*x^2)^(1/2),x)
```

output
```
asinh(x*(-a^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2)^(1/2)/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1+ax}{\sqrt{1-a^2x^2}}\,dx = \frac{asin(ax) - \sqrt{-a^2x^2+1}}{a}$$

input
```
int((a*x+1)/(-a^2*x^2+1)^(1/2),x)
```

output
```
(asin(a*x) - sqrt( - a**2*x**2 + 1))/a
```

3.32 $\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{\sqrt{1-a^2x^2}}{a}+\frac{\arcsin(ax)}{a}$$

output
```
-(-a^2*x^2+1)^(1/2)/a+arcsin(a*x)/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx = -\frac{\sqrt{1-a^2x^2}-2\arctan\left(\frac{ax}{-1+\sqrt{1-a^2x^2}}\right)}{a}$$

input
```
Integrate[Sqrt[1 - a^2*x^2]/(1 - a*x),x]
```

output
```
-((Sqrt[1 - a^2*x^2] - 2*ArcTan[(a*x)/(-1 + Sqrt[1 - a^2*x^2])])/a)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {466, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx$$

$$\Big\downarrow \text{466}$$

$$\int \frac{1}{\sqrt{1-a^2x^2}}dx-\frac{\sqrt{1-a^2x^2}}{a}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(ax)}{a}-\frac{\sqrt{1-a^2x^2}}{a}$$

input
```
Int[Sqrt[1 - a^2*x^2]/(1 - a*x),x]
```

output
```
-(Sqrt[1 - a^2*x^2]/a) + ArcSin[a*x]/a
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{a^2x^2-1}{a\sqrt{-a^2x^2+1}}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}}$	53
default	$-\frac{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}-\frac{a\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2\left(x-\frac{1}{a}\right)^2-2a\left(x-\frac{1}{a}\right)}}\right)}{\sqrt{a^2}}}{a}$	77

input
```
int((-a^2*x^2+1)^(1/2)/(-a*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a^2*x^2-1)/(-a^2*x^2+1)^(1/2)+1/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^
2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx=-\frac{\sqrt{-a^2x^2+1}+2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{a}$$

input
```
integrate((-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="fricas")
```

output
```
-(sqrt(-a^2*x^2 + 1) + 2*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a
```

Sympy [F]

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx = -\int \frac{\sqrt{-a^2x^2+1}}{ax-1}\,dx$$

input
```
integrate((-a**2*x**2+1)**(1/2)/(-a*x+1),x)
```

output
```
-Integral(sqrt(-a**2*x**2 + 1)/(a*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx = \frac{\arcsin(ax)}{a} - \frac{\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate((-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="maxima")
```

output
```
arcsin(a*x)/a - sqrt(-a^2*x^2 + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx = \frac{\arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{\sqrt{-a^2x^2+1}}{a}$$

input
```
integrate((-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="giac")
```

output
```
arcsin(a*x)*sgn(a)/abs(a) - sqrt(-a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx = \frac{\operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \frac{\sqrt{1-a^2\,x^2}}{a}$$

input
```
int(-(1 - a^2*x^2)^(1/2)/(a*x - 1),x)
```

output
```
asinh(x*(-a^2)^(1/2))/(-a^2)^(1/2) - (1 - a^2*x^2)^(1/2)/a
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx = \frac{asin(ax) - \sqrt{-a^2x^2+1}+1}{a}$$

input
```
int((-a^2*x^2+1)^(1/2)/(-a*x+1),x)
```

output
```
(asin(a*x) - sqrt( - a**2*x**2 + 1) + 1)/a
```

3.33 $\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = -\frac{(4c+dx)\sqrt{c^2-d^2x^2}}{2d}+\frac{3c^2\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{2d}$$

output
```
-1/2*(d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2*arctan(d*x/(-d^2*x^2+c^2)^(1
/2))/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = -\frac{(4c+dx)\sqrt{c^2-d^2x^2}+6c^2\arctan\left(\frac{dx}{\sqrt{c^2}-\sqrt{c^2-d^2x^2}}\right)}{2d}$$

input
```
Integrate[(c + d*x)^2/Sqrt[c^2 - d^2*x^2],x]
```

output
```
-1/2*((4*c + d*x)*Sqrt[c^2 - d^2*x^2] + 6*c^2*ArcTan[(d*x)/(Sqrt[c^2] - Sq
rt[c^2 - d^2*x^2])])/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}c\int \frac{c+dx}{\sqrt{c^2-d^2x^2}}dx-\frac{(c+dx)\sqrt{c^2-d^2x^2}}{2d}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\sqrt{c^2-d^2x^2}}dx-\frac{\sqrt{c^2-d^2x^2}}{d}\right)-\frac{(c+dx)\sqrt{c^2-d^2x^2}}{2d}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\frac{d^2x^2}{c^2-d^2x^2}+1}d\frac{x}{\sqrt{c^2-d^2x^2}}-\frac{\sqrt{c^2-d^2x^2}}{d}\right)-\frac{(c+dx)\sqrt{c^2-d^2x^2}}{2d}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}c\left(\frac{c\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d}-\frac{\sqrt{c^2-d^2x^2}}{d}\right)-\frac{(c+dx)\sqrt{c^2-d^2x^2}}{2d}$$

input
```
Int[(c + d*x)^2/Sqrt[c^2 - d^2*x^2],x]
```

output
```
-1/2*((c + d*x)*Sqrt[c^2 - d^2*x^2])/d + (3*c*(-(Sqrt[c^2 - d^2*x^2]/d) +
(c*ArcTan[(d*x)/Sqrt[c^2 - d^2*x^2]])/d))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{(dx+4c)\sqrt{-d^2x^2+c^2}}{2d}+\frac{3c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{2\sqrt{d^2}}$	60
default	$\frac{c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{\sqrt{d^2}}+d^2\left(-\frac{x\sqrt{-d^2x^2+c^2}}{2d^2}+\frac{c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{2d^2\sqrt{d^2}}\right)-\frac{2c\sqrt{-d^2x^2+c^2}}{d}$	113

input
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2/(d^2)^(1/2)*arctan((d^2)^(1/
2)*x/(-d^2*x^2+c^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = -\frac{6\,c^2\arctan\left(-\frac{c-\sqrt{-d^2x^2+c^2}}{dx}\right)+\sqrt{-d^2x^2+c^2}(dx+4\,c)}{2\,d}$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(6*c^2*arctan(-(c - sqrt(-d^2*x^2 + c^2))/(d*x)) + sqrt(-d^2*x^2 + c^
2)*(d*x + 4*c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(51) = 102.

Time = 0.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \begin{cases} \frac{3c^2\left(\begin{cases} \frac{\log\left(-2d^2x+2\sqrt{-d^2}\sqrt{c^2-d^2x^2}\right)}{\sqrt{-d^2}} & \text{for } c^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-d^2x^2}} & \text{otherwise} \end{cases}\right)}{2}+\sqrt{c^2-d^2x^2}\left(-\frac{2c}{d}-\frac{x}{2}\right) & \text{for } d^2\neq 0 \\ \frac{\begin{cases} c^2x & \text{for } d=0 \\ \frac{(c+dx)^3}{3d} & \text{otherwise} \end{cases}}{\sqrt{c^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**2/(-d**2*x**2+c**2)**(1/2),x)
```

output
```
Piecewise((3*c**2*Piecewise((log(-2*d**2*x + 2*sqrt(-d**2)*sqrt(c**2 - d**
2*x**2))/sqrt(-d**2), Ne(c**2, 0)), (x*log(x)/sqrt(-d**2*x**2), True))/2 +
 sqrt(c**2 - d**2*x**2)*(-2*c/d - x/2), Ne(d**2, 0)), (Piecewise((c**2*x,
Eq(d, 0)), ((c + d*x)**3/(3*d), True))/sqrt(c**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \frac{3\,c^2\arcsin\left(\frac{dx}{c}\right)}{2\,d} - \frac{1}{2}\,\sqrt{-d^2x^2+c^2}x - \frac{2\,\sqrt{-d^2x^2+c^2}c}{d}$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="maxima")
```

output
```
3/2*c^2*arcsin(d*x/c)/d - 1/2*sqrt(-d^2*x^2 + c^2)*x - 2*sqrt(-d^2*x^2 + c
^2)*c/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \frac{3\,c^2\arcsin\left(\frac{dx}{c}\right)\operatorname{sgn}(c)\operatorname{sgn}(d)}{2\,|d|} - \frac{1}{2}\,\sqrt{-d^2x^2+c^2}\left(x+\frac{4\,c}{d}\right)$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="giac")
```

output
```
3/2*c^2*arcsin(d*x/c)*sgn(c)*sgn(d)/abs(d) - 1/2*sqrt(-d^2*x^2 + c^2)*(x +
 4*c/d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \int \frac{{(c+d\,x)}^2}{\sqrt{c^2-d^2\,x^2}}\,dx$$

input
```
int((c + d*x)^2/(c^2 - d^2*x^2)^(1/2),x)
```

output
```
int((c + d*x)^2/(c^2 - d^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \frac{3asin\left(\frac{dx}{c}\right)c^2-4\sqrt{-d^2x^2+c^2}\,c-\sqrt{-d^2x^2+c^2}\,dx+4c^2}{2d}$$

input
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x)
```

output
```
(3*asin((d*x)/c)*c**2 - 4*sqrt(c**2 - d**2*x**2)*c - sqrt(c**2 - d**2*x**2
)*d*x + 4*c**2)/(2*d)
```

3.34 $\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 61

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2} dx = -\frac{(4c+dx)\sqrt{c^2-d^2x^2}}{2d} + \frac{3c^2 \arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{2d}$$

output
```
-1/2*(d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2*arctan(d*x/(-d^2*x^2+c^2)^(1
/2))/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2} dx = -\frac{(4c+dx)\sqrt{c^2-d^2x^2} + 6c^2 \arctan\left(\frac{dx}{\sqrt{c^2}-\sqrt{c^2-d^2x^2}}\right)}{2d}$$

input
```
Integrate[(c^2 - d^2*x^2)^(3/2)/(c - d*x)^2,x]
```

output
```
-1/2*((4*c + d*x)*Sqrt[c^2 - d^2*x^2] + 6*c^2*ArcTan[(d*x)/(Sqrt[c^2] - Sq
rt[c^2 - d^2*x^2])])/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {466, 466, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2}dx$$

$$\Big\downarrow \text{466}$$

$$\frac{3}{2}c\int \frac{\sqrt{c^2-d^2x^2}}{c-dx}dx-\frac{(c^2-d^2x^2)^{3/2}}{2d(c-dx)}$$

$$\Big\downarrow \text{466}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\sqrt{c^2-d^2x^2}}dx-\frac{\sqrt{c^2-d^2x^2}}{d}\right)-\frac{(c^2-d^2x^2)^{3/2}}{2d(c-dx)}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\frac{d^2x^2}{c^2-d^2x^2}+1}d\frac{x}{\sqrt{c^2-d^2x^2}}-\frac{\sqrt{c^2-d^2x^2}}{d}\right)-\frac{(c^2-d^2x^2)^{3/2}}{2d(c-dx)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}c\left(\frac{c\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d}-\frac{\sqrt{c^2-d^2x^2}}{d}\right)-\frac{(c^2-d^2x^2)^{3/2}}{2d(c-dx)}$$

input
```
Int[(c^2 - d^2*x^2)^(3/2)/(c - d*x)^2,x]
```

output
```
-1/2*(c^2 - d^2*x^2)^(3/2)/(d*(c - d*x)) + (3*c*(-(Sqrt[c^2 - d^2*x^2]/d)
+ (c*ArcTan[(d*x)/Sqrt[c^2 - d^2*x^2]])/d))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{(dx+4c)\sqrt{-d^{2}x^{2}+c^{2}}}{2d}+\frac{3c^{2}\arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2}x^{2}+c^{2}}}\right)}{2\sqrt{d^{2}}}$
default	$\frac{-\frac{\left(-d^{2}\left(x-\frac{c}{d}\right)^{2}-2cd\left(x-\frac{c}{d}\right)\right)^{\frac{5}{2}}}{cd\left(x-\frac{c}{d}\right)^{2}}-\frac{3d\left(\frac{\left(-d^{2}\left(x-\frac{c}{d}\right)^{2}-2cd\left(x-\frac{c}{d}\right)\right)^{\frac{3}{2}}}{3}-cd\left(-\frac{\left(-2d^{2}\left(x-\frac{c}{d}\right)-2cd\right)\sqrt{-d^{2}\left(x-\frac{c}{d}\right)^{2}-2cd\left(x-\frac{c}{d}\right)}}{4d^{2}}+\frac{c^{2}\arctan\left(\frac{}{\sqrt{}}\right.}{}\right.\right.}{c}}{d^{2}}$

input
```
int((-d^2*x^2+c^2)^(3/2)/(-d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2/(d^2)^(1/2)*arctan((d^2)^(1/
2)*x/(-d^2*x^2+c^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2}\,dx = -\frac{6\,c^2\arctan\left(-\frac{c-\sqrt{-d^2x^2+c^2}}{dx}\right)+\sqrt{-d^2x^2+c^2}(dx+4\,c)}{2\,d}$$

input
```
integrate((-d^2*x^2+c^2)^(3/2)/(-d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(6*c^2*arctan(-(c - sqrt(-d^2*x^2 + c^2))/(d*x)) + sqrt(-d^2*x^2 + c^
2)*(d*x + 4*c))/d
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2}\,dx = \int \frac{(-(-c+dx)(c+dx))^{\frac{3}{2}}}{(-c+dx)^2}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(3/2)/(-d*x+c)**2,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(3/2)/(-c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2}\,dx = \frac{3\,c^2\arcsin\left(\frac{dx}{c}\right)}{2\,d}+\frac{(-d^2x^2+c^2)^{\frac{3}{2}}}{2\,(d^2x-cd)}-\frac{3\,\sqrt{-d^2x^2+c^2}c}{2\,d}$$

input
```
integrate((-d^2*x^2+c^2)^(3/2)/(-d*x+c)^2,x, algorithm="maxima")
```

output
```
3/2*c^2*arcsin(d*x/c)/d + 1/2*(-d^2*x^2 + c^2)^(3/2)/(d^2*x - c*d) - 3/2*s
qrt(-d^2*x^2 + c^2)*c/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(53) = 106$.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.21

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2}\,dx = -\frac{\left(12\,c^3d^3\arctan\left(\sqrt{-\frac{2\,c}{dx-c}-1}\right)\operatorname{sgn}\left(\frac{1}{dx-c}\right)\operatorname{sgn}(d)-\frac{\left(5\,c^3d^3\left(-\frac{2\,c}{dx-c}-1\right)^{\frac{3}{2}}\operatorname{sgn}\left(\frac{1}{dx-c}\right)\operatorname{sgn}(d)+3\,c^3d^3\sqrt{-\frac{2\,c}{dx-c}-1}\operatorname{sgn}\left(\frac{1}{dx-\cdots}\right)\right.}{c^2}\right.}{4\,cd^5}$$

input
```
integrate((-d^2*x^2+c^2)^(3/2)/(-d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(12*c^3*d^3*arctan(sqrt(-2*c/(d*x - c) - 1))*sgn(1/(d*x - c))*sgn(d)
- (5*c^3*d^3*(-2*c/(d*x - c) - 1)^(3/2)*sgn(1/(d*x - c))*sgn(d) + 3*c^3*d^
3*sqrt(-2*c/(d*x - c) - 1)*sgn(1/(d*x - c))*sgn(d))*(d*x - c)^2/c^2)*abs(d
)/(c*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c-dx)^2}\,dx = \int \frac{{\left(c^2-d^2\,x^2\right)}^{3/2}}{{\left(c-d\,x\right)}^2}\,dx$$

input
```
int((c^2 - d^2*x^2)^(3/2)/(c - d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^(3/2)/(c - d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{(c^2 - d^2x^2)^{3/2}}{(c - dx)^2} \, dx = \frac{3asin\left(\frac{dx}{c}\right) c^2 - 4\sqrt{-d^2x^2 + c^2}\, c - \sqrt{-d^2x^2 + c^2}\, dx + 4c^2}{2d}$$

input
```
int((-d^2*x^2+c^2)^(3/2)/(-d*x+c)^2,x)
```

output
```
(3*asin((d*x)/c)*c**2 - 4*sqrt(c**2 - d**2*x**2)*c - sqrt(c**2 - d**2*x**2
)*d*x + 4*c**2)/(2*d)
```

3.35 $\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx=\frac{(4c-dx)\sqrt{c^2-d^2x^2}}{2d}+\frac{3c^2\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{2d}$$

output
```
1/2*(-d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2*arctan(d*x/(-d^2*x^2+c^2)^(1
/2))/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx=\frac{(4c-dx)\sqrt{c^2-d^2x^2}}{2d}-\frac{3c^2\arctan\left(\frac{dx}{\sqrt{c^2}-\sqrt{c^2-d^2x^2}}\right)}{d}$$

input
```
Integrate[(c - d*x)^2/Sqrt[c^2 - d^2*x^2],x]
```

output
```
((4*c - d*x)*Sqrt[c^2 - d^2*x^2])/(2*d) - (3*c^2*ArcTan[(d*x)/(Sqrt[c^2] -
 Sqrt[c^2 - d^2*x^2])])/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}c\int \frac{c-dx}{\sqrt{c^2-d^2x^2}}dx+\frac{\sqrt{c^2-d^2x^2}(c-dx)}{2d}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\sqrt{c^2-d^2x^2}}dx+\frac{\sqrt{c^2-d^2x^2}}{d}\right)+\frac{\sqrt{c^2-d^2x^2}(c-dx)}{2d}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\frac{d^2x^2}{c^2-d^2x^2}+1}d\frac{x}{\sqrt{c^2-d^2x^2}}+\frac{\sqrt{c^2-d^2x^2}}{d}\right)+\frac{\sqrt{c^2-d^2x^2}(c-dx)}{2d}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}c\left(\frac{c\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d}+\frac{\sqrt{c^2-d^2x^2}}{d}\right)+\frac{\sqrt{c^2-d^2x^2}(c-dx)}{2d}$$

input
```
Int[(c - d*x)^2/Sqrt[c^2 - d^2*x^2],x]
```

output
```
((c - d*x)*Sqrt[c^2 - d^2*x^2])/(2*d) + (3*c*(Sqrt[c^2 - d^2*x^2]/d + (c*A
rcTan[(d*x)/Sqrt[c^2 - d^2*x^2]])/d))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{(-dx+4c)\sqrt{-d^2x^2+c^2}}{2d}+\frac{3c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{2\sqrt{d^2}}$	61
default	$\frac{c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{\sqrt{d^2}}+d^2\left(-\frac{x\sqrt{-d^2x^2+c^2}}{2d^2}+\frac{c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{2d^2\sqrt{d^2}}\right)+\frac{2c\sqrt{-d^2x^2+c^2}}{d}$	113

input
```
int((-d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2/(d^2)^(1/2)*arctan((d^2)^(1/
2)*x/(-d^2*x^2+c^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = -\frac{6\,c^2\arctan\left(-\frac{c-\sqrt{-d^2x^2+c^2}}{dx}\right)+\sqrt{-d^2x^2+c^2}(dx-4\,c)}{2\,d}$$

input
```
integrate((-d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(6*c^2*arctan(-(c - sqrt(-d^2*x^2 + c^2))/(d*x)) + sqrt(-d^2*x^2 + c^
2)*(d*x - 4*c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \begin{cases} \frac{3c^2\left(\begin{cases} \frac{\log\left(-2d^2x+2\sqrt{-d^2}\sqrt{c^2-d^2x^2}\right)}{\sqrt{-d^2}} & \text{for}\ c^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-d^2x^2}} & \text{otherwise} \end{cases}\right)}{2}+\sqrt{c^2-d^2x^2}\cdot\left(\frac{2c}{d}-\frac{x}{2}\right) & \text{for}\ d^2\neq 0 \\ \frac{\begin{cases} c^2x & \text{for}\ d=0 \\ -\frac{(c-dx)^3}{3d} & \text{otherwise} \end{cases}}{\sqrt{c^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((-d*x+c)**2/(-d**2*x**2+c**2)**(1/2),x)
```

output
```
Piecewise((3*c**2*Piecewise((log(-2*d**2*x + 2*sqrt(-d**2)*sqrt(c**2 - d**
2*x**2))/sqrt(-d**2), Ne(c**2, 0)), (x*log(x)/sqrt(-d**2*x**2), True))/2 +
 sqrt(c**2 - d**2*x**2)*(2*c/d - x/2), Ne(d**2, 0)), (Piecewise((c**2*x, E
q(d, 0)), (-(c - d*x)**3/(3*d), True))/sqrt(c**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \frac{3\,c^2\arcsin\left(\frac{dx}{c}\right)}{2\,d} - \frac{1}{2}\sqrt{-d^2x^2+c^2}x + \frac{2\sqrt{-d^2x^2+c^2}c}{d}$$

input
```
integrate((-d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="maxima")
```

output
```
3/2*c^2*arcsin(d*x/c)/d - 1/2*sqrt(-d^2*x^2 + c^2)*x + 2*sqrt(-d^2*x^2 + c
^2)*c/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \frac{3\,c^2\arcsin\left(\frac{dx}{c}\right)\operatorname{sgn}(c)\operatorname{sgn}(d)}{2\,|d|} - \frac{1}{2}\sqrt{-d^2x^2+c^2}\left(x-\frac{4\,c}{d}\right)$$

input
```
integrate((-d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="giac")
```

output
```
3/2*c^2*arcsin(d*x/c)*sgn(c)*sgn(d)/abs(d) - 1/2*sqrt(-d^2*x^2 + c^2)*(x -
 4*c/d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \int \frac{(c-d\,x)^2}{\sqrt{c^2-d^2\,x^2}}\,dx$$

input
```
int((c - d*x)^2/(c^2 - d^2*x^2)^(1/2),x)
```

output
```
int((c - d*x)^2/(c^2 - d^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{(c-dx)^2}{\sqrt{c^2-d^2x^2}}\,dx = \frac{3asin\left(\frac{dx}{c}\right)c^2+4\sqrt{-d^2x^2+c^2}\,c-\sqrt{-d^2x^2+c^2}\,dx-4c^2}{2d}$$

input
```
int((-d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x)
```

output
```
(3*asin((d*x)/c)*c**2 + 4*sqrt(c**2 - d**2*x**2)*c - sqrt(c**2 - d**2*x**2
)*d*x - 4*c**2)/(2*d)
```

3.36 $\int \frac{\left(c^2-d^2x^2\right)^{3/2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\left(c^2-d^2x^2\right)^{3/2}}{(c+dx)^2}\,dx=\frac{(4c-dx)\sqrt{c^2-d^2x^2}}{2d}+\frac{3c^2\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{2d}$$

output
```
1/2*(-d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2*arctan(d*x/(-d^2*x^2+c^2)^(1
/2))/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \frac{\left(c^2-d^2x^2\right)^{3/2}}{(c+dx)^2}\,dx=\frac{(4c-dx)\sqrt{c^2-d^2x^2}}{2d}-\frac{3c^2\arctan\left(\frac{dx}{\sqrt{c^2}-\sqrt{c^2-d^2x^2}}\right)}{d}$$

input
```
Integrate[(c^2 - d^2*x^2)^(3/2)/(c + d*x)^2,x]
```

output
```
((4*c - d*x)*Sqrt[c^2 - d^2*x^2])/(2*d) - (3*c^2*ArcTan[(d*x)/(Sqrt[c^2] -
 Sqrt[c^2 - d^2*x^2])])/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {466, 466, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2-d^2x^2\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{466}$$

$$\frac{3}{2}c\int \frac{\sqrt{c^2-d^2x^2}}{c+dx}dx+\frac{\left(c^2-d^2x^2\right)^{3/2}}{2d(c+dx)}$$

$$\Big\downarrow \text{466}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\sqrt{c^2-d^2x^2}}dx+\frac{\sqrt{c^2-d^2x^2}}{d}\right)+\frac{\left(c^2-d^2x^2\right)^{3/2}}{2d(c+dx)}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\frac{d^2x^2}{c^2-d^2x^2}+1}d\frac{x}{\sqrt{c^2-d^2x^2}}+\frac{\sqrt{c^2-d^2x^2}}{d}\right)+\frac{\left(c^2-d^2x^2\right)^{3/2}}{2d(c+dx)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}c\left(\frac{c\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d}+\frac{\sqrt{c^2-d^2x^2}}{d}\right)+\frac{\left(c^2-d^2x^2\right)^{3/2}}{2d(c+dx)}$$

input
```
Int[(c^2 - d^2*x^2)^(3/2)/(c + d*x)^2,x]
```

output
```
(c^2 - d^2*x^2)^(3/2)/(2*d*(c + d*x)) + (3*c*(Sqrt[c^2 - d^2*x^2]/d + (c*A
rcTan[(d*x)/Sqrt[c^2 - d^2*x^2]])/d))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(-dx+4c)\sqrt{-d^2x^2+c^2}}{2d}+\frac{3c^2\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{2\sqrt{d^2}}$
default	$\frac{\frac{\left(-d^2\left(x+\frac{c}{d}\right)^2+2cd\left(x+\frac{c}{d}\right)\right)^{\frac{5}{2}}}{cd\left(x+\frac{c}{d}\right)^2}+\frac{3d\left(\frac{\left(-d^2\left(x+\frac{c}{d}\right)^2+2cd\left(x+\frac{c}{d}\right)\right)^{\frac{3}{2}}}{3}+cd\left(-\frac{\left(-2d^2\left(x+\frac{c}{d}\right)+2cd\right)\sqrt{-d^2\left(x+\frac{c}{d}\right)^2+2cd\left(x+\frac{c}{d}\right)}}{4d^2}+\frac{c^2\arctan\left(\frac{\ldots}{\sqrt{-d\ldots}}\right.}{\ldots}\right.\right.}{c}}{d^2}$

input
```
int((-d^2*x^2+c^2)^(3/2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-d*x+4*c)*(-d^2*x^2+c^2)^(1/2)/d+3/2*c^2/(d^2)^(1/2)*arctan((d^2)^(1/
2)*x/(-d^2*x^2+c^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c+dx)^2}\,dx = -\frac{6\,c^2\arctan\left(-\frac{c-\sqrt{-d^2x^2+c^2}}{dx}\right)+\sqrt{-d^2x^2+c^2}(dx-4\,c)}{2\,d}$$

input
```
integrate((-d^2*x^2+c^2)^(3/2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(6*c^2*arctan(-(c - sqrt(-d^2*x^2 + c^2))/(d*x)) + sqrt(-d^2*x^2 + c^
2)*(d*x - 4*c))/d
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{(-(-c+dx)\,(c+dx))^{\frac{3}{2}}}{(c+dx)^2}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(3/2)/(d*x+c)**2,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(3/2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c+dx)^2}\,dx = \frac{3\,c^2\arcsin\left(\frac{dx}{c}\right)}{2\,d}+\frac{(-d^2x^2+c^2)^{\frac{3}{2}}}{2\,(d^2x+cd)}+\frac{3\,\sqrt{-d^2x^2+c^2}c}{2\,d}$$

input
```
integrate((-d^2*x^2+c^2)^(3/2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
3/2*c^2*arcsin(d*x/c)/d + 1/2*(-d^2*x^2 + c^2)^(3/2)/(d^2*x + c*d) + 3/2*s
qrt(-d^2*x^2 + c^2)*c/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(53) = 106$.

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c+dx)^2}\,dx = -\frac{\left(12\,c^3d^3\arctan\left(\sqrt{\frac{2\,c}{dx+c}-1}\right)\operatorname{sgn}\left(\frac{1}{dx+c}\right)\operatorname{sgn}(d) - \frac{\left(5\,c^3d^3\left(\frac{2\,c}{dx+c}-1\right)^{\frac{3}{2}}\operatorname{sgn}\left(\frac{1}{dx+c}\right)\operatorname{sgn}(d)+3\,c^3d^3\sqrt{\frac{2\,c}{dx+c}-1}\operatorname{sgn}\left(\frac{1}{dx+c}\right)\operatorname{sgn}(\cdots\right.}{c^2}\cdots\right.}{4\,cd^5}$$

input
```
integrate((-d^2*x^2+c^2)^(3/2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(12*c^3*d^3*arctan(sqrt(2*c/(d*x + c) - 1))*sgn(1/(d*x + c))*sgn(d) -
 (5*c^3*d^3*(2*c/(d*x + c) - 1)^(3/2)*sgn(1/(d*x + c))*sgn(d) + 3*c^3*d^3*
sqrt(2*c/(d*x + c) - 1)*sgn(1/(d*x + c))*sgn(d))*(d*x + c)^2/c^2)*abs(d)/(
c*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{3/2}}{(c+dx)^2}\,dx = \int \frac{\left(c^2-d^2\,x^2\right)^{3/2}}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((c^2 - d^2*x^2)^(3/2)/(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^(3/2)/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{(c^2 - d^2x^2)^{3/2}}{(c + dx)^2}\, dx = \frac{3asin\left(\frac{dx}{c}\right) c^2 + 4\sqrt{-d^2x^2 + c^2}\, c - \sqrt{-d^2x^2 + c^2}\, dx - 4c^2}{2d}$$

input

```
int((-d^2*x^2+c^2)^(3/2)/(d*x+c)^2,x)
```

output

```
(3*asin((d*x)/c)*c**2 + 4*sqrt(c**2 - d**2*x**2)*c - sqrt(c**2 - d**2*x**2
)*d*x - 4*c**2)/(2*d)
```

3.37 $\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 80

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \frac{(4c+dx)\sqrt{-bc^2+bd^2x^2}}{2bd} + \frac{3c^2\text{arctanh}\left(\frac{\sqrt{b}dx}{\sqrt{-bc^2+bd^2x^2}}\right)}{2\sqrt{b}d}$$

output
```
1/2*(d*x+4*c)*(b*d^2*x^2-b*c^2)^(1/2)/b/d+3/2*c^2*arctanh(b^(1/2)*d*x/(b*d
^2*x^2-b*c^2)^(1/2))/b^(1/2)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(80) = 160.

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.10

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \frac{-4c^3-c^2dx+4cd^2x^2+d^3x^3+3c^2\sqrt{-c^2+d^2x^2}\log\left(\sqrt{-c^2}-dx-\sqrt{-c^2+d^2x^2}\right)-3c^2\sqrt{-c^2+d^2x^2}\,\text{lo}}{2d\sqrt{b\left(-c^2+d^2x^2\right)}}$$

input
```
Integrate[(c + d*x)^2/Sqrt[-(b*c^2) + b*d^2*x^2],x]
```

output
```
(-4*c^3 - c^2*d*x + 4*c*d^2*x^2 + d^3*x^3 + 3*c^2*Sqrt[-c^2 + d^2*x^2]*Log
[Sqrt[-c^2] - d*x - Sqrt[-c^2 + d^2*x^2]] - 3*c^2*Sqrt[-c^2 + d^2*x^2]*Log
[d*(Sqrt[-c^2] + d*x - Sqrt[-c^2 + d^2*x^2])])/(2*d*Sqrt[b*(-c^2 + d^2*x^2
)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {469, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt{bd^2x^2-bc^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}c\int \frac{c+dx}{\sqrt{bd^2x^2-bc^2}}dx+\frac{(c+dx)\sqrt{bd^2x^2-bc^2}}{2bd}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}c\left(c\int \frac{1}{\sqrt{bd^2x^2-bc^2}}dx+\frac{\sqrt{bd^2x^2-bc^2}}{bd}\right)+\frac{(c+dx)\sqrt{bd^2x^2-bc^2}}{2bd}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}c\left(c\int \frac{1}{1-\frac{bd^2x^2}{bd^2x^2-bc^2}}d\frac{x}{\sqrt{bd^2x^2-bc^2}}+\frac{\sqrt{bd^2x^2-bc^2}}{bd}\right)+\frac{(c+dx)\sqrt{bd^2x^2-bc^2}}{2bd}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{b}dx}{\sqrt{bd^2x^2-bc^2}}\right)}{\sqrt{b}d}+\frac{\sqrt{bd^2x^2-bc^2}}{bd}\right)+\frac{(c+dx)\sqrt{bd^2x^2-bc^2}}{2bd}$$

input
```
Int[(c + d*x)^2/Sqrt[-(b*c^2) + b*d^2*x^2],x]
```

output
```
((c + d*x)*Sqrt[-(b*c^2) + b*d^2*x^2])/(2*b*d) + (3*c*(Sqrt[-(b*c^2) + b*d
^2*x^2]/(b*d) + (c*ArcTanh[(Sqrt[b]*d*x)/Sqrt[-(b*c^2) + b*d^2*x^2]])/(Sqr
t[b]*d)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

method	result	size
risch	$-\frac{(dx+4c)\left(-d^2x^2+c^2\right)}{2d\sqrt{b\left(d^2x^2-c^2\right)}}+\frac{3c^2\ln\left(\frac{bx\,d^2}{\sqrt{b\,d^2}}+\sqrt{b\,x^2d^2-b\,c^2}\right)}{2\sqrt{b\,d^2}}$	87
default	$\frac{c^2\ln\left(\frac{bx\,d^2}{\sqrt{b\,d^2}}+\sqrt{b\,x^2d^2-b\,c^2}\right)}{\sqrt{b\,d^2}}+d^2\left(\frac{x\sqrt{b\,x^2d^2-b\,c^2}}{2b\,d^2}+\frac{c^2\ln\left(\frac{bx\,d^2}{\sqrt{b\,d^2}}+\sqrt{b\,x^2d^2-b\,c^2}\right)}{2d^2\sqrt{b\,d^2}}\right)+\frac{2c\sqrt{b\,x^2d^2-b\,c^2}}{db}$	149

input
```
int((d*x+c)^2/(b*d^2*x^2-b*c^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x+4*c)*(-d^2*x^2+c^2)/d/(b*(d^2*x^2-c^2))^(1/2)+3/2*c^2*ln(b*x*d^2
/(b*d^2)^(1/2)+(b*d^2*x^2-b*c^2)^(1/2))/(b*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \left[\frac{3\,\sqrt{b}c^2\log\left(2\,bd^2x^2-bc^2+2\,\sqrt{bd^2x^2-bc^2}\sqrt{b}dx\right)+2\,\sqrt{bd^2x^2-bc^2}(dx+4\,c)}{4\,bd}, -\frac{3\,\sqrt{-b}c^2\arctan\left(\frac{\sqrt{-b}dx}{\sqrt{bd^2x^2-bc^2}}\right)-\sqrt{bd^2x^2-bc^2}(dx+4\,c)}{2\,bd}\right]$$

input
```
integrate((d*x+c)^2/(b*d^2*x^2-b*c^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(3*sqrt(b)*c^2*log(2*b*d^2*x^2 - b*c^2 + 2*sqrt(b*d^2*x^2 - b*c^2)*sq
rt(b)*d*x) + 2*sqrt(b*d^2*x^2 - b*c^2)*(d*x + 4*c))/(b*d), -1/2*(3*sqrt(-b
)*c^2*arctan(sqrt(-b)*d*x/sqrt(b*d^2*x^2 - b*c^2)) - sqrt(b*d^2*x^2 - b*c^
2)*(d*x + 4*c))/(b*d)]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \begin{cases} \frac{3c^2\left(\begin{cases} \frac{\log\left(2bd^2x+2\sqrt{bd^2}\sqrt{-bc^2+bd^2x^2}\right)}{\sqrt{bd^2}} & \text{for } bc^2\neq 0 \\ \frac{x\log(x)}{\sqrt{bd^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \sqrt{-bc^2+bd^2x^2}\cdot\left(\frac{2c}{bd}+\frac{x}{2b}\right) & \text{for } bd^2\neq 0 \\ \frac{\begin{cases} c^2x & \text{for } d=0 \\ \frac{(c+dx)^3}{3d} & \text{otherwise} \end{cases}}{\sqrt{-bc^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**2/(b*d**2*x**2-b*c**2)**(1/2),x)
```

output
```
Piecewise((3*c**2*Piecewise((log(2*b*d**2*x + 2*sqrt(b*d**2)*sqrt(-b*c**2
+ b*d**2*x**2))/sqrt(b*d**2), Ne(b*c**2, 0)), (x*log(x)/sqrt(b*d**2*x**2),
 True))/2 + sqrt(-b*c**2 + b*d**2*x**2)*(2*c/(b*d) + x/(2*b)), Ne(b*d**2,
0)), (Piecewise((c**2*x, Eq(d, 0)), ((c + d*x)**3/(3*d), True))/sqrt(-b*c*
*2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \frac{3\,c^2\log\left(2\,bd^2x+2\,\sqrt{bd^2x^2-bc^2}\sqrt{b}d\right)}{2\,\sqrt{b}d} + \frac{\sqrt{bd^2x^2-bc^2}x}{2\,b} + \frac{2\,\sqrt{bd^2x^2-bc^2}c}{bd}$$

input
```
integrate((d*x+c)^2/(b*d^2*x^2-b*c^2)^(1/2),x, algorithm="maxima")
```

output
```
3/2*c^2*log(2*b*d^2*x + 2*sqrt(b*d^2*x^2 - b*c^2)*sqrt(b)*d)/(sqrt(b)*d) +
 1/2*sqrt(b*d^2*x^2 - b*c^2)*x/b + 2*sqrt(b*d^2*x^2 - b*c^2)*c/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \frac{1}{2}\sqrt{bd^2x^2-bc^2}\left(\frac{x}{b}+\frac{4\,c}{bd}\right)-\frac{3\,c^2\log\left(\left|-\sqrt{bd^2}x+\sqrt{bd^2x^2-bc^2}\right|\right)}{2\,\sqrt{b}|d|}$$

input
```
integrate((d*x+c)^2/(b*d^2*x^2-b*c^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*d^2*x^2 - b*c^2)*(x/b + 4*c/(b*d)) - 3/2*c^2*log(abs(-sqrt(b*d^
2)*x + sqrt(b*d^2*x^2 - b*c^2)))/(sqrt(b)*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \int \frac{(c+d\,x)^2}{\sqrt{b\,d^2\,x^2-b\,c^2}}\,dx$$

input
```
int((c + d*x)^2/(b*d^2*x^2 - b*c^2)^(1/2),x)
```

output
```
int((c + d*x)^2/(b*d^2*x^2 - b*c^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2}{\sqrt{-bc^2+bd^2x^2}}\,dx = \frac{\sqrt{b}\left(4\sqrt{d^2x^2-c^2}\,c+\sqrt{d^2x^2-c^2}\,dx+3\log\left(\frac{\sqrt{d^2x^2-c^2}+dx}{c}\right)c^2\right)}{2bd}$$

input
```
int((d*x+c)^2/(b*d^2*x^2-b*c^2)^(1/2),x)
```

output
```
(sqrt(b)*(4*sqrt( - c**2 + d**2*x**2)*c + sqrt( - c**2 + d**2*x**2)*d*x +
3*log((sqrt( - c**2 + d**2*x**2) + d*x)/c)*c**2))/(2*b*d)
```

3.38 $\int \frac{\left(-bc^2+bd^2x^2\right)^{3/2}}{b^2(c-dx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{\left(-bc^2+bd^2x^2\right)^{3/2}}{b^2(c-dx)^2}\,dx=\frac{(4c+dx)\sqrt{-bc^2+bd^2x^2}}{2bd}+\frac{3c^2\text{arctanh}\left(\frac{\sqrt{b}dx}{\sqrt{-bc^2+bd^2x^2}}\right)}{2\sqrt{b}d}$$

output
```
1/2*(d*x+4*c)*(b*d^2*x^2-b*c^2)^(1/2)/b/d+3/2*c^2*arctanh(b^(1/2)*d*x/(b*d
^2*x^2-b*c^2)^(1/2))/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.89

$$\int \frac{\left(-bc^2+bd^2x^2\right)^{3/2}}{b^2(c-dx)^2}\,dx=\frac{\left(b\left(-c^2+d^2x^2\right)\right)^{3/2}\left((4c+dx)\sqrt{-c^2+d^2x^2}+3c^2\log\left(\sqrt{-c^2}-dx-\sqrt{-c^2+d^2}\right.\right.}{2b^2d\left(-c^2+d^2x^2\right)^{3/2}}$$

input
```
Integrate[(-(b*c^2) + b*d^2*x^2)^(3/2)/(b^2*(c - d*x)^2),x]
```

output
```
((b*(-c^2 + d^2*x^2))^(3/2)*((4*c + d*x)*Sqrt[-c^2 + d^2*x^2] + 3*c^2*Log[
Sqrt[-c^2] - d*x - Sqrt[-c^2 + d^2*x^2]] - 3*c^2*Log[d*(Sqrt[-c^2] + d*x -
 Sqrt[-c^2 + d^2*x^2])]))/(2*b^2*d*(-c^2 + d^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {27, 466, 466, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bd^2x^2-bc^2\right)^{3/2}}{b^2(c-dx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bd^2x^2-bc^2\right)^{3/2}}{(c-dx)^2}dx}{b^2}$$

$$\Big\downarrow \text{466}$$

$$\frac{-\frac{3}{2}bc\int \frac{\sqrt{bd^2x^2-bc^2}}{c-dx}dx-\frac{\left(bd^2x^2-bc^2\right)^{3/2}}{2d(c-dx)}}{b^2}$$

$$\Big\downarrow \text{466}$$

$$\frac{-\frac{3}{2}bc\left(-bc\int \frac{1}{\sqrt{bd^2x^2-bc^2}}dx-\frac{\sqrt{bd^2x^2-bc^2}}{d}\right)-\frac{\left(bd^2x^2-bc^2\right)^{3/2}}{2d(c-dx)}}{b^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{-\frac{3}{2}bc\left(-bc\int \frac{1}{1-\frac{bd^2x^2}{bd^2x^2-bc^2}}d\frac{x}{\sqrt{bd^2x^2-bc^2}}-\frac{\sqrt{bd^2x^2-bc^2}}{d}\right)-\frac{\left(bd^2x^2-bc^2\right)^{3/2}}{2d(c-dx)}}{b^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{3}{2}bc\left(-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}dx}{\sqrt{bd^2x^2-bc^2}}\right)}{d}-\frac{\sqrt{bd^2x^2-bc^2}}{d}\right)-\frac{\left(bd^2x^2-bc^2\right)^{3/2}}{2d(c-dx)}}{b^2}$$

input
```
Int[(-(b*c^2) + b*d^2*x^2)^(3/2)/(b^2*(c - d*x)^2),x]
```

output
```
(-1/2*(-(b*c^2) + b*d^2*x^2)^(3/2)/(d*(c - d*x)) - (3*b*c*(-(Sqrt[-(b*c^2)
 + b*d^2*x^2]/d) - (Sqrt[b]*c*ArcTanh[(Sqrt[b]*d*x)/Sqrt[-(b*c^2) + b*d^2*
x^2]])/d))/2)/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{(dx+4c)\left(-d^2x^2+c^2\right)}{2d\sqrt{b(d^2x^2-c^2)}}+\frac{3c^2\ln\left(\frac{bx\,d^2}{\sqrt{b\,d^2}}+\sqrt{b\,x^2d^2-b\,c^2}\right)}{2\sqrt{b\,d^2}}$
default	$\frac{\frac{\left(b\,d^2\left(x-\frac{c}{d}\right)^2+2bcd\left(x-\frac{c}{d}\right)\right)^{\frac{5}{2}}}{bcd\left(x-\frac{c}{d}\right)^2}-\frac{3d\left(\frac{\left(b\,d^2\left(x-\frac{c}{d}\right)^2+2bcd\left(x-\frac{c}{d}\right)\right)^{\frac{3}{2}}}{3}+bcd\left(\frac{\left(2b\,d^2\left(x-\frac{c}{d}\right)+2dbc\right)\sqrt{b\,d^2\left(x-\frac{c}{d}\right)^2+2bcd\left(x-\frac{c}{d}\right)}}{4b\,d^2}-\frac{b\,c^2\ln\left(\frac{b\,d^2\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.}{c}}{b^2d^2}$

input
```
int((b*d^2*x^2-b*c^2)^(3/2)/b^2/(-d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x+4*c)*(-d^2*x^2+c^2)/d/(b*(d^2*x^2-c^2))^(1/2)+3/2*c^2*ln(b*x*d^2
/(b*d^2)^(1/2)+(b*d^2*x^2-b*c^2)^(1/2))/(b*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.95

$$\int \frac{(-bc^2+bd^2x^2)^{3/2}}{b^2(c-dx)^2}\,dx = \left[\frac{3\sqrt{b}c^2\log\left(2\,bd^2x^2-bc^2+2\sqrt{bd^2x^2-bc^2}\sqrt{b}dx\right)+2\sqrt{bd^2x^2-bc^2}(dx+4\,c)}{4\,bd}, -\frac{3\sqrt{-b}c^2\arctan\left(\frac{\sqrt{-b}dx}{\sqrt{bd^2x^2-bc^2}}\right)-\sqrt{bd^2x^2-bc^2}(dx+4\,c)}{2\,bd}\right]$$

input
```
integrate((b*d^2*x^2-b*c^2)^(3/2)/b^2/(-d*x+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(3*sqrt(b)*c^2*log(2*b*d^2*x^2 - b*c^2 + 2*sqrt(b*d^2*x^2 - b*c^2)*sq
rt(b)*d*x) + 2*sqrt(b*d^2*x^2 - b*c^2)*(d*x + 4*c))/(b*d), -1/2*(3*sqrt(-b
)*c^2*arctan(sqrt(-b)*d*x/sqrt(b*d^2*x^2 - b*c^2)) - sqrt(b*d^2*x^2 - b*c^
2)*(d*x + 4*c))/(b*d)]
```

Sympy [F]

$$\int \frac{(-bc^2+bd^2x^2)^{3/2}}{b^2(c-dx)^2}\,dx = \frac{\int\left(-\frac{bc^2\sqrt{-bc^2+bd^2x^2}}{c^2-2cdx+d^2x^2}\right)dx+\int\frac{bd^2x^2\sqrt{-bc^2+bd^2x^2}}{c^2-2cdx+d^2x^2}\,dx}{b^2}$$

input
```
integrate((b*d**2*x**2-b*c**2)**(3/2)/b**2/(-d*x+c)**2,x)
```

output
```
(Integral(-b*c**2*sqrt(-b*c**2 + b*d**2*x**2)/(c**2 - 2*c*d*x + d**2*x**2)
, x) + Integral(b*d**2*x**2*sqrt(-b*c**2 + b*d**2*x**2)/(c**2 - 2*c*d*x +
d**2*x**2), x))/b**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{(-bc^2+bd^2x^2)^{3/2}}{b^2(c-dx)^2}\,dx = \frac{\frac{3\,b^{\frac{3}{2}}c^2\log\left(2\,bdx+2\,\sqrt{bd^2x^2-bc^2}\sqrt{b}\right)}{d}+\frac{3\,\sqrt{bd^2x^2-bc^2}bc}{d}+\frac{(bd^2x^2-bc^2)^{\frac{3}{2}}}{d^2x-cd}}{2\,b^2}$$

input

```
integrate((b*d^2*x^2-b*c^2)^(3/2)/b^2/(-d*x+c)^2,x, algorithm="maxima")
```

output

```
1/2*(3*b^(3/2)*c^2*log(2*b*d*x + 2*sqrt(b*d^2*x^2 - b*c^2)*sqrt(b))/d + 3*
sqrt(b*d^2*x^2 - b*c^2)*b*c/d + (b*d^2*x^2 - b*c^2)^(3/2)/(d^2*x - c*d))/b
^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(68) = 136$.

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

$$\int \frac{(-bc^2+bd^2x^2)^{3/2}}{b^2(c-dx)^2}\,dx =$$

$$-\frac{\left(\frac{12\,b^2c^3d^3\arctan\left(\frac{\sqrt{b+\frac{2\,bc}{dx-c}}}{\sqrt{-b}}\right)\operatorname{sgn}\left(\frac{1}{dx-c}\right)\operatorname{sgn}(d)}{\sqrt{-b}}-\frac{\left(5\left(b+\frac{2\,bc}{dx-c}\right)^{\frac{3}{2}}b^2c^3d^3\operatorname{sgn}\left(\frac{1}{dx-c}\right)\operatorname{sgn}(d)-3\,\sqrt{b+\frac{2\,bc}{dx-c}}b^3c^3d^3\operatorname{sgn}\left(\frac{1}{dx-c}\right)\operatorname{sgn}(d)\right)(dx-c)^2}{b^2c^2}\right)|d|}{4\,b^2cd^5}$$

input

```
integrate((b*d^2*x^2-b*c^2)^(3/2)/b^2/(-d*x+c)^2,x, algorithm="giac")
```

output

```
-1/4*(12*b^2*c^3*d^3*arctan(sqrt(b + 2*b*c/(d*x - c))/sqrt(-b))*sgn(1/(d*x
 - c))*sgn(d)/sqrt(-b) - (5*(b + 2*b*c/(d*x - c))^(3/2)*b^2*c^3*d^3*sgn(1/
(d*x - c))*sgn(d) - 3*sqrt(b + 2*b*c/(d*x - c))*b^3*c^3*d^3*sgn(1/(d*x - c
))*sgn(d))*(d*x - c)^2/(b^2*c^2))*abs(d)/(b^2*c*d^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-bc^2+bd^2x^2)^{3/2}}{b^2(c-dx)^2}\,dx = \int \frac{\left(b\,d^2\,x^2-b\,c^2\right)^{3/2}}{b^2\,\left(c-d\,x\right)^2}\,dx$$

input
```
int((b*d^2*x^2 - b*c^2)^(3/2)/(b^2*(c - d*x)^2),x)
```

output
```
int((b*d^2*x^2 - b*c^2)^(3/2)/(b^2*(c - d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{(-bc^2+bd^2x^2)^{3/2}}{b^2(c-dx)^2}\,dx = \frac{\sqrt{b}\left(4\sqrt{d^2x^2-c^2}\,c+\sqrt{d^2x^2-c^2}\,dx+3\log\left(\frac{\sqrt{d^2x^2-c^2}+dx}{c}\right)c^2\right)}{2bd}$$

input
```
int((b*d^2*x^2-b*c^2)^(3/2)/b^2/(-d*x+c)^2,x)
```

output
```
(sqrt(b)*(4*sqrt( - c**2 + d**2*x**2)*c + sqrt( - c**2 + d**2*x**2)*d*x +
3*log((sqrt( - c**2 + d**2*x**2) + d*x)/c)*c**2))/(2*b*d)
```

3.39 $\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 150

$$\begin{aligned}\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx &= \frac{21}{16}a^4x\sqrt{a^2-b^2x^2}-\frac{3a(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{10b}\\ &\quad-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}\\ &\quad-\frac{7a^2(8a+3bx)\left(a^2-b^2x^2\right)^{3/2}}{40b}+\frac{21a^6\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{16b}\end{aligned}$$

output
```
21/16*a^4*x*(-b^2*x^2+a^2)^(1/2)-3/10*a*(b*x+a)^2*(-b^2*x^2+a^2)^(3/2)/b-1
/6*(b*x+a)^3*(-b^2*x^2+a^2)^(3/2)/b-7/40*a^2*(3*b*x+8*a)*(-b^2*x^2+a^2)^(3
/2)/b+21/16*a^6*arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.76

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx$$

$$=\frac{\sqrt{a^2-b^2x^2}(-448a^5-75a^4bx+256a^3b^2x^2+350a^2b^3x^3+192ab^4x^4+40b^5x^5)-630a^6\arctan\left(\frac{bx}{\sqrt{a^2-\sqrt{a^2\cdots}}}\right.}{240b}$$

input
```
Integrate[(a + b*x)^4*Sqrt[a^2 - b^2*x^2],x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-448*a^5 - 75*a^4*b*x + 256*a^3*b^2*x^2 + 350*a^2*b^
3*x^3 + 192*a*b^4*x^4 + 40*b^5*x^5) - 630*a^6*ArcTan[(b*x)/(Sqrt[a^2] - Sq
rt[a^2 - b^2*x^2])])/(240*b)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {469, 469, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}a\int (a+bx)^3\sqrt{a^2-b^2x^2}dx-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\int (a+bx)^2\sqrt{a^2-b^2x^2}dx-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}\right)-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\int (a+bx)\sqrt{a^2-b^2x^2}dx-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}\right)-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\int\sqrt{a^2-b^2x^2}dx-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}\right)-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}a^2\int\frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}\right)-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}a^2\int\frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}\right)-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{3}{2}a\left(\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}\right)-\frac{(a+bx)^3\left(a^2-b^2x^2\right)^{3/2}}{6b}$$

input
```
Int[(a + b*x)^4*Sqrt[a^2 - b^2*x^2],x]
```

output
```
-1/6*((a + b*x)^3*(a^2 - b^2*x^2)^(3/2))/b + (3*a*(-1/5*((a + b*x)^2*(a^2
- b^2*x^2)^(3/2))/b + (7*a*(-1/4*((a + b*x)*(a^2 - b^2*x^2)^(3/2))/b + (5*
a*(-1/3*(a^2 - b^2*x^2)^(3/2)/b + a*((x*Sqrt[a^2 - b^2*x^2])/2 + (a^2*ArcT
an[(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b))))/4))/5))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\left(-40 b^{5} x^{5}-192 a\,b^{4} x^{4}-350 a^{2} b^{3} x^{3}-256 a^{3} b^{2} x^{2}+75 a^{4} b x +448 a^{5}\right) \sqrt{-b^{2} x^{2}+a^{2}}}{240 b}+\frac{21 a^{6} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{16 \sqrt{b^{2}}}$
default	$a^{4} \left(\frac{x \sqrt{-b^{2} x^{2}+a^{2}}}{2}+\frac{a^{2} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{2 \sqrt{b^{2}}}\right)+b^{4} \left(-\frac{x^{3} \left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{6 b^{2}}+\frac{a^{2} \left(-\frac{x \left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{4 b^{2}}+\frac{a^{2} \left(\frac{x \sqrt{-b^{2} x^{2}+a^{2}}}{2}+\right.}{}\right)}{2 b^{2}}\right.$

input
```
int((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-40*b^5*x^5-192*a*b^4*x^4-350*a^2*b^3*x^3-256*a^3*b^2*x^2+75*a^4*b
*x+448*a^5)/b*(-b^2*x^2+a^2)^(1/2)+21/16*a^6/(b^2)^(1/2)*arctan((b^2)^(1/2
)*x/(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx =$$
$$-\frac{630\,a^6\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-\left(40\,b^5x^5+192\,ab^4x^4+350\,a^2b^3x^3+256\,a^3b^2x^2-75\,a^4bx-448\,a^5\right)\sqrt{}}{240\,b}$$

input
```
integrate((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/240*(630*a^6*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (40*b^5*x^5 +
192*a*b^4*x^4 + 350*a^2*b^3*x^3 + 256*a^3*b^2*x^2 - 75*a^4*b*x - 448*a^5)*
sqrt(-b^2*x^2 + a^2))/b
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx = \begin{cases} \frac{21a^6\left(\begin{cases} \frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for } a^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise} \end{cases}\right)}{16} + \sqrt{a^2-b^2x^2}\left(-\frac{28a^5}{15b}-\frac{5a^4x}{16}+\frac{16a^3bx^2}{15}+\frac{35a^2b^2x^3}{24}+\frac{4ab^3x^4}{5}\right. \\ \sqrt{a^2}\left(\begin{cases} a^4x & \text{for } b=0 \\ \frac{(a+bx)^5}{5b} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((b*x+a)**4*(-b**2*x**2+a**2)**(1/2),x)
```

output
```
Piecewise((21*a**6*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b*
*2*x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/16
 + sqrt(a**2 - b**2*x**2)*(-28*a**5/(15*b) - 5*a**4*x/16 + 16*a**3*b*x**2/
15 + 35*a**2*b**2*x**3/24 + 4*a*b**3*x**4/5 + b**4*x**5/6), Ne(b**2, 0)),
(sqrt(a**2)*Piecewise((a**4*x, Eq(b, 0)), ((a + b*x)**5/(5*b), True)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\begin{aligned}\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx = &-\frac{1}{6}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}b^2x^3+\frac{21\,a^6\arcsin\left(\frac{bx}{a}\right)}{16\,b}\\ &+\frac{21}{16}\sqrt{-b^2x^2+a^2}a^4x-\frac{4}{5}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}abx^2\\ &-\frac{13}{8}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^2x-\frac{28\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^3}{15\,b}\end{aligned}$$

input
```
integrate((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(-b^2*x^2 + a^2)^(3/2)*b^2*x^3 + 21/16*a^6*arcsin(b*x/a)/b + 21/16*sq
rt(-b^2*x^2 + a^2)*a^4*x - 4/5*(-b^2*x^2 + a^2)^(3/2)*a*b*x^2 - 13/8*(-b^2
*x^2 + a^2)^(3/2)*a^2*x - 28/15*(-b^2*x^2 + a^2)^(3/2)*a^3/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx = \frac{21\,a^6\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{16\,|b|} - \frac{1}{240}\left(\frac{448\,a^5}{b} + \left(75\,a^4 - 2\left(128\,a^3b + \left(175\,a^2b^2 + 4\left(5\,b^4x + 24\,ab^3\right)x\right)x\right)x\right)x\right)\sqrt{-b^2x^2+a^2}$$

input
```
integrate((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

output
```
21/16*a^6*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/240*(448*a^5/b + (75*a^4
- 2*(128*a^3*b + (175*a^2*b^2 + 4*(5*b^4*x + 24*a*b^3)*x)*x)*x)*x)*sqrt(-b
^2*x^2 + a^2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx = \int \sqrt{a^2-b^2\,x^2}\,{\left(a+b\,x\right)}^4\,dx$$

input
```
int((a^2 - b^2*x^2)^(1/2)*(a + b*x)^4,x)
```

output
```
int((a^2 - b^2*x^2)^(1/2)*(a + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.01

$$\int (a+bx)^4\sqrt{a^2-b^2x^2}\,dx$$
$$=\frac{315asin\left(\frac{bx}{a}\right)a^6-448\sqrt{-b^2x^2+a^2}\,a^5-75\sqrt{-b^2x^2+a^2}\,a^4bx+256\sqrt{-b^2x^2+a^2}\,a^3b^2x^2+350\sqrt{-b^2x^2+}}{240b}$$

input
```
int((b*x+a)^4*(-b^2*x^2+a^2)^(1/2),x)
```

output
```
(315*asin((b*x)/a)*a**6 - 448*sqrt(a**2 - b**2*x**2)*a**5 - 75*sqrt(a**2 -
 b**2*x**2)*a**4*b*x + 256*sqrt(a**2 - b**2*x**2)*a**3*b**2*x**2 + 350*sqr
t(a**2 - b**2*x**2)*a**2*b**3*x**3 + 192*sqrt(a**2 - b**2*x**2)*a*b**4*x**
4 + 40*sqrt(a**2 - b**2*x**2)*b**5*x**5 + 448*a**6)/(240*b)
```

3.40 $\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx = \frac{7}{8}a^3x\sqrt{a^2-b^2x^2}-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}-\frac{7a(8a+3bx)\left(a^2-b^2x^2\right)^{3/2}}{60b}+\frac{7a^5\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

output
```
7/8*a^3*x*(-b^2*x^2+a^2)^(1/2)-1/5*(b*x+a)^2*(-b^2*x^2+a^2)^(3/2)/b-7/60*a
*(3*b*x+8*a)*(-b^2*x^2+a^2)^(3/2)/b+7/8*a^5*arctan(b*x/(-b^2*x^2+a^2)^(1/2
))/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx = \frac{\sqrt{a^2-b^2x^2}(-136a^4+15a^3bx+112a^2b^2x^2+90ab^3x^3+24b^4x^4)}{120b}-\frac{7a^5\log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{8\sqrt{-b^2}}$$

input
```
Integrate[(a + b*x)^3*Sqrt[a^2 - b^2*x^2],x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-136*a^4 + 15*a^3*b*x + 112*a^2*b^2*x^2 + 90*a*b^3*x
^3 + 24*b^4*x^4))/(120*b) - (7*a^5*Log[-(Sqrt[-b^2]*x) + Sqrt[a^2 - b^2*x^
2]])/(8*Sqrt[-b^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {469, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}a\int (a+bx)^2\sqrt{a^2-b^2x^2}dx-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\int (a+bx)\sqrt{a^2-b^2x^2}dx-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\int \sqrt{a^2-b^2x^2}dx-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}a^2\int\frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 224$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}a^2\int\frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \; 216$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{3/2}}{5b}$$

input
```
Int[(a + b*x)^3*Sqrt[a^2 - b^2*x^2],x]
```

output
```
-1/5*((a + b*x)^2*(a^2 - b^2*x^2)^(3/2))/b + (7*a*(-1/4*((a + b*x)*(a^2 -
b^2*x^2)^(3/2))/b + (5*a*(-1/3*(a^2 - b^2*x^2)^(3/2)/b + a*((x*Sqrt[a^2 -
b^2*x^2])/2 + (a^2*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b))))/4))/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\left(-24 b^{4} x^{4}-90 a\,b^{3} x^{3}-112 a^{2} b^{2} x^{2}-15 a^{3} b x +136 a^{4}\right) \sqrt{-b^{2} x^{2}+a^{2}}}{120 b}+\frac{7 a^{5} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{8 \sqrt{b^{2}}}$
default	$a^{3} \left(\frac{x \sqrt{-b^{2} x^{2}+a^{2}}}{2}+\frac{a^{2} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{2 \sqrt{b^{2}}}\right)+b^{3} \left(-\frac{x^{2} \left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{5 b^{2}}-\frac{2 a^{2} \left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{15 b^{4}}\right)+3 a\,b^{2} \left(-\frac{x(-b^{2}\right.$

input
```
int((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*b^4*x^4-90*a*b^3*x^3-112*a^2*b^2*x^2-15*a^3*b*x+136*a^4)/b*(-b
^2*x^2+a^2)^(1/2)+7/8*a^5/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx =$$
$$-\frac{210\,a^5\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-(24\,b^4x^4+90\,ab^3x^3+112\,a^2b^2x^2+15\,a^3bx-136\,a^4)\sqrt{-b^2x^2+a^2}}{120\,b}$$

input
```
integrate((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(210*a^5*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (24*b^4*x^4 +
90*a*b^3*x^3 + 112*a^2*b^2*x^2 + 15*a^3*b*x - 136*a^4)*sqrt(-b^2*x^2 + a^2
))/b
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx$$
$$=\begin{cases}\frac{7a^5\left(\begin{cases}\frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for } a^2\neq 0\\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise}\end{cases}\right)}{8}+\sqrt{a^2-b^2x^2}\left(-\frac{17a^4}{15b}+\frac{a^3x}{8}+\frac{14a^2bx^2}{15}+\frac{3ab^2x^3}{4}+\frac{b^3x^4}{5}\right) & \text{fo}\\ \sqrt{a^2}\left(\begin{cases}a^3x & \text{for } b=0\\ \frac{(a+bx)^4}{4b} & \text{otherwise}\end{cases}\right) & \text{o}\end{cases}$$

input
```
integrate((b*x+a)**3*(-b**2*x**2+a**2)**(1/2),x)
```

output
```
Piecewise((7*a**5*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b**
2*x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/8 +
 sqrt(a**2 - b**2*x**2)*(-17*a**4/(15*b) + a**3*x/8 + 14*a**2*b*x**2/15 +
3*a*b**2*x**3/4 + b**3*x**4/5), Ne(b**2, 0)), (sqrt(a**2)*Piecewise((a**3*
x, Eq(b, 0)), ((a + b*x)**4/(4*b), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx = \frac{7\,a^5\arcsin\left(\frac{bx}{a}\right)}{8\,b} + \frac{7}{8}\sqrt{-b^2x^2+a^2}a^3x - \frac{1}{5}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}bx^2 - \frac{3}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}ax - \frac{17\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^2}{15\,b}$$

input
```
integrate((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

output
```
7/8*a^5*arcsin(b*x/a)/b + 7/8*sqrt(-b^2*x^2 + a^2)*a^3*x - 1/5*(-b^2*x^2 +
 a^2)^(3/2)*b*x^2 - 3/4*(-b^2*x^2 + a^2)^(3/2)*a*x - 17/15*(-b^2*x^2 + a^2
)^(3/2)*a^2/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx = \frac{7\,a^5\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{8\,|b|} - \frac{1}{120}\sqrt{-b^2x^2+a^2}\left(\frac{136\,a^4}{b} - \left(15\,a^3+2\left(56\,a^2b+3\left(4\,b^3x+15\,ab^2\right)x\right)x\right)x\right)$$

input
```
integrate((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

output
```
7/8*a^5*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/120*sqrt(-b^2*x^2 + a^2)*(1
36*a^4/b - (15*a^3 + 2*(56*a^2*b + 3*(4*b^3*x + 15*a*b^2)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx=\int \sqrt{a^2-b^2\,x^2}\,{\left(a+b\,x\right)}^3\,dx$$

input
```
int((a^2 - b^2*x^2)^(1/2)*(a + b*x)^3,x)
```

output
```
int((a^2 - b^2*x^2)^(1/2)*(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int (a+bx)^3\sqrt{a^2-b^2x^2}\,dx = \frac{105 asin\left(\frac{bx}{a}\right)a^5-136\sqrt{-b^2x^2+a^2}\,a^4+15\sqrt{-b^2x^2+a^2}\,a^3bx+112\sqrt{-b^2x^2+a^2}\,a^2b^2x^2+90\sqrt{-b^2x^2+}}{120b}$$

input
```
int((b*x+a)^3*(-b^2*x^2+a^2)^(1/2),x)
```

output
```
(105*asin((b*x)/a)*a**5 - 136*sqrt(a**2 - b**2*x**2)*a**4 + 15*sqrt(a**2 -
 b**2*x**2)*a**3*b*x + 112*sqrt(a**2 - b**2*x**2)*a**2*b**2*x**2 + 90*sqrt
(a**2 - b**2*x**2)*a*b**3*x**3 + 24*sqrt(a**2 - b**2*x**2)*b**4*x**4 + 136
*a**5)/(120*b)
```

3.41 $\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx=\frac{5}{8}a^2x\sqrt{a^2-b^2x^2}-\frac{(8a+3bx)\left(a^2-b^2x^2\right)^{3/2}}{12b}+\frac{5a^4\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

output
```
5/8*a^2*x*(-b^2*x^2+a^2)^(1/2)-1/12*(3*b*x+8*a)*(-b^2*x^2+a^2)^(3/2)/b+5/8
*a^4*arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx=\frac{\sqrt{a^2-b^2x^2}(-16a^3+9a^2bx+16ab^2x^2+6b^3x^3)-30a^4\arctan\left(\frac{bx}{\sqrt{a^2}-\sqrt{a^2-b^2x^2}}\right)}{24b}$$

input
```
Integrate[(a + b*x)^2*Sqrt[a^2 - b^2*x^2],x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-16*a^3 + 9*a^2*b*x + 16*a*b^2*x^2 + 6*b^3*x^3) - 30
*a^4*ArcTan[(b*x)/(Sqrt[a^2] - Sqrt[a^2 - b^2*x^2])])/(24*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{4}a\int (a+bx)\sqrt{a^2-b^2x^2}dx-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{455}$$

$$\frac{5}{4}a\left(a\int \sqrt{a^2-b^2x^2}dx-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}a^2\int \frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}a^2\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{4}a\left(a\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{3/2}}{4b}$$

input
```
Int[(a + b*x)^2*Sqrt[a^2 - b^2*x^2],x]
```

output
```
-1/4*((a + b*x)*(a^2 - b^2*x^2)^(3/2))/b + (5*a*(-1/3*(a^2 - b^2*x^2)^(3/2
)/b + a*((x*Sqrt[a^2 - b^2*x^2])/2 + (a^2*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]
])/(2*b))))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\left(-6 b^{3} x^{3}-16 a\, b^{2} x^{2}-9 a^{2} b x +16 a^{3}\right) \sqrt{-b^{2} x^{2}+a^{2}}}{24 b}+\frac{5 a^{4} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{8 \sqrt{b^{2}}}$
default	$a^{2} \left(\frac{x \sqrt{-b^{2} x^{2}+a^{2}}}{2}+\frac{a^{2} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{2 \sqrt{b^{2}}}\right)+b^{2} \left(-\frac{x \left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{4 b^{2}}+\frac{a^{2} \left(\frac{x \sqrt{-b^{2} x^{2}+a^{2}}}{2}+\frac{a^{2} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{2 \sqrt{b^{2}}}\right)}{4 b^{2}}\right.$

input
```
int((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*b^3*x^3-16*a*b^2*x^2-9*a^2*b*x+16*a^3)/b*(-b^2*x^2+a^2)^(1/2)+5/
8*a^4/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx$$
$$=-\frac{30\,a^4\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-\left(6\,b^3x^3+16\,ab^2x^2+9\,a^2bx-16\,a^3\right)\sqrt{-b^2x^2+a^2}}{24\,b}$$

input
```
integrate((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(30*a^4*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (6*b^3*x^3 + 16*
a*b^2*x^2 + 9*a^2*b*x - 16*a^3)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.60

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx = \begin{cases} \frac{5a^4\left(\begin{cases} \frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for } a^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise} \end{cases}\right)}{8} + \sqrt{a^2-b^2x^2}\left(-\frac{2a^3}{3b}+\frac{3a^2x}{8}+\frac{2abx^2}{3}+\frac{b^2x^3}{4}\right) & \text{for } b^2\neq 0 \\ \sqrt{a^2}\left(\begin{cases} a^2x & \text{for } b=0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**2*(-b**2*x**2+a**2)**(1/2),x)
```

output
```
Piecewise((5*a**4*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b**
2*x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/8 +
 sqrt(a**2 - b**2*x**2)*(-2*a**3/(3*b) + 3*a**2*x/8 + 2*a*b*x**2/3 + b**2*
x**3/4), Ne(b**2, 0)), (sqrt(a**2)*Piecewise((a**2*x, Eq(b, 0)), ((a + b*x
)**3/(3*b), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.85

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx = \frac{5\,a^4\arcsin\left(\frac{bx}{a}\right)}{8\,b} + \frac{5}{8}\,\sqrt{-b^2x^2+a^2}a^2x - \frac{1}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}x - \frac{2\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a}{3\,b}$$

input
```
integrate((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

output
```
5/8*a^4*arcsin(b*x/a)/b + 5/8*sqrt(-b^2*x^2 + a^2)*a^2*x - 1/4*(-b^2*x^2 +
 a^2)^(3/2)*x - 2/3*(-b^2*x^2 + a^2)^(3/2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx = \frac{5\,a^4\arcsin\left(\frac{bx}{a}\right)\mathrm{sgn}(a)\,\mathrm{sgn}(b)}{8\,|b|} - \frac{1}{24}\sqrt{-b^2x^2+a^2}\left(\frac{16\,a^3}{b}-\left(9\,a^2+2\left(3\,b^2x+8\,ab\right)x\right)x\right)$$

input
```
integrate((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

output
```
5/8*a^4*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/24*sqrt(-b^2*x^2 + a^2)*(16
*a^3/b - (9*a^2 + 2*(3*b^2*x + 8*a*b)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx = \int \sqrt{a^2-b^2\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

input
```
int((a^2 - b^2*x^2)^(1/2)*(a + b*x)^2,x)
```

output
```
int((a^2 - b^2*x^2)^(1/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int (a+bx)^2\sqrt{a^2-b^2x^2}\,dx$$
$$= \frac{15 asin\left(\frac{bx}{a}\right) a^4 - 16\sqrt{-b^2x^2+a^2}\,a^3 + 9\sqrt{-b^2x^2+a^2}\,a^2bx + 16\sqrt{-b^2x^2+a^2}\,a\,b^2x^2 + 6\sqrt{-b^2x^2+a^2}\,b^3x^3}{24b}$$

input
```
int((b*x+a)^2*(-b^2*x^2+a^2)^(1/2),x)
```

output
```
(15*asin((b*x)/a)*a**4 - 16*sqrt(a**2 - b**2*x**2)*a**3 + 9*sqrt(a**2 - b*
*2*x**2)*a**2*b*x + 16*sqrt(a**2 - b**2*x**2)*a*b**2*x**2 + 6*sqrt(a**2 -
b**2*x**2)*b**3*x**3 + 16*a**4)/(24*b)
```

3.42 $\int (a+bx)\sqrt{a^2-b^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx=\frac{1}{2}ax\sqrt{a^2-b^2x^2}-\frac{(a^2-b^2x^2)^{3/2}}{3b}+\frac{a^3\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

output
```
1/2*a*x*(-b^2*x^2+a^2)^(1/2)-1/3*(-b^2*x^2+a^2)^(3/2)/b+1/2*a^3*arctan(b*x
/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx=\frac{\sqrt{a^2-b^2x^2}(-2a^2+3abx+2b^2x^2)}{6b}-\frac{a^3\log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{2\sqrt{-b^2}}$$

input
```
Integrate[(a + b*x)*Sqrt[a^2 - b^2*x^2],x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-2*a^2 + 3*a*b*x + 2*b^2*x^2))/(6*b) - (a^3*Log[-(Sq
rt[-b^2]*x) + Sqrt[a^2 - b^2*x^2]])/(2*Sqrt[-b^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx$$

$$\Big\downarrow \text{455}$$

$$a\int \sqrt{a^2-b^2x^2}dx-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{1}{2}a^2\int \frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{2}a^2\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{216}$$

$$a\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)-\frac{\left(a^2-b^2x^2\right)^{3/2}}{3b}$$

input
```
Int[(a + b*x)*Sqrt[a^2 - b^2*x^2],x]
```

output
```
-1/3*(a^2 - b^2*x^2)^(3/2)/b + a*((x*Sqrt[a^2 - b^2*x^2])/2 + (a^2*ArcTan[
(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{\left(-2 b^{2} x^{2}-3 a b x+2 a^{2}\right) \sqrt{-b^{2} x^{2}+a^{2}}}{6 b}+\frac{a^{3} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{2 \sqrt{b^{2}}}$	72
default	$a \left(\frac{x \sqrt{-b^{2} x^{2}+a^{2}}}{2}+\frac{a^{2} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{2 \sqrt{b^{2}}}\right)-\frac{\left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{3 b}$	73

input
```
int((b*x+a)*(-b^2*x^2+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-2*b^2*x^2-3*a*b*x+2*a^2)/b*(-b^2*x^2+a^2)^(1/2)+1/2*a^3/(b^2)^(1/2)
*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx = -\frac{6\,a^3\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-(2\,b^2x^2+3\,abx-2\,a^2)\sqrt{-b^2x^2+a^2}}{6\,b}$$

input
```
integrate((b*x+a)*(-b^2*x^2+a^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(6*a^3*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - (2*b^2*x^2 + 3*a*b
*x - 2*a^2)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx = \begin{cases} \frac{a^3\left(\begin{cases} \frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for } a^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise} \end{cases}\right)}{2}+\sqrt{a^2-b^2x^2}\left(-\frac{a^2}{3b}+\frac{ax}{2}+\frac{bx^2}{3}\right) & \text{for } b^2\neq 0 \\ \left(ax+\frac{bx^2}{2}\right)\sqrt{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*(-b**2*x**2+a**2)**(1/2),x)
```

output
```
Piecewise((a**3*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b**2*
x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/2 + s
qrt(a**2 - b**2*x**2)*(-a**2/(3*b) + a*x/2 + b*x**2/3), Ne(b**2, 0)), ((a*
x + b*x**2/2)*sqrt(a**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx=\frac{a^3\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\,\sqrt{-b^2x^2+a^2}ax-\frac{\left(-b^2x^2+a^2\right)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((b*x+a)*(-b^2*x^2+a^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*a^3*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*x^2 + a^2)*a*x - 1/3*(-b^2*x^2 + a
^2)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx=\frac{a^3\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{2\,|b|}+\frac{1}{6}\,\sqrt{-b^2x^2+a^2}\left((2\,bx+3\,a)x-\frac{2\,a^2}{b}\right)$$

input
```
integrate((b*x+a)*(-b^2*x^2+a^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*a^3*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) + 1/6*sqrt(-b^2*x^2 + a^2)*((2*
b*x + 3*a)*x - 2*a^2/b)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx=\frac{a^3\,\ln\left(x\,\sqrt{-b^2}+\sqrt{a^2-b^2\,x^2}\right)}{2\,\sqrt{-b^2}}-\frac{{\left(a^2-b^2\,x^2\right)}^{3/2}}{3\,b}+\frac{a\,x\,\sqrt{a^2-b^2\,x^2}}{2}$$

input
```
int((a^2 - b^2*x^2)^(1/2)*(a + b*x),x)
```

output
```
(a^3*log(x*(-b^2)^(1/2) + (a^2 - b^2*x^2)^(1/2)))/(2*(-b^2)^(1/2)) - (a^2
- b^2*x^2)^(3/2)/(3*b) + (a*x*(a^2 - b^2*x^2)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int (a+bx)\sqrt{a^2-b^2x^2}\,dx$$
$$=\frac{3asin\left(\frac{bx}{a}\right)a^3-2\sqrt{-b^2x^2+a^2}\,a^2+3\sqrt{-b^2x^2+a^2}\,abx+2\sqrt{-b^2x^2+a^2}\,b^2x^2+2a^3}{6b}$$

input
```
int((b*x+a)*(-b^2*x^2+a^2)^(1/2),x)
```

output
```
(3*asin((b*x)/a)*a**3 - 2*sqrt(a**2 - b**2*x**2)*a**2 + 3*sqrt(a**2 - b**2
*x**2)*a*b*x + 2*sqrt(a**2 - b**2*x**2)*b**2*x**2 + 2*a**3)/(6*b)
```

3.43 $\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx=\frac{\sqrt{a^2-b^2x^2}}{b}+\frac{a\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

output
```
(-b^2*x^2+a^2)^(1/2)/b+a*arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx=\frac{\sqrt{a^2-b^2x^2}}{b}-\frac{a\log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}}$$

input
```
Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x),x]
```

output
```
Sqrt[a^2 - b^2*x^2]/b - (a*Log[-(Sqrt[-b^2]*x) + Sqrt[a^2 - b^2*x^2]])/Sqr
t[-b^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {466, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx$$

$$\Big\downarrow \ 466$$

$$a\int \frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{\sqrt{a^2-b^2x^2}}{b}$$

$$\Big\downarrow \ 224$$

$$a\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{\sqrt{a^2-b^2x^2}}{b}$$

$$\Big\downarrow \ 216$$

$$\frac{a\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}+\frac{\sqrt{a^2-b^2x^2}}{b}$$

input
```
Int[Sqrt[a^2 - b^2*x^2]/(a + b*x),x]
```

output
```
Sqrt[a^2 - b^2*x^2]/b + (a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{\sqrt{-b^2x^2+a^2}}{b}+\frac{a\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{\sqrt{b^2}}$	49
default	$\frac{\sqrt{-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)}+\frac{ab\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)}}\right)}{\sqrt{b^2}}}{b}$	78

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-b^2*x^2+a^2)^(1/2)/b+a/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}\,dx=-\frac{2\,a\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)-\sqrt{-b^2x^2+a^2}}{b}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
-(2*a*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) - sqrt(-b^2*x^2 + a^2))/b
```

Sympy [F]

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a + bx} \, dx = \int \frac{\sqrt{-(-a + bx)(a + bx)}}{a + bx} \, dx$$

input
```
integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a + bx} \, dx = \frac{a \arcsin\left(\frac{bx}{a}\right)}{b} + \frac{\sqrt{-b^2x^2 + a^2}}{b}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
a*arcsin(b*x/a)/b + sqrt(-b^2*x^2 + a^2)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a + bx} \, dx = \frac{a \arcsin\left(\frac{bx}{a}\right) \operatorname{sgn}(a) \operatorname{sgn}(b)}{|b|} + \frac{\sqrt{-b^2x^2 + a^2}}{b}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
a*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) + sqrt(-b^2*x^2 + a^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a + bx}\,dx = \int \frac{\sqrt{a^2 - b^2\,x^2}}{a + b\,x}\,dx$$

input
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x),x)
```

output
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a + bx}\,dx = \frac{asin\left(\frac{bx}{a}\right)a + \sqrt{-b^2x^2 + a^2} - a}{b}$$

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a),x)
```

output
```
(asin((b*x)/a)*a + sqrt(a**2 - b**2*x**2) - a)/b
```

3.44 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2} dx = -\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)} - \frac{\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

output
```
-2*(-b^2*x^2+a^2)^(1/2)/b/(b*x+a)-arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2} dx = \frac{2\left(-\frac{\sqrt{a^2-b^2x^2}}{a+bx} + \arctan\left(\frac{bx}{\sqrt{a^2}-\sqrt{a^2-b^2x^2}}\right)\right)}{b}$$

input
```
Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^2,x]
```

output
```
(2*(-(Sqrt[a^2 - b^2*x^2]/(a + b*x)) + ArcTan[(b*x)/(Sqrt[a^2] - Sqrt[a^2
 - b^2*x^2])]))/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {463, 25, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{ 463}$$

$$\int -\frac{1}{\sqrt{a^2-b^2x^2}}dx-\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\sqrt{a^2-b^2x^2}}dx-\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

$$\Big\downarrow \text{ 224}$$

$$-\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}-\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}-\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

input
```
Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^2,x]
```

output
```
(-2*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) - ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]]
/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(50) = 100$.

Time $= 0.22$ (sec) , antiderivative size $= 130$, normalized size of antiderivative $= 2.41$

method	result	size
default	$\frac{-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2 a b\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{a b\left(x+\frac{a}{b}\right)^{2}}-\frac{b\left(\sqrt{-b^{2}\left(x+\frac{a}{b}\right)^{2}+2 a b\left(x+\frac{a}{b}\right)}+\frac{a b \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2}\left(x+\frac{a}{b}\right)^{2}+2 a b\left(x+\frac{a}{b}\right)}}\right)}{\sqrt{b^{2}}}\right)}{a}}{b^{2}}$	130

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(-1/a/b/(x+a/b)^2*(-b^2*(x+a/b)^2+2*a*b*(x+a/b))^(3/2)-b/a*((-b^2*(x
+a/b)^2+2*a*b*(x+a/b))^(1/2)+a*b/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*(x
+a/b)^2+2*a*b*(x+a/b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}\,dx = -\frac{2\left(bx-(bx+a)\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+a+\sqrt{-b^2x^2+a^2}\right)}{b^2x+ab}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(b*x - (b*x + a)*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + a + sqrt(-
b^2*x^2 + a^2))/(b^2*x + a*b)
```

Sympy [F]

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}\,dx = \int \frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^2}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**2,x)
```

output
```
Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}\,dx = -\frac{\arcsin\left(\frac{bx}{a}\right)}{b} - \frac{2\sqrt{-b^2x^2+a^2}}{b^2x+ab}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-arcsin(b*x/a)/b - 2*sqrt(-b^2*x^2 + a^2)/(b^2*x + a*b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: abs(
sageVARb)*(-(2*atan(i)-2*i)/sageVARb^2*sign((sageVARb*sageVARx+sageVARa)^-
1)*sign(sageVARb)-2*sageVARa*(sqrt(2*sageVARa*sageVARb*(sageVARb*sageVARx+
sageVARa)^-1/sageVA
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}\,dx = \int \frac{\sqrt{a^2-b^2\,x^2}}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x)^2,x)
```

output
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}\,dx = \frac{-asin\left(\frac{bx}{a}\right)\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right) - asin\left(\frac{bx}{a}\right) + 4\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right)}{b\left(\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right)+1\right)}$$

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^2,x)
```

output
```
( - asin((b*x)/a)*tan(asin((b*x)/a)/2) - asin((b*x)/a) + 4*tan(asin((b*x)/
a)/2))/(b*(tan(asin((b*x)/a)/2) + 1))
```

3.45 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} dx = -\frac{(a^2-b^2x^2)^{3/2}}{3ab(a+bx)^3}$$

output
```
-1/3*(-b^2*x^2+a^2)^(3/2)/a/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} dx = \frac{(-a+bx)\sqrt{a^2-b^2x^2}}{3ab(a+bx)^2}$$

input
```
Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^3,x]
```

output
```
((-a + b*x)*Sqrt[a^2 - b^2*x^2])/(3*a*b*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3} \, dx$$
$$\Big\downarrow \text{ 460}$$
$$-\frac{(a^2-b^2x^2)^{3/2}}{3ab(a+bx)^3}$$

input
```
Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^3,x]
```

output
```
-1/3*(a^2 - b^2*x^2)^(3/2)/(a*b*(a + b*x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{(-bx+a)\sqrt{-b^2x^2+a^2}}{3(bx+a)^2ba}$	36
trager	$-\frac{(-bx+a)\sqrt{-b^2x^2+a^2}}{3(bx+a)^2ba}$	36
orering	$-\frac{(-bx+a)\sqrt{-b^2x^2+a^2}}{3(bx+a)^2ba}$	36
default	$-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{3b^4a\left(x+\frac{a}{b}\right)^3}$	46

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(b*x+a)^2*(-b*x+a)/b/a*(-b^2*x^2+a^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx = -\frac{b^2x^2+2\,abx+a^2-\sqrt{-b^2x^2+a^2}(bx-a)}{3\,(ab^3x^2+2\,a^2b^2x+a^3b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3*(b^2*x^2 + 2*a*b*x + a^2 - sqrt(-b^2*x^2 + a^2)*(b*x - a))/(a*b^3*x^2
 + 2*a^2*b^2*x + a^3*b)
```

Sympy [F]

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx = \int \frac{\sqrt{-(-a+bx)\,(a+bx)}}{(a+bx)^3}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**3,x)
```

output
```
Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx = -\frac{2\sqrt{-b^2x^2+a^2}}{3\,(b^3x^2+2\,ab^2x+a^2b)} + \frac{\sqrt{-b^2x^2+a^2}}{3\,(ab^2x+a^2b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-2/3*sqrt(-b^2*x^2 + a^2)/(b^3*x^2 + 2*a*b^2*x + a^2*b) + 1/3*sqrt(-b^2*x^
2 + a^2)/(a*b^2*x + a^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(29) = 58$.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx = \frac{2\left(\frac{3\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+1\right)}{3\,a\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^3|b|}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
2/3*(3*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 1)/(a*((a*b + sqr
t(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^3*abs(b))
```

Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx = -\frac{\sqrt{a^2-b^2\,x^2}\,(a-b\,x)}{3\,a\,b\,{(a+b\,x)}^2}$$

input
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x)^3,x)
```

output
```
-((a^2 - b^2*x^2)^(1/2)*(a - b*x))/(3*a*b*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}\,dx = \frac{7\sqrt{-b^2x^2+a^2}\,a+\sqrt{-b^2x^2+a^2}\,bx-7a^2-2abx-7b^2x^2}{9ab\left(\sqrt{-b^2x^2+a^2}\,a+\sqrt{-b^2x^2+a^2}\,bx-a^2-2abx-b^2x^2\right)}$$

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^3,x)
```

output
```
(7*sqrt(a**2 - b**2*x**2)*a + sqrt(a**2 - b**2*x**2)*b*x - 7*a**2 - 2*a*b*
x - 7*b**2*x**2)/(9*a*b*(sqrt(a**2 - b**2*x**2)*a + sqrt(a**2 - b**2*x**2)
*b*x - a**2 - 2*a*b*x - b**2*x**2))
```

3.46 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4} dx = -\frac{(a^2-b^2x^2)^{3/2}}{5ab(a+bx)^4} - \frac{(a^2-b^2x^2)^{3/2}}{15a^2b(a+bx)^3}$$

output
```
-1/5*(-b^2*x^2+a^2)^(3/2)/a/b/(b*x+a)^4-1/15*(-b^2*x^2+a^2)^(3/2)/a^2/b/(b
*x+a)^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4} dx = \frac{\sqrt{a^2-b^2x^2}(-4a^2+3abx+b^2x^2)}{15a^2b(a+bx)^3}$$

input
```
Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^4,x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-4*a^2 + 3*a*b*x + b^2*x^2))/(15*a^2*b*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4} \, dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}dx}{5a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}$$

$$\Big\downarrow \text{ 460}$$

$$-\frac{\left(a^2-b^2x^2\right)^{3/2}}{15a^2b(a+bx)^3}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}$$

input
```
Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^4,x]
```

output
```
-1/5*(a^2 - b^2*x^2)^(3/2)/(a*b*(a + b*x)^4) - (a^2 - b^2*x^2)^(3/2)/(15*a
^2*b*(a + b*x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(-bx+a)(bx+4a)\sqrt{-b^2x^2+a^2}}{15(bx+a)^3 a^2 b}$	43
orering	$-\frac{(-bx+a)(bx+4a)\sqrt{-b^2x^2+a^2}}{15(bx+a)^3 a^2 b}$	43
trager	$-\frac{(-b^2x^2-3abx+4a^2)\sqrt{-b^2x^2+a^2}}{15a^2(bx+a)^3 b}$	49
default	$\frac{-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{5ab\left(x+\frac{a}{b}\right)^4}-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{15a^2\left(x+\frac{a}{b}\right)^3}}{b^4}$	93

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-b*x+a)*(b*x+4*a)*(-b^2*x^2+a^2)^(1/2)/(b*x+a)^3/a^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx = -\frac{4\,b^3x^3+12\,ab^2x^2+12\,a^2bx+4\,a^3-(b^2x^2+3\,abx-4\,a^2)\sqrt{-b^2x^2+a^2}}{15\,(a^2b^4x^3+3\,a^3b^3x^2+3\,a^4b^2x+a^5b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/15*(4*b^3*x^3 + 12*a*b^2*x^2 + 12*a^2*b*x + 4*a^3 - (b^2*x^2 + 3*a*b*x
- 4*a^2)*sqrt(-b^2*x^2 + a^2))/(a^2*b^4*x^3 + 3*a^3*b^3*x^2 + 3*a^4*b^2*x
+ a^5*b)
```

Sympy [F]

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx = \int \frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^4}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**4,x)
```

output
```
Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(59) = 118$.

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx = -\frac{2\sqrt{-b^2x^2+a^2}}{5\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} + \frac{\sqrt{-b^2x^2+a^2}}{15\,(ab^3x^2+2\,a^2b^2x+a^3b)} + \frac{\sqrt{-b^2x^2+a^2}}{15\,(a^2b^2x+a^3b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-2/5*sqrt(-b^2*x^2 + a^2)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) +
1/15*sqrt(-b^2*x^2 + a^2)/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/15*sqrt(-b
^2*x^2 + a^2)/(a^2*b^2*x + a^3*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(59) = 118$.

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx = \frac{2\left(\frac{5\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{25\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{15\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{15\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4}+4\right)}{15\,a^2\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^5|b|}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x, algorithm="giac")
```

output
```
2/15*(5*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 25*(a*b + sqrt(-b^2*
x^2 + a^2)*abs(b))^2/(b^4*x^2) + 15*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^3/
(b^6*x^3) + 15*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) + 4)/(a^2*(
(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^5*abs(b))
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx = \frac{\sqrt{a^2-b^2\,x^2}\,\left(-4\,a^2+3\,a\,b\,x+b^2\,x^2\right)}{15\,a^2\,b\,{\left(a+b\,x\right)}^3}$$

input
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x)^4,x)
```

output
```
((a^2 - b^2*x^2)^(1/2)*(b^2*x^2 - 4*a^2 + 3*a*b*x))/(15*a^2*b*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}\,dx = \frac{\sqrt{-b^2x^2+a^2}\,(b^2x^2+3abx-4a^2)}{15a^2b\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^4,x)
```

output
```
(sqrt(a**2 - b**2*x**2)*( - 4*a**2 + 3*a*b*x + b**2*x**2))/(15*a**2*b*(a**
3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.47 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx = -\frac{(a^2-b^2x^2)^{3/2}}{7ab(a+bx)^5}-\frac{2(a^2-b^2x^2)^{3/2}}{35a^2b(a+bx)^4}-\frac{2(a^2-b^2x^2)^{3/2}}{105a^3b(a+bx)^3}$$

output
```
-1/7*(-b^2*x^2+a^2)^(3/2)/a/b/(b*x+a)^5-2/35*(-b^2*x^2+a^2)^(3/2)/a^2/b/(b
*x+a)^4-2/105*(-b^2*x^2+a^2)^(3/2)/a^3/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx = \frac{\sqrt{a^2-b^2x^2}(-23a^3+13a^2bx+8ab^2x^2+2b^3x^3)}{105a^3b(a+bx)^4}$$

input
```
Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^5,x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-23*a^3 + 13*a^2*b*x + 8*a*b^2*x^2 + 2*b^3*x^3))/(10
5*a^3*b*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx$$

$$\Big\downarrow \; 461$$

$$\frac{2\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}dx}{7a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}$$

$$\Big\downarrow \; 461$$

$$\frac{2\left(\frac{\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}dx}{5a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}\right)}{7a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}$$

$$\Big\downarrow \; 460$$

$$\frac{2\left(-\frac{\left(a^2-b^2x^2\right)^{3/2}}{15a^2b(a+bx)^3}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}\right)}{7a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}$$

input
```
Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^5,x]
```

output
```
-1/7*(a^2 - b^2*x^2)^(3/2)/(a*b*(a + b*x)^5) + (2*(-1/5*(a^2 - b^2*x^2)^(3
/2)/(a*b*(a + b*x)^4) - (a^2 - b^2*x^2)^(3/2)/(15*a^2*b*(a + b*x)^3)))/(7*
a)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{(-bx+a)\left(2b^{2}x^{2}+10abx+23a^{2}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{105(bx+a)^{4}a^{3}b}$	55
orering	$-\frac{(-bx+a)\left(2b^{2}x^{2}+10abx+23a^{2}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{105(bx+a)^{4}a^{3}b}$	55
trager	$-\frac{\left(-2b^{3}x^{3}-8a\,b^{2}x^{2}-13a^{2}bx+23a^{3}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{105a^{3}(bx+a)^{4}b}$	60
default	$\frac{-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{7ab\left(x+\frac{a}{b}\right)^{5}}+\frac{2b\left(-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{5ab\left(x+\frac{a}{b}\right)^{4}}-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{15a^{2}\left(x+\frac{a}{b}\right)^{3}}\right)}{7a}}{b^{5}}$	145

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-b*x+a)*(2*b^2*x^2+10*a*b*x+23*a^2)*(-b^2*x^2+a^2)^(1/2)/(b*x+a)^4
/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx = -\frac{23\,b^4x^4+92\,ab^3x^3+138\,a^2b^2x^2+92\,a^3bx+23\,a^4-(2\,b^3x^3+8\,ab^2x^2+13\,a^2bx-23\,a^3)\sqrt{-b^2x^2+a^2}}{105\,(a^3b^5x^4+4\,a^4b^4x^3+6\,a^5b^3x^2+4\,a^6b^2x+a^7b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/105*(23*b^4*x^4 + 92*a*b^3*x^3 + 138*a^2*b^2*x^2 + 92*a^3*b*x + 23*a^4
- (2*b^3*x^3 + 8*a*b^2*x^2 + 13*a^2*b*x - 23*a^3)*sqrt(-b^2*x^2 + a^2))/(a
^3*b^5*x^4 + 4*a^4*b^4*x^3 + 6*a^5*b^3*x^2 + 4*a^6*b^2*x + a^7*b)
```

Sympy [F]

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx = \int \frac{\sqrt{-(-a+bx)\,(a+bx)}}{(a+bx)^5}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**5,x)
```

output
```
Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx = -\frac{2\sqrt{-b^2x^2+a^2}}{7\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)} + \frac{\sqrt{-b^2x^2+a^2}}{35\,(ab^4x^3+3\,a^2b^3x^2+3\,a^3b^2x+a^4b)} + \frac{2\sqrt{-b^2x^2+a^2}}{105\,(a^2b^3x^2+2\,a^3b^2x+a^4b)} + \frac{2\sqrt{-b^2x^2+a^2}}{105\,(a^3b^2x+a^4b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
-2/7*sqrt(-b^2*x^2 + a^2)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b
^2*x + a^4*b) + 1/35*sqrt(-b^2*x^2 + a^2)/(a*b^4*x^3 + 3*a^2*b^3*x^2 + 3*a
^3*b^2*x + a^4*b) + 2/105*sqrt(-b^2*x^2 + a^2)/(a^2*b^3*x^2 + 2*a^3*b^2*x
+ a^4*b) + 2/105*sqrt(-b^2*x^2 + a^2)/(a^3*b^2*x + a^4*b)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx =$$

$$-\frac{1}{420}\left(\frac{\frac{3\left(5\left(\frac{2a}{bx+a}-1\right)^{\frac{7}{2}}+21\left(\frac{2a}{bx+a}-1\right)^{\frac{5}{2}}+35\left(\frac{2a}{bx+a}-1\right)^{\frac{3}{2}}+35\sqrt{\frac{2a}{bx+a}-1}\right)\operatorname{sgn}\left(\frac{1}{bx+a}\right)\operatorname{sgn}(b)}{a^2b^2}-\frac{7\left(3\left(\frac{2a}{bx+a}-1\right)^{\frac{5}{2}}+10\left(\frac{2a}{bx+a}-1\right)^{\frac{3}{2}}+\cdots}{a^2b^2}}{a}\right.$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/420*((3*(5*(2*a/(b*x + a) - 1)^(7/2) + 21*(2*a/(b*x + a) - 1)^(5/2) + 3
5*(2*a/(b*x + a) - 1)^(3/2) + 35*sqrt(2*a/(b*x + a) - 1))*sgn(1/(b*x + a))
*sgn(b)/(a^2*b^2) - 7*(3*(2*a/(b*x + a) - 1)^(5/2) + 10*(2*a/(b*x + a) - 1
)^(3/2) + 15*sqrt(2*a/(b*x + a) - 1))*sgn(1/(b*x + a))*sgn(b)/(a^2*b^2))/a
 + 8*I*sgn(1/(b*x + a))*sgn(b)/(a^3*b^2))*abs(b)
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx = \frac{\sqrt{a^2-b^2\,x^2}}{35\,a\,b\,{(a+b\,x)}^3}-\frac{2\,\sqrt{a^2-b^2\,x^2}}{7\,b\,{(a+b\,x)}^4}+\frac{2\,\sqrt{a^2-b^2\,x^2}}{105\,a^2\,b\,{(a+b\,x)}^2}+\frac{2\,\sqrt{a^2-b^2\,x^2}}{105\,a^3\,b\,\left(a+b\,x\right)}$$

input
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x)^5,x)
```

output
```
(a^2 - b^2*x^2)^(1/2)/(35*a*b*(a + b*x)^3) - (2*(a^2 - b^2*x^2)^(1/2))/(7*
b*(a + b*x)^4) + (2*(a^2 - b^2*x^2)^(1/2))/(105*a^2*b*(a + b*x)^2) + (2*(a
^2 - b^2*x^2)^(1/2))/(105*a^3*b*(a + b*x))
```

Reduce [F]

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}\,dx = \int \frac{\sqrt{-b^2x^2+a^2}}{(bx+a)^5}dx$$

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x)
```

output
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^5,x)
```

3.48 $\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = -\frac{(a^2-b^2x^2)^{3/2}}{9ab(a+bx)^6} - \frac{(a^2-b^2x^2)^{3/2}}{21a^2b(a+bx)^5} - \frac{2(a^2-b^2x^2)^{3/2}}{105a^3b(a+bx)^4} - \frac{2(a^2-b^2x^2)^{3/2}}{315a^4b(a+bx)^3}$$

output
```
-1/9*(-b^2*x^2+a^2)^(3/2)/a/b/(b*x+a)^6-1/21*(-b^2*x^2+a^2)^(3/2)/a^2/b/(b
*x+a)^5-2/105*(-b^2*x^2+a^2)^(3/2)/a^3/b/(b*x+a)^4-2/315*(-b^2*x^2+a^2)^(3
/2)/a^4/b/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = \frac{\sqrt{a^2-b^2x^2}(-58a^4+25a^3bx+21a^2b^2x^2+10ab^3x^3+2b^4x^4)}{315a^4b(a+bx)^5}$$

input
```
Integrate[Sqrt[a^2 - b^2*x^2]/(a + b*x)^6,x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-58*a^4 + 25*a^3*b*x + 21*a^2*b^2*x^2 + 10*a*b^3*x^3
 + 2*b^4*x^4))/(315*a^4*b*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^5}dx}{3a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{2\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^4}dx}{7a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}}{3a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{2\left(\frac{\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^3}dx}{5a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}\right)}{7a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}}{3a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}$$

$$\Big\downarrow \text{460}$$

$$\frac{\frac{2\left(-\frac{\left(a^2-b^2x^2\right)^{3/2}}{15a^2b(a+bx)^3}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{5ab(a+bx)^4}\right)}{7a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{7ab(a+bx)^5}}{3a}-\frac{\left(a^2-b^2x^2\right)^{3/2}}{9ab(a+bx)^6}$$

input
```
Int[Sqrt[a^2 - b^2*x^2]/(a + b*x)^6,x]
```

output
```
-1/9*(a^2 - b^2*x^2)^(3/2)/(a*b*(a + b*x)^6) + (-1/7*(a^2 - b^2*x^2)^(3/2)
/(a*b*(a + b*x)^5) + (2*(-1/5*(a^2 - b^2*x^2)^(3/2)/(a*b*(a + b*x)^4) - (a
^2 - b^2*x^2)^(3/2)/(15*a^2*b*(a + b*x)^3)))/(7*a))/(3*a)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.50

method	result
gosper	$-\frac{(-bx+a)\left(2b^{3}x^{3}+12a\,b^{2}x^{2}+33a^{2}bx+58a^{3}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{315(bx+a)^{5}a^{4}b}$
orering	$-\frac{(-bx+a)\left(2b^{3}x^{3}+12a\,b^{2}x^{2}+33a^{2}bx+58a^{3}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{315(bx+a)^{5}a^{4}b}$
trager	$-\frac{\left(-2b^{4}x^{4}-10a\,b^{3}x^{3}-21a^{2}b^{2}x^{2}-25a^{3}bx+58a^{4}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{315a^{4}(bx+a)^{5}b}$
default	$\frac{-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{9ab\left(x+\frac{a}{b}\right)^{6}}+\frac{b\left(-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{7ab\left(x+\frac{a}{b}\right)^{5}}+\frac{2b\left(-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{5ab\left(x+\frac{a}{b}\right)^{4}}-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{15a^{2}\left(x+\frac{a}{b}\right)^{3}}\right)}{7a}\right)}{3a}}{b^{6}}$

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(-b*x+a)*(2*b^3*x^3+12*a*b^2*x^2+33*a^2*b*x+58*a^3)*(-b^2*x^2+a^2)^
(1/2)/(b*x+a)^5/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = -\frac{58\,b^5x^5+290\,ab^4x^4+580\,a^2b^3x^3+580\,a^3b^2x^2+290\,a^4bx+58\,a^5-(2\,b^4x^4+10\,ab^3x^3+21\,a^2b^2x^2+}{315\,(a^4b^6x^5+5\,a^5b^5x^4+10\,a^6b^4x^3+10\,a^7b^3x^2+5\,a^8b^2x+a^9b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x, algorithm="fricas")
```

output
```
-1/315*(58*b^5*x^5 + 290*a*b^4*x^4 + 580*a^2*b^3*x^3 + 580*a^3*b^2*x^2 + 2
90*a^4*b*x + 58*a^5 - (2*b^4*x^4 + 10*a*b^3*x^3 + 21*a^2*b^2*x^2 + 25*a^3*
b*x - 58*a^4)*sqrt(-b^2*x^2 + a^2))/(a^4*b^6*x^5 + 5*a^5*b^5*x^4 + 10*a^6*
b^4*x^3 + 10*a^7*b^3*x^2 + 5*a^8*b^2*x + a^9*b)
```

Sympy [F]

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = \int \frac{\sqrt{-(-a+bx)(a+bx)}}{(a+bx)^6}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(1/2)/(b*x+a)**6,x)
```

output
```
Integral(sqrt(-(-a + b*x)*(a + b*x))/(a + b*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(117) = 234$.

Time = 0.04 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = -\frac{2\sqrt{-b^2x^2+a^2}}{9\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)} + \frac{\sqrt{-b^2x^2+a^2}}{63\,(ab^5x^4+4\,a^2b^4x^3+6\,a^3b^3x^2+4\,a^4b^2x+a^5b)} + \frac{\sqrt{-b^2x^2+a^2}}{105\,(a^2b^4x^3+3\,a^3b^3x^2+3\,a^4b^2x+a^5b)} + \frac{2\sqrt{-b^2x^2+a^2}}{315\,(a^3b^3x^2+2\,a^4b^2x+a^5b)} + \frac{2\sqrt{-b^2x^2+a^2}}{315\,(a^4b^2x+a^5b)}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x, algorithm="maxima")
```

output
```
-2/9*sqrt(-b^2*x^2 + a^2)/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3
*b^3*x^2 + 5*a^4*b^2*x + a^5*b) + 1/63*sqrt(-b^2*x^2 + a^2)/(a*b^5*x^4 + 4
*a^2*b^4*x^3 + 6*a^3*b^3*x^2 + 4*a^4*b^2*x + a^5*b) + 1/105*sqrt(-b^2*x^2
+ a^2)/(a^2*b^4*x^3 + 3*a^3*b^3*x^2 + 3*a^4*b^2*x + a^5*b) + 2/315*sqrt(-b
^2*x^2 + a^2)/(a^3*b^3*x^2 + 2*a^4*b^2*x + a^5*b) + 2/315*sqrt(-b^2*x^2 +
a^2)/(a^4*b^2*x + a^5*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(117) = 234$.

Time = 0.14 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = \frac{2\left(\frac{207\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x} + \frac{1143\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2} + \frac{2247\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3} + \frac{3843\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^4}{b^8x^4} + \frac{3465\left(ab\right.}{}\right.}{315\,a^4\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}\right.}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x, algorithm="giac")
```

output
```
2/315*(207*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1143*(a*b + sqrt(
-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 2247*(a*b + sqrt(-b^2*x^2 + a^2)*abs
(b))^3/(b^6*x^3) + 3843*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) +
3465*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 2625*(a*b + sqrt(-
b^2*x^2 + a^2)*abs(b))^6/(b^12*x^6) + 945*(a*b + sqrt(-b^2*x^2 + a^2)*abs(
b))^7/(b^14*x^7) + 315*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^8/(b^16*x^8) +
58)/(a^4*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^9*abs(b))
```

Mupad [B] (verification not implemented)

Time = 6.78 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = \frac{\sqrt{a^2-b^2\,x^2}}{63\,a\,b\,{(a+b\,x)}^4}-\frac{2\,\sqrt{a^2-b^2\,x^2}}{9\,b\,{(a+b\,x)}^5}+\frac{\sqrt{a^2-b^2\,x^2}}{105\,a^2\,b\,{(a+b\,x)}^3}+\frac{2\,\sqrt{a^2-b^2\,x^2}}{315\,a^3\,b\,{(a+b\,x)}^2}+\frac{2\,\sqrt{a^2-b^2\,x^2}}{315\,a^4\,b\,\left(a+b\,x\right)}$$

input
```
int((a^2 - b^2*x^2)^(1/2)/(a + b*x)^6,x)
```

output
```
(a^2 - b^2*x^2)^(1/2)/(63*a*b*(a + b*x)^4) - (2*(a^2 - b^2*x^2)^(1/2))/(9*
b*(a + b*x)^5) + (a^2 - b^2*x^2)^(1/2)/(105*a^2*b*(a + b*x)^3) + (2*(a^2 -
 b^2*x^2)^(1/2))/(315*a^3*b*(a + b*x)^2) + (2*(a^2 - b^2*x^2)^(1/2))/(315*
a^4*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^6}\,dx = \frac{70\sqrt{-b^2x^2+a^2}\,a^4+23\sqrt{-b^2x^2+a^2}\,a^3bx+51\sqrt{-b^2x^2+a^2}\,a^2b^2x^2+38\sqrt{-b^2x^2+a^2}\,a\,b^3x^3+10\sqrt{-b^2x}}{315a^4b\left(\sqrt{-b^2x^2+a^2}\,a^4+4\sqrt{-b^2x^2+a^2}\,a^3bx+6\sqrt{-b^2x^2+a^2}\,a^2b^2x^2+4\sqrt{-b^2x^2+a^2}\,a\,b^3x^3+\sqrt{}\right.}$$

input
```
int((-b^2*x^2+a^2)^(1/2)/(b*x+a)^6,x)
```

output
```
(70*sqrt(a**2 - b**2*x**2)*a**4 + 23*sqrt(a**2 - b**2*x**2)*a**3*b*x + 51*
sqrt(a**2 - b**2*x**2)*a**2*b**2*x**2 + 38*sqrt(a**2 - b**2*x**2)*a*b**3*x
**3 + 10*sqrt(a**2 - b**2*x**2)*b**4*x**4 - 70*a**5 + 23*a**4*b*x - 124*a*
*3*b**2*x**2 - 131*a**2*b**3*x**3 - 68*a*b**4*x**4 - 14*b**5*x**5)/(315*a*
*4*b*(sqrt(a**2 - b**2*x**2)*a**4 + 4*sqrt(a**2 - b**2*x**2)*a**3*b*x + 6*
sqrt(a**2 - b**2*x**2)*a**2*b**2*x**2 + 4*sqrt(a**2 - b**2*x**2)*a*b**3*x*
*3 + sqrt(a**2 - b**2*x**2)*b**4*x**4 - a**5 - 5*a**4*b*x - 10*a**3*b**2*x
**2 - 10*a**2*b**3*x**3 - 5*a*b**4*x**4 - b**5*x**5))
```

3.49 $\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{9}{16}a^5x\sqrt{a^2-b^2x^2}+\frac{3}{8}a^3x\left(a^2-b^2x^2\right)^{3/2} -\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b}-\frac{3a(12a+5bx)\left(a^2-b^2x^2\right)^{5/2}}{70b}+\frac{9a^7\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{16b}$$

output
```
9/16*a^5*x*(-b^2*x^2+a^2)^(1/2)+3/8*a^3*x*(-b^2*x^2+a^2)^(3/2)-1/7*(b*x+a)
^2*(-b^2*x^2+a^2)^(5/2)/b-3/70*a*(5*b*x+12*a)*(-b^2*x^2+a^2)^(5/2)/b+9/16*
a^7*arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{\sqrt{a^2-b^2x^2}(-368a^6+245a^5bx+656a^4b^2x^2+350a^3b^3x^3-208a^2b^4x^4-280ab^5x^5-80b^6x}{560b} -\frac{9a^7\log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{16\sqrt{-b^2}}$$

input
```
Integrate[(a + b*x)^3*(a^2 - b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-368*a^6 + 245*a^5*b*x + 656*a^4*b^2*x^2 + 350*a^3*b
^3*x^3 - 208*a^2*b^4*x^4 - 280*a*b^5*x^5 - 80*b^6*x^6))/(560*b) - (9*a^7*L
og[-(Sqrt[-b^2]*x) + Sqrt[a^2 - b^2*x^2]])/(16*Sqrt[-b^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {469, 469, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 469}$$

$$\frac{9}{7}a\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx-\frac{(a+bx)^2 \left(a^2-b^2x^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{ 469}$$

$$\frac{9}{7}a\left(\frac{7}{6}a\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} dx-\frac{(a+bx) \left(a^2-b^2x^2\right)^{5/2}}{6b}\right)-\frac{(a+bx)^2 \left(a^2-b^2x^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{9}{7}a\left(\frac{7}{6}a\left(a\int \left(a^2-b^2x^2\right)^{3/2} dx-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx) \left(a^2-b^2x^2\right)^{5/2}}{6b}\right)-\frac{(a+bx)^2 \left(a^2-b^2x^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{9}{7}a\left(\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\int\sqrt{a^2-b^2x^2}dx+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}\right)-\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 211

$$\frac{9}{7}a\left(\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx}{}\right.$$
$$\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 224

$$\frac{9}{7}a\left(\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)\right.$$
$$\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 216

$$\frac{9}{7}a\left(\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+}{}\right.$$
$$\frac{(a+bx)^2\left(a^2-b^2x^2\right)^{5/2}}{7b}$$

input
```
Int[(a + b*x)^3*(a^2 - b^2*x^2)^(3/2),x]
```

output
```
-1/7*((a + b*x)^2*(a^2 - b^2*x^2)^(5/2))/b + (9*a*(-1/6*((a + b*x)*(a^2 -
b^2*x^2)^(5/2))/b + (7*a*(-1/5*(a^2 - b^2*x^2)^(5/2)/b + a*((x*(a^2 - b^2*
x^2)^(3/2))/4 + (3*a^2*((x*Sqrt[a^2 - b^2*x^2])/2 + (a^2*ArcTan[(b*x)/Sqrt
[a^2 - b^2*x^2]])/(2*b)))/4)))/6))/7
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(80b^6x^6+280a\,b^5x^5+208a^2b^4x^4-350a^3b^3x^3-656a^4b^2x^2-245a^5bx+368a^6)\sqrt{-b^2x^2+a^2}}{560b}+\frac{9a^7\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{16\sqrt{b^2}}$
default	$a^3\left(\frac{x(-b^2x^2+a^2)^{\frac{3}{2}}}{4}+\frac{3a^2\left(\frac{x\sqrt{-b^2x^2+a^2}}{2}+\frac{a^2\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{2\sqrt{b^2}}\right)}{4}\right)+b^3\left(-\frac{x^2(-b^2x^2+a^2)^{\frac{5}{2}}}{7b^2}-\frac{2a^2(-b^2x^2+a^2)^{\frac{5}{2}}}{35b^4}\right.$

input
```
int((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/560*(80*b^6*x^6+280*a*b^5*x^5+208*a^2*b^4*x^4-350*a^3*b^3*x^3-656*a^4*b
^2*x^2-245*a^5*b*x+368*a^6)/b*(-b^2*x^2+a^2)^(1/2)+9/16*a^7/(b^2)^(1/2)*ar
ctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int (a+bx)^3\left(a^2-b^2x^2\right)^{3/2}\,dx =$$
$$-\frac{630\,a^7\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+(80\,b^6x^6+280\,ab^5x^5+208\,a^2b^4x^4-350\,a^3b^3x^3-656\,a^4b^2x^2-245\,a^5bx}{560\,b}$$

input
```
integrate((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/560*(630*a^7*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (80*b^6*x^6 +
280*a*b^5*x^5 + 208*a^2*b^4*x^4 - 350*a^3*b^3*x^3 - 656*a^4*b^2*x^2 - 245*
a^5*b*x + 368*a^6)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int (a+bx)^3 \left(a^2 -b^2x^2\right)^{3/2}\, dx = \begin{cases} \frac{9a^7\left(\begin{cases} \frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for } a^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise} \end{cases}\right)}{16} + \sqrt{a^2-b^2x^2}\left(-\frac{23a^6}{35b} + \frac{7a^5x}{16} + \frac{41a^4bx^2}{35} + \frac{5a^3\cdots}{} \right. \\ (a^2)^{\frac{3}{2}}\left(\begin{cases} a^3x & \text{for } b=0 \\ \frac{(a+bx)^4}{4b} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((b*x+a)**3*(-b**2*x**2+a**2)**(3/2),x)
```

output
```
Piecewise((9*a**7*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b**
2*x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/16
+ sqrt(a**2 - b**2*x**2)*(-23*a**6/(35*b) + 7*a**5*x/16 + 41*a**4*b*x**2/3
5 + 5*a**3*b**2*x**3/8 - 13*a**2*b**3*x**4/35 - a*b**4*x**5/2 - b**5*x**6/
7), Ne(b**2, 0)), ((a**2)**(3/2)*Piecewise((a**3*x, Eq(b, 0)), ((a + b*x)*
*4/(4*b), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2}\, dx = \frac{9\,a^7\arcsin\left(\frac{bx}{a}\right)}{16\,b} + \frac{9}{16}\sqrt{-b^2x^2+a^2}a^5x + \frac{3}{8}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^3x - \frac{1}{7}\left(-b^2x^2+a^2\right)^{\frac{5}{2}}bx^2 - \frac{1}{2}\left(-b^2x^2+a^2\right)^{\frac{5}{2}}ax - \frac{23\left(-b^2x^2+a^2\right)^{\frac{5}{2}}a^2}{35\,b}$$

input
```
integrate((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
9/16*a^7*arcsin(b*x/a)/b + 9/16*sqrt(-b^2*x^2 + a^2)*a^5*x + 3/8*(-b^2*x^2
 + a^2)^(3/2)*a^3*x - 1/7*(-b^2*x^2 + a^2)^(5/2)*b*x^2 - 1/2*(-b^2*x^2 + a
^2)^(5/2)*a*x - 23/35*(-b^2*x^2 + a^2)^(5/2)*a^2/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{9\,a^7 \arcsin\left(\frac{bx}{a}\right) \operatorname{sgn}(a) \operatorname{sgn}(b)}{16\,|b|}$$
$$-\frac{1}{560}\left(\frac{368\,a^6}{b} - \left(245\,a^5 + 2\left(328\,a^4b + \left(175\,a^3b^2 - 4\left(26\,a^2b^3 + 5\left(2\,b^5x + 7\,ab^4\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-b^2x^2+a^2}$$

input
```
integrate((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
9/16*a^7*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/560*(368*a^6/b - (245*a^5
+ 2*(328*a^4*b + (175*a^3*b^2 - 4*(26*a^2*b^3 + 5*(2*b^5*x + 7*a*b^4)*x)*x
)*x)*x)*x)*sqrt(-b^2*x^2 + a^2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2} \, dx = \int \left(a^2-b^2\,x^2\right)^{3/2}\,(a+b\,x)^3 \, dx$$

input
```
int((a^2 - b^2*x^2)^(3/2)*(a + b*x)^3,x)
```

output
```
int((a^2 - b^2*x^2)^(3/2)*(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.25

$$\int (a+bx)^3 \left(a^2-b^2x^2\right)^{3/2}\,dx = \frac{315asin\left(\frac{bx}{a}\right)a^7 - 368\sqrt{-b^2x^2+a^2}\,a^6 + 245\sqrt{-b^2x^2+a^2}\,a^5bx + 656\sqrt{-b^2x^2+a^2}\,a^4b^2x^2 -}{}$$

input
```
int((b*x+a)^3*(-b^2*x^2+a^2)^(3/2),x)
```

output
```
(315*asin((b*x)/a)*a**7 - 368*sqrt(a**2 - b**2*x**2)*a**6 + 245*sqrt(a**2
- b**2*x**2)*a**5*b*x + 656*sqrt(a**2 - b**2*x**2)*a**4*b**2*x**2 + 350*sq
rt(a**2 - b**2*x**2)*a**3*b**3*x**3 - 208*sqrt(a**2 - b**2*x**2)*a**2*b**4
*x**4 - 280*sqrt(a**2 - b**2*x**2)*a*b**5*x**5 - 80*sqrt(a**2 - b**2*x**2)
*b**6*x**6 + 368*a**7)/(560*b)
```

3.50 $\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx = \frac{7}{16}a^4x\sqrt{a^2-b^2x^2} + \frac{7}{24}a^2x\left(a^2-b^2x^2\right)^{3/2} - \frac{(12a+5bx)\left(a^2-b^2x^2\right)^{5/2}}{30b} + \frac{7a^6\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{16b}$$

output
```
7/16*a^4*x*(-b^2*x^2+a^2)^(1/2)+7/24*a^2*x*(-b^2*x^2+a^2)^(3/2)-1/30*(5*b*
x+12*a)*(-b^2*x^2+a^2)^(5/2)/b+7/16*a^6*arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx = \frac{\sqrt{a^2-b^2x^2}(-96a^5+135a^4bx+192a^3b^2x^2+10a^2b^3x^3-96ab^4x^4-40b^5x^5) - 210a^6 \text{arcta}}{240b}$$

input
```
Integrate[(a + b*x)^2*(a^2 - b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-96*a^5 + 135*a^4*b*x + 192*a^3*b^2*x^2 + 10*a^2*b^3
*x^3 - 96*a*b^4*x^4 - 40*b^5*x^5) - 210*a^6*ArcTan[(b*x)/(Sqrt[a^2] - Sqrt
[a^2 - b^2*x^2])])/(240*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {469, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{6}a\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} dx-\frac{(a+bx) \left(a^2-b^2x^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{6}a\left(a\int \left(a^2-b^2x^2\right)^{3/2} dx-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx) \left(a^2-b^2x^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\int \sqrt{a^2-b^2x^2}dx+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx) \left(a^2-b^2x^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx) \left(a^2-b^2x^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int\frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{6}a\left(a\left(\frac{3}{4}a^2\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}\right)-\frac{(a+bx)\left(a^2-b^2x^2\right)^{5/2}}{6b}$$

input
```
Int[(a + b*x)^2*(a^2 - b^2*x^2)^(3/2),x]
```

output
```
-1/6*((a + b*x)*(a^2 - b^2*x^2)^(5/2))/b + (7*a*(-1/5*(a^2 - b^2*x^2)^(5/2
)/b + a*((x*(a^2 - b^2*x^2)^(3/2))/4 + (3*a^2*((x*Sqrt[a^2 - b^2*x^2])/2 +
 (a^2*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b)))/4)))/6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\left(40 b^{5} x^{5}+96 a\,b^{4} x^{4}-10 a^{2} b^{3} x^{3}-192 a^{3} b^{2} x^{2}-135 a^{4} b x+96 a^{5}\right) \sqrt{-b^{2} x^{2}+a^{2}}}{240 b}+\frac{7 a^{6} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{16 \sqrt{b^{2}}}$
default	$a^{2} \left(\frac{x \left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{4}+\frac{3 a^{2} \left(\frac{x \sqrt{-b^{2} x^{2}+a^{2}}}{2}+\frac{a^{2} \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{2 \sqrt{b^{2}}}\right)}{4}\right)+b^{2} \left(-\frac{x \left(-b^{2} x^{2}+a^{2}\right)^{\frac{5}{2}}}{6 b^{2}}+\frac{a^{2} \left(\frac{x \left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{4}\right.}{}\right.$

input
```
int((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(40*b^5*x^5+96*a*b^4*x^4-10*a^2*b^3*x^3-192*a^3*b^2*x^2-135*a^4*b*x
+96*a^5)/b*(-b^2*x^2+a^2)^(1/2)+7/16*a^6/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/
(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx = -\frac{210\, a^6 \arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right) + (40\, b^5x^5 + 96\, ab^4x^4 - 10\, a^2b^3x^3 - 192\, a^3b^2x^2 - 135\, a^4bx + 96\, a^5)\sqrt{-b^2}}{240\, b}$$

input
```
integrate((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/240*(210*a^6*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (40*b^5*x^5 +
96*a*b^4*x^4 - 10*a^2*b^3*x^3 - 192*a^3*b^2*x^2 - 135*a^4*b*x + 96*a^5)*sq
rt(-b^2*x^2 + a^2))/b
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.48

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx = \begin{cases} \frac{7a^6\left(\begin{cases} \frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for } a^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise} \end{cases}\right)}{16} + \sqrt{a^2-b^2x^2}\left(-\frac{2a^5}{5b} + \frac{9a^4x}{16} + \frac{4a^3bx^2}{5} + \frac{a^2b^2x}{24}\right. \\ (a^2)^{\frac{3}{2}}\left(\begin{cases} a^2x & \text{for } b=0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((b*x+a)**2*(-b**2*x**2+a**2)**(3/2),x)
```

output
```
Piecewise((7*a**6*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b**
2*x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/16
+ sqrt(a**2 - b**2*x**2)*(-2*a**5/(5*b) + 9*a**4*x/16 + 4*a**3*b*x**2/5 +
a**2*b**2*x**3/24 - 2*a*b**3*x**4/5 - b**4*x**5/6), Ne(b**2, 0)), ((a**2)*
*(3/2)*Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3*b), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx = \frac{7\,a^6 \arcsin\left(\frac{bx}{a}\right)}{16\,b} + \frac{7}{16}\sqrt{-b^2x^2+a^2}a^4x + \frac{7}{24}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}a^2x - \frac{1}{6}\left(-b^2x^2+a^2\right)^{\frac{5}{2}}x - \frac{2\left(-b^2x^2+a^2\right)^{\frac{5}{2}}a}{5\,b}$$

input
```
integrate((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
7/16*a^6*arcsin(b*x/a)/b + 7/16*sqrt(-b^2*x^2 + a^2)*a^4*x + 7/24*(-b^2*x^
2 + a^2)^(3/2)*a^2*x - 1/6*(-b^2*x^2 + a^2)^(5/2)*x - 2/5*(-b^2*x^2 + a^2)
^(5/2)*a/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} dx = \frac{7\,a^6 \arcsin\left(\frac{bx}{a}\right)\mathrm{sgn}(a)\,\mathrm{sgn}(b)}{16\,|b|} - \frac{1}{240}\left(\frac{96\,a^5}{b} - \left(135\,a^4 + 2\left(96\,a^3b + \left(5\,a^2b^2 - 4\left(5\,b^4x + 12\,ab^3\right)x\right)x\right)x\right)x\right)\sqrt{-b^2x^2+a^2}$$

input
```
integrate((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
7/16*a^6*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/240*(96*a^5/b - (135*a^4 +
 2*(96*a^3*b + (5*a^2*b^2 - 4*(5*b^4*x + 12*a*b^3)*x)*x)*x)*x)*sqrt(-b^2*x
^2 + a^2)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} \, dx=\int \left(a^2-b^2\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2 \,dx$$

input
```
int((a^2 - b^2*x^2)^(3/2)*(a + b*x)^2,x)
```

output
```
int((a^2 - b^2*x^2)^(3/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.38

$$\int (a+bx)^2 \left(a^2-b^2x^2\right)^{3/2} \, dx=\frac{105 asin\left(\frac{bx}{a}\right) a^6-96\sqrt{-b^2x^2+a^2}\,a^5+135\sqrt{-b^2x^2+a^2}\,a^4bx+192\sqrt{-b^2x^2+a^2}\,a^3b^2x^2+}{240b}$$

input
```
int((b*x+a)^2*(-b^2*x^2+a^2)^(3/2),x)
```

output
```
(105*asin((b*x)/a)*a**6 - 96*sqrt(a**2 - b**2*x**2)*a**5 + 135*sqrt(a**2 -
 b**2*x**2)*a**4*b*x + 192*sqrt(a**2 - b**2*x**2)*a**3*b**2*x**2 + 10*sqrt
(a**2 - b**2*x**2)*a**2*b**3*x**3 - 96*sqrt(a**2 - b**2*x**2)*a*b**4*x**4
- 40*sqrt(a**2 - b**2*x**2)*b**5*x**5 + 96*a**6)/(240*b)
```

3.51 $\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{3}{8}a^3x\sqrt{a^2-b^2x^2} + \frac{1}{4}ax\left(a^2-b^2x^2\right)^{3/2} - \frac{\left(a^2-b^2x^2\right)^{5/2}}{5b} + \frac{3a^5\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{8b}$$

output
```
3/8*a^3*x*(-b^2*x^2+a^2)^(1/2)+1/4*a*x*(-b^2*x^2+a^2)^(3/2)-1/5*(-b^2*x^2+
a^2)^(5/2)/b+3/8*a^5*arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{\sqrt{a^2-b^2x^2}(-8a^4+25a^3bx+16a^2b^2x^2-10ab^3x^3-8b^4x^4)}{40b} - \frac{3a^5\log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{8\sqrt{-b^2}}$$

input
```
Integrate[(a + b*x)*(a^2 - b^2*x^2)^(3/2),x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-8*a^4 + 25*a^3*b*x + 16*a^2*b^2*x^2 - 10*a*b^3*x^3
- 8*b^4*x^4))/(40*b) - (3*a^5*Log[-(Sqrt[-b^2]*x) + Sqrt[a^2 - b^2*x^2]])/
(8*Sqrt[-b^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{455}$$

$$a\int \left(a^2-b^2x^2\right)^{3/2} dx-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{3}{4}a^2\int \sqrt{a^2-b^2x^2}dx+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{3}{4}a^2\left(\frac{1}{2}a^2\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{216}$$

$$a\left(\frac{3}{4}a^2\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{1}{4}x\left(a^2-b^2x^2\right)^{3/2}\right)-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5b}$$

input
```
Int[(a + b*x)*(a^2 - b^2*x^2)^(3/2),x]
```

output
```
-1/5*(a^2 - b^2*x^2)^(5/2)/b + a*((x*(a^2 - b^2*x^2)^(3/2))/4 + (3*a^2*((x
*Sqrt[a^2 - b^2*x^2])/2 + (a^2*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b)))/
4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{(8b^4x^4+10a\,b^3x^3-16a^2b^2x^2-25a^3bx+8a^4)\sqrt{-b^2x^2+a^2}}{40b}+\frac{3a^5\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{8\sqrt{b^2}}$	94
default	$a\left(\frac{x(-b^2x^2+a^2)^{\frac{3}{2}}}{4}+\frac{3a^2\left(\frac{x\sqrt{-b^2x^2+a^2}}{2}+\frac{a^2\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{2\sqrt{b^2}}\right)}{4}\right)-\frac{(-b^2x^2+a^2)^{\frac{5}{2}}}{5b}$	96

input
```
int((b*x+a)*(-b^2*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(8*b^4*x^4+10*a*b^3*x^3-16*a^2*b^2*x^2-25*a^3*b*x+8*a^4)/b*(-b^2*x^2
+a^2)^(1/2)+3/8*a^5/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int (a+bx)\left(a^2-b^2x^2\right)^{3/2}\,dx = -\frac{30\,a^5\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+(8\,b^4x^4+10\,ab^3x^3-16\,a^2b^2x^2-25\,a^3bx+8\,a^4)\sqrt{-b^2x^2+a^2}}{40\,b}$$

input
```
integrate((b*x+a)*(-b^2*x^2+a^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/40*(30*a^5*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (8*b^4*x^4 + 10*
a*b^3*x^3 - 16*a^2*b^2*x^2 - 25*a^3*b*x + 8*a^4)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int (a+bx)\left(a^2-b^2x^2\right)^{3/2}\,dx=\begin{cases}\frac{3a^5\left(\begin{cases}\frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for } a^2\neq 0\\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise}\end{cases}\right)}{8}+\sqrt{a^2-b^2x^2}\left(-\frac{a^4}{5b}+\frac{5a^3x}{8}+\frac{2a^2bx^2}{5}-\frac{ab^2x^3}{4}\right. \\ \left(ax+\frac{bx^2}{2}\right)\left(a^2\right)^{\frac{3}{2}} \end{cases}$$

input
```
integrate((b*x+a)*(-b**2*x**2+a**2)**(3/2),x)
```

output
```
Piecewise((3*a**5*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b**
2*x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/8 +
 sqrt(a**2 - b**2*x**2)*(-a**4/(5*b) + 5*a**3*x/8 + 2*a**2*b*x**2/5 - a*b*
*2*x**3/4 - b**3*x**4/5), Ne(b**2, 0)), ((a*x + b*x**2/2)*(a**2)**(3/2), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int (a+bx)\left(a^2-b^2x^2\right)^{3/2}\,dx=\frac{3\,a^5\arcsin\left(\frac{bx}{a}\right)}{8\,b}+\frac{3}{8}\sqrt{-b^2x^2+a^2}a^3x+\frac{1}{4}\left(-b^2x^2+a^2\right)^{\frac{3}{2}}ax-\frac{\left(-b^2x^2+a^2\right)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate((b*x+a)*(-b^2*x^2+a^2)^(3/2),x, algorithm="maxima")
```

output
```
3/8*a^5*arcsin(b*x/a)/b + 3/8*sqrt(-b^2*x^2 + a^2)*a^3*x + 1/4*(-b^2*x^2 +
 a^2)^(3/2)*a*x - 1/5*(-b^2*x^2 + a^2)^(5/2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{3\, a^5 \arcsin\left(\frac{bx}{a}\right) \operatorname{sgn}(a) \operatorname{sgn}(b)}{8\, |b|} - \frac{1}{40} \sqrt{-b^2x^2+a^2} \left(\frac{8\, a^4}{b} - \left(25\, a^3 + 2\left(8\, a^2 b - \left(4\, b^3 x + 5\, ab^2\right) x\right) x\right) x\right)$$

input
```
integrate((b*x+a)*(-b^2*x^2+a^2)^(3/2),x, algorithm="giac")
```

output
```
3/8*a^5*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/40*sqrt(-b^2*x^2 + a^2)*(8*
a^4/b - (25*a^3 + 2*(8*a^2*b - (4*b^3*x + 5*a*b^2)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{a\, x\, \left(a^2-b^2\, x^2\right)^{3/2} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2};\ \frac{3}{2};\ \frac{b^2\, x^2}{a^2}\right)}{\left(1-\frac{b^2\, x^2}{a^2}\right)^{3/2}} - \frac{\left(a^2-b^2\, x^2\right)^{5/2}}{5\, b}$$

input
```
int((a^2 - b^2*x^2)^(3/2)*(a + b*x),x)
```

output
```
(a*x*(a^2 - b^2*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, (b^2*x^2)/a^2))/(1
- (b^2*x^2)/a^2)^(3/2) - (a^2 - b^2*x^2)^(5/2)/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

$$\int (a+bx) \left(a^2-b^2x^2\right)^{3/2} \, dx = \frac{15 asin\left(\frac{bx}{a}\right) a^5 - 8\sqrt{-b^2x^2+a^2}\, a^4 + 25\sqrt{-b^2x^2+a^2}\, a^3 bx + 16\sqrt{-b^2x^2+a^2}\, a^2 b^2 x^2 - 10\sqrt{}}{40b}$$

input
```
int((b*x+a)*(-b^2*x^2+a^2)^(3/2),x)
```

output
```
(15*asin((b*x)/a)*a**5 - 8*sqrt(a**2 - b**2*x**2)*a**4 + 25*sqrt(a**2 - b*
*2*x**2)*a**3*b*x + 16*sqrt(a**2 - b**2*x**2)*a**2*b**2*x**2 - 10*sqrt(a**
2 - b**2*x**2)*a*b**3*x**3 - 8*sqrt(a**2 - b**2*x**2)*b**4*x**4 + 8*a**5)/
(40*b)
```

3.52 $\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx} dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx} dx = \frac{1}{2}ax\sqrt{a^2-b^2x^2} + \frac{(a^2-b^2x^2)^{3/2}}{3b} + \frac{a^3 \arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

output
```
1/2*a*x*(-b^2*x^2+a^2)^(1/2)+1/3*(-b^2*x^2+a^2)^(3/2)/b+1/2*a^3*arctan(b*x
/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx} dx = \frac{(2a^2+3abx-2b^2x^2)\sqrt{a^2-b^2x^2}}{6b} - \frac{a^3 \log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{2\sqrt{-b^2}}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x),x]
```

output
```
((2*a^2 + 3*a*b*x - 2*b^2*x^2)*Sqrt[a^2 - b^2*x^2])/(6*b) - (a^3*Log[-(Sqr
t[-b^2]*x) + Sqrt[a^2 - b^2*x^2]])/(2*Sqrt[-b^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {466, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx}\,dx$$

$$\Big\downarrow \text{466}$$

$$a\int \sqrt{a^2-b^2x^2}dx+\frac{(a^2-b^2x^2)^{3/2}}{3b}$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{1}{2}a^2\int \frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{(a^2-b^2x^2)^{3/2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{2}a^2\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{(a^2-b^2x^2)^{3/2}}{3b}$$

$$\Big\downarrow \text{216}$$

$$a\left(\frac{a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2-b^2x^2}\right)+\frac{(a^2-b^2x^2)^{3/2}}{3b}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x),x]
```

output
```
(a^2 - b^2*x^2)^(3/2)/(3*b) + a*((x*Sqrt[a^2 - b^2*x^2])/2 + (a^2*ArcTan[(
b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{\left(-2b^2x^2+3abx+2a^2\right)\sqrt{-b^2x^2+a^2}}{6b}+\frac{a^3\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{2\sqrt{b^2}}$	72
default	$\frac{\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{3}+ab\left(-\frac{\left(-2b^2\left(x+\frac{a}{b}\right)+2ab\right)\sqrt{-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)}}{4b^2}+\frac{a^2\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)}}\right)}{2\sqrt{b^2}}\right)}{b}$	136

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*b^2*x^2+3*a*b*x+2*a^2)/b*(-b^2*x^2+a^2)^(1/2)+1/2*a^3/(b^2)^(1/2)*
arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx}\,dx = -\frac{6\,a^3\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+(2\,b^2x^2-3\,abx-2\,a^2)\sqrt{-b^2x^2+a^2}}{6\,b}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/6*(6*a^3*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + (2*b^2*x^2 - 3*a*b
*x - 2*a^2)*sqrt(-b^2*x^2 + a^2))/b
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.76

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx}\,dx = a\left(\begin{cases}\frac{a^2\left(\begin{cases}\frac{\log\left(-2b^2x+2\sqrt{-b^2}\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}} & \text{for}\ a^2\neq 0\\ \frac{x\log(x)}{\sqrt{-b^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a^2-b^2x^2}}{2} & \text{for}\ b^2\neq 0\\ x\sqrt{a^2} & \text{otherwise}\end{cases}\right) - b\left(\begin{cases}\sqrt{a^2-b^2x^2}\left(-\frac{a^2}{3b^2}+\frac{x^2}{3}\right) & \text{for}\ b^2\neq 0\\ \frac{x^2\sqrt{a^2}}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a),x)
```

output
```
a*Piecewise((a**2*Piecewise((log(-2*b**2*x + 2*sqrt(-b**2)*sqrt(a**2 - b**
2*x**2))/sqrt(-b**2), Ne(a**2, 0)), (x*log(x)/sqrt(-b**2*x**2), True))/2 +
 x*sqrt(a**2 - b**2*x**2)/2, Ne(b**2, 0)), (x*sqrt(a**2), True)) - b*Piece
wise((sqrt(a**2 - b**2*x**2)*(-a**2/(3*b**2) + x**2/3), Ne(b**2, 0)), (x**
2*sqrt(a**2)/2, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx}\,dx = -\frac{i\,a^3\arcsin\left(\frac{bx}{a}+2\right)}{2\,b}+\frac{1}{2}\,\sqrt{b^2x^2+4\,abx+3\,a^2}ax + \frac{\sqrt{b^2x^2+4\,abx+3\,a^2}a^2}{b}+\frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*I*a^3*arcsin(b*x/a + 2)/b + 1/2*sqrt(b^2*x^2 + 4*a*b*x + 3*a^2)*a*x +
 sqrt(b^2*x^2 + 4*a*b*x + 3*a^2)*a^2/b + 1/3*(-b^2*x^2 + a^2)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx}\,dx = \frac{a^3\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{2\,|b|}-\frac{1}{6}\,\sqrt{-b^2x^2+a^2}\left((2\,bx-3\,a)x-\frac{2\,a^2}{b}\right)$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*a^3*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 1/6*sqrt(-b^2*x^2 + a^2)*((2*
b*x - 3*a)*x - 2*a^2/b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx}\,dx=\int \frac{(a^2-b^2\,x^2)^{3/2}}{a+b\,x}\,dx$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x),x)
```

output
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{(a^2-b^2x^2)^{3/2}}{a+bx}\,dx=\frac{3asin\left(\frac{bx}{a}\right)a^3+2\sqrt{-b^2x^2+a^2}\,a^2+3\sqrt{-b^2x^2+a^2}\,abx-2\sqrt{-b^2x^2+a^2}\,b^2x^2-2a^3}{6b}$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a),x)
```

output
```
(3*asin((b*x)/a)*a**3 + 2*sqrt(a**2 - b**2*x**2)*a**2 + 3*sqrt(a**2 - b**2
*x**2)*a*b*x - 2*sqrt(a**2 - b**2*x**2)*b**2*x**2 - 2*a**3)/(6*b)
```

3.53 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2} dx = \frac{(4a-bx)\sqrt{a^2-b^2x^2}}{2b} + \frac{3a^2\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{2b}$$

output
```
1/2*(-b*x+4*a)*(-b^2*x^2+a^2)^(1/2)/b+3/2*a^2*arctan(b*x/(-b^2*x^2+a^2)^(1
/2))/b
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2} dx = \frac{(4a-bx)\sqrt{a^2-b^2x^2}}{2b} - \frac{3a^2\arctan\left(\frac{bx}{\sqrt{a^2}-\sqrt{a^2-b^2x^2}}\right)}{b}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^2,x]
```

output
```
((4*a - b*x)*Sqrt[a^2 - b^2*x^2])/(2*b) - (3*a^2*ArcTan[(b*x)/(Sqrt[a^2] -
 Sqrt[a^2 - b^2*x^2])])/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {466, 466, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^2}dx$$

$$\Big\downarrow \text{466}$$

$$\frac{3}{2}a\int \frac{\sqrt{a^2-b^2x^2}}{a+bx}dx+\frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}$$

$$\Big\downarrow \text{466}$$

$$\frac{3}{2}a\left(a\int \frac{1}{\sqrt{a^2-b^2x^2}}dx+\frac{\sqrt{a^2-b^2x^2}}{b}\right)+\frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}a\left(a\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}+\frac{\sqrt{a^2-b^2x^2}}{b}\right)+\frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}a\left(\frac{a\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}+\frac{\sqrt{a^2-b^2x^2}}{b}\right)+\frac{\left(a^2-b^2x^2\right)^{3/2}}{2b(a+bx)}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^2,x]
```

output
```
(a^2 - b^2*x^2)^(3/2)/(2*b*(a + b*x)) + (3*a*(Sqrt[a^2 - b^2*x^2]/b + (a*A
rcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(-bx+4a)\sqrt{-b^2x^2+a^2}}{2b}+\frac{3a^2\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{2\sqrt{b^2}}$
default	$\frac{\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{ab\left(x+\frac{a}{b}\right)^2}+\frac{3b\left(\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{3}+ab\left(-\frac{\left(-2b^2\left(x+\frac{a}{b}\right)+2ab\right)\sqrt{-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)}}{4b^2}+\frac{a^2\arctan\left(\frac{\cdots}{\sqrt{-b\cdots}}\right)}{\cdots}\right)\right)}{a}}{b^2}$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x+4*a)*(-b^2*x^2+a^2)^(1/2)/b+3/2*a^2/(b^2)^(1/2)*arctan((b^2)^(1/
2)*x/(-b^2*x^2+a^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2}\,dx = -\frac{6\,a^2\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\sqrt{-b^2x^2+a^2}(bx-4\,a)}{2\,b}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(6*a^2*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + sqrt(-b^2*x^2 + a^
2)*(b*x - 4*a))/b
```

Sympy [F]

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2}\,dx = \int \frac{(-(-a+bx)(a+bx))^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**2,x)
```

output
```
Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2}\,dx = \frac{3\,a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{2\,(b^2x+ab)}+\frac{3\sqrt{-b^2x^2+a^2}a}{2\,b}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/2*a^2*arcsin(b*x/a)/b + 1/2*(-b^2*x^2 + a^2)^(3/2)/(b^2*x + a*b) + 3/2*s
qrt(-b^2*x^2 + a^2)*a/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(53) = 106$.

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{(a^2 - b^2x^2)^{3/2}}{(a + bx)^2} \, dx =$$

$$-\frac{\left(12\,a^3b^3\arctan\left(\sqrt{\frac{2\,a}{bx+a}-1}\right)\operatorname{sgn}\left(\frac{1}{bx+a}\right)\operatorname{sgn}(b) - \frac{\left(5\,a^3b^3\left(\frac{2\,a}{bx+a}-1\right)^{\frac{3}{2}}\operatorname{sgn}\left(\frac{1}{bx+a}\right)\operatorname{sgn}(b)+3\,a^3b^3\sqrt{\frac{2\,a}{bx+a}-1}\operatorname{sgn}\left(\frac{1}{bx+a}\right)\operatorname{sgn}(\right.}{a^2}\right.}{4\,ab^5}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*(12*a^3*b^3*arctan(sqrt(2*a/(b*x + a) - 1))*sgn(1/(b*x + a))*sgn(b) -
 (5*a^3*b^3*(2*a/(b*x + a) - 1)^(3/2)*sgn(1/(b*x + a))*sgn(b) + 3*a^3*b^3*
sqrt(2*a/(b*x + a) - 1)*sgn(1/(b*x + a))*sgn(b))*(b*x + a)^2/a^2)*abs(b)/(
a*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2 - b^2x^2)^{3/2}}{(a + bx)^2} \, dx = \int \frac{(a^2 - b^2\,x^2)^{3/2}}{(a + b\,x)^2} \, dx$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^2,x)
```

output
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^2}\,dx = \frac{3asin\left(\frac{bx}{a}\right)a^2+4\sqrt{-b^2x^2+a^2}\,a-\sqrt{-b^2x^2+a^2}\,bx-4a^2}{2b}$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^2,x)
```

output
```
(3*asin((b*x)/a)*a**2 + 4*sqrt(a**2 - b**2*x**2)*a - sqrt(a**2 - b**2*x**2
)*b*x - 4*a**2)/(2*b)
```

3.54 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = -\frac{\sqrt{a^2-b^2x^2}}{b} - \frac{4a\sqrt{a^2-b^2x^2}}{b(a+bx)} - \frac{3a\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

output
```
-(-b^2*x^2+a^2)^(1/2)/b-4*a*(-b^2*x^2+a^2)^(1/2)/b/(b*x+a)-3*a*arctan(b*x/
(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = \frac{(-5a-bx)\sqrt{a^2-b^2x^2}}{b(a+bx)} + \frac{3a\log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^3,x]
```

output
```
((-5*a - b*x)*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) + (3*a*Log[-(Sqrt[-b^2]*x
) + Sqrt[a^2 - b^2*x^2]])/Sqrt[-b^2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {463, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{ 463}$$

$$-\int \frac{3a-bx}{\sqrt{a^2-b^2x^2}}dx-\frac{4a\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

$$\Big\downarrow \text{ 455}$$

$$-3a\int \frac{1}{\sqrt{a^2-b^2x^2}}dx-\frac{4a\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

$$\Big\downarrow \text{ 224}$$

$$-3a\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}-\frac{4a\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{3a\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}-\frac{4a\sqrt{a^2-b^2x^2}}{b(a+bx)}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^3,x]
```

output
```
-(Sqrt[a^2 - b^2*x^2]/b) - (4*a*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) - (3*a*
ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\sqrt{-b^{2} x^{2}+a^{2}}}{b}-\frac{3 a \arctan\left(\frac{\sqrt{b^{2}}\, x}{\sqrt{-b^{2} x^{2}+a^{2}}}\right)}{\sqrt{b^{2}}}-\frac{4 a \sqrt{-b^{2} \left(x+\frac{a}{b}\right)^{2}+2 a b \left(x+\frac{a}{b}\right)}}{b^{2} \left(x+\frac{a}{b}\right)}$
default	$\frac{-\frac{\left(-b^{2} \left(x+\frac{a}{b}\right)^{2}+2 a b \left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{a b \left(x+\frac{a}{b}\right)^{3}}-\frac{2 b \left(\frac{\left(-b^{2} \left(x+\frac{a}{b}\right)^{2}+2 a b \left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{a b \left(x+\frac{a}{b}\right)^{2}}+\frac{3 b \left(\frac{\left(-b^{2} \left(x+\frac{a}{b}\right)^{2}+2 a b \left(x+\frac{a}{b}\right)\right)^{\frac{3}{2}}}{3}+a b \left(-\frac{\left(-2 b^{2} \left(x+\frac{a}{b}\right)+2 a b\right) \sqrt{-b^{2} \left(\cdots\right.}}{4 b^{2}} \cdots\right.\right.}{a}\cdots\right.}{a}}{b^{3}}$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-(-b^2*x^2+a^2)^(1/2)/b-3*a/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2
)^(1/2))-4*a/b^2/(x+a/b)*(-b^2*(x+a/b)^2+2*a*b*(x+a/b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = -\frac{5\,abx+5\,a^2-6\,(abx+a^2)\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\sqrt{-b^2x^2+a^2}(bx+5\,a)}{b^2x+ab}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-(5*a*b*x + 5*a^2 - 6*(a*b*x + a^2)*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*
x)) + sqrt(-b^2*x^2 + a^2)*(b*x + 5*a))/(b^2*x + a*b)
```

Sympy [F]

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = \int \frac{(-(-a+bx)\,(a+bx))^{\frac{3}{2}}}{(a+bx)^3}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**3,x)
```

output
```
Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = -\frac{3\,a\arcsin\left(\frac{bx}{a}\right)}{b}+\frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{b^3x^2+2\,ab^2x+a^2b}-\frac{6\sqrt{-b^2x^2+a^2}a}{b^2x+ab}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-3*a*arcsin(b*x/a)/b + (-b^2*x^2 + a^2)^(3/2)/(b^3*x^2 + 2*a*b^2*x + a^2*b
) - 6*sqrt(-b^2*x^2 + a^2)*a/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = -\frac{3\,a\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{|b|} - \frac{\sqrt{-b^2x^2+a^2}}{b}+\frac{8\,a}{\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)|b|}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-3*a*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - sqrt(-b^2*x^2 + a^2)/b + 8*a/(((
a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = \int \frac{(a^2-b^2\,x^2)^{3/2}}{(a+b\,x)^3}\,dx$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^3,x)
```

output
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.60

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^3}\,dx = \frac{-3\sqrt{-b^2x^2+a^2}\,asin\left(\frac{bx}{a}\right)a+3asin\left(\frac{bx}{a}\right)a^2+3asin\left(\frac{bx}{a}\right)abx+8\sqrt{-b^2x^2+a^2}\,a+\sqrt{}}{b\left(\sqrt{-b^2x^2+a^2}-a-bx\right)}$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^3,x)
```

output
```
( - 3*sqrt(a**2 - b**2*x**2)*asin((b*x)/a)*a + 3*asin((b*x)/a)*a**2 + 3*as
in((b*x)/a)*a*b*x + 8*sqrt(a**2 - b**2*x**2)*a + sqrt(a**2 - b**2*x**2)*b*
x - 8*a**2 + a*b*x + b**2*x**2)/(b*(sqrt(a**2 - b**2*x**2) - a - b*x))
```

3.55 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4} dx = \frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)} - \frac{2(a^2-b^2x^2)^{3/2}}{3b(a+bx)^3} + \frac{\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

output
```
2*(-b^2*x^2+a^2)^(1/2)/b/(b*x+a)-2/3*(-b^2*x^2+a^2)^(3/2)/b/(b*x+a)^3+arct
an(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4} dx = \frac{4(a+2bx)\sqrt{a^2-b^2x^2}}{3b(a+bx)^2} - \frac{2\arctan\left(\frac{bx}{\sqrt{a^2}-\sqrt{a^2-b^2x^2}}\right)}{b}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^4,x]
```

output
```
(4*(a + 2*b*x)*Sqrt[a^2 - b^2*x^2])/(3*b*(a + b*x)^2) - (2*ArcTan[(b*x)/(S
qrt[a^2] - Sqrt[a^2 - b^2*x^2])])/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {465, 463, 25, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4}dx$$

$$\Big\downarrow \text{465}$$

$$-\int \frac{\sqrt{a^2-b^2x^2}}{(a+bx)^2}dx-\frac{2(a^2-b^2x^2)^{3/2}}{3b(a+bx)^3}$$

$$\Big\downarrow \text{463}$$

$$-\int -\frac{1}{\sqrt{a^2-b^2x^2}}dx-\frac{2(a^2-b^2x^2)^{3/2}}{3b(a+bx)^3}+\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{\sqrt{a^2-b^2x^2}}dx-\frac{2(a^2-b^2x^2)^{3/2}}{3b(a+bx)^3}+\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}-\frac{2(a^2-b^2x^2)^{3/2}}{3b(a+bx)^3}+\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}-\frac{2(a^2-b^2x^2)^{3/2}}{3b(a+bx)^3}+\frac{2\sqrt{a^2-b^2x^2}}{b(a+bx)}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^4,x]
```

output
```
(2*Sqrt[a^2 - b^2*x^2])/(b*(a + b*x)) - (2*(a^2 - b^2*x^2)^(3/2))/(3*b*(a
+ b*x)^3) + ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]]/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(75) = 150$.

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.51

method	result
default	$\frac{-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{3ab\left(x+\frac{a}{b}\right)^{4}}-\frac{b\left(-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{ab\left(x+\frac{a}{b}\right)^{3}}-\frac{2b\left(\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{ab\left(x+\frac{a}{b}\right)^{2}}+\frac{3b\left(\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)\cdots}{3}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{b^{4}}$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(-1/3/a/b/(x+a/b)^4*(-b^2*(x+a/b)^2+2*a*b*(x+a/b))^(5/2)-1/3*b/a*(-1
/a/b/(x+a/b)^3*(-b^2*(x+a/b)^2+2*a*b*(x+a/b))^(5/2)-2*b/a*(1/a/b/(x+a/b)^2
*(-b^2*(x+a/b)^2+2*a*b*(x+a/b))^(5/2)+3*b/a*(1/3*(-b^2*(x+a/b)^2+2*a*b*(x+
a/b))^(3/2)+a*b*(-1/4*(-2*b^2*(x+a/b)+2*a*b)/b^2*(-b^2*(x+a/b)^2+2*a*b*(x+
a/b))^(1/2)+1/2*a^2/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*(x+a/b)^2+2*a*b
*(x+a/b))^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4}\,dx = \frac{2\left(2\,b^2x^2+4\,abx+2\,a^2-3\,(b^2x^2+2\,abx+a^2)\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+2\sqrt{-b^2x}\right.}{3\,(b^3x^2+2\,ab^2x+a^2b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
2/3*(2*b^2*x^2 + 4*a*b*x + 2*a^2 - 3*(b^2*x^2 + 2*a*b*x + a^2)*arctan(-(a
- sqrt(-b^2*x^2 + a^2))/(b*x)) + 2*sqrt(-b^2*x^2 + a^2)*(2*b*x + a))/(b^3*
x^2 + 2*a*b^2*x + a^2*b)
```

Sympy [F]

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4}\,dx = \int \frac{(-(-a+bx)\,(a+bx))^{\frac{3}{2}}}{(a+bx)^4}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**4,x)
```

output
```
Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4}\,dx = -\frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{3\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} - \frac{2\sqrt{-b^2x^2+a^2}\,a}{3\,(b^3x^2+2\,ab^2x+a^2b)} + \frac{\arcsin\left(\frac{bx}{a}\right)}{b} + \frac{7\sqrt{-b^2x^2+a^2}}{3\,(b^2x+ab)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(-b^2*x^2 + a^2)^(3/2)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
- 2/3*sqrt(-b^2*x^2 + a^2)*a/(b^3*x^2 + 2*a*b^2*x + a^2*b) + arcsin(b*x/a)
/b + 7/3*sqrt(-b^2*x^2 + a^2)/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4}\,dx = \frac{\arcsin\left(\frac{bx}{a}\right)\operatorname{sgn}(a)\operatorname{sgn}(b)}{|b|} - \frac{8\left(\frac{3\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+1\right)}{3\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^3|b|}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x, algorithm="giac")
```

output
```
arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - 8/3*(3*(a*b + sqrt(-b^2*x^2 + a^2)*ab
s(b))/(b^2*x) + 1)/(((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^3*ab
s(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4}\,dx = \int \frac{(a^2-b^2\,x^2)^{3/2}}{(a+b\,x)^4}\,dx$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^4,x)
```

output
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^4}\,dx = \frac{3asin\left(\frac{bx}{a}\right)\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right)^3+9asin\left(\frac{bx}{a}\right)\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right)^2+9asin\left(\frac{bx}{a}\right)\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right.}{3b\left(\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right)^3+3\tan\left(\frac{asin\left(\frac{bx}{a}\right)}{2}\right)^2+3\tan\left(\frac{asin(}{2}\right.\right.}$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^4,x)
```

output
```
(3*asin((b*x)/a)*tan(asin((b*x)/a)/2)**3 + 9*asin((b*x)/a)*tan(asin((b*x)/
a)/2)**2 + 9*asin((b*x)/a)*tan(asin((b*x)/a)/2) + 3*asin((b*x)/a) + 24*tan
(asin((b*x)/a)/2) + 8)/(3*b*(tan(asin((b*x)/a)/2)**3 + 3*tan(asin((b*x)/a)
/2)**2 + 3*tan(asin((b*x)/a)/2) + 1))
```

3.56 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = -\frac{(a^2-b^2x^2)^{5/2}}{5ab(a+bx)^5}$$

output
```
-1/5*(-b^2*x^2+a^2)^(5/2)/a/b/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = -\frac{(a-bx)^2\sqrt{a^2-b^2x^2}}{5ab(a+bx)^3}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^5,x]
```

output
```
-1/5*((a - b*x)^2*Sqrt[a^2 - b^2*x^2])/(a*b*(a + b*x)^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^5}\,dx$$

$$\Big\downarrow \text{ 460}$$

$$-\frac{\left(a^2-b^2x^2\right)^{5/2}}{5ab(a+bx)^5}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^5,x]
```

output
```
-1/5*(a^2 - b^2*x^2)^(5/2)/(a*b*(a + b*x)^5)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{(-bx+a)\left(-b^2x^2+a^2\right)^{\frac{3}{2}}}{5(bx+a)^4 ba}$	36
orering	$-\frac{(-bx+a)\left(-b^2x^2+a^2\right)^{\frac{3}{2}}}{5(bx+a)^4 ba}$	36
default	$-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{5b^6a\left(x+\frac{a}{b}\right)^5}$	46
trager	$-\frac{\left(b^2x^2-2abx+a^2\right)\sqrt{-b^2x^2+a^2}}{5a(bx+a)^3 b}$	46

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/5/(b*x+a)^4*(-b*x+a)/b/a*(-b^2*x^2+a^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = -\frac{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3+(b^2x^2-2\,abx+a^2)\sqrt{-b^2x^2+a^2}}{5\,(ab^4x^3+3\,a^2b^3x^2+3\,a^3b^2x+a^4b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/5*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3 + (b^2*x^2 - 2*a*b*x + a^2)*
sqrt(-b^2*x^2 + a^2))/(a*b^4*x^3 + 3*a^2*b^3*x^2 + 3*a^3*b^2*x + a^4*b)
```

Sympy [F]

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = \int \frac{\left(-(-a+bx)(a+bx)\right)^{\frac{3}{2}}}{(a+bx)^5}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**5,x)
```

output
```
Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 5.42

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = -\frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b} + \frac{6\sqrt{-b^2x^2+a^2}a}{5\,(b^4x^3+3\,ab^3x^2+3\,a^2b^2x+a^3b)} - \frac{\sqrt{-b^2x^2+a^2}}{5\,(b^3x^2+2\,ab^2x+a^2b)} - \frac{\sqrt{-b^2x^2+a^2}}{5\,(ab^2x+a^2b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
-(-b^2*x^2 + a^2)^(3/2)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2
*x + a^4*b) + 6/5*sqrt(-b^2*x^2 + a^2)*a/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^
2*x + a^3*b) - 1/5*sqrt(-b^2*x^2 + a^2)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/
5*sqrt(-b^2*x^2 + a^2)/(a*b^2*x + a^2*b)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.85

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx =$$
$$-\frac{1}{15}\left(-\frac{3i\,\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}(b)}{ab^2}+\frac{\left(3\left(\frac{2a}{bx+a}-1\right)^{\frac{5}{2}}+10\left(\frac{2a}{bx+a}-1\right)^{\frac{3}{2}}+15\sqrt{\frac{2a}{bx+a}-1}\right)\mathrm{sgn}\left(\frac{1}{bx+a}\right)\mathrm{sgn}(b)-10\,(\right.$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/15*(-3*I*sgn(1/(b*x + a))*sgn(b)/(a*b^2) + ((3*(2*a/(b*x + a) - 1)^(5/2
) + 10*(2*a/(b*x + a) - 1)^(3/2) + 15*sqrt(2*a/(b*x + a) - 1))*sgn(1/(b*x
+ a))*sgn(b) - 10*((2*a/(b*x + a) - 1)^(3/2) + 3*sqrt(2*a/(b*x + a) - 1))*
sgn(1/(b*x + a))*sgn(b) + 15*sqrt(2*a/(b*x + a) - 1)*sgn(1/(b*x + a))*sgn(
b))/(a*b^2))*abs(b)
```

Mupad [B] (verification not implemented)

Time = 6.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = -\frac{\sqrt{a^2-b^2\,x^2}\,(a-b\,x)^2}{5\,a\,b\,(a+b\,x)^3}$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^5,x)
```

output
```
-((a^2 - b^2*x^2)^(1/2)*(a - b*x)^2)/(5*a*b*(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}\,dx = \frac{\sqrt{-b^2x^2+a^2}\,(-b^2x^2+2abx-a^2)}{5ab\,(b^3x^3+3a\,b^2x^2+3a^2bx+a^3)}$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^5,x)
```

output
```
(sqrt(a**2 - b**2*x**2)*( - a**2 + 2*a*b*x - b**2*x**2))/(5*a*b*(a**3 + 3*
a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.57 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6} dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6} dx = -\frac{(a^2-b^2x^2)^{5/2}}{7ab(a+bx)^6} - \frac{(a^2-b^2x^2)^{5/2}}{35a^2b(a+bx)^5}$$

output
```
-1/7*(-b^2*x^2+a^2)^(5/2)/a/b/(b*x+a)^6-1/35*(-b^2*x^2+a^2)^(5/2)/a^2/b/(b
*x+a)^5
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6} dx = -\frac{(a-bx)^2(6a+bx)\sqrt{a^2-b^2x^2}}{35a^2b(a+bx)^4}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^6,x]
```

output
```
-1/35*((a - b*x)^2*(6*a + b*x)*Sqrt[a^2 - b^2*x^2])/(a^2*b*(a + b*x)^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^6}\,dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^5}dx}{7a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{7ab(a+bx)^6}$$

$$\Big\downarrow \text{ 460}$$

$$-\frac{\left(a^2-b^2x^2\right)^{5/2}}{35a^2b(a+bx)^5}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{7ab(a+bx)^6}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^6,x]
```

output
```
-1/7*(a^2 - b^2*x^2)^(5/2)/(a*b*(a + b*x)^6) - (a^2 - b^2*x^2)^(5/2)/(35*a
^2*b*(a + b*x)^5)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(-bx+a)(bx+6a)\left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{35(bx+a)^{5} a^{2} b}$	43
orering	$-\frac{(-bx+a)(bx+6a)\left(-b^{2} x^{2}+a^{2}\right)^{\frac{3}{2}}}{35(bx+a)^{5} a^{2} b}$	43
trager	$-\frac{\left(b^{3} x^{3}+4a\, b^{2} x^{2}-11a^{2} b x+6a^{3}\right)\sqrt{-b^{2} x^{2}+a^{2}}}{35a^{2} (bx+a)^{4} b}$	59
default	$\frac{-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{7ab\left(x+\frac{a}{b}\right)^{6}}-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{35a^{2}\left(x+\frac{a}{b}\right)^{5}}}{b^{6}}$	93

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(-b*x+a)*(b*x+6*a)*(-b^2*x^2+a^2)^(3/2)/(b*x+a)^5/a^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6}\,dx = -\frac{6\,b^4x^4+24\,ab^3x^3+36\,a^2b^2x^2+24\,a^3bx+6\,a^4+(b^3x^3+4\,ab^2x^2-11\,a^2bx+6\,a^3)\sqrt{-b^2x^2+a^2}}{35\,(a^2b^5x^4+4\,a^3b^4x^3+6\,a^4b^3x^2+4\,a^5b^2x+a^6b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x, algorithm="fricas")
```

output
```
-1/35*(6*b^4*x^4 + 24*a*b^3*x^3 + 36*a^2*b^2*x^2 + 24*a^3*b*x + 6*a^4 + (b
^3*x^3 + 4*a*b^2*x^2 - 11*a^2*b*x + 6*a^3)*sqrt(-b^2*x^2 + a^2))/(a^2*b^5*
x^4 + 4*a^3*b^4*x^3 + 6*a^4*b^3*x^2 + 4*a^5*b^2*x + a^6*b)
```

Sympy [F]

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6}\,dx = \int \frac{(-(-a+bx)\,(a+bx))^{\frac{3}{2}}}{(a+bx)^6}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**6,x)
```

output
```
Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.81

$$\int \frac{(a^2 - b^2x^2)^{3/2}}{(a + bx)^6}\,dx = -\frac{(-b^2x^2 + a^2)^{\frac{3}{2}}}{2\,(b^6x^5 + 5\,ab^5x^4 + 10\,a^2b^4x^3 + 10\,a^3b^3x^2 + 5\,a^4b^2x + a^5b)} + \frac{3\,\sqrt{-b^2x^2 + a^2}a}{7\,(b^5x^4 + 4\,ab^4x^3 + 6\,a^2b^3x^2 + 4\,a^3b^2x + a^4b)} - \frac{3\,\sqrt{-b^2x^2 + a^2}}{70\,(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b)} - \frac{\sqrt{-b^2x^2 + a^2}}{35\,(ab^3x^2 + 2\,a^2b^2x + a^3b)} - \frac{\sqrt{-b^2x^2 + a^2}}{35\,(a^2b^2x + a^3b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/2*(-b^2*x^2 + a^2)^(3/2)/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a
^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b) + 3/7*sqrt(-b^2*x^2 + a^2)*a/(b^5*x^4 +
4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 3/70*sqrt(-b^2*x^2 +
a^2)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/35*sqrt(-b^2*x^2 +
a^2)/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) - 1/35*sqrt(-b^2*x^2 + a^2)/(a^2*b^
2*x + a^3*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(59) = 118$.

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.39

$$\int \frac{(a^2 - b^2x^2)^{3/2}}{(a + bx)^6}\,dx = \frac{2\left(\frac{7\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x} + \frac{91\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2} + \frac{70\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3} + \frac{140\left(ab+\sqrt{-b^2x^2+a^2}\cdots\right.}{b^8x^4}\cdots\right.}{35\,a^2\left(\frac{ab+\sqrt{-b^2x^2+a^2}|b|}{b^2x}+1\right)^7|b\cdots}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x, algorithm="giac")
```

output
```
2/35*(7*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 91*(a*b + sqrt(-b^2*
x^2 + a^2)*abs(b))^2/(b^4*x^2) + 70*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^3/
(b^6*x^3) + 140*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) + 35*(a*b
+ sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 35*(a*b + sqrt(-b^2*x^2 + a^
2)*abs(b))^6/(b^12*x^6) + 6)/(a^2*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^
2*x) + 1)^7*abs(b))
```

Mupad [B] (verification not implemented)

Time = 7.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6}\,dx = \frac{16\sqrt{a^2-b^2\,x^2}}{35\,b\,{(a+b\,x)}^3}-\frac{4\,a\,\sqrt{a^2-b^2\,x^2}}{7\,b\,{(a+b\,x)}^4} - \frac{\sqrt{a^2-b^2\,x^2}}{35\,a\,b\,{(a+b\,x)}^2}-\frac{\sqrt{a^2-b^2\,x^2}}{35\,a^2\,b\,\left(a+b\,x\right)}$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^6,x)
```

output
```
(16*(a^2 - b^2*x^2)^(1/2))/(35*b*(a + b*x)^3) - (4*a*(a^2 - b^2*x^2)^(1/2)
)/(7*b*(a + b*x)^4) - (a^2 - b^2*x^2)^(1/2)/(35*a*b*(a + b*x)^2) - (a^2 -
b^2*x^2)^(1/2)/(35*a^2*b*(a + b*x))
```

Reduce [F]

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6}\,dx = \int \frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{(bx+a)^6}dx$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x)
```

output
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^6,x)
```

3.58 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7} dx = -\frac{(a^2-b^2x^2)^{5/2}}{9ab(a+bx)^7} - \frac{2(a^2-b^2x^2)^{5/2}}{63a^2b(a+bx)^6} - \frac{2(a^2-b^2x^2)^{5/2}}{315a^3b(a+bx)^5}$$

output
```
-1/9*(-b^2*x^2+a^2)^(5/2)/a/b/(b*x+a)^7-2/63*(-b^2*x^2+a^2)^(5/2)/a^2/b/(b
*x+a)^6-2/315*(-b^2*x^2+a^2)^(5/2)/a^3/b/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7} dx = -\frac{(a-bx)^2\sqrt{a^2-b^2x^2}(47a^2+14abx+2b^2x^2)}{315a^3b(a+bx)^5}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^7,x]
```

output
```
-1/315*((a - b*x)^2*Sqrt[a^2 - b^2*x^2]*(47*a^2 + 14*a*b*x + 2*b^2*x^2))/(
a^3*b*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^6}dx}{9a}-\frac{(a^2-b^2x^2)^{5/2}}{9ab(a+bx)^7}$$

$$\Big\downarrow \text{461}$$

$$\frac{2\left(\frac{\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^5}dx}{7a}-\frac{(a^2-b^2x^2)^{5/2}}{7ab(a+bx)^6}\right)}{9a}-\frac{(a^2-b^2x^2)^{5/2}}{9ab(a+bx)^7}$$

$$\Big\downarrow \text{460}$$

$$\frac{2\left(-\frac{(a^2-b^2x^2)^{5/2}}{35a^2b(a+bx)^5}-\frac{(a^2-b^2x^2)^{5/2}}{7ab(a+bx)^6}\right)}{9a}-\frac{(a^2-b^2x^2)^{5/2}}{9ab(a+bx)^7}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^7,x]
```

output
```
-1/9*(a^2 - b^2*x^2)^(5/2)/(a*b*(a + b*x)^7) + (2*(-1/7*(a^2 - b^2*x^2)^(5
/2)/(a*b*(a + b*x)^6) - (a^2 - b^2*x^2)^(5/2)/(35*a^2*b*(a + b*x)^5)))/(9*
a)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{(-bx+a)\left(2b^{2}x^{2}+14abx+47a^{2}\right)\left(-b^{2}x^{2}+a^{2}\right)^{\frac{3}{2}}}{315(bx+a)^{6}a^{3}b}$	55
orering	$-\frac{(-bx+a)\left(2b^{2}x^{2}+14abx+47a^{2}\right)\left(-b^{2}x^{2}+a^{2}\right)^{\frac{3}{2}}}{315(bx+a)^{6}a^{3}b}$	55
trager	$-\frac{\left(2b^{4}x^{4}+10a\,b^{3}x^{3}+21a^{2}b^{2}x^{2}-80a^{3}bx+47a^{4}\right)\sqrt{-b^{2}x^{2}+a^{2}}}{315a^{3}(bx+a)^{5}b}$	71
default	$\frac{-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{9ab\left(x+\frac{a}{b}\right)^{7}}+\frac{2b\left(-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{7ab\left(x+\frac{a}{b}\right)^{6}}-\frac{\left(-b^{2}\left(x+\frac{a}{b}\right)^{2}+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{35a^{2}\left(x+\frac{a}{b}\right)^{5}}\right)}{9a}}{b^{7}}$	145

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(-b*x+a)*(2*b^2*x^2+14*a*b*x+47*a^2)*(-b^2*x^2+a^2)^(3/2)/(b*x+a)^6
/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.70

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}\,dx = -\frac{47\,b^5x^5+235\,ab^4x^4+470\,a^2b^3x^3+470\,a^3b^2x^2+235\,a^4bx+47\,a^5+(2\,b^4x^4+10\,ab^3x^3+21\,a^2b^2x^2-80}{315\,(a^3b^6x^5+5\,a^4b^5x^4+10\,a^5b^4x^3+10\,a^6b^3x^2+5\,a^7b^2x+a^8b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/315*(47*b^5*x^5 + 235*a*b^4*x^4 + 470*a^2*b^3*x^3 + 470*a^3*b^2*x^2 + 2
35*a^4*b*x + 47*a^5 + (2*b^4*x^4 + 10*a*b^3*x^3 + 21*a^2*b^2*x^2 - 80*a^3*
b*x + 47*a^4)*sqrt(-b^2*x^2 + a^2))/(a^3*b^6*x^5 + 5*a^4*b^5*x^4 + 10*a^5*
b^4*x^3 + 10*a^6*b^3*x^2 + 5*a^7*b^2*x + a^8*b)
```

Sympy [F]

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}\,dx = \int \frac{(-(-a+bx)\,(a+bx))^{\frac{3}{2}}}{(a+bx)^7}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**7,x)
```

output
```
Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(88) = 176.

Time = 0.04 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.42

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}\,dx = -\frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{3\,(b^7x^6+6\,ab^6x^5+15\,a^2b^5x^4+20\,a^3b^4x^3+15\,a^4b^3x^2+6\,a^5b^2x+a^6b)} + \frac{2\sqrt{-b^2x^2+a^2}a}{9\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)} - \frac{\sqrt{-b^2x^2+a^2}}{63\,(b^5x^4+4\,ab^4x^3+6\,a^2b^3x^2+4\,a^3b^2x+a^4b)} - \frac{\sqrt{-b^2x^2+a^2}}{105\,(ab^4x^3+3\,a^2b^3x^2+3\,a^3b^2x+a^4b)} - \frac{2\sqrt{-b^2x^2+a^2}}{315\,(a^2b^3x^2+2\,a^3b^2x+a^4b)} - \frac{2\sqrt{-b^2x^2+a^2}}{315\,(a^3b^2x+a^4b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/3*(-b^2*x^2 + a^2)^(3/2)/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a
^3*b^4*x^3 + 15*a^4*b^3*x^2 + 6*a^5*b^2*x + a^6*b) + 2/9*sqrt(-b^2*x^2 + a
^2)*a/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2
*x + a^5*b) - 1/63*sqrt(-b^2*x^2 + a^2)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3
*x^2 + 4*a^3*b^2*x + a^4*b) - 1/105*sqrt(-b^2*x^2 + a^2)/(a*b^4*x^3 + 3*a^
2*b^3*x^2 + 3*a^3*b^2*x + a^4*b) - 2/315*sqrt(-b^2*x^2 + a^2)/(a^2*b^3*x^2
 + 2*a^3*b^2*x + a^4*b) - 2/315*sqrt(-b^2*x^2 + a^2)/(a^3*b^2*x + a^4*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(88) = 176.

Time = 0.14 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.89

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}\,dx = \frac{2\left(\frac{108\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x} + \frac{1062\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2} + \frac{1638\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3} + \frac{3402\left(ab+\sqrt{-\cdots}\right.}{b^8\cdots}\right.}{3\cdots}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x, algorithm="giac")
```

output
```
2/315*(108*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1062*(a*b + sqrt(
-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 1638*(a*b + sqrt(-b^2*x^2 + a^2)*abs
(b))^3/(b^6*x^3) + 3402*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4) +
2520*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 2310*(a*b + sqrt(-
b^2*x^2 + a^2)*abs(b))^6/(b^12*x^6) + 630*(a*b + sqrt(-b^2*x^2 + a^2)*abs(
b))^7/(b^14*x^7) + 315*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^8/(b^16*x^8) +
47)/(a^3*((a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^9*abs(b))
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.41

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}\,dx = \frac{20\sqrt{a^2-b^2\,x^2}}{63\,b\,(a+b\,x)^4} - \frac{4\,a\,\sqrt{a^2-b^2\,x^2}}{9\,b\,(a+b\,x)^5} - \frac{\sqrt{a^2-b^2\,x^2}}{105\,a\,b\,(a+b\,x)^3} - \frac{2\sqrt{a^2-b^2\,x^2}}{315\,a^2\,b\,(a+b\,x)^2} - \frac{2\sqrt{a^2-b^2\,x^2}}{315\,a^3\,b\,(a+b\,x)}$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^7,x)
```

output
```
(20*(a^2 - b^2*x^2)^(1/2))/(63*b*(a + b*x)^4) - (4*a*(a^2 - b^2*x^2)^(1/2)
)/(9*b*(a + b*x)^5) - (a^2 - b^2*x^2)^(1/2)/(105*a*b*(a + b*x)^3) - (2*(a^
2 - b^2*x^2)^(1/2))/(315*a^2*b*(a + b*x)^2) - (2*(a^2 - b^2*x^2)^(1/2))/(3
15*a^3*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.22

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^7}\,dx = \frac{70\sqrt{-b^2x^2+a^2}\,a^4+12\sqrt{-b^2x^2+a^2}\,a^3bx+159\sqrt{-b^2x^2+a^2}\,a^2b^2x^2+102\sqrt{-b^2x^2}}{315a^3b\left(\sqrt{-b^2x^2+a^2}\,a^4+4\sqrt{-b^2x^2+a^2}\,a^3bx+6\sqrt{-b^2x^2+a^2}\,a^2b^2x^2+4\sqrt{-}\right.}$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^7,x)
```

output
```
(70*sqrt(a**2 - b**2*x**2)*a**4 + 12*sqrt(a**2 - b**2*x**2)*a**3*b*x + 159
*sqrt(a**2 - b**2*x**2)*a**2*b**2*x**2 + 102*sqrt(a**2 - b**2*x**2)*a*b**3
*x**3 + 25*sqrt(a**2 - b**2*x**2)*b**4*x**4 - 70*a**5 + 12*a**4*b*x - 331*
a**3*b**2*x**2 - 219*a**2*b**3*x**3 - 107*a*b**4*x**4 - 21*b**5*x**5)/(315
*a**3*b*(sqrt(a**2 - b**2*x**2)*a**4 + 4*sqrt(a**2 - b**2*x**2)*a**3*b*x +
 6*sqrt(a**2 - b**2*x**2)*a**2*b**2*x**2 + 4*sqrt(a**2 - b**2*x**2)*a*b**3
*x**3 + sqrt(a**2 - b**2*x**2)*b**4*x**4 - a**5 - 5*a**4*b*x - 10*a**3*b**
2*x**2 - 10*a**2*b**3*x**3 - 5*a*b**4*x**4 - b**5*x**5))
```

3.59 $\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8} \, dx = -\frac{(a^2-b^2x^2)^{5/2}}{11ab(a+bx)^8} - \frac{(a^2-b^2x^2)^{5/2}}{33a^2b(a+bx)^7} - \frac{2(a^2-b^2x^2)^{5/2}}{231a^3b(a+bx)^6} - \frac{2(a^2-b^2x^2)^{5/2}}{1155a^4b(a+bx)^5}$$

output
```
-1/11*(-b^2*x^2+a^2)^(5/2)/a/b/(b*x+a)^8-1/33*(-b^2*x^2+a^2)^(5/2)/a^2/b/(
b*x+a)^7-2/231*(-b^2*x^2+a^2)^(5/2)/a^3/b/(b*x+a)^6-2/1155*(-b^2*x^2+a^2)^
(5/2)/a^4/b/(b*x+a)^5
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8} \, dx = -\frac{(a-bx)^2\sqrt{a^2-b^2x^2}(152a^3+61a^2bx+16ab^2x^2+2b^3x^3)}{1155a^4b(a+bx)^6}$$

input
```
Integrate[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^8,x]
```

output
```
-1/1155*((a - b*x)^2*Sqrt[a^2 - b^2*x^2]*(152*a^3 + 61*a^2*b*x + 16*a*b^2*
x^2 + 2*b^3*x^3))/(a^4*b*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^8}dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{3\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^7}dx}{11a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{11ab(a+bx)^8}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{3\left(\frac{2\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^6}dx}{9a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}\right)}{11a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{11ab(a+bx)^8}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{\left(a^2-b^2x^2\right)^{3/2}}{(a+bx)^5}dx}{7a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{7ab(a+bx)^6}\right)}{9a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}\right)}{11a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{11ab(a+bx)^8}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{3\left(\frac{2\left(-\frac{\left(a^2-b^2x^2\right)^{5/2}}{35a^2b(a+bx)^5}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{7ab(a+bx)^6}\right)}{9a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{9ab(a+bx)^7}\right)}{11a}-\frac{\left(a^2-b^2x^2\right)^{5/2}}{11ab(a+bx)^8}$$

input
```
Int[(a^2 - b^2*x^2)^(3/2)/(a + b*x)^8,x]
```

output
```
-1/11*(a^2 - b^2*x^2)^(5/2)/(a*b*(a + b*x)^8) + (3*(-1/9*(a^2 - b^2*x^2)^(
5/2)/(a*b*(a + b*x)^7) + (2*(-1/7*(a^2 - b^2*x^2)^(5/2)/(a*b*(a + b*x)^6)
- (a^2 - b^2*x^2)^(5/2)/(35*a^2*b*(a + b*x)^5)))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.50

method	result
gosper	$-\frac{(-bx+a)(2b^3x^3+16a\,b^2x^2+61a^2bx+152a^3)(-b^2x^2+a^2)^{\frac{3}{2}}}{1155(bx+a)^7a^4b}$
orering	$-\frac{(-bx+a)(2b^3x^3+16a\,b^2x^2+61a^2bx+152a^3)(-b^2x^2+a^2)^{\frac{3}{2}}}{1155(bx+a)^7a^4b}$
trager	$-\frac{(2b^5x^5+12a\,b^4x^4+31a^2b^3x^3+46a^3b^2x^2-243a^4bx+152a^5)\sqrt{-b^2x^2+a^2}}{1155a^4(bx+a)^6b}$
default	$\frac{-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{11ab\left(x+\frac{a}{b}\right)^8}+\frac{3b\left(-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{9ab\left(x+\frac{a}{b}\right)^7}+\frac{2b\left(-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{7ab\left(x+\frac{a}{b}\right)^6}-\frac{\left(-b^2\left(x+\frac{a}{b}\right)^2+2ab\left(x+\frac{a}{b}\right)\right)^{\frac{5}{2}}}{35a^2\left(x+\frac{a}{b}\right)^5}\right.}{9a}\right.}{11a}}{b^8}$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(-b*x+a)*(2*b^3*x^3+16*a*b^2*x^2+61*a^2*b*x+152*a^3)*(-b^2*x^2+a^2
)^(3/2)/(b*x+a)^7/a^4/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.53

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8}\,dx =$$
$$-\frac{152\,b^6x^6+912\,ab^5x^5+2280\,a^2b^4x^4+3040\,a^3b^3x^3+2280\,a^4b^2x^2+912\,a^5bx+152\,a^6+(2\,b^5x^5+12\,ab^4x}{1155\,(a^4b^7x^6+6\,a^5b^6x^5+15\,a^6b^5x^4+20\,a^7b^4x^3+15\,a^8b^3x^2}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x, algorithm="fricas")
```

output
```
-1/1155*(152*b^6*x^6 + 912*a*b^5*x^5 + 2280*a^2*b^4*x^4 + 3040*a^3*b^3*x^3
 + 2280*a^4*b^2*x^2 + 912*a^5*b*x + 152*a^6 + (2*b^5*x^5 + 12*a*b^4*x^4 +
31*a^2*b^3*x^3 + 46*a^3*b^2*x^2 - 243*a^4*b*x + 152*a^5)*sqrt(-b^2*x^2 + a
^2))/(a^4*b^7*x^6 + 6*a^5*b^6*x^5 + 15*a^6*b^5*x^4 + 20*a^7*b^4*x^3 + 15*a
^8*b^3*x^2 + 6*a^9*b^2*x + a^10*b)
```

Sympy [F]

$$\int \frac{(a^2 - b^2x^2)^{3/2}}{(a+bx)^8}\,dx = \int \frac{(-(-a+bx)(a+bx))^{\frac{3}{2}}}{(a+bx)^8}\,dx$$

input
```
integrate((-b**2*x**2+a**2)**(3/2)/(b*x+a)**8,x)
```

output
```
Integral((-(-a + b*x)*(a + b*x))**(3/2)/(a + b*x)**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(117) = 234$.

Time = 0.04 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.31

$$\int \frac{(a^2 - b^2x^2)^{3/2}}{(a+bx)^8}\,dx =$$
$$-\frac{(-b^2x^2+a^2)^{\frac{3}{2}}}{4\,(b^8x^7 + 7\,ab^7x^6 + 21\,a^2b^6x^5 + 35\,a^3b^5x^4 + 35\,a^4b^4x^3 + 21\,a^5b^3x^2 + 7\,a^6b^2x + a^7b)}$$
$$+\frac{3\sqrt{-b^2x^2+a^2}a}{22\,(b^7x^6 + 6\,ab^6x^5 + 15\,a^2b^5x^4 + 20\,a^3b^4x^3 + 15\,a^4b^3x^2 + 6\,a^5b^2x + a^6b)}$$
$$-\frac{\sqrt{-b^2x^2+a^2}}{132\,(b^6x^5 + 5\,ab^5x^4 + 10\,a^2b^4x^3 + 10\,a^3b^3x^2 + 5\,a^4b^2x + a^5b)}$$
$$-\frac{\sqrt{-b^2x^2+a^2}}{231\,(ab^5x^4 + 4\,a^2b^4x^3 + 6\,a^3b^3x^2 + 4\,a^4b^2x + a^5b)}$$
$$-\frac{\sqrt{-b^2x^2+a^2}}{385\,(a^2b^4x^3 + 3\,a^3b^3x^2 + 3\,a^4b^2x + a^5b)}$$
$$-\frac{2\sqrt{-b^2x^2+a^2}}{1155\,(a^3b^3x^2 + 2\,a^4b^2x + a^5b)} - \frac{2\sqrt{-b^2x^2+a^2}}{1155\,(a^4b^2x + a^5b)}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x, algorithm="maxima")
```

output
```
-1/4*(-b^2*x^2 + a^2)^(3/2)/(b^8*x^7 + 7*a*b^7*x^6 + 21*a^2*b^6*x^5 + 35*a
^3*b^5*x^4 + 35*a^4*b^4*x^3 + 21*a^5*b^3*x^2 + 7*a^6*b^2*x + a^7*b) + 3/22
*sqrt(-b^2*x^2 + a^2)*a/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a^3*b
^4*x^3 + 15*a^4*b^3*x^2 + 6*a^5*b^2*x + a^6*b) - 1/132*sqrt(-b^2*x^2 + a^2
)/(b^6*x^5 + 5*a*b^5*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x +
 a^5*b) - 1/231*sqrt(-b^2*x^2 + a^2)/(a*b^5*x^4 + 4*a^2*b^4*x^3 + 6*a^3*b^
3*x^2 + 4*a^4*b^2*x + a^5*b) - 1/385*sqrt(-b^2*x^2 + a^2)/(a^2*b^4*x^3 + 3
*a^3*b^3*x^2 + 3*a^4*b^2*x + a^5*b) - 2/1155*sqrt(-b^2*x^2 + a^2)/(a^3*b^3
*x^2 + 2*a^4*b^2*x + a^5*b) - 2/1155*sqrt(-b^2*x^2 + a^2)/(a^4*b^2*x + a^5
*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(117) = 234$.

Time = 0.14 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.64

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8}\,dx = \frac{2\left(\frac{517\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)}{b^2x}+\frac{4895\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^2}{b^4x^2}+\frac{11220\left(ab+\sqrt{-b^2x^2+a^2}|b|\right)^3}{b^6x^3}+\frac{27060\left(ab+\right.}{}\right.}{}$$

input
```
integrate((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x, algorithm="giac")
```

output
```
2/1155*(517*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))/(b^2*x) + 4895*(a*b + sqrt
(-b^2*x^2 + a^2)*abs(b))^2/(b^4*x^2) + 11220*(a*b + sqrt(-b^2*x^2 + a^2)*a
bs(b))^3/(b^6*x^3) + 27060*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^4/(b^8*x^4)
 + 32802*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^5/(b^10*x^5) + 37422*(a*b + s
qrt(-b^2*x^2 + a^2)*abs(b))^6/(b^12*x^6) + 23100*(a*b + sqrt(-b^2*x^2 + a^
2)*abs(b))^7/(b^14*x^7) + 13860*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^8/(b^1
6*x^8) + 3465*(a*b + sqrt(-b^2*x^2 + a^2)*abs(b))^9/(b^18*x^9) + 1155*(a*b
 + sqrt(-b^2*x^2 + a^2)*abs(b))^10/(b^20*x^10) + 152)/(a^4*((a*b + sqrt(-b
^2*x^2 + a^2)*abs(b))/(b^2*x) + 1)^11*abs(b))
```

Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.28

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8}\,dx = \frac{8\sqrt{a^2-b^2\,x^2}}{33\,b\,(a+b\,x)^5}-\frac{4\,a\,\sqrt{a^2-b^2\,x^2}}{11\,b\,(a+b\,x)^6}-\frac{\sqrt{a^2-b^2\,x^2}}{231\,a\,b\,(a+b\,x)^4} -\frac{\sqrt{a^2-b^2\,x^2}}{385\,a^2\,b\,(a+b\,x)^3}-\frac{2\sqrt{a^2-b^2\,x^2}}{1155\,a^3\,b\,(a+b\,x)^2}-\frac{2\sqrt{a^2-b^2\,x^2}}{1155\,a^4\,b\,(a+b\,x)}$$

input
```
int((a^2 - b^2*x^2)^(3/2)/(a + b*x)^8,x)
```

output
```
(8*(a^2 - b^2*x^2)^(1/2))/(33*b*(a + b*x)^5) - (4*a*(a^2 - b^2*x^2)^(1/2))
/(11*b*(a + b*x)^6) - (a^2 - b^2*x^2)^(1/2)/(231*a*b*(a + b*x)^4) - (a^2 -
 b^2*x^2)^(1/2)/(385*a^2*b*(a + b*x)^3) - (2*(a^2 - b^2*x^2)^(1/2))/(1155*
a^3*b*(a + b*x)^2) - (2*(a^2 - b^2*x^2)^(1/2))/(1155*a^4*b*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.95

$$\int \frac{(a^2-b^2x^2)^{3/2}}{(a+bx)^8}\,dx = \frac{210\sqrt{-b^2x^2+a^2}\,a^5+47\sqrt{-b^2x^2+a^2}\,a^4bx+626\sqrt{-b^2x^2+a^2}\,a^3b^2x^2+611\sqrt{-b^2x}}{1155a^4b\left(\sqrt{-b^2x^2+a^2}\,a^5+5\sqrt{-b^2x^2+a^2}\,a^4bx+10\sqrt{-b^2x^2+a^2}\,a^3b^2x^2+\right.}$$

input
```
int((-b^2*x^2+a^2)^(3/2)/(b*x+a)^8,x)
```

output
```
(210*sqrt(a**2 - b**2*x**2)*a**5 + 47*sqrt(a**2 - b**2*x**2)*a**4*b*x + 62
6*sqrt(a**2 - b**2*x**2)*a**3*b**2*x**2 + 611*sqrt(a**2 - b**2*x**2)*a**2*
b**3*x**3 + 302*sqrt(a**2 - b**2*x**2)*a*b**4*x**4 + 60*sqrt(a**2 - b**2*x
**2)*b**5*x**5 - 210*a**6 + 47*a**5*b*x - 1159*a**4*b**2*x**2 - 1145*a**3*
b**3*x**3 - 851*a**2*b**4*x**4 - 338*a*b**5*x**5 - 56*b**6*x**6)/(1155*a**
4*b*(sqrt(a**2 - b**2*x**2)*a**5 + 5*sqrt(a**2 - b**2*x**2)*a**4*b*x + 10*
sqrt(a**2 - b**2*x**2)*a**3*b**2*x**2 + 10*sqrt(a**2 - b**2*x**2)*a**2*b**
3*x**3 + 5*sqrt(a**2 - b**2*x**2)*a*b**4*x**4 + sqrt(a**2 - b**2*x**2)*b**
5*x**5 - a**6 - 6*a**5*b*x - 15*a**4*b**2*x**2 - 20*a**3*b**3*x**3 - 15*a*
*2*b**4*x**4 - 6*a*b**5*x**5 - b**6*x**6))
```

3.60 $\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} dx = \frac{91}{256}d^9x\sqrt{d^2-e^2x^2} + \frac{91}{384}d^7x\left(d^2-e^2x^2\right)^{3/2} + \frac{91}{480}d^5x\left(d^2-e^2x^2\right)^{5/2} + \frac{13}{80}d^3x\left(d^2-e^2x^2\right)^{7/2} - \frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e} - \frac{13d(20d+9ex)\left(d^2-e^2x^2\right)^{9/2}}{990e} + \frac{91d^{11}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e}$$

output

```
91/256*d^9*x*(-e^2*x^2+d^2)^(1/2)+91/384*d^7*x*(-e^2*x^2+d^2)^(3/2)+91/480
*d^5*x*(-e^2*x^2+d^2)^(5/2)+13/80*d^3*x*(-e^2*x^2+d^2)^(7/2)-1/11*(e*x+d)^
2*(-e^2*x^2+d^2)^(9/2)/e-13/990*d*(9*e*x+20*d)*(-e^2*x^2+d^2)^(9/2)/e+91/2
56*d^11*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.94

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} dx = \frac{\sqrt{d^2-e^2x^2}(-44800d^{10}+81675d^9ex+167680d^8e^2x^2+12210d^7e^3x^3-222720d^6e^4x^4-14}{1267} - \frac{91d^{11}\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{256\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)^3*(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-44800*d^10 + 81675*d^9*e*x + 167680*d^8*e^2*x^2 + 1
2210*d^7*e^3*x^3 - 222720*d^6*e^4*x^4 - 142296*d^5*e^5*x^5 + 110080*d^4*e^
6*x^6 + 131472*d^3*e^7*x^7 + 1280*d^2*e^8*x^8 - 38016*d*e^9*x^9 - 11520*e^
10*x^10))/(126720*e) - (91*d^11*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]]
)/(256*Sqrt[-e^2])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {469, 469, 455, 211, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} dx$$

$$\Big\downarrow \text{469}$$

$$\frac{13}{11}d\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} dx - \frac{(d+ex)^2 \left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$$\Big\downarrow \text{469}$$

$$\frac{13}{11}d\left(\frac{11}{10}d\int(d+ex)\left(d^2-e^2x^2\right)^{7/2}dx-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}\right)-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$\downarrow$ 455

$$\frac{13}{11}d\left(\frac{11}{10}d\left(d\int\left(d^2-e^2x^2\right)^{7/2}dx-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}\right)-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}\right)-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$\downarrow$ 211

$$\frac{13}{11}d\left(\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\int\left(d^2-e^2x^2\right)^{5/2}dx+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}\right)-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}\right)-\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$\downarrow$ 211

$$\frac{13}{11}d\left(\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\int\left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}\right)-\right.$$
$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$\downarrow$ 211

$$\frac{13}{11}d\left(\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/\cdots}\right.\right.\right.$$
$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$\downarrow$ 211

$$\frac{13}{11}d\left(\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^{2}\cdots\right.\right.\right.\right.$$
$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$\downarrow$ 224

$$\frac{13}{11}d\left(\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\right.\right.\right.\right.\right.$$
$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

$\Big\downarrow$ 216

$$\frac{13}{11}d\left(\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2\right.\right.\right.\right.\right.\right.$$
$$\frac{(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}{11e}$$

input
```
Int[(d + e*x)^3*(d^2 - e^2*x^2)^(7/2),x]
```

output
```
-1/11*((d + e*x)^2*(d^2 - e^2*x^2)^(9/2))/e + (13*d*(-1/10*((d + e*x)*(d^2
 - e^2*x^2)^(9/2))/e + (11*d*(-1/9*(d^2 - e^2*x^2)^(9/2)/e + d*((x*(d^2 -
e^2*x^2)^(7/2))/8 + (7*d^2*((x*(d^2 - e^2*x^2)^(5/2))/6 + (5*d^2*((x*(d^2
- e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4))/6))/8)))/10))/11
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{(11520e^{10}x^{10}+38016d\,e^{9}x^{9}-1280d^{2}e^{8}x^{8}-131472d^{3}e^{7}x^{7}-110080d^{4}e^{6}x^{6}+142296d^{5}e^{5}x^{5}+222720d^{6}e^{4}x^{4}-12210d^{7}e^{3}x^{3}-16768}{126720e}$
default	$d^{3}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{8}+\frac{7d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}{6}+\frac{5d^{2}\left(\frac{x\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}{4}+\frac{3d^{2}\left(\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2\sqrt{e^{2}}}\right)}{4}\right)}{6}\right)}{8}\right)+$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/126720*(11520*e^10*x^10+38016*d*e^9*x^9-1280*d^2*e^8*x^8-131472*d^3*e^7
*x^7-110080*d^4*e^6*x^6+142296*d^5*e^5*x^5+222720*d^6*e^4*x^4-12210*d^7*e^
3*x^3-167680*d^8*e^2*x^2-81675*d^9*e*x+44800*d^10)/e*(-e^2*x^2+d^2)^(1/2)+
91/256*d^11/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} \, dx = -\frac{90090\, d^{11} \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (11520\, e^{10}x^{10} + 38016\, de^9x^9 - 1280\, d^2e^8x^8 - 131472\, d^3e^7x^7 - 11008}{}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/126720*(90090*d^11*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (11520*e
^10*x^10 + 38016*d*e^9*x^9 - 1280*d^2*e^8*x^8 - 131472*d^3*e^7*x^7 - 11008
0*d^4*e^6*x^6 + 142296*d^5*e^5*x^5 + 222720*d^6*e^4*x^4 - 12210*d^7*e^3*x^
3 - 167680*d^8*e^2*x^2 - 81675*d^9*e*x + 44800*d^10)*sqrt(-e^2*x^2 + d^2))
/e
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.22

$$\int (d+ex)^3 \left(d^2 -e^2x^2\right)^{7/2} \, dx = \begin{cases} \frac{91d^{11}\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2 \neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{256} + \sqrt{d^2-e^2x^2}\left(-\frac{35d^{10}}{99e} + \frac{165d^9x}{256} + \frac{131d^8ex^2}{99} \right. \\ (d^2)^{\frac{7}{2}}\left(\begin{cases} d^3x & \text{for } e=0 \\ \frac{(d+ex)^4}{4e} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((e*x+d)**3*(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Piecewise((91*d**11*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2
56 + sqrt(d**2 - e**2*x**2)*(-35*d**10/(99*e) + 165*d**9*x/256 + 131*d**8*
e*x**2/99 + 37*d**7*e**2*x**3/384 - 58*d**6*e**3*x**4/33 - 539*d**5*e**4*x
**5/480 + 86*d**4*e**5*x**6/99 + 83*d**3*e**6*x**7/80 + d**2*e**7*x**8/99
- 3*d*e**8*x**9/10 - e**9*x**10/11), Ne(e**2, 0)), ((d**2)**(7/2)*Piecewis
e((d**3*x, Eq(e, 0)), ((d + e*x)**4/(4*e), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} \, dx = \frac{91\, d^{11} \arcsin\left(\frac{e^2 x}{d\sqrt{e^2}}\right)}{256\,\sqrt{e^2}} + \frac{91}{256}\,\sqrt{-e^2x^2+d^2}d^9x + \frac{91}{384}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^7x + \frac{91}{480}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^5x + \frac{13}{80}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^3x - \frac{1}{11}\left(-e^2x^2+d^2\right)^{\frac{9}{2}}ex^2 - \frac{3}{10}\left(-e^2x^2+d^2\right)^{\frac{9}{2}}dx - \frac{35\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^2}{99\,e}$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
91/256*d^11*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 91/256*sqrt(-e^2*x^2 +
 d^2)*d^9*x + 91/384*(-e^2*x^2 + d^2)^(3/2)*d^7*x + 91/480*(-e^2*x^2 + d^2
)^(5/2)*d^5*x + 13/80*(-e^2*x^2 + d^2)^(7/2)*d^3*x - 1/11*(-e^2*x^2 + d^2)
^(9/2)*e*x^2 - 3/10*(-e^2*x^2 + d^2)^(9/2)*d*x - 35/99*(-e^2*x^2 + d^2)^(9
/2)*d^2/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} \, dx = \frac{91\, d^{11} \arcsin\left(\frac{ex}{d}\right) \operatorname{sgn}(d) \operatorname{sgn}(e)}{256\, |e|} - \frac{1}{126720} \left(\frac{44800\, d^{10}}{e} - \left(81675\, d^9 + 2\left(83840\, d^8 e + \left(6105\, d^7 e^2 - 4\left(27840\, d^6 e^3 + \left(17787\, d^5 e^4 - 2\left(6880\, d^4 \right.\right.\right.\right.\right.\right.$$

input
```
integrate((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
91/256*d^11*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/126720*(44800*d^10/e -
(81675*d^9 + 2*(83840*d^8*e + (6105*d^7*e^2 - 4*(27840*d^6*e^3 + (17787*d^
5*e^4 - 2*(6880*d^4*e^5 + (8217*d^3*e^6 + 8*(10*d^2*e^7 - 9*(10*e^9*x + 33
*d*e^8)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(d^2-e^2x^2\right)^{7/2} \, dx = \int \left(d^2-e^2\,x^2\right)^{7/2}\,{\left(d+e\,x\right)}^3 \,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.44

$$\int (d+ex)^3 \left(d^2 -e^2x^2\right)^{7/2} \, dx = \frac{45045 asin\left(\frac{ex}{d}\right) d^{11} - 44800\sqrt{-e^2x^2+d^2}\, d^{10} + 81675\sqrt{-e^2x^2+d^2}\, d^9 ex + 167680\sqrt{-e^2x^2}}{}$$

input
```
int((e*x+d)^3*(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(45045*asin((e*x)/d)*d**11 - 44800*sqrt(d**2 - e**2*x**2)*d**10 + 81675*sq
rt(d**2 - e**2*x**2)*d**9*e*x + 167680*sqrt(d**2 - e**2*x**2)*d**8*e**2*x*
*2 + 12210*sqrt(d**2 - e**2*x**2)*d**7*e**3*x**3 - 222720*sqrt(d**2 - e**2
*x**2)*d**6*e**4*x**4 - 142296*sqrt(d**2 - e**2*x**2)*d**5*e**5*x**5 + 110
080*sqrt(d**2 - e**2*x**2)*d**4*e**6*x**6 + 131472*sqrt(d**2 - e**2*x**2)*
d**3*e**7*x**7 + 1280*sqrt(d**2 - e**2*x**2)*d**2*e**8*x**8 - 38016*sqrt(d
**2 - e**2*x**2)*d*e**9*x**9 - 11520*sqrt(d**2 - e**2*x**2)*e**10*x**10 +
44800*d**11)/(126720*e)
```

3.61 $\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} \, dx = \frac{77}{256}d^8x\sqrt{d^2-e^2x^2} + \frac{77}{384}d^6x\left(d^2-e^2x^2\right)^{3/2} + \frac{77}{480}d^4x\left(d^2-e^2x^2\right)^{5/2} + \frac{11}{80}d^2x\left(d^2-e^2x^2\right)^{7/2} - \frac{(20d+9ex)\left(d^2-e^2x^2\right)^{9/2}}{90e} + \frac{77d^{10}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e}$$

output
```
77/256*d^8*x*(-e^2*x^2+d^2)^(1/2)+77/384*d^6*x*(-e^2*x^2+d^2)^(3/2)+77/480
*d^4*x*(-e^2*x^2+d^2)^(5/2)+11/80*d^2*x*(-e^2*x^2+d^2)^(7/2)-1/90*(9*e*x+2
0*d)*(-e^2*x^2+d^2)^(9/2)/e+77/256*d^10*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} \, dx = -\frac{\sqrt{d^2-e^2x^2}(2560d^9 - 8055d^8ex - 10240d^7e^2x^2 + 6150d^6e^3x^3 + 15360d^5e^4x^4 + 312d^4e^5x^5 - 10240d^3e^6x}{11520e}$$

input
```
Integrate[(d + e*x)^2*(d^2 - e^2*x^2)^(7/2),x]
```

output
```
-1/11520*(Sqrt[d^2 - e^2*x^2]*(2560*d^9 - 8055*d^8*e*x - 10240*d^7*e^2*x^2
 + 6150*d^6*e^3*x^3 + 15360*d^5*e^4*x^4 + 312*d^4*e^5*x^5 - 10240*d^3*e^6*
x^6 - 3024*d^2*e^7*x^7 + 2560*d*e^8*x^8 + 1152*e^9*x^9) + 6930*d^10*ArcTan
[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {469, 455, 211, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} dx$$

$$\Big\downarrow \text{469}$$

$$\frac{11}{10}d\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}dx-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

$$\Big\downarrow \text{455}$$

$$\frac{11}{10}d\left(d\int \left(d^2-e^2x^2\right)^{7/2}dx-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}\right)-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

$$\Big\downarrow \text{211}$$

$$\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\int \left(d^2-e^2x^2\right)^{5/2}dx+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}\right)-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

$$\Big\downarrow \text{211}$$

$$\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\int\left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}\right)-\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

$$\Big\downarrow \text{211}$$

$$\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int\sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\right.$$
$$\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

$$\Big\downarrow \text{211}$$

$$\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)\right.\right.\right.$$
$$\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

$$\Big\downarrow \text{224}$$

$$\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-\right.\right.\right.\right.\right.$$
$$\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

$$\Big\downarrow \text{216}$$

$$\frac{11}{10}d\left(d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right.\right.\right.\right.$$
$$\frac{(d+ex)\left(d^2-e^2x^2\right)^{9/2}}{10e}$$

input
```
Int[(d + e*x)^2*(d^2 - e^2*x^2)^(7/2),x]
```

output
```
-1/10*((d + e*x)*(d^2 - e^2*x^2)^(9/2))/e + (11*d*(-1/9*(d^2 - e^2*x^2)^(9
/2)/e + d*((x*(d^2 - e^2*x^2)^(7/2))/8 + (7*d^2*((x*(d^2 - e^2*x^2)^(5/2))
/6 + (5*d^2*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])
/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4))/6))/8)))/10
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{(1152e^9x^9+2560d\,e^8x^8-3024d^2e^7x^7-10240d^3e^6x^6+312d^4e^5x^5+15360d^5e^4x^4+6150d^6e^3x^3-10240d^7e^2x^2-8055d^8ex+2560d^9}{11520e}$
default	$d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{7}{2}}}{8}+\frac{7d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{8}\right)+$

input
```
int((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/11520*(1152*e^9*x^9+2560*d*e^8*x^8-3024*d^2*e^7*x^7-10240*d^3*e^6*x^6+3
12*d^4*e^5*x^5+15360*d^5*e^4*x^4+6150*d^6*e^3*x^3-10240*d^7*e^2*x^2-8055*d
^8*e*x+2560*d^9)/e*(-e^2*x^2+d^2)^(1/2)+77/256*d^10/(e^2)^(1/2)*arctan((e^
2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} dx = -\frac{6930\,d^{10}\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(1152\,e^9x^9+2560\,de^8x^8-3024\,d^2e^7x^7-10240\,d^3e^6x^6+312\,d^4e^5x^5}{11520\,e}$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/11520*(6930*d^10*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (1152*e^9*
x^9 + 2560*d*e^8*x^8 - 3024*d^2*e^7*x^7 - 10240*d^3*e^6*x^6 + 312*d^4*e^5*
x^5 + 15360*d^5*e^4*x^4 + 6150*d^6*e^3*x^3 - 10240*d^7*e^2*x^2 - 8055*d^8*
e*x + 2560*d^9)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.39

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} dx = \begin{cases} \frac{77d^{10}\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{256}+\sqrt{d^2-e^2x^2}\left(-\frac{2d^9}{9e}+\frac{179d^8x}{256}+\frac{8d^7ex^2}{9}-\right. \\ \left(d^2\right)^{\frac{7}{2}}\left(\begin{cases} d^2x & \text{for } e=0\\ \frac{(d+ex)^3}{3e} & \text{otherwise}\end{cases}\right) \end{cases}$$

input
```
integrate((e*x+d)**2*(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Piecewise((77*d**10*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e
**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2
56 + sqrt(d**2 - e**2*x**2)*(-2*d**9/(9*e) + 179*d**8*x/256 + 8*d**7*e*x**
2/9 - 205*d**6*e**2*x**3/384 - 4*d**5*e**3*x**4/3 - 13*d**4*e**4*x**5/480
+ 8*d**3*e**5*x**6/9 + 21*d**2*e**6*x**7/80 - 2*d*e**7*x**8/9 - e**8*x**9/
10), Ne(e**2, 0)), ((d**2)**(7/2)*Piecewise((d**2*x, Eq(e, 0)), ((d + e*x)
**3/(3*e), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2}\, dx = \frac{77\, d^{10} \arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{256\,\sqrt{e^2}} + \frac{77}{256}\,\sqrt{-e^2x^2+d^2}d^8x + \frac{77}{384}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^6x + \frac{77}{480}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^4x + \frac{11}{80}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^2x - \frac{1}{10}\left(-e^2x^2+d^2\right)^{\frac{9}{2}}x - \frac{2\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d}{9\,e}$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
77/256*d^10*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 77/256*sqrt(-e^2*x^2 +
 d^2)*d^8*x + 77/384*(-e^2*x^2 + d^2)^(3/2)*d^6*x + 77/480*(-e^2*x^2 + d^2
)^(5/2)*d^4*x + 11/80*(-e^2*x^2 + d^2)^(7/2)*d^2*x - 1/10*(-e^2*x^2 + d^2)
^(9/2)*x - 2/9*(-e^2*x^2 + d^2)^(9/2)*d/e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.88

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} dx = \frac{77 d^{10} \arcsin\left(\frac{ex}{d}\right) \operatorname{sgn}(d) \operatorname{sgn}(e)}{256 |e|} - \frac{1}{11520}\left(\frac{2560 d^9}{e} - \left(8055 d^8 + 2\left(5120 d^7 e - \left(3075 d^6 e^2 + 4\left(1920 d^5 e^3 + \left(39 d^4 e^4 - 2\left(640 d^3 e^5 + \left(189 d\right.\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
77/256*d^10*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/11520*(2560*d^9/e - (80
55*d^8 + 2*(5120*d^7*e - (3075*d^6*e^2 + 4*(1920*d^5*e^3 + (39*d^4*e^4 - 2
*(640*d^3*e^5 + (189*d^2*e^6 - 8*(9*e^8*x + 20*d*e^7)*x)*x)*x)*x)*x)*x)*x)
*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(d^2-e^2x^2\right)^{7/2} dx = \int \left(d^2-e^2\,x^2\right)^{7/2} (d+e\,x)^2\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.57

$$\int (d+ex)^2 \left(d^2 -e^2x^2\right)^{7/2} dx = \frac{3465 asin\left(\frac{ex}{d}\right) d^{10} - 2560\sqrt{-e^2x^2+d^2}\, d^9 + 8055\sqrt{-e^2x^2+d^2}\, d^8 ex + 10240\sqrt{-e^2x^2+d^2}\, d}{}$$

input
```
int((e*x+d)^2*(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(3465*asin((e*x)/d)*d**10 - 2560*sqrt(d**2 - e**2*x**2)*d**9 + 8055*sqrt(d
**2 - e**2*x**2)*d**8*e*x + 10240*sqrt(d**2 - e**2*x**2)*d**7*e**2*x**2 -
6150*sqrt(d**2 - e**2*x**2)*d**6*e**3*x**3 - 15360*sqrt(d**2 - e**2*x**2)*
d**5*e**4*x**4 - 312*sqrt(d**2 - e**2*x**2)*d**4*e**5*x**5 + 10240*sqrt(d*
*2 - e**2*x**2)*d**3*e**6*x**6 + 3024*sqrt(d**2 - e**2*x**2)*d**2*e**7*x**
7 - 2560*sqrt(d**2 - e**2*x**2)*d*e**8*x**8 - 1152*sqrt(d**2 - e**2*x**2)*
e**9*x**9 + 2560*d**10)/(11520*e)
```

3.62 $\int (d+ex) \left(d^2-e^2x^2\right)^{7/2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int (d+ex) \left(d^2-e^2x^2\right)^{7/2} \, dx = \frac{35}{128}d^7x\sqrt{d^2-e^2x^2}+\frac{35}{192}d^5x\left(d^2-e^2x^2\right)^{3/2} +\frac{7}{48}d^3x\left(d^2-e^2x^2\right)^{5/2}+\frac{1}{8}dx\left(d^2-e^2x^2\right)^{7/2}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}+\frac{35d^9\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e}$$

output
```
35/128*d^7*x*(-e^2*x^2+d^2)^(1/2)+35/192*d^5*x*(-e^2*x^2+d^2)^(3/2)+7/48*d
^3*x*(-e^2*x^2+d^2)^(5/2)+1/8*d*x*(-e^2*x^2+d^2)^(7/2)-1/9*(-e^2*x^2+d^2)^
(9/2)/e+35/128*d^9*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int (d+ex) \left(d^2-e^2x^2\right)^{7/2} \, dx = \frac{\sqrt{d^2-e^2x^2}(-128d^8+837d^7ex+512d^6e^2x^2-978d^5e^3x^3-768d^4e^4x^4+600d^3e^5x^5+512}{1152e} - \frac{35d^9\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{128\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)*(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-128*d^8 + 837*d^7*e*x + 512*d^6*e^2*x^2 - 978*d^5*e
^3*x^3 - 768*d^4*e^4*x^4 + 600*d^3*e^5*x^5 + 512*d^2*e^6*x^6 - 144*d*e^7*x
^7 - 128*e^8*x^8))/(1152*e) - (35*d^9*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2
*x^2]])/(128*Sqrt[-e^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {455, 211, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}dx$$

$$\Big\downarrow \text{455}$$

$$d\int \left(d^2-e^2x^2\right)^{7/2}dx-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{7}{8}d^2\int \left(d^2-e^2x^2\right)^{5/2}dx+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\int \left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}x\left(d^2-e^2x^2\right)^{7/2}\right)-\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}x$$

$$\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

$\downarrow$ 224

$$d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)\right.\right.$$

$$\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

$\downarrow$ 216

$$d\left(\frac{7}{8}d^2\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{1}{8}\right.$$

$$\frac{\left(d^2-e^2x^2\right)^{9/2}}{9e}$$

input
```
Int[(d + e*x)*(d^2 - e^2*x^2)^(7/2),x]
```

output
```
-1/9*(d^2 - e^2*x^2)^(9/2)/e + d*((x*(d^2 - e^2*x^2)^(7/2))/8 + (7*d^2*((x
*(d^2 - e^2*x^2)^(5/2))/6 + (5*d^2*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*(
(x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))
)/4))/6))/8)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{(128e^8x^8+144d\,e^7x^7-512d^2e^6x^6-600d^3e^5x^5+768d^4e^4x^4+978d^5e^3x^3-512d^6e^2x^2-837d^7ex+128d^8)\sqrt{-e^2x^2+d^2}}{1152e}+\frac{35d^9\,\mathrm{arc}}{}$
default	$d\left(\frac{x(-e^2x^2+d^2)^{\frac{7}{2}}}{8}+\frac{7d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{5}{2}}}{6}+\frac{5d^2\left(\frac{x(-e^2x^2+d^2)^{\frac{3}{2}}}{4}+\frac{3d^2\left(\frac{x\sqrt{-e^2x^2+d^2}}{2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}\right)}{4}\right)}{6}\right)}{8}\right)-\frac{(}{}$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1152*(128*e^8*x^8+144*d*e^7*x^7-512*d^2*e^6*x^6-600*d^3*e^5*x^5+768*d^4
*e^4*x^4+978*d^5*e^3*x^3-512*d^6*e^2*x^2-837*d^7*e*x+128*d^8)/e*(-e^2*x^2+
d^2)^(1/2)+35/128*d^9/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx = -\frac{630\,d^9\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(128\,e^8x^8+144\,de^7x^7-512\,d^2e^6x^6-600\,d^3e^5x^5+768\,d^4e^4x^4+978\,d^5\right.}{1152\,e}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/1152*(630*d^9*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (128*e^8*x^8
+ 144*d*e^7*x^7 - 512*d^2*e^6*x^6 - 600*d^3*e^5*x^5 + 768*d^4*e^4*x^4 + 97
8*d^5*e^3*x^3 - 512*d^6*e^2*x^2 - 837*d^7*e*x + 128*d^8)*sqrt(-e^2*x^2 + d
^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.33

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx = \begin{cases} \frac{35d^9\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{128}+\sqrt{d^2-e^2x^2}\left(-\frac{d^8}{9e}+\frac{93d^7x}{128}+\frac{4d^6ex^2}{9}-\frac{163d}{1} \right. \\ \left(dx+\frac{ex^2}{2}\right)\left(d^2\right)^{\frac{7}{2}} \end{cases}$$

input
```
integrate((e*x+d)*(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Piecewise((35*d**9*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/12
8 + sqrt(d**2 - e**2*x**2)*(-d**8/(9*e) + 93*d**7*x/128 + 4*d**6*e*x**2/9
- 163*d**5*e**2*x**3/192 - 2*d**4*e**3*x**4/3 + 25*d**3*e**4*x**5/48 + 4*d
**2*e**5*x**6/9 - d*e**6*x**7/8 - e**7*x**8/9), Ne(e**2, 0)), ((d*x + e*x*
*2/2)*(d**2)**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx=\frac{35\,d^9\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{128\,\sqrt{e^2}}+\frac{35}{128}\,\sqrt{-e^2x^2+d^2}d^7x+\frac{35}{192}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^5x+\frac{7}{48}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^3x+\frac{1}{8}\left(-e^2x^2+d^2\right)^{\frac{7}{2}}dx-\frac{\left(-e^2x^2+d^2\right)^{\frac{9}{2}}}{9\,e}$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
35/128*d^9*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 35/128*sqrt(-e^2*x^2 +
d^2)*d^7*x + 35/192*(-e^2*x^2 + d^2)^(3/2)*d^5*x + 7/48*(-e^2*x^2 + d^2)^(
5/2)*d^3*x + 1/8*(-e^2*x^2 + d^2)^(7/2)*d*x - 1/9*(-e^2*x^2 + d^2)^(9/2)/e
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx=\frac{35\,d^9\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{128\,|e|}-\frac{1}{1152}\left(\frac{128\,d^8}{e}-\left(837\,d^7+2\left(256\,d^6e-\left(489\,d^5e^2+4\left(96\,d^4e^3-\left(75\,d^3e^4+2\left(32\,d^2e^5-\left(8\,e^7x+9\,de^6\right)x\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate((e*x+d)*(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
35/128*d^9*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/1152*(128*d^8/e - (837*d
^7 + 2*(256*d^6*e - (489*d^5*e^2 + 4*(96*d^4*e^3 - (75*d^3*e^4 + 2*(32*d^2
*e^5 - (8*e^7*x + 9*d*e^6)*x)*x)*x)*x)*x)*x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.45

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx=\frac{d\,x\,\left(d^2-e^2\,x^2\right)^{7/2}\,{}_2\mathrm{F}_1\left(-\frac{7}{2},\frac{1}{2};\ \frac{3}{2};\ \frac{e^2\,x^2}{d^2}\right)}{\left(1-\frac{e^2\,x^2}{d^2}\right)^{7/2}}-\frac{\left(d^2-e^2\,x^2\right)^{9/2}}{9\,e}$$

input
```
int((d^2 - e^2*x^2)^(7/2)*(d + e*x),x)
```

output
```
(d*x*(d^2 - e^2*x^2)^(7/2)*hypergeom([-7/2, 1/2], 3/2, (e^2*x^2)/d^2))/(1
- (e^2*x^2)/d^2)^(7/2) - (d^2 - e^2*x^2)^(9/2)/(9*e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.51

$$\int (d+ex)\left(d^2-e^2x^2\right)^{7/2}\,dx=\frac{315asin\left(\frac{ex}{d}\right)d^9-128\sqrt{-e^2x^2+d^2}\,d^8+837\sqrt{-e^2x^2+d^2}\,d^7ex+512\sqrt{-e^2x^2+d^2}\,d^6e^2x^2+\cdots}{}$$

input
```
int((e*x+d)*(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(315*asin((e*x)/d)*d**9 - 128*sqrt(d**2 - e**2*x**2)*d**8 + 837*sqrt(d**2
- e**2*x**2)*d**7*e*x + 512*sqrt(d**2 - e**2*x**2)*d**6*e**2*x**2 - 978*sq
rt(d**2 - e**2*x**2)*d**5*e**3*x**3 - 768*sqrt(d**2 - e**2*x**2)*d**4*e**4
*x**4 + 600*sqrt(d**2 - e**2*x**2)*d**3*e**5*x**5 + 512*sqrt(d**2 - e**2*x
**2)*d**2*e**6*x**6 - 144*sqrt(d**2 - e**2*x**2)*d*e**7*x**7 - 128*sqrt(d*
*2 - e**2*x**2)*e**8*x**8 + 128*d**9)/(1152*e)
```

3.63 $\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = \frac{5}{16}d^5x\sqrt{d^2-e^2x^2} + \frac{5}{24}d^3x(d^2-e^2x^2)^{3/2} + \frac{1}{6}dx(d^2-e^2x^2)^{5/2} + \frac{(d^2-e^2x^2)^{7/2}}{7e} + \frac{5d^7\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e}$$

output
```
5/16*d^5*x*(-e^2*x^2+d^2)^(1/2)+5/24*d^3*x*(-e^2*x^2+d^2)^(3/2)+1/6*d*x*(-
e^2*x^2+d^2)^(5/2)+1/7*(-e^2*x^2+d^2)^(7/2)/e+5/16*d^7*arctan(e*x/(-e^2*x^
2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = \frac{\sqrt{d^2-e^2x^2}(48d^6+231d^5ex-144d^4e^2x^2-182d^3e^3x^3+144d^2e^4x^4+56de^5x^5-48}{336e} - \frac{5d^7\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{16\sqrt{-e^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(48*d^6 + 231*d^5*e*x - 144*d^4*e^2*x^2 - 182*d^3*e^3
*x^3 + 144*d^2*e^4*x^4 + 56*d*e^5*x^5 - 48*e^6*x^6))/(336*e) - (5*d^7*Log[
-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(16*Sqrt[-e^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {466, 211, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{466}$$

$$d\int \left(d^2-e^2x^2\right)^{5/2}dx+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{5}{6}d^2\int \left(d^2-e^2x^2\right)^{3/2}dx+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}$$

$$\Big\downarrow \text{211}$$

$$d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}$$

$$\Big\downarrow \text{224}$$

$$d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}$$

$$\Big\downarrow \; 216$$

$$d\left(\frac{5}{6}d^2\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{1}{6}x\left(d^2-e^2x^2\right)^{5/2}\right)+\frac{\left(d^2-e^2x^2\right)^{7/2}}{7e}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x),x]
```

output
```
(d^2 - e^2*x^2)^(7/2)/(7*e) + d*((x*(d^2 - e^2*x^2)^(5/2))/6 + (5*d^2*((x*
(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (d^2*ArcTan
[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4))/6)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result
risch	$\frac{\left(-48 e^{6} x^{6}+56 d\, e^{5} x^{5}+144 d^{2} e^{4} x^{4}-182 d^{3} e^{3} x^{3}-144 d^{4} e^{2} x^{2}+231 d^{5} e x +48 d^{6}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{336 e}+\frac{5 d^{7} \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{16 \sqrt{e^{2}}}$
default	$\frac{\frac{\left(-e^{2} \left(x+\frac{d}{e}\right)^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7}+d e \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e \right) \left(-e^{2} \left(x+\frac{d}{e}\right)^{2}+2 d e \left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{12 e^{2}}+\frac{5 d^{2} \left(-\frac{\left(-2 e^{2} \left(x+\frac{d}{e}\right)+2 d e \right) \left(-e^{2} \left(x+\frac{d}{e}\right)^{2}+2 d e \left(x \right.\right.}{8 e^{2}}\right.}{}\right.}{e}$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/336*(-48*e^6*x^6+56*d*e^5*x^5+144*d^2*e^4*x^4-182*d^3*e^3*x^3-144*d^4*e^
2*x^2+231*d^5*e*x+48*d^6)/e*(-e^2*x^2+d^2)^(1/2)+5/16*d^7/(e^2)^(1/2)*arct
an((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx =$$
$$-\frac{210\,d^7\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(48\,e^6x^6-56\,de^5x^5-144\,d^2e^4x^4+182\,d^3e^3x^3+144\,d^4e^2x^2-231\,d^5ex}{336\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d),x, algorithm="fricas")
```

output
```
-1/336*(210*d^7*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (48*e^6*x^6 -
56*d*e^5*x^5 - 144*d^2*e^4*x^4 + 182*d^3*e^3*x^3 + 144*d^4*e^2*x^2 - 231*d
^5*e*x - 48*d^6)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.19

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d),x)
```

output
```
d**5*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - d**4
*e*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2, 0
)), (x**2*sqrt(d**2)/2, True)) - 2*d**3*e**2*Piecewise((d**4*Piecewise((lo
g(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2,
0)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*
(-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) + 2*
d**2*e**3*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2
/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True)) + d*e**4*Pi
ecewise((d**6*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x*
*2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/(16*e**
4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e**4) - d**2*x**3/(24*e**2) + x**
5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, True)) - e**5*Piecewise((sqrt(d**2
 - e**2*x**2)*(-8*d**6/(105*e**6) - 4*d**4*x**2/(105*e**4) - d**2*x**4/(35
*e**2) + x**6/7), Ne(e**2, 0)), (x**6*sqrt(d**2)/6, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = -\frac{5i\,d^7\arcsin\left(\frac{ex}{d}+2\right)}{16\,e} + \frac{5}{16}\sqrt{e^2x^2+4\,dex+3\,d^2}d^5x + \frac{5\sqrt{e^2x^2+4\,dex+3\,d^2}d^6}{8\,e} + \frac{5}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3x + \frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{5}{2}}dx + \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{7\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d),x, algorithm="maxima")
```

output
```
-5/16*I*d^7*arcsin(e*x/d + 2)/e + 5/16*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5
*x + 5/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^6/e + 5/24*(-e^2*x^2 + d^2)^(3/
2)*d^3*x + 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x + 1/7*(-e^2*x^2 + d^2)^(7/2)/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.83

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = \frac{5\,d^7\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{16\,|e|} + \frac{1}{336}\left(\frac{48\,d^6}{e} + \left(231\,d^5 - 2\left(72\,d^4e + \left(91\,d^3e^2 - 4\left(18\,d^2e^3 - \left(6\,e^5x - 7\,de^4\right)x\right)x\right)x\right)x\right)x\right)\sqrt{-e^2x^2+d^2}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d),x, algorithm="giac")
```

output
```
5/16*d^7*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/336*(48*d^6/e + (231*d^5 -
 2*(72*d^4*e + (91*d^3*e^2 - 4*(18*d^2*e^3 - (6*e^5*x - 7*d*e^4)*x)*x)*x)*
x)*x)*sqrt(-e^2*x^2 + d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = \int \frac{(d^2-e^2\,x^2)^{7/2}}{d+e\,x}\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x),x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.42

$$\int \frac{(d^2-e^2x^2)^{7/2}}{d+ex}\,dx = \frac{105asin\left(\frac{ex}{d}\right)d^7 + 48\sqrt{-e^2x^2+d^2}\,d^6 + 231\sqrt{-e^2x^2+d^2}\,d^5ex - 144\sqrt{-e^2x^2+d^2}\,d^4}{}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d),x)
```

output
```
(105*asin((e*x)/d)*d**7 + 48*sqrt(d**2 - e**2*x**2)*d**6 + 231*sqrt(d**2 -
 e**2*x**2)*d**5*e*x - 144*sqrt(d**2 - e**2*x**2)*d**4*e**2*x**2 - 182*sqr
t(d**2 - e**2*x**2)*d**3*e**3*x**3 + 144*sqrt(d**2 - e**2*x**2)*d**2*e**4*
x**4 + 56*sqrt(d**2 - e**2*x**2)*d*e**5*x**5 - 48*sqrt(d**2 - e**2*x**2)*e
**6*x**6 - 48*d**7)/(336*e)
```

3.64 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2} \, dx = \frac{7}{16}d^4x\sqrt{d^2-e^2x^2} + \frac{7}{24}d^2x\left(d^2-e^2x^2\right)^{3/2} + \frac{(12d-5ex)\left(d^2-e^2x^2\right)^{5/2}}{30e} + \frac{7d^6\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e}$$

output
```
7/16*d^4*x*(-e^2*x^2+d^2)^(1/2)+7/24*d^2*x*(-e^2*x^2+d^2)^(3/2)+1/30*(-5*e
*x+12*d)*(-e^2*x^2+d^2)^(5/2)/e+7/16*d^6*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/
e
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2} \, dx = \frac{\sqrt{d^2-e^2x^2}(96d^5+135d^4ex-192d^3e^2x^2+10d^2e^3x^3+96de^4x^4-40e^5x^5)-210d^6}{240e}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^2,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(96*d^5 + 135*d^4*e*x - 192*d^3*e^2*x^2 + 10*d^2*e^3*
x^3 + 96*d*e^4*x^4 - 40*e^5*x^5) - 210*d^6*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[
d^2 - e^2*x^2])])/(240*e)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {464, 469, 455, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{464}$$

$$\int (d-ex)^2\left(d^2-e^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{6}d\int (d-ex)\left(d^2-e^2x^2\right)^{3/2}dx+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{6}d\left(d\int \left(d^2-e^2x^2\right)^{3/2}dx+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{6}d\left(d\left(\frac{3}{4}d^2\int \sqrt{d^2-e^2x^2}dx+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{6}d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$\Big\downarrow$ 224

$$\frac{7}{6}d\left(d\left(\frac{3}{4}d^2\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

$\Big\downarrow$ 216

$$\frac{7}{6}d\left(d\left(\frac{3}{4}d^2\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{1}{4}x\left(d^2-e^2x^2\right)^{3/2}\right)+\frac{\left(d^2-e^2x^2\right)^{5/2}}{5e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}{6e}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^2,x]
```

output
```
((d - e*x)*(d^2 - e^2*x^2)^(5/2))/(6*e) + (7*d*((d^2 - e^2*x^2)^(5/2)/(5*e
) + d*((x*(d^2 - e^2*x^2)^(3/2))/4 + (3*d^2*((x*Sqrt[d^2 - e^2*x^2])/2 + (
d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e)))/4)))/6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
risch	$\frac{\left(-40e^{5}x^{5}+96d\,e^{4}x^{4}+10d^{2}e^{3}x^{3}-192d^{3}e^{2}x^{2}+135d^{4}ex+96d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{240e}+\frac{7d^{6}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{16\sqrt{e^{2}}}$
default	$\frac{\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{5de\left(x+\frac{d}{e}\right)^{2}}+7e\left(\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7}+de\left(-\frac{\left(-2e^{2}\left(x+\frac{d}{e}\right)+2de\right)\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}{12e^{2}}+\frac{5d^{2}\left(-\frac{\left(-2e^{2}\cdots\right.}{\cdots}\right.}{\cdots}\right.\right.}{\cdots}$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/240*(-40*e^5*x^5+96*d*e^4*x^4+10*d^2*e^3*x^3-192*d^3*e^2*x^2+135*d^4*e*x
+96*d^5)/e*(-e^2*x^2+d^2)^(1/2)+7/16*d^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2}\,dx =$$

$$-\frac{210\,d^{6}\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)+\left(40\,e^{5}x^{5}-96\,de^{4}x^{4}-10\,d^{2}e^{3}x^{3}+192\,d^{3}e^{2}x^{2}-135\,d^{4}ex-96\,d^{5}\right)\sqrt{-e^{2}\cdots}}{240\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/240*(210*d^6*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (40*e^5*x^5 -
96*d*e^4*x^4 - 10*d^2*e^3*x^3 + 192*d^3*e^2*x^2 - 135*d^4*e*x - 96*d^5)*sq
rt(-e^2*x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.98

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2}\,dx = d^4\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right) - 2d^3e\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right) + 2de^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right) - e^4\left(\begin{cases}\frac{d^6\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{16e^4}+\sqrt{d^2-e^2x^2}\left(-\frac{d^4x}{16e^4}-\frac{d^2x^3}{24e^2}+\frac{x^5}{6}\right) & \text{for } e^2\neq 0\\ \frac{x^5\sqrt{d^2}}{5} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**2,x)
```

output
```
d**4*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - 2*d*
*3*e*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2,
 0)), (x**2*sqrt(d**2)/2, True)) + 2*d*e**3*Piecewise((sqrt(d**2 - e**2*x*
*2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**2) + x**4/5), Ne(e**2, 0)), (x**
4*sqrt(d**2)/4, True)) - e**4*Piecewise((d**6*Piecewise((log(-2*e**2*x + 2
*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/
sqrt(-e**2*x**2), True))/(16*e**4) + sqrt(d**2 - e**2*x**2)*(-d**4*x/(16*e
**4) - d**2*x**3/(24*e**2) + x**5/6), Ne(e**2, 0)), (x**5*sqrt(d**2)/5, Tr
ue))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.26

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2}\,dx = -\frac{7i\,d^6\arcsin\left(\frac{ex}{d}+2\right)}{16\,e} + \frac{7}{16}\sqrt{e^2x^2+4\,dex+3\,d^2}d^4x + \frac{7\sqrt{e^2x^2+4\,dex+3\,d^2}d^5}{8\,e} + \frac{7}{24}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2x + \frac{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{6\left(e^2x+de\right)} + \frac{7\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}{30\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-7/16*I*d^6*arcsin(e*x/d + 2)/e + 7/16*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4
*x + 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^5/e + 7/24*(-e^2*x^2 + d^2)^(3/
2)*d^2*x + 1/6*(-e^2*x^2 + d^2)^(7/2)/(e^2*x + d*e) + 7/30*(-e^2*x^2 + d^2
)^(5/2)*d/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(94) = 188$.

Time = 0.17 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.25

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2}\,dx = -\frac{\left(6720\,d^7e^7\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+\frac{\left(105\,d^7e^7\left(\frac{2\,d}{ex+d}-1\right)^{\frac{11}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)+595\,d^7e^7\left(\frac{2\,d}{ex+d}-1\right)^{\frac{9}{2}}\operatorname{sgn}(\ldots\right.}{\ldots}\right.}{\ldots}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/7680*(6720*d^7*e^7*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn
(e) + (105*d^7*e^7*(2*d/(e*x + d) - 1)^(11/2)*sgn(1/(e*x + d))*sgn(e) + 59
5*d^7*e^7*(2*d/(e*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e) - 1686*d^7*e^7
*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x + d))*sgn(e) - 1386*d^7*e^7*(2*d/(e*
x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) - 595*d^7*e^7*(2*d/(e*x + d) - 1
)^(3/2)*sgn(1/(e*x + d))*sgn(e) - 105*d^7*e^7*sqrt(2*d/(e*x + d) - 1)*sgn(
1/(e*x + d))*sgn(e))*(e*x + d)^6/d^6)*abs(e)/(d*e^9)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^2}\,dx = \int \frac{(d^2-e^2\,x^2)^{7/2}}{(d+e\,x)^2}\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^2,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.38

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^2} \, dx = \frac{105 asin\left(\frac{ex}{d}\right) d^6 + 96\sqrt{-e^2x^2 + d^2}\, d^5 + 135\sqrt{-e^2x^2 + d^2}\, d^4 ex - 192\sqrt{-e^2x^2 + d^2}\, d^3}{2}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^2,x)
```

output
```
(105*asin((e*x)/d)*d**6 + 96*sqrt(d**2 - e**2*x**2)*d**5 + 135*sqrt(d**2 -
 e**2*x**2)*d**4*e*x - 192*sqrt(d**2 - e**2*x**2)*d**3*e**2*x**2 + 10*sqrt
(d**2 - e**2*x**2)*d**2*e**3*x**3 + 96*sqrt(d**2 - e**2*x**2)*d*e**4*x**4
- 40*sqrt(d**2 - e**2*x**2)*e**5*x**5 - 96*d**6)/(240*e)
```

3.65 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3} dx = \frac{7}{8}d^3x\sqrt{d^2-e^2x^2} + \frac{7}{12}dx(d^2-e^2x^2)^{3/2} + \frac{7(d^2-e^2x^2)^{5/2}}{15e} + \frac{2(d^2-e^2x^2)^{7/2}}{3e(d+ex)^2} + \frac{7d^5\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

output
```
7/8*d^3*x*(-e^2*x^2+d^2)^(1/2)+7/12*d*x*(-e^2*x^2+d^2)^(3/2)+7/15*(-e^2*x^
2+d^2)^(5/2)/e+2/3*(-e^2*x^2+d^2)^(7/2)/e/(e*x+d)^2+7/8*d^5*arctan(e*x/(-e
^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3} dx = \frac{\sqrt{d^2-e^2x^2}(136d^4+15d^3ex-112d^2e^2x^2+90de^3x^3-24e^4x^4)}{120e} - \frac{7d^5\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{8\sqrt{-e^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^3,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(136*d^4 + 15*d^3*e*x - 112*d^2*e^2*x^2 + 90*d*e^3*x^
3 - 24*e^4*x^4))/(120*e) - (7*d^5*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2
]])/(8*Sqrt[-e^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {464, 469, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{464}$$

$$\int (d-ex)^3\sqrt{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}d\int (d-ex)^2\sqrt{d^2-e^2x^2}dx+\frac{(d^2-e^2x^2)^{3/2}(d-ex)^2}{5e}$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}d\left(\frac{5}{4}d\int (d-ex)\sqrt{d^2-e^2x^2}dx+\frac{(d-ex)(d^2-e^2x^2)^{3/2}}{4e}\right)+\frac{(d^2-e^2x^2)^{3/2}(d-ex)^2}{5e}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{5}d\left(\frac{5}{4}d\left(d\int \sqrt{d^2-e^2x^2}dx+\frac{(d^2-e^2x^2)^{3/2}}{3e}\right)+\frac{(d-ex)(d^2-e^2x^2)^{3/2}}{4e}\right)+\frac{(d^2-e^2x^2)^{3/2}(d-ex)^2}{5e}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{5}d\left(\frac{5}{4}d\left(d\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{(d^2-e^2x^2)^{3/2}}{3e}\right)+\frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{4e}\right)+\frac{(d^2-e^2x^2)^{3/2}\,(d-ex)^2}{5e}$$

$\Big\downarrow$ 224

$$\frac{7}{5}d\left(\frac{5}{4}d\left(d\left(\frac{1}{2}d^2\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{(d^2-e^2x^2)^{3/2}}{3e}\right)+\frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{4e}\right)+\frac{(d^2-e^2x^2)^{3/2}\,(d-ex)^2}{5e}$$

$\Big\downarrow$ 216

$$\frac{7}{5}d\left(\frac{5}{4}d\left(d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{(d^2-e^2x^2)^{3/2}}{3e}\right)+\frac{(d-ex)\,(d^2-e^2x^2)^{3/2}}{4e}\right)+\frac{(d^2-e^2x^2)^{3/2}\,(d-ex)^2}{5e}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^3,x]
```

output
```
((d - e*x)^2*(d^2 - e^2*x^2)^(3/2))/(5*e) + (7*d*(((d - e*x)*(d^2 - e^2*x^
2)^(3/2))/(4*e) + (5*d*((d^2 - e^2*x^2)^(3/2)/(3*e) + d*((x*Sqrt[d^2 - e^2
*x^2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/4))/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

method	result
risch	$\frac{(-24e^4x^4+90d\,e^3x^3-112d^2e^2x^2+15d^3ex+136d^4)\sqrt{-e^2x^2+d^2}}{120e}+\frac{7d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}$
default	$\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{3de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{5de\left(x+\frac{d}{e}\right)^2}+\frac{7e\left(\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7}+de\left(-\frac{\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)\left(-e^2\left(x\cdots\right.\right.}{12e^2}\cdots\right.\right.}{\cdots}\right.}{\cdots}$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/120*(-24*e^4*x^4+90*d*e^3*x^3-112*d^2*e^2*x^2+15*d^3*e*x+136*d^4)/e*(-e^
2*x^2+d^2)^(1/2)+7/8*d^5/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3}\,dx =$$
$$-\frac{210\,d^5\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(24\,e^4x^4-90\,de^3x^3+112\,d^2e^2x^2-15\,d^3ex-136\,d^4)\sqrt{-e^2x^2+d^2}}{120\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/120*(210*d^5*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*e^4*x^4 -
90*d*e^3*x^3 + 112*d^2*e^2*x^2 - 15*d^3*e*x - 136*d^4)*sqrt(-e^2*x^2 + d^2
))/e
```

Sympy [A] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.43

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3}\,dx = d^3\left(\begin{cases}\frac{d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{d^2-e^2x^2}}{2} & \text{for } e^2\neq 0\\ x\sqrt{d^2} & \text{otherwise}\end{cases}\right) - 3d^2e\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{d^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } e^2\neq 0\\ \frac{x^2\sqrt{d^2}}{2} & \text{otherwise}\end{cases}\right) + 3de^2\left(\begin{cases}\frac{d^4\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{8e^2}+\sqrt{d^2-e^2x^2}\left(-\frac{d^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } e^2\neq 0\\ \frac{x^3\sqrt{d^2}}{3} & \text{otherwise}\end{cases}\right) - e^3\left(\begin{cases}\sqrt{d^2-e^2x^2}\left(-\frac{2d^4}{15e^4}-\frac{d^2x^2}{15e^2}+\frac{x^4}{5}\right) & \text{for } e^2\neq 0\\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**3,x)
```

output
```
d**3*Piecewise((d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 -
e**2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/
2 + x*sqrt(d**2 - e**2*x**2)/2, Ne(e**2, 0)), (x*sqrt(d**2), True)) - 3*d*
*2*e*Piecewise((sqrt(d**2 - e**2*x**2)*(-d**2/(3*e**2) + x**2/3), Ne(e**2,
 0)), (x**2*sqrt(d**2)/2, True)) + 3*d*e**2*Piecewise((d**4*Piecewise((log
(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**2))/sqrt(-e**2), Ne(d**2, 0
)), (x*log(x)/sqrt(-e**2*x**2), True))/(8*e**2) + sqrt(d**2 - e**2*x**2)*(
-d**2*x/(8*e**2) + x**3/4), Ne(e**2, 0)), (x**3*sqrt(d**2)/3, True)) - e**
3*Piecewise((sqrt(d**2 - e**2*x**2)*(-2*d**4/(15*e**4) - d**2*x**2/(15*e**
2) + x**4/5), Ne(e**2, 0)), (x**4*sqrt(d**2)/4, True))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.23

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3}\,dx = -\frac{7i\,d^5\arcsin\left(\frac{ex}{d}+2\right)}{8\,e} + \frac{7}{8}\sqrt{e^2x^2+4\,dex+3\,d^2}d^3x + \frac{7\sqrt{e^2x^2+4\,dex+3\,d^2}d^4}{4\,e} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{5\,(e^3x^2+2\,de^2x+d^2e)} + \frac{7\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{20\,(e^2x+de)} + \frac{7\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{12\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-7/8*I*d^5*arcsin(e*x/d + 2)/e + 7/8*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^3*x
 + 7/4*sqrt(e^2*x^2 + 4*d*e*x + 3*d^2)*d^4/e + 1/5*(-e^2*x^2 + d^2)^(7/2)/
(e^3*x^2 + 2*d*e^2*x + d^2*e) + 7/20*(-e^2*x^2 + d^2)^(5/2)*d/(e^2*x + d*e
) + 7/12*(-e^2*x^2 + d^2)^(3/2)*d^2/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3}\,dx = \frac{7\,d^5\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{8\,|e|} + \frac{1}{120}\sqrt{-e^2x^2+d^2}\left(\frac{136\,d^4}{e} + \left(15\,d^3 - 2\left(56\,d^2e + 3\left(4\,e^3x - 15\,de^2\right)x\right)x\right)x\right)$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
7/8*d^5*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/120*sqrt(-e^2*x^2 + d^2)*(1
36*d^4/e + (15*d^3 - 2*(56*d^2*e + 3*(4*e^3*x - 15*d*e^2)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3}\,dx=\int \frac{{\left(d^2-e^2\,x^2\right)}^{7/2}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^3,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^3}\,dx=\frac{105 asin\left(\frac{ex}{d}\right) d^5+136\sqrt{-e^2x^2+d^2}\,d^4+15\sqrt{-e^2x^2+d^2}\,d^3ex-112\sqrt{-e^2x^2+d^2}\,d^2}{120e}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^3,x)
```

output
```
(105*asin((e*x)/d)*d**5 + 136*sqrt(d**2 - e**2*x**2)*d**4 + 15*sqrt(d**2 -
 e**2*x**2)*d**3*e*x - 112*sqrt(d**2 - e**2*x**2)*d**2*e**2*x**2 + 90*sqrt
(d**2 - e**2*x**2)*d*e**3*x**3 - 24*sqrt(d**2 - e**2*x**2)*e**4*x**4 - 136
*d**5)/(120*e)
```

3.66 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4} dx = \frac{35}{8}d^2x\sqrt{d^2-e^2x^2} + \frac{7(8d-3ex)\,(d^2-e^2x^2)^{3/2}}{12e} + \frac{2(d^2-e^2x^2)^{7/2}}{e(d+ex)^3} + \frac{35d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

output
```
35/8*d^2*x*(-e^2*x^2+d^2)^(1/2)+7/12*(-3*e*x+8*d)*(-e^2*x^2+d^2)^(3/2)/e+2
*(-e^2*x^2+d^2)^(7/2)/e/(e*x+d)^3+35/8*d^4*arctan(e*x/(-e^2*x^2+d^2)^(1/2)
)/e
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4} dx = \frac{\sqrt{d^2-e^2x^2}(160d^3-81d^2ex+32de^2x^2-6e^3x^3)-210d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^4,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(160*d^3 - 81*d^2*e*x + 32*d*e^2*x^2 - 6*e^3*x^3) - 2
10*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(24*e)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 464, 469, 455, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^4}dx$$

$$\Big\downarrow \text{465}$$

$$7\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^2}dx+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}$$

$$\Big\downarrow \text{464}$$

$$7\int (d-ex)^2\sqrt{d^2-e^2x^2}dx+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}$$

$$\Big\downarrow \text{469}$$

$$7\left(\frac{5}{4}d\int (d-ex)\sqrt{d^2-e^2x^2}dx+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}$$

$$\Big\downarrow \text{455}$$

$$7\left(\frac{5}{4}d\left(d\int \sqrt{d^2-e^2x^2}dx+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}$$

$$\Big\downarrow \text{211}$$

$$7\left(\frac{5}{4}d\left(d\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}$$

$$\downarrow \text{224}$$

$$7\left(\frac{5}{4}d\left(d\left(\frac{1}{2}d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}$$

$$\downarrow \text{216}$$

$$7\left(\frac{5}{4}d\left(d\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)+\frac{\left(d^2-e^2x^2\right)^{3/2}}{3e}\right)+\frac{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}{4e}\right)+\frac{2\left(d^2-e^2x^2\right)^{7/2}}{e(d+ex)^3}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^4,x]
```

output
```
(2*(d^2 - e^2*x^2)^(7/2))/(e*(d + e*x)^3) + 7*(((d - e*x)*(d^2 - e^2*x^2)^
(3/2))/(4*e) + (5*d*((d^2 - e^2*x^2)^(3/2)/(3*e) + d*((x*Sqrt[d^2 - e^2*x^
2])/2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e))))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

method	result
risch	$\frac{\left(-6e^{3}x^{3}+32d\,e^{2}x^{2}-81d^{2}ex+160d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24e}+\frac{35d^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}}$
	$5e\left(\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{3de\left(x+\frac{d}{e}\right)^{3}}+\frac{2e\left(\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{5de\left(x+\frac{d}{e}\right)^{2}}+\frac{7e\left(\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}{7}\right.}{}\right.}{}\right.$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
1/24*(-6*e^3*x^3+32*d*e^2*x^2-81*d^2*e*x+160*d^3)/e*(-e^2*x^2+d^2)^(1/2)+3
5/8*d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4}\,dx = -\frac{210\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(6\,e^3x^3-32\,de^2x^2+81\,d^2ex-160\,d^3)\sqrt{-e^2x^2+d^2}}{24\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/24*(210*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 - 32
*d*e^2*x^2 + 81*d^2*e*x - 160*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4}\,dx = \int \frac{\left(-(-d+ex)\,(d+ex)\right)^{\frac{7}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**4,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.37

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4}\,dx = \frac{35\,d^4\arcsin\left(\frac{ex}{d}\right)}{8\,e} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{4\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} + \frac{7\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{12\,(e^3x^2+2\,de^2x+d^2e)} + \frac{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{24\,(e^2x+de)} + \frac{35\,\sqrt{-e^2x^2+d^2}d^3}{8\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
35/8*d^4*arcsin(e*x/d)/e + 1/4*(-e^2*x^2 + d^2)^(7/2)/(e^4*x^3 + 3*d*e^3*x
^2 + 3*d^2*e^2*x + d^3*e) + 7/12*(-e^2*x^2 + d^2)^(5/2)*d/(e^3*x^2 + 2*d*e
^2*x + d^2*e) + 35/24*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^2*x + d*e) + 35/8*sqrt
(-e^2*x^2 + d^2)*d^3/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^4}\,dx = \frac{35\,d^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} + \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\frac{160\,d^3}{e} - \left(81\,d^2+2\left(3\,e^2x-16\,de\right)x\right)x\right)$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
35/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/24*sqrt(-e^2*x^2 + d^2)*(1
60*d^3/e - (81*d^2 + 2*(3*e^2*x - 16*d*e)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^4} \, dx = \int \frac{\left(d^2 - e^2\,x^2\right)^{7/2}}{\left(d + e\,x\right)^4} \, dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^4,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^4} \, dx = \frac{105 asin\left(\frac{ex}{d}\right) d^4 + 160\sqrt{-e^2x^2 + d^2}\, d^3 - 81\sqrt{-e^2x^2 + d^2}\, d^2ex + 32\sqrt{-e^2x^2 + d^2}\, d\, e^{}}{24e}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^4,x)
```

output
```
(105*asin((e*x)/d)*d**4 + 160*sqrt(d**2 - e**2*x**2)*d**3 - 81*sqrt(d**2 -
 e**2*x**2)*d**2*e*x + 32*sqrt(d**2 - e**2*x**2)*d*e**2*x**2 - 6*sqrt(d**2
 - e**2*x**2)*e**3*x**3 - 160*d**4)/(24*e)
```

3.67 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5} dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5} dx = -\frac{12d^2\sqrt{d^2-e^2x^2}}{e} + \frac{5}{2}dx\sqrt{d^2-e^2x^2} - \frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)} + \frac{(d^2-e^2x^2)^{3/2}}{3e} - \frac{35d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
-12*d^2*(-e^2*x^2+d^2)^(1/2)/e+5/2*d*x*(-e^2*x^2+d^2)^(1/2)-16*d^3*(-e^2*x
^2+d^2)^(1/2)/e/(e*x+d)+1/3*(-e^2*x^2+d^2)^(3/2)/e-35/2*d^3*arctan(e*x/(-e
^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5} dx = \frac{\sqrt{d^2-e^2x^2}(-166d^3-55d^2ex+13de^2x^2-2e^3x^3)}{6e(d+ex)} + \frac{35d^3\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{2\sqrt{-e^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^5,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-166*d^3 - 55*d^2*e*x + 13*d*e^2*x^2 - 2*e^3*x^3))/(
6*e*(d + e*x)) + (35*d^3*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2*Sq
rt[-e^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {463, 2346, 27, 2346, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{463}$$

$$-\int \frac{15d^3-11exd^2+5e^2x^2d-e^3x^3}{\sqrt{d^2-e^2x^2}}dx-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int -\frac{5\left(3dx^2e^4-7d^2xe^3+9d^3e^2\right)}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\int \frac{3dx^2e^4-7d^2xe^3+9d^3e^2}{\sqrt{d^2-e^2x^2}}dx}{3e^2}-\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{5\left(-\frac{\int -\frac{7d^2e^4(3d-2ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}-\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{7}{2}d^2e^2\int\frac{3d-2ex}{\sqrt{d^2-e^2x^2}}dx-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}-\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{455}$$

$$-\frac{5\left(\frac{7}{2}d^2e^2\left(3d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}-\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{224}$$

$$-\frac{5\left(\frac{7}{2}d^2e^2\left(3d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}-\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \text{216}$$

$$-\frac{5\left(\frac{7}{2}d^2e^2\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}-\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}-\frac{16d^3\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^5,x]
```

output
```
-1/3*(e*x^2*Sqrt[d^2 - e^2*x^2]) - (16*d^3*Sqrt[d^2 - e^2*x^2])/(e*(d + e*
x)) - (5*((-3*d*e^2*x*Sqrt[d^2 - e^2*x^2])/2 + (7*d^2*e^2*((2*Sqrt[d^2 - e
^2*x^2])/e + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2))/(3*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{(2e^2x^2-15dex+70d^2)\sqrt{-e^2x^2+d^2}}{6e}-\frac{35d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{16d^3\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}{e^2\left(x+\frac{d}{e}\right)}$ $5e\left(\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{3de\left(x+\frac{d}{e}\right)^3}+\frac{2e\left(\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)}{5de\left(x+\frac{d}{e}\right)^2}\right.}{}\right.$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*e^2*x^2-15*d*e*x+70*d^2)/e*(-e^2*x^2+d^2)^(1/2)-35/2*d^3/(e^2)^(1/
2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-16*d^3/e^2/(x+d/e)*(-e^2*(x+
d/e)^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5}\,dx =$$
$$-\frac{166\,d^3ex+166\,d^4-210\,(d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(2\,e^3x^3-13\,de^2x^2+55\,d^2ex+166\,d^3)\sqrt{-}}{6\,(e^2x+de)}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
-1/6*(166*d^3*e*x + 166*d^4 - 210*(d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x
^2 + d^2))/(e*x)) + (2*e^3*x^3 - 13*d*e^2*x^2 + 55*d^2*e*x + 166*d^3)*sqrt
(-e^2*x^2 + d^2))/(e^2*x + d*e)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}{(d+ex)^5}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**5,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.50

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5}\,dx = -\frac{35\,d^3\arcsin\left(\frac{ex}{d}\right)}{2\,e} + \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{3\,(e^5x^4+4\,de^4x^3+6\,d^2e^3x^2+4\,d^3e^2x+d^4e)} + \frac{7\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{6\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} + \frac{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{6\,(e^3x^2+2\,de^2x+d^2e)} - \frac{35\,\sqrt{-e^2x^2+d^2}d^3}{e^2x+de}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
-35/2*d^3*arcsin(e*x/d)/e + 1/3*(-e^2*x^2 + d^2)^(7/2)/(e^5*x^4 + 4*d*e^4*
x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^4*e) + 7/6*(-e^2*x^2 + d^2)^(5/2)*d/
(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) + 35/6*(-e^2*x^2 + d^2)^(3/2
)*d^2/(e^3*x^2 + 2*d*e^2*x + d^2*e) - 35*sqrt(-e^2*x^2 + d^2)*d^3/(e^2*x +
 d*e)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.41

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5}\,dx = \frac{\left(840\,d^4e^4\arctan\left(\sqrt{\frac{2\,d}{ex+d}-1}\right)\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e) - 384\,d^4e^4\sqrt{\frac{2\,d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sg}\right.}{}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x, algorithm="giac")
```

output
```
1/24*(840*d^4*e^4*arctan(sqrt(2*d/(e*x + d) - 1))*sgn(1/(e*x + d))*sgn(e)
- 384*d^4*e^4*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e) - (87*d^4*e^
4*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e) + 136*d^4*e^4*(2*d/(e*
x + d) - 1)^(3/2)*sgn(1/(e*x + d))*sgn(e) + 57*d^4*e^4*sqrt(2*d/(e*x + d)
- 1)*sgn(1/(e*x + d))*sgn(e))*(e*x + d)^3/d^3)*abs(e)/(d*e^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5}\,dx = \int \frac{(d^2-e^2\,x^2)^{7/2}}{(d+e\,x)^5}\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^5,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.53

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^5}\,dx = \frac{-105\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^3+105asin\left(\frac{ex}{d}\right)d^4+105asin\left(\frac{ex}{d}\right)d^3ex+222\sqrt{-e^2x^2}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^5,x)
```

output
```
( - 105*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**3 + 105*asin((e*x)/d)*d**4
 + 105*asin((e*x)/d)*d**3*e*x + 222*sqrt(d**2 - e**2*x**2)*d**3 + 55*sqrt(
d**2 - e**2*x**2)*d**2*e*x - 13*sqrt(d**2 - e**2*x**2)*d*e**2*x**2 + 2*sqr
t(d**2 - e**2*x**2)*e**3*x**3 - 222*d**4 + 55*d**3*e*x + 68*d**2*e**2*x**2
 - 15*d*e**3*x**3 + 2*e**4*x**4)/(6*e*(sqrt(d**2 - e**2*x**2) - d - e*x))
```

3.68 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6} dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6} dx = \frac{28d\sqrt{d^2-e^2x^2}}{3e} - \frac{7}{6}x\sqrt{d^2-e^2x^2} + \frac{56d^2\sqrt{d^2-e^2x^2}}{3e(d+ex)} - \frac{2(d^2-e^2x^2)^{7/2}}{3e(d+ex)^5} + \frac{35d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
28/3*d*(-e^2*x^2+d^2)^(1/2)/e-7/6*x*(-e^2*x^2+d^2)^(1/2)+56/3*d^2*(-e^2*x^
2+d^2)^(1/2)/e/(e*x+d)-2/3*(-e^2*x^2+d^2)^(7/2)/e/(e*x+d)^5+35/2*d^2*arcta
n(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6} dx = \frac{\sqrt{d^2-e^2x^2}(164d^3+229d^2ex+30de^2x^2-3e^3x^3)}{6e(d+ex)^2} - \frac{35d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^6,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(164*d^3 + 229*d^2*e*x + 30*d*e^2*x^2 - 3*e^3*x^3))/(
6*e*(d + e*x)^2) - (35*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])]
)/e
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {465, 463, 25, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^6}dx$$

$$\Big\downarrow \text{ 465}$$

$$-\frac{7}{3}\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^4}dx-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \text{ 463}$$

$$-\frac{7}{3}\left(\int -\frac{7d^2-4exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{7}{3}\left(-\int \frac{7d^2-4exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \text{ 2346}$$

$$-\frac{7}{3}\left(\frac{\int -\frac{de^2(15d-8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{7}{3}\left(-\frac{\int \frac{de^2(15d-8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$\Big\downarrow$ 27

$$-\frac{7}{3}\left(-\frac{1}{2}d\int \frac{15d-8ex}{\sqrt{d^2-e^2x^2}}dx-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$\Big\downarrow$ 455

$$-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$\Big\downarrow$ 224

$$-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

$\Big\downarrow$ 216

$$-\frac{7}{3}\left(-\frac{1}{2}d\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{8\sqrt{d^2-e^2x^2}}{e}\right)-\frac{8d^2\sqrt{d^2-e^2x^2}}{e(d+ex)}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{3e(d+ex)^5}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^6,x]
```

output
```
(-2*(d^2 - e^2*x^2)^(7/2))/(3*e*(d + e*x)^5) - (7*((x*Sqrt[d^2 - e^2*x^2])
/2 - (8*d^2*Sqrt[d^2 - e^2*x^2])/(e*(d + e*x)) - (d*((8*Sqrt[d^2 - e^2*x^2
])/e + (15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

method	result
risch	$\frac{(-ex+12d)\sqrt{-e^2x^2+d^2}}{2e}+\frac{35d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{80d^2\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}{3e^2\left(x+\frac{d}{e}\right)}-\frac{16d^3\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right.}}{3e^3\left(x+\frac{d}{e}\right)^2}$ $5e\left(\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right.}{3de\left(x+\frac{d}{e}\right)^3}\right.$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e*x+12*d)/e*(-e^2*x^2+d^2)^(1/2)+35/2*d^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2))+80/3*d^2/e^2/(x+d/e)*(-e^2*(x+d/e)^2+2*d*e*(x
+d/e))^(1/2)-16/3*d^3/e^3/(x+d/e)^2*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6}\,dx = \frac{164\,d^2e^2x^2+328\,d^3ex+164\,d^4-210\,(d^2e^2x^2+2\,d^3ex+d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right.}{6\,(e^3x^2+2\,de^2x+d^2e)}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x, algorithm="fricas")
```

output
```
1/6*(164*d^2*e^2*x^2 + 328*d^3*e*x + 164*d^4 - 210*(d^2*e^2*x^2 + 2*d^3*e*
x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (3*e^3*x^3 - 30*d*e^2
*x^2 - 229*d^2*e*x - 164*d^3)*sqrt(-e^2*x^2 + d^2))/(e^3*x^2 + 2*d*e^2*x +
 d^2*e)
```

Sympy [F]

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6}\,dx = \int \frac{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}{(d+ex)^6}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**6,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(119) = 238$.

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.95

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6}\,dx = \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{2\,(e^6x^5+5\,de^5x^4+10\,d^2e^4x^3+10\,d^3e^3x^2+5\,d^4e^2x+d^5e)} + \frac{7\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{2\,(e^5x^4+4\,de^4x^3+6\,d^2e^3x^2+4\,d^3e^2x+d^4e)} - \frac{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{6\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} - \frac{35\,\sqrt{-e^2x^2+d^2}d^3}{3\,(e^3x^2+2\,de^2x+d^2e)} + \frac{35\,d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e} + \frac{245\,\sqrt{-e^2x^2+d^2}d^2}{6\,(e^2x+de)}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x, algorithm="maxima")
```

output
```
1/2*(-e^2*x^2 + d^2)^(7/2)/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^
3*e^3*x^2 + 5*d^4*e^2*x + d^5*e) + 7/2*(-e^2*x^2 + d^2)^(5/2)*d/(e^5*x^4 +
 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^4*e) - 35/6*(-e^2*x^2 + d^2
)^(3/2)*d^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) - 35/3*sqrt(-e^2
*x^2 + d^2)*d^3/(e^3*x^2 + 2*d*e^2*x + d^2*e) + 35/2*d^2*arcsin(e*x/d)/e +
 245/6*sqrt(-e^2*x^2 + d^2)*d^2/(e^2*x + d*e)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6}\,dx = \frac{35\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} - \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(x-\frac{12\,d}{e}\right) - \frac{32\left(4\,d^2+\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x}+\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2}\right)}{3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^3|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x, algorithm="giac")
```

output
```
35/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/2*sqrt(-e^2*x^2 + d^2)*(x
- 12*d/e) - 32/3*(4*d^2 + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2/(e^2*x
) + 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2))/(((d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6}\,dx=\int \frac{\left(d^2-e^2\,x^2\right)^{7/2}}{\left(d+e\,x\right)^6}\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^6,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.98

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^6}\,dx=\frac{105\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^3+105\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^2ex-105asin\left(\frac{ex}{d}\right)d^4-$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^6,x)
```

output
```
(105*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**3 + 105*sqrt(d**2 - e**2*x**2
)*asin((e*x)/d)*d**2*e*x - 105*asin((e*x)/d)*d**4 - 210*asin((e*x)/d)*d**3
*e*x - 105*asin((e*x)/d)*d**2*e**2*x**2 - 66*sqrt(d**2 - e**2*x**2)*d**3 -
 131*sqrt(d**2 - e**2*x**2)*d**2*e*x - 30*sqrt(d**2 - e**2*x**2)*d*e**2*x*
*2 + 3*sqrt(d**2 - e**2*x**2)*e**3*x**3 + 66*d**4 - 131*d**3*e*x - 297*d**
2*e**2*x**2 - 33*d*e**3*x**3 + 3*e**4*x**4)/(6*e*(sqrt(d**2 - e**2*x**2)*d
 + sqrt(d**2 - e**2*x**2)*e*x - d**2 - 2*d*e*x - e**2*x**2))
```

3.69 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7}\,dx = -\frac{7\sqrt{d^2-e^2x^2}}{3e} - \frac{28d\sqrt{d^2-e^2x^2}}{3e(d+ex)} + \frac{14(d^2-e^2x^2)^{5/2}}{15e(d+ex)^4} - \frac{2(d^2-e^2x^2)^{7/2}}{5e(d+ex)^6} - \frac{7d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
-7/3*(-e^2*x^2+d^2)^(1/2)/e-28/3*d*(-e^2*x^2+d^2)^(1/2)/e/(e*x+d)+14/15*(-
e^2*x^2+d^2)^(5/2)/e/(e*x+d)^4-2/5*(-e^2*x^2+d^2)^(7/2)/e/(e*x+d)^6-7*d*ar
ctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7}\,dx = \frac{\sqrt{d^2-e^2x^2}(-167d^3-381d^2ex-277de^2x^2-15e^3x^3)}{15e(d+ex)^3} + \frac{7d\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^7,x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-167*d^3 - 381*d^2*e*x - 277*d*e^2*x^2 - 15*e^3*x^3)
)/(15*e*(d + e*x)^3) + (7*d*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sq
rt[-e^2]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 465, 463, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^7}dx$$

$$\Big\downarrow \text{465}$$

$$-\frac{7}{5}\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^5}dx-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \text{465}$$

$$-\frac{7}{5}\left(-\frac{5}{3}\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{(d+ex)^3}dx-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^4}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \text{463}$$

$$-\frac{7}{5}\left(-\frac{5}{3}\left(-\int \frac{3d-ex}{\sqrt{d^2-e^2x^2}}dx-\frac{4d\sqrt{d^2-e^2x^2}}{e(d+ex)}\right)-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^4}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \text{455}$$

$$-\frac{7}{5}\left(-\frac{5}{3}\left(-3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{4d\sqrt{d^2-e^2x^2}}{e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^4}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \text{224}$$

$$-\frac{7}{5}\left(-\frac{5}{3}\left(-3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{4d\sqrt{d^2-e^2x^2}}{e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^4}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6}$$

$$\Big\downarrow \; 216$$

$$-\frac{7}{5}\left(-\frac{5}{3}\left(-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{4d\sqrt{d^2-e^2x^2}}{e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{2\left(d^2-e^2x^2\right)^{5/2}}{3e(d+ex)^4}\right)-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{5e(d+ex)^6}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^7,x]
```

output
```
(-2*(d^2 - e^2*x^2)^(7/2))/(5*e*(d + e*x)^6) - (7*((-2*(d^2 - e^2*x^2)^(5/
2))/(3*e*(d + e*x)^4) - (5*(-(Sqrt[d^2 - e^2*x^2]/e) - (4*d*Sqrt[d^2 - e^2
*x^2])/(e*(d + e*x)) - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/3))/5
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e}-\frac{7d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{232d\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}{15e^2\left(x+\frac{d}{e}\right)}+\frac{128d^2\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}{15e^3\left(x+\frac{d}{e}\right)^2}-\frac{16d}{}$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/e-7*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))-232/15*d/e^2/(x+d/e)*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(1/2)+128/15*
d^2/e^3/(x+d/e)^2*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(1/2)-16/5*d^3/e^4/(x+d/e
)^3*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.22

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^7}\,dx =$$

$$-\frac{167\,de^3x^3+501\,d^2e^2x^2+501\,d^3ex+167\,d^4-210\left(de^3x^3+3\,d^2e^2x^2+3\,d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+\cdots}}{ex}\right.\cdots}{15\left(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e\right)}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x, algorithm="fricas")
```

output
```
-1/15*(167*d*e^3*x^3 + 501*d^2*e^2*x^2 + 501*d^3*e*x + 167*d^4 - 210*(d*e^
3*x^3 + 3*d^2*e^2*x^2 + 3*d^3*e*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2)
)/(e*x)) + (15*e^3*x^3 + 277*d*e^2*x^2 + 381*d^2*e*x + 167*d^3)*sqrt(-e^2*
x^2 + d^2))/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)
```

Sympy [F]

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^7}\,dx = \int \frac{\left(-(-d+ex)\,(d+ex)\right)^{\frac{7}{2}}}{\left(d+ex\right)^7}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**7,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(123) = 246$.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.84

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7}\,dx = \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{e^7x^6+6\,de^6x^5+15\,d^2e^5x^4+20\,d^3e^4x^3+15\,d^4e^3x^2+6\,d^5e^2x+d^6e} - \frac{7\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{5\,(e^6x^5+5\,de^5x^4+10\,d^2e^4x^3+10\,d^3e^3x^2+5\,d^4e^2x+d^5e)} - \frac{7\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{e^5x^4+4\,de^4x^3+6\,d^2e^3x^2+4\,d^3e^2x+d^4e} + \frac{42\,\sqrt{-e^2x^2+d^2}d^3}{5\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} + \frac{7\,(-e^2x^2+d^2)^{\frac{3}{2}}d}{3\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} + \frac{49\,\sqrt{-e^2x^2+d^2}d^2}{15\,(e^3x^2+2\,de^2x+d^2e)} - \frac{7\,d\arcsin\left(\frac{ex}{d}\right)}{e} - \frac{266\,\sqrt{-e^2x^2+d^2}d}{15\,(e^2x+de)}$$

input

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x, algorithm="maxima")
```

output

```
(-e^2*x^2 + d^2)^(7/2)/(e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5*x^4 + 20*d^3*e^
4*x^3 + 15*d^4*e^3*x^2 + 6*d^5*e^2*x + d^6*e) - 7/5*(-e^2*x^2 + d^2)^(5/2)
*d/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x
+ d^5*e) - 7*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3
*x^2 + 4*d^3*e^2*x + d^4*e) + 42/5*sqrt(-e^2*x^2 + d^2)*d^3/(e^4*x^3 + 3*d
*e^3*x^2 + 3*d^2*e^2*x + d^3*e) + 7/3*(-e^2*x^2 + d^2)^(3/2)*d/(e^4*x^3 +
3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) + 49/15*sqrt(-e^2*x^2 + d^2)*d^2/(e^3*x
^2 + 2*d*e^2*x + d^2*e) - 7*d*arcsin(e*x/d)/e - 266/15*sqrt(-e^2*x^2 + d^2
)*d/(e^2*x + d*e)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.46

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7}\,dx = -\frac{7\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{\sqrt{-e^2x^2+d^2}}{e} + \frac{16\left(19\,d + \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{130\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2} + \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d}{e^8x^4}\right)}{15\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x, algorithm="giac")
```

output
```
-7*d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - sqrt(-e^2*x^2 + d^2)/e + 16/15*(
19*d + 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 130*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) + 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*d/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e^8*x^4))
/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7}\,dx = \int \frac{(d^2-e^2\,x^2)^{7/2}}{(d+e\,x)^7}\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^7,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.72

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^7}\,dx = \frac{105atan\left(\frac{\sqrt{-e^2x^2+d^2}\,d^2-2\sqrt{-e^2x^2+d^2}\,e^2x^2}{-2e^3x^3+2d^2ex}\right)d^4+315atan\left(\frac{\sqrt{-e^2x^2+d^2}\,d^2-2\sqrt{-e^2x^2+d^2}\,e^2x^2}{-2e^3x^3+2d^2ex}\right)d}{}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^7,x)
```

output
```
(105*atan((sqrt(d**2 - e**2*x**2)*d**2 - 2*sqrt(d**2 - e**2*x**2)*e**2*x**
2)/(2*d**2*e*x - 2*e**3*x**3))*d**4 + 315*atan((sqrt(d**2 - e**2*x**2)*d**
2 - 2*sqrt(d**2 - e**2*x**2)*e**2*x**2)/(2*d**2*e*x - 2*e**3*x**3))*d**3*e
*x + 315*atan((sqrt(d**2 - e**2*x**2)*d**2 - 2*sqrt(d**2 - e**2*x**2)*e**2
*x**2)/(2*d**2*e*x - 2*e**3*x**3))*d**2*e**2*x**2 + 105*atan((sqrt(d**2 -
e**2*x**2)*d**2 - 2*sqrt(d**2 - e**2*x**2)*e**2*x**2)/(2*d**2*e*x - 2*e**3
*x**3))*d*e**3*x**3 - 334*sqrt(d**2 - e**2*x**2)*d**3 - 762*sqrt(d**2 - e*
*2*x**2)*d**2*e*x - 554*sqrt(d**2 - e**2*x**2)*d*e**2*x**2 - 30*sqrt(d**2
- e**2*x**2)*e**3*x**3)/(30*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x*
*3))
```

3.70 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^8} dx = \frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)} - \frac{2(d^2-e^2x^2)^{3/2}}{3e(d+ex)^3} + \frac{2(d^2-e^2x^2)^{5/2}}{5e(d+ex)^5} - \frac{2(d^2-e^2x^2)^{7/2}}{7e(d+ex)^7} + \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2*(-e^2*x^2+d^2)^(1/2)/e/(e*x+d)-2/3*(-e^2*x^2+d^2)^(3/2)/e/(e*x+d)^3+2/5*
(-e^2*x^2+d^2)^(5/2)/e/(e*x+d)^5-2/7*(-e^2*x^2+d^2)^(7/2)/e/(e*x+d)^7+arct
an(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^8} dx = \frac{8\sqrt{d^2-e^2x^2}(19d^3+76d^2ex+71de^2x^2+44e^3x^3)}{105e(d+ex)^4} - \frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^8,x]
```

output
```
(8*Sqrt[d^2 - e^2*x^2]*(19*d^3 + 76*d^2*e*x + 71*d*e^2*x^2 + 44*e^3*x^3))/
(105*e*(d + e*x)^4) - (2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/
e
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 465, 465, 463, 25, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^8}dx$$

$$\big\downarrow \text{465}$$

$$-\int \frac{\left(d^2-e^2x^2\right)^{5/2}}{(d+ex)^6}dx-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}$$

$$\big\downarrow \text{465}$$

$$\int \frac{\left(d^2-e^2x^2\right)^{3/2}}{(d+ex)^4}dx-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}$$

$$\big\downarrow \text{465}$$

$$-\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}dx-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}-\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\big\downarrow \text{463}$$

$$-\int -\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}-\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\big\downarrow \text{25}$$

$$\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}-\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \; 224$$

$$\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}-\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{2\left(d^2-e^2x^2\right)^{7/2}}{7e(d+ex)^7}+\frac{2\left(d^2-e^2x^2\right)^{5/2}}{5e(d+ex)^5}-\frac{2\left(d^2-e^2x^2\right)^{3/2}}{3e(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}}{e(d+ex)}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^8,x]
```

output
```
(2*Sqrt[d^2 - e^2*x^2])/(e*(d + e*x)) - (2*(d^2 - e^2*x^2)^(3/2))/(3*e*(d
+ e*x)^3) + (2*(d^2 - e^2*x^2)^(5/2))/(5*e*(d + e*x)^5) - (2*(d^2 - e^2*x^
2)^(7/2))/(7*e*(d + e*x)^7) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(127) = 254$.

Time = 1.19 (sec) , antiderivative size = 611, normalized size of antiderivative = 4.27

method	result

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
1/e^8*(-1/7/d/e/(x+d/e)^8*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(9/2)-1/7*e/d*(-1
/5/d/e/(x+d/e)^7*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(9/2)-2/5*e/d*(-1/3/d/e/(x
+d/e)^6*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(9/2)-e/d*(-1/d/e/(x+d/e)^5*(-e^2*(
x+d/e)^2+2*d*e*(x+d/e))^(9/2)-4*e/d*(1/d/e/(x+d/e)^4*(-e^2*(x+d/e)^2+2*d*e
*(x+d/e))^(9/2)+5*e/d*(1/3/d/e/(x+d/e)^3*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(9
/2)+2*e/d*(1/5/d/e/(x+d/e)^2*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(9/2)+7/5*e/d*
(1/7*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(7/2)+d*e*(-1/12*(-2*e^2*(x+d/e)+2*d*e
)/e^2*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(5/2)+5/6*d^2*(-1/8*(-2*e^2*(x+d/e)+2
*d*e)/e^2*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(3/2)+3/4*d^2*(-1/4*(-2*e^2*(x+d/
e)+2*d*e)/e^2*(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(1/2)+1/2*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)*x/(-e^2*(x+d/e)^2+2*d*e*(x+d/e))^(1/2)))))))))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.40

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^8}\,dx = \frac{2\,(76\,e^4x^4+304\,de^3x^3+456\,d^2e^2x^2+304\,d^3ex+76\,d^4-105\,(e^4x^4+4\,de^3x^3+6\,d}{105\,(e^5x^4+4\,e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x, algorithm="fricas")
```

output
```
2/105*(76*e^4*x^4 + 304*d*e^3*x^3 + 456*d^2*e^2*x^2 + 304*d^3*e*x + 76*d^4
 - 105*(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*arctan(-(
d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 4*(44*e^3*x^3 + 71*d*e^2*x^2 + 76*d^2*e
*x + 19*d^3)*sqrt(-e^2*x^2 + d^2))/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2
+ 4*d^3*e^2*x + d^4*e)
```

Sympy [F]

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^8} \, dx = \int \frac{(-(-d + ex)(d + ex))^{\frac{7}{2}}}{(d + ex)^8} \, dx$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**8,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**(7/2)/(d + e*x)**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(127) = 254$.

Time = 0.13 (sec) , antiderivative size = 623, normalized size of antiderivative = 4.36

$$\begin{aligned}
&\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^8} \, dx = \\
&-\frac{(-e^2x^2 + d^2)^{\frac{7}{2}}}{7\,(e^8x^7 + 7\,de^7x^6 + 21\,d^2e^6x^5 + 35\,d^3e^5x^4 + 35\,d^4e^4x^3 + 21\,d^5e^3x^2 + 7\,d^6e^2x + d^7e)} \\
&-\frac{(-e^2x^2 + d^2)^{\frac{5}{2}}d}{e^7x^6 + 6\,de^6x^5 + 15\,d^2e^5x^4 + 20\,d^3e^4x^3 + 15\,d^4e^3x^2 + 6\,d^5e^2x + d^6e} \\
&+\frac{5\,(-e^2x^2 + d^2)^{\frac{3}{2}}d^2}{2\,(e^6x^5 + 5\,de^5x^4 + 10\,d^2e^4x^3 + 10\,d^3e^3x^2 + 5\,d^4e^2x + d^5e)} \\
&-\frac{15\,\sqrt{-e^2x^2 + d^2}d^3}{7\,(e^5x^4 + 4\,de^4x^3 + 6\,d^2e^3x^2 + 4\,d^3e^2x + d^4e)} \\
&+\frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{5\,(e^6x^5 + 5\,de^5x^4 + 10\,d^2e^4x^3 + 10\,d^3e^3x^2 + 5\,d^4e^2x + d^5e)} \\
&+\frac{(-e^2x^2 + d^2)^{\frac{3}{2}}d}{e^5x^4 + 4\,de^4x^3 + 6\,d^2e^3x^2 + 4\,d^3e^2x + d^4e} \\
&-\frac{69\,\sqrt{-e^2x^2 + d^2}d^2}{70\,(e^4x^3 + 3\,de^3x^2 + 3\,d^2e^2x + d^3e)} - \frac{(-e^2x^2 + d^2)^{\frac{3}{2}}}{3\,(e^4x^3 + 3\,de^3x^2 + 3\,d^2e^2x + d^3e)} \\
&-\frac{34\,\sqrt{-e^2x^2 + d^2}d}{105\,(e^3x^2 + 2\,de^2x + d^2e)} + \frac{\arcsin\left(\frac{ex}{d}\right)}{e} + \frac{281\,\sqrt{-e^2x^2 + d^2}}{105\,(e^2x + de)}
\end{aligned}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x, algorithm="maxima")
```

output
```
-1/7*(-e^2*x^2 + d^2)^(7/2)/(e^8*x^7 + 7*d*e^7*x^6 + 21*d^2*e^6*x^5 + 35*d
^3*e^5*x^4 + 35*d^4*e^4*x^3 + 21*d^5*e^3*x^2 + 7*d^6*e^2*x + d^7*e) - (-e^
2*x^2 + d^2)^(5/2)*d/(e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5*x^4 + 20*d^3*e^4*
x^3 + 15*d^4*e^3*x^2 + 6*d^5*e^2*x + d^6*e) + 5/2*(-e^2*x^2 + d^2)^(3/2)*d
^2/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x
+ d^5*e) - 15/7*sqrt(-e^2*x^2 + d^2)*d^3/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^
3*x^2 + 4*d^3*e^2*x + d^4*e) + 1/5*(-e^2*x^2 + d^2)^(5/2)/(e^6*x^5 + 5*d*e
^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e) + (-e^2*x^
2 + d^2)^(3/2)*d/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^
4*e) - 69/70*sqrt(-e^2*x^2 + d^2)*d^2/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x
 + d^3*e) - 1/3*(-e^2*x^2 + d^2)^(3/2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*
x + d^3*e) - 34/105*sqrt(-e^2*x^2 + d^2)*d/(e^3*x^2 + 2*d*e^2*x + d^2*e) +
 arcsin(e*x/d)/e + 281/105*sqrt(-e^2*x^2 + d^2)/(e^2*x + d*e)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.47

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^8}\,dx = \frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{16\left(\frac{133\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{294\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{490\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + \frac{175\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + \frac{105\left(de+\sqrt{\right.}}{}\right.}{105\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^7|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x, algorithm="giac")
```

output
```
arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 16/105*(133*(d*e + sqrt(-e^2*x^2 + d^
2)*abs(e))/(e^2*x) + 294*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) +
 490*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) + 175*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 105*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^5/(e^10*x^5) + 19)/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^7*a
bs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^8}\,dx = \int \frac{(d^2-e^2\,x^2)^{7/2}}{(d+e\,x)^8}\,dx$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^8,x)
```

output
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^8, x)
```

Reduce [F]

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^8}\,dx = \int \frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{(ex+d)^8}dx$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x)
```

output
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^8,x)
```

3.71 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9} dx$

Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9} dx = -\frac{(d^2-e^2x^2)^{9/2}}{9de(d+ex)^9}$$

output
```
-1/9*(-e^2*x^2+d^2)^(9/2)/d/e/(e*x+d)^9
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9} dx = -\frac{(d-ex)^4\sqrt{d^2-e^2x^2}}{9de(d+ex)^5}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^9,x]
```

output
```
-1/9*((d - e*x)^4*Sqrt[d^2 - e^2*x^2])/(d*e*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9}\,dx$$

$$\Big\downarrow \text{460}$$

$$-\frac{(d^2-e^2x^2)^{9/2}}{9de(d+ex)^9}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^9,x]
```

output
```
-1/9*(d^2 - e^2*x^2)^(9/2)/(d*e*(d + e*x)^9)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{(-ex+d)\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9(ex+d)^{8}de}$	36
orering	$-\frac{(-ex+d)\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{9(ex+d)^{8}de}$	36
default	$-\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{9e^{10}d\left(x+\frac{d}{e}\right)^{9}}$	46
trager	$-\frac{\left(e^{4}x^{4}-4d\,e^{3}x^{3}+6d^{2}e^{2}x^{2}-4d^{3}ex+d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{9d(ex+d)^{5}e}$	68

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

output
```
-1/9/(e*x+d)^8*(-e*x+d)/d/e*(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.91

$$\int \frac{\left(d^{2}-e^{2}x^{2}\right)^{7/2}}{(d+ex)^{9}}\,dx = -\frac{e^{5}x^{5}+5\,de^{4}x^{4}+10\,d^{2}e^{3}x^{3}+10\,d^{3}e^{2}x^{2}+5\,d^{4}ex+d^{5}+\left(e^{4}x^{4}-4\,de^{3}x^{3}+6\,d^{2}e^{2}x^{2}-4\,d^{3}ex+d^{4}\right)\sqrt{-e^{2}\cdots}}{9\left(de^{6}x^{5}+5\,d^{2}e^{5}x^{4}+10\,d^{3}e^{4}x^{3}+10\,d^{4}e^{3}x^{2}+5\,d^{5}e^{2}x+d^{6}e\right)}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x, algorithm="fricas")
```

output
```
-1/9*(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x
+ d^5 + (e^4*x^4 - 4*d*e^3*x^3 + 6*d^2*e^2*x^2 - 4*d^3*e*x + d^4)*sqrt(-e^
2*x^2 + d^2))/(d*e^6*x^5 + 5*d^2*e^5*x^4 + 10*d^3*e^4*x^3 + 10*d^4*e^3*x^2
 + 5*d^5*e^2*x + d^6*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**9,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 539, normalized size of antiderivative = 16.33

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9}\,dx = -\frac{(-e^2x^2+d^2)^{\frac{7}{2}}}{e^9x^8+8\,de^8x^7+28\,d^2e^7x^6+56\,d^3e^6x^5+70\,d^4e^5x^4+56\,d^5e^4x^3+28\,d^6e^3x^2+8\,d^7e^2x+d^8e} + \frac{7\,(-e^2x^2+d^2)^{\frac{5}{2}}d}{2\,(e^8x^7+7\,de^7x^6+21\,d^2e^6x^5+35\,d^3e^5x^4+35\,d^4e^4x^3+21\,d^5e^3x^2+7\,d^6e^2x+d^7e)} - \frac{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{6\,(e^7x^6+6\,de^6x^5+15\,d^2e^5x^4+20\,d^3e^4x^3+15\,d^4e^3x^2+6\,d^5e^2x+d^6e)} + \frac{35\,\sqrt{-e^2x^2+d^2}d^3}{9\,(e^6x^5+5\,de^5x^4+10\,d^2e^4x^3+10\,d^3e^3x^2+5\,d^4e^2x+d^5e)} - \frac{5\,\sqrt{-e^2x^2+d^2}d^2}{18\,(e^5x^4+4\,de^4x^3+6\,d^2e^3x^2+4\,d^3e^2x+d^4e)} - \frac{\sqrt{-e^2x^2+d^2}d}{6\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+d^3e)} - \frac{\sqrt{-e^2x^2+d^2}}{9\,(e^3x^2+2\,de^2x+d^2e)} - \frac{\sqrt{-e^2x^2+d^2}}{9\,(de^2x+d^2e)}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x, algorithm="maxima")
```

output
```
-(-e^2*x^2 + d^2)^(7/2)/(e^9*x^8 + 8*d*e^8*x^7 + 28*d^2*e^7*x^6 + 56*d^3*e
^6*x^5 + 70*d^4*e^5*x^4 + 56*d^5*e^4*x^3 + 28*d^6*e^3*x^2 + 8*d^7*e^2*x +
d^8*e) + 7/2*(-e^2*x^2 + d^2)^(5/2)*d/(e^8*x^7 + 7*d*e^7*x^6 + 21*d^2*e^6*
x^5 + 35*d^3*e^5*x^4 + 35*d^4*e^4*x^3 + 21*d^5*e^3*x^2 + 7*d^6*e^2*x + d^7
*e) - 35/6*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5*
x^4 + 20*d^3*e^4*x^3 + 15*d^4*e^3*x^2 + 6*d^5*e^2*x + d^6*e) + 35/9*sqrt(-
e^2*x^2 + d^2)*d^3/(e^6*x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^
2 + 5*d^4*e^2*x + d^5*e) - 5/18*sqrt(-e^2*x^2 + d^2)*d^2/(e^5*x^4 + 4*d*e^
4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^4*e) - 1/6*sqrt(-e^2*x^2 + d^2)*d/
(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e) - 1/9*sqrt(-e^2*x^2 + d^2)/(
e^3*x^2 + 2*d*e^2*x + d^2*e) - 1/9*sqrt(-e^2*x^2 + d^2)/(d*e^2*x + d^2*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(29) = 58$.

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.06

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9}\,dx = \frac{2\left(\frac{36\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{126\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+\frac{84\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^6}{e^{12}x^6}+\frac{9\left(de+\sqrt{-e^2x^2\cdots\right.}{e^{16}x^8}\cdots\right.}{9\,d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^9|e|}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x, algorithm="giac")
```

output
```
2/9*(36*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 126*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 84*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^6/(e^12*x^6) + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8) + 1)/
(d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^9*abs(e))
```

Mupad [B] (verification not implemented)

Time = 7.99 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.27

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9}\,dx = \frac{8\sqrt{d^2-e^2\,x^2}}{9\,e\,(d+e\,x)^2} - \frac{8\,d\,\sqrt{d^2-e^2\,x^2}}{3\,e\,(d+e\,x)^3} - \frac{\sqrt{d^2-e^2\,x^2}}{9\,d\,e\,(d+e\,x)} + \frac{32\,d^2\,\sqrt{d^2-e^2\,x^2}}{9\,e\,(d+e\,x)^4} - \frac{16\,d^3\,\sqrt{d^2-e^2\,x^2}}{9\,e\,(d+e\,x)^5}$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^9,x)
```

output
```
(8*(d^2 - e^2*x^2)^(1/2))/(9*e*(d + e*x)^2) - (8*d*(d^2 - e^2*x^2)^(1/2))/
(3*e*(d + e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(9*d*e*(d + e*x)) + (32*d^2*(d^2
 - e^2*x^2)^(1/2))/(9*e*(d + e*x)^4) - (16*d^3*(d^2 - e^2*x^2)^(1/2))/(9*e
*(d + e*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 7.64

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^9}\,dx = \frac{\frac{2\sqrt{-e^2x^2+d^2}\,d^4}{9} + \frac{4\sqrt{-e^2x^2+d^2}\,d^2e^2x^2}{3} + 2\sqrt{}}{de\left(\sqrt{-e^2x^2+d^2}\,d^4 + 4\sqrt{-e^2x^2+d^2}\,d^3ex + 6\sqrt{-e^2x^2+d^2}\,d^2e^2x^2 + 4\sqrt{-e^2x^2+d}\right.}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^9,x)
```

output
```
(2*(sqrt(d**2 - e**2*x**2)*d**4 + 6*sqrt(d**2 - e**2*x**2)*d**2*e**2*x**2
+ sqrt(d**2 - e**2*x**2)*e**4*x**4 - d**5 - 10*d**3*e**2*x**2 - 5*d*e**4*x
**4))/(9*d*e*(sqrt(d**2 - e**2*x**2)*d**4 + 4*sqrt(d**2 - e**2*x**2)*d**3*
e*x + 6*sqrt(d**2 - e**2*x**2)*d**2*e**2*x**2 + 4*sqrt(d**2 - e**2*x**2)*d
*e**3*x**3 + sqrt(d**2 - e**2*x**2)*e**4*x**4 - d**5 - 5*d**4*e*x - 10*d**
3*e**2*x**2 - 10*d**2*e**3*x**3 - 5*d*e**4*x**4 - e**5*x**5))
```

3.72 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx = -\frac{(d^2-e^2x^2)^{9/2}}{11de(d+ex)^{10}} - \frac{(d^2-e^2x^2)^{9/2}}{99d^2e(d+ex)^9}$$

output
```
-1/11*(-e^2*x^2+d^2)^(9/2)/d/e/(e*x+d)^10-1/99*(-e^2*x^2+d^2)^(9/2)/d^2/e/
(e*x+d)^9
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx = -\frac{(d-ex)^4(10d+ex)\sqrt{d^2-e^2x^2}}{99d^2e(d+ex)^6}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^10,x]
```

output
```
-1/99*((d - e*x)^4*(10*d + e*x)*Sqrt[d^2 - e^2*x^2])/(d^2*e*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{10}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^9}dx}{11d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{11de(d+ex)^{10}}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\left(d^2-e^2x^2\right)^{9/2}}{99d^2e(d+ex)^9}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{11de(d+ex)^{10}}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^10,x]
```

output
```
-1/11*(d^2 - e^2*x^2)^(9/2)/(d*e*(d + e*x)^10) - (d^2 - e^2*x^2)^(9/2)/(99
*d^2*e*(d + e*x)^9)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(-ex+d)(ex+10d)\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{99(ex+d)^{9}d^{2}e}$	43
orering	$-\frac{(-ex+d)(ex+10d)\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{99(ex+d)^{9}d^{2}e}$	43
trager	$-\frac{\left(e^{5}x^{5}+6d\,e^{4}x^{4}-34d^{2}e^{3}x^{3}+56d^{3}e^{2}x^{2}-39d^{4}ex+10d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{99d^{2}(ex+d)^{6}e}$	81
default	$\frac{-\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{11de\left(x+\frac{d}{e}\right)^{10}}-\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{99d^{2}\left(x+\frac{d}{e}\right)^{9}}}{e^{10}}$	93

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/99*(-e*x+d)*(e*x+10*d)*(-e^2*x^2+d^2)^(7/2)/(e*x+d)^9/d^2/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(59) = 118$.

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.01

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d+ex)^{10}}\, dx = -\frac{10\, e^6x^6 + 60\, de^5x^5 + 150\, d^2e^4x^4 + 200\, d^3e^3x^3 + 150\, d^4e^2x^2 + 60\, d^5ex + 10\, d^6 + (e^5x^5 + 6\, de^4x^4 - 34\, d^2e}{99\,(d^2e^7x^6 + 6\, d^3e^6x^5 + 15\, d^4e^5x^4 + 20\, d^5e^4x^3 + 15\, d^6e^3x^2 + 6\, d^7e}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x, algorithm="fricas")
```

output
```
-1/99*(10*e^6*x^6 + 60*d*e^5*x^5 + 150*d^2*e^4*x^4 + 200*d^3*e^3*x^3 + 150
*d^4*e^2*x^2 + 60*d^5*e*x + 10*d^6 + (e^5*x^5 + 6*d*e^4*x^4 - 34*d^2*e^3*x
^3 + 56*d^3*e^2*x^2 - 39*d^4*e*x + 10*d^5)*sqrt(-e^2*x^2 + d^2))/(d^2*e^7*
x^6 + 6*d^3*e^6*x^5 + 15*d^4*e^5*x^4 + 20*d^5*e^4*x^3 + 15*d^6*e^3*x^2 + 6
*d^7*e^2*x + d^8*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d+ex)^{10}}\, dx = \text{Timed out}$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**10,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 659, normalized size of antiderivative = 9.84

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^{10}}\,dx =$$

$$-\frac{(-e^2x^2 + d^2)^{\frac{7}{2}}}{2\,(e^{10}x^9 + 9\,de^9x^8 + 36\,d^2e^8x^7 + 84\,d^3e^7x^6 + 126\,d^4e^6x^5 + 126\,d^5e^5x^4 + 84\,d^6e^4x^3 + 36\,d^7e^3x^2 + 9\,d^8e^2x}$$
$$+\frac{7\,(-e^2x^2 + d^2)^{\frac{5}{2}}d}{6\,(e^9x^8 + 8\,de^8x^7 + 28\,d^2e^7x^6 + 56\,d^3e^6x^5 + 70\,d^4e^5x^4 + 56\,d^5e^4x^3 + 28\,d^6e^3x^2 + 8\,d^7e^2x + d^8e)}$$
$$-\frac{35\,(-e^2x^2 + d^2)^{\frac{3}{2}}d^2}{24\,(e^8x^7 + 7\,de^7x^6 + 21\,d^2e^6x^5 + 35\,d^3e^5x^4 + 35\,d^4e^4x^3 + 21\,d^5e^3x^2 + 7\,d^6e^2x + d^7e)}$$
$$+\frac{35\,\sqrt{-e^2x^2 + d^2}d^3}{44\,(e^7x^6 + 6\,de^6x^5 + 15\,d^2e^5x^4 + 20\,d^3e^4x^3 + 15\,d^4e^3x^2 + 6\,d^5e^2x + d^6e)}$$
$$-\frac{35\,\sqrt{-e^2x^2 + d^2}d^2}{792\,(e^6x^5 + 5\,de^5x^4 + 10\,d^2e^4x^3 + 10\,d^3e^3x^2 + 5\,d^4e^2x + d^5e)}$$
$$-\frac{5\,\sqrt{-e^2x^2 + d^2}d}{198\,(e^5x^4 + 4\,de^4x^3 + 6\,d^2e^3x^2 + 4\,d^3e^2x + d^4e)}$$
$$-\frac{\sqrt{-e^2x^2 + d^2}}{66\,(e^4x^3 + 3\,de^3x^2 + 3\,d^2e^2x + d^3e)} - \frac{\sqrt{-e^2x^2 + d^2}}{99\,(de^3x^2 + 2\,d^2e^2x + d^3e)} - \frac{\sqrt{-e^2x^2 + d^2}}{99\,(d^2e^2x + d^3e)}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x, algorithm="maxima")
```

output

```
-1/2*(-e^2*x^2 + d^2)^(7/2)/(e^10*x^9 + 9*d*e^9*x^8 + 36*d^2*e^8*x^7 + 84*
d^3*e^7*x^6 + 126*d^4*e^6*x^5 + 126*d^5*e^5*x^4 + 84*d^6*e^4*x^3 + 36*d^7*
e^3*x^2 + 9*d^8*e^2*x + d^9*e) + 7/6*(-e^2*x^2 + d^2)^(5/2)*d/(e^9*x^8 + 8
*d*e^8*x^7 + 28*d^2*e^7*x^6 + 56*d^3*e^6*x^5 + 70*d^4*e^5*x^4 + 56*d^5*e^4
*x^3 + 28*d^6*e^3*x^2 + 8*d^7*e^2*x + d^8*e) - 35/24*(-e^2*x^2 + d^2)^(3/2
)*d^2/(e^8*x^7 + 7*d*e^7*x^6 + 21*d^2*e^6*x^5 + 35*d^3*e^5*x^4 + 35*d^4*e^
4*x^3 + 21*d^5*e^3*x^2 + 7*d^6*e^2*x + d^7*e) + 35/44*sqrt(-e^2*x^2 + d^2)
*d^3/(e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5*x^4 + 20*d^3*e^4*x^3 + 15*d^4*e^3
*x^2 + 6*d^5*e^2*x + d^6*e) - 35/792*sqrt(-e^2*x^2 + d^2)*d^2/(e^6*x^5 + 5
*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e) - 5/19
8*sqrt(-e^2*x^2 + d^2)*d/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^
2*x + d^4*e) - 1/66*sqrt(-e^2*x^2 + d^2)/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^
2*x + d^3*e) - 1/99*sqrt(-e^2*x^2 + d^2)/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e)
 - 1/99*sqrt(-e^2*x^2 + d^2)/(d^2*e^2*x + d^3*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(59) = 118$.

Time = 0.15 (sec) , antiderivative size = 351, normalized size of antiderivative = 5.24

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx = \frac{2\left(\frac{11\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{451\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{396\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + \frac{2376\left(de+\sqrt{-e^2\cdots}\right.}{e^8x^4}\cdots\right.}{\cdots}$$

input

```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x, algorithm="giac")
```

output

```
2/99*(11*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 451*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 396*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/(e^6*x^3) + 2376*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 1386
*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) + 3234*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^6/(e^12*x^6) + 924*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
7/(e^14*x^7) + 1254*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8) + 99*
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9/(e^18*x^9) + 99*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^10/(e^20*x^10) + 10)/(d^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))/(e^2*x) + 1)^11*abs(e))
```

Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx = \frac{16\sqrt{d^2-e^2\,x^2}}{33\,e\,(d+e\,x)^3}-\frac{184\,d\,\sqrt{d^2-e^2\,x^2}}{99\,e\,(d+e\,x)^4}-\frac{\sqrt{d^2-e^2\,x^2}}{99\,d\,e\,(d+e\,x)^2} -\frac{\sqrt{d^2-e^2\,x^2}}{99\,d^2\,e\,\left(d+e\,x\right)}+\frac{272\,d^2\,\sqrt{d^2-e^2\,x^2}}{99\,e\,(d+e\,x)^5}-\frac{16\,d^3\,\sqrt{d^2-e^2\,x^2}}{11\,e\,(d+e\,x)^6}$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^10,x)
```

output
```
(16*(d^2 - e^2*x^2)^(1/2))/(33*e*(d + e*x)^3) - (184*d*(d^2 - e^2*x^2)^(1/
2))/(99*e*(d + e*x)^4) - (d^2 - e^2*x^2)^(1/2)/(99*d*e*(d + e*x)^2) - (d^2
 - e^2*x^2)^(1/2)/(99*d^2*e*(d + e*x)) + (272*d^2*(d^2 - e^2*x^2)^(1/2))/(
99*e*(d + e*x)^5) - (16*d^3*(d^2 - e^2*x^2)^(1/2))/(11*e*(d + e*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 390, normalized size of antiderivative = 5.82

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{10}}\,dx = \frac{18\sqrt{-e^2x^2+d^2}\,d^5+\sqrt{-e^2x^2+d^2}\,d^4ex+136\sqrt{-e^2x^2+d^2}\,d^3e^2x^2+46\sqrt{-e^2x^2+\cdots}}{99d^2e\left(\sqrt{-e^2x^2+d^2}\,d^5+5\sqrt{-e^2x^2+d^2}\,d^4ex+10\sqrt{-e^2x^2+d^2}\,d^3e^2x^2+10\sqrt{-e^2\cdots}\right.}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^10,x)
```

output
```
(18*sqrt(d**2 - e**2*x**2)*d**5 + sqrt(d**2 - e**2*x**2)*d**4*e*x + 136*sq
rt(d**2 - e**2*x**2)*d**3*e**2*x**2 + 46*sqrt(d**2 - e**2*x**2)*d**2*e**3*
x**3 + 46*sqrt(d**2 - e**2*x**2)*d*e**4*x**4 + 9*sqrt(d**2 - e**2*x**2)*e*
*5*x**5 - 18*d**6 + d**5*e*x - 215*d**4*e**2*x**2 - 70*d**3*e**3*x**3 - 16
0*d**2*e**4*x**4 - 43*d*e**5*x**5 - 7*e**6*x**6)/(99*d**2*e*(sqrt(d**2 - e
**2*x**2)*d**5 + 5*sqrt(d**2 - e**2*x**2)*d**4*e*x + 10*sqrt(d**2 - e**2*x
**2)*d**3*e**2*x**2 + 10*sqrt(d**2 - e**2*x**2)*d**2*e**3*x**3 + 5*sqrt(d*
*2 - e**2*x**2)*d*e**4*x**4 + sqrt(d**2 - e**2*x**2)*e**5*x**5 - d**6 - 6*
d**5*e*x - 15*d**4*e**2*x**2 - 20*d**3*e**3*x**3 - 15*d**2*e**4*x**4 - 6*d
*e**5*x**5 - e**6*x**6))
```

3.73 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = -\frac{(d^2-e^2x^2)^{9/2}}{13de(d+ex)^{11}} - \frac{2(d^2-e^2x^2)^{9/2}}{143d^2e(d+ex)^{10}} - \frac{2(d^2-e^2x^2)^{9/2}}{1287d^3e(d+ex)^9}$$

output
```
-1/13*(-e^2*x^2+d^2)^(9/2)/d/e/(e*x+d)^11-2/143*(-e^2*x^2+d^2)^(9/2)/d^2/e
/(e*x+d)^10-2/1287*(-e^2*x^2+d^2)^(9/2)/d^3/e/(e*x+d)^9
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = -\frac{(d-ex)^4\sqrt{d^2-e^2x^2}(119d^2+22dex+2e^2x^2)}{1287d^3e(d+ex)^7}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^11,x]
```

output
```
-1/1287*((d - e*x)^4*Sqrt[d^2 - e^2*x^2]*(119*d^2 + 22*d*e*x + 2*e^2*x^2))
/(d^3*e*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{11}}dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{10}}dx}{13d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2\left(\frac{\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^9}dx}{11d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{11de(d+ex)^{10}}\right)}{13d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{2\left(-\frac{\left(d^2-e^2x^2\right)^{9/2}}{99d^2e(d+ex)^9}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{11de(d+ex)^{10}}\right)}{13d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^11,x]
```

output
```
-1/13*(d^2 - e^2*x^2)^(9/2)/(d*e*(d + e*x)^11) + (2*(-1/11*(d^2 - e^2*x^2)
^(9/2)/(d*e*(d + e*x)^10) - (d^2 - e^2*x^2)^(9/2)/(99*d^2*e*(d + e*x)^9)))
/(13*d)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{(-ex+d)\left(2e^{2}x^{2}+22dex+119d^{2}\right)\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{1287(ex+d)^{10}d^{3}e}$	55
orering	$-\frac{(-ex+d)\left(2e^{2}x^{2}+22dex+119d^{2}\right)\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}{1287(ex+d)^{10}d^{3}e}$	55
trager	$-\frac{\left(2e^{6}x^{6}+14d\,e^{5}x^{5}+43d^{2}e^{4}x^{4}-352d^{3}e^{3}x^{3}+628d^{4}e^{2}x^{2}-454d^{5}ex+119d^{6}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{1287d^{3}(ex+d)^{7}e}$	93
default	$\frac{-\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{13de\left(x+\frac{d}{e}\right)^{11}}+\frac{2e\left(-\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{11de\left(x+\frac{d}{e}\right)^{10}}-\frac{\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{99d^{2}\left(x+\frac{d}{e}\right)^{9}}\right)}{13d}}{e^{11}}$	145

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x,method=_RETURNVERBOSE)
```

output
```
-1/1287*(-e*x+d)*(2*e^2*x^2+22*d*e*x+119*d^2)*(-e^2*x^2+d^2)^(7/2)/(e*x+d)
^10/d^3/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(88) = 176.

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.36

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = -\frac{119\,e^7x^7+833\,de^6x^6+2499\,d^2e^5x^5+4165\,d^3e^4x^4+4165\,d^4e^3x^3+2499\,d^5e^2x^2+833\,d^6ex+119\,d^7+(2}{1287\,(d^3e^8x^7+7\,d^4e^7x^6+21\,d^5e^6x^5+35\,d^6e^5x^4+3}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x, algorithm="fricas")
```

output
```
-1/1287*(119*e^7*x^7 + 833*d*e^6*x^6 + 2499*d^2*e^5*x^5 + 4165*d^3*e^4*x^4
 + 4165*d^4*e^3*x^3 + 2499*d^5*e^2*x^2 + 833*d^6*e*x + 119*d^7 + (2*e^6*x^
6 + 14*d*e^5*x^5 + 43*d^2*e^4*x^4 - 352*d^3*e^3*x^3 + 628*d^4*e^2*x^2 - 45
4*d^5*e*x + 119*d^6)*sqrt(-e^2*x^2 + d^2))/(d^3*e^8*x^7 + 7*d^4*e^7*x^6 +
21*d^5*e^6*x^5 + 35*d^6*e^5*x^4 + 35*d^7*e^4*x^3 + 21*d^8*e^3*x^2 + 7*d^9*
e^2*x + d^10*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**11,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(88) = 176$.

Time = 0.04 (sec) , antiderivative size = 790, normalized size of antiderivative = 7.90

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = \text{Too large to display}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x, algorithm="maxima")
```

output
```
-1/3*(-e^2*x^2 + d^2)^(7/2)/(e^11*x^10 + 10*d*e^10*x^9 + 45*d^2*e^9*x^8 +
120*d^3*e^8*x^7 + 210*d^4*e^7*x^6 + 252*d^5*e^6*x^5 + 210*d^6*e^5*x^4 + 12
0*d^7*e^4*x^3 + 45*d^8*e^3*x^2 + 10*d^9*e^2*x + d^10*e) + 7/12*(-e^2*x^2 +
 d^2)^(5/2)*d/(e^10*x^9 + 9*d*e^9*x^8 + 36*d^2*e^8*x^7 + 84*d^3*e^7*x^6 +
126*d^4*e^6*x^5 + 126*d^5*e^5*x^4 + 84*d^6*e^4*x^3 + 36*d^7*e^3*x^2 + 9*d^
8*e^2*x + d^9*e) - 7/12*(-e^2*x^2 + d^2)^(3/2)*d^2/(e^9*x^8 + 8*d*e^8*x^7
+ 28*d^2*e^7*x^6 + 56*d^3*e^6*x^5 + 70*d^4*e^5*x^4 + 56*d^5*e^4*x^3 + 28*d
^6*e^3*x^2 + 8*d^7*e^2*x + d^8*e) + 7/26*sqrt(-e^2*x^2 + d^2)*d^3/(e^8*x^7
 + 7*d*e^7*x^6 + 21*d^2*e^6*x^5 + 35*d^3*e^5*x^4 + 35*d^4*e^4*x^3 + 21*d^5
*e^3*x^2 + 7*d^6*e^2*x + d^7*e) - 7/572*sqrt(-e^2*x^2 + d^2)*d^2/(e^7*x^6
+ 6*d*e^6*x^5 + 15*d^2*e^5*x^4 + 20*d^3*e^4*x^3 + 15*d^4*e^3*x^2 + 6*d^5*e
^2*x + d^6*e) - 35/5148*sqrt(-e^2*x^2 + d^2)*d/(e^6*x^5 + 5*d*e^5*x^4 + 10
*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e) - 5/1287*sqrt(-e^2*x^
2 + d^2)/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x + d^4*e) - 1
/429*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e
) - 2/1287*sqrt(-e^2*x^2 + d^2)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2/12
87*sqrt(-e^2*x^2 + d^2)/(d^3*e^2*x + d^4*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(88) = 176$.

Time = 0.15 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.13

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = \frac{2\left(\frac{260\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{6708\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{11726\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{52481\left(de+\right.}{}\right.}{}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x, algorithm="giac")
```

output
```
2/1287*(260*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 6708*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 11726*(d*e + sqrt(-e^2*x^2 + d^2)*a
bs(e))^3/(e^6*x^3) + 52481*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4)
 + 61776*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) + 120120*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^12*x^6) + 84084*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^7/(e^14*x^7) + 91377*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^
16*x^8) + 32604*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9/(e^18*x^9) + 22308*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^10/(e^20*x^10) + 2574*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^11/(e^22*x^11) + 1287*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^12/(e^24*x^12) + 119)/(d^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x)
 + 1)^13*abs(e))
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.99

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = \frac{424\sqrt{d^2-e^2\,x^2}}{1287\,e\,{(d+e\,x)}^4}-\frac{1832\,d\,\sqrt{d^2-e^2\,x^2}}{1287\,e\,{(d+e\,x)}^5}-\frac{\sqrt{d^2-e^2\,x^2}}{429\,d\,e\,{(d+e\,x)}^3}-\frac{2\,\sqrt{d^2-e^2\,x^2}}{1287\,d^2\,e\,{(d+e\,x)}^2}-\frac{2\,\sqrt{d^2-e^2\,x^2}}{1287\,d^3\,e\,(d+e\,x)}+\frac{320\,d^2\,\sqrt{d^2-e^2\,x^2}}{143\,e\,{(d+e\,x)}^6}-\frac{16\,d^3\,\sqrt{d^2-e^2\,x^2}}{13\,e\,{(d+e\,x)}^7}$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^11,x)
```

output
```
(424*(d^2 - e^2*x^2)^(1/2))/(1287*e*(d + e*x)^4) - (1832*d*(d^2 - e^2*x^2)
^(1/2))/(1287*e*(d + e*x)^5) - (d^2 - e^2*x^2)^(1/2)/(429*d*e*(d + e*x)^3)
 - (2*(d^2 - e^2*x^2)^(1/2))/(1287*d^2*e*(d + e*x)^2) - (2*(d^2 - e^2*x^2)
^(1/2))/(1287*d^3*e*(d + e*x)) + (320*d^2*(d^2 - e^2*x^2)^(1/2))/(143*e*(d
 + e*x)^6) - (16*d^3*(d^2 - e^2*x^2)^(1/2))/(13*e*(d + e*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 462, normalized size of antiderivative = 4.62

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{11}}\,dx = \frac{198\sqrt{-e^2x^2+d^2}\,d^6+20\sqrt{-e^2x^2+d^2}\,d^5ex+1813\sqrt{-e^2x^2+d^2}\,d^4e^2x^2+1228\sqrt{-e}}{1287d^3e\left(\sqrt{-e^2x^2+d^2}\,d^6+6\sqrt{-e^2x^2+d^2}\,d^5ex+15\sqrt{-e^2x^2+d^2}\,d^4e^2\right.}$$

input

```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^11,x)
```

output

```
(198*sqrt(d**2 - e**2*x**2)*d**6 + 20*sqrt(d**2 - e**2*x**2)*d**5*e*x + 18
13*sqrt(d**2 - e**2*x**2)*d**4*e**2*x**2 + 1228*sqrt(d**2 - e**2*x**2)*d**
3*e**3*x**3 + 1228*sqrt(d**2 - e**2*x**2)*d**2*e**4*x**4 + 488*sqrt(d**2 -
 e**2*x**2)*d*e**5*x**5 + 81*sqrt(d**2 - e**2*x**2)*e**6*x**6 - 198*d**7 +
 20*d**6*e*x - 2741*d**5*e**2*x**2 - 1785*d**4*e**3*x**3 - 3160*d**3*e**4*
x**4 - 1630*d**2*e**5*x**5 - 541*d*e**6*x**6 - 77*e**7*x**7)/(1287*d**3*e*
(sqrt(d**2 - e**2*x**2)*d**6 + 6*sqrt(d**2 - e**2*x**2)*d**5*e*x + 15*sqrt
(d**2 - e**2*x**2)*d**4*e**2*x**2 + 20*sqrt(d**2 - e**2*x**2)*d**3*e**3*x*
*3 + 15*sqrt(d**2 - e**2*x**2)*d**2*e**4*x**4 + 6*sqrt(d**2 - e**2*x**2)*d
*e**5*x**5 + sqrt(d**2 - e**2*x**2)*e**6*x**6 - d**7 - 7*d**6*e*x - 21*d**
5*e**2*x**2 - 35*d**4*e**3*x**3 - 35*d**3*e**4*x**4 - 21*d**2*e**5*x**5 -
7*d*e**6*x**6 - e**7*x**7))
```

3.74 $\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}}\,dx = -\frac{(d^2-e^2x^2)^{9/2}}{15de(d+ex)^{12}} - \frac{(d^2-e^2x^2)^{9/2}}{65d^2e(d+ex)^{11}} - \frac{2(d^2-e^2x^2)^{9/2}}{715d^3e(d+ex)^{10}} - \frac{2(d^2-e^2x^2)^{9/2}}{6435d^4e(d+ex)^9}$$

output
```
-1/15*(-e^2*x^2+d^2)^(9/2)/d/e/(e*x+d)^12-1/65*(-e^2*x^2+d^2)^(9/2)/d^2/e/
(e*x+d)^11-2/715*(-e^2*x^2+d^2)^(9/2)/d^3/e/(e*x+d)^10-2/6435*(-e^2*x^2+d^
2)^(9/2)/d^4/e/(e*x+d)^9
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}}\,dx = -\frac{(d-ex)^4\sqrt{d^2-e^2x^2}(548d^3+141d^2ex+24de^2x^2+2e^3x^3)}{6435d^4e(d+ex)^8}$$

input
```
Integrate[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^12,x]
```

output
```
-1/6435*((d - e*x)^4*Sqrt[d^2 - e^2*x^2]*(548*d^3 + 141*d^2*e*x + 24*d*e^2
*x^2 + 2*e^3*x^3))/(d^4*e*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{12}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{11}}dx}{5d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{2\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{10}}dx}{13d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}}{5d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}$$

$$\Big\downarrow \text{461}$$

$$\frac{\frac{2\left(\frac{\int \frac{\left(d^2-e^2x^2\right)^{7/2}}{(d+ex)^{9}}dx}{11d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{11de(d+ex)^{10}}\right)}{13d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}}{5d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}$$

$$\Big\downarrow \text{460}$$

$$\frac{\frac{2\left(-\frac{\left(d^2-e^2x^2\right)^{9/2}}{99d^2e(d+ex)^{9}}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{11de(d+ex)^{10}}\right)}{13d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{13de(d+ex)^{11}}}{5d}-\frac{\left(d^2-e^2x^2\right)^{9/2}}{15de(d+ex)^{12}}$$

input
```
Int[(d^2 - e^2*x^2)^(7/2)/(d + e*x)^12,x]
```

output
```
-1/15*(d^2 - e^2*x^2)^(9/2)/(d*e*(d + e*x)^12) + (-1/13*(d^2 - e^2*x^2)^(9
/2)/(d*e*(d + e*x)^11) + (2*(-1/11*(d^2 - e^2*x^2)^(9/2)/(d*e*(d + e*x)^10
) - (d^2 - e^2*x^2)^(9/2)/(99*d^2*e*(d + e*x)^9)))/(13*d))/(5*d)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 6.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.50

method	result
gosper	$-\frac{(-ex+d)\left(2e^3x^3+24d\,e^2x^2+141d^2ex+548d^3\right)\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{6435(ex+d)^{11}d^4e}$
orering	$-\frac{(-ex+d)\left(2e^3x^3+24d\,e^2x^2+141d^2ex+548d^3\right)\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}{6435(ex+d)^{11}d^4e}$
trager	$-\frac{\left(2e^7x^7+16d\,e^6x^6+57d^2e^5x^5+120d^3e^4x^4-1440d^4e^3x^3+2748d^5e^2x^2-2051d^6ex+548d^7\right)\sqrt{-e^2x^2+d^2}}{6435d^4(ex+d)^8e}$
default	$\frac{-\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{15de\left(x+\frac{d}{e}\right)^{12}}+\frac{e\left(-\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{13de\left(x+\frac{d}{e}\right)^{11}}+\frac{2e\left(-\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{11de\left(x+\frac{d}{e}\right)^{10}}-\frac{\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}{99d^2\left(x+\frac{d}{e}\right)^9}\right)}{13d}\right)}{5d}}{e^{12}}$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x,method=_RETURNVERBOSE)
```

output
```
-1/6435*(-e*x+d)*(2*e^3*x^3+24*d*e^2*x^2+141*d^2*e*x+548*d^3)*(-e^2*x^2+d^
2)^(7/2)/(e*x+d)^11/d^4/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(117) = 234$.

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.02

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^{12}}\,dx = -\frac{548\,e^8x^8 + 4384\,de^7x^7 + 15344\,d^2e^6x^6 + 30688\,d^3e^5x^5 + 38360\,d^4e^4x^4 + 30688\,d^5e^3x^3 + 15344\,d^6e^2x^2 + 4}{6435\,(d^4e^9x^8 + 8\,d^5e^8x^7 + 28\,d^6e^7x^6 + 5}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x, algorithm="fricas")
```

output
```
-1/6435*(548*e^8*x^8 + 4384*d*e^7*x^7 + 15344*d^2*e^6*x^6 + 30688*d^3*e^5*
x^5 + 38360*d^4*e^4*x^4 + 30688*d^5*e^3*x^3 + 15344*d^6*e^2*x^2 + 4384*d^7
*e*x + 548*d^8 + (2*e^7*x^7 + 16*d*e^6*x^6 + 57*d^2*e^5*x^5 + 120*d^3*e^4*
x^4 - 1440*d^4*e^3*x^3 + 2748*d^5*e^2*x^2 - 2051*d^6*e*x + 548*d^7)*sqrt(-
e^2*x^2 + d^2))/(d^4*e^9*x^8 + 8*d^5*e^8*x^7 + 28*d^6*e^7*x^6 + 56*d^7*e^6
*x^5 + 70*d^8*e^5*x^4 + 56*d^9*e^4*x^3 + 28*d^10*e^3*x^2 + 8*d^11*e^2*x +
d^12*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^{12}}\,dx = \text{Timed out}$$

input
```
integrate((-e**2*x**2+d**2)**(7/2)/(e*x+d)**12,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(117) = 234$.

Time = 0.04 (sec) , antiderivative size = 932, normalized size of antiderivative = 7.01

$$\int \frac{(d^2 - e^2x^2)^{7/2}}{(d + ex)^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x, algorithm="maxima")
```

output
```
-1/4*(-e^2*x^2 + d^2)^(7/2)/(e^12*x^11 + 11*d*e^11*x^10 + 55*d^2*e^10*x^9
+ 165*d^3*e^9*x^8 + 330*d^4*e^8*x^7 + 462*d^5*e^7*x^6 + 462*d^6*e^6*x^5 +
330*d^7*e^5*x^4 + 165*d^8*e^4*x^3 + 55*d^9*e^3*x^2 + 11*d^10*e^2*x + d^11*
e) + 7/20*(-e^2*x^2 + d^2)^(5/2)*d/(e^11*x^10 + 10*d*e^10*x^9 + 45*d^2*e^9
*x^8 + 120*d^3*e^8*x^7 + 210*d^4*e^7*x^6 + 252*d^5*e^6*x^5 + 210*d^6*e^5*x
^4 + 120*d^7*e^4*x^3 + 45*d^8*e^3*x^2 + 10*d^9*e^2*x + d^10*e) - 7/24*(-e^
2*x^2 + d^2)^(3/2)*d^2/(e^10*x^9 + 9*d*e^9*x^8 + 36*d^2*e^8*x^7 + 84*d^3*e
^7*x^6 + 126*d^4*e^6*x^5 + 126*d^5*e^5*x^4 + 84*d^6*e^4*x^3 + 36*d^7*e^3*x
^2 + 9*d^8*e^2*x + d^9*e) + 7/60*sqrt(-e^2*x^2 + d^2)*d^3/(e^9*x^8 + 8*d*e
^8*x^7 + 28*d^2*e^7*x^6 + 56*d^3*e^6*x^5 + 70*d^4*e^5*x^4 + 56*d^5*e^4*x^3
 + 28*d^6*e^3*x^2 + 8*d^7*e^2*x + d^8*e) - 7/1560*sqrt(-e^2*x^2 + d^2)*d^2
/(e^8*x^7 + 7*d*e^7*x^6 + 21*d^2*e^6*x^5 + 35*d^3*e^5*x^4 + 35*d^4*e^4*x^3
 + 21*d^5*e^3*x^2 + 7*d^6*e^2*x + d^7*e) - 7/2860*sqrt(-e^2*x^2 + d^2)*d/(
e^7*x^6 + 6*d*e^6*x^5 + 15*d^2*e^5*x^4 + 20*d^3*e^4*x^3 + 15*d^4*e^3*x^2 +
 6*d^5*e^2*x + d^6*e) - 7/5148*sqrt(-e^2*x^2 + d^2)/(e^6*x^5 + 5*d*e^5*x^4
 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e) - 1/1287*sqrt(-e
^2*x^2 + d^2)/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*x^2 + 4*d^4*e^2*x + d
^5*e) - 1/2145*sqrt(-e^2*x^2 + d^2)/(d^2*e^4*x^3 + 3*d^3*e^3*x^2 + 3*d^4*e
^2*x + d^5*e) - 2/6435*sqrt(-e^2*x^2 + d^2)/(d^3*e^3*x^2 + 2*d^4*e^2*x + d
^5*e) - 2/6435*sqrt(-e^2*x^2 + d^2)/(d^4*e^2*x + d^5*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(117) = 234$.

Time = 0.14 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.57

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}}\,dx = \frac{2\left(\frac{1785\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{38235\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{99190\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{426270\left(\right.}{}\right.}{}$$

input
```
integrate((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x, algorithm="giac")
```

output
```
2/6435*(1785*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 38235*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 99190*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^3/(e^6*x^3) + 426270*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x
^4) + 735735*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) + 1492205*(d
*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^12*x^6) + 1621620*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^7/(e^14*x^7) + 1904760*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^8/(e^16*x^8) + 1250535*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^9/(e^18*x^9
) + 909909*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^10/(e^20*x^10) + 321750*(d*
e + sqrt(-e^2*x^2 + d^2)*abs(e))^11/(e^22*x^11) + 150150*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^12/(e^24*x^12) + 19305*(d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))^13/(e^26*x^13) + 6435*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^14/(e^28*x^1
4) + 548)/(d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^15*abs(e)
)
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.71

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}}\,dx = \frac{320\sqrt{d^2-e^2\,x^2}}{1287\,e\,{(d+e\,x)}^5}-\frac{824\,d\,\sqrt{d^2-e^2\,x^2}}{715\,e\,{(d+e\,x)}^6} - \frac{\sqrt{d^2-e^2\,x^2}}{1287\,d\,e\,{(d+e\,x)}^4}-\frac{\sqrt{d^2-e^2\,x^2}}{2145\,d^2\,e\,{(d+e\,x)}^3}-\frac{2\sqrt{d^2-e^2\,x^2}}{6435\,d^3\,e\,{(d+e\,x)}^2} - \frac{2\sqrt{d^2-e^2\,x^2}}{6435\,d^4\,e\,\left(d+e\,x\right)}+\frac{368\,d^2\sqrt{d^2-e^2\,x^2}}{195\,e\,{(d+e\,x)}^7}-\frac{16\,d^3\sqrt{d^2-e^2\,x^2}}{15\,e\,{(d+e\,x)}^8}$$

input
```
int((d^2 - e^2*x^2)^(7/2)/(d + e*x)^12,x)
```

output
```
(320*(d^2 - e^2*x^2)^(1/2))/(1287*e*(d + e*x)^5) - (824*d*(d^2 - e^2*x^2)^
(1/2))/(715*e*(d + e*x)^6) - (d^2 - e^2*x^2)^(1/2)/(1287*d*e*(d + e*x)^4)
- (d^2 - e^2*x^2)^(1/2)/(2145*d^2*e*(d + e*x)^3) - (2*(d^2 - e^2*x^2)^(1/2
))/(6435*d^3*e*(d + e*x)^2) - (2*(d^2 - e^2*x^2)^(1/2))/(6435*d^4*e*(d + e
*x)) + (368*d^2*(d^2 - e^2*x^2)^(1/2))/(195*e*(d + e*x)^7) - (16*d^3*(d^2
- e^2*x^2)^(1/2))/(15*e*(d + e*x)^8)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.00

$$\int \frac{(d^2-e^2x^2)^{7/2}}{(d+ex)^{12}}\,dx = \frac{858\sqrt{-e^2x^2+d^2}\,d^7+119\sqrt{-e^2x^2+d^2}\,d^6ex+9258\sqrt{-e^2x^2+d^2}\,d^5e^2x^2+9410\sqrt{-}}{6435d^4e\left(\sqrt{-e^2x^2+d^2}\,d^7+7\sqrt{-e^2x^2+d^2}\,d^6ex+21\sqrt{-e^2x^2}\right.}$$

input
```
int((-e^2*x^2+d^2)^(7/2)/(e*x+d)^12,x)
```

output
```
(858*sqrt(d**2 - e**2*x**2)*d**7 + 119*sqrt(d**2 - e**2*x**2)*d**6*e*x + 9
258*sqrt(d**2 - e**2*x**2)*d**5*e**2*x**2 + 9410*sqrt(d**2 - e**2*x**2)*d*
*4*e**3*x**3 + 10970*sqrt(d**2 - e**2*x**2)*d**3*e**4*x**4 + 6567*sqrt(d**
2 - e**2*x**2)*d**2*e**5*x**5 + 2186*sqrt(d**2 - e**2*x**2)*d*e**6*x**6 +
312*sqrt(d**2 - e**2*x**2)*e**7*x**7 - 858*d**8 + 119*d**7*e*x - 13479*d**
6*e**2*x**2 - 13172*d**5*e**3*x**3 - 23260*d**4*e**4*x**4 - 17297*d**3*e**
5*x**5 - 8639*d**2*e**6*x**6 - 2466*d*e**7*x**7 - 308*e**8*x**8)/(6435*d**
4*e*(sqrt(d**2 - e**2*x**2)*d**7 + 7*sqrt(d**2 - e**2*x**2)*d**6*e*x + 21*
sqrt(d**2 - e**2*x**2)*d**5*e**2*x**2 + 35*sqrt(d**2 - e**2*x**2)*d**4*e**
3*x**3 + 35*sqrt(d**2 - e**2*x**2)*d**3*e**4*x**4 + 21*sqrt(d**2 - e**2*x*
*2)*d**2*e**5*x**5 + 7*sqrt(d**2 - e**2*x**2)*d*e**6*x**6 + sqrt(d**2 - e*
*2*x**2)*e**7*x**7 - d**8 - 8*d**7*e*x - 28*d**6*e**2*x**2 - 56*d**5*e**3*
x**3 - 70*d**4*e**4*x**4 - 56*d**3*e**5*x**5 - 28*d**2*e**6*x**6 - 8*d*e**
7*x**7 - e**8*x**8))
```

3.75 $\int (1+x)^3\sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int (1+x)^3\sqrt{1-x^2}\,dx = \frac{7}{8}x\sqrt{1-x^2}-\frac{1}{5}(1+x)^2\left(1-x^2\right)^{3/2} -\frac{7}{60}(8+3x)\left(1-x^2\right)^{3/2}+\frac{7\arcsin(x)}{8}$$

output
```
7/8*x*(-x^2+1)^(1/2)-1/5*(1+x)^2*(-x^2+1)^(3/2)-7/60*(8+3*x)*(-x^2+1)^(3/2
)+7/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int (1+x)^3\sqrt{1-x^2}\,dx = \frac{1}{120}\sqrt{1-x^2}\left(-136+15x+112x^2+90x^3+24x^4\right) -\frac{7}{4}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + x)^3*Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-136 + 15*x + 112*x^2 + 90*x^3 + 24*x^4))/120 - (7*ArcTan[
Sqrt[1 - x^2]/(1 + x)])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {469, 469, 455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^3\sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\int (x+1)^2\sqrt{1-x^2}dx-\frac{1}{5}(x+1)^2\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\left(\frac{5}{4}\int (x+1)\sqrt{1-x^2}dx-\frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}\right)-\frac{1}{5}(x+1)^2\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\int \sqrt{1-x^2}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}\right)-\frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}\right)-\frac{1}{5}(x+1)^2\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)-\frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}\right)-\frac{1}{5}(x+1)^2\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\frac{\arcsin(x)}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)-\frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}\right)-\frac{1}{5}(x+1)^2\left(1-x^2\right)^{3/2}$$

input
```
Int[(1 + x)^3*Sqrt[1 - x^2],x]
```

output
```
-1/5*((1 + x)^2*(1 - x^2)^(3/2)) + (7*(-1/4*((1 + x)*(1 - x^2)^(3/2)) + (5
*((x*Sqrt[1 - x^2])/2 - (1 - x^2)^(3/2)/3 + ArcSin[x]/2))/4))/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{(24x^4+90x^3+112x^2+15x-136)(x^2-1)}{120\sqrt{-x^2+1}}+\frac{7\arcsin(x)}{8}$
default	$\frac{7x\sqrt{-x^2+1}}{8}+\frac{7\arcsin(x)}{8}-\frac{x^2(-x^2+1)^{\frac{3}{2}}}{5}-\frac{17(-x^2+1)^{\frac{3}{2}}}{15}-\frac{3x(-x^2+1)^{\frac{3}{2}}}{4}$
trager	$\left(\frac{1}{5}x^4+\frac{3}{4}x^3+\frac{14}{15}x^2+\frac{1}{8}x-\frac{17}{15}\right)\sqrt{-x^2+1}+\frac{7\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{8}$
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\arcsin(x)\right)}{4\sqrt{\pi}}-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(-x^2+1\right)^{\frac{3}{2}}\left(3x^2+2\right)}{15}}{4\sqrt{\pi}}-\frac{3i\left(-\frac{i\sqrt{\pi}\,x\left(-6x^2+3\right)\sqrt{-x^2+1}}{6}+\frac{i\sqrt{\pi}\arcsin(x)}{2}\right)}{4\sqrt{\pi}}+$

input
```
int((x+1)^3*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(24*x^4+90*x^3+112*x^2+15*x-136)*(x^2-1)/(-x^2+1)^(1/2)+7/8*arcsin(
x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int (1+x)^3\sqrt{1-x^2}\,dx = \frac{1}{120}\left(24\,x^4+90\,x^3+112\,x^2+15\,x-136\right)\sqrt{-x^2+1} - \frac{7}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1+x)^3*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(24*x^4 + 90*x^3 + 112*x^2 + 15*x - 136)*sqrt(-x^2 + 1) - 7/4*arctan
((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int (1+x)^3\sqrt{1-x^2}\,dx = \frac{x^4\sqrt{1-x^2}}{5} + \frac{3x^3\sqrt{1-x^2}}{4} + \frac{14x^2\sqrt{1-x^2}}{15} + \frac{x\sqrt{1-x^2}}{8} - \frac{17\sqrt{1-x^2}}{15} + \frac{7\,\mathrm{asin}\,(x)}{8}$$

input
```
integrate((1+x)**3*(-x**2+1)**(1/2),x)
```

output
```
x**4*sqrt(1 - x**2)/5 + 3*x**3*sqrt(1 - x**2)/4 + 14*x**2*sqrt(1 - x**2)/1
5 + x*sqrt(1 - x**2)/8 - 17*sqrt(1 - x**2)/15 + 7*asin(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int (1+x)^3\sqrt{1-x^2}\,dx = -\frac{1}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{3}{4}\left(-x^2+1\right)^{\frac{3}{2}}x - \frac{17}{15}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{7}{8}\sqrt{-x^2+1}x + \frac{7}{8}\arcsin(x)$$

input
```
integrate((1+x)^3*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x - 17/15*(-x^2 + 1)^(3/2
) + 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int (1+x)^3\sqrt{1-x^2}\,dx = \frac{1}{120}\left((2\left(3\left(4\,x+15\right)x+56\right)x+15)x-136\right)\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

input
```
integrate((1+x)^3*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/120*((2*(3*(4*x + 15)*x + 56)*x + 15)*x - 136)*sqrt(-x^2 + 1) + 7/8*arcs
in(x)
```

Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int (1+x)^3\sqrt{1-x^2}\,dx = \frac{7\,\mathrm{asin}(x)}{8} + \sqrt{1-x^2}\left(\frac{x^4}{5} + \frac{3\,x^3}{4} + \frac{14\,x^2}{15} + \frac{x}{8} - \frac{17}{15}\right)$$

input
```
int((1 - x^2)^(1/2)*(x + 1)^3,x)
```

output
```
(7*asin(x))/8 + (1 - x^2)^(1/2)*(x/8 + (14*x^2)/15 + (3*x^3)/4 + x^4/5 - 1
7/15)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int (1+x)^3\sqrt{1-x^2}\,dx = \frac{7asin(x)}{8} + \frac{\sqrt{-x^2+1}\,x^4}{5} + \frac{3\sqrt{-x^2+1}\,x^3}{4} + \frac{14\sqrt{-x^2+1}\,x^2}{15} + \frac{\sqrt{-x^2+1}\,x}{8} - \frac{17\sqrt{-x^2+1}}{15} + \frac{17}{15}$$

input
```
int((1+x)^3*(-x^2+1)^(1/2),x)
```

output
```
(105*asin(x) + 24*sqrt( - x**2 + 1)*x**4 + 90*sqrt( - x**2 + 1)*x**3 + 112
*sqrt( - x**2 + 1)*x**2 + 15*sqrt( - x**2 + 1)*x - 136*sqrt( - x**2 + 1) +
 136)/120
```

3.76 $\int (1+x)^2\sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int (1+x)^2\sqrt{1-x^2}\,dx=\frac{5}{8}x\sqrt{1-x^2}-\frac{1}{12}(8+3x)\left(1-x^2\right)^{3/2}+\frac{5\arcsin(x)}{8}$$

output
```
5/8*x*(-x^2+1)^(1/2)-1/12*(8+3*x)*(-x^2+1)^(3/2)+5/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int (1+x)^2\sqrt{1-x^2}\,dx=\frac{1}{24}\sqrt{1-x^2}\left(-16+9x+16x^2+6x^3\right)-\frac{5}{4}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + x)^2*Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-16 + 9*x + 16*x^2 + 6*x^3))/24 - (5*ArcTan[Sqrt[1 - x^2]/
(1 + x)])/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {469, 455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^2 \sqrt{1-x^2} \, dx$$

$$\Big\downarrow \text{ 469}$$

$$\frac{5}{4} \int (x+1)\sqrt{1-x^2}dx - \frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{5}{4}\left(\int \sqrt{1-x^2}dx - \frac{1}{3}\left(1-x^2\right)^{3/2}\right) - \frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx - \frac{1}{3}\left(1-x^2\right)^{3/2} + \frac{1}{2}x\sqrt{1-x^2}\right) - \frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{5}{4}\left(\frac{\arcsin(x)}{2} - \frac{1}{3}\left(1-x^2\right)^{3/2} + \frac{1}{2}x\sqrt{1-x^2}\right) - \frac{1}{4}(x+1)\left(1-x^2\right)^{3/2}$$

input
```
Int[(1 + x)^2*Sqrt[1 - x^2],x]
```

output
```
-1/4*((1 + x)*(1 - x^2)^(3/2)) + (5*((x*Sqrt[1 - x^2])/2 - (1 - x^2)^(3/2)
/3 + ArcSin[x]/2))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	siz
risch	$-\frac{\left(6x^3+16x^2+9x-16\right)\left(x^2-1\right)}{24\sqrt{-x^2+1}}+\frac{5\arcsin(x)}{8}$	37
default	$\frac{5x\sqrt{-x^2+1}}{8}+\frac{5\arcsin(x)}{8}-\frac{x\left(-x^2+1\right)^{\frac{3}{2}}}{4}-\frac{2\left(-x^2+1\right)^{\frac{3}{2}}}{3}$	41
trager	$\left(\frac{1}{4}x^3+\frac{2}{3}x^2+\frac{3}{8}x-\frac{2}{3}\right)\sqrt{-x^2+1}+\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{8}$	54
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}-\frac{i\left(-\frac{i\sqrt{\pi}\,x\left(-6x^2+3\right)\sqrt{-x^2+1}}{6}+\frac{i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{4\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(-2x^2+2\right)\sqrt{-x^2+1}}{3}}{2\sqrt{\pi}}$	10

input
```
int((x+1)^2*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*x^3+16*x^2+9*x-16)*(x^2-1)/(-x^2+1)^(1/2)+5/8*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int (1+x)^2\sqrt{1-x^2}\,dx = \frac{1}{24}\left(6\,x^3+16\,x^2+9\,x-16\right)\sqrt{-x^2+1} - \frac{5}{4}\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1+x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(-x^2 + 1) - 5/4*arctan((sqrt(-x^2 +
1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int (1+x)^2\sqrt{1-x^2}\,dx = \sqrt{1-x^2}\left(\frac{x^3}{4}+\frac{2x^2}{3}+\frac{3x}{8}-\frac{2}{3}\right)+\frac{5\,\mathrm{asin}\,(x)}{8}$$

input
```
integrate((1+x)**2*(-x**2+1)**(1/2),x)
```

output
```
sqrt(1 - x**2)*(x**3/4 + 2*x**2/3 + 3*x/8 - 2/3) + 5*asin(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int (1+x)^2\sqrt{1-x^2}\,dx = -\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x - \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{5}{8}\sqrt{-x^2+1}x + \frac{5}{8}\arcsin(x)$$

input
```
integrate((1+x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(-x^2 + 1)^(3/2)*x - 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/
8*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int (1+x)^2\sqrt{1-x^2}\,dx = \frac{1}{24}\left((2\,(3\,x+8)x+9)x-16\right)\sqrt{-x^2+1} + \frac{5}{8}\arcsin(x)$$

input
```
integrate((1+x)^2*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x + 8)*x + 9)*x - 16)*sqrt(-x^2 + 1) + 5/8*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int (1+x)^2\sqrt{1-x^2}\,dx = \frac{5\,\mathrm{asin}(x)}{8} + \sqrt{1-x^2}\left(\frac{x^3}{4} + \frac{2\,x^2}{3} + \frac{3\,x}{8} - \frac{2}{3}\right)$$

input
```
int((1 - x^2)^(1/2)*(x + 1)^2,x)
```

output
```
(5*asin(x))/8 + (1 - x^2)^(1/2)*((3*x)/8 + (2*x^2)/3 + x^3/4 - 2/3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int (1+x)^2\sqrt{1-x^2}\,dx = \frac{5asin(x)}{8} + \frac{\sqrt{-x^2+1}\,x^3}{4} + \frac{2\sqrt{-x^2+1}\,x^2}{3} + \frac{3\sqrt{-x^2+1}\,x}{8} - \frac{2\sqrt{-x^2+1}}{3} + \frac{2}{3}$$

input
```
int((1+x)^2*(-x^2+1)^(1/2),x)
```

output
```
(15*asin(x) + 6*sqrt( - x**2 + 1)*x**3 + 16*sqrt( - x**2 + 1)*x**2 + 9*sqr
t( - x**2 + 1)*x - 16*sqrt( - x**2 + 1) + 16)/24
```

3.77 $\int (1+x)\sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int (1+x)\sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2}-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{\arcsin(x)}{2}$$

output
```
1/2*x*(-x^2+1)^(1/2)-1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (1+x)\sqrt{1-x^2}\,dx = \frac{1}{6}\sqrt{1-x^2}\left(-2+3x+2x^2\right)-\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + x)*Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(-2 + 3*x + 2*x^2))/6 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)\sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{455}$$

$$\int \sqrt{1-x^2}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(x)}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

input
```
Int[(1 + x)*Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 - (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3}+\frac{\arcsin(x)}{2}$	29
risch	$-\frac{(2x^2+3x-2)(x^2-1)}{6\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	32
trager	$\left(\frac{1}{3}x^2+\frac{1}{2}x-\frac{1}{3}\right)\sqrt{-x^2+1}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	49
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}+\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(-2x^2+2\right)\sqrt{-x^2+1}}{3}}{4\sqrt{\pi}}$	65

input
```
int((x+1)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-x^2+1)^(1/2)-1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (1+x)\sqrt{1-x^2}\,dx = \frac{1}{6}\left(2\,x^2+3\,x-2\right)\sqrt{-x^2+1}-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1+x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^2 + 3*x - 2)*sqrt(-x^2 + 1) - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int (1+x)\sqrt{1-x^2}\,dx = \frac{x^2\sqrt{1-x^2}}{3} + \frac{x\sqrt{1-x^2}}{2} - \frac{\sqrt{1-x^2}}{3} + \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((1+x)*(-x**2+1)**(1/2),x)
```

output
```
x**2*sqrt(1 - x**2)/3 + x*sqrt(1 - x**2)/2 - sqrt(1 - x**2)/3 + asin(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int (1+x)\sqrt{1-x^2}\,dx = -\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1+x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int (1+x)\sqrt{1-x^2}\,dx = \frac{1}{6}\left((2\,x+3)x-2\right)\sqrt{-x^2+1} + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1+x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/6*((2*x + 3)*x - 2)*sqrt(-x^2 + 1) + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int (1+x)\sqrt{1-x^2}\,dx = \frac{\operatorname{asin}(x)}{2} + \sqrt{1-x^2}\left(\frac{x^2}{3}+\frac{x}{2}-\frac{1}{3}\right)$$

input
```
int((1 - x^2)^(1/2)*(x + 1),x)
```

output
```
asin(x)/2 + (1 - x^2)^(1/2)*(x/2 + x^2/3 - 1/3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (1+x)\sqrt{1-x^2}\,dx = \frac{asin(x)}{2} + \frac{\sqrt{-x^2+1}\,x^2}{3} + \frac{\sqrt{-x^2+1}\,x}{2} - \frac{\sqrt{-x^2+1}}{3} + \frac{1}{3}$$

input
```
int((1+x)*(-x^2+1)^(1/2),x)
```

output
```
(3*asin(x) + 2*sqrt( - x**2 + 1)*x**2 + 3*sqrt( - x**2 + 1)*x - 2*sqrt( -
x**2 + 1) + 2)/6
```

3.78 $\int \frac{\sqrt{1-x^2}}{1+x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{\sqrt{1-x^2}}{1+x} \, dx = \sqrt{1-x^2} + \arcsin(x)$$

output
```
(-x^2+1)^(1/2)+arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{1-x^2}}{1+x} \, dx = \sqrt{1-x^2} - 2 \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/(1 + x),x]
```

output
```
Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {466, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{x+1} \, dx$$

$$\Big\downarrow \text{466}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx + \sqrt{1-x^2}$$

$$\Big\downarrow \text{223}$$

$$\arcsin(x) + \sqrt{1-x^2}$$

input
```
Int[Sqrt[1 - x^2]/(1 + x),x]
```

output
```
Sqrt[1 - x^2] + ArcSin[x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

method	result	size
default	$\sqrt{-\left(x+1\right)^{2}+2x+2}+\arcsin\left(x\right)$	18
risch	$-\frac{x^{2}-1}{\sqrt{-x^{2}+1}}+\arcsin\left(x\right)$	20
trager	$\sqrt{-x^{2}+1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	37

input
```
int((-x^2+1)^(1/2)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
(-(x+1)^2+2*x+2)^(1/2)+arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1-x^2}}{1+x}\,dx=\sqrt{-x^{2}+1}-2\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1-x^2}}{1+x}\,dx = \begin{cases} \sqrt{1-x^2} + \operatorname{asin}(x) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate((-x**2+1)**(1/2)/(1+x),x)
```

output
```
Piecewise((sqrt(1 - x**2) + asin(x), (x > -1) & (x < 1)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{1-x^2}}{1+x}\,dx = \sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{1-x^2}}{1+x}\,dx = \sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2)/(1+x),x, algorithm="giac")
```

output
```
sqrt(-x^2 + 1) + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{1-x^2}}{1+x}\,dx = \operatorname{asin}(x) + \sqrt{1-x^2}$$

input
```
int((1 - x^2)^(1/2)/(x + 1),x)
```

output
```
asin(x) + (1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{1-x^2}}{1+x}\,dx = asin(x) + \sqrt{-x^2+1} - 1$$

input
```
int((-x^2+1)^(1/2)/(1+x),x)
```

output
```
asin(x) + sqrt( - x**2 + 1) - 1
```

3.79 $\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx = -\frac{2\sqrt{1-x^2}}{1+x} - \arcsin(x)$$

output
```
-2*(-x^2+1)^(1/2)/(1+x)-arcsin(x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx = -\frac{2\sqrt{1-x^2}}{1+x} + 2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/(1 + x)^2,x]
```

output
```
(-2*Sqrt[1 - x^2])/(1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {463, 25, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(x+1)^2} \, dx$$
$$\Big\downarrow \text{463}$$
$$\int -\frac{1}{\sqrt{1-x^2}}dx-\frac{2\sqrt{1-x^2}}{x+1}$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{1}{\sqrt{1-x^2}}dx-\frac{2\sqrt{1-x^2}}{x+1}$$
$$\Big\downarrow \text{223}$$
$$-\arcsin(x)-\frac{2\sqrt{1-x^2}}{x+1}$$

input
```
Int[Sqrt[1 - x^2]/(1 + x)^2,x]
```

output
```
(-2*Sqrt[1 - x^2])/(1 + x) - ArcSin[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{2x-2}{\sqrt{-x^{2}+1}}-\arcsin\left(x\right)$	20
default	$-\frac{\left(-\left(x+1\right)^{2}+2x+2\right)^{\frac{3}{2}}}{\left(x+1\right)^{2}}-\sqrt{-\left(x+1\right)^{2}+2x+2}-\arcsin\left(x\right)$	43
trager	$-\frac{2\sqrt{-x^{2}+1}}{x+1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	45

input
```
int((-x^2+1)^(1/2)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(x-1)/(-x^2+1)^(1/2)-arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int\frac{\sqrt{1-x^2}}{(1+x)^2}\,dx=\frac{2\left((x+1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-x-\sqrt{-x^2+1}-1\right)}{x+1}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

output
```
2*((x + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - x - sqrt(-x^2 + 1) - 1)/(x + 1
)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{(x+1)^2}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1+x)**2,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x + 1)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx = -\frac{2\sqrt{-x^2+1}}{x+1} - \arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + 1)/(x + 1) - arcsin(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^2,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: -(2*
atan(i)-2*i)*sign((sageVARx+1)^-1)-2*(sqrt(2*(sageVARx+1)^-1-1)*sign((sage
VARx+1)^-1)-sign((sageVARx+1)^-1)*atan(sqrt(2*(sageVARx+1)^-1-1)))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx = -\mathrm{asin}(x) - \frac{2\sqrt{1-x^2}}{x+1}$$

input

```
int((1 - x^2)^(1/2)/(x + 1)^2,x)
```

output

```
- asin(x) - (2*(1 - x^2)^(1/2))/(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{1-x^2}}{(1+x)^2}\,dx = \frac{-asin(x)\tan\left(\frac{asin(x)}{2}\right) - asin(x) + 4\tan\left(\frac{asin(x)}{2}\right)}{\tan\left(\frac{asin(x)}{2}\right) + 1}$$

input

```
int((-x^2+1)^(1/2)/(1+x)^2,x)
```

output

```
( - asin(x)*tan(asin(x)/2) - asin(x) + 4*tan(asin(x)/2))/(tan(asin(x)/2) +
 1)
```

3.80 $\int \frac{\sqrt{1-x^2}}{(1+x)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3} \, dx = -\frac{(1-x^2)^{3/2}}{3(1+x)^3}$$

output
```
-1/3*(-x^2+1)^(3/2)/(1+x)^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3} \, dx = \frac{(-1+x)\sqrt{1-x^2}}{3(1+x)^2}$$

input
```
Integrate[Sqrt[1 - x^2]/(1 + x)^3,x]
```

output
```
((-1 + x)*Sqrt[1 - x^2])/(3*(1 + x)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(x+1)^3} \, dx$$

$$\Big\downarrow \text{460}$$

$$-\frac{\left(1-x^2\right)^{3/2}}{3(x+1)^3}$$

input
```
Int[Sqrt[1 - x^2]/(1 + x)^3,x]
```

output
```
-1/3*(1 - x^2)^(3/2)/(1 + x)^3
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(x-1)\sqrt{-x^2+1}}{3(x+1)^2}$	20
trager	$\frac{(x-1)\sqrt{-x^2+1}}{3(x+1)^2}$	20
orering	$\frac{(x-1)\sqrt{-x^2+1}}{3(x+1)^2}$	20
default	$-\frac{\left(-(x+1)^2+2x+2\right)^{\frac{3}{2}}}{3(x+1)^3}$	22
risch	$-\frac{x^2-2x+1}{3(x+1)\sqrt{-x^2+1}}$	25

input
```
int((-x^2+1)^(1/2)/(x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x-1)/(x+1)^2*(-x^2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3}\,dx = -\frac{x^2-\sqrt{-x^2+1}(x-1)+2\,x+1}{3\,(x^2+2\,x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^3,x, algorithm="fricas")
```

output
```
-1/3*(x^2 - sqrt(-x^2 + 1)*(x - 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{(x+1)^3}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1+x)**3,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x + 1)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3}\,dx = -\frac{2\sqrt{-x^2+1}}{3(x^2+2x+1)} + \frac{\sqrt{-x^2+1}}{3(x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^3,x, algorithm="maxima")
```

output
```
-2/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(16) = 32.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3}\,dx = \frac{2\left(\frac{3\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{3\left(\frac{\sqrt{-x^2+1}-1}{x}-1\right)^3}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^3,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/((sqrt(-x^2 + 1) - 1)/x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3}\,dx = \frac{\sqrt{1-x^2}\,(x-1)}{3\,(x+1)^2}$$

input
```
int((1 - x^2)^(1/2)/(x + 1)^3,x)
```

output
```
((1 - x^2)^(1/2)*(x - 1))/(3*(x + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.20

$$\int \frac{\sqrt{1-x^2}}{(1+x)^3}\,dx = \frac{\sqrt{-x^2+1}\,x+7\sqrt{-x^2+1}-7x^2-2x-7}{9\sqrt{-x^2+1}\,x+9\sqrt{-x^2+1}-9x^2-18x-9}$$

input
```
int((-x^2+1)^(1/2)/(1+x)^3,x)
```

output
```
(sqrt( - x**2 + 1)*x + 7*sqrt( - x**2 + 1) - 7*x**2 - 2*x - 7)/(9*(sqrt( -
 x**2 + 1)*x + sqrt( - x**2 + 1) - x**2 - 2*x - 1))
```

3.81 $\int \frac{\sqrt{1-x^2}}{(1+x)^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4} dx = -\frac{(1-x^2)^{3/2}}{5(1+x)^4} - \frac{(1-x^2)^{3/2}}{15(1+x)^3}$$

output
```
-1/5*(-x^2+1)^(3/2)/(1+x)^4-1/15*(-x^2+1)^(3/2)/(1+x)^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4} dx = \frac{\sqrt{1-x^2}(-4+3x+x^2)}{15(1+x)^3}$$

input
```
Integrate[Sqrt[1 - x^2]/(1 + x)^4,x]
```

output
```
(Sqrt[1 - x^2]*(-4 + 3*x + x^2))/(15*(1 + x)^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(x+1)^4} dx$$

$$\Big\downarrow \text{461}$$

$$\frac{1}{5}\int \frac{\sqrt{1-x^2}}{(x+1)^3}dx-\frac{\left(1-x^2\right)^{3/2}}{5(x+1)^4}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\left(1-x^2\right)^{3/2}}{15(x+1)^3}-\frac{\left(1-x^2\right)^{3/2}}{5(x+1)^4}$$

input
```
Int[Sqrt[1 - x^2]/(1 + x)^4,x]
```

output
```
-1/5*(1 - x^2)^(3/2)/(1 + x)^4 - (1 - x^2)^(3/2)/(15*(1 + x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{(x-1)(x+4)\sqrt{-x^2+1}}{15(x+1)^3}$	23
orering	$\frac{(x-1)(x+4)\sqrt{-x^2+1}}{15(x+1)^3}$	23
trager	$\frac{(x^2+3x-4)\sqrt{-x^2+1}}{15(x+1)^3}$	25
risch	$-\frac{x^3+2x^2-7x+4}{15(x+1)^2\sqrt{-x^2+1}}$	30
default	$-\frac{\left(-(x+1)^2+2x+2\right)^{\frac{3}{2}}}{5(x+1)^4}-\frac{\left(-(x+1)^2+2x+2\right)^{\frac{3}{2}}}{15(x+1)^3}$	44

input
```
int((-x^2+1)^(1/2)/(x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*(x-1)*(x+4)*(-x^2+1)^(1/2)/(x+1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4}\,dx = -\frac{4\,x^3+12\,x^2-(x^2+3\,x-4)\sqrt{-x^2+1}+12\,x+4}{15\,(x^3+3\,x^2+3\,x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^4,x, algorithm="fricas")
```

output
```
-1/15*(4*x^3 + 12*x^2 - (x^2 + 3*x - 4)*sqrt(-x^2 + 1) + 12*x + 4)/(x^3 +
3*x^2 + 3*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{(x+1)^4}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1+x)**4,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x + 1)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4}\,dx = -\frac{2\sqrt{-x^2+1}}{5\,(x^3+3\,x^2+3\,x+1)} + \frac{\sqrt{-x^2+1}}{15\,(x^2+2\,x+1)} + \frac{\sqrt{-x^2+1}}{15\,(x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^4,x, algorithm="maxima")
```

output
```
-2/5*sqrt(-x^2 + 1)/(x^3 + 3*x^2 + 3*x + 1) + 1/15*sqrt(-x^2 + 1)/(x^2 + 2
*x + 1) + 1/15*sqrt(-x^2 + 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4}\,dx = -\frac{2\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{25\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{15\left(\sqrt{-x^2+1}-1\right)^3}{x^3} - \frac{15\left(\sqrt{-x^2+1}-1\right)^4}{x^4} - 4\right)}{15\left(\frac{\sqrt{-x^2+1}-1}{x}-1\right)^5}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^4,x, algorithm="giac")
```

output
```
-2/15*(5*(sqrt(-x^2 + 1) - 1)/x - 25*(sqrt(-x^2 + 1) - 1)^2/x^2 + 15*(sqrt
(-x^2 + 1) - 1)^3/x^3 - 15*(sqrt(-x^2 + 1) - 1)^4/x^4 - 4)/((sqrt(-x^2 + 1
) - 1)/x - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4}\,dx = \frac{\sqrt{1-x^2}\,(x^2+3\,x-4)}{15\,(x+1)^3}$$

input
```
int((1 - x^2)^(1/2)/(x + 1)^4,x)
```

output
```
((1 - x^2)^(1/2)*(3*x + x^2 - 4))/(15*(x + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{1-x^2}}{(1+x)^4}\,dx = \frac{\sqrt{-x^2+1}\,(x^2+3x-4)}{15x^3+45x^2+45x+15}$$

input
```
int((-x^2+1)^(1/2)/(1+x)^4,x)
```

output
```
(sqrt( - x**2 + 1)*(x**2 + 3*x - 4))/(15*(x**3 + 3*x**2 + 3*x + 1))
```

3.82 $\int \frac{\sqrt{1-x^2}}{(1+x)^5} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5} \, dx = -\frac{(1-x^2)^{3/2}}{7(1+x)^5} - \frac{2(1-x^2)^{3/2}}{35(1+x)^4} - \frac{2(1-x^2)^{3/2}}{105(1+x)^3}$$

output
```
-1/7*(-x^2+1)^(3/2)/(1+x)^5-2/35*(-x^2+1)^(3/2)/(1+x)^4-2/105*(-x^2+1)^(3/
2)/(1+x)^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5} \, dx = \frac{\sqrt{1-x^2}(-23+13x+8x^2+2x^3)}{105(1+x)^4}$$

input
```
Integrate[Sqrt[1 - x^2]/(1 + x)^5,x]
```

output
```
(Sqrt[1 - x^2]*(-23 + 13*x + 8*x^2 + 2*x^3))/(105*(1 + x)^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(x+1)^5}dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2}{7}\int \frac{\sqrt{1-x^2}}{(x+1)^4}dx-\frac{\left(1-x^2\right)^{3/2}}{7(x+1)^5}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2}{7}\left(\frac{1}{5}\int \frac{\sqrt{1-x^2}}{(x+1)^3}dx-\frac{\left(1-x^2\right)^{3/2}}{5(x+1)^4}\right)-\frac{\left(1-x^2\right)^{3/2}}{7(x+1)^5}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{2}{7}\left(-\frac{\left(1-x^2\right)^{3/2}}{15(x+1)^3}-\frac{\left(1-x^2\right)^{3/2}}{5(x+1)^4}\right)-\frac{\left(1-x^2\right)^{3/2}}{7(x+1)^5}$$

input
```
Int[Sqrt[1 - x^2]/(1 + x)^5,x]
```

output
```
-1/7*(1 - x^2)^(3/2)/(1 + x)^5 + (2*(-1/5*(1 - x^2)^(3/2)/(1 + x)^4 - (1 -
 x^2)^(3/2)/(15*(1 + x)^3)))/7
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{(x-1)\left(2x^{2}+10x+23\right)\sqrt{-x^{2}+1}}{105(x+1)^{4}}$	30
orering	$\frac{(x-1)\left(2x^{2}+10x+23\right)\sqrt{-x^{2}+1}}{105(x+1)^{4}}$	30
trager	$\frac{\left(2x^{3}+8x^{2}+13x-23\right)\sqrt{-x^{2}+1}}{105(x+1)^{4}}$	32
risch	$-\frac{2x^{4}+6x^{3}+5x^{2}-36x+23}{105(x+1)^{3}\sqrt{-x^{2}+1}}$	37
default	$-\frac{\left(-(x+1)^{2}+2x+2\right)^{\frac{3}{2}}}{7(x+1)^{5}}-\frac{2\left(-(x+1)^{2}+2x+2\right)^{\frac{3}{2}}}{35(x+1)^{4}}-\frac{2\left(-(x+1)^{2}+2x+2\right)^{\frac{3}{2}}}{105(x+1)^{3}}$	65

input
```
int((-x^2+1)^(1/2)/(x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/105*(x-1)*(2*x^2+10*x+23)*(-x^2+1)^(1/2)/(x+1)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5}\,dx = -\frac{23\,x^4+92\,x^3+138\,x^2-(2\,x^3+8\,x^2+13\,x-23)\sqrt{-x^2+1}+92\,x+23}{105\,(x^4+4\,x^3+6\,x^2+4\,x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^5,x, algorithm="fricas")
```

output
```
-1/105*(23*x^4 + 92*x^3 + 138*x^2 - (2*x^3 + 8*x^2 + 13*x - 23)*sqrt(-x^2
+ 1) + 92*x + 23)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{(x+1)^5}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1+x)**5,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x + 1)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5}\,dx = -\frac{2\sqrt{-x^2+1}}{7\,(x^4+4\,x^3+6\,x^2+4\,x+1)} + \frac{\sqrt{-x^2+1}}{35\,(x^3+3\,x^2+3\,x+1)} + \frac{2\sqrt{-x^2+1}}{105\,(x^2+2\,x+1)} + \frac{2\sqrt{-x^2+1}}{105\,(x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^5,x, algorithm="maxima")
```

output
```
-2/7*sqrt(-x^2 + 1)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) + 1/35*sqrt(-x^2 + 1)/
(x^3 + 3*x^2 + 3*x + 1) + 2/105*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 2/105*sqr
t(-x^2 + 1)/(x + 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5}\,dx = -\frac{1}{140}\left(5\left(\frac{2}{x+1}-1\right)^{\frac{7}{2}}+21\left(\frac{2}{x+1}-1\right)^{\frac{5}{2}}+35\left(\frac{2}{x+1}-1\right)^{\frac{3}{2}}+35\sqrt{\frac{2}{x+1}-1}\right)\mathrm{sgn}\left(\frac{1}{x+1}\right) + \frac{1}{60}\left(3\left(\frac{2}{x+1}-1\right)^{\frac{5}{2}}+10\left(\frac{2}{x+1}-1\right)^{\frac{3}{2}}+15\sqrt{\frac{2}{x+1}-1}\right)\mathrm{sgn}\left(\frac{1}{x+1}\right) - \frac{2}{105}i\,\mathrm{sgn}\left(\frac{1}{x+1}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^5,x, algorithm="giac")
```

output
```
-1/140*(5*(2/(x + 1) - 1)^(7/2) + 21*(2/(x + 1) - 1)^(5/2) + 35*(2/(x + 1)
 - 1)^(3/2) + 35*sqrt(2/(x + 1) - 1))*sgn(1/(x + 1)) + 1/60*(3*(2/(x + 1)
- 1)^(5/2) + 10*(2/(x + 1) - 1)^(3/2) + 15*sqrt(2/(x + 1) - 1))*sgn(1/(x +
 1)) - 2/105*I*sgn(1/(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5}\,dx = \frac{\sqrt{1-x^2}\,(2\,x^3+8\,x^2+13\,x-23)}{105\,{(x+1)}^4}$$

input
```
int((1 - x^2)^(1/2)/(x + 1)^5,x)
```

output
```
((1 - x^2)^(1/2)*(13*x + 8*x^2 + 2*x^3 - 23))/(105*(x + 1)^4)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^2}}{(1+x)^5}\,dx = \int \frac{\sqrt{-x^2+1}}{(x+1)^5}dx$$

input
```
int((-x^2+1)^(1/2)/(1+x)^5,x)
```

output
```
int((-x^2+1)^(1/2)/(1+x)^5,x)
```

3.83 $\int (1-x)^3 \sqrt{1-x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int (1-x)^3 \sqrt{1-x^2} \, dx = \frac{7}{8} x \sqrt{1-x^2} + \frac{7}{60} (8-3x) \left(1-x^2\right)^{3/2} + \frac{1}{5} (1-x)^2 \left(1-x^2\right)^{3/2} + \frac{7 \arcsin(x)}{8}$$

output
```
7/8*x*(-x^2+1)^(1/2)+7/60*(8-3*x)*(-x^2+1)^(3/2)+1/5*(1-x)^2*(-x^2+1)^(3/2
)+7/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int (1-x)^3 \sqrt{1-x^2} \, dx = -\frac{1}{120} \sqrt{1-x^2} \left(-136-15x+112x^2-90x^3+24x^4\right) - \frac{7}{4} \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 - x)^3*Sqrt[1 - x^2],x]
```

output
```
-1/120*(Sqrt[1 - x^2]*(-136 - 15*x + 112*x^2 - 90*x^3 + 24*x^4)) - (7*ArcT
an[Sqrt[1 - x^2]/(1 + x)])/4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {469, 469, 455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^3 \sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\int (1-x)^2\sqrt{1-x^2}dx+\frac{1}{5}\left(1-x^2\right)^{3/2}(1-x)^2$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\left(\frac{5}{4}\int (1-x)\sqrt{1-x^2}dx+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}\right)+\frac{1}{5}\left(1-x^2\right)^{3/2}(1-x)^2$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\int \sqrt{1-x^2}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}\right)+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}\right)+\frac{1}{5}\left(1-x^2\right)^{3/2}(1-x)^2$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}\right)+\frac{1}{5}\left(1-x^2\right)^{3/2}(1-x)^2$$

$$\Big\downarrow \text{223}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}\right)+\frac{1}{5}\left(1-x^2\right)^{3/2}(1-x)^2$$

input
```
Int[(1 - x)^3*Sqrt[1 - x^2],x]
```

output
```
((1 - x)^2*(1 - x^2)^(3/2))/5 + (7*(((1 - x)*(1 - x^2)^(3/2))/4 + (5*((x*S
qrt[1 - x^2])/2 + (1 - x^2)^(3/2)/3 + ArcSin[x]/2))/4))/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result
risch	$\frac{\left(24 x^{4}-90 x^{3}+112 x^{2}-15 x -136\right) \left(x^{2}-1\right)}{120 \sqrt{-x^{2}+1}}+\frac{7 \arcsin \left(x \right)}{8}$
default	$\frac{x^{2} \left(-x^{2}+1\right)^{\frac{3}{2}}}{5}+\frac{17 \left(-x^{2}+1\right)^{\frac{3}{2}}}{15}+\frac{7 x \sqrt{-x^{2}+1}}{8}+\frac{7 \arcsin \left(x \right)}{8}-\frac{3 x \left(-x^{2}+1\right)^{\frac{3}{2}}}{4}$
trager	$\left(-\frac{1}{5} x^{4}+\frac{3}{4} x^{3}-\frac{14}{15} x^{2}+\frac{1}{8} x +\frac{17}{15}\right) \sqrt{-x^{2}+1}+\frac{7 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ln \left(\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-x^{2}+1}+x \right)}{8}$
meijerg	$\frac{i \left(-2 i \sqrt{\pi}\, x \sqrt{-x^{2}+1}-2 i \sqrt{\pi}\, \arcsin \left(x \right)\right)}{4 \sqrt{\pi}}-\frac{3 \left(\frac{4 \sqrt{\pi}}{3}-\frac{2 \sqrt{\pi}\, \left(-2 x^{2}+2\right) \sqrt{-x^{2}+1}}{3}\right)}{4 \sqrt{\pi}}-\frac{3 i \left(-\frac{i \sqrt{\pi}\, x \left(-6 x^{2}+3\right) \sqrt{-x^{2}+1}}{6}+\frac{i \sqrt{\pi}\, \arcsin \left(x \right)}{2}\right)}{4 \sqrt{\pi}}$

input
```
int((1-x)^3*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*x^4-90*x^3+112*x^2-15*x-136)*(x^2-1)/(-x^2+1)^(1/2)+7/8*arcsin(x
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int (1-x)^3\sqrt{1-x^2}\,dx = -\frac{1}{120}\left(24\,x^4-90\,x^3+112\,x^2-15\,x-136\right)\sqrt{-x^2+1} - \frac{7}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1-x)^3*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(-x^2 + 1) - 7/4*arcta
n((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int (1-x)^3\sqrt{1-x^2}\,dx = -\frac{x^4\sqrt{1-x^2}}{5} + \frac{3x^3\sqrt{1-x^2}}{4} - \frac{14x^2\sqrt{1-x^2}}{15} + \frac{x\sqrt{1-x^2}}{8} + \frac{17\sqrt{1-x^2}}{15} + \frac{7\operatorname{asin}(x)}{8}$$

input
```
integrate((1-x)**3*(-x**2+1)**(1/2),x)
```

output
```
-x**4*sqrt(1 - x**2)/5 + 3*x**3*sqrt(1 - x**2)/4 - 14*x**2*sqrt(1 - x**2)/
15 + x*sqrt(1 - x**2)/8 + 17*sqrt(1 - x**2)/15 + 7*asin(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int (1-x)^3\sqrt{1-x^2}\,dx = \frac{1}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2 - \frac{3}{4}\left(-x^2+1\right)^{\frac{3}{2}}x + \frac{17}{15}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{7}{8}\sqrt{-x^2+1}x + \frac{7}{8}\arcsin(x)$$

input
```
integrate((1-x)^3*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x + 17/15*(-x^2 + 1)^(3/2)
 + 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int (1-x)^3\sqrt{1-x^2}\,dx = -\frac{1}{120}\left((2\left(3\left(4x-15\right)x+56\right)x-15)x-136\right)\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

input
```
integrate((1-x)^3*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/120*((2*(3*(4*x - 15)*x + 56)*x - 15)*x - 136)*sqrt(-x^2 + 1) + 7/8*arc
sin(x)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int (1-x)^3\sqrt{1-x^2}\,dx = \frac{7\,\mathrm{asin}(x)}{8} + \sqrt{1-x^2}\left(-\frac{x^4}{5} + \frac{3\,x^3}{4} - \frac{14\,x^2}{15} + \frac{x}{8} + \frac{17}{15}\right)$$

input
```
int(-(1 - x^2)^(1/2)*(x - 1)^3,x)
```

output
```
(7*asin(x))/8 + (1 - x^2)^(1/2)*(x/8 - (14*x^2)/15 + (3*x^3)/4 - x^4/5 + 1
7/15)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int (1-x)^3\sqrt{1-x^2}\,dx = \frac{7asin(x)}{8} - \frac{\sqrt{-x^2+1}\,x^4}{5} + \frac{3\sqrt{-x^2+1}\,x^3}{4} - \frac{14\sqrt{-x^2+1}\,x^2}{15} + \frac{\sqrt{-x^2+1}\,x}{8} + \frac{17\sqrt{-x^2+1}}{15} - \frac{17}{15}$$

input
```
int((1-x)^3*(-x^2+1)^(1/2),x)
```

output
```
(105*asin(x) - 24*sqrt( - x**2 + 1)*x**4 + 90*sqrt( - x**2 + 1)*x**3 - 112
*sqrt( - x**2 + 1)*x**2 + 15*sqrt( - x**2 + 1)*x + 136*sqrt( - x**2 + 1) -
 136)/120
```

3.84 $\int (1-x)^2\sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (1-x)^2\sqrt{1-x^2}\,dx=\frac{5}{8}x\sqrt{1-x^2}+\frac{1}{12}(8-3x)\left(1-x^2\right)^{3/2}+\frac{5\arcsin(x)}{8}$$

output
```
5/8*x*(-x^2+1)^(1/2)+1/12*(8-3*x)*(-x^2+1)^(3/2)+5/8*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int (1-x)^2\sqrt{1-x^2}\,dx=\frac{1}{24}\sqrt{1-x^2}\left(16+9x-16x^2+6x^3\right)-\frac{5}{4}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 - x)^2*Sqrt[1 - x^2],x]
```

output
```
(Sqrt[1 - x^2]*(16 + 9*x - 16*x^2 + 6*x^3))/24 - (5*ArcTan[Sqrt[1 - x^2]/(
1 + x)])/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {469, 455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^2\sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{4}\int (1-x)\sqrt{1-x^2}dx+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{455}$$

$$\frac{5}{4}\left(\int \sqrt{1-x^2}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}\right)+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{5}{4}\left(\frac{\arcsin(x)}{2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}\right)+\frac{1}{4}(1-x)\left(1-x^2\right)^{3/2}$$

input
```
Int[(1 - x)^2*Sqrt[1 - x^2],x]
```

output
```
((1 - x)*(1 - x^2)^(3/2))/4 + (5*((x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3/2)/3
+ ArcSin[x]/2))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	siz
risch	$-\frac{\left(6 x^{3}-16 x^{2}+9 x +16\right) \left(x^{2}-1\right)}{24 \sqrt{-x^{2}+1}}+\frac{5 \arcsin \left(x \right)}{8}$	37
default	$\frac{5 x \sqrt{-x^{2}+1}}{8}+\frac{5 \arcsin \left(x \right)}{8}-\frac{x \left(-x^{2}+1\right)^{\frac{3}{2}}}{4}+\frac{2 \left(-x^{2}+1\right)^{\frac{3}{2}}}{3}$	41
trager	$\left(\frac{1}{4} x^{3}-\frac{2}{3} x^{2}+\frac{3}{8} x +\frac{2}{3}\right) \sqrt{-x^{2}+1}+\frac{5 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ln \left(\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-x^{2}+1}+x \right)}{8}$	54
meijerg	$\frac{i \left(-2 i \sqrt{\pi}\, x \sqrt{-x^{2}+1}-2 i \sqrt{\pi}\, \arcsin \left(x \right)\right)}{4 \sqrt{\pi}}-\frac{\frac{4 \sqrt{\pi}}{3}-\frac{2 \sqrt{\pi}\, \left(-2 x^{2}+2\right) \sqrt{-x^{2}+1}}{3}}{2 \sqrt{\pi}}-\frac{i \left(-\frac{i \sqrt{\pi}\, x \left(-6 x^{2}+3\right) \sqrt{-x^{2}+1}}{6}+\frac{i \sqrt{\pi}\, \arcsin \left(x \right)}{2}\right)}{4 \sqrt{\pi}}$	10

input
```
int((1-x)^2*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*x^3-16*x^2+9*x+16)*(x^2-1)/(-x^2+1)^(1/2)+5/8*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int (1-x)^2\sqrt{1-x^2}\,dx = \frac{1}{24}\left(6\,x^3-16\,x^2+9\,x+16\right)\sqrt{-x^2+1} - \frac{5}{4}\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1-x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(-x^2 + 1) - 5/4*arctan((sqrt(-x^2 +
1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int (1-x)^2\sqrt{1-x^2}\,dx = \sqrt{1-x^2}\left(\frac{x^3}{4}-\frac{2x^2}{3}+\frac{3x}{8}+\frac{2}{3}\right)+\frac{5\operatorname{asin}(x)}{8}$$

input
```
integrate((1-x)**2*(-x**2+1)**(1/2),x)
```

output
```
sqrt(1 - x**2)*(x**3/4 - 2*x**2/3 + 3*x/8 + 2/3) + 5*asin(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int (1-x)^2\sqrt{1-x^2}\,dx = -\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}x+\frac{5}{8}\arcsin(x)$$

input
```
integrate((1-x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(-x^2 + 1)^(3/2)*x + 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/
8*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int (1-x)^2\sqrt{1-x^2}\,dx = \frac{1}{24}\left((2\,(3\,x-8)x+9)x+16\right)\sqrt{-x^2+1}+\frac{5}{8}\arcsin(x)$$

input
```
integrate((1-x)^2*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x - 8)*x + 9)*x + 16)*sqrt(-x^2 + 1) + 5/8*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int (1-x)^2\sqrt{1-x^2}\,dx = \frac{5\,\mathrm{asin}(x)}{8}+\sqrt{1-x^2}\left(\frac{x^3}{4}-\frac{2\,x^2}{3}+\frac{3\,x}{8}+\frac{2}{3}\right)$$

input
```
int((1 - x^2)^(1/2)*(x - 1)^2,x)
```

output
```
(5*asin(x))/8 + (1 - x^2)^(1/2)*((3*x)/8 - (2*x^2)/3 + x^3/4 + 2/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int (1-x)^2\sqrt{1-x^2}\,dx = \frac{5asin(x)}{8} + \frac{\sqrt{-x^2+1}\,x^3}{4} - \frac{2\sqrt{-x^2+1}\,x^2}{3} + \frac{3\sqrt{-x^2+1}\,x}{8} + \frac{2\sqrt{-x^2+1}}{3} - \frac{2}{3}$$

input
```
int((1-x)^2*(-x^2+1)^(1/2),x)
```

output
```
(15*asin(x) + 6*sqrt( - x**2 + 1)*x**3 - 16*sqrt( - x**2 + 1)*x**2 + 9*sqr
t( - x**2 + 1)*x + 16*sqrt( - x**2 + 1) - 16)/24
```

3.85 $\int (1-x)\sqrt{1-x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int (1-x)\sqrt{1-x^2}\,dx=\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{\arcsin(x)}{2}$$

output
```
1/2*x*(-x^2+1)^(1/2)+1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (1-x)\sqrt{1-x^2}\,dx=-\frac{1}{6}\sqrt{1-x^2}\left(-2-3x+2x^2\right)-\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 - x)*Sqrt[1 - x^2],x]
```

output
```
-1/6*(Sqrt[1 - x^2]*(-2 - 3*x + 2*x^2)) - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)\sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{455}$$

$$\int \sqrt{1-x^2}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(x)}{2}+\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}x\sqrt{1-x^2}$$

input
```
Int[(1 - x)*Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + (1 - x^2)^(3/2)/3 + ArcSin[x]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3}+\frac{\arcsin\left(x\right)}{2}$	29
risch	$\frac{\left(2x^2-3x-2\right)\left(x^2-1\right)}{6\sqrt{-x^2+1}}+\frac{\arcsin\left(x\right)}{2}$	32
trager	$\left(-\frac{1}{3}x^2+\frac{1}{2}x+\frac{1}{3}\right)\sqrt{-x^2+1}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	49
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin\left(x\right)\right)}{4\sqrt{\pi}}-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(-2x^2+2\right)\sqrt{-x^2+1}}{3}}{4\sqrt{\pi}}$	65

input
```
int((1-x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-x^2+1)^(1/2)+1/3*(-x^2+1)^(3/2)+1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int(1-x)\sqrt{1-x^2}\,dx=-\frac{1}{6}\left(2\,x^2-3\,x-2\right)\sqrt{-x^2+1}-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1-x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*x^2 - 3*x - 2)*sqrt(-x^2 + 1) - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int (1-x)\sqrt{1-x^2}\,dx = -\frac{x^2\sqrt{1-x^2}}{3} + \frac{x\sqrt{1-x^2}}{2} + \frac{\sqrt{1-x^2}}{3} + \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((1-x)*(-x**2+1)**(1/2),x)
```

output
```
-x**2*sqrt(1 - x**2)/3 + x*sqrt(1 - x**2)/2 + sqrt(1 - x**2)/3 + asin(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int (1-x)\sqrt{1-x^2}\,dx = \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int (1-x)\sqrt{1-x^2}\,dx = -\frac{1}{6}\left((2x-3)x-2\right)\sqrt{-x^2+1} + \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/6*((2*x - 3)*x - 2)*sqrt(-x^2 + 1) + 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int (1-x)\sqrt{1-x^2}\,dx = \frac{\operatorname{asin}(x)}{2} + \sqrt{1-x^2}\left(-\frac{x^2}{3} + \frac{x}{2} + \frac{1}{3}\right)$$

input
```
int(-(1 - x^2)^(1/2)*(x - 1),x)
```

output
```
asin(x)/2 + (1 - x^2)^(1/2)*(x/2 - x^2/3 + 1/3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (1-x)\sqrt{1-x^2}\,dx = \frac{asin(x)}{2} - \frac{\sqrt{-x^2+1}\,x^2}{3} + \frac{\sqrt{-x^2+1}\,x}{2} + \frac{\sqrt{-x^2+1}}{3} - \frac{1}{3}$$

input
```
int((1-x)*(-x^2+1)^(1/2),x)
```

output
```
(3*asin(x) - 2*sqrt( - x**2 + 1)*x**2 + 3*sqrt( - x**2 + 1)*x + 2*sqrt( -
x**2 + 1) - 2)/6
```

3.86 $\int \frac{\sqrt{1-x^2}}{1-x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{\sqrt{1-x^2}}{1-x} \, dx = -\sqrt{1-x^2} + \arcsin(x)$$

output
```
-(-x^2+1)^(1/2)+arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(16) = 32.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{1-x^2}}{1-x} \, dx = -\sqrt{1-x^2} - 2 \arctan \left(\frac{\sqrt{1-x^2}}{1+x} \right)$$

input
```
Integrate[Sqrt[1 - x^2]/(1 - x),x]
```

output
```
-Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {466, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{1-x} \, dx$$

$$\Big\downarrow \text{ 466}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx - \sqrt{1-x^2}$$

$$\Big\downarrow \text{ 223}$$

$$\arcsin(x) - \sqrt{1-x^2}$$

input
```
Int[Sqrt[1 - x^2]/(1 - x),x]
```

output
```
-Sqrt[1 - x^2] + ArcSin[x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

method	result	size
risch	$\frac{x^2-1}{\sqrt{-x^2+1}}+\arcsin(x)$	19
default	$-\sqrt{-(x-1)^2-2x+2}+\arcsin(x)$	20
trager	$-\sqrt{-x^2+1}+\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)$	39

input
```
int((-x^2+1)^(1/2)/(1-x),x,method=_RETURNVERBOSE)
```

output
```
(x^2-1)/(-x^2+1)^(1/2)+arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{1-x^2}}{1-x}\,dx = -\sqrt{-x^2+1}-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(1-x),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{1-x^2}}{1-x}\,dx = -\begin{cases} \sqrt{1-x^2} - \operatorname{asin}(x) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate((-x**2+1)**(1/2)/(1-x),x)
```

output
```
-Piecewise((sqrt(1 - x**2) - asin(x), (x > -1) & (x < 1)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{1-x^2}}{1-x}\,dx = -\sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2)/(1-x),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{1-x^2}}{1-x}\,dx = -\sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2)/(1-x),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1) + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{1-x^2}}{1-x}\,dx = \operatorname{asin}(x) - \sqrt{1-x^2}$$

input
```
int(-(1 - x^2)^(1/2)/(x - 1),x)
```

output
```
asin(x) - (1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{1-x^2}}{1-x}\,dx = asin(x) - \sqrt{-x^2+1} + 1$$

input
```
int((-x^2+1)^(1/2)/(1-x),x)
```

output
```
asin(x) - sqrt( - x**2 + 1) + 1
```

3.87 $\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx=\frac{2\sqrt{1-x^2}}{1-x}-\arcsin(x)$$

output
```
2*(-x^2+1)^(1/2)/(1-x)-arcsin(x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx=-\frac{2\sqrt{1-x^2}}{-1+x}+2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/(1 - x)^2,x]
```

output
```
(-2*Sqrt[1 - x^2])/(-1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {463, 25, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2} \, dx$$

$$\Big\downarrow \ 463$$

$$\int -\frac{1}{\sqrt{1-x^2}}dx+\frac{2\sqrt{1-x^2}}{1-x}$$

$$\Big\downarrow \ 25$$

$$\frac{2\sqrt{1-x^2}}{1-x}-\int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \ 223$$

$$\frac{2\sqrt{1-x^2}}{1-x}-\arcsin(x)$$

input
```
Int[Sqrt[1 - x^2]/(1 - x)^2,x]
```

output
```
(2*Sqrt[1 - x^2])/(1 - x) - ArcSin[x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{2+2x}{\sqrt{-x^2+1}}-\arcsin\left(x\right)$	20
default	$\frac{\left(-(x-1)^2-2x+2\right)^{\frac{3}{2}}}{(x-1)^2}+\sqrt{-\left(x-1\right)^2-2x+2}-\arcsin\left(x\right)$	40
trager	$-\frac{2\sqrt{-x^2+1}}{x-1}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	45

input
```
int((-x^2+1)^(1/2)/(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(x+1)/(-x^2+1)^(1/2)-arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx=\frac{2\left((x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+x-\sqrt{-x^2+1}-1\right)}{x-1}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^2,x, algorithm="fricas")
```

output
```
2*((x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + x - sqrt(-x^2 + 1) - 1)/(x - 1
)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{(x-1)^2}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1-x)**2,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x - 1)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx = -\frac{2\sqrt{-x^2+1}}{x-1} - \arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^2,x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^2,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: -(2*
atan(i)-2*i)*sign((sageVARx-1)^-1)+2*(-sqrt(-2*(sageVARx-1)^-1-1)*sign((sa
geVARx-1)^-1)+sign((sageVARx-1)^-1)*atan(sqrt(-2*(sageVARx-1)^-1-1)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx = -\mathrm{asin}(x) - \frac{2\sqrt{1-x^2}}{x-1}$$

input
```
int((1 - x^2)^(1/2)/(x - 1)^2,x)
```

output
```
- asin(x) - (2*(1 - x^2)^(1/2))/(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1-x^2}}{(1-x)^2}\,dx = \frac{-asin(x)\tan\left(\frac{asin(x)}{2}\right) + asin(x) - 4\tan\left(\frac{asin(x)}{2}\right)}{\tan\left(\frac{asin(x)}{2}\right) - 1}$$

input
```
int((-x^2+1)^(1/2)/(1-x)^2,x)
```

output
```
( - asin(x)*tan(asin(x)/2) + asin(x) - 4*tan(asin(x)/2))/(tan(asin(x)/2) -
 1)
```

3.88 $\int \frac{\sqrt{1-x^2}}{(1-x)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3} \, dx = \frac{(1-x^2)^{3/2}}{3(1-x)^3}$$

output
```
1/3*(-x^2+1)^(3/2)/(1-x)^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3} \, dx = -\frac{(-1-x)\sqrt{1-x^2}}{3(-1+x)^2}$$

input
```
Integrate[Sqrt[1 - x^2]/(1 - x)^3,x]
```

output
```
-1/3*((-1 - x)*Sqrt[1 - x^2])/(-1 + x)^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3} \, dx$$

$$\Big\downarrow \; 460$$

$$\frac{(1-x^2)^{3/2}}{3(1-x)^3}$$

input
```
Int[Sqrt[1 - x^2]/(1 - x)^3,x]
```

output
```
(1 - x^2)^(3/2)/(3*(1 - x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{(x+1)\sqrt{-x^2+1}}{3(x-1)^2}$	20
trager	$\frac{(x+1)\sqrt{-x^2+1}}{3(x-1)^2}$	20
default	$-\frac{\left(-(x-1)^2-2x+2\right)^{\frac{3}{2}}}{3(x-1)^3}$	22
risch	$-\frac{x^2+2x+1}{3(x-1)\sqrt{-x^2+1}}$	25
orering	$-\frac{(x-1)(x+1)\sqrt{-x^2+1}}{3(1-x)^3}$	25

input
```
int((-x^2+1)^(1/2)/(1-x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x+1)*(-x^2+1)^(1/2)/(x-1)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx = \frac{x^2+\sqrt{-x^2+1}(x+1)-2\,x+1}{3\,(x^2-2\,x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^3,x, algorithm="fricas")
```

output
```
1/3*(x^2 + sqrt(-x^2 + 1)*(x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx = -\int \frac{\sqrt{1-x^2}}{x^3-3x^2+3x-1}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1-x)**3,x)
```

output
```
-Integral(sqrt(1 - x**2)/(x**3 - 3*x**2 + 3*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx = \frac{2\sqrt{-x^2+1}}{3(x^2-2x+1)} + \frac{\sqrt{-x^2+1}}{3(x-1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^3,x, algorithm="maxima")
```

output
```
2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(16) = 32$.

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx = \frac{2\left(\frac{3\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{3\left(\frac{\sqrt{-x^2+1}-1}{x}+1\right)^3}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^3,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/((sqrt(-x^2 + 1) - 1)/x + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx = \frac{\sqrt{1-x^2}\,(x+1)}{3\,(x-1)^2}$$

input
```
int(-(1 - x^2)^(1/2)/(x - 1)^3,x)
```

output
```
((1 - x^2)^(1/2)*(x + 1))/(3*(x - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\sqrt{1-x^2}}{(1-x)^3}\,dx = \frac{-\sqrt{-x^2+1}\,x+7\sqrt{-x^2+1}-7x^2+2x-7}{9\sqrt{-x^2+1}\,x-9\sqrt{-x^2+1}+9x^2-18x+9}$$

input
```
int((-x^2+1)^(1/2)/(1-x)^3,x)
```

output
```
( - sqrt( - x**2 + 1)*x + 7*sqrt( - x**2 + 1) - 7*x**2 + 2*x - 7)/(9*(sqrt
( - x**2 + 1)*x - sqrt( - x**2 + 1) + x**2 - 2*x + 1))
```

3.89 $\int \frac{\sqrt{1-x^2}}{(1-x)^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4} \, dx = \frac{(1-x^2)^{3/2}}{5(1-x)^4} + \frac{(1-x^2)^{3/2}}{15(1-x)^3}$$

output
```
1/5*(-x^2+1)^(3/2)/(1-x)^4+1/15*(-x^2+1)^(3/2)/(1-x)^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4} \, dx = \frac{\sqrt{1-x^2}(-4-3x+x^2)}{15(-1+x)^3}$$

input
```
Integrate[Sqrt[1 - x^2]/(1 - x)^4,x]
```

output
```
(Sqrt[1 - x^2]*(-4 - 3*x + x^2))/(15*(-1 + x)^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4} \, dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{1}{5} \int \frac{\sqrt{1-x^2}}{(1-x)^3} dx + \frac{\left(1-x^2\right)^{3/2}}{5(1-x)^4}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{\left(1-x^2\right)^{3/2}}{15(1-x)^3} + \frac{\left(1-x^2\right)^{3/2}}{5(1-x)^4}$$

input
```
Int[Sqrt[1 - x^2]/(1 - x)^4,x]
```

output
```
(1 - x^2)^(3/2)/(5*(1 - x)^4) + (1 - x^2)^(3/2)/(15*(1 - x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{(x+1)(x-4)\sqrt{-x^2+1}}{15(x-1)^3}$	23
trager	$\frac{(x^2-3x-4)\sqrt{-x^2+1}}{15(x-1)^3}$	25
orering	$\frac{(x-4)(x-1)(x+1)\sqrt{-x^2+1}}{15(1-x)^4}$	28
risch	$-\frac{x^3-2x^2-7x-4}{15(x-1)^2\sqrt{-x^2+1}}$	30
default	$\frac{\left(-(x-1)^2-2x+2\right)^{\frac{3}{2}}}{5(x-1)^4}-\frac{\left(-(x-1)^2-2x+2\right)^{\frac{3}{2}}}{15(x-1)^3}$	44

input
```
int((-x^2+1)^(1/2)/(1-x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*(x+1)*(x-4)*(-x^2+1)^(1/2)/(x-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4}\,dx = \frac{4\,x^3-12\,x^2+(x^2-3\,x-4)\sqrt{-x^2+1}+12\,x-4}{15\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

output
```
1/15*(4*x^3 - 12*x^2 + (x^2 - 3*x - 4)*sqrt(-x^2 + 1) + 12*x - 4)/(x^3 - 3
*x^2 + 3*x - 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{(x-1)^4}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1-x)**4,x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/(x - 1)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4}\,dx = -\frac{2\sqrt{-x^2+1}}{5\,(x^3-3\,x^2+3\,x-1)} - \frac{\sqrt{-x^2+1}}{15\,(x^2-2\,x+1)} + \frac{\sqrt{-x^2+1}}{15\,(x-1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

output
```
-2/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) + 1/15*sqrt(-x^2 + 1)/(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(33) = 66.

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4}\,dx = \frac{2\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} + \frac{25\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + \frac{15\left(\sqrt{-x^2+1}-1\right)^3}{x^3} + \frac{15\left(\sqrt{-x^2+1}-1\right)^4}{x^4} + 4\right)}{15\left(\frac{\sqrt{-x^2+1}-1}{x}+1\right)^5}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^4,x, algorithm="giac")
```

output
```
2/15*(5*(sqrt(-x^2 + 1) - 1)/x + 25*(sqrt(-x^2 + 1) - 1)^2/x^2 + 15*(sqrt(
-x^2 + 1) - 1)^3/x^3 + 15*(sqrt(-x^2 + 1) - 1)^4/x^4 + 4)/((sqrt(-x^2 + 1)
 - 1)/x + 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4} \, dx = -\frac{\sqrt{1-x^2}\,(-x^2+3\,x+4)}{15\,(x-1)^3}$$

input
```
int((1 - x^2)^(1/2)/(x - 1)^4,x)
```

output
```
-((1 - x^2)^(1/2)*(3*x - x^2 + 4))/(15*(x - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{1-x^2}}{(1-x)^4} \, dx = \frac{\sqrt{-x^2+1}\,(x^2-3x-4)}{15x^3-45x^2+45x-15}$$

input
```
int((-x^2+1)^(1/2)/(1-x)^4,x)
```

output
```
(sqrt( - x**2 + 1)*(x**2 - 3*x - 4))/(15*(x**3 - 3*x**2 + 3*x - 1))
```

3.90 $\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx=\frac{(1-x^2)^{3/2}}{7(1-x)^5}+\frac{2(1-x^2)^{3/2}}{35(1-x)^4}+\frac{2(1-x^2)^{3/2}}{105(1-x)^3}$$

output
```
1/7*(-x^2+1)^(3/2)/(1-x)^5+2/35*(-x^2+1)^(3/2)/(1-x)^4+2/105*(-x^2+1)^(3/2
)/(1-x)^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx=-\frac{\sqrt{1-x^2}(-23-13x+8x^2-2x^3)}{105(-1+x)^4}$$

input
```
Integrate[Sqrt[1 - x^2]/(1 - x)^5,x]
```

output
```
-1/105*(Sqrt[1 - x^2]*(-23 - 13*x + 8*x^2 - 2*x^3))/(-1 + x)^4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5} \, dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2}{7} \int \frac{\sqrt{1-x^2}}{(1-x)^4} dx + \frac{\left(1-x^2\right)^{3/2}}{7(1-x)^5}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2}{7} \left(\frac{1}{5} \int \frac{\sqrt{1-x^2}}{(1-x)^3} dx + \frac{\left(1-x^2\right)^{3/2}}{5(1-x)^4} \right) + \frac{\left(1-x^2\right)^{3/2}}{7(1-x)^5}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{\left(1-x^2\right)^{3/2}}{7(1-x)^5} + \frac{2}{7} \left(\frac{\left(1-x^2\right)^{3/2}}{15(1-x)^3} + \frac{\left(1-x^2\right)^{3/2}}{5(1-x)^4} \right)$$

input
```
Int[Sqrt[1 - x^2]/(1 - x)^5,x]
```

output
```
(1 - x^2)^(3/2)/(7*(1 - x)^5) + (2*((1 - x^2)^(3/2)/(5*(1 - x)^4) + (1 - x
^2)^(3/2)/(15*(1 - x)^3)))/7
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

method	result	size
gosper	$\frac{(x+1)(2x^2-10x+23)\sqrt{-x^2+1}}{105(x-1)^4}$	30
trager	$\frac{(2x^3-8x^2+13x+23)\sqrt{-x^2+1}}{105(x-1)^4}$	32
orering	$-\frac{(2x^2-10x+23)(x-1)(x+1)\sqrt{-x^2+1}}{105(1-x)^5}$	35
risch	$-\frac{2x^4-6x^3+5x^2+36x+23}{105(x-1)^3\sqrt{-x^2+1}}$	37
default	$-\frac{\left(-(x-1)^2-2x+2\right)^{\frac{3}{2}}}{7(x-1)^5}+\frac{2\left(-(x-1)^2-2x+2\right)^{\frac{3}{2}}}{35(x-1)^4}-\frac{2\left(-(x-1)^2-2x+2\right)^{\frac{3}{2}}}{105(x-1)^3}$	65

input
```
int((-x^2+1)^(1/2)/(1-x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/105*(x+1)*(2*x^2-10*x+23)*(-x^2+1)^(1/2)/(x-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx = \frac{23\,x^4-92\,x^3+138\,x^2+(2\,x^3-8\,x^2+13\,x+23)\sqrt{-x^2+1}-92\,x+23}{105\,(x^4-4\,x^3+6\,x^2-4\,x+1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^5,x, algorithm="fricas")
```

output
```
1/105*(23*x^4 - 92*x^3 + 138*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(-x^2 +
 1) - 92*x + 23)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx = -\int \frac{\sqrt{1-x^2}}{x^5-5x^4+10x^3-10x^2+5x-1}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1-x)**5,x)
```

output
```
-Integral(sqrt(1 - x**2)/(x**5 - 5*x**4 + 10*x**3 - 10*x**2 + 5*x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx = \frac{2\sqrt{-x^2+1}}{7\,(x^4-4\,x^3+6\,x^2-4\,x+1)} + \frac{\sqrt{-x^2+1}}{35\,(x^3-3\,x^2+3\,x-1)} - \frac{2\sqrt{-x^2+1}}{105\,(x^2-2\,x+1)} + \frac{2\sqrt{-x^2+1}}{105\,(x-1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^5,x, algorithm="maxima")
```

output
```
2/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(-x^2 + 1)/(
x^3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/105*sqrt
(-x^2 + 1)/(x - 1)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx$$
$$= \frac{1}{140}\left(5\left(\frac{2}{x-1}+1\right)^3\sqrt{-\frac{2}{x-1}-1}-21\left(\frac{2}{x-1}+1\right)^2\sqrt{-\frac{2}{x-1}-1}-35\left(-\frac{2}{x-1}-1\right)^{\frac{3}{2}}-35\sqrt{-\right.$$
$$+\frac{1}{60}\left(3\left(\frac{2}{x-1}+1\right)^2\sqrt{-\frac{2}{x-1}-1}+10\left(-\frac{2}{x-1}-1\right)^{\frac{3}{2}}+15\sqrt{-\frac{2}{x-1}-1}\right)\mathrm{sgn}\left(\frac{1}{x-1}\right)$$
$$-\frac{2}{105}i\,\mathrm{sgn}\left(\frac{1}{x-1}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^5,x, algorithm="giac")
```

output
```
1/140*(5*(2/(x - 1) + 1)^3*sqrt(-2/(x - 1) - 1) - 21*(2/(x - 1) + 1)^2*sqr
t(-2/(x - 1) - 1) - 35*(-2/(x - 1) - 1)^(3/2) - 35*sqrt(-2/(x - 1) - 1))*s
gn(1/(x - 1)) + 1/60*(3*(2/(x - 1) + 1)^2*sqrt(-2/(x - 1) - 1) + 10*(-2/(x
 - 1) - 1)^(3/2) + 15*sqrt(-2/(x - 1) - 1))*sgn(1/(x - 1)) - 2/105*I*sgn(1
/(x - 1))
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx = \frac{\sqrt{1-x^2}\,(2\,x^3-8\,x^2+13\,x+23)}{105\,(x-1)^4}$$

input
```
int(-(1 - x^2)^(1/2)/(x - 1)^5,x)
```

output
```
((1 - x^2)^(1/2)*(13*x - 8*x^2 + 2*x^3 + 23))/(105*(x - 1)^4)
```

Reduce [F]

$$\int \frac{\sqrt{1-x^2}}{(1-x)^5}\,dx = \int \frac{\sqrt{-x^2+1}}{(1-x)^5}dx$$

input
```
int((-x^2+1)^(1/2)/(1-x)^5,x)
```

output
```
int((-x^2+1)^(1/2)/(1-x)^5,x)
```

3.91 $\int (1+x)^3\sqrt{-1+x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int (1+x)^3\sqrt{-1+x^2}\,dx=\frac{7}{8}x\sqrt{-1+x^2}+\frac{1}{5}(1+x)^2\left(-1+x^2\right)^{3/2}+\frac{7}{60}(8+3x)\left(-1+x^2\right)^{3/2}-\frac{7}{8}\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
7/8*x*(x^2-1)^(1/2)+1/5*(1+x)^2*(x^2-1)^(3/2)+7/60*(8+3*x)*(x^2-1)^(3/2)-7
/8*arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int (1+x)^3\sqrt{-1+x^2}\,dx=\frac{1}{120}\sqrt{-1+x^2}\left(-136+15x+112x^2+90x^3+24x^4\right)-\frac{7}{4}\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^3*Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(-136 + 15*x + 112*x^2 + 90*x^3 + 24*x^4))/120 - (7*ArcTan
h[Sqrt[-1 + x^2]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {469, 469, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^3\sqrt{x^2-1}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\int (x+1)^2\sqrt{x^2-1}dx+\frac{1}{5}\left(x^2-1\right)^{3/2}(x+1)^2$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\left(\frac{5}{4}\int (x+1)\sqrt{x^2-1}dx+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}\right)+\frac{1}{5}\left(x^2-1\right)^{3/2}(x+1)^2$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\int \sqrt{x^2-1}dx+\frac{1}{3}\left(x^2-1\right)^{3/2}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}\right)+\frac{1}{5}\left(x^2-1\right)^{3/2}(x+1)^2$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{x^2-1}}dx+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}\right)+\frac{1}{5}\left(x^2-1\right)^{3/2}(x+1)^2$$

$$\Big\downarrow \text{224}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}\right)+\frac{1}{5}\left(x^2-1\right)^{3/2}(x+1)^2$$

$$\Big\downarrow \text{219}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(-\frac{1}{2}\operatorname{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}\right)+\frac{1}{5}\left(x^2-1\right)^{3/2}(x+1)^2$$

input
```
Int[(1 + x)^3*Sqrt[-1 + x^2],x]
```

output
```
((1 + x)^2*(-1 + x^2)^(3/2))/5 + (7*(((1 + x)*(-1 + x^2)^(3/2))/4 + (5*((x
*Sqrt[-1 + x^2])/2 + (-1 + x^2)^(3/2)/3 - ArcTanh[x/Sqrt[-1 + x^2]]/2))/4)
)/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result
trager	$\left(\frac{1}{5}x^4+\frac{3}{4}x^3+\frac{14}{15}x^2+\frac{1}{8}x-\frac{17}{15}\right)\sqrt{x^2-1}-\frac{7\ln\left(x+\sqrt{x^2-1}\right)}{8}$
risch	$\frac{(24x^4+90x^3+112x^2+15x-136)\sqrt{x^2-1}}{120}-\frac{7\ln\left(x+\sqrt{x^2-1}\right)}{8}$
default	$\frac{7x\sqrt{x^2-1}}{8}-\frac{7\ln\left(x+\sqrt{x^2-1}\right)}{8}+\frac{x^2(x^2-1)^{\frac{3}{2}}}{5}+\frac{17(x^2-1)^{\frac{3}{2}}}{15}+\frac{3x(x^2-1)^{\frac{3}{2}}}{4}$
meijerg	$\frac{i\sqrt{\mathrm{signum}(x^2-1)}\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}\,\sqrt{-\mathrm{signum}(x^2-1)}}-\frac{\sqrt{\mathrm{signum}(x^2-1)}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(-x^2+1)^{\frac{3}{2}}(3x^2+2)}{15}\right)}{4\sqrt{\pi}\,\sqrt{-\mathrm{signum}(x^2-1)}}-\frac{3i\sqrt{\mathrm{signum}}$

input
```
int((x+1)^3*(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/5*x^4+3/4*x^3+14/15*x^2+1/8*x-17/15)*(x^2-1)^(1/2)-7/8*ln(x+(x^2-1)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int (1+x)^3\sqrt{-1+x^2}\,dx = \frac{1}{120}\left(24\,x^4+90\,x^3+112\,x^2+15\,x-136\right)\sqrt{x^2-1} + \frac{7}{8}\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((1+x)^3*(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(24*x^4 + 90*x^3 + 112*x^2 + 15*x - 136)*sqrt(x^2 - 1) + 7/8*log(-x
+ sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int (1+x)^3\sqrt{-1+x^2}\,dx = \frac{x^4\sqrt{x^2-1}}{5}+\frac{3x^3\sqrt{x^2-1}}{4}+\frac{14x^2\sqrt{x^2-1}}{15}+\frac{x\sqrt{x^2-1}}{8}-\frac{17\sqrt{x^2-1}}{15}-\frac{7\log\left(x+\sqrt{x^2-1}\right)}{8}$$

input
```
integrate((1+x)**3*(x**2-1)**(1/2),x)
```

output
```
x**4*sqrt(x**2 - 1)/5 + 3*x**3*sqrt(x**2 - 1)/4 + 14*x**2*sqrt(x**2 - 1)/1
5 + x*sqrt(x**2 - 1)/8 - 17*sqrt(x**2 - 1)/15 - 7*log(x + sqrt(x**2 - 1))/
8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int (1+x)^3\sqrt{-1+x^2}\,dx = \frac{1}{5}\left(x^2-1\right)^{\frac{3}{2}}x^2+\frac{3}{4}\left(x^2-1\right)^{\frac{3}{2}}x+\frac{17}{15}\left(x^2-1\right)^{\frac{3}{2}}+\frac{7}{8}\sqrt{x^2-1}x-\frac{7}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((1+x)^3*(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(x^2 - 1)^(3/2)*x^2 + 3/4*(x^2 - 1)^(3/2)*x + 17/15*(x^2 - 1)^(3/2) +
7/8*sqrt(x^2 - 1)*x - 7/8*log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int (1+x)^3\sqrt{-1+x^2}\,dx = \frac{1}{120}\left(\left(2\left(3\left(4x+15\right)x+56\right)x+15\right)x-136\right)\sqrt{x^2-1} + \frac{7}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((1+x)^3*(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/120*((2*(3*(4*x + 15)*x + 56)*x + 15)*x - 136)*sqrt(x^2 - 1) + 7/8*log(a
bs(-x + sqrt(x^2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int (1+x)^3\sqrt{-1+x^2}\,dx = \int \sqrt{x^2-1}\,(x+1)^3\,dx$$

input
```
int((x^2 - 1)^(1/2)*(x + 1)^3,x)
```

output
```
int((x^2 - 1)^(1/2)*(x + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int (1+x)^3\sqrt{-1+x^2}\,dx = \frac{\sqrt{x^2-1}\,x^4}{5} + \frac{3\sqrt{x^2-1}\,x^3}{4} + \frac{14\sqrt{x^2-1}\,x^2}{15} + \frac{\sqrt{x^2-1}\,x}{8} - \frac{17\sqrt{x^2-1}}{15} - \frac{7\log\left(\sqrt{x^2-1}+x\right)}{8}$$

input
```
int((1+x)^3*(x^2-1)^(1/2),x)
```

output
```
(24*sqrt(x**2 - 1)*x**4 + 90*sqrt(x**2 - 1)*x**3 + 112*sqrt(x**2 - 1)*x**2
 + 15*sqrt(x**2 - 1)*x - 136*sqrt(x**2 - 1) - 105*log(sqrt(x**2 - 1) + x))
/120
```

3.92 $\int (1+x)^2\sqrt{-1+x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int (1+x)^2\sqrt{-1+x^2}\,dx=\frac{5}{8}x\sqrt{-1+x^2}+\frac{1}{12}(8+3x)\left(-1+x^2\right)^{3/2}-\frac{5}{8}\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
5/8*x*(x^2-1)^(1/2)+1/12*(8+3*x)*(x^2-1)^(3/2)-5/8*arctanh(x/(x^2-1)^(1/2)
)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int (1+x)^2\sqrt{-1+x^2}\,dx=\frac{1}{24}\sqrt{-1+x^2}\left(-16+9x+16x^2+6x^3\right)-\frac{5}{4}\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^2*Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(-16 + 9*x + 16*x^2 + 6*x^3))/24 - (5*ArcTanh[Sqrt[-1 + x^
2]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {469, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)^2\sqrt{x^2-1}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{4}\int (x+1)\sqrt{x^2-1}dx+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{455}$$

$$\frac{5}{4}\left(\int \sqrt{x^2-1}dx+\frac{1}{3}\left(x^2-1\right)^{3/2}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{x^2-1}}dx+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{4}\left(-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)+\frac{1}{4}(x+1)\left(x^2-1\right)^{3/2}$$

input
```
Int[(1 + x)^2*Sqrt[-1 + x^2],x]
```

output
```
((1 + x)*(-1 + x^2)^(3/2))/4 + (5*((x*Sqrt[-1 + x^2])/2 + (-1 + x^2)^(3/2)
/3 - ArcTanh[x/Sqrt[-1 + x^2]]/2))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result
trager	$\left(\frac{1}{4}x^3+\frac{2}{3}x^2+\frac{3}{8}x-\frac{2}{3}\right)\sqrt{x^2-1}-\frac{5\ln\left(x+\sqrt{x^2-1}\right)}{8}$
risch	$\frac{(6x^3+16x^2+9x-16)\sqrt{x^2-1}}{24}-\frac{5\ln\left(x+\sqrt{x^2-1}\right)}{8}$
default	$\frac{5x\sqrt{x^2-1}}{8}-\frac{5\ln\left(x+\sqrt{x^2-1}\right)}{8}+\frac{x(x^2-1)^{\frac{3}{2}}}{4}+\frac{2(x^2-1)^{\frac{3}{2}}}{3}$
meijerg	$\frac{i\sqrt{\mathrm{signum}(x^2-1)}\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}\,\sqrt{-\mathrm{signum}(x^2-1)}}-\frac{i\sqrt{\mathrm{signum}(x^2-1)}\left(-\frac{i\sqrt{\pi}\,x\left(-6x^2+3\right)\sqrt{-x^2+1}}{6}+\frac{i\sqrt{\pi}\,\arcsin(x)}{2}\right)}{4\sqrt{\pi}\,\sqrt{-\mathrm{signum}(x^2-1)}}+$

input
```
int((x+1)^2*(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/4*x^3+2/3*x^2+3/8*x-2/3)*(x^2-1)^(1/2)-5/8*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int (1+x)^2\sqrt{-1+x^2}\,dx=\frac{1}{24}\left(6\,x^3+16\,x^2+9\,x-16\right)\sqrt{x^2-1}+\frac{5}{8}\,\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((1+x)^2*(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(x^2 - 1) + 5/8*log(-x + sqrt(x^2 - 1
))
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int (1+x)^2\sqrt{-1+x^2}\,dx=\sqrt{x^2-1}\left(\frac{x^3}{4}+\frac{2x^2}{3}+\frac{3x}{8}-\frac{2}{3}\right)-\frac{5\log\left(2x+2\sqrt{x^2-1}\right)}{8}$$

input
```
integrate((1+x)**2*(x**2-1)**(1/2),x)
```

output
```
sqrt(x**2 - 1)*(x**3/4 + 2*x**2/3 + 3*x/8 - 2/3) - 5*log(2*x + 2*sqrt(x**2
 - 1))/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int (1+x)^2\sqrt{-1+x^2}\,dx = \frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(x^2-1\right)^{\frac{3}{2}} + \frac{5}{8}\sqrt{x^2-1}x-\frac{5}{8}\log\left(2\,x+2\sqrt{x^2-1}\right)$$

input
```
integrate((1+x)^2*(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 - 1)^(3/2)*x + 2/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - 5/8*lo
g(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int (1+x)^2\sqrt{-1+x^2}\,dx = \frac{1}{24}\left((2\,(3\,x+8)x+9)x-16\right)\sqrt{x^2-1}+\frac{5}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((1+x)^2*(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x + 8)*x + 9)*x - 16)*sqrt(x^2 - 1) + 5/8*log(abs(-x + sqrt(x^
2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int (1+x)^2\sqrt{-1+x^2}\,dx = \int \sqrt{x^2-1}\,(x+1)^2\,dx$$

input
```
int((x^2 - 1)^(1/2)*(x + 1)^2,x)
```

output
```
int((x^2 - 1)^(1/2)*(x + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int (1+x)^2\sqrt{-1+x^2}\,dx = \frac{\sqrt{x^2-1}\,x^3}{4} + \frac{2\sqrt{x^2-1}\,x^2}{3} + \frac{3\sqrt{x^2-1}\,x}{8} - \frac{2\sqrt{x^2-1}}{3} - \frac{5\log\left(\sqrt{x^2-1}+x\right)}{8}$$

input
```
int((1+x)^2*(x^2-1)^(1/2),x)
```

output
```
(6*sqrt(x**2 - 1)*x**3 + 16*sqrt(x**2 - 1)*x**2 + 9*sqrt(x**2 - 1)*x - 16*
sqrt(x**2 - 1) - 15*log(sqrt(x**2 - 1) + x))/24
```

3.93 $\int (1+x)\sqrt{-1+x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{1}{2}x\sqrt{-1+x^2}+\frac{1}{3}\left(-1+x^2\right)^{3/2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
1/2*x*(x^2-1)^(1/2)+1/3*(x^2-1)^(3/2)-1/2*arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{1}{6}\sqrt{-1+x^2}\left(-2+3x+2x^2\right)-\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)*Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(-2 + 3*x + 2*x^2))/6 - ArcTanh[Sqrt[-1 + x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)\sqrt{x^2-1}\,dx$$

$$\Big\downarrow \text{455}$$

$$\int \sqrt{x^2-1}dx+\frac{1}{3}\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{x^2-1}}dx+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}$$

input
```
Int[(1 + x)*Sqrt[-1 + x^2],x]
```

output
```
(x*Sqrt[-1 + x^2])/2 + (-1 + x^2)^(3/2)/3 - ArcTanh[x/Sqrt[-1 + x^2]]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{x\sqrt{x^2-1}}{2}-\frac{\ln\left(x+\sqrt{x^2-1}\right)}{2}+\frac{(x^2-1)^{\frac{3}{2}}}{3}$	33
risch	$\frac{(2x^2+3x-2)\sqrt{x^2-1}}{6}-\frac{\ln\left(x+\sqrt{x^2-1}\right)}{2}$	33
trager	$\left(\frac{1}{3}x^2+\frac{1}{2}x-\frac{1}{3}\right)\sqrt{x^2-1}+\frac{\ln\left(x-\sqrt{x^2-1}\right)}{2}$	34
meijerg	$\frac{i\sqrt{\operatorname{signum}(x^2-1)}\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}(x^2-1)}}+\frac{\sqrt{\operatorname{signum}(x^2-1)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(-2x^2+2)\sqrt{-x^2+1}}{3}\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}(x^2-1)}}$	101

input
```
int((x+1)*(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x^2-1)^(1/2)-1/2*ln(x+(x^2-1)^(1/2))+1/3*(x^2-1)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{1}{6}\left(2\,x^2+3\,x-2\right)\sqrt{x^2-1}+\frac{1}{2}\,\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((1+x)*(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^2 + 3*x - 2)*sqrt(x^2 - 1) + 1/2*log(-x + sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{x^2\sqrt{x^2-1}}{3}+\frac{x\sqrt{x^2-1}}{2}-\frac{\sqrt{x^2-1}}{3}-\frac{\log\left(x+\sqrt{x^2-1}\right)}{2}$$

input
```
integrate((1+x)*(x**2-1)**(1/2),x)
```

output
```
x**2*sqrt(x**2 - 1)/3 + x*sqrt(x**2 - 1)/2 - sqrt(x**2 - 1)/3 - log(x + sq
rt(x**2 - 1))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{1}{2}\,\sqrt{x^2-1}x-\frac{1}{2}\,\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((1+x)*(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(x^2 - 1)^(3/2) + 1/2*sqrt(x^2 - 1)*x - 1/2*log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{1}{6}\left((2\,x+3)x-2\right)\sqrt{x^2-1}+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((1+x)*(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/6*((2*x + 3)*x - 2)*sqrt(x^2 - 1) + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{x\,\sqrt{x^2-1}}{2}-\frac{\ln\left(x+\sqrt{x^2-1}\right)}{2}+\frac{\left(x^2-1\right)^{3/2}}{3}$$

input
```
int((x^2 - 1)^(1/2)*(x + 1),x)
```

output
```
(x*(x^2 - 1)^(1/2))/2 - log(x + (x^2 - 1)^(1/2))/2 + (x^2 - 1)^(3/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int (1+x)\sqrt{-1+x^2}\,dx = \frac{\sqrt{x^2-1}\,x^2}{3}+\frac{\sqrt{x^2-1}\,x}{2}-\frac{\sqrt{x^2-1}}{3}-\frac{\log\left(\sqrt{x^2-1}+x\right)}{2}$$

input
```
int((1+x)*(x^2-1)^(1/2),x)
```

output
```
(2*sqrt(x**2 - 1)*x**2 + 3*sqrt(x**2 - 1)*x - 2*sqrt(x**2 - 1) - 3*log(sqr
t(x**2 - 1) + x))/6
```

3.94 $\int \frac{\sqrt{-1+x^2}}{1+x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx = \sqrt{-1+x^2} - \operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
(x^2-1)^(1/2)-arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx = \sqrt{-1+x^2} + \log\left(-x+\sqrt{-1+x^2}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 + x),x]
```

output
```
Sqrt[-1 + x^2] + Log[-x + Sqrt[-1 + x^2]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {466, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{x+1} \, dx$$

$$\Big\downarrow \text{466}$$

$$\sqrt{x^2-1} - \int \frac{1}{\sqrt{x^2-1}}dx$$

$$\Big\downarrow \text{224}$$

$$\sqrt{x^2-1} - \int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}$$

$$\Big\downarrow \text{219}$$

$$\sqrt{x^2-1} - \text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

input
```
Int[Sqrt[-1 + x^2]/(1 + x),x]
```

output
```
Sqrt[-1 + x^2] - ArcTanh[x/Sqrt[-1 + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
trager	$\sqrt{x^2-1}+\ln\left(x-\sqrt{x^2-1}\right)$	21
risch	$\sqrt{x^2-1}-\ln\left(x+\sqrt{x^2-1}\right)$	21
default	$\sqrt{\left(x+1\right)^2-2x-2}-\ln\left(x+\sqrt{\left(x+1\right)^2-2x-2}\right)$	31

input
```
int((x^2-1)^(1/2)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
(x^2-1)^(1/2)+ln(x-(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx=\sqrt{x^2-1}+\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
sqrt(x^2 - 1) + log(-x + sqrt(x^2 - 1))
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx = \int \frac{\sqrt{(x-1)(x+1)}}{x+1}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1+x),x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx = \sqrt{x^2-1} - \log\left(2\,x + 2\,\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
sqrt(x^2 - 1) - log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx = \sqrt{x^2-1} + \log\left(\left|-x + \sqrt{x^2-1}\right|\right)$$

input
```
integrate((x^2-1)^(1/2)/(1+x),x, algorithm="giac")
```

output
```
sqrt(x^2 - 1) + log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx = \int \frac{\sqrt{x^2-1}}{x+1}\,dx$$

input
```
int((x^2 - 1)^(1/2)/(x + 1),x)
```

output
```
int((x^2 - 1)^(1/2)/(x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-1+x^2}}{1+x}\,dx = \sqrt{x^2-1} - \log\left(\sqrt{x^2-1}+x\right)$$

input
```
int((x^2-1)^(1/2)/(1+x),x)
```

output
```
sqrt(x**2 - 1) - log(sqrt(x**2 - 1) + x)
```

3.95 $\int \frac{\sqrt{-1+x^2}}{(1+x)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2} dx = -\frac{2\sqrt{-1+x^2}}{1+x} + \operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
-2*(x^2-1)^(1/2)/(1+x)+arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2} dx = -\frac{2\sqrt{-1+x^2}}{1+x} + 2\operatorname{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 + x)^2,x]
```

output
```
(-2*Sqrt[-1 + x^2])/(1 + x) + 2*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {463, 25, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(x+1)^2}dx$$

$$\Big\downarrow \text{463}$$

$$-\int -\frac{1}{\sqrt{x^2-1}}dx-\frac{2\sqrt{x^2-1}}{x+1}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{\sqrt{x^2-1}}dx-\frac{2\sqrt{x^2-1}}{x+1}$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}-\frac{2\sqrt{x^2-1}}{x+1}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{2\sqrt{x^2-1}}{x+1}$$

input
```
Int[Sqrt[-1 + x^2]/(1 + x)^2,x]
```

output
```
(-2*Sqrt[-1 + x^2])/(1 + x) + ArcTanh[x/Sqrt[-1 + x^2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{2(x-1)}{\sqrt{x^2-1}}+\ln\left(x+\sqrt{x^2-1}\right)$	24
trager	$-\frac{2\sqrt{x^2-1}}{x+1}+\ln\left(x+\sqrt{x^2-1}\right)$	26
default	$\frac{\left((x+1)^2-2x-2\right)^{\frac{3}{2}}}{(x+1)^2}-\sqrt{(x+1)^2-2x-2}+\ln\left(x+\sqrt{(x+1)^2-2x-2}\right)$	49

input
```
int((x^2-1)^(1/2)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(x-1)/(x^2-1)^(1/2)+ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2}\,dx = -\frac{(x+1)\log\left(-x+\sqrt{x^2-1}\right)+2\,x+2\,\sqrt{x^2-1}+2}{x+1}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

output
```
-((x + 1)*log(-x + sqrt(x^2 - 1)) + 2*x + 2*sqrt(x^2 - 1) + 2)/(x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}}{(x+1)^2}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1+x)**2,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x + 1)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2}\,dx = -\frac{2\,\sqrt{x^2-1}}{x+1}+\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

output
```
-2*sqrt(x^2 - 1)/(x + 1) + log(2*x + 2*sqrt(x^2 - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2}\,dx = \log\left(\sqrt{-\frac{2}{x+1}+1}+1\right)\operatorname{sgn}\left(\frac{1}{x+1}\right) - \log\left(\left|\sqrt{-\frac{2}{x+1}+1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{x+1}\right) - 2\sqrt{-\frac{2}{x+1}+1}\operatorname{sgn}\left(\frac{1}{x+1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^2,x, algorithm="giac")
```

output
```
log(sqrt(-2/(x + 1) + 1) + 1)*sgn(1/(x + 1)) - log(abs(sqrt(-2/(x + 1) + 1
) - 1))*sgn(1/(x + 1)) - 2*sqrt(-2/(x + 1) + 1)*sgn(1/(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2}\,dx = \int \frac{\sqrt{x^2-1}}{(x+1)^2}\,dx$$

input
```
int((x^2 - 1)^(1/2)/(x + 1)^2,x)
```

output
```
int((x^2 - 1)^(1/2)/(x + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^2}\,dx = \frac{-2\sqrt{x^2-1}+\log\left(\sqrt{x^2-1}+x\right)x+\log\left(\sqrt{x^2-1}+x\right)-2x-2}{x+1}$$

input
```
int((x^2-1)^(1/2)/(1+x)^2,x)
```

output
```
( - 2*sqrt(x**2 - 1) + log(sqrt(x**2 - 1) + x)*x + log(sqrt(x**2 - 1) + x)
 - 2*x - 2)/(x + 1)
```

3.96 $\int \frac{\sqrt{-1+x^2}}{(1+x)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3} \, dx = \frac{(-1+x^2)^{3/2}}{3(1+x)^3}$$

output
```
1/3*(x^2-1)^(3/2)/(1+x)^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3} \, dx = \frac{(-1+x)\sqrt{-1+x^2}}{3(1+x)^2}$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 + x)^3,x]
```

output
```
((-1 + x)*Sqrt[-1 + x^2])/(3*(1 + x)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(x+1)^3}\,dx$$

$$\Big\downarrow \text{ 460}$$

$$\frac{\left(x^2-1\right)^{3/2}}{3(x+1)^3}$$

input
```
Int[Sqrt[-1 + x^2]/(1 + x)^3,x]
```

output
```
(-1 + x^2)^(3/2)/(3*(1 + x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{(x-1)\sqrt{x^2-1}}{3(x+1)^2}$	18
trager	$\frac{(x-1)\sqrt{x^2-1}}{3(x+1)^2}$	18
orering	$\frac{(x-1)\sqrt{x^2-1}}{3(x+1)^2}$	18
default	$\frac{\left((x+1)^2-2x-2\right)^{\frac{3}{2}}}{3(x+1)^3}$	20
risch	$\frac{x^2-2x+1}{3(x+1)\sqrt{x^2-1}}$	23

input
```
int((x^2-1)^(1/2)/(x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x-1)/(x+1)^2*(x^2-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3}\,dx = \frac{x^2+\sqrt{x^2-1}(x-1)+2\,x+1}{3\,(x^2+2\,x+1)}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^3,x, algorithm="fricas")
```

output
```
1/3*(x^2 + sqrt(x^2 - 1)*(x - 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3}\,dx = \int \frac{\sqrt{(x-1)(x+1)}}{(x+1)^3}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1+x)**3,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x + 1)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3}\,dx = -\frac{2\sqrt{x^2-1}}{3(x^2+2x+1)} + \frac{\sqrt{x^2-1}}{3(x+1)}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^3,x, algorithm="maxima")
```

output
```
-2/3*sqrt(x^2 - 1)/(x^2 + 2*x + 1) + 1/3*sqrt(x^2 - 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3}\,dx = \frac{2\left(3\left(x-\sqrt{x^2-1}\right)^2+1\right)}{3\left(x-\sqrt{x^2-1}+1\right)^3}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^3,x, algorithm="giac")
```

output
```
2/3*(3*(x - sqrt(x^2 - 1))^2 + 1)/(x - sqrt(x^2 - 1) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3}\,dx = \frac{\sqrt{x^2-1}\,(x-1)}{3\,{(x+1)}^2}$$

input
```
int((x^2 - 1)^(1/2)/(x + 1)^3,x)
```

output
```
((x^2 - 1)^(1/2)*(x - 1))/(3*(x + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^3}\,dx = \frac{9\sqrt{x^2-1}\,x-9\sqrt{x^2-1}+8x^2+16x+8}{27x^2+54x+27}$$

input
```
int((x^2-1)^(1/2)/(1+x)^3,x)
```

output
```
(9*sqrt(x**2 - 1)*x - 9*sqrt(x**2 - 1) + 8*x**2 + 16*x + 8)/(27*(x**2 + 2*
x + 1))
```

3.97 $\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = \frac{\left(-1+x^2\right)^{3/2}}{5(1+x)^4} + \frac{\left(-1+x^2\right)^{3/2}}{15(1+x)^3}$$

output
```
1/5*(x^2-1)^(3/2)/(1+x)^4+1/15*(x^2-1)^(3/2)/(1+x)^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = \frac{\sqrt{-1+x^2}(-4+3x+x^2)}{15(1+x)^3}$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 + x)^4,x]
```

output
```
(Sqrt[-1 + x^2]*(-4 + 3*x + x^2))/(15*(1 + x)^3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(x+1)^4}\,dx$$

$$\Big\downarrow \text{461}$$

$$\frac{1}{5}\int \frac{\sqrt{x^2-1}}{(x+1)^3}dx+\frac{\left(x^2-1\right)^{3/2}}{5(x+1)^4}$$

$$\Big\downarrow \text{460}$$

$$\frac{\left(x^2-1\right)^{3/2}}{15(x+1)^3}+\frac{\left(x^2-1\right)^{3/2}}{5(x+1)^4}$$

input
```
Int[Sqrt[-1 + x^2]/(1 + x)^4,x]
```

output
```
(-1 + x^2)^(3/2)/(5*(1 + x)^4) + (-1 + x^2)^(3/2)/(15*(1 + x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{(x-1)(x+4)\sqrt{x^2-1}}{15(x+1)^3}$	21
orering	$\frac{(x-1)(x+4)\sqrt{x^2-1}}{15(x+1)^3}$	21
trager	$\frac{(x^2+3x-4)\sqrt{x^2-1}}{15(x+1)^3}$	23
risch	$\frac{x^3+2x^2-7x+4}{15(x+1)^2\sqrt{x^2-1}}$	28
default	$\frac{\left((x+1)^2-2x-2\right)^{\frac{3}{2}}}{5(x+1)^4}+\frac{\left((x+1)^2-2x-2\right)^{\frac{3}{2}}}{15(x+1)^3}$	40

input
```
int((x^2-1)^(1/2)/(x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*(x-1)*(x+4)*(x^2-1)^(1/2)/(x+1)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = \frac{x^3+3\,x^2+(x^2+3\,x-4)\sqrt{x^2-1}+3\,x+1}{15\,(x^3+3\,x^2+3\,x+1)}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^4,x, algorithm="fricas")
```

output
```
1/15*(x^3 + 3*x^2 + (x^2 + 3*x - 4)*sqrt(x^2 - 1) + 3*x + 1)/(x^3 + 3*x^2
+ 3*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = \int \frac{\sqrt{(x-1)(x+1)}}{(x+1)^4}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1+x)**4,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x + 1)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = -\frac{2\sqrt{x^2-1}}{5(x^3+3x^2+3x+1)} + \frac{\sqrt{x^2-1}}{15(x^2+2x+1)} + \frac{\sqrt{x^2-1}}{15(x+1)}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^4,x, algorithm="maxima")
```

output
```
-2/5*sqrt(x^2 - 1)/(x^3 + 3*x^2 + 3*x + 1) + 1/15*sqrt(x^2 - 1)/(x^2 + 2*x
 + 1) + 1/15*sqrt(x^2 - 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = \frac{2\left(15\left(x-\sqrt{x^2-1}\right)^3 - 5\left(x-\sqrt{x^2-1}\right)^2 + 5x - 5\sqrt{x^2-1} + 1\right)}{15\left(x-\sqrt{x^2-1}+1\right)^5}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^4,x, algorithm="giac")
```

output
```
2/15*(15*(x - sqrt(x^2 - 1))^3 - 5*(x - sqrt(x^2 - 1))^2 + 5*x - 5*sqrt(x^
2 - 1) + 1)/(x - sqrt(x^2 - 1) + 1)^5
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = \frac{\sqrt{x^2-1}\,(x^2+3\,x-4)}{15\,(x+1)^3}$$

input
```
int((x^2 - 1)^(1/2)/(x + 1)^4,x)
```

output
```
((x^2 - 1)^(1/2)*(3*x + x^2 - 4))/(15*(x + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^4}\,dx = \frac{\sqrt{x^2-1}\,x^2+3\sqrt{x^2-1}\,x-4\sqrt{x^2-1}-x^3-3x^2-3x-1}{15x^3+45x^2+45x+15}$$

input
```
int((x^2-1)^(1/2)/(1+x)^4,x)
```

output
```
(sqrt(x**2 - 1)*x**2 + 3*sqrt(x**2 - 1)*x - 4*sqrt(x**2 - 1) - x**3 - 3*x*
*2 - 3*x - 1)/(15*(x**3 + 3*x**2 + 3*x + 1))
```

3.98 $\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx=\frac{(-1+x^2)^{3/2}}{7(1+x)^5}+\frac{2(-1+x^2)^{3/2}}{35(1+x)^4}+\frac{2(-1+x^2)^{3/2}}{105(1+x)^3}$$

output
```
1/7*(x^2-1)^(3/2)/(1+x)^5+2/35*(x^2-1)^(3/2)/(1+x)^4+2/105*(x^2-1)^(3/2)/(
1+x)^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx=\frac{\sqrt{-1+x^2}(-23+13x+8x^2+2x^3)}{105(1+x)^4}$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 + x)^5,x]
```

output
```
(Sqrt[-1 + x^2]*(-23 + 13*x + 8*x^2 + 2*x^3))/(105*(1 + x)^4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(x+1)^5}\,dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2}{7}\int \frac{\sqrt{x^2-1}}{(x+1)^4}dx+\frac{\left(x^2-1\right)^{3/2}}{7(x+1)^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{2}{7}\left(\frac{1}{5}\int \frac{\sqrt{x^2-1}}{(x+1)^3}dx+\frac{\left(x^2-1\right)^{3/2}}{5(x+1)^4}\right)+\frac{\left(x^2-1\right)^{3/2}}{7(x+1)^5}$$

$$\Big\downarrow \text{460}$$

$$\frac{\left(x^2-1\right)^{3/2}}{7(x+1)^5}+\frac{2}{7}\left(\frac{\left(x^2-1\right)^{3/2}}{15(x+1)^3}+\frac{\left(x^2-1\right)^{3/2}}{5(x+1)^4}\right)$$

input
```
Int[Sqrt[-1 + x^2]/(1 + x)^5,x]
```

output
```
(-1 + x^2)^(3/2)/(7*(1 + x)^5) + (2*((-1 + x^2)^(3/2)/(5*(1 + x)^4) + (-1
+ x^2)^(3/2)/(15*(1 + x)^3)))/7
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{(x-1)(2x^2+10x+23)\sqrt{x^2-1}}{105(x+1)^4}$	28
orering	$\frac{(x-1)(2x^2+10x+23)\sqrt{x^2-1}}{105(x+1)^4}$	28
trager	$\frac{(2x^3+8x^2+13x-23)\sqrt{x^2-1}}{105(x+1)^4}$	30
risch	$\frac{2x^4+6x^3+5x^2-36x+23}{105(x+1)^3\sqrt{x^2-1}}$	35
default	$\frac{\left((x+1)^2-2x-2\right)^{\frac{3}{2}}}{7(x+1)^5}+\frac{2\left((x+1)^2-2x-2\right)^{\frac{3}{2}}}{35(x+1)^4}+\frac{2\left((x+1)^2-2x-2\right)^{\frac{3}{2}}}{105(x+1)^3}$	59

input
```
int((x^2-1)^(1/2)/(x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/105*(x-1)*(2*x^2+10*x+23)*(x^2-1)^(1/2)/(x+1)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx = \frac{2\,x^4+8\,x^3+12\,x^2+(2\,x^3+8\,x^2+13\,x-23)\sqrt{x^2-1}+8\,x+2}{105\,(x^4+4\,x^3+6\,x^2+4\,x+1)}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^5,x, algorithm="fricas")
```

output
```
1/105*(2*x^4 + 8*x^3 + 12*x^2 + (2*x^3 + 8*x^2 + 13*x - 23)*sqrt(x^2 - 1)
+ 8*x + 2)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}}{(x+1)^5}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1+x)**5,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x + 1)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx = -\frac{2\,\sqrt{x^2-1}}{7\,(x^4+4\,x^3+6\,x^2+4\,x+1)}+\frac{\sqrt{x^2-1}}{35\,(x^3+3\,x^2+3\,x+1)} + \frac{2\,\sqrt{x^2-1}}{105\,(x^2+2\,x+1)}+\frac{2\,\sqrt{x^2-1}}{105\,(x+1)}$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^5,x, algorithm="maxima")
```

output
```
-2/7*sqrt(x^2 - 1)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) + 1/35*sqrt(x^2 - 1)/(x
^3 + 3*x^2 + 3*x + 1) + 2/105*sqrt(x^2 - 1)/(x^2 + 2*x + 1) + 2/105*sqrt(x
^2 - 1)/(x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx =$$
$$-\frac{1}{140}\left(5\left(\frac{2}{x+1}-1\right)^3\sqrt{-\frac{2}{x+1}+1}+21\left(\frac{2}{x+1}-1\right)^2\sqrt{-\frac{2}{x+1}+1}-35\left(-\frac{2}{x+1}+1\right)^{\frac{3}{2}}+35\sqrt{}\right.$$
$$+\frac{1}{60}\left(3\left(\frac{2}{x+1}-1\right)^2\sqrt{-\frac{2}{x+1}+1}-10\left(-\frac{2}{x+1}+1\right)^{\frac{3}{2}}+15\sqrt{-\frac{2}{x+1}+1}\right)\operatorname{sgn}\left(\frac{1}{x+1}\right)$$
$$-\frac{2}{105}\operatorname{sgn}\left(\frac{1}{x+1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1+x)^5,x, algorithm="giac")
```

output
```
-1/140*(5*(2/(x + 1) - 1)^3*sqrt(-2/(x + 1) + 1) + 21*(2/(x + 1) - 1)^2*sq
rt(-2/(x + 1) + 1) - 35*(-2/(x + 1) + 1)^(3/2) + 35*sqrt(-2/(x + 1) + 1))*
sgn(1/(x + 1)) + 1/60*(3*(2/(x + 1) - 1)^2*sqrt(-2/(x + 1) + 1) - 10*(-2/(
x + 1) + 1)^(3/2) + 15*sqrt(-2/(x + 1) + 1))*sgn(1/(x + 1)) - 2/105*sgn(1/
(x + 1))
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx = \frac{2\sqrt{x^2-1}}{105\,(x+1)}+\frac{2\sqrt{x^2-1}}{105\,(x+1)^2}+\frac{\sqrt{x^2-1}}{35\,(x+1)^3}-\frac{2\sqrt{x^2-1}}{7\,(x+1)^4}$$

input
```
int((x^2 - 1)^(1/2)/(x + 1)^5,x)
```

output
```
(2*(x^2 - 1)^(1/2))/(105*(x + 1)) + (2*(x^2 - 1)^(1/2))/(105*(x + 1)^2) +
(x^2 - 1)^(1/2)/(35*(x + 1)^3) - (2*(x^2 - 1)^(1/2))/(7*(x + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{-1+x^2}}{(1+x)^5}\,dx = \frac{2\sqrt{x^2-1}\,x^3+8\sqrt{x^2-1}\,x^2+13\sqrt{x^2-1}\,x-23\sqrt{x^2-1}-2x^4-8x^3-12x^2-8x-2}{105x^4+420x^3+630x^2+420x+105}$$

input
```
int((x^2-1)^(1/2)/(1+x)^5,x)
```

output
```
(2*sqrt(x**2 - 1)*x**3 + 8*sqrt(x**2 - 1)*x**2 + 13*sqrt(x**2 - 1)*x - 23*
sqrt(x**2 - 1) - 2*x**4 - 8*x**3 - 12*x**2 - 8*x - 2)/(105*(x**4 + 4*x**3
+ 6*x**2 + 4*x + 1))
```

3.99 $\int (1-x)^3\sqrt{-1+x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (1-x)^3\sqrt{-1+x^2}\,dx=\frac{7}{8}x\sqrt{-1+x^2}-\frac{7}{60}(8-3x)\left(-1+x^2\right)^{3/2}-\frac{1}{5}(1-x)^2\left(-1+x^2\right)^{3/2}-\frac{7}{8}\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
7/8*x*(x^2-1)^(1/2)-7/60*(8-3*x)*(x^2-1)^(3/2)-1/5*(1-x)^2*(x^2-1)^(3/2)-7
/8*arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int (1-x)^3\sqrt{-1+x^2}\,dx=-\frac{1}{120}\sqrt{-1+x^2}\left(-136-15x+112x^2-90x^3+24x^4\right)-\frac{7}{4}\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^3*Sqrt[-1 + x^2],x]
```

output
```
-1/120*(Sqrt[-1 + x^2]*(-136 - 15*x + 112*x^2 - 90*x^3 + 24*x^4)) - (7*Arc
Tanh[Sqrt[-1 + x^2]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {469, 469, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^3\sqrt{x^2-1}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\int (1-x)^2\sqrt{x^2-1}dx-\frac{1}{5}(1-x)^2\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}\left(\frac{5}{4}\int (1-x)\sqrt{x^2-1}dx-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}\right)-\frac{1}{5}(1-x)^2\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(\int \sqrt{x^2-1}dx-\frac{1}{3}\left(x^2-1\right)^{3/2}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}\right)-\frac{1}{5}(1-x)^2\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{x^2-1}}dx-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}\right)-\frac{1}{5}(1-x)^2\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}\right)-\frac{1}{5}(1-x)^2\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{7}{5}\left(\frac{5}{4}\left(-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}\right)-\frac{1}{5}(1-x)^2\left(x^2-1\right)^{3/2}$$

input
```
Int[(1 - x)^3*Sqrt[-1 + x^2],x]
```

output
```
-1/5*((1 - x)^2*(-1 + x^2)^(3/2)) + (7*(-1/4*((1 - x)*(-1 + x^2)^(3/2)) +
(5*((x*Sqrt[-1 + x^2])/2 - (-1 + x^2)^(3/2)/3 - ArcTanh[x/Sqrt[-1 + x^2]]/
2))/4))/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{(24x^4-90x^3+112x^2-15x-136)\sqrt{x^2-1}}{120}-\frac{7\ln\left(x+\sqrt{x^2-1}\right)}{8}$
trager	$\left(-\frac{1}{5}x^4+\frac{3}{4}x^3-\frac{14}{15}x^2+\frac{1}{8}x+\frac{17}{15}\right)\sqrt{x^2-1}+\frac{7\ln\left(x-\sqrt{x^2-1}\right)}{8}$
default	$-\frac{x^2(x^2-1)^{\frac{3}{2}}}{5}-\frac{17(x^2-1)^{\frac{3}{2}}}{15}+\frac{7x\sqrt{x^2-1}}{8}-\frac{7\ln\left(x+\sqrt{x^2-1}\right)}{8}+\frac{3x(x^2-1)^{\frac{3}{2}}}{4}$
meijerg	$\frac{i\sqrt{\operatorname{signum}(x^2-1)}\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\arcsin(x)\right)}{4\sqrt{\pi}\sqrt{-\operatorname{signum}(x^2-1)}}-\frac{3\sqrt{\operatorname{signum}(x^2-1)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(-2x^2+2)\sqrt{-x^2+1}}{3}\right)}{4\sqrt{\pi}\sqrt{-\operatorname{signum}(x^2-1)}}-\frac{3i\sqrt{\operatorname{signum}(}{}$

input
```
int((1-x)^3*(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(24*x^4-90*x^3+112*x^2-15*x-136)*(x^2-1)^(1/2)-7/8*ln(x+(x^2-1)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int (1-x)^3\sqrt{-1+x^2}\,dx = -\frac{1}{120}\left(24\,x^4-90\,x^3+112\,x^2-15\,x-136\right)\sqrt{x^2-1} + \frac{7}{8}\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((1-x)^3*(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(x^2 - 1) + 7/8*log(-x
 + sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int (1-x)^3\sqrt{-1+x^2}\,dx = -\frac{x^4\sqrt{x^2-1}}{5}+\frac{3x^3\sqrt{x^2-1}}{4}-\frac{14x^2\sqrt{x^2-1}}{15}+\frac{x\sqrt{x^2-1}}{8}+\frac{17\sqrt{x^2-1}}{15}-\frac{7\log\left(x+\sqrt{x^2-1}\right)}{8}$$

input
```
integrate((1-x)**3*(x**2-1)**(1/2),x)
```

output
```
-x**4*sqrt(x**2 - 1)/5 + 3*x**3*sqrt(x**2 - 1)/4 - 14*x**2*sqrt(x**2 - 1)/
15 + x*sqrt(x**2 - 1)/8 + 17*sqrt(x**2 - 1)/15 - 7*log(x + sqrt(x**2 - 1))
/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int (1-x)^3\sqrt{-1+x^2}\,dx = -\frac{1}{5}\left(x^2-1\right)^{\frac{3}{2}}x^2+\frac{3}{4}\left(x^2-1\right)^{\frac{3}{2}}x-\frac{17}{15}\left(x^2-1\right)^{\frac{3}{2}}+\frac{7}{8}\sqrt{x^2-1}x-\frac{7}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((1-x)^3*(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*(x^2 - 1)^(3/2)*x^2 + 3/4*(x^2 - 1)^(3/2)*x - 17/15*(x^2 - 1)^(3/2) +
 7/8*sqrt(x^2 - 1)*x - 7/8*log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int (1-x)^3\sqrt{-1+x^2}\,dx = -\frac{1}{120}\left((2\left(3\left(4\,x-15\right)x+56\right)x-15\right)x-136\right)\sqrt{x^2-1} + \frac{7}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((1-x)^3*(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-1/120*((2*(3*(4*x - 15)*x + 56)*x - 15)*x - 136)*sqrt(x^2 - 1) + 7/8*log(
abs(-x + sqrt(x^2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^3\sqrt{-1+x^2}\,dx = \int -\sqrt{x^2-1}\,(x-1)^3\,dx$$

input
```
int(-(x^2 - 1)^(1/2)*(x - 1)^3,x)
```

output
```
int(-(x^2 - 1)^(1/2)*(x - 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int (1-x)^3\sqrt{-1+x^2}\,dx = -\frac{\sqrt{x^2-1}\,x^4}{5} + \frac{3\sqrt{x^2-1}\,x^3}{4} - \frac{14\sqrt{x^2-1}\,x^2}{15} + \frac{\sqrt{x^2-1}\,x}{8} + \frac{17\sqrt{x^2-1}}{15} - \frac{7\log\left(\sqrt{x^2-1}+x\right)}{8}$$

input
```
int((1-x)^3*(x^2-1)^(1/2),x)
```

output
```
( - 24*sqrt(x**2 - 1)*x**4 + 90*sqrt(x**2 - 1)*x**3 - 112*sqrt(x**2 - 1)*x
**2 + 15*sqrt(x**2 - 1)*x + 136*sqrt(x**2 - 1) - 105*log(sqrt(x**2 - 1) +
x))/120
```

3.100 $\int (1-x)^2\sqrt{-1+x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int (1-x)^2\sqrt{-1+x^2}\,dx = \frac{5}{8}x\sqrt{-1+x^2}-\frac{1}{12}(8-3x)\left(-1+x^2\right)^{3/2} -\frac{5}{8}\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
5/8*x*(x^2-1)^(1/2)-1/12*(8-3*x)*(x^2-1)^(3/2)-5/8*arctanh(x/(x^2-1)^(1/2)
)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int (1-x)^2\sqrt{-1+x^2}\,dx = \frac{1}{24}\sqrt{-1+x^2}\left(16+9x-16x^2+6x^3\right)-\frac{5}{4}\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)^2*Sqrt[-1 + x^2],x]
```

output
```
(Sqrt[-1 + x^2]*(16 + 9*x - 16*x^2 + 6*x^3))/24 - (5*ArcTanh[Sqrt[-1 + x^2
]/(-1 + x)])/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {469, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)^2\sqrt{x^2-1}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{4}\int (1-x)\sqrt{x^2-1}dx-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{455}$$

$$\frac{5}{4}\left(\int \sqrt{x^2-1}dx-\frac{1}{3}\left(x^2-1\right)^{3/2}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{x^2-1}}dx-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{4}\left(-\frac{1}{2}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{4}\left(-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}\right)-\frac{1}{4}(1-x)\left(x^2-1\right)^{3/2}$$

input
```
Int[(1 - x)^2*Sqrt[-1 + x^2],x]
```

output
```
-1/4*((1 - x)*(-1 + x^2)^(3/2)) + (5*((x*Sqrt[-1 + x^2])/2 - (-1 + x^2)^(3
/2)/3 - ArcTanh[x/Sqrt[-1 + x^2]]/2))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result
risch	$\frac{(6x^3-16x^2+9x+16)\sqrt{x^2-1}}{24}-\frac{5\ln\left(x+\sqrt{x^2-1}\right)}{8}$
trager	$\left(\frac{1}{4}x^3-\frac{2}{3}x^2+\frac{3}{8}x+\frac{2}{3}\right)\sqrt{x^2-1}+\frac{5\ln\left(x-\sqrt{x^2-1}\right)}{8}$
default	$\frac{5x\sqrt{x^2-1}}{8}-\frac{5\ln\left(x+\sqrt{x^2-1}\right)}{8}+\frac{x(x^2-1)^{\frac{3}{2}}}{4}-\frac{2(x^2-1)^{\frac{3}{2}}}{3}$
meijerg	$\frac{i\sqrt{\operatorname{signum}(x^2-1)}\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}(x^2-1)}}-\frac{\sqrt{\operatorname{signum}(x^2-1)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(-2x^2+2\right)\sqrt{-x^2+1}}{3}\right)}{2\sqrt{\pi}\,\sqrt{-\operatorname{signum}(x^2-1)}}-\frac{i\sqrt{\operatorname{signum}(x^2}{}$

input
```
int((1-x)^2*(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*x^3-16*x^2+9*x+16)*(x^2-1)^(1/2)-5/8*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int (1-x)^2\sqrt{-1+x^2}\,dx=\frac{1}{24}\left(6\,x^3-16\,x^2+9\,x+16\right)\sqrt{x^2-1}+\frac{5}{8}\,\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((1-x)^2*(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(x^2 - 1) + 5/8*log(-x + sqrt(x^2 - 1
))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int (1-x)^2\sqrt{-1+x^2}\,dx=\sqrt{x^2-1}\left(\frac{x^3}{4}-\frac{2x^2}{3}+\frac{3x}{8}+\frac{2}{3}\right)-\frac{5\log\left(2x+2\sqrt{x^2-1}\right)}{8}$$

input
```
integrate((1-x)**2*(x**2-1)**(1/2),x)
```

output
```
sqrt(x**2 - 1)*(x**3/4 - 2*x**2/3 + 3*x/8 + 2/3) - 5*log(2*x + 2*sqrt(x**2
 - 1))/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int (1-x)^2\sqrt{-1+x^2}\,dx = \frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x - \frac{2}{3}\left(x^2-1\right)^{\frac{3}{2}} + \frac{5}{8}\sqrt{x^2-1}x - \frac{5}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((1-x)^2*(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 - 1)^(3/2)*x - 2/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - 5/8*lo
g(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int (1-x)^2\sqrt{-1+x^2}\,dx = \frac{1}{24}\left((2\,(3\,x-8)x+9)x+16\right)\sqrt{x^2-1} + \frac{5}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((1-x)^2*(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x - 8)*x + 9)*x + 16)*sqrt(x^2 - 1) + 5/8*log(abs(-x + sqrt(x^
2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int (1-x)^2\sqrt{-1+x^2}\,dx=\int \sqrt{x^2-1}\,(x-1)^2\,dx$$

input
```
int((x^2 - 1)^(1/2)*(x - 1)^2,x)
```

output
```
int((x^2 - 1)^(1/2)*(x - 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int (1-x)^2\sqrt{-1+x^2}\,dx=\frac{\sqrt{x^2-1}\,x^3}{4}-\frac{2\sqrt{x^2-1}\,x^2}{3}+\frac{3\sqrt{x^2-1}\,x}{8}+\frac{2\sqrt{x^2-1}}{3}-\frac{5\log\left(\sqrt{x^2-1}+x\right)}{8}$$

input
```
int((1-x)^2*(x^2-1)^(1/2),x)
```

output
```
(6*sqrt(x**2 - 1)*x**3 - 16*sqrt(x**2 - 1)*x**2 + 9*sqrt(x**2 - 1)*x + 16*
sqrt(x**2 - 1) - 15*log(sqrt(x**2 - 1) + x))/24
```

3.101 $\int (1-x)\sqrt{-1+x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int (1-x)\sqrt{-1+x^2}\,dx = \frac{1}{2}x\sqrt{-1+x^2}-\frac{1}{3}\left(-1+x^2\right)^{3/2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
1/2*x*(x^2-1)^(1/2)-1/3*(x^2-1)^(3/2)-1/2*arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int (1-x)\sqrt{-1+x^2}\,dx = -\frac{1}{6}\sqrt{-1+x^2}\left(-2-3x+2x^2\right)-\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[(1 - x)*Sqrt[-1 + x^2],x]
```

output
```
-1/6*(Sqrt[-1 + x^2]*(-2 - 3*x + 2*x^2)) - ArcTanh[Sqrt[-1 + x^2]/(-1 + x)
]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-x)\sqrt{x^2-1}\,dx$$

$$\Big\downarrow \ 455$$

$$\int \sqrt{x^2-1}dx-\frac{1}{3}\left(x^2-1\right)^{3/2}$$

$$\Big\downarrow \ 211$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{x^2-1}}dx-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}$$

$$\Big\downarrow \ 224$$

$$-\frac{1}{2}\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}$$

$$\Big\downarrow \ 219$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)-\frac{1}{3}\left(x^2-1\right)^{3/2}+\frac{1}{2}x\sqrt{x^2-1}$$

input
```
Int[(1 - x)*Sqrt[-1 + x^2],x]
```

output
```
(x*Sqrt[-1 + x^2])/2 - (-1 + x^2)^(3/2)/3 - ArcTanh[x/Sqrt[-1 + x^2]]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\left(x^{2}-1\right)^{\frac{3}{2}}}{3}+\frac{x\sqrt{x^{2}-1}}{2}-\frac{\ln\left(x+\sqrt{x^{2}-1}\right)}{2}$	33
risch	$-\frac{\left(2x^{2}-3x-2\right)\sqrt{x^{2}-1}}{6}-\frac{\ln\left(x+\sqrt{x^{2}-1}\right)}{2}$	33
trager	$\left(-\frac{1}{3}x^{2}+\frac{1}{2}x+\frac{1}{3}\right)\sqrt{x^{2}-1}+\frac{\ln\left(x-\sqrt{x^{2}-1}\right)}{2}$	34
meijerg	$\frac{i\sqrt{\operatorname{signum}\left(x^{2}-1\right)}\left(-2i\sqrt{\pi}\,x\sqrt{-x^{2}+1}-2i\sqrt{\pi}\,\arcsin\left(x\right)\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}\left(x^{2}-1\right)}}-\frac{\sqrt{\operatorname{signum}\left(x^{2}-1\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(-2x^{2}+2\right)\sqrt{-x^{2}+1}}{3}\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}\left(x^{2}-1\right)}}$	101

input
```
int((1-x)*(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x^2-1)^(3/2)+1/2*x*(x^2-1)^(1/2)-1/2*ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (1-x)\sqrt{-1+x^2}\,dx = -\frac{1}{6}\left(2\,x^2-3\,x-2\right)\sqrt{x^2-1}+\frac{1}{2}\,\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((1-x)*(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*x^2 - 3*x - 2)*sqrt(x^2 - 1) + 1/2*log(-x + sqrt(x^2 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int (1-x)\sqrt{-1+x^2}\,dx = -\frac{x^2\sqrt{x^2-1}}{3}+\frac{x\sqrt{x^2-1}}{2}+\frac{\sqrt{x^2-1}}{3}-\frac{\log\left(x+\sqrt{x^2-1}\right)}{2}$$

input
```
integrate((1-x)*(x**2-1)**(1/2),x)
```

output
```
-x**2*sqrt(x**2 - 1)/3 + x*sqrt(x**2 - 1)/2 + sqrt(x**2 - 1)/3 - log(x + s
qrt(x**2 - 1))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int (1-x)\sqrt{-1+x^2}\,dx = -\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{1}{2}\,\sqrt{x^2-1}x-\frac{1}{2}\,\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((1-x)*(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*(x^2 - 1)^(3/2) + 1/2*sqrt(x^2 - 1)*x - 1/2*log(2*x + 2*sqrt(x^2 - 1)
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (1-x)\sqrt{-1+x^2}\,dx = -\frac{1}{6}\left((2\,x-3)x-2\right)\sqrt{x^2-1}+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((1-x)*(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
-1/6*((2*x - 3)*x - 2)*sqrt(x^2 - 1) + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int (1-x)\sqrt{-1+x^2}\,dx = \frac{x\,\sqrt{x^2-1}}{2}-\frac{\ln\left(x+\sqrt{x^2-1}\right)}{2}-\frac{\left(x^2-1\right)^{3/2}}{3}$$

input
```
int(-(x^2 - 1)^(1/2)*(x - 1),x)
```

output
```
(x*(x^2 - 1)^(1/2))/2 - log(x + (x^2 - 1)^(1/2))/2 - (x^2 - 1)^(3/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int (1-x)\sqrt{-1+x^2}\,dx = -\frac{\sqrt{x^2-1}\,x^2}{3}+\frac{\sqrt{x^2-1}\,x}{2}+\frac{\sqrt{x^2-1}}{3}-\frac{\log\left(\sqrt{x^2-1}+x\right)}{2}$$

input
```
int((1-x)*(x^2-1)^(1/2),x)
```

output
```
( - 2*sqrt(x**2 - 1)*x**2 + 3*sqrt(x**2 - 1)*x + 2*sqrt(x**2 - 1) - 3*log(
sqrt(x**2 - 1) + x))/6
```

3.102 $\int \frac{\sqrt{-1+x^2}}{1-x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{\sqrt{-1+x^2}}{1-x} \, dx = -\sqrt{-1+x^2} - \text{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
-(x^2-1)^(1/2)-arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{-1+x^2}}{1-x} \, dx = -\sqrt{-1+x^2} - 2\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 - x),x]
```

output
```
-Sqrt[-1 + x^2] - 2*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {466, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{1-x}dx$$

$$\Big\downarrow \text{466}$$

$$-\int \frac{1}{\sqrt{x^2-1}}dx-\sqrt{x^2-1}$$

$$\Big\downarrow \text{224}$$

$$-\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}-\sqrt{x^2-1}$$

$$\Big\downarrow \text{219}$$

$$-\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}$$

input
```
Int[Sqrt[-1 + x^2]/(1 - x),x]
```

output
```
-Sqrt[-1 + x^2] - ArcTanh[x/Sqrt[-1 + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
trager	$-\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)$	23
risch	$-\sqrt{x^{2}-1}-\ln\left(x+\sqrt{x^{2}-1}\right)$	23
default	$-\sqrt{\left(x-1\right)^{2}+2x-2}-\ln\left(x+\sqrt{\left(x-1\right)^{2}+2x-2}\right)$	33

input
```
int((x^2-1)^(1/2)/(1-x),x,method=_RETURNVERBOSE)
```

output
```
-(x^2-1)^(1/2)-ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{-1+x^2}}{1-x}\,dx=-\sqrt{x^2-1}+\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1-x),x, algorithm="fricas")
```

output
```
-sqrt(x^2 - 1) + log(-x + sqrt(x^2 - 1))
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{1-x}\,dx = -\int \frac{\sqrt{x^2-1}}{x-1}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1-x),x)
```

output
```
-Integral(sqrt(x**2 - 1)/(x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+x^2}}{1-x}\,dx = -\sqrt{x^2-1} - \log\left(2\,x + 2\,\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1-x),x, algorithm="maxima")
```

output
```
-sqrt(x^2 - 1) - log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{-1+x^2}}{1-x}\,dx = -\sqrt{x^2-1} + \log\left(\left|-x + \sqrt{x^2-1}\right|\right)$$

input
```
integrate((x^2-1)^(1/2)/(1-x),x, algorithm="giac")
```

output
```
-sqrt(x^2 - 1) + log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{1-x}\,dx = -\int \frac{\sqrt{x^2-1}}{x-1}\,dx$$

input
```
int(-(x^2 - 1)^(1/2)/(x - 1),x)
```

output
```
-int((x^2 - 1)^(1/2)/(x - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+x^2}}{1-x}\,dx = -\sqrt{x^2-1} - \log\left(\sqrt{x^2-1}+x\right)$$

input
```
int((x^2-1)^(1/2)/(1-x),x)
```

output
```
- (sqrt(x**2 - 1) + log(sqrt(x**2 - 1) + x))
```

3.103 $\int \frac{\sqrt{-1+x^2}}{(1-x)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2} \, dx = \frac{2\sqrt{-1+x^2}}{1-x} + \operatorname{arctanh}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

output
```
2*(x^2-1)^(1/2)/(1-x)+arctanh(x/(x^2-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2} \, dx = -\frac{2\sqrt{-1+x^2}}{-1+x} + 2\operatorname{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 - x)^2,x]
```

output
```
(-2*Sqrt[-1 + x^2])/(-1 + x) + 2*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {463, 25, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(1-x)^2}dx$$

$$\Big\downarrow \ 463$$

$$\frac{2\sqrt{x^2-1}}{1-x}-\int -\frac{1}{\sqrt{x^2-1}}dx$$

$$\Big\downarrow \ 25$$

$$\int \frac{1}{\sqrt{x^2-1}}dx+\frac{2\sqrt{x^2-1}}{1-x}$$

$$\Big\downarrow \ 224$$

$$\int \frac{1}{1-\frac{x^2}{x^2-1}}d\frac{x}{\sqrt{x^2-1}}+\frac{2\sqrt{x^2-1}}{1-x}$$

$$\Big\downarrow \ 219$$

$$\text{arctanh}\left(\frac{x}{\sqrt{x^2-1}}\right)+\frac{2\sqrt{x^2-1}}{1-x}$$

input
```
Int[Sqrt[-1 + x^2]/(1 - x)^2,x]
```

output
```
(2*Sqrt[-1 + x^2])/(1 - x) + ArcTanh[x/Sqrt[-1 + x^2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 463
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(-c)^(-n - 2))*d^(2*n + 3)*(Sqrt[a + b*x^2]/(2^(n + 1)*b^(n + 2)*(c + d*x
))), x] - Simp[d^(2*n + 2)/b^(n + 1)   Int[(1/Sqrt[a + b*x^2])*ExpandToSum[
(2^(-n - 1)*(-c)^(-n - 1) - (-c + d*x)^(-n - 1))/(c + d*x), x], x], x] /; F
reeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[n, 0] && EqQ[n + p,
-3/2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{2(x+1)}{\sqrt{x^2-1}}+\ln\left(x+\sqrt{x^2-1}\right)$	24
trager	$-\frac{2\sqrt{x^2-1}}{x-1}-\ln\left(x-\sqrt{x^2-1}\right)$	30
default	$-\frac{\left((x-1)^2+2x-2\right)^{\frac{3}{2}}}{(x-1)^2}+\sqrt{(x-1)^2+2x-2}+\ln\left(x+\sqrt{(x-1)^2+2x-2}\right)$	48

input
```
int((x^2-1)^(1/2)/(1-x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(x+1)/(x^2-1)^(1/2)+ln(x+(x^2-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2}\,dx = -\frac{(x-1)\log\left(-x+\sqrt{x^2-1}\right)+2\,x+2\,\sqrt{x^2-1}-2}{x-1}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^2,x, algorithm="fricas")
```

output
```
-((x - 1)*log(-x + sqrt(x^2 - 1)) + 2*x + 2*sqrt(x^2 - 1) - 2)/(x - 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2}\,dx = \int \frac{\sqrt{(x-1)\,(x+1)}}{\left(x-1\right)^2}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1-x)**2,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x - 1)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2}\,dx = -\frac{2\,\sqrt{x^2-1}}{x-1}+\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^2,x, algorithm="maxima")
```

output
```
-2*sqrt(x^2 - 1)/(x - 1) + log(2*x + 2*sqrt(x^2 - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2}\,dx = \log\left(\sqrt{\frac{2}{x-1}+1}+1\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) - \log\left(\left|\sqrt{\frac{2}{x-1}+1}-1\right|\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) - 2\sqrt{\frac{2}{x-1}+1}\operatorname{sgn}\left(\frac{1}{x-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^2,x, algorithm="giac")
```

output
```
log(sqrt(2/(x - 1) + 1) + 1)*sgn(1/(x - 1)) - log(abs(sqrt(2/(x - 1) + 1)
- 1))*sgn(1/(x - 1)) - 2*sqrt(2/(x - 1) + 1)*sgn(1/(x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2}\,dx = \int \frac{\sqrt{x^2-1}}{(x-1)^2}\,dx$$

input
```
int((x^2 - 1)^(1/2)/(x - 1)^2,x)
```

output
```
int((x^2 - 1)^(1/2)/(x - 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^2}\,dx = \frac{-2\sqrt{x^2-1}+\log\left(\sqrt{x^2-1}+x\right)x-\log\left(\sqrt{x^2-1}+x\right)-2x+2}{x-1}$$

input
```
int((x^2-1)^(1/2)/(1-x)^2,x)
```

output
```
( - 2*sqrt(x**2 - 1) + log(sqrt(x**2 - 1) + x)*x - log(sqrt(x**2 - 1) + x)
 - 2*x + 2)/(x - 1)
```

3.104 $\int \frac{\sqrt{-1+x^2}}{(1-x)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3} dx = -\frac{(-1+x^2)^{3/2}}{3(1-x)^3}$$

output
```
-1/3*(x^2-1)^(3/2)/(1-x)^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3} dx = -\frac{(-1-x)\sqrt{-1+x^2}}{3(-1+x)^2}$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 - x)^3,x]
```

output
```
-1/3*((-1 - x)*Sqrt[-1 + x^2])/(-1 + x)^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(1-x)^3} \, dx$$

$$\Big\downarrow \text{ 460}$$

$$-\frac{\left(x^2-1\right)^{3/2}}{3(1-x)^3}$$

input
```
Int[Sqrt[-1 + x^2]/(1 - x)^3,x]
```

output
```
-1/3*(-1 + x^2)^(3/2)/(1 - x)^3
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{(x+1)\sqrt{x^2-1}}{3(x-1)^2}$	18
trager	$\frac{(x+1)\sqrt{x^2-1}}{3(x-1)^2}$	18
default	$\frac{\left((x-1)^2+2x-2\right)^{\frac{3}{2}}}{3(x-1)^3}$	20
risch	$\frac{x^2+2x+1}{3(x-1)\sqrt{x^2-1}}$	23
orering	$-\frac{(x-1)(x+1)\sqrt{x^2-1}}{3(1-x)^3}$	23

input
```
int((x^2-1)^(1/2)/(1-x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(x+1)*(x^2-1)^(1/2)/(x-1)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3}\,dx = \frac{x^2+\sqrt{x^2-1}(x+1)-2\,x+1}{3\,(x^2-2\,x+1)}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^3,x, algorithm="fricas")
```

output
```
1/3*(x^2 + sqrt(x^2 - 1)*(x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3}\,dx = -\int \frac{\sqrt{x^2-1}}{x^3-3x^2+3x-1}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1-x)**3,x)
```

output
```
-Integral(sqrt(x**2 - 1)/(x**3 - 3*x**2 + 3*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3}\,dx = \frac{2\sqrt{x^2-1}}{3\,(x^2-2\,x+1)} + \frac{\sqrt{x^2-1}}{3\,(x-1)}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^3,x, algorithm="maxima")
```

output
```
2/3*sqrt(x^2 - 1)/(x^2 - 2*x + 1) + 1/3*sqrt(x^2 - 1)/(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3}\,dx = -\frac{2\left(3\left(x-\sqrt{x^2-1}\right)^2+1\right)}{3\left(x-\sqrt{x^2-1}-1\right)^3}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^3,x, algorithm="giac")
```

output
```
-2/3*(3*(x - sqrt(x^2 - 1))^2 + 1)/(x - sqrt(x^2 - 1) - 1)^3
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3}\,dx = \frac{\sqrt{x^2-1}\,(x+1)}{3\,(x-1)^2}$$

input
```
int(-(x^2 - 1)^(1/2)/(x - 1)^3,x)
```

output
```
((x^2 - 1)^(1/2)*(x + 1))/(3*(x - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^3}\,dx = \frac{9\sqrt{x^2-1}\,x+9\sqrt{x^2-1}+8x^2-16x+8}{27x^2-54x+27}$$

input
```
int((x^2-1)^(1/2)/(1-x)^3,x)
```

output
```
(9*sqrt(x**2 - 1)*x + 9*sqrt(x**2 - 1) + 8*x**2 - 16*x + 8)/(27*(x**2 - 2*
x + 1))
```

3.105 $\int \frac{\sqrt{-1+x^2}}{(1-x)^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4} \, dx = -\frac{(-1+x^2)^{3/2}}{5(1-x)^4} - \frac{(-1+x^2)^{3/2}}{15(1-x)^3}$$

output
```
-1/5*(x^2-1)^(3/2)/(1-x)^4-1/15*(x^2-1)^(3/2)/(1-x)^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4} \, dx = \frac{\sqrt{-1+x^2}(-4-3x+x^2)}{15(-1+x)^3}$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 - x)^4,x]
```

output
```
(Sqrt[-1 + x^2]*(-4 - 3*x + x^2))/(15*(-1 + x)^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(1-x)^4} \, dx$$

$$\Big\downarrow \text{461}$$

$$\frac{1}{5}\int \frac{\sqrt{x^2-1}}{(1-x)^3}dx-\frac{\left(x^2-1\right)^{3/2}}{5(1-x)^4}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\left(x^2-1\right)^{3/2}}{15(1-x)^3}-\frac{\left(x^2-1\right)^{3/2}}{5(1-x)^4}$$

input
```
Int[Sqrt[-1 + x^2]/(1 - x)^4,x]
```

output
```
-1/5*(-1 + x^2)^(3/2)/(1 - x)^4 - (-1 + x^2)^(3/2)/(15*(1 - x)^3)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{(x+1)(x-4)\sqrt{x^2-1}}{15(x-1)^3}$	21
trager	$\frac{(x^2-3x-4)\sqrt{x^2-1}}{15(x-1)^3}$	23
orering	$\frac{(x-4)(x-1)(x+1)\sqrt{x^2-1}}{15(1-x)^4}$	26
risch	$\frac{x^3-2x^2-7x-4}{15(x-1)^2\sqrt{x^2-1}}$	28
default	$-\frac{\left((x-1)^2+2x-2\right)^{\frac{3}{2}}}{5(x-1)^4}+\frac{\left((x-1)^2+2x-2\right)^{\frac{3}{2}}}{15(x-1)^3}$	40

input
```
int((x^2-1)^(1/2)/(1-x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*(x+1)*(x-4)*(x^2-1)^(1/2)/(x-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4}\,dx = \frac{x^3-3\,x^2+(x^2-3\,x-4)\sqrt{x^2-1}+3\,x-1}{15\,(x^3-3\,x^2+3\,x-1)}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^4,x, algorithm="fricas")
```

output
```
1/15*(x^3 - 3*x^2 + (x^2 - 3*x - 4)*sqrt(x^2 - 1) + 3*x - 1)/(x^3 - 3*x^2
+ 3*x - 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4}\,dx = \int \frac{\sqrt{(x-1)(x+1)}}{(x-1)^4}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1-x)**4,x)
```

output
```
Integral(sqrt((x - 1)*(x + 1))/(x - 1)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4}\,dx = -\frac{2\sqrt{x^2-1}}{5(x^3-3x^2+3x-1)} - \frac{\sqrt{x^2-1}}{15(x^2-2x+1)} + \frac{\sqrt{x^2-1}}{15(x-1)}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^4,x, algorithm="maxima")
```

output
```
-2/5*sqrt(x^2 - 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(x^2 - 1)/(x^2 - 2*x
 + 1) + 1/15*sqrt(x^2 - 1)/(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4}\,dx = \frac{2\left(15\left(x-\sqrt{x^2-1}\right)^3 + 5\left(x-\sqrt{x^2-1}\right)^2 + 5x - 5\sqrt{x^2-1} - 1\right)}{15\left(x-\sqrt{x^2-1}-1\right)^5}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^4,x, algorithm="giac")
```

output
```
2/15*(15*(x - sqrt(x^2 - 1))^3 + 5*(x - sqrt(x^2 - 1))^2 + 5*x - 5*sqrt(x^
2 - 1) - 1)/(x - sqrt(x^2 - 1) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4}\,dx = -\frac{\sqrt{x^2-1}\,(-x^2+3\,x+4)}{15\,(x-1)^3}$$

input
```
int((x^2 - 1)^(1/2)/(x - 1)^4,x)
```

output
```
-((x^2 - 1)^(1/2)*(3*x - x^2 + 4))/(15*(x - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^4}\,dx = \frac{\sqrt{x^2-1}\,x^2-3\sqrt{x^2-1}\,x-4\sqrt{x^2-1}-x^3+3x^2-3x+1}{15x^3-45x^2+45x-15}$$

input
```
int((x^2-1)^(1/2)/(1-x)^4,x)
```

output
```
(sqrt(x**2 - 1)*x**2 - 3*sqrt(x**2 - 1)*x - 4*sqrt(x**2 - 1) - x**3 + 3*x*
*2 - 3*x + 1)/(15*(x**3 - 3*x**2 + 3*x - 1))
```

3.106 $\int \frac{\sqrt{-1+x^2}}{(1-x)^5} dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5} dx = -\frac{(-1+x^2)^{3/2}}{7(1-x)^5} - \frac{2(-1+x^2)^{3/2}}{35(1-x)^4} - \frac{2(-1+x^2)^{3/2}}{105(1-x)^3}$$

output
```
-1/7*(x^2-1)^(3/2)/(1-x)^5-2/35*(x^2-1)^(3/2)/(1-x)^4-2/105*(x^2-1)^(3/2)/
(1-x)^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5} dx = -\frac{\sqrt{-1+x^2}(-23-13x+8x^2-2x^3)}{105(-1+x)^4}$$

input
```
Integrate[Sqrt[-1 + x^2]/(1 - x)^5,x]
```

output
```
-1/105*(Sqrt[-1 + x^2]*(-23 - 13*x + 8*x^2 - 2*x^3))/(-1 + x)^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-1}}{(1-x)^5} \, dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2}{7} \int \frac{\sqrt{x^2-1}}{(1-x)^4}dx-\frac{\left(x^2-1\right)^{3/2}}{7(1-x)^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{2}{7} \left(\frac{1}{5} \int \frac{\sqrt{x^2-1}}{(1-x)^3}dx-\frac{\left(x^2-1\right)^{3/2}}{5(1-x)^4}\right)-\frac{\left(x^2-1\right)^{3/2}}{7(1-x)^5}$$

$$\Big\downarrow \text{460}$$

$$\frac{2}{7} \left(-\frac{\left(x^2-1\right)^{3/2}}{15(1-x)^3}-\frac{\left(x^2-1\right)^{3/2}}{5(1-x)^4}\right)-\frac{\left(x^2-1\right)^{3/2}}{7(1-x)^5}$$

input
```
Int[Sqrt[-1 + x^2]/(1 - x)^5,x]
```

output
```
-1/7*(-1 + x^2)^(3/2)/(1 - x)^5 + (2*(-1/5*(-1 + x^2)^(3/2)/(1 - x)^4 - (-
1 + x^2)^(3/2)/(15*(1 - x)^3)))/7
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{(x+1)\left(2x^2-10x+23\right)\sqrt{x^2-1}}{105(x-1)^4}$	28
trager	$\frac{\left(2x^3-8x^2+13x+23\right)\sqrt{x^2-1}}{105(x-1)^4}$	30
orering	$-\frac{\left(2x^2-10x+23\right)(x-1)(x+1)\sqrt{x^2-1}}{105(1-x)^5}$	33
risch	$\frac{2x^4-6x^3+5x^2+36x+23}{105(x-1)^3\sqrt{x^2-1}}$	35
default	$\frac{\left((x-1)^2+2x-2\right)^{\frac{3}{2}}}{7(x-1)^5}-\frac{2\left((x-1)^2+2x-2\right)^{\frac{3}{2}}}{35(x-1)^4}+\frac{2\left((x-1)^2+2x-2\right)^{\frac{3}{2}}}{105(x-1)^3}$	59

input
```
int((x^2-1)^(1/2)/(1-x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/105*(x+1)*(2*x^2-10*x+23)*(x^2-1)^(1/2)/(x-1)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5}\,dx = \frac{2\,x^4 - 8\,x^3 + 12\,x^2 + (2\,x^3 - 8\,x^2 + 13\,x + 23)\sqrt{x^2-1} - 8\,x + 2}{105\,(x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1)}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^5,x, algorithm="fricas")
```

output
```
1/105*(2*x^4 - 8*x^3 + 12*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(x^2 - 1)
- 8*x + 2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5}\,dx = -\int \frac{\sqrt{x^2-1}}{x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1}\,dx$$

input
```
integrate((x**2-1)**(1/2)/(1-x)**5,x)
```

output
```
-Integral(sqrt(x**2 - 1)/(x**5 - 5*x**4 + 10*x**3 - 10*x**2 + 5*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5}\,dx = \frac{2\sqrt{x^2-1}}{7\,(x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1)} + \frac{\sqrt{x^2-1}}{35\,(x^3 - 3\,x^2 + 3\,x - 1)} - \frac{2\sqrt{x^2-1}}{105\,(x^2 - 2\,x + 1)} + \frac{2\sqrt{x^2-1}}{105\,(x-1)}$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^5,x, algorithm="maxima")
```

output
```
2/7*sqrt(x^2 - 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(x^2 - 1)/(x^
3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(x^2 - 1)/(x^2 - 2*x + 1) + 2/105*sqrt(x^
2 - 1)/(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5}\,dx = \frac{1}{140}\left(5\left(\frac{2}{x-1}+1\right)^{\frac{7}{2}} - 21\left(\frac{2}{x-1}+1\right)^{\frac{5}{2}} + 35\left(\frac{2}{x-1}+1\right)^{\frac{3}{2}} - 35\sqrt{\frac{2}{x-1}+1}\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) + \frac{1}{60}\left(3\left(\frac{2}{x-1}+1\right)^{\frac{5}{2}} - 10\left(\frac{2}{x-1}+1\right)^{\frac{3}{2}} + 15\sqrt{\frac{2}{x-1}+1}\right)\operatorname{sgn}\left(\frac{1}{x-1}\right) - \frac{2}{105}\operatorname{sgn}\left(\frac{1}{x-1}\right)$$

input
```
integrate((x^2-1)^(1/2)/(1-x)^5,x, algorithm="giac")
```

output
```
1/140*(5*(2/(x - 1) + 1)^(7/2) - 21*(2/(x - 1) + 1)^(5/2) + 35*(2/(x - 1)
+ 1)^(3/2) - 35*sqrt(2/(x - 1) + 1))*sgn(1/(x - 1)) + 1/60*(3*(2/(x - 1) +
 1)^(5/2) - 10*(2/(x - 1) + 1)^(3/2) + 15*sqrt(2/(x - 1) + 1))*sgn(1/(x -
1)) - 2/105*sgn(1/(x - 1))
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5}\,dx = \frac{2\sqrt{x^2-1}}{105\,(x-1)} - \frac{2\sqrt{x^2-1}}{105\,(x-1)^2} + \frac{\sqrt{x^2-1}}{35\,(x-1)^3} + \frac{2\sqrt{x^2-1}}{7\,(x-1)^4}$$

input
```
int(-(x^2 - 1)^(1/2)/(x - 1)^5,x)
```

output
```
(2*(x^2 - 1)^(1/2))/(105*(x - 1)) - (2*(x^2 - 1)^(1/2))/(105*(x - 1)^2) +
(x^2 - 1)^(1/2)/(35*(x - 1)^3) + (2*(x^2 - 1)^(1/2))/(7*(x - 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{-1+x^2}}{(1-x)^5}\,dx = \frac{2\sqrt{x^2-1}\,x^3-8\sqrt{x^2-1}\,x^2+13\sqrt{x^2-1}\,x+23\sqrt{x^2-1}-2x^4+8x^3-12x^2+8x-2}{105x^4-420x^3+630x^2-420x+105}$$

input
```
int((x^2-1)^(1/2)/(1-x)^5,x)
```

output
```
(2*sqrt(x**2 - 1)*x**3 - 8*sqrt(x**2 - 1)*x**2 + 13*sqrt(x**2 - 1)*x + 23*
sqrt(x**2 - 1) - 2*x**4 + 8*x**3 - 12*x**2 + 8*x - 2)/(105*(x**4 - 4*x**3
+ 6*x**2 - 4*x + 1))
```

3.107 $\int \frac{\sqrt{a^2-b^2x^2}}{a-bx} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{\sqrt{a^2-b^2x^2}}{a-bx} \, dx = -\frac{\sqrt{a^2-b^2x^2}}{b}+\frac{a\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}$$

output
```
-(-b^2*x^2+a^2)^(1/2)/b+a*arctan(b*x/(-b^2*x^2+a^2)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a^2-b^2x^2}}{a-bx} \, dx = -\frac{\sqrt{a^2-b^2x^2}}{b}-\frac{a\log\left(-\sqrt{-b^2}x+\sqrt{a^2-b^2x^2}\right)}{\sqrt{-b^2}}$$

input
```
Integrate[Sqrt[a^2 - b^2*x^2]/(a - b*x),x]
```

output
```
-(Sqrt[a^2 - b^2*x^2]/b) - (a*Log[-(Sqrt[-b^2]*x) + Sqrt[a^2 - b^2*x^2]])/
Sqrt[-b^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {466, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2-b^2x^2}}{a-bx}dx$$

$$\Big\downarrow \text{466}$$

$$a\int \frac{1}{\sqrt{a^2-b^2x^2}}dx-\frac{\sqrt{a^2-b^2x^2}}{b}$$

$$\Big\downarrow \text{224}$$

$$a\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{a\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)}{b}-\frac{\sqrt{a^2-b^2x^2}}{b}$$

input
```
Int[Sqrt[a^2 - b^2*x^2]/(a - b*x),x]
```

output
```
-(Sqrt[a^2 - b^2*x^2]/b) + (a*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{\sqrt{-b^2x^2+a^2}}{b}+\frac{a\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+a^2}}\right)}{\sqrt{b^2}}$	50
default	$-\frac{\sqrt{-b^2\left(x-\frac{a}{b}\right)^2-2ab\left(x-\frac{a}{b}\right)}-\frac{ab\arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2\left(x-\frac{a}{b}\right)^2-2ab\left(x-\frac{a}{b}\right)}}\right)}{\sqrt{b^2}}}{b}$	84

input
```
int((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(-b^2*x^2+a^2)^(1/2)/b+a/(b^2)^(1/2)*arctan((b^2)^(1/2)*x/(-b^2*x^2+a^2)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a^2-b^2x^2}}{a-bx}\,dx=-\frac{2\,a\arctan\left(-\frac{a-\sqrt{-b^2x^2+a^2}}{bx}\right)+\sqrt{-b^2x^2+a^2}}{b}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x, algorithm="fricas")
```

output
```
-(2*a*arctan(-(a - sqrt(-b^2*x^2 + a^2))/(b*x)) + sqrt(-b^2*x^2 + a^2))/b
```

Sympy [F]

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a - bx} \, dx = -\int \frac{\sqrt{a^2 - b^2x^2}}{-a + bx} \, dx$$

input
```
integrate((-b**2*x**2+a**2)**(1/2)/(-b*x+a),x)
```

output
```
-Integral(sqrt(a**2 - b**2*x**2)/(-a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a - bx} \, dx = \frac{a \arcsin\left(\frac{bx}{a}\right)}{b} - \frac{\sqrt{-b^2x^2 + a^2}}{b}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x, algorithm="maxima")
```

output
```
a*arcsin(b*x/a)/b - sqrt(-b^2*x^2 + a^2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a^2 - b^2x^2}}{a - bx} \, dx = \frac{a \arcsin\left(\frac{bx}{a}\right) \operatorname{sgn}(a) \operatorname{sgn}(b)}{|b|} - \frac{\sqrt{-b^2x^2 + a^2}}{b}$$

input
```
integrate((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x, algorithm="giac")
```

output
```
a*arcsin(b*x/a)*sgn(a)*sgn(b)/abs(b) - sqrt(-b^2*x^2 + a^2)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2-b^2x^2}}{a-bx}\,dx = \int \frac{\sqrt{a^2-b^2\,x^2}}{a-b\,x}\,dx$$

input
```
int((a^2 - b^2*x^2)^(1/2)/(a - b*x),x)
```

output
```
int((a^2 - b^2*x^2)^(1/2)/(a - b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a^2-b^2x^2}}{a-bx}\,dx = \frac{asin\left(\frac{bx}{a}\right)a-\sqrt{-b^2x^2+a^2}+a}{b}$$

input
```
int((-b^2*x^2+a^2)^(1/2)/(-b*x+a),x)
```

output
```
(asin((b*x)/a)*a - sqrt(a**2 - b**2*x**2) + a)/b
```

3.108 $\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx=\frac{1}{2}ax\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{b\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{3c}-\frac{a^3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{-\frac{a^2c}{b^2}+cx^2}}\right)}{2b^2}$$

output
```
1/2*a*x*(-a^2*c/b^2+c*x^2)^(1/2)+1/3*b*(-a^2*c/b^2+c*x^2)^(3/2)/c-1/2*a^3*
c^(1/2)*arctanh(c^(1/2)*x/(-a^2*c/b^2+c*x^2)^(1/2))/b^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx=\frac{\sqrt{c\left(-\frac{a^2}{b^2}+x^2\right)}\left(b\sqrt{-\frac{a^2}{b^2}+x^2}(-2a^2+3abx+2b^2x^2)+3a^3\log\left(-x+\sqrt{-\frac{a^2}{b^2}+x^2}\right)\right)}{6b^2\sqrt{-\frac{a^2}{b^2}+x^2}}$$

input
```
Integrate[(a + b*x)*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

output
```
(Sqrt[c*(-(a^2/b^2) + x^2)]*(b*Sqrt[-(a^2/b^2) + x^2]*(-2*a^2 + 3*a*b*x +
2*b^2*x^2) + 3*a^3*Log[-x + Sqrt[-(a^2/b^2) + x^2]]))/(6*b^2*Sqrt[-(a^2/b^
2) + x^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\sqrt{cx^2-\frac{a^2c}{b^2}}dx$$

$$\Big\downarrow \text{455}$$

$$a\int \sqrt{cx^2-\frac{a^2c}{b^2}}dx+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2c\int \frac{1}{\sqrt{cx^2-\frac{a^2c}{b^2}}}dx}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2c\int \frac{1}{1-\frac{cx^2}{cx^2-\frac{a^2c}{b^2}}}d\frac{x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{219}$$

$$a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}\right)}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}$$

input
```
Int[(a + b*x)*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

output
```
(b*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(3*c) + a*((x*Sqrt[-((a^2*c)/b^2) + c*x
^2])/2 - (a^2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[-((a^2*c)/b^2) + c*x^2]])/(
2*b^2))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

method	result
default	$a\left(\frac{x\sqrt{-\frac{a^2c}{b^2}+c\,x^2}}{2}-\frac{\sqrt{c}\,a^2\ln\left(\sqrt{c}\,x+\sqrt{-\frac{a^2c}{b^2}+c\,x^2}\right)}{2b^2}\right)+\frac{b\left(-\frac{c\left(-b^2x^2+a^2\right)}{b^2}\right)^{\frac{3}{2}}}{3c}$
risch	$-\frac{\left(-2b^2x^2-3abx+2a^2\right)\sqrt{-\frac{c\left(-b^2x^2+a^2\right)}{b^2}}\,\sqrt{-c\left(-b^2x^2+a^2\right)}}{6b\sqrt{c\left(b^2x^2-a^2\right)}}+\frac{a^3\ln\left(\frac{b^2cx}{\sqrt{b^2c}}+\sqrt{b^2c\,x^2-a^2c}\right)\sqrt{-\frac{c\left(-b^2x^2+a^2\right)}{b^2}}\,\sqrt{-c\left(-b^2x^2+a^2\right)}}{2\sqrt{b^2c}\left(-b^2x^2+a^2\right)}$

input
```
int((b*x+a)*(-a^2*c/b^2+c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*x*(-a^2*c/b^2+c*x^2)^(1/2)-1/2*c^(1/2)*a^2/b^2*ln(c^(1/2)*x+(-a^2*c
/b^2+c*x^2)^(1/2)))+1/3*b/c*(-c*(-b^2*x^2+a^2)/b^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.27

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$
$$=\left[\frac{3\,a^3\sqrt{c}\log\left(2\,b^2cx^2-2\,b^2\sqrt{c}x\sqrt{\frac{b^2cx^2-a^2c}{b^2}}-a^2c\right)+2\left(2\,b^3x^2+3\,ab^2x-2\,a^2b\right)\sqrt{\frac{b^2cx^2-a^2c}{b^2}}}{12\,b^2},\ \frac{3\,a^3\sqrt{-c}\,\mathrm{arc}\ldots}{\ldots}\right.$$

input
```
integrate((b*x+a)*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*a^3*sqrt(c)*log(2*b^2*c*x^2 - 2*b^2*sqrt(c)*x*sqrt((b^2*c*x^2 - a
^2*c)/b^2) - a^2*c) + 2*(2*b^3*x^2 + 3*a*b^2*x - 2*a^2*b)*sqrt((b^2*c*x^2
- a^2*c)/b^2))/b^2, 1/6*(3*a^3*sqrt(-c)*arctan(b^2*sqrt(-c)*x*sqrt((b^2*c*
x^2 - a^2*c)/b^2)/(b^2*c*x^2 - a^2*c)) + (2*b^3*x^2 + 3*a*b^2*x - 2*a^2*b)
*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$
$$= \begin{cases} -\frac{a^3c\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{-\frac{a^2c}{b^2}+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{a^2c}{b^2}\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right)}{2b^2}+\sqrt{-\frac{a^2c}{b^2}+cx^2}\left(-\frac{a^2}{3b}+\frac{ax}{2}+\frac{bx^2}{3}\right) & \text{for } c\neq 0\\ \sqrt{-\frac{a^2c}{b^2}}\left(ax+\frac{bx^2}{2}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((b*x+a)*(-a**2*c/b**2+c*x**2)**(1/2),x)
```

output
```
Piecewise((-a**3*c*Piecewise((log(2*sqrt(c)*sqrt(-a**2*c/b**2 + c*x**2) +
2*c*x)/sqrt(c), Ne(a**2*c/b**2, 0)), (x*log(x)/sqrt(c*x**2), True))/(2*b**
2) + sqrt(-a**2*c/b**2 + c*x**2)*(-a**2/(3*b) + a*x/2 + b*x**2/3), Ne(c, 0
)), (sqrt(-a**2*c/b**2)*(a*x + b*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{1}{2}\sqrt{cx^2-\frac{a^2c}{b^2}}ax-\frac{a^3\sqrt{c}\log\left(2\,cx+2\,\sqrt{cx^2-\frac{a^2c}{b^2}}\sqrt{c}\right)}{2\,b^2}+\frac{\left(cx^2-\frac{a^2c}{b^2}\right)^{\frac{3}{2}}b}{3\,c}$$

input
```
integrate((b*x+a)*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 - a^2*c/b^2)*a*x - 1/2*a^3*sqrt(c)*log(2*c*x + 2*sqrt(c*x^2
 - a^2*c/b^2)*sqrt(c))/b^2 + 1/3*(c*x^2 - a^2*c/b^2)^(3/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{\left(\frac{3\,a^3\sqrt{c}\log\left(\left|-\sqrt{b^2c}x+\sqrt{b^2cx^2-a^2c}\right|\right)}{|b|}+\sqrt{b^2cx^2-a^2c}\left((2\,bx+3\,a)x-\frac{2\,a^2}{b}\right)\right)|b|}{6\,b^2}$$

input
```
integrate((b*x+a)*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/6*(3*a^3*sqrt(c)*log(abs(-sqrt(b^2*c)*x + sqrt(b^2*c*x^2 - a^2*c)))/abs(
b) + sqrt(b^2*c*x^2 - a^2*c)*((2*b*x + 3*a)*x - 2*a^2/b))*abs(b)/b^2
```

Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{a\,x\,\sqrt{c\,x^2-\frac{a^2\,c}{b^2}}}{2}-\frac{(a^2-b^2\,x^2)\,\sqrt{c\,x^2-\frac{a^2\,c}{b^2}}}{3\,b} - \frac{a^3\,\sqrt{c}\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2-\frac{a^2\,c}{b^2}}\right)}{2\,b^2}$$

input
```
int((c*x^2 - (a^2*c)/b^2)^(1/2)*(a + b*x),x)
```

output
```
(a*x*(c*x^2 - (a^2*c)/b^2)^(1/2))/2 - ((a^2 - b^2*x^2)*(c*x^2 - (a^2*c)/b^
2)^(1/2))/(3*b) - (a^3*c^(1/2)*log(c^(1/2)*x + (c*x^2 - (a^2*c)/b^2)^(1/2)
))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int (a+bx)\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$
$$=\frac{\sqrt{c}\left(-2\sqrt{b^2x^2-a^2}\,a^2+3\sqrt{b^2x^2-a^2}\,abx+2\sqrt{b^2x^2-a^2}\,b^2x^2-3\log\left(\frac{\sqrt{b^2x^2-a^2}+bx}{a}\right)a^3\right)}{6b^2}$$

input
```
int((b*x+a)*(-a^2*c/b^2+c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt( - a**2 + b**2*x**2)*a**2 + 3*sqrt( - a**2 + b**2*x**2
)*a*b*x + 2*sqrt( - a**2 + b**2*x**2)*b**2*x**2 - 3*log((sqrt( - a**2 + b*
*2*x**2) + b*x)/a)*a**3))/(6*b**2)
```

3.109 $\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx=\frac{5}{8}a^2x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{b(8a+3bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{12c}-\frac{5a^4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{-\frac{a^2c}{b^2}+cx^2}}\right)}{8b^2}$$

output
```
5/8*a^2*x*(-a^2*c/b^2+c*x^2)^(1/2)+1/12*b*(3*b*x+8*a)*(-a^2*c/b^2+c*x^2)^(
3/2)/c-5/8*a^4*c^(1/2)*arctanh(c^(1/2)*x/(-a^2*c/b^2+c*x^2)^(1/2))/b^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx=\frac{\sqrt{c\left(-\frac{a^2}{b^2}+x^2\right)}\left(b\sqrt{-\frac{a^2}{b^2}+x^2}(-16a^3+9a^2bx+16ab^2x^2+6b^3x^3)+15a^4\log\left(-x+\sqrt{-\frac{a^2}{b^2}+x^2}\right)\right)}{24b^2\sqrt{-\frac{a^2}{b^2}+x^2}}$$

input
```
Integrate[(a + b*x)^2*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

output
```
(Sqrt[c*(-(a^2/b^2) + x^2)]*(b*Sqrt[-(a^2/b^2) + x^2]*(-16*a^3 + 9*a^2*b*x
 + 16*a*b^2*x^2 + 6*b^3*x^3) + 15*a^4*Log[-x + Sqrt[-(a^2/b^2) + x^2]]))/(
24*b^2*Sqrt[-(a^2/b^2) + x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {469, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{cx^2-\frac{a^2c}{b^2}}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{4}a\int (a+bx)\sqrt{cx^2-\frac{a^2c}{b^2}}dx+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{455}$$

$$\frac{5}{4}a\left(a\int \sqrt{cx^2-\frac{a^2c}{b^2}}dx+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2c\int \frac{1}{\sqrt{cx^2-\frac{a^2c}{b^2}}}dx}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2c\int\frac{1}{1-\frac{cx^2}{cx^2-\frac{a^2c}{b^2}}}d\frac{x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{5}{4}a\left(a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}\right)}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}$$

input
```
Int[(a + b*x)^2*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

output
```
(b*(a + b*x)*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(4*c) + (5*a*((b*(-((a^2*c)/b
^2) + c*x^2)^(3/2))/(3*c) + a*((x*Sqrt[-((a^2*c)/b^2) + c*x^2])/2 - (a^2*S
qrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[-((a^2*c)/b^2) + c*x^2]])/(2*b^2))))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))    Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(89) = 178$.

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.72

method	result
default	$a^{2}\left(\frac{x\sqrt{-\frac{a^{2}c}{b^{2}}+c\,x^{2}}}{2}-\frac{\sqrt{c}\,a^{2}\ln\left(\sqrt{c}\,x+\sqrt{-\frac{a^{2}c}{b^{2}}+c\,x^{2}}\right)}{2b^{2}}\right)+b^{2}\left(\frac{x\left(-\frac{a^{2}c}{b^{2}}+c\,x^{2}\right)^{\frac{3}{2}}}{4c}+\frac{a^{2}\left(\frac{x\sqrt{-\frac{a^{2}c}{b^{2}}+c\,x^{2}}}{2}-\frac{\sqrt{c}\,a^{2}\ln\left(\sqrt{c}\,x+\sqrt{}\right)}{2b^{2}}\right)}{4b^{2}}\right)$
risch	$-\frac{\left(-6b^{3}x^{3}-16a\,b^{2}x^{2}-9a^{2}bx+16a^{3}\right)\sqrt{-\frac{c\left(-b^{2}x^{2}+a^{2}\right)}{b^{2}}}\,\sqrt{-c\left(-b^{2}x^{2}+a^{2}\right)}}{24b\sqrt{c\left(b^{2}x^{2}-a^{2}\right)}}+\frac{5a^{4}\ln\left(\frac{b^{2}cx}{\sqrt{b^{2}c}}+\sqrt{b^{2}c\,x^{2}-a^{2}c}\right)\sqrt{-\frac{c\left(-b^{2}x^{2}+a^{2}\right)}{b^{2}}}\,\sqrt{-}}{8\sqrt{b^{2}c}\left(-b^{2}x^{2}+a^{2}\right)}$

input
```
int((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*x*(-a^2*c/b^2+c*x^2)^(1/2)-1/2*c^(1/2)*a^2/b^2*ln(c^(1/2)*x+(-a^2
*c/b^2+c*x^2)^(1/2)))+b^2*(1/4*x*(-a^2*c/b^2+c*x^2)^(3/2)/c+1/4*a^2/b^2*(1
/2*x*(-a^2*c/b^2+c*x^2)^(1/2)-1/2*c^(1/2)*a^2/b^2*ln(c^(1/2)*x+(-a^2*c/b^2
+c*x^2)^(1/2))))+2/3*a*b/c*(-c*(-b^2*x^2+a^2)/b^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.27

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \left[\frac{15\,a^4\sqrt{c}\log\left(2\,b^2cx^2-2\,b^2\sqrt{c}x\sqrt{\frac{b^2cx^2-a^2c}{b^2}}-a^2c\right)+2\,(6\,b^4x^3+16\,ab^3x^2+9\,a^2b^2x-16\,a^3b)\sqrt{\frac{b^2cx^2-a^2c}{b^2}}}{48\,b^2}\right.$$

input
```
integrate((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(15*a^4*sqrt(c)*log(2*b^2*c*x^2 - 2*b^2*sqrt(c)*x*sqrt((b^2*c*x^2 -
a^2*c)/b^2) - a^2*c) + 2*(6*b^4*x^3 + 16*a*b^3*x^2 + 9*a^2*b^2*x - 16*a^3*
b)*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2, 1/24*(15*a^4*sqrt(-c)*arctan(b^2*sq
rt(-c)*x*sqrt((b^2*c*x^2 - a^2*c)/b^2)/(b^2*c*x^2 - a^2*c)) + (6*b^4*x^3 +
 16*a*b^3*x^2 + 9*a^2*b^2*x - 16*a^3*b)*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2
]
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \begin{cases} -\frac{5a^4c\left(\begin{cases}\frac{\log\left(2\sqrt{c}\sqrt{-\frac{a^2c}{b^2}+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{a^2c}{b^2}\neq 0\\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{-\frac{a^2c}{b^2}+cx^2}\left(-\frac{2a^3}{3b}+\frac{3a^2x}{8}+\frac{2abx^2}{3}+\frac{b^2x^3}{4}\right) & \text{for } c\neq 0\\ \sqrt{-\frac{a^2c}{b^2}}\left(\begin{cases}a^2x & \text{for } b=0\\ \frac{(a+bx)^3}{3b} & \text{otherwise}\end{cases}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((b*x+a)**2*(-a**2*c/b**2+c*x**2)**(1/2),x)
```

output
```
Piecewise((-5*a**4*c*Piecewise((log(2*sqrt(c)*sqrt(-a**2*c/b**2 + c*x**2)
+ 2*c*x)/sqrt(c), Ne(a**2*c/b**2, 0)), (x*log(x)/sqrt(c*x**2), True))/(8*b
**2) + sqrt(-a**2*c/b**2 + c*x**2)*(-2*a**3/(3*b) + 3*a**2*x/8 + 2*a*b*x**
2/3 + b**2*x**3/4), Ne(c, 0)), (sqrt(-a**2*c/b**2)*Piecewise((a**2*x, Eq(b
, 0)), ((a + b*x)**3/(3*b), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{5}{8}\sqrt{cx^2-\frac{a^2c}{b^2}}a^2x + \frac{\left(cx^2-\frac{a^2c}{b^2}\right)^{\frac{3}{2}}b^2x}{4\,c} - \frac{5\,a^4\sqrt{c}\log\left(2\,cx+2\,\sqrt{cx^2-\frac{a^2c}{b^2}}\sqrt{c}\right)}{8\,b^2} + \frac{2\left(cx^2-\frac{a^2c}{b^2}\right)^{\frac{3}{2}}ab}{3\,c}$$

input
```
integrate((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
5/8*sqrt(c*x^2 - a^2*c/b^2)*a^2*x + 1/4*(c*x^2 - a^2*c/b^2)^(3/2)*b^2*x/c
- 5/8*a^4*sqrt(c)*log(2*c*x + 2*sqrt(c*x^2 - a^2*c/b^2)*sqrt(c))/b^2 + 2/3
*(c*x^2 - a^2*c/b^2)^(3/2)*a*b/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{\left(\frac{15\,a^4\sqrt{c}\log\left(\left|-\sqrt{b^2c}x+\sqrt{b^2cx^2-a^2c}\right|\right)}{|b|} - \sqrt{b^2cx^2-a^2c}\left(\frac{16\,a^3}{b}-(9\,a^2+2\,(3\,b^2x+8\,ab)x)x\right)\right)|b|}{24\,b^2}$$

input
```
integrate((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/24*(15*a^4*sqrt(c)*log(abs(-sqrt(b^2*c)*x + sqrt(b^2*c*x^2 - a^2*c)))/ab
s(b) - sqrt(b^2*c*x^2 - a^2*c)*(16*a^3/b - (9*a^2 + 2*(3*b^2*x + 8*a*b)*x)
*x))*abs(b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx=\int \sqrt{c\,x^2-\frac{a^2\,c}{b^2}}\,(a+b\,x)^2\,dx$$

input
```
int((c*x^2 - (a^2*c)/b^2)^(1/2)*(a + b*x)^2,x)
```

output
```
int((c*x^2 - (a^2*c)/b^2)^(1/2)*(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int (a+bx)^2\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$
$$=\frac{\sqrt{c}\left(-16\sqrt{b^2x^2-a^2}\,a^3+9\sqrt{b^2x^2-a^2}\,a^2bx+16\sqrt{b^2x^2-a^2}\,a\,b^2x^2+6\sqrt{b^2x^2-a^2}\,b^3x^3-15\log\left(\frac{\sqrt{b^2x^2-c}}{a}\right.\right.}{24b^2}$$

input
```
int((b*x+a)^2*(-a^2*c/b^2+c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 16*sqrt( - a**2 + b**2*x**2)*a**3 + 9*sqrt( - a**2 + b**2*x**
2)*a**2*b*x + 16*sqrt( - a**2 + b**2*x**2)*a*b**2*x**2 + 6*sqrt( - a**2 +
b**2*x**2)*b**3*x**3 - 15*log((sqrt( - a**2 + b**2*x**2) + b*x)/a)*a**4))/
(24*b**2)
```

3.110 $\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx=\frac{7}{8}a^3x\sqrt{-\frac{a^2c}{b^2}+cx^2}+\frac{b(a+bx)^2\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{5c}+\frac{7ab(8a+3bx)\left(-\frac{a^2c}{b^2}+cx^2\right)^{3/2}}{60c}-\frac{7a^5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{-\frac{a^2c}{b^2}+cx^2}}\right)}{8b^2}$$

output
```
7/8*a^3*x*(-a^2*c/b^2+c*x^2)^(1/2)+1/5*b*(b*x+a)^2*(-a^2*c/b^2+c*x^2)^(3/2
)/c+7/60*a*b*(3*b*x+8*a)*(-a^2*c/b^2+c*x^2)^(3/2)/c-7/8*a^5*c^(1/2)*arctan
h(c^(1/2)*x/(-a^2*c/b^2+c*x^2)^(1/2))/b^2
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$
$$=\frac{\sqrt{c\left(-\frac{a^2}{b^2}+x^2\right)}\left(b\sqrt{-\frac{a^2}{b^2}+x^2}(-136a^4+15a^3bx+112a^2b^2x^2+90ab^3x^3+24b^4x^4)+105a^5\log\left(-x+\sqrt{\right.\right.}{120b^2\sqrt{-\frac{a^2}{b^2}+x^2}}$$

input
```
Integrate[(a + b*x)^3*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

output
```
(Sqrt[c*(-(a^2/b^2) + x^2)]*(b*Sqrt[-(a^2/b^2) + x^2]*(-136*a^4 + 15*a^3*b
*x + 112*a^2*b^2*x^2 + 90*a*b^3*x^3 + 24*b^4*x^4) + 105*a^5*Log[-x + Sqrt[
-(a^2/b^2) + x^2]]))/(120*b^2*Sqrt[-(a^2/b^2) + x^2])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {469, 469, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3\sqrt{cx^2-\frac{a^2c}{b^2}}\,dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}a\int (a+bx)^2\sqrt{cx^2-\frac{a^2c}{b^2}}dx+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{5}a\left(\frac{5}{4}a\int (a+bx)\sqrt{cx^2-\frac{a^2c}{b^2}}dx+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}\right)+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}$$

$$\Big\downarrow \; 455$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\int\sqrt{cx^2-\frac{a^2c}{b^2}}dx+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}\right)+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}$$

$$\Big\downarrow \; 211$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2c\int\frac{1}{\sqrt{cx^2-\frac{a^2c}{b^2}}}dx}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}\right)+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}$$

$$\Big\downarrow \; 224$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2c\int\frac{1}{1-\frac{cx^2}{cx^2-\frac{a^2c}{b^2}}}d\frac{x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}\right)+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}$$

$$\Big\downarrow \; 219$$

$$\frac{7}{5}a\left(\frac{5}{4}a\left(a\left(\frac{1}{2}x\sqrt{cx^2-\frac{a^2c}{b^2}}-\frac{a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{cx^2-\frac{a^2c}{b^2}}}\right)}{2b^2}\right)+\frac{b\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{3c}\right)+\frac{b(a+bx)\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{4c}\right)+\frac{b(a+bx)^2\left(cx^2-\frac{a^2c}{b^2}\right)^{3/2}}{5c}$$

input
```
Int[(a + b*x)^3*Sqrt[-((a^2*c)/b^2) + c*x^2],x]
```

output
```
(b*(a + b*x)^2*(-((a^2*c)/b^2) + c*x^2)^(3/2))/(5*c) + (7*a*((b*(a + b*x)*
(-((a^2*c)/b^2) + c*x^2)^(3/2))/(4*c) + (5*a*((b*(-((a^2*c)/b^2) + c*x^2)^
(3/2))/(3*c) + a*((x*Sqrt[-((a^2*c)/b^2) + c*x^2])/2 - (a^2*Sqrt[c]*ArcTan
h[(Sqrt[c]*x)/Sqrt[-((a^2*c)/b^2) + c*x^2]])/(2*b^2))))/4))/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{\left(-24 b^4 x^4-90 a\, b^3 x^3-112 a^2 b^2 x^2-15 a^3 b x+136 a^4\right) \sqrt{-\frac{c\left(-b^2 x^2+a^2\right)}{b^2}}\, \sqrt{-c\left(-b^2 x^2+a^2\right)}}{120 b \sqrt{c\left(b^2 x^2-a^2\right)}}+\frac{7 a^5 \ln\left(\frac{b^2 c x}{\sqrt{b^2 c}}+\sqrt{b^2 c\, x^2-a^2 c}\right) \sqrt{-\frac{c(}{}}}{8 \sqrt{b^2 c}\,(-b^2 x}$
default	$a^3 \left(\frac{x \sqrt{-\frac{a^2 c}{b^2}+c\, x^2}}{2}-\frac{\sqrt{c}\, a^2 \ln\left(\sqrt{c}\, x+\sqrt{-\frac{a^2 c}{b^2}+c\, x^2}\right)}{2 b^2}\right)+b^3 \left(\frac{x^2 \left(-\frac{a^2 c}{b^2}+c\, x^2\right)^{\frac{3}{2}}}{5 c}+\frac{2 a^2 \left(-\frac{a^2 c}{b^2}+c\, x^2\right)^{\frac{3}{2}}}{15 c\, b^2}\right)+3 a\, b^2 \Bigg($

input
```
int((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*b^4*x^4-90*a*b^3*x^3-112*a^2*b^2*x^2-15*a^3*b*x+136*a^4)/b/(c*
(b^2*x^2-a^2))^(1/2)*(-c*(-b^2*x^2+a^2)/b^2)^(1/2)*(-c*(-b^2*x^2+a^2))^(1/
2)+7/8*a^5*ln(b^2*c*x/(b^2*c)^(1/2)+(b^2*c*x^2-a^2*c)^(1/2))/(b^2*c)^(1/2)
*(-c*(-b^2*x^2+a^2)/b^2)^(1/2)*(-c*(-b^2*x^2+a^2))^(1/2)/(-b^2*x^2+a^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.86

$$\int (a+bx)^3 \sqrt{-\frac{a^2 c}{b^2}+cx^2}\, dx$$
$$= \left[\frac{105\, a^5 \sqrt{c} \log\left(2\, b^2 c x^2-2\, b^2 \sqrt{c} x \sqrt{\frac{b^2 c x^2-a^2 c}{b^2}}-a^2 c\right)+2\,(24\, b^5 x^4+90\, a b^4 x^3+112\, a^2 b^3 x^2+15\, a^3 b^2 x-}{240\, b^2}\right.$$

input
```
integrate((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/240*(105*a^5*sqrt(c)*log(2*b^2*c*x^2 - 2*b^2*sqrt(c)*x*sqrt((b^2*c*x^2
- a^2*c)/b^2) - a^2*c) + 2*(24*b^5*x^4 + 90*a*b^4*x^3 + 112*a^2*b^3*x^2 +
15*a^3*b^2*x - 136*a^4*b)*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2, 1/120*(105*a
^5*sqrt(-c)*arctan(b^2*sqrt(-c)*x*sqrt((b^2*c*x^2 - a^2*c)/b^2)/(b^2*c*x^2
 - a^2*c)) + (24*b^5*x^4 + 90*a*b^4*x^3 + 112*a^2*b^3*x^2 + 15*a^3*b^2*x -
 136*a^4*b)*sqrt((b^2*c*x^2 - a^2*c)/b^2))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$
$$= \begin{cases} -\frac{7a^5c\left(\begin{cases} \frac{\log\left(2\sqrt{c}\sqrt{-\frac{a^2c}{b^2}+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ \frac{a^2c}{b^2}\neq 0 \\ \frac{x\log(x)}{\sqrt{cx^2}} & \text{otherwise} \end{cases}\right)}{8b^2} + \sqrt{-\frac{a^2c}{b^2}+cx^2}\left(-\frac{17a^4}{15b}+\frac{a^3x}{8}+\frac{14a^2bx^2}{15}+\frac{3ab^2x^3}{4}+\frac{b^3x^4}{5}\right) \\ \sqrt{-\frac{a^2c}{b^2}}\left(\begin{cases} a^3x & \text{for}\ b=0 \\ \frac{(a+bx)^4}{4b} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((b*x+a)**3*(-a**2*c/b**2+c*x**2)**(1/2),x)
```

output
```
Piecewise((-7*a**5*c*Piecewise((log(2*sqrt(c)*sqrt(-a**2*c/b**2 + c*x**2)
+ 2*c*x)/sqrt(c), Ne(a**2*c/b**2, 0)), (x*log(x)/sqrt(c*x**2), True))/(8*b
**2) + sqrt(-a**2*c/b**2 + c*x**2)*(-17*a**4/(15*b) + a**3*x/8 + 14*a**2*b
*x**2/15 + 3*a*b**2*x**3/4 + b**3*x**4/5), Ne(c, 0)), (sqrt(-a**2*c/b**2)*
Piecewise((a**3*x, Eq(b, 0)), ((a + b*x)**4/(4*b), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{\left(cx^2-\frac{a^2c}{b^2}\right)^{\frac{3}{2}}b^3x^2}{5\,c}+\frac{7}{8}\sqrt{cx^2-\frac{a^2c}{b^2}}a^3x + \frac{3\left(cx^2-\frac{a^2c}{b^2}\right)^{\frac{3}{2}}ab^2x}{4\,c} - \frac{7\,a^5\sqrt{c}\log\left(2\,cx+2\sqrt{cx^2-\frac{a^2c}{b^2}}\sqrt{c}\right)}{8\,b^2} + \frac{17\left(cx^2-\frac{a^2c}{b^2}\right)^{\frac{3}{2}}a^2b}{15\,c}$$

input
```
integrate((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(c*x^2 - a^2*c/b^2)^(3/2)*b^3*x^2/c + 7/8*sqrt(c*x^2 - a^2*c/b^2)*a^3*
x + 3/4*(c*x^2 - a^2*c/b^2)^(3/2)*a*b^2*x/c - 7/8*a^5*sqrt(c)*log(2*c*x +
2*sqrt(c*x^2 - a^2*c/b^2)*sqrt(c))/b^2 + 17/15*(c*x^2 - a^2*c/b^2)^(3/2)*a
^2*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx = \frac{\left(\frac{105\,a^5\sqrt{c}\log\left(\left|-\sqrt{b^2c}x+\sqrt{b^2cx^2-a^2c}\right|\right)}{|b|}-\sqrt{b^2cx^2-a^2c}\left(\frac{136\,a^4}{b}-(15\,a^3+2\,(56\,a^2b+3\,(4\,b^3x+15\,ab^2)x)x)x\right)\right)}{120\,b^2}$$

input
```
integrate((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/120*(105*a^5*sqrt(c)*log(abs(-sqrt(b^2*c)*x + sqrt(b^2*c*x^2 - a^2*c)))/
abs(b) - sqrt(b^2*c*x^2 - a^2*c)*(136*a^4/b - (15*a^3 + 2*(56*a^2*b + 3*(4
*b^3*x + 15*a*b^2)*x)*x)*x))*abs(b)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx=\int \sqrt{c\,x^2-\frac{a^2\,c}{b^2}}\,{\left(a+b\,x\right)}^3\,dx$$

input
```
int((c*x^2 - (a^2*c)/b^2)^(1/2)*(a + b*x)^3,x)
```

output
```
int((c*x^2 - (a^2*c)/b^2)^(1/2)*(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int (a+bx)^3\sqrt{-\frac{a^2c}{b^2}+cx^2}\,dx$$
$$=\frac{\sqrt{c}\left(-136\sqrt{b^2x^2-a^2}\,a^4+15\sqrt{b^2x^2-a^2}\,a^3bx+112\sqrt{b^2x^2-a^2}\,a^2b^2x^2+90\sqrt{b^2x^2-a^2}\,a\,b^3x^3+24\sqrt{b^2x}\right.}{120b^2}$$

input
```
int((b*x+a)^3*(-a^2*c/b^2+c*x^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 136*sqrt( - a**2 + b**2*x**2)*a**4 + 15*sqrt( - a**2 + b**2*x
**2)*a**3*b*x + 112*sqrt( - a**2 + b**2*x**2)*a**2*b**2*x**2 + 90*sqrt( -
a**2 + b**2*x**2)*a*b**3*x**3 + 24*sqrt( - a**2 + b**2*x**2)*b**4*x**4 - 1
05*log((sqrt( - a**2 + b**2*x**2) + b*x)/a)*a**5))/(120*b**2)
```

3.111 $\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{7d(d+ex)^2\sqrt{d^2-e^2x^2}}{12e}-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}-\frac{35d^2(4d+ex)\sqrt{d^2-e^2x^2}}{24e}+\frac{35d^4\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e}$$

output
```
-7/12*d*(e*x+d)^2*(-e^2*x^2+d^2)^(1/2)/e-1/4*(e*x+d)^3*(-e^2*x^2+d^2)^(1/2
)/e-35/24*d^2*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/e+35/8*d^4*arctan(e*x/(-e^2*x
^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(160d^3+81d^2ex+32de^2x^2+6e^3x^3)+210d^4\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{24e}$$

input
```
Integrate[(d + e*x)^4/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/24*(Sqrt[d^2 - e^2*x^2]*(160*d^3 + 81*d^2*e*x + 32*d*e^2*x^2 + 6*e^3*x^
3) + 210*d^4*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {469, 469, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{4}d\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}dx-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{4}d\left(\frac{5}{3}d\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}$$

$$\Big\downarrow \text{469}$$

$$\frac{7}{4}d\left(\frac{5}{3}d\left(\frac{3}{2}d\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{4}d\left(\frac{5}{3}d\left(\frac{3}{2}d\left(d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}$$

$$\Big\downarrow \text{224}$$

$$\frac{7}{4}d\left(\frac{5}{3}d\left(\frac{3}{2}d\left(d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}$$

$$\Big\downarrow \text{216}$$

$$\frac{7}{4}d\left(\frac{5}{3}d\left(\frac{3}{2}d\left(\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}\right)-\frac{(d+ex)^3\sqrt{d^2-e^2x^2}}{4e}$$

input
```
Int[(d + e*x)^4/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/4*((d + e*x)^3*Sqrt[d^2 - e^2*x^2])/e + (7*d*(-1/3*((d + e*x)^2*Sqrt[d^
2 - e^2*x^2])/e + (5*d*(-1/2*((d + e*x)*Sqrt[d^2 - e^2*x^2])/e + (3*d*(-(S
qrt[d^2 - e^2*x^2]/e) + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2))/3))/
4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\left(6e^{3}x^{3}+32d\,e^{2}x^{2}+81d^{2}ex+160d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{24e}+\frac{35d^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{8\sqrt{e^{2}}}$
default	$\frac{d^{4}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}+e^{4}\left(-\frac{x^{3}\sqrt{-e^{2}x^{2}+d^{2}}}{4e^{2}}+\frac{3d^{2}\left(-\frac{x\sqrt{-e^{2}x^{2}+d^{2}}}{2e^{2}}+\frac{d^{2}\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{2e^{2}\sqrt{e^{2}}}\right)}{4e^{2}}\right)+4d\,e^{3}\left(-\frac{x^{2}\sqrt{}}{}\right.$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*e^3*x^3+32*d*e^2*x^2+81*d^2*e*x+160*d^3)/e*(-e^2*x^2+d^2)^(1/2)+3
5/8*d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{210\,d^4\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,e^3x^3+32\,de^2x^2+81\,d^2ex+160\,d^3\right)\sqrt{-e^2x^2+d^2}}{24\,e}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(210*d^4*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (6*e^3*x^3 + 32
*d*e^2*x^2 + 81*d^2*e*x + 160*d^3)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{35d^4\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for}\ d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{8} + \sqrt{d^2-e^2x^2}\left(-\frac{20d^3}{3e}-\frac{27d^2x}{8}-\frac{4dex^2}{3}-\frac{e^2x^3}{4}\right) & \text{for}\ e^2\neq \\ \frac{\begin{cases} d^4x & \text{for}\ e=0 \\ \frac{(d+ex)^5}{5e} & \text{otherwise} \end{cases}}{\sqrt{d^2}} & \text{otherwis} \end{cases}$$

input
```
integrate((e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((35*d**4*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e*
*2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/8
+ sqrt(d**2 - e**2*x**2)*(-20*d**3/(3*e) - 27*d**2*x/8 - 4*d*e*x**2/3 - e*
*2*x**3/4), Ne(e**2, 0)), (Piecewise((d**4*x, Eq(e, 0)), ((d + e*x)**5/(5*
e), True))/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}\sqrt{-e^2x^2+d^2}e^2x^3 - \frac{4}{3}\sqrt{-e^2x^2+d^2}dex^2 + \frac{35\,d^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\sqrt{e^2}} - \frac{27}{8}\sqrt{-e^2x^2+d^2}d^2x - \frac{20\sqrt{-e^2x^2+d^2}d^3}{3\,e}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-e^2*x^2 + d^2)*e^2*x^3 - 4/3*sqrt(-e^2*x^2 + d^2)*d*e*x^2 + 35/
8*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 27/8*sqrt(-e^2*x^2 + d^2)*d^
2*x - 20/3*sqrt(-e^2*x^2 + d^2)*d^3/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx = \frac{35\,d^4\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{8\,|e|} - \frac{1}{24}\sqrt{-e^2x^2+d^2}\left(\frac{160\,d^3}{e}+\left(81\,d^2+2\,\left(3\,e^2x+16\,de\right)x\right)x\right)$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
35/8*d^4*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/24*sqrt(-e^2*x^2 + d^2)*(1
60*d^3/e + (81*d^2 + 2*(3*e^2*x + 16*d*e)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^4}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^4}{\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{105 asin\left(\frac{ex}{d}\right) d^4 - 160\sqrt{-e^2x^2+d^2}\, d^3 - 81\sqrt{-e^2x^2+d^2}\, d^2ex - 32\sqrt{-e^2x^2+d^2}\, d\, e^2x^2 - 6\sqrt{-e^2x^2+d^2}}{24e}$$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(105*asin((e*x)/d)*d**4 - 160*sqrt(d**2 - e**2*x**2)*d**3 - 81*sqrt(d**2 -
 e**2*x**2)*d**2*e*x - 32*sqrt(d**2 - e**2*x**2)*d*e**2*x**2 - 6*sqrt(d**2
 - e**2*x**2)*e**3*x**3 + 160*d**4)/(24*e)
```

3.112 $\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}} \, dx = -\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e} - \frac{5d(4d+ex)\sqrt{d^2-e^2x^2}}{6e} + \frac{5d^3 \arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
-1/3*(e*x+d)^2*(-e^2*x^2+d^2)^(1/2)/e-5/6*d*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)
/e+5/2*d^3*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}} \, dx = \frac{(-22d^2-9dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{6e} - \frac{5d^3 \arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(d + e*x)^3/Sqrt[d^2 - e^2*x^2],x]
```

output
```
((-22*d^2 - 9*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(6*e) - (5*d^3*ArcTa
n[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {469, 469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{3}d\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{469}$$

$$\frac{5}{3}d\left(\frac{3}{2}d\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{455}$$

$$\frac{5}{3}d\left(\frac{3}{2}d\left(d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{3}d\left(\frac{3}{2}d\left(d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{3}d\left(\frac{3}{2}d\left(\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}\right)-\frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{3e}$$

input
```
Int[(d + e*x)^3/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/3*((d + e*x)^2*Sqrt[d^2 - e^2*x^2])/e + (5*d*(-1/2*((d + e*x)*Sqrt[d^2
- e^2*x^2])/e + (3*d*(-(Sqrt[d^2 - e^2*x^2]/e) + (d*ArcTan[(e*x)/Sqrt[d^2
- e^2*x^2]])/e))/2))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(2e^2x^2+9dex+22d^2)\sqrt{-e^2x^2+d^2}}{6e}+\frac{5d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}$
default	$\frac{d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^3\left(-\frac{x^2\sqrt{-e^2x^2+d^2}}{3e^2}-\frac{2d^2\sqrt{-e^2x^2+d^2}}{3e^4}\right)+3d\,e^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right.$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*e^2*x^2+9*d*e*x+22*d^2)/e*(-e^2*x^2+d^2)^(1/2)+5/2*d^3/(e^2)^(1/2)
*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{30\,d^3\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,e^2x^2+9\,dex+22\,d^2\right)\sqrt{-e^2x^2+d^2}}{6\,e}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(30*d^3*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (2*e^2*x^2 + 9*d*
e*x + 22*d^2)*sqrt(-e^2*x^2 + d^2))/e
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{5d^3\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right)}{2} + \sqrt{d^2-e^2x^2}\left(-\frac{11d^2}{3e}-\frac{3dx}{2}-\frac{ex^2}{3}\right) & \text{for } e^2\neq 0 \\ \frac{\begin{cases} d^3x & \text{for } e=0 \\ \frac{(d+ex)^4}{4e} & \text{otherwise} \end{cases}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((5*d**3*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 +
 sqrt(d**2 - e**2*x**2)*(-11*d**2/(3*e) - 3*d*x/2 - e*x**2/3), Ne(e**2, 0)
), (Piecewise((d**3*x, Eq(e, 0)), ((d + e*x)**4/(4*e), True))/sqrt(d**2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{3}\sqrt{-e^2x^2+d^2}ex^2 + \frac{5\,d^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} - \frac{3}{2}\sqrt{-e^2x^2+d^2}dx - \frac{11\sqrt{-e^2x^2+d^2}d^2}{3\,e}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)*e*x^2 + 5/2*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt
(e^2) - 3/2*sqrt(-e^2*x^2 + d^2)*d*x - 11/3*sqrt(-e^2*x^2 + d^2)*d^2/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx = \frac{5\,d^3\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{2\,|e|} - \frac{1}{6}\sqrt{-e^2x^2+d^2}\left((2\,ex+9\,d)x+\frac{22\,d^2}{e}\right)$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
5/2*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/6*sqrt(-e^2*x^2 + d^2)*((2*
e*x + 9*d)*x + 22*d^2/e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^3}{\sqrt{d^2-e^2x^2}}\,dx = \frac{15asin\left(\frac{ex}{d}\right)d^3 - 22\sqrt{-e^2x^2+d^2}\,d^2 - 9\sqrt{-e^2x^2+d^2}\,dex - 2\sqrt{-e^2x^2+d^2}\,e^2x^2 + 22d^3}{6e}$$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(15*asin((e*x)/d)*d**3 - 22*sqrt(d**2 - e**2*x**2)*d**2 - 9*sqrt(d**2 - e*
*2*x**2)*d*e*x - 2*sqrt(d**2 - e**2*x**2)*e**2*x**2 + 22*d**3)/(6*e)
```

3.113 $\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(4d+ex)\sqrt{d^2-e^2x^2}}{2e}+\frac{3d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
-1/2*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/e+3/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1
/2))/e
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{(4d+ex)\sqrt{d^2-e^2x^2}+6d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{2e}$$

input
```
Integrate[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/2*((4*d + e*x)*Sqrt[d^2 - e^2*x^2] + 6*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sq
rt[d^2 - e^2*x^2])])/e
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {469, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{469}$$

$$\frac{3}{2}d\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$\Big\downarrow \text{455}$$

$$\frac{3}{2}d\left(d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}d\left(d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}d\left(\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}\right)-\frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

input
```
Int[(d + e*x)^2/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-1/2*((d + e*x)*Sqrt[d^2 - e^2*x^2])/e + (3*d*(-(Sqrt[d^2 - e^2*x^2]/e) +
(d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 469
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
((n + p)/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x], x] /; Fr
eeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[n, 0] && NeQ[n + 2*
p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{(ex+4d)\sqrt{-e^2x^2+d^2}}{2e}+\frac{3d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}$	60
default	$\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+e^2\left(-\frac{x\sqrt{-e^2x^2+d^2}}{2e^2}+\frac{d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2e^2\sqrt{e^2}}\right)-\frac{2d\sqrt{-e^2x^2+d^2}}{e}$	113

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/e+3/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{6\,d^2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex+4\,d)}{2\,e}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(6*d^2*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^
2)*(e*x + 4*d))/e
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} \frac{3d^2\left(\begin{cases}\frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0\\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise}\end{cases}\right)}{2}+\sqrt{d^2-e^2x^2}\left(-\frac{2d}{e}-\frac{x}{2}\right) & \text{for } e^2\neq 0\\ \frac{\begin{cases} d^2x & \text{for } e=0\\ \frac{(d+ex)^3}{3e} & \text{otherwise}\end{cases}}{\sqrt{d^2}} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((3*d**2*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**
2*x**2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True))/2 +
 sqrt(d**2 - e**2*x**2)*(-2*d/e - x/2), Ne(e**2, 0)), (Piecewise((d**2*x,
Eq(e, 0)), ((d + e*x)**3/(3*e), True))/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}} - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}x - \frac{2\,\sqrt{-e^2x^2+d^2}d}{e}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
3/2*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*x
 - 2*sqrt(-e^2*x^2 + d^2)*d/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} - \frac{1}{2}\,\sqrt{-e^2x^2+d^2}\left(x+\frac{4\,d}{e}\right)$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
3/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/2*sqrt(-e^2*x^2 + d^2)*(x +
 4*d/e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \int \frac{\left(d+e\,x\right)^2}{\sqrt{d^2-e^2\,x^2}}\,dx$$

input
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}}\,dx = \frac{3asin\left(\frac{ex}{d}\right)d^2-4\sqrt{-e^2x^2+d^2}\,d-\sqrt{-e^2x^2+d^2}\,ex+4d^2}{2e}$$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(3*asin((e*x)/d)*d**2 - 4*sqrt(d**2 - e**2*x**2)*d - sqrt(d**2 - e**2*x**2
)*e*x + 4*d**2)/(2*e)
```

3.114 $\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}}{e}+\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
-(-e^2*x^2+d^2)^(1/2)/e+d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx=-\frac{\sqrt{d^2-e^2x^2}+2d\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(d + e*x)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-((Sqrt[d^2 - e^2*x^2] + 2*d*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2]
)])/e)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 455}$$

$$d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}}{e}$$

$$\Big\downarrow \text{ 224}$$

$$d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}}{e}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}}{e}$$

input
```
Int[(d + e*x)/Sqrt[d^2 - e^2*x^2],x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/e) + (d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{d \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2 x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2 x^2+d^2}}{e}$	50
risch	$\frac{d \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{-e^2 x^2+d^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2 x^2+d^2}}{e}$	50

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-(-e^2*x^2+d^2)^(1
/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,d \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + \sqrt{-e^2x^2+d^2}}{e}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*d*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(-e^2*x^2 + d^2))/e
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(36) = 72$.

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx = \begin{cases} d\left(\begin{cases} \frac{\log\left(-2e^2x+2\sqrt{-e^2}\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}} & \text{for } d^2\neq 0 \\ \frac{x\log(x)}{\sqrt{-e^2x^2}} & \text{otherwise} \end{cases}\right) - \frac{\sqrt{d^2-e^2x^2}}{e} & \text{for } e^2\neq 0 \\ \frac{dx+\frac{ex^2}{2}}{\sqrt{d^2}} & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Piecewise((d*Piecewise((log(-2*e**2*x + 2*sqrt(-e**2)*sqrt(d**2 - e**2*x**
2))/sqrt(-e**2), Ne(d**2, 0)), (x*log(x)/sqrt(-e**2*x**2), True)) - sqrt(d
**2 - e**2*x**2)/e, Ne(e**2, 0)), ((d*x + e*x**2/2)/sqrt(d**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}}{e}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
d*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - sqrt(-e^2*x^2 + d^2)/e
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{\sqrt{-e^2x^2+d^2}}{e}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - sqrt(-e^2*x^2 + d^2)/e
```

Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx = \frac{d\,\ln\left(x\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}} - \frac{\sqrt{d^2-e^2\,x^2}}{e}$$

input
```
int((d + e*x)/(d^2 - e^2*x^2)^(1/2),x)
```

output
```
(d*log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (d^2 - e^2*
x^2)^(1/2)/e
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{d+ex}{\sqrt{d^2-e^2x^2}}\,dx = \frac{asin\left(\frac{ex}{d}\right)d-\sqrt{-e^2x^2+d^2}}{e}$$

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(asin((e*x)/d)*d - sqrt(d**2 - e**2*x**2))/e
```

3.115 $\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

output
```
-(-e^2*x^2+d^2)^(1/2)/d/e/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

input
```
Integrate[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$$

$$\Big\downarrow \text{ 460}$$

$$-\frac{\sqrt{d^2-e^2x^2}}{de(d+ex)}$$

input
```
Int[1/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-(Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{-ex+d}{de\sqrt{-e^{2}x^{2}+d^{2}}}$	29
orering	$-\frac{-ex+d}{de\sqrt{-e^{2}x^{2}+d^{2}}}$	29
trager	$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}}{de(ex+d)}$	30
default	$-\frac{\sqrt{-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)}}{e^{2}d\left(x+\frac{d}{e}\right)}$	46

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx=-\frac{ex+d+\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-(e*x + d + sqrt(-e^2*x^2 + d^2))/(d*e^2*x + d^2*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{de^2x+d^2e}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-e^2*x^2 + d^2)/(d*e^2*x + d^2*e)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{2}{d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)|e|}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
2/(d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}}{d\,e\,\left(d+e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

output
```
-(d^2 - e^2*x^2)^(1/2)/(d*e*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx = \frac{2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)}{de\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)+1\right)}$$

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(2*tan(asin((e*x)/d)/2))/(d*e*(tan(asin((e*x)/d)/2) + 1))
```

3.116 $\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \frac{x}{3d^2\sqrt{d^2-e^2x^2}} - \frac{2}{3e(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
1/3*x/d^2/(-e^2*x^2+d^2)^(1/2)-2/3/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \frac{(-2d-ex)\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)^2}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((-2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}$$

$$\Big\downarrow \text{460}$$

$$-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}$$

input
```
Int[1/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/3*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*
(d + e*x))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

method	result	size
trager	$-\frac{(ex+2d)\sqrt{-e^{2}x^{2}+d^{2}}}{3d^{2}(ex+d)^{2}e}$	37
gosper	$-\frac{(-ex+d)(ex+2d)}{3(ex+d)d^{2}e\sqrt{-e^{2}x^{2}+d^{2}}}$	43
orering	$-\frac{(-ex+d)(ex+2d)}{3(ex+d)d^{2}e\sqrt{-e^{2}x^{2}+d^{2}}}$	43
default	$\frac{-\frac{\sqrt{-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)}}{3de\left(x+\frac{d}{e}\right)^{2}}-\frac{\sqrt{-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)}}{3d^{2}\left(x+\frac{d}{e}\right)}}{e^{2}}$	93

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(e*x+2*d)/d^2/(e*x+d)^2/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{2\,e^2x^2+4\,dex+2\,d^2+\sqrt{-e^2x^2+d^2}(ex+2\,d)}{3\,(d^2e^3x^2+2\,d^3e^2x+d^4e)}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*e^2*x^2 + 4*d*e*x + 2*d^2 + sqrt(-e^2*x^2 + d^2)*(e*x + 2*d))/(d^2
*e^3*x^2 + 2*d^3*e^2*x + d^4*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{3\,(de^3x^2+2\,d^2e^2x+d^3e)} - \frac{\sqrt{-e^2x^2+d^2}}{3\,(d^2e^2x+d^3e)}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)/(d*e^3*x^2 + 2*d^2*e^2*x + d^3*e) - 1/3*sqrt(-e^
2*x^2 + d^2)/(d^2*e^2*x + d^3*e)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \frac{i\,\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}(e)}{3\,d^2|e|} - \frac{\left(\frac{2\,d}{ex+d}-1\right)^{\frac{3}{2}}+3\,\sqrt{\frac{2\,d}{ex+d}-1}}{6\,d^2|e|\mathrm{sgn}\left(\frac{1}{ex+d}\right)\mathrm{sgn}\left(e\right)}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*I*sgn(1/(e*x + d))*sgn(e)/(d^2*abs(e)) - 1/6*((2*d/(e*x + d) - 1)^(3/2
) + 3*sqrt(2*d/(e*x + d) - 1))/(d^2*abs(e)*sgn(1/(e*x + d))*sgn(e))
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,(2\,d+e\,x)}{3\,d^2\,e\,(d+e\,x)^2}$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(2*d + e*x))/(3*d^2*e*(d + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx = \frac{\frac{2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3}{3}-\frac{2}{3}}{d^2e\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3+3\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2+3\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)+1\right)}$$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(2*(tan(asin((e*x)/d)/2)**3 - 1))/(3*d**2*e*(tan(asin((e*x)/d)/2)**3 + 3*t
an(asin((e*x)/d)/2)**2 + 3*tan(asin((e*x)/d)/2) + 1))
```

3.117 $\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{2x}{15d^3\sqrt{d^2-e^2x^2}} - \frac{2}{5e(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{1}{15de(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
2/15*x/d^3/(-e^2*x^2+d^2)^(1/2)-2/5/e/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2)-1/15/
d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{(-7d^2-6dex-2e^2x^2)\sqrt{d^2-e^2x^2}}{15d^3e(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
((-7*d^2 - 6*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x)^3
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

$$\Big\downarrow \text{461}$$

$$\frac{2\left(\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

$$\Big\downarrow \text{460}$$

$$\frac{2\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}$$

input
```
Int[1/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/5*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^3) + (2*(-1/3*Sqrt[d^2 - e^2*x^2]/
(d*e*(d + e*x)^2) - Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d + e*x))))/(5*d)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

method	result	size
trager	$-\frac{\left(2 e^{2} x^{2}+6 d e x+7 d^{2}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{15 d^{3} \left(e x+d\right)^{3} e}$	49
gosper	$-\frac{\left(-e x+d\right) \left(2 e^{2} x^{2}+6 d e x+7 d^{2}\right)}{15 \left(e x+d\right)^{2} d^{3} e \sqrt{-e^{2} x^{2}+d^{2}}}$	55
orering	$-\frac{\left(-e x+d\right) \left(2 e^{2} x^{2}+6 d e x+7 d^{2}\right)}{15 \left(e x+d\right)^{2} d^{3} e \sqrt{-e^{2} x^{2}+d^{2}}}$	55
default	$\frac{-\frac{\sqrt{-e^{2} \left(x+\frac{d}{e}\right)^{2}+2 d e \left(x+\frac{d}{e}\right)}}{5 d e \left(x+\frac{d}{e}\right)^{3}}+\frac{2 e \left(-\frac{\sqrt{-e^{2} \left(x+\frac{d}{e}\right)^{2}+2 d e \left(x+\frac{d}{e}\right)}}{3 d e \left(x+\frac{d}{e}\right)^{2}}-\frac{\sqrt{-e^{2} \left(x+\frac{d}{e}\right)^{2}+2 d e \left(x+\frac{d}{e}\right)}}{3 d^{2} \left(x+\frac{d}{e}\right)}\right)}{5 d}}{e^{3}}$	145

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(2*e^2*x^2+6*d*e*x+7*d^2)/d^3/(e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{7\,e^3x^3+21\,de^2x^2+21\,d^2ex+7\,d^3+(2\,e^2x^2+6\,dex+7\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^4x^3+3\,d^4e^3x^2+3\,d^5e^2x+d^6e)}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(7*e^3*x^3 + 21*d*e^2*x^2 + 21*d^2*e*x + 7*d^3 + (2*e^2*x^2 + 6*d*e*
x + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + 3*d^4*e^3*x^2 + 3*d^5*e^2*
x + d^6*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)\,(d+ex)}\,(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{5\,(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e)} - \frac{2\,\sqrt{-e^2x^2+d^2}}{15\,(d^2e^3x^2+2\,d^3e^2x+d^4e)} - \frac{2\,\sqrt{-e^2x^2+d^2}}{15\,(d^3e^2x+d^4e)}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e
) - 2/15*sqrt(-e^2*x^2 + d^2)/(d^2*e^3*x^2 + 2*d^3*e^2*x + d^4*e) - 2/15*s
qrt(-e^2*x^2 + d^2)/(d^3*e^2*x + d^4*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(76) = 152.

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.88

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x}+\frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3}+\frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^5|e|}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 40*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 7)/(d^3*
((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(7\,d^2+6\,d\,e\,x+2\,e^2\,x^2\right)}{15\,d^3\,e\,\left(d+e\,x\right)^3}$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 + 6*d*e*x))/(15*d^3*e*(d + e*x)
^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$$
$$= \frac{\frac{2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^5}{5}-\frac{4\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2}{3}-\frac{2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)}{3}-\frac{8}{15}}{d^3e\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^5+5\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^4+10\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3+10\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2+5\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)+1\right.}$$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(2*(3*tan(asin((e*x)/d)/2)**5 - 10*tan(asin((e*x)/d)/2)**2 - 5*tan(asin((e
*x)/d)/2) - 4))/(15*d**3*e*(tan(asin((e*x)/d)/2)**5 + 5*tan(asin((e*x)/d)/
2)**4 + 10*tan(asin((e*x)/d)/2)**3 + 10*tan(asin((e*x)/d)/2)**2 + 5*tan(as
in((e*x)/d)/2) + 1))
```

3.118 $\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{x}{35d^2(d^2-e^2x^2)^{3/2}} - \frac{2}{35e(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2x}{35d^4\sqrt{d^2-e^2x^2}} - \frac{2}{7e(d+ex)^3\sqrt{d^2-e^2x^2}}$$

output
```
1/35*x/d^2/(-e^2*x^2+d^2)^(3/2)-2/35/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/35*x
/d^4/(-e^2*x^2+d^2)^(1/2)-2/7/e/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-12d^3-13d^2ex-8de^2x^2-2e^3x^3)}{35d^4e(d+ex)^4}$$

input
```
Integrate[1/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-12*d^3 - 13*d^2*e*x - 8*d*e^2*x^2 - 2*e^3*x^3))/(35
*d^4*e*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{3\int \frac{1}{(d+ex)^3\sqrt{d^2-e^2x^2}}dx}{7d}-\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}$$

$$\Big\downarrow \text{461}$$

$$\frac{3\left(\frac{2\int \frac{1}{(d+ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}\right)}{7d}-\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}$$

$$\Big\downarrow \text{461}$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}dx}{3d}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}\right)}{7d}-\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}$$

$$\Big\downarrow \text{460}$$

$$\frac{3\left(\frac{2\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d+ex)}-\frac{\sqrt{d^2-e^2x^2}}{3de(d+ex)^2}\right)}{5d}-\frac{\sqrt{d^2-e^2x^2}}{5de(d+ex)^3}\right)}{7d}-\frac{\sqrt{d^2-e^2x^2}}{7de(d+ex)^4}$$

input
```
Int[1/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

output
```
-1/7*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^4) + (3*(-1/5*Sqrt[d^2 - e^2*x^2]/
(d*e*(d + e*x)^3) + (2*(-1/3*Sqrt[d^2 - e^2*x^2]/(d*e*(d + e*x)^2) - Sqrt[
d^2 - e^2*x^2]/(3*d^2*e*(d + e*x))))/(5*d)))/(7*d)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.55

method	result	si
trager	$-\frac{\left(2 e^{3} x^{3}+8 d\,e^{2} x^{2}+13 d^{2} e x +12 d^{3}\right) \sqrt{-e^{2} x^{2}+d^{2}}}{35 d^{4} \left(e x +d \right)^{4} e}$	60
gosper	$-\frac{\left(-e x +d \right) \left(2 e^{3} x^{3}+8 d\,e^{2} x^{2}+13 d^{2} e x +12 d^{3}\right)}{35 \left(e x +d \right)^{3} d^{4} e \sqrt{-e^{2} x^{2}+d^{2}}}$	60
orering	$-\frac{\left(-e x +d \right) \left(2 e^{3} x^{3}+8 d\,e^{2} x^{2}+13 d^{2} e x +12 d^{3}\right)}{35 \left(e x +d \right)^{3} d^{4} e \sqrt{-e^{2} x^{2}+d^{2}}}$	60
default	$\frac{-\frac{\sqrt{-e^{2} \left(x +\frac{d}{e}\right)^{2}+2 d e \left(x +\frac{d}{e}\right)}}{7 d e \left(x +\frac{d}{e}\right)^{4}}+\frac{3 e \left(-\frac{\sqrt{-e^{2} \left(x +\frac{d}{e}\right)^{2}+2 d e \left(x +\frac{d}{e}\right)}}{5 d e \left(x +\frac{d}{e}\right)^{3}}+\frac{2 e \left(-\frac{\sqrt{-e^{2} \left(x +\frac{d}{e}\right)^{2}+2 d e \left(x +\frac{d}{e}\right)}}{3 d e \left(x +\frac{d}{e}\right)^{2}}-\frac{\sqrt{-e^{2} \left(x +\frac{d}{e}\right)^{2}+2 d e \left(x +\frac{d}{e}\right)}}{3 d^{2} \left(x +\frac{d}{e}\right)}\right)}{5 d}\right)}{7 d}}{e^{4}}$	19

input
```
int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(2*e^3*x^3+8*d*e^2*x^2+13*d^2*e*x+12*d^3)/d^4/(e*x+d)^4/e*(-e^2*x^2+
d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.26

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{12\,e^4x^4+48\,de^3x^3+72\,d^2e^2x^2+48\,d^3ex+12\,d^4+(2\,e^3x^3+8\,de^2x^2+13\,d^2ex+12\,d^3)\sqrt{-e^2x^2+d^2}}{35\,(d^4e^5x^4+4\,d^5e^4x^3+6\,d^6e^3x^2+4\,d^7e^2x+d^8e)}$$

input
```
integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/35*(12*e^4*x^4 + 48*d*e^3*x^3 + 72*d^2*e^2*x^2 + 48*d^3*e*x + 12*d^4 +
(2*e^3*x^3 + 8*d*e^2*x^2 + 13*d^2*e*x + 12*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4
*e^5*x^4 + 4*d^5*e^4*x^3 + 6*d^6*e^3*x^2 + 4*d^7*e^2*x + d^8*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \int \frac{1}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)^4}\,dx$$

input
```
integrate(1/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(93) = 186.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.77

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}}{7\,(de^5x^4+4\,d^2e^4x^3+6\,d^3e^3x^2+4\,d^4e^2x+d^5e)} - \frac{3\,\sqrt{-e^2x^2+d^2}}{35\,(d^2e^4x^3+3\,d^3e^3x^2+3\,d^4e^2x+d^5e)} - \frac{2\,\sqrt{-e^2x^2+d^2}}{35\,(d^3e^3x^2+2\,d^4e^2x+d^5e)} - \frac{2\,\sqrt{-e^2x^2+d^2}}{35\,(d^4e^2x+d^5e)}$$

input
```
integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/7*sqrt(-e^2*x^2 + d^2)/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*x^2 + 4*d
^4*e^2*x + d^5*e) - 3/35*sqrt(-e^2*x^2 + d^2)/(d^2*e^4*x^3 + 3*d^3*e^3*x^2
 + 3*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)/(d^3*e^3*x^2 + 2*d^4*e
^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)/(d^4*e^2*x + d^5*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(93) = 186$.

Time = 0.13 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.08

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{2\left(\frac{49\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} + \frac{147\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{210\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} + \frac{210\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + \frac{105\left(de+\sqrt{-\cdots}\right.}{e^{1\cdots}} \cdots\right.}{35\,d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}+1\right)^7|e|}$$

input
```
integrate(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

output
```
2/35*(49*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) + 147*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 210*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/(e^6*x^3) + 210*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 105*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) + 35*(d*e + sqrt(-e^2*x^2
+ d^2)*abs(e))^6/(e^12*x^6) + 12)/(d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e)
)/(e^2*x) + 1)^7*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}}{7\,d\,e\,{(d+e\,x)}^4}-\frac{3\,\sqrt{d^2-e^2\,x^2}}{35\,d^2\,e\,{(d+e\,x)}^3} -\frac{2\,\sqrt{d^2-e^2\,x^2}}{35\,d^3\,e\,{(d+e\,x)}^2}-\frac{2\,\sqrt{d^2-e^2\,x^2}}{35\,d^4\,e\,\left(d+e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^4),x)
```

output
```
- (d^2 - e^2*x^2)^(1/2)/(7*d*e*(d + e*x)^4) - (3*(d^2 - e^2*x^2)^(1/2))/(3
5*d^2*e*(d + e*x)^3) - (2*(d^2 - e^2*x^2)^(1/2))/(35*d^3*e*(d + e*x)^2) -
(2*(d^2 - e^2*x^2)^(1/2))/(35*d^4*e*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}\,(-2e^3x^3-8d\,e^2x^2-13d^2ex-12d^3)}{35d^4e\,(e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4)}$$

input
```
int(1/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x)
```

output
```
(sqrt(d**2 - e**2*x**2)*( - 12*d**3 - 13*d**2*e*x - 8*d*e**2*x**2 - 2*e**3
*x**3))/(35*d**4*e*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 +
 e**4*x**4))
```

3.119 $\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}} + \frac{12d^2\sqrt{d^2-e^2x^2}}{e} + \frac{5}{2}dx\sqrt{d^2-e^2x^2} - \frac{(d^2-e^2x^2)^{3/2}}{3e} - \frac{35d^3\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
16*d^3*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+12*d^2*(-e^2*x^2+d^2)^(1/2)/e+5/2*d*
x*(-e^2*x^2+d^2)^(1/2)-1/3*(-e^2*x^2+d^2)^(3/2)/e-35/2*d^3*arctan(e*x/(-e^
2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(-166d^3+55d^2ex+13de^2x^2+2e^3x^3)}{6e(d-ex)} + \frac{35d^3\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{2\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)^5/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/6*(Sqrt[d^2 - e^2*x^2]*(-166*d^3 + 55*d^2*e*x + 13*d*e^2*x^2 + 2*e^3*x^
3))/(e*(d - e*x)) + (35*d^3*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/(2
*Sqrt[-e^2])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {462, 2346, 27, 2346, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{462}$$

$$\frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}} - \int \frac{15d^3+11exd^2+5e^2x^2d+e^3x^3}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int -\frac{5(3dx^2e^4+7d^2xe^3+9d^3e^2)}{\sqrt{d^2-e^2x^2}}dx}{3e^2} + \frac{1}{3}ex^2\sqrt{d^2-e^2x^2} + \frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\int \frac{3dx^2e^4+7d^2xe^3+9d^3e^2}{\sqrt{d^2-e^2x^2}}dx}{3e^2} + \frac{1}{3}ex^2\sqrt{d^2-e^2x^2} + \frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{5\left(-\frac{\int -\frac{7d^2e^4(3d+2ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2} - \frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2} + \frac{1}{3}ex^2\sqrt{d^2-e^2x^2} + \frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{7}{2}d^2e^2\int\frac{3d+2ex}{\sqrt{d^2-e^2x^2}}dx-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}+\frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$\big\downarrow$ 455

$$-\frac{5\left(\frac{7}{2}d^2e^2\left(3d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}+\frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$\big\downarrow$ 224

$$-\frac{5\left(\frac{7}{2}d^2e^2\left(3d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}+\frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$\big\downarrow$ 216

$$-\frac{5\left(\frac{7}{2}d^2e^2\left(\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{2\sqrt{d^2-e^2x^2}}{e}\right)-\frac{3}{2}de^2x\sqrt{d^2-e^2x^2}\right)}{3e^2}+\frac{1}{3}ex^2\sqrt{d^2-e^2x^2}+\frac{16d^3(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

input
```
Int[(d + e*x)^5/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
(16*d^3*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) + (e*x^2*Sqrt[d^2 - e^2*x^2])/3
 - (5*((-3*d*e^2*x*Sqrt[d^2 - e^2*x^2])/2 + (7*d^2*e^2*((-2*Sqrt[d^2 - e^2
*x^2])/e + (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2))/(3*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

method	result
risch	$\frac{(2e^2x^2+15dex+70d^2)\sqrt{-e^2x^2+d^2}}{6e}-\frac{35d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{16d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{e^2\left(x-\frac{d}{e}\right)}$
default	$\frac{d^3x}{\sqrt{-e^2x^2+d^2}}+e^5\left(-\frac{x^4}{3e^2\sqrt{-e^2x^2+d^2}}+\frac{4d^2\left(-\frac{x^2}{e^2\sqrt{-e^2x^2+d^2}}+\frac{2d^2}{e^4\sqrt{-e^2x^2+d^2}}\right)}{3e^2}\right)+5d\,e^4\left(-\frac{x^3}{2e^2\sqrt{-e^2x^2+d^2}}+\frac{3d^2}{}\right.$

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*e^2*x^2+15*d*e*x+70*d^2)/e*(-e^2*x^2+d^2)^(1/2)-35/2*d^3/(e^2)^(1/2
)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-16*d^3/e^2/(x-d/e)*(-(x-d/e)^
2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{166\,d^3ex - 166\,d^4 + 210\,(d^3ex - d^4)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (2\,e^3x^3 + 13\,de^2x^2 +}{6\,(e^2x - de)}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(166*d^3*e*x - 166*d^4 + 210*(d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (2*e^3*x^3 + 13*d*e^2*x^2 + 55*d^2*e*x - 166*d^3)*sqrt(
-e^2*x^2 + d^2))/(e^2*x - d*e)
```

Sympy [F]

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^5}{(-(-d+ex)\,(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**5/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**5/(-(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{e^3x^4}{3\sqrt{-e^2x^2+d^2}} - \frac{5\,de^2x^3}{2\sqrt{-e^2x^2+d^2}} - \frac{34\,d^2ex^2}{3\sqrt{-e^2x^2+d^2}} + \frac{37\,d^3x}{2\sqrt{-e^2x^2+d^2}} - \frac{35\,d^3\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + \frac{83\,d^4}{3\sqrt{-e^2x^2+d^2}e}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*e^3*x^4/sqrt(-e^2*x^2 + d^2) - 5/2*d*e^2*x^3/sqrt(-e^2*x^2 + d^2) - 3
4/3*d^2*e*x^2/sqrt(-e^2*x^2 + d^2) + 37/2*d^3*x/sqrt(-e^2*x^2 + d^2) - 35/
2*d^3*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 83/3*d^4/(sqrt(-e^2*x^2 + d^
2)*e)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{35\,d^3\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{1}{6}\sqrt{-e^2x^2+d^2}\left((2\,ex+15\,d)x + \frac{70\,d^2}{e}\right) + \frac{32\,d^3}{\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)|e|}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-35/2*d^3*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/6*sqrt(-e^2*x^2 + d^2)*((
2*e*x + 15*d)*x + 70*d^2/e) + 32*d^3/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
/(e^2*x) - 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^5}{(d^2-e^2\,x^2)^{3/2}}\,dx$$

input
```
int((d + e*x)^5/(d^2 - e^2*x^2)^(3/2),x)
```

output
```
int((d + e*x)^5/(d^2 - e^2*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{-105\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^3+105asin\left(\frac{ex}{d}\right)d^4-105asin\left(\frac{ex}{d}\right)d^3ex-222\sqrt{-e^2x^2}}{}$$

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
( - 105*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**3 + 105*asin((e*x)/d)*d**4
 - 105*asin((e*x)/d)*d**3*e*x - 222*sqrt(d**2 - e**2*x**2)*d**3 + 55*sqrt(
d**2 - e**2*x**2)*d**2*e*x + 13*sqrt(d**2 - e**2*x**2)*d*e**2*x**2 + 2*sqr
t(d**2 - e**2*x**2)*e**3*x**3 + 222*d**4 + 55*d**3*e*x - 68*d**2*e**2*x**2
 - 15*d*e**3*x**3 - 2*e**4*x**4)/(6*e*(sqrt(d**2 - e**2*x**2) - d + e*x))
```

3.120 $\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{4d\sqrt{d^2-e^2x^2}}{e}+\frac{1}{2}x\sqrt{d^2-e^2x^2}-\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
8*d^2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+4*d*(-e^2*x^2+d^2)^(1/2)/e+1/2*x*(-e^
2*x^2+d^2)^(1/2)-15/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{\sqrt{d^2-e^2x^2}(-24d^2+7dex+e^2x^2)}{2e(d-ex)}+\frac{15d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(d + e*x)^4/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[d^2 - e^2*x^2]*(-24*d^2 + 7*d*e*x + e^2*x^2))/(e*(d - e*x)) + (
15*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/e
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {462, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{462}$$

$$\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int \frac{7d^2+4exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int -\frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}d\int \frac{15d+8ex}{\sqrt{d^2-e^2x^2}}dx+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{455}$$

$$-\frac{1}{2}d\left(15d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{224}$$

$$-\frac{1}{2}d\left(15d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{1}{2}d\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}$$

input
```
Int[(d + e*x)^4/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
(8*d^2*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) + (x*Sqrt[d^2 - e^2*x^2])/2 - (d
*((-8*Sqrt[d^2 - e^2*x^2])/e + (15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e)
)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

method	result
risch	$\frac{(ex+8d)\sqrt{-e^2x^2+d^2}}{2e}-\frac{15d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{8d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{e^2\left(x-\frac{d}{e}\right)}$
default	$\frac{d^2x}{\sqrt{-e^2x^2+d^2}}+e^4\left(-\frac{x^3}{2e^2\sqrt{-e^2x^2+d^2}}+\frac{3d^2\left(\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}\right)}{2e^2}\right)+4d\,e^3\left(-\frac{x^2}{e^2\sqrt{-e^2x^2+d^2}}+\right.$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+8*d)/e*(-e^2*x^2+d^2)^(1/2)-15/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))-8*d^2/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{24\,d^2ex - 24\,d^3 + 30\,(d^2ex - d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (e^2x^2 + 7\,dex - 24\,d^2)\sqrt{}}{2\,(e^2x - de)}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(24*d^2*e*x - 24*d^3 + 30*(d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (e^2*x^2 + 7*d*e*x - 24*d^2)*sqrt(-e^2*x^2 + d^2))/(e^2*x
- d*e)
```

Sympy [F]

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^4}{(-(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**4/(-(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{e^2x^3}{2\sqrt{-e^2x^2+d^2}} - \frac{4\,dex^2}{\sqrt{-e^2x^2+d^2}} + \frac{17\,d^2x}{2\sqrt{-e^2x^2+d^2}} - \frac{15\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}} + \frac{12\,d^3}{\sqrt{-e^2x^2+d^2}e}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*e^2*x^3/sqrt(-e^2*x^2 + d^2) - 4*d*e*x^2/sqrt(-e^2*x^2 + d^2) + 17/2*
d^2*x/sqrt(-e^2*x^2 + d^2) - 15/2*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2
) + 12*d^3/(sqrt(-e^2*x^2 + d^2)*e)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{15\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(x+\frac{8\,d}{e}\right) + \frac{16\,d^2}{\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)|e|}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-15/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*sqrt(-e^2*x^2 + d^2)*(x
 + 8*d/e) + 16*d^2/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)*abs(
e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^4}{(d^2-e^2\,x^2)^{3/2}}\,dx$$

input
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(3/2),x)
```

output
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.59

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{-15\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^2+15asin\left(\frac{ex}{d}\right)d^3-15asin\left(\frac{ex}{d}\right)d^2ex-34\sqrt{-e^2x^2+d}}{2e\left(\sqrt{-e^2x^2+d}\right.}$$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
( - 15*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**2 + 15*asin((e*x)/d)*d**3 -
 15*asin((e*x)/d)*d**2*e*x - 34*sqrt(d**2 - e**2*x**2)*d**2 + 7*sqrt(d**2
- e**2*x**2)*d*e*x + sqrt(d**2 - e**2*x**2)*e**2*x**2 + 34*d**3 + 7*d**2*e
*x - 8*d*e**2*x**2 - e**3*x**3)/(2*e*(sqrt(d**2 - e**2*x**2) - d + e*x))
```

3.121 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
4*d*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+(-e^2*x^2+d^2)^(1/2)/e-3*d*arctan(e*x/(
-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=-\frac{(-5d+ex)\sqrt{d^2-e^2x^2}}{e(d-ex)}+\frac{3d\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
-(((-5*d + e*x)*Sqrt[d^2 - e^2*x^2])/(e*(d - e*x))) + (3*d*Log[-(Sqrt[-e^2
]*x) + Sqrt[d^2 - e^2*x^2]])/Sqrt[-e^2]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {462, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 462$$

$$\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int \frac{3d+ex}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \; 455$$

$$-3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}$$

$$\Big\downarrow \; 224$$

$$-3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}$$

$$\Big\downarrow \; 216$$

$$-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}$$

input
```
Int[(d + e*x)^3/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
(4*d*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e - (3*d*Arc
Tan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

method	result
risch	$\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e}-\frac{3 d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{4 d \sqrt{-\left(x-\frac{d}{e}\right)^{2} e^{2}-2 d e \left(x-\frac{d}{e}\right)}}{e^{2} \left(x-\frac{d}{e}\right)}$
default	$\frac{dx}{\sqrt{-e^{2} x^{2}+d^{2}}}+e^{3}\left(-\frac{x^{2}}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}+\frac{2 d^{2}}{e^{4} \sqrt{-e^{2} x^{2}+d^{2}}}\right)+3 d\, e^{2}\left(\frac{x}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}\right)+\frac{}{e\sqrt{-}}$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e-3*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))-4*d/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{5\,dex-5\,d^2+6\,(dex-d^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}(ex-5\,d)}{e^2x-de}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
(5*d*e*x - 5*d^2 + 6*(d*e*x - d^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x
)) + sqrt(-e^2*x^2 + d^2)*(e*x - 5*d))/(e^2*x - d*e)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{ex^2}{\sqrt{-e^2x^2+d^2}}+\frac{4\,dx}{\sqrt{-e^2x^2+d^2}} - \frac{3\,d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}+\frac{5\,d^2}{\sqrt{-e^2x^2+d^2}e}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-e*x^2/sqrt(-e^2*x^2 + d^2) + 4*d*x/sqrt(-e^2*x^2 + d^2) - 3*d*arcsin(e^2*
x/(d*sqrt(e^2)))/sqrt(e^2) + 5*d^2/(sqrt(-e^2*x^2 + d^2)*e)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{3\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{\sqrt{-e^2x^2+d^2}}{e} + \frac{8\,d}{\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)|e|}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-3*d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + sqrt(-e^2*x^2 + d^2)/e + 8*d/(((
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{(d^2-e^2\,x^2)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(3/2),x)
```

output
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{-3\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d+3asin\left(\frac{ex}{d}\right)d^2-3asin\left(\frac{ex}{d}\right)dex-8\sqrt{-e^2x^2+d^2}\,d+\sqrt{}}{e\left(\sqrt{-e^2x^2+d^2}-d+ex\right)}$$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
( - 3*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d + 3*asin((e*x)/d)*d**2 - 3*as
in((e*x)/d)*d*e*x - 8*sqrt(d**2 - e**2*x**2)*d + sqrt(d**2 - e**2*x**2)*e*
x + 8*d**2 + d*e*x - e**2*x**2)/(e*(sqrt(d**2 - e**2*x**2) - d + e*x))
```

3.122 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2\left(\frac{\sqrt{d^2-e^2x^2}}{d-ex}+\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)\right)}{e}$$

input
```
Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
(2*(Sqrt[d^2 - e^2*x^2]/(d - e*x) + ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e
^2*x^2])]))/e
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 457$$

$$\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \int \frac{1}{\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \; 224$$

$$\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \; 216$$

$$\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Int[(d + e*x)^2/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
(2*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) - ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/
e
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.81

method	result	size
default	$\frac{x}{\sqrt{-e^{2} x^{2}+d^{2}}}+e^{2} \left(\frac{x}{e^{2} \sqrt{-e^{2} x^{2}+d^{2}}}-\frac{\arctan \left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{e^{2} \sqrt{e^{2}}}\right)+\frac{2 d}{e \sqrt{-e^{2} x^{2}+d^{2}}}$	94

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(-e^2*x^2+d^2)^(1/2)+e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))+2*d/e/(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2\left(ex+(ex-d)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-d-\sqrt{-e^2x^2+d^2}\right)}{e^2x-de}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
2*(e*x + (e*x - d)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - d - sqrt(-e
^2*x^2 + d^2))/(e^2*x - d*e)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{2\,x}{\sqrt{-e^2x^2+d^2}} - \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{2\,d}{\sqrt{-e^2x^2+d^2}e}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
2*x/sqrt(-e^2*x^2 + d^2) - arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 2*d/(sq
rt(-e^2*x^2 + d^2)*e)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{\arcsin\left(\frac{ex}{d}\right)\mathrm{sgn}(d)\,\mathrm{sgn}(e)}{|e|} + \frac{4}{\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)|e|}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
-arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 4/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(
e))/(e^2*x) - 1)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{2\,x}{\sqrt{d^2-e^2\,x^2}}-\frac{\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}+\frac{2\,d}{e\,\sqrt{d^2-e^2\,x^2}}$$

input
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(3/2),x)
```

output
```
(2*x)/(d^2 - e^2*x^2)^(1/2) - log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2))/
(-e^2)^(1/2) + (2*d)/(e*(d^2 - e^2*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{-asin\left(\frac{ex}{d}\right)\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)+asin\left(\frac{ex}{d}\right)-4\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)}{e\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)-1\right)}$$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
( - asin((e*x)/d)*tan(asin((e*x)/d)/2) + asin((e*x)/d) - 4*tan(asin((e*x)/
d)/2))/(e*(tan(asin((e*x)/d)/2) - 1))
```

3.123 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{d+ex}{de\sqrt{d^2-e^2x^2}}$$

output
```
(e*x+d)/d/e/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}}{de(d-ex)}$$

input
```
Integrate[(d + e*x)/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(d*e*(d - e*x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 453}$$

$$\frac{d+ex}{de\sqrt{d^2-e^2x^2}}$$

input
```
Int[(d + e*x)/(d^2 - e^2*x^2)^(3/2),x]
```

output
```
(d + e*x)/(d*e*Sqrt[d^2 - e^2*x^2])
```

Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
trager	$\frac{\sqrt{-e^2x^2+d^2}}{de(-ex+d)}$	30
gosper	$\frac{(ex+d)^2(-ex+d)}{de(-e^2x^2+d^2)^{\frac{3}{2}}}$	35
orering	$\frac{(ex+d)^2(-ex+d)}{de(-e^2x^2+d^2)^{\frac{3}{2}}}$	35
default	$\frac{x}{d\sqrt{-e^2x^2+d^2}}+\frac{1}{e\sqrt{-e^2x^2+d^2}}$	39

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d/e/(-e*x+d)*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx=\frac{ex-d-\sqrt{-e^2x^2+d^2}}{de^2x-d^2e}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
(e*x - d - sqrt(-e^2*x^2 + d^2))/(d*e^2*x - d^2*e)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx = d\left(\begin{cases} -\frac{ix}{d^3\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{x}{d^3\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} \frac{1}{e^2\sqrt{d^2-e^2x^2}} & \text{for } e\neq 0 \\ \frac{x^2}{2(d^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
d*Piecewise((-I*x/(d**3*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (x/(d**3*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piecewise((1/(e**2*sqrt
(d**2 - e**2*x**2)), Ne(e, 0)), (x**2/(2*(d**2)**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{x}{\sqrt{-e^2x^2+d^2}d} + \frac{1}{\sqrt{-e^2x^2+d^2}e}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
x/(sqrt(-e^2*x^2 + d^2)*d) + 1/(sqrt(-e^2*x^2 + d^2)*e)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{2}{d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)|e|}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
2/(d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}}{d\,e\,(d-e\,x)}$$

input
```
int((d + e*x)/(d^2 - e^2*x^2)^(3/2),x)
```

output
```
(d^2 - e^2*x^2)^(1/2)/(d*e*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{d+ex}{(d^2-e^2x^2)^{3/2}}\,dx = -\frac{2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)}{de\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)-1\right)}$$

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
( - 2*tan(asin((e*x)/d)/2))/(d*e*(tan(asin((e*x)/d)/2) - 1))
```

3.124 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{2x}{3d^3\sqrt{d^2-e^2x^2}} - \frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

output
```
2/3*x/d^3/(-e^2*x^2+d^2)^(1/2)-1/3/d/e/(e*x+d)/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-d^2+2dex+2e^2x^2)}{3d^3e(d-ex)(d+ex)^2}$$

input
```
Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-d^2 + 2*d*e*x + 2*e^2*x^2))/(3*d^3*e*(d - e*x)*(d + 
 e*x)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {470, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{470}$$

$$\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/((d + e*x)*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2]
)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

method	result	size
gosper	$-\frac{(-ex+d)(-2e^2x^2-2dex+d^2)}{3d^3e(-e^2x^2+d^2)^{\frac{3}{2}}}$	46
orering	$-\frac{(-ex+d)(-2e^2x^2-2dex+d^2)}{3d^3e(-e^2x^2+d^2)^{\frac{3}{2}}}$	46
trager	$-\frac{(-2e^2x^2-2dex+d^2)\sqrt{-e^2x^2+d^2}}{3d^3(ex+d)^2e(-ex+d)}$	55
default	$\frac{-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e\,d^3\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}}{e}$	104

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e*x+d)*(-2*e^2*x^2-2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(50) = 100.

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{e^3x^3+de^2x^2-d^2ex-d^3+\left(2\,e^2x^2+2\,dex-d^2\right)\sqrt{-e^2x^2+d^2}}{3\left(d^3e^4x^3+d^4e^3x^2-d^5e^2x-d^6e\right)}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3 + (2*e^2*x^2 + 2*d*e*x - d^2)*sq
rt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 + d^4*e^3*x^2 - d^5*e^2*x - d^6*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = -\frac{1}{3\left(\sqrt{-e^2x^2+d^2}de^2x+\sqrt{-e^2x^2+d^2}d^2e\right)}+\frac{2\,x}{3\,\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/3/(sqrt(-e^2*x^2 + d^2)*d*e^2*x + sqrt(-e^2*x^2 + d^2)*d^2*e) + 2/3*x/(
sqrt(-e^2*x^2 + d^2)*d^3)
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}\left(ex+d\right)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-d^2+2\,d\,e\,x+2\,e^2\,x^2\right)}{3\,d^3\,e\,{\left(d+e\,x\right)}^2\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*e^2*x^2 - d^2 + 2*d*e*x))/(3*d^3*e*(d + e*x)^2*(
d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{2\sqrt{-e^2x^2+d^2}\,d+2\sqrt{-e^2x^2+d^2}\,ex-d^2+2dex+2e^2x^2}{3\sqrt{-e^2x^2+d^2}\,d^3e\,(ex+d)}$$

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
(2*sqrt(d**2 - e**2*x**2)*d + 2*sqrt(d**2 - e**2*x**2)*e*x - d**2 + 2*d*e*
x + 2*e**2*x**2)/(3*sqrt(d**2 - e**2*x**2)*d**3*e*(d + e*x))
```

3.125 $\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}-\frac{2}{5e(d+ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

output
```
1/5*x/d^2/(-e^2*x^2+d^2)^(3/2)-2/5/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+2/5*x/d^
4/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-2d^3+d^2ex+4de^2x^2+2e^3x^3)}{5d^4e(d-ex)(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-2*d^3 + d^2*e*x + 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*
e*(d - e*x)*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {461, 470, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{3\int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}}dx}{5d}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{3\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{3\left(\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
-1/5*1/(d*e*(d + e*x)^2*Sqrt[d^2 - e^2*x^2]) + (3*((2*x)/(3*d^3*Sqrt[d^2 -
 e^2*x^2]) - 1/(3*d*e*(d + e*x)*Sqrt[d^2 - e^2*x^2])))/(5*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{(-ex+d)\left(-2e^{3}x^{3}-4d\,e^{2}x^{2}-d^{2}ex+2d^{3}\right)}{5(ex+d)d^{4}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}$	66
orering	$-\frac{(-ex+d)\left(-2e^{3}x^{3}-4d\,e^{2}x^{2}-d^{2}ex+2d^{3}\right)}{5(ex+d)d^{4}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}$	66
trager	$-\frac{\left(-2e^{3}x^{3}-4d\,e^{2}x^{2}-d^{2}ex+2d^{3}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{5d^{4}(ex+d)^{3}e(-ex+d)}$	68
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)^{2}\sqrt{-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)}}-\frac{-2e^{2}\left(x+\frac{d}{e}\right)+2de}{3e\,d^{3}\sqrt{-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)}}\right)}{5d}}{e^{2}}$	156

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-e*x+d)*(-2*e^3*x^3-4*d*e^2*x^2-d^2*e*x+2*d^3)/(e*x+d)/d^4/e/(-e^2*x
^2+d^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.46

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = -\frac{2 e^4x^4 + 4 de^3x^3 - 4 d^3ex - 2 d^4 + (2 e^3x^3 + 4 de^2x^2 + d^2ex - 2 d^3)\sqrt{-e^2x^2 + d^2}}{5 (d^4e^5x^4 + 2 d^5e^4x^3 - 2 d^7e^2x - d^8e)}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(2*e^4*x^4 + 4*d*e^3*x^3 - 4*d^3*e*x - 2*d^4 + (2*e^3*x^3 + 4*d*e^2*x
^2 + d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 + 2*d^5*e^4*x^3 -
 2*d^7*e^2*x - d^8*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}} dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{3}{2}} (d+ex)^2} dx$$

input
```
integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(67) = 134$.

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = -\frac{1}{5\left(\sqrt{-e^2x^2+d^2}de^3x^2+2\sqrt{-e^2x^2+d^2}d^2e^2x+\sqrt{-e^2x^2+d^2}d^3e\right)} - \frac{1}{5\left(\sqrt{-e^2x^2+d^2}d^2e^2x+\sqrt{-e^2x^2+d^2}d^3e\right)} + \frac{2x}{5\sqrt{-e^2x^2+d^2}d^4}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/5/(sqrt(-e^2*x^2 + d^2)*d*e^3*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d^2*e^2*x +
sqrt(-e^2*x^2 + d^2)*d^3*e) - 1/5/(sqrt(-e^2*x^2 + d^2)*d^2*e^2*x + sqrt(-
e^2*x^2 + d^2)*d^3*e) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.44

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{e^3\left(\frac{5}{d^4e^3\sqrt{\frac{2d}{ex+d}-1}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{d^{16}e^{12}\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)^4\operatorname{sgn}(e)^4+5d^{16}e^{12}\left(\frac{2d}{ex+d}\right.}{d^{20}e^{15}\mathrm{s}}\right.}{40\,|}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
1/40*(e^3*(5/(d^4*e^3*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))*sgn(e)) - (
d^16*e^12*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 5*d^16*e
^12*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^4*sgn(e)^4 + 15*d^16*e^12*s
qrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^4*sgn(e)^4)/(d^20*e^15*sgn(1/(e*x
+ d))^5*sgn(e)^5)) + 16*I*sgn(1/(e*x + d))*sgn(e)/d^4)/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(-2\,d^3+d^2\,e\,x+4\,d\,e^2\,x^2+2\,e^3\,x^3\right)}{5\,d^4\,e\,{\left(d+e\,x\right)}^3\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*e^3*x^3 - 2*d^3 + 4*d*e^2*x^2 + d^2*e*x))/(5*d^4
*e*(d + e*x)^3*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.59

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}\,d^2+2\sqrt{-e^2x^2+d^2}\,dex+\sqrt{-e^2x^2+d^2}\,e^2x^2-4d^3+2d^2ex}{10\sqrt{-e^2x^2+d^2}\,d^4e\left(e^2x^2+2dex+d^2\right)}$$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
(sqrt(d**2 - e**2*x**2)*d**2 + 2*sqrt(d**2 - e**2*x**2)*d*e*x + sqrt(d**2
- e**2*x**2)*e**2*x**2 - 4*d**3 + 2*d**2*e*x + 8*d*e**2*x**2 + 4*e**3*x**3
)/(10*sqrt(d**2 - e**2*x**2)*d**4*e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.126 $\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{4x}{35d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{2}{7e(d+ex)^2\left(d^2-e^2x^2\right)^{3/2}} - \frac{3}{35de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{35d^5\sqrt{d^2-e^2x^2}}$$

output
```
4/35*x/d^3/(-e^2*x^2+d^2)^(3/2)-2/7/e/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2)-3/35/
d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+8/35*x/d^5/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-13d^4-4d^3ex+20d^2e^2x^2+24de^3x^3+8e^4x^4)}{35d^5e(d-ex)(d+ex)^4}$$

input
```
Integrate[1/((d + e*x)^3*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-13*d^4 - 4*d^3*e*x + 20*d^2*e^2*x^2 + 24*d*e^3*x^3
+ 8*e^4*x^4))/(35*d^5*e*(d - e*x)*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {461, 461, 470, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{3/2}} dx$$

$$\Big\downarrow \text{461}$$

$$\frac{4\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{3/2}}dx}{7d} - \frac{1}{7de(d+ex)^3\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{4\left(\frac{3\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{3/2}}dx}{5d} - \frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}\right)}{7d} - \frac{1}{7de(d+ex)^3\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d} - \frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5d} - \frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}\right)}{7d} - \frac{1}{7de(d+ex)^3\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4\left(\frac{3\left(\frac{2x}{3d^3\sqrt{d^2-e^2x^2}} - \frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}\right)}{5d} - \frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}}\right)}{7d} - \frac{1}{7de(d+ex)^3\sqrt{d^2-e^2x^2}}$$

input
```
Int[1/((d + e*x)^3*(d^2 - e^2*x^2)^(3/2)),x]
```

output
```
-1/7*1/(d*e*(d + e*x)^3*Sqrt[d^2 - e^2*x^2]) + (4*(-1/5*1/(d*e*(d + e*x)^2
*Sqrt[d^2 - e^2*x^2]) + (3*((2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]) - 1/(3*d*e*(
d + e*x)*Sqrt[d^2 - e^2*x^2])))/(5*d)))/(7*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

method	result
gosper	$-\frac{(-ex+d)(-8e^4x^4-24d\,e^3x^3-20d^2e^2x^2+4d^3ex+13d^4)}{35(ex+d)^2d^5e(-e^2x^2+d^2)^{\frac{3}{2}}}$
orering	$-\frac{(-ex+d)(-8e^4x^4-24d\,e^3x^3-20d^2e^2x^2+4d^3ex+13d^4)}{35(ex+d)^2d^5e(-e^2x^2+d^2)^{\frac{3}{2}}}$
trager	$-\frac{(-8e^4x^4-24d\,e^3x^3-20d^2e^2x^2+4d^3ex+13d^4)\sqrt{-e^2x^2+d^2}}{35d^5(ex+d)^4e(-ex+d)}$
default	$\frac{-\frac{1}{7de\left(x+\frac{d}{e}\right)^3\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}+\frac{4e\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)^2\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}+\frac{3e\left(-\frac{1}{3de\left(x+\frac{d}{e}\right)\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}-\frac{}{3e\,d}\right.}{5d}\right.}{7d}}{e^3}$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(-e*x+d)*(-8*e^4*x^4-24*d*e^3*x^3-20*d^2*e^2*x^2+4*d^3*e*x+13*d^4)/(
e*x+d)^2/d^5/e/(-e^2*x^2+d^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d+ex)^3\,(d^2-e^2x^2)^{3/2}}\,dx =$$
$$-\frac{13\,e^5x^5+39\,de^4x^4+26\,d^2e^3x^3-26\,d^3e^2x^2-39\,d^4ex-13\,d^5+(8\,e^4x^4+24\,de^3x^3+20\,d^2e^2x^2-4\,d^3ex}{35\,(d^5e^6x^5+3\,d^6e^5x^4+2\,d^7e^4x^3-2\,d^8e^3x^2-3\,d^9e^2x-d^{10}e)}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(13*e^5*x^5 + 39*d*e^4*x^4 + 26*d^2*e^3*x^3 - 26*d^3*e^2*x^2 - 39*d^
4*e*x - 13*d^5 + (8*e^4*x^4 + 24*d*e^3*x^3 + 20*d^2*e^2*x^2 - 4*d^3*e*x -
13*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + 3*d^6*e^5*x^4 + 2*d^7*e^4*x^3
 - 2*d^8*e^3*x^2 - 3*d^9*e^2*x - d^10*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(3/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(96) = 192.

Time = 0.04 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.07

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{3/2}}\,dx =$$
$$-\frac{1}{7\left(\sqrt{-e^2x^2+d^2}de^4x^3+3\sqrt{-e^2x^2+d^2}d^2e^3x^2+3\sqrt{-e^2x^2+d^2}d^3e^2x+\sqrt{-e^2x^2+d^2}d^4e\right)}$$
$$-\frac{4}{35\left(\sqrt{-e^2x^2+d^2}d^2e^3x^2+2\sqrt{-e^2x^2+d^2}d^3e^2x+\sqrt{-e^2x^2+d^2}d^4e\right)}$$
$$-\frac{4}{35\left(\sqrt{-e^2x^2+d^2}d^3e^2x+\sqrt{-e^2x^2+d^2}d^4e\right)}+\frac{8x}{35\sqrt{-e^2x^2+d^2}d^5}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/7/(sqrt(-e^2*x^2 + d^2)*d*e^4*x^3 + 3*sqrt(-e^2*x^2 + d^2)*d^2*e^3*x^2
+ 3*sqrt(-e^2*x^2 + d^2)*d^3*e^2*x + sqrt(-e^2*x^2 + d^2)*d^4*e) - 4/35/(s
qrt(-e^2*x^2 + d^2)*d^2*e^3*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d^3*e^2*x + sqrt(
-e^2*x^2 + d^2)*d^4*e) - 4/35/(sqrt(-e^2*x^2 + d^2)*d^3*e^2*x + sqrt(-e^2*
x^2 + d^2)*d^4*e) + 8/35*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{3/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{3}{2}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^3), x)
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{3/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{8\,x}{35\,d^5}-\frac{29}{280\,d^4\,e}\right)}{(d+e\,x)\,(d-e\,x)} - \frac{\sqrt{d^2-e^2\,x^2}}{14\,d^2\,e\,(d+e\,x)^4} - \frac{13\,\sqrt{d^2-e^2\,x^2}}{140\,d^3\,e\,(d+e\,x)^3} - \frac{29\,\sqrt{d^2-e^2\,x^2}}{280\,d^4\,e\,(d+e\,x)^2}$$

input
```
int(1/((d^2 - e^2*x^2)^(3/2)*(d + e*x)^3),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((8*x)/(35*d^5) - 29/(280*d^4*e)))/((d + e*x)*(d -
e*x)) - (d^2 - e^2*x^2)^(1/2)/(14*d^2*e*(d + e*x)^4) - (13*(d^2 - e^2*x^2)
^(1/2))/(140*d^3*e*(d + e*x)^3) - (29*(d^2 - e^2*x^2)^(1/2))/(280*d^4*e*(d
 + e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{3/2}}\,dx = \frac{-4\sqrt{-e^2x^2+d^2}\,d^3-12\sqrt{-e^2x^2+d^2}\,d^2ex-12\sqrt{-e^2x^2+d^2}\,d\,e^2x^2-4\sqrt{}}{105\sqrt{-e^2x^2+d^2}\,d^5e\,(e^3x^3+}$$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(3/2),x)
```

output
```
( - 4*sqrt(d**2 - e**2*x**2)*d**3 - 12*sqrt(d**2 - e**2*x**2)*d**2*e*x - 1
2*sqrt(d**2 - e**2*x**2)*d*e**2*x**2 - 4*sqrt(d**2 - e**2*x**2)*e**3*x**3
- 39*d**4 - 12*d**3*e*x + 60*d**2*e**2*x**2 + 72*d*e**3*x**3 + 24*e**4*x**
4)/(105*sqrt(d**2 - e**2*x**2)*d**5*e*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 +
 e**3*x**3))
```

3.127 $\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx=\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{56d^2(d+ex)}{3e\sqrt{d^2-e^2x^2}}-\frac{28d\sqrt{d^2-e^2x^2}}{3e}-\frac{7}{6}x\sqrt{d^2-e^2x^2}+\frac{35d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
2/3*(e*x+d)^5/e/(-e^2*x^2+d^2)^(3/2)-56/3*d^2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/
2)-28/3*d*(-e^2*x^2+d^2)^(1/2)/e-7/6*x*(-e^2*x^2+d^2)^(1/2)+35/2*d^2*arcta
n(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx=-\frac{\frac{\sqrt{d^2-e^2x^2}(164d^3-229d^2ex+30de^2x^2+3e^3x^3)}{(d-ex)^2}+210d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{6e}$$

input
```
Integrate[(d + e*x)^6/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
-1/6*((Sqrt[d^2 - e^2*x^2]*(164*d^3 - 229*d^2*e*x + 30*d*e^2*x^2 + 3*e^3*x
^3))/(d - e*x)^2 + 210*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])]
)/e
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {468, 462, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{462}$$

$$\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\left(\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int \frac{7d^2+4exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx\right)$$

$$\Big\downarrow \text{2346}$$

$$\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\left(\frac{\int -\frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\left(-\frac{\int \frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\int \frac{15d+8ex}{\sqrt{d^2-e^2x^2}}dx+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)$$

$\downarrow$ 224

$$\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)$$

$\downarrow$ 216

$$\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)$$

input
```
Int[(d + e*x)^6/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^5)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (7*((8*d^2*(d + e*x))/(e*Sqr
t[d^2 - e^2*x^2]) + (x*Sqrt[d^2 - e^2*x^2])/2 - (d*((-8*Sqrt[d^2 - e^2*x^2
])/e + (15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{(ex+12d)\sqrt{-e^2x^2+d^2}}{2e}+\frac{35d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{80d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{3e^2\left(x-\frac{d}{e}\right)}+\frac{16d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\right.}}{3e^3\left(x-\frac{d}{e}\right)^2}$
default	$d^6\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)+e^6\left(-\frac{x^5}{2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{5d^2\left(\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan}{}}{e^2}\right.}{2e^2}\right.$

input
```
int((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x+12*d)/e*(-e^2*x^2+d^2)^(1/2)+35/2*d^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2))+80/3*d^2/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x
-d/e))^(1/2)+16/3*d^3/e^3/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int\frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx=$$

$$-\frac{164\,d^2e^2x^2-328\,d^3ex+164\,d^4+210\left(d^2e^2x^2-2\,d^3ex+d^4\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(3\,e^3x^3+30\,de^2x\right.}{6\left(e^3x^2-2\,de^2x+d^2e\right)}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(164*d^2*e^2*x^2 - 328*d^3*e*x + 164*d^4 + 210*(d^2*e^2*x^2 - 2*d^3*e
*x + d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^3*x^3 + 30*d*e^
2*x^2 - 229*d^2*e*x + 164*d^3)*sqrt(-e^2*x^2 + d^2))/(e^3*x^2 - 2*d*e^2*x
+ d^2*e)
```

Sympy [F]

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^6}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**6/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**6/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{35}{6}\,d^2e^4x\left(\frac{3\,x^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e^4}\right) - \frac{e^4x^5}{2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \frac{6\,de^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{44\,d^3ex^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{16\,d^4x}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \frac{82\,d^5}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} - \frac{61\,d^2x}{6\sqrt{-e^2x^2+d^2}} + \frac{35\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
35/6*d^2*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^
2)^(3/2)*e^4)) - 1/2*e^4*x^5/(-e^2*x^2 + d^2)^(3/2) - 6*d*e^3*x^4/(-e^2*x^
2 + d^2)^(3/2) + 44*d^3*e*x^2/(-e^2*x^2 + d^2)^(3/2) + 16/3*d^4*x/(-e^2*x^
2 + d^2)^(3/2) - 82/3*d^5/((-e^2*x^2 + d^2)^(3/2)*e) - 61/6*d^2*x/sqrt(-e^
2*x^2 + d^2) + 35/2*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{35\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} - \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(x+\frac{12\,d}{e}\right) - \frac{32\left(4\,d^2 - \frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x} + \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2}\right)}{3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^3|e|}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
35/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 1/2*sqrt(-e^2*x^2 + d^2)*(x
+ 12*d/e) - 32/3*(4*d^2 - 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2/(e^2*x
) + 3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2))/(((d*e + sqrt(-
e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^6}{(d^2-e^2\,x^2)^{5/2}}\,dx$$

input
```
int((d + e*x)^6/(d^2 - e^2*x^2)^(5/2),x)
```

output
```
int((d + e*x)^6/(d^2 - e^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{105\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^3-105\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^2ex-105asin\left(\frac{ex}{d}\right)d^4+}{}$$

input
```
int((e*x+d)^6/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
(105*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**3 - 105*sqrt(d**2 - e**2*x**2
)*asin((e*x)/d)*d**2*e*x - 105*asin((e*x)/d)*d**4 + 210*asin((e*x)/d)*d**3
*e*x - 105*asin((e*x)/d)*d**2*e**2*x**2 + 66*sqrt(d**2 - e**2*x**2)*d**3 -
 131*sqrt(d**2 - e**2*x**2)*d**2*e*x + 30*sqrt(d**2 - e**2*x**2)*d*e**2*x*
*2 + 3*sqrt(d**2 - e**2*x**2)*e**3*x**3 - 66*d**4 - 131*d**3*e*x + 297*d**
2*e**2*x**2 - 33*d*e**3*x**3 - 3*e**4*x**4)/(6*e*(sqrt(d**2 - e**2*x**2)*d
 - sqrt(d**2 - e**2*x**2)*e*x - d**2 + 2*d*e*x - e**2*x**2))
```

3.128 $\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}} - \frac{20d(d+ex)}{3e\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{3e} + \frac{5d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2/3*(e*x+d)^4/e/(-e^2*x^2+d^2)^(3/2)-20/3*d*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)
-5/3*(-e^2*x^2+d^2)^(1/2)/e+5*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{(-23d^2+34dex-3e^2x^2)\sqrt{d^2-e^2x^2}}{3e(-d+ex)^2} - \frac{5d\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)^5/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
((-23*d^2 + 34*d*e*x - 3*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*e*(-d + e*x)^2)
- (5*d*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sqrt[-e^2]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {468, 462, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{462}$$

$$\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int \frac{3d+ex}{\sqrt{d^2-e^2x^2}}dx\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(-3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(-3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)$$

input
```
Int[(d + e*x)^5/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^4)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (5*((4*d*(d + e*x))/(e*Sqrt[
d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/e))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\sqrt{-e^2x^2+d^2}}{e}+\frac{5d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{28d\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{3e^2\left(x-\frac{d}{e}\right)}+\frac{8d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{3e^3\left(x-\frac{d}{e}\right)^2}$
default	$d^5\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)+e^5\left(-\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{4d^2\left(\frac{x^2}{e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{2d^2}{3e^4\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}\right)}{e^2}\right)$

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-e^2*x^2+d^2)^(1/2)/e+5*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))+28/3*d/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)+8/3*d^2/e
^3/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx =$$
$$-\frac{23\,de^2x^2-46\,d^2ex+23\,d^3+30\,(de^2x^2-2\,d^2ex+d^3)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+(3\,e^2x^2-34\,dex+23\,d}{3\,(e^3x^2-2\,de^2x+d^2e)}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(23*d*e^2*x^2 - 46*d^2*e*x + 23*d^3 + 30*(d*e^2*x^2 - 2*d^2*e*x + d^3
)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (3*e^2*x^2 - 34*d*e*x + 23*d
^2)*sqrt(-e^2*x^2 + d^2))/(e^3*x^2 - 2*d*e^2*x + d^2*e)
```

Sympy [F]

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^5}{\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**5/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**5/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{5}{3}\,de^4x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2}-\frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) - \frac{e^3x^4}{(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{14\,d^2ex^2}{(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{11\,d^3x}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{23\,d^4}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e} - \frac{13\,dx}{3\,\sqrt{-e^2x^2+d^2}} + \frac{5\,d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
5/3*d*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^
(3/2)*e^4)) - e^3*x^4/(-e^2*x^2 + d^2)^(3/2) + 14*d^2*e*x^2/(-e^2*x^2 + d^
2)^(3/2) + 11/3*d^3*x/(-e^2*x^2 + d^2)^(3/2) - 23/3*d^4/((-e^2*x^2 + d^2)^
(3/2)*e) - 13/3*d*x/sqrt(-e^2*x^2 + d^2) + 5*d*arcsin(e^2*x/(d*sqrt(e^2)))
/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{5\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{\sqrt{-e^2x^2+d^2}}{e} - \frac{8\left(5\,d-\frac{12\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x}+\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2}\right)}{3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^3|e|}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
5*d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - sqrt(-e^2*x^2 + d^2)/e - 8/3*(5*d
 - 12*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 3*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2*d/(e^4*x^2))/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^
2*x) - 1)^3*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^5}{(d^2-e^2\,x^2)^{5/2}}\,dx$$

input
```
int((d + e*x)^5/(d^2 - e^2*x^2)^(5/2),x)
```

output
```
int((d + e*x)^5/(d^2 - e^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.17

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{15\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^2 - 15\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)dex - 15asin\left(\frac{ex}{d}\right)d^3 + 30a\cdots}{3e\left(\cdots\right.}$$

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
(15*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**2 - 15*sqrt(d**2 - e**2*x**2)*
asin((e*x)/d)*d*e*x - 15*asin((e*x)/d)*d**3 + 30*asin((e*x)/d)*d**2*e*x -
15*asin((e*x)/d)*d*e**2*x**2 + 8*sqrt(d**2 - e**2*x**2)*d**2 - 19*sqrt(d**
2 - e**2*x**2)*d*e*x + 3*sqrt(d**2 - e**2*x**2)*e**2*x**2 - 8*d**3 - 19*d*
*2*e*x + 46*d*e**2*x**2 - 3*e**3*x**3)/(3*e*(sqrt(d**2 - e**2*x**2)*d - sq
rt(d**2 - e**2*x**2)*e*x - d**2 + 2*d*e*x - e**2*x**2))
```

3.129 $\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx=\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2/3*(e*x+d)^3/e/(-e^2*x^2+d^2)^(3/2)-2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+arct
an(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx=\frac{2\left(-\frac{2(d-2ex)\sqrt{d^2-e^2x^2}}{(d-ex)^2}-3\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)\right)}{3e}$$

input
```
Integrate[(d + e*x)^4/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(2*((-2*(d - 2*e*x)*Sqrt[d^2 - e^2*x^2])/(d - e*x)^2 - 3*ArcTan[(e*x)/(Sqr
t[d^2] - Sqrt[d^2 - e^2*x^2])]))/(3*e)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {468, 457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 468}$$

$$\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}-\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 457}$$

$$\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}-\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}-\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}-\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

input
```
Int[(d + e*x)^4/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^3)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (2*(d + e*x))/(e*Sqrt[d^2 -
e^2*x^2]) + ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(73) = 146$.

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.47

method	result
default	$d^4\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)+e^4\left(\frac{x^3}{3e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{\frac{x}{e^2\sqrt{-e^2x^2+d^2}}-\frac{\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{e^2\sqrt{e^2}}}{e^2}\right)+4d\,e$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d^4*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))+e^4*(1
/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1/e^2/(e
^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))))+4*d*e^3*(x^2/e^2/(-
e^2*x^2+d^2)^(3/2)-2/3*d^2/e^4/(-e^2*x^2+d^2)^(3/2))+6*d^2*e^2*(1/2*x/e^2/
(-e^2*x^2+d^2)^(3/2)-1/2*d^2/e^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4
/(-e^2*x^2+d^2)^(1/2)))+4/3*d^3/e/(-e^2*x^2+d^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx = -\frac{2\left(2\,e^2x^2-4\,dex+2\,d^2+3\,(e^2x^2-2\,dex+d^2)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-2\,\sqrt{-e^2x^2+d^2}(2\,ex-d)\right)}{3\,(e^3x^2-2\,de^2x+d^2e)}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*e^2*x^2 - 4*d*e*x + 2*d^2 + 3*(e^2*x^2 - 2*d*e*x + d^2)*arctan(-(d
 - sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*sqrt(-e^2*x^2 + d^2)*(2*e*x - d))/(e^3
*x^2 - 2*d*e^2*x + d^2*e)
```

Sympy [F]

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^4}{(-(-d+ex)\,(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**4/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{1}{3}\,e^4x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{4\,dex^2}{(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{7\,d^2x}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{4\,d^3}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e} - \frac{5\,x}{3\,\sqrt{-e^2x^2+d^2}} + \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3
/2)*e^4)) + 4*d*e*x^2/(-e^2*x^2 + d^2)^(3/2) + 7/3*d^2*x/(-e^2*x^2 + d^2)^
(3/2) - 4/3*d^3/((-e^2*x^2 + d^2)^(3/2)*e) - 5/3*x/sqrt(-e^2*x^2 + d^2) +
arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{8\left(\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - 1\right)}{3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^3|e|}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 8/3*(3*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))/(e^2*x) - 1)/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^3*ab
s(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^4}{(d^2-e^2\,x^2)^{5/2}}\,dx$$

input
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(5/2),x)
```

output
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{3asin\left(\frac{ex}{d}\right)\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3 - 9asin\left(\frac{ex}{d}\right)\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2 + 9asin\left(\frac{ex}{d}\right)\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right) -}{3e\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3 - 3\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2 + 3\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)\right.}$$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
(3*asin((e*x)/d)*tan(asin((e*x)/d)/2)**3 - 9*asin((e*x)/d)*tan(asin((e*x)/
d)/2)**2 + 9*asin((e*x)/d)*tan(asin((e*x)/d)/2) - 3*asin((e*x)/d) - 24*tan
(asin((e*x)/d)/2) + 8)/(3*e*(tan(asin((e*x)/d)/2)**3 - 3*tan(asin((e*x)/d)
/2)**2 + 3*tan(asin((e*x)/d)/2) - 1))
```

3.130 $\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{(d+ex)^3}{3de\,(d^2-e^2x^2)^{3/2}}$$

output
```
1/3*(e*x+d)^3/d/e/(-e^2*x^2+d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{(d+ex)\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}$$

input
```
Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
((d + e*x)*Sqrt[d^2 - e^2*x^2])/(3*d*e*(d - e*x)^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 460}$$

$$\frac{(d+ex)^3}{3de\,(d^2-e^2x^2)^{3/2}}$$

input
```
Int[(d + e*x)^3/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d + e*x)^3/(3*d*e*(d^2 - e^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result
gosper	$\frac{(ex+d)^{4}(-ex+d)}{3de(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}$
trager	$\frac{(ex+d)\sqrt{-e^{2}x^{2}+d^{2}}}{3d(-ex+d)^{2}e}$
orering	$\frac{(ex+d)^{4}(-ex+d)}{3de(-e^{2}x^{2}+d^{2})^{\frac{5}{2}}}$
default	$d^{3}\left(\frac{x}{3d^{2}(-e^{2}x^{2}+d^{2})^{\frac{3}{2}}}+\frac{2x}{3d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}\right)+e^{3}\left(\frac{x^{2}}{e^{2}(-e^{2}x^{2}+d^{2})^{\frac{3}{2}}}-\frac{2d^{2}}{3e^{4}(-e^{2}x^{2}+d^{2})^{\frac{3}{2}}}\right)+3d\,e^{2}\left(\frac{x}{2e^{2}(-e^{2}x^{2}+d^{2})^{\frac{3}{2}}}\right.$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(e*x+d)^4*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{e^2x^2-2\,dex+d^2+\sqrt{-e^2x^2+d^2}(ex+d)}{3\,(de^3x^2-2\,d^2e^2x+d^3e)}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(e^2*x^2 - 2*d*e*x + d^2 + sqrt(-e^2*x^2 + d^2)*(e*x + d))/(d*e^3*x^2
- 2*d^2*e^2*x + d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^3}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.42

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{ex^2}{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{4\,dx}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} + \frac{d^2}{3\left(-e^2x^2+d^2\right)^{\frac{3}{2}}e} - \frac{x}{3\sqrt{-e^2x^2+d^2}\,d}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
e*x^2/(-e^2*x^2 + d^2)^(3/2) + 4/3*d*x/(-e^2*x^2 + d^2)^(3/2) + 1/3*d^2/((
-e^2*x^2 + d^2)^(3/2)*e) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(29) = 58$.

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{2\left(\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+1\right)}{3\,d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^3|e|}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 1)/(d*((d*e + sqr
t(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^3*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)}{3\,d\,e\,(d-e\,x)^2}$$

input
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(5/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(d + e*x))/(3*d*e*(d - e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.67

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{-\frac{2\sqrt{-e^2x^2+d^2}\,d}{3}+\frac{2d^2}{3}+\frac{2e^2x^2}{3}}{de\left(\sqrt{-e^2x^2+d^2}\,d-\sqrt{-e^2x^2+d^2}\,ex-d^2+2dex-e^2x^2\right)}$$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
(2*( - sqrt(d**2 - e**2*x**2)*d + d**2 + e**2*x**2))/(3*d*e*(sqrt(d**2 - e
**2*x**2)*d - sqrt(d**2 - e**2*x**2)*e*x - d**2 + 2*d*e*x - e**2*x**2))
```

3.131 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 53

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{2(d+ex)}{3e\left(d^2-e^2x^2\right)^{3/2}}+\frac{x}{3d^2\sqrt{d^2-e^2x^2}}$$

output
```
2/3*(e*x+d)/e/(-e^2*x^2+d^2)^(3/2)+1/3*x/d^2/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{(2d-ex)\sqrt{d^2-e^2x^2}}{3d^2e(d-ex)^2}$$

input
```
Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
((2*d - e*x)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e*(d - e*x)^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {457, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{457}$$

$$\frac{1}{3}\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx+\frac{2(d+ex)}{3e\,(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{3d^2\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)}{3e\,(d^2-e^2x^2)^{3/2}}$$

input
```
Int[(d + e*x)^2/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(2*(d + e*x))/(3*e*(d^2 - e^2*x^2)^(3/2)) + x/(3*d^2*Sqrt[d^2 - e^2*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

method	result
trager	$\frac{(-ex+2d)\sqrt{-e^2x^2+d^2}}{3d^2(-ex+d)^2e}$
gosper	$\frac{(ex+d)^3(-ex+d)(-ex+2d)}{3d^2e(-e^2x^2+d^2)^{\frac{5}{2}}}$
orering	$\frac{(ex+d)^3(-ex+d)(-ex+2d)}{3d^2e(-e^2x^2+d^2)^{\frac{5}{2}}}$
default	$d^2\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)+e^2\left(\frac{x}{2e^2(-e^2x^2+d^2)^{\frac{3}{2}}}-\frac{d^2\left(\frac{x}{3d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)}{2e^2}\right)+$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-e*x+2*d)/d^2/(-e*x+d)^2/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{2\,e^2x^2-4\,dex+2\,d^2-\sqrt{-e^2x^2+d^2}(ex-2\,d)}{3\,(d^2e^3x^2-2\,d^3e^2x+d^4e)}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*e^2*x^2 - 4*d*e*x + 2*d^2 - sqrt(-e^2*x^2 + d^2)*(e*x - 2*d))/(d^2*
e^3*x^2 - 2*d^3*e^2*x + d^4*e)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{2\,x}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{2\,d}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e} + \frac{x}{3\,\sqrt{-e^2x^2+d^2}d^2}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/(-e^2*x^2 + d^2)^(3/2) + 2/3*d/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*x/(s
qrt(-e^2*x^2 + d^2)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(45) = 90.

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx = -\frac{2\left(\frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{3\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} - 2\right)}{3\,d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^3|e|}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 3*(d*e + sqrt(-e^2*x
^2 + d^2)*abs(e))^2/(e^4*x^2) - 2)/(d^2*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))/(e^2*x) - 1)^3*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(2\,d-e\,x)}{3\,d^2\,e\,(d-e\,x)^2}$$

input
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(5/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d - e*x))/(3*d^2*e*(d - e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{-2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3-2}{3d^2e\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3-3\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2+3\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)-1\right)}$$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
( - 2*(tan(asin((e*x)/d)/2)**3 + 1))/(3*d**2*e*(tan(asin((e*x)/d)/2)**3 -
3*tan(asin((e*x)/d)/2)**2 + 3*tan(asin((e*x)/d)/2) - 1))
```

3.132 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{d+ex}{3de\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}$$

output
```
1/3*(e*x+d)/d/e/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^3/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{5/2}}\,dx=\frac{\left(d^2+2dex-2e^2x^2\right)\sqrt{d^2-e^2x^2}}{3d^3e(d-ex)^2(d+ex)}$$

input
```
Integrate[(d + e*x)/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
((d^2 + 2*d*e*x - 2*e^2*x^2)*Sqrt[d^2 - e^2*x^2])/(3*d^3*e*(d - e*x)^2*(d
+ e*x))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 454}$$

$$\frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{3d}+\frac{d+ex}{3de\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{d+ex}{3de\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}$$

input
```
Int[(d + e*x)/(d^2 - e^2*x^2)^(5/2),x]
```

output
```
(d + e*x)/(3*d*e*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^3*Sqrt[d^2 - e^2*x^2]
)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{(ex+d)^2(-ex+d)(-2e^2x^2+2dex+d^2)}{3d^3e(-e^2x^2+d^2)^{\frac{5}{2}}}$	53
orering	$\frac{(ex+d)^2(-ex+d)(-2e^2x^2+2dex+d^2)}{3d^3e(-e^2x^2+d^2)^{\frac{5}{2}}}$	53
trager	$\frac{(-2e^2x^2+2dex+d^2)\sqrt{-e^2x^2+d^2}}{3d^3(-ex+d)^2e(ex+d)}$	55
default	$d\left(\frac{x}{3d^2(-e^2x^2+d^2)^{\frac{3}{2}}}+\frac{2x}{3d^4\sqrt{-e^2x^2+d^2}}\right)+\frac{1}{3e(-e^2x^2+d^2)^{\frac{3}{2}}}$	64

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(e*x+d)^2*(-e*x+d)*(-2*e^2*x^2+2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(48) = 96.

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.82

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{e^3x^3-de^2x^2-d^2ex+d^3-(2\,e^2x^2-2\,dex-d^2)\sqrt{-e^2x^2+d^2}}{3\,(d^3e^4x^3-d^4e^3x^2-d^5e^2x+d^6e)}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^3 - (2*e^2*x^2 - 2*d*e*x - d^2)*sqr
t(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - d^4*e^3*x^2 - d^5*e^2*x + d^6*e)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 296, normalized size of antiderivative = 5.29

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx = d\left(\begin{cases} \frac{3id^2x}{-3d^7\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{2ie^2x^3}{-3d^7\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{3d^2x}{-3d^7\sqrt{1-\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{2e^2x^3}{-3d^7\sqrt{1-\frac{e^2x^2}{d^2}}+3d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e\left(\begin{cases} -\frac{1}{-3d^2e^2\sqrt{d^2-e^2x^2}+3e^4x^2\sqrt{d^2-e^2x^2}} & \text{for } e \neq 0 \\ \frac{x^2}{2(d^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
d*Piecewise((3*I*d**2*x/(-3*d**7*sqrt(-1 + e**2*x**2/d**2) + 3*d**5*e**2*x
**2*sqrt(-1 + e**2*x**2/d**2)) - 2*I*e**2*x**3/(-3*d**7*sqrt(-1 + e**2*x**
2/d**2) + 3*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (-3*d**2*x/(-3*d**7*sqrt(1 - e**2*x**2/d**2) + 3*d**5*e**2*x**2*sqr
t(1 - e**2*x**2/d**2)) + 2*e**2*x**3/(-3*d**7*sqrt(1 - e**2*x**2/d**2) + 3
*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piecewise((-1/(-3*d*
*2*e**2*sqrt(d**2 - e**2*x**2) + 3*e**4*x**2*sqrt(d**2 - e**2*x**2)), Ne(e
, 0)), (x**2/(2*(d**2)**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{x}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{1}{3\,(-e^2x^2+d^2)^{\frac{3}{2}}e} + \frac{2\,x}{3\,\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
1/3*x/((-e^2*x^2 + d^2)^(3/2)*d) + 1/3/((-e^2*x^2 + d^2)^(3/2)*e) + 2/3*x/
(sqrt(-e^2*x^2 + d^2)*d^3)
```

Giac [F]

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(-e^2*x^2 + d^2)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(d^2+2\,d\,e\,x-2\,e^2\,x^2\right)}{3\,d^3\,e\,\left(d+e\,x\right)\,\left(d-e\,x\right)^2}$$

input
```
int((d + e*x)/(d^2 - e^2*x^2)^(5/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(d^2 - 2*e^2*x^2 + 2*d*e*x))/(3*d^3*e*(d + e*x)*(d
- e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{d+ex}{(d^2-e^2x^2)^{5/2}}\,dx = \frac{-2\sqrt{-e^2x^2+d^2}\,d+2\sqrt{-e^2x^2+d^2}\,ex+d^2+2dex-2e^2x^2}{3\sqrt{-e^2x^2+d^2}\,d^3e\left(-ex+d\right)}$$

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
( - 2*sqrt(d**2 - e**2*x**2)*d + 2*sqrt(d**2 - e**2*x**2)*e*x + d**2 + 2*d
*e*x - 2*e**2*x**2)/(3*sqrt(d**2 - e**2*x**2)*d**3*e*(d - e*x))
```

3.133 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}} - \frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

output
```
4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)-1/5/d/e/(e*x+d)/(-e^2*x^2+d^2)^(3/2)+8/15*
x/d^5/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}\left(-3d^4+12d^3ex+12d^2e^2x^2-8de^3x^3-8e^4x^4\right)}{15d^5e(d-ex)^2(d+ex)^3}$$

input
```
Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-3*d^4 + 12*d^3*e*x + 12*d^2*e^2*x^2 - 8*d*e^3*x^3 -
 8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {470, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{470}$$

$$\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
-1/5*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2
)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{(-ex+d)\left(8e^4x^4+8d\,e^3x^3-12d^2e^2x^2-12d^3ex+3d^4\right)}{15d^5e\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$	70
orering	$-\frac{(-ex+d)\left(8e^4x^4+8d\,e^3x^3-12d^2e^2x^2-12d^3ex+3d^4\right)}{15d^5e\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$	70
trager	$-\frac{\left(8e^4x^4+8d\,e^3x^3-12d^2e^2x^2-12d^3ex+3d^4\right)\sqrt{-e^2x^2+d^2}}{15d^5(ex+d)^3(-ex+d)^2e}$	79
default	$\frac{-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{6d^2e^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{3e^2d^4\sqrt{-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)}}\right)}{5d}}{e}$	164

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5
/e/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = -\frac{3\,e^5x^5+3\,de^4x^4-6\,d^2e^3x^3-6\,d^3e^2x^2+3\,d^4ex+3\,d^5+\left(8\,e^4x^4+8\,de^3x^3-12\,d^2e^2x^2-12\,d^3ex+3\,d^4\right)}{15\,(d^5e^6x^5+d^6e^5x^4-2\,d^7e^4x^3-2\,d^8e^3x^2+d^9e^2x+d^{10}e)}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(3*e^5*x^5 + 3*d*e^4*x^4 - 6*d^2*e^3*x^3 - 6*d^3*e^2*x^2 + 3*d^4*e*x
 + 3*d^5 + (8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2 - 12*d^3*e*x + 3*d^4)
*sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 + d^6*e^5*x^4 - 2*d^7*e^4*x^3 - 2*d^8*
e^3*x^2 + d^9*e^2*x + d^10*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)\,(d^2-e^2x^2)^{5/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{1}{5\left(\left(-e^2x^2+d^2\right)^{\frac{3}{2}}de^2x+\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2e\right)} + \frac{4\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3} + \frac{8\,x}{15\sqrt{-e^2x^2+d^2}\,d^5}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/5/((-e^2*x^2 + d^2)^(3/2)*d*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^2*e) + 4/1
5*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{5}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = -\frac{\sqrt{d^2-e^2\,x^2}\,\left(3\,d^4-12\,d^3\,e\,x-12\,d^2\,e^2\,x^2+8\,d\,e^3\,x^3+8\,e^4\,x^4\right)}{15\,d^5\,e\,{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^2}$$

input
```
int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)),x)
```

output
```
-((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 + 8*d*e^3*x^3 - 12*d^2*e^2*x^2
- 12*d^3*e*x))/(15*d^5*e*(d + e*x)^3*(d - e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.12

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{12\sqrt{-e^2x^2+d^2}\,d^3+12\sqrt{-e^2x^2+d^2}\,d^2ex-12\sqrt{-e^2x^2+d^2}\,d\,e^2x^2-12\sqrt{}}{15\sqrt{-e^2x^2+d^2}\,d^5e\left(-e^3x^3-d\right.}$$

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
(12*sqrt(d**2 - e**2*x**2)*d**3 + 12*sqrt(d**2 - e**2*x**2)*d**2*e*x - 12*
sqrt(d**2 - e**2*x**2)*d*e**2*x**2 - 12*sqrt(d**2 - e**2*x**2)*e**3*x**3 -
 3*d**4 + 12*d**3*e*x + 12*d**2*e**2*x**2 - 8*d*e**3*x**3 - 8*e**4*x**4)/(
15*sqrt(d**2 - e**2*x**2)*d**5*e*(d**3 + d**2*e*x - d*e**2*x**2 - e**3*x**
3))
```

3.134 $\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{x}{7d^2\left(d^2-e^2x^2\right)^{5/2}} - \frac{2}{7e(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{4x}{21d^4\left(d^2-e^2x^2\right)^{3/2}} + \frac{8x}{21d^6\sqrt{d^2-e^2x^2}}$$

output
```
1/7*x/d^2/(-e^2*x^2+d^2)^(5/2)-2/7/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+4/21*x/d
^4/(-e^2*x^2+d^2)^(3/2)+8/21*x/d^6/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-6d^5+9d^4ex+24d^3e^2x^2+4d^2e^3x^3-16de^4x^4-8e^5x^5)}{21d^6e(d-ex)^2(d+ex)^4}$$

input
```
Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-6*d^5 + 9*d^4*e*x + 24*d^3*e^2*x^2 + 4*d^2*e^3*x^3
- 16*d*e^4*x^4 - 8*e^5*x^5))/(21*d^6*e*(d - e*x)^2*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {461, 470, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{461}$$

$$\frac{5\int \frac{1}{(d+ex)(d^2-e^2x^2)^{5/2}}dx}{7d}-\frac{1}{7de(d+ex)^2 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{5\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}-\frac{1}{5de(d+ex)(d^2-e^2x^2)^{3/2}}\right)}{7d}-\frac{1}{7de(d+ex)^2 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{5\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d}-\frac{1}{5de(d+ex)(d^2-e^2x^2)^{3/2}}\right)}{7d}-\frac{1}{7de(d+ex)^2 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{5\left(\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)(d^2-e^2x^2)^{3/2}}\right)}{7d}-\frac{1}{7de(d+ex)^2 (d^2-e^2x^2)^{3/2}}$$

input
```
Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
-1/7*1/(d*e*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)) + (5*(-1/5*1/(d*e*(d + e*x)
*(d^2 - e^2*x^2)^(3/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d
^4*Sqrt[d^2 - e^2*x^2])))/(5*d)))/(7*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result
gosper	$-\frac{(-ex+d)\left(8e^5x^5+16d\,e^4x^4-4d^2e^3x^3-24d^3e^2x^2-9d^4ex+6d^5\right)}{21(ex+d)d^6e\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
orering	$-\frac{(-ex+d)\left(8e^5x^5+16d\,e^4x^4-4d^2e^3x^3-24d^3e^2x^2-9d^4ex+6d^5\right)}{21(ex+d)d^6e\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}$
trager	$-\frac{\left(8e^5x^5+16d\,e^4x^4-4d^2e^3x^3-24d^3e^2x^2-9d^4ex+6d^5\right)\sqrt{-e^2x^2+d^2}}{21d^6(ex+d)^4(-ex+d)^2e}$
default	$\frac{-\frac{1}{7de\left(x+\frac{d}{e}\right)^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{5e\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{4e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{6d^2e^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\cdots\right.}{5d}\right.}{7d}}{e^2}$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-e*x+d)*(8*e^5*x^5+16*d*e^4*x^4-4*d^2*e^3*x^3-24*d^3*e^2*x^2-9*d^4*
e*x+6*d^5)/(e*x+d)/d^6/e/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(87) = 174.

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.97

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx =$$
$$-\frac{6\,e^6x^6+12\,de^5x^5-6\,d^2e^4x^4-24\,d^3e^3x^3-6\,d^4e^2x^2+12\,d^5ex+6\,d^6+\left(8\,e^5x^5+16\,de^4x^4-4\,d^2e^3x^3-\cdots\right.}{21\left(d^6e^7x^6+2\,d^7e^6x^5-d^8e^5x^4-4\,d^9e^4x^3-d^{10}e^3x^2+2\,d^{11}e^2x+d^{1}\cdots\right.}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/21*(6*e^6*x^6 + 12*d*e^5*x^5 - 6*d^2*e^4*x^4 - 24*d^3*e^3*x^3 - 6*d^4*e
^2*x^2 + 12*d^5*e*x + 6*d^6 + (8*e^5*x^5 + 16*d*e^4*x^4 - 4*d^2*e^3*x^3 -
24*d^3*e^2*x^2 - 9*d^4*e*x + 6*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*x^6 + 2
*d^7*e^6*x^5 - d^8*e^5*x^4 - 4*d^9*e^4*x^3 - d^10*e^3*x^2 + 2*d^11*e^2*x +
 d^12*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{5/2}}\,dx = \int \frac{1}{\left(-(-d+ex)(d+ex)\right)^{\frac{5}{2}}(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{5/2}}\,dx = -\frac{1}{7\left((-e^2x^2+d^2)^{\frac{3}{2}}de^3x^2+2(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2x+(-e^2x^2+d^2)^{\frac{3}{2}}d^3e\right)} - \frac{1}{7\left((-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2x+(-e^2x^2+d^2)^{\frac{3}{2}}d^3e\right)} + \frac{4x}{21(-e^2x^2+d^2)^{\frac{3}{2}}d^4} + \frac{8x}{21\sqrt{-e^2x^2+d^2}d^6}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/7/((-e^2*x^2 + d^2)^(3/2)*d*e^3*x^2 + 2*(-e^2*x^2 + d^2)^(3/2)*d^2*e^2*
x + (-e^2*x^2 + d^2)^(3/2)*d^3*e) - 1/7/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2*x
+ (-e^2*x^2 + d^2)^(3/2)*d^3*e) + 4/21*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 8/
21*x/(sqrt(-e^2*x^2 + d^2)*d^6)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.35

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{e^5\left(\frac{14\left(\frac{15d}{ex+d}-7\right)}{d^6e^5\left(\frac{2d}{ex+d}-1\right)^{\frac{3}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{3d^{36}e^{30}\left(\frac{2d}{ex+d}-1\right)^{\frac{7}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)^6\operatorname{sgn}(e)^6+21d^{36}e^{30}\left(\right.}{}\right.}{}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
1/672*(e^5*(14*(15*d/(e*x + d) - 7)/(d^6*e^5*(2*d/(e*x + d) - 1)^(3/2)*sgn
(1/(e*x + d))*sgn(e)) - (3*d^36*e^30*(2*d/(e*x + d) - 1)^(7/2)*sgn(1/(e*x
+ d))^6*sgn(e)^6 + 21*d^36*e^30*(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))
^6*sgn(e)^6 + 70*d^36*e^30*(2*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^6*sg
n(e)^6 + 210*d^36*e^30*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^6*sgn(e)^6
)/(d^42*e^35*sgn(1/(e*x + d))^7*sgn(e)^7)) + 256*I*sgn(1/(e*x + d))*sgn(e)
/d^6)/abs(e)
```

Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{11\,x}{42\,d^4}-\frac{5}{28\,d^3\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} - \frac{\sqrt{d^2-e^2\,x^2}}{28\,d^3\,e\,{\left(d+e\,x\right)}^4} - \frac{\sqrt{d^2-e^2\,x^2}}{14\,d^4\,e\,{\left(d+e\,x\right)}^3} + \frac{8\,x\,\sqrt{d^2-e^2\,x^2}}{21\,d^6\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((11*x)/(42*d^4) - 5/(28*d^3*e)))/((d + e*x)^2*(d -
 e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(28*d^3*e*(d + e*x)^4) - (d^2 - e^2*x^2)^
(1/2)/(14*d^4*e*(d + e*x)^3) + (8*x*(d^2 - e^2*x^2)^(1/2))/(21*d^6*(d + e*
x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{1}{(d+ex)^2 \left(d^2-e^2x^2\right)^{5/2}} \, dx = \frac{9\sqrt{-e^2x^2+d^2}\, d^4+18\sqrt{-e^2x^2+d^2}\, d^3ex-18\sqrt{-e^2x^2+d^2}\, d\, e^3x^3-9\sqrt{-}}{42\sqrt{-e^2x^2+d^2}\, d^6e\left(-e^4\right.}$$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
(9*sqrt(d**2 - e**2*x**2)*d**4 + 18*sqrt(d**2 - e**2*x**2)*d**3*e*x - 18*s
qrt(d**2 - e**2*x**2)*d*e**3*x**3 - 9*sqrt(d**2 - e**2*x**2)*e**4*x**4 - 1
2*d**5 + 18*d**4*e*x + 48*d**3*e**2*x**2 + 8*d**2*e**3*x**3 - 32*d*e**4*x*
*4 - 16*e**5*x**5)/(42*sqrt(d**2 - e**2*x**2)*d**6*e*(d**4 + 2*d**3*e*x -
2*d*e**3*x**3 - e**4*x**4))
```

3.135 $\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}} \, dx = \frac{2x}{21d^3 (d^2-e^2x^2)^{5/2}} - \frac{2}{9e(d+ex)^2 (d^2-e^2x^2)^{5/2}} - \frac{5}{63de(d+ex) (d^2-e^2x^2)^{5/2}} + \frac{8x}{63d^5 (d^2-e^2x^2)^{3/2}} + \frac{16x}{63d^7\sqrt{d^2-e^2x^2}}$$

output
```
2/21*x/d^3/(-e^2*x^2+d^2)^(5/2)-2/9/e/(e*x+d)^2/(-e^2*x^2+d^2)^(5/2)-5/63/
d/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+8/63*x/d^5/(-e^2*x^2+d^2)^(3/2)+16/63*x/d
^7/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(-19d^6+6d^5ex+66d^4e^2x^2+56d^3e^3x^3-24d^2e^4x^4-48de^5x^5}{63d^7e(d-ex)^2(d+ex)^5}$$

input
```
Integrate[1/((d + e*x)^3*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-19*d^6 + 6*d^5*e*x + 66*d^4*e^2*x^2 + 56*d^3*e^3*x^
3 - 24*d^2*e^4*x^4 - 48*d*e^5*x^5 - 16*e^6*x^6))/(63*d^7*e*(d - e*x)^2*(d
+ e*x)^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {461, 461, 470, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{5/2}}dx}{3d}-\frac{1}{9de(d+ex)^3 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{2\left(\frac{5\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{5/2}}dx}{7d}-\frac{1}{7de(d+ex)^2(d^2-e^2x^2)^{3/2}}\right)}{3d}-\frac{1}{9de(d+ex)^3 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{2\left(\frac{5\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\right)}{7d}-\frac{1}{7de(d+ex)^2(d^2-e^2x^2)^{3/2}}\right)}{3d}-\frac{1}{9de(d+ex)^3 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{2\left(\frac{5\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\right)}{7d}-\frac{1}{7de(d+ex)^2(d^2-e^2x^2)^{3/2}}\right)}{3d}-\frac{1}{9de(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \color{blue}{208}$$

$$\frac{2\left(\frac{5\left(\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}-\frac{1}{5de(d+ex)\left(d^2-e^2x^2\right)^{3/2}}\right)}{7d}-\frac{1}{7de(d+ex)^2(d^2-e^2x^2)^{3/2}}\right)}{3d}-\frac{1}{9de(d+ex)^3\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)^3*(d^2 - e^2*x^2)^(5/2)),x]
```

output
```
-1/9*1/(d*e*(d + e*x)^3*(d^2 - e^2*x^2)^(3/2)) + (2*(-1/7*1/(d*e*(d + e*x)
^2*(d^2 - e^2*x^2)^(3/2)) + (5*(-1/5*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(3/2
)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2
])))/(5*d)))/(7*d)))/(3*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

method	result
gosper	$-\frac{(-ex+d)(16e^6x^6+48d\,e^5x^5+24d^2e^4x^4-56d^3e^3x^3-66d^4e^2x^2-6d^5ex+19d^6)}{63(ex+d)^2d^7e(-e^2x^2+d^2)^{\frac{5}{2}}}$
orering	$-\frac{(-ex+d)(16e^6x^6+48d\,e^5x^5+24d^2e^4x^4-56d^3e^3x^3-66d^4e^2x^2-6d^5ex+19d^6)}{63(ex+d)^2d^7e(-e^2x^2+d^2)^{\frac{5}{2}}}$
trager	$-\frac{(16e^6x^6+48d\,e^5x^5+24d^2e^4x^4-56d^3e^3x^3-66d^4e^2x^2-6d^5ex+19d^6)\sqrt{-e^2x^2+d^2}}{63d^7(ex+d)^5(-ex+d)^2e}$
default	$\frac{-\frac{1}{9de\left(x+\frac{d}{e}\right)^3\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{2e\left(-\frac{1}{7de\left(x+\frac{d}{e}\right)^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}+\frac{5e\left(-\frac{1}{5de\left(x+\frac{d}{e}\right)\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)}\right.}{}\right)}{3d}}{e^3}$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/63*(-e*x+d)*(16*e^6*x^6+48*d*e^5*x^5+24*d^2*e^4*x^4-56*d^3*e^3*x^3-66*d
^4*e^2*x^2-6*d^5*e*x+19*d^6)/(e*x+d)^2/d^7/e/(-e^2*x^2+d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(116) = 232$.

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.72

$$\int \frac{1}{(d+ex)^3(d^2-e^2x^2)^{5/2}}\,dx =$$
$$-\frac{19\,e^7x^7+57\,de^6x^6+19\,d^2e^5x^5-95\,d^3e^4x^4-95\,d^4e^3x^3+19\,d^5e^2x^2+57\,d^6ex+19\,d^7+(16\,e^6x^6+48\,de\ldots}{63\,(d^7e^8x^7+3\,d^8e^7x^6+d^9e^6x^5-5\,d^{10}e^5x^4-5\,d^{11}e^4x^3+\ldots}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/63*(19*e^7*x^7 + 57*d*e^6*x^6 + 19*d^2*e^5*x^5 - 95*d^3*e^4*x^4 - 95*d^
4*e^3*x^3 + 19*d^5*e^2*x^2 + 57*d^6*e*x + 19*d^7 + (16*e^6*x^6 + 48*d*e^5*
x^5 + 24*d^2*e^4*x^4 - 56*d^3*e^3*x^3 - 66*d^4*e^2*x^2 - 6*d^5*e*x + 19*d^
6)*sqrt(-e^2*x^2 + d^2))/(d^7*e^8*x^7 + 3*d^8*e^7*x^6 + d^9*e^6*x^5 - 5*d^
10*e^5*x^4 - 5*d^11*e^4*x^3 + d^12*e^3*x^2 + 3*d^13*e^2*x + d^14*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}}\,dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{5}{2}}(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(5/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)**3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(116) = 232.

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.85

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}}\,dx =$$
$$-\frac{1}{9\left((-e^2x^2+d^2)^{\frac{3}{2}}de^4x^3+3(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^3x^2+3(-e^2x^2+d^2)^{\frac{3}{2}}d^3e^2x+(-e^2x^2+d^2)^{\frac{3}{2}}d^4e\right)}$$
$$-\frac{2}{21\left((-e^2x^2+d^2)^{\frac{3}{2}}d^2e^3x^2+2(-e^2x^2+d^2)^{\frac{3}{2}}d^3e^2x+(-e^2x^2+d^2)^{\frac{3}{2}}d^4e\right)}$$
$$-\frac{2}{21\left((-e^2x^2+d^2)^{\frac{3}{2}}d^3e^2x+(-e^2x^2+d^2)^{\frac{3}{2}}d^4e\right)}$$
$$+\frac{8x}{63(-e^2x^2+d^2)^{\frac{3}{2}}d^5}+\frac{16x}{63\sqrt{-e^2x^2+d^2}d^7}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/9/((-e^2*x^2 + d^2)^(3/2)*d*e^4*x^3 + 3*(-e^2*x^2 + d^2)^(3/2)*d^2*e^3*
x^2 + 3*(-e^2*x^2 + d^2)^(3/2)*d^3*e^2*x + (-e^2*x^2 + d^2)^(3/2)*d^4*e) -
 2/21/((-e^2*x^2 + d^2)^(3/2)*d^2*e^3*x^2 + 2*(-e^2*x^2 + d^2)^(3/2)*d^3*e
^2*x + (-e^2*x^2 + d^2)^(3/2)*d^4*e) - 2/21/((-e^2*x^2 + d^2)^(3/2)*d^3*e^
2*x + (-e^2*x^2 + d^2)^(3/2)*d^4*e) + 8/63*x/((-e^2*x^2 + d^2)^(3/2)*d^5)
+ 16/63*x/(sqrt(-e^2*x^2 + d^2)*d^7)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}} dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{5}{2}}(ex+d)^3} dx$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3), x)
```

Mupad [B] (verification not implemented)

Time = 6.82 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}} dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{197\,x}{1008\,d^5}-\frac{155}{1008\,d^4\,e}\right)}{(d+e\,x)^2\,(d-e\,x)^2} - \frac{\sqrt{d^2-e^2\,x^2}}{36\,d^3\,e\,(d+e\,x)^5} - \frac{13\,\sqrt{d^2-e^2\,x^2}}{252\,d^4\,e\,(d+e\,x)^4} - \frac{23\,\sqrt{d^2-e^2\,x^2}}{336\,d^5\,e\,(d+e\,x)^3} + \frac{16\,x\,\sqrt{d^2-e^2\,x^2}}{63\,d^7\,(d+e\,x)\,(d-e\,x)}$$

input
```
int(1/((d^2 - e^2*x^2)^(5/2)*(d + e*x)^3),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((197*x)/(1008*d^5) - 155/(1008*d^4*e)))/((d + e*x)
^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(36*d^3*e*(d + e*x)^5) - (13*(d^2
- e^2*x^2)^(1/2))/(252*d^4*e*(d + e*x)^4) - (23*(d^2 - e^2*x^2)^(1/2))/(33
6*d^5*e*(d + e*x)^3) + (16*x*(d^2 - e^2*x^2)^(1/2))/(63*d^7*(d + e*x)*(d -
 e*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.96

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{5/2}}\,dx = \frac{2\sqrt{-e^2x^2+d^2}\,d^5+6\sqrt{-e^2x^2+d^2}\,d^4ex+4\sqrt{-e^2x^2+d^2}\,d^3e^2x^2-4\sqrt{-e^2}}{63\sqrt{-}}$$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(5/2),x)
```

output
```
(2*sqrt(d**2 - e**2*x**2)*d**5 + 6*sqrt(d**2 - e**2*x**2)*d**4*e*x + 4*sqr
t(d**2 - e**2*x**2)*d**3*e**2*x**2 - 4*sqrt(d**2 - e**2*x**2)*d**2*e**3*x*
*3 - 6*sqrt(d**2 - e**2*x**2)*d*e**4*x**4 - 2*sqrt(d**2 - e**2*x**2)*e**5*
x**5 - 19*d**6 + 6*d**5*e*x + 66*d**4*e**2*x**2 + 56*d**3*e**3*x**3 - 24*d
**2*e**4*x**4 - 48*d*e**5*x**5 - 16*e**6*x**6)/(63*sqrt(d**2 - e**2*x**2)*
d**7*e*(d**5 + 3*d**4*e*x + 2*d**3*e**2*x**2 - 2*d**2*e**3*x**3 - 3*d*e**4
*x**4 - e**5*x**5))
```

3.136 $\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}}-\frac{6(d+ex)^5}{5e(d^2-e^2x^2)^{3/2}}+\frac{168d^2(d+ex)}{5e\sqrt{d^2-e^2x^2}}+\frac{84d\sqrt{d^2-e^2x^2}}{5e}+\frac{21}{10}x\sqrt{d^2-e^2x^2}-\frac{63d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
2/5*(e*x+d)^7/e/(-e^2*x^2+d^2)^(5/2)-6/5*(e*x+d)^5/e/(-e^2*x^2+d^2)^(3/2)+
168/5*d^2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+84/5*d*(-e^2*x^2+d^2)^(1/2)/e+21/
10*x*(-e^2*x^2+d^2)^(1/2)-63/2*d^2*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\frac{\sqrt{d^2-e^2x^2}(496d^4-1163d^3ex+801d^2e^2x^2-65de^3x^3-5e^4x^4)}{(d-ex)^3}+630d^2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{10e}$$

input
```
Integrate[(d + e*x)^8/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(496*d^4 - 1163*d^3*e*x + 801*d^2*e^2*x^2 - 65*d*e^3
*x^3 - 5*e^4*x^4))/(d - e*x)^3 + 630*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^
2 - e^2*x^2])])/(10*e)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {468, 468, 462, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}}-\frac{9}{5}\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{3/2}}dx\right)$$

$$\Big\downarrow \text{462}$$

$$\frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\left(\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int \frac{7d^2+4exd+e^2x^2}{\sqrt{d^2-e^2x^2}}dx\right)\right)$$

$$\Big\downarrow \text{2346}$$

$$\frac{2(d+ex)^7}{5e(d^2-e^2x^2)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e(d^2-e^2x^2)^{3/2}}-\frac{7}{3}\left(\frac{\int -\frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{\int\frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\int\frac{15d+8ex}{\sqrt{d^2-e^2x^2}}dx+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)\right)$$

$$\Big\downarrow \; 455$$

$$\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)\right)$$

$$\Big\downarrow \; 224$$

$$\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-}\right.\right.$$

$$\Big\downarrow \; 216$$

$$\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{8\sqrt{d^2-e^2x^2}}{e}\right)+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{1}{2}x\sqrt{d^2-e^2x^2}\right)\right.$$

input
```
Int[(d + e*x)^8/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(2*(d + e*x)^7)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (9*((2*(d + e*x)^5)/(3*e*(d^
2 - e^2*x^2)^(3/2)) - (7*((8*d^2*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) + (x*S
qrt[d^2 - e^2*x^2])/2 - (d*((-8*Sqrt[d^2 - e^2*x^2])/e + (15*d*ArcTan[(e*x
)/Sqrt[d^2 - e^2*x^2]])/e))/2))/3))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(ex+16d)\sqrt{-e^2x^2+d^2}}{2e}-\frac{63d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{288d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{5e^2\left(x-\frac{d}{e}\right)}-\frac{112d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\right.}}{5e^3\left(x-\frac{d}{e}\right)^2}$
default	$d^8\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^8\left(-\frac{x^7}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{7d^2\left(\frac{x^5}{5e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\right.}{}\right.$

input
```
int((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+16*d)/e*(-e^2*x^2+d^2)^(1/2)-63/2*d^2/(e^2)^(1/2)*arctan((e^2)^(1
/2)*x/(-e^2*x^2+d^2)^(1/2))-288/5*d^2/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x
-d/e))^(1/2)-112/5*d^3/e^3/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-
32/5*d^4/e^4/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{496\,d^2e^3x^3 - 1488\,d^3e^2x^2 + 1488\,d^4ex - 496\,d^5 + 630\,(d^2e^3x^3 - 3\,d^3e^2x^2 + 3\,d^4ex - }{10\,(e^4x^3 - }$$

input
```
integrate((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/10*(496*d^2*e^3*x^3 - 1488*d^3*e^2*x^2 + 1488*d^4*e*x - 496*d^5 + 630*(d
^2*e^3*x^3 - 3*d^3*e^2*x^2 + 3*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 +
 d^2))/(e*x)) + (5*e^4*x^4 + 65*d*e^3*x^3 - 801*d^2*e^2*x^2 + 1163*d^3*e*x
 - 496*d^4)*sqrt(-e^2*x^2 + d^2))/(e^4*x^3 - 3*d*e^3*x^2 + 3*d^2*e^2*x - d
^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^8}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**8/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**8/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(143) = 286.

Time = 0.12 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{e^6x^7}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{21}{10}\,d^2e^6x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{8\,de^5x^6}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{21}{2}\,d^2e^4x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{104\,d^3e^3x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{35\,d^4e^2x^3}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{120\,d^5ex^2}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{76\,d^6x}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{248\,d^7}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{69\,d^4x}{5\,(-e^2x^2+d^2)^{\frac{3}{2}}} - \frac{39\,d^2x}{10\,\sqrt{-e^2x^2+d^2}} - \frac{63\,d^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}}$$

input
```
integrate((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*e^6*x^7/(-e^2*x^2 + d^2)^(5/2) + 21/10*d^2*e^6*x*(15*x^4/((-e^2*x^2 +
 d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*
x^2 + d^2)^(5/2)*e^6)) - 8*d*e^5*x^6/(-e^2*x^2 + d^2)^(5/2) - 21/2*d^2*e^4
*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4
)) + 104*d^3*e^3*x^4/(-e^2*x^2 + d^2)^(5/2) + 35*d^4*e^2*x^3/(-e^2*x^2 + d
^2)^(5/2) - 120*d^5*e*x^2/(-e^2*x^2 + d^2)^(5/2) - 76/5*d^6*x/(-e^2*x^2 +
d^2)^(5/2) + 248/5*d^7/((-e^2*x^2 + d^2)^(5/2)*e) + 69/5*d^4*x/(-e^2*x^2 +
 d^2)^(3/2) - 39/10*d^2*x/sqrt(-e^2*x^2 + d^2) - 63/2*d^2*arcsin(e^2*x/(d*
sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{63\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{1}{2}\,\sqrt{-e^2x^2+d^2}\left(x+\frac{16\,d}{e}\right) + \frac{32\left(13\,d^2 - \frac{55\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x} + \frac{85\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2} - \frac{45\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3} + \frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d^2}{e^8x^4}\right.}{5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-63/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*sqrt(-e^2*x^2 + d^2)*(x
 + 16*d/e) + 32/5*(13*d^2 - 55*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d^2/(e^
2*x) + 85*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2) - 45*(d*e +
sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 10*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))^4*d^2/(e^8*x^4))/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x)
 - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(d+e\,x)^8}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((d + e*x)^8/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((d + e*x)^8/(d^2 - e^2*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{-315\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^4+630\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^3ex-315\sqrt{-e^2x^2+d^2}\cdots}{\cdots}$$

input
```
int((e*x+d)^8/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
( - 315*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**4 + 630*sqrt(d**2 - e**2*x
**2)*asin((e*x)/d)*d**3*e*x - 315*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**
2*e**2*x**2 + 315*asin((e*x)/d)*d**5 - 945*asin((e*x)/d)*d**4*e*x + 945*as
in((e*x)/d)*d**3*e**2*x**2 - 315*asin((e*x)/d)*d**2*e**3*x**3 - 130*sqrt(d
**2 - e**2*x**2)*d**4 + 431*sqrt(d**2 - e**2*x**2)*d**3*e*x - 435*sqrt(d**
2 - e**2*x**2)*d**2*e**2*x**2 + 65*sqrt(d**2 - e**2*x**2)*d*e**3*x**3 + 5*
sqrt(d**2 - e**2*x**2)*e**4*x**4 + 130*d**5 + 431*d**4*e*x - 1460*d**3*e**
2*x**2 + 1102*d**2*e**3*x**3 - 70*d*e**4*x**4 - 5*e**5*x**5)/(10*e*(sqrt(d
**2 - e**2*x**2)*d**2 - 2*sqrt(d**2 - e**2*x**2)*d*e*x + sqrt(d**2 - e**2*
x**2)*e**2*x**2 - d**3 + 3*d**2*e*x - 3*d*e**2*x**2 + e**3*x**3))
```

3.137 $\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{2(d+ex)^6}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{14(d+ex)^4}{15e\left(d^2-e^2x^2\right)^{3/2}}+\frac{28d(d+ex)}{3e\sqrt{d^2-e^2x^2}}+\frac{7\sqrt{d^2-e^2x^2}}{3e}-\frac{7d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2/5*(e*x+d)^6/e/(-e^2*x^2+d^2)^(5/2)-14/15*(e*x+d)^4/e/(-e^2*x^2+d^2)^(3/2
)+28/3*d*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+7/3*(-e^2*x^2+d^2)^(1/2)/e-7*d*arc
tan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-167d^3+381d^2ex-277de^2x^2+15e^3x^3)}{15e(-d+ex)^3}+\frac{7d\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)^7/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-167*d^3 + 381*d^2*e*x - 277*d*e^2*x^2 + 15*e^3*x^3)
)/(15*e*(-d + e*x)^3) + (7*d*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/S
qrt[-e^2]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {468, 468, 462, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}dx\right)$$

$$\Big\downarrow \text{462}$$

$$\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int \frac{3d+ex}{\sqrt{d^2-e^2x^2}}dx\right)\right)$$

$$\Big\downarrow \text{455}$$

$$\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(-3d\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}+\frac{\sqrt{d^2-e^2x^2}}{e}\right)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{2(d+ex)^6}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{5}\left(\frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{5}{3}\left(-3d\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}} + \frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e}\right)\right)$$

$$\Big\downarrow \ 216$$

$$\frac{2(d+ex)^6}{5e\left(d^2-e^2x^2\right)^{5/2}} - \frac{7}{5}\left(\frac{2(d+ex)^4}{3e\left(d^2-e^2x^2\right)^{3/2}} - \frac{5}{3}\left(-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e} + \frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e}\right)\right)$$

input
```
Int[(d + e*x)^7/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(2*(d + e*x)^6)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (7*((2*(d + e*x)^4)/(3*e*(d^
2 - e^2*x^2)^(3/2)) - (5*((4*d*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) + Sqrt[d
^2 - e^2*x^2]/e - (3*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/3))/5
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.38

method	result
risch	$\frac{\sqrt{-e^2x^2+d^2}}{e}-\frac{7d\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}-\frac{232d\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{15e^2\left(x-\frac{d}{e}\right)}-\frac{128d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{15e^3\left(x-\frac{d}{e}\right)^2}-\frac{16d^3\sqrt{}}{}$
default	$d^7\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^7\left(-\frac{x^6}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{6d^2\left(\frac{x^4}{e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{4d}{}\right.}{}\right.$

input
```
int((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e-7*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)
^(1/2))-232/15*d/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-128/15*d
^2/e^3/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-16/5*d^3/e^4/(x-d/e)
^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{167\,de^3x^3 - 501\,d^2e^2x^2 + 501\,d^3ex - 167\,d^4 + 210\,(de^3x^3 - 3\,d^2e^2x^2 + 3\,d^3ex - d^4)}{15\,(e^4x^3 - 3\,de^3x^2}$$

input
```
integrate((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(167*d*e^3*x^3 - 501*d^2*e^2*x^2 + 501*d^3*e*x - 167*d^4 + 210*(d*e^3
*x^3 - 3*d^2*e^2*x^2 + 3*d^3*e*x - d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))
/(e*x)) + (15*e^3*x^3 - 277*d*e^2*x^2 + 381*d^2*e*x - 167*d^3)*sqrt(-e^2*x
^2 + d^2))/(e^4*x^3 - 3*d*e^3*x^2 + 3*d^2*e^2*x - d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^7}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**7/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**7/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(121) = 242$.

Time = 0.14 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{7}{15}\,de^6x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{e^5x^6}{(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{7}{3}\,de^4x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{27\,d^2e^3x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{35\,d^3e^2x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{73\,d^4ex^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{61\,d^5x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{167\,d^6}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{127\,d^3x}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{22\,dx}{15\,\sqrt{-e^2x^2+d^2}} - \frac{7\,d\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}$$

input
```
integrate((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
7/15*d*e^6*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2
+ d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e^5*x^6/(-e^2*x^
2 + d^2)^(5/2) - 7/3*d*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/(
(-e^2*x^2 + d^2)^(3/2)*e^4)) + 27*d^2*e^3*x^4/(-e^2*x^2 + d^2)^(5/2) + 35/
2*d^3*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) - 73/3*d^4*e*x^2/(-e^2*x^2 + d^2)^(5/
2) - 61/10*d^5*x/(-e^2*x^2 + d^2)^(5/2) + 167/15*d^6/((-e^2*x^2 + d^2)^(5/
2)*e) + 127/30*d^3*x/(-e^2*x^2 + d^2)^(3/2) + 22/15*d*x/sqrt(-e^2*x^2 + d^
2) - 7*d*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{7\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{\sqrt{-e^2x^2+d^2}}{e} + \frac{16\left(19\,d - \frac{80\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{130\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2} - \frac{60\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{e^6x^3} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4d}{e^8x^4}\right)}{15\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-7*d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + sqrt(-e^2*x^2 + d^2)/e + 16/15*(
19*d - 80*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 130*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) - 60*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^3*d/(e^6*x^3) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d/(e^8*x^4))
/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+e\,x)^7}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((d + e*x)^7/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((d + e*x)^7/(d^2 - e^2*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.55

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{-105\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^3+210\sqrt{-e^2x^2+d^2}\,asin\left(\frac{ex}{d}\right)d^2ex-105\sqrt{-e^2x^2+d}$$

input
```
int((e*x+d)^7/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
( - 105*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d**3 + 210*sqrt(d**2 - e**2*x
**2)*asin((e*x)/d)*d**2*e*x - 105*sqrt(d**2 - e**2*x**2)*asin((e*x)/d)*d*e
**2*x**2 + 105*asin((e*x)/d)*d**4 - 315*asin((e*x)/d)*d**3*e*x + 315*asin(
(e*x)/d)*d**2*e**2*x**2 - 105*asin((e*x)/d)*d*e**3*x**3 - 48*sqrt(d**2 - e
**2*x**2)*d**3 + 143*sqrt(d**2 - e**2*x**2)*d**2*e*x - 158*sqrt(d**2 - e**
2*x**2)*d*e**2*x**2 + 15*sqrt(d**2 - e**2*x**2)*e**3*x**3 + 48*d**4 + 143*
d**3*e*x - 461*d**2*e**2*x**2 + 381*d*e**3*x**3 - 15*e**4*x**4)/(15*e*(sqr
t(d**2 - e**2*x**2)*d**2 - 2*sqrt(d**2 - e**2*x**2)*d*e*x + sqrt(d**2 - e*
*2*x**2)*e**2*x**2 - d**3 + 3*d**2*e*x - 3*d*e**2*x**2 + e**3*x**3))
```

3.138 $\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2(d+ex)^5}{5e(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)^3}{3e(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2/5*(e*x+d)^5/e/(-e^2*x^2+d^2)^(5/2)-2/3*(e*x+d)^3/e/(-e^2*x^2+d^2)^(3/2)+
2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(\frac{\sqrt{d^2-e^2x^2}(13d^2-24dex+23e^2x^2)}{(d-ex)^3} + 15\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)\right)}{15e}$$

input
```
Integrate[(d + e*x)^6/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(2*((Sqrt[d^2 - e^2*x^2]*(13*d^2 - 24*d*e*x + 23*e^2*x^2))/(d - e*x)^3 + 1
5*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])]))/(15*e)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {468, 468, 457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}}-\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{3/2}}dx+\frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}}-\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{457}$$

$$-\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}}-\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{224}$$

$$-\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}}-\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}}-\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

input
```
Int[(d + e*x)^6/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(2*(d + e*x)^5)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*x)^3)/(3*e*(d^2 -
e^2*x^2)^(3/2)) + (2*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2]) - ArcTan[(e*x)/Sqr
t[d^2 - e^2*x^2]]/e
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(100) = 200$.

Time = 0.40 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.18

method	result
default	$d^6\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^6\left(\frac{x^5}{5e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{\frac{x^3}{3e^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}-\frac{x}{e^2\sqrt{-e^2}}}{}\right.$

input
```
int((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
d^6*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2
)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))+e^6*(1/5*x^5/e^2/(-e^2*x^2+d^2)^(5/2)-1
/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*x^2+d^2)^(1/2)-1
/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)))))+6*d*e^5*(x^
4/e^2/(-e^2*x^2+d^2)^(5/2)-4*d^2/e^2*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/1
5*d^2/e^4/(-e^2*x^2+d^2)^(5/2)))+15*d^2*e^4*(1/2*x^3/e^2/(-e^2*x^2+d^2)^(5
/2)-3/2*d^2/e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^2/e^2*(1/5*x/d^2/(-e
^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*
x^2+d^2)^(1/2)))))+20*d^3*e^3*(1/3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)-2/15*d^2/e
^4/(-e^2*x^2+d^2)^(5/2))+15*d^4*e^2*(1/4*x/e^2/(-e^2*x^2+d^2)^(5/2)-1/4*d^
2/e^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3
/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2))))+6/5*d^5/e/(-e^2*x^2+d^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(13\,e^3x^3-39\,de^2x^2+39\,d^2ex-13\,d^3+15\left(e^3x^3-3\,de^2x^2+3\,d^2ex-d^3\right)\arctan\right.}{15\left(e^4x^3-3\,de^3x^2+3\,d^2e^2x\right.}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(13*e^3*x^3 - 39*d*e^2*x^2 + 39*d^2*e*x - 13*d^3 + 15*(e^3*x^3 - 3*d*
e^2*x^2 + 3*d^2*e*x - d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (23
*e^2*x^2 - 24*d*e*x + 13*d^2)*sqrt(-e^2*x^2 + d^2))/(e^4*x^3 - 3*d*e^3*x^2
 + 3*d^2*e^2*x - d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^6}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**6/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**6/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(100) = 200$.

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.67

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{1}{15}e^6x\left(\frac{15\,x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20\,d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8\,d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) - \frac{1}{3}e^4x\left(\frac{3\,x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2\,d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{6\,de^3x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{15\,d^2e^2x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{4\,d^3ex^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{13\,d^4x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{26\,d^5}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{31\,d^2x}{30\,(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{16\,x}{15\,\sqrt{-e^2x^2+d^2}} - \frac{\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/15*e^6*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - 1/3*e^4*x*(3*x^2/(
(-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 6*d*e^
3*x^4/(-e^2*x^2 + d^2)^(5/2) + 15/2*d^2*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) - 4
/3*d^3*e*x^2/(-e^2*x^2 + d^2)^(5/2) - 13/10*d^4*x/(-e^2*x^2 + d^2)^(5/2) +
 26/15*d^5/((-e^2*x^2 + d^2)^(5/2)*e) + 31/30*d^2*x/(-e^2*x^2 + d^2)^(3/2)
 + 16/15*x/sqrt(-e^2*x^2 + d^2) - arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{4\left(\frac{50\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{100\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 13\right)}{15\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 4/15*(50*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))/(e^2*x) - 100*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 3
0*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) - 15*(d*e + sqrt(-e^2*x^
2 + d^2)*abs(e))^4/(e^8*x^4) - 13)/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(
e^2*x) - 1)^5*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx=\int \frac{(d+e\,x)^6}{(d^2-e^2\,x^2)^{7/2}}\,dx$$

input
```
int((d + e*x)^6/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
int((d + e*x)^6/(d^2 - e^2*x^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}\,dx=\frac{-15asin\left(\frac{ex}{d}\right)\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^5+75asin\left(\frac{ex}{d}\right)\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^4-150asin\left(\frac{ex}{d}\right)\tan\left(\frac{asin(}{2}\right.}{15e\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^5\right.}$$

input
```
int((e*x+d)^6/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
( - 15*asin((e*x)/d)*tan(asin((e*x)/d)/2)**5 + 75*asin((e*x)/d)*tan(asin((
e*x)/d)/2)**4 - 150*asin((e*x)/d)*tan(asin((e*x)/d)/2)**3 + 150*asin((e*x)
/d)*tan(asin((e*x)/d)/2)**2 - 75*asin((e*x)/d)*tan(asin((e*x)/d)/2) + 15*a
sin((e*x)/d) - 12*tan(asin((e*x)/d)/2)**5 - 280*tan(asin((e*x)/d)/2)**2 +
140*tan(asin((e*x)/d)/2) - 40)/(15*e*(tan(asin((e*x)/d)/2)**5 - 5*tan(asin
((e*x)/d)/2)**4 + 10*tan(asin((e*x)/d)/2)**3 - 10*tan(asin((e*x)/d)/2)**2
+ 5*tan(asin((e*x)/d)/2) - 1))
```

3.139 $\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^5}{5de\,(d^2-e^2x^2)^{5/2}}$$

output
```
1/5*(e*x+d)^5/d/e/(-e^2*x^2+d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{(d+ex)^2\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

input
```
Integrate[(d + e*x)^5/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
((d + e*x)^2*Sqrt[d^2 - e^2*x^2])/(5*d*e*(d - e*x)^3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{460\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}} \, dx$$

$$\Big\downarrow \; 460$$

$$\frac{(d+ex)^5}{5de\,(d^2-e^2x^2)^{5/2}}$$

input
```
Int[(d + e*x)^5/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d + e*x)^5/(5*d*e*(d^2 - e^2*x^2)^(5/2))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result
gosper	$\frac{(ex+d)^6(-ex+d)}{5de(-e^2x^2+d^2)^{\frac{7}{2}}}$
orering	$\frac{(ex+d)^6(-ex+d)}{5de(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(e^2x^2+2dex+d^2)\sqrt{-e^2x^2+d^2}}{5d(-ex+d)^3e}$
default	$d^5\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^5\left(\frac{x^4}{e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{4d^2\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{}{15e^4}\right.}{e^2}\right.$

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(e*x+d)^6*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.03

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{e^3x^3-3\,de^2x^2+3\,d^2ex-d^3-(e^2x^2+2\,dex+d^2)\sqrt{-e^2x^2+d^2}}{5\,(de^4x^3-3\,d^2e^3x^2+3\,d^3e^2x-d^4e)}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(e^3*x^3 - 3*d*e^2*x^2 + 3*d^2*e*x - d^3 - (e^2*x^2 + 2*d*e*x + d^2)*s
qrt(-e^2*x^2 + d^2))/(d*e^4*x^3 - 3*d^2*e^3*x^2 + 3*d^3*e^2*x - d^4*e)
```

Sympy [F]

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^5}{(-(-d+ex)(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**5/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**5/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{e^3x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{5\,de^2x^3}{2\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{2\,d^2ex^2}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{7\,d^3x}{10\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{d^4}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{dx}{10\,(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{x}{5\,\sqrt{-e^2x^2+d^2}d}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
e^3*x^4/(-e^2*x^2 + d^2)^(5/2) + 5/2*d*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) + 2*
d^2*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 7/10*d^3*x/(-e^2*x^2 + d^2)^(5/2) + 1/5
*d^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/10*d*x/(-e^2*x^2 + d^2)^(3/2) + 1/5*x/
(sqrt(-e^2*x^2 + d^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(29) = 58$.

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.18

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\left(\frac{10\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+1\right)}{5\,d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
2/5*(10*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 5*(d*e + sqrt(-e
^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 1)/(d*((d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(d+e\,x)^2}{5\,d\,e\,(d-e\,x)^3}$$

input
```
int((d + e*x)^5/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2)/(5*d*e*(d - e*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.42

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{-\frac{2\sqrt{-e^2x^2+d^2}\,d^2}{5}-\frac{2\sqrt{-e^2x^2+d^2}\,e^2x^2}{5}+\frac{2d^3}{5}+\frac{6d\,e^2x^2}{5}}{de\left(\sqrt{-e^2x^2+d^2}\,d^2-2\sqrt{-e^2x^2+d^2}\,dex+\sqrt{-e^2x^2+d^2}\,e^2x^2-d^3+3d^2ex-3d\,e\right.}$$

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(2*( - sqrt(d**2 - e**2*x**2)*d**2 - sqrt(d**2 - e**2*x**2)*e**2*x**2 + d*
*3 + 3*d*e**2*x**2))/(5*d*e*(sqrt(d**2 - e**2*x**2)*d**2 - 2*sqrt(d**2 - e
**2*x**2)*d*e*x + sqrt(d**2 - e**2*x**2)*e**2*x**2 - d**3 + 3*d**2*e*x - 3
*d*e**2*x**2 + e**3*x**3))
```

3.140 $\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{(d+ex)^4}{5de\,(d^2-e^2x^2)^{5/2}} + \frac{(d+ex)^3}{15d^2e\,(d^2-e^2x^2)^{3/2}}$$

output
```
1/5*(e*x+d)^4/d/e/(-e^2*x^2+d^2)^(5/2)+1/15*(e*x+d)^3/d^2/e/(-e^2*x^2+d^2)
^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(4d^2+3dex-e^2x^2)}{15d^2e(d-ex)^3}$$

input
```
Integrate[(d + e*x)^4/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(4*d^2 + 3*d*e*x - e^2*x^2))/(15*d^2*e*(d - e*x)^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{461}$$

$$\frac{(d+ex)^4}{3de\,(d^2-e^2x^2)^{5/2}}-\frac{\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{7/2}}dx}{3d}$$

$$\Big\downarrow \text{460}$$

$$\frac{(d+ex)^4}{3de\,(d^2-e^2x^2)^{5/2}}-\frac{(d+ex)^5}{15d^2e\,(d^2-e^2x^2)^{5/2}}$$

input
```
Int[(d + e*x)^4/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d + e*x)^4/(3*d*e*(d^2 - e^2*x^2)^(5/2)) - (d + e*x)^5/(15*d^2*e*(d^2 - e
^2*x^2)^(5/2))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result
gosper	$\frac{(ex+d)^5(-ex+d)(-ex+4d)}{15d^2e(-e^2x^2+d^2)^{\frac{7}{2}}}$
orering	$\frac{(ex+d)^5(-ex+d)(-ex+4d)}{15d^2e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(-e^2x^2+3dex+4d^2)\sqrt{-e^2x^2+d^2}}{15d^2(-ex+d)^3e}$
default	$d^4\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^4\left(\frac{x^3}{2e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{3d^2\left(\frac{x}{4e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{d^2 \cdots}{\cdots}\right.}{\cdots}\right.$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(e*x+d)^5*(-e*x+d)*(-e*x+4*d)/d^2/e/(-e^2*x^2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{4\,e^3x^3-12\,de^2x^2+12\,d^2ex-4\,d^3+(e^2x^2-3\,dex-4\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^2e^4x^3-3\,d^3e^3x^2+3\,d^4e^2x-d^5e)}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(4*e^3*x^3 - 12*d*e^2*x^2 + 12*d^2*e*x - 4*d^3 + (e^2*x^2 - 3*d*e*x -
 4*d^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^4*x^3 - 3*d^3*e^3*x^2 + 3*d^4*e^2*x -
 d^5*e)
```

Sympy [F]

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^4}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**4/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{e^2x^3}{2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{4\,dex^2}{3\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{11\,d^2x}{10\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{4\,d^3}{15\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} - \frac{x}{30\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \frac{x}{15\sqrt{-e^2x^2+d^2}d^2}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/2*e^2*x^3/(-e^2*x^2 + d^2)^(5/2) + 4/3*d*e*x^2/(-e^2*x^2 + d^2)^(5/2) +
11/10*d^2*x/(-e^2*x^2 + d^2)^(5/2) + 4/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e) -
 1/30*x/(-e^2*x^2 + d^2)^(3/2) - 1/15*x/(sqrt(-e^2*x^2 + d^2)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(59) = 118$.

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\left(\frac{5\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{25\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 4\right)}{15\,d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^5|e|}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(5*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 25*(d*e + sqrt(-e^2
*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3
/(e^6*x^3) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 4)/(d^2*
((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(4\,d^2+3\,d\,e\,x-e^2\,x^2\right)}{15\,d^2\,e\,{\left(d-e\,x\right)}^3}$$

input
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(4*d^2 - e^2*x^2 + 3*d*e*x))/(15*d^2*e*(d - e*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.67

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{-6\sqrt{-e^2x^2+d^2}\,d^2+\sqrt{-e^2x^2+d^2}\,dex-\sqrt{-e^2x^2+d^2}\,e^2x^2+6d^3+d^2ex+8d\,e}{15d^2e\left(\sqrt{-e^2x^2+d^2}\,d^2-2\sqrt{-e^2x^2+d^2}\,dex+\sqrt{-e^2x^2+d^2}\,e^2x^2-d^3+3d^2ex-3\right.}$$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
( - 6*sqrt(d**2 - e**2*x**2)*d**2 + sqrt(d**2 - e**2*x**2)*d*e*x - sqrt(d*
*2 - e**2*x**2)*e**2*x**2 + 6*d**3 + d**2*e*x + 8*d*e**2*x**2 - 3*e**3*x**
3)/(15*d**2*e*(sqrt(d**2 - e**2*x**2)*d**2 - 2*sqrt(d**2 - e**2*x**2)*d*e*
x + sqrt(d**2 - e**2*x**2)*e**2*x**2 - d**3 + 3*d**2*e*x - 3*d*e**2*x**2 +
 e**3*x**3))
```

3.141 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{2(d+ex)^2}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{d+ex}{15de\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{15d^3\sqrt{d^2-e^2x^2}}$$

output
```
2/5*(e*x+d)^2/e/(-e^2*x^2+d^2)^(5/2)+1/15*(e*x+d)/d/e/(-e^2*x^2+d^2)^(3/2)
+2/15*x/d^3/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(7d^2-6dex+2e^2x^2)}{15d^3e(d-ex)^3}$$

input
```
Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(7*d^2 - 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d - e*x)^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {464, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{ 464}$$

$$\int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{2\left(\frac{\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}dx}{3d}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{2\left(\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d-ex)}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}$$

input
```
Int[(d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*(Sqrt[d^2 - e^2*x^2]/(3*d*e*(
d - e*x)^2) + Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d - e*x))))/(5*d)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

method	result
trager	$\frac{\left(2e^2x^2-6dex+7d^2\right)\sqrt{-e^2x^2+d^2}}{15d^3\left(-ex+d\right)^3 e}$
gosper	$\frac{\left(ex+d\right)^4\left(-ex+d\right)\left(2e^2x^2-6dex+7d^2\right)}{15d^3e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
orering	$\frac{\left(ex+d\right)^4\left(-ex+d\right)\left(2e^2x^2-6dex+7d^2\right)}{15d^3e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
default	$d^3\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^3\left(\frac{x^2}{3e^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}-\frac{2d^2}{15e^4\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}\right)+3$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/(-e*x+d)^3/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{7\,e^3x^3-21\,de^2x^2+21\,d^2ex-7\,d^3-(2\,e^2x^2-6\,dex+7\,d^2)\sqrt{-e^2x^2+d^2}}{15\,(d^3e^4x^3-3\,d^4e^3x^2+3\,d^5e^2x-d^6e)}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(7*e^3*x^3 - 21*d*e^2*x^2 + 21*d^2*e*x - 7*d^3 - (2*e^2*x^2 - 6*d*e*x
 + 7*d^2)*sqrt(-e^2*x^2 + d^2))/(d^3*e^4*x^3 - 3*d^4*e^3*x^2 + 3*d^5*e^2*x
 - d^6*e)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{ex^2}{3\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{4\,dx}{5\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{7\,d^2}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{x}{15\,(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{2\,x}{15\,\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d
^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x
/(sqrt(-e^2*x^2 + d^2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(74) = 148$.

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = -\frac{2\left(\frac{20\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{40\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{30\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{15\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} - 7\right)}{15\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right)^5|e|}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(20*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 40*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 30*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
3/(e^6*x^3) - 15*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) - 7)/(d^3
*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^5*abs(e))
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(7\,d^2-6\,d\,e\,x+2\,e^2\,x^2)}{15\,d^3\,e\,(d-e\,x)^3}$$

input
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(7*d^2 + 2*e^2*x^2 - 6*d*e*x))/(15*d^3*e*(d - e*x)^
3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{-\frac{2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^5}{5}-\frac{4\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2}{3}+\frac{2\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)}{3}-\frac{8}{15}}{d^3e\left(\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^5-5\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^4+10\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^3-10\tan\left(\frac{asin\left(\frac{ex}{d}\right)}{2}\right)^2+\right.}$$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(2*( - 3*tan(asin((e*x)/d)/2)**5 - 10*tan(asin((e*x)/d)/2)**2 + 5*tan(asin
((e*x)/d)/2) - 4))/(15*d**3*e*(tan(asin((e*x)/d)/2)**5 - 5*tan(asin((e*x)/
d)/2)**4 + 10*tan(asin((e*x)/d)/2)**3 - 10*tan(asin((e*x)/d)/2)**2 + 5*tan
(asin((e*x)/d)/2) - 1))
```

3.142 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{2(d+ex)}{5e\left(d^2-e^2x^2\right)^{5/2}}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

output
```
2/5*(e*x+d)/e/(-e^2*x^2+d^2)^(5/2)+1/5*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/5*x/d^
4/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(2d^3+d^2ex-4de^2x^2+2e^3x^3)}{5d^4e(d-ex)^3(d+ex)}$$

input
```
Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(2*d^3 + d^2*e*x - 4*d*e^2*x^2 + 2*e^3*x^3))/(5*d^4*e
*(d - e*x)^3*(d + e*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{457}$$

$$\frac{3}{5}\int \frac{1}{(d^2-e^2x^2)^{5/2}}dx+\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{3}{5}\left(\frac{2\int \frac{1}{(d^2-e^2x^2)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\,(d^2-e^2x^2)^{3/2}}\right)+\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2(d+ex)}{5e\,(d^2-e^2x^2)^{5/2}}+\frac{3}{5}\left(\frac{x}{3d^2\,(d^2-e^2x^2)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)$$

input
```
Int[(d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(2*(d + e*x))/(5*e*(d^2 - e^2*x^2)^(5/2)) + (3*(x/(3*d^2*(d^2 - e^2*x^2)^(
3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/5
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{(ex+d)^3(-ex+d)(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)}{5d^4e(-e^2x^2+d^2)^{\frac{7}{2}}}$
orering	$\frac{(ex+d)^3(-ex+d)(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)}{5d^4e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$\frac{(2e^3x^3-4d\,e^2x^2+d^2ex+2d^3)\sqrt{-e^2x^2+d^2}}{5d^4(-ex+d)^3e(ex+d)}$
default	$d^2\left(\frac{x}{5d^2(-e^2x^2+d^2)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}}{d^2}\right)+e^2\left(\frac{x}{4e^2(-e^2x^2+d^2)^{\frac{5}{2}}}-\frac{d^2\left(\frac{x}{5d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{}{15d^{}}\right.}{}\right.$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(e*x+d)^3*(-e*x+d)*(2*e^3*x^3-4*d*e^2*x^2+d^2*e*x+2*d^3)/d^4/e/(-e^2*x
^2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,e^4x^4-4\,de^3x^3+4\,d^3ex-2\,d^4-\left(2\,e^3x^3-4\,de^2x^2+d^2ex+2\,d^3\right)\sqrt{-e^2x^2+d^2}}{5\,\left(d^4e^5x^4-2\,d^5e^4x^3+2\,d^7e^2x-d^8e\right)}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(2*e^4*x^4 - 4*d*e^3*x^3 + 4*d^3*e*x - 2*d^4 - (2*e^3*x^3 - 4*d*e^2*x^
2 + d^2*e*x + 2*d^3)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^4 - 2*d^5*e^4*x^3 +
2*d^7*e^2*x - d^8*e)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{2\,x}{5\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{2\,d}{5\,\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{x}{5\,\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^2} + \frac{2\,x}{5\,\sqrt{-e^2x^2+d^2}d^4}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
2/5*x/(-e^2*x^2 + d^2)^(5/2) + 2/5*d/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*x/((
-e^2*x^2 + d^2)^(3/2)*d^2) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d^4)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(2\,d^3+d^2\,e\,x-4\,d\,e^2\,x^2+2\,e^3\,x^3)}{5\,d^4\,e\,(d+e\,x)\,(d-e\,x)^3}$$

input
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(2*d^3 + 2*e^3*x^3 - 4*d*e^2*x^2 + d^2*e*x))/(5*d^4
*e*(d + e*x)*(d - e*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{-\sqrt{-e^2x^2+d^2}\,d^2+2\sqrt{-e^2x^2+d^2}\,dex-\sqrt{-e^2x^2+d^2}\,e^2x^2+4d^3+2d^2ex-8d\,e^2}{10\sqrt{-e^2x^2+d^2}\,d^4e\left(e^2x^2-2dex+d^2\right)}$$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
( - sqrt(d**2 - e**2*x**2)*d**2 + 2*sqrt(d**2 - e**2*x**2)*d*e*x - sqrt(d*
*2 - e**2*x**2)*e**2*x**2 + 4*d**3 + 2*d**2*e*x - 8*d*e**2*x**2 + 4*e**3*x
**3)/(10*sqrt(d**2 - e**2*x**2)*d**4*e*(d**2 - 2*d*e*x + e**2*x**2))
```

3.143 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{d+ex}{5de\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{15d^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

output
```
1/5*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/15*x/d^3/(-e^2*x^2+d^2)^(3/2)+8/15*
x/d^5/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(3d^4+12d^3ex-12d^2e^2x^2-8de^3x^3+8e^4x^4)}{15d^5e(d-ex)^3(d+ex)^2}$$

input
```
Integrate[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 +
8*e^4*x^4))/(15*d^5*e*(d - e*x)^3*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}dx$$

$$\Big\downarrow \; 454$$

$$\frac{4\int \frac{1}{(d^2-e^2x^2)^{5/2}}dx}{5d}+\frac{d+ex}{5de\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2(d^2-e^2x^2)^{3/2}}\right)}{5d}+\frac{d+ex}{5de\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{d+ex}{5de\,(d^2-e^2x^2)^{5/2}}+\frac{4\left(\frac{x}{3d^2(d^2-e^2x^2)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}$$

input
```
Int[(d + e*x)/(d^2 - e^2*x^2)^(7/2),x]
```

output
```
(d + e*x)/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/
2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(ex+d)^{2}(-ex+d)\left(8e^{4}x^{4}-8d\,e^{3}x^{3}-12d^{2}e^{2}x^{2}+12d^{3}ex+3d^{4}\right)}{15d^{5}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$	77
orering	$\frac{(ex+d)^{2}(-ex+d)\left(8e^{4}x^{4}-8d\,e^{3}x^{3}-12d^{2}e^{2}x^{2}+12d^{3}ex+3d^{4}\right)}{15d^{5}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}$	77
trager	$\frac{\left(8e^{4}x^{4}-8d\,e^{3}x^{3}-12d^{2}e^{2}x^{2}+12d^{3}ex+3d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{15d^{5}(-ex+d)^{3}(ex+d)^{2}e}$	79
default	$d\left(\frac{x}{5d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}}{d^{2}}\right)+\frac{1}{5e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}$	90

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(e*x+d)^2*(-e*x+d)*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3
*d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.14

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{3\,e^5x^5 - 3\,de^4x^4 - 6\,d^2e^3x^3 + 6\,d^3e^2x^2 + 3\,d^4ex - 3\,d^5 - (8\,e^4x^4 - 8\,de^3x^3 - 12\,d^2e}{15\,(d^5e^6x^5 - d^6e^5x^4 - 2\,d^7e^4x^3 + 2\,d^8e^3x^2 + d^9e^2x - }$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(3*e^5*x^5 - 3*d*e^4*x^4 - 6*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 3*d^4*e*x
- 3*d^5 - (8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 + 12*d^3*e*x + 3*d^4)*
sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - d^6*e^5*x^4 - 2*d^7*e^4*x^3 + 2*d^8*e
^3*x^2 + d^9*e^2*x - d^10*e)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.37 (sec) , antiderivative size = 604, normalized size of antiderivative = 7.55

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = d\left(\begin{cases} -\frac{15id^4x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{20id^2e}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1} \\ \frac{15d^4x}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{20d^2e^2x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15} \end{cases}\right.$$
$$+ e\left(\begin{cases} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for } e \neq 0 \\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
d*Piecewise((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/
d**2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9
*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x*
*2/d**2)) - 8*I*e**4*x**5/(15*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/
d**2)), Abs(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/
d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqr
t(1 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(1 - e**2*x**2/d*
*2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(
1 - e**2*x**2/d**2)) + 8*e**4*x**5/(15*d**11*sqrt(1 - e**2*x**2/d**2) - 30
*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2
*x**2/d**2)), True)) + e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2)
- 10*d**2*e**4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*
x**2)), Ne(e, 0)), (x**2/(2*(d**2)**(7/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{x}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d}+\frac{1}{5\left(-e^2x^2+d^2\right)^{\frac{5}{2}}e} + \frac{4\,x}{15\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^3}+\frac{8\,x}{15\sqrt{-e^2x^2+d^2}\,d^5}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x
/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Giac [F]

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(-e^2*x^2 + d^2)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,(3\,d^4+12\,d^3\,e\,x-12\,d^2\,e^2\,x^2-8\,d\,e^3\,x^3+8\,e^4\,x^4)}{15\,d^5\,e\,(d+e\,x)^2\,(d-e\,x)^3}$$

input
```
int((d + e*x)/(d^2 - e^2*x^2)^(7/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*(3*d^4 + 8*e^4*x^4 - 8*d*e^3*x^3 - 12*d^2*e^2*x^2 +
 12*d^3*e*x))/(15*d^5*e*(d + e*x)^2*(d - e*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.18

$$\int \frac{d+ex}{(d^2-e^2x^2)^{7/2}}\,dx = \frac{-12\sqrt{-e^2x^2+d^2}\,d^3+12\sqrt{-e^2x^2+d^2}\,d^2ex+12\sqrt{-e^2x^2+d^2}\,d\,e^2x^2-12\sqrt{-e^2x^2}}{15\sqrt{-e^2x^2+d^2}\,d^5e\,(e^3x^3-d\,e^2x^2-}$$

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
( - 12*sqrt(d**2 - e**2*x**2)*d**3 + 12*sqrt(d**2 - e**2*x**2)*d**2*e*x +
12*sqrt(d**2 - e**2*x**2)*d*e**2*x**2 - 12*sqrt(d**2 - e**2*x**2)*e**3*x**
3 + 3*d**4 + 12*d**3*e*x - 12*d**2*e**2*x**2 - 8*d*e**3*x**3 + 8*e**4*x**4
)/(15*sqrt(d**2 - e**2*x**2)*d**5*e*(d**3 - d**2*e*x - d*e**2*x**2 + e**3*
x**3))
```

3.144 $\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{6x}{35d^3\left(d^2-e^2x^2\right)^{5/2}} - \frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}} + \frac{8x}{35d^5\left(d^2-e^2x^2\right)^{3/2}} + \frac{16x}{35d^7\sqrt{d^2-e^2x^2}}$$

output
```
6/35*x/d^3/(-e^2*x^2+d^2)^(5/2)-1/7/d/e/(e*x+d)/(-e^2*x^2+d^2)^(5/2)+8/35*
x/d^5/(-e^2*x^2+d^2)^(3/2)+16/35*x/d^7/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-5d^6+30d^5ex+30d^4e^2x^2-40d^3e^3x^3-40d^2e^4x^4+16de^5x^5}{35d^7e(d-ex)^3(d+ex)^4}$$

input
```
Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-5*d^6 + 30*d^5*e*x + 30*d^4*e^2*x^2 - 40*d^3*e^3*x^
3 - 40*d^2*e^4*x^4 + 16*d*e^5*x^5 + 16*e^6*x^6))/(35*d^7*e*(d - e*x)^3*(d
+ e*x)^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {470, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 470}$$

$$\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{6\left(\frac{x}{5d^2(d^2-e^2x^2)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/((d + e*x)*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
-1/7*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2
)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e
^2*x^2])))/(5*d^2)))/(7*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{(-ex+d)\left(-16e^6x^6-16d\,e^5x^5+40d^2e^4x^4+40d^3e^3x^3-30d^4e^2x^2-30d^5ex+5d^6\right)}{35d^7e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
orering	$-\frac{(-ex+d)\left(-16e^6x^6-16d\,e^5x^5+40d^2e^4x^4+40d^3e^3x^3-30d^4e^2x^2-30d^5ex+5d^6\right)}{35d^7e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(-16e^6x^6-16d\,e^5x^5+40d^2e^4x^4+40d^3e^3x^3-30d^4e^2x^2-30d^5ex+5d^6\right)\sqrt{-e^2x^2+d^2}}{35d^7(ex+d)^4(-ex+d)^3e}$
default	$\frac{-\frac{1}{7de\left(x+\frac{d}{e}\right)\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{6e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{10d^2e^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{-\frac{2\left(-2e^2\left(x+\frac{d}{e}\right)+2de\right)}{15d^2e^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}}-\frac{4(}{15e^2d^4\sqrt{}}}{d^2}\right.}{7d}}{e}$

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(-e*x+d)*(-16*e^6*x^6-16*d*e^5*x^5+40*d^2*e^4*x^4+40*d^3*e^3*x^3-30*
d^4*e^2*x^2-30*d^5*e*x+5*d^6)/d^7/e/(-e^2*x^2+d^2)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(90) = 180.

Time = 0.16 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.23

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx =$$
$$-\frac{5\,e^7x^7+5\,de^6x^6-15\,d^2e^5x^5-15\,d^3e^4x^4+15\,d^4e^3x^3+15\,d^5e^2x^2-5\,d^6ex-5\,d^7+\left(16\,e^6x^6+16\,de^5x^5\right.}{35\left(d^7e^8x^7+d^8e^7x^6-3\,d^9e^6x^5-3\,d^{10}e^5x^4+3\,d^{11}e^4x^3+3\,d\right.}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/35*(5*e^7*x^7 + 5*d*e^6*x^6 - 15*d^2*e^5*x^5 - 15*d^3*e^4*x^4 + 15*d^4*
e^3*x^3 + 15*d^5*e^2*x^2 - 5*d^6*e*x - 5*d^7 + (16*e^6*x^6 + 16*d*e^5*x^5
- 40*d^2*e^4*x^4 - 40*d^3*e^3*x^3 + 30*d^4*e^2*x^2 + 30*d^5*e*x - 5*d^6)*s
qrt(-e^2*x^2 + d^2))/(d^7*e^8*x^7 + d^8*e^7*x^6 - 3*d^9*e^6*x^5 - 3*d^10*e
^5*x^4 + 3*d^11*e^4*x^3 + 3*d^12*e^3*x^2 - d^13*e^2*x - d^14*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{7}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{7/2}}\,dx = -\frac{1}{7\left((-e^2x^2+d^2)^{\frac{5}{2}}de^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^2e\right)} + \frac{6\,x}{35\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3} + \frac{8\,x}{35\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5} + \frac{16\,x}{35\,\sqrt{-e^2x^2+d^2}d^7}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/7/((-e^2*x^2 + d^2)^(5/2)*d*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^2*e) + 6/3
5*x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 8/35*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 1
6/35*x/(sqrt(-e^2*x^2 + d^2)*d^7)
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx=\int \frac{1}{\left(-e^2x^2+d^2\right)^{\frac{7}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{17\,x}{70\,d^3}-\frac{1}{7\,d^2\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{8\,x}{35\,d^5}+\frac{1}{56\,d^4\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}-\frac{\sqrt{d^2-e^2\,x^2}}{56\,d^4\,e\,{\left(d+e\,x\right)}^4}+\frac{16\,x\,\sqrt{d^2-e^2\,x^2}}{35\,d^7\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((17*x)/(70*d^3) - 1/(7*d^2*e)))/((d + e*x)^3*(d -
e*x)^3) + ((d^2 - e^2*x^2)^(1/2)*((8*x)/(35*d^5) + 1/(56*d^4*e)))/((d + e*
x)^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(56*d^4*e*(d + e*x)^4) + (16*x*(
d^2 - e^2*x^2)^(1/2))/(35*d^7*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.49

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}\,dx=\frac{30\sqrt{-e^2x^2+d^2}\,d^5+30\sqrt{-e^2x^2+d^2}\,d^4ex-60\sqrt{-e^2x^2+d^2}\,d^3e^2x^2-60\sqrt{}}{35}$$

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(30*sqrt(d**2 - e**2*x**2)*d**5 + 30*sqrt(d**2 - e**2*x**2)*d**4*e*x - 60*
sqrt(d**2 - e**2*x**2)*d**3*e**2*x**2 - 60*sqrt(d**2 - e**2*x**2)*d**2*e**
3*x**3 + 30*sqrt(d**2 - e**2*x**2)*d*e**4*x**4 + 30*sqrt(d**2 - e**2*x**2)
*e**5*x**5 - 5*d**6 + 30*d**5*e*x + 30*d**4*e**2*x**2 - 40*d**3*e**3*x**3
- 40*d**2*e**4*x**4 + 16*d*e**5*x**5 + 16*e**6*x**6)/(35*sqrt(d**2 - e**2*
x**2)*d**7*e*(d**5 + d**4*e*x - 2*d**3*e**2*x**2 - 2*d**2*e**3*x**3 + d*e*
*4*x**4 + e**5*x**5))
```

3.145 $\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx = \frac{x}{9d^2(d^2-e^2x^2)^{7/2}} - \frac{2}{9e(d+ex)(d^2-e^2x^2)^{7/2}} + \frac{2x}{15d^4(d^2-e^2x^2)^{5/2}} + \frac{8x}{45d^6(d^2-e^2x^2)^{3/2}} + \frac{16x}{45d^8\sqrt{d^2-e^2x^2}}$$

output
```
1/9*x/d^2/(-e^2*x^2+d^2)^(7/2)-2/9/e/(e*x+d)/(-e^2*x^2+d^2)^(7/2)+2/15*x/d
^4/(-e^2*x^2+d^2)^(5/2)+8/45*x/d^6/(-e^2*x^2+d^2)^(3/2)+16/45*x/d^8/(-e^2*
x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-10d^7+25d^6ex+60d^5e^2x^2-10d^4e^3x^3-80d^3e^4x^4-24d^2e^5}{45d^8e(d-ex)^3(d+ex)^5}$$

input
```
Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-10*d^7 + 25*d^6*e*x + 60*d^5*e^2*x^2 - 10*d^4*e^3*x
^3 - 80*d^3*e^4*x^4 - 24*d^2*e^5*x^5 + 32*d*e^6*x^6 + 16*e^7*x^7))/(45*d^8
*e*(d - e*x)^3*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {461, 470, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{7/2}} \, dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{7 \int \frac{1}{(d+ex)(d^2-e^2x^2)^{7/2}}dx}{9d} - \frac{1}{9de(d+ex)^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 470}$$

$$\frac{7\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d} - \frac{1}{7de(d+ex)(d^2-e^2x^2)^{5/2}}\right)}{9d} - \frac{1}{9de(d+ex)^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{7\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2} + \frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d} - \frac{1}{7de(d+ex)(d^2-e^2x^2)^{5/2}}\right)}{9d} - \frac{1}{9de(d+ex)^2 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)(d^2-e^2x^2)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{7\left(\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}-\frac{1}{7de(d+ex)(d^2-e^2x^2)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
-1/9*1/(d*e*(d + e*x)^2*(d^2 - e^2*x^2)^(5/2)) + (7*(-1/7*1/(d*e*(d + e*x)
*(d^2 - e^2*x^2)^(5/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d
^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/
(7*d)))/(9*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

method	result
gosper	$-\frac{(-ex+d)\left(-16e^7x^7-32d\,e^6x^6+24d^2e^5x^5+80d^3e^4x^4+10d^4e^3x^3-60d^5e^2x^2-25d^6ex+10d^7\right)}{45(ex+d)d^8e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
orering	$-\frac{(-ex+d)\left(-16e^7x^7-32d\,e^6x^6+24d^2e^5x^5+80d^3e^4x^4+10d^4e^3x^3-60d^5e^2x^2-25d^6ex+10d^7\right)}{45(ex+d)d^8e\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}$
trager	$-\frac{\left(-16e^7x^7-32d\,e^6x^6+24d^2e^5x^5+80d^3e^4x^4+10d^4e^3x^3-60d^5e^2x^2-25d^6ex+10d^7\right)\sqrt{-e^2x^2+d^2}}{45d^8(ex+d)^5(-ex+d)^3e}$
default	$\frac{-\frac{1}{9de\left(x+\frac{d}{e}\right)^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{7e\left(-\frac{1}{7de\left(x+\frac{d}{e}\right)\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\frac{6e\left(-\frac{-2e^2\left(x+\frac{d}{e}\right)+2de}{10d^2e^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+\cdots\right)}{\cdots}\right)}{9d}}{e^2}$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(-e*x+d)*(-16*e^7*x^7-32*d*e^6*x^6+24*d^2*e^5*x^5+80*d^3*e^4*x^4+10*
d^4*e^3*x^3-60*d^5*e^2*x^2-25*d^6*e*x+10*d^7)/(e*x+d)/d^8/e/(-e^2*x^2+d^2)
^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(107) = 214$.

Time = 0.23 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.95

$$\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}\,dx = -\frac{10\,e^8x^8+20\,de^7x^7-20\,d^2e^6x^6-60\,d^3e^5x^5+60\,d^5e^3x^3+20\,d^6e^2x^2-20\,d^7ex-10\,d^8+(16\,e^7x^7+32\,de\cdots}{45\,(d^8e^9x^8+2\,d^9e^8x^7-2\,d^{10}e^7x^6-6\,d^{11}e^6x^5+6\,d^{13}e\cdots}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/45*(10*e^8*x^8 + 20*d*e^7*x^7 - 20*d^2*e^6*x^6 - 60*d^3*e^5*x^5 + 60*d^
5*e^3*x^3 + 20*d^6*e^2*x^2 - 20*d^7*e*x - 10*d^8 + (16*e^7*x^7 + 32*d*e^6*
x^6 - 24*d^2*e^5*x^5 - 80*d^3*e^4*x^4 - 10*d^4*e^3*x^3 + 60*d^5*e^2*x^2 +
25*d^6*e*x - 10*d^7)*sqrt(-e^2*x^2 + d^2))/(d^8*e^9*x^8 + 2*d^9*e^8*x^7 -
2*d^10*e^7*x^6 - 6*d^11*e^6*x^5 + 6*d^13*e^4*x^3 + 2*d^14*e^3*x^2 - 2*d^15
*e^2*x - d^16*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{7}{2}}\left(d+ex\right)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = -\frac{1}{9\left((-e^2x^2+d^2)^{\frac{5}{2}}de^3x^2+2\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^3e\right)} - \frac{1}{9\left((-e^2x^2+d^2)^{\frac{5}{2}}d^2e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^3e\right)} + \frac{2\,x}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4} + \frac{8\,x}{45\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6} + \frac{16\,x}{45\,\sqrt{-e^2x^2+d^2}d^8}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/9/((-e^2*x^2 + d^2)^(5/2)*d*e^3*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^2*e^2*
x + (-e^2*x^2 + d^2)^(5/2)*d^3*e) - 1/9/((-e^2*x^2 + d^2)^(5/2)*d^2*e^2*x
+ (-e^2*x^2 + d^2)^(5/2)*d^3*e) + 2/15*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 8/
45*x/((-e^2*x^2 + d^2)^(3/2)*d^6) + 16/45*x/(sqrt(-e^2*x^2 + d^2)*d^8)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.31

$$\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{e^7\left(\frac{3\left(315\left(\frac{2d}{ex+d}-1\right)^2+\frac{70d}{ex+d}-32\right)}{d^8e^7\left(\frac{2d}{ex+d}-1\right)^{\frac{5}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{5d^{64}e^{56}\left(\frac{2d}{ex+d}-1\right)^{\frac{9}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)^8\operatorname{sgn}(e)^8+45d^{64}e^{56}\left(\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
1/5760*(e^7*(3*(315*(2*d/(e*x + d) - 1)^2 + 70*d/(e*x + d) - 32)/(d^8*e^7*
(2*d/(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))*sgn(e)) - (5*d^64*e^56*(2*d/(e*
x + d) - 1)^(9/2)*sgn(1/(e*x + d))^8*sgn(e)^8 + 45*d^64*e^56*(2*d/(e*x + d
) - 1)^(7/2)*sgn(1/(e*x + d))^8*sgn(e)^8 + 189*d^64*e^56*(2*d/(e*x + d) -
1)^(5/2)*sgn(1/(e*x + d))^8*sgn(e)^8 + 525*d^64*e^56*(2*d/(e*x + d) - 1)^(
3/2)*sgn(1/(e*x + d))^8*sgn(e)^8 + 1575*d^64*e^56*sqrt(2*d/(e*x + d) - 1)*
sgn(1/(e*x + d))^8*sgn(e)^8)/(d^72*e^63*sgn(1/(e*x + d))^9*sgn(e)^9)) + 20
48*I*sgn(1/(e*x + d))*sgn(e)/d^8)/abs(e)
```

Mupad [B] (verification not implemented)

Time = 6.63 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{31\,x}{120\,d^4}-\frac{5}{24\,d^3\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} \\ &+ \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{8\,x}{45\,d^6}+\frac{5}{144\,d^5\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} - \frac{\sqrt{d^2-e^2\,x^2}}{72\,d^4\,e\,{\left(d+e\,x\right)}^5} \\ &- \frac{5\,\sqrt{d^2-e^2\,x^2}}{144\,d^5\,e\,{\left(d+e\,x\right)}^4} + \frac{16\,x\,\sqrt{d^2-e^2\,x^2}}{45\,d^8\,\left(d+e\,x\right)\,\left(d-e\,x\right)}\end{aligned}$$

input
```
int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((31*x)/(120*d^4) - 5/(24*d^3*e)))/((d + e*x)^3*(d
- e*x)^3) + ((d^2 - e^2*x^2)^(1/2)*((8*x)/(45*d^6) + 5/(144*d^5*e)))/((d +
 e*x)^2*(d - e*x)^2) - (d^2 - e^2*x^2)^(1/2)/(72*d^4*e*(d + e*x)^5) - (5*(
d^2 - e^2*x^2)^(1/2))/(144*d^5*e*(d + e*x)^4) + (16*x*(d^2 - e^2*x^2)^(1/2
))/(45*d^8*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.46

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{7/2}}\,dx = \frac{25\sqrt{-e^2x^2+d^2}\,d^6+50\sqrt{-e^2x^2+d^2}\,d^5ex-25\sqrt{-e^2x^2+d^2}\,d^4e^2x^2-100}{}$$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(25*sqrt(d**2 - e**2*x**2)*d**6 + 50*sqrt(d**2 - e**2*x**2)*d**5*e*x - 25*
sqrt(d**2 - e**2*x**2)*d**4*e**2*x**2 - 100*sqrt(d**2 - e**2*x**2)*d**3*e*
*3*x**3 - 25*sqrt(d**2 - e**2*x**2)*d**2*e**4*x**4 + 50*sqrt(d**2 - e**2*x
**2)*d*e**5*x**5 + 25*sqrt(d**2 - e**2*x**2)*e**6*x**6 - 20*d**7 + 50*d**6
*e*x + 120*d**5*e**2*x**2 - 20*d**4*e**3*x**3 - 160*d**3*e**4*x**4 - 48*d*
*2*e**5*x**5 + 64*d*e**6*x**6 + 32*e**7*x**7)/(90*sqrt(d**2 - e**2*x**2)*d
**8*e*(d**6 + 2*d**5*e*x - d**4*e**2*x**2 - 4*d**3*e**3*x**3 - d**2*e**4*x
**4 + 2*d*e**5*x**5 + e**6*x**6))
```

3.146 $\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 160

$$\begin{aligned}\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx &= \frac{8x}{99d^3\left(d^2-e^2x^2\right)^{7/2}} \\ &- \frac{2}{11e(d+ex)^2\left(d^2-e^2x^2\right)^{7/2}} - \frac{7}{99de(d+ex)\left(d^2-e^2x^2\right)^{7/2}} \\ &+ \frac{16x}{165d^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{64x}{495d^7\left(d^2-e^2x^2\right)^{3/2}} + \frac{128x}{495d^9\sqrt{d^2-e^2x^2}}\end{aligned}$$

output
```
8/99*x/d^3/(-e^2*x^2+d^2)^(7/2)-2/11/e/(e*x+d)^2/(-e^2*x^2+d^2)^(7/2)-7/99
/d/e/(e*x+d)/(-e^2*x^2+d^2)^(7/2)+16/165*x/d^5/(-e^2*x^2+d^2)^(5/2)+64/495
*x/d^7/(-e^2*x^2+d^2)^(3/2)+128/495*x/d^9/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d+ex)^3\left(d^2-e^2x^2\right)^{7/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-125d^8+120d^7ex+680d^6e^2x^2+400d^5e^3x^3-720d^4e^4x^4-}{495d^9e(d-ex)^3(d+ex)^6}$$

input
```
Integrate[1/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-125*d^8 + 120*d^7*e*x + 680*d^6*e^2*x^2 + 400*d^5*e
^3*x^3 - 720*d^4*e^4*x^4 - 832*d^3*e^5*x^5 + 64*d^2*e^6*x^6 + 384*d*e^7*x^
7 + 128*e^8*x^8))/(495*d^9*e*(d - e*x)^3*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {461, 461, 470, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}} \, dx$$

$$\Big\downarrow \text{461}$$

$$\frac{8\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{7/2}}dx}{11d}-\frac{1}{11de(d+ex)^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{8\left(\frac{7\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{7/2}}dx}{9d}-\frac{1}{9de(d+ex)^2(d^2-e^2x^2)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{8\left(\frac{7\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2(d^2-e^2x^2)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3 (d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2(d^2-e^2x^2)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 209

$$\frac{8\left(\frac{7\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2(d^2-e^2x^2)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

$\Big\downarrow$ 208

$$\frac{8\left(\frac{7\left(\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d}-\frac{1}{7de(d+ex)\left(d^2-e^2x^2\right)^{5/2}}\right)}{9d}-\frac{1}{9de(d+ex)^2(d^2-e^2x^2)^{5/2}}\right)}{11d}-\frac{1}{11de(d+ex)^3\left(d^2-e^2x^2\right)^{5/2}}$$

input
```
Int[1/((d + e*x)^3*(d^2 - e^2*x^2)^(7/2)),x]
```

output
```
-1/11*1/(d*e*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2)) + (8*(-1/9*1/(d*e*(d + e*x
)^2*(d^2 - e^2*x^2)^(5/2)) + (7*(-1/7*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(5/
2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(
3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d)))/(9*d)))/(11*
d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

method	result
gosper	$-\frac{(-ex+d)\left(-128e^8x^8-384d\,e^7x^7-64d^2e^6x^6+832d^3e^5x^5+720d^4e^4x^4-400d^5e^3x^3-680d^6e^2x^2-120d^7ex+125d^8\right)}{495(ex+d)^2d^9e(-e^2x^2+d^2)^{\frac{7}{2}}}$
orering	$-\frac{(-ex+d)\left(-128e^8x^8-384d\,e^7x^7-64d^2e^6x^6+832d^3e^5x^5+720d^4e^4x^4-400d^5e^3x^3-680d^6e^2x^2-120d^7ex+125d^8\right)}{495(ex+d)^2d^9e(-e^2x^2+d^2)^{\frac{7}{2}}}$
trager	$-\frac{\left(-128e^8x^8-384d\,e^7x^7-64d^2e^6x^6+832d^3e^5x^5+720d^4e^4x^4-400d^5e^3x^3-680d^6e^2x^2-120d^7ex+125d^8\right)\sqrt{-e^2x^2+d^2}}{495d^9(ex+d)^6(-ex+d)^3e}$
default	$-\frac{1}{11de\left(x+\frac{d}{e}\right)^3\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+8e\left(-\frac{1}{9de\left(x+\frac{d}{e}\right)^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}}}+7e\left(-\frac{1}{7de\left(x+\frac{d}{e}\right)\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right.\right.}\right.\right.$ … e^3

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/495*(-e*x+d)*(-128*e^8*x^8-384*d*e^7*x^7-64*d^2*e^6*x^6+832*d^3*e^5*x^5
+720*d^4*e^4*x^4-400*d^5*e^3*x^3-680*d^6*e^2*x^2-120*d^7*e*x+125*d^8)/(e*x
+d)^2/d^9/e/(-e^2*x^2+d^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.62

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx =$$

$$-\frac{125\,e^9x^9+375\,de^8x^8-1000\,d^3e^6x^6-750\,d^4e^5x^5+750\,d^5e^4x^4+1000\,d^6e^3x^3-375\,d^8ex-125\,d^9+(128}{495\,(d^9e^{10}x^9+3\,d^{10}e^9x^8-8\,d^{12}e^7x^6-}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="fricas")
```

output
```
-1/495*(125*e^9*x^9 + 375*d*e^8*x^8 - 1000*d^3*e^6*x^6 - 750*d^4*e^5*x^5 +
 750*d^5*e^4*x^4 + 1000*d^6*e^3*x^3 - 375*d^8*e*x - 125*d^9 + (128*e^8*x^8
 + 384*d*e^7*x^7 + 64*d^2*e^6*x^6 - 832*d^3*e^5*x^5 - 720*d^4*e^4*x^4 + 40
0*d^5*e^3*x^3 + 680*d^6*e^2*x^2 + 120*d^7*e*x - 125*d^8)*sqrt(-e^2*x^2 + d
^2))/(d^9*e^10*x^9 + 3*d^10*e^9*x^8 - 8*d^12*e^7*x^6 - 6*d^13*e^6*x^5 + 6*
d^14*e^5*x^4 + 8*d^15*e^4*x^3 - 3*d^17*e^2*x - d^18*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-(-d+ex)\,(d+ex))^{\frac{7}{2}}\,(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(7/2)*(d + e*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.70

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = -\frac{1}{11\left((-e^2x^2+d^2)^{\frac{5}{2}}de^4x^3+3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2e^3x^2+3\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^4e\right)} - \frac{8}{99\left((-e^2x^2+d^2)^{\frac{5}{2}}d^2e^3x^2+2\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^4e\right)} - \frac{8}{99\left((-e^2x^2+d^2)^{\frac{5}{2}}d^3e^2x+(-e^2x^2+d^2)^{\frac{5}{2}}d^4e\right)} + \frac{16\,x}{165\,(-e^2x^2+d^2)^{\frac{5}{2}}d^5} + \frac{64\,x}{495\,(-e^2x^2+d^2)^{\frac{3}{2}}d^7} + \frac{128\,x}{495\,\sqrt{-e^2x^2+d^2}d^9}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="maxima")
```

output
```
-1/11/((-e^2*x^2 + d^2)^(5/2)*d*e^4*x^3 + 3*(-e^2*x^2 + d^2)^(5/2)*d^2*e^3
*x^2 + 3*(-e^2*x^2 + d^2)^(5/2)*d^3*e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e)
- 8/99/((-e^2*x^2 + d^2)^(5/2)*d^2*e^3*x^2 + 2*(-e^2*x^2 + d^2)^(5/2)*d^3*
e^2*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e) - 8/99/((-e^2*x^2 + d^2)^(5/2)*d^3*e
^2*x + (-e^2*x^2 + d^2)^(5/2)*d^4*e) + 16/165*x/((-e^2*x^2 + d^2)^(5/2)*d^
5) + 64/495*x/((-e^2*x^2 + d^2)^(3/2)*d^7) + 128/495*x/(sqrt(-e^2*x^2 + d^
2)*d^9)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{7}{2}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```

Mupad [B] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.33

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}} \, dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{64\,x}{495\,d^7}+\frac{67}{1584\,d^6\,e}\right)}{{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{631\,x}{2640\,d^5}-\frac{113}{528\,d^4\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}}{88\,d^4\,e\,{\left(d+e\,x\right)}^6} - \frac{43\,\sqrt{d^2-e^2\,x^2}}{1584\,d^5\,e\,{\left(d+e\,x\right)}^5} - \frac{67\,\sqrt{d^2-e^2\,x^2}}{1584\,d^6\,e\,{\left(d+e\,x\right)}^4} + \frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{495\,d^9\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(7/2)*(d + e*x)^3),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((64*x)/(495*d^7) + 67/(1584*d^6*e)))/((d + e*x)^2*
(d - e*x)^2) + ((d^2 - e^2*x^2)^(1/2)*((631*x)/(2640*d^5) - 113/(528*d^4*e
)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(88*d^4*e*(d + e*x)^
6) - (43*(d^2 - e^2*x^2)^(1/2))/(1584*d^5*e*(d + e*x)^5) - (67*(d^2 - e^2*
x^2)^(1/2))/(1584*d^6*e*(d + e*x)^4) + (128*x*(d^2 - e^2*x^2)^(1/2))/(495*
d^9*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.22

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{7/2}} \, dx = \frac{40\sqrt{-e^2x^2+d^2}\,d^7+120\sqrt{-e^2x^2+d^2}\,d^6ex+40\sqrt{-e^2x^2+d^2}\,d^5e^2x^2-20}{}$$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)
```

output
```
(40*sqrt(d**2 - e**2*x**2)*d**7 + 120*sqrt(d**2 - e**2*x**2)*d**6*e*x + 40
*sqrt(d**2 - e**2*x**2)*d**5*e**2*x**2 - 200*sqrt(d**2 - e**2*x**2)*d**4*e
**3*x**3 - 200*sqrt(d**2 - e**2*x**2)*d**3*e**4*x**4 + 40*sqrt(d**2 - e**2
*x**2)*d**2*e**5*x**5 + 120*sqrt(d**2 - e**2*x**2)*d*e**6*x**6 + 40*sqrt(d
**2 - e**2*x**2)*e**7*x**7 - 125*d**8 + 120*d**7*e*x + 680*d**6*e**2*x**2
+ 400*d**5*e**3*x**3 - 720*d**4*e**4*x**4 - 832*d**3*e**5*x**5 + 64*d**2*e
**6*x**6 + 384*d*e**7*x**7 + 128*e**8*x**8)/(495*sqrt(d**2 - e**2*x**2)*d*
*9*e*(d**7 + 3*d**6*e*x + d**5*e**2*x**2 - 5*d**4*e**3*x**3 - 5*d**3*e**4*
x**4 + d**2*e**5*x**5 + 3*d*e**6*x**6 + e**7*x**7))
```

3.147 $\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 227

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}} dx = \frac{2(d+ex)^{11}}{9e(d^2-e^2x^2)^{9/2}} - \frac{26(d+ex)^9}{63e(d^2-e^2x^2)^{7/2}} + \frac{286(d+ex)^7}{315e(d^2-e^2x^2)^{5/2}} - \frac{286(d+ex)^5}{105e(d^2-e^2x^2)^{3/2}} + \frac{1144d^2(d+ex)}{15e\sqrt{d^2-e^2x^2}} + \frac{572d\sqrt{d^2-e^2x^2}}{15e} + \frac{143}{30}x\sqrt{d^2-e^2x^2} - \frac{143d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

output
```
2/9*(e*x+d)^11/e/(-e^2*x^2+d^2)^(9/2)-26/63*(e*x+d)^9/e/(-e^2*x^2+d^2)^(7/
2)+286/315*(e*x+d)^7/e/(-e^2*x^2+d^2)^(5/2)-286/105*(e*x+d)^5/e/(-e^2*x^2+
d^2)^(3/2)+1144/15*d^2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+572/15*d*(-e^2*x^2+d
^2)^(1/2)/e+143/30*x*(-e^2*x^2+d^2)^(1/2)-143/2*d^2*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.73 (sec), antiderivative size = 133, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\frac{\sqrt{d^2-e^2x^2}(70808d^6-308365d^5ex+518889d^4e^2x^2-401890d^3e^3x^3+135818d^2e^4x^4-5985de^5x^5-315e^6x^6)}{(d-ex)^5}+90}{630e}$$

input
```
Integrate[(d + e*x)^12/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
((Sqrt[d^2 - e^2*x^2]*(70808*d^6 - 308365*d^5*e*x + 518889*d^4*e^2*x^2 - 4
01890*d^3*e^3*x^3 + 135818*d^2*e^4*x^4 - 5985*d*e^5*x^5 - 315*e^6*x^6))/(d
 - e*x)^5 + 90090*d^2*ArcTan[(e*x)/(Sqrt[d^2] - Sqrt[d^2 - e^2*x^2])])/(63
0*e)
```

Rubi [A] (verified)

Time = 0.70 (sec), antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {468, 468, 468, 468, 462, 2346, 25, 27, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^{11}}{9e(d^2-e^2x^2)^{9/2}}-\frac{13}{9}\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{9/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^{11}}{9e(d^2-e^2x^2)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e(d^2-e^2x^2)^{7/2}}-\frac{11}{7}\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{7/2}}dx\right)$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\int\frac{(d+ex)^6}{\left(d^2-e^2x^2\right)^{5/2}}dx\right)\right)$$

$\downarrow$ 468

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\int\frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{3/2}}dx\right)\right)\right)$$

$\downarrow$ 462

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int\frac{7d^2+4exd+e^2}{\sqrt{d^2-e^2x^2}}\right.\right.\right.\right.$$

$\downarrow$ 2346

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(\frac{\int-\frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}\right.\right.\right.\right.$$

$\downarrow$ 25

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{\int\frac{de^2(15d+8ex)}{\sqrt{d^2-e^2x^2}}dx}{2e^2}+\frac{8d^2(d+ex)}{e\sqrt{d^2-e^2x^2}}\right.\right.\right.\right.$$

$\downarrow$ 27

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\int\frac{15d+8ex}{\sqrt{d^2-e^2x^2}}dx+\frac{8d^2(d+}{e\sqrt{d^2-e}}\right.\right.\right.\right.$$

$\downarrow$ 455

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{8\sqrt{}}{}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 224$$

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(15d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 216$$

$$\frac{2(d+ex)^{11}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{13}{9}\left(\frac{2(d+ex)^9}{7e\left(d^2-e^2x^2\right)^{7/2}}-\frac{11}{7}\left(\frac{2(d+ex)^7}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{9}{5}\left(\frac{2(d+ex)^5}{3e\left(d^2-e^2x^2\right)^{3/2}}-\frac{7}{3}\left(-\frac{1}{2}d\left(\frac{15d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{8}{}\right.\right.\right.\right.\right.$$

input
```
Int[(d + e*x)^12/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(2*(d + e*x)^11)/(9*e*(d^2 - e^2*x^2)^(9/2)) - (13*((2*(d + e*x)^9)/(7*e*(
d^2 - e^2*x^2)^(7/2)) - (11*((2*(d + e*x)^7)/(5*e*(d^2 - e^2*x^2)^(5/2)) -
 (9*((2*(d + e*x)^5)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (7*((8*d^2*(d + e*x))/(
e*Sqrt[d^2 - e^2*x^2]) + (x*Sqrt[d^2 - e^2*x^2])/2 - (d*((-8*Sqrt[d^2 - e^
2*x^2])/e + (15*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e))/2))/3))/5))/7))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

method	result
risch	$\frac{(ex+24d)\sqrt{-e^2x^2+d^2}}{2e}-\frac{143d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}-\frac{50584d^2\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de\left(x-\frac{d}{e}\right)}}{315e^2\left(x-\frac{d}{e}\right)}-\frac{37616d^3\sqrt{-\left(x-\frac{d}{e}\right)^2e^2-2de}}{315e^3\left(x-\frac{d}{e}\right)^2}$
default	Expression too large to display

input
```
int((e*x+d)^12/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*x+24*d)/e*(-e^2*x^2+d^2)^(1/2)-143/2*d^2/(e^2)^(1/2)*arctan((e^2)^(
1/2)*x/(-e^2*x^2+d^2)^(1/2))-50584/315*d^2/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d
*e*(x-d/e))^(1/2)-37616/315*d^3/e^3/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e
))^(1/2)-10592/105*d^4/e^4/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-
3520/63*d^5/e^5/(x-d/e)^4*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-128/9*d^6/e
^6/(x-d/e)^5*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{70808\,d^2e^5x^5-354040\,d^3e^4x^4+708080\,d^4e^3x^3-708080\,d^5e^2x^2+354040\,d^6ex-7}{}$$

input
```
integrate((e*x+d)^12/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/630*(70808*d^2*e^5*x^5 - 354040*d^3*e^4*x^4 + 708080*d^4*e^3*x^3 - 70808
0*d^5*e^2*x^2 + 354040*d^6*e*x - 70808*d^7 + 90090*(d^2*e^5*x^5 - 5*d^3*e^
4*x^4 + 10*d^4*e^3*x^3 - 10*d^5*e^2*x^2 + 5*d^6*e*x - d^7)*arctan(-(d - sq
rt(-e^2*x^2 + d^2))/(e*x)) + (315*e^6*x^6 + 5985*d*e^5*x^5 - 135818*d^2*e^
4*x^4 + 401890*d^3*e^3*x^3 - 518889*d^4*e^2*x^2 + 308365*d^5*e*x - 70808*d
^6)*sqrt(-e^2*x^2 + d^2))/(e^6*x^5 - 5*d*e^5*x^4 + 10*d^2*e^4*x^3 - 10*d^3
*e^3*x^2 + 5*d^4*e^2*x - d^5*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^{12}}{(-(-d+ex)(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**12/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**12/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(195) = 390.

Time = 0.17 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.55

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^12/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
-1/2*e^10*x^11/(-e^2*x^2 + d^2)^(9/2) - 12*d*e^9*x^10/(-e^2*x^2 + d^2)^(9/
2) + 143/630*(315*x^8/((-e^2*x^2 + d^2)^(9/2)*e^2) - 840*d^2*x^6/((-e^2*x^
2 + d^2)^(9/2)*e^4) + 1008*d^4*x^4/((-e^2*x^2 + d^2)^(9/2)*e^6) - 576*d^6*
x^2/((-e^2*x^2 + d^2)^(9/2)*e^8) + 128*d^8/((-e^2*x^2 + d^2)^(9/2)*e^10))*
d^2*e^10*x + 340*d^3*e^7*x^8/(-e^2*x^2 + d^2)^(9/2) + 495/2*d^4*e^6*x^7/(-
e^2*x^2 + d^2)^(9/2) - 143/70*d^2*e^8*x*(35*x^6/((-e^2*x^2 + d^2)^(7/2)*e^
2) - 70*d^2*x^4/((-e^2*x^2 + d^2)^(7/2)*e^4) + 56*d^4*x^2/((-e^2*x^2 + d^2
)^(7/2)*e^6) - 16*d^6/((-e^2*x^2 + d^2)^(7/2)*e^8)) - 1928/3*d^5*e^5*x^6/(
-e^2*x^2 + d^2)^(9/2) - 1617/8*d^6*e^4*x^5/(-e^2*x^2 + d^2)^(9/2) + 143/30
*d^2*e^6*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 +
d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) + 4648/5*d^7*e^3*x^4
/(-e^2*x^2 + d^2)^(9/2) + 286/3*d^4*e^4*x^5/(-e^2*x^2 + d^2)^(7/2) + 4015/
16*d^8*e^2*x^3/(-e^2*x^2 + d^2)^(9/2) - 143/6*d^2*e^4*x*(3*x^2/((-e^2*x^2
+ d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) - 17492/35*d^9*e*x
^2/(-e^2*x^2 + d^2)^(9/2) - 5291/30*d^6*e^2*x^3/(-e^2*x^2 + d^2)^(7/2) - 1
0973/144*d^10*x/(-e^2*x^2 + d^2)^(9/2) - 143/2*d^4*e^2*x^3/(-e^2*x^2 + d^2
)^(5/2) + 35404/315*d^11/((-e^2*x^2 + d^2)^(9/2)*e) + 509753/5040*d^8*x/(-
e^2*x^2 + d^2)^(7/2) + 40861/840*d^6*x/(-e^2*x^2 + d^2)^(5/2) - 8098/315*d
^4*x/(-e^2*x^2 + d^2)^(3/2) - 77437/630*d^2*x/sqrt(-e^2*x^2 + d^2) - 143/2
*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{143\,d^2\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{2\,|e|} + \frac{1}{2}\sqrt{-e^2x^2+d^2}\left(x+\frac{24\,d}{e}\right)$$
$$+\frac{16\left(3953\,d^2 - \frac{32742\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d^2}{e^2x} + \frac{117738\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d^2}{e^4x^2} - \frac{235662\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d^2}{e^6x^3} + \frac{289548\left(de+\right.}{}\cdots\right.}{315\left(\cdots\right.}$$

input
```
integrate((e*x+d)^12/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
-143/2*d^2*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + 1/2*sqrt(-e^2*x^2 + d^2)*(
x + 24*d/e) + 16/315*(3953*d^2 - 32742*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
*d^2/(e^2*x) + 117738*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d^2/(e^4*x^2)
- 235662*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3*d^2/(e^6*x^3) + 289548*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^4*d^2/(e^8*x^4) - 208530*(d*e + sqrt(-e^2*
x^2 + d^2)*abs(e))^5*d^2/(e^10*x^5) + 96390*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^6*d^2/(e^12*x^6) - 24570*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7*d^2/(
e^14*x^7) + 2835*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8*d^2/(e^16*x^8))/(((
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^9*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx=\int \frac{(d+e\,x)^{12}}{(d^2-e^2\,x^2)^{11/2}}\,dx$$

input
```
int((d + e*x)^12/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
int((d + e*x)^12/(d^2 - e^2*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{12}}{(d^2-e^2x^2)^{11/2}}\,dx=\int \frac{(ex+d)^{12}}{(-e^2x^2+d^2)^{\frac{11}{2}}}dx$$

input
```
int((e*x+d)^12/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^12/(-e^2*x^2+d^2)^(11/2),x)
```

3.148 $\int \frac{(d+ex)^{11}}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{(d+ex)^{11}}{\left(d^2-e^2x^2\right)^{11/2}}\,dx=\frac{2(d+ex)^{10}}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{22(d+ex)^8}{63e\left(d^2-e^2x^2\right)^{7/2}}+\frac{22(d+ex)^6}{35e\left(d^2-e^2x^2\right)^{5/2}}-\frac{22(d+ex)^4}{15e\left(d^2-e^2x^2\right)^{3/2}}+\frac{44d(d+ex)}{3e\sqrt{d^2-e^2x^2}}+\frac{11\sqrt{d^2-e^2x^2}}{3e}-\frac{11d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2/9*(e*x+d)^10/e/(-e^2*x^2+d^2)^(9/2)-22/63*(e*x+d)^8/e/(-e^2*x^2+d^2)^(7/
2)+22/35*(e*x+d)^6/e/(-e^2*x^2+d^2)^(5/2)-22/15*(e*x+d)^4/e/(-e^2*x^2+d^2)
^(3/2)+44/3*d*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)+11/3*(-e^2*x^2+d^2)^(1/2)/e-1
1*d*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{11}}{\left(d^2-e^2x^2\right)^{11/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(-5471d^5+23575d^4ex-40278d^3e^2x^2+30430d^2e^3x^3-10811de^4x^4+}{315e(-d+ex)^5}+\frac{11d\log\left(-\sqrt{-e^2}x+\sqrt{d^2-e^2x^2}\right)}{\sqrt{-e^2}}$$

input
```
Integrate[(d + e*x)^11/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-5471*d^5 + 23575*d^4*e*x - 40278*d^3*e^2*x^2 + 3043
0*d^2*e^3*x^3 - 10811*d*e^4*x^4 + 315*e^5*x^5))/(315*e*(-d + e*x)^5) + (11
*d*Log[-(Sqrt[-e^2]*x) + Sqrt[d^2 - e^2*x^2]])/Sqrt[-e^2]
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {468, 468, 468, 468, 462, 455, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11}}{(d^2-e^2x^2)^{11/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-\frac{11}{9}\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{9/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-\frac{11}{9}\left(\frac{2(d+ex)^8}{7e(d^2-e^2x^2)^{7/2}}-\frac{9}{7}\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{7/2}}dx\right)$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-\frac{11}{9}\left(\frac{2(d+ex)^8}{7e(d^2-e^2x^2)^{7/2}}-\frac{9}{7}\left(\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{5/2}}dx\right)\right)$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-\frac{11}{9}\left(\frac{2(d+ex)^8}{7e(d^2-e^2x^2)^{7/2}}-\frac{9}{7}\left(\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{3/2}}dx\right)\right)\right)$$

$$\Big\downarrow \text{462}$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-$$
$$\frac{11}{9}\left(\frac{2(d+ex)^8}{7e(d^2-e^2x^2)^{7/2}}-\frac{9}{7}\left(\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(\frac{4d(d+ex)}{e\sqrt{d^2-e^2x^2}}-\int\frac{3d+ex}{\sqrt{d^2-e^2x^2}}dx\right)\right.\right.\right.$$

$$\Big\downarrow \; 455$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-$$
$$\frac{11}{9}\left(\frac{2(d+ex)^8}{7e(d^2-e^2x^2)^{7/2}}-\frac{9}{7}\left(\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(-3d\int\frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{4d(d+ex}{e\sqrt{d^2-e^2}}\right.\right.\right.\right.$$

$$\Big\downarrow \; 224$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-$$
$$\frac{11}{9}\left(\frac{2(d+ex)^8}{7e(d^2-e^2x^2)^{7/2}}-\frac{9}{7}\left(\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(-3d\int\frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\right.\right.\right.\right.$$

$$\Big\downarrow \; 216$$

$$\frac{2(d+ex)^{10}}{9e(d^2-e^2x^2)^{9/2}}-$$
$$\frac{11}{9}\left(\frac{2(d+ex)^8}{7e(d^2-e^2x^2)^{7/2}}-\frac{9}{7}\left(\frac{2(d+ex)^6}{5e(d^2-e^2x^2)^{5/2}}-\frac{7}{5}\left(\frac{2(d+ex)^4}{3e(d^2-e^2x^2)^{3/2}}-\frac{5}{3}\left(-\frac{3d\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{4d(d+}{e\sqrt{d^2-}}\right.\right.\right.\right.$$

input
```
Int[(d + e*x)^11/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(2*(d + e*x)^10)/(9*e*(d^2 - e^2*x^2)^(9/2)) - (11*((2*(d + e*x)^8)/(7*e*(
d^2 - e^2*x^2)^(7/2)) - (9*((2*(d + e*x)^6)/(5*e*(d^2 - e^2*x^2)^(5/2)) -
(7*((2*(d + e*x)^4)/(3*e*(d^2 - e^2*x^2)^(3/2)) - (5*((4*d*(d + e*x))/(e*S
qrt[d^2 - e^2*x^2]) + Sqrt[d^2 - e^2*x^2]/e - (3*d*ArcTan[(e*x)/Sqrt[d^2 -
 e^2*x^2]])/e))/3))/5))/7))/9
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 462
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp
[(-2^(n - 1))*d*c^(n - 2)*((c + d*x)/(b*Sqrt[a + b*x^2])), x] + Simp[d^2/b
  Int[(1/Sqrt[a + b*x^2])*ExpandToSum[(2^(n - 1)*c^(n - 1) - (c + d*x)^(n -
 1))/(c - d*x), x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2,
0] && IGtQ[n, 2]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.45

method	result
risch	$\frac{\sqrt{-e^{2} x^{2}+d^{2}}}{e}-\frac{11 d \arctan\left(\frac{\sqrt{e^{2}}\, x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}}-\frac{9236 d \sqrt{-\left(x-\frac{d}{e}\right)^{2} e^{2}-2 d e \left(x-\frac{d}{e}\right)}}{315 e^{2} \left(x-\frac{d}{e}\right)}-\frac{9664 d^{2} \sqrt{-\left(x-\frac{d}{e}\right)^{2} e^{2}-2 d e \left(x-\frac{d}{e}\right)}}{315 e^{3} \left(x-\frac{d}{e}\right)^{2}}-\ 35$
default	Expression too large to display

input
```
int((e*x+d)^11/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
(-e^2*x^2+d^2)^(1/2)/e-11*d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2
)^(1/2))-9236/315*d/e^2/(x-d/e)*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-9664/
315*d^2/e^3/(x-d/e)^2*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-3568/105*d^3/e^
4/(x-d/e)^3*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-1472/63*d^4/e^5/(x-d/e)^4
*(-(x-d/e)^2*e^2-2*d*e*(x-d/e))^(1/2)-64/9*d^5/e^6/(x-d/e)^5*(-(x-d/e)^2*e
^2-2*d*e*(x-d/e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^{11}}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{5471\,de^5x^5 - 27355\,d^2e^4x^4 + 54710\,d^3e^3x^3 - 54710\,d^4e^2x^2 + 27355\,d^5ex - 5471\,d^6}{}$$

input
```
integrate((e*x+d)^11/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(5471*d*e^5*x^5 - 27355*d^2*e^4*x^4 + 54710*d^3*e^3*x^3 - 54710*d^4*
e^2*x^2 + 27355*d^5*e*x - 5471*d^6 + 6930*(d*e^5*x^5 - 5*d^2*e^4*x^4 + 10*
d^3*e^3*x^3 - 10*d^4*e^2*x^2 + 5*d^5*e*x - d^6)*arctan(-(d - sqrt(-e^2*x^2
 + d^2))/(e*x)) + (315*e^5*x^5 - 10811*d*e^4*x^4 + 30430*d^2*e^3*x^3 - 402
78*d^3*e^2*x^2 + 23575*d^4*e*x - 5471*d^5)*sqrt(-e^2*x^2 + d^2))/(e^6*x^5
- 5*d*e^5*x^4 + 10*d^2*e^4*x^3 - 10*d^3*e^3*x^2 + 5*d^4*e^2*x - d^5*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{11}}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^{11}}{(-(-d+ex)(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**11/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**11/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(173) = 346$.

Time = 0.17 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.87

$$\int \frac{(d+ex)^{11}}{(d^2-e^2x^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^11/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
-e^9*x^10/(-e^2*x^2 + d^2)^(9/2) + 11/315*(315*x^8/((-e^2*x^2 + d^2)^(9/2)
*e^2) - 840*d^2*x^6/((-e^2*x^2 + d^2)^(9/2)*e^4) + 1008*d^4*x^4/((-e^2*x^2
 + d^2)^(9/2)*e^6) - 576*d^6*x^2/((-e^2*x^2 + d^2)^(9/2)*e^8) + 128*d^8/((
-e^2*x^2 + d^2)^(9/2)*e^10))*d*e^10*x + 65*d^2*e^7*x^8/(-e^2*x^2 + d^2)^(9
/2) + 165/2*d^3*e^6*x^7/(-e^2*x^2 + d^2)^(9/2) - 11/35*d*e^8*x*(35*x^6/((-
e^2*x^2 + d^2)^(7/2)*e^2) - 70*d^2*x^4/((-e^2*x^2 + d^2)^(7/2)*e^4) + 56*d
^4*x^2/((-e^2*x^2 + d^2)^(7/2)*e^6) - 16*d^6/((-e^2*x^2 + d^2)^(7/2)*e^8))
 - 190/3*d^4*e^5*x^6/(-e^2*x^2 + d^2)^(9/2) - 231/8*d^5*e^4*x^5/(-e^2*x^2
+ d^2)^(9/2) + 11/15*d*e^6*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2
*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) +
842/5*d^6*e^3*x^4/(-e^2*x^2 + d^2)^(9/2) + 44/3*d^3*e^4*x^5/(-e^2*x^2 + d^
2)^(7/2) + 1265/16*d^7*e^2*x^3/(-e^2*x^2 + d^2)^(9/2) - 11/3*d*e^4*x*(3*x^
2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) - 254
3/35*d^8*e*x^2/(-e^2*x^2 + d^2)^(9/2) - 407/15*d^5*e^2*x^3/(-e^2*x^2 + d^2
)^(7/2) - 2899/144*d^9*x/(-e^2*x^2 + d^2)^(9/2) - 11*d^3*e^2*x^3/(-e^2*x^2
 + d^2)^(5/2) + 5471/315*d^10/((-e^2*x^2 + d^2)^(9/2)*e) + 85087/5040*d^7*
x/(-e^2*x^2 + d^2)^(7/2) + 7619/840*d^5*x/(-e^2*x^2 + d^2)^(5/2) - 1159/63
0*d^3*x/(-e^2*x^2 + d^2)^(3/2) - 4624/315*d*x/sqrt(-e^2*x^2 + d^2) - 11*d*
arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^{11}}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{11\,d\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} + \frac{\sqrt{-e^2x^2+d^2}}{e}$$

$$+ \frac{8\left(1289\,d - \frac{10656\left(de+\sqrt{-e^2x^2+d^2}|e|\right)d}{e^2x} + \frac{38844\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2d}{e^4x^2} - \frac{76776\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3d}{e^6x^3} + \frac{96894\left(de+\sqrt{-e^2x^2\cdots}\right.}{e^8x^4}\cdots\right.}{315\left(\frac{de+\sqrt{-e\cdots}}{e\cdots}\right.\cdots}$$

input
```
integrate((e*x+d)^11/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
-11*d*arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) + sqrt(-e^2*x^2 + d^2)/e + 8/315*
(1289*d - 10656*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))*d/(e^2*x) + 38844*(d*e
 + sqrt(-e^2*x^2 + d^2)*abs(e))^2*d/(e^4*x^2) - 76776*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^3*d/(e^6*x^3) + 96894*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^
4*d/(e^8*x^4) - 65520*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5*d/(e^10*x^5) +
 31500*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6*d/(e^12*x^6) - 7560*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^7*d/(e^14*x^7) + 945*(d*e + sqrt(-e^2*x^2 + d^2
)*abs(e))^8*d/(e^16*x^8))/(((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) -
1)^9*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{11}}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+e\,x)^{11}}{(d^2-e^2\,x^2)^{11/2}}\,dx$$

input
```
int((d + e*x)^11/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
int((d + e*x)^11/(d^2 - e^2*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{11}}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^{11}}{(-e^2x^2+d^2)^{\frac{11}{2}}}dx$$

input
```
int((e*x+d)^11/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^11/(-e^2*x^2+d^2)^(11/2),x)
```

3.149 $\int \frac{(d+ex)^{10}}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{2(d+ex)^9}{9e\,(d^2-e^2x^2)^{9/2}} - \frac{2(d+ex)^7}{7e\,(d^2-e^2x^2)^{7/2}} + \frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}} - \frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

output
```
2/9*(e*x+d)^9/e/(-e^2*x^2+d^2)^(9/2)-2/7*(e*x+d)^7/e/(-e^2*x^2+d^2)^(7/2)+
2/5*(e*x+d)^5/e/(-e^2*x^2+d^2)^(5/2)-2/3*(e*x+d)^3/e/(-e^2*x^2+d^2)^(3/2)+
2*(e*x+d)/e/(-e^2*x^2+d^2)^(1/2)-arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{2\sqrt{d^2-e^2x^2}(263d^4-1000d^3ex+1974d^2e^2x^2-1240de^3x^3+563e^4x^4)}{315e(-d+ex)^5} + \frac{2\arctan\left(\frac{ex}{\sqrt{d^2}-\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(d + e*x)^10/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(-2*Sqrt[d^2 - e^2*x^2]*(263*d^4 - 1000*d^3*e*x + 1974*d^2*e^2*x^2 - 1240*
d*e^3*x^3 + 563*e^4*x^4))/(315*e*(-d + e*x)^5) + (2*ArcTan[(e*x)/(Sqrt[d^2
] - Sqrt[d^2 - e^2*x^2])])/e
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {468, 468, 468, 468, 457, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2(d+ex)^9}{9e\,(d^2-e^2x^2)^{9/2}}-\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{9/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{7/2}}dx+\frac{2(d+ex)^9}{9e\,(d^2-e^2x^2)^{9/2}}-\frac{2(d+ex)^7}{7e\,(d^2-e^2x^2)^{7/2}}$$

$$\Big\downarrow \text{468}$$

$$-\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{5/2}}dx+\frac{2(d+ex)^9}{9e\,(d^2-e^2x^2)^{9/2}}-\frac{2(d+ex)^7}{7e\,(d^2-e^2x^2)^{7/2}}+\frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}}$$

$$\Big\downarrow \text{468}$$

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{3/2}}dx+\frac{2(d+ex)^9}{9e\,(d^2-e^2x^2)^{9/2}}-\frac{2(d+ex)^7}{7e\,(d^2-e^2x^2)^{7/2}}+\frac{2(d+ex)^5}{5e\,(d^2-e^2x^2)^{5/2}}-\frac{2(d+ex)^3}{3e\,(d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{457}$$

$$-\int \frac{1}{\sqrt{d^2-e^2x^2}}dx+\frac{2(d+ex)^9}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d+ex)^7}{7e\left(d^2-e^2x^2\right)^{7/2}}+\frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{224}$$

$$-\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}+\frac{2(d+ex)^9}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d+ex)^7}{7e\left(d^2-e^2x^2\right)^{7/2}}+\frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}+\frac{2(d+ex)^9}{9e\left(d^2-e^2x^2\right)^{9/2}}-\frac{2(d+ex)^7}{7e\left(d^2-e^2x^2\right)^{7/2}}+\frac{2(d+ex)^5}{5e\left(d^2-e^2x^2\right)^{5/2}}-\frac{2(d+ex)^3}{3e\left(d^2-e^2x^2\right)^{3/2}}+\frac{2(d+ex)}{e\sqrt{d^2-e^2x^2}}$$

input
```
Int[(d + e*x)^10/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(2*(d + e*x)^9)/(9*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d + e*x)^7)/(7*e*(d^2 -
e^2*x^2)^(7/2)) + (2*(d + e*x)^5)/(5*e*(d^2 - e^2*x^2)^(5/2)) - (2*(d + e*
x)^3)/(3*e*(d^2 - e^2*x^2)^(3/2)) + (2*(d + e*x))/(e*Sqrt[d^2 - e^2*x^2])
- ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/e
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1519 vs. $2(152) = 304$.

Time = 1.16 (sec) , antiderivative size = 1520, normalized size of antiderivative = 8.84

method	result	size
default	Expression too large to display	1520

input
```
int((e*x+d)^10/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
d^10*(1/9*x/d^2/(-e^2*x^2+d^2)^(9/2)+8/9/d^2*(1/7*x/d^2/(-e^2*x^2+d^2)^(7/
2)+6/7/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^
2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2)))))+e^10*(1/9*x^9/e^2/(-e^2*x^2+d^
2)^(9/2)-1/e^2*(1/7*x^7/e^2/(-e^2*x^2+d^2)^(7/2)-1/e^2*(1/5*x^5/e^2/(-e^2*
x^2+d^2)^(5/2)-1/e^2*(1/3*x^3/e^2/(-e^2*x^2+d^2)^(3/2)-1/e^2*(x/e^2/(-e^2*
x^2+d^2)^(1/2)-1/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)
))))))+10*d*e^9*(x^8/e^2/(-e^2*x^2+d^2)^(9/2)-8*d^2/e^2*(1/3*x^6/e^2/(-e^2
*x^2+d^2)^(9/2)-2*d^2/e^2*(1/5*x^4/e^2/(-e^2*x^2+d^2)^(9/2)-4/5*d^2/e^2*(1
/7*x^2/e^2/(-e^2*x^2+d^2)^(9/2)-2/63*d^2/e^4/(-e^2*x^2+d^2)^(9/2)))))+45*d
^2*e^8*(1/2*x^7/e^2/(-e^2*x^2+d^2)^(9/2)-7/2*d^2/e^2*(1/4*x^5/e^2/(-e^2*x^
2+d^2)^(9/2)-5/4*d^2/e^2*(1/6*x^3/e^2/(-e^2*x^2+d^2)^(9/2)-1/2*d^2/e^2*(1/
8*x/e^2/(-e^2*x^2+d^2)^(9/2)-1/8*d^2/e^2*(1/9*x/d^2/(-e^2*x^2+d^2)^(9/2)+8
/9/d^2*(1/7*x/d^2/(-e^2*x^2+d^2)^(7/2)+6/7/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(
5/2)+4/5/d^2*(1/3*x/d^2/(-e^2*x^2+d^2)^(3/2)+2/3*x/d^4/(-e^2*x^2+d^2)^(1/2
)))))))))+120*d^3*e^7*(1/3*x^6/e^2/(-e^2*x^2+d^2)^(9/2)-2*d^2/e^2*(1/5*x^4
/e^2/(-e^2*x^2+d^2)^(9/2)-4/5*d^2/e^2*(1/7*x^2/e^2/(-e^2*x^2+d^2)^(9/2)-2/
63*d^2/e^4/(-e^2*x^2+d^2)^(9/2))))+210*d^4*e^6*(1/4*x^5/e^2/(-e^2*x^2+d^2)
^(9/2)-5/4*d^2/e^2*(1/6*x^3/e^2/(-e^2*x^2+d^2)^(9/2)-1/2*d^2/e^2*(1/8*x/e^
2/(-e^2*x^2+d^2)^(9/2)-1/8*d^2/e^2*(1/9*x/d^2/(-e^2*x^2+d^2)^(9/2)+8/9/d^2
*(1/7*x/d^2/(-e^2*x^2+d^2)^(7/2)+6/7/d^2*(1/5*x/d^2/(-e^2*x^2+d^2)^(5/2...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{2\left(263\,e^5x^5-1315\,de^4x^4+2630\,d^2e^3x^3-2630\,d^3e^2x^2+1315\,d^4ex-263\,d^5+315\right.}{}$$

input
```
integrate((e*x+d)^10/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
2/315*(263*e^5*x^5 - 1315*d*e^4*x^4 + 2630*d^2*e^3*x^3 - 2630*d^3*e^2*x^2
+ 1315*d^4*e*x - 263*d^5 + 315*(e^5*x^5 - 5*d*e^4*x^4 + 10*d^2*e^3*x^3 - 1
0*d^3*e^2*x^2 + 5*d^4*e*x - d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
 - (563*e^4*x^4 - 1240*d*e^3*x^3 + 1974*d^2*e^2*x^2 - 1000*d^3*e*x + 263*d
^4)*sqrt(-e^2*x^2 + d^2))/(e^6*x^5 - 5*d*e^5*x^4 + 10*d^2*e^4*x^3 - 10*d^3
*e^3*x^2 + 5*d^4*e^2*x - d^5*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^{10}}{(-(-d+ex)\,(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**10/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**10/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(152) = 304$.

Time = 0.17 (sec) , antiderivative size = 741, normalized size of antiderivative = 4.31

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^10/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/315*(315*x^8/((-e^2*x^2 + d^2)^(9/2)*e^2) - 840*d^2*x^6/((-e^2*x^2 + d^2
)^(9/2)*e^4) + 1008*d^4*x^4/((-e^2*x^2 + d^2)^(9/2)*e^6) - 576*d^6*x^2/((-
e^2*x^2 + d^2)^(9/2)*e^8) + 128*d^8/((-e^2*x^2 + d^2)^(9/2)*e^10))*e^10*x
+ 10*d*e^7*x^8/(-e^2*x^2 + d^2)^(9/2) + 45/2*d^2*e^6*x^7/(-e^2*x^2 + d^2)^
(9/2) - 1/35*e^8*x*(35*x^6/((-e^2*x^2 + d^2)^(7/2)*e^2) - 70*d^2*x^4/((-e^
2*x^2 + d^2)^(7/2)*e^4) + 56*d^4*x^2/((-e^2*x^2 + d^2)^(7/2)*e^6) - 16*d^6
/((-e^2*x^2 + d^2)^(7/2)*e^8)) + 40/3*d^3*e^5*x^6/(-e^2*x^2 + d^2)^(9/2) +
 105/8*d^4*e^4*x^5/(-e^2*x^2 + d^2)^(9/2) + 1/15*e^6*x*(15*x^4/((-e^2*x^2
+ d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2
*x^2 + d^2)^(5/2)*e^6)) + 172/5*d^5*e^3*x^4/(-e^2*x^2 + d^2)^(9/2) + 4/3*d
^2*e^4*x^5/(-e^2*x^2 + d^2)^(7/2) + 385/16*d^6*e^2*x^3/(-e^2*x^2 + d^2)^(9
/2) - 1/3*e^4*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d
^2)^(3/2)*e^4)) - 88/35*d^7*e*x^2/(-e^2*x^2 + d^2)^(9/2) - 37/15*d^4*e^2*x
^3/(-e^2*x^2 + d^2)^(7/2) - 419/144*d^8*x/(-e^2*x^2 + d^2)^(9/2) - d^2*e^2
*x^3/(-e^2*x^2 + d^2)^(5/2) + 526/315*d^9/((-e^2*x^2 + d^2)^(9/2)*e) + 916
7/5040*d^6*x/(-e^2*x^2 + d^2)^(7/2) + 979/840*d^4*x/(-e^2*x^2 + d^2)^(5/2)
 + 181/630*d^2*x/(-e^2*x^2 + d^2)^(3/2) - 134/315*x/sqrt(-e^2*x^2 + d^2) -
 arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{\arcsin\left(\frac{ex}{d}\right)\operatorname{sgn}(d)\operatorname{sgn}(e)}{|e|} - \frac{4\left(\frac{2052\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{8208\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{14532\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{21798\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + \frac{1134\cdots}{\cdots}\right.}{315\left(\frac{de+\sqrt{-e^2x^2+d^2}\cdots}{e^2x}\right.}$$

input
```
integrate((e*x+d)^10/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
-arcsin(e*x/d)*sgn(d)*sgn(e)/abs(e) - 4/315*(2052*(d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))/(e^2*x) - 8208*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2)
 + 14532*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^3/(e^6*x^3) - 21798*(d*e + sq
rt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 11340*(d*e + sqrt(-e^2*x^2 + d^2)
*abs(e))^5/(e^10*x^5) - 7560*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^6/(e^12*x
^6) + 1260*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e^14*x^7) - 315*(d*e + s
qrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8) - 263)/(((d*e + sqrt(-e^2*x^2 + d
^2)*abs(e))/(e^2*x) - 1)^9*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+e\,x)^{10}}{(d^2-e^2\,x^2)^{11/2}}\,dx$$

input
```
int((d + e*x)^10/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
int((d + e*x)^10/(d^2 - e^2*x^2)^(11/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{10}}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^{10}}{(-e^2x^2+d^2)^{\frac{11}{2}}}dx$$

input
```
int((e*x+d)^10/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^10/(-e^2*x^2+d^2)^(11/2),x)
```

3.150 $\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{(d+ex)^9}{9de\,(d^2-e^2x^2)^{9/2}}$$

output
```
1/9*(e*x+d)^9/d/e/(-e^2*x^2+d^2)^(9/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{(d+ex)^4\sqrt{d^2-e^2x^2}}{9de(d-ex)^5}$$

input
```
Integrate[(d + e*x)^9/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
((d + e*x)^4*Sqrt[d^2 - e^2*x^2])/(9*d*e*(d - e*x)^5)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx$$

$$\Big\downarrow \; 460$$

$$\frac{(d+ex)^9}{9de\,(d^2-e^2x^2)^{9/2}}$$

input
```
Int[(d + e*x)^9/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(d + e*x)^9/(9*d*e*(d^2 - e^2*x^2)^(9/2))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{(ex+d)^{10}(-ex+d)}{9de(-e^2x^2+d^2)^{\frac{11}{2}}}$	36
orering	$\frac{(ex+d)^{10}(-ex+d)}{9de(-e^2x^2+d^2)^{\frac{11}{2}}}$	36
trager	$\frac{(e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4)\sqrt{-e^2x^2+d^2}}{9d(-ex+d)^5 e}$	69
default	Expression too large to display	1347

input
```
int((e*x+d)^9/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(e*x+d)^10*(-e*x+d)/d/e/(-e^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(29) = 58.

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.03

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{e^5x^5 - 5\,de^4x^4 + 10\,d^2e^3x^3 - 10\,d^3e^2x^2 + 5\,d^4ex - d^5 - (e^4x^4 + 4\,de^3x^3 + 6\,d^2e^2x^2}{9\,(de^6x^5 - 5\,d^2e^5x^4 + 10\,d^3e^4x^3 - 10\,d^4e^3x^2 + 5\,d^5e^2x - }$$

input
```
integrate((e*x+d)^9/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/9*(e^5*x^5 - 5*d*e^4*x^4 + 10*d^2*e^3*x^3 - 10*d^3*e^2*x^2 + 5*d^4*e*x -
 d^5 - (e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)*sqrt(-e^2
*x^2 + d^2))/(d*e^6*x^5 - 5*d^2*e^5*x^4 + 10*d^3*e^4*x^3 - 10*d^4*e^3*x^2
+ 5*d^5*e^2*x - d^6*e)
```

Sympy [F]

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^9}{(-(-d+ex)(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**9/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**9/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 288, normalized size of antiderivative = 8.73

$$\begin{aligned}\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx &= \frac{e^7x^8}{(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{9\,de^6x^7}{2\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{28\,d^2e^5x^6}{3\,(-e^2x^2+d^2)^{\frac{9}{2}}} \\ &+ \frac{105\,d^3e^4x^5}{8\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{14\,d^4e^3x^4}{(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{161\,d^5e^2x^3}{16\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{4\,d^6ex^2}{(-e^2x^2+d^2)^{\frac{9}{2}}} \\ &+ \frac{109\,d^7x}{144\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{d^8}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}e} + \frac{5\,d^5x}{144\,(-e^2x^2+d^2)^{\frac{7}{2}}} \\ &+ \frac{d^3x}{24\,(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{dx}{18\,(-e^2x^2+d^2)^{\frac{3}{2}}} + \frac{x}{9\,\sqrt{-e^2x^2+d^2}d}\end{aligned}$$

input
```
integrate((e*x+d)^9/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
e^7*x^8/(-e^2*x^2 + d^2)^(9/2) + 9/2*d*e^6*x^7/(-e^2*x^2 + d^2)^(9/2) + 28
/3*d^2*e^5*x^6/(-e^2*x^2 + d^2)^(9/2) + 105/8*d^3*e^4*x^5/(-e^2*x^2 + d^2)
^(9/2) + 14*d^4*e^3*x^4/(-e^2*x^2 + d^2)^(9/2) + 161/16*d^5*e^2*x^3/(-e^2*
x^2 + d^2)^(9/2) + 4*d^6*e*x^2/(-e^2*x^2 + d^2)^(9/2) + 109/144*d^7*x/(-e^
2*x^2 + d^2)^(9/2) + 1/9*d^8/((-e^2*x^2 + d^2)^(9/2)*e) + 5/144*d^5*x/(-e^
2*x^2 + d^2)^(7/2) + 1/24*d^3*x/(-e^2*x^2 + d^2)^(5/2) + 1/18*d*x/(-e^2*x^
2 + d^2)^(3/2) + 1/9*x/(sqrt(-e^2*x^2 + d^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(29) = 58$.

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 5.06

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{2\left(\frac{36\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2}+\frac{126\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4}+\frac{84\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^6}{e^{12}x^6}+\frac{9\left(de+\sqrt{-e^2x^{2}}\right.}{e^{16}x^{8}}\right.}{9\,d\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x}-1\right)^9|e|}$$

input
```
integrate((e*x+d)^9/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
2/9*(36*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 126*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 84*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e
))^6/(e^12*x^6) + 9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8) + 1)/
(d*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^9*abs(e))
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.42

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{8\,d\,\sqrt{d^2-e^2\,x^2}}{3\,e\,(d-e\,x)^3}-\frac{8\,\sqrt{d^2-e^2\,x^2}}{9\,e\,(d-e\,x)^2} + \frac{\sqrt{d^2-e^2\,x^2}}{9\,d\,e\,(d-e\,x)}-\frac{32\,d^2\,\sqrt{d^2-e^2\,x^2}}{9\,e\,(d-e\,x)^4}+\frac{16\,d^3\,\sqrt{d^2-e^2\,x^2}}{9\,e\,(d-e\,x)^5}$$

input
```
int((d + e*x)^9/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
(8*d*(d^2 - e^2*x^2)^(1/2))/(3*e*(d - e*x)^3) - (8*(d^2 - e^2*x^2)^(1/2))/
(9*e*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(9*d*e*(d - e*x)) - (32*d^2*(d^2
 - e^2*x^2)^(1/2))/(9*e*(d - e*x)^4) + (16*d^3*(d^2 - e^2*x^2)^(1/2))/(9*e
*(d - e*x)^5)
```

Reduce [F]

$$\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^9}{(-e^2x^2+d^2)^{\frac{11}{2}}}dx$$

input
```
int((e*x+d)^9/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^9/(-e^2*x^2+d^2)^(11/2),x)
```

3.151 $\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx=\frac{(d+ex)^8}{9de\,(d^2-e^2x^2)^{9/2}}+\frac{(d+ex)^7}{63d^2e\,(d^2-e^2x^2)^{7/2}}$$

output
```
1/9*(e*x+d)^8/d/e/(-e^2*x^2+d^2)^(9/2)+1/63*(e*x+d)^7/d^2/e/(-e^2*x^2+d^2)
^(7/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx=\frac{(8d-ex)(d+ex)^3\sqrt{d^2-e^2x^2}}{63d^2e(d-ex)^5}$$

input
```
Integrate[(d + e*x)^8/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
((8*d - e*x)*(d + e*x)^3*Sqrt[d^2 - e^2*x^2])/(63*d^2*e*(d - e*x)^5)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx$$

$$\Big\downarrow \; 461$$

$$\frac{(d+ex)^8}{7de\,(d^2-e^2x^2)^{9/2}}-\frac{\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}dx}{7d}$$

$$\Big\downarrow \; 460$$

$$\frac{(d+ex)^8}{7de\,(d^2-e^2x^2)^{9/2}}-\frac{(d+ex)^9}{63d^2e\,(d^2-e^2x^2)^{9/2}}$$

input
```
Int[(d + e*x)^8/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(d + e*x)^8/(7*d*e*(d^2 - e^2*x^2)^(9/2)) - (d + e*x)^9/(63*d^2*e*(d^2 - e
^2*x^2)^(9/2))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{(ex+d)^9(-ex+d)(-ex+8d)}{63d^2e(-e^2x^2+d^2)^{\frac{11}{2}}}$	44
orering	$\frac{(ex+d)^9(-ex+d)(-ex+8d)}{63d^2e(-e^2x^2+d^2)^{\frac{11}{2}}}$	44
trager	$\frac{(-e^4x^4+5d\,e^3x^3+21d^2e^2x^2+23d^3ex+8d^4)\sqrt{-e^2x^2+d^2}}{63d^2(-ex+d)^5e}$	72
default	Expression too large to display	1203

input
```
int((e*x+d)^8/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/63*(e*x+d)^9*(-e*x+d)*(-e*x+8*d)/d^2/e/(-e^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{8\,e^5x^5 - 40\,de^4x^4 + 80\,d^2e^3x^3 - 80\,d^3e^2x^2 + 40\,d^4ex - 8\,d^5 + (e^4x^4 - 5\,de^3x^3 - 21}{63\,(d^2e^6x^5 - 5\,d^3e^5x^4 + 10\,d^4e^4x^3 - 10\,d^5e^3x^2 + 5\,d^6e}$$

input
```
integrate((e*x+d)^8/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/63*(8*e^5*x^5 - 40*d*e^4*x^4 + 80*d^2*e^3*x^3 - 80*d^3*e^2*x^2 + 40*d^4*
e*x - 8*d^5 + (e^4*x^4 - 5*d*e^3*x^3 - 21*d^2*e^2*x^2 - 23*d^3*e*x - 8*d^4
)*sqrt(-e^2*x^2 + d^2))/(d^2*e^6*x^5 - 5*d^3*e^5*x^4 + 10*d^4*e^4*x^3 - 10
*d^5*e^3*x^2 + 5*d^6*e^2*x - d^7*e)
```

Sympy [F]

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^8}{(-(-d+ex)\,(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**8/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**8/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(59) = 118$.

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.93

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{e^6x^7}{2\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{8\,de^5x^6}{3\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{49\,d^2e^4x^5}{8\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{8\,d^3e^3x^4}{\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{105\,d^4e^2x^3}{16\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{24\,d^5ex^2}{7\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{149\,d^6x}{144\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{8\,d^7}{63\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e} - \frac{5\,d^4x}{1008\left(-e^2x^2+d^2\right)^{\frac{7}{2}}} - \frac{d^2x}{168\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} - \frac{x}{126\left(-e^2x^2+d^2\right)^{\frac{3}{2}}} - \frac{x}{63\sqrt{-e^2x^2+d^2}\,d^2}$$

input
```
integrate((e*x+d)^8/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/2*e^6*x^7/(-e^2*x^2 + d^2)^(9/2) + 8/3*d*e^5*x^6/(-e^2*x^2 + d^2)^(9/2)
+ 49/8*d^2*e^4*x^5/(-e^2*x^2 + d^2)^(9/2) + 8*d^3*e^3*x^4/(-e^2*x^2 + d^2)
^(9/2) + 105/16*d^4*e^2*x^3/(-e^2*x^2 + d^2)^(9/2) + 24/7*d^5*e*x^2/(-e^2*
x^2 + d^2)^(9/2) + 149/144*d^6*x/(-e^2*x^2 + d^2)^(9/2) + 8/63*d^7/((-e^2*
x^2 + d^2)^(9/2)*e) - 5/1008*d^4*x/(-e^2*x^2 + d^2)^(7/2) - 1/168*d^2*x/(-
e^2*x^2 + d^2)^(5/2) - 1/126*x/(-e^2*x^2 + d^2)^(3/2) - 1/63*x/(sqrt(-e^2*
x^2 + d^2)*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(59) = 118$.

Time = 0.16 (sec) , antiderivative size = 289, normalized size of antiderivative = 4.31

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{2\left(\frac{9\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{225\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{189\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{693\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + \frac{315\left(de+\sqrt{-e\right.}}{e^{10}}\right.}{63\,d^2\left(\frac{de+\sqrt{-e^2x^2+d^2}|e|}{e^2x} - 1\right.}$$

input
```
integrate((e*x+d)^8/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
-2/63*(9*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 225*(d*e + sqrt(-e^
2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 189*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))
^3/(e^6*x^3) - 693*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) + 315*(
d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) - 483*(d*e + sqrt(-e^2*x^2
 + d^2)*abs(e))^6/(e^12*x^6) + 63*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^7/(e
^14*x^7) - 63*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8) - 8)/(d^2*(
(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^9*abs(e))
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{10\sqrt{d^2-e^2x^2}}{21\,e\,(d-e\,x)^3} - \frac{76\,d\sqrt{d^2-e^2x^2}}{63\,e\,(d-e\,x)^4} - \frac{\sqrt{d^2-e^2x^2}}{63\,d\,e\,(d-e\,x)^2} - \frac{\sqrt{d^2-e^2x^2}}{63\,d^2\,e\,(d-e\,x)} + \frac{8\,d^2\sqrt{d^2-e^2x^2}}{9\,e\,(d-e\,x)^5}$$

input
```
int((d + e*x)^8/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
(10*(d^2 - e^2*x^2)^(1/2))/(21*e*(d - e*x)^3) - (76*d*(d^2 - e^2*x^2)^(1/2
))/(63*e*(d - e*x)^4) - (d^2 - e^2*x^2)^(1/2)/(63*d*e*(d - e*x)^2) - (d^2
- e^2*x^2)^(1/2)/(63*d^2*e*(d - e*x)) + (8*d^2*(d^2 - e^2*x^2)^(1/2))/(9*e
*(d - e*x)^5)
```

Reduce [F]

$$\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^8}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

input
```
int((e*x+d)^8/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^8/(-e^2*x^2+d^2)^(11/2),x)
```

3.152 $\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}} dx = \frac{(d+ex)^7}{9de(d^2-e^2x^2)^{9/2}} + \frac{2(d+ex)^6}{63d^2e(d^2-e^2x^2)^{7/2}} + \frac{2(d+ex)^5}{315d^3e(d^2-e^2x^2)^{5/2}}$$

output
```
1/9*(e*x+d)^7/d/e/(-e^2*x^2+d^2)^(9/2)+2/63*(e*x+d)^6/d^2/e/(-e^2*x^2+d^2)
^(7/2)+2/315*(e*x+d)^5/d^3/e/(-e^2*x^2+d^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}} dx = \frac{(d+ex)^2\sqrt{d^2-e^2x^2}(47d^2-14dex+2e^2x^2)}{315d^3e(d-ex)^5}$$

input
```
Integrate[(d + e*x)^7/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
((d + e*x)^2*Sqrt[d^2 - e^2*x^2]*(47*d^2 - 14*d*e*x + 2*e^2*x^2))/(315*d^3
*e*(d - e*x)^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{(d+ex)^7}{5de(d^2-e^2x^2)^{9/2}}-\frac{2\int \frac{(d+ex)^8}{(d^2-e^2x^2)^{11/2}}dx}{5d}$$

$$\Big\downarrow \text{461}$$

$$\frac{(d+ex)^7}{5de(d^2-e^2x^2)^{9/2}}-\frac{2\left(\frac{(d+ex)^8}{7de(d^2-e^2x^2)^{9/2}}-\frac{\int \frac{(d+ex)^9}{(d^2-e^2x^2)^{11/2}}dx}{7d}\right)}{5d}$$

$$\Big\downarrow \text{460}$$

$$\frac{(d+ex)^7}{5de(d^2-e^2x^2)^{9/2}}-\frac{2\left(\frac{(d+ex)^8}{7de(d^2-e^2x^2)^{9/2}}-\frac{(d+ex)^9}{63d^2e(d^2-e^2x^2)^{9/2}}\right)}{5d}$$

input
```
Int[(d + e*x)^7/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(d + e*x)^7/(5*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*((d + e*x)^8/(7*d*e*(d^2 -
e^2*x^2)^(9/2)) - (d + e*x)^9/(63*d^2*e*(d^2 - e^2*x^2)^(9/2))))/(5*d)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{(ex+d)^{8}(-ex+d)\left(2e^{2}x^{2}-14dex+47d^{2}\right)}{315d^{3}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$	55
orering	$\frac{(ex+d)^{8}(-ex+d)\left(2e^{2}x^{2}-14dex+47d^{2}\right)}{315d^{3}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$	55
trager	$\frac{\left(2e^{4}x^{4}-10d\,e^{3}x^{3}+21d^{2}e^{2}x^{2}+80d^{3}ex+47d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{315d^{3}(-ex+d)^{5}e}$	72
default	Expression too large to display	954

input
```
int((e*x+d)^7/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(e*x+d)^8*(-e*x+d)*(2*e^2*x^2-14*d*e*x+47*d^2)/d^3/e/(-e^2*x^2+d^2)^
(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{47\,e^5x^5 - 235\,de^4x^4 + 470\,d^2e^3x^3 - 470\,d^3e^2x^2 + 235\,d^4ex - 47\,d^5 - (2\,e^4x^4 - 10\,d}{315\,(d^3e^6x^5 - 5\,d^4e^5x^4 + 10\,d^5e^4x^3 - 10\,d^6e^3x^2 +}$$

input
```
integrate((e*x+d)^7/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(47*e^5*x^5 - 235*d*e^4*x^4 + 470*d^2*e^3*x^3 - 470*d^3*e^2*x^2 + 23
5*d^4*e*x - 47*d^5 - (2*e^4*x^4 - 10*d*e^3*x^3 + 21*d^2*e^2*x^2 + 80*d^3*e
*x + 47*d^4)*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^5 - 5*d^4*e^5*x^4 + 10*d^5*e
^4*x^3 - 10*d^6*e^3*x^2 + 5*d^7*e^2*x - d^8*e)
```

Sympy [F]

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^7}{(-(-d+ex)\,(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**7/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**7/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(88) = 176.

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.39

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{e^5x^6}{3\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{7\,de^4x^5}{4\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{19\,d^2e^3x^4}{5\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{35\,d^3e^2x^3}{8\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{99\,d^4ex^2}{35\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{71\,d^5x}{72\left(-e^2x^2+d^2\right)^{\frac{9}{2}}} + \frac{47\,d^6}{315\left(-e^2x^2+d^2\right)^{\frac{9}{2}}e} + \frac{d^3x}{504\left(-e^2x^2+d^2\right)^{\frac{7}{2}}} + \frac{dx}{420\left(-e^2x^2+d^2\right)^{\frac{5}{2}}} + \frac{x}{315\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d} + \frac{2\,x}{315\sqrt{-e^2x^2+d^2}d^3}$$

input
```
integrate((e*x+d)^7/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/3*e^5*x^6/(-e^2*x^2 + d^2)^(9/2) + 7/4*d*e^4*x^5/(-e^2*x^2 + d^2)^(9/2)
+ 19/5*d^2*e^3*x^4/(-e^2*x^2 + d^2)^(9/2) + 35/8*d^3*e^2*x^3/(-e^2*x^2 + d
^2)^(9/2) + 99/35*d^4*e*x^2/(-e^2*x^2 + d^2)^(9/2) + 71/72*d^5*x/(-e^2*x^2
 + d^2)^(9/2) + 47/315*d^6/((-e^2*x^2 + d^2)^(9/2)*e) + 1/504*d^3*x/(-e^2*
x^2 + d^2)^(7/2) + 1/420*d*x/(-e^2*x^2 + d^2)^(5/2) + 1/315*x/((-e^2*x^2 +
 d^2)^(3/2)*d) + 2/315*x/(sqrt(-e^2*x^2 + d^2)*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(88) = 176.

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.89

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{2\left(\frac{108\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{1062\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{1638\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{3402\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + \frac{2520\left(de\cdots\right.}{\cdots}\cdots\right.}{315\,d^3\left(\frac{de+\sqrt{-e^2x^2+d^2}|e\cdots}{e^2x}\cdots\right.}$$

input
```
integrate((e*x+d)^7/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
-2/315*(108*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1062*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 1638*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3/(e^6*x^3) - 3402*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) +
 2520*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) - 2310*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^6/(e^12*x^6) + 630*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^7/(e^14*x^7) - 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8) -
 47)/(d^3*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^9*abs(e))
```

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{4\,d\,\sqrt{d^2-e^2\,x^2}}{9\,e\,{(d-e\,x)}^5} - \frac{20\,\sqrt{d^2-e^2\,x^2}}{63\,e\,{(d-e\,x)}^4} + \frac{\sqrt{d^2-e^2\,x^2}}{105\,d\,e\,{(d-e\,x)}^3} + \frac{2\,\sqrt{d^2-e^2\,x^2}}{315\,d^2\,e\,{(d-e\,x)}^2} + \frac{2\,\sqrt{d^2-e^2\,x^2}}{315\,d^3\,e\,\left(d-e\,x\right)}$$

input
```
int((d + e*x)^7/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
(4*d*(d^2 - e^2*x^2)^(1/2))/(9*e*(d - e*x)^5) - (20*(d^2 - e^2*x^2)^(1/2))
/(63*e*(d - e*x)^4) + (d^2 - e^2*x^2)^(1/2)/(105*d*e*(d - e*x)^3) + (2*(d^
2 - e^2*x^2)^(1/2))/(315*d^2*e*(d - e*x)^2) + (2*(d^2 - e^2*x^2)^(1/2))/(3
15*d^3*e*(d - e*x))
```

Reduce [F]

$$\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^7}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

input
```
int((e*x+d)^7/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^7/(-e^2*x^2+d^2)^(11/2),x)
```

3.153 $\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}} \, dx = \frac{(d+ex)^6}{9de\left(d^2-e^2x^2\right)^{9/2}} + \frac{(d+ex)^5}{21d^2e\left(d^2-e^2x^2\right)^{7/2}} + \frac{2(d+ex)^4}{105d^3e\left(d^2-e^2x^2\right)^{5/2}} + \frac{2(d+ex)^3}{315d^4e\left(d^2-e^2x^2\right)^{3/2}}$$

output
```
1/9*(e*x+d)^6/d/e/(-e^2*x^2+d^2)^(9/2)+1/21*(e*x+d)^5/d^2/e/(-e^2*x^2+d^2)
^(7/2)+2/105*(e*x+d)^4/d^3/e/(-e^2*x^2+d^2)^(5/2)+2/315*(e*x+d)^3/d^4/e/(-
e^2*x^2+d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(58d^4+25d^3ex-21d^2e^2x^2+10de^3x^3-2e^4x^4)}{315d^4e(d-ex)^5}$$

input
```
Integrate[(d + e*x)^6/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(58*d^4 + 25*d^3*e*x - 21*d^2*e^2*x^2 + 10*d*e^3*x^3
- 2*e^4*x^4))/(315*d^4*e*(d - e*x)^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}}\,dx$$

$$\Big\downarrow \text{461}$$

$$\frac{(d+ex)^6}{3de\,(d^2-e^2x^2)^{9/2}}-\frac{\int \frac{(d+ex)^7}{(d^2-e^2x^2)^{11/2}}dx}{d}$$

$$\Big\downarrow \text{461}$$

$$\frac{(d+ex)^6}{3de\,(d^2-e^2x^2)^{9/2}}-\frac{\frac{(d+ex)^7}{5de(d^2-e^2x^2)^{9/2}}-\frac{2\int \frac{(d+ex)^8}{\left(d^2-e^2x^2\right)^{11/2}}dx}{5d}}{d}$$

$$\Big\downarrow \text{461}$$

$$\frac{(d+ex)^6}{3de\,(d^2-e^2x^2)^{9/2}}-\frac{\frac{(d+ex)^7}{5de(d^2-e^2x^2)^{9/2}}-\frac{2\left(\frac{(d+ex)^8}{7de\left(d^2-e^2x^2\right)^{9/2}}-\frac{\int \frac{(d+ex)^9}{\left(d^2-e^2x^2\right)^{11/2}}dx}{7d}\right)}{5d}}{d}$$

$$\Big\downarrow \text{460}$$

$$\frac{(d+ex)^6}{3de\,(d^2-e^2x^2)^{9/2}}-\frac{\frac{(d+ex)^7}{5de(d^2-e^2x^2)^{9/2}}-\frac{2\left(\frac{(d+ex)^8}{7de\left(d^2-e^2x^2\right)^{9/2}}-\frac{(d+ex)^9}{63d^2e\left(d^2-e^2x^2\right)^{9/2}}\right)}{5d}}{d}$$

input
```
Int[(d + e*x)^6/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(d + e*x)^6/(3*d*e*(d^2 - e^2*x^2)^(9/2)) - ((d + e*x)^7/(5*d*e*(d^2 - e^2
*x^2)^(9/2)) - (2*((d + e*x)^8/(7*d*e*(d^2 - e^2*x^2)^(9/2)) - (d + e*x)^9
/(63*d^2*e*(d^2 - e^2*x^2)^(9/2))))/(5*d))/d
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.50

method	result
gosper	$\frac{(ex+d)^{7}(-ex+d)\left(-2e^{3}x^{3}+12d\,e^{2}x^{2}-33d^{2}ex+58d^{3}\right)}{315d^{4}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
orering	$\frac{(ex+d)^{7}(-ex+d)\left(-2e^{3}x^{3}+12d\,e^{2}x^{2}-33d^{2}ex+58d^{3}\right)}{315d^{4}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
trager	$\frac{\left(-2e^{4}x^{4}+10d\,e^{3}x^{3}-21d^{2}e^{2}x^{2}+25d^{3}ex+58d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{315d^{4}(-ex+d)^{5}e}$
default	$d^{6}\left(\frac{x}{[illegible]}+\frac{\frac{8x}{63d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{7d^{2}}\right)}{9d^{2}}}{[illegible]}\right)+e^{6}$ [illegible]

input
```
int((e*x+d)^6/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(e*x+d)^7*(-e*x+d)*(-2*e^3*x^3+12*d*e^2*x^2-33*d^2*e*x+58*d^3)/d^4/e
/(-e^2*x^2+d^2)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{58\,e^5x^5 - 290\,de^4x^4 + 580\,d^2e^3x^3 - 580\,d^3e^2x^2 + 290\,d^4ex - 58\,d^5 + (2\,e^4x^4 - 10\,d}{315\,(d^4e^6x^5 - 5\,d^5e^5x^4 + 10\,d^6e^4x^3 - 10\,d^7e^3x^2 +}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(58*e^5*x^5 - 290*d*e^4*x^4 + 580*d^2*e^3*x^3 - 580*d^3*e^2*x^2 + 29
0*d^4*e*x - 58*d^5 + (2*e^4*x^4 - 10*d*e^3*x^3 + 21*d^2*e^2*x^2 - 25*d^3*e
*x - 58*d^4)*sqrt(-e^2*x^2 + d^2))/(d^4*e^6*x^5 - 5*d^5*e^5*x^4 + 10*d^6*e
^4*x^3 - 10*d^7*e^3*x^2 + 5*d^8*e^2*x - d^9*e)
```

Sympy [F]

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^6}{(-(-d+ex)\,(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**6/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**6/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{e^4x^5}{4\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{6\,de^3x^4}{5\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{55\,d^2e^2x^3}{24\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{76\,d^3ex^2}{35\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{73\,d^4x}{72\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{58\,d^5}{315\,(-e^2x^2+d^2)^{\frac{9}{2}}e} - \frac{d^2x}{504\,(-e^2x^2+d^2)^{\frac{7}{2}}} - \frac{x}{420\,(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{x}{315\,(-e^2x^2+d^2)^{\frac{3}{2}}d^2} - \frac{2\,x}{315\,\sqrt{-e^2x^2+d^2}d^4}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/4*e^4*x^5/(-e^2*x^2 + d^2)^(9/2) + 6/5*d*e^3*x^4/(-e^2*x^2 + d^2)^(9/2)
+ 55/24*d^2*e^2*x^3/(-e^2*x^2 + d^2)^(9/2) + 76/35*d^3*e*x^2/(-e^2*x^2 + d
^2)^(9/2) + 73/72*d^4*x/(-e^2*x^2 + d^2)^(9/2) + 58/315*d^5/((-e^2*x^2 + d
^2)^(9/2)*e) - 1/504*d^2*x/(-e^2*x^2 + d^2)^(7/2) - 1/420*x/(-e^2*x^2 + d^
2)^(5/2) - 1/315*x/((-e^2*x^2 + d^2)^(3/2)*d^2) - 2/315*x/(sqrt(-e^2*x^2 +
 d^2)*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(117) = 234.

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.17

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{2\left(\frac{207\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{1143\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{2247\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{3843\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + \frac{3465\,(de \cdots}{\cdots}\right.}{315\,d^4\left(\frac{de+\sqrt{-e^2x^2+d^2}|e \cdots}{e^2x}\cdots\right.}$$

input
```
integrate((e*x+d)^6/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
-2/315*(207*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1143*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 2247*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3/(e^6*x^3) - 3843*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) +
 3465*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) - 2625*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^6/(e^12*x^6) + 945*(d*e + sqrt(-e^2*x^2 + d^2)*abs
(e))^7/(e^14*x^7) - 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8) -
 58)/(d^4*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^9*abs(e))
```

Mupad [B] (verification not implemented)

Time = 7.70 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{2\sqrt{d^2-e^2x^2}}{9e(d-ex)^5} - \frac{\sqrt{d^2-e^2x^2}}{63de(d-ex)^4} - \frac{\sqrt{d^2-e^2x^2}}{105d^2e(d-ex)^3} - \frac{2\sqrt{d^2-e^2x^2}}{315d^3e(d-ex)^2} - \frac{2\sqrt{d^2-e^2x^2}}{315d^4e(d-ex)}$$

input
```
int((d + e*x)^6/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
(2*(d^2 - e^2*x^2)^(1/2))/(9*e*(d - e*x)^5) - (d^2 - e^2*x^2)^(1/2)/(63*d*
e*(d - e*x)^4) - (d^2 - e^2*x^2)^(1/2)/(105*d^2*e*(d - e*x)^3) - (2*(d^2 -
 e^2*x^2)^(1/2))/(315*d^3*e*(d - e*x)^2) - (2*(d^2 - e^2*x^2)^(1/2))/(315*
d^4*e*(d - e*x))
```

Reduce [F]

$$\int \frac{(d+ex)^6}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^6}{(-e^2x^2+d^2)^{\frac{11}{2}}}dx$$

input
```
int((e*x+d)^6/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^6/(-e^2*x^2+d^2)^(11/2),x)
```

3.154 $\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{2(d+ex)^4}{9e\,(d^2-e^2x^2)^{9/2}}+\frac{2(d+ex)^2}{63e\,(d^2-e^2x^2)^{7/2}}+\frac{d+ex}{105de\,(d^2-e^2x^2)^{5/2}}+\frac{4x}{315d^3\,(d^2-e^2x^2)^{3/2}}+\frac{8x}{315d^5\sqrt{d^2-e^2x^2}}$$

output
```
2/9*(e*x+d)^4/e/(-e^2*x^2+d^2)^(9/2)+2/63*(e*x+d)^2/e/(-e^2*x^2+d^2)^(7/2)
+1/105*(e*x+d)/d/e/(-e^2*x^2+d^2)^(5/2)+4/315*x/d^3/(-e^2*x^2+d^2)^(3/2)+8
/315*x/d^5/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(83d^4-100d^3ex+84d^2e^2x^2-40de^3x^3+8e^4x^4)}{315d^5e(d-ex)^5}$$

input
```
Integrate[(d + e*x)^5/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(83*d^4 - 100*d^3*e*x + 84*d^2*e^2*x^2 - 40*d*e^3*x^3
 + 8*e^4*x^4))/(315*d^5*e*(d - e*x)^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {464, 461, 461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}dx$$

$$\Big\downarrow \text{464}$$

$$\int \frac{1}{(d-ex)^5\sqrt{d^2-e^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{4\int \frac{1}{(d-ex)^4\sqrt{d^2-e^2x^2}}dx}{9d}+\frac{\sqrt{d^2-e^2x^2}}{9de(d-ex)^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{4\left(\frac{3\int \frac{1}{(d-ex)^3\sqrt{d^2-e^2x^2}}dx}{7d}+\frac{\sqrt{d^2-e^2x^2}}{7de(d-ex)^4}\right)}{9d}+\frac{\sqrt{d^2-e^2x^2}}{9de(d-ex)^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{4\left(\frac{3\left(\frac{2\int \frac{1}{(d-ex)^2\sqrt{d^2-e^2x^2}}dx}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}\right)}{7d}+\frac{\sqrt{d^2-e^2x^2}}{7de(d-ex)^4}\right)}{9d}+\frac{\sqrt{d^2-e^2x^2}}{9de(d-ex)^5}$$

$$\Big\downarrow \text{461}$$

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\int \frac{1}{(d-ex)\sqrt{d^2-e^2x^2}}dx}{3d}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}\right)}{7d}+\frac{\sqrt{d^2-e^2x^2}}{7de(d-ex)^4}\right)}{9d}+\frac{\sqrt{d^2-e^2x^2}}{9de(d-ex)^5}$$

$$\Big\downarrow \ 460$$

$$\frac{4\left(\frac{3\left(\frac{2\left(\frac{\sqrt{d^2-e^2x^2}}{3d^2e(d-ex)}+\frac{\sqrt{d^2-e^2x^2}}{3de(d-ex)^2}\right)}{5d}+\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3}\right)}{7d}+\frac{\sqrt{d^2-e^2x^2}}{7de(d-ex)^4}\right)}{9d}+\frac{\sqrt{d^2-e^2x^2}}{9de(d-ex)^5}$$

input
```
Int[(d + e*x)^5/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
Sqrt[d^2 - e^2*x^2]/(9*d*e*(d - e*x)^5) + (4*(Sqrt[d^2 - e^2*x^2]/(7*d*e*(
d - e*x)^4) + (3*(Sqrt[d^2 - e^2*x^2]/(5*d*e*(d - e*x)^3) + (2*(Sqrt[d^2 -
 e^2*x^2]/(3*d*e*(d - e*x)^2) + Sqrt[d^2 - e^2*x^2]/(3*d^2*e*(d - e*x))))/
(5*d)))/(7*d)))/(9*d)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

method	result
trager	$\frac{\left(8e^{4}x^{4}-40d\,e^{3}x^{3}+84d^{2}e^{2}x^{2}-100d^{3}ex+83d^{4}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{315d^{5}\left(-ex+d\right)^{5}e}$
gosper	$\frac{\left(ex+d\right)^{6}\left(-ex+d\right)\left(8e^{4}x^{4}-40d\,e^{3}x^{3}+84d^{2}e^{2}x^{2}-100d^{3}ex+83d^{4}\right)}{315d^{5}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
orering	$\frac{\left(ex+d\right)^{6}\left(-ex+d\right)\left(8e^{4}x^{4}-40d\,e^{3}x^{3}+84d^{2}e^{2}x^{2}-100d^{3}ex+83d^{4}\right)}{315d^{5}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
default	$d^{5}\left(\frac{x}{9d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{7d^{2}}\right)}{9d^{2}}}{d^{2}}\right)+e^{5}\Bigg($

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(8*e^4*x^4-40*d*e^3*x^3+84*d^2*e^2*x^2-100*d^3*e*x+83*d^4)/d^5/(-e*x
+d)^5/e*(-e^2*x^2+d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{83\,e^5x^5-415\,de^4x^4+830\,d^2e^3x^3-830\,d^3e^2x^2+415\,d^4ex-83\,d^5-(8\,e^4x^4-40\,d}{315\,(d^5e^6x^5-5\,d^6e^5x^4+10\,d^7e^4x^3-10\,d^8e^3x^2-}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(83*e^5*x^5 - 415*d*e^4*x^4 + 830*d^2*e^3*x^3 - 830*d^3*e^2*x^2 + 41
5*d^4*e*x - 83*d^5 - (8*e^4*x^4 - 40*d*e^3*x^3 + 84*d^2*e^2*x^2 - 100*d^3*
e*x + 83*d^4)*sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^5 - 5*d^6*e^5*x^4 + 10*d^7*
e^4*x^3 - 10*d^8*e^3*x^2 + 5*d^9*e^2*x - d^10*e)
```

Sympy [F]

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^5}{(-(-d+ex)\,(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**5/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**5/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{e^3x^4}{5\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{5\,de^2x^3}{6\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{46\,d^2ex^2}{35\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{17\,d^3x}{18\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{83\,d^4}{315\,(-e^2x^2+d^2)^{\frac{9}{2}}e} + \frac{dx}{126\,(-e^2x^2+d^2)^{\frac{7}{2}}} + \frac{x}{105\,(-e^2x^2+d^2)^{\frac{5}{2}}d} + \frac{4\,x}{315\,(-e^2x^2+d^2)^{\frac{3}{2}}d^3} + \frac{8\,x}{315\,\sqrt{-e^2x^2+d^2}d^5}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/5*e^3*x^4/(-e^2*x^2 + d^2)^(9/2) + 5/6*d*e^2*x^3/(-e^2*x^2 + d^2)^(9/2)
+ 46/35*d^2*e*x^2/(-e^2*x^2 + d^2)^(9/2) + 17/18*d^3*x/(-e^2*x^2 + d^2)^(9
/2) + 83/315*d^4/((-e^2*x^2 + d^2)^(9/2)*e) + 1/126*d*x/(-e^2*x^2 + d^2)^(
7/2) + 1/105*x/((-e^2*x^2 + d^2)^(5/2)*d) + 4/315*x/((-e^2*x^2 + d^2)^(3/2
)*d^3) + 8/315*x/(sqrt(-e^2*x^2 + d^2)*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(120) = 240.

Time = 0.15 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.06

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{2\left(\frac{432\left(de+\sqrt{-e^2x^2+d^2}|e|\right)}{e^2x} - \frac{1728\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^2}{e^4x^2} + \frac{3612\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^3}{e^6x^3} - \frac{5418\left(de+\sqrt{-e^2x^2+d^2}|e|\right)^4}{e^8x^4} + \frac{5040\left(de \cdots\right.}{\cdots}\right.}{315\,d^5\left(\frac{de+\sqrt{-e^2x^2+d^2}|e \cdots}{e^2x}\right.}$$

input
```
integrate((e*x+d)^5/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
-2/315*(432*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1728*(d*e + sqrt
(-e^2*x^2 + d^2)*abs(e))^2/(e^4*x^2) + 3612*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^3/(e^6*x^3) - 5418*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^4/(e^8*x^4) +
 5040*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^5/(e^10*x^5) - 3360*(d*e + sqrt(
-e^2*x^2 + d^2)*abs(e))^6/(e^12*x^6) + 1260*(d*e + sqrt(-e^2*x^2 + d^2)*ab
s(e))^7/(e^14*x^7) - 315*(d*e + sqrt(-e^2*x^2 + d^2)*abs(e))^8/(e^16*x^8)
- 83)/(d^5*((d*e + sqrt(-e^2*x^2 + d^2)*abs(e))/(e^2*x) - 1)^9*abs(e))
```

Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}}{9\,d\,e\,(d-e\,x)^5} + \frac{4\,\sqrt{d^2-e^2\,x^2}}{63\,d^2\,e\,(d-e\,x)^4} + \frac{4\,\sqrt{d^2-e^2\,x^2}}{105\,d^3\,e\,(d-e\,x)^3} + \frac{8\,\sqrt{d^2-e^2\,x^2}}{315\,d^4\,e\,(d-e\,x)^2} + \frac{8\,\sqrt{d^2-e^2\,x^2}}{315\,d^5\,e\,(d-e\,x)}$$

input
```
int((d + e*x)^5/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
(d^2 - e^2*x^2)^(1/2)/(9*d*e*(d - e*x)^5) + (4*(d^2 - e^2*x^2)^(1/2))/(63*
d^2*e*(d - e*x)^4) + (4*(d^2 - e^2*x^2)^(1/2))/(105*d^3*e*(d - e*x)^3) + (
8*(d^2 - e^2*x^2)^(1/2))/(315*d^4*e*(d - e*x)^2) + (8*(d^2 - e^2*x^2)^(1/2
))/(315*d^5*e*(d - e*x))
```

Reduce [F]

$$\int \frac{(d+ex)^5}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^5}{(-e^2x^2+d^2)^{\frac{11}{2}}}dx$$

input
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
int((e*x+d)^5/(-e^2*x^2+d^2)^(11/2),x)
```

3.155 $\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx=\frac{2(d+ex)^3}{9e\left(d^2-e^2x^2\right)^{9/2}}+\frac{2(d+ex)}{21e\left(d^2-e^2x^2\right)^{7/2}}+\frac{x}{21d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4x}{63d^4\left(d^2-e^2x^2\right)^{3/2}}+\frac{8x}{63d^6\sqrt{d^2-e^2x^2}}$$

output
```
2/9*(e*x+d)^3/e/(-e^2*x^2+d^2)^(9/2)+2/21*(e*x+d)/e/(-e^2*x^2+d^2)^(7/2)+1
/21*x/d^2/(-e^2*x^2+d^2)^(5/2)+4/63*x/d^4/(-e^2*x^2+d^2)^(3/2)+8/63*x/d^6/
(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx=\frac{\sqrt{d^2-e^2x^2}(20d^5-17d^4ex-16d^3e^2x^2+44d^2e^3x^3-32de^4x^4+8e^5x^5)}{63d^6e(d-ex)^5(d+ex)}$$

input
```
Integrate[(d + e*x)^4/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(20*d^5 - 17*d^4*e*x - 16*d^3*e^2*x^2 + 44*d^2*e^3*x^
3 - 32*d*e^4*x^4 + 8*e^5*x^5))/(63*d^6*e*(d - e*x)^5*(d + e*x))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {464, 461, 461, 461, 470, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{\left(d^2-e^2x^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{464}$$

$$\int \frac{1}{(d-ex)^4\left(d^2-e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{5\int \frac{1}{(d-ex)^3\left(d^2-e^2x^2\right)^{3/2}}dx}{9d}+\frac{1}{9de(d-ex)^4\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{5\left(\frac{4\int \frac{1}{(d-ex)^2\left(d^2-e^2x^2\right)^{3/2}}dx}{7d}+\frac{1}{7de(d-ex)^3\sqrt{d^2-e^2x^2}}\right)}{9d}+\frac{1}{9de(d-ex)^4\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{5\left(\frac{4\left(\frac{3\int \frac{1}{(d-ex)\left(d^2-e^2x^2\right)^{3/2}}dx}{5d}+\frac{1}{5de(d-ex)^2\sqrt{d^2-e^2x^2}}\right)}{7d}+\frac{1}{7de(d-ex)^3\sqrt{d^2-e^2x^2}}\right)}{9d}+\frac{1}{9de(d-ex)^4\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{5\left(\frac{4\left(\frac{3\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d}+\frac{1}{3de(d-ex)\sqrt{d^2-e^2x^2}}\right)}{5d}+\frac{1}{5de(d-ex)^2\sqrt{d^2-e^2x^2}}\right)}{7d}+\frac{1}{7de(d-ex)^3\sqrt{d^2-e^2x^2}}\right)}{9d}+\frac{1}{9de(d-ex)^4\sqrt{d^2-e^2x^2}}$$

$$\Big\downarrow \ 208$$

$$\frac{1}{9de(d-ex)^4\sqrt{d^2-e^2x^2}}+\frac{5\left(\frac{1}{7de(d-ex)^3\sqrt{d^2-e^2x^2}}+\frac{4\left(\frac{1}{5de(d-ex)^2\sqrt{d^2-e^2x^2}}+\frac{3\left(\frac{1}{3de(d-ex)\sqrt{d^2-e^2x^2}}+\frac{2x}{3d^3\sqrt{d^2-e^2x^2}}\right)}{5d}\right)}{7d}\right)}{9d}$$

input
```
Int[(d + e*x)^4/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
1/(9*d*e*(d - e*x)^4*Sqrt[d^2 - e^2*x^2]) + (5*(1/(7*d*e*(d - e*x)^3*Sqrt[
d^2 - e^2*x^2]) + (4*(1/(5*d*e*(d - e*x)^2*Sqrt[d^2 - e^2*x^2]) + (3*((2*x
)/(3*d^3*Sqrt[d^2 - e^2*x^2]) + 1/(3*d*e*(d - e*x)*Sqrt[d^2 - e^2*x^2])))/
(5*d)))/(7*d)))/(9*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67

method	result
gosper	$\frac{(ex+d)^{5}(-ex+d)\left(8e^{5}x^{5}-32d\,e^{4}x^{4}+44d^{2}e^{3}x^{3}-16d^{3}e^{2}x^{2}-17d^{4}ex+20d^{5}\right)}{63d^{6}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
orering	$\frac{(ex+d)^{5}(-ex+d)\left(8e^{5}x^{5}-32d\,e^{4}x^{4}+44d^{2}e^{3}x^{3}-16d^{3}e^{2}x^{2}-17d^{4}ex+20d^{5}\right)}{63d^{6}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
trager	$\frac{\left(8e^{5}x^{5}-32d\,e^{4}x^{4}+44d^{2}e^{3}x^{3}-16d^{3}e^{2}x^{2}-17d^{4}ex+20d^{5}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{63d^{6}(-ex+d)^{5}e(ex+d)}$
default	$d^{4}\left(\frac{x}{9d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{7d^{2}}\right)}{9d^{2}}}{d^{2}}\right)+e^{4}$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/63*(e*x+d)^5*(-e*x+d)*(8*e^5*x^5-32*d*e^4*x^4+44*d^2*e^3*x^3-16*d^3*e^2*
x^2-17*d^4*e*x+20*d^5)/d^6/e/(-e^2*x^2+d^2)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{20\,e^6x^6 - 80\,de^5x^5 + 100\,d^2e^4x^4 - 100\,d^4e^2x^2 + 80\,d^5ex - 20\,d^6 - (8\,e^5x^5 - 32\,de^4}{63\,(d^6e^7x^6 - 4\,d^7e^6x^5 + 5\,d^8e^5x^4 - 5\,d^{10}e^3x}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/63*(20*e^6*x^6 - 80*d*e^5*x^5 + 100*d^2*e^4*x^4 - 100*d^4*e^2*x^2 + 80*d
^5*e*x - 20*d^6 - (8*e^5*x^5 - 32*d*e^4*x^4 + 44*d^2*e^3*x^3 - 16*d^3*e^2*
x^2 - 17*d^4*e*x + 20*d^5)*sqrt(-e^2*x^2 + d^2))/(d^6*e^7*x^6 - 4*d^7*e^6*
x^5 + 5*d^8*e^5*x^4 - 5*d^10*e^3*x^2 + 4*d^11*e^2*x - d^12*e)
```

Sympy [F]

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^4}{(-(-d+ex)(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**4/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{e^2x^3}{6\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{4\,dex^2}{7\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{13\,d^2x}{18\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{20\,d^3}{63\,(-e^2x^2+d^2)^{\frac{9}{2}}e} + \frac{5\,x}{126\,(-e^2x^2+d^2)^{\frac{7}{2}}} + \frac{x}{21\,(-e^2x^2+d^2)^{\frac{5}{2}}d^2} + \frac{4\,x}{63\,(-e^2x^2+d^2)^{\frac{3}{2}}d^4} + \frac{8\,x}{63\,\sqrt{-e^2x^2+d^2}d^6}$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/6*e^2*x^3/(-e^2*x^2 + d^2)^(9/2) + 4/7*d*e*x^2/(-e^2*x^2 + d^2)^(9/2) +
13/18*d^2*x/(-e^2*x^2 + d^2)^(9/2) + 20/63*d^3/((-e^2*x^2 + d^2)^(9/2)*e)
+ 5/126*x/(-e^2*x^2 + d^2)^(7/2) + 1/21*x/((-e^2*x^2 + d^2)^(5/2)*d^2) + 4
/63*x/((-e^2*x^2 + d^2)^(3/2)*d^4) + 8/63*x/(sqrt(-e^2*x^2 + d^2)*d^6)
```

Giac [F]

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^4}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)^4/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^4/(-e^2*x^2 + d^2)^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{8\,x}{63\,d^6}+\frac{65}{1008\,d^5\,e}\right)}{(d+e\,x)\,(d-e\,x)}+\frac{\sqrt{d^2-e^2\,x^2}}{18\,d^2\,e\,(d-e\,x)^5}+\frac{17\,\sqrt{d^2-e^2\,x^2}}{252\,d^3\,e\,(d-e\,x)^4}+\frac{11\,\sqrt{d^2-e^2\,x^2}}{168\,d^4\,e\,(d-e\,x)^3}+\frac{65\,\sqrt{d^2-e^2\,x^2}}{1008\,d^5\,e\,(d-e\,x)^2}$$

input
```
int((d + e*x)^4/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((8*x)/(63*d^6) + 65/(1008*d^5*e)))/((d + e*x)*(d -
 e*x)) + (d^2 - e^2*x^2)^(1/2)/(18*d^2*e*(d - e*x)^5) + (17*(d^2 - e^2*x^2
)^(1/2))/(252*d^3*e*(d - e*x)^4) + (11*(d^2 - e^2*x^2)^(1/2))/(168*d^4*e*(
d - e*x)^3) + (65*(d^2 - e^2*x^2)^(1/2))/(1008*d^5*e*(d - e*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^4}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{-e^2x^2+d^2}\,(8e^5x^5-32d\,e^4x^4+44d^2e^3x^3-16d^3e^2x^2-17d^4ex+20d^5)}{63d^6e\,(-e^6x^6+4d\,e^5x^5-5d^2e^4x^4+5d^4e^2x^2-4d^5ex+d^6)}$$

input
```
int((e*x+d)^4/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
(sqrt(d**2 - e**2*x**2)*(20*d**5 - 17*d**4*e*x - 16*d**3*e**2*x**2 + 44*d*
*2*e**3*x**3 - 32*d*e**4*x**4 + 8*e**5*x**5))/(63*d**6*e*(d**6 - 4*d**5*e*
x + 5*d**4*e**2*x**2 - 5*d**2*e**4*x**4 + 4*d*e**5*x**5 - e**6*x**6))
```

3.156 $\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{11/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{11/2}} \, dx = \frac{2(d+ex)^2}{9e\left(d^2-e^2x^2\right)^{9/2}}+\frac{5(d+ex)}{63de\left(d^2-e^2x^2\right)^{7/2}} +\frac{2x}{21d^3\left(d^2-e^2x^2\right)^{5/2}}+\frac{8x}{63d^5\left(d^2-e^2x^2\right)^{3/2}}+\frac{16x}{63d^7\sqrt{d^2-e^2x^2}}$$

output
```
2/9*(e*x+d)^2/e/(-e^2*x^2+d^2)^(9/2)+5/63*(e*x+d)/d/e/(-e^2*x^2+d^2)^(7/2)
+2/21*x/d^3/(-e^2*x^2+d^2)^(5/2)+8/63*x/d^5/(-e^2*x^2+d^2)^(3/2)+16/63*x/d
^7/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^3}{\left(d^2-e^2x^2\right)^{11/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(19d^6+6d^5ex-66d^4e^2x^2+56d^3e^3x^3+24d^2e^4x^4-48de^5x^5+16e^6x^6)}{63d^7e(d-ex)^5(d+ex)^2}$$

input
```
Integrate[(d + e*x)^3/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(19*d^6 + 6*d^5*e*x - 66*d^4*e^2*x^2 + 56*d^3*e^3*x^3
 + 24*d^2*e^4*x^4 - 48*d*e^5*x^5 + 16*e^6*x^6))/(63*d^7*e*(d - e*x)^5*(d +
 e*x)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {464, 461, 461, 470, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{11/2}}dx$$

$$\Big\downarrow \text{464}$$

$$\int \frac{1}{(d-ex)^3 (d^2-e^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{2\int \frac{1}{(d-ex)^2(d^2-e^2x^2)^{5/2}}dx}{3d}+\frac{1}{9de(d-ex)^3 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{461}$$

$$\frac{2\left(\frac{5\int \frac{1}{(d-ex)\left(d^2-e^2x^2\right)^{5/2}}dx}{7d}+\frac{1}{7de(d-ex)^2(d^2-e^2x^2)^{3/2}}\right)}{3d}+\frac{1}{9de(d-ex)^3 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{2\left(\frac{5\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d}+\frac{1}{5de(d-ex)\left(d^2-e^2x^2\right)^{3/2}}\right)}{7d}+\frac{1}{7de(d-ex)^2(d^2-e^2x^2)^{3/2}}\right)}{3d}+\frac{1}{9de(d-ex)^3 (d^2-e^2x^2)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{2\left(\frac{5\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d}+\frac{1}{5de(d-ex)\left(d^2-e^2x^2\right)^{3/2}}\right)}{7d}+\frac{1}{7de(d-ex)^2(d^2-e^2x^2)^{3/2}}\right)}{3d}+\frac{1}{9de(d-ex)^3\left(d^2-e^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{1}{9de(d-ex)^3\left(d^2-e^2x^2\right)^{3/2}}+\frac{2\left(\frac{1}{7de(d-ex)^2(d^2-e^2x^2)^{3/2}}+\frac{5\left(\frac{1}{5de(d-ex)\left(d^2-e^2x^2\right)^{3/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d}\right)}{7d}\right)}{3d}$$

input
```
Int[(d + e*x)^3/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
1/(9*d*e*(d - e*x)^3*(d^2 - e^2*x^2)^(3/2)) + (2*(1/(7*d*e*(d - e*x)^2*(d^
2 - e^2*x^2)^(3/2)) + (5*(1/(5*d*e*(d - e*x)*(d^2 - e^2*x^2)^(3/2)) + (4*(
x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d
)))/(7*d)))/(3*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 464
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
a + b*x^2)^(n + p)/(a/c + b*(x/d))^n, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c^2 + a*d^2, 0] && IntegerQ[n] && RationalQ[p] && (LtQ[0, -n, p] || LtQ[p,
 -n, 0]) && NeQ[n, 2] && NeQ[n, -1]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.74

method	result
gosper	$\frac{(ex+d)^{4}(-ex+d)\left(16e^{6}x^{6}-48d\,e^{5}x^{5}+24d^{2}e^{4}x^{4}+56d^{3}e^{3}x^{3}-66d^{4}e^{2}x^{2}+6d^{5}ex+19d^{6}\right)}{63d^{7}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
orering	$\frac{(ex+d)^{4}(-ex+d)\left(16e^{6}x^{6}-48d\,e^{5}x^{5}+24d^{2}e^{4}x^{4}+56d^{3}e^{3}x^{3}-66d^{4}e^{2}x^{2}+6d^{5}ex+19d^{6}\right)}{63d^{7}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
trager	$\frac{\left(16e^{6}x^{6}-48d\,e^{5}x^{5}+24d^{2}e^{4}x^{4}+56d^{3}e^{3}x^{3}-66d^{4}e^{2}x^{2}+6d^{5}ex+19d^{6}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{63d^{7}(-ex+d)^{5}(ex+d)^{2}e}$
default	$d^{3}\left(\frac{x}{9d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{7d^{2}}\right)}{9d^{2}}}{d^{2}}\right)+e^{3}\Big($

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/63*(e*x+d)^4*(-e*x+d)*(16*e^6*x^6-48*d*e^5*x^5+24*d^2*e^4*x^4+56*d^3*e^3
*x^3-66*d^4*e^2*x^2+6*d^5*e*x+19*d^6)/d^7/e/(-e^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(114) = 228$.

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{19\,e^7x^7 - 57\,de^6x^6 + 19\,d^2e^5x^5 + 95\,d^3e^4x^4 - 95\,d^4e^3x^3 - 19\,d^5e^2x^2 + 57\,d^6ex - 19}{63\,(d^7e^8x^7 - 3\,d^8e^7x^6 + d^9e^6x^5 + 5\,d}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/63*(19*e^7*x^7 - 57*d*e^6*x^6 + 19*d^2*e^5*x^5 + 95*d^3*e^4*x^4 - 95*d^4
*e^3*x^3 - 19*d^5*e^2*x^2 + 57*d^6*e*x - 19*d^7 - (16*e^6*x^6 - 48*d*e^5*x
^5 + 24*d^2*e^4*x^4 + 56*d^3*e^3*x^3 - 66*d^4*e^2*x^2 + 6*d^5*e*x + 19*d^6
)*sqrt(-e^2*x^2 + d^2))/(d^7*e^8*x^7 - 3*d^8*e^7*x^6 + d^9*e^6*x^5 + 5*d^1
0*e^5*x^4 - 5*d^11*e^4*x^3 - d^12*e^3*x^2 + 3*d^13*e^2*x - d^14*e)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^3}{(-(-d+ex)\,(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{11/2}}\,dx &= \frac{ex^2}{7\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{4\,dx}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}} \\ &+ \frac{19\,d^2}{63\,(-e^2x^2+d^2)^{\frac{9}{2}}e} + \frac{5\,x}{63\,(-e^2x^2+d^2)^{\frac{7}{2}}d} + \frac{2\,x}{21\,(-e^2x^2+d^2)^{\frac{5}{2}}d^3} \\ &+ \frac{8\,x}{63\,(-e^2x^2+d^2)^{\frac{3}{2}}d^5} + \frac{16\,x}{63\,\sqrt{-e^2x^2+d^2}\,d^7}\end{aligned}$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/7*e*x^2/(-e^2*x^2 + d^2)^(9/2) + 4/9*d*x/(-e^2*x^2 + d^2)^(9/2) + 19/63*
d^2/((-e^2*x^2 + d^2)^(9/2)*e) + 5/63*x/((-e^2*x^2 + d^2)^(7/2)*d) + 2/21*
x/((-e^2*x^2 + d^2)^(5/2)*d^3) + 8/63*x/((-e^2*x^2 + d^2)^(3/2)*d^5) + 16/
63*x/(sqrt(-e^2*x^2 + d^2)*d^7)
```

Giac [F]

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^3}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(-e^2*x^2 + d^2)^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{197\,x}{1008\,d^5}+\frac{155}{1008\,d^4\,e}\right)}{(d+e\,x)^2\,(d-e\,x)^2}+\frac{\sqrt{d^2-e^2\,x^2}}{36\,d^3\,e\,(d-e\,x)^5}+\frac{13\,\sqrt{d^2-e^2\,x^2}}{252\,d^4\,e\,(d-e\,x)^4}+\frac{23\,\sqrt{d^2-e^2\,x^2}}{336\,d^5\,e\,(d-e\,x)^3}+\frac{16\,x\,\sqrt{d^2-e^2\,x^2}}{63\,d^7\,(d+e\,x)\,(d-e\,x)}$$

input
```
int((d + e*x)^3/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((197*x)/(1008*d^5) + 155/(1008*d^4*e)))/((d + e*x)
^2*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(36*d^3*e*(d - e*x)^5) + (13*(d^2
- e^2*x^2)^(1/2))/(252*d^4*e*(d - e*x)^4) + (23*(d^2 - e^2*x^2)^(1/2))/(33
6*d^5*e*(d - e*x)^3) + (16*x*(d^2 - e^2*x^2)^(1/2))/(63*d^7*(d + e*x)*(d -
 e*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.99

$$\int \frac{(d+ex)^3}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{-2\sqrt{-e^2x^2+d^2}\,d^5+6\sqrt{-e^2x^2+d^2}\,d^4ex-4\sqrt{-e^2x^2+d^2}\,d^3e^2x^2-4\sqrt{-e^2x^2+d}}{63\sqrt{-e^2x^2+}}$$

input
```
int((e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
( - 2*sqrt(d**2 - e**2*x**2)*d**5 + 6*sqrt(d**2 - e**2*x**2)*d**4*e*x - 4*
sqrt(d**2 - e**2*x**2)*d**3*e**2*x**2 - 4*sqrt(d**2 - e**2*x**2)*d**2*e**3
*x**3 + 6*sqrt(d**2 - e**2*x**2)*d*e**4*x**4 - 2*sqrt(d**2 - e**2*x**2)*e*
*5*x**5 + 19*d**6 + 6*d**5*e*x - 66*d**4*e**2*x**2 + 56*d**3*e**3*x**3 + 2
4*d**2*e**4*x**4 - 48*d*e**5*x**5 + 16*e**6*x**6)/(63*sqrt(d**2 - e**2*x**
2)*d**7*e*(d**5 - 3*d**4*e*x + 2*d**3*e**2*x**2 + 2*d**2*e**3*x**3 - 3*d*e
**4*x**4 + e**5*x**5))
```

3.157 $\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{11/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{11/2}} \, dx = \frac{2(d+ex)}{9e\left(d^2-e^2x^2\right)^{9/2}}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{7/2}} +\frac{2x}{15d^4\left(d^2-e^2x^2\right)^{5/2}}+\frac{8x}{45d^6\left(d^2-e^2x^2\right)^{3/2}}+\frac{16x}{45d^8\sqrt{d^2-e^2x^2}}$$

output
```
2/9*(e*x+d)/e/(-e^2*x^2+d^2)^(9/2)+1/9*x/d^2/(-e^2*x^2+d^2)^(7/2)+2/15*x/d
^4/(-e^2*x^2+d^2)^(5/2)+8/45*x/d^6/(-e^2*x^2+d^2)^(3/2)+16/45*x/d^8/(-e^2*
x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{\left(d^2-e^2x^2\right)^{11/2}} \, dx = \frac{\sqrt{d^2-e^2x^2}(10d^7+25d^6ex-60d^5e^2x^2-10d^4e^3x^3+80d^3e^4x^4-24d^2e^5x^5-32de^6}{45d^8e(d-ex)^5(d+ex)^3}$$

input
```
Integrate[(d + e*x)^2/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(10*d^7 + 25*d^6*e*x - 60*d^5*e^2*x^2 - 10*d^4*e^3*x^
3 + 80*d^3*e^4*x^4 - 24*d^2*e^5*x^5 - 32*d*e^6*x^6 + 16*e^7*x^7))/(45*d^8*
e*(d - e*x)^5*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {457, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{11/2}} dx$$

$$\Big\downarrow \text{457}$$

$$\frac{7}{9}\int \frac{1}{(d^2-e^2x^2)^{9/2}}dx+\frac{2(d+ex)}{9e\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{7}{9}\left(\frac{6\int \frac{1}{(d^2-e^2x^2)^{7/2}}dx}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)+\frac{2(d+ex)}{9e\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{7}{9}\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2(d^2-e^2x^2)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)+\frac{2(d+ex)}{9e\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{7}{9}\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2(d^2-e^2x^2)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)+$$

$$\frac{2(d+ex)}{9e\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2(d+ex)}{9e\left(d^2-e^2x^2\right)^{9/2}}+$$

$$\frac{7}{9}\left(\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}+\frac{6\left(\frac{x}{5d^2(d^2-e^2x^2)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d^2}\right)$$

input
```
Int[(d + e*x)^2/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(2*(d + e*x))/(9*e*(d^2 - e^2*x^2)^(9/2)) + (7*(x/(7*d^2*(d^2 - e^2*x^2)^(
7/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)
^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d^2)))/9
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 457
```
Int[((c_) + (d_.)*(x_))^2*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[d*(
c + d*x)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((p + 2)/(b*(p +
1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[
b*c^2 + a*d^2, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{(ex+d)^{3}(-ex+d)\left(16e^{7}x^{7}-32d\,e^{6}x^{6}-24d^{2}e^{5}x^{5}+80d^{3}e^{4}x^{4}-10d^{4}e^{3}x^{3}-60d^{5}e^{2}x^{2}+25d^{6}ex+10d^{7}\right)}{45d^{8}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
orering	$\frac{(ex+d)^{3}(-ex+d)\left(16e^{7}x^{7}-32d\,e^{6}x^{6}-24d^{2}e^{5}x^{5}+80d^{3}e^{4}x^{4}-10d^{4}e^{3}x^{3}-60d^{5}e^{2}x^{2}+25d^{6}ex+10d^{7}\right)}{45d^{8}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
trager	$\frac{\left(16e^{7}x^{7}-32d\,e^{6}x^{6}-24d^{2}e^{5}x^{5}+80d^{3}e^{4}x^{4}-10d^{4}e^{3}x^{3}-60d^{5}e^{2}x^{2}+25d^{6}ex+10d^{7}\right)\sqrt{-e^{2}x^{2}+d^{2}}}{45d^{8}(-ex+d)^{5}(ex+d)^{3}e}$
default	$d^{2}\left(\frac{x}{9d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d^{2}\left(-e^{2}x^{2}+d^{2}\right)^{\frac{3}{2}}}+\frac{8x}{15d^{4}\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{7d^{2}}\right)}{9d^{2}}}{d^{2}}\right)+e^{2}\Bigg($

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*(e*x+d)^3*(-e*x+d)*(16*e^7*x^7-32*d*e^6*x^6-24*d^2*e^5*x^5+80*d^3*e^4
*x^4-10*d^4*e^3*x^3-60*d^5*e^2*x^2+25*d^6*e*x+10*d^7)/d^8/e/(-e^2*x^2+d^2)
^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(105) = 210$.

Time = 0.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.99

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{10\,e^8x^8 - 20\,de^7x^7 - 20\,d^2e^6x^6 + 60\,d^3e^5x^5 - 60\,d^5e^3x^3 + 20\,d^6e^2x^2 + 20\,d^7ex - 10}{45\,(d^8e^9x^8 - 2\,d^9e^8x^7 - 2\,d^{10}e^7x}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/45*(10*e^8*x^8 - 20*d*e^7*x^7 - 20*d^2*e^6*x^6 + 60*d^3*e^5*x^5 - 60*d^5
*e^3*x^3 + 20*d^6*e^2*x^2 + 20*d^7*e*x - 10*d^8 - (16*e^7*x^7 - 32*d*e^6*x
^6 - 24*d^2*e^5*x^5 + 80*d^3*e^4*x^4 - 10*d^4*e^3*x^3 - 60*d^5*e^2*x^2 + 2
5*d^6*e*x + 10*d^7)*sqrt(-e^2*x^2 + d^2))/(d^8*e^9*x^8 - 2*d^9*e^8*x^7 - 2
*d^10*e^7*x^6 + 6*d^11*e^6*x^5 - 6*d^13*e^4*x^3 + 2*d^14*e^3*x^2 + 2*d^15*
e^2*x - d^16*e)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(d+ex)^2}{(-(-d+ex)\,(d+ex))^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral((d + e*x)**2/(-(-d + e*x)*(d + e*x))**(11/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{2\,x}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}} + \frac{2\,d}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}e} + \frac{x}{9\,(-e^2x^2+d^2)^{\frac{7}{2}}d^2} + \frac{2\,x}{15\,(-e^2x^2+d^2)^{\frac{5}{2}}d^4} + \frac{8\,x}{45\,(-e^2x^2+d^2)^{\frac{3}{2}}d^6} + \frac{16\,x}{45\,\sqrt{-e^2x^2+d^2}d^8}$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
2/9*x/(-e^2*x^2 + d^2)^(9/2) + 2/9*d/((-e^2*x^2 + d^2)^(9/2)*e) + 1/9*x/((
-e^2*x^2 + d^2)^(7/2)*d^2) + 2/15*x/((-e^2*x^2 + d^2)^(5/2)*d^4) + 8/45*x/
((-e^2*x^2 + d^2)^(3/2)*d^6) + 16/45*x/(sqrt(-e^2*x^2 + d^2)*d^8)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{(ex+d)^2}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(-e^2*x^2 + d^2)^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{31\,x}{120\,d^4}+\frac{5}{24\,d^3\,e}\right)}{(d+e\,x)^3\,(d-e\,x)^3} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{8\,x}{45\,d^6}-\frac{5}{144\,d^5\,e}\right)}{(d+e\,x)^2\,(d-e\,x)^2} + \frac{\sqrt{d^2-e^2\,x^2}}{72\,d^4\,e\,(d-e\,x)^5} + \frac{5\,\sqrt{d^2-e^2\,x^2}}{144\,d^5\,e\,(d-e\,x)^4} + \frac{16\,x\,\sqrt{d^2-e^2\,x^2}}{45\,d^8\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int((d + e*x)^2/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((31*x)/(120*d^4) + 5/(24*d^3*e)))/((d + e*x)^3*(d
- e*x)^3) + ((d^2 - e^2*x^2)^(1/2)*((8*x)/(45*d^6) - 5/(144*d^5*e)))/((d +
 e*x)^2*(d - e*x)^2) + (d^2 - e^2*x^2)^(1/2)/(72*d^4*e*(d - e*x)^5) + (5*(
d^2 - e^2*x^2)^(1/2))/(144*d^5*e*(d - e*x)^4) + (16*x*(d^2 - e^2*x^2)^(1/2
))/(45*d^8*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.50

$$\int \frac{(d+ex)^2}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{-25\sqrt{-e^2x^2+d^2}\,d^6+50\sqrt{-e^2x^2+d^2}\,d^5ex+25\sqrt{-e^2x^2+d^2}\,d^4e^2x^2-100\sqrt{-e^2x^2}}{}$$

input
```
int((e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
( - 25*sqrt(d**2 - e**2*x**2)*d**6 + 50*sqrt(d**2 - e**2*x**2)*d**5*e*x +
25*sqrt(d**2 - e**2*x**2)*d**4*e**2*x**2 - 100*sqrt(d**2 - e**2*x**2)*d**3
*e**3*x**3 + 25*sqrt(d**2 - e**2*x**2)*d**2*e**4*x**4 + 50*sqrt(d**2 - e**
2*x**2)*d*e**5*x**5 - 25*sqrt(d**2 - e**2*x**2)*e**6*x**6 + 20*d**7 + 50*d
**6*e*x - 120*d**5*e**2*x**2 - 20*d**4*e**3*x**3 + 160*d**3*e**4*x**4 - 48
*d**2*e**5*x**5 - 64*d*e**6*x**6 + 32*e**7*x**7)/(90*sqrt(d**2 - e**2*x**2
)*d**8*e*(d**6 - 2*d**5*e*x - d**4*e**2*x**2 + 4*d**3*e**3*x**3 - d**2*e**
4*x**4 - 2*d*e**5*x**5 + e**6*x**6))
```

3.158 $\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{11/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{11/2}}\,dx = \frac{d+ex}{9de\left(d^2-e^2x^2\right)^{9/2}} + \frac{8x}{63d^3\left(d^2-e^2x^2\right)^{7/2}} + \frac{16x}{105d^5\left(d^2-e^2x^2\right)^{5/2}} + \frac{64x}{315d^7\left(d^2-e^2x^2\right)^{3/2}} + \frac{128x}{315d^9\sqrt{d^2-e^2x^2}}$$

output
```
1/9*(e*x+d)/d/e/(-e^2*x^2+d^2)^(9/2)+8/63*x/d^3/(-e^2*x^2+d^2)^(7/2)+16/10
5*x/d^5/(-e^2*x^2+d^2)^(5/2)+64/315*x/d^7/(-e^2*x^2+d^2)^(3/2)+128/315*x/d
^9/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(35d^8+280d^7ex-280d^6e^2x^2-560d^5e^3x^3+560d^4e^4x^4+448d^3e^5x^5-}{315d^9e(d-ex)^5(d+ex)^4}$$

input
```
Integrate[(d + e*x)/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(35*d^8 + 280*d^7*e*x - 280*d^6*e^2*x^2 - 560*d^5*e^3
*x^3 + 560*d^4*e^4*x^4 + 448*d^3*e^5*x^5 - 448*d^2*e^6*x^6 - 128*d*e^7*x^7
 + 128*e^8*x^8))/(315*d^9*e*(d - e*x)^5*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {454, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(d^2-e^2x^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{ 454}$$

$$\frac{8\int \frac{1}{(d^2-e^2x^2)^{9/2}}dx}{9d}+\frac{d+ex}{9de\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{8\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}+\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}\right)}{9d}+\frac{d+ex}{9de\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{8\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}\right)}{9d}+\frac{d+ex}{9de\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}\right)}{9d}+\frac{d+ex}{9de\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{d+ex}{9de\left(d^2-e^2x^2\right)^{9/2}}+\frac{8\left(\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}+\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d^2}\right)}{9d}$$

input
```
Int[(d + e*x)/(d^2 - e^2*x^2)^(11/2),x]
```

output
```
(d + e*x)/(9*d*e*(d^2 - e^2*x^2)^(9/2)) + (8*(x/(7*d^2*(d^2 - e^2*x^2)^(7/
2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(
3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d^2)))/(9*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result
gosper	$\frac{(ex+d)^2(-ex+d)\left(128e^8x^8-128d\,e^7x^7-448d^2e^6x^6+448d^3e^5x^5+560d^4e^4x^4-560d^5e^3x^3-280d^6e^2x^2+280d^7ex+35d^8\right)}{315d^9e\left(-e^2x^2+d^2\right)^{\frac{11}{2}}}$
orering	$\frac{(ex+d)^2(-ex+d)\left(128e^8x^8-128d\,e^7x^7-448d^2e^6x^6+448d^3e^5x^5+560d^4e^4x^4-560d^5e^3x^3-280d^6e^2x^2+280d^7ex+35d^8\right)}{315d^9e\left(-e^2x^2+d^2\right)^{\frac{11}{2}}}$
trager	$\frac{\left(128e^8x^8-128d\,e^7x^7-448d^2e^6x^6+448d^3e^5x^5+560d^4e^4x^4-560d^5e^3x^3-280d^6e^2x^2+280d^7ex+35d^8\right)\sqrt{-e^2x^2+d^2}}{315d^9(-ex+d)^5(ex+d)^4e}$
default	$d\left(\frac{x}{9d^2(-e^2x^2+d^2)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}\right)}{7d^2}\right)}{9d^2}}{d^2}\right)+\frac{}{9e(-e^2}$

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(e*x+d)^2*(-e*x+d)*(128*e^8*x^8-128*d*e^7*x^7-448*d^2*e^6*x^6+448*d^
3*e^5*x^5+560*d^4*e^4*x^4-560*d^5*e^3*x^3-280*d^6*e^2*x^2+280*d^7*e*x+35*d
^8)/d^9/e/(-e^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(108) = 216$.

Time = 0.37 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.37

$$\int \frac{d+ex}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{35\,e^9x^9 - 35\,de^8x^8 - 140\,d^2e^7x^7 + 140\,d^3e^6x^6 + 210\,d^4e^5x^5 - 210\,d^5e^4x^4 - 140\,d^6e}{315\,(d^9e^{10}x^9 - d}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
1/315*(35*e^9*x^9 - 35*d*e^8*x^8 - 140*d^2*e^7*x^7 + 140*d^3*e^6*x^6 + 210
*d^4*e^5*x^5 - 210*d^5*e^4*x^4 - 140*d^6*e^3*x^3 + 140*d^7*e^2*x^2 + 35*d^
8*e*x - 35*d^9 - (128*e^8*x^8 - 128*d*e^7*x^7 - 448*d^2*e^6*x^6 + 448*d^3*
e^5*x^5 + 560*d^4*e^4*x^4 - 560*d^5*e^3*x^3 - 280*d^6*e^2*x^2 + 280*d^7*e*
x + 35*d^8)*sqrt(-e^2*x^2 + d^2))/(d^9*e^10*x^9 - d^10*e^9*x^8 - 4*d^11*e^
8*x^7 + 4*d^12*e^7*x^6 + 6*d^13*e^6*x^5 - 6*d^14*e^5*x^4 - 4*d^15*e^4*x^3
+ 4*d^16*e^3*x^2 + d^17*e^2*x - d^18*e)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.02 (sec) , antiderivative size = 1549, normalized size of antiderivative = 12.10

$$\int \frac{d+ex}{(d^2-e^2x^2)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
d*Piecewise((-315*I*d**8*x/(315*d**19*sqrt(-1 + e**2*x**2/d**2) - 1260*d**
17*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**15*e**4*x**4*sqrt(-1 + e*
*2*x**2/d**2) - 1260*d**13*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**11
*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 840*I*d**6*e**2*x**3/(315*d**19*sq
rt(-1 + e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) +
 1890*d**15*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**13*e**6*x**6*sqr
t(-1 + e**2*x**2/d**2) + 315*d**11*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) -
1008*I*d**4*e**4*x**5/(315*d**19*sqrt(-1 + e**2*x**2/d**2) - 1260*d**17*e*
*2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**15*e**4*x**4*sqrt(-1 + e**2*x*
*2/d**2) - 1260*d**13*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**8
*x**8*sqrt(-1 + e**2*x**2/d**2)) + 576*I*d**2*e**6*x**7/(315*d**19*sqrt(-1
 + e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890
*d**15*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**13*e**6*x**6*sqrt(-1
+ e**2*x**2/d**2) + 315*d**11*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 128*I
*e**8*x**9/(315*d**19*sqrt(-1 + e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqr
t(-1 + e**2*x**2/d**2) + 1890*d**15*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) -
1260*d**13*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**8*x**8*sqrt(
-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (315*d**8*x/(315*d**19*sq
rt(1 - e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 1
890*d**15*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**13*e**6*x**6*sqr...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int \frac{d+ex}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{x}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}d}+\frac{1}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}e}+\frac{8\,x}{63\,(-e^2x^2+d^2)^{\frac{7}{2}}d^3}+\frac{16\,x}{105\,(-e^2x^2+d^2)^{\frac{5}{2}}d^5}+\frac{64\,x}{315\,(-e^2x^2+d^2)^{\frac{3}{2}}d^7}+\frac{128\,x}{315\,\sqrt{-e^2x^2+d^2}\,d^9}$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/9*x/((-e^2*x^2 + d^2)^(9/2)*d) + 1/9/((-e^2*x^2 + d^2)^(9/2)*e) + 8/63*x
/((-e^2*x^2 + d^2)^(7/2)*d^3) + 16/105*x/((-e^2*x^2 + d^2)^(5/2)*d^5) + 64
/315*x/((-e^2*x^2 + d^2)^(3/2)*d^7) + 128/315*x/(sqrt(-e^2*x^2 + d^2)*d^9)
```

Giac [F]

$$\int \frac{d+ex}{(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{ex+d}{(-e^2x^2+d^2)^{\frac{11}{2}}}\,dx$$

input
```
integrate((e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(-e^2*x^2 + d^2)^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.78 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.49

$$\int \frac{d+ex}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{53\,x}{252\,d^3}+\frac{5}{36\,d^2\,e}\right)}{{\left(d+e\,x\right)}^4\,{\left(d-e\,x\right)}^4}+\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{733\,x}{5040\,d^5}-\frac{5}{144\,d^4\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} + \frac{\sqrt{d^2-e^2\,x^2}}{144\,d^5\,e\,{\left(d-e\,x\right)}^5}+\frac{64\,x\,\sqrt{d^2-e^2\,x^2}}{315\,d^7\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2}+\frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{315\,d^9\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int((d + e*x)/(d^2 - e^2*x^2)^(11/2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((53*x)/(252*d^3) + 5/(36*d^2*e)))/((d + e*x)^4*(d
- e*x)^4) + ((d^2 - e^2*x^2)^(1/2)*((733*x)/(5040*d^5) - 5/(144*d^4*e)))/(
(d + e*x)^3*(d - e*x)^3) + (d^2 - e^2*x^2)^(1/2)/(144*d^5*e*(d - e*x)^5) +
 (64*x*(d^2 - e^2*x^2)^(1/2))/(315*d^7*(d + e*x)^2*(d - e*x)^2) + (128*x*(
d^2 - e^2*x^2)^(1/2))/(315*d^9*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.80

$$\int \frac{d+ex}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{-280\sqrt{-e^2x^2+d^2}\,d^7+280\sqrt{-e^2x^2+d^2}\,d^6ex+840\sqrt{-e^2x^2+d^2}\,d^5e^2x^2-840\sqrt{-}}{}$$

input
```
int((e*x+d)/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
( - 280*sqrt(d**2 - e**2*x**2)*d**7 + 280*sqrt(d**2 - e**2*x**2)*d**6*e*x
+ 840*sqrt(d**2 - e**2*x**2)*d**5*e**2*x**2 - 840*sqrt(d**2 - e**2*x**2)*d
**4*e**3*x**3 - 840*sqrt(d**2 - e**2*x**2)*d**3*e**4*x**4 + 840*sqrt(d**2
- e**2*x**2)*d**2*e**5*x**5 + 280*sqrt(d**2 - e**2*x**2)*d*e**6*x**6 - 280
*sqrt(d**2 - e**2*x**2)*e**7*x**7 + 35*d**8 + 280*d**7*e*x - 280*d**6*e**2
*x**2 - 560*d**5*e**3*x**3 + 560*d**4*e**4*x**4 + 448*d**3*e**5*x**5 - 448
*d**2*e**6*x**6 - 128*d*e**7*x**7 + 128*e**8*x**8)/(315*sqrt(d**2 - e**2*x
**2)*d**9*e*(d**7 - d**6*e*x - 3*d**5*e**2*x**2 + 3*d**4*e**3*x**3 + 3*d**
3*e**4*x**4 - 3*d**2*e**5*x**5 - d*e**6*x**6 + e**7*x**7))
```

3.159 $\int \frac{1}{(d^2-e^2x^2)^{11/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{1}{(d^2-e^2x^2)^{11/2}}\,dx=\frac{x}{9d^2(d^2-e^2x^2)^{9/2}}+\frac{8x}{63d^4(d^2-e^2x^2)^{7/2}}+\frac{16x}{105d^6(d^2-e^2x^2)^{5/2}}+\frac{64x}{315d^8(d^2-e^2x^2)^{3/2}}+\frac{128x}{315d^{10}\sqrt{d^2-e^2x^2}}$$

output
```
1/9*x/d^2/(-e^2*x^2+d^2)^(9/2)+8/63*x/d^4/(-e^2*x^2+d^2)^(7/2)+16/105*x/d^
6/(-e^2*x^2+d^2)^(5/2)+64/315*x/d^8/(-e^2*x^2+d^2)^(3/2)+128/315*x/d^10/(-
e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d^2-e^2x^2)^{11/2}}\,dx=\frac{x\sqrt{d^2-e^2x^2}(315d^8-840d^6e^2x^2+1008d^4e^4x^4-576d^2e^6x^6+128e^8x^8)}{315d^{10}(d-ex)^5(d+ex)^5}$$

input
```
Integrate[(d^2 - e^2*x^2)^(-11/2),x]
```

output
```
(x*Sqrt[d^2 - e^2*x^2]*(315*d^8 - 840*d^6*e^2*x^2 + 1008*d^4*e^4*x^4 - 576
*d^2*e^6*x^6 + 128*e^8*x^8))/(315*d^10*(d - e*x)^5*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {209, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d^2-e^2x^2)^{11/2}}dx$$

$$\Big\downarrow \text{ 209}$$

$$\frac{8\int \frac{1}{(d^2-e^2x^2)^{9/2}}dx}{9d^2}+\frac{x}{9d^2(d^2-e^2x^2)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{8\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}+\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2(d^2-e^2x^2)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{8\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2(d^2-e^2x^2)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}+\frac{8\left(\frac{x}{7d^2(d^2-e^2x^2)^{7/2}}+\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d^2}\right)}{9d^2}$$

input
```
Int[(d^2 - e^2*x^2)^(-11/2),x]
```

output
```
x/(9*d^2*(d^2 - e^2*x^2)^(9/2)) + (8*(x/(7*d^2*(d^2 - e^2*x^2)^(7/2)) + (6
*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) +
(2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d^2)))/(9*d^2)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

method	result	size
trager	$\frac{(128e^8x^8-576d^2e^6x^6+1008d^4e^4x^4-840d^6e^2x^2+315d^8)x}{315d^{10}(-e^2x^2+d^2)^{\frac{9}{2}}}$	68
pseudoelliptic	$\frac{(128e^8x^8-576d^2e^6x^6+1008d^4e^4x^4-840d^6e^2x^2+315d^8)x}{315d^{10}(-e^2x^2+d^2)^{\frac{9}{2}}}$	68
gosper	$\frac{(ex+d)(-ex+d)x(128e^8x^8-576d^2e^6x^6+1008d^4e^4x^4-840d^6e^2x^2+315d^8)}{315d^{10}(-e^2x^2+d^2)^{\frac{11}{2}}}$	79
orering	$\frac{(ex+d)(-ex+d)x(128e^8x^8-576d^2e^6x^6+1008d^4e^4x^4-840d^6e^2x^2+315d^8)}{315d^{10}(-e^2x^2+d^2)^{\frac{11}{2}}}$	79
default	$\frac{x}{9d^2(-e^2x^2+d^2)^{\frac{9}{2}}}+\frac{\frac{8x}{63d^2\left(-e^2x^2+d^2\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d^2\left(-e^2x^2+d^2\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d^2\left(-e^2x^2+d^2\right)^{\frac{3}{2}}}+\frac{8x}{15d^4\sqrt{-e^2x^2+d^2}}\right)}{7d^2}\right)}{9d^2}}{d^2}$	120

input
```
int(1/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(128*e^8*x^8-576*d^2*e^6*x^6+1008*d^4*e^4*x^4-840*d^6*e^2*x^2+315*d^
8)*x/d^10/(-e^2*x^2+d^2)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(d^2-e^2x^2\right)^{11/2}}\,dx = -\frac{\left(128\,e^8x^9-576\,d^2e^6x^7+1008\,d^4e^4x^5-840\,d^6e^2x^3+315\,d^8x\right)\sqrt{-e^2x^2+d^2}}{315\left(d^{10}e^{10}x^{10}-5\,d^{12}e^8x^8+10\,d^{14}e^6x^6-10\,d^{16}e^4x^4+5\,d^{18}e^2x^2-d^{20}\right)}$$

input
```
integrate(1/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
-1/315*(128*e^8*x^9 - 576*d^2*e^6*x^7 + 1008*d^4*e^4*x^5 - 840*d^6*e^2*x^3
 + 315*d^8*x)*sqrt(-e^2*x^2 + d^2)/(d^10*e^10*x^10 - 5*d^12*e^8*x^8 + 10*d
^14*e^6*x^6 - 10*d^16*e^4*x^4 + 5*d^18*e^2*x^2 - d^20)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 1411, normalized size of antiderivative = 11.66

$$\int \frac{1}{\left(d^2-e^2x^2\right)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Piecewise((-315*I*d**8*x/(315*d**19*sqrt(-1 + e**2*x**2/d**2) - 1260*d**17
*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**15*e**4*x**4*sqrt(-1 + e**2
*x**2/d**2) - 1260*d**13*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e
**8*x**8*sqrt(-1 + e**2*x**2/d**2)) + 840*I*d**6*e**2*x**3/(315*d**19*sqrt
(-1 + e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1
890*d**15*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**13*e**6*x**6*sqrt(
-1 + e**2*x**2/d**2) + 315*d**11*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 10
08*I*d**4*e**4*x**5/(315*d**19*sqrt(-1 + e**2*x**2/d**2) - 1260*d**17*e**2
*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d**15*e**4*x**4*sqrt(-1 + e**2*x**2
/d**2) - 1260*d**13*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**8*x
**8*sqrt(-1 + e**2*x**2/d**2)) + 576*I*d**2*e**6*x**7/(315*d**19*sqrt(-1 +
 e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 1890*d
**15*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 1260*d**13*e**6*x**6*sqrt(-1 +
e**2*x**2/d**2) + 315*d**11*e**8*x**8*sqrt(-1 + e**2*x**2/d**2)) - 128*I*e
**8*x**9/(315*d**19*sqrt(-1 + e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqrt(
-1 + e**2*x**2/d**2) + 1890*d**15*e**4*x**4*sqrt(-1 + e**2*x**2/d**2) - 12
60*d**13*e**6*x**6*sqrt(-1 + e**2*x**2/d**2) + 315*d**11*e**8*x**8*sqrt(-1
 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (315*d**8*x/(315*d**19*sqrt
(1 - e**2*x**2/d**2) - 1260*d**17*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 189
0*d**15*e**4*x**4*sqrt(1 - e**2*x**2/d**2) - 1260*d**13*e**6*x**6*sqrt(...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{x}{9\,(-e^2x^2+d^2)^{\frac{9}{2}}d^2} + \frac{8\,x}{63\,(-e^2x^2+d^2)^{\frac{7}{2}}d^4} + \frac{16\,x}{105\,(-e^2x^2+d^2)^{\frac{5}{2}}d^6} + \frac{64\,x}{315\,(-e^2x^2+d^2)^{\frac{3}{2}}d^8} + \frac{128\,x}{315\,\sqrt{-e^2x^2+d^2}d^{10}}$$

input
```
integrate(1/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
1/9*x/((-e^2*x^2 + d^2)^(9/2)*d^2) + 8/63*x/((-e^2*x^2 + d^2)^(7/2)*d^4) +
 16/105*x/((-e^2*x^2 + d^2)^(5/2)*d^6) + 64/315*x/((-e^2*x^2 + d^2)^(3/2)*
d^8) + 128/315*x/(sqrt(-e^2*x^2 + d^2)*d^10)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{1}{(d^2-e^2x^2)^{11/2}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}\left(8\left(2\left(4\,x^2\left(\frac{2\,e^8x^2}{d^{10}}-\frac{9\,e^6}{d^8}\right)+\frac{63\,e^4}{d^6}\right)x^2-\frac{105\,e^2}{d^4}\right)x^2+\frac{315}{d^2}\right)x}{315\,(e^2x^2-d^2)^5}$$

input

```
integrate(1/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output

```
-1/315*sqrt(-e^2*x^2 + d^2)*(8*(2*(4*x^2*(2*e^8*x^2/d^10 - 9*e^6/d^8) + 63
*e^4/d^6)*x^2 - 105*e^2/d^4)*x^2 + 315/d^2)*x/(e^2*x^2 - d^2)^5
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{x}{9\,d^2\,(d^2-e^2\,x^2)^{9/2}}+\frac{8\,x}{63\,d^4\,(d^2-e^2\,x^2)^{7/2}}+\frac{16\,x}{105\,d^6\,(d^2-e^2\,x^2)^{5/2}}+\frac{64\,x}{315\,d^8\,(d^2-e^2\,x^2)^{3/2}}+\frac{128\,x}{315\,d^{10}\,\sqrt{d^2-e^2\,x^2}}$$

input

```
int(1/(d^2 - e^2*x^2)^(11/2),x)
```

output

```
x/(9*d^2*(d^2 - e^2*x^2)^(9/2)) + (8*x)/(63*d^4*(d^2 - e^2*x^2)^(7/2)) + (
16*x)/(105*d^6*(d^2 - e^2*x^2)^(5/2)) + (64*x)/(315*d^8*(d^2 - e^2*x^2)^(3
/2)) + (128*x)/(315*d^10*(d^2 - e^2*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d^2-e^2x^2)^{11/2}}\,dx = \frac{x(128e^8x^8-576d^2e^6x^6+1008d^4e^4x^4-840d^6e^2x^2+315d^8)}{315\sqrt{-e^2x^2+d^2}\,d^{10}\,(e^8x^8-4d^2e^6x^6+6d^4e^4x^4-4d^6e^2x^2+d^8)}$$

input
```
int(1/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
(x*(315*d**8 - 840*d**6*e**2*x**2 + 1008*d**4*e**4*x**4 - 576*d**2*e**6*x*
*6 + 128*e**8*x**8))/(315*sqrt(d**2 - e**2*x**2)*d**10*(d**8 - 4*d**6*e**2
*x**2 + 6*d**4*e**4*x**4 - 4*d**2*e**6*x**6 + e**8*x**8))
```

3.160 $\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}\,dx = \frac{10x}{99d^3(d^2-e^2x^2)^{9/2}} - \frac{1}{11de(d+ex)(d^2-e^2x^2)^{9/2}} + \frac{80x}{693d^5(d^2-e^2x^2)^{7/2}} + \frac{32x}{231d^7(d^2-e^2x^2)^{5/2}} + \frac{128x}{693d^9(d^2-e^2x^2)^{3/2}} + \frac{256x}{693d^{11}\sqrt{d^2-e^2x^2}}$$

output
```
10/99*x/d^3/(-e^2*x^2+d^2)^(9/2)-1/11/d/e/(e*x+d)/(-e^2*x^2+d^2)^(9/2)+80/
693*x/d^5/(-e^2*x^2+d^2)^(7/2)+32/231*x/d^7/(-e^2*x^2+d^2)^(5/2)+128/693*x
/d^9/(-e^2*x^2+d^2)^(3/2)+256/693*x/d^11/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-63d^{10}+630d^9ex+630d^8e^2x^2-1680d^7e^3x^3-1680d^6e^4x^4 \cdots}{693d^{11}e(\cdots}$$

input
```
Integrate[1/((d + e*x)*(d^2 - e^2*x^2)^(11/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-63*d^10 + 630*d^9*e*x + 630*d^8*e^2*x^2 - 1680*d^7*
e^3*x^3 - 1680*d^6*e^4*x^4 + 2016*d^5*e^5*x^5 + 2016*d^4*e^6*x^6 - 1152*d^
3*e^7*x^7 - 1152*d^2*e^8*x^8 + 256*d*e^9*x^9 + 256*e^10*x^10))/(693*d^11*e
*(d - e*x)^5*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {470, 209, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{11/2}}dx$$

$$\Big\downarrow \text{ 470}$$

$$\frac{10\int \frac{1}{\left(d^2-e^2x^2\right)^{11/2}}dx}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{10\left(\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{10\left(\frac{8\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 209}$$

$$\frac{10\left(\frac{8\left(\frac{6\left(\frac{4\int\frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2(d^2-e^2x^2)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{10\left(\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2(d^2-e^2x^2)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{10\left(\frac{x}{9d^2(d^2-e^2x^2)^{9/2}}+\frac{8\left(\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}+\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d^2}\right)}{9d^2}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}$$

input
```
Int[1/((d + e*x)*(d^2 - e^2*x^2)^(11/2)),x]
```

output
```
-1/11*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^(9/2)) + (10*(x/(9*d^2*(d^2 - e^2*x
^2)^(9/2)) + (8*(x/(7*d^2*(d^2 - e^2*x^2)^(7/2)) + (6*(x/(5*d^2*(d^2 - e^2
*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2
 - e^2*x^2])))/(5*d^2)))/(7*d^2)))/(9*d^2)))/(11*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

method	result
gosper	$-\frac{(-ex+d)\left(-256e^{10}x^{10}-256d\,e^{9}x^{9}+1152d^{2}e^{8}x^{8}+1152d^{3}e^{7}x^{7}-2016d^{4}e^{6}x^{6}-2016d^{5}e^{5}x^{5}+1680d^{6}e^{4}x^{4}+1680d^{7}e^{3}x^{3}-630d^{8}e^{2}x^{2}\right.}{693d^{11}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
orering	$-\frac{(-ex+d)\left(-256e^{10}x^{10}-256d\,e^{9}x^{9}+1152d^{2}e^{8}x^{8}+1152d^{3}e^{7}x^{7}-2016d^{4}e^{6}x^{6}-2016d^{5}e^{5}x^{5}+1680d^{6}e^{4}x^{4}+1680d^{7}e^{3}x^{3}-630d^{8}e^{2}x^{2}\right.}{693d^{11}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
trager	$-\frac{\left(-256e^{10}x^{10}-256d\,e^{9}x^{9}+1152d^{2}e^{8}x^{8}+1152d^{3}e^{7}x^{7}-2016d^{4}e^{6}x^{6}-2016d^{5}e^{5}x^{5}+1680d^{6}e^{4}x^{4}+1680d^{7}e^{3}x^{3}-630d^{8}e^{2}x^{2}-630d^{9}\right.}{693d^{11}(ex+d)^{6}(-ex+d)^{5}e}$
default	$\frac{-\frac{1}{11de\left(x+\frac{d}{e}\right)\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}+\frac{10e\left(-\frac{-2e^{2}\left(x+\frac{d}{e}\right)+2de}{18d^{2}e^{2}\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}+\frac{-\frac{4\left(-2e^{2}\left(x+\frac{d}{e}\right)+2de\right)}{63d^{2}e^{2}\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}}+\frac{8\left(-\right.}{3}}{}\right.}{}}{e}$

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/693*(-e*x+d)*(-256*e^10*x^10-256*d*e^9*x^9+1152*d^2*e^8*x^8+1152*d^3*e^
7*x^7-2016*d^4*e^6*x^6-2016*d^5*e^5*x^5+1680*d^6*e^4*x^4+1680*d^7*e^3*x^3-
630*d^8*e^2*x^2-630*d^9*e*x+63*d^10)/d^11/e/(-e^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(130) = 260$.

Time = 0.74 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.39

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}\,dx = -\frac{63\,e^{11}x^{11}+63\,de^{10}x^{10}-315\,d^2e^9x^9-315\,d^3e^8x^8+630\,d^4e^7x^7+630\,d^5e^6x^6-630\,d^6e^5x^5-630\,d^7e^4x^4+}{693\,(d^{11}e^{12}x^{11}+d^{12}e}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
-1/693*(63*e^11*x^11 + 63*d*e^10*x^10 - 315*d^2*e^9*x^9 - 315*d^3*e^8*x^8
+ 630*d^4*e^7*x^7 + 630*d^5*e^6*x^6 - 630*d^6*e^5*x^5 - 630*d^7*e^4*x^4 +
315*d^8*e^3*x^3 + 315*d^9*e^2*x^2 - 63*d^10*e*x - 63*d^11 + (256*e^10*x^10
 + 256*d*e^9*x^9 - 1152*d^2*e^8*x^8 - 1152*d^3*e^7*x^7 + 2016*d^4*e^6*x^6
+ 2016*d^5*e^5*x^5 - 1680*d^6*e^4*x^4 - 1680*d^7*e^3*x^3 + 630*d^8*e^2*x^2
 + 630*d^9*e*x - 63*d^10)*sqrt(-e^2*x^2 + d^2))/(d^11*e^12*x^11 + d^12*e^1
1*x^10 - 5*d^13*e^10*x^9 - 5*d^14*e^9*x^8 + 10*d^15*e^8*x^7 + 10*d^16*e^7*
x^6 - 10*d^17*e^6*x^5 - 10*d^18*e^5*x^4 + 5*d^19*e^4*x^3 + 5*d^20*e^3*x^2
- d^21*e^2*x - d^22*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{11}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(11/2)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}\,dx = -\frac{1}{11\left((-e^2x^2+d^2)^{\frac{9}{2}}de^2x+(-e^2x^2+d^2)^{\frac{9}{2}}d^2e\right)} + \frac{10\,x}{99\,(-e^2x^2+d^2)^{\frac{9}{2}}d^3} + \frac{80\,x}{693\,(-e^2x^2+d^2)^{\frac{7}{2}}d^5} + \frac{32\,x}{231\,(-e^2x^2+d^2)^{\frac{5}{2}}d^7} + \frac{128\,x}{693\,(-e^2x^2+d^2)^{\frac{3}{2}}d^9} + \frac{256\,x}{693\,\sqrt{-e^2x^2+d^2}d^{11}}$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
-1/11/((-e^2*x^2 + d^2)^(9/2)*d*e^2*x + (-e^2*x^2 + d^2)^(9/2)*d^2*e) + 10
/99*x/((-e^2*x^2 + d^2)^(9/2)*d^3) + 80/693*x/((-e^2*x^2 + d^2)^(7/2)*d^5)
 + 32/231*x/((-e^2*x^2 + d^2)^(5/2)*d^7) + 128/693*x/((-e^2*x^2 + d^2)^(3/
2)*d^9) + 256/693*x/(sqrt(-e^2*x^2 + d^2)*d^11)
```

Giac [F]

$$\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{11}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(11/2)*(e*x + d)), x)
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{19\,x}{99\,d^3}-\frac{3}{22\,d^2\,e}\right)}{\left(d+e\,x\right)^5\,\left(d-e\,x\right)^5} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{32\,x}{231\,d^7}-\frac{1}{352\,d^6\,e}\right)}{\left(d+e\,x\right)^3\,\left(d-e\,x\right)^3} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{1091\,x}{11088\,d^5}+\frac{9}{176\,d^4\,e}\right)}{\left(d+e\,x\right)^4\,\left(d-e\,x\right)^4} - \frac{\sqrt{d^2-e^2\,x^2}}{352\,d^6\,e\,\left(d+e\,x\right)^6} + \frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{693\,d^9\,\left(d+e\,x\right)^2\,\left(d-e\,x\right)^2} + \frac{256\,x\,\sqrt{d^2-e^2\,x^2}}{693\,d^{11}\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(11/2)*(d + e*x)),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((19*x)/(99*d^3) - 3/(22*d^2*e)))/((d + e*x)^5*(d -
 e*x)^5) + ((d^2 - e^2*x^2)^(1/2)*((32*x)/(231*d^7) - 1/(352*d^6*e)))/((d
+ e*x)^3*(d - e*x)^3) + ((d^2 - e^2*x^2)^(1/2)*((1091*x)/(11088*d^5) + 9/(
176*d^4*e)))/((d + e*x)^4*(d - e*x)^4) - (d^2 - e^2*x^2)^(1/2)/(352*d^6*e*
(d + e*x)^6) + (128*x*(d^2 - e^2*x^2)^(1/2))/(693*d^9*(d + e*x)^2*(d - e*x
)^2) + (256*x*(d^2 - e^2*x^2)^(1/2))/(693*d^11*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.91

$$\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{11/2}}\,dx = \frac{630\sqrt{-e^2x^2+d^2}\,d^9+630\sqrt{-e^2x^2+d^2}\,d^8ex-2520\sqrt{-e^2x^2+d^2}\,d^7e^2x^2 \cdots}{}$$

input
```
int(1/(e*x+d)/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
(630*sqrt(d**2 - e**2*x**2)*d**9 + 630*sqrt(d**2 - e**2*x**2)*d**8*e*x - 2
520*sqrt(d**2 - e**2*x**2)*d**7*e**2*x**2 - 2520*sqrt(d**2 - e**2*x**2)*d*
*6*e**3*x**3 + 3780*sqrt(d**2 - e**2*x**2)*d**5*e**4*x**4 + 3780*sqrt(d**2
 - e**2*x**2)*d**4*e**5*x**5 - 2520*sqrt(d**2 - e**2*x**2)*d**3*e**6*x**6
- 2520*sqrt(d**2 - e**2*x**2)*d**2*e**7*x**7 + 630*sqrt(d**2 - e**2*x**2)*
d*e**8*x**8 + 630*sqrt(d**2 - e**2*x**2)*e**9*x**9 - 63*d**10 + 630*d**9*e
*x + 630*d**8*e**2*x**2 - 1680*d**7*e**3*x**3 - 1680*d**6*e**4*x**4 + 2016
*d**5*e**5*x**5 + 2016*d**4*e**6*x**6 - 1152*d**3*e**7*x**7 - 1152*d**2*e*
*8*x**8 + 256*d*e**9*x**9 + 256*e**10*x**10)/(693*sqrt(d**2 - e**2*x**2)*d
**11*e*(d**9 + d**8*e*x - 4*d**7*e**2*x**2 - 4*d**6*e**3*x**3 + 6*d**5*e**
4*x**4 + 6*d**4*e**5*x**5 - 4*d**3*e**6*x**6 - 4*d**2*e**7*x**7 + d*e**8*x
**8 + e**9*x**9))
```

3.161 $\int \frac{1}{(d+ex)^2 \left(d^2-e^2x^2\right)^{11/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}} \, dx = \frac{x}{13d^2 (d^2-e^2x^2)^{11/2}} - \frac{2}{13e(d+ex) (d^2-e^2x^2)^{11/2}} + \frac{10x}{117d^4 (d^2-e^2x^2)^{9/2}} + \frac{80x}{819d^6 (d^2-e^2x^2)^{7/2}} + \frac{32x}{273d^8 (d^2-e^2x^2)^{5/2}} + \frac{128x}{819d^{10} (d^2-e^2x^2)^{3/2}} + \frac{256x}{819d^{12}\sqrt{d^2-e^2x^2}}$$

output

```
1/13*x/d^2/(-e^2*x^2+d^2)^(11/2)-2/13/e/(e*x+d)/(-e^2*x^2+d^2)^(11/2)+10/1
17*x/d^4/(-e^2*x^2+d^2)^(9/2)+80/819*x/d^6/(-e^2*x^2+d^2)^(7/2)+32/273*x/d
^8/(-e^2*x^2+d^2)^(5/2)+128/819*x/d^10/(-e^2*x^2+d^2)^(3/2)+256/819*x/d^12
/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-126d^{11}+567d^{10}ex+1260d^9e^2x^2-1050d^8e^3x^3-3360d^7e^{\cdots}}{8\cdots}$$

input
```
Integrate[1/((d + e*x)^2*(d^2 - e^2*x^2)^(11/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-126*d^11 + 567*d^10*e*x + 1260*d^9*e^2*x^2 - 1050*d
^8*e^3*x^3 - 3360*d^7*e^4*x^4 + 336*d^6*e^5*x^5 + 4032*d^5*e^6*x^6 + 864*d
^4*e^7*x^7 - 2304*d^3*e^8*x^8 - 896*d^2*e^9*x^9 + 512*d*e^10*x^10 + 256*e^
11*x^11))/(819*d^12*e*(d - e*x)^5*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {461, 470, 209, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{11\int \frac{1}{(d+ex)(d^2-e^2x^2)^{11/2}}dx}{13d}-\frac{1}{13de(d+ex)^2 (d^2-e^2x^2)^{9/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{11\left(\frac{10\int \frac{1}{(d^2-e^2x^2)^{11/2}}dx}{11d}-\frac{1}{11de(d+ex)(d^2-e^2x^2)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2 (d^2-e^2x^2)^{9/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{11\left(\frac{10\left(\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)(d^2-e^2x^2)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{11\left(\frac{10\left(\frac{8\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)(d^2-e^2x^2)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{11\left(\frac{10\left(\frac{8\left(\frac{6\left(\frac{4\int \frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)(d^2-e^2x^2)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}$$

$$\Big\downarrow \ 209$$

$$\frac{11\left(\frac{10\left(\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)(d^2}\right.}{13d}$$

$$\frac{1}{13de(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}$$

$\downarrow$ 208

$$\frac{11\left(\frac{10\left(\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}+\frac{8\left(\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}+\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d^2}\right)}{9d^2}\right)}{11d}-\frac{1}{11de(d+ex)(d^2-e^2x^2)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2\left(d^2-e^2x^2\right)^{9/2}}$$

input `Int[1/((d + e*x)^2*(d^2 - e^2*x^2)^(11/2)),x]`

output
```
-1/13*1/(d*e*(d + e*x)^2*(d^2 - e^2*x^2)^(9/2)) + (11*(-1/11*1/(d*e*(d + e
*x)*(d^2 - e^2*x^2)^(9/2)) + (10*(x/(9*d^2*(d^2 - e^2*x^2)^(9/2)) + (8*(x/
(7*d^2*(d^2 - e^2*x^2)^(7/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(
x/(3*d^2*(d^2 - e^2*x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d
^2)))/(7*d^2)))/(9*d^2)))/(11*d)))/(13*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.88

method	result
gosper	$-\frac{(-ex+d)\left(-256e^{11}x^{11}-512d\,e^{10}x^{10}+896d^{2}e^{9}x^{9}+2304d^{3}e^{8}x^{8}-864d^{4}e^{7}x^{7}-4032d^{5}e^{6}x^{6}-336d^{6}e^{5}x^{5}+3360d^{7}e^{4}x^{4}+1050d^{8}e^{3}x^{3}}{819(ex+d)d^{12}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
orering	$-\frac{(-ex+d)\left(-256e^{11}x^{11}-512d\,e^{10}x^{10}+896d^{2}e^{9}x^{9}+2304d^{3}e^{8}x^{8}-864d^{4}e^{7}x^{7}-4032d^{5}e^{6}x^{6}-336d^{6}e^{5}x^{5}+3360d^{7}e^{4}x^{4}+1050d^{8}e^{3}x^{3}}{819(ex+d)d^{12}e\left(-e^{2}x^{2}+d^{2}\right)^{\frac{11}{2}}}$
trager	$-\frac{\left(-256e^{11}x^{11}-512d\,e^{10}x^{10}+896d^{2}e^{9}x^{9}+2304d^{3}e^{8}x^{8}-864d^{4}e^{7}x^{7}-4032d^{5}e^{6}x^{6}-336d^{6}e^{5}x^{5}+3360d^{7}e^{4}x^{4}+1050d^{8}e^{3}x^{3}-1260d^{9}}{819d^{12}(ex+d)^{7}(-ex+d)^{5}e}$
default	$\frac{-\frac{1}{13de\left(x+\frac{d}{e}\right)^{2}\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}+\frac{11e\left(-\frac{1}{11de\left(x+\frac{d}{e}\right)\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}+\frac{10e\left(-\frac{-2e^{2}\left(x+\frac{d}{e}\right)+2de}{18d^{2}e^{2}\left(-e^{2}\left(x+\frac{d}{e}\right)^{2}+2de\left(x+\frac{d}{e}\right)\right)}\right.}{}\right.}{}}{}$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/819*(-e*x+d)*(-256*e^11*x^11-512*d*e^10*x^10+896*d^2*e^9*x^9+2304*d^3*e
^8*x^8-864*d^4*e^7*x^7-4032*d^5*e^6*x^6-336*d^6*e^5*x^5+3360*d^7*e^4*x^4+1
050*d^8*e^3*x^3-1260*d^9*e^2*x^2-567*d^10*e*x+126*d^11)/(e*x+d)/d^12/e/(-e
^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(147) = 294$.

Time = 1.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.17

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}}\,dx =$$

$$-\frac{126\,e^{12}x^{12} + 252\,de^{11}x^{11} - 504\,d^2e^{10}x^{10} - 1260\,d^3e^9x^9 + 630\,d^4e^8x^8 + 2520\,d^5e^7x^7 - 2520\,d^7e^5x^5 - 630\,d}{819\,(d^{12}e}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
-1/819*(126*e^12*x^12 + 252*d*e^11*x^11 - 504*d^2*e^10*x^10 - 1260*d^3*e^9
*x^9 + 630*d^4*e^8*x^8 + 2520*d^5*e^7*x^7 - 2520*d^7*e^5*x^5 - 630*d^8*e^4
*x^4 + 1260*d^9*e^3*x^3 + 504*d^10*e^2*x^2 - 252*d^11*e*x - 126*d^12 + (25
6*e^11*x^11 + 512*d*e^10*x^10 - 896*d^2*e^9*x^9 - 2304*d^3*e^8*x^8 + 864*d
^4*e^7*x^7 + 4032*d^5*e^6*x^6 + 336*d^6*e^5*x^5 - 3360*d^7*e^4*x^4 - 1050*
d^8*e^3*x^3 + 1260*d^9*e^2*x^2 + 567*d^10*e*x - 126*d^11)*sqrt(-e^2*x^2 +
d^2))/(d^12*e^13*x^12 + 2*d^13*e^12*x^11 - 4*d^14*e^11*x^10 - 10*d^15*e^10
*x^9 + 5*d^16*e^9*x^8 + 20*d^17*e^8*x^7 - 20*d^19*e^6*x^5 - 5*d^20*e^5*x^4
 + 10*d^21*e^4*x^3 + 4*d^22*e^3*x^2 - 2*d^23*e^2*x - d^24*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}}\,dx = \int \frac{1}{(-(-d+ex)(d+ex))^{\frac{11}{2}}(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(11/2)*(d + e*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.23

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}}\,dx = -\frac{1}{13\left((-e^2x^2+d^2)^{\frac{9}{2}}de^3x^2+2\,(-e^2x^2+d^2)^{\frac{9}{2}}d^2e^2x+(-e^2x^2+d^2)^{\frac{9}{2}}d^3e\right)} - \frac{1}{13\left((-e^2x^2+d^2)^{\frac{9}{2}}d^2e^2x+(-e^2x^2+d^2)^{\frac{9}{2}}d^3e\right)} + \frac{10\,x}{117\,(-e^2x^2+d^2)^{\frac{9}{2}}d^4} + \frac{80\,x}{819\,(-e^2x^2+d^2)^{\frac{7}{2}}d^6} + \frac{32\,x}{273\,(-e^2x^2+d^2)^{\frac{5}{2}}d^8} + \frac{128\,x}{819\,(-e^2x^2+d^2)^{\frac{3}{2}}d^{10}} + \frac{256\,x}{819\,\sqrt{-e^2x^2+d^2}\,d^{12}}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
-1/13/((-e^2*x^2 + d^2)^(9/2)*d*e^3*x^2 + 2*(-e^2*x^2 + d^2)^(9/2)*d^2*e^2
*x + (-e^2*x^2 + d^2)^(9/2)*d^3*e) - 1/13/((-e^2*x^2 + d^2)^(9/2)*d^2*e^2*
x + (-e^2*x^2 + d^2)^(9/2)*d^3*e) + 10/117*x/((-e^2*x^2 + d^2)^(9/2)*d^4)
+ 80/819*x/((-e^2*x^2 + d^2)^(7/2)*d^6) + 32/273*x/((-e^2*x^2 + d^2)^(5/2)
*d^8) + 128/819*x/((-e^2*x^2 + d^2)^(3/2)*d^10) + 256/819*x/(sqrt(-e^2*x^2
 + d^2)*d^12)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.27

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}}\,dx = \frac{e^{11}\left(\frac{13\left(20790\left(\frac{2\,d}{ex+d}-1\right)^4+3465\left(\frac{2\,d}{ex+d}-1\right)^3+693\left(\frac{2\,d}{ex+d}-1\right)^2+\frac{198\,d}{ex+d}-92\right)}{d^{12}e^{11}\left(\frac{2\,d}{ex+d}-1\right)^{\frac{9}{2}}\operatorname{sgn}\left(\frac{1}{ex+d}\right)\operatorname{sgn}(e)} - \frac{63\,d^{144}e^{132}\left(\ldots\right.}{\ldots}\right.}{\ldots}$$

input
```
integrate(1/(e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
1/1677312*(e^11*(13*(20790*(2*d/(e*x + d) - 1)^4 + 3465*(2*d/(e*x + d) - 1
)^3 + 693*(2*d/(e*x + d) - 1)^2 + 198*d/(e*x + d) - 92)/(d^12*e^11*(2*d/(e
*x + d) - 1)^(9/2)*sgn(1/(e*x + d))*sgn(e)) - (63*d^144*e^132*(2*d/(e*x +
d) - 1)^(13/2)*sgn(1/(e*x + d))^12*sgn(e)^12 + 819*d^144*e^132*(2*d/(e*x +
 d) - 1)^(11/2)*sgn(1/(e*x + d))^12*sgn(e)^12 + 5005*d^144*e^132*(2*d/(e*x
 + d) - 1)^(9/2)*sgn(1/(e*x + d))^12*sgn(e)^12 + 19305*d^144*e^132*(2*d/(e
*x + d) - 1)^(7/2)*sgn(1/(e*x + d))^12*sgn(e)^12 + 54054*d^144*e^132*(2*d/
(e*x + d) - 1)^(5/2)*sgn(1/(e*x + d))^12*sgn(e)^12 + 126126*d^144*e^132*(2
*d/(e*x + d) - 1)^(3/2)*sgn(1/(e*x + d))^12*sgn(e)^12 + 378378*d^144*e^132
*sqrt(2*d/(e*x + d) - 1)*sgn(1/(e*x + d))^12*sgn(e)^12)/(d^156*e^143*sgn(1
/(e*x + d))^13*sgn(e)^13)) + 524288*I*sgn(1/(e*x + d))*sgn(e)/d^12)/abs(e)
```

Mupad [B] (verification not implemented)

Time = 7.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.51

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}} \, dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{25\,x}{504\,d^6}+\frac{7}{52\,d^5\,e}\right)}{{\left(d+e\,x\right)}^4\,{\left(d-e\,x\right)}^4} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{139\,x}{468\,d^4}-\frac{7}{26\,d^3\,e}\right)}{{\left(d+e\,x\right)}^5\,{\left(d-e\,x\right)}^5} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{32\,x}{273\,d^8}-\frac{7}{832\,d^7\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}}{416\,d^6\,e\,{\left(d+e\,x\right)}^7} - \frac{7\,\sqrt{d^2-e^2\,x^2}}{832\,d^7\,e\,{\left(d+e\,x\right)}^6} + \frac{128\,x\,\sqrt{d^2-e^2\,x^2}}{819\,d^{10}\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{256\,x\,\sqrt{d^2-e^2\,x^2}}{819\,d^{12}\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(11/2)*(d + e*x)^2),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((25*x)/(504*d^6) + 7/(52*d^5*e)))/((d + e*x)^4*(d
- e*x)^4) + ((d^2 - e^2*x^2)^(1/2)*((139*x)/(468*d^4) - 7/(26*d^3*e)))/((d
 + e*x)^5*(d - e*x)^5) + ((d^2 - e^2*x^2)^(1/2)*((32*x)/(273*d^8) - 7/(832
*d^7*e)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(416*d^6*e*(d
+ e*x)^7) - (7*(d^2 - e^2*x^2)^(1/2))/(832*d^7*e*(d + e*x)^6) + (128*x*(d^
2 - e^2*x^2)^(1/2))/(819*d^10*(d + e*x)^2*(d - e*x)^2) + (256*x*(d^2 - e^2
*x^2)^(1/2))/(819*d^12*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.83

$$\int \frac{1}{(d+ex)^2 (d^2-e^2x^2)^{11/2}}\,dx = \frac{567\sqrt{-e^2x^2+d^2}\,d^{10}+1134\sqrt{-e^2x^2+d^2}\,d^9ex-1701\sqrt{-e^2x^2+d^2}\,d^8e^2x}{}$$

input
```
int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
(567*sqrt(d**2 - e**2*x**2)*d**10 + 1134*sqrt(d**2 - e**2*x**2)*d**9*e*x -
 1701*sqrt(d**2 - e**2*x**2)*d**8*e**2*x**2 - 4536*sqrt(d**2 - e**2*x**2)*
d**7*e**3*x**3 + 1134*sqrt(d**2 - e**2*x**2)*d**6*e**4*x**4 + 6804*sqrt(d*
*2 - e**2*x**2)*d**5*e**5*x**5 + 1134*sqrt(d**2 - e**2*x**2)*d**4*e**6*x**
6 - 4536*sqrt(d**2 - e**2*x**2)*d**3*e**7*x**7 - 1701*sqrt(d**2 - e**2*x**
2)*d**2*e**8*x**8 + 1134*sqrt(d**2 - e**2*x**2)*d*e**9*x**9 + 567*sqrt(d**
2 - e**2*x**2)*e**10*x**10 - 252*d**11 + 1134*d**10*e*x + 2520*d**9*e**2*x
**2 - 2100*d**8*e**3*x**3 - 6720*d**7*e**4*x**4 + 672*d**6*e**5*x**5 + 806
4*d**5*e**6*x**6 + 1728*d**4*e**7*x**7 - 4608*d**3*e**8*x**8 - 1792*d**2*e
**9*x**9 + 1024*d*e**10*x**10 + 512*e**11*x**11)/(1638*sqrt(d**2 - e**2*x*
*2)*d**12*e*(d**10 + 2*d**9*e*x - 3*d**8*e**2*x**2 - 8*d**7*e**3*x**3 + 2*
d**6*e**4*x**4 + 12*d**5*e**5*x**5 + 2*d**4*e**6*x**6 - 8*d**3*e**7*x**7 -
 3*d**2*e**8*x**8 + 2*d*e**9*x**9 + e**10*x**10))
```

3.162 $\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\begin{aligned}\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}} \, dx &= \frac{4x}{65d^3 (d^2-e^2x^2)^{11/2}} \\ &- \frac{2}{15e(d+ex)^2 (d^2-e^2x^2)^{11/2}} - \frac{11}{195de(d+ex) (d^2-e^2x^2)^{11/2}} \\ &+ \frac{8x}{117d^5 (d^2-e^2x^2)^{9/2}} + \frac{64x}{819d^7 (d^2-e^2x^2)^{7/2}} + \frac{128x}{1365d^9 (d^2-e^2x^2)^{5/2}} \\ &+ \frac{512x}{4095d^{11} (d^2-e^2x^2)^{3/2}} + \frac{1024x}{4095d^{13}\sqrt{d^2-e^2x^2}}\end{aligned}$$

output
```
4/65*x/d^3/(-e^2*x^2+d^2)^(11/2)-2/15/e/(e*x+d)^2/(-e^2*x^2+d^2)^(11/2)-11
/195/d/e/(e*x+d)/(-e^2*x^2+d^2)^(11/2)+8/117*x/d^5/(-e^2*x^2+d^2)^(9/2)+64
/819*x/d^7/(-e^2*x^2+d^2)^(7/2)+128/1365*x/d^9/(-e^2*x^2+d^2)^(5/2)+512/40
95*x/d^11/(-e^2*x^2+d^2)^(3/2)+1024/4095*x/d^13/(-e^2*x^2+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2x^2}(-777d^{12}+1764d^{11}ex+7308d^{10}e^2x^2+840d^9e^3x^3-17640d^8}{}$$

input
```
Integrate[1/((d + e*x)^3*(d^2 - e^2*x^2)^(11/2)),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-777*d^12 + 1764*d^11*e*x + 7308*d^10*e^2*x^2 + 840*
d^9*e^3*x^3 - 17640*d^8*e^4*x^4 - 12096*d^7*e^5*x^5 + 17472*d^6*e^6*x^6 +
19584*d^5*e^7*x^7 - 5760*d^4*e^8*x^8 - 12800*d^3*e^9*x^9 - 1536*d^2*e^10*x
^10 + 3072*d*e^11*x^11 + 1024*e^12*x^12))/(4095*d^13*e*(d - e*x)^5*(d + e*
x)^8)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {461, 461, 470, 209, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}}\,dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{4\int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{11/2}}dx}{5d}-\frac{1}{15de(d+ex)^3(d^2-e^2x^2)^{9/2}}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{4\left(\frac{11\int \frac{1}{(d+ex)\left(d^2-e^2x^2\right)^{11/2}}dx}{13d}-\frac{1}{13de(d+ex)^2(d^2-e^2x^2)^{9/2}}\right)}{5d}-\frac{1}{15de(d+ex)^3(d^2-e^2x^2)^{9/2}}$$

$$\Big\downarrow \text{ 470}$$

$$\frac{4\left(\frac{11\left(\frac{10\int \frac{1}{\left(d^2-e^2x^2\right)^{11/2}}dx}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2(d^2-e^2x^2)^{9/2}}\right)}{5d}-\frac{1}{15de(d+ex)^3\left(d^2-e^2x^2\right)^{9/2}}$$

$\downarrow$ 209

$$\frac{4\left(\frac{11\left(\frac{10\left(\frac{8\int \frac{1}{\left(d^2-e^2x^2\right)^{9/2}}dx}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2(d^2-e^2x^2)^{9/2}}\right)}{5d}-\frac{1}{15de(d+ex)^3\left(d^2-e^2x^2\right)^{9/2}}$$

$\downarrow$ 209

$$\frac{4\left(\frac{11\left(\frac{10\left(\frac{8\left(\frac{6\int \frac{1}{\left(d^2-e^2x^2\right)^{7/2}}dx}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}\right)}{13d}-\frac{1}{13de(d+ex)^2(d^2-e^2x^2)^{9/2}}\right)}{5d}-\frac{1}{15de(d+ex)^3\left(d^2-e^2x^2\right)^{9/2}}$$

$\big\downarrow$ 209

$$\frac{4\left(\frac{11\left(\frac{10\left(\frac{8\left(\frac{6\left(\frac{4\int\frac{1}{\left(d^2-e^2x^2\right)^{5/2}}dx}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)^{9/2}}\right)}{13d}-\frac{\ }{13de(d+ \cdots}\right)}{5d}$$

$$\frac{1}{15de(d+ex)^3\left(d^2-e^2x^2\right)^{9/2}}$$

↓ 209

$$4\left(\frac{11\left(\frac{10\left(\frac{8\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(d^2-e^2x^2\right)^{3/2}}dx}{3d^2}+\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}\right)}{5d^2}+\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}\right)}{7d^2}+\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}\right)}{9d^2}+\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-\right.}}{13d}\right.$$

↓ 208

$$4\left(\frac{11\left(\frac{10\left(\frac{x}{9d^2\left(d^2-e^2x^2\right)^{9/2}}+\frac{8\left(\frac{x}{7d^2\left(d^2-e^2x^2\right)^{7/2}}+\frac{6\left(\frac{x}{5d^2\left(d^2-e^2x^2\right)^{5/2}}+\frac{4\left(\frac{x}{3d^2\left(d^2-e^2x^2\right)^{3/2}}+\frac{2x}{3d^4\sqrt{d^2-e^2x^2}}\right)}{5d^2}\right)}{7d^2}\right)}{9d^2}\right)}{11d}-\frac{1}{11de(d+ex)\left(d^2-e^2x^2\right)}}{13d}\right.$$

input
```
Int[1/((d + e*x)^3*(d^2 - e^2*x^2)^(11/2)),x]
```

output
```
-1/15*1/(d*e*(d + e*x)^3*(d^2 - e^2*x^2)^(9/2)) + (4*(-1/13*1/(d*e*(d + e*
x)^2*(d^2 - e^2*x^2)^(9/2)) + (11*(-1/11*1/(d*e*(d + e*x)*(d^2 - e^2*x^2)^
(9/2)) + (10*(x/(9*d^2*(d^2 - e^2*x^2)^(9/2)) + (8*(x/(7*d^2*(d^2 - e^2*x^
2)^(7/2)) + (6*(x/(5*d^2*(d^2 - e^2*x^2)^(5/2)) + (4*(x/(3*d^2*(d^2 - e^2*
x^2)^(3/2)) + (2*x)/(3*d^4*Sqrt[d^2 - e^2*x^2])))/(5*d^2)))/(7*d^2)))/(9*d
^2)))/(11*d)))/(13*d)))/(5*d)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.79

method	result
gosper	$-\frac{(-ex+d)(-1024e^{12}x^{12}-3072e^{11}x^{11}d+1536d^2e^{10}x^{10}+12800e^9x^9d^3+5760d^4e^8x^8-19584e^7x^7d^5-17472d^6e^6x^6+12096e^5x^5d^7-}{4095(ex+d)^2d^{13}e(-e^2x^2+d^2)^{\frac{11}{2}}}$
orering	$-\frac{(-ex+d)(-1024e^{12}x^{12}-3072e^{11}x^{11}d+1536d^2e^{10}x^{10}+12800e^9x^9d^3+5760d^4e^8x^8-19584e^7x^7d^5-17472d^6e^6x^6+12096e^5x^5d^7-}{4095(ex+d)^2d^{13}e(-e^2x^2+d^2)^{\frac{11}{2}}}$
trager	$-\frac{(-1024e^{12}x^{12}-3072e^{11}x^{11}d+1536d^2e^{10}x^{10}+12800e^9x^9d^3+5760d^4e^8x^8-19584e^7x^7d^5-17472d^6e^6x^6+12096e^5x^5d^7+17640d^8}{4095d^{13}(ex+d)^8(-ex+d)^5e}$
	$4e\left(-\frac{1}{13de\left(x+\frac{d}{e}\right)^2\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{9}{2}}}+\frac{11e\left(-\frac{1}{11de\left(x+\frac{d}{e}\right)\left(-e^2\left(x+\frac{d}{e}\right)^2+2de\left(x-\right.\right.}\right.}{}\right.$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4095*(-e*x+d)*(-1024*e^12*x^12-3072*d*e^11*x^11+1536*d^2*e^10*x^10+1280
0*d^3*e^9*x^9+5760*d^4*e^8*x^8-19584*d^5*e^7*x^7-17472*d^6*e^6*x^6+12096*d
^7*e^5*x^5+17640*d^8*e^4*x^4-840*d^9*e^3*x^3-7308*d^10*e^2*x^2-1764*d^11*e
*x+777*d^12)/(e*x+d)^2/d^13/e/(-e^2*x^2+d^2)^(11/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(176) = 352$.

Time = 2.06 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.09

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}}\,dx =$$
$$-\frac{777\,e^{13}x^{13}+2331\,de^{12}x^{12}-1554\,d^2e^{11}x^{11}-10878\,d^3e^{10}x^{10}-3885\,d^4e^9x^9+19425\,d^5e^8x^8+15540\,d^6e^7x}{}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x, algorithm="fricas")
```

output
```
-1/4095*(777*e^13*x^13 + 2331*d*e^12*x^12 - 1554*d^2*e^11*x^11 - 10878*d^3
*e^10*x^10 - 3885*d^4*e^9*x^9 + 19425*d^5*e^8*x^8 + 15540*d^6*e^7*x^7 - 15
540*d^7*e^6*x^6 - 19425*d^8*e^5*x^5 + 3885*d^9*e^4*x^4 + 10878*d^10*e^3*x^
3 + 1554*d^11*e^2*x^2 - 2331*d^12*e*x - 777*d^13 + (1024*e^12*x^12 + 3072*
d*e^11*x^11 - 1536*d^2*e^10*x^10 - 12800*d^3*e^9*x^9 - 5760*d^4*e^8*x^8 +
19584*d^5*e^7*x^7 + 17472*d^6*e^6*x^6 - 12096*d^7*e^5*x^5 - 17640*d^8*e^4*
x^4 + 840*d^9*e^3*x^3 + 7308*d^10*e^2*x^2 + 1764*d^11*e*x - 777*d^12)*sqrt
(-e^2*x^2 + d^2))/(d^13*e^14*x^13 + 3*d^14*e^13*x^12 - 2*d^15*e^12*x^11 -
14*d^16*e^11*x^10 - 5*d^17*e^10*x^9 + 25*d^18*e^9*x^8 + 20*d^19*e^8*x^7 -
20*d^20*e^7*x^6 - 25*d^21*e^6*x^5 + 5*d^22*e^5*x^4 + 14*d^23*e^4*x^3 + 2*d
^24*e^3*x^2 - 3*d^25*e^2*x - d^26*e)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}}\,dx = \int \frac{1}{\left(-\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{11}{2}}\left(d+ex\right)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(11/2),x)
```

output
```
Integral(1/((-(-d + e*x)*(d + e*x))**(11/2)*(d + e*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.50

$$\begin{aligned}
&\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}}\,dx = \\
&-\frac{1}{15\left(\left(-e^2x^2+d^2\right)^{\frac{9}{2}}de^4x^3+3\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^2e^3x^2+3\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^3e^2x+\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^4e\right)} \\
&-\frac{4}{65\left(\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^2e^3x^2+2\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^3e^2x+\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^4e\right)} \\
&-\frac{4}{65\left(\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^3e^2x+\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^4e\right)} \\
&+\frac{8\,x}{117\left(-e^2x^2+d^2\right)^{\frac{9}{2}}d^5}+\frac{64\,x}{819\left(-e^2x^2+d^2\right)^{\frac{7}{2}}d^7}+\frac{128\,x}{1365\left(-e^2x^2+d^2\right)^{\frac{5}{2}}d^9} \\
&+\frac{512\,x}{4095\left(-e^2x^2+d^2\right)^{\frac{3}{2}}d^{11}}+\frac{1024\,x}{4095\sqrt{-e^2x^2+d^2}\,d^{13}}
\end{aligned}$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x, algorithm="maxima")
```

output
```
-1/15/((-e^2*x^2 + d^2)^(9/2)*d*e^4*x^3 + 3*(-e^2*x^2 + d^2)^(9/2)*d^2*e^3
*x^2 + 3*(-e^2*x^2 + d^2)^(9/2)*d^3*e^2*x + (-e^2*x^2 + d^2)^(9/2)*d^4*e)
- 4/65/((-e^2*x^2 + d^2)^(9/2)*d^2*e^3*x^2 + 2*(-e^2*x^2 + d^2)^(9/2)*d^3*
e^2*x + (-e^2*x^2 + d^2)^(9/2)*d^4*e) - 4/65/((-e^2*x^2 + d^2)^(9/2)*d^3*e
^2*x + (-e^2*x^2 + d^2)^(9/2)*d^4*e) + 8/117*x/((-e^2*x^2 + d^2)^(9/2)*d^5
) + 64/819*x/((-e^2*x^2 + d^2)^(7/2)*d^7) + 128/1365*x/((-e^2*x^2 + d^2)^(
5/2)*d^9) + 512/4095*x/((-e^2*x^2 + d^2)^(3/2)*d^11) + 1024/4095*x/(sqrt(-
e^2*x^2 + d^2)*d^13)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}}\,dx = \int \frac{1}{(-e^2x^2+d^2)^{\frac{11}{2}}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x, algorithm="giac")
```

output
```
integrate(1/((-e^2*x^2 + d^2)^(11/2)*(e*x + d)^3), x)
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}}\,dx = \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{355\,x}{936\,d^5}-\frac{19}{52\,d^4\,e}\right)}{{\left(d+e\,x\right)}^5\,{\left(d-e\,x\right)}^5} - \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{x}{504\,d^7}-\frac{89}{416\,d^6\,e}\right)}{{\left(d+e\,x\right)}^4\,{\left(d-e\,x\right)}^4} + \frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{128\,x}{1365\,d^9}-\frac{121}{8320\,d^8\,e}\right)}{{\left(d+e\,x\right)}^3\,{\left(d-e\,x\right)}^3} - \frac{\sqrt{d^2-e^2\,x^2}}{480\,d^6\,e\,{\left(d+e\,x\right)}^8} - \frac{89\,\sqrt{d^2-e^2\,x^2}}{12480\,d^7\,e\,{\left(d+e\,x\right)}^7} - \frac{121\,\sqrt{d^2-e^2\,x^2}}{8320\,d^8\,e\,{\left(d+e\,x\right)}^6} + \frac{512\,x\,\sqrt{d^2-e^2\,x^2}}{4095\,d^{11}\,{\left(d+e\,x\right)}^2\,{\left(d-e\,x\right)}^2} + \frac{1024\,x\,\sqrt{d^2-e^2\,x^2}}{4095\,d^{13}\,\left(d+e\,x\right)\,\left(d-e\,x\right)}$$

input
```
int(1/((d^2 - e^2*x^2)^(11/2)*(d + e*x)^3),x)
```

output
```
((d^2 - e^2*x^2)^(1/2)*((355*x)/(936*d^5) - 19/(52*d^4*e)))/((d + e*x)^5*(
d - e*x)^5) - ((d^2 - e^2*x^2)^(1/2)*(x/(504*d^7) - 89/(416*d^6*e)))/((d +
 e*x)^4*(d - e*x)^4) + ((d^2 - e^2*x^2)^(1/2)*((128*x)/(1365*d^9) - 121/(8
320*d^8*e)))/((d + e*x)^3*(d - e*x)^3) - (d^2 - e^2*x^2)^(1/2)/(480*d^6*e*
(d + e*x)^8) - (89*(d^2 - e^2*x^2)^(1/2))/(12480*d^7*e*(d + e*x)^7) - (121
*(d^2 - e^2*x^2)^(1/2))/(8320*d^8*e*(d + e*x)^6) + (512*x*(d^2 - e^2*x^2)^
(1/2))/(4095*d^11*(d + e*x)^2*(d - e*x)^2) + (1024*x*(d^2 - e^2*x^2)^(1/2)
)/(4095*d^13*(d + e*x)*(d - e*x))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.61

$$\int \frac{1}{(d+ex)^3 (d^2-e^2x^2)^{11/2}} dx = \frac{588\sqrt{-e^2x^2+d^2}\, d^{11} + 1024e^{12}x^{12} + 1764\sqrt{-e^2x^2+d^2}\, d^{10}ex - 588\sqrt{-e}$$

input
```
int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(11/2),x)
```

output
```
(588*sqrt(d**2 - e**2*x**2)*d**11 + 1764*sqrt(d**2 - e**2*x**2)*d**10*e*x
- 588*sqrt(d**2 - e**2*x**2)*d**9*e**2*x**2 - 6468*sqrt(d**2 - e**2*x**2)*
d**8*e**3*x**3 - 3528*sqrt(d**2 - e**2*x**2)*d**7*e**4*x**4 + 8232*sqrt(d*
*2 - e**2*x**2)*d**6*e**5*x**5 + 8232*sqrt(d**2 - e**2*x**2)*d**5*e**6*x**
6 - 3528*sqrt(d**2 - e**2*x**2)*d**4*e**7*x**7 - 6468*sqrt(d**2 - e**2*x**
2)*d**3*e**8*x**8 - 588*sqrt(d**2 - e**2*x**2)*d**2*e**9*x**9 + 1764*sqrt(
d**2 - e**2*x**2)*d*e**10*x**10 + 588*sqrt(d**2 - e**2*x**2)*e**11*x**11 -
 777*d**12 + 1764*d**11*e*x + 7308*d**10*e**2*x**2 + 840*d**9*e**3*x**3 -
17640*d**8*e**4*x**4 - 12096*d**7*e**5*x**5 + 17472*d**6*e**6*x**6 + 19584
*d**5*e**7*x**7 - 5760*d**4*e**8*x**8 - 12800*d**3*e**9*x**9 - 1536*d**2*e
**10*x**10 + 3072*d*e**11*x**11 + 1024*e**12*x**12)/(4095*sqrt(d**2 - e**2
*x**2)*d**13*e*(d**11 + 3*d**10*e*x - d**9*e**2*x**2 - 11*d**8*e**3*x**3 -
 6*d**7*e**4*x**4 + 14*d**6*e**5*x**5 + 14*d**5*e**6*x**6 - 6*d**4*e**7*x*
*7 - 11*d**3*e**8*x**8 - d**2*e**9*x**9 + 3*d*e**10*x**10 + e**11*x**11))
```

3.163 $\int \frac{1+x}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1+x}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2} + \arcsin(x)$$

output
```
-(-x^2+1)^(1/2)+arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(16) = 32.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{1+x}{\sqrt{1-x^2}} \, dx = -\sqrt{1-x^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + x)/Sqrt[1 - x^2],x]
```

output
```
-Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \text{ 455}$$
$$\int \frac{1}{\sqrt{1-x^2}}dx-\sqrt{1-x^2}$$
$$\Big\downarrow \text{ 223}$$
$$\arcsin(x)-\sqrt{1-x^2}$$

input
```
Int[(1 + x)/Sqrt[1 - x^2],x]
```

output
```
-Sqrt[1 - x^2] + ArcSin[x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$-\sqrt{-x^{2}+1}+\arcsin\left(x\right)$	15
risch	$\frac{x^{2}-1}{\sqrt{-x^{2}+1}}+\arcsin\left(x\right)$	19
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{2}+1}}{2\sqrt{\pi}}+\arcsin\left(x\right)$	29
trager	$-\sqrt{-x^{2}+1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	39

input
```
int((x+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x^2+1)^(1/2)+arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{1+x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^{2}+1}-2\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

input
```
integrate((1+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1+x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + \operatorname{asin}(x)$$

input
```
integrate((1+x)/(-x**2+1)**(1/2),x)
```

output
```
-sqrt(1 - x**2) + asin(x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((1+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{\sqrt{1-x^2}}\,dx = -\sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((1+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1) + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1+x}{\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) - \sqrt{1-x^2}$$

input
```
int((x + 1)/(1 - x^2)^(1/2),x)
```

output
```
asin(x) - (1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1+x}{\sqrt{1-x^2}}\,dx = asin(x) - \sqrt{-x^2+1}$$

input
```
int((1+x)/(-x^2+1)^(1/2),x)
```

output
```
asin(x) - sqrt( - x**2 + 1)
```

3.164 $\int \frac{1-x}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{1-x}{\sqrt{1-x^2}} \, dx = \sqrt{1-x^2} + \arcsin(x)$$

output
```
(-x^2+1)^(1/2)+arcsin(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1-x}{\sqrt{1-x^2}} \, dx = \sqrt{1-x^2} - 2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 - x)/Sqrt[1 - x^2],x]
```

output
```
Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{\sqrt{1-x^2}} \, dx$$

$$\Big\downarrow \; 455$$

$$\int \frac{1}{\sqrt{1-x^2}}dx + \sqrt{1-x^2}$$

$$\Big\downarrow \; 223$$

$$\arcsin(x) + \sqrt{1-x^2}$$

input
```
Int[(1 - x)/Sqrt[1 - x^2],x]
```

output
```
Sqrt[1 - x^2] + ArcSin[x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\sqrt{-x^{2}+1}+\arcsin\left(x\right)$	13
risch	$-\frac{x^{2}-1}{\sqrt{-x^{2}+1}}+\arcsin\left(x\right)$	20
meijerg	$\arcsin\left(x\right)+\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{2}+1}}{2\sqrt{\pi}}$	29
trager	$\sqrt{-x^{2}+1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	37

input
```
int((1-x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-x^2+1)^(1/2)+arcsin(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx=\sqrt{-x^2+1}-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1-x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(-x^2 + 1) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx = \sqrt{1-x^2} + \operatorname{asin}(x)$$

input
```
integrate((1-x)/(-x**2+1)**(1/2),x)
```

output
```
sqrt(1 - x**2) + asin(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx = \sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((1-x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-x^2 + 1) + arcsin(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx = \sqrt{-x^2+1} + \arcsin(x)$$

input
```
integrate((1-x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sqrt(-x^2 + 1) + arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) + \sqrt{1-x^2}$$

input
```
int(-(x - 1)/(1 - x^2)^(1/2),x)
```

output
```
asin(x) + (1 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx = asin(x) + \sqrt{-x^2+1}$$

input
```
int((1-x)/(-x^2+1)^(1/2),x)
```

output
```
asin(x) + sqrt( - x**2 + 1)
```

3.165 $\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx=-\frac{256d^3(cd^2-ce^2x^2)^{3/2}}{315ce(d+ex)^{3/2}}-\frac{64d^2(cd^2-ce^2x^2)^{3/2}}{105ce\sqrt{d+ex}}-\frac{8d\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}{21ce}-\frac{2(d+ex)^{3/2}\,(cd^2-ce^2x^2)^{3/2}}{9ce}$$

output
```
-256/315*d^3*(-c*e^2*x^2+c*d^2)^(3/2)/c/e/(e*x+d)^(3/2)-64/105*d^2*(-c*e^2
*x^2+c*d^2)^(3/2)/c/e/(e*x+d)^(1/2)-8/21*d*(e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2
)^(3/2)/c/e-2/9*(e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2)/c/e
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx = -\frac{2\sqrt{c\,(d^2-e^2x^2)}(319d^4+2d^3ex-156d^2e^2x^2-130de^3x^3-35e^4x^4)}{315e\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*Sqrt[c*(d^2 - e^2*x^2)]*(319*d^4 + 2*d^3*e*x - 156*d^2*e^2*x^2 - 130*d
*e^3*x^3 - 35*e^4*x^4))/(315*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \text{459}$$

$$\frac{4}{3}d\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}dx-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{4}{3}d\left(\frac{8}{7}d\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}dx-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}\right)-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{4}{3}d\left(\frac{8}{7}d\left(\frac{4}{5}d\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}\right)-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}$$

$$\Big\downarrow \text{458}$$

$$\frac{4}{3}d\left(\frac{8}{7}d\left(-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}-\frac{8d\left(cd^2-ce^2x^2\right)^{3/2}}{15ce(d+ex)^{3/2}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}\right)-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}{9ce}$$

input
```
Int[(d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2))/(9*c*e) + (4*d*((-2*Sqrt[d
+ e*x]*(c*d^2 - c*e^2*x^2)^(3/2))/(7*c*e) + (8*d*((-8*d*(c*d^2 - c*e^2*x^2
)^(3/2))/(15*c*e*(d + e*x)^(3/2)) - (2*(c*d^2 - c*e^2*x^2)^(3/2))/(5*c*e*S
qrt[d + e*x])))/7))/3
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

method	result	size
default	$-\frac{2\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\,\left(-e x +d \right) \left(35 e^{3} x^{3}+165 d\,e^{2} x^{2}+321 d^{2} e x +319 d^{3}\right)}{315 \sqrt{e x +d}\, e}$	65
gosper	$-\frac{2 \left(-e x +d \right) \left(35 e^{3} x^{3}+165 d\,e^{2} x^{2}+321 d^{2} e x +319 d^{3}\right) \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}{315 e \sqrt{e x +d}}$	66
orering	$-\frac{2 \left(-e x +d \right) \left(35 e^{3} x^{3}+165 d\,e^{2} x^{2}+321 d^{2} e x +319 d^{3}\right) \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}{315 e \sqrt{e x +d}}$	66
risch	$-\frac{2 \sqrt{-\frac{c \left(e^{2} x^{2}-d^{2}\right)}{e x +d}}\, \sqrt{e x +d}\, c \left(-35 e^{4} x^{4}-130 d\,e^{3} x^{3}-156 d^{2} e^{2} x^{2}+2 d^{3} e x +319 d^{4}\right) \left(-e x +d \right)}{315 \sqrt{-c \left(e^{2} x^{2}-d^{2}\right)}\, e \sqrt{-c \left(e x -d \right)}}$	116

input
```
int((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)*(-e*x+d)*(35*e^3*x^3+165*d*e
^2*x^2+321*d^2*e*x+319*d^3)/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.49

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\,(35\,e^4x^4+130\,de^3x^3+156\,d^2e^2x^2-2\,d^3ex-319\,d^4)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{315\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*e^4*x^4 + 130*d*e^3*x^3 + 156*d^2*e^2*x^2 - 2*d^3*e*x - 319*d^4)
*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F]

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx = \int \sqrt{-c\,(-d+ex)\,(d+ex)}(d+ex)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x+d)**(5/2)*(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.51

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\left(35\sqrt{c}e^4x^4+130\sqrt{c}de^3x^3+156\sqrt{c}d^2e^2x^2-2\sqrt{c}d^3ex-319\sqrt{c}d^4\right)(ex+d)\sqrt{}}{315\left(e^2x+de\right)}$$

input
```
integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*sqrt(c)*e^4*x^4 + 130*sqrt(c)*d*e^3*x^3 + 156*sqrt(c)*d^2*e^2*x^
2 - 2*sqrt(c)*d^3*e*x - 319*sqrt(c)*d^4)*(e*x + d)*sqrt(-e*x + d)/(e^2*x +
 d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(136) = 272.

Time = 0.12 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.74

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx =$$

$$-\frac{2\left(315\sqrt{-cex+cd}d^4+\frac{210\left(3\sqrt{-cex+cd}cd-(-cex+cd)^{\frac{3}{2}}\right)d^3}{c}-\frac{18\left(35\sqrt{-cex+cd}c^3d^3-35(-cex+cd)^{\frac{3}{2}}c^2d^2+21(cex-cd)^2\sqrt{-c}\right.}{c^3}}{}$$

input
```
integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
-2/315*(315*sqrt(-c*e*x + c*d)*d^4 + 210*(3*sqrt(-c*e*x + c*d)*c*d - (-c*e
*x + c*d)^(3/2))*d^3/c - 18*(35*sqrt(-c*e*x + c*d)*c^3*d^3 - 35*(-c*e*x +
c*d)^(3/2)*c^2*d^2 + 21*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c*d + 5*(c*e*x
- c*d)^3*sqrt(-c*e*x + c*d))*d/c^3 - (315*sqrt(-c*e*x + c*d)*c^4*d^4 - 420
*(-c*e*x + c*d)^(3/2)*c^3*d^3 + 378*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c^2
*d^2 + 180*(c*e*x - c*d)^3*sqrt(-c*e*x + c*d)*c*d + 35*(c*e*x - c*d)^4*sqr
t(-c*e*x + c*d))/c^4)/e
```

Mupad [B] (verification not implemented)

Time = 7.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.64

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx=\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{104\,d^2\,x^2\,\sqrt{d+e\,x}}{105}-\frac{638\,d^4\,\sqrt{d+e\,x}}{315\,e^2}+\frac{2\,e^2\,x^4\,\sqrt{d+e\,x}}{9}+\frac{52\,d\,e\,x^3\,\sqrt{d+e\,x}}{63}-\frac{4\,d^3}{}\right.}{x+\frac{d}{e}}$$

input
```
int((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(5/2),x)
```

output
```
((c*d^2 - c*e^2*x^2)^(1/2)*((104*d^2*x^2*(d + e*x)^(1/2))/105 - (638*d^4*(
d + e*x)^(1/2))/(315*e^2) + (2*e^2*x^4*(d + e*x)^(1/2))/9 + (52*d*e*x^3*(d
 + e*x)^(1/2))/63 - (4*d^3*x*(d + e*x)^(1/2))/(315*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.34

$$\int (d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}\,dx=\frac{2\sqrt{c}\,\sqrt{-ex+d}\,(35e^4x^4+130d\,e^3x^3+156d^2e^2x^2-2d^3ex-319d^4)}{315e}$$

input
```
int((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*( - 319*d**4 - 2*d**3*e*x + 156*d**2*e**2*x**2 +
130*d*e**3*x**3 + 35*e**4*x**4))/(315*e)
```

3.166 $\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx=-\frac{64d^2(cd^2-ce^2x^2)^{3/2}}{105ce(d+ex)^{3/2}}-\frac{16d(cd^2-ce^2x^2)^{3/2}}{35ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}{7ce}$$

output
```
-64/105*d^2*(-c*e^2*x^2+c*d^2)^(3/2)/c/e/(e*x+d)^(3/2)-16/35*d*(-c*e^2*x^2
+c*d^2)^(3/2)/c/e/(e*x+d)^(1/2)-2/7*(e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2)
/c/e
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.54

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx=\frac{2\sqrt{c(d^2-e^2x^2)}(-71d^3+17d^2ex+39de^2x^2+15e^3x^3)}{105e\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(2*Sqrt[c*(d^2 - e^2*x^2)]*(-71*d^3 + 17*d^2*e*x + 39*d*e^2*x^2 + 15*e^3*x
^3))/(105*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{7}d\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}dx-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{7}d\left(\frac{4}{5}d\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}$$

$$\Big\downarrow \text{458}$$

$$\frac{8}{7}d\left(-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}-\frac{8d\left(cd^2-ce^2x^2\right)^{3/2}}{15ce(d+ex)^{3/2}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}{7ce}$$

input
```
Int[(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2))/(7*c*e) + (8*d*((-8*d*(c*d^2
- c*e^2*x^2)^(3/2))/(15*c*e*(d + e*x)^(3/2)) - (2*(c*d^2 - c*e^2*x^2)^(3/2
))/(5*c*e*Sqrt[d + e*x])))/7
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

method	result	size
default	$-\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\,\left(-ex+d\right)\left(15e^{2}x^{2}+54dex+71d^{2}\right)}{105\sqrt{ex+d}\,e}$	54
gosper	$-\frac{2\left(-ex+d\right)\left(15e^{2}x^{2}+54dex+71d^{2}\right)\sqrt{-c\,e^{2}x^{2}+c\,d^{2}}}{105\sqrt{ex+d}\,e}$	55
orering	$-\frac{2\left(-ex+d\right)\left(15e^{2}x^{2}+54dex+71d^{2}\right)\sqrt{-c\,e^{2}x^{2}+c\,d^{2}}}{105\sqrt{ex+d}\,e}$	55
risch	$-\frac{2\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}\,c\left(-15e^{3}x^{3}-39d\,e^{2}x^{2}-17d^{2}ex+71d^{3}\right)\left(-ex+d\right)}{105\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,e\sqrt{-c\left(ex-d\right)}}$	105

input
```
int((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)*(-e*x+d)*(15*e^2*x^2+54*d*e*
x+71*d^2)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\,(15\,e^3x^3+39\,de^2x^2+17\,d^2ex-71\,d^3)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{105\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*e^3*x^3 + 39*d*e^2*x^2 + 17*d^2*e*x - 71*d^3)*sqrt(-c*e^2*x^2 +
c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F]

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx = \int \sqrt{-c\,(-d+ex)\,(d+ex)}(d+ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\left(15\,\sqrt{c}e^3x^3+39\,\sqrt{c}de^2x^2+17\,\sqrt{c}d^2ex-71\,\sqrt{c}d^3\right)(ex+d)\sqrt{-ex+d}}{105\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*sqrt(c)*e^3*x^3 + 39*sqrt(c)*d*e^2*x^2 + 17*sqrt(c)*d^2*e*x - 71
*sqrt(c)*d^3)*(e*x + d)*sqrt(-e*x + d)/(e^2*x + d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(101) = 202$.

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.83

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx = -\frac{2\left(105\sqrt{-cex+cd}d^3+\frac{35\left(3\sqrt{-cex+cd}cd-(-cex+cd)^{\frac{3}{2}}\right)d^2}{c}-\frac{7\left(15\sqrt{-cex+cd}c^2d^2-10(-cex+cd)^{\frac{3}{2}}cd+3(cex-cd)^2\sqrt{-cex+cd}\right)}{c^2}\right.}{105e}$$

input
```
integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
-2/105*(105*sqrt(-c*e*x + c*d)*d^3 + 35*(3*sqrt(-c*e*x + c*d)*c*d - (-c*e*
x + c*d)^(3/2))*d^2/c - 7*(15*sqrt(-c*e*x + c*d)*c^2*d^2 - 10*(-c*e*x + c*
d)^(3/2)*c*d + 3*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d))*d/c^2 - 3*(35*sqrt(-c
*e*x + c*d)*c^3*d^3 - 35*(-c*e*x + c*d)^(3/2)*c^2*d^2 + 21*(c*e*x - c*d)^2
*sqrt(-c*e*x + c*d)*c*d + 5*(c*e*x - c*d)^3*sqrt(-c*e*x + c*d))/c^3)/e
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx = \frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{26\,d\,x^2\,\sqrt{d+e\,x}}{35}-\frac{142\,d^3\,\sqrt{d+e\,x}}{105\,e^2}+\frac{2\,e\,x^3\,\sqrt{d+e\,x}}{7}+\frac{34\,d^2\,x\,\sqrt{d+e\,x}}{105\,e}\right)}{x+\frac{d}{e}}$$

input
```
int((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(3/2),x)
```

output
```
((c*d^2 - c*e^2*x^2)^(1/2)*((26*d*x^2*(d + e*x)^(1/2))/35 - (142*d^3*(d +
e*x)^(1/2))/(105*e^2) + (2*e*x^3*(d + e*x)^(1/2))/7 + (34*d^2*x*(d + e*x)^
(1/2))/(105*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.37

$$\int (d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,(15e^3x^3+39d\,e^2x^2+17d^2ex-71d^3)}{105e}$$

input
```
int((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*( - 71*d**3 + 17*d**2*e*x + 39*d*e**2*x**2 + 15*e
**3*x**3))/(105*e)
```

3.167 $\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = -\frac{8d(cd^2-ce^2x^2)^{3/2}}{15ce(d+ex)^{3/2}}-\frac{2(cd^2-ce^2x^2)^{3/2}}{5ce\sqrt{d+ex}}$$

output
```
-8/15*d*(-c*e^2*x^2+c*d^2)^(3/2)/c/e/(e*x+d)^(3/2)-2/5*(-c*e^2*x^2+c*d^2)^
(3/2)/c/e/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = -\frac{2(7d^2-4dex-3e^2x^2)\sqrt{c(d^2-e^2x^2)}}{15e\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*(7*d^2 - 4*d*e*x - 3*e^2*x^2)*Sqrt[c*(d^2 - e^2*x^2)])/(15*e*Sqrt[d +
e*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx$$

$$\Big\downarrow \text{459}$$

$$\frac{4}{5}d\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}$$

$$\Big\downarrow \text{458}$$

$$-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{5ce\sqrt{d+ex}}-\frac{8d\left(cd^2-ce^2x^2\right)^{3/2}}{15ce(d+ex)^{3/2}}$$

input
```
Int[Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-8*d*(c*d^2 - c*e^2*x^2)^(3/2))/(15*c*e*(d + e*x)^(3/2)) - (2*(c*d^2 - c*
e^2*x^2)^(3/2))/(5*c*e*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(-e x+d\right) \left(3 e x+7 d\right)}{15 \sqrt{e x+d}\, e}$	43
gosper	$-\frac{2 \left(-e x+d\right) \left(3 e x+7 d\right) \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}{15 e \sqrt{e x+d}}$	44
orering	$-\frac{2 \left(-e x+d\right) \left(3 e x+7 d\right) \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}{15 e \sqrt{e x+d}}$	44
risch	$-\frac{2 \sqrt{-\frac{c\left(e^{2} x^{2}-d^{2}\right)}{e x+d}}\, \sqrt{e x+d}\, c\left(-3 e^{2} x^{2}-4 d e x+7 d^{2}\right) \left(-e x+d\right)}{15 \sqrt{-c\left(e^{2} x^{2}-d^{2}\right)}\, e \sqrt{-c\left(e x-d\right)}}$	94

input
```
int((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)*(-e*x+d)*(3*e*x+7*d)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\,\sqrt{-ce^2x^2+cd^2}(3\,e^2x^2+4\,dex-7\,d^2)\sqrt{ex+d}}{15\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*sqrt(-c*e^2*x^2 + c*d^2)*(3*e^2*x^2 + 4*d*e*x - 7*d^2)*sqrt(e*x + d)/
(e^2*x + d*e)
```

Sympy [F]

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = \int \sqrt{-c(-d+ex)(d+ex)}\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-d + e*x)*(d + e*x))*sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\left(3\sqrt{c}e^2x^2+4\sqrt{c}dex-7\sqrt{c}d^2\right)(ex+d)\sqrt{-ex+d}}{15\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*sqrt(c)*e^2*x^2 + 4*sqrt(c)*d*e*x - 7*sqrt(c)*d^2)*(e*x + d)*sqrt(
-e*x + d)/(e^2*x + d*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = -\frac{2\left(15\sqrt{-cex+cd}d^2-\frac{15\sqrt{-cex+cd}c^2d^2-10(-cex+cd)^{\frac{3}{2}}cd+3(cex-cd)^2\sqrt{-cex+cd}}{c^2}\right)}{15\,e}$$

input
```
integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(15*sqrt(-c*e*x + c*d)*d^2 - (15*sqrt(-c*e*x + c*d)*c^2*d^2 - 10*(-c
*e*x + c*d)^(3/2)*c*d + 3*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d))/c^2)/e
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = \frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{2\,x^2\,\sqrt{d+e\,x}}{5}-\frac{14\,d^2\,\sqrt{d+e\,x}}{15\,e^2}+\frac{8\,d\,x\,\sqrt{d+e\,x}}{15\,e}\right)}{x+\frac{d}{e}}$$

input
```
int((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(1/2),x)
```

output
```
((c*d^2 - c*e^2*x^2)^(1/2)*((2*x^2*(d + e*x)^(1/2))/5 - (14*d^2*(d + e*x)^
(1/2))/(15*e^2) + (8*d*x*(d + e*x)^(1/2))/(15*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.42

$$\int \sqrt{d+ex}\sqrt{cd^2-ce^2x^2}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,(3e^2x^2+4dex-7d^2)}{15e}$$

input
```
int((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*( - 7*d**2 + 4*d*e*x + 3*e**2*x**2))/(15*e)
```

3.168 $\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 38

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx=-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3ce(d+ex)^{3/2}}$$

output
```
-2/3*(-c*e^2*x^2+c*d^2)^(3/2)/c/e/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx=-\frac{2(d-ex)\sqrt{c\left(d^2-e^2x^2\right)}}{3e\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[c*d^2 - c*e^2*x^2]/Sqrt[d + e*x],x]
```

output
```
(-2*(d - e*x)*Sqrt[c*(d^2 - e^2*x^2)])/(3*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{458}$$

$$-\frac{2\left(cd^2-ce^2x^2\right)^{3/2}}{3ce(d+ex)^{3/2}}$$

input
```
Int[Sqrt[c*d^2 - c*e^2*x^2]/Sqrt[d + e*x],x]
```

output
```
(-2*(c*d^2 - c*e^2*x^2)^(3/2))/(3*c*e*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{2\sqrt{c(-e^2x^2+d^2)}\,(-ex+d)}{3\sqrt{ex+d}\,e}$	35
gosper	$-\frac{2(-ex+d)\sqrt{-c\,e^2x^2+c\,d^2}}{3e\sqrt{ex+d}}$	36
orering	$-\frac{2(-ex+d)\sqrt{-c\,e^2x^2+c\,d^2}}{3e\sqrt{ex+d}}$	36
risch	$-\frac{2\sqrt{-\frac{c\left(e^2x^2-d^2\right)}{ex+d}}\,\sqrt{ex+d}\,c(-ex+d)^2}{3\sqrt{-c(e^2x^2-d^2)}\,e\sqrt{-c(ex-d)}}$	77

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(c*(-e^2*x^2+d^2))^(1/2)/(e*x+d)^(1/2)*(-e*x+d)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx=\frac{2\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}(ex-d)}{3\left(e^2x+de\right)}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*(e*x - d)/(e^2*x + d*e)
```

Sympy [F]

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt(-c*(-d + e*x)*(d + e*x))/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{\sqrt{d + ex}}\, dx = \frac{2\left(\sqrt{c}ex - \sqrt{c}d\right)\sqrt{-ex + d}}{3\, e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(sqrt(c)*e*x - sqrt(c)*d)*sqrt(-e*x + d)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{\sqrt{d + ex}}\, dx = -\frac{2\left(3\sqrt{-cex + cd}d - \frac{3\sqrt{-cex+cd}cd - (-cex+cd)^{\frac{3}{2}}}{c}\right)}{3\, e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(3*sqrt(-c*e*x + c*d)*d - (3*sqrt(-c*e*x + c*d)*c*d - (-c*e*x + c*d)^
(3/2))/c)/e
```

Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{\sqrt{d + ex}}\, dx = \frac{\sqrt{c\, d^2 - c\, e^2\, x^2}\left(\frac{2\, x}{3} - \frac{2\, d}{3\, e}\right)}{\sqrt{d + e\, x}}$$

input
```
int((c*d^2 - c*e^2*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

output
```
((c*d^2 - c*e^2*x^2)^(1/2)*((2*x)/3 - (2*d)/(3*e)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{\sqrt{d + ex}}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex + d}\,(ex - d)}{3e}$$

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*( - d + e*x))/(3*e)
```

3.169 $\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 99

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx=\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{e}$$

output
```
2*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(1/2)-2*2^(1/2)*c^(1/2)*d^(1/2)*arcta
nh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx=\frac{2\sqrt{c(d^2-e^2x^2)}\left(\frac{1}{\sqrt{d+ex}}-\frac{\sqrt{2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(3/2),x]
```

output
```
(2*Sqrt[c*(d^2 - e^2*x^2)]*(1/Sqrt[d + e*x] - (Sqrt[2]*Sqrt[d]*ArcTanh[(Sq
rt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/Sqrt[d^2 - e^2*x^2]))/e
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{466}$$

$$2cd\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx + \frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{471}$$

$$4cde\int \frac{1}{\frac{e^2(cd^2-ce^2x^2)}{d+ex} - 2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}} + \frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} - \frac{2\sqrt{2}\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}$$

input
```
Int[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(3/2),x]
```

output
```
(2*Sqrt[c*d^2 - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2]*Sqrt[c]*Sqrt[d]
*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])
/e
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{2\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(c d \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right)-\sqrt{c\left(-e x+d\right)}\, \sqrt{c d}\right)}{\sqrt{e x+d}\, \sqrt{c\left(-e x+d\right)}\, e \sqrt{c d}}$	89
risch	$\frac{2\left(-e x+d\right) \sqrt{-\frac{c\left(e^{2} x^{2}-d^{2}\right)}{e x+d}}\, \sqrt{e x+d}\, c}{e \sqrt{-c\left(e x-d\right)}\, \sqrt{-c\left(e^{2} x^{2}-d^{2}\right)}}-\frac{2 d \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-c e x+c d}\, \sqrt{2}}{2 \sqrt{c d}}\right) \sqrt{-\frac{c\left(e^{2} x^{2}-d^{2}\right)}{e x+d}}\, \sqrt{e x+d}\, c}{e \sqrt{c d}\, \sqrt{-c\left(e^{2} x^{2}-d^{2}\right)}}$	163

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*(-e^2*x^2+d^2))^(1/2)/(e*x+d)^(1/2)*(c*d*2^(1/2)*arctanh(1/2*(c*(-e*
x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))-(c*(-e*x+d))^(1/2)*(c*d)^(1/2))/(c*(-e*x+
d))^(1/2)/e/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{3/2}}\,dx = \left[\frac{\sqrt{2}\sqrt{cd}(ex+d)\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)+2\,\sqrt{-ce^2x^2+c}}{e^2x+de}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
[(sqrt(2)*sqrt(c*d)*(e*x + d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sq
rt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x
 + d^2)) + 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(e^2*x + d*e), 2*(sqr
t(2)*sqrt(-c*d)*(e*x + d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt
(-c*d)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) + sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*
x + d))/(e^2*x + d*e)]
```

Sympy [F]

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{-c\,(-d+ex)\,(d+ex)}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(3/2),x)
```

output
```
Integral(sqrt(-c*(-d + e*x)*(d + e*x))/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{-ce^2x^2+cd^2}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{2}d\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\,\sqrt{-cd}}\right)}{\sqrt{-cd}}+\frac{\sqrt{-(ex+d)c+2\,cd}}{c}\right)c}{e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
2*(sqrt(2)*d*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))/sqr
t(-c*d) + sqrt(-(e*x + d)*c + 2*c*d)/c)*c/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{c\,d^2-c\,e^2\,x^2}}{{\left(d+e\,x\right)}^{3/2}} \,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(1/2)/(d + e*x)^(3/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(1/2)/(d + e*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{c}\left(\sqrt{-ex+d}+\sqrt{d}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)\right)-\sqrt{d}\,\sqrt{2}\right)}{e}$$

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(c)*(sqrt(d - e*x) + sqrt(d)*sqrt(2)*log(tan(asin(sqrt(d + e*x)/(sq
rt(d)*sqrt(2)))/2)) - sqrt(d)*sqrt(2)))/e
```

3.170 $\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 98

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx=-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{\sqrt{2}\sqrt{d}e}$$

output
```
-(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(3/2)+1/2*c^(1/2)*arctanh(2^(1/2)*c^(1
/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/d^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx=\frac{\sqrt{c\left(d^2-e^2x^2\right)}\left(-\frac{2}{(d+ex)^{3/2}}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d}\sqrt{d^2-e^2x^2}}\right)}{2e}$$

input
```
Integrate[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(5/2),x]
```

output
```
(Sqrt[c*(d^2 - e^2*x^2)]*(-2/(d + e*x)^(3/2) + (Sqrt[2]*ArcTanh[(Sqrt[2]*S
qrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/(Sqrt[d]*Sqrt[d^2 - e^2*x^2]))
)/(2*e)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {465, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \ 465$$

$$-\frac{1}{2}c\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 471$$

$$-ce\int \frac{1}{\frac{e^2(cd^2-ce^2x^2)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}\sqrt{d}e}-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}$$

input
```
Int[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(5/2),x]
```

output
```
-(Sqrt[c*d^2 - c*e^2*x^2]/(e*(d + e*x)^(3/2))) + (Sqrt[c]*ArcTanh[Sqrt[c*d
^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[2]*Sqrt[d]
*e)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) c e x+c d \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right)-2 \sqrt{c\left(-e x+d\right)}\, \sqrt{c d}\right)}{2 \left(e x+d\right)^{\frac{3}{2}} \sqrt{c\left(-e x+d\right)}\, e \sqrt{c d}}$	117

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*(-e^2*x^2+d^2))^(1/2)*(2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/
2)/(c*d)^(1/2))*c*e*x+c*d*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(
c*d)^(1/2))-2*(c*(-e*x+d))^(1/2)*(c*d)^(1/2))/(e*x+d)^(3/2)/(c*(-e*x+d))^(
1/2)/e/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx = \left[\frac{\sqrt{\frac{1}{2}}\left(e^2x^2+2\,dex+d^2\right)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2-4\,\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}d\sqrt{\frac{c}{d}}}{e^2x^2+2\,dex+d^2}\right)-2}{2\left(e^3x^2+2\,de^2x+d^2e\right)}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(1/2)*(e^2*x^2 + 2*d*e*x + d^2)*sqrt(c/d)*log(-(c*e^2*x^2 - 2*c*
d*e*x - 3*c*d^2 - 4*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqr
t(c/d))/(e^2*x^2 + 2*d*e*x + d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x +
 d))/(e^3*x^2 + 2*d*e^2*x + d^2*e), (sqrt(1/2)*(e^2*x^2 + 2*d*e*x + d^2)*s
qrt(-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt
(-c/d)/(c*e^2*x^2 - c*d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(e^3
*x^2 + 2*d*e^2*x + d^2*e)]
```

Sympy [F]

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}}{\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(5/2),x)
```

output
```
Integral(sqrt(-c*(-d + e*x)*(d + e*x))/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{-ce^2x^2 + cd^2}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{5/2}}\,dx = -\frac{\frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\,\sqrt{-cd}}\right)}{\sqrt{-cd}} + \frac{2\,\sqrt{-(ex+d)c+2\,cd}}{ex+d}}{2\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))/
sqrt(-c*d) + 2*sqrt(-(e*x + d)*c + 2*c*d)/(e*x + d))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{c\,d^2 - c\,e^2\,x^2}}{{(d+e\,x)}^{5/2}}\,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(1/2)/(d + e*x)^(5/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(1/2)/(d + e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{5/2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{-ex+d}\,d - \sqrt{d}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)\right)d - \sqrt{d}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\sqrt{\right.}\right.}{2}\right.\right.}{2de\,(ex+d)}$$

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(5/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(d - e*x)*d - sqrt(d)*sqrt(2)*log(tan(asin(sqrt(d + e*x
)/(sqrt(d)*sqrt(2)))/2))*d - sqrt(d)*sqrt(2)*log(tan(asin(sqrt(d + e*x)/(s
qrt(d)*sqrt(2)))/2))*e*x))/(2*d*e*(d + e*x))
```

3.171 $\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 141

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx=-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}+\frac{\sqrt{cd^2-ce^2x^2}}{8de(d+ex)^{3/2}}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{8\sqrt{2}d^{3/2}e}$$

output
```
-1/2*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(5/2)+1/8*(-c*e^2*x^2+c*d^2)^(1/2)
/d/e/(e*x+d)^(3/2)+1/16*c^(1/2)*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1
/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/d^(3/2)/e
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\frac{\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{2\sqrt{d}(-3d+ex)}{(d+ex)^{5/2}}+\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d^2-e^2x^2}}\right)}{16d^{3/2}e}$$

input
```
Integrate[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(7/2),x]
```

output
```
(Sqrt[c*(d^2 - e^2*x^2)]*((2*Sqrt[d]*(-3*d + e*x))/(d + e*x)^(5/2) + (Sqrt
[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/Sqrt[d^2
 - e^2*x^2]))/(16*d^(3/2)*e)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {465, 470, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \; 465$$

$$-\frac{1}{4}c\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}dx-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 470$$

$$-\frac{1}{4}c\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{4d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 471$$

$$-\frac{1}{4}c\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{2d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$-\frac{1}{4}c\left(-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}$$

input
```
Int[Sqrt[c*d^2 - c*e^2*x^2]/(d + e*x)^(7/2),x]
```

output
```
-1/2*Sqrt[c*d^2 - c*e^2*x^2]/(e*(d + e*x)^(5/2)) - (c*(-1/2*Sqrt[c*d^2 - c
*e^2*x^2]/(c*d*e*(d + e*x)^(3/2)) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[
2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(2*Sqrt[2]*Sqrt[c]*d^(3/2)*e)))/4
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.25

method	result
default	$\frac{\sqrt{c(-e^2x^2+d^2)}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)c\,e^2x^2+2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)cdex+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)c\,d^2+2\right.}{16(ex+d)^{\frac{5}{2}}\sqrt{c(-ex+d)}\,ed\sqrt{cd}}$

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(c*(-e^2*x^2+d^2))^(1/2)*(2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1
/2)/(c*d)^(1/2))*c*e^2*x^2+2*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2
)/(c*d)^(1/2))*c*d*e*x+2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d
)^(1/2))*c*d^2+2*e*x*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)-6*(c*(-e*x+d))^(1/2)*(
c*d)^(1/2)*d)/(e*x+d)^(5/2)/(c*(-e*x+d))^(1/2)/e/d/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx = \left[\frac{\sqrt{\frac{1}{2}}(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2-4\,\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right.}{16\,(de^4x^3+3\,d^2e^3x^2+3\,d^3e^2x+d^4e)}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
[1/16*(sqrt(1/2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c/d)*log(-
(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 - 4*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sq
rt(e*x + d)*d*sqrt(c/d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e^2*x^2 +
c*d^2)*sqrt(e*x + d)*(e*x - 3*d))/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x
 + d^4*e), 1/8*(sqrt(1/2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(-
c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(-c/d
)/(c*e^2*x^2 - c*d^2)) + sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*(e*x - 3*d
))/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e^2*x + d^4*e)]
```

Sympy [F]

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{7/2}}\,dx = \int \frac{\sqrt{-c\,(-d+ex)\,(d+ex)}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(1/2)/(e*x+d)**(7/2),x)
```

output
```
Integral(sqrt(-c*(-d + e*x)*(d + e*x))/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{7/2}}\,dx = \int \frac{\sqrt{-ce^2x^2 + cd^2}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*e^2*x^2 + c*d^2)/(e*x + d)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{cd^2 - ce^2x^2}}{(d+ex)^{7/2}}\,dx = -\frac{\frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\,\sqrt{-cd}}\right)}{\sqrt{-cd}d} + \frac{2\left(2\,\sqrt{-(ex+d)c+2\,cd}c^2d+(-(ex+d)c+2\,cd)^{\frac{3}{2}}c\right)}{(ex+d)^2c^2d}}{16\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-1/16*(sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))
/(sqrt(-c*d)*d) + 2*(2*sqrt(-(e*x + d)*c + 2*c*d)*c^2*d + (-(e*x + d)*c +
2*c*d)^(3/2)*c)/((e*x + d)^2*c^2*d))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\int \frac{\sqrt{c\,d^2-c\,e^2\,x^2}}{\left(d+e\,x\right)^{7/2}}\,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(1/2)/(d + e*x)^(7/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(1/2)/(d + e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}\,dx=\frac{\sqrt{d}\,\sqrt{c}\,\sqrt{2}\left(-8\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)\right)\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)^4+\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)^8\right.}{128\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)^4d^2e}$$

input
```
int((-c*e^2*x^2+c*d^2)^(1/2)/(e*x+d)^(7/2),x)
```

output
```
(sqrt(d)*sqrt(c)*sqrt(2)*( - 8*log(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)
))/2))*tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2)**4 + tan(asin(sqrt(d +
 e*x)/(sqrt(d)*sqrt(2)))/2)**8 - 1))/(128*tan(asin(sqrt(d + e*x)/(sqrt(d)*
sqrt(2)))/2)**4*d**2*e)
```

3.172 $\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} dx$

Optimal result

Integrand size = 29, antiderivative size = 201

$$\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} dx = -\frac{4096d^4(cd^2-ce^2x^2)^{5/2}}{15015ce(d+ex)^{5/2}} - \frac{1024d^3(cd^2-ce^2x^2)^{5/2}}{3003ce(d+ex)^{3/2}} - \frac{128d^2(cd^2-ce^2x^2)^{5/2}}{429ce\sqrt{d+ex}} - \frac{32d\sqrt{d+ex}(cd^2-ce^2x^2)^{5/2}}{143ce} - \frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}$$

output

```
-4096/15015*d^4*(-c*e^2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(5/2)-1024/3003*d^3*(
-c*e^2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(3/2)-128/429*d^2*(-c*e^2*x^2+c*d^2)^(
5/2)/c/e/(e*x+d)^(1/2)-32/143*d*(e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(5/2)/c/e
-2/13*(e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(5/2)/c/e
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx = -\frac{2c(d-ex)^2\sqrt{c(d^2-e^2x^2)}(9683d^4+16700d^3ex+14210d^2e^2x^2+6300de^3x^3+1155e^4x^4)}{15015e\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)]*(9683*d^4 + 16700*d^3*e*x + 1421
0*d^2*e^2*x^2 + 6300*d*e^3*x^3 + 1155*e^4*x^4))/(15015*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {459, 459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{459}$$

$$\frac{16}{13}d\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2}dx-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{16}{13}d\left(\frac{12}{11}d\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}dx-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}\right)-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{16}{13}d\left(\frac{12}{11}d\left(\frac{8}{9}d\int\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}\right)-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}$$

$\big\downarrow$ 459

$$\frac{16}{13}d\left(\frac{12}{11}d\left(\frac{8}{9}d\left(\frac{4}{7}d\int\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}\right)-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}$$

$\big\downarrow$ 458

$$\frac{16}{13}d\left(\frac{12}{11}d\left(\frac{8}{9}d\left(-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{35ce(d+ex)^{5/2}}\right)-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}\right)-\frac{2(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}{13ce}$$

input
```
Int[(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(5/2))/(13*c*e) + (16*d*((-2*Sqrt[
d + e*x]*(c*d^2 - c*e^2*x^2)^(5/2))/(11*c*e) + (12*d*((-2*(c*d^2 - c*e^2*x
^2)^(5/2))/(9*c*e*Sqrt[d + e*x]) + (8*d*((-8*d*(c*d^2 - c*e^2*x^2)^(5/2))/
(35*c*e*(d + e*x)^(5/2)) - (2*(c*d^2 - c*e^2*x^2)^(5/2))/(7*c*e*(d + e*x)^
(3/2))))/9))/11))/13
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.38

method	result	s
gosper	$-\frac{2(-ex+d)\left(1155e^{4}x^{4}+6300d\,e^{3}x^{3}+14210d^{2}e^{2}x^{2}+16700d^{3}ex+9683d^{4}\right)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{15015e\left(ex+d\right)^{\frac{3}{2}}}$	7
orering	$-\frac{2(-ex+d)\left(1155e^{4}x^{4}+6300d\,e^{3}x^{3}+14210d^{2}e^{2}x^{2}+16700d^{3}ex+9683d^{4}\right)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{15015e\left(ex+d\right)^{\frac{3}{2}}}$	7
default	$-\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\,c\left(-ex+d\right)^{2}\left(1155e^{4}x^{4}+6300d\,e^{3}x^{3}+14210d^{2}e^{2}x^{2}+16700d^{3}ex+9683d^{4}\right)}{15015\sqrt{ex+d}\,e}$	7
risch	$-\frac{2\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}\,c^{2}\left(1155e^{6}x^{6}+3990d\,e^{5}x^{5}+2765d^{2}e^{4}x^{4}-5420d^{3}e^{3}x^{3}-9507d^{4}e^{2}x^{2}-2666d^{5}ex+9683d^{6}\right)\left(-ex+d\right)}{15015\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,e\sqrt{-c\left(ex-d\right)}}$	1

input
```
int((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15015*(-e*x+d)*(1155*e^4*x^4+6300*d*e^3*x^3+14210*d^2*e^2*x^2+16700*d^3
*e*x+9683*d^4)*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.53

$$\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx = -\frac{2 \left(1155 \, ce^6x^6+3990 \, cde^5x^5+2765 \, cd^2e^4x^4-5420 \, cd^3e^3x^3-9507 \, cd^4e^2x^2-2666 \, cd^5ex+9683 \, cd^6\right)\sqrt{-}}{15015 \left(e^2x+de\right)}$$

input
```
integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/15015*(1155*c*e^6*x^6 + 3990*c*d*e^5*x^5 + 2765*c*d^2*e^4*x^4 - 5420*c*
d^3*e^3*x^3 - 9507*c*d^4*e^2*x^2 - 2666*c*d^5*e*x + 9683*c*d^6)*sqrt(-c*e^
2*x^2 + c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F]

$$\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx = \int \left(-c(-d+ex)\left(d+ex\right)\right)^{\frac{3}{2}} \left(d+ex\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x+d)**(5/2)*(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.55

$$\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx = -\frac{2 \left(1155 \, c^{\frac{3}{2}}e^6x^6+3990 \, c^{\frac{3}{2}}de^5x^5+2765 \, c^{\frac{3}{2}}d^2e^4x^4-5420 \, c^{\frac{3}{2}}d^3e^3x^3-9507 \, c^{\frac{3}{2}}d^4e^2x^2-2666 \, c^{\frac{3}{2}}d^5ex+9683\right.}{15015 \left(e^2x+de\right)}$$

input
```
integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
-2/15015*(1155*c^(3/2)*e^6*x^6 + 3990*c^(3/2)*d*e^5*x^5 + 2765*c^(3/2)*d^2
*e^4*x^4 - 5420*c^(3/2)*d^3*e^3*x^3 - 9507*c^(3/2)*d^4*e^2*x^2 - 2666*c^(3
/2)*d^5*e*x + 9683*c^(3/2)*d^6)*(e*x + d)*sqrt(-e*x + d)/(e^2*x + d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(171) = 342$.

Time = 0.14 (sec) , antiderivative size = 687, normalized size of antiderivative = 3.42

$$\int (d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{2\left(45045\sqrt{-cex+cd}cd^6+30030\left(3\sqrt{-cex+cd}cd-(-cex+cd)^{\frac{3}{2}}\right)d^5-\frac{3003\left(15\sqrt{-cex+cd}c^2d^2-10\,(-cex+c\right.}{c}\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
-2/45045*(45045*sqrt(-c*e*x + c*d)*c*d^6 + 30030*(3*sqrt(-c*e*x + c*d)*c*d
 - (-c*e*x + c*d)^(3/2))*d^5 - 3003*(15*sqrt(-c*e*x + c*d)*c^2*d^2 - 10*(-
c*e*x + c*d)^(3/2)*c*d + 3*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d))*d^4/c - 514
8*(35*sqrt(-c*e*x + c*d)*c^3*d^3 - 35*(-c*e*x + c*d)^(3/2)*c^2*d^2 + 21*(c
*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c*d + 5*(c*e*x - c*d)^3*sqrt(-c*e*x + c*d
))*d^3/c^2 - 143*(315*sqrt(-c*e*x + c*d)*c^4*d^4 - 420*(-c*e*x + c*d)^(3/2
)*c^3*d^3 + 378*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c^2*d^2 + 180*(c*e*x -
c*d)^3*sqrt(-c*e*x + c*d)*c*d + 35*(c*e*x - c*d)^4*sqrt(-c*e*x + c*d))*d^2
/c^3 + 130*(693*sqrt(-c*e*x + c*d)*c^5*d^5 - 1155*(-c*e*x + c*d)^(3/2)*c^4
*d^4 + 1386*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c^3*d^3 + 990*(c*e*x - c*d)
^3*sqrt(-c*e*x + c*d)*c^2*d^2 + 385*(c*e*x - c*d)^4*sqrt(-c*e*x + c*d)*c*d
 + 63*(c*e*x - c*d)^5*sqrt(-c*e*x + c*d))*d/c^4 + 15*(3003*sqrt(-c*e*x + c
*d)*c^6*d^6 - 6006*(-c*e*x + c*d)^(3/2)*c^5*d^5 + 9009*(c*e*x - c*d)^2*sqr
t(-c*e*x + c*d)*c^4*d^4 + 8580*(c*e*x - c*d)^3*sqrt(-c*e*x + c*d)*c^3*d^3
+ 5005*(c*e*x - c*d)^4*sqrt(-c*e*x + c*d)*c^2*d^2 + 1638*(c*e*x - c*d)^5*s
qrt(-c*e*x + c*d)*c*d + 231*(c*e*x - c*d)^6*sqrt(-c*e*x + c*d))/c^5)/e
```

Mupad [B] (verification not implemented)

Time = 7.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int (d+ex)^{5/2} \left(cd^2-ce^2x^2\right)^{3/2} dx = -\frac{16384\,c\,d^6\,\sqrt{c\,d^2-c\,e^2\,x^2}}{15015\,e\,\sqrt{d+e\,x}} - \frac{2\,c\,\sqrt{c\,d^2-c\,e^2\,x^2}\,\sqrt{d+e\,x}\,(1491\,d^5-4157\,d^4\,e\,x-5350\,d^3\,e^2\,x^2-70\,d^2\,e^3\,x^3+2835\,d\,e^4\,x^4+1155\,e^5\,x}{15015\,e}$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)*(d + e*x)^(5/2),x)
```

output
```
- (16384*c*d^6*(c*d^2 - c*e^2*x^2)^(1/2))/(15015*e*(d + e*x)^(1/2)) - (2*c
*(c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(1/2)*(1491*d^5 + 1155*e^5*x^5 + 2835
*d*e^4*x^4 - 5350*d^3*e^2*x^2 - 70*d^2*e^3*x^3 - 4157*d^4*e*x))/(15015*e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.39

$$\int (d+ex)^{5/2} \left(cd^2 -ce^2x^2\right)^{3/2} dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,c(-1155e^6x^6-3990d\,e^5x^5-2765d^2e^4x^4+5420d^3e^3x^3+9507d^4e^2x^2+2}{15015e}$$

input
```
int((e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*c*( - 9683*d**6 + 2666*d**5*e*x + 9507*d**4*e**2*
x**2 + 5420*d**3*e**3*x**3 - 2765*d**2*e**4*x**4 - 3990*d*e**5*x**5 - 1155
*e**6*x**6))/(15015*e)
```

3.173 $\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx = -\frac{256d^3(cd^2-ce^2x^2)^{5/2}}{1155ce(d+ex)^{5/2}} - \frac{64d^2(cd^2-ce^2x^2)^{5/2}}{231ce(d+ex)^{3/2}} - \frac{8d(cd^2-ce^2x^2)^{5/2}}{33ce\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(cd^2-ce^2x^2)^{5/2}}{11ce}$$

output
```
-256/1155*d^3*(-c*e^2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(5/2)-64/231*d^2*(-c*e^
2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(3/2)-8/33*d*(-c*e^2*x^2+c*d^2)^(5/2)/c/e/(
e*x+d)^(1/2)-2/11*(e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(5/2)/c/e
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.46

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} \, dx = -\frac{2c(d-ex)^2\sqrt{c(d^2-e^2x^2)}(533d^3+755d^2ex+455de^2x^2+105e^3x^3)}{1155e\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)]*(533*d^3 + 755*d^2*e*x + 455*d*e
^2*x^2 + 105*e^3*x^3))/(1155*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 459}$$

$$\frac{12}{11}d\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}dx-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}$$

$$\Big\downarrow \text{ 459}$$

$$\frac{12}{11}d\left(\frac{8}{9}d\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}$$

$$\Big\downarrow \text{ 459}$$

$$\frac{12}{11}d\left(\frac{8}{9}d\left(\frac{4}{7}d\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}$$

$$\Big\downarrow \text{ 458}$$

$$\frac{12}{11}d\left(\frac{8}{9}d\left(-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{35ce(d+ex)^{5/2}}\right)-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}\right)-\frac{2\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}{11ce}$$

input
```
Int[(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(5/2))/(11*c*e) + (12*d*((-2*(c*d^2
- c*e^2*x^2)^(5/2))/(9*c*e*Sqrt[d + e*x]) + (8*d*((-8*d*(c*d^2 - c*e^2*x^2
)^(5/2))/(35*c*e*(d + e*x)^(5/2)) - (2*(c*d^2 - c*e^2*x^2)^(5/2))/(7*c*e*(
d + e*x)^(3/2))))/9))/11
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.41

method	result	size
gosper	$-\frac{2(-ex+d)\left(105e^{3}x^{3}+455d\,e^{2}x^{2}+755d^{2}ex+533d^{3}\right)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{1155e\left(ex+d\right)^{\frac{3}{2}}}$	66
orering	$-\frac{2(-ex+d)\left(105e^{3}x^{3}+455d\,e^{2}x^{2}+755d^{2}ex+533d^{3}\right)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{1155e\left(ex+d\right)^{\frac{3}{2}}}$	66
default	$-\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\,c\left(-ex+d\right)^{2}\left(105e^{3}x^{3}+455d\,e^{2}x^{2}+755d^{2}ex+533d^{3}\right)}{1155\sqrt{ex+d}\,e}$	68
risch	$-\frac{2\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}\,c^{2}\left(105e^{5}x^{5}+245d\,e^{4}x^{4}-50d^{2}e^{3}x^{3}-522d^{3}e^{2}x^{2}-311d^{4}ex+533d^{5}\right)\left(-ex+d\right)}{1155\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,e\sqrt{-c\left(ex-d\right)}}$	129

input
```
int((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(-e*x+d)*(105*e^3*x^3+455*d*e^2*x^2+755*d^2*e*x+533*d^3)*(-c*e^2*x
^2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} dx = -\frac{2 \left(105 \, ce^5x^5+245 \, cde^4x^4-50 \, cd^2e^3x^3-522 \, cd^3e^2x^2-311 \, cd^4ex+533 \, cd^5\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{1155 \left(e^2x+de\right)}$$

input
```
integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/1155*(105*c*e^5*x^5 + 245*c*d*e^4*x^4 - 50*c*d^2*e^3*x^3 - 522*c*d^3*e^
2*x^2 - 311*c*d^4*e*x + 533*c*d^5)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/
(e^2*x + d*e)
```

Sympy [F]

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} dx = \int \left(-c(-d+ex)\left(d+ex\right)\right)^{\frac{3}{2}} \left(d+ex\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*x+d)**(3/2)*(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.60

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} dx =$$

$$-\frac{2\left(105\,c^{\frac{3}{2}}e^5x^5+245\,c^{\frac{3}{2}}de^4x^4-50\,c^{\frac{3}{2}}d^2e^3x^3-522\,c^{\frac{3}{2}}d^3e^2x^2-311\,c^{\frac{3}{2}}d^4ex+533\,c^{\frac{3}{2}}d^5\right)(ex+d)\sqrt{-ex+d}}{1155\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
-2/1155*(105*c^(3/2)*e^5*x^5 + 245*c^(3/2)*d*e^4*x^4 - 50*c^(3/2)*d^2*e^3*
x^3 - 522*c^(3/2)*d^3*e^2*x^2 - 311*c^(3/2)*d^4*e*x + 533*c^(3/2)*d^5)*(e*
x + d)*sqrt(-e*x + d)/(e^2*x + d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. 2(136) = 272.

Time = 0.13 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.12

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} dx =$$

$$-\frac{2\left(3465\sqrt{-cex+cd}cd^5+1155\left(3\sqrt{-cex+cd}cd-(-cex+cd)^{\frac{3}{2}}\right)d^4-\frac{462\left(15\sqrt{-cex+cd}c^2d^2-10\,(-cex+cd)^{\frac{3}{2}}c\right.}{c}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output

```
-2/3465*(3465*sqrt(-c*e*x + c*d)*c*d^5 + 1155*(3*sqrt(-c*e*x + c*d)*c*d -
(-c*e*x + c*d)^(3/2))*d^4 - 462*(15*sqrt(-c*e*x + c*d)*c^2*d^2 - 10*(-c*e*
x + c*d)^(3/2)*c*d + 3*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d))*d^3/c - 198*(35
*sqrt(-c*e*x + c*d)*c^3*d^3 - 35*(-c*e*x + c*d)^(3/2)*c^2*d^2 + 21*(c*e*x
- c*d)^2*sqrt(-c*e*x + c*d)*c*d + 5*(c*e*x - c*d)^3*sqrt(-c*e*x + c*d))*d^
2/c^2 + 11*(315*sqrt(-c*e*x + c*d)*c^4*d^4 - 420*(-c*e*x + c*d)^(3/2)*c^3*
d^3 + 378*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c^2*d^2 + 180*(c*e*x - c*d)^3
*sqrt(-c*e*x + c*d)*c*d + 35*(c*e*x - c*d)^4*sqrt(-c*e*x + c*d))*d/c^3 + 5
*(693*sqrt(-c*e*x + c*d)*c^5*d^5 - 1155*(-c*e*x + c*d)^(3/2)*c^4*d^4 + 138
6*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c^3*d^3 + 990*(c*e*x - c*d)^3*sqrt(-c
*e*x + c*d)*c^2*d^2 + 385*(c*e*x - c*d)^4*sqrt(-c*e*x + c*d)*c*d + 63*(c*e
*x - c*d)^5*sqrt(-c*e*x + c*d))/c^4)/e
```

Mupad [B] (verification not implemented)

Time = 7.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\left(\frac{1066\,c\,d^5\,\sqrt{d+e\,x}}{1155\,e^2}-\frac{348\,c\,d^3\,x^2\,\sqrt{d+e\,x}}{385}+\frac{2\,c\,e^3\,x^5\,\sqrt{d+e\,x}}{11}-\frac{20\,c\,d^2\,e\,x^3\,\sqrt{d+e\,x}}{231}-\frac{622\,c\,d^4\,x\,\sqrt{d+e\,x}}{1155\,e}+\frac{14\,c\,d\,e^2 \cdots}{}\right.}{x+\frac{d}{e}}$$

input

```
int((c*d^2 - c*e^2*x^2)^(3/2)*(d + e*x)^(3/2),x)
```

output

```
-((c*d^2 - c*e^2*x^2)^(1/2)*((1066*c*d^5*(d + e*x)^(1/2))/(1155*e^2) - (34
8*c*d^3*x^2*(d + e*x)^(1/2))/385 + (2*c*e^3*x^5*(d + e*x)^(1/2))/11 - (20*
c*d^2*e*x^3*(d + e*x)^(1/2))/231 - (622*c*d^4*x*(d + e*x)^(1/2))/(1155*e)
+ (14*c*d*e^2*x^4*(d + e*x)^(1/2))/33))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

$$\int (d+ex)^{3/2} \left(cd^2-ce^2x^2\right)^{3/2} dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,c(-105e^5x^5-245d\,e^4x^4+50d^2e^3x^3+522d^3e^2x^2+311d^4ex-533d^5)}{1155e}$$

input
```
int((e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*c*( - 533*d**5 + 311*d**4*e*x + 522*d**3*e**2*x**
2 + 50*d**2*e**3*x**3 - 245*d*e**4*x**4 - 105*e**5*x**5))/(1155*e)
```

3.174 $\int \sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}\,dx=-\frac{64d^2(cd^2-ce^2x^2)^{5/2}}{315ce(d+ex)^{5/2}}-\frac{16d(cd^2-ce^2x^2)^{5/2}}{63ce(d+ex)^{3/2}}-\frac{2(cd^2-ce^2x^2)^{5/2}}{9ce\sqrt{d+ex}}$$

output
```
-64/315*d^2*(-c*e^2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(5/2)-16/63*d*(-c*e^2*x^2
+c*d^2)^(5/2)/c/e/(e*x+d)^(3/2)-2/9*(-c*e^2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.52

$$\int \sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}\,dx=-\frac{2c(d-ex)^2\sqrt{c\,(d^2-e^2x^2)}(107d^2+110dex+35e^2x^2)}{315e\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)]*(107*d^2 + 110*d*e*x + 35*e^2*x^
2))/(315*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{9}d\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{9}d\left(\frac{4}{7}d\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}dx-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}\right)-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}$$

$$\Big\downarrow \text{458}$$

$$\frac{8}{9}d\left(-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{7ce(d+ex)^{3/2}}-\frac{8d\left(cd^2-ce^2x^2\right)^{5/2}}{35ce(d+ex)^{5/2}}\right)-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{9ce\sqrt{d+ex}}$$

input
```
Int[Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*(c*d^2 - c*e^2*x^2)^(5/2))/(9*c*e*Sqrt[d + e*x]) + (8*d*((-8*d*(c*d^2
- c*e^2*x^2)^(5/2))/(35*c*e*(d + e*x)^(5/2)) - (2*(c*d^2 - c*e^2*x^2)^(5/2
))/(7*c*e*(d + e*x)^(3/2))))/9
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{2(-ex+d)\left(35e^{2}x^{2}+110dex+107d^{2}\right)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{315e(ex+d)^{\frac{3}{2}}}$	55
orering	$-\frac{2(-ex+d)\left(35e^{2}x^{2}+110dex+107d^{2}\right)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{315e(ex+d)^{\frac{3}{2}}}$	55
default	$-\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\,c(-ex+d)^{2}\left(35e^{2}x^{2}+110dex+107d^{2}\right)}{315\sqrt{ex+d}\,e}$	57
risch	$-\frac{2\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}\,c^{2}\left(35e^{4}x^{4}+40d\,e^{3}x^{3}-78d^{2}e^{2}x^{2}-104d^{3}ex+107d^{4}\right)(-ex+d)}{315\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,e\sqrt{-c(ex-d)}}$	118

input
```
int((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(-e*x+d)*(35*e^2*x^2+110*d*e*x+107*d^2)*(-c*e^2*x^2+c*d^2)^(3/2)/e/
(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx = -\frac{2\,(35\,ce^4x^4+40\,cde^3x^3-78\,cd^2e^2x^2-104\,cd^3ex+107\,cd^4)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{315\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/315*(35*c*e^4*x^4 + 40*c*d*e^3*x^3 - 78*c*d^2*e^2*x^2 - 104*c*d^3*e*x +
 107*c*d^4)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx = \int \left(-c(-d+ex)\,(d+ex)\right)^{\frac{3}{2}}\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)*sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx = -\frac{2\left(35\,c^{\frac{3}{2}}e^4x^4+40\,c^{\frac{3}{2}}de^3x^3-78\,c^{\frac{3}{2}}d^2e^2x^2-104\,c^{\frac{3}{2}}d^3ex+107\,c^{\frac{3}{2}}d^4\right)(ex+d)\sqrt{-ex+d}}{315\,(e^2x+de)}$$

input
```
integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
-2/315*(35*c^(3/2)*e^4*x^4 + 40*c^(3/2)*d*e^3*x^3 - 78*c^(3/2)*d^2*e^2*x^2
 - 104*c^(3/2)*d^3*e*x + 107*c^(3/2)*d^4)*(e*x + d)*sqrt(-e*x + d)/(e^2*x
+ d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(101) = 202$.

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.79

$$\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{2\left(315\sqrt{-cex+cd}cd^4-\frac{42\left(15\sqrt{-cex+cd}c^2d^2-10\left(-cex+cd\right)^{\frac{3}{2}}cd+3\left(cex-cd\right)^2\sqrt{-cex+cd}\right)d^2}{c}+\frac{315\sqrt{-cex+cd}c^4d^4-420\left(-cex\right.}{}\right.}{315\,e}$$

input
```
integrate((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
-2/315*(315*sqrt(-c*e*x + c*d)*c*d^4 - 42*(15*sqrt(-c*e*x + c*d)*c^2*d^2 -
 10*(-c*e*x + c*d)^(3/2)*c*d + 3*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d))*d^2/c
 + (315*sqrt(-c*e*x + c*d)*c^4*d^4 - 420*(-c*e*x + c*d)^(3/2)*c^3*d^3 + 37
8*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d)*c^2*d^2 + 180*(c*e*x - c*d)^3*sqrt(-c
*e*x + c*d)*c*d + 35*(c*e*x - c*d)^4*sqrt(-c*e*x + c*d))/c^3)/e
```

Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{214\,c\,d^4\,\sqrt{d+e\,x}}{315\,e^2}-\frac{52\,c\,d^2\,x^2\,\sqrt{d+e\,x}}{105}+\frac{2\,c\,e^2\,x^4\,\sqrt{d+e\,x}}{9}-\frac{208\,c\,d^3\,x\,\sqrt{d+e\,x}}{315\,e}+\frac{16\,c\,d\,e\,x^3\,\sqrt{d+e\,x}}{63}\right)}{x+\frac{d}{e}}$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)*(d + e*x)^(1/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((214*c*d^4*(d + e*x)^(1/2))/(315*e^2) - (52*c
*d^2*x^2*(d + e*x)^(1/2))/105 + (2*c*e^2*x^4*(d + e*x)^(1/2))/9 - (208*c*d
^3*x*(d + e*x)^(1/2))/(315*e) + (16*c*d*e*x^3*(d + e*x)^(1/2))/63))/(x + d
/e)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

$$\int \sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}\,dx=\frac{2\sqrt{c}\,\sqrt{-ex+d}\,c(-35e^4x^4-40d\,e^3x^3+78d^2e^2x^2+104d^3ex-107d^4)}{315e}$$

input
```
int((e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*c*( - 107*d**4 + 104*d**3*e*x + 78*d**2*e**2*x**2
 - 40*d*e**3*x**3 - 35*e**4*x**4))/(315*e)
```

3.175 $\int \frac{(cd^2-ce^2x^2)^{3/2}}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{\sqrt{d+ex}}\,dx=-\frac{8d(cd^2-ce^2x^2)^{5/2}}{35ce(d+ex)^{5/2}}-\frac{2(cd^2-ce^2x^2)^{5/2}}{7ce(d+ex)^{3/2}}$$

output
```
-8/35*d*(-c*e^2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(5/2)-2/7*(-c*e^2*x^2+c*d^2)^
(5/2)/c/e/(e*x+d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{\sqrt{d+ex}}\,dx=-\frac{2c(d-ex)^2(9d+5ex)\sqrt{c(d^2-e^2x^2)}}{35e\sqrt{d+ex}}$$

input
```
Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/Sqrt[d + e*x],x]
```

output
```
(-2*c*(d - e*x)^2*(9*d + 5*e*x)*Sqrt[c*(d^2 - e^2*x^2)])/(35*e*Sqrt[d + e*
x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{ 459}$$

$$\frac{4}{7}d\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{3/2}}dx-\frac{2(cd^2-ce^2x^2)^{5/2}}{7ce(d+ex)^{3/2}}$$

$$\Big\downarrow \text{ 458}$$

$$-\frac{2(cd^2-ce^2x^2)^{5/2}}{7ce(d+ex)^{3/2}}-\frac{8d(cd^2-ce^2x^2)^{5/2}}{35ce(d+ex)^{5/2}}$$

input
```
Int[(c*d^2 - c*e^2*x^2)^(3/2)/Sqrt[d + e*x],x]
```

output
```
(-8*d*(c*d^2 - c*e^2*x^2)^(5/2))/(35*c*e*(d + e*x)^(5/2)) - (2*(c*d^2 - c*
e^2*x^2)^(5/2))/(7*c*e*(d + e*x)^(3/2))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{2(-ex+d)(5ex+9d)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{35e(ex+d)^{\frac{3}{2}}}$	44
orering	$-\frac{2(-ex+d)(5ex+9d)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{35e(ex+d)^{\frac{3}{2}}}$	44
default	$-\frac{2\sqrt{c(-e^{2}x^{2}+d^{2})}\,c(-ex+d)^{2}(5ex+9d)}{35\sqrt{ex+d}\,e}$	46
risch	$-\frac{2\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}\,c^{2}\left(5e^{3}x^{3}-d\,e^{2}x^{2}-13d^{2}ex+9d^{3}\right)(-ex+d)}{35\sqrt{-c(e^{2}x^{2}-d^{2})}\,e\sqrt{-c(ex-d)}}$	107

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-e*x+d)*(5*e*x+9*d)*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{\sqrt{d+ex}}\,dx = -\frac{2\left(5\,ce^3x^3-cde^2x^2-13\,cd^2ex+9\,cd^3\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{35\left(e^2x+de\right)}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
-2/35*(5*c*e^3*x^3 - c*d*e^2*x^2 - 13*c*d^2*e*x + 9*c*d^3)*sqrt(-c*e^2*x^2
 + c*d^2)*sqrt(e*x + d)/(e^2*x + d*e)
```

Sympy [F]

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=\int \frac{\left(-c(-d+ex)\,(d+ex)\right)^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(1/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/sqrt(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=-\frac{2\left(5\,c^{\frac{3}{2}}e^3x^3-c^{\frac{3}{2}}de^2x^2-13\,c^{\frac{3}{2}}d^2ex+9\,c^{\frac{3}{2}}d^3\right)\sqrt{-ex+d}}{35\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
-2/35*(5*c^(3/2)*e^3*x^3 - c^(3/2)*d*e^2*x^2 - 13*c^(3/2)*d^2*e*x + 9*c^(3
/2)*d^3)*sqrt(-e*x + d)/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(66) = 132.

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.77

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{\sqrt{d+ex}}\,dx=$$
$$-\frac{2\left(105\sqrt{-cex+cd}cd^3-35\left(3\sqrt{-cex+cd}cd-(-cex+cd)^{\frac{3}{2}}\right)d^2-\frac{7\left(15\sqrt{-cex+cd}c^2d^2-10\,(-cex+cd)^{\frac{3}{2}}cd+3\,(c\cdots}{c}\cdots\right)}{105\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
-2/105*(105*sqrt(-c*e*x + c*d)*c*d^3 - 35*(3*sqrt(-c*e*x + c*d)*c*d - (-c*
e*x + c*d)^(3/2))*d^2 - 7*(15*sqrt(-c*e*x + c*d)*c^2*d^2 - 10*(-c*e*x + c*
d)^(3/2)*c*d + 3*(c*e*x - c*d)^2*sqrt(-c*e*x + c*d))*d/c + 3*(35*sqrt(-c*e
*x + c*d)*c^3*d^3 - 35*(-c*e*x + c*d)^(3/2)*c^2*d^2 + 21*(c*e*x - c*d)^2*s
qrt(-c*e*x + c*d)*c*d + 5*(c*e*x - c*d)^3*sqrt(-c*e*x + c*d))/c^2)/e
```

Mupad [B] (verification not implemented)

Time = 7.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{\sqrt{d + ex}}\,dx = -\frac{\sqrt{c\,d^2 - c\,e^2\,x^2}\,\left(\frac{18\,c\,d^3}{35\,e} + \frac{2\,c\,e^2\,x^3}{7} - \frac{26\,c\,d^2\,x}{35} - \frac{2\,c\,d\,e\,x^2}{35}\right)}{\sqrt{d + e\,x}}$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(1/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((18*c*d^3)/(35*e) + (2*c*e^2*x^3)/7 - (26*c*d
^2*x)/35 - (2*c*d*e*x^2)/35))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{\sqrt{d + ex}}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex + d}\,c(-5e^3x^3 + d\,e^2x^2 + 13d^2ex - 9d^3)}{35e}$$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*c*( - 9*d**3 + 13*d**2*e*x + d*e**2*x**2 - 5*e**3
*x**3))/(35*e)
```

3.176 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 38

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=-\frac{2\left(cd^2-ce^2x^2\right)^{5/2}}{5ce(d+ex)^{5/2}}$$

output
```
-2/5*(-c*e^2*x^2+c*d^2)^(5/2)/c/e/(e*x+d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx=-\frac{2c(d-ex)^2\sqrt{c\left(d^2-e^2x^2\right)}}{5e\sqrt{d+ex}}$$

input
```
Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(-2*c*(d - e*x)^2*Sqrt[c*(d^2 - e^2*x^2)])/(5*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 458}$$

$$-\frac{2(cd^2 - ce^2x^2)^{5/2}}{5ce(d+ex)^{5/2}}$$

input
```
Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(-2*(c*d^2 - c*e^2*x^2)^(5/2))/(5*c*e*(d + e*x)^(5/2))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{2(-ex+d)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{5e(ex+d)^{\frac{3}{2}}}$	36
orering	$-\frac{2(-ex+d)\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}{5e(ex+d)^{\frac{3}{2}}}$	36
default	$-\frac{2\sqrt{c(-e^{2}x^{2}+d^{2})}\,c(-ex+d)^{2}}{5\sqrt{ex+d}\,e}$	38
risch	$-\frac{2\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}\,c^{2}\left(e^{2}x^{2}-2dex+d^{2}\right)(-ex+d)}{5\sqrt{-c(e^{2}x^{2}-d^{2})}\,e\sqrt{-c(ex-d)}}$	93

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-e*x+d)*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{2\,(ce^2x^2-2\,cdex+cd^2)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{5\,(e^2x+de)}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(c*e^2*x^2 - 2*c*d*e*x + c*d^2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d
)/(e^2*x + d*e)
```

Sympy [F]

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(-c(-d+ex)\,(d+ex))^{\frac{3}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(3/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{3/2}}\, dx = -\frac{2\left(c^{\frac{3}{2}}e^2x^2 - 2\,c^{\frac{3}{2}}dex + c^{\frac{3}{2}}d^2\right)\sqrt{-ex + d}}{5\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
-2/5*(c^(3/2)*e^2*x^2 - 2*c^(3/2)*d*e*x + c^(3/2)*d^2)*sqrt(-e*x + d)/e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(32) = 64.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{3/2}}\, dx = -\frac{2\left(5\left(-(ex + d)c + 2\,cd\right)^{\frac{3}{2}}d - \frac{5\left(-(ex+d)c+2\,cd\right)^{\frac{3}{2}}cd - 3\left((ex+d)c-2\,cd\right)^2\sqrt{-(ex+d)c+2\,cd}}{c}\right)}{15\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
-2/15*(5*(-(e*x + d)*c + 2*c*d)^(3/2)*d - (5*(-(e*x + d)*c + 2*c*d)^(3/2)*
c*d - 3*((e*x + d)*c - 2*c*d)^2*sqrt(-(e*x + d)*c + 2*c*d))/c)/e
```

Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{3/2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{2\,c\,d^2}{5\,e}-\frac{4\,c\,d\,x}{5}+\frac{2\,c\,e\,x^2}{5}\right)}{\sqrt{d+e\,x}}$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((2*c*d^2)/(5*e) - (4*c*d*x)/5 + (2*c*e*x^2)/5
))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,c(-e^2x^2+2dex-d^2)}{5e}$$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*c*( - d**2 + 2*d*e*x - e**2*x**2))/(5*e)
```

3.177 $\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 136

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{4cd\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} + \frac{2(cd^2-ce^2x^2)^{3/2}}{3e(d+ex)^{3/2}} - \frac{4\sqrt{2}c^{3/2}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{e}$$

output
```
4*c*d*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(1/2)+2/3*(-c*e^2*x^2+c*d^2)^(3/2
)/e/(e*x+d)^(3/2)-4*2^(1/2)*c^(3/2)*d^(3/2)*arctanh(2^(1/2)*c^(1/2)*d^(1/2
)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2c\sqrt{c(d^2-e^2x^2)}\left(\frac{7d-ex}{\sqrt{d+ex}} - \frac{6\sqrt{2}d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d^2-e^2x^2}}\right)}{3e}$$

input
```
Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

output
```
(2*c*Sqrt[c*(d^2 - e^2*x^2)]*((7*d - e*x)/Sqrt[d + e*x] - (6*Sqrt[2]*d^(3/
2)*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/Sqrt[d^2
- e^2*x^2]))/(3*e)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {466, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{466}$$

$$2cd\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}dx+\frac{2(cd^2-ce^2x^2)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{466}$$

$$2cd\left(2cd\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx+\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}\right)+\frac{2(cd^2-ce^2x^2)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{471}$$

$$2cd\left(4cde\int \frac{1}{\frac{e^2(cd^2-ce^2x^2)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}+\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}\right)+\frac{2(cd^2-ce^2x^2)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$2cd\left(\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}\right)+\frac{2(cd^2-ce^2x^2)^{3/2}}{3e(d+ex)^{3/2}}$$

input
```
Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

output
```
(2*(c*d^2 - c*e^2*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) + 2*c*d*((2*Sqrt[c*d^2
 - c*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2]*Sqrt[c]*Sqrt[d]*ArcTanh[Sqrt
[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{2\sqrt{c(-e^2x^2+d^2)}\,c\left(6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)c\,d^2+ex\sqrt{c(-ex+d)}\,\sqrt{cd}-7\sqrt{c(-ex+d)}\,\sqrt{cd}\,d\right)}{3\sqrt{ex+d}\,\sqrt{c(-ex+d)}\,e\sqrt{cd}}$	112
risch	$\frac{2(-ex+7d)(-ex+d)\sqrt{-\frac{c\left(e^2x^2-d^2\right)}{ex+d}}\,\sqrt{ex+d}\,c^2}{3e\sqrt{-c(ex-d)}\,\sqrt{-c(e^2x^2-d^2)}}-\frac{4d^2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-cex+cd}\,\sqrt{2}}{2\sqrt{cd}}\right)\sqrt{-\frac{c\left(e^2x^2-d^2\right)}{ex+d}}\,\sqrt{ex+d}\,c^2}{e\sqrt{cd}\,\sqrt{-c(e^2x^2-d^2)}}$	177

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(c*(-e^2*x^2+d^2))^(1/2)*c*(6*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*
2^(1/2)/(c*d)^(1/2))*c*d^2+e*x*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)-7*(c*(-e*x+d
))^(1/2)*(c*d)^(1/2)*d)/(e*x+d)^(1/2)/(c*(-e*x+d))^(1/2)/e/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.96

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \left[\frac{2\left(3\sqrt{2}(cdex+cd^2)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-\sqrt{-}\right.}{3\,(e^2x+de)}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
[2/3*(3*sqrt(2)*(c*d*e*x + c*d^2)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*e*x -
3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2
*x^2 + 2*d*e*x + d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*(c*e*x - 7*c*d)*sqrt(e*x
 + d))/(e^2*x + d*e), 2/3*(6*sqrt(2)*(c*d*e*x + c*d^2)*sqrt(-c*d)*arctan(1
/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*d*e*x + c*
d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*(c*e*x - 7*c*d)*sqrt(e*x + d))/(e^2*x + d
*e)]
```

Sympy [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(-c(-d+ex)\,(d+ex))^{\frac{3}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(5/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{6\sqrt{2}cd^2\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} + \frac{6\sqrt{-(ex+d)c+2cd}c^3d+(-(ex+d)c+2cd)^{\frac{3}{2}}c^2}{c^3}\right)c}{3e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
2/3*(6*sqrt(2)*c*d^2*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c
*d))/sqrt(-c*d) + (6*sqrt(-(e*x + d)*c + 2*c*d)*c^3*d + (-(e*x + d)*c + 2*
c*d)^(3/2)*c^2)/c^3)*c/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(c\,d^2 - c\,e^2\,x^2)^{3/2}}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(5/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{c}\,c\left(7\sqrt{-ex+d}\,d - \sqrt{-ex+d}\,ex + 6\sqrt{d}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)\right)d - 8\sqrt{}\right.}{3e}$$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(5/2),x)
```

output
```
(2*sqrt(c)*c*(7*sqrt(d - e*x)*d - sqrt(d - e*x)*e*x + 6*sqrt(d)*sqrt(2)*lo
g(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2))*d - 8*sqrt(d)*sqrt(2)*d))/
(3*e)
```

3.178 $\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 133

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx = -\frac{3c\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}} - \frac{(cd^2-ce^2x^2)^{3/2}}{e(d+ex)^{5/2}} + \frac{3\sqrt{2}c^{3/2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{e}$$

output
```
-3*c*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(1/2)-(-c*e^2*x^2+c*d^2)^(3/2)/e/(
e*x+d)^(5/2)+3*2^(1/2)*c^(3/2)*d^(1/2)*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*
x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx = \frac{c\sqrt{c(d^2-e^2x^2)}\left(-\frac{2(2d+ex)}{(d+ex)^{3/2}} + \frac{3\sqrt{2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d^2-e^2x^2}}\right)}{e}$$

input
```
Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

output
```
(c*Sqrt[c*(d^2 - e^2*x^2)]*((-2*(2*d + e*x))/(d + e*x)^(3/2) + (3*Sqrt[2]*
Sqrt[d]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/Sqrt
[d^2 - e^2*x^2]))/e
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {465, 466, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{ 465}$$

$$-\frac{3}{2}c\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{3/2}}dx-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 466}$$

$$-\frac{3}{2}c\left(2cd\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx+\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 471}$$

$$-\frac{3}{2}c\left(4cde\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}+\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{3}{2}c\left(\frac{2\sqrt{cd^2-ce^2x^2}}{e\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{c}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{e}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{e(d+ex)^{5/2}}$$

input
```
Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

output
```
-((c*d^2 - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(5/2))) - (3*c*((2*Sqrt[c*d^2 - c
*e^2*x^2])/(e*Sqrt[d + e*x]) - (2*Sqrt[2]*Sqrt[c]*Sqrt[d]*ArcTanh[Sqrt[c*d
^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/e))/2
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 466
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] - Simp[2*b*c*(p/(d^
2*(n + 2*p + 1)))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; Fr
eeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LeQ[-2, n, 0
] || EqQ[n + p + 1, 0]) && NeQ[n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

method	result
default	$\frac{\sqrt{c(-e^2x^2+d^2)}\,c\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)cdex+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)c\,d^2-2ex\sqrt{c(-ex+d)}\,\sqrt{cd}-4\sqrt{c(-ex+d)}\right.}{(ex+d)^{\frac{3}{2}}\sqrt{c(-ex+d)}\,e\sqrt{cd}}$
risch	$-\frac{2(-ex+d)\sqrt{-\frac{c\left(e^2x^2-d^2\right)}{ex+d}}\,\sqrt{ex+d}\,c^2}{e\sqrt{-c(ex-d)}\,\sqrt{-c(e^2x^2-d^2)}}-\frac{4d\left(-\frac{\sqrt{-cex+cd}}{2(-cex-cd)}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-cex+cd}\,\sqrt{2}}{2\sqrt{cd}}\right)}{4\sqrt{cd}}\right)\sqrt{-\frac{c\left(e^2x^2-d^2\right)}{ex+d}}\,\sqrt{ex+d}\,c^2}{e\sqrt{-c(e^2x^2-d^2)}}$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(c*(-e^2*x^2+d^2))^(1/2)/(e*x+d)^(3/2)*c*(3*2^(1/2)*arctanh(1/2*(c*(-e*x+d
))^(1/2)*2^(1/2)/(c*d)^(1/2))*c*d*e*x+3*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(
1/2)*2^(1/2)/(c*d)^(1/2))*c*d^2-2*e*x*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)-4*(c*
(-e*x+d))^(1/2)*(c*d)^(1/2)*d)/(c*(-e*x+d))^(1/2)/e/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.31

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx = \left[\frac{3\sqrt{2}(ce^2x^2+2\,cdex+cd^2)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2-2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\,\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)}{2\,(e^3x^2+2\,de^2x+d^2e)}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
[1/2*(3*sqrt(2)*(c*e^2*x^2 + 2*c*d*e*x + c*d^2)*sqrt(c*d)*log(-(c*e^2*x^2
- 2*c*d*e*x - 3*c*d^2 - 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(
e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*(c*e*x +
 2*c*d)*sqrt(e*x + d))/(e^3*x^2 + 2*d*e^2*x + d^2*e), -(3*sqrt(2)*(c*e^2*x
^2 + 2*c*d*e*x + c*d^2)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*
d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) + 2*sqrt(-c*e^2*x^2 + c*d
^2)*(c*e*x + 2*c*d)*sqrt(e*x + d))/(e^3*x^2 + 2*d*e^2*x + d^2*e)]
```

Sympy [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx = \int \frac{(-c(-d+ex)\,(d+ex))^{\frac{3}{2}}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(7/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx = \int \frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{7/2}}\,dx = -\frac{\frac{3\sqrt{2}c^2d\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} + 2\sqrt{-(ex+d)c+2\,cd}\,c + \frac{2\sqrt{-(ex+d)c+2\,cd}\,cd}{ex+d}}{e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-(3*sqrt(2)*c^2*d*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d)
)/sqrt(-c*d) + 2*sqrt(-(e*x + d)*c + 2*c*d)*c + 2*sqrt(-(e*x + d)*c + 2*c*
d)*c*d/(e*x + d))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{7/2}} \, dx = \int \frac{(c\,d^2 - c\,e^2\,x^2)^{3/2}}{(d + e\,x)^{7/2}} \,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(7/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{7/2}} \, dx = \frac{\sqrt{c}\,c\left(-16\sqrt{-ex+d}\,d - 8\sqrt{-ex+d}\,ex - 12\sqrt{d}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)\right)d - \right.}{4e\,(ex+d)}$$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(7/2),x)
```

output
```
(sqrt(c)*c*( - 16*sqrt(d - e*x)*d - 8*sqrt(d - e*x)*e*x - 12*sqrt(d)*sqrt(
2)*log(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2))*d - 12*sqrt(d)*sqrt(2
)*log(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2))*e*x + 9*sqrt(d)*sqrt(2
)*d + 9*sqrt(d)*sqrt(2)*e*x))/(4*e*(d + e*x))
```

3.179 $\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 139

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \frac{3c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{3/2}} - \frac{(cd^2-ce^2x^2)^{3/2}}{2e(d+ex)^{7/2}} - \frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{4\sqrt{2}\sqrt{d}e}$$

output
```
3/4*c*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(3/2)-1/2*(-c*e^2*x^2+c*d^2)^(3/2
)/e/(e*x+d)^(7/2)-3/8*c^(3/2)*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2
)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/d^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \frac{c\sqrt{c\left(d^2-e^2x^2\right)}\left(\frac{2(d+5ex)}{(d+ex)^{5/2}} - \frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{d}\sqrt{d^2-e^2x^2}}\right)}{8e}$$

input
```
Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

output
```
(c*Sqrt[c*(d^2 - e^2*x^2)]*((2*(d + 5*e*x))/(d + e*x)^(5/2) - (3*Sqrt[2]*A
rcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/(Sqrt[d]*Sqrt
[d^2 - e^2*x^2])))/(8*e)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {465, 465, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx$$

$$\Big\downarrow \text{465}$$

$$-\frac{3}{4}c\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{5/2}}dx-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{465}$$

$$-\frac{3}{4}c\left(-\frac{1}{2}c\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{471}$$

$$-\frac{3}{4}c\left(-ce\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}\sqrt{d}e}-\frac{\sqrt{cd^2-ce^2x^2}}{e(d+ex)^{3/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{2e(d+ex)^{7/2}}$$

input
```
Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

output
```
-1/2*(c*d^2 - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(7/2)) - (3*c*(-(Sqrt[c*d^2 -
c*e^2*x^2]/(e*(d + e*x)^(3/2))) + (Sqrt[c]*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]
/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])])/(Sqrt[2]*Sqrt[d]*e)))/4
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, c \left(3 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) c\, e^{2} x^{2}+6 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) c d e x+3 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) c}{8 \left(e x+d\right)^{\frac{5}{2}} \sqrt{c\left(-e x+d\right)}\, e \sqrt{c d}}$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*(-e^2*x^2+d^2))^(1/2)*c*(3*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*
2^(1/2)/(c*d)^(1/2))*c*e^2*x^2+6*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^
(1/2)/(c*d)^(1/2))*c*d*e*x+3*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2
)/(c*d)^(1/2))*c*d^2-10*e*x*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)-2*(c*(-e*x+d))^
(1/2)*(c*d)^(1/2)*d)/(e*x+d)^(5/2)/(c*(-e*x+d))^(1/2)/e/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.64

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \left[\frac{3\sqrt{\frac{1}{2}}(ce^3x^3+3\,cde^2x^2+3\,cd^2ex+cd^3)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+4\sqrt{\frac{1}{2}}\sqrt{-ce^2x^2-}}{e^2x^2+2\,dex+d^2}\right.}{8\,(e^4x^3+3\,de^3x^2+3\,d^2e^2x+}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(1/2)*(c*e^3*x^3 + 3*c*d*e^2*x^2 + 3*c*d^2*e*x + c*d^3)*sqrt(c
/d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 4*sqrt(1/2)*sqrt(-c*e^2*x^2 +
c*d^2)*sqrt(e*x + d)*d*sqrt(c/d))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*sqrt(-c*e
^2*x^2 + c*d^2)*(5*c*e*x + c*d)*sqrt(e*x + d))/(e^4*x^3 + 3*d*e^3*x^2 + 3*
d^2*e^2*x + d^3*e), -1/4*(3*sqrt(1/2)*(c*e^3*x^3 + 3*c*d*e^2*x^2 + 3*c*d^2
*e*x + c*d^3)*sqrt(-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(
e*x + d)*d*sqrt(-c/d)/(c*e^2*x^2 - c*d^2)) - sqrt(-c*e^2*x^2 + c*d^2)*(5*c
*e*x + c*d)*sqrt(e*x + d))/(e^4*x^3 + 3*d*e^3*x^2 + 3*d^2*e^2*x + d^3*e)]
```

Sympy [F]

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(-c(-d+ex)\,(d+ex))^{\frac{3}{2}}}{(d+ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(9/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{9/2}}\, dx = \int \frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{(ex + d)^{\frac{9}{2}}}\, dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{9/2}}\, dx = \frac{\frac{3\sqrt{2}c^2 \arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}} + \frac{2\left(6\sqrt{-(ex+d)c+2\,cd}c^3d - 5\,(-(ex+d)c+2\,cd)^{\frac{3}{2}}c^2\right)}{(ex+d)^2c^2}}{8\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d
))/sqrt(-c*d) + 2*(6*sqrt(-(e*x + d)*c + 2*c*d)*c^3*d - 5*(-(e*x + d)*c +
2*c*d)^(3/2)*c^2)/((e*x + d)^2*c^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(c\,d^2-c\,e^2\,x^2)^{3/2}}{(d+e\,x)^{9/2}}\,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(9/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \frac{\sqrt{c}\,c\left(2\sqrt{-ex+d}\,d^2+10\sqrt{-ex+d}\,dex+3\sqrt{d}\,\sqrt{2}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)\right)d^2+\right.}{8de\,(e^2x^2+}$$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(9/2),x)
```

output
```
(sqrt(c)*c*(2*sqrt(d - e*x)*d**2 + 10*sqrt(d - e*x)*d*e*x + 3*sqrt(d)*sqrt
(2)*log(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2))*d**2 + 6*sqrt(d)*sqr
t(2)*log(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2))*d*e*x + 3*sqrt(d)*s
qrt(2)*log(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2))*e**2*x**2))/(8*d*
e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.180 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 178

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx = \frac{c\sqrt{cd^2-ce^2x^2}}{4e(d+ex)^{5/2}}-\frac{c\sqrt{cd^2-ce^2x^2}}{16de(d+ex)^{3/2}} -\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}-\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{16\sqrt{2}d^{3/2}e}$$

output
```
1/4*c*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(5/2)-1/16*c*(-c*e^2*x^2+c*d^2)^(
1/2)/d/e/(e*x+d)^(3/2)-1/3*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(9/2)-1/32*c
^(3/2)*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1
/2))*2^(1/2)/d^(3/2)/e
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.85

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}\,dx = -\frac{\left(c\left(d^2-e^2x^2\right)\right)^{3/2}\left(2\sqrt{d}\sqrt{d^2-e^2x^2}\left(7d^2-22dex+3e^2x^2\right)+3\sqrt{2}(d+ex)^{7/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)\right)}{96d^{3/2}e(d+ex)^{7/2}\left(d^2-e^2x^2\right)^{3/2}}$$

input
```
Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

output
```
-1/96*((c*(d^2 - e^2*x^2))^(3/2)*(2*Sqrt[d]*Sqrt[d^2 - e^2*x^2]*(7*d^2 - 2
2*d*e*x + 3*e^2*x^2) + 3*Sqrt[2]*(d + e*x)^(7/2)*ArcTanh[(Sqrt[2]*Sqrt[d]*
Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]]))/(d^(3/2)*e*(d + e*x)^(7/2)*(d^2 - e^
2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {465, 465, 470, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{11/2}}dx$$

$$\Big\downarrow \text{465}$$

$$-\frac{1}{2}c\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{7/2}}dx-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{465}$$

$$-\frac{1}{2}c\left(-\frac{1}{4}c\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}dx-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{470}$$

$$-\frac{1}{2}c\left(-\frac{1}{4}c\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{4d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{471}$$

$$-\frac{1}{2}c\left(-\frac{1}{4}c\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{2d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

$$\Big\downarrow \ 221$$

$$-\frac{1}{2}c\left(-\frac{1}{4}c\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{2e(d+ex)^{5/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{3e(d+ex)^{9/2}}$$

input
```
Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(11/2),x]
```

output
```
-1/3*(c*d^2 - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(9/2)) - (c*(-1/2*Sqrt[c*d^2 -
 c*e^2*x^2]/(e*(d + e*x)^(5/2)) - (c*(-1/2*Sqrt[c*d^2 - c*e^2*x^2]/(c*d*e*
(d + e*x)^(3/2)) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d
]*Sqrt[d + e*x])]/(2*Sqrt[2]*Sqrt[c]*d^(3/2)*e)))/4))/2
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\,c\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)c\,e^3x^3+9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)cd\,e^2x^2+9\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right.\right.}{96(ex+d)^{\frac{7}{2}}\sqrt{c(-e}}$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*(c*(-e^2*x^2+d^2))^(1/2)*c*(3*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)
*2^(1/2)/(c*d)^(1/2))*c*e^3*x^3+9*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2
^(1/2)/(c*d)^(1/2))*c*d*e^2*x^2+9*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2
^(1/2)/(c*d)^(1/2))*c*d^2*e*x+3*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(
1/2)/(c*d)^(1/2))*c*d^3+6*e^2*x^2*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)-44*d*e*x*
(c*(-e*x+d))^(1/2)*(c*d)^(1/2)+14*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)*d^2)/(e*x
+d)^(7/2)/(c*(-e*x+d))^(1/2)/e/d/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.49

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{11/2}}\,dx = \left[\frac{3\sqrt{\frac{1}{2}}(ce^4x^4 + 4\,cde^3x^3 + 6\,cd^2e^2x^2 + 4\,cd^3ex + cd^4)\sqrt{\frac{c}{d}}\log\left(-\frac{ce^2x^2 - 2\,cdex - 3\,cd^2 +}{e^2}\right.}{96\,(de^5x^4 + 4\,d^2e^4x^3 +}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

output
```
[1/96*(3*sqrt(1/2)*(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*c*d^3*
e*x + c*d^4)*sqrt(c/d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 4*sqrt(1/2)
*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(c/d))/(e^2*x^2 + 2*d*e*x +
d^2)) - 2*(3*c*e^2*x^2 - 22*c*d*e*x + 7*c*d^2)*sqrt(-c*e^2*x^2 + c*d^2)*sq
rt(e*x + d))/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*x^2 + 4*d^4*e^2*x + d^
5*e), -1/48*(3*sqrt(1/2)*(c*e^4*x^4 + 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 4*
c*d^3*e*x + c*d^4)*sqrt(-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*
sqrt(e*x + d)*d*sqrt(-c/d)/(c*e^2*x^2 - c*d^2)) + (3*c*e^2*x^2 - 22*c*d*e*
x + 7*c*d^2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(d*e^5*x^4 + 4*d^2*e^
4*x^3 + 6*d^3*e^3*x^2 + 4*d^4*e^2*x + d^5*e)]
```

Sympy [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{11/2}}\,dx = \int \frac{(-c(-d+ex)\,(d+ex))^{\frac{3}{2}}}{(d+ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(11/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{11/2}}\,dx = \int \frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{11/2}}\,dx = \frac{\frac{3\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}d} + \frac{2\left(12\sqrt{-(ex+d)c+2\,cd}c^4d^2 - 16\left(-(ex+d)c+2\,cd\right)^{\frac{3}{2}}c^3d - 3\left((ex+d)c-\right.}{(ex+d)^3c^3d}}{96\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

output
```
1/96*(3*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*
d))/(sqrt(-c*d)*d) + 2*(12*sqrt(-(e*x + d)*c + 2*c*d)*c^4*d^2 - 16*(-(e*x
+ d)*c + 2*c*d)^(3/2)*c^3*d - 3*((e*x + d)*c - 2*c*d)^2*sqrt(-(e*x + d)*c
+ 2*c*d)*c^2)/((e*x + d)^3*c^3*d))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{11/2}}\,dx = \int \frac{(c\,d^2 - c\,e^2\,x^2)^{3/2}}{(d+e\,x)^{11/2}}\,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(11/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.17

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d + ex)^{11/2}} \, dx = \frac{\sqrt{c}\, c\Big(-14\sqrt{-ex + d}\, d^3 + 44\sqrt{-ex + d}\, d^2ex - 6\sqrt{-ex + d}\, d\, e^2x^2 + 3\sqrt{d}\,\sqrt{2}\log\Big(}{}$$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(11/2),x)
```

output
```
(sqrt(c)*c*( - 14*sqrt(d - e*x)*d**3 + 44*sqrt(d - e*x)*d**2*e*x - 6*sqrt(
d - e*x)*d*e**2*x**2 + 3*sqrt(d)*sqrt(2)*log(tan(asin(sqrt(d + e*x)/(sqrt(
d)*sqrt(2)))/2))*d**3 + 9*sqrt(d)*sqrt(2)*log(tan(asin(sqrt(d + e*x)/(sqrt
(d)*sqrt(2)))/2))*d**2*e*x + 9*sqrt(d)*sqrt(2)*log(tan(asin(sqrt(d + e*x)/
(sqrt(d)*sqrt(2)))/2))*d*e**2*x**2 + 3*sqrt(d)*sqrt(2)*log(tan(asin(sqrt(d
 + e*x)/(sqrt(d)*sqrt(2)))/2))*e**3*x**3))/(96*d**2*e*(d**3 + 3*d**2*e*x +
 3*d*e**2*x**2 + e**3*x**3))
```

3.181 $\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 217

$$\int \frac{\left(cd^2-ce^2x^2\right)^{3/2}}{(d+ex)^{13/2}}\,dx=\frac{c\sqrt{cd^2-ce^2x^2}}{8e(d+ex)^{7/2}}-\frac{c\sqrt{cd^2-ce^2x^2}}{64de(d+ex)^{5/2}}-\frac{3c\sqrt{cd^2-ce^2x^2}}{256d^2e(d+ex)^{3/2}}-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}}-\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{256\sqrt{2}d^{5/2}e}$$

output
```
1/8*c*(-c*e^2*x^2+c*d^2)^(1/2)/e/(e*x+d)^(7/2)-1/64*c*(-c*e^2*x^2+c*d^2)^(
1/2)/d/e/(e*x+d)^(5/2)-3/256*c*(-c*e^2*x^2+c*d^2)^(1/2)/d^2/e/(e*x+d)^(3/2
)-1/4*(-c*e^2*x^2+c*d^2)^(3/2)/e/(e*x+d)^(11/2)-3/512*c^(3/2)*arctanh(2^(1
/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/d^(5/2
)/e
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.71

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \frac{(c(d^2-e^2x^2))^{3/2}\left(-\frac{2\sqrt{d}\sqrt{d^2-e^2x^2}(39d^3-79d^2ex+13de^2x^2+3e^3x^3)}{(d+ex)^{9/2}} - 3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d-}}{\sqrt{d^2-e^2}}\right.\right.}{512d^{5/2}e\,(d^2-e^2x^2)^{3/2}}$$

input
```
Integrate[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(13/2),x]
```

output
```
((c*(d^2 - e^2*x^2))^(3/2)*((-2*Sqrt[d]*Sqrt[d^2 - e^2*x^2]*(39*d^3 - 79*d
^2*e*x + 13*d*e^2*x^2 + 3*e^3*x^3))/(d + e*x)^(9/2) - 3*Sqrt[2]*ArcTanh[(S
qrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]]))/(512*d^(5/2)*e*(d^2 -
 e^2*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {465, 465, 470, 470, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{13/2}}dx$$

$$\Big\downarrow \text{465}$$

$$-\frac{3}{8}c\int \frac{\sqrt{cd^2-ce^2x^2}}{(d+ex)^{9/2}}dx - \frac{(cd^2-ce^2x^2)^{3/2}}{4e(d+ex)^{11/2}}$$

$$\Big\downarrow \text{465}$$

$$-\frac{3}{8}c\left(-\frac{1}{6}c\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}dx - \frac{\sqrt{cd^2-ce^2x^2}}{3e(d+ex)^{7/2}}\right) - \frac{(cd^2-ce^2x^2)^{3/2}}{4e(d+ex)^{11/2}}$$

$$\Big\downarrow \text{470}$$

$$-\frac{3}{8}c\left(-\frac{1}{6}c\left(\frac{3\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}dx}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{3e(d+ex)^{7/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

$$\Big\downarrow \textbf{470}$$

$$-\frac{3}{8}c\left(-\frac{1}{6}c\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{4d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{3e(d+ex)^{7/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

$$\Big\downarrow \textbf{471}$$

$$-\frac{3}{8}c\left(-\frac{1}{6}c\left(\frac{3\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{2d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{3e(d+ex)^{7/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{3}{8}c\left(-\frac{1}{6}c\left(\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}\right)-\frac{\sqrt{cd^2-ce^2x^2}}{3e(d+ex)^{7/2}}\right)-\frac{\left(cd^2-ce^2x^2\right)^{3/2}}{4e(d+ex)^{11/2}}$$

input
```
Int[(c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(13/2),x]
```

output
```
-1/4*(c*d^2 - c*e^2*x^2)^(3/2)/(e*(d + e*x)^(11/2)) - (3*c*(-1/3*Sqrt[c*d^
2 - c*e^2*x^2]/(e*(d + e*x)^(7/2)) - (c*(-1/4*Sqrt[c*d^2 - c*e^2*x^2]/(c*d
*e*(d + e*x)^(5/2)) + (3*(-1/2*Sqrt[c*d^2 - c*e^2*x^2]/(c*d*e*(d + e*x)^(3
/2)) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e
*x])]/(2*Sqrt[2]*Sqrt[c]*d^(3/2)*e)))/(8*d)))/6))/8
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 465
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + p + 1))), x] - Simp[b*(p/(d^2*(n +
 p + 1)))   Int[(c + d*x)^(n + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && GtQ[p, 0] && (LtQ[n, -2] || EqQ[n
+ 2*p + 1, 0]) && NeQ[n + p + 1, 0] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.40

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\,c\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)c\,e^4x^4+12\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)cd\,e^3x^3+18\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{}}{2\sqrt{cd}}\right.\right.}{}$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/512*(c*(-e^2*x^2+d^2))^(1/2)*c*(3*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2
)*2^(1/2)/(c*d)^(1/2))*c*e^4*x^4+12*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)
*2^(1/2)/(c*d)^(1/2))*c*d*e^3*x^3+18*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2
)*2^(1/2)/(c*d)^(1/2))*c*d^2*e^2*x^2+12*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(
1/2)*2^(1/2)/(c*d)^(1/2))*c*d^3*e*x+6*e^3*x^3*(c*(-e*x+d))^(1/2)*(c*d)^(1/
2)+3*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*c*d^4+26*
d*e^2*x^2*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)-158*d^2*e*x*(c*(-e*x+d))^(1/2)*(c
*d)^(1/2)+78*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)*d^3)/(e*x+d)^(9/2)/(c*(-e*x+d)
)^(1/2)/e/d^2/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.39

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \left[\frac{3\sqrt{\frac{1}{2}}(ce^5x^5+5\,cde^4x^4+10\,cd^2e^3x^3+10\,cd^3e^2x^2+5\,cd^4ex+cd^5)\sqrt{\frac{c}{d}}\log\left(-\frac{ce}{}\right.}{512\,(d^2e^6x^5+5\,}\right.$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="fricas")
```

output
```
[1/512*(3*sqrt(1/2)*(c*e^5*x^5 + 5*c*d*e^4*x^4 + 10*c*d^2*e^3*x^3 + 10*c*d
^3*e^2*x^2 + 5*c*d^4*e*x + c*d^5)*sqrt(c/d)*log(-(c*e^2*x^2 - 2*c*d*e*x -
3*c*d^2 + 4*sqrt(1/2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(c/d))/
(e^2*x^2 + 2*d*e*x + d^2)) - 2*(3*c*e^3*x^3 + 13*c*d*e^2*x^2 - 79*c*d^2*e*
x + 39*c*d^3)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(d^2*e^6*x^5 + 5*d^3
*e^5*x^4 + 10*d^4*e^4*x^3 + 10*d^5*e^3*x^2 + 5*d^6*e^2*x + d^7*e), -1/256*
(3*sqrt(1/2)*(c*e^5*x^5 + 5*c*d*e^4*x^4 + 10*c*d^2*e^3*x^3 + 10*c*d^3*e^2*
x^2 + 5*c*d^4*e*x + c*d^5)*sqrt(-c/d)*arctan(2*sqrt(1/2)*sqrt(-c*e^2*x^2 +
 c*d^2)*sqrt(e*x + d)*d*sqrt(-c/d)/(c*e^2*x^2 - c*d^2)) + (3*c*e^3*x^3 + 1
3*c*d*e^2*x^2 - 79*c*d^2*e*x + 39*c*d^3)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x
 + d))/(d^2*e^6*x^5 + 5*d^3*e^5*x^4 + 10*d^4*e^4*x^3 + 10*d^5*e^3*x^2 + 5*
d^6*e^2*x + d^7*e)]
```

Sympy [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \int \frac{(-c(-d+ex)\,(d+ex))^{\frac{3}{2}}}{(d+ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-c*e**2*x**2+c*d**2)**(3/2)/(e*x+d)**(13/2),x)
```

output
```
Integral((-c*(-d + e*x)*(d + e*x))**(3/2)/(d + e*x)**(13/2), x)
```

Maxima [F]

$$\int \frac{(cd^2 - ce^2x^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \int \frac{(-ce^2x^2 + cd^2)^{\frac{3}{2}}}{(ex+d)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="maxima")
```

output
```
integrate((-c*e^2*x^2 + c*d^2)^(3/2)/(e*x + d)^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.83

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \frac{\frac{3\sqrt{2}c^2\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}d^2}+\frac{2\left(24\sqrt{-(ex+d)c+2\,cd}c^5d^3-44\,(-(ex+d)c+2\,cd)^{\frac{3}{2}}c^4d^2-22\,((ex+d)c\right.}{(ex+c}}{512\,e}$$

input
```
integrate((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x, algorithm="giac")
```

output
```
1/512*(3*sqrt(2)*c^2*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c
*d))/(sqrt(-c*d)*d^2) + 2*(24*sqrt(-(e*x + d)*c + 2*c*d)*c^5*d^3 - 44*(-(e
*x + d)*c + 2*c*d)^(3/2)*c^4*d^2 - 22*((e*x + d)*c - 2*c*d)^2*sqrt(-(e*x +
 d)*c + 2*c*d)*c^3*d - 3*((e*x + d)*c - 2*c*d)^3*sqrt(-(e*x + d)*c + 2*c*d
)*c^2)/((e*x + d)^4*c^4*d^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \int \frac{\left(c\,d^2-c\,e^2\,x^2\right)^{3/2}}{{\left(d+e\,x\right)}^{13/2}}\,dx$$

input
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(13/2),x)
```

output
```
int((c*d^2 - c*e^2*x^2)^(3/2)/(d + e*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.68

$$\int \frac{(cd^2-ce^2x^2)^{3/2}}{(d+ex)^{13/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,\sqrt{2}\,c\left(48\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)\right)\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)^8+\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right.\right.}{8192\tan\left(\frac{asin\left(\frac{\sqrt{ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)^8 d}$$

input
```
int((-c*e^2*x^2+c*d^2)^(3/2)/(e*x+d)^(13/2),x)
```

output
```
(sqrt(d)*sqrt(c)*sqrt(2)*c*(48*log(tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)
))/2))*tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2)**8 + tan(asin(sqrt(d +
 e*x)/(sqrt(d)*sqrt(2)))/2)**16 - 8*tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2
)))/2)**12 + 8*tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2)**4 - 1))/(8192
*tan(asin(sqrt(d + e*x)/(sqrt(d)*sqrt(2)))/2)**8*d**3*e)
```

3.182 $\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{256d^3\sqrt{cd^2-ce^2x^2}}{35ce\sqrt{d+ex}} - \frac{64d^2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{35ce} - \frac{24d(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{35ce} - \frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}$$

output
```
-256/35*d^3*(-c*e^2*x^2+c*d^2)^(1/2)/c/e/(e*x+d)^(1/2)-64/35*d^2*(e*x+d)^(
1/2)*(-c*e^2*x^2+c*d^2)^(1/2)/c/e-24/35*d*(e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)
^(1/2)/c/e-2/7*(e*x+d)^(5/2)*(-c*e^2*x^2+c*d^2)^(1/2)/c/e
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{c(d^2-e^2x^2)}(177d^3+71d^2ex+27de^2x^2+5e^3x^3)}{35ce\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(7/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*Sqrt[c*(d^2 - e^2*x^2)]*(177*d^3 + 71*d^2*e*x + 27*d*e^2*x^2 + 5*e^3*x
^3))/(35*c*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{459}$$

$$\frac{12}{7}d\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}dx-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{12}{7}d\left(\frac{8}{5}d\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}dx-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}\right)-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{12}{7}d\left(\frac{8}{5}d\left(\frac{4}{3}d\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}dx-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}\right)-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}\right)-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}$$

$$\Big\downarrow \text{458}$$

$$\frac{12}{7}d\left(\frac{8}{5}d\left(-\frac{8d\sqrt{cd^2-ce^2x^2}}{3ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}\right)-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}\right)-\frac{2(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}{7ce}$$

input
```
Int[(d + e*x)^(7/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*(d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2])/(7*c*e) + (12*d*((-2*(d + e*x
)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2])/(5*c*e) + (8*d*((-8*d*Sqrt[c*d^2 - c*e^2*
x^2])/(3*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2])/(3
*c*e)))/5))/7
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{2\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\,\left(5 e^{3} x^{3}+27 d\,e^{2} x^{2}+71 d^{2} e x +177 d^{3}\right)}{35 \sqrt{e x +d}\, c e}$	62
gosper	$-\frac{2\left(-e x +d \right) \left(5 e^{3} x^{3}+27 d\,e^{2} x^{2}+71 d^{2} e x +177 d^{3}\right) \sqrt{e x +d}}{35 e \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}$	66
orering	$-\frac{2\left(-e x +d \right) \left(5 e^{3} x^{3}+27 d\,e^{2} x^{2}+71 d^{2} e x +177 d^{3}\right) \sqrt{e x +d}}{35 e \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}$	66
risch	$-\frac{2 \sqrt{-\frac{c \left(e^{2} x^{2}-d^{2}\right)}{e x +d}}\, \sqrt{e x +d}\, \left(5 e^{3} x^{3}+27 d\,e^{2} x^{2}+71 d^{2} e x +177 d^{3}\right) \left(-e x +d \right)}{35 \sqrt{-c \left(e^{2} x^{2}-d^{2}\right)}\, e \sqrt{-c \left(e x -d \right)}}$	104

input
```
int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)/c*(5*e^3*x^3+27*d*e^2*x^2+71*
d^2*e*x+177*d^3)/e
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\,(5\,e^3x^3+27\,de^2x^2+71\,d^2ex+177\,d^3)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{35\,(ce^2x+cde)}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/35*(5*e^3*x^3 + 27*d*e^2*x^2 + 71*d^2*e*x + 177*d^3)*sqrt(-c*e^2*x^2 +
c*d^2)*sqrt(e*x + d)/(c*e^2*x + c*d*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{(d+ex)^{\frac{7}{2}}}{\sqrt{-c\,(-d+ex)\,(d+ex)}}\,dx$$

input
```
integrate((e*x+d)**(7/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(7/2)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \frac{2\,(5\,e^4x^4+22\,de^3x^3+44\,d^2e^2x^2+106\,d^3ex-177\,d^4)}{35\,\sqrt{-ex+d}\sqrt{c}e}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*e^4*x^4 + 22*d*e^3*x^3 + 44*d^2*e^2*x^2 + 106*d^3*e*x - 177*d^4)/(
sqrt(-e*x + d)*sqrt(c)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\left(\frac{280\sqrt{-(ex+d)c+2\,cd}d^3}{c}-\frac{140\,(-(ex+d)c+2\,cd)^{\frac{3}{2}}c^2d^2-42\,((ex+d)c-2\,cd)^2\sqrt{-(ex+d)c+2\,cd}cd-5\,((ex+d)c-2\,cd)^3\sqrt{-(ex+d)c+2\,cd}}{c^4}\right)}{35\,e}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
-2/35*(280*sqrt(-(e*x + d)*c + 2*c*d)*d^3/c - (140*(-(e*x + d)*c + 2*c*d)^
(3/2)*c^2*d^2 - 42*((e*x + d)*c - 2*c*d)^2*sqrt(-(e*x + d)*c + 2*c*d)*c*d
- 5*((e*x + d)*c - 2*c*d)^3*sqrt(-(e*x + d)*c + 2*c*d))/c^4)/e
```

Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.61

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\left(\frac{354\,d^3\,\sqrt{d+e\,x}}{35\,c\,e^2}+\frac{54\,d\,x^2\,\sqrt{d+e\,x}}{35\,c}+\frac{2\,e\,x^3\,\sqrt{d+e\,x}}{7\,c}+\frac{142\,d^2\,x\,\sqrt{d+e\,x}}{35\,c\,e}\right)}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(1/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((354*d^3*(d + e*x)^(1/2))/(35*c*e^2) + (54*d*
x^2*(d + e*x)^(1/2))/(35*c) + (2*e*x^3*(d + e*x)^(1/2))/(7*c) + (142*d^2*x
*(d + e*x)^(1/2))/(35*c*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.29

$$\int \frac{(d+ex)^{7/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,(-5e^3x^3-27d\,e^2x^2-71d^2ex-177d^3)}{35ce}$$

input
```
int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*( - 177*d**3 - 71*d**2*e*x - 27*d*e**2*x**2 - 5*e
**3*x**3))/(35*c*e)
```

3.183 $\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 119

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{64d^2\sqrt{cd^2-ce^2x^2}}{15ce\sqrt{d+ex}} - \frac{16d\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{15ce} - \frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}$$

output
```
-64/15*d^2*(-c*e^2*x^2+c*d^2)^(1/2)/c/e/(e*x+d)^(1/2)-16/15*d*(e*x+d)^(1/2
)*(-c*e^2*x^2+c*d^2)^(1/2)/c/e-2/5*(e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2)/
c/e
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{c(d^2-e^2x^2)}(43d^2+14dex+3e^2x^2)}{15ce\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(5/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*Sqrt[c*(d^2 - e^2*x^2)]*(43*d^2 + 14*d*e*x + 3*e^2*x^2))/(15*c*e*Sqrt[
d + e*x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{5}d\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}dx-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{5}d\left(\frac{4}{3}d\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}dx-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}\right)-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}$$

$$\Big\downarrow \text{458}$$

$$\frac{8}{5}d\left(-\frac{8d\sqrt{cd^2-ce^2x^2}}{3ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}\right)-\frac{2(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5ce}$$

input
```
Int[(d + e*x)^(5/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2])/(5*c*e) + (8*d*((-8*d*Sqrt[c*
d^2 - c*e^2*x^2])/(3*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sqrt[c*d^2 - c*
e^2*x^2])/(3*c*e)))/5
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.43

method	result	size
default	$-\frac{2\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(3 e^{2} x^{2}+14 d e x +43 d^{2}\right)}{15 \sqrt{e x +d}\, c e}$	51
gosper	$-\frac{2\left(-e x +d \right) \left(3 e^{2} x^{2}+14 d e x +43 d^{2}\right) \sqrt{e x +d}}{15 e \sqrt{-c \,e^{2} x^{2}+c \,d^{2}}}$	55
orering	$-\frac{2\left(-e x +d \right) \left(3 e^{2} x^{2}+14 d e x +43 d^{2}\right) \sqrt{e x +d}}{15 e \sqrt{-c \,e^{2} x^{2}+c \,d^{2}}}$	55
risch	$-\frac{2 \sqrt{-\frac{c \left(e^{2} x^{2}-d^{2}\right)}{e x +d}}\, \sqrt{e x +d}\, \left(3 e^{2} x^{2}+14 d e x +43 d^{2}\right) \left(-e x +d \right)}{15 \sqrt{-c \left(e^{2} x^{2}-d^{2}\right)}\, e \sqrt{-c \left(e x -d \right)}}$	93

input
```
int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)/c*(3*e^2*x^2+14*d*e*x+43*d^2)
/e
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{-ce^2x^2+cd^2}(3\,e^2x^2+14\,dex+43\,d^2)\sqrt{ex+d}}{15\,(ce^2x+cde)}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*sqrt(-c*e^2*x^2 + c*d^2)*(3*e^2*x^2 + 14*d*e*x + 43*d^2)*sqrt(e*x +
d)/(c*e^2*x + c*d*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{\sqrt{-c\,(-d+ex)\,(d+ex)}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(5/2)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \frac{2\left(3\sqrt{c}e^3x^3+11\sqrt{c}de^2x^2+29\sqrt{c}d^2ex-43\sqrt{c}d^3\right)}{15\sqrt{-ex+d}ce}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*sqrt(c)*e^3*x^3 + 11*sqrt(c)*d*e^2*x^2 + 29*sqrt(c)*d^2*e*x - 43*s
qrt(c)*d^3)/(sqrt(-e*x + d)*c*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\left(\frac{60\sqrt{-(ex+d)c+2\,cd}d^2}{c}-\frac{20\,(-(ex+d)c+2\,cd)^{\frac{3}{2}}cd-3\,((ex+d)c-2\,cd)^2\sqrt{-(ex+d)c+2\,cd}}{c^3}\right)}{15\,e}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(60*sqrt(-(e*x + d)*c + 2*c*d)*d^2/c - (20*(-(e*x + d)*c + 2*c*d)^(3
/2)*c*d - 3*((e*x + d)*c - 2*c*d)^2*sqrt(-(e*x + d)*c + 2*c*d))/c^3)/e
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\left(\frac{2\,x^2\,\sqrt{d+e\,x}}{5\,c}+\frac{86\,d^2\,\sqrt{d+e\,x}}{15\,c\,e^2}+\frac{28\,d\,x\,\sqrt{d+e\,x}}{15\,c\,e}\right)}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(1/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((2*x^2*(d + e*x)^(1/2))/(5*c) + (86*d^2*(d +
e*x)^(1/2))/(15*c*e^2) + (28*d*x*(d + e*x)^(1/2))/(15*c*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

$$\int \frac{(d+ex)^{5/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,(-3e^2x^2-14dex-43d^2)}{15ce}$$

input
```
int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*( - 43*d**2 - 14*d*e*x - 3*e**2*x**2))/(15*c*e)
```

3.184 $\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 78

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{8d\sqrt{cd^2-ce^2x^2}}{3ce\sqrt{d+ex}} - \frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}$$

output
```
-8/3*d*(-c*e^2*x^2+c*d^2)^(1/2)/c/e/(e*x+d)^(1/2)-2/3*(e*x+d)^(1/2)*(-c*e^
2*x^2+c*d^2)^(1/2)/c/e
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2(5d+ex)\sqrt{c(d^2-e^2x^2)}}{3ce\sqrt{d+ex}}$$

input
```
Integrate[(d + e*x)^(3/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*(5*d + e*x)*Sqrt[c*(d^2 - e^2*x^2)])/(3*c*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{459}$$

$$\frac{4}{3}d\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}dx-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}$$

$$\Big\downarrow \text{458}$$

$$-\frac{8d\sqrt{cd^2-ce^2x^2}}{3ce\sqrt{d+ex}}-\frac{2\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3ce}$$

input
```
Int[(d + e*x)^(3/2)/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-8*d*Sqrt[c*d^2 - c*e^2*x^2])/(3*c*e*Sqrt[d + e*x]) - (2*Sqrt[d + e*x]*Sq
rt[c*d^2 - c*e^2*x^2])/(3*c*e)
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{2\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(e x +5 d \right)}{3 \sqrt{e x +d}\, c e}$	39
gosper	$-\frac{2\left(-e x +d \right) \left(e x +5 d \right) \sqrt{e x +d}}{3 e \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}$	43
orering	$-\frac{2\left(-e x +d \right) \left(e x +5 d \right) \sqrt{e x +d}}{3 e \sqrt{-c\,e^{2} x^{2}+c\,d^{2}}}$	43
risch	$-\frac{2\sqrt{-\frac{c\left(e^{2} x^{2}-d^{2}\right)}{e x +d}}\, \sqrt{e x +d}\, \left(e x +5 d \right) \left(-e x +d \right)}{3 \sqrt{-c\left(e^{2} x^{2}-d^{2}\right)}\, e \sqrt{-c\left(e x -d \right)}}$	81

input
```
int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)/c*(e*x+5*d)/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx=-\frac{2\,\sqrt{-ce^2x^2+cd^2}(ex+5\,d)\sqrt{ex+d}}{3\,(ce^2x+cde)}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-c*e^2*x^2 + c*d^2)*(e*x + 5*d)*sqrt(e*x + d)/(c*e^2*x + c*d*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{-c\,(-d+ex)\,(d+ex)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.44

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \frac{2\,(e^2x^2+4\,dex-5\,d^2)}{3\,\sqrt{-ex+d}\sqrt{c}e}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(e^2*x^2 + 4*d*e*x - 5*d^2)/(sqrt(-e*x + d)*sqrt(c)*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\left(\frac{6\,\sqrt{-(ex+d)c+2\,cd}d}{c}-\frac{(-(ex+d)c+2\,cd)^{\frac{3}{2}}}{c^2}\right)}{3\,e}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(6*sqrt(-(e*x + d)*c + 2*c*d)*d/c - (-(e*x + d)*c + 2*c*d)^(3/2)/c^2)
/e
```

Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{10\,d\,\sqrt{d+e\,x}}{3\,c\,e^2}+\frac{2\,x\,\sqrt{d+e\,x}}{3\,c\,e}\right)}{x+\frac{d}{e}}$$

input
```
int((d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(1/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((10*d*(d + e*x)^(1/2))/(3*c*e^2) + (2*x*(d +
e*x)^(1/2))/(3*c*e)))/(x + d/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.32

$$\int \frac{(d+ex)^{3/2}}{\sqrt{cd^2-ce^2x^2}}\,dx = \frac{2\sqrt{c}\,\sqrt{-ex+d}\,(-ex-5d)}{3ce}$$

input
```
int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(d - e*x)*( - 5*d - e*x))/(3*c*e)
```

3.185 $\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 36

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{cd^2-ce^2x^2}}{ce\sqrt{d+ex}}$$

output
```
-2*(-c*e^2*x^2+c*d^2)^(1/2)/c/e/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{c(d^2-e^2x^2)}}{ce\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*Sqrt[c*(d^2 - e^2*x^2)])/(c*e*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{ 458}$$

$$-\frac{2\sqrt{cd^2-ce^2x^2}}{ce\sqrt{d+ex}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[c*d^2 - c*e^2*x^2],x]
```

output
```
(-2*Sqrt[c*d^2 - c*e^2*x^2])/(c*e*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2\sqrt{c(-e^2x^2+d^2)}}{\sqrt{ex+d}\,ce}$	32
gosper	$-\frac{2(-ex+d)\sqrt{ex+d}}{e\sqrt{-c\,e^2x^2+c\,d^2}}$	36
orering	$-\frac{2(-ex+d)\sqrt{ex+d}}{e\sqrt{-c\,e^2x^2+c\,d^2}}$	36
risch	$-\frac{2\sqrt{-\frac{c\left(e^2x^2-d^2\right)}{ex+d}}\,\sqrt{ex+d}\,(-ex+d)}{\sqrt{-c(e^2x^2-d^2)}\,e\sqrt{-c(ex-d)}}$	74

input
```
int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)/c/e
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{ce^2x+cde}$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(c*e^2*x + c*d*e)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx=\int \frac{\sqrt{d+ex}}{\sqrt{-c\left(-d+ex\right)\left(d+ex\right)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt(-c*(-d + e*x)*(d + e*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx=\frac{2\left(\sqrt{c}ex-\sqrt{c}d\right)}{\sqrt{-ex+d}ce}$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(c)*e*x - sqrt(c)*d)/(sqrt(-e*x + d)*c*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx=-\frac{2\sqrt{-(ex+d)c+2\,cd}}{ce}$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(-(e*x + d)*c + 2*c*d)/(c*e)
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{c\,d^2-c\,e^2\,x^2}}{c\,e\,\sqrt{d+e\,x}}$$

input
```
int((d + e*x)^(1/2)/(c*d^2 - c*e^2*x^2)^(1/2),x)
```

output
```
-(2*(c*d^2 - c*e^2*x^2)^(1/2))/(c*e*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{2\sqrt{c}\,\sqrt{-ex+d}}{ce}$$

input
```
int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(d - e*x))/(c*e)
```

3.186 $\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx=-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{\sqrt{c}\sqrt{de}}$$

output
```
-2^(1/2)*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^
(1/2))/c^(1/2)/d^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx=-\frac{\sqrt{2}\sqrt{d^2-e^2x^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{\sqrt{de}\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

output
```
-((Sqrt[2]*Sqrt[d^2 - e^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqr
t[d^2 - e^2*x^2]])/(Sqrt[d]*e*Sqrt[c*(d^2 - e^2*x^2)]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{471}$$

$$2e\int \frac{1}{\frac{e^2(cd^2-ce^2x^2)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{c}\sqrt{d}e}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d
 + e*x])])/(Sqrt[c]*Sqrt[d]*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)}{\sqrt{ex+d}\,\sqrt{c(-ex+d)}\,e\sqrt{cd}}$	68

input
```
int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)/(c*(-e*x+d))^(1/2)/e*2^(1/2)/(c*
d)^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx = \left[\frac{\sqrt{2}\sqrt{\frac{1}{cd}}\log\left(-\frac{e^2x^2-2\,dex+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\,\sqrt{ex+d}\,d\sqrt{\frac{1}{cd}}-3\,d^2}{e^2x^2+2\,dex+d^2}\right)}{2\,e}, -\frac{\sqrt{2}\sqrt{-\frac{1}{cd}}\arctan\left(\frac{\sqrt{2}\sqrt{-ce^2x^2+cd^2}\,\sqrt{ex+d}\,d\sqrt{-\frac{1}{cd}}}{e^2x^2-d^2}\right)}{e}\right]$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(1/(c*d))*log(-(e^2*x^2 - 2*d*e*x + 2*sqrt(2)*sqrt(-c*e^2
*x^2 + c*d^2)*sqrt(e*x + d)*d*sqrt(1/(c*d)) - 3*d^2)/(e^2*x^2 + 2*d*e*x +
d^2))/e, -sqrt(2)*sqrt(-1/(c*d))*arctan(sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*s
qrt(e*x + d)*d*sqrt(-1/(c*d))/(e^2*x^2 - d^2))/e]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{-c(-d+ex)(d+ex)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-c*(-d + e*x)*(d + e*x))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}e}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))/(sqrt(-c
*d)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{c\,d^2-c\,e^2\,x^2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,\sqrt{2}\left(\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)-\log\left(\sqrt{-ex+d}+\sqrt{d}\,\sqrt{2}\right)\right)}{2cde}$$

input
```
int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*sqrt(2)*(log(sqrt(d - e*x) - sqrt(d)*sqrt(2)) - log(sqrt(
d - e*x) + sqrt(d)*sqrt(2))))/(2*c*d*e)
```

3.187 $\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}$$

output
```
-1/2*(-c*e^2*x^2+c*d^2)^(1/2)/c/d/e/(e*x+d)^(3/2)-1/4*arctanh(2^(1/2)*c^(1
/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(1/2)/d^(3/2
)/e
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{-2\sqrt{d}(d-ex)-\sqrt{2}\sqrt{d+ex}\sqrt{d^2-e^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{4d^{3/2}e\sqrt{d+ex}\sqrt{c\,(d^2-e^2x^2)}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

output
```
(-2*Sqrt[d]*(d - e*x) - Sqrt[2]*Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]*ArcTanh[
(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/(4*d^(3/2)*e*Sqrt[d
+ e*x]*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {470, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \text{470}$$

$$\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{4d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}$$

$$\Big\downarrow \text{471}$$

$$\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{2d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}$$

input
```
Int[1/((d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

output
```
-1/2*Sqrt[c*d^2 - c*e^2*x^2]/(c*d*e*(d + e*x)^(3/2)) - ArcTanh[Sqrt[c*d^2
- c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(2*Sqrt[2]*Sqrt[c]*d
^(3/2)*e)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)cex+cd\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)+2\sqrt{c(-ex+d)}\,\sqrt{cd}\right)}{4(ex+d)^{\frac{3}{2}}c\sqrt{c(-ex+d)}\,ed\sqrt{cd}}$	123

input
```
int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(e*x+d)^(3/2)*(c*(-e^2*x^2+d^2))^(1/2)/c*(2^(1/2)*arctanh(1/2*(c*(-e*
x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*c*e*x+c*d*2^(1/2)*arctanh(1/2*(c*(-e*x+d)
)^(1/2)*2^(1/2)/(c*d)^(1/2))+2*(c*(-e*x+d))^(1/2)*(c*d)^(1/2))/(c*(-e*x+d)
)^(1/2)/e/d/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.72

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = \left[\frac{\sqrt{2}(e^2x^2+2\,dex+d^2)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex}}{e^2x^2+2\,dex+d^2}\right.}{8\,(cd^2e^3x^2+2\,cd^3e^2x+cd^4e)}\right.$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(2)*(e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*
e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d)
)/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d)
/(c*d^2*e^3*x^2 + 2*c*d^3*e^2*x + c*d^4*e), 1/4*(sqrt(2)*(e^2*x^2 + 2*d*e*
x + d^2)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)
*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x +
d)*d)/(c*d^2*e^3*x^2 + 2*c*d^3*e^2*x + c*d^4*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{-c\,(-d+ex)\,(d+ex)}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{-ce^2x^2+cd^2}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*(e*x + d)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{\frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\,\sqrt{-cd}}\right)}{\sqrt{-cd}d} - \frac{2\,\sqrt{-(ex+d)c+2\,cd}}{(ex+d)d}}{4\,ce}$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))/(
sqrt(-c*d)*d) - 2*sqrt(-(e*x + d)*c + 2*c*d)/((e*x + d)*d))/(c*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{c\,d^2-c\,e^2\,x^2}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{\sqrt{c}\left(-4\sqrt{-ex+d}\,d+\sqrt{d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)d+\sqrt{d}\,\sqrt{2}\log\left(\right.\right.}{}$$

input
```
int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 4*sqrt(d - e*x)*d + sqrt(d)*sqrt(2)*log(sqrt(d - e*x) - sqrt(
d)*sqrt(2))*d + sqrt(d)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt(2))*e*x -
 sqrt(d)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d - sqrt(d)*sqrt(2)*
log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*e*x))/(8*c*d**2*e*(d + e*x))
```

3.188 $\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 150

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = -\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}} - \frac{3\sqrt{cd^2-ce^2x^2}}{16cd^2e(d+ex)^{3/2}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{16\sqrt{2}\sqrt{c}d^{5/2}e}$$

output
```
-1/4*(-c*e^2*x^2+c*d^2)^(1/2)/c/d/e/(e*x+d)^(5/2)-3/16*(-c*e^2*x^2+c*d^2)^
(1/2)/c/d^2/e/(e*x+d)^(3/2)-3/32*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(
1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(1/2)/d^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{2\sqrt{d}(-7d^2+4dex+3e^2x^2) - 3\sqrt{2}(d+ex)^{3/2}\sqrt{d^2-e^2x^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{}}{\sqrt{d^2-}}\right.}{32d^{5/2}e(d+ex)^{3/2}\sqrt{c(d^2-e^2x^2)}}$$

input
```
Integrate[1/((d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

output
```
(2*Sqrt[d]*(-7*d^2 + 4*d*e*x + 3*e^2*x^2) - 3*Sqrt[2]*(d + e*x)^(3/2)*Sqrt
[d^2 - e^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2
]])/(32*d^(5/2)*e*(d + e*x)^(3/2)*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {470, 470, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}dx$$

$$\Big\downarrow \ 470$$

$$\frac{3\int \frac{1}{(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}dx}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}$$

$$\Big\downarrow \ 470$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{4d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}$$

$$\Big\downarrow \ 471$$

$$\frac{3\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{2d}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{3\left(-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{2\sqrt{2}\sqrt{c}d^{3/2}e}-\frac{\sqrt{cd^2-ce^2x^2}}{2cde(d+ex)^{3/2}}\right)}{8d}-\frac{\sqrt{cd^2-ce^2x^2}}{4cde(d+ex)^{5/2}}$$

input
```
Int[1/((d + e*x)^(5/2)*Sqrt[c*d^2 - c*e^2*x^2]),x]
```

output
```
-1/4*Sqrt[c*d^2 - c*e^2*x^2]/(c*d*e*(d + e*x)^(5/2)) + (3*(-1/2*Sqrt[c*d^2
 - c*e^2*x^2]/(c*d*e*(d + e*x)^(3/2)) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(S
qrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(2*Sqrt[2]*Sqrt[c]*d^(3/2)*e)))/(8*
d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.21

method	result
default	$-\frac{\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(3 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) c\, e^{2} x^{2}+6 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) c d e x+3 \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) c\, d}{32 \left(e x+d\right)^{\frac{5}{2}} c \sqrt{c\left(-e x+d\right)}\, e\, d^{2} \sqrt{c d}}$

input
```
int(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/(e*x+d)^(5/2)*(c*(-e^2*x^2+d^2))^(1/2)/c*(3*2^(1/2)*arctanh(1/2*(c*(
-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*c*e^2*x^2+6*2^(1/2)*arctanh(1/2*(c*(-e
*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*c*d*e*x+3*2^(1/2)*arctanh(1/2*(c*(-e*x+d
))^(1/2)*2^(1/2)/(c*d)^(1/2))*c*d^2+6*e*x*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)+1
4*(c*(-e*x+d))^(1/2)*(c*d)^(1/2)*d)/(c*(-e*x+d))^(1/2)/e/d^2/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.43

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = \left[\frac{3\sqrt{2}(e^3x^3+3\,de^2x^2+3\,d^2ex+d^3)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce}}{e^2x^2+2\,dex+c}\right.}{64\,(cd^3e^4x^3+3\,cd^4e^3x^2+3}\right.$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/64*(3*sqrt(2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*d)*log(-
(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt
(c*d)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^
2)*(3*d*e*x + 7*d^2)*sqrt(e*x + d))/(c*d^3*e^4*x^3 + 3*c*d^4*e^3*x^2 + 3*c
*d^5*e^2*x + c*d^6*e), 1/32*(3*sqrt(2)*(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x
+ d^3)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*s
qrt(e*x + d)/(c*d*e*x + c*d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*(3*d*e*x + 7*
d^2)*sqrt(e*x + d))/(c*d^3*e^4*x^3 + 3*c*d^4*e^3*x^2 + 3*c*d^5*e^2*x + c*d
^6*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{-c\,(-d+ex)\,(d+ex)}\,(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(1/2),x)
```

output
```
Integral(1/(sqrt(-c*(-d + e*x)*(d + e*x))*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{-ce^2x^2+cd^2}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-c*e^2*x^2 + c*d^2)*(e*x + d)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{\frac{3\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}d^2} - \frac{2\left(10\sqrt{-(ex+d)c+2\,cd}c^2d-3\left(-(ex+d)c+2\,cd\right)^{\frac{3}{2}}c\right)}{(ex+d)^2c^2d^2}}{32\,ce}$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x, algorithm="giac")
```

output
```
1/32*(3*sqrt(2)*c*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d)
)/(sqrt(-c*d)*d^2) - 2*(10*sqrt(-(e*x + d)*c + 2*c*d)*c^2*d - 3*(-(e*x + d
)*c + 2*c*d)^(3/2)*c)/((e*x + d)^2*c^2*d^2))/(c*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = \int \frac{1}{\sqrt{c\,d^2-c\,e^2\,x^2}\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(5/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.35

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{cd^2-ce^2x^2}}\,dx = \frac{\sqrt{c}\left(-28\sqrt{-ex+d}\,d^2-12\sqrt{-ex+d}\,dex+3\sqrt{d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{\right.\right.}{}$$

input
```
int(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(1/2),x)
```

output
```
(sqrt(c)*( - 28*sqrt(d - e*x)*d**2 - 12*sqrt(d - e*x)*d*e*x + 3*sqrt(d)*sq
rt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt(2))*d**2 + 6*sqrt(d)*sqrt(2)*log(sq
rt(d - e*x) - sqrt(d)*sqrt(2))*d*e*x + 3*sqrt(d)*sqrt(2)*log(sqrt(d - e*x)
 - sqrt(d)*sqrt(2))*e**2*x**2 - 3*sqrt(d)*sqrt(2)*log(sqrt(d - e*x) + sqrt
(d)*sqrt(2))*d**2 - 6*sqrt(d)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))
*d*e*x - 3*sqrt(d)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*e**2*x**2)
)/(64*c*d**3*e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.189 $\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 155

$$\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{3/2}} \, dx = \frac{2(d+ex)^{7/2}}{ce\sqrt{cd^2-ce^2x^2}} + \frac{128d^2\sqrt{cd^2-ce^2x^2}}{5c^2e\sqrt{d+ex}} + \frac{32d\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{5c^2e} + \frac{12(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}{5c^2e}$$

output
```
2*(e*x+d)^(7/2)/c/e/(-c*e^2*x^2+c*d^2)^(1/2)+128/5*d^2*(-c*e^2*x^2+c*d^2)^
(1/2)/c^2/e/(e*x+d)^(1/2)+32/5*d*(e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2)/c^
2/e+12/5*(e*x+d)^(3/2)*(-c*e^2*x^2+c*d^2)^(1/2)/c^2/e
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{3/2}} \, dx = -\frac{2\sqrt{d+ex}\left(-91d^3+43d^2ex+7de^2x^2+e^3x^3\right)}{5ce\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[(d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(-91*d^3 + 43*d^2*e*x + 7*d*e^2*x^2 + e^3*x^3))/(5*c*e*S
qrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 459}$$

$$\frac{12}{5}d\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}dx-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{ 459}$$

$$\frac{12}{5}d\left(\frac{8}{3}d\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}dx-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}\right)-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{ 459}$$

$$\frac{12}{5}d\left(\frac{8}{3}d\left(4d\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}dx-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}\right)-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}\right)-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{ 458}$$

$$\frac{12}{5}d\left(\frac{8}{3}d\left(\frac{8d\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}\right)-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}\right)-\frac{2(d+ex)^{7/2}}{5ce\sqrt{cd^2-ce^2x^2}}$$

input
```
Int[(d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x)^(7/2))/(5*c*e*Sqrt[c*d^2 - c*e^2*x^2]) + (12*d*((-2*(d + e*x
)^(5/2))/(3*c*e*Sqrt[c*d^2 - c*e^2*x^2]) + (8*d*((8*d*Sqrt[d + e*x])/(c*e*
Sqrt[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(3/2))/(c*e*Sqrt[c*d^2 - c*e^2*x^2
])))/3))/5
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

method	result	size
gosper	$\frac{2(-ex+d)\left(-e^{3}x^{3}-7d\,e^{2}x^{2}-43d^{2}ex+91d^{3}\right)(ex+d)^{\frac{3}{2}}}{5e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}$	66
orering	$\frac{2(-ex+d)\left(-e^{3}x^{3}-7d\,e^{2}x^{2}-43d^{2}ex+91d^{3}\right)(ex+d)^{\frac{3}{2}}}{5e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}$	66
default	$\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\,\left(-e^{3}x^{3}-7d\,e^{2}x^{2}-43d^{2}ex+91d^{3}\right)}{5\sqrt{ex+d}\,c^{2}(-ex+d)e}$	70
risch	$\frac{2\left(e^{2}x^{2}+8dex+51d^{2}\right)(-ex+d)\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{5e\sqrt{-c(ex-d)}\,\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c}+\frac{16d^{3}\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{e\sqrt{c(-ex+d)}\,\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c}$	167

input
```
int((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(-e*x+d)*(-e^3*x^3-7*d*e^2*x^2-43*d^2*e*x+91*d^3)*(e*x+d)^(3/2)/e/(-c*
e^2*x^2+c*d^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\,(e^3x^3+7\,de^2x^2+43\,d^2ex-91\,d^3)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{5\,(c^2e^3x^2-c^2d^2e)}$$

input
```
integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
2/5*(e^3*x^3 + 7*d*e^2*x^2 + 43*d^2*e*x - 91*d^3)*sqrt(-c*e^2*x^2 + c*d^2)
*sqrt(e*x + d)/(c^2*e^3*x^2 - c^2*d^2*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{9}{2}}}{(-c\,(-d+ex)\,(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(9/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(9/2)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.29

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = -\frac{2\,(e^3x^3+7\,de^2x^2+43\,d^2ex-91\,d^3)}{5\,\sqrt{-ex+d}c^{\frac{3}{2}}e}$$

input
```
integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
-2/5*(e^3*x^3 + 7*d*e^2*x^2 + 43*d^2*e*x - 91*d^3)/(sqrt(-e*x + d)*c^(3/2)
*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\left(\frac{40\,d^3}{\sqrt{-(ex+d)c+2\,cd}c}+\frac{60\,\sqrt{-(ex+d)c+2\,cd}c^{18}d^2-10\,(-(ex+d)c+2\,cd)^{\frac{3}{2}}c^{17}d+((ex+d)c-2\,cd)^2\sqrt{-(ex+}}{c^{20}}\right.}{5\,e}$$

input
```
integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
2/5*(40*d^3/(sqrt(-(e*x + d)*c + 2*c*d)*c) + (60*sqrt(-(e*x + d)*c + 2*c*d
)*c^18*d^2 - 10*(-(e*x + d)*c + 2*c*d)^(3/2)*c^17*d + ((e*x + d)*c - 2*c*d
)^2*sqrt(-(e*x + d)*c + 2*c*d)*c^16)/c^20)/e
```

Mupad [B] (verification not implemented)

Time = 6.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{2\,x^3\,\sqrt{d+e\,x}}{5\,c^2}-\frac{182\,d^3\,\sqrt{d+e\,x}}{5\,c^2\,e^3}+\frac{14\,d\,x^2\,\sqrt{d+e\,x}}{5\,c^2\,e}+\frac{86\,d^2\,x\,\sqrt{d+e\,x}}{5\,c^2\,e^2}\right)}{x^2-\frac{d^2}{e^2}}$$

input
```
int((d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(3/2),x)
```

output
```
((c*d^2 - c*e^2*x^2)^(1/2)*((2*x^3*(d + e*x)^(1/2))/(5*c^2) - (182*d^3*(d
+ e*x)^(1/2))/(5*c^2*e^3) + (14*d*x^2*(d + e*x)^(1/2))/(5*c^2*e) + (86*d^2
*x*(d + e*x)^(1/2))/(5*c^2*e^2)))/(x^2 - d^2/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,(-e^3x^3-7d\,e^2x^2-43d^2ex+91d^3)}{5\sqrt{-ex+d}\,c^2e}$$

input
```
int((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(2*sqrt(c)*(91*d**3 - 43*d**2*e*x - 7*d*e**2*x**2 - e**3*x**3))/(5*sqrt(d
- e*x)*c**2*e)
```

3.190 $\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2(d+ex)^{5/2}}{ce\sqrt{cd^2-ce^2x^2}} + \frac{32d\sqrt{cd^2-ce^2x^2}}{3c^2e\sqrt{d+ex}} + \frac{8\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3c^2e}$$

output
```
2*(e*x+d)^(5/2)/c/e/(-c*e^2*x^2+c*d^2)^(1/2)+32/3*d*(-c*e^2*x^2+c*d^2)^(1/
2)/c^2/e/(e*x+d)^(1/2)+8/3*(e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2)/c^2/e
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}(-23d^2+10dex+e^2x^2)}{3ce\sqrt{c(d^2-e^2x^2)}}$$

input
```
Integrate[(d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(-23*d^2 + 10*d*e*x + e^2*x^2))/(3*c*e*Sqrt[c*(d^2 - e^2
*x^2)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{3}d\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}dx-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{459}$$

$$\frac{8}{3}d\left(4d\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}dx-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}\right)-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{458}$$

$$\frac{8}{3}d\left(\frac{8d\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}\right)-\frac{2(d+ex)^{5/2}}{3ce\sqrt{cd^2-ce^2x^2}}$$

input
```
Int[(d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x)^(5/2))/(3*c*e*Sqrt[c*d^2 - c*e^2*x^2]) + (8*d*((8*d*Sqrt[d +
 e*x])/(c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(3/2))/(c*e*Sqrt[c*d^2
 - c*e^2*x^2])))/3
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

method	result	size
gosper	$\frac{2(-ex+d)\left(-e^{2}x^{2}-10dex+23d^{2}\right)(ex+d)^{\frac{3}{2}}}{3e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}$	55
orering	$\frac{2(-ex+d)\left(-e^{2}x^{2}-10dex+23d^{2}\right)(ex+d)^{\frac{3}{2}}}{3e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{3}{2}}}$	55
default	$\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\left(-e^{2}x^{2}-10dex+23d^{2}\right)}{3\sqrt{ex+d}\,c^{2}(-ex+d)e}$	59
risch	$\frac{2(ex+11d)(-ex+d)\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{3e\sqrt{-c(ex-d)}\,\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c}+\frac{8d^{2}\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{e\sqrt{c(-ex+d)}\,\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c}$	156

input
```
int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-e*x+d)*(-e^2*x^2-10*d*e*x+23*d^2)*(e*x+d)^(3/2)/e/(-c*e^2*x^2+c*d^2)
^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{-ce^2x^2+cd^2}(e^2x^2+10\,dex-23\,d^2)\sqrt{ex+d}}{3\,(c^2e^3x^2-c^2d^2e)}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-c*e^2*x^2 + c*d^2)*(e^2*x^2 + 10*d*e*x - 23*d^2)*sqrt(e*x + d)/(
c^2*e^3*x^2 - c^2*d^2*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{7}{2}}}{(-c\,(-d+ex)\,(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(7/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(7/2)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.38

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = -\frac{2\left(\sqrt{c}e^2x^2+10\,\sqrt{c}dex-23\,\sqrt{c}d^2\right)}{3\,\sqrt{-ex+d}c^2e}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
-2/3*(sqrt(c)*e^2*x^2 + 10*sqrt(c)*d*e*x - 23*sqrt(c)*d^2)/(sqrt(-e*x + d)
*c^2*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\left(\frac{12\,d^2}{\sqrt{-(ex+d)c+2\,cd}c} + \frac{12\,\sqrt{-(ex+d)c+2\,cd}c^7d-(-(ex+d)c+2\,cd)^{\frac{3}{2}}c^6}{c^9}\right)}{3\,e}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
2/3*(12*d^2/(sqrt(-(e*x + d)*c + 2*c*d)*c) + (12*sqrt(-(e*x + d)*c + 2*c*d
)*c^7*d - (-(e*x + d)*c + 2*c*d)^(3/2)*c^6)/c^9)/e
```

Mupad [B] (verification not implemented)

Time = 6.97 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{\sqrt{c\,d^2-c\,e^2\,x^2}\left(\frac{2\,x^2\,\sqrt{d+e\,x}}{3\,c^2\,e} - \frac{46\,d^2\,\sqrt{d+e\,x}}{3\,c^2\,e^3} + \frac{20\,d\,x\,\sqrt{d+e\,x}}{3\,c^2\,e^2}\right)}{x^2-\frac{d^2}{e^2}}$$

input
```
int((d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(3/2),x)
```

output
```
((c*d^2 - c*e^2*x^2)^(1/2)*((2*x^2*(d + e*x)^(1/2))/(3*c^2*e) - (46*d^2*(d
 + e*x)^(1/2))/(3*c^2*e^3) + (20*d*x*(d + e*x)^(1/2))/(3*c^2*e^2)))/(x^2 -
 d^2/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.33

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,(-e^2x^2-10dex+23d^2)}{3\sqrt{-ex+d}\,c^2e}$$

input
```
int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(2*sqrt(c)*(23*d**2 - 10*d*e*x - e**2*x**2))/(3*sqrt(d - e*x)*c**2*e)
```

3.191 $\int \frac{(d+ex)^{5/2}}{\left(cd^2-ce^2x^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 74

$$\int \frac{(d+ex)^{5/2}}{\left(cd^2-ce^2x^2\right)^{3/2}} \, dx = \frac{4d\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}} + \frac{2\sqrt{cd^2-ce^2x^2}}{c^2e\sqrt{d+ex}}$$

output
```
4*d*(e*x+d)^(1/2)/c/e/(-c*e^2*x^2+c*d^2)^(1/2)+2*(-c*e^2*x^2+c*d^2)^(1/2)/
c^2/e/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex)^{5/2}}{\left(cd^2-ce^2x^2\right)^{3/2}} \, dx = \frac{2(3d-ex)\sqrt{d+ex}}{ce\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[(d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(2*(3*d - e*x)*Sqrt[d + e*x])/(c*e*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 459}$$

$$4d\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}dx-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{ 458}$$

$$\frac{8d\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}-\frac{2(d+ex)^{3/2}}{ce\sqrt{cd^2-ce^2x^2}}$$

input
```
Int[(d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(8*d*Sqrt[d + e*x])/(c*e*Sqrt[c*d^2 - c*e^2*x^2]) - (2*(d + e*x)^(3/2))/(c
*e*Sqrt[c*d^2 - c*e^2*x^2])
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))    Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{2(-ex+d)(-ex+3d)(ex+d)^{\frac{3}{2}}}{e(-c\,e^{2}x^{2}+c\,d^{2})^{\frac{3}{2}}}$	44
orering	$\frac{2(-ex+d)(-ex+3d)(ex+d)^{\frac{3}{2}}}{e(-c\,e^{2}x^{2}+c\,d^{2})^{\frac{3}{2}}}$	44
default	$\frac{2\sqrt{c(-e^{2}x^{2}+d^{2})}\,(-ex+3d)}{\sqrt{ex+d}\,c^{2}(-ex+d)e}$	48
risch	$\frac{2(-ex+d)\sqrt{-\frac{c(e^{2}x^{2}-d^{2})}{ex+d}}\,\sqrt{ex+d}}{e\sqrt{-c(ex-d)}\,\sqrt{-c(e^{2}x^{2}-d^{2})}\,c}+\frac{4d\sqrt{-\frac{c(e^{2}x^{2}-d^{2})}{ex+d}}\,\sqrt{ex+d}}{e\sqrt{c(-ex+d)}\,\sqrt{-c(e^{2}x^{2}-d^{2})}\,c}$	147

input
```
int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-e*x+d)*(-e*x+3*d)*(e*x+d)^(3/2)/e/(-c*e^2*x^2+c*d^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx=\frac{2\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}(ex-3\,d)}{c^2e^3x^2-c^2d^2e}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*(e*x - 3*d)/(c^2*e^3*x^2 - c^2*d^
2*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{(-c(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(5/2)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.31

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = -\frac{2(ex-3d)}{\sqrt{-ex+d}c^{\frac{3}{2}}e}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
-2*(e*x - 3*d)/(sqrt(-e*x + d)*c^(3/2)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\left(\frac{2d}{\sqrt{-(ex+d)c+2cd}}+\frac{\sqrt{-(ex+d)c+2cd}}{c}\right)}{ce}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
2*(2*d/sqrt(-(e*x + d)*c + 2*c*d) + sqrt(-(e*x + d)*c + 2*c*d)/c)/(c*e)
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{6\,d\,\sqrt{d+e\,x}}{c^2\,e^3}-\frac{2\,x\,\sqrt{d+e\,x}}{c^2\,e^2}\right)}{x^2-\frac{d^2}{e^2}}$$

input
```
int((d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(3/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((6*d*(d + e*x)^(1/2))/(c^2*e^3) - (2*x*(d + e
*x)^(1/2))/(c^2*e^2)))/(x^2 - d^2/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,(-ex+3d)}{\sqrt{-ex+d}\,c^2e}$$

input
```
int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(2*sqrt(c)*(3*d - e*x))/(sqrt(d - e*x)*c**2*e)
```

3.192 $\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 36

$$\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx=\frac{2\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}$$

output
```
2*(e*x+d)^(1/2)/c/e/(-c*e^2*x^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^{3/2}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx=\frac{2\sqrt{d+ex}}{ce\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[(d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x])/(c*e*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 458}$$

$$\frac{2\sqrt{d+ex}}{ce\sqrt{cd^2-ce^2x^2}}$$

input
```
Int[(d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(2*Sqrt[d + e*x])/(c*e*Sqrt[c*d^2 - c*e^2*x^2])
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{2(-ex+d)(ex+d)^{\frac{3}{2}}}{e(-c\,e^{2}x^{2}+c\,d^{2})^{\frac{3}{2}}}$	36
orering	$\frac{2(-ex+d)(ex+d)^{\frac{3}{2}}}{e(-c\,e^{2}x^{2}+c\,d^{2})^{\frac{3}{2}}}$	36
default	$\frac{2\sqrt{c(-e^{2}x^{2}+d^{2})}}{\sqrt{ex+d}\,c^{2}(-ex+d)e}$	40

input
```
int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-e*x+d)*(e*x+d)^(3/2)/e/(-c*e^2*x^2+c*d^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = -\frac{2\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{c^2e^3x^2-c^2d^2e}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(c^2*e^3*x^2 - c^2*d^2*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(-c(-d+ex)(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.44

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2}{\sqrt{-ex+d}c^{\frac{3}{2}}e}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
2/(sqrt(-e*x + d)*c^(3/2)*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2}{\sqrt{-(ex+d)c+2\,cd}ce}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
2/(sqrt(-(e*x + d)*c + 2*c*d)*c*e)
```

Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{c\,d^2-c\,e^2\,x^2}\,\sqrt{d+e\,x}}{e\,\left(c^2\,d^2-c^2\,e^2\,x^2\right)}$$

input
```
int((d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(3/2),x)
```

output
```
(2*(c*d^2 - c*e^2*x^2)^(1/2)*(d + e*x)^(1/2))/(e*(c^2*d^2 - c^2*e^2*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{2\sqrt{c}}{\sqrt{-ex+d}\,c^2e}$$

input
```
int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(2*sqrt(c))/(sqrt(d - e*x)*c**2*e)
```

3.193 $\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 104

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx=\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}$$

output
```
(e*x+d)^(1/2)/c/d/e/(-c*e^2*x^2+c*d^2)^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*d
^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(3/2)/d^(3/2)/e
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx=\frac{2\sqrt{d}(d+ex)-\sqrt{2}\sqrt{d+ex}\sqrt{d^2-e^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{2cd^{3/2}e\sqrt{d+ex}\sqrt{c(d^2-e^2x^2)}}$$

input
```
Integrate[Sqrt[d + e*x]/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
(2*Sqrt[d]*(d + e*x) - Sqrt[2]*Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]*ArcTanh[(
Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/(2*c*d^(3/2)*e*Sqrt[d
 + e*x]*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{467}$$

$$\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{471}$$

$$\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}$$

input
```
Int[Sqrt[d + e*x]/(c*d^2 - c*e^2*x^2)^(3/2),x]
```

output
```
Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2 - c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2 - c*e^2
*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(3/2)*e)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) \sqrt{c\left(-e x+d\right)}-2 \sqrt{c d}\right)}{2 \sqrt{e x+d}\, c^{2} \left(-e x+d\right) e d \sqrt{c d}}$	91

input
```
int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(e*x+d)^(1/2)*(c*(-e^2*x^2+d^2))^(1/2)*(2^(1/2)*arctanh(1/2*(c*(-e*x+
d))^(1/2)*2^(1/2)/(c*d)^(1/2))*(c*(-e*x+d))^(1/2)-2*(c*d)^(1/2))/c^2/(-e*x
+d)/e/d/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \left[\frac{\sqrt{2}(e^2x^2-d^2)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right)-4\,\sqrt{-ce^2x}}{4\,(c^2d^2e^3x^2-c^2d^4e)}\right.$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*(e^2*x^2 - d^2)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*
d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2*x^2
 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d)/(c^2*d^2*
e^3*x^2 - c^2*d^4*e), 1/2*(sqrt(2)*(e^2*x^2 - d^2)*sqrt(-c*d)*arctan(1/2*s
qrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*d*e*x + c*d^2)
) - 2*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)*d)/(c^2*d^2*e^3*x^2 - c^2*d^4
*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{d+ex}}{(-c\,(-d+ex)\,(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral(sqrt(d + e*x)/(-c*(-d + e*x)*(d + e*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}}{(-ce^2x^2+cd^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(-c*e^2*x^2 + c*d^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\,\sqrt{-cd}}\right)}{\sqrt{-cd}cd} + \frac{2}{\sqrt{-(ex+d)c+2\,cd}cd}}{2\,e}$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))/(sq
rt(-c*d)*c*d) + 2/(sqrt(-(e*x + d)*c + 2*c*d)*c*d))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{d+e\,x}}{(c\,d^2-c\,e^2\,x^2)^{3/2}}\,dx$$

input
```
int((d + e*x)^(1/2)/(c*d^2 - c*e^2*x^2)^(3/2),x)
```

output
```
int((d + e*x)^(1/2)/(c*d^2 - c*e^2*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)-\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex\right.\right.}{4\sqrt{-ex+d}\,c^2d^2e}$$

input
```
int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(sqrt(c)*(sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt(2
)) - sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2)) +
4*d))/(4*sqrt(d - e*x)*c**2*d**2*e)
```

3.194 $\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 115

$$\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx=\frac{d+3ex}{4cd^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}-\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{4\sqrt{2}c^{3/2}d^{5/2}e}$$

output
```
1/4*(3*e*x+d)/c/d^2/e/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2)-3/8*arctanh(2
^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(
3/2)/d^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx=\frac{2\sqrt{d}(d+3ex)-3\sqrt{2}\sqrt{d+ex}\sqrt{d^2-e^2x^2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right)}{8cd^{5/2}e\sqrt{d+ex}\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[d]*(d + 3*e*x) - 3*Sqrt[2]*Sqrt[d + e*x]*Sqrt[d^2 - e^2*x^2]*ArcTa
nh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/(8*c*d^(5/2)*e*Sq
rt[d + e*x]*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {470, 467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}dx$$

$$\Big\downarrow \; 470$$

$$\frac{3\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}dx}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \; 467$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \; 471$$

$$\frac{3\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{3\left(\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}$$

input
```
Int[1/(Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```

output
```
-1/2*1/(c*d*e*Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (3*(Sqrt[d + e*x]/(
c*d*e*Sqrt[c*d^2 - c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*
Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(3/2)*e)))/(4*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\left(3\sqrt{2}\,\sqrt{c(-ex+d)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)ex+3\sqrt{2}\,\sqrt{c(-ex+d)}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)d-6\sqrt{cd}\,ex-2\sqrt{c}\right.}{8(ex+d)^{\frac{3}{2}}c^2(-ex+d)e\,d^2\sqrt{cd}}$

input
```
int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(e*x+d)^(3/2)*(c*(-e^2*x^2+d^2))^(1/2)/c^2*(3*2^(1/2)*(c*(-e*x+d))^(1
/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*e*x+3*2^(1/2)*(c*(
-e*x+d))^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d-6*(c*
d)^(1/2)*e*x-2*(c*d)^(1/2)*d)/(-e*x+d)/e/d^2/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.30

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}\,dx = \left[\frac{3\sqrt{2}(e^3x^3+de^2x^2-d^2ex-d^3)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\sqrt{2}\sqrt{-ce^2x^2}}{e^2x^2+2\,dex+d^2}\right.}{16\,(c^2d^3e^4x^3+c^2d^4e^3x^2-c^2d}\right.$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(2)*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3)*sqrt(c*d)*log(-(c*e
^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d
)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*(
3*d*e*x + d^2)*sqrt(e*x + d))/(c^2*d^3*e^4*x^3 + c^2*d^4*e^3*x^2 - c^2*d^5
*e^2*x - c^2*d^6*e), 1/8*(3*sqrt(2)*(e^3*x^3 + d*e^2*x^2 - d^2*e*x - d^3)*
sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x
 + d)/(c*d*e*x + c*d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*(3*d*e*x + d^2)*sqrt
(e*x + d))/(c^2*d^3*e^4*x^3 + c^2*d^4*e^3*x^2 - c^2*d^5*e^2*x - c^2*d^6*e)
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}\,dx=\int \frac{1}{(-c\,(-d+ex)\,(d+ex))^{\frac{3}{2}}\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral(1/((-c*(-d + e*x)*(d + e*x))**(3/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}\,dx=\int \frac{1}{(-ce^2x^2+cd^2)^{\frac{3}{2}}\,\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*e^2*x^2 + c*d^2)^(3/2)*sqrt(e*x + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}\,dx=\frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}cd^2}+\frac{2\,(3\,(ex+d)c-2\,cd)}{\left(2\sqrt{-(ex+d)c+2\,cd}cd-(-(ex+d)c+2\,cd)^{\frac{3}{2}}\right)cd^2}}{8\,e}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))/(
sqrt(-c*d)*c*d^2) + 2*(3*(e*x + d)*c - 2*c*d)/((2*sqrt(-(e*x + d)*c + 2*c*
d)*c*d - (-(e*x + d)*c + 2*c*d)^(3/2))*c*d^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{1}{(c\,d^2-c\,e^2\,x^2)^{3/2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(3/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(3/2)*(d + e*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(3\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)d+3\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{}\right.}{}$$

input
```
int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output
```
(sqrt(c)*(3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt
(2))*d + 3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt(
2))*e*x - 3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt
(2))*d - 3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(
2))*e*x + 4*d**2 + 12*d*e*x))/(16*sqrt(d - e*x)*c**2*d**3*e*(d + e*x))
```

3.195 $\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 156

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = -\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}} + \frac{5(d+3ex)}{32cd^3e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}} - \frac{15\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{32\sqrt{2}c^{3/2}d^{7/2}e}$$

output
```
-1/4/c/d/e/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(1/2)+5/32*(3*e*x+d)/c/d^3/e/(
e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2)-15/64*arctanh(2^(1/2)*c^(1/2)*d^(1/2
)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(3/2)/d^(7/2)/e
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = \frac{2\sqrt{d}(-3d^2+20dex+15e^2x^2)-15\sqrt{2}(d+ex)^{3/2}\sqrt{d^2-e^2x^2}\operatorname{arctanh}}{64cd^{7/2}e(d+ex)^{3/2}\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[d]*(-3*d^2 + 20*d*e*x + 15*e^2*x^2) - 15*Sqrt[2]*(d + e*x)^(3/2)*S
qrt[d^2 - e^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*
x^2]])/(64*c*d^(7/2)*e*(d + e*x)^(3/2)*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {470, 470, 467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 470}$$

$$\frac{5\int \frac{1}{\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}dx}{8d}-\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{ 470}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}dx}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{8d}-\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{ 467}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{8d}-\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow \text{ 471}$$

$$\frac{5\left(\frac{3\left(\frac{e\int \frac{1}{\frac{e^2(cd^2-ce^2x^2)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{8d}-\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}$$

$$\Big\downarrow 221$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{8d}-\frac{1}{4cde(d+ex)^{3/2}\sqrt{cd^2-ce^2x^2}}$$

input
```
Int[1/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2)),x]
```

output
```
-1/4*1/(c*d*e*(d + e*x)^(3/2)*Sqrt[c*d^2 - c*e^2*x^2]) + (5*(-1/2*1/(c*d*e
*Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (3*(Sqrt[d + e*x]/(c*d*e*Sqrt[c*
d^2 - c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[
d]*Sqrt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(3/2)*e)))/(4*d)))/(8*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\left(15\sqrt{c(-ex+d)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)e^2x^2+30\sqrt{c(-ex+d)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)dex+15\sqrt{c(-\right.}{64(ex+d)^{\frac{5}{2}}c^2(-ex+d)e\,d^3\sqrt{cd}}$

input
```
int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/(e*x+d)^(5/2)*(c*(-e^2*x^2+d^2))^(1/2)/c^2*(15*(c*(-e*x+d))^(1/2)*2^
(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*e^2*x^2+30*(c*(-
e*x+d))^(1/2)*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*
d*e*x+15*(c*(-e*x+d))^(1/2)*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)
/(c*d)^(1/2))*d^2-30*(c*d)^(1/2)*e^2*x^2-40*(c*d)^(1/2)*d*e*x+6*(c*d)^(1/2
)*d^2)/(-e*x+d)/e/d^3/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.62

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx = \left[\frac{15\sqrt{2}(e^4x^4+2\,de^3x^3-2\,d^3ex-d^4)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\sqrt{2}}{e^2x^2+2}\right.}{128\left(c^2d^4e^5x^4+2\,c^2\right.}\right.$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(15*sqrt(2)*(e^4*x^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4)*sqrt(c*d)*log
(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sq
rt(c*d)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*
d^2)*(15*d*e^2*x^2 + 20*d^2*e*x - 3*d^3)*sqrt(e*x + d))/(c^2*d^4*e^5*x^4 +
 2*c^2*d^5*e^4*x^3 - 2*c^2*d^7*e^2*x - c^2*d^8*e), 1/64*(15*sqrt(2)*(e^4*x
^4 + 2*d*e^3*x^3 - 2*d^3*e*x - d^4)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*
e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - 2*sqrt(-c*e
^2*x^2 + c*d^2)*(15*d*e^2*x^2 + 20*d^2*e*x - 3*d^3)*sqrt(e*x + d))/(c^2*d^
4*e^5*x^4 + 2*c^2*d^5*e^4*x^3 - 2*c^2*d^7*e^2*x - c^2*d^8*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(3/2),x)
```

output
```
Integral(1/((-c*(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-ce^2x^2+cd^2\right)^{\frac{3}{2}}\left(ex+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*e^2*x^2 + c*d^2)^(3/2)*(e*x + d)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int \frac{1}{(d+ex)^{3/2}\,(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}cd^3}+\frac{16}{\sqrt{-(ex+d)c+2\,cd}cd^3}-\frac{2\left(18\sqrt{-(ex+d)c+2\,cd}cd-7\right.}{(ex+d)^2c^3}}{64\,e}$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x, algorithm="giac")
```

output
```
1/64*(15*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))
/(sqrt(-c*d)*c*d^3) + 16/(sqrt(-(e*x + d)*c + 2*c*d)*c*d^3) - 2*(18*sqrt(-
(e*x + d)*c + 2*c*d)*c*d - 7*(-(e*x + d)*c + 2*c*d)^(3/2))/((e*x + d)^2*c^
3*d^3))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\,(cd^2-ce^2x^2)^{3/2}}\,dx = \int \frac{1}{\left(c\,d^2-c\,e^2\,x^2\right)^{3/2}\,\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(3/2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(3/2)*(d + e*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.61

$$\int \frac{1}{(d+ex)^{3/2}\,(cd^2-ce^2x^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(15\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)d^2+30\sqrt{d}\,\sqrt{-ex\right.}{}$$

input
```
int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2),x)
```

output

```
(sqrt(c)*(15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqr
t(2))*d**2 + 30*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*
sqrt(2))*d*e*x + 15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt
(d)*sqrt(2))*e**2*x**2 - 15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x
) + sqrt(d)*sqrt(2))*d**2 - 30*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d -
e*x) + sqrt(d)*sqrt(2))*d*e*x - 15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(
d - e*x) + sqrt(d)*sqrt(2))*e**2*x**2 - 12*d**3 + 80*d**2*e*x + 60*d*e**2*
x**2))/(128*sqrt(d - e*x)*c**2*d**4*e*(d**2 + 2*d*e*x + e**2*x**2))
```

3.196 $\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 152

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2(d+ex)^{9/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}} - \frac{4(d+ex)^{5/2}}{c^2e\sqrt{cd^2-ce^2x^2}} - \frac{64d\sqrt{cd^2-ce^2x^2}}{3c^3e\sqrt{d+ex}} - \frac{16\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}{3c^3e}$$

output
```
2/3*(e*x+d)^(9/2)/c/e/(-c*e^2*x^2+c*d^2)^(3/2)-4*(e*x+d)^(5/2)/c^2/e/(-c*e
^2*x^2+c*d^2)^(1/2)-64/3*d*(-c*e^2*x^2+c*d^2)^(1/2)/c^3/e/(e*x+d)^(1/2)-16
/3*(e*x+d)^(1/2)*(-c*e^2*x^2+c*d^2)^(1/2)/c^3/e
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{d+ex}(45d^3-69d^2ex+15de^2x^2+e^3x^3)}{3c^2e(-d+ex)\sqrt{c\,(d^2-e^2x^2)}}$$

input
```
Integrate[(d + e*x)^(11/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*Sqrt[d + e*x]*(45*d^3 - 69*d^2*e*x + 15*d*e^2*x^2 + e^3*x^3))/(3*c^2*e*
(-d + e*x)*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{459}$$

$$4d\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}dx-\frac{2(d+ex)^{9/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{459}$$

$$4d\left(8d\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{5/2}}dx-\frac{2(d+ex)^{7/2}}{ce\,(cd^2-ce^2x^2)^{3/2}}\right)-\frac{2(d+ex)^{9/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{459}$$

$$4d\left(8d\left(\frac{2(d+ex)^{5/2}}{ce\,(cd^2-ce^2x^2)^{3/2}}-4d\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}dx\right)-\frac{2(d+ex)^{7/2}}{ce\,(cd^2-ce^2x^2)^{3/2}}\right)-\frac{2(d+ex)^{9/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{458}$$

$$4d\left(8d\left(\frac{2(d+ex)^{5/2}}{ce\,(cd^2-ce^2x^2)^{3/2}}-\frac{8d(d+ex)^{3/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}\right)-\frac{2(d+ex)^{7/2}}{ce\,(cd^2-ce^2x^2)^{3/2}}\right)-\frac{2(d+ex)^{9/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}$$

input
```
Int[(d + e*x)^(11/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(9/2))/(3*c*e*(c*d^2 - c*e^2*x^2)^(3/2)) + 4*d*((-2*(d + e*x
)^(7/2))/(c*e*(c*d^2 - c*e^2*x^2)^(3/2)) + 8*d*((-8*d*(d + e*x)^(3/2))/(3*
c*e*(c*d^2 - c*e^2*x^2)^(3/2)) + (2*(d + e*x)^(5/2))/(c*e*(c*d^2 - c*e^2*x
^2)^(3/2))))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.43

method	result	size
gosper	$-\frac{2(-ex+d)\left(e^{3}x^{3}+15d\,e^{2}x^{2}-69d^{2}ex+45d^{3}\right)(ex+d)^{\frac{5}{2}}}{3e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{5}{2}}}$	65
orering	$-\frac{2(-ex+d)\left(e^{3}x^{3}+15d\,e^{2}x^{2}-69d^{2}ex+45d^{3}\right)(ex+d)^{\frac{5}{2}}}{3e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{5}{2}}}$	65
default	$-\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\,\left(e^{3}x^{3}+15d\,e^{2}x^{2}-69d^{2}ex+45d^{3}\right)}{3\sqrt{ex+d}\,c^{3}(-ex+d)^{2}e}$	69
risch	$-\frac{2(ex+17d)(-ex+d)\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{3e\sqrt{-c(ex-d)}\,\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c^{2}}-\frac{8d^{2}(-9ex+7d)\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{3e\sqrt{c(-ex+d)}\,(-ex+d)\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c^{2}}$	172

input
```
int((e*x+d)^(11/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-e*x+d)*(e^3*x^3+15*d*e^2*x^2-69*d^2*e*x+45*d^3)*(e*x+d)^(5/2)/e/(-c
*e^2*x^2+c*d^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2\left(e^3x^3+15\,de^2x^2-69\,d^2ex+45\,d^3\right)\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{3\left(c^3e^4x^3-c^3de^3x^2-c^3d^2e^2x+c^3d^3e\right)}$$

input
```
integrate((e*x+d)^(11/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(e^3*x^3 + 15*d*e^2*x^2 - 69*d^2*e*x + 45*d^3)*sqrt(-c*e^2*x^2 + c*d^
2)*sqrt(e*x + d)/(c^3*e^4*x^3 - c^3*d*e^3*x^2 - c^3*d^2*e^2*x + c^3*d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{11}{2}}}{(-c(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(11/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**(11/2)/(-c*(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\,(e^3x^3+15\,de^2x^2-69\,d^2ex+45\,d^3)}{3\left(c^{\frac{5}{2}}e^2x-c^{\frac{5}{2}}de\right)\sqrt{-ex+d}}$$

input
```
integrate((e*x+d)^(11/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(e^3*x^3 + 15*d*e^2*x^2 - 69*d^2*e*x + 45*d^3)/((c^(5/2)*e^2*x - c^(5/
2)*d*e)*sqrt(-e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2\left(\frac{4\,(2\,cd^3+9\,((ex+d)c-2\,cd)d^2)}{((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}c^2}+\frac{18\,\sqrt{-(ex+d)c+2\,cd}c^9d-(-(ex+d)c+2\,cd)^{\frac{3}{2}}c^8}{c^{12}}\right)}{3\,e}$$

input
```
integrate((e*x+d)^(11/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(4*(2*c*d^3 + 9*((e*x + d)*c - 2*c*d)*d^2)/((((e*x + d)*c - 2*c*d)*sqr
t(-(e*x + d)*c + 2*c*d)*c^2) + (18*sqrt(-(e*x + d)*c + 2*c*d)*c^9*d - (-(e
*x + d)*c + 2*c*d)^(3/2)*c^8)/c^12)/e
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{30\,d^3\,\sqrt{d+e\,x}}{c^3\,e^4}+\frac{2\,x^3\,\sqrt{d+e\,x}}{3\,c^3\,e}+\frac{10\,d\,x^2\,\sqrt{d+e\,x}}{c^3\,e^2}-\frac{46\,d^2\,x\,\sqrt{d+e\,x}}{c^3\,e^3}\right)}{x^3+\frac{d^3}{e^3}-\frac{d\,x^2}{e}-\frac{d^2\,x}{e^2}}$$

input
```
int((d + e*x)^(11/2)/(c*d^2 - c*e^2*x^2)^(5/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((30*d^3*(d + e*x)^(1/2))/(c^3*e^4) + (2*x^3*(
d + e*x)^(1/2))/(3*c^3*e) + (10*d*x^2*(d + e*x)^(1/2))/(c^3*e^2) - (46*d^2
*x*(d + e*x)^(1/2))/(c^3*e^3)))/(x^3 + d^3/e^3 - (d*x^2)/e - (d^2*x)/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

$$\int \frac{(d+ex)^{11/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,(-e^3x^3-15d\,e^2x^2+69d^2ex-45d^3)}{3\sqrt{-ex+d}\,c^3e\,(-ex+d)}$$

input
```
int((e*x+d)^(11/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 45*d**3 + 69*d**2*e*x - 15*d*e**2*x**2 - e**3*x**3))/(3*sqr
t(d - e*x)*c**3*e*(d - e*x))
```

3.197 $\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 116

$$\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{2(d+ex)^{7/2}}{3ce\left(cd^2-ce^2x^2\right)^{3/2}}-\frac{16d\sqrt{d+ex}}{3c^2e\sqrt{cd^2-ce^2x^2}}-\frac{8\sqrt{cd^2-ce^2x^2}}{3c^3e\sqrt{d+ex}}$$

output
```
2/3*(e*x+d)^(7/2)/c/e/(-c*e^2*x^2+c*d^2)^(3/2)-16/3*d*(e*x+d)^(1/2)/c^2/e/
(-c*e^2*x^2+c*d^2)^(1/2)-8/3*(-c*e^2*x^2+c*d^2)^(1/2)/c^3/e/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^{9/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{2\sqrt{d+ex}\left(11d^2-18dex+3e^2x^2\right)}{3c^2e(-d+ex)\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[(d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*Sqrt[d + e*x]*(11*d^2 - 18*d*e*x + 3*e^2*x^2))/(3*c^2*e*(-d + e*x)*Sqrt
[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{459}$$

$$8d\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{5/2}}dx-\frac{2(d+ex)^{7/2}}{ce\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{459}$$

$$8d\left(\frac{2(d+ex)^{5/2}}{ce\left(cd^2-ce^2x^2\right)^{3/2}}-4d\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}dx\right)-\frac{2(d+ex)^{7/2}}{ce\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{458}$$

$$8d\left(\frac{2(d+ex)^{5/2}}{ce\left(cd^2-ce^2x^2\right)^{3/2}}-\frac{8d(d+ex)^{3/2}}{3ce\left(cd^2-ce^2x^2\right)^{3/2}}\right)-\frac{2(d+ex)^{7/2}}{ce\left(cd^2-ce^2x^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(7/2))/(c*e*(c*d^2 - c*e^2*x^2)^(3/2)) + 8*d*((-8*d*(d + e*x
)^(3/2))/(3*c*e*(c*d^2 - c*e^2*x^2)^(3/2)) + (2*(d + e*x)^(5/2))/(c*e*(c*d
^2 - c*e^2*x^2)^(3/2)))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2(-ex+d)\left(3e^{2}x^{2}-18dex+11d^{2}\right)(ex+d)^{\frac{5}{2}}}{3e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{5}{2}}}$	55
orering	$-\frac{2(-ex+d)\left(3e^{2}x^{2}-18dex+11d^{2}\right)(ex+d)^{\frac{5}{2}}}{3e\left(-c\,e^{2}x^{2}+c\,d^{2}\right)^{\frac{5}{2}}}$	55
default	$-\frac{2\sqrt{c\left(-e^{2}x^{2}+d^{2}\right)}\,\left(3e^{2}x^{2}-18dex+11d^{2}\right)}{3\sqrt{ex+d}\,c^{3}(-ex+d)^{2}e}$	59
risch	$-\frac{2(-ex+d)\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{e\sqrt{-c(ex-d)}\,\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c^{2}}-\frac{8d(-3ex+2d)\sqrt{-\frac{c\left(e^{2}x^{2}-d^{2}\right)}{ex+d}}\,\sqrt{ex+d}}{3e\sqrt{c(-ex+d)}\,(-ex+d)\sqrt{-c\left(e^{2}x^{2}-d^{2}\right)}\,c^{2}}$	163

input
```
int((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-e*x+d)*(3*e^2*x^2-18*d*e*x+11*d^2)*(e*x+d)^(5/2)/e/(-c*e^2*x^2+c*d^
2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2\sqrt{-ce^2x^2+cd^2}(3\,e^2x^2-18\,dex+11\,d^2)\sqrt{ex+d}}{3\,(c^3e^4x^3-c^3de^3x^2-c^3d^2e^2x+c^3d^3e)}$$

input
```
integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-c*e^2*x^2 + c*d^2)*(3*e^2*x^2 - 18*d*e*x + 11*d^2)*sqrt(e*x + d
)/(c^3*e^4*x^3 - c^3*d*e^3*x^2 - c^3*d^2*e^2*x + c^3*d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{9}{2}}}{(-c\,(-d+ex)\,(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(9/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**(9/2)/(-c*(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\left(3\sqrt{c}e^2x^2-18\sqrt{c}dex+11\sqrt{c}d^2\right)}{3\,(c^3e^2x-c^3de)\sqrt{-ex+d}}$$

input
```
integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(c)*e^2*x^2 - 18*sqrt(c)*d*e*x + 11*sqrt(c)*d^2)/((c^3*e^2*x -
c^3*d*e)*sqrt(-e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2\left(\frac{3\sqrt{-(ex+d)c+2\,cd}}{c^3}+\frac{4\,(cd^2+3\,((ex+d)c-2\,cd)d)}{((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}c^2}\right)}{3\,e}$$

input
```
integrate((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*sqrt(-(e*x + d)*c + 2*c*d)/c^3 + 4*(c*d^2 + 3*((e*x + d)*c - 2*c*d
)*d)/((((e*x + d)*c - 2*c*d)*sqrt(-(e*x + d)*c + 2*c*d)*c^2))/e
```

Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\left(\frac{22\,d^2\,\sqrt{d+e\,x}}{3\,c^3\,e^4}+\frac{2\,x^2\,\sqrt{d+e\,x}}{c^3\,e^2}-\frac{12\,d\,x\,\sqrt{d+e\,x}}{c^3\,e^3}\right)}{x^3+\frac{d^3}{e^3}-\frac{d\,x^2}{e}-\frac{d^2\,x}{e^2}}$$

input
```
int((d + e*x)^(9/2)/(c*d^2 - c*e^2*x^2)^(5/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((22*d^2*(d + e*x)^(1/2))/(3*c^3*e^4) + (2*x^2
*(d + e*x)^(1/2))/(c^3*e^2) - (12*d*x*(d + e*x)^(1/2))/(c^3*e^3)))/(x^3 +
d^3/e^3 - (d*x^2)/e - (d^2*x)/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^{9/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,(-3e^2x^2+18dex-11d^2)}{3\sqrt{-ex+d}\,c^3e\,(-ex+d)}$$

input
```
int((e*x+d)^(9/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 11*d**2 + 18*d*e*x - 3*e**2*x**2))/(3*sqrt(d - e*x)*c**3*e*
(d - e*x))
```

3.198 $\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{2(d+ex)^{5/2}}{3ce\left(cd^2-ce^2x^2\right)^{3/2}}-\frac{4\sqrt{d+ex}}{3c^2e\sqrt{cd^2-ce^2x^2}}$$

output
```
2/3*(e*x+d)^(5/2)/c/e/(-c*e^2*x^2+c*d^2)^(3/2)-4/3*(e*x+d)^(1/2)/c^2/e/(-c
*e^2*x^2+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{2(d-3ex)\sqrt{d+ex}}{3c^2e(-d+ex)\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[(d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*(d - 3*e*x)*Sqrt[d + e*x])/(3*c^2*e*(-d + e*x)*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{459}$$

$$\frac{2(d+ex)^{5/2}}{ce\,(cd^2-ce^2x^2)^{3/2}}-4d\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{458}$$

$$\frac{2(d+ex)^{5/2}}{ce\,(cd^2-ce^2x^2)^{3/2}}-\frac{8d(d+ex)^{3/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}$$

input
```
Int[(d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(-8*d*(d + e*x)^(3/2))/(3*c*e*(c*d^2 - c*e^2*x^2)^(3/2)) + (2*(d + e*x)^(5
/2))/(c*e*(c*d^2 - c*e^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2(-ex+d)(-3ex+d)(ex+d)^{\frac{5}{2}}}{3e(-c\,e^{2}x^{2}+c\,d^{2})^{\frac{5}{2}}}$	42
orering	$-\frac{2(-ex+d)(-3ex+d)(ex+d)^{\frac{5}{2}}}{3e(-c\,e^{2}x^{2}+c\,d^{2})^{\frac{5}{2}}}$	42
default	$-\frac{2\sqrt{c(-e^{2}x^{2}+d^{2})}\,(-3ex+d)}{3\sqrt{ex+d}\,c^{3}(-ex+d)^{2}e}$	46

input
```
int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-e*x+d)*(-3*e*x+d)*(e*x+d)^(5/2)/e/(-c*e^2*x^2+c*d^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{-ce^2x^2+cd^2}(3\,ex-d)\sqrt{ex+d}}{3\left(c^3e^4x^3-c^3de^3x^2-c^3d^2e^2x+c^3d^3e\right)}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-c*e^2*x^2 + c*d^2)*(3*e*x - d)*sqrt(e*x + d)/(c^3*e^4*x^3 - c^3*
d*e^3*x^2 - c^3*d^2*e^2*x + c^3*d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{7}{2}}}{(-c(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(7/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**(7/2)/(-c*(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2\,(3\,ex-d)}{3\left(c^{\frac{5}{2}}e^2x-c^{\frac{5}{2}}de\right)\sqrt{-ex+d}}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(3*e*x - d)/((c^(5/2)*e^2*x - c^(5/2)*d*e)*sqrt(-e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{7/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2\,(3\,(ex+d)c-4\,cd)}{3\,((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}c^2e}$$

input
```
integrate((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*(e*x + d)*c - 4*c*d)/((((e*x + d)*c - 2*c*d)*sqrt(-(e*x + d)*c + 2*
c*d)*c^2*e)
```

Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = -\frac{\sqrt{c\,d^2-c\,e^2\,x^2}\,\left(\frac{2\,d\,\sqrt{d+e\,x}}{3\,c^3\,e^4}-\frac{2\,x\,\sqrt{d+e\,x}}{c^3\,e^3}\right)}{x^3+\frac{d^3}{e^3}-\frac{d\,x^2}{e}-\frac{d^2\,x}{e^2}}$$

input
```
int((d + e*x)^(7/2)/(c*d^2 - c*e^2*x^2)^(5/2),x)
```

output
```
-((c*d^2 - c*e^2*x^2)^(1/2)*((2*d*(d + e*x)^(1/2))/(3*c^3*e^4) - (2*x*(d +
 e*x)^(1/2))/(c^3*e^3)))/(x^3 + d^3/e^3 - (d*x^2)/e - (d^2*x)/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\int \frac{(d+ex)^{7/2}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \frac{2\sqrt{c}\,(3ex-d)}{3\sqrt{-ex+d}\,c^3e\,(-ex+d)}$$

input
```
int((e*x+d)^(7/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(2*sqrt(c)*( - d + 3*e*x))/(3*sqrt(d - e*x)*c**3*e*(d - e*x))
```

3.199 $\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 38

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}$$

output
```
2/3*(e*x+d)^(3/2)/c/e/(-c*e^2*x^2+c*d^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2(d+ex)^{3/2}}{3ce\,(c\,(d^2-e^2x^2))^{3/2}}$$

input
```
Integrate[(d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^(3/2))/(3*c*e*(c*(d^2 - e^2*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = $\{458\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx$$

$$\Big\downarrow \, 458$$

$$\frac{2(d+ex)^{3/2}}{3ce\,(cd^2-ce^2x^2)^{3/2}}$$

input
```
Int[(d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^(3/2))/(3*c*e*(c*d^2 - c*e^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{2(-ex+d)(ex+d)^{\frac{5}{2}}}{3e(-c\,e^2x^2+c\,d^2)^{\frac{5}{2}}}$	36
orering	$\frac{2(-ex+d)(ex+d)^{\frac{5}{2}}}{3e(-c\,e^2x^2+c\,d^2)^{\frac{5}{2}}}$	36
default	$\frac{2\sqrt{c(-e^2x^2+d^2)}}{3\sqrt{ex+d}\,c^3(-ex+d)^2e}$	40

input
```
int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-e*x+d)*(e*x+d)^(5/2)/e/(-c*e^2*x^2+c*d^2)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{-ce^2x^2+cd^2}\sqrt{ex+d}}{3\,(c^3e^4x^3-c^3de^3x^2-c^3d^2e^2x+c^3d^3e)}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d)/(c^3*e^4*x^3 - c^3*d*e^3*x^2 -
c^3*d^2*e^2*x + c^3*d^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{(-c\,(-d+ex)\,(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**(5/2)/(-c*(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2\sqrt{c}}{3\,(c^3e^2x-c^3de)\sqrt{-ex+d}}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(c)/((c^3*e^2*x - c^3*d*e)*sqrt(-e*x + d))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{2}{3\,((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}ce}$$

input
```
integrate((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
-2/3/(((e*x + d)*c - 2*c*d)*sqrt(-(e*x + d)*c + 2*c*d)*c*e)
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\,(d+e\,x)^{3/2}}{3\,c\,e\,(c\,d^2-c\,e^2\,x^2)^{3/2}}$$

input
```
int((d + e*x)^(5/2)/(c*d^2 - c*e^2*x^2)^(5/2),x)
```

output
```
(2*(d + e*x)^(3/2))/(3*c*e*(c*d^2 - c*e^2*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex)^{5/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{c}}{3\sqrt{-ex+d}\,c^3e\,(-ex+d)}$$

input
```
int((e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(2*sqrt(c))/(3*sqrt(d - e*x)*c**3*e*(d - e*x))
```

3.200 $\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 153

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{d+ex}}{3ce\,(cd^2-ce^2x^2)^{3/2}} + \frac{d+3ex}{6c^2d^2e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{2\sqrt{2}c^{5/2}d^{5/2}e}$$

output
```
2/3*(e*x+d)^(1/2)/c/e/(-c*e^2*x^2+c*d^2)^(3/2)+1/6*(3*e*x+d)/c^2/d^2/e/(e*
x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2)-1/4*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e
*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(5/2)/d^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{d}(5d^2+2dex-3e^2x^2)-3\sqrt{2}(d-ex)\sqrt{d+ex}\sqrt{d^2-e^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{d+ex}}{\sqrt{d^2-e^2x^2}}\right.}{12c^2d^{5/2}e(d-ex)\sqrt{d+ex}\sqrt{c\,(d^2-e^2x^2)}}$$

input
```
Integrate[(d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*Sqrt[d]*(5*d^2 + 2*d*e*x - 3*e^2*x^2) - 3*Sqrt[2]*(d - e*x)*Sqrt[d + e*
x]*Sqrt[d^2 - e^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 -
e^2*x^2]])/(12*c^2*d^(5/2)*e*(d - e*x)*Sqrt[d + e*x]*Sqrt[c*(d^2 - e^2*x^2
)])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {468, 470, 467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{468}$$

$$\frac{2\int \frac{1}{\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}dx}{3c}+\frac{2\sqrt{d+ex}}{3ce(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{2\left(\frac{3\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{3/2}}dx}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{3c}+\frac{2\sqrt{d+ex}}{3ce(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{467}$$

$$\frac{2\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{3c}+\frac{2\sqrt{d+ex}}{3ce(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{471}$$

$$\frac{2\left(\frac{3\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{3c}+\frac{2\sqrt{d+ex}}{3ce\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{2\left(\frac{3\left(\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{3c}+\frac{2\sqrt{d+ex}}{3ce\left(cd^2-ce^2x^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*Sqrt[d + e*x])/(3*c*e*(c*d^2 - c*e^2*x^2)^(3/2)) + (2*(-1/2*1/(c*d*e*Sq
rt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (3*(Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2
 - c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*
Sqrt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(3/2)*e)))/(4*d)))/(3*c)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 468
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] - Simp[d^2*((n +
p)/(b*(p + 1)))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && GtQ[n, 1] && I
ntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

method	result
default	$-\frac{\sqrt{c\left(-e^{2} x^{2}+d^{2}\right)}\, \left(-3 \sqrt{2}\, \sqrt{c\left(-e x+d\right)}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) e x+3 \sqrt{2}\, \sqrt{c\left(-e x+d\right)}\, \operatorname{arctanh}\left(\frac{\sqrt{c\left(-e x+d\right)}\, \sqrt{2}}{2 \sqrt{c d}}\right) d+6 \sqrt{c d}\, e x-10}{12 c^{3} \sqrt{e x+d}\, \left(-e x+d\right)^{2} e\, d^{2} \sqrt{c d}}$

input
```
int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(c*(-e^2*x^2+d^2))^(1/2)/c^3*(-3*2^(1/2)*(c*(-e*x+d))^(1/2)*arctanh(
1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*e*x+3*2^(1/2)*(c*(-e*x+d))^(1/
2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d+6*(c*d)^(1/2)*e*x
-10*(c*d)^(1/2)*d)/(e*x+d)^(1/2)/(-e*x+d)^2/e/d^2/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.49

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \Bigg[\frac{3\sqrt{2}(e^3x^3-de^2x^2-d^2ex+d^3)\sqrt{cd}\log\Big(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{e}}{e^2x^2+2\,dex+d^2}}{24\,(c^3d^3e^4x^3-c^3d^4e^3x^2-c^3d^5e^2x+c}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(2)*(e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^3)*sqrt(c*d)*log(-(c*e
^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d
)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*(
3*d*e*x - 5*d^2)*sqrt(e*x + d))/(c^3*d^3*e^4*x^3 - c^3*d^4*e^3*x^2 - c^3*d
^5*e^2*x + c^3*d^6*e), 1/12*(3*sqrt(2)*(e^3*x^3 - d*e^2*x^2 - d^2*e*x + d^
3)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(
e*x + d)/(c*d*e*x + c*d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*(3*d*e*x - 5*d^2)
*sqrt(e*x + d))/(c^3*d^3*e^4*x^3 - c^3*d^4*e^3*x^2 - c^3*d^5*e^2*x + c^3*d
^6*e)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(-c\,(-d+ex)\,(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(-c*(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(-ce^2x^2+cd^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(-c*e^2*x^2 + c*d^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}c^2d^2} + \frac{2\,(3\,(ex+d)c-8\,cd)}{((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}c^2d^2}}{12\,e}$$

input
```
integrate((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
1/12*(3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))/
(sqrt(-c*d)*c^2*d^2) + 2*(3*(e*x + d)*c - 8*c*d)/(((e*x + d)*c - 2*c*d)*sq
rt(-(e*x + d)*c + 2*c*d)*c^2*d^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{{(d+e\,x)}^{3/2}}{{(c\,d^2-c\,e^2\,x^2)}^{5/2}}\,dx$$

input
```
int((d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(3/2)/(c*d^2 - c*e^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^{3/2}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(3\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)d-3\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{\right.\right.}{}$$

input
```
int((e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(sqrt(c)*(3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt
(2))*d - 3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt(
2))*e*x - 3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt
(2))*d + 3*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(
2))*e*x + 20*d**2 - 12*d*e*x))/(24*sqrt(d - e*x)*c**3*d**3*e*(d - e*x))
```

3.201 $\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 156

$$\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}+\frac{5(d+3ex)}{24c^2d^3e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}-\frac{5\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{8\sqrt{2}c^{5/2}d^{7/2}e}$$

output
```
1/3*(e*x+d)^(1/2)/c/d/e/(-c*e^2*x^2+c*d^2)^(3/2)+5/24*(3*e*x+d)/c^2/d^3/e/
(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2)-5/16*arctanh(2^(1/2)*c^(1/2)*d^(1/2
)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(5/2)/d^(7/2)/e
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{2\sqrt{d}(13d^2+10dex-15e^2x^2)-15\sqrt{2}(d-ex)\sqrt{d+ex}\sqrt{d^2-e^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\cdots}{\sqrt{d^2-\cdots}}\right.}{48c^2d^{7/2}e(d-ex)\sqrt{d+ex}\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[Sqrt[d + e*x]/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output
```
(2*Sqrt[d]*(13*d^2 + 10*d*e*x - 15*e^2*x^2) - 15*Sqrt[2]*(d - e*x)*Sqrt[d
+ e*x]*Sqrt[d^2 - e^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^
2 - e^2*x^2]])/(48*c^2*d^(7/2)*e*(d - e*x)*Sqrt[d + e*x]*Sqrt[c*(d^2 - e^2
*x^2)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {467, 470, 467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{467}$$

$$\frac{5\int \frac{1}{\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}dx}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}dx}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{467}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{471}$$

$$\frac{5\left(\frac{3\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}$$

input

```
Int[Sqrt[d + e*x]/(c*d^2 - c*e^2*x^2)^(5/2),x]
```

output

```
Sqrt[d + e*x]/(3*c*d*e*(c*d^2 - c*e^2*x^2)^(3/2)) + (5*(-1/2*1/(c*d*e*Sqrt
[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2]) + (3*(Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2 -
 c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sq
rt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(3/2)*e)))/(4*d)))/(6*c*d)
```

Defintions of rubi rules used

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\left(-15\sqrt{c(-ex+d)}\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{c(-ex+d)}\sqrt{2}}{2\sqrt{cd}}\right)e^2x^2+15\sqrt{c(-ex+d)}\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{c(-ex+d)}\sqrt{2}}{2\sqrt{cd}}\right)d^2+30\sqrt{cd}\right.}{48c^3(ex+d)^{\frac{3}{2}}(-ex+d)^2e\,d^3\sqrt{cd}}$

input
```
int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(c*(-e^2*x^2+d^2))^(1/2)/c^3*(-15*(c*(-e*x+d))^(1/2)*2^(1/2)*arctanh
(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*e^2*x^2+15*(c*(-e*x+d))^(1/2)
*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d^2+30*(c*d)^
(1/2)*e^2*x^2-20*(c*d)^(1/2)*d*e*x-26*(c*d)^(1/2)*d^2)/(e*x+d)^(3/2)/(-e*x
+d)^2/e/d^3/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \left[\frac{15\sqrt{2}(e^4x^4-2\,d^2e^2x^2+d^4)\sqrt{cd}\log\left(-\frac{ce^2x^2-2\,cdex-3\,cd^2+2\,\sqrt{2}\sqrt{-ce^2x^2+cd^2}\sqrt{cd}\sqrt{ex+d}}{e^2x^2+2\,dex+d^2}\right.}{96\,(c^3d^4e^5x^4-2\,c^3d^6e^3x^2+}\right.$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(15*sqrt(2)*(e^4*x^4 - 2*d^2*e^2*x^2 + d^4)*sqrt(c*d)*log(-(c*e^2*x^
2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqr
t(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*sqrt(-c*e^2*x^2 + c*d^2)*(15*d*
e^2*x^2 - 10*d^2*e*x - 13*d^3)*sqrt(e*x + d))/(c^3*d^4*e^5*x^4 - 2*c^3*d^6
*e^3*x^2 + c^3*d^8*e), 1/48*(15*sqrt(2)*(e^4*x^4 - 2*d^2*e^2*x^2 + d^4)*sq
rt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e*x +
 d)/(c*d*e*x + c*d^2)) - 2*sqrt(-c*e^2*x^2 + c*d^2)*(15*d*e^2*x^2 - 10*d^2
*e*x - 13*d^3)*sqrt(e*x + d))/(c^3*d^4*e^5*x^4 - 2*c^3*d^6*e^3*x^2 + c^3*d
^8*e)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{\sqrt{d+ex}}{(-c(-d+ex)(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral(sqrt(d + e*x)/(-c*(-d + e*x)*(d + e*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}}{(-ce^2x^2+cd^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(-c*e^2*x^2 + c*d^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}c^2d^3} + \frac{8\,(3\,(ex+d)c-7\,cd)}{((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}c^2d^3} - \frac{6\,\sqrt{-(ex+d)c+2\,cd}}{(ex+d)c^3d^3}}{48\,e}$$

input
```
integrate((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
1/48*(15*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d))
/(sqrt(-c*d)*c^2*d^3) + 8*(3*(e*x + d)*c - 7*c*d)/((((e*x + d)*c - 2*c*d)*s
qrt(-(e*x + d)*c + 2*c*d)*c^2*d^3) - 6*sqrt(-(e*x + d)*c + 2*c*d)/((e*x +
d)*c^3*d^3))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{\sqrt{d+e\,x}}{{\left(c\,d^2-c\,e^2\,x^2\right)}^{5/2}} \,d x$$

input
```
int((d + e*x)^(1/2)/(c*d^2 - c*e^2*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(1/2)/(c*d^2 - c*e^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(15\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)d^2-15\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\right.}{}$$

input
```
int((e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output

```
(sqrt(c)*(15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqr
t(2))*d**2 - 15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*
sqrt(2))*e**2*x**2 - 15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) +
sqrt(d)*sqrt(2))*d**2 + 15*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x)
 + sqrt(d)*sqrt(2))*e**2*x**2 + 52*d**3 + 40*d**2*e*x - 60*d*e**2*x**2))/(
96*sqrt(d - e*x)*c**3*d**4*e*(d**2 - e**2*x**2))
```

3.202 $\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 162

$$\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{d+7ex}{24cd^2e\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}+\frac{35(d+3ex)}{192c^2d^4e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}-\frac{35\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{64\sqrt{2}c^{5/2}d^{9/2}e}$$

output
```
1/24*(7*e*x+d)/c/d^2/e/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2)+35/192*(3*e*
x+d)/c^2/d^4/e/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2)-35/128*arctanh(2^(1/
2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(5/2)
/d^(9/2)/e
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{2\sqrt{d}(43d^3+161d^2ex-35de^2x^2-105e^3x^3)-105\sqrt{2}\sqrt{d+ex}(d^2-e^2x^2}{384c^2d^{9/2}e(d-ex)(d+ex)^{3/2}\sqrt{c\left(d^2-e^2x^2\right)}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(5/2)),x]
```

output
```
(2*Sqrt[d]*(43*d^3 + 161*d^2*e*x - 35*d*e^2*x^2 - 105*e^3*x^3) - 105*Sqrt[
2]*Sqrt[d + e*x]*(d^2 - e^2*x^2)^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e
*x])/Sqrt[d^2 - e^2*x^2]])/(384*c^2*d^(9/2)*e*(d - e*x)*(d + e*x)^(3/2)*Sq
rt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {470, 467, 470, 467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{470}$$

$$\frac{7\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{5/2}}dx}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{467}$$

$$\frac{7\left(\frac{5\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}dx}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}dx}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{467}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde(cd^2-ce^2x^2)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{471}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{e\int \frac{1}{\frac{e^2(cd^2-ce^2x^2)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde(cd^2-ce^2x^2)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde(cd^2-ce^2x^2)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\,(cd^2-ce^2x^2)^{3/2}}$$

input
```
Int[1/(Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(5/2)),x]
```

output
```
-1/4*1/(c*d*e*Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2)) + (7*(Sqrt[d + e*x]
/(3*c*d*e*(c*d^2 - c*e^2*x^2)^(3/2)) + (5*(-1/2*1/(c*d*e*Sqrt[d + e*x]*Sqr
t[c*d^2 - c*e^2*x^2]) + (3*(Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2 - c*e^2*x^2])
- ArcTanh[Sqrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]
/(Sqrt[2]*c^(3/2)*d^(3/2)*e)))/(4*d)))/(6*c*d)))/(8*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.62

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\left(-105\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)e^3x^3\sqrt{c(-ex+d)}-105\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)d\,e^2x^2\sqrt{c(-ex+d)}+1}{38}$

input
```
int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(c*(-e^2*x^2+d^2))^(1/2)/c^3*(-105*2^(1/2)*arctanh(1/2*(c*(-e*x+d))
^(1/2)*2^(1/2)/(c*d)^(1/2))*e^3*x^3*(c*(-e*x+d))^(1/2)-105*2^(1/2)*arctanh
(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d*e^2*x^2*(c*(-e*x+d))^(1/2)+
105*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d^2*e*x*(c
*(-e*x+d))^(1/2)+210*(c*d)^(1/2)*e^3*x^3+105*2^(1/2)*arctanh(1/2*(c*(-e*x+
d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d^3*(c*(-e*x+d))^(1/2)+70*(c*d)^(1/2)*d*e^2
*x^2-322*(c*d)^(1/2)*d^2*e*x-86*(c*d)^(1/2)*d^3)/(e*x+d)^(5/2)/(-e*x+d)^2/
e/d^4/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{5/2}}\,dx = \left[\frac{105\,\sqrt{2}(e^5x^5+de^4x^4-2\,d^2e^3x^3-2\,d^3e^2x^2+d^4ex+d^5)\sqrt{cd}\log\left(-\frac{ce^2}{}\right.}{768\,(c^3d^5e^6x^5+}\right.$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(105*sqrt(2)*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 +
 d^4*e*x + d^5)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2
)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2*x^2 + 2*d*e*x + d
^2)) - 4*(105*d*e^3*x^3 + 35*d^2*e^2*x^2 - 161*d^3*e*x - 43*d^4)*sqrt(-c*e
^2*x^2 + c*d^2)*sqrt(e*x + d))/(c^3*d^5*e^6*x^5 + c^3*d^6*e^5*x^4 - 2*c^3*
d^7*e^4*x^3 - 2*c^3*d^8*e^3*x^2 + c^3*d^9*e^2*x + c^3*d^10*e), 1/384*(105*
sqrt(2)*(e^5*x^5 + d*e^4*x^4 - 2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 + d^4*e*x + d
^5)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt
(e*x + d)/(c*d*e*x + c*d^2)) - 2*(105*d*e^3*x^3 + 35*d^2*e^2*x^2 - 161*d^3
*e*x - 43*d^4)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(c^3*d^5*e^6*x^5 +
c^3*d^6*e^5*x^4 - 2*c^3*d^7*e^4*x^3 - 2*c^3*d^8*e^3*x^2 + c^3*d^9*e^2*x +
c^3*d^10*e)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{1}{(-c\,(-d+ex)\,(d+ex))^{\frac{5}{2}}\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral(1/((-c*(-d + e*x)*(d + e*x))**(5/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{1}{(-ce^2x^2+cd^2)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*e^2*x^2 + c*d^2)^(5/2)*sqrt(e*x + d)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{\frac{105\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}c^2d^4} + \frac{16\,(9\,(ex+d)c-20\,cd)}{((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}c^2d^4} - \frac{6\left(26\sqrt{-(ex+$$

$$384\,e$$

input
```
integrate(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
1/384*(105*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*d
))/(sqrt(-c*d)*c^2*d^4) + 16*(9*(e*x + d)*c - 20*c*d)/(((e*x + d)*c - 2*c*
d)*sqrt(-(e*x + d)*c + 2*c*d)*c^2*d^4) - 6*(26*sqrt(-(e*x + d)*c + 2*c*d)*
c*d - 11*(-(e*x + d)*c + 2*c*d)^(3/2))/((e*x + d)^2*c^4*d^4))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{5/2}}\,dx = \int \frac{1}{\left(c\,d^2-c\,e^2\,x^2\right)^{5/2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(5/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(5/2)*(d + e*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.14

$$\int \frac{1}{\sqrt{d+ex}\,(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(105\sqrt{d}\sqrt{-ex+d}\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\sqrt{2}\right)d^3+105\sqrt{d}\sqrt{-ex$$

input
```
int(1/(e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(sqrt(c)*(105*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sq
rt(2))*d**3 + 105*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d
)*sqrt(2))*d**2*e*x - 105*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x)
- sqrt(d)*sqrt(2))*d*e**2*x**2 - 105*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqr
t(d - e*x) - sqrt(d)*sqrt(2))*e**3*x**3 - 105*sqrt(d)*sqrt(d - e*x)*sqrt(2
)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d**3 - 105*sqrt(d)*sqrt(d - e*x)*sq
rt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d**2*e*x + 105*sqrt(d)*sqrt(d -
 e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d*e**2*x**2 + 105*sqrt(
d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*e**3*x**3 +
172*d**4 + 644*d**3*e*x - 140*d**2*e**2*x**2 - 420*d*e**3*x**3))/(768*sqrt
(d - e*x)*c**3*d**5*e*(d**3 + d**2*e*x - d*e**2*x**2 - e**3*x**3))
```

3.203 $\int \frac{1}{(d+ex)^{3/2}(cd^2-ce^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 203

$$\int \frac{1}{(d+ex)^{3/2}(cd^2-ce^2x^2)^{5/2}}\,dx = -\frac{1}{6cde(d+ex)^{3/2}(cd^2-ce^2x^2)^{3/2}}+\frac{d+7ex}{32cd^3e\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}+\frac{35(d+3ex)}{256c^2d^5e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}-\frac{105\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{256\sqrt{2}c^{5/2}d^{11/2}e}$$

output
```
-1/6/c/d/e/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(3/2)+1/32*(7*e*x+d)/c/d^3/e/(
e*x+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(3/2)+35/256*(3*e*x+d)/c^2/d^5/e/(e*x+d)^(
1/2)/(-c*e^2*x^2+c*d^2)^(1/2)-105/512*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x
+d)^(1/2)/(-c*e^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(5/2)/d^(11/2)/e
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex)^{3/2}(cd^2-ce^2x^2)^{5/2}}\,dx = \frac{2\sqrt{d}(d^4+612d^3ex+378d^2e^2x^2-420de^3x^3-315e^4x^4)-315\sqrt{2}(d-}{1536c^2d^{11/2}e(d-ex)(d+ex)^{5/2}\sqrt{}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(5/2)),x]
```

output
```
(2*Sqrt[d]*(d^4 + 612*d^3*e*x + 378*d^2*e^2*x^2 - 420*d*e^3*x^3 - 315*e^4*
x^4) - 315*Sqrt[2]*(d - e*x)*(d + e*x)^(5/2)*Sqrt[d^2 - e^2*x^2]*ArcTanh[(
Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^2*x^2]])/(1536*c^2*d^(11/2)*e*
(d - e*x)*(d + e*x)^(5/2)*Sqrt[c*(d^2 - e^2*x^2)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.49, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {470, 470, 467, 470, 467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}(cd^2-ce^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 470}$$

$$\frac{3\int \frac{1}{\sqrt{d+ex}(cd^2-ce^2x^2)^{5/2}}dx}{4d}-\frac{1}{6cde(d+ex)^{3/2}(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{ 470}$$

$$\frac{3\left(\frac{7\int \frac{\sqrt{d+ex}}{(cd^2-ce^2x^2)^{5/2}}dx}{8d}-\frac{1}{4cde\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}\right)}{4d}-\frac{1}{6cde(d+ex)^{3/2}(cd^2-ce^2x^2)^{3/2}}$$

$$\Big\downarrow \text{ 467}$$

$$\frac{3\left(\frac{7\left(\frac{5\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}dx}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}\right)}{4d}-\frac{1}{6cde(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 470}$$

$$\frac{3\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}dx}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}\right)}{4d}-\frac{1}{6cde(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 467}$$

$$\frac{3\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}\right)}{4d}-\frac{1}{6cde(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{471}$$

$$3\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{e\int \frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}(cd^2}\right.$$

$$\overline{\qquad 4d \qquad}$$

$$\frac{1}{6cde(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\downarrow 221$$

$$3\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}(cd^2-ce^2x^2)^{3/2}}\right)$$

$$\overline{\qquad 4d \qquad}$$

$$\frac{1}{6cde(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(5/2)),x]
```

output
```
-1/6*1/(c*d*e*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2)) + (3*(-1/4*1/(c*d
*e*Sqrt[d + e*x]*(c*d^2 - c*e^2*x^2)^(3/2)) + (7*(Sqrt[d + e*x]/(3*c*d*e*(
c*d^2 - c*e^2*x^2)^(3/2)) + (5*(-1/2*1/(c*d*e*Sqrt[d + e*x]*Sqrt[c*d^2 - c
*e^2*x^2]) + (3*(Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2 - c*e^2*x^2]) - ArcTanh[S
qrt[c*d^2 - c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(Sqrt[2]*c
^(3/2)*d^(3/2)*e)))/(4*d)))/(6*c*d)))/(8*d)))/(4*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.37

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\left(-315\,\text{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)\sqrt{2}\,e^4x^4\sqrt{c(-ex+d)}-630\,\text{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)\sqrt{2}\,d\,e^3x^3\sqrt{c(-ex+d)}+\right.}{}$

input
```
int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1536*(c*(-e^2*x^2+d^2))^(1/2)/c^3*(-315*arctanh(1/2*(c*(-e*x+d))^(1/2)*
2^(1/2)/(c*d)^(1/2))*2^(1/2)*e^4*x^4*(c*(-e*x+d))^(1/2)-630*arctanh(1/2*(c
*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*2^(1/2)*d*e^3*x^3*(c*(-e*x+d))^(1/2)
+630*(c*d)^(1/2)*e^4*x^4+630*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^
(1/2))*2^(1/2)*d^3*e*x*(c*(-e*x+d))^(1/2)+840*(c*d)^(1/2)*d*e^3*x^3+315*ar
ctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*2^(1/2)*d^4*(c*(-e*x+d))
^(1/2)-756*(c*d)^(1/2)*d^2*e^2*x^2-1224*(c*d)^(1/2)*d^3*e*x-2*(c*d)^(1/2)*
d^4)/(e*x+d)^(7/2)/(-e*x+d)^2/e/d^5/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.94

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \left[\frac{315\sqrt{2}(e^6x^6+2\,de^5x^5-d^2e^4x^4-4\,d^3e^3x^3-d^4e^2x^2+2\,d^5ex+d^6)}{3072\,(c^3d^6}\right.$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/3072*(315*sqrt(2)*(e^6*x^6 + 2*d*e^5*x^5 - d^2*e^4*x^4 - 4*d^3*e^3*x^3
- d^4*e^2*x^2 + 2*d^5*e*x + d^6)*sqrt(c*d)*log(-(c*e^2*x^2 - 2*c*d*e*x - 3
*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sqrt(e*x + d))/(e^2*
x^2 + 2*d*e*x + d^2)) - 4*(315*d*e^4*x^4 + 420*d^2*e^3*x^3 - 378*d^3*e^2*x
^2 - 612*d^4*e*x - d^5)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(c^3*d^6*e
^7*x^6 + 2*c^3*d^7*e^6*x^5 - c^3*d^8*e^5*x^4 - 4*c^3*d^9*e^4*x^3 - c^3*d^1
0*e^3*x^2 + 2*c^3*d^11*e^2*x + c^3*d^12*e), 1/1536*(315*sqrt(2)*(e^6*x^6 +
 2*d*e^5*x^5 - d^2*e^4*x^4 - 4*d^3*e^3*x^3 - d^4*e^2*x^2 + 2*d^5*e*x + d^6
)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(-c*d)*sqrt(e
*x + d)/(c*d*e*x + c*d^2)) - 2*(315*d*e^4*x^4 + 420*d^2*e^3*x^3 - 378*d^3*
e^2*x^2 - 612*d^4*e*x - d^5)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(c^3*
d^6*e^7*x^6 + 2*c^3*d^7*e^6*x^5 - c^3*d^8*e^5*x^4 - 4*c^3*d^9*e^4*x^3 - c^
3*d^10*e^3*x^2 + 2*c^3*d^11*e^2*x + c^3*d^12*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \int \frac{1}{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral(1/((-c*(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\int \frac{1}{\left(-ce^2x^2+cd^2\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*e^2*x^2 + c*d^2)^(5/2)*(e*x + d)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\frac{\frac{315\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}c^2d^5}+\frac{2\left(256\,c^4d^4-1152\,((ex+d)c-2\,cd)c^3d^3-2772\,((ex+d)c\right.}{\left(2\sqrt{-(ex+d)c+2\,cd}\right.}}{1536\,e}$$

input
```
integrate(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
1/1536*(315*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-c*
d))/(sqrt(-c*d)*c^2*d^5) + 2*(256*c^4*d^4 - 1152*((e*x + d)*c - 2*c*d)*c^3
*d^3 - 2772*((e*x + d)*c - 2*c*d)^2*c^2*d^2 - 1680*((e*x + d)*c - 2*c*d)^3
*c*d - 315*((e*x + d)*c - 2*c*d)^4)/((2*sqrt(-(e*x + d)*c + 2*c*d)*c*d - (
-(e*x + d)*c + 2*c*d)^(3/2))^3*c^2*d^5))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\int \frac{1}{{\left(c\,d^2-c\,e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(5/2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(5/2)*(d + e*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.76

$$\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(315\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)d^4+630\sqrt{d}\,\sqrt{-\cdots}\right.}{}$$

input
```
int(1/(e*x+d)^(3/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(sqrt(c)*(315*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sq
rt(2))*d**4 + 630*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d
)*sqrt(2))*d**3*e*x - 630*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x)
- sqrt(d)*sqrt(2))*d*e**3*x**3 - 315*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqr
t(d - e*x) - sqrt(d)*sqrt(2))*e**4*x**4 - 315*sqrt(d)*sqrt(d - e*x)*sqrt(2
)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d**4 - 630*sqrt(d)*sqrt(d - e*x)*sq
rt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d**3*e*x + 630*sqrt(d)*sqrt(d -
 e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d*e**3*x**3 + 315*sqrt(
d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*e**4*x**4 +
4*d**5 + 2448*d**4*e*x + 1512*d**3*e**2*x**2 - 1680*d**2*e**3*x**3 - 1260*
d*e**4*x**4))/(3072*sqrt(d - e*x)*c**3*d**6*e*(d**4 + 2*d**3*e*x - 2*d*e**
3*x**3 - e**4*x**4))
```

3.204 $\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 244

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=-\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}-\frac{11}{96cd^2e(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}+\frac{11(d+7ex)}{512cd^4e\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}+\frac{385(d+3ex)}{4096c^2d^6e\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}-\frac{1155\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}{\sqrt{cd^2-ce^2x^2}}\right)}{4096\sqrt{2}c^{5/2}d^{13/2}e}$$

output
```
-1/8/c/d/e/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(3/2)-11/96/c/d^2/e/(e*x+d)^(3
/2)/(-c*e^2*x^2+c*d^2)^(3/2)+11/512*(7*e*x+d)/c/d^4/e/(e*x+d)^(1/2)/(-c*e^
2*x^2+c*d^2)^(3/2)+385/4096*(3*e*x+d)/c^2/d^6/e/(e*x+d)^(1/2)/(-c*e^2*x^2+
c*d^2)^(1/2)-1155/8192*arctanh(2^(1/2)*c^(1/2)*d^(1/2)*(e*x+d)^(1/2)/(-c*e
^2*x^2+c*d^2)^(1/2))*2^(1/2)/c^(5/2)/d^(13/2)/e
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.76

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \frac{-2\sqrt{d}(1525d^5-6743d^4ex-10890d^3e^2x^2+462d^2e^3x^3+8085de^4x^4+}{24576c^2d^{13/2}e(d-ex)}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2)^(5/2)),x]
```

output
```
(-2*Sqrt[d]*(1525*d^5 - 6743*d^4*e*x - 10890*d^3*e^2*x^2 + 462*d^2*e^3*x^3
 + 8085*d*e^4*x^4 + 3465*e^5*x^5) - 3465*Sqrt[2]*(d - e*x)*(d + e*x)^(7/2)
*Sqrt[d^2 - e^2*x^2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sqrt[d + e*x])/Sqrt[d^2 - e^
2*x^2]])/(24576*c^2*d^(13/2)*e*(d - e*x)*(d + e*x)^(7/2)*Sqrt[c*(d^2 - e^2
*x^2)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.44, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {470, 470, 470, 467, 470, 467, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 470$$

$$\frac{11\int \frac{1}{(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{5/2}}dx}{16d}-\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \ 470$$

$$\frac{11\left(\frac{3\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{5/2}}dx}{4d}-\frac{1}{6cde(d+ex)^{3/2}\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{16d}-\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{11\left(\frac{3\left(\frac{7\int \frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{5/2}}dx}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{4d}-\frac{1}{6cde(d+ex)^{3/2}(cd^2-ce^2x^2)^{3/2}}\right)}{16d}-\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{467}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int \frac{1}{\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}dx}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{4d}-\frac{1}{6cde(d+ex)^{3/2}(cd^2-ce^2x^2)^{3/2}}\right)}{16d}-\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{470}$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{\sqrt{d+ex}}{\left(cd^2-ce^2x^2\right)^{3/2}}dx}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{4d}-\frac{1}{6cde(d+ex)^{3/2}(cd^2-\cdots}\cdots\right)}{16d}$$

$$\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \; 467$$

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}dx}{2cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{4d} - \cdots\right.}{16d}$$

$$\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{471}$$

$$11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{e\int\frac{1}{\frac{e^2\left(cd^2-ce^2x^2\right)}{d+ex}-2cde^2}d\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{d+ex}}}{cd}+\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2\cdots\right.}}{4d}$$

$\big\downarrow$ 221

$$\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{d+ex}}{cde\sqrt{cd^2-ce^2x^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cd^2-ce^2x^2}}{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d+ex}}\right)}{\sqrt{2}c^{3/2}d^{3/2}e}\right)}{4d}-\frac{1}{2cde\sqrt{d+ex}\sqrt{cd^2-ce^2x^2}}\right)}{6cd}+\frac{\sqrt{d+ex}}{3cde\left(cd^2-ce^2x^2\right)^{3/2}}\right)}{8d}-\frac{1}{4cde\sqrt{d+ex}\left(cd^2-ce^2x^2\right)^{3/2}}\cdots}{4d}\cdots\right)}{16d}\cdots$$

$$\frac{1}{8cde(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{3/2}}$$

input
```
Int[1/((d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2)^(5/2)),x]
```

output
```
-1/8*1/(c*d*e*(d + e*x)^(5/2)*(c*d^2 - c*e^2*x^2)^(3/2)) + (11*(-1/6*1/(c*
d*e*(d + e*x)^(3/2)*(c*d^2 - c*e^2*x^2)^(3/2)) + (3*(-1/4*1/(c*d*e*Sqrt[d
+ e*x]*(c*d^2 - c*e^2*x^2)^(3/2)) + (7*(Sqrt[d + e*x]/(3*c*d*e*(c*d^2 - c*
e^2*x^2)^(3/2)) + (5*(-1/2*1/(c*d*e*Sqrt[d + e*x]*Sqrt[c*d^2 - c*e^2*x^2])
 + (3*(Sqrt[d + e*x]/(c*d*e*Sqrt[c*d^2 - c*e^2*x^2]) - ArcTanh[Sqrt[c*d^2
- c*e^2*x^2]/(Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d + e*x])]/(Sqrt[2]*c^(3/2)*d^(
3/2)*e)))/(4*d)))/(6*c*d)))/(8*d)))/(4*d)))/(16*d)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 467
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-c)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*a*d*(p + 1))), x] + Simp[c*((n + 2
*p + 2)/(2*a*(p + 1)))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^(p + 1), x], x]
/; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[p, -1] && LtQ[0,
n, 1] && IntegerQ[2*p]
```

rule 470
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[(n +
2*p + 2)/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
 FreeQ[{a, b, c, d, p}, x] && EqQ[b*c^2 + a*d^2, 0] && LtQ[n, 0] && NeQ[n +
 p + 1, 0] && IntegerQ[2*p]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.58

method	result
default	$-\frac{\sqrt{c(-e^2x^2+d^2)}\left(-3465\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)e^5x^5\sqrt{c(-ex+d)}-10395\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(-ex+d)}\,\sqrt{2}}{2\sqrt{cd}}\right)d\,e^4x^4\sqrt{c(-ex+d)}\right.}{}$

input
```
int(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24576*(c*(-e^2*x^2+d^2))^(1/2)/c^3*(-3465*2^(1/2)*arctanh(1/2*(c*(-e*x+
d))^(1/2)*2^(1/2)/(c*d)^(1/2))*e^5*x^5*(c*(-e*x+d))^(1/2)-10395*2^(1/2)*ar
ctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d*e^4*x^4*(c*(-e*x+d))^(
1/2)-6930*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d^2*
e^3*x^3*(c*(-e*x+d))^(1/2)+6930*(c*d)^(1/2)*e^5*x^5+6930*2^(1/2)*arctanh(1
/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d^3*e^2*x^2*(c*(-e*x+d))^(1/2)+
16170*(c*d)^(1/2)*d*e^4*x^4+10395*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2
^(1/2)/(c*d)^(1/2))*d^4*e*x*(c*(-e*x+d))^(1/2)+924*(c*d)^(1/2)*d^2*e^3*x^3
+3465*2^(1/2)*arctanh(1/2*(c*(-e*x+d))^(1/2)*2^(1/2)/(c*d)^(1/2))*d^5*(c*(
-e*x+d))^(1/2)-21780*(c*d)^(1/2)*d^3*e^2*x^2-13486*(c*d)^(1/2)*d^4*e*x+305
0*(c*d)^(1/2)*d^5)/(e*x+d)^(9/2)/(-e*x+d)^2/e/d^6/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.71

$$\int \frac{1}{(d+ex)^{5/2}\,(cd^2-ce^2x^2)^{5/2}}\,dx = \left[\frac{3465\sqrt{2}(e^7x^7+3\,de^6x^6+d^2e^5x^5-5\,d^3e^4x^4-5\,d^4e^3x^3+d^5e^2x^2+}{}\right.$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/49152*(3465*sqrt(2)*(e^7*x^7 + 3*d*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^
4 - 5*d^4*e^3*x^3 + d^5*e^2*x^2 + 3*d^6*e*x + d^7)*sqrt(c*d)*log(-(c*e^2*x
^2 - 2*c*d*e*x - 3*c*d^2 + 2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(c*d)*sq
rt(e*x + d))/(e^2*x^2 + 2*d*e*x + d^2)) - 4*(3465*d*e^5*x^5 + 8085*d^2*e^4
*x^4 + 462*d^3*e^3*x^3 - 10890*d^4*e^2*x^2 - 6743*d^5*e*x + 1525*d^6)*sqrt
(-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(c^3*d^7*e^8*x^7 + 3*c^3*d^8*e^7*x^6 +
 c^3*d^9*e^6*x^5 - 5*c^3*d^10*e^5*x^4 - 5*c^3*d^11*e^4*x^3 + c^3*d^12*e^3*
x^2 + 3*c^3*d^13*e^2*x + c^3*d^14*e), 1/24576*(3465*sqrt(2)*(e^7*x^7 + 3*d
*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^4 - 5*d^4*e^3*x^3 + d^5*e^2*x^2 + 3*d
^6*e*x + d^7)*sqrt(-c*d)*arctan(1/2*sqrt(2)*sqrt(-c*e^2*x^2 + c*d^2)*sqrt(
-c*d)*sqrt(e*x + d)/(c*d*e*x + c*d^2)) - 2*(3465*d*e^5*x^5 + 8085*d^2*e^4*
x^4 + 462*d^3*e^3*x^3 - 10890*d^4*e^2*x^2 - 6743*d^5*e*x + 1525*d^6)*sqrt(
-c*e^2*x^2 + c*d^2)*sqrt(e*x + d))/(c^3*d^7*e^8*x^7 + 3*c^3*d^8*e^7*x^6 +
c^3*d^9*e^6*x^5 - 5*c^3*d^10*e^5*x^4 - 5*c^3*d^11*e^4*x^3 + c^3*d^12*e^3*x
^2 + 3*c^3*d^13*e^2*x + c^3*d^14*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\int \frac{1}{\left(-c\left(-d+ex\right)\left(d+ex\right)\right)^{\frac{5}{2}}\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(5/2)/(-c*e**2*x**2+c*d**2)**(5/2),x)
```

output
```
Integral(1/((-c*(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx=\int \frac{1}{\left(-ce^2x^2+cd^2\right)^{\frac{5}{2}}\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((-c*e^2*x^2 + c*d^2)^(5/2)*(e*x + d)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \frac{\frac{3465\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-(ex+d)c+2\,cd}}{2\sqrt{-cd}}\right)}{\sqrt{-cd}c^2d^6} + \frac{256\,(15\,(ex+d)c-32\,cd)}{((ex+d)c-2\,cd)\sqrt{-(ex+d)c+2\,cd}c^2d^6} - \frac{2\left(18360\ldots\right.}{\ldots}}{\ldots}$$

input
```
integrate(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x, algorithm="giac")
```

output
```
1/24576*(3465*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-(e*x + d)*c + 2*c*d)/sqrt(-
c*d))/(sqrt(-c*d)*c^2*d^6) + 256*(15*(e*x + d)*c - 32*c*d)/(((e*x + d)*c -
 2*c*d)*sqrt(-(e*x + d)*c + 2*c*d)*c^2*d^6) - 2*(18360*sqrt(-(e*x + d)*c +
 2*c*d)*c^3*d^3 - 23420*(-(e*x + d)*c + 2*c*d)^(3/2)*c^2*d^2 + 10306*((e*x
 + d)*c - 2*c*d)^2*sqrt(-(e*x + d)*c + 2*c*d)*c*d + 1545*((e*x + d)*c - 2*
c*d)^3*sqrt(-(e*x + d)*c + 2*c*d))/((e*x + d)^4*c^6*d^6))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \int \frac{1}{{\left(c\,d^2-c\,e^2\,x^2\right)}^{5/2}\,{\left(d+e\,x\right)}^{5/2}} \,dx$$

input
```
int(1/((c*d^2 - c*e^2*x^2)^(5/2)*(d + e*x)^(5/2)),x)
```

output
```
int(1/((c*d^2 - c*e^2*x^2)^(5/2)*(d + e*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.20

$$\int \frac{1}{(d+ex)^{5/2}\left(cd^2-ce^2x^2\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(3465\sqrt{d}\,\sqrt{-ex+d}\,\sqrt{2}\log\left(\sqrt{-ex+d}-\sqrt{d}\,\sqrt{2}\right)d^5+10395\sqrt{d}\,\cdots\right.}{}$$

input
```
int(1/(e*x+d)^(5/2)/(-c*e^2*x^2+c*d^2)^(5/2),x)
```

output
```
(sqrt(c)*(3465*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*s
qrt(2))*d**5 + 10395*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqr
t(d)*sqrt(2))*d**4*e*x + 6930*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e
*x) - sqrt(d)*sqrt(2))*d**3*e**2*x**2 - 6930*sqrt(d)*sqrt(d - e*x)*sqrt(2)
*log(sqrt(d - e*x) - sqrt(d)*sqrt(2))*d**2*e**3*x**3 - 10395*sqrt(d)*sqrt(
d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt(2))*d*e**4*x**4 - 3465*s
qrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) - sqrt(d)*sqrt(2))*e**5*x**
5 - 3465*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2)
)*d**5 - 10395*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*s
qrt(2))*d**4*e*x - 6930*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) +
sqrt(d)*sqrt(2))*d**3*e**2*x**2 + 6930*sqrt(d)*sqrt(d - e*x)*sqrt(2)*log(s
qrt(d - e*x) + sqrt(d)*sqrt(2))*d**2*e**3*x**3 + 10395*sqrt(d)*sqrt(d - e*
x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*d*e**4*x**4 + 3465*sqrt(d)
*sqrt(d - e*x)*sqrt(2)*log(sqrt(d - e*x) + sqrt(d)*sqrt(2))*e**5*x**5 - 61
00*d**6 + 26972*d**5*e*x + 43560*d**4*e**2*x**2 - 1848*d**3*e**3*x**3 - 32
340*d**2*e**4*x**4 - 13860*d*e**5*x**5))/(49152*sqrt(d - e*x)*c**3*d**7*e*
(d**5 + 3*d**4*e*x + 2*d**3*e**2*x**2 - 2*d**2*e**3*x**3 - 3*d*e**4*x**4 -
 e**5*x**5))
```

3.205 $\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-1+x}}{\sqrt{1-x^2}}\right)$$

output
```
-2^(1/2)*arctan(2^(1/2)*(-1+x)^(1/2)/(-x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = -\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-1+x}}{\sqrt{1-x^2}}\right)$$

input
```
Integrate[1/(Sqrt[-1 + x]*Sqrt[1 - x^2]),x]
```

output
```
-(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[-1 + x])/Sqrt[1 - x^2]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {471, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{471}$$
$$2\int \frac{1}{\frac{1-x^2}{x-1}+2}d\frac{\sqrt{1-x^2}}{\sqrt{x-1}}$$
$$\Big\downarrow \text{216}$$
$$\sqrt{2}\arctan\left(\frac{\sqrt{1-x^2}}{\sqrt{2}\sqrt{x-1}}\right)$$

input
```
Int[1/(Sqrt[-1 + x]*Sqrt[1 - x^2]),x]
```

output
```
Sqrt[2]*ArcTan[Sqrt[1 - x^2]/(Sqrt[2]*Sqrt[-1 + x])]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{-x^{2}+1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-x-1}\,\sqrt{2}}{2}\right)}{\sqrt{x-1}\,\sqrt{-x-1}}$	39

input
```
int(1/(x-1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(x-1)^(1/2)*(-x^2+1)^(1/2)/(-x-1)^(1/2)*2^(1/2)*arctan(1/2*(-x-1)^(1/2)*
2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2\,\sqrt{x-1}}\right)$$

input
```
integrate(1/(x-1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/sqrt(x - 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-(x-1)\,(x+1)}\sqrt{x-1}}\,dx$$

input
```
integrate(1/(x-1)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(x - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{x-1}}\,dx$$

input
```
integrate(1/(x-1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(x - 1)), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = \frac{1}{2}i\sqrt{2}\log\left(\sqrt{2}+\sqrt{x+1}\right) - \frac{1}{2}i\sqrt{2}\log\left(-\sqrt{2}+\sqrt{x+1}\right)$$

input
```
integrate(1/(x-1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*I*sqrt(2)*log(sqrt(2) + sqrt(x + 1)) - 1/2*I*sqrt(2)*log(-sqrt(2) + sq
rt(x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{x-1}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(x - 1)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(x - 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{-1+x}\sqrt{1-x^2}}\,dx = \sqrt{2}\,atanh\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)i$$

input
```
int(1/(x-1)^(1/2)/(-x^2+1)^(1/2),x)
```

output
```
sqrt(2)*atanh(sqrt(x + 1)/sqrt(2))*i
```

3.206 $\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = -\frac{128(2-ex)^{3/2}}{\sqrt{3}e} + \frac{96\sqrt{3}(2-ex)^{5/2}}{5e} - \frac{24\sqrt{3}(2-ex)^{7/2}}{7e} + \frac{2(2-ex)^{9/2}}{3\sqrt{3}e}$$

output
```
-128/3*(-e*x+2)^(3/2)*3^(1/2)/e+96/5*3^(1/2)*(-e*x+2)^(5/2)/e-24/7*3^(1/2)
*(-e*x+2)^(7/2)/e+2/9*(-e*x+2)^(9/2)*3^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = \frac{2(-2+ex)\sqrt{4-e^2x^2}(2552+1284ex+330e^2x^2+35e^3x^3)}{105e\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(2*(-2 + e*x)*Sqrt[4 - e^2*x^2]*(2552 + 1284*e*x + 330*e^2*x^2 + 35*e^3*x^
3))/(105*e*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex+2)^{5/2}\sqrt{12-3e^2x^2}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \sqrt{6-3ex}(ex+2)^3dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{1}{27}(6-3ex)^{7/2}+\frac{4}{3}(6-3ex)^{5/2}-16(6-3ex)^{3/2}+64\sqrt{6-3ex}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(2-ex)^{9/2}}{3\sqrt{3}e}-\frac{24\sqrt{3}(2-ex)^{7/2}}{7e}+\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{128(2-ex)^{3/2}}{\sqrt{3}e}$$

input
```
Int[(2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-128*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (96*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) -
(24*Sqrt[3]*(2 - e*x)^(7/2))/(7*e) + (2*(2 - e*x)^(9/2))/(3*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2(ex-2)\left(35e^{3}x^{3}+330e^{2}x^{2}+1284ex+2552\right)\sqrt{-3e^{2}x^{2}+12}}{315e\sqrt{ex+2}}$	52
default	$\frac{2(ex-2)\left(35e^{3}x^{3}+330e^{2}x^{2}+1284ex+2552\right)\sqrt{-3e^{2}x^{2}+12}}{315e\sqrt{ex+2}}$	52
orering	$\frac{2(ex-2)\left(35e^{3}x^{3}+330e^{2}x^{2}+1284ex+2552\right)\sqrt{-3e^{2}x^{2}+12}}{315e\sqrt{ex+2}}$	52
risch	$-\frac{2\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(35e^{4}x^{4}+260e^{3}x^{3}+624e^{2}x^{2}-16ex-5104\right)(ex-2)}{105\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	88

input
```
int((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(e*x-2)*(35*e^3*x^3+330*e^2*x^2+1284*e*x+2552)*(-3*e^2*x^2+12)^(1/2)
/e/(e*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = \frac{2\,(35\,e^4x^4+260\,e^3x^3+624\,e^2x^2-16\,ex-5104)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{315\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*e^4*x^4 + 260*e^3*x^3 + 624*e^2*x^2 - 16*e*x - 5104)*sqrt(-3*e^2
*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F(-1)]

Timed out.

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+2)**(5/2)*(-3*e**2*x**2+12)**(1/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = -\frac{2\left(-35i\sqrt{3}e^4x^4-260i\sqrt{3}e^3x^3-624i\sqrt{3}e^2x^2+16i\sqrt{3}ex+5104i\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{315\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
-2/315*(-35*I*sqrt(3)*e^4*x^4 - 260*I*sqrt(3)*e^3*x^3 - 624*I*sqrt(3)*e^2*
x^2 + 16*I*sqrt(3)*e*x + 5104*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x +
2*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = \frac{2\sqrt{3}\Big(35\,(ex-2)^4\sqrt{-ex+2}+540\,(ex-2)^3\sqrt{-ex+2}+3024\,(ex-2)^2\sqrt{-ex}}{315\,e}$$

input
```
integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
2/315*sqrt(3)*(35*(e*x - 2)^4*sqrt(-e*x + 2) + 540*(e*x - 2)^3*sqrt(-e*x +
 2) + 3024*(e*x - 2)^2*sqrt(-e*x + 2) - 6720*(-e*x + 2)^(3/2))/e
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = \frac{2\sqrt{12-3\,e^2\,x^2}\,\left(35\,e^4\,x^4+260\,e^3\,x^3+624\,e^2\,x^2-16\,e\,x-5104\right)}{315\,e\,\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(5/2),x)
```

output
```
(2*(12 - 3*e^2*x^2)^(1/2)*(624*e^2*x^2 - 16*e*x + 260*e^3*x^3 + 35*e^4*x^4
 - 5104))/(315*e*(e*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int (2+ex)^{5/2}\sqrt{12-3e^2x^2}\,dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(35e^4x^4+260e^3x^3+624e^2x^2-16ex-5104)}{315e}$$

input
```
int((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*(35*e**4*x**4 + 260*e**3*x**3 + 624*e**2*x**2
- 16*e*x - 5104))/(315*e)
```

3.207 $\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx=-\frac{32(2-ex)^{3/2}}{\sqrt{3}e}+\frac{16\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{2\sqrt{3}(2-ex)^{7/2}}{7e}$$

output
```
-32/3*(-e*x+2)^(3/2)*3^(1/2)/e+16/5*3^(1/2)*(-e*x+2)^(5/2)/e-2/7*3^(1/2)*(
-e*x+2)^(7/2)/e
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx=\frac{2(-2+ex)\sqrt{4-e^2x^2}(284+108ex+15e^2x^2)}{35e\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(2*(-2 + e*x)*Sqrt[4 - e^2*x^2]*(284 + 108*e*x + 15*e^2*x^2))/(35*e*Sqrt[6
 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex+2)^{3/2}\sqrt{12-3e^2x^2}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \sqrt{6-3ex}(ex+2)^2dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{9}(6-3ex)^{5/2}-\frac{8}{3}(6-3ex)^{3/2}+16\sqrt{6-3ex}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{3}(2-ex)^{7/2}}{7e}+\frac{16\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{32(2-ex)^{3/2}}{\sqrt{3}e}$$

input
```
Int[(2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-32*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (16*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) - (
2*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{2(ex-2)\left(15e^{2}x^{2}+108ex+284\right)\sqrt{-3e^{2}x^{2}+12}}{105\sqrt{ex+2}\,e}$	44
default	$\frac{2(ex-2)\left(15e^{2}x^{2}+108ex+284\right)\sqrt{-3e^{2}x^{2}+12}}{105\sqrt{ex+2}\,e}$	44
orering	$\frac{2(ex-2)\left(15e^{2}x^{2}+108ex+284\right)\sqrt{-3e^{2}x^{2}+12}}{105\sqrt{ex+2}\,e}$	44
risch	$-\frac{2\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(15e^{3}x^{3}+78e^{2}x^{2}+68ex-568\right)(ex-2)}{35\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	80

input
```
int((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(e*x-2)*(15*e^2*x^2+108*e*x+284)*(-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2)
/e
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx = \frac{2\left(15\,e^3x^3+78\,e^2x^2+68\,ex-568\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{105\left(e^2x+2\,e\right)}$$

input
```
integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*e^3*x^3 + 78*e^2*x^2 + 68*e*x - 568)*sqrt(-3*e^2*x^2 + 12)*sqrt(
e*x + 2)/(e^2*x + 2*e)
```

Sympy [F]

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx = \sqrt{3}\left(\int 2\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx + \int ex\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx\right)$$

input
```
integrate((e*x+2)**(3/2)*(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*(Integral(2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x) + Integral(e*x*
sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx = -\frac{2\left(-15i\sqrt{3}e^3x^3-78i\sqrt{3}e^2x^2-68i\sqrt{3}ex+568i\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{105\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
-2/105*(-15*I*sqrt(3)*e^3*x^3 - 78*I*sqrt(3)*e^2*x^2 - 68*I*sqrt(3)*e*x +
568*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x + 2*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx = \frac{2\sqrt{3}\left(15\,(ex-2)^3\sqrt{-ex+2}+168\,(ex-2)^2\sqrt{-ex+2}-560\,(-ex+2)^{\frac{3}{2}}\right)}{105\,e}$$

input
```
integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
2/105*sqrt(3)*(15*(e*x - 2)^3*sqrt(-e*x + 2) + 168*(e*x - 2)^2*sqrt(-e*x +
 2) - 560*(-e*x + 2)^(3/2))/e
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx = \frac{\sqrt{12-3\,e^2\,x^2}\,\left(\frac{52\,x^2\,\sqrt{e\,x+2}}{35}-\frac{1136\,\sqrt{e\,x+2}}{105\,e^2}+\frac{136\,x\,\sqrt{e\,x+2}}{105\,e}+\frac{2\,e\,x^3\,\sqrt{e\,x+2}}{7}\right)}{x+\frac{2}{e}}$$

input
```
int((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(3/2),x)
```

output
```
((12 - 3*e^2*x^2)^(1/2)*((52*x^2*(e*x + 2)^(1/2))/35 - (1136*(e*x + 2)^(1/
2))/(105*e^2) + (136*x*(e*x + 2)^(1/2))/(105*e) + (2*e*x^3*(e*x + 2)^(1/2)
)/7))/(x + 2/e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

$$\int (2+ex)^{3/2}\sqrt{12-3e^2x^2}\,dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(15e^3x^3+78e^2x^2+68ex-568)}{105e}$$

input
```
int((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*(15*e**3*x**3 + 78*e**2*x**2 + 68*e*x - 568))/
(105*e)
```

3.208 $\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = -\frac{8(2-ex)^{3/2}}{\sqrt{3}e}+\frac{2\sqrt{3}(2-ex)^{5/2}}{5e}$$

output
```
-8/3*(-e*x+2)^(3/2)*3^(1/2)/e+2/5*3^(1/2)*(-e*x+2)^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = \frac{2(-2+ex)(14+3ex)\sqrt{4-e^2x^2}}{5e\sqrt{6+3ex}}$$

input
```
Integrate[Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(2*(-2 + e*x)*(14 + 3*e*x)*Sqrt[4 - e^2*x^2])/(5*e*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex+2}\sqrt{12-3e^2x^2}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \sqrt{6-3ex}(ex+2)dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(4\sqrt{6-3ex}-\frac{1}{3}(6-3ex)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{3}(2-ex)^{5/2}}{5e}-\frac{8(2-ex)^{3/2}}{\sqrt{3}e}$$

input
```
Int[Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-8*(2 - e*x)^(3/2))/(Sqrt[3]*e) + (2*Sqrt[3]*(2 - e*x)^(5/2))/(5*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{2(ex-2)(3ex+14)\sqrt{-3e^2x^2+12}}{15e\sqrt{ex+2}}$	36
default	$\frac{2(ex-2)(3ex+14)\sqrt{-3e^2x^2+12}}{15e\sqrt{ex+2}}$	36
orering	$\frac{2(ex-2)(3ex+14)\sqrt{-3e^2x^2+12}}{15e\sqrt{ex+2}}$	36
risch	$-\frac{2\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}\,(3e^2x^2+8ex-28)(ex-2)}{5\sqrt{-3e^2x^2+12}\,e\sqrt{-3ex+6}}$	72

input
```
int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(e*x-2)*(3*e*x+14)*(-3*e^2*x^2+12)^(1/2)/e/(e*x+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = \frac{2\,(3\,e^2x^2+8\,ex-28)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{15\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*e^2*x^2 + 8*e*x - 28)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x +
 2*e)
```

Sympy [F]

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = \sqrt{3}\int \sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx$$

input
```
integrate((e*x+2)**(1/2)*(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = -\frac{2\left(-3i\sqrt{3}e^2x^2-8i\sqrt{3}ex+28i\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{15\left(e^2x+2e\right)}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(-3*I*sqrt(3)*e^2*x^2 - 8*I*sqrt(3)*e*x + 28*I*sqrt(3))*(e*x + 2)*sq
rt(e*x - 2)/(e^2*x + 2*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = \frac{2\sqrt{3}\left(3\,(ex-2)^2\sqrt{-ex+2}-20\,(-ex+2)^{\frac{3}{2}}\right)}{15\,e}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
2/15*sqrt(3)*(3*(e*x - 2)^2*sqrt(-e*x + 2) - 20*(-e*x + 2)^(3/2))/e
```

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = \frac{\sqrt{12-3\,e^2\,x^2}\,\left(\frac{2\,x^2\,\sqrt{e\,x+2}}{5}-\frac{56\,\sqrt{e\,x+2}}{15\,e^2}+\frac{16\,x\,\sqrt{e\,x+2}}{15\,e}\right)}{x+\frac{2}{e}}$$

input
```
int((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(1/2),x)
```

output
```
((12 - 3*e^2*x^2)^(1/2)*((2*x^2*(e*x + 2)^(1/2))/5 - (56*(e*x + 2)^(1/2))/
(15*e^2) + (16*x*(e*x + 2)^(1/2))/(15*e)))/(x + 2/e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \sqrt{2+ex}\sqrt{12-3e^2x^2}\,dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(3e^2x^2+8ex-28)}{15e}$$

input
```
int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*(3*e**2*x**2 + 8*e*x - 28))/(15*e)
```

3.209 $\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = -\frac{2(2-ex)^{3/2}}{\sqrt{3}e}$$

output
```
-2/3*(-e*x+2)^(3/2)*3^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = -\frac{2(4-e^2x^2)^{3/2}}{\sqrt{3}e(2+ex)^{3/2}}$$

input
```
Integrate[Sqrt[12 - 3*e^2*x^2]/Sqrt[2 + e*x],x]
```

output
```
(-2*(4 - e^2*x^2)^(3/2))/(Sqrt[3]*e*(2 + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{ex+2}}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \sqrt{6-3ex}dx$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{2(2-ex)^{3/2}}{\sqrt{3}e}$$

input
```
Int[Sqrt[12 - 3*e^2*x^2]/Sqrt[2 + e*x],x]
```

output
```
(-2*(2 - e*x)^(3/2))/(Sqrt[3]*e)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

method	result	size
gosper	$\frac{2(ex-2)\sqrt{-3e^2x^2+12}}{3e\sqrt{ex+2}}$	30
default	$\frac{2(ex-2)\sqrt{-3e^2x^2+12}}{3e\sqrt{ex+2}}$	30
orering	$\frac{2(ex-2)\sqrt{-3e^2x^2+12}}{3e\sqrt{ex+2}}$	30
risch	$-\frac{2\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}\,(ex-2)^2}{\sqrt{-3e^2x^2+12}\,e\sqrt{-3ex+6}}$	60

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x-2)*(-3*e^2*x^2+12)^(1/2)/e/(e*x+2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \frac{2\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}(ex-2)}{3\,(e^2x+2\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)*(e*x - 2)/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \sqrt{3}\int \frac{\sqrt{-e^2x^2+4}}{\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(-e**2*x**2 + 4)/sqrt(e*x + 2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = -\frac{2\left(-i\sqrt{3}ex+2i\sqrt{3}\right)\sqrt{ex-2}}{3\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(-I*sqrt(3)*e*x + 2*I*sqrt(3))*sqrt(e*x - 2)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = -\frac{2\sqrt{3}(-ex+2)^{\frac{3}{2}}}{3\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*(-e*x + 2)^(3/2)/e
```

Mupad [B] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \frac{\left(\frac{2\,x}{3}-\frac{4}{3\,e}\right)\,\sqrt{12-3\,e^2\,x^2}}{\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(1/2)/(e*x + 2)^(1/2),x)
```

output
```
(((2*x)/3 - 4/(3*e))*(12 - 3*e^2*x^2)^(1/2))/(e*x + 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(ex-2)}{3e}$$

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(1/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*(e*x - 2))/(3*e)
```

3.210 $\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 46

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx=\frac{2\sqrt{3}\sqrt{2-ex}}{e}-\frac{4\sqrt{3}\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

output
```
2*3^(1/2)*(-e*x+2)^(1/2)/e-4*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx=\frac{2\sqrt{3}\left(\frac{\sqrt{4-e^2x^2}}{\sqrt{2+ex}}-2\text{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)\right)}{e}$$

input
```
Integrate[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(3/2),x]
```

output
```
(2*Sqrt[3]*(Sqrt[4 - e^2*x^2]/Sqrt[2 + e*x] - 2*ArcTanh[(2*Sqrt[2 + e*x])/
Sqrt[4 - e^2*x^2]]))/e
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {456, 60, 27, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{12-3e^2x^2}}{(ex+2)^{3/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{\sqrt{6-3ex}}{ex+2}dx$$

$$\Big\downarrow \text{60}$$

$$12\int \frac{1}{\sqrt{3}\sqrt{2-ex}(ex+2)}dx+\frac{2\sqrt{3}\sqrt{2-ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$4\sqrt{3}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx+\frac{2\sqrt{3}\sqrt{2-ex}}{e}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{3}\sqrt{2-ex}}{e}-\frac{8\sqrt{3}\int \frac{1}{ex+2}d\sqrt{2-ex}}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{3}\sqrt{2-ex}}{e}-\frac{4\sqrt{3}\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

input
```
Int[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(3/2),x]
```

output
```
(2*Sqrt[3]*Sqrt[2 - e*x])/e - (4*Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/2])/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

method	result	size
default	$-\frac{2\sqrt{-e^{2}x^{2}+4}\,\left(2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)-\sqrt{-3ex+6}\right)\sqrt{3}}{\sqrt{ex+2}\,\sqrt{-3ex+6}\,e}$	66
risch	$-\frac{6(ex-2)\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}}{e\sqrt{-3ex+6}\,\sqrt{-3e^{2}x^{2}+12}}-\frac{4\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}}{e\sqrt{-3e^{2}x^{2}+12}}$	120

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-e^2*x^2+4)^(1/2)*(2*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))-(-3
*e*x+6)^(1/2))/(e*x+2)^(1/2)/(-3*e*x+6)^(1/2)*3^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(36) = 72.

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx=\frac{2\left(\sqrt{3}(ex+2)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)+\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}\right)}{e^2x+2\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(3)*(e*x + 2)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2
+ 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) + sqrt(-3*e^2*x^2 + 12)*s
qrt(e*x + 2))/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \sqrt{3}\int \frac{\sqrt{-e^2x^2+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(3/2),x)
```

output
```
sqrt(3)*Integral(sqrt(-e**2*x**2 + 4)/(e*x*sqrt(e*x + 2) + 2*sqrt(e*x + 2)
), x)
```

Maxima [F]

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \int \frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \frac{2\sqrt{3}\left(\sqrt{-ex+2}-\log\left(\sqrt{-ex+2}+2\right)+\log\left(-\sqrt{-ex+2}+2\right)\right)}{e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(3)*(sqrt(-e*x + 2) - log(sqrt(-e*x + 2) + 2) + log(-sqrt(-e*x + 2)
+ 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \int \frac{\sqrt{12-3\,e^2\,x^2}}{{\left(e\,x+2\right)}^{3/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(1/2)/(e*x + 2)^(3/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(1/2)/(e*x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \frac{2\sqrt{3}\left(\sqrt{-ex+2}+2\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)-2\right)}{e}$$

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(3/2),x)
```

output
```
(2*sqrt(3)*(sqrt( - e*x + 2) + 2*log(tan(asin(sqrt(e*x + 2)/2)/2)) - 2))/e
```

3.211 $\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = -\frac{\sqrt{3}\sqrt{2-ex}}{e(2+ex)}+\frac{\sqrt{3}\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2e}$$

output
```
-3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+2)+1/2*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/
e
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \frac{\sqrt{3}\left(-\frac{2\sqrt{4-e^2x^2}}{(2+ex)^{3/2}}+\text{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)\right)}{2e}$$

input
```
Integrate[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(5/2),x]
```

output
```
(Sqrt[3]*((-2*Sqrt[4 - e^2*x^2])/(2 + e*x)^(3/2) + ArcTanh[(2*Sqrt[2 + e*x
])/Sqrt[4 - e^2*x^2]]))/(2*e)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {456, 51, 27, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{12-3e^2x^2}}{(ex+2)^{5/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{\sqrt{6-3ex}}{(ex+2)^2}dx$$

$$\Big\downarrow \text{51}$$

$$-\frac{3}{2}\int \frac{1}{\sqrt{3}\sqrt{2-ex}(ex+2)}dx-\frac{\sqrt{3}\sqrt{2-ex}}{e(ex+2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\sqrt{3}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx-\frac{\sqrt{3}\sqrt{2-ex}}{e(ex+2)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{3}\int \frac{1}{ex+2}d\sqrt{2-ex}}{e}-\frac{\sqrt{3}\sqrt{2-ex}}{e(ex+2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{3}\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2e}-\frac{\sqrt{3}\sqrt{2-ex}}{e(ex+2)}$$

input
```
Int[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(5/2),x]
```

output
```
-((Sqrt[3]*Sqrt[2 - e*x])/(e*(2 + e*x))) + (Sqrt[3]*ArcTanh[Sqrt[2 - e*x]/
2])/(2*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{\sqrt{-e^{2} x^{2}+4}\, \left(\sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{-3 e x +6}\, \sqrt{3}}{6}\right) e x +2 \sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{-3 e x +6}\, \sqrt{3}}{6}\right)-2 \sqrt{-3 e x +6}\right) \sqrt{3}}{2 \left(e x +2\right)^{\frac{3}{2}} \sqrt{-3 e x +6}\, e}$	86

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-e^2*x^2+4)^(1/2)*(3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e*x+
2*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))-2*(-3*e*x+6)^(1/2))/(e*x+2
)^(3/2)/(-3*e*x+6)^(1/2)*3^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(43) = 86$.

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \frac{\sqrt{3}(e^2x^2+4\,ex+4)\log\left(-\frac{3\,e^2x^2-12\,ex-4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-4\,\sqrt{-3\,e^2x^2+12}\sqrt{}}{4\,(e^3x^2+4\,e^2x+4\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(3)*(e^2*x^2 + 4*e*x + 4)*log(-(3*e^2*x^2 - 12*e*x - 4*sqrt(3)*sq
rt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*sqrt(-3
*e^2*x^2 + 12)*sqrt(e*x + 2))/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F]

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \sqrt{3}\int \frac{\sqrt{-e^2x^2+4}}{e^2x^2\sqrt{ex+2}+4ex\sqrt{ex+2}+4\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(5/2),x)
```

output
```
sqrt(3)*Integral(sqrt(-e**2*x**2 + 4)/(e**2*x**2*sqrt(e*x + 2) + 4*e*x*sqr
t(e*x + 2) + 4*sqrt(e*x + 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \int \frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = -\frac{\sqrt{3}\left(\frac{4\sqrt{-ex+2}}{ex+2} - \log\left(\sqrt{-ex+2}+2\right) + \log\left(-\sqrt{-ex+2}+2\right)\right)}{4\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*(4*sqrt(-e*x + 2)/(e*x + 2) - log(sqrt(-e*x + 2) + 2) + log(-
sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \int \frac{\sqrt{12-3\,e^2\,x^2}}{(e\,x+2)^{5/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(1/2)/(e*x + 2)^(5/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(1/2)/(e*x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \frac{\sqrt{3}\left(-2\sqrt{-ex+2}-\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)ex-2\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)\right)}{2e\,(ex+2)}$$

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(5/2),x)
```

output
```
(sqrt(3)*( - 2*sqrt( - e*x + 2) - log(tan(asin(sqrt(e*x + 2)/2)/2))*e*x -
2*log(tan(asin(sqrt(e*x + 2)/2)/2))))/(2*e*(e*x + 2))
```

3.212 $\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx=-\frac{\sqrt{3}\sqrt{2-ex}}{2e(2+ex)^2}+\frac{\sqrt{3}\sqrt{2-ex}}{16e(2+ex)}+\frac{\sqrt{3}\,\mathrm{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32e}$$

output
```
-1/2*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+2)^2+1/16*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x
+2)+1/32*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx=\frac{\sqrt{3}\left(\frac{4(-6+ex)\sqrt{4-e^2x^2}}{(2+ex)^{5/2}}-\log\left(e\left(-2\sqrt{2+ex}+\sqrt{4-e^2x^2}\right)\right)+\log\left(2\sqrt{2+ex}+\sqrt{4-e^2x^2}\right)\right)}{64e}$$

input
```
Integrate[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(7/2),x]
```

output
```
(Sqrt[3]*((4*(-6 + e*x)*Sqrt[4 - e^2*x^2])/(2 + e*x)^(5/2) - Log[e*(-2*Sqr
t[2 + e*x] + Sqrt[4 - e^2*x^2])] + Log[2*Sqrt[2 + e*x] + Sqrt[4 - e^2*x^2]
]))/(64*e)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {456, 51, 27, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{12-3e^2x^2}}{(ex+2)^{7/2}}\,dx$$

$$\Big\downarrow \ 456$$

$$\int \frac{\sqrt{6-3ex}}{(ex+2)^3}dx$$

$$\Big\downarrow \ 51$$

$$-\frac{3}{4}\int \frac{1}{\sqrt{3}\sqrt{2-ex}(ex+2)^2}dx-\frac{\sqrt{3}\sqrt{2-ex}}{2e(ex+2)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{4}\sqrt{3}\int \frac{1}{\sqrt{2-ex}(ex+2)^2}dx-\frac{\sqrt{3}\sqrt{2-ex}}{2e(ex+2)^2}$$

$$\Big\downarrow \ 52$$

$$-\frac{1}{4}\sqrt{3}\left(\frac{1}{8}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{3}\sqrt{2-ex}}{2e(ex+2)^2}$$

$$\Big\downarrow \ 73$$

$$-\frac{1}{4}\sqrt{3}\left(-\frac{\int \frac{1}{ex+2}d\sqrt{2-ex}}{4e}-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{3}\sqrt{2-ex}}{2e(ex+2)^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{1}{4}\sqrt{3}\left(-\frac{\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{3}\sqrt{2-ex}}{2e(ex+2)^2}$$

input
```
Int[Sqrt[12 - 3*e^2*x^2]/(2 + e*x)^(7/2),x]
```

output
```
-1/2*(Sqrt[3]*Sqrt[2 - e*x])/(e*(2 + e*x)^2) - (Sqrt[3]*(-1/4*Sqrt[2 - e*x
]/(e*(2 + e*x)) - ArcTanh[Sqrt[2 - e*x]/2]/(8*e)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{-e^2x^2+4}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)e^2x^2+4\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)ex+2ex\sqrt{-3ex+6}+4\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)-12\sqrt{\ldots}\right.}{32(ex+2)^{\frac{5}{2}}\sqrt{-3ex+6}\,e}$

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*(-e^2*x^2+4)^(1/2)*(3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e^2
*x^2+4*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e*x+2*e*x*(-3*e*x+6)^
(1/2)+4*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))-12*(-3*e*x+6)^(1/2))
*3^(1/2)/(e*x+2)^(5/2)/(-3*e*x+6)^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(66) = 132.

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \frac{\sqrt{3}(e^3x^3+6\,e^2x^2+12\,ex+8)\log\left(-\frac{3\,e^2x^2-12\,ex-4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)+4\sqrt{-3\,e^{\ldots}}}{64\,(e^4x^3+6\,e^3x^2+12\,e^2x+8\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x, algorithm="fricas")
```

output
```
1/64*(sqrt(3)*(e^3*x^3 + 6*e^2*x^2 + 12*e*x + 8)*log(-(3*e^2*x^2 - 12*e*x
- 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4
)) + 4*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)*(e*x - 6))/(e^4*x^3 + 6*e^3*x^2
 + 12*e^2*x + 8*e)
```

Sympy [F]

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \sqrt{3}\int \frac{\sqrt{-e^2x^2+4}}{e^3x^3\sqrt{ex+2}+6e^2x^2\sqrt{ex+2}+12ex\sqrt{ex+2}+8\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/2)/(e*x+2)**(7/2),x)
```

output
```
sqrt(3)*Integral(sqrt(-e**2*x**2 + 4)/(e**3*x**3*sqrt(e*x + 2) + 6*e**2*x*
*2*sqrt(e*x + 2) + 12*e*x*sqrt(e*x + 2) + 8*sqrt(e*x + 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \int \frac{\sqrt{-3\,e^2x^2+12}}{(ex+2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-3*e^2*x^2 + 12)/(e*x + 2)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = -\frac{\sqrt{3}\left(\frac{4\left((-ex+2)^{\frac{3}{2}}+4\sqrt{-ex+2}\right)}{(ex+2)^2}-\log\left(\sqrt{-ex+2}+2\right)+\log\left(-\sqrt{-ex+2}+2\right)\right)}{64\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x, algorithm="giac")
```

output
```
-1/64*sqrt(3)*(4*((-e*x + 2)^(3/2) + 4*sqrt(-e*x + 2))/(e*x + 2)^2 - log(s
qrt(-e*x + 2) + 2) + log(-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \int \frac{\sqrt{12-3\,e^2\,x^2}}{{\left(e\,x+2\right)}^{7/2}} \,dx$$

input
```
int((12 - 3*e^2*x^2)^(1/2)/(e*x + 2)^(7/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(1/2)/(e*x + 2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \frac{\sqrt{3}\left(2\sqrt{-ex+2}\,ex-12\sqrt{-ex+2}-\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)e^2x^2-4\log\left(\tan\left(\cdots\right.\right.\right.}{32e\left(e^2x^2+4ex+4\right)}$$

input
```
int((-3*e^2*x^2+12)^(1/2)/(e*x+2)^(7/2),x)
```

output
```
(sqrt(3)*(2*sqrt( - e*x + 2)*e*x - 12*sqrt( - e*x + 2) - log(tan(asin(sqrt
(e*x + 2)/2)/2))*e**2*x**2 - 4*log(tan(asin(sqrt(e*x + 2)/2)/2))*e*x - 4*l
og(tan(asin(sqrt(e*x + 2)/2)/2))))/(32*e*(e**2*x**2 + 4*e*x + 4))
```

3.213 $\int (2+ex)^{5/2} \left(12-3e^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int (2+ex)^{5/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{1536\sqrt{3}(2-ex)^{5/2}}{5e} + \frac{1536\sqrt{3}(2-ex)^{7/2}}{7e} - \frac{64\sqrt{3}(2-ex)^{9/2}}{e} + \frac{96\sqrt{3}(2-ex)^{11/2}}{11e} - \frac{6\sqrt{3}(2-ex)^{13/2}}{13e}$$

output
```
-1536/5*3^(1/2)*(-e*x+2)^(5/2)/e+1536/7*3^(1/2)*(-e*x+2)^(7/2)/e-64*(-e*x+
2)^(9/2)*3^(1/2)/e+96/11*3^(1/2)*(-e*x+2)^(11/2)/e-6/13*3^(1/2)*(-e*x+2)^(
13/2)/e
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

$$\int (2+ex)^{5/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{2(-2+ex)^2\sqrt{12-3e^2x^2}\left(154928+133600ex+56840e^2x^2+12600e^3x^3+1155e^4x^4\right)}{5005e\sqrt{2+ex}}$$

input
```
Integrate[(2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-2*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2]*(154928 + 133600*e*x + 56840*e^2*x^2
 + 12600*e^3*x^3 + 1155*e^4*x^4))/(5005*e*Sqrt[2 + e*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex+2)^{5/2}\left(12-3e^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int (6-3ex)^{3/2}(ex+2)^4dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{81}(6-3ex)^{11/2}-\frac{16}{27}(6-3ex)^{9/2}+\frac{32}{3}(6-3ex)^{7/2}-\frac{256}{3}(6-3ex)^{5/2}+256(6-3ex)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\sqrt{3}(2-ex)^{13/2}}{13e}+\frac{96\sqrt{3}(2-ex)^{11/2}}{11e}-\frac{64\sqrt{3}(2-ex)^{9/2}}{e}+\frac{1536\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{1536\sqrt{3}(2-ex)^{5/2}}{5e}$$

input
```
Int[(2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-1536*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (1536*Sqrt[3]*(2 - e*x)^(7/2))/(7*
e) - (64*Sqrt[3]*(2 - e*x)^(9/2))/e + (96*Sqrt[3]*(2 - e*x)^(11/2))/(11*e)
 - (6*Sqrt[3]*(2 - e*x)^(13/2))/(13*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{2(ex-2)\left(1155e^{4}x^{4}+12600e^{3}x^{3}+56840e^{2}x^{2}+133600ex+154928\right)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{15015e\left(ex+2\right)^{\frac{3}{2}}}$	60
orering	$\frac{2(ex-2)\left(1155e^{4}x^{4}+12600e^{3}x^{3}+56840e^{2}x^{2}+133600ex+154928\right)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{15015e\left(ex+2\right)^{\frac{3}{2}}}$	60
default	$-\frac{2\sqrt{-3e^{2}x^{2}+12}\,(ex-2)^{2}\left(1155e^{4}x^{4}+12600e^{3}x^{3}+56840e^{2}x^{2}+133600ex+154928\right)}{5005\sqrt{ex+2}\,e}$	62
risch	$\frac{6\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(1155e^{6}x^{6}+7980e^{5}x^{5}+11060e^{4}x^{4}-43360e^{3}x^{3}-152112e^{2}x^{2}-85312ex+619712\right)(ex-2)}{5005\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	104

input
```
int((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15015*(e*x-2)*(1155*e^4*x^4+12600*e^3*x^3+56840*e^2*x^2+133600*e*x+15492
8)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int (2+ex)^{5/2} \left(12-3e^2x^2\right)^{3/2} dx = -\frac{2\,(1155\,e^6x^6+7980\,e^5x^5+11060\,e^4x^4-43360\,e^3x^3-152112\,e^2x^2-85312\,ex+619712)\sqrt{-3\,e^2x^2+12}}{5005\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
-2/5005*(1155*e^6*x^6 + 7980*e^5*x^5 + 11060*e^4*x^4 - 43360*e^3*x^3 - 152
112*e^2*x^2 - 85312*e*x + 619712)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2
*x + 2*e)
```

Sympy [F(-1)]

Timed out.

$$\int (2+ex)^{5/2} \left(12-3e^2x^2\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((e*x+2)**(5/2)*(-3*e**2*x**2+12)**(3/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int (2+ex)^{5/2} (12 -3e^2x^2)^{3/2} dx = \frac{2\left(-1155i\sqrt{3}e^6x^6-7980i\sqrt{3}e^5x^5-11060i\sqrt{3}e^4x^4+43360i\sqrt{3}e^3x^3+152112i\sqrt{3}e^2x^2\right.}{5005\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
2/5005*(-1155*I*sqrt(3)*e^6*x^6 - 7980*I*sqrt(3)*e^5*x^5 - 11060*I*sqrt(3)
*e^4*x^4 + 43360*I*sqrt(3)*e^3*x^3 + 152112*I*sqrt(3)*e^2*x^2 + 85312*I*sq
rt(3)*e*x - 619712*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x + 2*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int (2+ex)^{5/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{2\sqrt{3}\left(1155\,(ex-2)^6\sqrt{-ex+2}+21840\,(ex-2)^5\sqrt{-ex+2}+160160\,(ex-2)^4\sqrt{-ex+2}+549120\,(ex\right.}{5005\,e}$$

input
```
integrate((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
-2/5005*sqrt(3)*(1155*(e*x - 2)^6*sqrt(-e*x + 2) + 21840*(e*x - 2)^5*sqrt(
-e*x + 2) + 160160*(e*x - 2)^4*sqrt(-e*x + 2) + 549120*(e*x - 2)^3*sqrt(-e
*x + 2) + 768768*(e*x - 2)^2*sqrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int (2+ex)^{5/2} (12 -3e^2x^2)^{3/2} \, dx = \frac{2\sqrt{12-3\,e^2\,x^2}\,\sqrt{e\,x+2}\,\left(-1155\,e^5\,x^5-5670\,e^4\,x^4+280\,e^3\,x^3+42800\,e^2\,x^2+66512\,e\,x\right.}{5005\,e} - \frac{1048576\sqrt{12-3\,e^2\,x^2}}{5005\,e\,\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(3/2)*(e*x + 2)^(5/2),x)
```

output
```
(2*(12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(1/2)*(66512*e*x + 42800*e^2*x^2 + 280
*e^3*x^3 - 5670*e^4*x^4 - 1155*e^5*x^5 - 47712))/(5005*e) - (1048576*(12 -
 3*e^2*x^2)^(1/2))/(5005*e*(e*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.55

$$\int (2+ex)^{5/2} \left(12 -3e^2x^2\right)^{3/2} \, dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(-1155e^6x^6 - 7980e^5x^5 - 11060e^4x^4 + 43360e^3x^3 + 152112e^2x^2 + 85312}{5005e}$$

input
```
int((e*x+2)^(5/2)*(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*( - 1155*e**6*x**6 - 7980*e**5*x**5 - 11060*e*
*4*x**4 + 43360*e**3*x**3 + 152112*e**2*x**2 + 85312*e*x - 619712))/(5005*
e)
```

3.214 $\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{384\sqrt{3}(2-ex)^{5/2}}{5e} + \frac{288\sqrt{3}(2-ex)^{7/2}}{7e} - \frac{8\sqrt{3}(2-ex)^{9/2}}{e} + \frac{6\sqrt{3}(2-ex)^{11/2}}{11e}$$

output
```
-384/5*3^(1/2)*(-e*x+2)^(5/2)/e+288/7*3^(1/2)*(-e*x+2)^(7/2)/e-8*(-e*x+2)^
(9/2)*3^(1/2)/e+6/11*3^(1/2)*(-e*x+2)^(11/2)/e
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

$$\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{2(-2+ex)^2\sqrt{12-3e^2x^2}(4264+3020ex+910e^2x^2+105e^3x^3)}{385e\sqrt{2+ex}}$$

input
```
Integrate[(2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-2*(-2 + e*x)^2*Sqrt[12 - 3*e^2*x^2]*(4264 + 3020*e*x + 910*e^2*x^2 + 105
*e^3*x^3))/(385*e*Sqrt[2 + e*x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex+2)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{456}$$

$$\int (6-3ex)^{3/2}(ex+2)^3dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(-\frac{1}{27}(6-3ex)^{9/2}+\frac{4}{3}(6-3ex)^{7/2}-16(6-3ex)^{5/2}+64(6-3ex)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\sqrt{3}(2-ex)^{11/2}}{11e}-\frac{8\sqrt{3}(2-ex)^{9/2}}{e}+\frac{288\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{384\sqrt{3}(2-ex)^{5/2}}{5e}$$

input
```
Int[(2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-384*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (288*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
 - (8*Sqrt[3]*(2 - e*x)^(9/2))/e + (6*Sqrt[3]*(2 - e*x)^(11/2))/(11*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{2(ex-2)\left(105e^{3}x^{3}+910e^{2}x^{2}+3020ex+4264\right)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{1155e\left(ex+2\right)^{\frac{3}{2}}}$	52
orering	$\frac{2(ex-2)\left(105e^{3}x^{3}+910e^{2}x^{2}+3020ex+4264\right)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{1155e\left(ex+2\right)^{\frac{3}{2}}}$	52
default	$-\frac{2\sqrt{-3e^{2}x^{2}+12}\,(ex-2)^{2}\left(105e^{3}x^{3}+910e^{2}x^{2}+3020ex+4264\right)}{385\sqrt{ex+2}\,e}$	54
risch	$\frac{6\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(105e^{5}x^{5}+490e^{4}x^{4}-200e^{3}x^{3}-4176e^{2}x^{2}-4976ex+17056\right)(ex-2)}{385\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	96

input
```
int((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(e*x-2)*(105*e^3*x^3+910*e^2*x^2+3020*e*x+4264)*(-3*e^2*x^2+12)^(3/
2)/e/(e*x+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{2\,(105\,e^5x^5+490\,e^4x^4-200\,e^3x^3-4176\,e^2x^2-4976\,ex+17056)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{385\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
-2/385*(105*e^5*x^5 + 490*e^4*x^4 - 200*e^3*x^3 - 4176*e^2*x^2 - 4976*e*x
+ 17056)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F]

$$\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx = 3\sqrt{3}\left(\int 8\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx+\int 4ex\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx+\int \left(-2e^2x^2\sqrt{ex+2}\right.\right.$$

input
```
integrate((e*x+2)**(3/2)*(-3*e**2*x**2+12)**(3/2),x)
```

output
```
3*sqrt(3)*(Integral(8*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x) + Integral(4*
e*x*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x) + Integral(-2*e**2*x**2*sqrt(e*
x + 2)*sqrt(-e**2*x**2 + 4), x) + Integral(-e**3*x**3*sqrt(e*x + 2)*sqrt(-
e**2*x**2 + 4), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int (2+ex)^{3/2} (12 -3e^2x^2)^{3/2} \, dx = \frac{2 \left(-105i \sqrt{3}e^5x^5 - 490i \sqrt{3}e^4x^4 + 200i \sqrt{3}e^3x^3 + 4176i \sqrt{3}e^2x^2 + 4976i \sqrt{3}ex - 17056i \right.}{385 \left(e^2x + 2e\right)}$$

input
```
integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
2/385*(-105*I*sqrt(3)*e^5*x^5 - 490*I*sqrt(3)*e^4*x^4 + 200*I*sqrt(3)*e^3*
x^3 + 4176*I*sqrt(3)*e^2*x^2 + 4976*I*sqrt(3)*e*x - 17056*I*sqrt(3))*(e*x
+ 2)*sqrt(e*x - 2)/(e^2*x + 2*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{2 \sqrt{3} \left(105 (ex-2)^5 \sqrt{-ex+2} + 1540 (ex-2)^4 \sqrt{-ex+2} + 7920 (ex-2)^3 \sqrt{-ex+2} + 14784 (ex-2)^2 \right.}{385 e}$$

input
```
integrate((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
-2/385*sqrt(3)*(105*(e*x - 2)^5*sqrt(-e*x + 2) + 1540*(e*x - 2)^4*sqrt(-e*
x + 2) + 7920*(e*x - 2)^3*sqrt(-e*x + 2) + 14784*(e*x - 2)^2*sqrt(-e*x + 2
))/e
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx = -\frac{2\sqrt{12-3\,e^2\,x^2}\,\left(e\,x-2\right)^2\,\left(105\,e^3\,x^3+910\,e^2\,x^2+3020\,e\,x+4264\right)}{385\,e\,\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(3/2)*(e*x + 2)^(3/2),x)
```

output
```
-(2*(12 - 3*e^2*x^2)^(1/2)*(e*x - 2)^2*(3020*e*x + 910*e^2*x^2 + 105*e^3*x
^3 + 4264))/(385*e*(e*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int (2+ex)^{3/2} \left(12-3e^2x^2\right)^{3/2} \, dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,\left(-105e^5x^5-490e^4x^4+200e^3x^3+4176e^2x^2+4976ex-17056\right)}{385e}$$

input
```
int((e*x+2)^(3/2)*(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*( - 105*e**5*x**5 - 490*e**4*x**4 + 200*e**3*x
**3 + 4176*e**2*x**2 + 4976*e*x - 17056))/(385*e)
```

3.215 $\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx=-\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{48\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{2(2-ex)^{9/2}}{\sqrt{3}e}$$

output
```
-96/5*3^(1/2)*(-e*x+2)^(5/2)/e+48/7*3^(1/2)*(-e*x+2)^(7/2)/e-2/3*(-e*x+2)^
(9/2)*3^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx=-\frac{2(-2+ex)^2\sqrt{4-e^2x^2}\left(428+220ex+35e^2x^2\right)}{35e\sqrt{6+3ex}}$$

input
```
Integrate[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-2*(-2 + e*x)^2*Sqrt[4 - e^2*x^2]*(428 + 220*e*x + 35*e^2*x^2))/(35*e*Sqr
t[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex+2}\left(12-3e^2x^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 456}$$

$$\int (6-3ex)^{3/2}(ex+2)^2dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(\frac{1}{9}(6-3ex)^{7/2}-\frac{8}{3}(6-3ex)^{5/2}+16(6-3ex)^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(2-ex)^{9/2}}{\sqrt{3}e}+\frac{48\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{96\sqrt{3}(2-ex)^{5/2}}{5e}$$

input
```
Int[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-96*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (48*Sqrt[3]*(2 - e*x)^(7/2))/(7*e) -
 (2*(2 - e*x)^(9/2))/(Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{2(ex-2)(35e^2x^2+220ex+428)\left(-3e^2x^2+12\right)^{\frac{3}{2}}}{315e(ex+2)^{\frac{3}{2}}}$	44
orering	$\frac{2(ex-2)(35e^2x^2+220ex+428)\left(-3e^2x^2+12\right)^{\frac{3}{2}}}{315e(ex+2)^{\frac{3}{2}}}$	44
default	$-\frac{2\sqrt{-3e^2x^2+12}\,(ex-2)^2(35e^2x^2+220ex+428)}{105\sqrt{ex+2}\,e}$	46
risch	$\frac{2\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}\,(35e^4x^4+80e^3x^3-312e^2x^2-832ex+1712)(ex-2)}{35\sqrt{-3e^2x^2+12}\,e\sqrt{-3ex+6}}$	88

input
```
int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(e*x-2)*(35*e^2*x^2+220*e*x+428)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/
2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx = -\frac{2\,(35\,e^4x^4+80\,e^3x^3-312\,e^2x^2-832\,ex+1712)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{105\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
-2/105*(35*e^4*x^4 + 80*e^3*x^3 - 312*e^2*x^2 - 832*e*x + 1712)*sqrt(-3*e^
2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F]

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx = 3\sqrt{3}\left(\int 4\sqrt{ex+2}\sqrt{-e^2x^2+4}\,dx + \int \left(-e^2x^2\sqrt{ex+2}\sqrt{-e^2x^2+4}\right)\,dx\right)$$

input
```
integrate((e*x+2)**(1/2)*(-3*e**2*x**2+12)**(3/2),x)
```

output
```
3*sqrt(3)*(Integral(4*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x) + Integral(-e
**2*x***2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx = \frac{2\left(-35i\sqrt{3}e^4x^4-80i\sqrt{3}e^3x^3+312i\sqrt{3}e^2x^2+832i\sqrt{3}ex-1712i\sqrt{3}\right)(ex+2)\sqrt{ex-2}}{105\left(e^2x+2e\right)}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
2/105*(-35*I*sqrt(3)*e^4*x^4 - 80*I*sqrt(3)*e^3*x^3 + 312*I*sqrt(3)*e^2*x^
2 + 832*I*sqrt(3)*e*x - 1712*I*sqrt(3))*(e*x + 2)*sqrt(e*x - 2)/(e^2*x + 2
*e)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx = -\frac{2\sqrt{3}\left(35\,(ex-2)^4\sqrt{-ex+2}+360\,(ex-2)^3\sqrt{-ex+2}+1008\,(ex-2)^2\sqrt{-ex+2}\right)}{105\,e}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
-2/105*sqrt(3)*(35*(e*x - 2)^4*sqrt(-e*x + 2) + 360*(e*x - 2)^3*sqrt(-e*x
+ 2) + 1008*(e*x - 2)^2*sqrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx = \frac{2\sqrt{12-3\,e^2\,x^2}\,\sqrt{e\,x+2}\,\left(-35\,e^3\,x^3-10\,e^2\,x^2+332\,e\,x+168\right)}{105\,e} - \frac{4096\sqrt{12-3\,e^2\,x^2}}{105\,e\,\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(3/2)*(e*x + 2)^(1/2),x)
```

output
```
(2*(12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(1/2)*(332*e*x - 10*e^2*x^2 - 35*e^3*x
^3 + 168))/(105*e) - (4096*(12 - 3*e^2*x^2)^(1/2))/(105*e*(e*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}\,dx=\frac{2\sqrt{-ex+2}\,\sqrt{3}\,(-35e^4x^4-80e^3x^3+312e^2x^2+832ex-1712)}{105e}$$

input
```
int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*( - 35*e**4*x**4 - 80*e**3*x**3 + 312*e**2*x**
2 + 832*e*x - 1712))/(105*e)
```

3.216 $\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = -\frac{24\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{6\sqrt{3}(2-ex)^{7/2}}{7e}$$

output
```
-24/5*3^(1/2)*(-e*x+2)^(5/2)/e+6/7*3^(1/2)*(-e*x+2)^(7/2)/e
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = -\frac{6(-2+ex)^2(18+5ex)\sqrt{12-3e^2x^2}}{35e\sqrt{2+ex}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(3/2)/Sqrt[2 + e*x],x]
```

output
```
(-6*(-2 + e*x)^2*(18 + 5*e*x)*Sqrt[12 - 3*e^2*x^2])/(35*e*Sqrt[2 + e*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{ex+2}}dx$$
$$\Big\downarrow \text{456}$$
$$\int (6-3ex)^{3/2}(ex+2)dx$$
$$\Big\downarrow \text{53}$$
$$\int \left(4(6-3ex)^{3/2}-\frac{1}{3}(6-3ex)^{5/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{6\sqrt{3}(2-ex)^{7/2}}{7e}-\frac{24\sqrt{3}(2-ex)^{5/2}}{5e}$$

input
```
Int[(12 - 3*e^2*x^2)^(3/2)/Sqrt[2 + e*x],x]
```

output
```
(-24*Sqrt[3]*(2 - e*x)^(5/2))/(5*e) + (6*Sqrt[3]*(2 - e*x)^(7/2))/(7*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{2(ex-2)(5ex+18)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{35e\left(ex+2\right)^{\frac{3}{2}}}$	36
orering	$\frac{2(ex-2)(5ex+18)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{35e\left(ex+2\right)^{\frac{3}{2}}}$	36
default	$-\frac{6\sqrt{-3e^{2}x^{2}+12}\,\left(ex-2\right)^{2}(5ex+18)}{35\sqrt{ex+2}\,e}$	38
risch	$\frac{18\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(5e^{3}x^{3}-2e^{2}x^{2}-52ex+72\right)(ex-2)}{35\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	80

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(e*x-2)*(5*e*x+18)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = -\frac{6\,(5\,e^3x^3-2\,e^2x^2-52\,ex+72)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{35\,(e^2x+2\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-6/35*(5*e^3*x^3 - 2*e^2*x^2 - 52*e*x + 72)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x
 + 2)/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = 3\sqrt{3}\left(\int \frac{4\sqrt{-e^2x^2+4}}{\sqrt{ex+2}}\,dx + \int \left(-\frac{e^2x^2\sqrt{-e^2x^2+4}}{\sqrt{ex+2}}\right)dx\right)$$

input
```
integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(1/2),x)
```

output
```
3*sqrt(3)*(Integral(4*sqrt(-e**2*x**2 + 4)/sqrt(e*x + 2), x) + Integral(-e
**2*x**2*sqrt(-e**2*x**2 + 4)/sqrt(e*x + 2), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = \frac{6\left(-5i\sqrt{3}e^3x^3 + 2i\sqrt{3}e^2x^2 + 52i\sqrt{3}ex - 72i\sqrt{3}\right)\sqrt{ex-2}}{35\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x, algorithm="maxima")
```

output
```
6/35*(-5*I*sqrt(3)*e^3*x^3 + 2*I*sqrt(3)*e^2*x^2 + 52*I*sqrt(3)*e*x - 72*I
*sqrt(3))*sqrt(e*x - 2)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = -\frac{6\sqrt{3}\left(5\,(ex-2)^3\sqrt{-ex+2}+28\,(ex-2)^2\sqrt{-ex+2}\right)}{35\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x, algorithm="giac")
```

output
```
-6/35*sqrt(3)*(5*(e*x - 2)^3*sqrt(-e*x + 2) + 28*(e*x - 2)^2*sqrt(-e*x + 2
))/e
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = \frac{\sqrt{12-3\,e^2\,x^2}\,\left(\frac{312\,x}{35}+\frac{12\,e\,x^2}{35}-\frac{432}{35\,e}-\frac{6\,e^2\,x^3}{7}\right)}{\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(1/2),x)
```

output
```
((12 - 3*e^2*x^2)^(1/2)*((312*x)/35 + (12*e*x^2)/35 - 432/(35*e) - (6*e^2*
x^3)/7))/(e*x + 2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{(12-3e^2x^2)^{3/2}}{\sqrt{2+ex}}\,dx = \frac{6\sqrt{-ex+2}\,\sqrt{3}\,(-5e^3x^3+2e^2x^2+52ex-72)}{35e}$$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(1/2),x)
```

output
```
(6*sqrt( - e*x + 2)*sqrt(3)*( - 5*e**3*x**3 + 2*e**2*x**2 + 52*e*x - 72))/
(35*e)
```

3.217 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 22

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx = -\frac{6\sqrt{3}(2-ex)^{5/2}}{5e}$$

output
```
-6/5*3^(1/2)*(-e*x+2)^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx = -\frac{6\sqrt{3}(4-e^2x^2)^{5/2}}{5e(2+ex)^{5/2}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(3/2),x]
```

output
```
(-6*Sqrt[3]*(4 - e^2*x^2)^(5/2))/(5*e*(2 + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(ex+2)^{3/2}}dx$$
$$\Big\downarrow \text{456}$$
$$\int (6-3ex)^{3/2}dx$$
$$\Big\downarrow \text{17}$$
$$-\frac{6\sqrt{3}(2-ex)^{5/2}}{5e}$$

input
```
Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(3/2),x]
```

output
```
(-6*Sqrt[3]*(2 - e*x)^(5/2))/(5*e)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

method	result	size
gosper	$\frac{2(ex-2)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{5e(ex+2)^{\frac{3}{2}}}$	30
orering	$\frac{2(ex-2)\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}{5e(ex+2)^{\frac{3}{2}}}$	30
default	$-\frac{6\sqrt{-3e^{2}x^{2}+12}\,(ex-2)^{2}}{5\sqrt{ex+2}\,e}$	32
risch	$\frac{18\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(e^{2}x^{2}-4ex+4\right)(ex-2)}{5\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	71

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(e*x-2)*(-3*e^2*x^2+12)^(3/2)/e/(e*x+2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(16) = 32.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{\left(12-3e^2x^2\right)^{3/2}}{(2+ex)^{3/2}}\,dx = -\frac{6\left(e^2x^2-4\,ex+4\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{5\left(e^2x+2\,e\right)}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-6/5*(e^2*x^2 - 4*e*x + 4)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*
e)
```

Sympy [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx = 3\sqrt{3}\left(\int \frac{4\sqrt{-e^2x^2+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\,dx + \int \left(-\frac{e^2x^2\sqrt{-e^2x^2+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\right)dx\right)$$

input
```
integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(3/2),x)
```

output
```
3*sqrt(3)*(Integral(4*sqrt(-e**2*x**2 + 4)/(e*x*sqrt(e*x + 2) + 2*sqrt(e*x
 + 2)), x) + Integral(-e**2*x**2*sqrt(-e**2*x**2 + 4)/(e*x*sqrt(e*x + 2) +
 2*sqrt(e*x + 2)), x))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx = \frac{6\left(-i\sqrt{3}e^2x^2+4i\sqrt{3}ex-4i\sqrt{3}\right)\sqrt{ex-2}}{5e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x, algorithm="maxima")
```

output
```
6/5*(-I*sqrt(3)*e^2*x^2 + 4*I*sqrt(3)*e*x - 4*I*sqrt(3))*sqrt(e*x - 2)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx = -\frac{6\sqrt{3}(ex-2)^2\sqrt{-ex+2}}{5e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x, algorithm="giac")
```

output
```
-6/5*sqrt(3)*(e*x - 2)^2*sqrt(-e*x + 2)/e
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx = -\frac{\sqrt{12-3\,e^2\,x^2}\,\left(\frac{6\,e\,x^2}{5}-\frac{24\,x}{5}+\frac{24}{5\,e}\right)}{\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(3/2),x)
```

output
```
-((12 - 3*e^2*x^2)^(1/2)*((6*e*x^2)/5 - (24*x)/5 + 24/(5*e)))/(e*x + 2)^(1
/2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{3/2}}\,dx = \frac{6\sqrt{-ex+2}\,\sqrt{3}\,(-e^2x^2+4ex-4)}{5e}$$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(3/2),x)
```

output
```
(6*sqrt( - e*x + 2)*sqrt(3)*( - e**2*x**2 + 4*e*x - 4))/(5*e)
```

3.218 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx=\frac{24\sqrt{3}\sqrt{2-ex}}{e}+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}-\frac{48\sqrt{3}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

output
```
24*3^(1/2)*(-e*x+2)^(1/2)/e+2*(-e*x+2)^(3/2)*3^(1/2)/e-48*3^(1/2)*arctanh(
1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx=-\frac{2\sqrt{3}\left((-14+ex)\sqrt{4-e^2x^2}+24\sqrt{2+ex}\operatorname{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)\right)}{e\sqrt{2+ex}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(5/2),x]
```

output
```
(-2*Sqrt[3]*((-14 + e*x)*Sqrt[4 - e^2*x^2] + 24*Sqrt[2 + e*x]*ArcTanh[(2*S
qrt[2 + e*x])/Sqrt[4 - e^2*x^2]]))/(e*Sqrt[2 + e*x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {456, 60, 27, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(ex+2)^{5/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(6-3ex)^{3/2}}{ex+2}dx$$

$$\Big\downarrow \text{60}$$

$$12\int \frac{\sqrt{3}\sqrt{2-ex}}{ex+2}dx+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}$$

$$\Big\downarrow \text{27}$$

$$12\sqrt{3}\int \frac{\sqrt{2-ex}}{ex+2}dx+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}$$

$$\Big\downarrow \text{60}$$

$$12\sqrt{3}\left(4\int \frac{1}{\sqrt{2-ex}(ex+2)}dx+\frac{2\sqrt{2-ex}}{e}\right)+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}$$

$$\Big\downarrow \text{73}$$

$$12\sqrt{3}\left(\frac{2\sqrt{2-ex}}{e}-\frac{8\int \frac{1}{ex+2}d\sqrt{2-ex}}{e}\right)+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}$$

$$\Big\downarrow \text{219}$$

$$12\sqrt{3}\left(\frac{2\sqrt{2-ex}}{e}-\frac{4\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}\right)+\frac{2\sqrt{3}(2-ex)^{3/2}}{e}$$

input
```
Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(5/2),x]
```

output
```
(2*Sqrt[3]*(2 - e*x)^(3/2))/e + 12*Sqrt[3]*((2*Sqrt[2 - e*x])/e - (4*ArcTa
nh[Sqrt[2 - e*x]/2])/e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{2\sqrt{-e^2x^2+4}\left(ex\sqrt{-3ex+6}+24\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)-14\sqrt{-3ex+6}\right)\sqrt{3}}{\sqrt{ex+2}\,\sqrt{-3ex+6}\,e}$	77
risch	$\frac{6(ex-14)(ex-2)\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}}{e\sqrt{-3ex+6}\,\sqrt{-3e^2x^2+12}}-\frac{48\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}}{e\sqrt{-3e^2x^2+12}}$	125

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-e^2*x^2+4)^(1/2)*(e*x*(-3*e*x+6)^(1/2)+24*3^(1/2)*arctanh(1/6*(-3*e*x
+6)^(1/2)*3^(1/2))-14*(-3*e*x+6)^(1/2))*3^(1/2)/(e*x+2)^(1/2)/(-3*e*x+6)^(
1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(52) = 104.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.61

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx = \frac{2\left(12\sqrt{3}(ex+2)\log\left(-\frac{3e^2x^2-12ex+4\sqrt{3}\sqrt{-3e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4ex+4}\right)-\sqrt{-3e^2x^2+12}\sqrt{ex}\right.}{e^2x+2e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x, algorithm="fricas")
```

output
```
2*(12*sqrt(3)*(e*x + 2)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x
^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - sqrt(-3*e^2*x^2 + 12
)*sqrt(e*x + 2)*(e*x - 14))/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx = 3\sqrt{3}\left(\int \frac{4\sqrt{-e^2x^2+4}}{e^2x^2\sqrt{ex+2}+4ex\sqrt{ex+2}+4\sqrt{ex+2}}\,dx + \int \left(-\frac{e^2x^2\sqrt{-}}{e^2x^2\sqrt{ex+2}+4ex}\right.\right.$$

input
```
integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(5/2),x)
```

output
```
3*sqrt(3)*(Integral(4*sqrt(-e**2*x**2 + 4)/(e**2*x**2*sqrt(e*x + 2) + 4*e*
x*sqrt(e*x + 2) + 4*sqrt(e*x + 2)), x) + Integral(-e**2*x**2*sqrt(-e**2*x*
*2 + 4)/(e**2*x**2*sqrt(e*x + 2) + 4*e*x*sqrt(e*x + 2) + 4*sqrt(e*x + 2)),
 x))
```

Maxima [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{3}{2}}}{(ex+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx = \frac{2\sqrt{3}\left((-ex+2)^{\frac{3}{2}}+12\sqrt{-ex+2}-12\log\left(\sqrt{-ex+2}+2\right)+12\log\left(-\sqrt{-ex+}\right.\right.}{e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x, algorithm="giac")
```

output
```
2*sqrt(3)*((-e*x + 2)^(3/2) + 12*sqrt(-e*x + 2) - 12*log(sqrt(-e*x + 2) +
2) + 12*log(-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx = \int \frac{(12-3\,e^2\,x^2)^{3/2}}{(e\,x+2)^{5/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(5/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{5/2}}\,dx = \frac{2\sqrt{3}\left(-\sqrt{-ex+2}\,ex+14\sqrt{-ex+2}+24\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)-32\right)}{e}$$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(5/2),x)
```

output
```
(2*sqrt(3)*( - sqrt( - e*x + 2)*e*x + 14*sqrt( - e*x + 2) + 24*log(tan(asi
n(sqrt(e*x + 2)/2)/2)) - 32))/e
```

3.219 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = -\frac{9\sqrt{3}\sqrt{2-ex}}{e} - \frac{3\sqrt{3}(2-ex)^{3/2}}{e(2+ex)} + \frac{18\sqrt{3}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}$$

output
```
-9*3^(1/2)*(-e*x+2)^(1/2)/e-3*3^(1/2)*(-e*x+2)^(3/2)/e/(e*x+2)+18*3^(1/2)*
arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = \frac{6\sqrt{3}\left(-\frac{(4+ex)\sqrt{4-e^2x^2}}{(2+ex)^{3/2}} + 3\operatorname{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)\right)}{e}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(7/2),x]
```

output
```
(6*Sqrt[3]*(-(((4 + e*x)*Sqrt[4 - e^2*x^2])/(2 + e*x)^(3/2)) + 3*ArcTanh[(
2*Sqrt[2 + e*x])/Sqrt[4 - e^2*x^2]]))/e
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {456, 51, 27, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(ex+2)^{7/2}}dx$$
$$\Big\downarrow \text{456}$$
$$\int \frac{(6-3ex)^{3/2}}{(ex+2)^2}dx$$
$$\Big\downarrow \text{51}$$
$$-\frac{9}{2}\int \frac{\sqrt{3}\sqrt{2-ex}}{ex+2}dx-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(ex+2)}$$
$$\Big\downarrow \text{27}$$
$$-\frac{9}{2}\sqrt{3}\int \frac{\sqrt{2-ex}}{ex+2}dx-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(ex+2)}$$
$$\Big\downarrow \text{60}$$
$$-\frac{9}{2}\sqrt{3}\left(4\int \frac{1}{\sqrt{2-ex}(ex+2)}dx+\frac{2\sqrt{2-ex}}{e}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(ex+2)}$$
$$\Big\downarrow \text{73}$$
$$-\frac{9}{2}\sqrt{3}\left(\frac{2\sqrt{2-ex}}{e}-\frac{8\int \frac{1}{ex+2}d\sqrt{2-ex}}{e}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(ex+2)}$$
$$\Big\downarrow \text{219}$$
$$-\frac{9}{2}\sqrt{3}\left(\frac{2\sqrt{2-ex}}{e}-\frac{4\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{e}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{e(ex+2)}$$

input
```
Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(7/2),x]
```

output
```
(-3*Sqrt[3]*(2 - e*x)^(3/2))/(e*(2 + e*x)) - (9*Sqrt[3]*((2*Sqrt[2 - e*x])
/e - (4*ArcTanh[Sqrt[2 - e*x]/2])/e))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

method	result	size
default	$\frac{6\sqrt{-e^2x^2+4}\left(3\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)ex-ex\sqrt{-3ex+6}+6\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)-4\sqrt{-3ex+6}\right)\sqrt{3}}{(ex+2)^{\frac{3}{2}}\sqrt{-3ex+6}\,e}$	99
risch	$\frac{18(ex-2)\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}}{e\sqrt{-3ex+6}\,\sqrt{-3e^2x^2+12}}-\frac{72\left(-\frac{\sqrt{-3ex+6}}{2(-3ex-6)}-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)}{4}\right)\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}}{e\sqrt{-3e^2x^2+12}}$	141

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
6*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e*x-
e*x*(-3*e*x+6)^(1/2)+6*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))-4*(-3
*e*x+6)^(1/2))/(e*x+2)^(3/2)/(-3*e*x+6)^(1/2)*3^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(59) = 118.

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = \frac{3\left(3\sqrt{3}(e^2x^2+4\,ex+4)\log\left(-\frac{3\,e^2x^2-12\,ex-4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-2\sqrt{-3\,e^2x^2}\right.}{e^3x^2+4\,e^2x+4\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x, algorithm="fricas")
```

output
```
3*(3*sqrt(3)*(e^2*x^2 + 4*e*x + 4)*log(-(3*e^2*x^2 - 12*e*x - 4*sqrt(3)*sq
rt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 2*sqrt(-3
*e^2*x^2 + 12)*(e*x + 4)*sqrt(e*x + 2))/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = 3\sqrt{3}\left(\int \frac{4\sqrt{-e^2x^2+4}}{e^3x^3\sqrt{ex+2}+6e^2x^2\sqrt{ex+2}+12ex\sqrt{ex+2}+8\sqrt{ex+2}}\,dx+\int\left(-\frac{}{e^3}\right.\right.$$

input
```
integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(7/2),x)
```

output
```
3*sqrt(3)*(Integral(4*sqrt(-e**2*x**2 + 4)/(e**3*x**3*sqrt(e*x + 2) + 6*e*
*2*x**2*sqrt(e*x + 2) + 12*e*x*sqrt(e*x + 2) + 8*sqrt(e*x + 2)), x) + Inte
gral(-e**2*x**2*sqrt(-e**2*x**2 + 4)/(e**3*x**3*sqrt(e*x + 2) + 6*e**2*x**
2*sqrt(e*x + 2) + 12*e*x*sqrt(e*x + 2) + 8*sqrt(e*x + 2)), x))
```

Maxima [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{3}{2}}}{(ex+2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = -\frac{3\sqrt{3}\left(2\sqrt{-ex+2}+\frac{4\sqrt{-ex+2}}{ex+2}-3\log\left(\sqrt{-ex+2}+2\right)+3\log\left(-\sqrt{-ex+2}+2\right)\right)}{e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x, algorithm="giac")
```

output
```
-3*sqrt(3)*(2*sqrt(-e*x + 2) + 4*sqrt(-e*x + 2)/(e*x + 2) - 3*log(sqrt(-e*
x + 2) + 2) + 3*log(-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = \int \frac{(12-3\,e^2\,x^2)^{3/2}}{(e\,x+2)^{7/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(7/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{7/2}}\,dx = \frac{3\sqrt{3}\left(-4\sqrt{-ex+2}\,ex-16\sqrt{-ex+2}-12\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)ex-24\log\left(\right.\right.}{2e\,(ex+2)}$$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(7/2),x)
```

output
```
(3*sqrt(3)*( - 4*sqrt( - e*x + 2)*e*x - 16*sqrt( - e*x + 2) - 12*log(tan(a
sin(sqrt(e*x + 2)/2)/2))*e*x - 24*log(tan(asin(sqrt(e*x + 2)/2)/2)) + 9*e*
x + 18))/(2*e*(e*x + 2))
```

3.220 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = -\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(2+ex)^2} + \frac{9\sqrt{3}\sqrt{2-ex}}{4e(2+ex)} - \frac{9\sqrt{3}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}$$

output
```
-3/2*3^(1/2)*(-e*x+2)^(3/2)/e/(e*x+2)^2+9/4*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+
2)-9/8*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = \frac{3\sqrt{3}\left(\frac{2(2+5ex)\sqrt{4-e^2x^2}}{(2+ex)^{5/2}} - 3\operatorname{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)\right)}{8e}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(9/2),x]
```

output
```
(3*Sqrt[3]*((2*(2 + 5*e*x)*Sqrt[4 - e^2*x^2])/(2 + e*x)^(5/2) - 3*ArcTanh[
(2*Sqrt[2 + e*x])/Sqrt[4 - e^2*x^2]]))/(8*e)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {456, 51, 27, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(ex+2)^{9/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(6-3ex)^{3/2}}{(ex+2)^3}dx$$

$$\Big\downarrow \text{51}$$

$$-\frac{9}{4}\int \frac{\sqrt{3}\sqrt{2-ex}}{(ex+2)^2}dx-\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(ex+2)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{4}\sqrt{3}\int \frac{\sqrt{2-ex}}{(ex+2)^2}dx-\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(ex+2)^2}$$

$$\Big\downarrow \text{51}$$

$$-\frac{9}{4}\sqrt{3}\left(-\frac{1}{2}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx-\frac{\sqrt{2-ex}}{e(ex+2)}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(ex+2)^2}$$

$$\Big\downarrow \text{73}$$

$$-\frac{9}{4}\sqrt{3}\left(\frac{\int \frac{1}{ex+2}d\sqrt{2-ex}}{e}-\frac{\sqrt{2-ex}}{e(ex+2)}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(ex+2)^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{9}{4}\sqrt{3}\left(\frac{\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2e}-\frac{\sqrt{2-ex}}{e(ex+2)}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{2e(ex+2)^2}$$

input

```
Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(9/2),x]
```

output
```
(-3*Sqrt[3]*(2 - e*x)^(3/2))/(2*e*(2 + e*x)^2) - (9*Sqrt[3]*(-(Sqrt[2 - e*
x]/(e*(2 + e*x))) + ArcTanh[Sqrt[2 - e*x]/2]/(2*e)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

method	result
default	$-\frac{3\sqrt{-e^2x^2+4}\left(3\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)e^2x^2+12\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)ex-10ex\sqrt{-3ex+6}+12\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{}}{6}\right.\right.}{8(ex+2)^{\frac{5}{2}}\sqrt{-3ex+6}\,e}$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-3/8*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e
^2*x^2+12*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e*x-10*e*x*(-3*e*x
+6)^(1/2)+12*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))-4*(-3*e*x+6)^(1
/2))*3^(1/2)/(e*x+2)^(5/2)/(-3*e*x+6)^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(66) = 132.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.62

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = \frac{3\left(3\sqrt{3}(e^3x^3+6\,e^2x^2+12\,ex+8)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)+4\right.}{16\,(e^4x^3+6\,e^3x^2+12\,e^2x+8\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x, algorithm="fricas")
```

output
```
3/16*(3*sqrt(3)*(e^3*x^3 + 6*e^2*x^2 + 12*e*x + 8)*log(-(3*e^2*x^2 - 12*e*
x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x +
 4)) + 4*sqrt(-3*e^2*x^2 + 12)*(5*e*x + 2)*sqrt(e*x + 2))/(e^4*x^3 + 6*e^3
*x^2 + 12*e^2*x + 8*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{3}{2}}}{(ex+2)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = -\frac{3\sqrt{3}\left(\frac{4\left(5\,(-ex+2)^{\frac{3}{2}}-12\sqrt{-ex+2}\right)}{(ex+2)^2}+3\log\left(\sqrt{-ex+2}+2\right)-3\log\left(-\sqrt{-ex+2}+2\right)\right)}{16\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x, algorithm="giac")
```

output
```
-3/16*sqrt(3)*(4*(5*(-e*x + 2)^(3/2) - 12*sqrt(-e*x + 2))/(e*x + 2)^2 + 3*
log(sqrt(-e*x + 2) + 2) - 3*log(-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = \int \frac{(12-3\,e^2\,x^2)^{3/2}}{(e\,x+2)^{9/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(9/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{9/2}}\,dx = \frac{3\sqrt{3}\left(10\sqrt{-ex+2}\,ex+4\sqrt{-ex+2}+3\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)e^2x^2+12\log\left(\mathrm{t}\right.\right.}{8e\left(e^2x^2+4ex+4\right)}$$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(9/2),x)
```

output
```
(3*sqrt(3)*(10*sqrt( - e*x + 2)*e*x + 4*sqrt( - e*x + 2) + 3*log(tan(asin(
sqrt(e*x + 2)/2)/2))*e**2*x**2 + 12*log(tan(asin(sqrt(e*x + 2)/2)/2))*e*x
+ 12*log(tan(asin(sqrt(e*x + 2)/2)/2))))/(8*e*(e**2*x**2 + 4*e*x + 4))
```

3.221 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = -\frac{\sqrt{3}(2-ex)^{3/2}}{e(2+ex)^3}+\frac{3\sqrt{3}\sqrt{2-ex}}{4e(2+ex)^2} -\frac{3\sqrt{3}\sqrt{2-ex}}{32e(2+ex)}-\frac{3\sqrt{3}\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{64e}$$

output
```
-3^(1/2)*(-e*x+2)^(3/2)/e/(e*x+2)^3+3/4*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+2)^2
-3/32*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+2)-3/64*3^(1/2)*arctanh(1/2*(-e*x+2)^(
1/2))/e
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = \frac{\sqrt{3}\left(-\frac{2\sqrt{4-e^2x^2}(28-44ex+3e^2x^2)}{(2+ex)^{7/2}}-3\operatorname{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)\right)}{64e}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(11/2),x]
```

output
```
(Sqrt[3]*((-2*Sqrt[4 - e^2*x^2]*(28 - 44*e*x + 3*e^2*x^2))/(2 + e*x)^(7/2)
 - 3*ArcTanh[(2*Sqrt[2 + e*x])/Sqrt[4 - e^2*x^2]]))/(64*e)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {456, 51, 27, 51, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(ex+2)^{11/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(6-3ex)^{3/2}}{(ex+2)^4}dx$$

$$\Big\downarrow \text{51}$$

$$-\frac{3}{2}\int \frac{\sqrt{3}\sqrt{2-ex}}{(ex+2)^3}dx-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}\sqrt{3}\int \frac{\sqrt{2-ex}}{(ex+2)^3}dx-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}$$

$$\Big\downarrow \text{51}$$

$$-\frac{3}{2}\sqrt{3}\left(-\frac{1}{4}\int \frac{1}{\sqrt{2-ex}(ex+2)^2}dx-\frac{\sqrt{2-ex}}{2e(ex+2)^2}\right)-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}$$

$$\Big\downarrow \text{52}$$

$$-\frac{3}{2}\sqrt{3}\left(\frac{1}{4}\left(\frac{\sqrt{2-ex}}{4e(ex+2)}-\frac{1}{8}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx\right)-\frac{\sqrt{2-ex}}{2e(ex+2)^2}\right)-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}$$

$$\Big\downarrow \text{73}$$

$$-\frac{3}{2}\sqrt{3}\left(\frac{1}{4}\left(\frac{\int \frac{1}{ex+2}d\sqrt{2-ex}}{4e}+\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{2-ex}}{2e(ex+2)^2}\right)-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{3}{2}\sqrt{3}\left(\frac{1}{4}\left(\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}+\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{2-ex}}{2e(ex+2)^2}\right)-\frac{\sqrt{3}(2-ex)^{3/2}}{e(ex+2)^3}$$

input
```
Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(11/2),x]
```

output
```
-((Sqrt[3]*(2 - e*x)^(3/2))/(e*(2 + e*x)^3)) - (3*Sqrt[3]*(-1/2*Sqrt[2 - e
*x]/(e*(2 + e*x)^2) + (Sqrt[2 - e*x]/(4*e*(2 + e*x)) + ArcTanh[Sqrt[2 - e*
x]/2]/(8*e))/4))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{-e^{2} x^{2}+4}\, \left(3 \sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{-3 e x+6}\, \sqrt{3}}{6}\right) e^{3} x^{3}+18 \sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{-3 e x+6}\, \sqrt{3}}{6}\right) e^{2} x^{2}+6 e^{2} x^{2} \sqrt{-3 e x+6}+36 \sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{-3 e x+6}}{6}\right.\right.}{64 \left(e x+2\right)^{\frac{7}{2}} \sqrt{-3 e x+6}\, e}$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*
e^3*x^3+18*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e^2*x^2+6*e^2*x^2
*(-3*e*x+6)^(1/2)+36*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e*x-88*
e*x*(-3*e*x+6)^(1/2)+24*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))+56*(
-3*e*x+6)^(1/2))*3^(1/2)/(e*x+2)^(7/2)/(-3*e*x+6)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.44

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = \frac{3\sqrt{3}(e^4x^4+8\,e^3x^3+24\,e^2x^2+32\,ex+16)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-}{e^2x^2+4\,ex+4}\right.}{128\,(e^5x^4+8\,e^4x^3+24\,e^3x^2+32\,e^2x}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x, algorithm="fricas")
```

output
```
1/128*(3*sqrt(3)*(e^4*x^4 + 8*e^3*x^3 + 24*e^2*x^2 + 32*e*x + 16)*log(-(3*
e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^
2*x^2 + 4*e*x + 4)) - 4*(3*e^2*x^2 - 44*e*x + 28)*sqrt(-3*e^2*x^2 + 12)*sq
rt(e*x + 2))/(e^5*x^4 + 8*e^4*x^3 + 24*e^3*x^2 + 32*e^2*x + 16*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{3}{2}}}{(ex+2)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = -\frac{\sqrt{3}\left(\frac{4\left(3\,(ex-2)^2\sqrt{-ex+2}+32\,(-ex+2)^{\frac{3}{2}}-48\,\sqrt{-ex+2}\right)}{(ex+2)^3}+3\,\log\left(\sqrt{-ex+2}+2\right)-3\,\log\left(-\sqrt{-ex+2}+2\right)\right)}{128\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x, algorithm="giac")
```

output
```
-1/128*sqrt(3)*(4*(3*(e*x - 2)^2*sqrt(-e*x + 2) + 32*(-e*x + 2)^(3/2) - 48
*sqrt(-e*x + 2))/(e*x + 2)^3 + 3*log(sqrt(-e*x + 2) + 2) - 3*log(-sqrt(-e*
x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = \int \frac{(12-3\,e^2\,x^2)^{3/2}}{(e\,x+2)^{11/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(11/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.24

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{11/2}}\,dx = \frac{\sqrt{3}\left(-6\sqrt{-ex+2}\,e^2x^2+88\sqrt{-ex+2}\,ex-56\sqrt{-ex+2}+3\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right.\right.\right.}{6}$$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(11/2),x)
```

output
```
(sqrt(3)*( - 6*sqrt( - e*x + 2)*e**2*x**2 + 88*sqrt( - e*x + 2)*e*x - 56*s
qrt( - e*x + 2) + 3*log(tan(asin(sqrt(e*x + 2)/2)/2))*e**3*x**3 + 18*log(t
an(asin(sqrt(e*x + 2)/2)/2))*e**2*x**2 + 36*log(tan(asin(sqrt(e*x + 2)/2)/
2))*e*x + 24*log(tan(asin(sqrt(e*x + 2)/2)/2))))/(64*e*(e**3*x**3 + 6*e**2
*x**2 + 12*e*x + 8))
```

3.222 $\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx = -\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(2+ex)^4}+\frac{3\sqrt{3}\sqrt{2-ex}}{8e(2+ex)^3} - \frac{3\sqrt{3}\sqrt{2-ex}}{128e(2+ex)^2}-\frac{9\sqrt{3}\sqrt{2-ex}}{1024e(2+ex)}-\frac{9\sqrt{3}\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{2048e}$$

output
```
-3/4*3^(1/2)*(-e*x+2)^(3/2)/e/(e*x+2)^4+3/8*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+
2)^3-3/128*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+2)^2-9/1024*3^(1/2)*(-e*x+2)^(1/2
)/e/(e*x+2)-9/2048*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx = \frac{3\sqrt{3}\left(-\frac{2\sqrt{4-e^2x^2}(312-316ex+26e^2x^2+3e^3x^3)}{(2+ex)^{9/2}}-3\text{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)\right)}{2048e}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(13/2),x]
```

output
```
(3*Sqrt[3]*((-2*Sqrt[4 - e^2*x^2]*(312 - 316*e*x + 26*e^2*x^2 + 3*e^3*x^3)
)/(2 + e*x)^(9/2) - 3*ArcTanh[(2*Sqrt[2 + e*x])/Sqrt[4 - e^2*x^2]]))/(2048
*e)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {456, 51, 27, 51, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(ex+2)^{13/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(6-3ex)^{3/2}}{(ex+2)^5}dx$$

$$\Big\downarrow \text{51}$$

$$-\frac{9}{8}\int \frac{\sqrt{3}\sqrt{2-ex}}{(ex+2)^4}dx-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{9}{8}\sqrt{3}\int \frac{\sqrt{2-ex}}{(ex+2)^4}dx-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}$$

$$\Big\downarrow \text{51}$$

$$-\frac{9}{8}\sqrt{3}\left(-\frac{1}{6}\int \frac{1}{\sqrt{2-ex}(ex+2)^3}dx-\frac{\sqrt{2-ex}}{3e(ex+2)^3}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}$$

$$\Big\downarrow \text{52}$$

$$-\frac{9}{8}\sqrt{3}\left(\frac{1}{6}\left(\frac{\sqrt{2-ex}}{8e(ex+2)^2}-\frac{3}{16}\int \frac{1}{\sqrt{2-ex}(ex+2)^2}dx\right)-\frac{\sqrt{2-ex}}{3e(ex+2)^3}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}$$

$$\Big\downarrow \text{52}$$

$$-\frac{9}{8}\sqrt{3}\left(\frac{1}{6}\left(\frac{\sqrt{2-ex}}{8e(ex+2)^2}-\frac{3}{16}\left(\frac{1}{8}\int\frac{1}{\sqrt{2-ex}(ex+2)}dx-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)\right)-\frac{\sqrt{2-ex}}{3e(ex+2)^3}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{9}{8}\sqrt{3}\left(\frac{1}{6}\left(\frac{\sqrt{2-ex}}{8e(ex+2)^2}-\frac{3}{16}\left(-\frac{\int\frac{1}{ex+2}d\sqrt{2-ex}}{4e}-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)\right)-\frac{\sqrt{2-ex}}{3e(ex+2)^3}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}$$

$$\Big\downarrow \text{219}$$

$$-\frac{9}{8}\sqrt{3}\left(\frac{1}{6}\left(\frac{\sqrt{2-ex}}{8e(ex+2)^2}-\frac{3}{16}\left(-\frac{\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)\right)-\frac{\sqrt{2-ex}}{3e(ex+2)^3}\right)-\frac{3\sqrt{3}(2-ex)^{3/2}}{4e(ex+2)^4}$$

input
```
Int[(12 - 3*e^2*x^2)^(3/2)/(2 + e*x)^(13/2),x]
```

output
```
(-3*Sqrt[3]*(2 - e*x)^(3/2))/(4*e*(2 + e*x)^4) - (9*Sqrt[3]*(-1/3*Sqrt[2 -
 e*x]/(e*(2 + e*x)^3) + (Sqrt[2 - e*x]/(8*e*(2 + e*x)^2) - (3*(-1/4*Sqrt[2
 - e*x]/(e*(2 + e*x)) - ArcTanh[Sqrt[2 - e*x]/2]/(8*e)))/16)/6))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.43

method	result
default	$-\frac{3\sqrt{-e^{2}x^{2}+4}\,\left(3\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)e^{4}x^{4}+24\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)e^{3}x^{3}+6e^{3}x^{3}\sqrt{-3ex+6}+72\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+}}{6}\right.\right.}{2048(ex+}$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-3/2048*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2)
)*e^4*x^4+24*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e^3*x^3+6*e^3*x
^3*(-3*e*x+6)^(1/2)+72*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e^2*x
^2+52*e^2*x^2*(-3*e*x+6)^(1/2)+96*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(
1/2))*e*x-632*e*x*(-3*e*x+6)^(1/2)+48*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)
*3^(1/2))+624*(-3*e*x+6)^(1/2))*3^(1/2)/(e*x+2)^(9/2)/(-3*e*x+6)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.30

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx = \frac{3\left(3\sqrt{3}(e^5x^5+10\,e^4x^4+40\,e^3x^3+80\,e^2x^2+80\,ex+32)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-}}{e^2x^2+4}\right.\right.}{4096\,(e^6x^5+10\,e^5x^4+40\,e^4x}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x, algorithm="fricas")
```

output
```
3/4096*(3*sqrt(3)*(e^5*x^5 + 10*e^4*x^4 + 40*e^3*x^3 + 80*e^2*x^2 + 80*e*x
 + 32)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x
 + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*(3*e^3*x^3 + 26*e^2*x^2 - 316*e*x +
 312)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2))/(e^6*x^5 + 10*e^5*x^4 + 40*e^4*
x^3 + 80*e^3*x^2 + 80*e^2*x + 32*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((-3*e**2*x**2+12)**(3/2)/(e*x+2)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{3}{2}}}{(ex+2)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(3/2)/(e*x + 2)^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.70

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx =$$
$$-\frac{3\sqrt{3}\left(\frac{4\left(3\,(ex-2)^3\sqrt{-ex+2}+44\,(ex-2)^2\sqrt{-ex+2}+176\,(-ex+2)^{\frac{3}{2}}-192\,\sqrt{-ex+2}\right)}{(ex+2)^4}+3\,\log\left(\sqrt{-ex+2}+2\right)-3\,\log\left(-\sqrt{-e\right.\right.}{4096\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x, algorithm="giac")
```

output
```
-3/4096*sqrt(3)*(4*(3*(e*x - 2)^3*sqrt(-e*x + 2) + 44*(e*x - 2)^2*sqrt(-e*
x + 2) + 176*(-e*x + 2)^(3/2) - 192*sqrt(-e*x + 2))/(e*x + 2)^4 + 3*log(sq
rt(-e*x + 2) + 2) - 3*log(-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx = \int \frac{{\left(12-3\,e^2\,x^2\right)}^{3/2}}{{\left(e\,x+2\right)}^{13/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(13/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(3/2)/(e*x + 2)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.68

$$\int \frac{(12-3e^2x^2)^{3/2}}{(2+ex)^{13/2}}\,dx = \frac{3\sqrt{3}\left(48\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)\right)\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)^{8}+\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)^{16}-8\,\mathrm{t}\cdots}{32768\tan\left(\frac{asin\left(\frac{\sqrt{ex+2}}{2}\right)}{2}\right)^{8}e}$$

input
```
int((-3*e^2*x^2+12)^(3/2)/(e*x+2)^(13/2),x)
```

output
```
(3*sqrt(3)*(48*log(tan(asin(sqrt(e*x + 2)/2)/2))*tan(asin(sqrt(e*x + 2)/2)
/2)**8 + tan(asin(sqrt(e*x + 2)/2)/2)**16 - 8*tan(asin(sqrt(e*x + 2)/2)/2)
**12 + 8*tan(asin(sqrt(e*x + 2)/2)/2)**4 - 1))/(32768*tan(asin(sqrt(e*x +
2)/2)/2)**8*e)
```

3.223 $\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}} dx = -\frac{128\sqrt{2-ex}}{\sqrt{3}e} + \frac{32(2-ex)^{3/2}}{\sqrt{3}e} - \frac{8\sqrt{3}(2-ex)^{5/2}}{5e} + \frac{2(2-ex)^{7/2}}{7\sqrt{3}e}$$

output
```
-128/3*3^(1/2)*(-e*x+2)^(1/2)/e+32/3*(-e*x+2)^(3/2)*3^(1/2)/e-8/5*3^(1/2)*
(-e*x+2)^(5/2)/e+2/21*3^(1/2)*(-e*x+2)^(7/2)/e
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}} dx = -\frac{2\sqrt{4-e^2x^2}(1416+284ex+54e^2x^2+5e^3x^3)}{35e\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(7/2)/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-2*Sqrt[4 - e^2*x^2]*(1416 + 284*e*x + 54*e^2*x^2 + 5*e^3*x^3))/(35*e*Sqr
t[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{7/2}}{\sqrt{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{(ex+2)^3}{\sqrt{6-3ex}}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(-\frac{1}{27}(6-3ex)^{5/2}+\frac{4}{3}(6-3ex)^{3/2}-16\sqrt{6-3ex}+\frac{64}{\sqrt{6-3ex}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(2-ex)^{7/2}}{7\sqrt{3}e}-\frac{8\sqrt{3}(2-ex)^{5/2}}{5e}+\frac{32(2-ex)^{3/2}}{\sqrt{3}e}-\frac{128\sqrt{2-ex}}{\sqrt{3}e}$$

input
```
Int[(2 + e*x)^(7/2)/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-128*Sqrt[2 - e*x])/(Sqrt[3]*e) + (32*(2 - e*x)^(3/2))/(Sqrt[3]*e) - (8*S
qrt[3]*(2 - e*x)^(5/2))/(5*e) + (2*(2 - e*x)^(7/2))/(7*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2\sqrt{-3e^{2}x^{2}+12}\,\left(5e^{3}x^{3}+54e^{2}x^{2}+284ex+1416\right)}{105\sqrt{ex+2}\,e}$	47
gosper	$\frac{2(ex-2)\left(5e^{3}x^{3}+54e^{2}x^{2}+284ex+1416\right)\sqrt{ex+2}}{35e\sqrt{-3e^{2}x^{2}+12}}$	52
orering	$\frac{2(ex-2)\left(5e^{3}x^{3}+54e^{2}x^{2}+284ex+1416\right)\sqrt{ex+2}}{35e\sqrt{-3e^{2}x^{2}+12}}$	52
risch	$\frac{2\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(5e^{3}x^{3}+54e^{2}x^{2}+284ex+1416\right)(ex-2)}{35\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	80

input
```
int((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105/(e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2)*(5*e^3*x^3+54*e^2*x^2+284*e*x+1
416)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx=-\frac{2\,(5\,e^3x^3+54\,e^2x^2+284\,ex+1416)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{105\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(5*e^3*x^3 + 54*e^2*x^2 + 284*e*x + 1416)*sqrt(-3*e^2*x^2 + 12)*sqr
t(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+2)**(7/2)/(-3*e**2*x**2+12)**(1/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.53

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2i\,\sqrt{3}(5\,e^4x^4+44\,e^3x^3+176\,e^2x^2+848\,ex-2832)}{105\,\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
-2/105*I*sqrt(3)*(5*e^4*x^4 + 44*e^3*x^3 + 176*e^2*x^2 + 848*e*x - 2832)/(
sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\sqrt{3}\left(5\,(ex-2)^3\sqrt{-ex+2}+84\,(ex-2)^2\sqrt{-ex+2}-560\,(-ex+2)^{\frac{3}{2}}+2240\,\sqrt{-ex+2}\right)}{105\,e}$$

input
```
integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
-2/105*sqrt(3)*(5*(e*x - 2)^3*sqrt(-e*x + 2) + 84*(e*x - 2)^2*sqrt(-e*x +
2) - 560*(-e*x + 2)^(3/2) + 2240*sqrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{\sqrt{12-3\,e^2\,x^2}\,\left(\frac{944\,\sqrt{e\,x+2}}{35\,e^2}+\frac{36\,x^2\,\sqrt{e\,x+2}}{35}+\frac{568\,x\,\sqrt{e\,x+2}}{105\,e}+\frac{2\,e\,x^3\,\sqrt{e\,x+2}}{21}\right)}{x+\frac{2}{e}}$$

input
```
int((e*x + 2)^(7/2)/(12 - 3*e^2*x^2)^(1/2),x)
```

output
```
-((12 - 3*e^2*x^2)^(1/2)*((944*(e*x + 2)^(1/2))/(35*e^2) + (36*x^2*(e*x +
2)^(1/2))/35 + (568*x*(e*x + 2)^(1/2))/(105*e) + (2*e*x^3*(e*x + 2)^(1/2))
/21))/(x + 2/e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

$$\int \frac{(2+ex)^{7/2}}{\sqrt{12-3e^2x^2}}\,dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(-5e^3x^3-54e^2x^2-284ex-1416)}{105e}$$

input
```
int((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*( - 5*e**3*x**3 - 54*e**2*x**2 - 284*e*x - 141
6))/(105*e)
```

3.224 $\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}} dx = -\frac{32\sqrt{2-ex}}{\sqrt{3}e} + \frac{16(2-ex)^{3/2}}{3\sqrt{3}e} - \frac{2(2-ex)^{5/2}}{5\sqrt{3}e}$$

output
```
-32/3*3^(1/2)*(-e*x+2)^(1/2)/e+16/9*(-e*x+2)^(3/2)*3^(1/2)/e-2/15*3^(1/2)*
(-e*x+2)^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}} dx = -\frac{2\sqrt{4-e^2x^2}(172+28ex+3e^2x^2)}{15e\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(5/2)/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-2*Sqrt[4 - e^2*x^2]*(172 + 28*e*x + 3*e^2*x^2))/(15*e*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{5/2}}{\sqrt{12-3e^2x^2}}dx$$

$$\Big\downarrow \ 456$$

$$\int \frac{(ex+2)^2}{\sqrt{6-3ex}}dx$$

$$\Big\downarrow \ 53$$

$$\int \left(\frac{1}{9}(6-3ex)^{3/2}-\frac{8}{3}\sqrt{6-3ex}+\frac{16}{\sqrt{6-3ex}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2(2-ex)^{5/2}}{5\sqrt{3}e}+\frac{16(2-ex)^{3/2}}{3\sqrt{3}e}-\frac{32\sqrt{2-ex}}{\sqrt{3}e}$$

input
```
Int[(2 + e*x)^(5/2)/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-32*Sqrt[2 - e*x])/(Sqrt[3]*e) + (16*(2 - e*x)^(3/2))/(3*Sqrt[3]*e) - (2*
(2 - e*x)^(5/2))/(5*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{-3e^{2}x^{2}+12}\,\left(3e^{2}x^{2}+28ex+172\right)}{45\sqrt{ex+2}\,e}$	39
gosper	$\frac{2(ex-2)\left(3e^{2}x^{2}+28ex+172\right)\sqrt{ex+2}}{15e\sqrt{-3e^{2}x^{2}+12}}$	44
orering	$\frac{2(ex-2)\left(3e^{2}x^{2}+28ex+172\right)\sqrt{ex+2}}{15e\sqrt{-3e^{2}x^{2}+12}}$	44
risch	$\frac{2\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,\left(3e^{2}x^{2}+28ex+172\right)(ex-2)}{15\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	72

input
```
int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/(e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2)*(3*e^2*x^2+28*e*x+172)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx=-\frac{2\,(3\,e^{2}x^{2}+28\,ex+172)\sqrt{-3\,e^{2}x^{2}+12}\sqrt{ex+2}}{45\,(e^{2}x+2\,e)}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(3*e^2*x^2 + 28*e*x + 172)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*
x + 2*e)
```

Sympy [F]

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\left(\int \frac{4\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx + \int \frac{4ex\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx + \int \frac{e^2x^2\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx\right)}{3}$$

input
```
integrate((e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*(Integral(4*sqrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x) + Integral(4*e*
x*sqrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x) + Integral(e**2*x**2*sqrt(e*x + 2
)/sqrt(-e**2*x**2 + 4), x))/3
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx = \frac{2\left(-3i\sqrt{3}e^3x^3 - 22i\sqrt{3}e^2x^2 - 116i\sqrt{3}ex + 344i\sqrt{3}\right)}{45\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
2/45*(-3*I*sqrt(3)*e^3*x^3 - 22*I*sqrt(3)*e^2*x^2 - 116*I*sqrt(3)*e*x + 34
4*I*sqrt(3))/(sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\sqrt{3}\left(3\,(ex-2)^2\sqrt{-ex+2}-40\,(-ex+2)^{\frac{3}{2}}+240\,\sqrt{-ex+2}\right)}{45\,e}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
-2/45*sqrt(3)*(3*(e*x - 2)^2*sqrt(-e*x + 2) - 40*(-e*x + 2)^(3/2) + 240*sq
rt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{\sqrt{12-3\,e^2\,x^2}\,\left(\frac{344\,\sqrt{e\,x+2}}{45\,e^2}+\frac{2\,x^2\,\sqrt{e\,x+2}}{15}+\frac{56\,x\,\sqrt{e\,x+2}}{45\,e}\right)}{x+\frac{2}{e}}$$

input
```
int((e*x + 2)^(5/2)/(12 - 3*e^2*x^2)^(1/2),x)
```

output
```
-((12 - 3*e^2*x^2)^(1/2)*((344*(e*x + 2)^(1/2))/(45*e^2) + (2*x^2*(e*x + 2
)^(1/2))/15 + (56*x*(e*x + 2)^(1/2))/(45*e)))/(x + 2/e)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.43

$$\int \frac{(2+ex)^{5/2}}{\sqrt{12-3e^2x^2}}\,dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(-3e^2x^2-28ex-172)}{45e}$$

input
```
int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*( - 3*e**2*x**2 - 28*e*x - 172))/(45*e)
```

3.225 $\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{8\sqrt{2-ex}}{\sqrt{3}e}+\frac{2(2-ex)^{3/2}}{3\sqrt{3}e}$$

output
```
-8/3*3^(1/2)*(-e*x+2)^(1/2)/e+2/9*(-e*x+2)^(3/2)*3^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2(10+ex)\sqrt{4-e^2x^2}}{3e\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(3/2)/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-2*(10 + e*x)*Sqrt[4 - e^2*x^2])/(3*e*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{3/2}}{\sqrt{12-3e^2x^2}}dx$$
$$\Big\downarrow \text{ 456}$$
$$\int \frac{ex+2}{\sqrt{6-3ex}}dx$$
$$\Big\downarrow \text{ 53}$$
$$\int \left(\frac{4}{\sqrt{6-3ex}}-\frac{1}{3}\sqrt{6-3ex}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2(2-ex)^{3/2}}{3\sqrt{3}e}-\frac{8\sqrt{2-ex}}{\sqrt{3}e}$$

input
```
Int[(2 + e*x)^(3/2)/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-8*Sqrt[2 - e*x])/(Sqrt[3]*e) + (2*(2 - e*x)^(3/2))/(3*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{2\sqrt{-3e^{2}x^{2}+12}\,(ex+10)}{9\sqrt{ex+2}\,e}$	30
gosper	$\frac{2(ex-2)(ex+10)\sqrt{ex+2}}{3e\sqrt{-3e^{2}x^{2}+12}}$	35
orering	$\frac{2(ex-2)(ex+10)\sqrt{ex+2}}{3e\sqrt{-3e^{2}x^{2}+12}}$	35
risch	$\frac{2\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}\,(ex+10)(ex-2)}{3\sqrt{-3e^{2}x^{2}+12}\,e\sqrt{-3ex+6}}$	63

input
```
int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/(e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2)*(e*x+10)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\,\sqrt{-3\,e^2x^2+12}(ex+10)\sqrt{ex+2}}{9\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(-3*e^2*x^2 + 12)*(e*x + 10)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\left(\int \frac{2\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx + \int \frac{ex\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx\right)}{3}$$

input
```
integrate((e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*(Integral(2*sqrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x) + Integral(e*x*
sqrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x))/3
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2i\sqrt{3}(e^2x^2+8\,ex-20)}{9\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
-2/9*I*sqrt(3)*(e^2*x^2 + 8*e*x - 20)/(sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = \frac{2\sqrt{3}\left((-ex+2)^{\frac{3}{2}} - 12\sqrt{-ex+2}\right)}{9\,e}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*((-e*x + 2)^(3/2) - 12*sqrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{\left(\frac{20\,\sqrt{e\,x+2}}{9\,e^2}+\frac{2\,x\,\sqrt{e\,x+2}}{9\,e}\right)\,\sqrt{12-3\,e^2\,x^2}}{x+\frac{2}{e}}$$

input
```
int((e*x + 2)^(3/2)/(12 - 3*e^2*x^2)^(1/2),x)
```

output
```
-(((20*(e*x + 2)^(1/2))/(9*e^2) + (2*x*(e*x + 2)^(1/2))/(9*e))*(12 - 3*e^2
*x^2)^(1/2))/(x + 2/e)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.47

$$\int \frac{(2+ex)^{3/2}}{\sqrt{12-3e^2x^2}}\,dx = \frac{2\sqrt{-ex+2}\,\sqrt{3}\,(-ex-10)}{9e}$$

input
```
int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(2*sqrt( - e*x + 2)*sqrt(3)*( - e*x - 10))/(9*e)
```

3.226 $\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\sqrt{2-ex}}{\sqrt{3}e}$$

output
```
-2/3*3^(1/2)*(-e*x+2)^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\sqrt{4-e^2x^2}}{e\sqrt{6+3ex}}$$

input
```
Integrate[Sqrt[2 + e*x]/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-2*Sqrt[4 - e^2*x^2])/(e*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex+2}}{\sqrt{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{\sqrt{6-3ex}}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{2\sqrt{2-ex}}{\sqrt{3}e}$$

input
```
Int[Sqrt[2 + e*x]/Sqrt[12 - 3*e^2*x^2],x]
```

output
```
(-2*Sqrt[2 - e*x])/(Sqrt[3]*e)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{2\sqrt{-3e^{2}x^{2}+12}}{3\sqrt{ex+2}\,e}$	25
gosper	$\frac{2(ex-2)\sqrt{ex+2}}{e\sqrt{-3e^{2}x^{2}+12}}$	30
orering	$\frac{2(ex-2)\sqrt{ex+2}}{e\sqrt{-3e^{2}x^{2}+12}}$	30
risch	$\frac{2(ex-2)\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}}{e\sqrt{-3ex+6}\,\sqrt{-3e^{2}x^{2}+12}}$	58

input
```
int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx=-\frac{2\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{3\,(e^2x+2\,e)}$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\int \frac{\sqrt{ex+2}}{\sqrt{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate((e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*Integral(sqrt(e*x + 2)/sqrt(-e**2*x**2 + 4), x)/3
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx = \frac{2\left(-i\sqrt{3}ex+2i\sqrt{3}\right)}{3\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(-I*sqrt(3)*e*x + 2*I*sqrt(3))/(sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\sqrt{3}\sqrt{-ex+2}}{3\,e}$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*sqrt(-e*x + 2)/e
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\,\sqrt{12-3\,e^2\,x^2}}{3\,e\,\sqrt{e\,x+2}}$$

input
```
int((e*x + 2)^(1/2)/(12 - 3*e^2*x^2)^(1/2),x)
```

output
```
-(2*(12 - 3*e^2*x^2)^(1/2))/(3*e*(e*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{2+ex}}{\sqrt{12-3e^2x^2}}\,dx = -\frac{2\sqrt{-ex+2}\,\sqrt{3}}{3e}$$

input
```
int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x)
```

output
```
( - 2*sqrt( - e*x + 2)*sqrt(3))/(3*e)
```

3.227 $\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{\sqrt{3}e}$$

output
```
-1/3*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = \frac{\log\left(-2\sqrt{2+ex}+\sqrt{4-e^2x^2}\right)-\log\left(e\left(2\sqrt{2+ex}+\sqrt{4-e^2x^2}\right)\right)}{2\sqrt{3}e}$$

input
```
Integrate[1/(Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2]),x]
```

output
```
(Log[-2*Sqrt[2 + e*x] + Sqrt[4 - e^2*x^2]] - Log[e*(2*Sqrt[2 + e*x] + Sqrt
[4 - e^2*x^2])])/(2*Sqrt[3]*e)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{456, 73, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex+2}\sqrt{12-3e^2x^2}}dx$$
$$\Big\downarrow \; 456$$
$$\int \frac{1}{\sqrt{6-3ex}(ex+2)}dx$$
$$\Big\downarrow \; 73$$
$$-\frac{2\int \frac{1}{\frac{1}{3}(3ex-6)+4}d\sqrt{6-3ex}}{3e}$$
$$\Big\downarrow \; 219$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{6-3ex}}{2\sqrt{3}}\right)}{\sqrt{3}e}$$

input
```
Int[1/(Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2]),x]
```

output
```
-(ArcTanh[Sqrt[6 - 3*e*x]/(2*Sqrt[3])]/(Sqrt[3]*e))
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(19) = 38$.

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

method	result	size
default	$-\frac{\sqrt{-e^{2} x^{2}+4}\, \sqrt{3}\, \operatorname{arctanh}\left(\frac{\sqrt{-3 e x +6}\, \sqrt{3}}{6}\right)}{3 \sqrt{e x +2}\, \sqrt{-e x +2}\, e}$	50

input
```
int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-e^2*x^2+4)^(1/2)*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))/(e*x
+2)^(1/2)/(-e*x+2)^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)}{6\,e}$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqr
t(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4))/e
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\int \frac{1}{\sqrt{ex+2}\sqrt{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate(1/(e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*Integral(1/(sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4)), x)/3
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = \int \frac{1}{\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}\,dx$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = -\frac{\sqrt{3}\left(\log\left(\sqrt{-ex+2}+2\right)-\log\left(-\sqrt{-ex+2}+2\right)\right)}{6\,e}$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*(log(sqrt(-e*x + 2) + 2) - log(-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = \int \frac{1}{\sqrt{12-3\,e^2\,x^2}\,\sqrt{e\,x+2}}\,dx$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(1/2)),x)
```

output
```
int(1/((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{2+ex}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\left(\log\left(\sqrt{-ex+2}-2\right)-\log\left(\sqrt{-ex+2}+2\right)\right)}{6e}$$

input
```
int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(sqrt(3)*(log(sqrt( - e*x + 2) - 2) - log(sqrt( - e*x + 2) + 2)))/(6*e)
```

3.228 $\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 57

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = -\frac{\sqrt{2-ex}}{4\sqrt{3}e(2+ex)} - \frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8\sqrt{3}e}$$

output
```
-1/12*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+2)-1/24*3^(1/2)*arctanh(1/2*(-e*x+2)^(
1/2))/e
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = -\frac{2\sqrt{4-e^2x^2}+(2+ex)^{3/2}\operatorname{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)}{8\sqrt{3}e(2+ex)^{3/2}}$$

input
```
Integrate[1/((2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

output
```
-1/8*(2*Sqrt[4 - e^2*x^2] + (2 + e*x)^(3/2)*ArcTanh[(2*Sqrt[2 + e*x])/Sqrt
[4 - e^2*x^2]])/(Sqrt[3]*e*(2 + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {456, 52, 27, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex+2)^{3/2}\sqrt{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{\sqrt{6-3ex}(ex+2)^2}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{8}\int \frac{1}{\sqrt{3}\sqrt{2-ex}(ex+2)}dx-\frac{\sqrt{2-ex}}{4\sqrt{3}e(ex+2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{2-ex}(ex+2)}dx}{8\sqrt{3}}-\frac{\sqrt{2-ex}}{4\sqrt{3}e(ex+2)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{ex+2}d\sqrt{2-ex}}{4\sqrt{3}e}-\frac{\sqrt{2-ex}}{4\sqrt{3}e(ex+2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8\sqrt{3}e}-\frac{\sqrt{2-ex}}{4\sqrt{3}e(ex+2)}$$

input
```
Int[1/((2 + e*x)^(3/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

output
```
-1/4*Sqrt[2 - e*x]/(Sqrt[3]*e*(2 + e*x)) - ArcTanh[Sqrt[2 - e*x]/2]/(8*Sqr
t[3]*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

method	result	size
default	$-\frac{\sqrt{-e^{2} x^{2}+4}\,\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3 e x +6}\,\sqrt{3}}{6}\right) e x +2 \sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3 e x +6}\,\sqrt{3}}{6}\right)+2 \sqrt{-3 e x +6}\right) \sqrt{3}}{24 \left(e x +2\right)^{\frac{3}{2}} \sqrt{-3 e x +6}\, e}$	86

input
```
int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-e^2*x^2+4)^(1/2)*(3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e*
x+2*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))+2*(-3*e*x+6)^(1/2))/(e*x
+2)^(3/2)/(-3*e*x+6)^(1/2)*3^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(43) = 86$.

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}(e^2x^2+4\,ex+4)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-4\,\sqrt{-3\,e}}{48\,(e^3x^2+4\,e^2x+4\,e)}$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(sqrt(3)*(e^2*x^2 + 4*e*x + 4)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*s
qrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*sqrt(-
3*e^2*x^2 + 12)*sqrt(e*x + 2))/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F]

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\int \frac{1}{ex\sqrt{ex+2}\sqrt{-e^2x^2+4}+2\sqrt{ex+2}\sqrt{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate(1/(e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*Integral(1/(e*x*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4) + 2*sqrt(e*x +
2)*sqrt(-e**2*x**2 + 4)), x)/3
```

Maxima [F]

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = \int \frac{1}{\sqrt{-3\,e^2x^2+12}(ex+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-3*e^2*x^2 + 12)*(e*x + 2)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = -\frac{\sqrt{3}\left(\frac{4\sqrt{-ex+2}}{ex+2}+\log\left(\sqrt{-ex+2}+2\right)-\log\left(-\sqrt{-ex+2}+2\right)\right)}{48\,e}$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
-1/48*sqrt(3)*(4*sqrt(-e*x + 2)/(e*x + 2) + log(sqrt(-e*x + 2) + 2) - log(
-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = \int \frac{1}{\sqrt{12-3\,e^2\,x^2}\,{\left(e\,x+2\right)}^{3/2}}\,dx$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(3/2)),x)
```

output
```
int(1/((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{1}{(2+ex)^{3/2}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\left(-4\sqrt{-ex+2}+\log\left(\sqrt{-ex+2}-2\right)ex+2\log\left(\sqrt{-ex+2}-2\right)-\mathrm{lo}\right.}{48e\,(ex+2)}$$

input
```
int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(sqrt(3)*( - 4*sqrt( - e*x + 2) + log(sqrt( - e*x + 2) - 2)*e*x + 2*log(sq
rt( - e*x + 2) - 2) - log(sqrt( - e*x + 2) + 2)*e*x - 2*log(sqrt( - e*x +
2) + 2)))/(48*e*(e*x + 2))
```

3.229 $\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx=-\frac{\sqrt{2-ex}}{8\sqrt{3}e(2+ex)^2}-\frac{\sqrt{3}\sqrt{2-ex}}{64e(2+ex)}-\frac{\sqrt{3}\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{128e}$$

output
```
-1/24*3^(1/2)*(-e*x+2)^(1/2)/e/(e*x+2)^2-1/64*3^(1/2)*(-e*x+2)^(1/2)/e/(e*
x+2)-1/128*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx=\frac{-2(14+3ex)\sqrt{4-e^2x^2}-3(2+ex)^{5/2}\text{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)}{128\sqrt{3}e(2+ex)^{5/2}}$$

input
```
Integrate[1/((2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

output
```
(-2*(14 + 3*e*x)*Sqrt[4 - e^2*x^2] - 3*(2 + e*x)^(5/2)*ArcTanh[(2*Sqrt[2 +
 e*x])/Sqrt[4 - e^2*x^2]])/(128*Sqrt[3]*e*(2 + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {456, 52, 27, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex+2)^{5/2}\sqrt{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{1}{\sqrt{6-3ex}(ex+2)^3}dx$$

$$\Big\downarrow \text{ 52}$$

$$\frac{3}{16}\int \frac{1}{\sqrt{3}\sqrt{2-ex}(ex+2)^2}dx-\frac{\sqrt{2-ex}}{8\sqrt{3}e(ex+2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{16}\sqrt{3}\int \frac{1}{\sqrt{2-ex}(ex+2)^2}dx-\frac{\sqrt{2-ex}}{8\sqrt{3}e(ex+2)^2}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{16}\sqrt{3}\left(\frac{1}{8}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{2-ex}}{8\sqrt{3}e(ex+2)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{16}\sqrt{3}\left(-\frac{\int \frac{1}{ex+2}d\sqrt{2-ex}}{4e}-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{2-ex}}{8\sqrt{3}e(ex+2)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{16}\sqrt{3}\left(-\frac{\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{8e}-\frac{\sqrt{2-ex}}{4e(ex+2)}\right)-\frac{\sqrt{2-ex}}{8\sqrt{3}e(ex+2)^2}$$

input
```
Int[1/((2 + e*x)^(5/2)*Sqrt[12 - 3*e^2*x^2]),x]
```

output
```
-1/8*Sqrt[2 - e*x]/(Sqrt[3]*e*(2 + e*x)^2) + (Sqrt[3]*(-1/4*Sqrt[2 - e*x]/
(e*(2 + e*x)) - ArcTanh[Sqrt[2 - e*x]/2]/(8*e)))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

method	result
default	$-\frac{\sqrt{-e^2x^2+4}\left(3\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)e^2x^2+12\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)ex+6ex\sqrt{-3ex+6}+12\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\right.}{384(ex+2)^{\frac{5}{2}}\sqrt{-3ex+6}\,e}$

input
```
int(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(-e^2*x^2+4)^(1/2)*(3*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))
*e^2*x^2+12*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))*e*x+6*e*x*(-3*e*
x+6)^(1/2)+12*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)*3^(1/2))+28*(-3*e*x+6)^
(1/2))/(e*x+2)^(5/2)/(-3*e*x+6)^(1/2)*3^(1/2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(66) = 132.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.62

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx = \frac{3\,\sqrt{3}(e^3x^3+6\,e^2x^2+12\,ex+8)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right.}{768\,(e^4x^3+6\,e^3x^2+12\,e^2x+8\,e)}$$

input
```
integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="fricas")
```

output
```
1/768*(3*sqrt(3)*(e^3*x^3 + 6*e^2*x^2 + 12*e*x + 8)*log(-(3*e^2*x^2 - 12*e
*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x
+ 4)) - 4*sqrt(-3*e^2*x^2 + 12)*(3*e*x + 14)*sqrt(e*x + 2))/(e^4*x^3 + 6*e
^3*x^2 + 12*e^2*x + 8*e)
```

Sympy [F]

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\int \frac{1}{e^2x^2\sqrt{ex+2}\sqrt{-e^2x^2+4}+4ex\sqrt{ex+2}\sqrt{-e^2x^2+4}+4\sqrt{ex+2}\sqrt{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate(1/(e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/2),x)
```

output
```
sqrt(3)*Integral(1/(e**2*x**2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4) + 4*e*x*s
qrt(e*x + 2)*sqrt(-e**2*x**2 + 4) + 4*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4)),
 x)/3
```

Maxima [F]

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx = \int \frac{1}{\sqrt{-3\,e^2x^2+12}(ex+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-3*e^2*x^2 + 12)*(e*x + 2)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx = \frac{\sqrt{3}\left(\frac{4\left(3(-ex+2)^{\frac{3}{2}}-20\sqrt{-ex+2}\right)}{(ex+2)^2} - 3\log\left(\sqrt{-ex+2}+2\right) + 3\log\left(-\sqrt{-ex+\right.\right.}{768\,e}$$

input
```
integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x, algorithm="giac")
```

output
```
1/768*sqrt(3)*(4*(3*(-e*x + 2)^(3/2) - 20*sqrt(-e*x + 2))/(e*x + 2)^2 - 3*
log(sqrt(-e*x + 2) + 2) + 3*log(-sqrt(-e*x + 2) + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx=\int \frac{1}{\sqrt{12-3\,e^2\,x^2}\,{\left(e\,x+2\right)}^{5/2}}\,dx$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(5/2)),x)
```

output
```
int(1/((12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.52

$$\int \frac{1}{(2+ex)^{5/2}\sqrt{12-3e^2x^2}}\,dx=\frac{\sqrt{3}\left(-12\sqrt{-ex+2}\,ex-56\sqrt{-ex+2}+3\log\left(\sqrt{-ex+2}-2\right)e^2x^2+12\right.}{}$$

input
```
int(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/2),x)
```

output
```
(sqrt(3)*( - 12*sqrt( - e*x + 2)*e*x - 56*sqrt( - e*x + 2) + 3*log(sqrt( -
 e*x + 2) - 2)*e**2*x**2 + 12*log(sqrt( - e*x + 2) - 2)*e*x + 12*log(sqrt(
 - e*x + 2) - 2) - 3*log(sqrt( - e*x + 2) + 2)*e**2*x**2 - 12*log(sqrt( -
e*x + 2) + 2)*e*x - 12*log(sqrt( - e*x + 2) + 2)))/(768*e*(e**2*x**2 + 4*e
*x + 4))
```

3.230 $\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{512}{3\sqrt{3}e\sqrt{2-ex}}+\frac{512\sqrt{2-ex}}{3\sqrt{3}e} - \frac{64(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{32(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{2(2-ex)^{7/2}}{21\sqrt{3}e}$$

output
```
512/9*3^(1/2)/e/(-e*x+2)^(1/2)+512/9*3^(1/2)*(-e*x+2)^(1/2)/e-64/9*(-e*x+2
)^(3/2)*3^(1/2)/e+32/45*3^(1/2)*(-e*x+2)^(5/2)/e-2/63*3^(1/2)*(-e*x+2)^(7/
2)/e
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{4-e^2x^2}(-23216+5664ex+568e^2x^2+72e^3x^3+5e^4x^4)}{105e(-2+ex)\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(11/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(2*Sqrt[4 - e^2*x^2]*(-23216 + 5664*e*x + 568*e^2*x^2 + 72*e^3*x^3 + 5*e^4
*x^4))/(105*e*(-2 + e*x)*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{(ex+2)^4}{(6-3ex)^{3/2}}dx$$

$$\Big\downarrow \; 53$$

$$\int \left(\frac{1}{81}(6-3ex)^{5/2}-\frac{16}{27}(6-3ex)^{3/2}+\frac{32}{3}\sqrt{6-3ex}-\frac{256}{3\sqrt{6-3ex}}+\frac{256}{(6-3ex)^{3/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2(2-ex)^{7/2}}{21\sqrt{3}e}+\frac{32(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{64(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{512\sqrt{2-ex}}{3\sqrt{3}e}+\frac{512}{3\sqrt{3}e\sqrt{2-ex}}$$

input
```
Int[(2 + e*x)^(11/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
512/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (512*Sqrt[2 - e*x])/(3*Sqrt[3]*e) - (64*
(2 - e*x)^(3/2))/(3*Sqrt[3]*e) + (32*(2 - e*x)^(5/2))/(15*Sqrt[3]*e) - (2*
(2 - e*x)^(7/2))/(21*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.54

method	result	size
gosper	$\frac{2(ex-2)\left(5e^{4}x^{4}+72e^{3}x^{3}+568e^{2}x^{2}+5664ex-23216\right)(ex+2)^{\frac{3}{2}}}{35e\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}$	60
orering	$\frac{2(ex-2)\left(5e^{4}x^{4}+72e^{3}x^{3}+568e^{2}x^{2}+5664ex-23216\right)(ex+2)^{\frac{3}{2}}}{35e\left(-3e^{2}x^{2}+12\right)^{\frac{3}{2}}}$	60
default	$\frac{2\sqrt{-3e^{2}x^{2}+12}\,\left(5e^{4}x^{4}+72e^{3}x^{3}+568e^{2}x^{2}+5664ex-23216\right)}{315\sqrt{ex+2}\,(ex-2)e}$	62
risch	$-\frac{2\left(5e^{3}x^{3}+82e^{2}x^{2}+732ex+7128\right)(ex-2)\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}}{105e\sqrt{-3ex+6}\,\sqrt{-3e^{2}x^{2}+12}}+\frac{512\sqrt{\frac{-3e^{2}x^{2}+12}{ex+2}}\,\sqrt{ex+2}}{3e\sqrt{-3ex+6}\,\sqrt{-3e^{2}x^{2}+12}}$	133

input
```
int((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(e*x-2)*(5*e^4*x^4+72*e^3*x^3+568*e^2*x^2+5664*e*x-23216)*(e*x+2)^(3/
2)/e/(-3*e^2*x^2+12)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\,(5\,e^4x^4+72\,e^3x^3+568\,e^2x^2+5664\,ex-23216)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{315\,(e^3x^2-4\,e)}$$

input
```
integrate((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
2/315*(5*e^4*x^4 + 72*e^3*x^3 + 568*e^2*x^2 + 5664*e*x - 23216)*sqrt(-3*e^
2*x^2 + 12)*sqrt(e*x + 2)/(e^3*x^2 - 4*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+2)**(11/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.52

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{2\left(-5i\,\sqrt{3}e^4x^4-72i\,\sqrt{3}e^3x^3-568i\,\sqrt{3}e^2x^2-5664i\,\sqrt{3}ex+23216i\,\sqrt{3}\right)}{315\,\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
-2/315*(-5*I*sqrt(3)*e^4*x^4 - 72*I*sqrt(3)*e^3*x^3 - 568*I*sqrt(3)*e^2*x^
2 - 5664*I*sqrt(3)*e*x + 23216*I*sqrt(3))/(sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\left(5\sqrt{3}(ex-2)^3\sqrt{-ex+2}+112\sqrt{3}(ex-2)^2\sqrt{-ex+2}-1120\sqrt{3}(-ex+2)^{\frac{3}{2}}-\right.}{315\,e}$$

input
```
integrate((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
2/315*(5*sqrt(3)*(e*x - 2)^3*sqrt(-e*x + 2) + 112*sqrt(3)*(e*x - 2)^2*sqrt
(-e*x + 2) - 1120*sqrt(3)*(-e*x + 2)^(3/2) + 8960*sqrt(3)*sqrt(-e*x + 2) +
 8960*sqrt(3)/sqrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{\sqrt{12-3\,e^2\,x^2}\,\left(\frac{16\,x^3\,\sqrt{e\,x+2}}{35}-\frac{46432\,\sqrt{e\,x+2}}{315\,e^3}+\frac{3776\,x\,\sqrt{e\,x+2}}{105\,e^2}+\frac{2\,e\,x^4\,\sqrt{e\,x+2}}{63}+\frac{1136\,x^2\,\sqrt{e\,x+2}}{315\,e}\right)}{\frac{4}{e^2}-x^2}$$

input
```
int((e*x + 2)^(11/2)/(12 - 3*e^2*x^2)^(3/2),x)
```

output
```
-((12 - 3*e^2*x^2)^(1/2)*((16*x^3*(e*x + 2)^(1/2))/35 - (46432*(e*x + 2)^(
1/2))/(315*e^3) + (3776*x*(e*x + 2)^(1/2))/(105*e^2) + (2*e*x^4*(e*x + 2)^
(1/2))/63 + (1136*x^2*(e*x + 2)^(1/2))/(315*e)))/(4/e^2 - x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int \frac{(2+ex)^{11/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{3}\,(-5e^4x^4-72e^3x^3-568e^2x^2-5664ex+23216)}{315\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(11/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(2*sqrt(3)*( - 5*e**4*x**4 - 72*e**3*x**3 - 568*e**2*x**2 - 5664*e*x + 232
16))/(315*sqrt( - e*x + 2)*e)
```

3.231 $\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx=\frac{128}{3\sqrt{3}e\sqrt{2-ex}}+\frac{32\sqrt{2-ex}}{\sqrt{3}e}-\frac{8(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{2(2-ex)^{5/2}}{15\sqrt{3}e}$$

output
```
128/9*3^(1/2)/e/(-e*x+2)^(1/2)+32/3*3^(1/2)*(-e*x+2)^(1/2)/e-8/9*(-e*x+2)^
(3/2)*3^(1/2)/e+2/45*3^(1/2)*(-e*x+2)^(5/2)/e
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx=\frac{2\sqrt{4-e^2x^2}(-728+172ex+14e^2x^2+e^3x^3)}{15e(-2+ex)\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(9/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(2*Sqrt[4 - e^2*x^2]*(-728 + 172*e*x + 14*e^2*x^2 + e^3*x^3))/(15*e*(-2 +
e*x)*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{9/2}}{(12-3e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{(ex+2)^3}{(6-3ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(-\frac{1}{27}(6-3ex)^{3/2}+\frac{4}{3}\sqrt{6-3ex}-\frac{16}{\sqrt{6-3ex}}+\frac{64}{(6-3ex)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(2-ex)^{5/2}}{15\sqrt{3}e}-\frac{8(2-ex)^{3/2}}{3\sqrt{3}e}+\frac{32\sqrt{2-ex}}{\sqrt{3}e}+\frac{128}{3\sqrt{3}e\sqrt{2-ex}}$$

input
```
Int[(2 + e*x)^(9/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
128/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (32*Sqrt[2 - e*x])/(Sqrt[3]*e) - (8*(2 -
 e*x)^(3/2))/(3*Sqrt[3]*e) + (2*(2 - e*x)^(5/2))/(15*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{2(ex-2)\left(e^{3} x^{3}+14 e^{2} x^{2}+172 e x -728\right) \left(e x +2\right)^{\frac{3}{2}}}{5 e \left(-3 e^{2} x^{2}+12\right)^{\frac{3}{2}}}$	51
orering	$\frac{2(ex-2)\left(e^{3} x^{3}+14 e^{2} x^{2}+172 e x -728\right) \left(e x +2\right)^{\frac{3}{2}}}{5 e \left(-3 e^{2} x^{2}+12\right)^{\frac{3}{2}}}$	51
default	$\frac{2 \sqrt{-3 e^{2} x^{2}+12}\, \left(e^{3} x^{3}+14 e^{2} x^{2}+172 e x -728\right)}{45 \sqrt{e x +2}\, \left(e x -2\right) e}$	53
risch	$-\frac{2 \left(e^{2} x^{2}+16 e x +204\right) \left(e x -2\right) \sqrt{\frac{-3 e^{2} x^{2}+12}{e x +2}}\, \sqrt{e x +2}}{15 e \sqrt{-3 e x +6}\, \sqrt{-3 e^{2} x^{2}+12}}+\frac{128 \sqrt{\frac{-3 e^{2} x^{2}+12}{e x +2}}\, \sqrt{e x +2}}{3 e \sqrt{-3 e x +6}\, \sqrt{-3 e^{2} x^{2}+12}}$	124

input
```
int((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(e*x-2)*(e^3*x^3+14*e^2*x^2+172*e*x-728)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+1
2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\left(e^3x^3+14\,e^2x^2+172\,ex-728\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{45\left(e^3x^2-4\,e\right)}$$

input
```
integrate((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
2/45*(e^3*x^3 + 14*e^2*x^2 + 172*e*x - 728)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x
 + 2)/(e^3*x^2 - 4*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+2)**(9/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2i\,\sqrt{3}(e^3x^3+14\,e^2x^2+172\,ex-728)}{45\,\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
2/45*I*sqrt(3)*(e^3*x^3 + 14*e^2*x^2 + 172*e*x - 728)/(sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\left(\sqrt{3}(ex-2)^2\sqrt{-ex+2}-20\sqrt{3}(-ex+2)^{\frac{3}{2}}+240\sqrt{3}\sqrt{-ex+2}+\frac{320\sqrt{3}}{\sqrt{-ex+2}}\right)}{45\,e}$$

input
```
integrate((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
2/45*(sqrt(3)*(e*x - 2)^2*sqrt(-e*x + 2) - 20*sqrt(3)*(-e*x + 2)^(3/2) + 2
40*sqrt(3)*sqrt(-e*x + 2) + 320*sqrt(3)/sqrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{\sqrt{12-3\,e^2\,x^2}\left(\frac{2\,x^3\,\sqrt{e\,x+2}}{45}-\frac{1456\,\sqrt{e\,x+2}}{45\,e^3}+\frac{344\,x\,\sqrt{e\,x+2}}{45\,e^2}+\frac{28\,x^2\,\sqrt{e\,x+2}}{45\,e}\right)}{\frac{4}{e^2}-x^2}$$

input
```
int((e*x + 2)^(9/2)/(12 - 3*e^2*x^2)^(3/2),x)
```

output
```
-((12 - 3*e^2*x^2)^(1/2)*((2*x^3*(e*x + 2)^(1/2))/45 - (1456*(e*x + 2)^(1/
2))/(45*e^3) + (344*x*(e*x + 2)^(1/2))/(45*e^2) + (28*x^2*(e*x + 2)^(1/2))
/(45*e)))/(4/e^2 - x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{(2+ex)^{9/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{3}\,(-e^3x^3-14e^2x^2-172ex+728)}{45\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(9/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output

```
(2*sqrt(3)*( - e**3*x**3 - 14*e**2*x**2 - 172*e*x + 728))/(45*sqrt( - e*x
+ 2)*e)
```

3.232 $\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{32}{3\sqrt{3}e\sqrt{2-ex}} + \frac{16\sqrt{2-ex}}{3\sqrt{3}e} - \frac{2(2-ex)^{3/2}}{9\sqrt{3}e}$$

output
```
32/9*3^(1/2)/e/(-e*x+2)^(1/2)+16/9*3^(1/2)*(-e*x+2)^(1/2)/e-2/27*(-e*x+2)^
(3/2)*3^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{4-e^2x^2}(-92+20ex+e^2x^2)}{9e(-2+ex)\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(7/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(2*Sqrt[4 - e^2*x^2]*(-92 + 20*e*x + e^2*x^2))/(9*e*(-2 + e*x)*Sqrt[6 + 3*
e*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{7/2}}{(12-3e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(ex+2)^2}{(6-3ex)^{3/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{9}\sqrt{6-3ex}-\frac{8}{3\sqrt{6-3ex}}+\frac{16}{(6-3ex)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(2-ex)^{3/2}}{9\sqrt{3}e}+\frac{16\sqrt{2-ex}}{3\sqrt{3}e}+\frac{32}{3\sqrt{3}e\sqrt{2-ex}}$$

input
```
Int[(2 + e*x)^(7/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
32/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (16*Sqrt[2 - e*x])/(3*Sqrt[3]*e) - (2*(2
- e*x)^(3/2))/(9*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2(ex-2)\left(e^{2} x^{2}+20 e x -92\right)(e x +2)^{\frac{3}{2}}}{3 e \left(-3 e^{2} x^{2}+12\right)^{\frac{3}{2}}}$	43
orering	$\frac{2(ex-2)\left(e^{2} x^{2}+20 e x -92\right)(e x +2)^{\frac{3}{2}}}{3 e \left(-3 e^{2} x^{2}+12\right)^{\frac{3}{2}}}$	43
default	$\frac{2 \sqrt{-3 e^{2} x^{2}+12}\, \left(e^{2} x^{2}+20 e x -92\right)}{27 \sqrt{e x +2}\, \left(e x -2\right) e}$	45
risch	$-\frac{2(e x +22)(e x -2) \sqrt{\frac{-3 e^{2} x^{2}+12}{e x +2}}\, \sqrt{e x +2}}{9 e \sqrt{-3 e x +6}\, \sqrt{-3 e^{2} x^{2}+12}}+\frac{32 \sqrt{\frac{-3 e^{2} x^{2}+12}{e x +2}}\, \sqrt{e x +2}}{3 e \sqrt{-3 e x +6}\, \sqrt{-3 e^{2} x^{2}+12}}$	116

input
```
int((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(e*x-2)*(e^2*x^2+20*e*x-92)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\left(e^2x^2+20\,ex-92\right)\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{27\left(e^3x^2-4\,e\right)}$$

input
```
integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
2/27*(e^2*x^2 + 20*e*x - 92)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^3*x^2
- 4*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+2)**(7/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.54

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{2\left(-i\sqrt{3}e^2x^2-20i\sqrt{3}ex+92i\sqrt{3}\right)}{27\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
-2/27*(-I*sqrt(3)*e^2*x^2 - 20*I*sqrt(3)*e*x + 92*I*sqrt(3))/(sqrt(e*x - 2
)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{2\left(\sqrt{3}(-ex+2)^{\frac{3}{2}} - 24\sqrt{3}\sqrt{-ex+2} - \frac{48\sqrt{3}}{\sqrt{-ex+2}}\right)}{27e}$$

input
```
integrate((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
-2/27*(sqrt(3)*(-e*x + 2)^(3/2) - 24*sqrt(3)*sqrt(-e*x + 2) - 48*sqrt(3)/s
qrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{12-3e^2x^2}\sqrt{ex+2}\,(e^2x^2+20ex-92)}{27e\,(e^2x^2-4)}$$

input
```
int((e*x + 2)^(7/2)/(12 - 3*e^2*x^2)^(3/2),x)
```

output
```
(2*(12 - 3*e^2*x^2)^(1/2)*(e*x + 2)^(1/2)*(20*e*x + e^2*x^2 - 92))/(27*e*(
e^2*x^2 - 4))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

$$\int \frac{(2+ex)^{7/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{3}\,(-e^2x^2-20ex+92)}{27\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(7/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(2*sqrt(3)*( - e**2*x**2 - 20*e*x + 92))/(27*sqrt( - e*x + 2)*e)
```

3.233 $\int \frac{(2+ex)^{5/2}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{(2+ex)^{5/2}}{(12-3e^2x^2)^{3/2}}\,dx=\frac{8}{3\sqrt{3}e\sqrt{2-ex}}+\frac{2\sqrt{2-ex}}{3\sqrt{3}e}$$

output
```
8/9*3^(1/2)/e/(-e*x+2)^(1/2)+2/9*3^(1/2)*(-e*x+2)^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{(2+ex)^{5/2}}{(12-3e^2x^2)^{3/2}}\,dx=\frac{2(-6+ex)\sqrt{4-e^2x^2}}{3e(-2+ex)\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(2*(-6 + e*x)*Sqrt[4 - e^2*x^2])/(3*e*(-2 + e*x)*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{5/2}}{(12-3e^2x^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{ex+2}{(6-3ex)^{3/2}}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{4}{(6-3ex)^{3/2}}-\frac{1}{3\sqrt{6-3ex}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{2-ex}}{3\sqrt{3}e}+\frac{8}{3\sqrt{3}e\sqrt{2-ex}}$$

input
```
Int[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
8/(3*Sqrt[3]*e*Sqrt[2 - e*x]) + (2*Sqrt[2 - e*x])/(3*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2(ex-2)(ex-6)(ex+2)^{\frac{3}{2}}}{e(-3e^2x^2+12)^{\frac{3}{2}}}$	35
orering	$\frac{2(ex-2)(ex-6)(ex+2)^{\frac{3}{2}}}{e(-3e^2x^2+12)^{\frac{3}{2}}}$	35
default	$\frac{2\sqrt{-3e^2x^2+12}\,(ex-6)}{9\sqrt{ex+2}\,(ex-2)e}$	37
risch	$-\frac{2(ex-2)\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}}{3e\sqrt{-3ex+6}\,\sqrt{-3e^2x^2+12}}+\frac{8\sqrt{\frac{-3e^2x^2+12}{ex+2}}\,\sqrt{ex+2}}{3e\sqrt{-3ex+6}\,\sqrt{-3e^2x^2+12}}$	111

input
```
int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(e*x-2)*(e*x-6)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{(2+ex)^{5/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}(ex-6)}{9\,(e^3x^2-4\,e)}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)*(e*x - 6)/(e^3*x^2 - 4*e)
```

Sympy [F]

$$\int \frac{(2+ex)^{5/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{\sqrt{3}\left(\int \frac{4\sqrt{ex+2}}{-e^2x^2\sqrt{-e^2x^2+4}+4\sqrt{-e^2x^2+4}}\,dx + \int \frac{4ex\sqrt{ex+2}}{-e^2x^2\sqrt{-e^2x^2+4}+4\sqrt{-e^2x^2+4}}\,dx + \int \frac{e^2x^2}{-e^2x^2\sqrt{-e^2x^2}}\right.}{9}$$

input
```
integrate((e*x+2)**(5/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
sqrt(3)*(Integral(4*sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e**2*x**2 + 4) + 4*sqr
t(-e**2*x**2 + 4)), x) + Integral(4*e*x*sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e*
*2*x**2 + 4) + 4*sqrt(-e**2*x**2 + 4)), x) + Integral(e**2*x**2*sqrt(e*x +
 2)/(-e**2*x**2*sqrt(-e**2*x**2 + 4) + 4*sqrt(-e**2*x**2 + 4)), x))/9
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

$$\int \frac{(2+ex)^{5/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2i\sqrt{3}(ex-6)}{9\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
2/9*I*sqrt(3)*(e*x - 6)/(sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{(2+ex)^{5/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx = \frac{2\left(\sqrt{3}\sqrt{-ex+2}+\frac{4\sqrt{3}}{\sqrt{-ex+2}}\right)}{9\,e}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
2/9*(sqrt(3)*sqrt(-e*x + 2) + 4*sqrt(3)/sqrt(-e*x + 2))/e
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{(2+ex)^{5/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx = \frac{\left(\frac{4\sqrt{e\,x+2}}{3\,e^3}-\frac{2\,x\,\sqrt{e\,x+2}}{9\,e^2}\right)\,\sqrt{12-3\,e^2\,x^2}}{\frac{4}{e^2}-x^2}$$

input
```
int((e*x + 2)^(5/2)/(12 - 3*e^2*x^2)^(3/2),x)
```

output
```
(((4*(e*x + 2)^(1/2))/(3*e^3) - (2*x*(e*x + 2)^(1/2))/(9*e^2))*(12 - 3*e^2
*x^2)^(1/2))/(4/e^2 - x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{(2+ex)^{5/2}}{\left(12-3e^2x^2\right)^{3/2}}\,dx = \frac{2\sqrt{3}\,(-ex+6)}{9\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(2*sqrt(3)*( - e*x + 6))/(9*sqrt( - e*x + 2)*e)
```

3.234 $\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 22

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2}{3\sqrt{3}e\sqrt{2-ex}}$$

output
```
2/9*3^(1/2)/e/(-e*x+2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{2\sqrt{4-e^2x^2}}{3e(-2+ex)\sqrt{6+3ex}}$$

input
```
Integrate[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[4 - e^2*x^2])/(3*e*(-2 + e*x)*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{3/2}}{(12-3e^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(6-3ex)^{3/2}}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{2}{3\sqrt{3}e\sqrt{2-ex}}$$

input
```
Int[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
2/(3*Sqrt[3]*e*Sqrt[2 - e*x])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

method	result	size
gosper	$-\frac{2(ex-2)(ex+2)^{\frac{3}{2}}}{e(-3e^2x^2+12)^{\frac{3}{2}}}$	30
orering	$-\frac{2(ex-2)(ex+2)^{\frac{3}{2}}}{e(-3e^2x^2+12)^{\frac{3}{2}}}$	30
default	$-\frac{2\sqrt{-3e^2x^2+12}}{9\sqrt{ex+2}\,(ex-2)e}$	32

input
```
int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(e*x-2)*(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{2\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}}{9\,(e^3x^2-4\,e)}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2)/(e^3*x^2 - 4*e)
```

Sympy [F]

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{\sqrt{3}\left(\int \frac{2\sqrt{ex+2}}{-e^2x^2\sqrt{-e^2x^2+4}+4\sqrt{-e^2x^2+4}}\,dx + \int \frac{ex\sqrt{ex+2}}{-e^2x^2\sqrt{-e^2x^2+4}+4\sqrt{-e^2x^2+4}}\,dx\right)}{9}$$

input
```
integrate((e*x+2)**(3/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
sqrt(3)*(Integral(2*sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e**2*x**2 + 4) + 4*sqr
t(-e**2*x**2 + 4)), x) + Integral(e*x*sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e**2
*x**2 + 4) + 4*sqrt(-e**2*x**2 + 4)), x))/9
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{2i\sqrt{3}}{9\sqrt{ex-2}e}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
-2/9*I*sqrt(3)/(sqrt(e*x - 2)*e)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{3}}{9\sqrt{-ex+2}e}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
2/9*sqrt(3)/(sqrt(-e*x + 2)*e)
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{e\,x+2}}{3\,e\,\sqrt{12-3\,e^2\,x^2}}$$

input
```
int((e*x + 2)^(3/2)/(12 - 3*e^2*x^2)^(3/2),x)
```

output
```
(2*(e*x + 2)^(1/2))/(3*e*(12 - 3*e^2*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{(2+ex)^{3/2}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{2\sqrt{3}}{9\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(2*sqrt(3))/(9*sqrt( - e*x + 2)*e)
```

3.235 $\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx=\frac{1}{6\sqrt{3}e\sqrt{2-ex}}-\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{12\sqrt{3}e}$$

output
```
1/18*3^(1/2)/e/(-e*x+2)^(1/2)-1/36*3^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx=\frac{-2\sqrt{4-e^2x^2}-(-2+ex)\sqrt{2+ex}\operatorname{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)}{12e(-2+ex)\sqrt{6+3ex}}$$

input
```
Integrate[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[4 - e^2*x^2] - (-2 + e*x)*Sqrt[2 + e*x]*ArcTanh[(2*Sqrt[2 + e*x])
/Sqrt[4 - e^2*x^2]])/(12*e*(-2 + e*x)*Sqrt[6 + 3*e*x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {456, 61, 27, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex+2}}{(12-3e^2x^2)^{3/2}}dx$$
$$\Big\downarrow \text{456}$$
$$\int \frac{1}{(6-3ex)^{3/2}(ex+2)}dx$$
$$\Big\downarrow \text{61}$$
$$\frac{1}{12}\int \frac{1}{\sqrt{3}\sqrt{2-ex}(ex+2)}dx+\frac{1}{6\sqrt{3}e\sqrt{2-ex}}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{\sqrt{2-ex}(ex+2)}dx}{12\sqrt{3}}+\frac{1}{6\sqrt{3}e\sqrt{2-ex}}$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{6\sqrt{3}e\sqrt{2-ex}}-\frac{\int \frac{1}{ex+2}d\sqrt{2-ex}}{6\sqrt{3}e}$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{6\sqrt{3}e\sqrt{2-ex}}-\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{12\sqrt{3}e}$$

input
```
Int[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(3/2),x]
```

output
```
1/(6*Sqrt[3]*e*Sqrt[2 - e*x]) - ArcTanh[Sqrt[2 - e*x]/2]/(12*Sqrt[3]*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{\sqrt{-3e^{2}x^{2}+12}\,\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\sqrt{-3ex+6}-6\right)}{108\sqrt{ex+2}\,(ex-2)e}$	60

input
```
int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/108/(e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/2)*(3^(1/2)*arctanh(1/6*(-3*e*x+6)^
(1/2)*3^(1/2))*(-3*e*x+6)^(1/2)-6)/(e*x-2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{\sqrt{3}(e^2x^2-4)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)-4\,\sqrt{-3\,e^2x^2+12}\sqrt{ex+}}{72\,(e^3x^2-4\,e)}$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
1/72*(sqrt(3)*(e^2*x^2 - 4)*log(-(3*e^2*x^2 - 12*e*x + 4*sqrt(3)*sqrt(-3*e
^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x + 4)) - 4*sqrt(-3*e^2*x^
2 + 12)*sqrt(e*x + 2))/(e^3*x^2 - 4*e)
```

Sympy [F]

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{\sqrt{3}\int \frac{\sqrt{ex+2}}{-e^2x^2\sqrt{-e^2x^2+4}+4\sqrt{-e^2x^2+4}}\,dx}{9}$$

input
```
integrate((e*x+2)**(1/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
sqrt(3)*Integral(sqrt(e*x + 2)/(-e**2*x**2*sqrt(-e**2*x**2 + 4) + 4*sqrt(-
e**2*x**2 + 4)), x)/9
```

Maxima [F]

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+2}}{(-3\,e^2x^2+12)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + 2)/(-3*e^2*x^2 + 12)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx = -\frac{\sqrt{3}\log\left(\sqrt{-ex+2}+2\right) - \sqrt{3}\log\left(-\sqrt{-ex+2}+2\right) - \frac{4\sqrt{3}}{\sqrt{-ex+2}}}{72\,e}$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
-1/72*(sqrt(3)*log(sqrt(-e*x + 2) + 2) - sqrt(3)*log(-sqrt(-e*x + 2) + 2)
- 4*sqrt(3)/sqrt(-e*x + 2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx = \int \frac{\sqrt{e\,x+2}}{(12-3\,e^2\,x^2)^{3/2}}\,dx$$

input
```
int((e*x + 2)^(1/2)/(12 - 3*e^2*x^2)^(3/2),x)
```

output
```
int((e*x + 2)^(1/2)/(12 - 3*e^2*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{2+ex}}{(12-3e^2x^2)^{3/2}}\,dx = \frac{\sqrt{3}\left(\sqrt{-ex+2}\,\log\left(\sqrt{-ex+2}-2\right)-\sqrt{-ex+2}\,\log\left(\sqrt{-ex+2}+2\right)+4\right)}{72\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(sqrt(3)*(sqrt( - e*x + 2)*log(sqrt( - e*x + 2) - 2) - sqrt( - e*x + 2)*lo
g(sqrt( - e*x + 2) + 2) + 4))/(72*sqrt( - e*x + 2)*e)
```

3.236 $\int \frac{1}{\sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{1}{\sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}}\,dx=\frac{1}{16\sqrt{3}e\sqrt{2-ex}}-\frac{1}{12\sqrt{3}e\sqrt{2-ex}(2+ex)}-\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{32\sqrt{3}e}$$

output
```
1/48*3^(1/2)/e/(-e*x+2)^(1/2)-1/36*3^(1/2)/e/(-e*x+2)^(1/2)/(e*x+2)-1/96*3
^(1/2)*arctanh(1/2*(-e*x+2)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{2+ex}\left(12-3e^2x^2\right)^{3/2}}\,dx=\frac{4+6ex-3\sqrt{2+ex}\sqrt{4-e^2x^2}\operatorname{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)}{96e\sqrt{2+ex}\sqrt{12-3e^2x^2}}$$

input
```
Integrate[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2)),x]
```

output
```
(4 + 6*e*x - 3*Sqrt[2 + e*x]*Sqrt[4 - e^2*x^2]*ArcTanh[(2*Sqrt[2 + e*x])/S
qrt[4 - e^2*x^2]])/(96*e*Sqrt[2 + e*x]*Sqrt[12 - 3*e^2*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {456, 52, 27, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex+2}\left(12-3e^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{1}{(6-3ex)^{3/2}(ex+2)^2}dx$$

$$\Big\downarrow \text{ 52}$$

$$\frac{3}{8}\int \frac{1}{3\sqrt{3}(2-ex)^{3/2}(ex+2)}dx-\frac{1}{12\sqrt{3}e\sqrt{2-ex}(ex+2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{(2-ex)^{3/2}(ex+2)}dx}{8\sqrt{3}}-\frac{1}{12\sqrt{3}e\sqrt{2-ex}(ex+2)}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{\frac{1}{4}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx+\frac{1}{2e\sqrt{2-ex}}}{8\sqrt{3}}-\frac{1}{12\sqrt{3}e\sqrt{2-ex}(ex+2)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{1}{2e\sqrt{2-ex}}-\frac{\int \frac{1}{ex+2}d\sqrt{2-ex}}{2e}}{8\sqrt{3}}-\frac{1}{12\sqrt{3}e\sqrt{2-ex}(ex+2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2e\sqrt{2-ex}}-\frac{\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{4e}}{8\sqrt{3}}-\frac{1}{12\sqrt{3}e\sqrt{2-ex}(ex+2)}$$

input
```
Int[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(3/2)),x]
```

output
```
-1/12*1/(Sqrt[3]*e*Sqrt[2 - e*x]*(2 + e*x)) + (1/(2*e*Sqrt[2 - e*x]) - Arc
Tanh[Sqrt[2 - e*x]/2]/(4*e))/(8*Sqrt[3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\sqrt{-3e^{2}x^{2}+12}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\sqrt{-3ex+6}\,ex+2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\sqrt{-3ex+6}-6ex-4\right)}{288(ex+2)^{\frac{3}{2}}(ex-2)e}$	93

input
```
int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/288/(e*x+2)^(3/2)*(-3*e^2*x^2+12)^(1/2)*(3^(1/2)*arctanh(1/6*(-3*e*x+6)^
(1/2)*3^(1/2))*(-3*e*x+6)^(1/2)*e*x+2*3^(1/2)*arctanh(1/6*(-3*e*x+6)^(1/2)
*3^(1/2))*(-3*e*x+6)^(1/2)-6*e*x-4)/(e*x-2)/e
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(59) = 118$.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{2+ex}\,(12-3e^2x^2)^{3/2}}\,dx=\frac{3\sqrt{3}(e^3x^3+2\,e^2x^2-4\,ex-8)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex+2}-36}{e^2x^2+4\,ex+4}\right)}{576\,(e^4x^3+2\,e^3x^2-4\,e^2x-8\,e)}$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
1/576*(3*sqrt(3)*(e^3*x^3 + 2*e^2*x^2 - 4*e*x - 8)*log(-(3*e^2*x^2 - 12*e*
x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e*x +
 4)) - 4*sqrt(-3*e^2*x^2 + 12)*(3*e*x + 2)*sqrt(e*x + 2))/(e^4*x^3 + 2*e^3
*x^2 - 4*e^2*x - 8*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+ex}\,(12-3e^2x^2)^{3/2}}\,dx = \frac{\sqrt{3}\int \frac{1}{-e^2x^2\sqrt{ex+2}\sqrt{-e^2x^2+4}+4\sqrt{ex+2}\sqrt{-e^2x^2+4}}\,dx}{9}$$

input
```
integrate(1/(e*x+2)**(1/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
sqrt(3)*Integral(1/(-e**2*x**2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4) + 4*sqrt
(e*x + 2)*sqrt(-e**2*x**2 + 4)), x)/9
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+ex}\,(12-3e^2x^2)^{3/2}}\,dx = \int \frac{1}{(-3\,e^2x^2+12)^{\frac{3}{2}}\sqrt{ex+2}}\,dx$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-3*e^2*x^2 + 12)^(3/2)*sqrt(e*x + 2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{2+ex}\,(12-3e^2x^2)^{3/2}}\,dx = -\frac{3\sqrt{3}\log\left(\sqrt{-ex+2}+2\right)-3\sqrt{3}\log\left(-\sqrt{-ex+2}+2\right)+\frac{4\sqrt{3}(3\,ex+2)}{(-ex+2)^{\frac{3}{2}}-4\sqrt{-ex+2}}}{576\,e}$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
-1/576*(3*sqrt(3)*log(sqrt(-e*x + 2) + 2) - 3*sqrt(3)*log(-sqrt(-e*x + 2)
+ 2) + 4*sqrt(3)*(3*e*x + 2)/((-e*x + 2)^(3/2) - 4*sqrt(-e*x + 2)))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+ex}\,(12-3e^2x^2)^{3/2}}\,dx = \int \frac{1}{(12-3\,e^2\,x^2)^{3/2}\,\sqrt{e\,x+2}}\,dx$$

input
```
int(1/((12 - 3*e^2*x^2)^(3/2)*(e*x + 2)^(1/2)),x)
```

output
```
int(1/((12 - 3*e^2*x^2)^(3/2)*(e*x + 2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{2+ex}\,(12-3e^2x^2)^{3/2}}\,dx = \frac{\sqrt{3}\,\left(3\sqrt{-ex+2}\log\left(\sqrt{-ex+2}-2\right)ex+6\sqrt{-ex+2}\log\left(\sqrt{-ex+2}-2\right.\right.}{576\sqrt{}}$$

input
```
int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(sqrt(3)*(3*sqrt( - e*x + 2)*log(sqrt( - e*x + 2) - 2)*e*x + 6*sqrt( - e*x
 + 2)*log(sqrt( - e*x + 2) - 2) - 3*sqrt( - e*x + 2)*log(sqrt( - e*x + 2)
+ 2)*e*x - 6*sqrt( - e*x + 2)*log(sqrt( - e*x + 2) + 2) + 12*e*x + 8))/(57
6*sqrt( - e*x + 2)*e*(e*x + 2))
```

3.237 $\int \frac{1}{(2+ex)^{3/2}(12-3e^2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{1}{(2+ex)^{3/2}(12-3e^2x^2)^{3/2}}\,dx = \frac{5}{256\sqrt{3}e\sqrt{2-ex}} - \frac{1}{24\sqrt{3}e\sqrt{2-ex}(2+ex)^2} - \frac{5}{192\sqrt{3}e\sqrt{2-ex}(2+ex)} - \frac{5\text{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{512\sqrt{3}e}$$

output
```
5/768*3^(1/2)/e/(-e*x+2)^(1/2)-1/72*3^(1/2)/e/(-e*x+2)^(1/2)/(e*x+2)^2-5/5
76*3^(1/2)/e/(-e*x+2)^(1/2)/(e*x+2)-5/1536*3^(1/2)*arctanh(1/2*(-e*x+2)^(1
/2))/e
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{1}{(2+ex)^{3/2}(12-3e^2x^2)^{3/2}}\,dx = \frac{-\frac{2\sqrt{4-e^2x^2}(-12+40ex+15e^2x^2)}{(-2+ex)(2+ex)^{5/2}} - 15\text{arctanh}\left(\frac{2\sqrt{2+ex}}{\sqrt{4-e^2x^2}}\right)}{1536\sqrt{3}e}$$

input
```
Integrate[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2)),x]
```

output
```
((-2*Sqrt[4 - e^2*x^2]*(-12 + 40*e*x + 15*e^2*x^2))/((-2 + e*x)*(2 + e*x)^
(5/2)) - 15*ArcTanh[(2*Sqrt[2 + e*x])/Sqrt[4 - e^2*x^2]])/(1536*Sqrt[3]*e)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {456, 52, 27, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex+2)^{3/2}\,(12-3e^2x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{(6-3ex)^{3/2}(ex+2)^3}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{5}{16}\int \frac{1}{3\sqrt{3}(2-ex)^{3/2}(ex+2)^2}dx-\frac{1}{24\sqrt{3}e\sqrt{2-ex}(ex+2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{1}{(2-ex)^{3/2}(ex+2)^2}dx}{48\sqrt{3}}-\frac{1}{24\sqrt{3}e\sqrt{2-ex}(ex+2)^2}$$

$$\Big\downarrow \text{52}$$

$$\frac{5\left(\frac{3}{8}\int \frac{1}{(2-ex)^{3/2}(ex+2)}dx-\frac{1}{4e\sqrt{2-ex}(ex+2)}\right)}{48\sqrt{3}}-\frac{1}{24\sqrt{3}e\sqrt{2-ex}(ex+2)^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{5\left(\frac{3}{8}\left(\frac{1}{4}\int \frac{1}{\sqrt{2-ex}(ex+2)}dx+\frac{1}{2e\sqrt{2-ex}}\right)-\frac{1}{4e\sqrt{2-ex}(ex+2)}\right)}{48\sqrt{3}}-\frac{1}{24\sqrt{3}e\sqrt{2-ex}(ex+2)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{5\left(\frac{3}{8}\left(\frac{1}{2e\sqrt{2-ex}}-\frac{\int\frac{1}{ex+2}d\sqrt{2-ex}}{2e}\right)-\frac{1}{4e\sqrt{2-ex}(ex+2)}\right)}{48\sqrt{3}}-\frac{1}{24\sqrt{3}e\sqrt{2-ex}(ex+2)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{3}{8}\left(\frac{1}{2e\sqrt{2-ex}}-\frac{\operatorname{arctanh}\left(\frac{1}{2}\sqrt{2-ex}\right)}{4e}\right)-\frac{1}{4e\sqrt{2-ex}(ex+2)}\right)}{48\sqrt{3}}-\frac{1}{24\sqrt{3}e\sqrt{2-ex}(ex+2)^2}$$

input
```
Int[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(3/2)),x]
```

output
```
-1/24*1/(Sqrt[3]*e*Sqrt[2 - e*x]*(2 + e*x)^2) + (5*(-1/4*1/(e*Sqrt[2 - e*x
]*(2 + e*x)) + (3*(1/(2*e*Sqrt[2 - e*x]) - ArcTanh[Sqrt[2 - e*x]/2]/(4*e))
)/8))/(48*Sqrt[3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

method	result
default	$\frac{\sqrt{-3e^2x^2+12}\left(5\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\sqrt{-3ex+6}\,e^2x^2+20\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3ex+6}\,\sqrt{3}}{6}\right)\sqrt{-3ex+6}\,ex-30e^2x^2+20\sqrt{3}\,\operatorname{arctanh}\left(\right.\right.}{4608(ex+2)^{\frac{5}{2}}(ex-2)e}$

input
```
int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4608/(e*x+2)^(5/2)*(-3*e^2*x^2+12)^(1/2)*(5*3^(1/2)*arctanh(1/6*(-3*e*x+
6)^(1/2)*3^(1/2))*(-3*e*x+6)^(1/2)*e^2*x^2+20*3^(1/2)*arctanh(1/6*(-3*e*x+
6)^(1/2)*3^(1/2))*(-3*e*x+6)^(1/2)*e*x-30*e^2*x^2+20*3^(1/2)*arctanh(1/6*(
-3*e*x+6)^(1/2)*3^(1/2))*(-3*e*x+6)^(1/2)-80*e*x+24)/(e*x-2)/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.36

$$\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx = \frac{15\sqrt{3}(e^4x^4+4\,e^3x^3-16\,ex-16)\log\left(-\frac{3\,e^2x^2-12\,ex+4\,\sqrt{3}\sqrt{-3\,e^2x^2+12}\sqrt{ex-}}{e^2x^2+4\,ex+4}\right.}{9216\left(e^5x^4+4\,e^4x^3-16\right.}$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="fricas")
```

output
```
1/9216*(15*sqrt(3)*(e^4*x^4 + 4*e^3*x^3 - 16*e*x - 16)*log(-(3*e^2*x^2 - 1
2*e*x + 4*sqrt(3)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2) - 36)/(e^2*x^2 + 4*e
*x + 4)) - 4*(15*e^2*x^2 + 40*e*x - 12)*sqrt(-3*e^2*x^2 + 12)*sqrt(e*x + 2
))/(e^5*x^4 + 4*e^4*x^3 - 16*e^2*x - 16*e)
```

Sympy [F]

$$\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{3}\int \frac{1}{-e^3x^3\sqrt{ex+2}\sqrt{-e^2x^2+4}-2e^2x^2\sqrt{ex+2}\sqrt{-e^2x^2+4}+4ex\sqrt{ex+2}\sqrt{-e^2x^2+4}+8\sqrt{ex+2}\sqrt{}}}{9}$$

input
```
integrate(1/(e*x+2)**(3/2)/(-3*e**2*x**2+12)**(3/2),x)
```

output
```
sqrt(3)*Integral(1/(-e**3*x**3*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4) - 2*e**2
*x**2*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4) + 4*e*x*sqrt(e*x + 2)*sqrt(-e**2*
x**2 + 4) + 8*sqrt(e*x + 2)*sqrt(-e**2*x**2 + 4)), x)/9
```

Maxima [F]

$$\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{3}{2}}(ex+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-3*e^2*x^2 + 12)^(3/2)*(e*x + 2)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx = -\frac{15\sqrt{3}\log\left(\sqrt{-ex+2}+2\right)-15\sqrt{3}\log\left(-\sqrt{-ex+2}+2\right)-\frac{32\sqrt{3}}{\sqrt{-ex+2}}-\frac{4\left(7\sqrt{3}(-ex+2)^{\frac{3}{2}}-36\sqrt{3}\sqrt{-ex+2}\right)}{(ex+2)^2}}{9216\,e}$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x, algorithm="giac")
```

output
```
-1/9216*(15*sqrt(3)*log(sqrt(-e*x + 2) + 2) - 15*sqrt(3)*log(-sqrt(-e*x +
2) + 2) - 32*sqrt(3)/sqrt(-e*x + 2) - 4*(7*sqrt(3)*(-e*x + 2)^(3/2) - 36*s
qrt(3)*sqrt(-e*x + 2))/(e*x + 2)^2)/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx = \int \frac{1}{\left(12-3\,e^2\,x^2\right)^{3/2}\,\left(e\,x+2\right)^{3/2}}\,dx$$

input
```
int(1/((12 - 3*e^2*x^2)^(3/2)*(e*x + 2)^(3/2)),x)
```

output
```
int(1/((12 - 3*e^2*x^2)^(3/2)*(e*x + 2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

$$\int \frac{1}{(2+ex)^{3/2}\left(12-3e^2x^2\right)^{3/2}}\,dx = \frac{\sqrt{3}\left(15\sqrt{-ex+2}\log\left(\sqrt{-ex+2}-2\right)e^2x^2+60\sqrt{-ex+2}\log\left(\sqrt{-ex+\right.\right.}{}$$

input
```
int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(3/2),x)
```

output
```
(sqrt(3)*(15*sqrt( - e*x + 2)*log(sqrt( - e*x + 2) - 2)*e**2*x**2 + 60*sqr
t( - e*x + 2)*log(sqrt( - e*x + 2) - 2)*e*x + 60*sqrt( - e*x + 2)*log(sqrt
( - e*x + 2) - 2) - 15*sqrt( - e*x + 2)*log(sqrt( - e*x + 2) + 2)*e**2*x**
2 - 60*sqrt( - e*x + 2)*log(sqrt( - e*x + 2) + 2)*e*x - 60*sqrt( - e*x + 2
)*log(sqrt( - e*x + 2) + 2) + 60*e**2*x**2 + 160*e*x - 48))/(9216*sqrt( -
e*x + 2)*e*(e**2*x**2 + 4*e*x + 4))
```

3.238 $\int \frac{1}{\sqrt{1-x}(1+x)} dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{\sqrt{1-x}(1+x)} dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

output
```
-2^(1/2)*arctanh(1/2*(1-x)^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x}(1+x)} dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

input
```
Integrate[1/(Sqrt[1 - x]*(1 + x)),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}(x+1)}\,dx$$
$$\Big\downarrow \text{73}$$
$$-2\int \frac{1}{x+1}d\sqrt{1-x}$$
$$\Big\downarrow \text{219}$$
$$-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

input
```
Int[1/(Sqrt[1 - x]*(1 + x)),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1-x}\,\sqrt{2}}{2}\right)$	19
default	$-\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1-x}\,\sqrt{2}}{2}\right)$	19
pseudoelliptic	$-\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{1-x}\,\sqrt{2}}{2}\right)$	19
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right) \ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2-2\right) x+4\sqrt{1-x}+3\operatorname{RootOf}\left(\_Z^2-2\right)}{x+1}\right)}{2}$	43

input
```
int(1/(1-x)^(1/2)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)*arctanh(1/2*(1-x)^(1/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{1-x}(1+x)}\, dx = \frac{1}{2}\,\sqrt{2}\log\left(\frac{x+2\,\sqrt{2}\sqrt{-x+1}-3}{x+1}\right)$$

input
```
integrate(1/(1-x)^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((x + 2*sqrt(2)*sqrt(-x + 1) - 3)/(x + 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1-x}(1+x)}\,dx = \begin{cases} -\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{for}\ \frac{1}{|x+1|} > \frac{1}{2} \\ \sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-x)**(1/2)/(1+x),x)
```

output
```
Piecewise((-sqrt(2)*acosh(sqrt(2)/sqrt(x + 1)), 1/Abs(x + 1) > 1/2), (sqrt
(2)*I*asin(sqrt(2)/sqrt(x + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{1-x}(1+x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{2}+\sqrt{-x+1}}\right)$$

input
```
integrate(1/(1-x)^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(sqrt(2) - sqrt(-x + 1))/(sqrt(2) + sqrt(-x + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{1-x}(1+x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x+1}\right) + \frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{2}+\sqrt{-x+1}\right|\right)$$

input
```
integrate(1/(1-x)^(1/2)/(1+x),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(sqrt(2) + sqrt(-x + 1)) + 1/2*sqrt(2)*log(abs(-sqrt(2) +
sqrt(-x + 1)))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1-x}(1+x)}\,dx = -\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\,\sqrt{1-x}}{2}\right)$$

input
```
int(1/((1 - x)^(1/2)*(x + 1)),x)
```

output
```
-2^(1/2)*atanh((2^(1/2)*(1 - x)^(1/2))/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{1-x}(1+x)}\,dx = \frac{\sqrt{2}\left(\log\left(\sqrt{1-x}-\sqrt{2}\right)-\log\left(\sqrt{1-x}+\sqrt{2}\right)\right)}{2}$$

input
```
int(1/(1-x)^(1/2)/(1+x),x)
```

output
```
(sqrt(2)*(log(sqrt( - x + 1) - sqrt(2)) - log(sqrt( - x + 1) + sqrt(2))))/
2
```

3.239 $\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}} \, dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

output
```
-2^(1/2)*arctanh(1/2*(1-x)^(1/2)*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}} \, dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{1-x^2}}\right)$$

input
```
Integrate[1/(Sqrt[1 + x]*Sqrt[1 - x^2]),x]
```

output
```
-(Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[1 + x])/Sqrt[1 - x^2]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {456, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 456}$$

$$\int \frac{1}{\sqrt{1-x}(x+1)}dx$$

$$\Big\downarrow \text{ 73}$$

$$-2\int \frac{1}{x+1}d\sqrt{1-x}$$

$$\Big\downarrow \text{ 219}$$

$$-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

input
```
Int[1/(Sqrt[1 + x]*Sqrt[1 - x^2]),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[1 - x]/Sqrt[2]])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(18) = 36$.

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

method	result	size
default	$-\frac{\sqrt{-x^2+1}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{1-x}\,\sqrt{2}}{2}\right)}{\sqrt{x+1}\,\sqrt{1-x}}$	40

input
```
int(1/(x+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(x+1)^(1/2)*(-x^2+1)^(1/2)/(1-x)^(1/2)*2^(1/2)*arctanh(1/2*(1-x)^(1/2)*
2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx = \frac{1}{2}\,\sqrt{2}\log\left(-\frac{x^2+2\,\sqrt{2}\sqrt{-x^2+1}\sqrt{x+1}-2\,x-3}{x^2+2\,x+1}\right)$$

input
```
integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(x + 1) - 2*x - 3)/(x
^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1)*sqrt(x + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x+1}\right)+\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}-\sqrt{-x+1}\right)$$

input
```
integrate(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(sqrt(2) + sqrt(-x + 1)) + 1/2*sqrt(2)*log(sqrt(2) - sqrt(
-x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{x+1}}\,dx$$

input
```
int(1/((1 - x^2)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int(1/((1 - x^2)^(1/2)*(x + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{1+x}\sqrt{1-x^2}}\,dx = \frac{\sqrt{2}\left(\log\left(\sqrt{1-x}-\sqrt{2}\right)-\log\left(\sqrt{1-x}+\sqrt{2}\right)\right)}{2}$$

input
```
int(1/(1+x)^(1/2)/(-x^2+1)^(1/2),x)
```

output
```
(sqrt(2)*(log(sqrt( - x + 1) - sqrt(2)) - log(sqrt( - x + 1) + sqrt(2))))/
2
```

3.240 $\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

output
```
-2^(1/2)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

input
```
Integrate[1/(Sqrt[1 - a*x]*(1 + a*x)),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-ax}(ax+1)}dx$$
$$\Big\downarrow \text{73}$$
$$-\frac{2\int \frac{1}{ax+1}d\sqrt{1-ax}}{a}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

input
```
Int[1/(Sqrt[1 - a*x]*(1 + a*x)),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-ax+1}\, \sqrt{2}}{2}\right)}{a}$	23
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-ax+1}\, \sqrt{2}}{2}\right)}{a}$	23
pseudoelliptic	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-ax+1}\, \sqrt{2}}{2}\right)}{a}$	23

input
```
int(1/(-a*x+1)^(1/2)/(a*x+1),x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = \frac{\sqrt{2}\log\left(\frac{ax+2\,\sqrt{2}\sqrt{-ax+1}-3}{ax+1}\right)}{2\,a}$$

input
```
integrate(1/(-a*x+1)^(1/2)/(a*x+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((a*x + 2*sqrt(2)*sqrt(-a*x + 1) - 3)/(a*x + 1))/a
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = \begin{cases} \frac{\sqrt{2}\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{2a} & \text{for } a\neq 0 \\ \begin{cases} x & \text{for } a=0 \\ \frac{\log(ax+1)}{a} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-a*x+1)**(1/2)/(a*x+1),x)
```

output
```
Piecewise((sqrt(2)*(log(sqrt(-a*x + 1) - sqrt(2)) - log(sqrt(-a*x + 1) + s
qrt(2)))/(2*a), Ne(a, 0)), (Piecewise((x, Eq(a, 0)), (log(a*x + 1)/a, True
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = \frac{\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{-ax+1}}{\sqrt{2}+\sqrt{-ax+1}}\right)}{2\,a}$$

input
```
integrate(1/(-a*x+1)^(1/2)/(a*x+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*log(-(sqrt(2) - sqrt(-a*x + 1))/(sqrt(2) + sqrt(-a*x + 1)))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{-ax+1}\right|}{2\left(\sqrt{2}+\sqrt{-ax+1}\right)}\right)}{2\,a}$$

input
```
integrate(1/(-a*x+1)^(1/2)/(a*x+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(-a*x + 1))/(sqrt(2) + sqrt(-a*
x + 1)))/a
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2-2\,a\,x}}{2}\right)}{a}$$

input
```
int(1/((1 - a*x)^(1/2)*(a*x + 1)),x)
```

output
```
-(2^(1/2)*atanh((2 - 2*a*x)^(1/2)/2))/a
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{1-ax}(1+ax)}\,dx = \frac{\sqrt{2}\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{2a}$$

input
```
int(1/(-a*x+1)^(1/2)/(a*x+1),x)
```

output
```
(sqrt(2)*(log(sqrt( - a*x + 1) - sqrt(2)) - log(sqrt( - a*x + 1) + sqrt(2)
)))/(2*a)
```

3.241 $\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

output
```
-2^(1/2)*arctanh(1/2*(-a*x+1)^(1/2)*2^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2+2ax}}{\sqrt{1-a^2x^2}}\right)}{a}$$

input
```
Integrate[1/(Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[2 + 2*a*x]/Sqrt[1 - a^2*x^2]])/a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {456, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+1}\sqrt{1-a^2x^2}}dx$$

$$\Big\downarrow \; 456$$

$$\int \frac{1}{\sqrt{1-ax}(ax+1)}dx$$

$$\Big\downarrow \; 73$$

$$-\frac{2\int \frac{1}{ax+1}d\sqrt{1-ax}}{a}$$

$$\Big\downarrow \; 219$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{a}$$

input
```
Int[1/(Sqrt[1 + a*x]*Sqrt[1 - a^2*x^2]),x]
```

output
```
-((Sqrt[2]*ArcTanh[Sqrt[1 - a*x]/Sqrt[2]])/a)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(22) = 44$.

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

method	result	size
default	$-\frac{\sqrt{-a^{2} x^{2}+1}\, \sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{-a x+1}\, \sqrt{2}}{2}\right)}{\sqrt{a x+1}\, \sqrt{-a x+1}\, a}$	50

input
```
int(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*x+1)^(1/2)*(-a^2*x^2+1)^(1/2)/(-a*x+1)^(1/2)*2^(1/2)*arctanh(1/2*(-a
*x+1)^(1/2)*2^(1/2))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(22) = 44$.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\, dx = \frac{\sqrt{2} \log\left(-\frac{a^{2} x^{2} - 2 \, a x + 2 \, \sqrt{2} \sqrt{-a^{2} x^{2} + 1} \sqrt{a x + 1} - 3}{a^{2} x^{2} + 2 \, a x + 1}\right)}{2 \, a}$$

input
```
integrate(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(a^2*x^2 - 2*a*x + 2*sqrt(2)*sqrt(-a^2*x^2 + 1)*sqrt(a*x
+ 1) - 3)/(a^2*x^2 + 2*a*x + 1))/a
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{-(ax-1)(ax+1)}\sqrt{ax+1}}\,dx$$

input
```
integrate(1/(a*x+1)**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*sqrt(a*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\sqrt{ax+1}}\,dx$$

input
```
integrate(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(a*x + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{2}\log\left(\sqrt{2}+\sqrt{-ax+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-ax+1}\right)}{2\,a}$$

input
```
integrate(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*log(sqrt(2) + sqrt(-a*x + 1)) - sqrt(2)*log(sqrt(2) - sqrt(-
a*x + 1)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{1-a^2\,x^2}\,\sqrt{a\,x+1}}\,dx$$

input
```
int(1/((1 - a^2*x^2)^(1/2)*(a*x + 1)^(1/2)),x)
```

output
```
int(1/((1 - a^2*x^2)^(1/2)*(a*x + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{1+ax}\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{2}\left(\log\left(\sqrt{-ax+1}-\sqrt{2}\right)-\log\left(\sqrt{-ax+1}+\sqrt{2}\right)\right)}{2a}$$

input
```
int(1/(a*x+1)^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

output
```
(sqrt(2)*(log(sqrt( - a*x + 1) - sqrt(2)) - log(sqrt( - a*x + 1) + sqrt(2)
)))/(2*a)
```

3.242 $\int (c+dx)^3 \sqrt[3]{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 378

$$\int (c+dx)^3 \sqrt[3]{c^2-d^2x^2}\,dx$$
$$= \frac{12}{11}c^3x\sqrt[3]{c^2-d^2x^2} - \frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d} - \frac{15c(11c+4dx)\left(c^2-d^2x^2\right)^{4/3}}{154d}$$
$$+ \frac{8\;3^{3/4}\sqrt{2-\sqrt{3}}c^5\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/\cdots}}{\left(1-\sqrt{3}\right)c^{2/\cdots}}\right.\right.}{11d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output

```
12/11*c^3*x*(-d^2*x^2+c^2)^(1/3)-3/14*(d*x+c)^2*(-d^2*x^2+c^2)^(4/3)/d-15/
154*c*(4*d*x+11*c)*(-d^2*x^2+c^2)^(4/3)/d+8/11*3^(3/4)*(1/2*6^(1/2)-1/2*2^
(1/2))*c^5*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)
^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)
^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c
^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2
*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.71 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.28

$$\int (c+dx)^3\sqrt[3]{c^2-d^2x^2}\,dx = \frac{1}{154}\sqrt[3]{c^2-d^2x^2}\left(3\left(-\frac{66c^4}{d}-42c^3x+55c^2dx^2+42cd^2x^3+11d^3x^4\right)+\frac{280c^3x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[3]{1-\frac{d^2x^2}{c^2}}}\right)$$

input
```
Integrate[(c + d*x)^3*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
((c^2 - d^2*x^2)^(1/3)*(3*((-66*c^4)/d - 42*c^3*x + 55*c^2*d*x^2 + 42*c*d^
2*x^3 + 11*d^3*x^4) + (280*c^3*x*Hypergeometric2F1[-1/3, 1/2, 3/2, (d^2*x^
2)/c^2])/(1 - (d^2*x^2)/c^2)^(1/3)))/154
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {497, 27, 497, 27, 455, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sqrt[3]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \ 497$$

$$-\frac{3\int -\frac{20}{3}cd^2(c+dx)^2\sqrt[3]{c^2-d^2x^2}dx}{14d^2}-\frac{3\left(c^2-d^2x^2\right)^{4/3}(c+dx)^2}{14d}$$

$$\Big\downarrow \; 27$$

$$\frac{10}{7}c\int (c+dx)^2\sqrt[3]{c^2-d^2x^2}dx-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d}$$

$$\Big\downarrow \; 497$$

$$\frac{10}{7}c\left(-\frac{3\int -\frac{14}{3}cd^2(c+dx)\sqrt[3]{c^2-d^2x^2}dx}{11d^2}-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d}$$

$$\Big\downarrow \; 27$$

$$\frac{10}{7}c\left(\frac{14}{11}c\int (c+dx)\sqrt[3]{c^2-d^2x^2}dx-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d}$$

$$\Big\downarrow \; 455$$

$$\frac{10}{7}c\left(\frac{14}{11}c\left(c\int \sqrt[3]{c^2-d^2x^2}dx-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d}$$

$$\Big\downarrow \; 211$$

$$\frac{10}{7}c\left(\frac{14}{11}c\left(c\left(\frac{2}{5}c^2\int \frac{1}{\left(c^2-d^2x^2\right)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}\right)-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d}$$

$$\Big\downarrow \; 234$$

$$\frac{10}{7}c\left(\frac{14}{11}c\left(c\left(\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}-\frac{3c^2\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{5d^2x}\right)-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d}$$

$$\Big\downarrow \; 760$$

$$\frac{10}{7}c\left(\frac{14}{11}c\left(c\left(\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}c^2\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(1}{(1}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.\right.\right.$$

$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{4/3}}{14d}$$

input
```
Int[(c + d*x)^3*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c + d*x)^2*(c^2 - d^2*x^2)^(4/3))/(14*d) + (10*c*((-3*(c + d*x)*(c^2
- d^2*x^2)^(4/3))/(11*d) + (14*c*((-3*(c^2 - d^2*x^2)^(4/3))/(8*d) + c*((3
*x*(c^2 - d^2*x^2)^(1/3))/5 + (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*c^2*(c^(2/3) -
(c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^
2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3
])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*d^2*x*Sqrt[-((c^
(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d
^2*x^2)^(1/3))^2)])))/11))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (dx+c)^3 \left(-d^2x^2+c^2\right)^{\frac{1}{3}} dx$$

input
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)^3 \sqrt[3]{c^2-d^2x^2}\, dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{3}} (dx+c)^3\, dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-d^2*x^2 + c^2)^(1/3),
 x)
```

Sympy [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.73

$$\int (c+dx)^3\sqrt[3]{c^2-d^2x^2}\,dx = c^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + c^{\frac{5}{3}}d^2x^3\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + 3c^2d\left(\begin{cases}\frac{x^2\sqrt[3]{c^2}}{2} & \text{for } d^2=0\\ -\frac{3(c^2-d^2x^2)^{\frac{4}{3}}}{8d^2} & \text{otherwise}\end{cases}\right) + d^3\left(\begin{cases}\frac{9c^{\frac{26}{3}}\sqrt[3]{-1+\frac{d^2x^2}{c^2}}e^{\frac{i\pi}{3}}}{-56c^4d^4+56c^2d^6x^2} - \frac{9c^{\frac{26}{3}}}{-56c^4d^4+56c^2d^6x^2} - \frac{6c^{\frac{20}{3}}d^2x^2\sqrt[3]{-1+\frac{d^2x^2}{c^2}}e^{\frac{i\pi}{3}}}{-56c^4d^4+56c^2d^6x^2} + \frac{9c^{\frac{20}{3}}d^2x^2}{-56c^4d^4+56c^2d^6x^2} - \frac{15c^{\frac{14}{3}}d^4x^4\sqrt[3]{\ldots}}{-56c^4d^4\ldots} \\ \frac{9c^{\frac{26}{3}}\sqrt[3]{1-\frac{d^2x^2}{c^2}}}{-56c^4d^4+56c^2d^6x^2} - \frac{9c^{\frac{26}{3}}}{-56c^4d^4+56c^2d^6x^2} - \frac{6c^{\frac{20}{3}}d^2x^2\sqrt[3]{1-\frac{d^2x^2}{c^2}}}{-56c^4d^4+56c^2d^6x^2} + \frac{9c^{\frac{20}{3}}d^2x^2}{-56c^4d^4+56c^2d^6x^2} - \frac{15c^{\frac{14}{3}}d^4x^4\sqrt[3]{1-\frac{d^2x^2}{c^2}}}{-56c^4d^4+56c^2d^6x^2} \ldots\end{cases}\right.$$

input
```
integrate((d*x+c)**3*(-d**2*x**2+c**2)**(1/3),x)
```

output
```
c**(11/3)*x*hyper((-1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
 c**(5/3)*d**2*x**3*hyper((-1/3, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)
/c**2) + 3*c**2*d*Piecewise((x**2*(c**2)**(1/3)/2, Eq(d**2, 0)), (-3*(c**2
 - d**2*x**2)**(4/3)/(8*d**2), True)) + d**3*Piecewise((9*c**(26/3)*(-1 +
d**2*x**2/c**2)**(1/3)*exp(I*pi/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2) - 9
*c**(26/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2) - 6*c**(20/3)*d**2*x**2*(-1
 + d**2*x**2/c**2)**(1/3)*exp(I*pi/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2)
+ 9*c**(20/3)*d**2*x**2/(-56*c**4*d**4 + 56*c**2*d**6*x**2) - 15*c**(14/3)
*d**4*x**4*(-1 + d**2*x**2/c**2)**(1/3)*exp(I*pi/3)/(-56*c**4*d**4 + 56*c*
*2*d**6*x**2) + 12*c**(8/3)*d**6*x**6*(-1 + d**2*x**2/c**2)**(1/3)*exp(I*p
i/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2), Abs(d**2*x**2/c**2) > 1), (9*c**
(26/3)*(1 - d**2*x**2/c**2)**(1/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2) - 9
*c**(26/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2) - 6*c**(20/3)*d**2*x**2*(1
- d**2*x**2/c**2)**(1/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2) + 9*c**(20/3)
*d**2*x**2/(-56*c**4*d**4 + 56*c**2*d**6*x**2) - 15*c**(14/3)*d**4*x**4*(1
 - d**2*x**2/c**2)**(1/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2) + 12*c**(8/3
)*d**6*x**6*(1 - d**2*x**2/c**2)**(1/3)/(-56*c**4*d**4 + 56*c**2*d**6*x**2
), True))
```

Maxima [F]

$$\int (c+dx)^3\sqrt[3]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^3\,dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^3, x)
```

Giac [F]

$$\int (c+dx)^3\sqrt[3]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^3\,dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\sqrt[3]{c^2-d^2x^2}\,dx=\int \left(c^2-d^2\,x^2\right)^{1/3}\,(c+d\,x)^3\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x)^3,x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3\sqrt[3]{c^2-d^2x^2}\,dx$$
$$=\frac{-198(-d^2x^2+c^2)^{\frac{1}{3}}c^4+42(-d^2x^2+c^2)^{\frac{1}{3}}c^3dx+165(-d^2x^2+c^2)^{\frac{1}{3}}c^2d^2x^2+126(-d^2x^2+c^2)^{\frac{1}{3}}cd^3x^3+}{154d}$$

input
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
( - 198*(c**2 - d**2*x**2)**(1/3)*c**4 + 42*(c**2 - d**2*x**2)**(1/3)*c**3
*d*x + 165*(c**2 - d**2*x**2)**(1/3)*c**2*d**2*x**2 + 126*(c**2 - d**2*x**
2)**(1/3)*c*d**3*x**3 + 33*(c**2 - d**2*x**2)**(1/3)*d**4*x**4 + 112*int((
c**2 - d**2*x**2)**(1/3)/(c**2 - d**2*x**2),x)*c**5*d)/(154*d)
```

3.243 $\int (c+dx)^2 \sqrt[3]{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 347

$$\int (c+dx)^2 \sqrt[3]{c^2-d^2x^2}\,dx = \frac{42}{55}c^2x\sqrt[3]{c^2-d^2x^2} - \frac{3(11c+4dx)\left(c^2-d^2x^2\right)^{4/3}}{44d} + \frac{28\ 3^{3/4}\sqrt{2-\sqrt{3}}c^4\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c}{\left(1-\sqrt{3}\right)c}\right.\right.}{55d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
42/55*c^2*x*(-d^2*x^2+c^2)^(1/3)-3/44*(4*d*x+11*c)*(-d^2*x^2+c^2)^(4/3)/d+
28/55*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^4*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))
*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))
*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d
^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2
))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d
^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.27

$$\int (c+dx)^2 \sqrt[3]{c^2-d^2x^2}\,dx = \frac{1}{44}\sqrt[3]{c^2-d^2x^2}\left(-\frac{33c^3}{d}+33cdx^2+12x\left(-c^2+d^2x^2\right)+\frac{56c^2x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[3]{1-\frac{d^2x^2}{c^2}}}\right)$$

input
```
Integrate[(c + d*x)^2*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
((c^2 - d^2*x^2)^(1/3)*((-33*c^3)/d + 33*c*d*x^2 + 12*x*(-c^2 + d^2*x^2) +
 (56*c^2*x*Hypergeometric2F1[-1/3, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2
)/c^2)^(1/3)))/44
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {497, 27, 455, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sqrt[3]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \ 497$$

$$-\frac{3\int -\frac{14}{3}cd^2(c+dx)\sqrt[3]{c^2-d^2x^2}dx}{11d^2}-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}$$

$$\Big\downarrow \ 27$$

$$\frac{14}{11}c\int (c+dx)\sqrt[3]{c^2-d^2x^2}dx-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}$$

$$\Big\downarrow \text{455}$$

$$\frac{14}{11}c\left(c\int \sqrt[3]{c^2-d^2x^2}dx-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}$$

$$\Big\downarrow \text{211}$$

$$\frac{14}{11}c\left(c\left(\frac{2}{5}c^2\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}\right)-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}$$

$$\Big\downarrow \text{234}$$

$$\frac{14}{11}c\left(c\left(\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}-\frac{3c^2\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{5d^2x}\right)-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}$$

$$\Big\downarrow \text{760}$$

$$\frac{14}{11}c\left(c\left(\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}c^2\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c\cdots}{\left(1-\sqrt{3}\right)c\cdots}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.\right.$$

$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{4/3}}{11d}$$

input
```
Int[(c + d*x)^2*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c + d*x)*(c^2 - d^2*x^2)^(4/3))/(11*d) + (14*c*((-3*(c^2 - d^2*x^2)^(
4/3))/(8*d) + c*((3*x*(c^2 - d^2*x^2)^(1/3))/5 + (2*3^(3/4)*Sqrt[2 - Sqrt[
3]]*c^2*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d
^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2
*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^
(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(5*d^2*x*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])
*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]))))/11
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (dx+c)^2 \left(-d^2x^2+c^2\right)^{\frac{1}{3}} dx$$

input
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)^2 \sqrt[3]{c^2-d^2x^2}\, dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{3}} (dx+c)^2\, dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(-d^2*x^2 + c^2)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.32

$$\int (c+dx)^2 \sqrt[3]{c^2-d^2x^2}\,dx = c^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + \frac{c^{\frac{2}{3}}d^2x^3\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{3} + 2cd\left(\begin{cases} \frac{x^2\sqrt[3]{c^2}}{2} & \text{for } d^2=0 \\ -\frac{3(c^2-d^2x^2)^{\frac{4}{3}}}{8d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**2*(-d**2*x**2+c**2)**(1/3),x)
```

output
```
c**(8/3)*x*hyper((-1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
c**(2/3)*d**2*x**3*hyper((-1/3, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)/
c**2)/3 + 2*c*d*Piecewise((x**2*(c**2)**(1/3)/2, Eq(d**2, 0)), (-3*(c**2 -
 d**2*x**2)**(4/3)/(8*d**2), True))
```

Maxima [F]

$$\int (c+dx)^2 \sqrt[3]{c^2-d^2x^2}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^2, x)
```

Giac [F]

$$\int (c+dx)^2\sqrt[3]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\sqrt[3]{c^2-d^2x^2}\,dx=\int \left(c^2-d^2\,x^2\right)^{1/3}\,(c+d\,x)^2\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\sqrt[3]{c^2-d^2x^2}\,dx$$
$$=\frac{-165\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^3+108\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^2dx+165\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c\,d^2x^2+60\left(-d^2x^2+c^2\right)^{\frac{1}{3}}d^3x^3+1}{220d}$$

input
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
( - 165*(c**2 - d**2*x**2)**(1/3)*c**3 + 108*(c**2 - d**2*x**2)**(1/3)*c**
2*d*x + 165*(c**2 - d**2*x**2)**(1/3)*c*d**2*x**2 + 60*(c**2 - d**2*x**2)*
*(1/3)*d**3*x**3 + 112*int((c**2 - d**2*x**2)**(1/3)/(c**2 - d**2*x**2),x)
*c**4*d)/(220*d)
```

3.244 $\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 337

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx = \frac{3}{5}cx\sqrt[3]{c^2-d^2x^2}-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}$$
$$+\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}c^3\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/}}{\left(1-\sqrt{3}\right)c^{2/}}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/5*c*x*(-d^2*x^2+c^2)^(1/3)-3/8*(-d^2*x^2+c^2)^(4/3)/d+2/5*3^(3/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*c^3*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(
-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c
^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/(
(1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(
c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.24

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx = -\frac{3(c^2-d^2x^2)^{4/3}}{8d} + \frac{cx\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[3]{1-\frac{d^2x^2}{c^2}}}$$

input
```
Integrate[(c + d*x)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(4/3))/(8*d) + (c*x*(c^2 - d^2*x^2)^(1/3)*Hypergeometr
ic2F1[-1/3, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {455, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \sqrt[3]{c^2-d^2x^2}dx-\frac{3(c^2-d^2x^2)^{4/3}}{8d}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{2}{5}c^2\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}\right)-\frac{3(c^2-d^2x^2)^{4/3}}{8d}$$

$$\Big\downarrow \ 234$$

$$c\left(\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}-\frac{3c^2\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{5d^2x}\right)-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}$$

$$\Big\downarrow \ 760$$

$$c\left(\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}c^2\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{}}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.$$

$$\frac{3\left(c^2-d^2x^2\right)^{4/3}}{8d}$$

input
```
Int[(c + d*x)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(4/3))/(8*d) + c*((3*x*(c^2 - d^2*x^2)^(1/3))/5 + (2*3
^(3/4)*Sqrt[2 - Sqrt[3]]*c^2*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/
3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])
*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/
3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3)
)], -7 + 4*Sqrt[3]])/(5*d^2*x*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(
1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (dx+c)\left(-d^2x^2+c^2\right)^{\frac{1}{3}}dx$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)*(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/3)*(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.20

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx = c^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)+d\left(\begin{cases} \frac{x^2\sqrt[3]{c^2}}{2} & \text{for } d^2=0 \\ -\frac{3(c^2-d^2x^2)^{\frac{4}{3}}}{8d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)*(-d**2*x**2+c**2)**(1/3),x)
```

output
```
c**(5/3)*x*hyper((-1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
d*Piecewise((x**2*(c**2)**(1/3)/2, Eq(d**2, 0)), (-3*(c**2 - d**2*x**2)**(
4/3)/(8*d**2), True))
```

Maxima [F]

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.20

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx=\frac{c\,x\,(c^2-d^2\,x^2)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{1/3}}-\frac{3\,(c^2-d^2\,x^2)^{4/3}}{8\,d}$$

input
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x),x)
```

output
```
(c*x*(c^2 - d^2*x^2)^(1/3)*hypergeom([-1/3, 1/2], 3/2, (d^2*x^2)/c^2))/(1
- (d^2*x^2)/c^2)^(1/3) - (3*(c^2 - d^2*x^2)^(4/3))/(8*d)
```

Reduce [F]

$$\int (c+dx)\sqrt[3]{c^2-d^2x^2}\,dx$$
$$=\frac{-15(-d^2x^2+c^2)^{\frac{1}{3}}c^2+24(-d^2x^2+c^2)^{\frac{1}{3}}cdx+15(-d^2x^2+c^2)^{\frac{1}{3}}d^2x^2+16\left(\int\frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)c^3d}{40d}$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
( - 15*(c**2 - d**2*x**2)**(1/3)*c**2 + 24*(c**2 - d**2*x**2)**(1/3)*c*d*x
 + 15*(c**2 - d**2*x**2)**(1/3)*d**2*x**2 + 16*int((c**2 - d**2*x**2)**(1/
3)/(c**2 - d**2*x**2),x)*c**3*d)/(40*d)
```

3.245 $\int \sqrt[3]{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 313

$$\int \sqrt[3]{c^2-d^2x^2}\,dx = \frac{3}{5}x\sqrt[3]{c^2-d^2x^2} + \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}c^2\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/\cdots}}{\left(1-\sqrt{3}\right)c^{2/\cdots}}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/5*x*(-d^2*x^2+c^2)^(1/3)+2/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^2*(c^(2
/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2
+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF
(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2
+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))
/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.18

$$\int \sqrt[3]{c^2-d^2x^2}\,dx=\frac{x\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[3]{1-\frac{d^2x^2}{c^2}}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(x*(c^2 - d^2*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, (d^2*x^2)/c^2])
/(1 - (d^2*x^2)/c^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{2}{5}c^2\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}$$

$$\Big\downarrow \text{234}$$

$$\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}-\frac{3c^2\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{5d^2x}$$

$$\Big\downarrow \text{760}$$

$$\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}c^2\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$\frac{3}{5}x\sqrt[3]{c^2-d^2x^2}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(3*x*(c^2 - d^2*x^2)^(1/3))/5 + (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*c^2*(c^(2/3)
- (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (
c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt
[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*d^2*x*Sqrt[-((
c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 -
 d^2*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}} dx$$

input
```
int((-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{c^2-d^2x^2}\, dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{3}} dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.10

$$\int \sqrt[3]{c^2-d^2x^2}\, dx = c^{\frac{2}{3}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{d^2x^2e^{2i\pi}}{c^2}\right)$$

input
```
integrate((-d**2*x**2+c**2)**(1/3),x)
```

output
```
c**(2/3)*x*hyper((-1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)
```

Maxima [F]

$$\int \sqrt[3]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.15

$$\int \sqrt[3]{c^2-d^2x^2}\,dx=\frac{x\,\left(c^2-d^2\,x^2\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{1/3}}$$

input
```
int((c^2 - d^2*x^2)^(1/3),x)
```

output
```
(x*(c^2 - d^2*x^2)^(1/3)*hypergeom([-1/3, 1/2], 3/2, (d^2*x^2)/c^2))/(1 -
(d^2*x^2)/c^2)^(1/3)
```

Reduce [F]

$$\int \sqrt[3]{c^2-d^2x^2}\,dx=\frac{3(-d^2x^2+c^2)^{\frac{1}{3}}\,x}{5}+\frac{2\left(\int\frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)c^2}{5}$$

input
```
int((-d^2*x^2+c^2)^(1/3),x)
```

output
```
(3*(c**2 - d**2*x**2)**(1/3)*x + 2*int((c**2 - d**2*x**2)**(1/3)/(c**2 - d
**2*x**2),x)*c**2)/5
```

3.246 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 310

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx = \frac{3\sqrt[3]{c^2-d^2x^2}}{2d} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-}{\left(1-\sqrt{3}\right)c^{2/3}-}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/2*(-d^2*x^2+c^2)^(1/3)/d+3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c*(c^(2/3)-(-
d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^
(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^
(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.41 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx$$
$$=-\frac{3(c-dx)\left(1+\frac{dx}{c}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},\frac{c-dx}{2c}\right)}{4\;2^{2/3}d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(2/3)*(c^2 - d^2*x^2)^(1/3)*Hypergeometric2F1[
2/3, 4/3, 7/3, (c - d*x)/(2*c)])/(4*2^(2/3)*d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {504, 234, 241, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}dx$$
$$\Big\downarrow \text{ 504}$$
$$c\int \frac{1}{\left(c^2-d^2x^2\right)^{2/3}}dx-d\int \frac{x}{\left(c^2-d^2x^2\right)^{2/3}}dx$$
$$\Big\downarrow \text{ 234}$$
$$-\frac{3c\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-d\int \frac{x}{\left(c^2-d^2x^2\right)^{2/3}}dx$$
$$\Big\downarrow \text{ 241}$$

$$\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}-\frac{3c\sqrt{-d^2x^2}\int\frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}$$

$$\Big\downarrow \text{760}$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x),x]
```

output
```
(3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*c*(c^(2/3) -
(c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^
2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3
])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(d^2*x*Sqrt[-((c^(2
/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2
*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/3)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\sqrt[3]{-\left(-c+dx\right)\left(c+dx\right)}}{c+dx}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx = \int \frac{\left(c^2-d^2\,x^2\right)^{1/3}}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx = \int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c),x)
```

output
```
int((c**2 - d**2*x**2)**(1/3)/(c + d*x),x)
```

3.247 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 319

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx = -\frac{3\sqrt[3]{c^2-d^2x^2}}{2d(c+dx)} - \frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{\cdots}}\right.\right.\cdots}{2d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/2*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)-1/2*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*
(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^
2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*Elli
pticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^
2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(
1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.90 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx$$
$$= -\frac{3(c-dx)\left(1+\frac{dx}{c}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{7}{3},\frac{c-dx}{2c}\right)}{8\ 2^{2/3}cd(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^2,x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(2/3)*(c^2 - d^2*x^2)^(1/3)*Hypergeometric2F1[
4/3, 5/3, 7/3, (c - d*x)/(2*c)])/(8*2^(2/3)*c*d*(c + d*x))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.20, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{506}$$
$$-\frac{\left(\frac{1}{c+dx}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}\int \frac{\sqrt[3]{1-\frac{2c}{c+dx}}}{\left(\frac{1}{c+dx}\right)^{2/3}}d\frac{1}{c+dx}}{d\sqrt[3]{1-\frac{2c}{c+dx}}}$$
$$\Big\downarrow \text{74}$$

$$-\frac{3\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},\frac{2c}{c+dx}\right)}{d(c+dx)\sqrt[3]{1-\frac{2c}{c+dx}}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^2,x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, (2*c)/(c + d*x
)])/(d*(c + d*x)*(1 - (2*c)/(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^2}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^2,x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx=\int \frac{\sqrt[3]{-\left(-c+dx\right)\left(c+dx\right)}}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c)**2,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx=\int \frac{\left(c^2-d^2\,x^2\right)^{1/3}}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^2}\,dx$$
$$=\frac{-3(-d^2x^2+c^2)^{\frac{1}{3}}-2\left(\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right)c\,d^2-2\left(\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right)d^3x}{3d\,(dx+c)}$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^2,x)
```

output
```
( - 3*(c**2 - d**2*x**2)**(1/3) - 2*int(((c**2 - d**2*x**2)**(1/3)*x)/(c**
3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*c*d**2 - 2*int(((c**2 - d**2*x*
*2)**(1/3)*x)/(c**3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*d**3*x)/(3*d*
(c + d*x))
```

3.248 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 355

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx = -\frac{3\sqrt[3]{c^2-d^2x^2}}{5d(c+dx)^2}+\frac{3\sqrt[3]{c^2-d^2x^2}}{20cd(c+dx)}$$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{}}\right.\right.}{20cd^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output

```
-3/5*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^2+3/20*(-d^2*x^2+c^2)^(1/3)/c/d/(d*x+c
)-1/20*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((
c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^
(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*
x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/
c/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^
2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.82 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx = -\frac{3(c-dx)\left(1+\frac{dx}{c}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{7}{3},\frac{c-dx}{2c}\right)}{16\ 2^{2/3}c^2d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^3,x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(2/3)*(c^2 - d^2*x^2)^(1/3)*Hypergeometric2F1[
4/3, 8/3, 7/3, (c - d*x)/(2*c)])/(16*2^(2/3)*c^2*d*(c + d*x))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.19, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{506}$$

$$-\frac{\left(\frac{1}{c+dx}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}\int \sqrt[3]{\frac{1}{c+dx}}\sqrt[3]{1-\frac{2c}{c+dx}}d\frac{1}{c+dx}}{d\sqrt[3]{1-\frac{2c}{c+dx}}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{3\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},\frac{2c}{c+dx}\right)}{4d(c+dx)^2\sqrt[3]{1-\frac{2c}{c+dx}}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^3,x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, (2*c)/(c + d*x
)])/(4*d*(c + d*x)^2*(1 - (2*c)/(c + d*x))^(1/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{1}{3}}}{\left(d x +c \right)^{3}}d x$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^3,x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{\left(dx+c\right)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/3)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3),
 x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx=\int \frac{\sqrt[3]{-\left(-c+dx\right)\left(c+dx\right)}}{\left(c+dx\right)^3}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c)**3,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{\left(dx+c\right)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx=\int \frac{\left(c^2-d^2\,x^2\right)^{1/3}}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^3,x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^3}\,dx = \frac{-3\left(-d^2x^2+c^2\right)^{\frac{1}{3}}-2\left(\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c^2d^2-4\left(\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c\,d^3x-2\left(\int \frac{}{-d}\right.}{6d\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^3,x)
```

output
```
( - 3*(c**2 - d**2*x**2)**(1/3) - 2*int(((c**2 - d**2*x**2)**(1/3)*x)/(c**
4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c**2*d**2 - 4*int(((c**2 -
d**2*x**2)**(1/3)*x)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c*
d**3*x - 2*int(((c**2 - d**2*x**2)**(1/3)*x)/(c**4 + 2*c**3*d*x - 2*c*d**3
*x**3 - d**4*x**4),x)*d**4*x**2)/(6*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.249 $\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx$

Optimal result

Integrand size = 24, antiderivative size = 707

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx = \frac{66}{91}c^3x\left(c^2-d^2x^2\right)^{2/3} - \frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{5/3}}{16d}$$
$$- \frac{33c(13c+5dx)\left(c^2-d^2x^2\right)^{5/3}}{520d} - \frac{264c^5x}{91\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$
$$- \frac{132\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{17/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}\right.\right.}{91d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$
$$+ \frac{88\sqrt{2}3^{3/4}c^{17/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}\right.\right.}{91d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
66/91*c^3*x*(-d^2*x^2+c^2)^(2/3)-3/16*(d*x+c)^2*(-d^2*x^2+c^2)^(5/3)/d-33/
520*c*(5*d*x+13*c)*(-d^2*x^2+c^2)^(5/3)/d-264*c^5*x/(91*(1-3^(1/2))*c^(2/3
)-91*(-d^2*x^2+c^2)^(1/3))-132/91*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*c^(17/
3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(
-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*E
llipticE(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(
-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2
)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)+88/91*2^(1/2)
*3^(3/4)*c^(17/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x
^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1
/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(
1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3
)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.98 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.15

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx = \frac{\left(c^2-d^2x^2\right)^{2/3}\left(-\frac{1053c^4}{d}-720c^3x+858c^2dx^2+720cd^2x^3+195d^3x^4+\frac{1760c^3x\,\text{Hypergeometric}}{\left(1-\frac{d^2x^2}{c^2}\right.}\right.}{1040}$$

input
```
Integrate[(c + d*x)^3*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
((c^2 - d^2*x^2)^(2/3)*((-1053*c^4)/d - 720*c^3*x + 858*c^2*d*x^2 + 720*c*
d^2*x^3 + 195*d^3*x^4 + (1760*c^3*x*Hypergeometric2F1[-2/3, 1/2, 3/2, (d^2
*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(2/3)))/1040
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {497, 27, 497, 27, 455, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx$$

$$\Big\downarrow \text{ 497}$$

$$-\frac{3\int -\frac{22}{3}cd^2(c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} dx}{16d^2}-\frac{3\left(c^2-d^2x^2\right)^{5/3}(c+dx)^2}{16d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{11}{8}c\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} dx-\frac{3(c+dx)^2 \left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$$\Big\downarrow \text{ 497}$$

$$\frac{11}{8}c\left(-\frac{3\int -\frac{16}{3}cd^2(c+dx) \left(c^2-d^2x^2\right)^{2/3} dx}{13d^2}-\frac{3(c+dx) \left(c^2-d^2x^2\right)^{5/3}}{13d}\right)-\frac{3(c+dx)^2 \left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{11}{8}c\left(\frac{16}{13}c\int (c+dx) \left(c^2-d^2x^2\right)^{2/3} dx-\frac{3(c+dx) \left(c^2-d^2x^2\right)^{5/3}}{13d}\right)-\frac{3(c+dx)^2 \left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$$\Big\downarrow \text{ 455}$$

$$\frac{11}{8}c\left(\frac{16}{13}c\left(c\int \left(c^2-d^2x^2\right)^{2/3} dx-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}\right)-\frac{3(c+dx) \left(c^2-d^2x^2\right)^{5/3}}{13d}\right)-\frac{3(c+dx)^2 \left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{11}{8}c\left(\frac{16}{13}c\left(c\left(\frac{4}{7}c^2\int\frac{1}{\sqrt[3]{c^2-d^2x^2}}dx+\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}\right)-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$\big\downarrow$ 233

$$\frac{11}{8}c\left(\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\int\frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{7d^2x}\right)-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}\right)-\frac{3(c+dx)}{}\right.$$
$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$\big\downarrow$ 833

$$\frac{11}{8}c\left(\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int\frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2}}{\sqrt{-d^2x^2}}\right.}{7d^2x}\right.\right.\right.$$
$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$\big\downarrow$ 760

$$\frac{11}{8}c\left(\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\int\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}}{}\right.}{}\right.\right.\right.$$
$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{5/3}}{16d}$$

$$\Big\downarrow \text{2418}$$

$$\frac{11}{8}c\left(\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{\cdots}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\cdots}\right.}}}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c\cdots\right.}{\left(\left(1-\sqrt{3}\right)\cdots\right.}}}\right.}{\cdots}\right.\right.\right.$$

$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{5/3}}{16d}$$

input
```
Int[(c + d*x)^3*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c + d*x)^2*(c^2 - d^2*x^2)^(5/3))/(16*d) + (11*c*((-3*(c + d*x)*(c^2
- d^2*x^2)^(5/3))/(13*d) + (16*c*((-3*(c^2 - d^2*x^2)^(5/3))/(10*d) + c*((
3*x*(c^2 - d^2*x^2)^(2/3))/7 - (6*c^2*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^2
)])/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sq
rt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(
c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^
2 - d^2*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^
2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqr
t[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))
)/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[
3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3)
+ c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^
(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3)
- (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))],
 -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^
2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]))
)/(7*d^2*x))))/13))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int (dx+c)^3 \left(-d^2x^2+c^2\right)^{\frac{2}{3}} dx$$

input
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}} (dx+c)^3 \, dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-d^2*x^2 + c^2)^(2/3),
 x)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.28

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx = c^{\frac{13}{3}} x\,{}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + c^{\frac{7}{3}} d^2 x^3\,{}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + 3c^2 d \left(\begin{cases} \frac{x^2(c^2)^{\frac{2}{3}}}{2} & \text{for } d^2 = 0 \\ -\frac{3(c^2-d^2x^2)^{\frac{5}{3}}}{10d^2} & \text{otherwise} \end{cases}\right) + d^3 \left(\begin{cases} -\frac{9c^4(c^2-d^2x^2)^{\frac{2}{3}}}{80d^4} - \frac{3c^2x^2(c^2-d^2x^2)^{\frac{2}{3}}}{40d^2} + \frac{3x^4(c^2-d^2x^2)^{\frac{2}{3}}}{16} & \text{for } d \neq 0 \\ \frac{x^4(c^2)^{\frac{2}{3}}}{4} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**3*(-d**2*x**2+c**2)**(2/3),x)
```

output
```
c**(13/3)*x*hyper((-2/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
 c**(7/3)*d**2*x**3*hyper((-2/3, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)
/c**2) + 3*c**2*d*Piecewise((x**2*(c**2)**(2/3)/2, Eq(d**2, 0)), (-3*(c**2
 - d**2*x**2)**(5/3)/(10*d**2), True)) + d**3*Piecewise((-9*c**4*(c**2 - d
**2*x**2)**(2/3)/(80*d**4) - 3*c**2*x**2*(c**2 - d**2*x**2)**(2/3)/(40*d**
2) + 3*x**4*(c**2 - d**2*x**2)**(2/3)/16, Ne(d, 0)), (x**4*(c**2)**(2/3)/4
, True))
```

Maxima [F]

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}} (dx+c)^3 \, dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^3, x)
```

Giac [F]

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)^3 \, dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(c^2-d^2\,x^2\right)^{2/3}(c+d\,x)^3 \, dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)*(c + d*x)^3,x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \left(c^2 -d^2x^2\right)^{2/3} dx = \frac{-7371\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c^4+240\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c^3dx+6006\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c^2d^2x^2+5040(-d}{7280d}$$

input
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 7371*(c**2 - d**2*x**2)**(2/3)*c**4 + 240*(c**2 - d**2*x**2)**(2/3)*c*
*3*d*x + 6006*(c**2 - d**2*x**2)**(2/3)*c**2*d**2*x**2 + 5040*(c**2 - d**2
*x**2)**(2/3)*c*d**3*x**3 + 1365*(c**2 - d**2*x**2)**(2/3)*d**4*x**4 + 704
0*int((c**2 - d**2*x**2)**(2/3)/(c**2 - d**2*x**2),x)*c**5*d)/(7280*d)
```

3.250 $\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} dx$

Optimal result

Integrand size = 24, antiderivative size = 676

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} dx = \frac{48}{91}c^2x\left(c^2-d^2x^2\right)^{2/3}$$

$$-\frac{3(13c+5dx)\left(c^2-d^2x^2\right)^{5/3}}{65d}-\frac{192c^4x}{91\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$-\frac{96\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{14/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}\right.\right.}{91d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+\frac{64\sqrt{2}3^{3/4}c^{14/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}\right.\right.}{91d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
48/91*c^2*x*(-d^2*x^2+c^2)^(2/3)-3/65*(5*d*x+13*c)*(-d^2*x^2+c^2)^(5/3)/d-
192*c^4*x/(91*(1-3^(1/2))*c^(2/3)-91*(-d^2*x^2+c^2)^(1/3))-96/91*3^(1/4)*(
1/2*6^(1/2)+1/2*2^(1/2))*c^(14/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)
+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(
-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2
)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(
-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2
)^(1/3))^2)^(1/2)+64/91*2^(1/2)*3^(3/4)*c^(14/3)*(c^(2/3)-(-d^2*x^2+c^2)^(
1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(
1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3
)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3
^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3
)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.76 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.14

$$\int (c+dx)^2\left(c^2-d^2x^2\right)^{2/3}\,dx=\frac{1}{65}\left(c^2-d^2x^2\right)^{2/3}\left(-\frac{39c^3}{d}+39cdx^2+15x\left(-c^2+d^2x^2\right)+\frac{80c^2x\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}}\right)$$

input
```
Integrate[(c + d*x)^2*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
((c^2 - d^2*x^2)^(2/3)*((-39*c^3)/d + 39*c*d*x^2 + 15*x*(-c^2 + d^2*x^2) +
 (80*c^2*x*Hypergeometric2F1[-2/3, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2
)/c^2)^(2/3)))/65
```

Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {497, 27, 455, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} \, dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{3\int -\frac{16}{3}cd^2(c+dx)\left(c^2-d^2x^2\right)^{2/3}dx}{13d^2}-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

$$\Big\downarrow \text{27}$$

$$\frac{16}{13}c\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}dx-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

$$\Big\downarrow \text{455}$$

$$\frac{16}{13}c\left(c\int \left(c^2-d^2x^2\right)^{2/3}dx-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

$$\Big\downarrow \text{211}$$

$$\frac{16}{13}c\left(c\left(\frac{4}{7}c^2\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx+\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}\right)-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

$$\Big\downarrow \text{233}$$

$$\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{7d^2x}\right)-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

$$\Big\downarrow \text{833}$$

$$\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int\frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{\right.}{7d^2x}\right.\right.$$

$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

$\downarrow$ 760

$$\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\int\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\right.}{}\right.}{}\right.\right.$$

$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

$\downarrow$ 2418

$$\frac{16}{13}c\left(c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right.}}}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{}\right.}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{}\right.}}}\right.}{}\right.\right.$$

$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{5/3}}{13d}$$

input
```
Int[(c + d*x)^2*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c + d*x)*(c^2 - d^2*x^2)^(5/3))/(13*d) + (16*c*((-3*(c^2 - d^2*x^2)^(
5/3))/(10*d) + c*((3*x*(c^2 - d^2*x^2)^(2/3))/7 - (6*c^2*Sqrt[-(d^2*x^2)]*
((-2*Sqrt[-(d^2*x^2)])/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3)) + (
3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(
c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqr
t[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*
c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^
2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])
- (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1
/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3)
)/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1
+ Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2
- d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(d^2*x^2)]*Sqrt[-((c^(
2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^
2*x^2)^(1/3))^2)])))/(7*d^2*x))))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int (dx+c)^2 \left(-d^2x^2+c^2\right)^{\frac{2}{3}} dx$$

input
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}} (dx+c)^2 dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(-d^2*x^2 + c^2)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.16

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} dx = c^{\frac{10}{3}} x {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + \frac{c^{\frac{4}{3}} d^2 x^3 {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{3} + 2cd \left(\begin{cases} \frac{x^2(c^2)^{\frac{2}{3}}}{2} & \text{for } d^2 = 0 \\ -\frac{3(c^2-d^2x^2)^{\frac{5}{3}}}{10d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**2*(-d**2*x**2+c**2)**(2/3),x)
```

output
```
c**(10/3)*x*hyper((-2/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
 c**(4/3)*d**2*x**3*hyper((-2/3, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)
/c**2)/3 + 2*c*d*Piecewise((x**2*(c**2)**(2/3)/2, Eq(d**2, 0)), (-3*(c**2
- d**2*x**2)**(5/3)/(10*d**2), True))
```

Maxima [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}} (dx+c)^2 \, dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^2, x)
```

Giac [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} \, dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)^2 \, dx$$

input

```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output

```
integrate((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} \, dx = \int \left(c^2-d^2\,x^2\right)^{2/3}(c+d\,x)^2 \, dx$$

input

```
int((c^2 - d^2*x^2)^(2/3)*(c + d*x)^2,x)
```

output

```
int((c^2 - d^2*x^2)^(2/3)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/3} \, dx = \frac{-273\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c^3+135\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c^2dx+273\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c\,d^2x^2+105\left(-d^2x^2\right.}{455d}$$

input

```
int((d*x+c)^2*(-d^2*x^2+c^2)^(2/3),x)
```

output

```
( - 273*(c**2 - d**2*x**2)**(2/3)*c**3 + 135*(c**2 - d**2*x**2)**(2/3)*c**
2*d*x + 273*(c**2 - d**2*x**2)**(2/3)*c*d**2*x**2 + 105*(c**2 - d**2*x**2)
**(2/3)*d**3*x**3 + 320*int((c**2 - d**2*x**2)**(2/3)/(c**2 - d**2*x**2),x
)*c**4*d)/(455*d)
```

3.251 $\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 666

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}\,dx=\frac{3}{7}cx\left(c^2-d^2x^2\right)^{2/3}$$

$$-\frac{3(c^2-d^2x^2)^{5/3}}{10d}-\frac{12c^3x}{7\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$-\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{11/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\cdots}}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+\frac{4\sqrt{2}3^{3/4}c^{11/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2\cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2\cdots}}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/7*c*x*(-d^2*x^2+c^2)^(2/3)-3/10*(-d^2*x^2+c^2)^(5/3)/d-12*c^3*x/(7*(1-3^
(1/2))*c^(2/3)-7*(-d^2*x^2+c^2)^(1/3))-6/7*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2
))*c^(11/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2
)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2
)^(1/2)*EllipticE(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*
c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^
2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)+4/7*
2^(1/2)*3^(3/4)*c^(11/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*
(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+
c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/
((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*
(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^
2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.12

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}\,dx=-\frac{3(c^2-d^2x^2)^{5/3}}{10d}+\frac{cx(c^2-d^2x^2)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}}$$

input
```
Integrate[(c + d*x)*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(5/3))/(10*d) + (c*x*(c^2 - d^2*x^2)^(2/3)*Hypergeomet
ric2F1[-2/3, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(2/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {455, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(c^2-d^2x^2\right)^{2/3}dx-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}$$

$$\Big\downarrow \text{211}$$

$$c\left(\frac{4}{7}c^2\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx+\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}\right)-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}$$

$$\Big\downarrow \text{233}$$

$$c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{7d^2x}\right)-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}$$

$$\Big\downarrow \text{833}$$

$$c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-c\right.$$

$$\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}$$

$$\Big\downarrow \text{760}$$

$$c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\int\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-\right.}{}\right.}{}\right.$$

$$\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}$$

$\downarrow$ 2418

$$c\left(\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{E}}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-\right.}{}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\right.}{}}}}\right.}{}\right.$$

$$\frac{3\left(c^2-d^2x^2\right)^{5/3}}{10d}$$

input
```
Int[(c + d*x)*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(5/3))/(10*d) + c*((3*x*(c^2 - d^2*x^2)^(2/3))/7 - (6*
c^2*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^2)])/((1 - Sqrt[3])*c^(2/3) - (c^2
- d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d
^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*
x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/
3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c
^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 -
d^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3)
 - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) +
(c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqr
t[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(
d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3]
)*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])))/(7*d^2*x))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int (dx + c) \left(-d^2x^2 + c^2\right)^{\frac{2}{3}} dx$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((d*x+c)*(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/3} \, dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}} (dx+c) \, dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/3)*(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.10

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/3} \, dx = c^{\frac{7}{3}} x {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + d \left(\begin{cases} \frac{x^2(c^2)^{\frac{2}{3}}}{2} & \text{for } d^2 = 0 \\ -\frac{3(c^2-d^2x^2)^{\frac{5}{3}}}{10d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)*(-d**2*x**2+c**2)**(2/3),x)
```

output
```
c**(7/3)*x*hyper((-2/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
d*Piecewise((x**2*(c**2)**(2/3)/2, Eq(d**2, 0)), (-3*(c**2 - d**2*x**2)**(
5/3)/(10*d**2), True))
```

Maxima [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)*(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.10

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/3}\,dx=\frac{c\,x\,\left(c^2-d^2\,x^2\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{2/3}}-\frac{3\,\left(c^2-d^2\,x^2\right)^{5/3}}{10\,d}$$

input
```
int((c^2 - d^2*x^2)^(2/3)*(c + d*x),x)
```

output
```
(c*x*(c^2 - d^2*x^2)^(2/3)*hypergeom([-2/3, 1/2], 3/2, (d^2*x^2)/c^2))/(1
- (d^2*x^2)/c^2)^(2/3) - (3*(c^2 - d^2*x^2)^(5/3))/(10*d)
```

Reduce [F]

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/3} dx = \frac{-21(-d^2x^2+c^2)^{\frac{2}{3}} c^2+30(-d^2x^2+c^2)^{\frac{2}{3}} cdx+21(-d^2x^2+c^2)^{\frac{2}{3}} d^2x^2+40\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}} c\right.}{70d}$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 21*(c**2 - d**2*x**2)**(2/3)*c**2 + 30*(c**2 - d**2*x**2)**(2/3)*c*d*x
 + 21*(c**2 - d**2*x**2)**(2/3)*d**2*x**2 + 40*int((c**2 - d**2*x**2)**(2/
3)/(c**2 - d**2*x**2),x)*c**3*d)/(70*d)
```

3.252 $\int (c^2 - d^2x^2)^{2/3}\, dx$

Optimal result

Integrand size = 16, antiderivative size = 642

$$\int \left(c^2-d^2x^2\right)^{2/3} dx = \frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3} - \frac{12c^2x}{7\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$- \frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{8/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+ \frac{4\sqrt{2}3^{3/4}c^{8/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/7*x*(-d^2*x^2+c^2)^(2/3)-12*c^2*x/(7*(1-3^(1/2))*c^(2/3)-7*(-d^2*x^2+c^2
)^(1/3))-6/7*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*c^(8/3)*(c^(2/3)-(-d^2*x^2+
c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/(
(1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*
c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2
*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*
c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)+4/7*2^(1/2)*3^(3/4)*c^(8/3)*(c^(2/3
)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c
^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF((
(1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c
^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/(
(1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.94 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.09

$$\int \left(c^2-d^2x^2\right)^{2/3} dx = \frac{x(c^2-d^2x^2)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(x*(c^2 - d^2*x^2)^(2/3)*Hypergeometric2F1[-2/3, 1/2, 3/2, (d^2*x^2)/c^2])
/(1 - (d^2*x^2)/c^2)^(2/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2-d^2x^2\right)^{2/3}dx$$

$\downarrow$ 211

$$\frac{4}{7}c^2\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx+\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}$$

$\downarrow$ 233

$$\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{7d^2x}$$

$\downarrow$ 833

$$\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}\right)}{7d^2x}$$

$\downarrow$ 760

$$\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\right.}}}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-}}\right.}{7d^2x}$$

$\downarrow$ 2418

$$\frac{3}{7}x\left(c^2-d^2x^2\right)^{2/3}-\frac{6c^2\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}}{\left(1-\sqrt{3}\right)c^{2/3}}\right.\right.}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.}{}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(3*x*(c^2 - d^2*x^2)^(2/3))/7 - (6*c^2*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^
2)])/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + S
qrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*
(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c
^2 - d^2*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d
^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)
))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt
[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3)
 + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c
^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3)
 - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c
^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])
))/(7*d^2*x)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{3}}d x$$

input
```
int((-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \left(c^2-d^2x^2\right)^{2/3}\,dx=\int \left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{3}}d x$$

input
```
integrate((-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.05

$$\int \left(c^2-d^2x^2\right)^{2/3} dx = c^{\frac{4}{3}} x {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)$$

input
```
integrate((-d**2*x**2+c**2)**(2/3),x)
```

output
```
c**(4/3)*x*hyper((-2/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)
```

Maxima [F]

$$\int \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}} dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3), x)
```

Giac [F]

$$\int \left(c^2-d^2x^2\right)^{2/3} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{3}} dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.07

$$\int \left(c^2-d^2x^2\right)^{2/3}\,dx=\frac{x\,\left(c^2-d^2\,x^2\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{{\left(1-\frac{d^2\,x^2}{c^2}\right)}^{2/3}}$$

input
```
int((c^2 - d^2*x^2)^(2/3),x)
```

output
```
(x*(c^2 - d^2*x^2)^(2/3)*hypergeom([-2/3, 1/2], 3/2, (d^2*x^2)/c^2))/(1 -
(d^2*x^2)/c^2)^(2/3)
```

Reduce [F]

$$\int \left(c^2-d^2x^2\right)^{2/3}\,dx=\frac{3(-d^2x^2+c^2)^{\frac{2}{3}}x}{7}+\frac{4\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)c^2}{7}$$

input
```
int((-d^2*x^2+c^2)^(2/3),x)
```

output
```
(3*(c**2 - d**2*x**2)**(2/3)*x + 4*int((c**2 - d**2*x**2)**(2/3)/(c**2 - d
**2*x**2),x)*c**2)/7
```

3.253 $\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 637

$$\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx = \frac{3(c^2-d^2x^2)^{2/3}}{4d} - \frac{3cx}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}$$

$$- \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,c^{5/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\ldots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\ldots}}\right.\right.\ldots}{2d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+ \frac{\sqrt{2}\,3^{3/4}c^{5/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\ldots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\ldots}}\right.\right.\ldots}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/4*(-d^2*x^2+c^2)^(2/3)/d-3*c*x/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)
)-3/2*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*c^(5/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1
/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1
/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^(2/3)
-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^
(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)
-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)+2^(1/2)*3^(3/4)*c^(5/3)*(c^(2/3)-(-d^2*x^2
+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/
((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))
*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),
2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))
*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.12

$$\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx = -\frac{3(c-dx)\sqrt[3]{1+\frac{dx}{c}}(c^2-d^2x^2)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},\frac{c-dx}{2c}\right)}{5\sqrt[3]{2}d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(1/3)*(c^2 - d^2*x^2)^(2/3)*Hypergeometric2F1[
1/3, 5/3, 8/3, (c - d*x)/(2*c)])/(5*2^(1/3)*d*(c + d*x))
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {504, 233, 241, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{c+dx}dx$$

$$\Big\downarrow \text{504}$$

$$c\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx-d\int \frac{x}{\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{233}$$

$$-d\int \frac{x}{\sqrt[3]{c^2-d^2x^2}}dx-\frac{3c\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}-\frac{3c\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}$$

$$\Big\downarrow \text{833}$$

$$\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}-\frac{3c\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}\right)}{2d^2x}$$

$$\Big\downarrow \text{760}$$

$$\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}-\frac{3c\sqrt{-d^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}\cdots}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\cdots\right.}}\cdots}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\cdots}}\cdots\right)}{2d^2x}$$

$\Big\downarrow$ 2418

$$\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}-\frac{3c\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\cdots}{\left(1-\sqrt{3}\right)c^{2/3}-\cdots}\right.\right.\cdots}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\cdots\right)}{\cdots}$$

input

```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x),x]
```

output

```
(3*(c^2 - d^2*x^2)^(2/3))/(4*d) - (3*c*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^
2)])/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + S
qrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*
(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c
^2 - d^2*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d
^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)
))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt
[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3)
 + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c
^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3)
 - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c
^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])
))/(2*d^2*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/3)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx = \int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{3}}}{c+dx}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c), x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{c+dx}\,dx=\int \frac{(c^2-d^2\,x^2)^{2/3}}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{c + dx}\,dx = \int \frac{(-d^2x^2 + c^2)^{\frac{2}{3}}}{dx + c}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c),x)
```

output
```
int((c**2 - d**2*x**2)**(2/3)/(c + d*x),x)
```

3.254 $\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 640

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{d(c+dx)} + \frac{6x}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}$$
$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$
$$-\frac{2\sqrt{2}3^{3/4}c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3*(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)+6*x/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^
(1/3))+3*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)
^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^(2
/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I
*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2
/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)-2*2^(1/2)*3^(3/4)*c^(2/3)*(c^(2/3)-(-d^
2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2
/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(
1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1
/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(
1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.12

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx = -\frac{3(c-dx)\sqrt[3]{1+\frac{dx}{c}}(c^2-d^2x^2)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},\frac{c-dx}{2c}\right)}{10\sqrt[3]{2}cd(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^2,x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(1/3)*(c^2 - d^2*x^2)^(2/3)*Hypergeometric2F1[
4/3, 5/3, 8/3, (c - d*x)/(2*c)])/(10*2^(1/3)*c*d*(c + d*x))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \ 506$$

$$-\frac{\left(\frac{1}{c+dx}\right)^{4/3}(c^2-d^2x^2)^{2/3}\int \frac{\left(1-\frac{2c}{c+dx}\right)^{2/3}}{\left(\frac{1}{c+dx}\right)^{4/3}}d\frac{1}{c+dx}}{d\left(1-\frac{2c}{c+dx}\right)^{2/3}}$$

$$\Big\downarrow \ 74$$

$$\frac{3(c^2-d^2x^2)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},\frac{2c}{c+dx}\right)}{d(c+dx)\left(1-\frac{2c}{c+dx}\right)^{2/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^2,x]
```

output
```
(3*(c^2 - d^2*x^2)^(2/3)*Hypergeometric2F1[-2/3, -1/3, 2/3, (2*c)/(c + d*x
)])/(d*(c + d*x)*(1 - (2*c)/(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{3}}}{\left(d x +c \right)^{2}}d x$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^2,x)
```

output
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx = \int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{3}}}{\left(d x +c \right)^{2}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx = \int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{3}}}{(c+dx)^2}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**2,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx=\int \frac{{\left(c^2-d^2\,x^2\right)}^{2/3}}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^2}\,dx=\frac{-3(-d^2x^2+c^2)^{\frac{2}{3}}-4\left(\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right)c\,d^2-4\left(\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right)}{3d\,(dx+c)}$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^2,x)
```

output
```
( - 3*(c**2 - d**2*x**2)**(2/3) - 4*int(((c**2 - d**2*x**2)**(2/3)*x)/(c**
3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*c*d**2 - 4*int(((c**2 - d**2*x*
*2)**(2/3)*x)/(c**3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*d**3*x)/(3*d*
(c + d*x))
```

3.255 $\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 684

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{4d(c+dx)^2}$$

$$+\frac{3(c^2-d^2x^2)^{2/3}}{4cd(c+dx)}-\frac{3x}{4c\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right)\right.}{8\sqrt[3]{c}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+\frac{3^{3/4}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right)\right.}{2\sqrt{2}\sqrt[3]{c}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/4*(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)^2+3/4*(-d^2*x^2+c^2)^(2/3)/c/d/(d*x+c)
-3/4*x/c/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))-3/8*3^(1/4)*(1/2*6^(1/
2)+1/2*2^(1/2))*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2
+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3
))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/
2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/c^(1/3)/d^2/x/(-c^(2/3)*(
c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2
)^(1/2)+1/4*3^(3/4)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2
*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^
(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/c^(1/3)/d^2/x
/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c
^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.60 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.12

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx = -\frac{3(c-dx)\sqrt[3]{1+\frac{dx}{c}}(c^2-d^2x^2)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{7}{3},\frac{8}{3},\frac{c-dx}{2c}\right)}{20\sqrt[3]{2}c^2d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^3,x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(1/3)*(c^2 - d^2*x^2)^(2/3)*Hypergeometric2F1[
5/3, 7/3, 8/3, (c - d*x)/(2*c)])/(20*2^(1/3)*c^2*d*(c + d*x))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3} \, dx$$

$$\Big\downarrow \ 506$$

$$-\frac{\left(\frac{1}{c+dx}\right)^{4/3} (c^2-d^2x^2)^{2/3} \int \frac{\left(1-\frac{2c}{c+dx}\right)^{2/3}}{\sqrt[3]{\frac{1}{c+dx}}} d\frac{1}{c+dx}}{d\left(1-\frac{2c}{c+dx}\right)^{2/3}}$$

$$\Big\downarrow \ 74$$

$$-\frac{3(c^2-d^2x^2)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},\frac{2c}{c+dx}\right)}{2d(c+dx)^2\left(1-\frac{2c}{c+dx}\right)^{2/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^3,x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, (2*c)/(c + d*x
)])/(2*d*(c + d*x)^2*(1 - (2*c)/(c + d*x))^(2/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^3}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^3,x)
```

output
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/3)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3),
 x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx=\int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{3}}}{(c+dx)^3}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**3,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx = \int \frac{\left(c^2-d^2\,x^2\right)^{2/3}}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^3,x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^3}\,dx = \frac{-3(-d^2x^2+c^2)^{\frac{2}{3}}-4\left(\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c^2d^2-8\left(\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}\right.}{6d\,(d^2x^2+2cdx+c^2)}$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^3,x)
```

output
```
( - 3*(c**2 - d**2*x**2)**(2/3) - 4*int(((c**2 - d**2*x**2)**(2/3)*x)/(c**
4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c**2*d**2 - 8*int(((c**2 -
d**2*x**2)**(2/3)*x)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c*
d**3*x - 4*int(((c**2 - d**2*x**2)**(2/3)*x)/(c**4 + 2*c**3*d*x - 2*c*d**3
*x**3 - d**4*x**4),x)*d**4*x**2)/(6*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.256 $\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 683

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d} - \frac{12c(7c+2dx)\left(c^2-d^2x^2\right)^{2/3}}{35d} - \frac{48c^3x}{7\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$-\frac{24\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{11/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c}}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+\frac{16\sqrt{2}3^{3/4}c^{11/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-}{\left(1-\sqrt{3}\right)c^{2/3}-}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/10*(d*x+c)^2*(-d^2*x^2+c^2)^(2/3)/d-12/35*c*(2*d*x+7*c)*(-d^2*x^2+c^2)^
(2/3)/d-48*c^3*x/(7*(1-3^(1/2))*c^(2/3)-7*(-d^2*x^2+c^2)^(1/3))-24/7*3^(1/
4)*(1/2*6^(1/2)+1/2*2^(1/2))*c^(11/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(
4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/
3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^(2/3)-(-d^2*x^2
+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2
/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2
+c^2)^(1/3))^2)^(1/2)+16/7*2^(1/2)*3^(3/4)*c^(11/3)*(c^(2/3)-(-d^2*x^2+c^2
)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-
3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(
2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-
I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(
2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.16

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx$$
$$= \frac{3(-63c^4-30c^3dx+56c^2d^2x^2+30cd^3x^3+7d^4x^4)+160c^3dx\sqrt[3]{1-\frac{d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d}{\ }\right.}{70d\sqrt[3]{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)^3/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(3*(-63*c^4 - 30*c^3*d*x + 56*c^2*d^2*x^2 + 30*c*d^3*x^3 + 7*d^4*x^4) + 16
0*c^3*d*x*(1 - (d^2*x^2)/c^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (d^2*
x^2)/c^2])/(70*d*(c^2 - d^2*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {497, 27, 497, 27, 455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{3\int -\frac{16cd^2(c+dx)^2}{3\sqrt[3]{c^2-d^2x^2}}dx}{10d^2}-\frac{3\left(c^2-d^2x^2\right)^{2/3}(c+dx)^2}{10d}$$

$$\Big\downarrow \text{27}$$

$$\frac{8}{5}c\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}dx-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

$$\Big\downarrow \text{497}$$

$$\frac{8}{5}c\left(-\frac{3\int -\frac{10cd^2(c+dx)}{3\sqrt[3]{c^2-d^2x^2}}dx}{7d^2}-\frac{3\left(c^2-d^2x^2\right)^{2/3}(c+dx)}{7d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

$$\Big\downarrow \text{27}$$

$$\frac{8}{5}c\left(\frac{10}{7}c\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}dx-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

$$\Big\downarrow \text{455}$$

$$\frac{8}{5}c\left(\frac{10}{7}c\left(c\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}\right)-\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

$$\Big\downarrow \text{233}$$

$$\frac{8}{5}c\left(\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}\right)-$$

$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

$\downarrow$ 833

$$\frac{8}{5}c\left(\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}\right)}{2d^2x}-\frac{3(c\right.\right.$$

$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

$\downarrow$ 760

$$\frac{8}{5}c\left(\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/}}{}}}{\sqrt[4]{}}\right.}{2d^2x}\right.\right.$$

$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

$\downarrow$ 2418

$$\frac{8}{5}c\left(\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\text{arcsin}\right.}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.}{}\right.\right.$$

$$\frac{3(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}{10d}$$

input
```
Int[(c + d*x)^3/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c + d*x)^2*(c^2 - d^2*x^2)^(2/3))/(10*d) + (8*c*((-3*(c + d*x)*(c^2 -
 d^2*x^2)^(2/3))/(7*d) + (10*c*((-3*(c^2 - d^2*x^2)^(2/3))/(4*d) - (3*c*Sq
rt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^2)])/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*
x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2
)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(
2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticE[ArcSin[
((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (
c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)
*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^
2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3) - (c^
2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 -
 d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*Ellipt
icF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*
c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(d^2*x^
2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2
/3) - (c^2 - d^2*x^2)^(1/3))^2)])))/(2*d^2*x)))/7))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(d^2*x^2 + 2*c*d*x + c^2)*(-d^2*x^2 + c^2)^(2/3)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx = c^{\frac{7}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + \sqrt[3]{c}d^2x^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + 3c^2d\left(\begin{cases}\frac{x^2}{2\sqrt[3]{c^2}} & \text{for } d^2=0\\ -\frac{3(c^2-d^2x^2)^{\frac{2}{3}}}{4d^2} & \text{otherwise}\end{cases}\right) + d^3\left(\left(\begin{cases}-\frac{9c^{\frac{22}{3}}\left(-1+\frac{d^2x^2}{c^2}\right)^{\frac{2}{3}}}{-20c^4d^4e^{\frac{i\pi}{3}}+20c^2d^6x^2e^{\frac{i\pi}{3}}} - \frac{9c^{\frac{22}{3}}}{-20c^4d^4+20c^2d^6x^2} + \frac{3c^{\frac{16}{3}}d^2x^2\left(-1+\frac{d^2x^2}{c^2}\right)^{\frac{2}{3}}}{-20c^4d^4e^{\frac{i\pi}{3}}+20c^2d^6x^2e^{\frac{i\pi}{3}}} + \frac{9c^{\frac{16}{3}}d^2x^2}{-20c^4d^4+20c^2d^6x^2} + \frac{6c^{\frac{10}{3}}d^4x^4(-\\ \frac{9c^{\frac{22}{3}}\left(1-\frac{d^2x^2}{c^2}\right)^{\frac{2}{3}}}{-20c^4d^4+20c^2d^6x^2} - \frac{9c^{\frac{22}{3}}}{-20c^4d^4+20c^2d^6x^2} - \frac{3c^{\frac{16}{3}}d^2x^2\left(1-\frac{d^2x^2}{c^2}\right)^{\frac{2}{3}}}{-20c^4d^4+20c^2d^6x^2} + \frac{9c^{\frac{16}{3}}d^2x^2}{-20c^4d^4+20c^2d^6x^2} - \frac{6c^{\frac{10}{3}}d^4x^4\left(1-\frac{d^2x^2}{c^2}\right)^{\frac{2}{3}}}{-20c^4d^4+20c^2d^6x^2}\end{cases}\right.\right.$$

input
```
integrate((d*x+c)**3/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
c**(7/3)*x*hyper((1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + c
**(1/3)*d**2*x**3*hyper((1/3, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)/c*
*2) + 3*c**2*d*Piecewise((x**2/(2*(c**2)**(1/3)), Eq(d**2, 0)), (-3*(c**2
- d**2*x**2)**(2/3)/(4*d**2), True)) + d**3*Piecewise((-9*c**(22/3)*(-1 +
d**2*x**2/c**2)**(2/3)/(-20*c**4*d**4*exp(I*pi/3) + 20*c**2*d**6*x**2*exp(
I*pi/3)) - 9*c**(22/3)/(-20*c**4*d**4 + 20*c**2*d**6*x**2) + 3*c**(16/3)*d
**2*x**2*(-1 + d**2*x**2/c**2)**(2/3)/(-20*c**4*d**4*exp(I*pi/3) + 20*c**2
*d**6*x**2*exp(I*pi/3)) + 9*c**(16/3)*d**2*x**2/(-20*c**4*d**4 + 20*c**2*d
**6*x**2) + 6*c**(10/3)*d**4*x**4*(-1 + d**2*x**2/c**2)**(2/3)/(-20*c**4*d
**4*exp(I*pi/3) + 20*c**2*d**6*x**2*exp(I*pi/3)), Abs(d**2*x**2/c**2) > 1)
, (9*c**(22/3)*(1 - d**2*x**2/c**2)**(2/3)/(-20*c**4*d**4 + 20*c**2*d**6*x
**2) - 9*c**(22/3)/(-20*c**4*d**4 + 20*c**2*d**6*x**2) - 3*c**(16/3)*d**2*
x**2*(1 - d**2*x**2/c**2)**(2/3)/(-20*c**4*d**4 + 20*c**2*d**6*x**2) + 9*c
**(16/3)*d**2*x**2/(-20*c**4*d**4 + 20*c**2*d**6*x**2) - 6*c**(10/3)*d**4*
x**4*(1 - d**2*x**2/c**2)**(2/3)/(-20*c**4*d**4 + 20*c**2*d**6*x**2), True
))
```

Maxima [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3/(-d^2*x^2 + c^2)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(-d^2*x^2 + c^2)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(c+d\,x)^3}{(c^2-d^2\,x^2)^{1/3}}\,dx$$

input
```
int((c + d*x)^3/(c^2 - d^2*x^2)^(1/3),x)
```

output
```
int((c + d*x)^3/(c^2 - d^2*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{c^2-d^2x^2}}\,dx = \left(\int \frac{x^3}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)d^3 + 3\left(\int \frac{x^2}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)c\,d^2 + 3\left(\int \frac{x}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)c^2d + \left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)c^3$$

input

```
int((d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x)
```

output

```
int(x**3/(c**2 - d**2*x**2)**(1/3),x)*d**3 + 3*int(x**2/(c**2 - d**2*x**2)
**(1/3),x)*c*d**2 + 3*int(x/(c**2 - d**2*x**2)**(1/3),x)*c**2*d + int(1/(c
**2 - d**2*x**2)**(1/3),x)*c**3
```

3.257 $\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 652

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(7c+2dx)\left(c^2-d^2x^2\right)^{2/3}}{14d} - \frac{30c^2x}{7\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{8/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+\frac{10\sqrt{2}3^{3/4}c^{8/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{}}\right.\right.}{7d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/14*(2*d*x+7*c)*(-d^2*x^2+c^2)^(2/3)/d-30*c^2*x/(7*(1-3^(1/2))*c^(2/3)-7
*(-d^2*x^2+c^2)^(1/3))-15/7*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*c^(8/3)*(c^(
2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^
2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*Elliptic
E(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^
2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3)
)/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)+10/7*2^(1/2)*3^(3/4)
*c^(8/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(
1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(
1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(
2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x
^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.15

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx$$
$$=\frac{-21c^3-6c^2dx+21cd^2x^2+6d^3x^3+20c^2dx\sqrt[3]{1-\frac{d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{14d\sqrt[3]{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)^2/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-21*c^3 - 6*c^2*d*x + 21*c*d^2*x^2 + 6*d^3*x^3 + 20*c^2*d*x*(1 - (d^2*x^2
)/c^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (d^2*x^2)/c^2])/(14*d*(c^2 -
 d^2*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {497, 27, 455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \ 497$$

$$-\frac{3\int -\frac{10cd^2(c+dx)}{3\sqrt[3]{c^2-d^2x^2}}dx}{7d^2}-\frac{3\left(c^2-d^2x^2\right)^{2/3}(c+dx)}{7d}$$

$$\Big\downarrow \ 27$$

$$\frac{10}{7}c\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}dx-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}$$

$$\Big\downarrow \ 455$$

$$\frac{10}{7}c\left(c\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}\right)-\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}$$

$$\Big\downarrow \ 233$$

$$\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}\right)-$$
$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}$$

$$\Big\downarrow \ 833$$

$$\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}\right)}{2d^2x}-\frac{3(c^2-}{4}\right.$$
$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}$$

$\downarrow$ 760

$$\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2 \ldots}{\left(\left(1- \ldots\right.\right.}}}{\sqrt[4]{3}\sqrt{- \ldots}}\right.}{2d^2x}\right.$$

$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}$$

$\downarrow$ 2418

$$\frac{10}{7}c\left(-\frac{3c\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+ \ldots\right.}{\left(1- \ldots\right.}\right.\right.}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.}{\ldots}\right.$$

$$\frac{3(c+dx)\left(c^2-d^2x^2\right)^{2/3}}{7d}$$

input
```
Int[(c + d*x)^2/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c + d*x)*(c^2 - d^2*x^2)^(2/3))/(7*d) + (10*c*((-3*(c^2 - d^2*x^2)^(2
/3))/(4*d) - (3*c*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^2)])/((1 - Sqrt[3])*c
^(2/3) - (c^2 - d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(2/3)*(c^(2
/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3)
 + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^
2]*EllipticE[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 -
Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(d^2*x
^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(
2/3) - (c^2 - d^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*c^
(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2
*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1
/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3
^(1/4)*Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))
/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])))/(2*d^2*x)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(2/3)*(d*x + c)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.16

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx = c^{\frac{4}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + 2cd\left(\begin{cases}\frac{x^2}{2\sqrt[3]{c^2}} & \text{for } d^2=0\\ -\frac{3(c^2-d^2x^2)^{\frac{2}{3}}}{4d^2} & \text{otherwise}\end{cases}\right) + \frac{d^2x^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)}{3c^{\frac{2}{3}}}$$

input
```
integrate((d*x+c)**2/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
c**(4/3)*x*hyper((1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + 2
*c*d*Piecewise((x**2/(2*(c**2)**(1/3)), Eq(d**2, 0)), (-3*(c**2 - d**2*x**
2)**(2/3)/(4*d**2), True)) + d**2*x**3*hyper((1/3, 3/2), (5/2,), d**2*x**2
*exp_polar(2*I*pi)/c**2)/(3*c**(2/3))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(-d^2*x^2 + c^2)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(-d^2*x^2 + c^2)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(c+d\,x)^2}{(c^2-d^2\,x^2)^{1/3}}\,dx$$

input
```
int((c + d*x)^2/(c^2 - d^2*x^2)^(1/3),x)
```

output
```
int((c + d*x)^2/(c^2 - d^2*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{c^2-d^2x^2}}\,dx = \left(\int \frac{x^2}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)d^2 + 2\left(\int \frac{x}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)cd + \left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)c^2$$

input
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(x**2/(c**2 - d**2*x**2)**(1/3),x)*d**2 + 2*int(x/(c**2 - d**2*x**2)**(
1/3),x)*c*d + int(1/(c**2 - d**2*x**2)**(1/3),x)*c**2
```

3.258 $\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 637

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{4d} - \frac{3cx}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{5/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}\right.\right.}{2d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$+\frac{\sqrt{2}3^{3/4}c^{5/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/4*(-d^2*x^2+c^2)^(2/3)/d-3*c*x/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3
))-3/2*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*c^(5/3)*(c^(2/3)-(-d^2*x^2+c^2)^(
1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(
1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^(2/3
)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3
^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3
)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)+2^(1/2)*3^(3/4)*c^(5/3)*(c^(2/3)-(-d^2*x^
2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))
/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2)
)*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))
,2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2)
)*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.62 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.13

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{4d} + \frac{cx\sqrt[3]{1-\frac{d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[3]{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3))/(4*d) + (c*x*(1 - (d^2*x^2)/c^2)^(1/3)*Hypergeo
metric2F1[1/3, 1/2, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(1/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {455, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{ 455}$$

$$c\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}$$

$$\Big\downarrow \text{ 233}$$

$$-\frac{3c\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}$$

$$\Big\downarrow \text{ 833}$$

$$-\frac{3c\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}\right)}{2d^2x}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{3c\sqrt{-d^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}\right.}}}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{\cdots}}\right.}{2d^2x}$$

$$\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}$$

$$\Big\downarrow \text{ 2418}$$

$$-\frac{3c\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/\cdots}}{\left(1-\sqrt{3}\right)c^{2/\cdots}}\cdots\right.\right.}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\cdots\right.}{\cdots}$$

$$\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4d}$$

input
```
Int[(c + d*x)/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3))/(4*d) - (3*c*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x
^2)])/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 +
Sqrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)
*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (
c^2 - d^2*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 -
d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*S
qrt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3
)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqr
t[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3
) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*
c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3
) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (
c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]
)))/(2*d^2*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{dx + c}{(-d^2x^2 + c^2)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{dx+c}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(2/3)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.10

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx = \sqrt[3]{c}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + d\left(\begin{cases}\frac{x^2}{2\sqrt[3]{c^2}} & \text{for } d^2=0\\ -\frac{3\left(c^2-d^2x^2\right)^{\frac{2}{3}}}{4d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
c**(1/3)*x*hyper((1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + d
*Piecewise((x**2/(2*(c**2)**(1/3)), Eq(d**2, 0)), (-3*(c**2 - d**2*x**2)**
(2/3)/(4*d**2), True))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{dx+c}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(1/3), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx=\int \frac{dx+c}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.11

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx=\frac{c\,x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(c^2-d^2\,x^2\right)^{1/3}}-\frac{3\,\left(c^2-d^2\,x^2\right)^{2/3}}{4\,d}$$

input
```
int((c + d*x)/(c^2 - d^2*x^2)^(1/3),x)
```

output
```
(c*x*(1 - (d^2*x^2)/c^2)^(1/3)*hypergeom([1/3, 1/2], 3/2, (d^2*x^2)/c^2))/
(c^2 - d^2*x^2)^(1/3) - (3*(c^2 - d^2*x^2)^(2/3))/(4*d)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt[3]{c^2-d^2x^2}}\,dx=\left(\int \frac{x}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}dx\right)d+\left(\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}dx\right)c$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(x/(c**2 - d**2*x**2)**(1/3),x)*d + int(1/(c**2 - d**2*x**2)**(1/3),x)*
c
```

3.259 $\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 613

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3x}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}} - \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}\right.\right.}{2d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}} + \frac{\sqrt{2}3^{3/4}c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2 \cdots}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3*x/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))-3/2*3^(1/4)*(1/2*6^(1/2)+1
/2*2^(1/2))*c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2
*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^
(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2
/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1
/2)+2^(1/2)*3^(3/4)*c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/
3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x
^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3
))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/
3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3
))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx=\frac{x\sqrt[3]{1-\frac{d^2x^2}{c^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[3]{c^2-d^2x^2}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(-1/3),x]
```

output
```
(x*(1 - (d^2*x^2)/c^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (d^2*x^2)/c^
2])/(c^2 - d^2*x^2)^(1/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx$$

$\big\downarrow$ 233

$$-\frac{3\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}$$

$\big\downarrow$ 833

$$-\frac{3\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}\right)}{2d^2x}$$

$\big\downarrow$ 760

$$-\frac{3\sqrt{-d^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/\cdots}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\cdots\right.}}}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\cdots}}\cdots\right.}{2d^2x}$$

$\big\downarrow$ 2418

$$-\frac{3\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}\cdots}{\left(1-\sqrt{3}\right)c^{2/3}\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\cdots\right.}{\cdots}$$

input

```
Int[(c^2 - d^2*x^2)^(-1/3),x]
```

output
```
(-3*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^2)])/((1 - Sqrt[3])*c^(2/3) - (c^2
- d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d
^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*
x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/
3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c
^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 -
d^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3)
 - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) +
(c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqr
t[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(
d^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3]
)*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])))/(2*d^2*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx=\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(2/3)/(d^2*x^2 - c^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)}{c^{\frac{2}{3}}}$$

input
```
integrate(1/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
x*hyper((1/3, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)/c**(2/3)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx=\frac{x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(c^2-d^2\,x^2\right)^{1/3}}$$

input
```
int(1/(c^2 - d^2*x^2)^(1/3),x)
```

output
```
(x*(1 - (d^2*x^2)/c^2)^(1/3)*hypergeom([1/3, 1/2], 3/2, (d^2*x^2)/c^2))/(c
^2 - d^2*x^2)^(1/3)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}\,dx=\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/(c**2 - d**2*x**2)**(1/3),x)
```

3.260 $\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 652

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{2cd(c+dx)} + \frac{3x}{2c\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2}}\right.\right.}{4\sqrt[3]{c}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$-\frac{3^{3/4}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right.\right.}{\sqrt{2}\sqrt[3]{c}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/2*(-d^2*x^2+c^2)^(2/3)/c/d/(d*x+c)+3/2*x/c/((1-3^(1/2))*c^(2/3)-(-d^2*x
^2+c^2)^(1/3))+3/4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(c^(2/3)-(-d^2*x^2+c^
2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1
-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*c^
(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I
-I*3^(1/2))/c^(1/3)/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(
1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)-1/2*3^(3/4)*(c^(2/3)-(-d^2*x^
2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))
/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2)
)*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))
,2*I-I*3^(1/2))*2^(1/2)/c^(1/3)/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1
/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.11

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c-dx)\sqrt[3]{1+\frac{dx}{c}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},\frac{c-dx}{2c}\right)}{4\sqrt[3]{2}cd\sqrt[3]{c^2-d^2x^2}}$$

input
```
Integrate[1/((c + d*x)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(1/3)*Hypergeometric2F1[2/3, 4/3, 5/3, (c - d*
x)/(2*c)])/(4*2^(1/3)*c*d*(c^2 - d^2*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {504, 215, 233, 241, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}dx$$

$\downarrow$ 504

$$c\int \frac{1}{(c^2-d^2x^2)^{4/3}}dx-d\int \frac{x}{(c^2-d^2x^2)^{4/3}}dx$$

$\downarrow$ 215

$$c\left(\frac{3x}{2c^2\sqrt[3]{c^2-d^2x^2}}-\frac{\int \frac{1}{\sqrt[3]{c^2-d^2x^2}}dx}{2c^2}\right)-d\int \frac{x}{(c^2-d^2x^2)^{4/3}}dx$$

$\downarrow$ 233

$$c\left(\frac{3\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{4c^2d^2x}+\frac{3x}{2c^2\sqrt[3]{c^2-d^2x^2}}\right)-d\int \frac{x}{(c^2-d^2x^2)^{4/3}}dx$$

$\downarrow$ 241

$$c\left(\frac{3\sqrt{-d^2x^2}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{4c^2d^2x}+\frac{3x}{2c^2\sqrt[3]{c^2-d^2x^2}}\right)-\frac{3}{2d\sqrt[3]{c^2-d^2x^2}}$$

$\downarrow$ 833

$$c\left(\frac{3\sqrt{-d^2x^2}\left(\left(1+\sqrt{3}\right)c^{2/3}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}\right)}{4c^2d^2x}+\frac{3x}{2c^2\sqrt[3]{c^2-d^2}}\right.$$

$$\frac{3}{2d\sqrt[3]{c^2-d^2x^2}}$$

$\downarrow$ 760

$$c\left(\frac{3\sqrt{-d^2x^2}\left(-\int \frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}\right.}}}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{}}\right.}{4c^2d^2x}\right.$$

$$\frac{3}{2d\sqrt[3]{c^2-d^2x^2}}$$

$\downarrow$ 2418

$$c\left(\frac{3\sqrt{-d^2x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/}}{\left(1-\sqrt{3}\right)c^{2/}}\right.\right.}{\sqrt[4]{3}\sqrt{-d^2x^2}\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.}{}\right.$$

$$\frac{3}{2d\sqrt[3]{c^2-d^2x^2}}$$

input
```
Int[1/((c + d*x)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
-3/(2*d*(c^2 - d^2*x^2)^(1/3)) + c*((3*x)/(2*c^2*(c^2 - d^2*x^2)^(1/3)) +
(3*Sqrt[-(d^2*x^2)]*((-2*Sqrt[-(d^2*x^2)])/((1 - Sqrt[3])*c^(2/3) - (c^2 -
 d^2*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(2/3)*(c^(2/3) - (c^2 - d^
2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x
^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3
) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-(d^2*x^2)]*Sqrt[-((c^
(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d
^2*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*c^(2/3)*(c^(2/3)
- (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (
c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt
[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-(d
^2*x^2)]*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])
*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])))/(4*c^2*d^2*x))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/(d*x+c)/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(2/3)/(d^3*x^3 + c*d^2*x^2 - c^2*d*x - c^3), x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-(-c+dx)(c+dx)}(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(c^2-d^2\,x^2)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}\,c+(-d^2x^2+c^2)^{\frac{1}{3}}\,dx}dx$$

input
```
int(1/(d*x+c)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c**2 - d**2*x**2)**(1/3)*c + (c**2 - d**2*x**2)**(1/3)*d*x),x)
```

3.261 $\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 675

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx$$

$$= \frac{3x}{8c^2\sqrt[3]{c^2-d^2x^2}} - \frac{3}{4d(c+dx)\sqrt[3]{c^2-d^2x^2}} + \frac{3x}{8c^2\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2}}\right.\right.}{16c^{4/3}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$- \frac{3^{3/4}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right.\right.}{4\sqrt{2}c^{4/3}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/8*x/c^2/(-d^2*x^2+c^2)^(1/3)-3/4/d/(d*x+c)/(-d^2*x^2+c^2)^(1/3)+3/8*x/c^
2/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))+3/16*3^(1/4)*(1/2*6^(1/2)+1/2
*2^(1/2))*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^
(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^
(1/2)*EllipticE(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^
(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/c^(4/3)/d^2/x/(-c^(2/3)*(c^(2/3
)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2
)-1/8*3^(3/4)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c
^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))
^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2)
)*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/c^(4/3)/d^2/x/(-c^(
2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1
/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.97 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.11

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c-dx)\sqrt[3]{1+\frac{dx}{c}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{5}{3},\frac{c-dx}{2c}\right)}{8\sqrt[3]{2}c^2d\sqrt[3]{c^2-d^2x^2}}$$

input
```
Integrate[1/((c + d*x)^2*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(1/3)*Hypergeometric2F1[2/3, 7/3, 5/3, (c - d*
x)/(2*c)])/(8*2^(1/3)*c^2*d*(c^2 - d^2*x^2)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{506}$$

$$-\frac{\sqrt[3]{1-\frac{2c}{c+dx}}\int \frac{\left(\frac{1}{c+dx}\right)^{2/3}}{\sqrt[3]{1-\frac{2c}{c+dx}}}d\frac{1}{c+dx}}{d\left(\frac{1}{c+dx}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{3\sqrt[3]{1-\frac{2c}{c+dx}}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},\frac{2c}{c+dx}\right)}{5d(c+dx)\sqrt[3]{c^2-d^2x^2}}$$

input
```
Int[1/((c + d*x)^2*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(1 - (2*c)/(c + d*x))^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, (2*c)/(c
+ d*x)])/(5*d*(c + d*x)*(c^2 - d^2*x^2)^(1/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{1}{(dx+c)^2 (-d^2x^2+c^2)^{\frac{1}{3}}} dx$$

input
```
int(1/(d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/(d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}} dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}} (dx+c)^2} dx$$

input
```
integrate(1/(d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(2/3)/(d^4*x^4 + 2*c*d^3*x^3 - 2*c^3*d*x - c^4)
, x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-(-c+dx)(c+dx)}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\left(c^2-d^2\,x^2\right)^{1/3}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^2),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^2+2\left(-d^2x^2+c^2\right)^{\frac{1}{3}}cdx+\left(-d^2x^2+c^2\right)^{\frac{1}{3}}d^2x^2}dx$$

input
```
int(1/(d*x+c)^2/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c**2 - d**2*x**2)**(1/3)*c**2 + 2*(c**2 - d**2*x**2)**(1/3)*c*d*x
+ (c**2 - d**2*x**2)**(1/3)*d**2*x**2),x)
```

3.262 $\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 708

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx = \frac{15x}{112c^3\sqrt[3]{c^2-d^2x^2}} - \frac{3}{7d(c+dx)^2\sqrt[3]{c^2-d^2x^2}}$$

$$- \frac{3}{56cd(c+dx)\sqrt[3]{c^2-d^2x^2}} + \frac{15x}{112c^3\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}$$

$$+ \frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\cdots}}\right.\right.}{224c^{7/3}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$- \frac{5\,3^{3/4}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2\cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2\cdots}}\right.\right.}{56\sqrt{2}c^{7/3}d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
15/112*x/c^3/(-d^2*x^2+c^2)^(1/3)-3/7/d/(d*x+c)^2/(-d^2*x^2+c^2)^(1/3)-3/5
6/c/d/(d*x+c)/(-d^2*x^2+c^2)^(1/3)+15/112*x/c^3/((1-3^(1/2))*c^(2/3)-(-d^2
*x^2+c^2)^(1/3))+15/224*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(c^(2/3)-(-d^2*x
^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3)
)/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2
))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)
),2*I-I*3^(1/2))/c^(7/3)/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((
1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)-5/112*3^(3/4)*(c^(2/3)-(
-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)
^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+
3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)
^(1/3)),2*I-I*3^(1/2))*2^(1/2)/c^(7/3)/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+
c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.10

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c-dx)\sqrt[3]{1+\frac{dx}{c}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{10}{3},\frac{5}{3},\frac{c-dx}{2c}\right)}{16\sqrt[3]{2}c^3d\sqrt[3]{c^2-d^2x^2}}$$

input
```
Integrate[1/((c + d*x)^3*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(1/3)*Hypergeometric2F1[2/3, 10/3, 5/3, (c - d
*x)/(2*c)])/(16*2^(1/3)*c^3*d*(c^2 - d^2*x^2)^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.09, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{506}$$

$$-\frac{\sqrt[3]{1-\frac{2c}{c+dx}}\int \frac{\left(\frac{1}{c+dx}\right)^{5/3}}{\sqrt[3]{1-\frac{2c}{c+dx}}}d\frac{1}{c+dx}}{d\left(\frac{1}{c+dx}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{3\sqrt[3]{1-\frac{2c}{c+dx}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},\frac{2c}{c+dx}\right)}{8d(c+dx)^2\sqrt[3]{c^2-d^2x^2}}$$

input
```
Int[1/((c + d*x)^3*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(1 - (2*c)/(c + d*x))^(1/3)*Hypergeometric2F1[1/3, 8/3, 11/3, (2*c)/(c
 + d*x)])/(8*d*(c + d*x)^2*(c^2 - d^2*x^2)^(1/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{1}{(dx+c)^3 (-d^2x^2+c^2)^{\frac{1}{3}}} dx$$

input
```
int(1/(d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/(d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{c^2-d^2x^2}} dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}} (dx+c)^3} dx$$

input
```
integrate(1/(d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(2/3)/(d^5*x^5 + 3*c*d^4*x^4 + 2*c^2*d^3*x^3 -
2*c^3*d^2*x^2 - 3*c^4*d*x - c^5), x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-(-c+dx)(c+dx)}\,(c+dx)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\left(c^2-d^2\,x^2\right)^{1/3}\,\left(c+d\,x\right)^3}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^3),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{c^2-d^2x^2}}\,dx$$
$$= \int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^3+3\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^2dx+3\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c\,d^2x^2+\left(-d^2x^2+c^2\right)^{\frac{1}{3}}d^3x^3}dx$$

input
```
int(1/(d*x+c)^3/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c**2 - d**2*x**2)**(1/3)*c**3 + 3*(c**2 - d**2*x**2)**(1/3)*c**2*d
*x + 3*(c**2 - d**2*x**2)**(1/3)*c*d**2*x**2 + (c**2 - d**2*x**2)**(1/3)*d
**3*x**3),x)
```

3.263 $\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 353

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3(c+dx)^2\sqrt[3]{c^2-d^2x^2}}{8d} - \frac{21c(5c+dx)\sqrt[3]{c^2-d^2x^2}}{20d}$$
$$+\frac{14\,3^{3/4}\sqrt{2-\sqrt{3}}c^3\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-}{\left(1-\sqrt{3}\right)c^{2/3}-}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/8*(d*x+c)^2*(-d^2*x^2+c^2)^(1/3)/d-21/20*c*(d*x+5*c)*(-d^2*x^2+c^2)^(1/
3)/d+14/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c^3*(c^(2/3)-(-d^2*x^2+c^2)^(1
/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1
/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)
-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^
(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)
-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.31

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3(-75c^4-24c^3dx+70c^2d^2x^2+24cd^3x^3+5d^4x^4)+112c^3dx\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}\text{Hyperge}}{40d\,(c^2-d^2x^2)^{2/3}}$$

input
```
Integrate[(c + d*x)^3/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(3*(-75*c^4 - 24*c^3*d*x + 70*c^2*d^2*x^2 + 24*c*d^3*x^3 + 5*d^4*x^4) + 11
2*c^3*d*x*(1 - (d^2*x^2)/c^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, (d^2*
x^2)/c^2])/(40*d*(c^2 - d^2*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {497, 27, 497, 27, 455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{3\int -\frac{14cd^2(c+dx)^2}{3(c^2-d^2x^2)^{2/3}}dx}{8d^2}-\frac{3\sqrt[3]{c^2-d^2x^2}(c+dx)^2}{8d}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}c\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)^2\sqrt[3]{c^2-d^2x^2}}{8d}$$

$$\Big\downarrow \text{497}$$

$$\frac{7}{4}c\left(-\frac{3\int -\frac{8cd^2(c+dx)}{3(c^2-d^2x^2)^{2/3}}dx}{5d^2}-\frac{3\sqrt[3]{c^2-d^2x^2}(c+dx)}{5d}\right)-\frac{3(c+dx)^2\sqrt[3]{c^2-d^2x^2}}{8d}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}c\left(\frac{8}{5}c\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)\sqrt[3]{c^2-d^2x^2}}{5d}\right)-\frac{3(c+dx)^2\sqrt[3]{c^2-d^2x^2}}{8d}$$

$$\Big\downarrow \text{455}$$

$$\frac{7}{4}c\left(\frac{8}{5}c\left(c\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx-\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}\right)-\frac{3(c+dx)\sqrt[3]{c^2-d^2x^2}}{5d}\right)-\frac{3(c+dx)^2\sqrt[3]{c^2-d^2x^2}}{8d}$$

$$\Big\downarrow \text{234}$$

$$\frac{7}{4}c\left(\frac{8}{5}c\left(-\frac{3c\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}\right)-\frac{3(c+dx)\sqrt[3]{c^2-d^2x^2}}{5d}\right)-\frac{3(c+dx)^2\sqrt[3]{c^2-d^2x^2}}{8d}$$

$$\Big\downarrow \text{760}$$

$$\frac{7}{4}c\left(\frac{8}{5}c\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}}{\left(1-\sqrt{3}\right)c^{2/3}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.\right.$$

$$\frac{3(c+dx)^2\sqrt[3]{c^2-d^2x^2}}{8d}$$

input
```
Int[(c + d*x)^3/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c + d*x)^2*(c^2 - d^2*x^2)^(1/3))/(8*d) + (7*c*((-3*(c + d*x)*(c^2 -
d^2*x^2)^(1/3))/(5*d) + (8*c*((-3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (3^(3/4)*
Sqrt[2 - Sqrt[3]]*c*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2
/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3)
- (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2
 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(d^2*x*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1
- Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])))/5))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(d^2*x^2 + 2*c*d*x + c^2)*(-d^2*x^2 + c^2)^(1/3)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = c^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)+3c^2d\left(\begin{cases}\frac{x^2}{2(c^2)^{\frac{2}{3}}} & \text{for } d^2=0\\ -\frac{3\sqrt[3]{c^2-d^2x^2}}{2d^2} & \text{otherwise}\end{cases}\right)$$
$$+d^3\left(\begin{cases}-\frac{9c^{\frac{20}{3}}\sqrt[3]{-1+\frac{d^2x^2}{c^2}}}{-8c^4d^4e^{\frac{2i\pi}{3}}+8c^2d^6x^2e^{\frac{2i\pi}{3}}}-\frac{9c^{\frac{20}{3}}}{-8c^4d^4+8c^2d^6x^2}+\frac{6c^{\frac{14}{3}}d^2x^2\sqrt[3]{-1+\frac{d^2x^2}{c^2}}}{-8c^4d^4e^{\frac{2i\pi}{3}}+8c^2d^6x^2e^{\frac{2i\pi}{3}}}+\frac{9c^{\frac{14}{3}}d^2x^2}{-8c^4d^4+8c^2d^6x^2}+\frac{3c^{\frac{8}{3}}d^4x^4\sqrt[3]{-1+\frac{d^2x^2}{c^2}}}{-8c^4d^4e^{\frac{2i\pi}{3}}+8c^2d^6x^2e^{\frac{2i\pi}{3}}} \\ \frac{9c^{\frac{20}{3}}\sqrt[3]{1-\frac{d^2x^2}{c^2}}}{-8c^4d^4+8c^2d^6x^2}-\frac{9c^{\frac{20}{3}}}{-8c^4d^4+8c^2d^6x^2}-\frac{6c^{\frac{14}{3}}d^2x^2\sqrt[3]{1-\frac{d^2x^2}{c^2}}}{-8c^4d^4+8c^2d^6x^2}+\frac{9c^{\frac{14}{3}}d^2x^2}{-8c^4d^4+8c^2d^6x^2}-\frac{3c^{\frac{8}{3}}d^4x^4\sqrt[3]{1-\frac{d^2x^2}{c^2}}}{-8c^4d^4+8c^2d^6x^2}\end{cases}\right)$$
$$+\frac{d^2x^3\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)}{\sqrt[3]{c}}$$

input
```
integrate((d*x+c)**3/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
c**(5/3)*x*hyper((1/2, 2/3), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + 3
*c**2*d*Piecewise((x**2/(2*(c**2)**(2/3)), Eq(d**2, 0)), (-3*(c**2 - d**2*
x**2)**(1/3)/(2*d**2), True)) + d**3*Piecewise((-9*c**(20/3)*(-1 + d**2*x*
*2/c**2)**(1/3)/(-8*c**4*d**4*exp(2*I*pi/3) + 8*c**2*d**6*x**2*exp(2*I*pi/
3)) - 9*c**(20/3)/(-8*c**4*d**4 + 8*c**2*d**6*x**2) + 6*c**(14/3)*d**2*x**
2*(-1 + d**2*x**2/c**2)**(1/3)/(-8*c**4*d**4*exp(2*I*pi/3) + 8*c**2*d**6*x
**2*exp(2*I*pi/3)) + 9*c**(14/3)*d**2*x**2/(-8*c**4*d**4 + 8*c**2*d**6*x**
2) + 3*c**(8/3)*d**4*x**4*(-1 + d**2*x**2/c**2)**(1/3)/(-8*c**4*d**4*exp(2
*I*pi/3) + 8*c**2*d**6*x**2*exp(2*I*pi/3)), Abs(d**2*x**2/c**2) > 1), (9*c
**(20/3)*(1 - d**2*x**2/c**2)**(1/3)/(-8*c**4*d**4 + 8*c**2*d**6*x**2) - 9
*c**(20/3)/(-8*c**4*d**4 + 8*c**2*d**6*x**2) - 6*c**(14/3)*d**2*x**2*(1 -
d**2*x**2/c**2)**(1/3)/(-8*c**4*d**4 + 8*c**2*d**6*x**2) + 9*c**(14/3)*d**
2*x**2/(-8*c**4*d**4 + 8*c**2*d**6*x**2) - 3*c**(8/3)*d**4*x**4*(1 - d**2*
x**2/c**2)**(1/3)/(-8*c**4*d**4 + 8*c**2*d**6*x**2), True)) + d**2*x**3*hy
per((2/3, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)/c**(1/3)
```

Maxima [F]

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3/(-d^2*x^2 + c^2)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(c+d\,x)^3}{(c^2-d^2\,x^2)^{2/3}}\,dx$$

input
```
int((c + d*x)^3/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
int((c + d*x)^3/(c^2 - d^2*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{-9(-d^2x^2+c^2)^{\frac{1}{3}}\,c^2+2\left(\int \frac{x^3}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)d^4+6\left(\int \frac{x^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)c\,d^3+2\left(\int \frac{}{(-}\right.}{2d}$$

input
```
int((d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 9*(c**2 - d**2*x**2)**(1/3)*c**2 + 2*int(x**3/(c**2 - d**2*x**2)**(2/3
),x)*d**4 + 6*int(x**2/(c**2 - d**2*x**2)**(2/3),x)*c*d**3 + 2*int(1/(c**2
 - d**2*x**2)**(2/3),x)*c**3*d)/(2*d)
```

3.264 $\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 322

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3(5c+dx)\sqrt[3]{c^2-d^2x^2}}{5d} + \frac{8\ 3^{3/4}\sqrt{2-\sqrt{3}}c^2\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\cdots}{\left(1-\sqrt{3}\right)c^{2/3}-\cdots}\right.\right.}{5d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/5*(d*x+5*c)*(-d^2*x^2+c^2)^(1/3)/d+8/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2)
)*c^2*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3
)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2
)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3
)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+
c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.31

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3(-5c^3-c^2dx+5cd^2x^2+d^3x^3)+8c^2dx\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2}\right.}{5d\,(c^2-d^2x^2)^{2/3}}$$

input
```
Integrate[(c + d*x)^2/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(3*(-5*c^3 - c^2*d*x + 5*c*d^2*x^2 + d^3*x^3) + 8*c^2*d*x*(1 - (d^2*x^2)/c
^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, (d^2*x^2)/c^2])/(5*d*(c^2 - d^2
*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {497, 27, 455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{497}$$

$$-\frac{3\int -\frac{8cd^2(c+dx)}{3(c^2-d^2x^2)^{2/3}}dx}{5d^2}-\frac{3\sqrt[3]{c^2-d^2x^2}(c+dx)}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{8}{5}c\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)\sqrt[3]{c^2-d^2x^2}}{5d}$$

$$\Big\downarrow \text{455}$$

$$\frac{8}{5}c\left(c\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx-\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}\right)-\frac{3(c+dx)\sqrt[3]{c^2-d^2x^2}}{5d}$$

$$\Big\downarrow \text{234}$$

$$\frac{8}{5}c\left(-\frac{3c\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}\right)-\frac{3(c+dx)\sqrt[3]{c^2-d^2x^2}}{5d}$$

$$\Big\downarrow \text{760}$$

$$\frac{8}{5}c\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.$$

$$\frac{3(c+dx)\sqrt[3]{c^2-d^2x^2}}{5d}$$

input
```
Int[(c + d*x)^2/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c + d*x)*(c^2 - d^2*x^2)^(1/3))/(5*d) + (8*c*((-3*(c^2 - d^2*x^2)^(1/
3))/(2*d) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*c*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))
*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((
1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sq
rt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^
2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(d^2*x*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 -
d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(1/3)*(d*x + c)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.33

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = c^{\frac{2}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + 2cd\left(\begin{cases}\frac{x^2}{2(c^2)^{\frac{2}{3}}} & \text{for } d^2=0\\ -\frac{3\sqrt[3]{c^2-d^2x^2}}{2d^2} & \text{otherwise}\end{cases}\right) + \frac{d^2x^3\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)}{3c^{\frac{4}{3}}}$$

input
```
integrate((d*x+c)**2/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
c**(2/3)*x*hyper((1/2, 2/3), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + 2
*c*d*Piecewise((x**2/(2*(c**2)**(2/3)), Eq(d**2, 0)), (-3*(c**2 - d**2*x**
2)**(1/3)/(2*d**2), True)) + d**2*x**3*hyper((2/3, 3/2), (5/2,), d**2*x**2
*exp_polar(2*I*pi)/c**2)/(3*c**(4/3))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(-d^2*x^2 + c^2)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(c+d\,x)^2}{(c^2-d^2\,x^2)^{2/3}}\,dx$$

input
```
int((c + d*x)^2/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
int((c + d*x)^2/(c^2 - d^2*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{-3(-d^2x^2+c^2)^{\frac{1}{3}}c+\left(\int \frac{x^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)d^3+\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)c^2d}{d}$$

input
```
int((d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 3*(c**2 - d**2*x**2)**(1/3)*c + int(x**2/(c**2 - d**2*x**2)**(2/3),x)*
d**3 + int(1/(c**2 - d**2*x**2)**(2/3),x)*c**2*d)/d
```

3.265 $\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 310

$$\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\sqrt[3]{c^2-d^2x^2}}{2d} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right) \ldots}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/2*(-d^2*x^2+c^2)^(1/3)/d+3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c*(c^(2/3)-(
-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)
^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+
3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)
^(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-
3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

$$\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}+\frac{cx\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{(c^2-d^2x^2)^{2/3}}$$

input
```
Integrate[(c + d*x)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (c*x*(1 - (d^2*x^2)/c^2)^(2/3)*Hypergeo
metric2F1[1/2, 2/3, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx-\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}$$

$$\Big\downarrow \text{234}$$

$$-\frac{3c\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}$$

$$\Big\downarrow \text{760}$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d\ldots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d\ldots}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}$$

input
```
Int[(c + d*x)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*c*(c^(2/3) -
 (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c
^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[
3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(d^2*x*Sqrt[-((c^(
2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^
2*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{dx + c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{c + dx}{(c^2 - d^2x^2)^{2/3}}\, dx = \int \frac{dx + c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{3}}}\, dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(1/3)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.21

$$\int \frac{c+dx}{\left(c^2-d^2x^2\right)^{2/3}}\,dx = d\left(\begin{cases} \frac{x^2}{2(c^2)^{\frac{2}{3}}} & \text{for } d^2=0 \\ -\frac{3\sqrt[3]{c^2-d^2x^2}}{2d^2} & \text{otherwise} \end{cases}\right) + \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2},\frac{2}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{\sqrt[3]{c}}$$

input
```
integrate((d*x+c)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
d*Piecewise((x**2/(2*(c**2)**(2/3)), Eq(d**2, 0)), (-3*(c**2 - d**2*x**2)*
*(1/3)/(2*d**2), True)) + x*hyper((1/2, 2/3), (3/2,), d**2*x**2*exp_polar(
2*I*pi)/c**2)/c**(1/3)
```

Maxima [F]

$$\int \frac{c+dx}{\left(c^2-d^2x^2\right)^{2/3}}\,dx = \int \frac{dx+c}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(2/3), x)
```

Giac [F]

$$\int \frac{c+dx}{\left(c^2-d^2x^2\right)^{2/3}}\,dx = \int \frac{dx+c}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{c\,x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{(c^2-d^2\,x^2)^{2/3}} - \frac{3\,(c^2-d^2\,x^2)^{1/3}}{2\,d}$$

input
```
int((c + d*x)/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
(c*x*(1 - (d^2*x^2)/c^2)^(2/3)*hypergeom([1/2, 2/3], 3/2, (d^2*x^2)/c^2))/
(c^2 - d^2*x^2)^(2/3) - (3*(c^2 - d^2*x^2)^(1/3))/(2*d)
```

Reduce [F]

$$\int \frac{c+dx}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{-3(-d^2x^2+c^2)^{\frac{1}{3}}+2\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)cd}{2d}$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 3*(c**2 - d**2*x**2)**(1/3) + 2*int(1/(c**2 - d**2*x**2)**(2/3),x)*c*d
)/(2*d)
```

3.266 $\int \frac{1}{\left(c^2-d^2x^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 285

$$\int \frac{1}{\left(c^2-d^2x^2\right)^{2/3}}\,dx=\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\text{arc}\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)
+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(
-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2
)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/d^2/x/(
-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2
)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.19

$$\int \frac{1}{(c^2-d^2x^2)^{2/3}}\,dx=\frac{x\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{(c^2-d^2x^2)^{2/3}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(-2/3),x]
```

output
```
(x*(1 - (d^2*x^2)/c^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, (d^2*x^2)/c^
2])/(c^2 - d^2*x^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{234}$$

$$-\frac{3\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}$$

$$\Big\downarrow \text{760}$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2\cdots}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2\cdots}}\cdots\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

input
```
Int[(c^2 - d^2*x^2)^(-2/3),x]
```

output
```
(3^(3/4)*Sqrt[2 - Sqrt[3]]*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3)
 + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c
^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3)
 - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(d^2*x*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)
))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(c^2-d^2x^2)^{2/3}}\,dx=\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(1/3)/(d^2*x^2 - c^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int \frac{1}{(c^2-d^2x^2)^{2/3}}\,dx=\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)}{c^{\frac{4}{3}}}$$

input
```
integrate(1/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
x*hyper((1/2, 2/3), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)/c**(4/3)
```

Maxima [F]

$$\int \frac{1}{(c^2 - d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2 + c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(c^2 - d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2 + c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 7.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.16

$$\int \frac{1}{(c^2 - d^2x^2)^{2/3}}\,dx = \frac{x\left(1 - \frac{d^2x^2}{c^2}\right)^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3};\ \frac{3}{2};\ \frac{d^2x^2}{c^2}\right)}{(c^2 - d^2x^2)^{2/3}}$$

input
```
int(1/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
(x*(1 - (d^2*x^2)/c^2)^(2/3)*hypergeom([1/2, 2/3], 3/2, (d^2*x^2)/c^2))/(c
^2 - d^2*x^2)^(2/3)
```

Reduce [F]

$$\int \frac{1}{(c^2 - d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2 + c^2)^{\frac{2}{3}}}dx$$

input
```
int(1/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(c**2 - d**2*x**2)**(2/3),x)
```

3.267 $\int \frac{1}{(c+dx)\left(c^2-d^2x^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 325

$$\int \frac{1}{(c+dx)\left(c^2-d^2x^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{c^2-d^2x^2}}{4cd(c+dx)} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}\right.\right.}{4cd^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
-3/4*(-d^2*x^2+c^2)^(1/3)/c/d/(d*x+c)+1/4*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2)
)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-
d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*El
lipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-
d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/c/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^
2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.74 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int \frac{1}{(c+dx)(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3(c-dx)\left(1+\frac{dx}{c}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{4}{3},\frac{c-dx}{2c}\right)}{2\,2^{2/3}cd\,(c^2-d^2x^2)^{2/3}}$$

input
```
Integrate[1/((c + d*x)*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, (c - d*
x)/(2*c)])/(2*2^(2/3)*c*d*(c^2 - d^2*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {504, 215, 234, 241, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(c^2-d^2x^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{504}$$

$$c\int \frac{1}{(c^2-d^2x^2)^{5/3}}dx - d\int \frac{x}{(c^2-d^2x^2)^{5/3}}dx$$

$$\Big\downarrow \text{215}$$

$$c\left(\frac{\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx}{4c^2}+\frac{3x}{4c^2(c^2-d^2x^2)^{2/3}}\right) - d\int \frac{x}{(c^2-d^2x^2)^{5/3}}dx$$

$$\Big\downarrow \text{234}$$

$$c\left(\frac{3x}{4c^2\left(c^2-d^2x^2\right)^{2/3}}-\frac{3\sqrt{-d^2x^2}\int\frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{8c^2d^2x}\right)-d\int\frac{x}{\left(c^2-d^2x^2\right)^{5/3}}dx$$

$$\Big\downarrow\ 241$$

$$c\left(\frac{3x}{4c^2\left(c^2-d^2x^2\right)^{2/3}}-\frac{3\sqrt{-d^2x^2}\int\frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{8c^2d^2x}\right)-\frac{3}{4d\left(c^2-d^2x^2\right)^{2/3}}$$

$$\Big\downarrow\ 760$$

$$c\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+\left(c^2-d^2x^2\right)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\right.}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-\right.}\right.\right.}{4c^2d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}\right.$$

$$\frac{3}{4d\left(c^2-d^2x^2\right)^{2/3}}$$

input
```
Int[1/((c + d*x)*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
-3/(4*d*(c^2 - d^2*x^2)^(2/3)) + c*((3*x)/(4*c^2*(c^2 - d^2*x^2)^(2/3)) +
(3^(3/4)*Sqrt[2 - Sqrt[3]]*(c^(2/3) - (c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3)
 + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c
^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*c^(2/3)
 - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(4*c^2*d^2*x*Sqrt[-((c^(2/3)*(c^(2/3) - (c^2 - d^2*x^2)
^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2)]))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(c+dx)(c^2-d^2x^2)^{2/3}}\,dx=\int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(1/3)/(d^3*x^3 + c*d^2*x^2 - c^2*d*x - c^3), x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)(c^2-d^2x^2)^{2/3}}\,dx=\int \frac{1}{(-(-c+dx)(c+dx))^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(2/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)(c^2-d^2x^2)^{2/3}}\,dx=\int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\left(c^2-d^2x^2\right)^{2/3}}\,dx = \int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(c^2-d^2x^2\right)^{2/3}}\,dx = \int \frac{1}{\left(c^2-d^2\,x^2\right)^{2/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\left(c^2-d^2x^2\right)^{2/3}}\,dx = \int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c+\left(-d^2x^2+c^2\right)^{\frac{2}{3}}dx}dx$$

input
```
int(1/(d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/((c**2 - d**2*x**2)**(2/3)*c + (c**2 - d**2*x**2)**(2/3)*d*x),x)
```

3.268 $\int \frac{1}{(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 346

$$\int \frac{1}{(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}\,dx = \frac{3x}{10c^2\left(c^2-d^2x^2\right)^{2/3}} - \frac{3}{5d(c+dx)\left(c^2-d^2x^2\right)^{2/3}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2}}\right.\right.}{10c^2d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/10*x/c^2/(-d^2*x^2+c^2)^(2/3)-3/5/d/(d*x+c)/(-d^2*x^2+c^2)^(2/3)+1/10*3^
(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c
^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d
^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^
(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/c^2/d^2/x
/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c
^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.21

$$\int \frac{1}{(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}\,dx=-\frac{3(c-dx)\left(1+\frac{dx}{c}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{4}{3},\frac{c-dx}{2c}\right)}{4\,2^{2/3}c^2d\left(c^2-d^2x^2\right)^{2/3}}$$

input
```
Integrate[1/((c + d*x)^2*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, (c - d*
x)/(2*c)])/(4*2^(2/3)*c^2*d*(c^2 - d^2*x^2)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.19, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}dx$$

$$\Big\downarrow \text{506}$$

$$-\frac{\left(1-\frac{2c}{c+dx}\right)^{2/3}\int \frac{\left(\frac{1}{c+dx}\right)^{4/3}}{\left(1-\frac{2c}{c+dx}\right)^{2/3}}d\frac{1}{c+dx}}{d\left(\frac{1}{c+dx}\right)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}$$

$$\Big\downarrow \text{74}$$

$$-\frac{3\left(1-\frac{2c}{c+dx}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},\frac{2c}{c+dx}\right)}{7d(c+dx)\left(c^2-d^2x^2\right)^{2/3}}$$

input
```
Int[1/((c + d*x)^2*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(-3*(1 - (2*c)/(c + d*x))^(2/3)*Hypergeometric2F1[2/3, 7/3, 10/3, (2*c)/(c
 + d*x)])/(7*d*(c + d*x)*(c^2 - d^2*x^2)^(2/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{2}\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(c+dx)^2 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(1/3)/(d^4*x^4 + 2*c*d^3*x^3 - 2*c^3*d*x - c^4)
, x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-(-c+dx)(c+dx))^{\frac{2}{3}}(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(2/3)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(c^2-d^2\,x^2)^{2/3}\,(c+d\,x)^2}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^2),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}\,c^2+2\,(-d^2x^2+c^2)^{\frac{2}{3}}\,cdx+(-d^2x^2+c^2)^{\frac{2}{3}}\,d^2x^2}dx$$

input
```
int(1/(d*x+c)^2/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/((c**2 - d**2*x**2)**(2/3)*c**2 + 2*(c**2 - d**2*x**2)**(2/3)*c*d*x
+ (c**2 - d**2*x**2)**(2/3)*d**2*x**2),x)
```

3.269 $\int \frac{1}{(c+dx)^3\left(c^2-d^2x^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 379

$$\int \frac{1}{(c+dx)^3\left(c^2-d^2x^2\right)^{2/3}}\,dx = \frac{21x}{160c^3\left(c^2-d^2x^2\right)^{2/3}} - \frac{3}{8d(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}} - \frac{3}{40cd(c+dx)\left(c^2-d^2x^2\right)^{2/3}} + \frac{7\,3^{3/4}\sqrt{2-\sqrt{3}}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+\left(c^2-d^2x^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c}}\right.\right.}{160c^3d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
21/160*x/c^3/(-d^2*x^2+c^2)^(2/3)-3/8/d/(d*x+c)^2/(-d^2*x^2+c^2)^(2/3)-3/4
0/c/d/(d*x+c)/(-d^2*x^2+c^2)^(2/3)+7/160*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))
*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d
^2*x^2+c^2)^(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*Ell
ipticF(((1+3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d
^2*x^2+c^2)^(1/3)),2*I-I*3^(1/2))/c^3/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c
^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.42 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.19

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}\,dx = -\frac{3(c-dx)\left(1+\frac{dx}{c}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{11}{3},\frac{4}{3},\frac{c-dx}{2c}\right)}{8\ 2^{2/3}c^3d\left(c^2-d^2x^2\right)^{2/3}}$$

input
```
Integrate[1/((c + d*x)^3*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(2/3)*Hypergeometric2F1[1/3, 11/3, 4/3, (c - d
*x)/(2*c)])/(8*2^(2/3)*c^3*d*(c^2 - d^2*x^2)^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {506, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{ 506}$$

$$-\frac{\left(1-\frac{2c}{c+dx}\right)^{2/3}\int \frac{\left(\frac{1}{c+dx}\right)^{7/3}}{\left(1-\frac{2c}{c+dx}\right)^{2/3}}d\frac{1}{c+dx}}{d\left(\frac{1}{c+dx}\right)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}$$

$$\Big\downarrow \text{ 74}$$

$$-\frac{3\left(1-\frac{2c}{c+dx}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{10}{3},\frac{13}{3},\frac{2c}{c+dx}\right)}{10d(c+dx)^2\left(c^2-d^2x^2\right)^{2/3}}$$

input
```
Int[1/((c + d*x)^3*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(-3*(1 - (2*c)/(c + d*x))^(2/3)*Hypergeometric2F1[2/3, 10/3, 13/3, (2*c)/(
c + d*x)])/(10*d*(c + d*x)^2*(c^2 - d^2*x^2)^(2/3))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 506
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-a/b, 2]}, Simp[(-(a + b*x^2)^p)*((1/(c + d*x))^(2*p)/(d*(1 - (c -
d*q)/(c + d*x))^p*(1 - (c + d*q)/(c + d*x))^p))   Subst[Int[(1 - (c - d*q)*
x)^p*((1 - (c + d*q)*x)^p/x^(n + 2*p + 2)), x], x, 1/(c + d*x)], x]] /; Fre
eQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && NegQ[a/b]
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{3}\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(1/3)/(d^5*x^5 + 3*c*d^4*x^4 + 2*c^2*d^3*x^3 -
2*c^3*d^2*x^2 - 3*c^4*d*x - c^5), x)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-(-c+dx)(c+dx))^{\frac{2}{3}}(c+dx)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(2/3)*(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^3), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(c^2-d^2\,x^2)^{2/3}\,(c+d\,x)^3}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^3),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^3 (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}\,c^3+3\,(-d^2x^2+c^2)^{\frac{2}{3}}\,c^2dx+3\,(-d^2x^2+c^2)^{\frac{2}{3}}\,c\,d^2x^2+(-}$$

input
```
int(1/(d*x+c)^3/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/((c**2 - d**2*x**2)**(2/3)*c**3 + 3*(c**2 - d**2*x**2)**(2/3)*c**2*d
*x + 3*(c**2 - d**2*x**2)**(2/3)*c*d**2*x**2 + (c**2 - d**2*x**2)**(2/3)*d
**3*x**3),x)
```

3.270 $\int \frac{c-dx}{(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\int \frac{c-dx}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3\sqrt[3]{c^2-d^2x^2}}{2d} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}}\right),-7+4\sqrt{3}\right)}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/2*(-d^2*x^2+c^2)^(1/3)/d+3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c*(c^(2/3)-(-
d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^
(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^
(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

$$\int \frac{c-dx}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3\sqrt[3]{c^2-d^2x^2}}{2d} + \frac{cx\left(1-\frac{d^2x^2}{c^2}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{(c^2-d^2x^2)^{2/3}}$$

input
```
Integrate[(c - d*x)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (c*x*(1 - (d^2*x^2)/c^2)^(2/3)*Hypergeom
etric2F1[1/2, 2/3, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {455, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{(c^2-d^2x^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx + \frac{3\sqrt[3]{c^2-d^2x^2}}{2d}$$

$$\Big\downarrow \text{234}$$

$$\frac{3\sqrt[3]{c^2-d^2x^2}}{2d} - \frac{3c\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}$$

$$\Big\downarrow \text{760}$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-c}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-c}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}$$

input
```
Int[(c - d*x)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*c*(c^(2/3) -
(c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^
2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3
])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(d^2*x*Sqrt[-((c^(2
/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2
*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{-dx+c}{\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}dx$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((-d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [F]

$$\int \frac{c-dx}{\left(c^{2}-d^{2}x^{2}\right)^{2/3}}\,dx=\int -\frac{dx-c}{\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/3)/(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.21

$$\int \frac{c-dx}{(c^2-d^2x^2)^{2/3}}\,dx = -d\left(\begin{cases} \frac{x^2}{2(c^2)^{\frac{2}{3}}} & \text{for } d^2=0 \\ -\frac{3\sqrt[3]{c^2-d^2x^2}}{2d^2} & \text{otherwise} \end{cases}\right) + \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2},\frac{2}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{\sqrt[3]{c}}$$

input
```
integrate((-d*x+c)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
-d*Piecewise((x**2/(2*(c**2)**(2/3)), Eq(d**2, 0)), (-3*(c**2 - d**2*x**2)
**(1/3)/(2*d**2), True)) + x*hyper((1/2, 2/3), (3/2,), d**2*x**2*exp_polar
(2*I*pi)/c**2)/c**(1/3)
```

Maxima [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{2/3}}\,dx = \int -\frac{dx-c}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
-integrate((d*x - c)/(-d^2*x^2 + c^2)^(2/3), x)
```

Giac [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{2/3}}\,dx = \int -\frac{dx-c}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate(-(d*x - c)/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{c-dx}{\left(c^2-d^2x^2\right)^{2/3}}\,dx=\frac{3\,{\left(c^2-d^2\,x^2\right)}^{1/3}}{2\,d}+\frac{c\,x\,{\left(1-\frac{d^2\,x^2}{c^2}\right)}^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{{\left(c^2-d^2\,x^2\right)}^{2/3}}$$

input
```
int((c - d*x)/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
(3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (c*x*(1 - (d^2*x^2)/c^2)^(2/3)*hypergeom
([1/2, 2/3], 3/2, (d^2*x^2)/c^2))/(c^2 - d^2*x^2)^(2/3)
```

Reduce [F]

$$\int \frac{c-dx}{\left(c^2-d^2x^2\right)^{2/3}}\,dx=\frac{3(-d^2x^2+c^2)^{\frac{1}{3}}+2\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)cd}{2d}$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
(3*(c**2 - d**2*x**2)**(1/3) + 2*int(1/(c**2 - d**2*x**2)**(2/3),x)*c*d)/(
2*d)
```

3.271 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 310

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx = \frac{3\sqrt[3]{c^2-d^2x^2}}{2d} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}\,c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-}{\left(1-\sqrt{3}\right)c^{2/3}-}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3/2*(-d^2*x^2+c^2)^(1/3)/d+3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*c*(c^(2/3)-(-
d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^2)^
(2/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*EllipticF(((1+3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^
(1/3)),2*I-I*3^(1/2))/d^2/x/(-c^(2/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/((1-3
^(1/2))*c^(2/3)-(-d^2*x^2+c^2)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx$$
$$= -\frac{3(c-dx)\left(1+\frac{dx}{c}\right)^{2/3}\sqrt[3]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},\frac{c-dx}{2c}\right)}{4\,2^{2/3}d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x),x]
```

output
```
(-3*(c - d*x)*(1 + (d*x)/c)^(2/3)*(c^2 - d^2*x^2)^(1/3)*Hypergeometric2F1[
2/3, 4/3, 7/3, (c - d*x)/(2*c)])/(4*2^(2/3)*d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {504, 234, 241, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx$$

$$\Big\downarrow \text{504}$$

$$c\int \frac{1}{(c^2-d^2x^2)^{2/3}}dx-d\int \frac{x}{(c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{234}$$

$$-\frac{3c\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}-d\int \frac{x}{(c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{241}$$

$$\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}-\frac{3c\sqrt{-d^2x^2}\int \frac{1}{\sqrt{-d^2x^2}}d\sqrt[3]{c^2-d^2x^2}}{2d^2x}$$

$\Big\downarrow$ 760

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}c\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+(c^2-d^2x^2)^{2/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}{\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-}}\right.\right.}{d^2x\sqrt{-\frac{c^{2/3}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(\left(1-\sqrt{3}\right)c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

$$\frac{3\sqrt[3]{c^2-d^2x^2}}{2d}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x),x]
```

output
```
(3*(c^2 - d^2*x^2)^(1/3))/(2*d) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*c*(c^(2/3) -
(c^2 - d^2*x^2)^(1/3))*Sqrt[(c^(4/3) + c^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^
2 - d^2*x^2)^(2/3))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))^2]*Ell
ipticF[ArcSin[((1 + Sqrt[3])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))/((1 - Sqrt[3
])*c^(2/3) - (c^2 - d^2*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(d^2*x*Sqrt[-((c^(2
/3)*(c^(2/3) - (c^2 - d^2*x^2)^(1/3)))/((1 - Sqrt[3])*c^(2/3) - (c^2 - d^2
*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(-d^2x^2 + c^2)^{\frac{1}{3}}}{dx + c} dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{c^2 - d^2x^2}}{c + dx} dx = \int \frac{(-d^2x^2 + c^2)^{\frac{1}{3}}}{dx + c} dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/3)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2 - d^2x^2}}{c + dx}\,dx = \int \frac{\sqrt[3]{-(-c + dx)(c + dx)}}{c + dx}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2 - d^2x^2}}{c + dx}\,dx = \int \frac{(-d^2x^2 + c^2)^{\frac{1}{3}}}{dx + c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2 - d^2x^2}}{c + dx}\,dx = \int \frac{(-d^2x^2 + c^2)^{\frac{1}{3}}}{dx + c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(c^2-d^2\,x^2\right)^{1/3}}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c),x)
```

output
```
int((c**2 - d**2*x**2)**(1/3)/(c + d*x),x)
```

3.272 $\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx=-\frac{486c^3(c^2-d^2x^2)^{4/3}}{455d(c+dx)^{4/3}}-\frac{324c^2(c^2-d^2x^2)^{4/3}}{455d\sqrt[3]{c+dx}}-\frac{27c(c+dx)^{2/3}(c^2-d^2x^2)^{4/3}}{65d}-\frac{3(c+dx)^{5/3}(c^2-d^2x^2)^{4/3}}{13d}$$

output
```
-486/455*c^3*(-d^2*x^2+c^2)^(4/3)/d/(d*x+c)^(4/3)-324/455*c^2*(-d^2*x^2+c^
2)^(4/3)/d/(d*x+c)^(1/3)-27/65*c*(d*x+c)^(2/3)*(-d^2*x^2+c^2)^(4/3)/d-3/13
*(d*x+c)^(5/3)*(-d^2*x^2+c^2)^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx=\frac{3(-c+dx)\sqrt[3]{c^2-d^2x^2}(368c^3+339c^2dx+168cd^2x^2+35d^3x^3)}{455d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(8/3)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(3*(-c + d*x)*(c^2 - d^2*x^2)^(1/3)*(368*c^3 + 339*c^2*d*x + 168*c*d^2*x^2
 + 35*d^3*x^3))/(455*d*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \text{459}$$

$$\frac{18}{13}c\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}dx-\frac{3(c+dx)^{5/3}\left(c^2-d^2x^2\right)^{4/3}}{13d}$$

$$\Big\downarrow \text{459}$$

$$\frac{18}{13}c\left(\frac{6}{5}c\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}dx-\frac{3(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{4/3}}{10d}\right)-\frac{3(c+dx)^{5/3}\left(c^2-d^2x^2\right)^{4/3}}{13d}$$

$$\Big\downarrow \text{459}$$

$$\frac{18}{13}c\left(\frac{6}{5}c\left(\frac{6}{7}c\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}dx-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{7d\sqrt[3]{c+dx}}\right)-\frac{3(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{4/3}}{10d}\right)-\frac{3(c+dx)^{5/3}\left(c^2-d^2x^2\right)^{4/3}}{13d}$$

$$\Big\downarrow \text{458}$$

$$\frac{18}{13}c\left(\frac{6}{5}c\left(-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{7d\sqrt[3]{c+dx}}-\frac{9c\left(c^2-d^2x^2\right)^{4/3}}{14d(c+dx)^{4/3}}\right)-\frac{3(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{4/3}}{10d}\right)-\frac{3(c+dx)^{5/3}\left(c^2-d^2x^2\right)^{4/3}}{13d}$$

input
```
Int[(c + d*x)^(8/3)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c + d*x)^(5/3)*(c^2 - d^2*x^2)^(4/3))/(13*d) + (18*c*((-3*(c + d*x)^(
2/3)*(c^2 - d^2*x^2)^(4/3))/(10*d) + (6*c*((-9*c*(c^2 - d^2*x^2)^(4/3))/(1
4*d*(c + d*x)^(4/3)) - (3*(c^2 - d^2*x^2)^(4/3))/(7*d*(c + d*x)^(1/3))))/5
))/13
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3(-dx+c)\left(35d^{3}x^{3}+168c\,d^{2}x^{2}+339c^{2}dx+368c^{3}\right)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{1}{3}}}{455d\left(dx+c\right)^{\frac{1}{3}}}$	63
orering	$-\frac{3(-dx+c)\left(35d^{3}x^{3}+168c\,d^{2}x^{2}+339c^{2}dx+368c^{3}\right)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{1}{3}}}{455d\left(dx+c\right)^{\frac{1}{3}}}$	63
risch	$-\frac{3\left(\left(-d^{2}x^{2}+c^{2}\right)^{2}\right)^{\frac{1}{3}}\left(\frac{\left(d^{2}x^{2}-c^{2}\right)^{2}}{\left(dx+c\right)^{2}}\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}\left(-35d^{4}x^{4}-133d^{3}c\,x^{3}-171c^{2}x^{2}d^{2}-29c^{3}dx+368c^{4}\right)\left(-dx+c\right)}{455\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}\left(\left(d^{2}x^{2}-c^{2}\right)^{2}\right)^{\frac{1}{3}}d\left(\left(dx-c\right)^{2}\right)^{\frac{1}{3}}}$	143

input
```
int((d*x+c)^(8/3)*(-d^2*x^2+c^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/455*(-d*x+c)*(35*d^3*x^3+168*c*d^2*x^2+339*c^2*d*x+368*c^3)*(-d^2*x^2+c
^2)^(1/3)/d/(d*x+c)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{3\,(35\,d^4x^4+133\,cd^3x^3+171\,c^2d^2x^2+29\,c^3dx-368\,c^4)(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{455\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(8/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
3/455*(35*d^4*x^4 + 133*c*d^3*x^3 + 171*c^2*d^2*x^2 + 29*c^3*d*x - 368*c^4
)*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)/(d^2*x + c*d)
```

Sympy [F]

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx = \int \sqrt[3]{-(-c+dx)\,(c+dx)}(c+dx)^{\frac{8}{3}}\,dx$$

input
```
integrate((d*x+c)**(8/3)*(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(8/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx=\frac{3\,(35\,d^4x^4+133\,cd^3x^3+171\,c^2d^2x^2+29\,c^3dx-368\,c^4)(dx+c)(-dx+c)^{\frac{1}{3}}}{455\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(8/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
3/455*(35*d^4*x^4 + 133*c*d^3*x^3 + 171*c^2*d^2*x^2 + 29*c^3*d*x - 368*c^4
)*(d*x + c)*(-d*x + c)^(1/3)/(d^2*x + c*d)
```

Giac [F]

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^{\frac{8}{3}}\,dx$$

input
```
integrate((d*x+c)^(8/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.74

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx=\frac{(c^2-d^2\,x^2)^{1/3}\left(\frac{513\,c^2\,x^2\,(c+d\,x)^{2/3}}{455}-\frac{1104\,c^4\,(c+d\,x)^{2/3}}{455\,d^2}+\frac{3\,d^2\,x^4\,(c+d\,x)^{2/3}}{13}+\frac{57\,c\,d\,x^3\,(c+d\,x)^{2/}}{65}\right.}{x+\frac{c}{d}}$$

input
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(8/3),x)
```

output
```
((c^2 - d^2*x^2)^(1/3)*((513*c^2*x^2*(c + d*x)^(2/3))/455 - (1104*c^4*(c +
d*x)^(2/3))/(455*d^2) + (3*d^2*x^4*(c + d*x)^(2/3))/13 + (57*c*d*x^3*(c +
d*x)^(2/3))/65 + (87*c^3*x*(c + d*x)^(2/3))/(455*d)))/(x + c/d)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.40

$$\int (c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{3(-dx+c)^{\frac{1}{3}}\left(35d^4x^4+133c\,d^3x^3+171c^2d^2x^2+29c^3dx-368c^4\right)}{455d}$$

input
```
int((d*x+c)^(8/3)*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
(3*(c - d*x)**(1/3)*( - 368*c**4 + 29*c**3*d*x + 171*c**2*d**2*x**2 + 133*
c*d**3*x**3 + 35*d**4*x**4))/(455*d)
```

3.273 $\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx=-\frac{27c^2(c^2-d^2x^2)^{4/3}}{35d(c+dx)^{4/3}}-\frac{18c(c^2-d^2x^2)^{4/3}}{35d\sqrt[3]{c+dx}}-\frac{3(c+dx)^{2/3}\,(c^2-d^2x^2)^{4/3}}{10d}$$

output
```
-27/35*c^2*(-d^2*x^2+c^2)^(4/3)/d/(d*x+c)^(4/3)-18/35*c*(-d^2*x^2+c^2)^(4/
3)/d/(d*x+c)^(1/3)-3/10*(d*x+c)^(2/3)*(-d^2*x^2+c^2)^(4/3)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx=\frac{3(-c+dx)\sqrt[3]{c^2-d^2x^2}(37c^2+26cdx+7d^2x^2)}{70d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/3)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(3*(-c + d*x)*(c^2 - d^2*x^2)^(1/3)*(37*c^2 + 26*c*d*x + 7*d^2*x^2))/(70*d
*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \ 459$$

$$\frac{6}{5}c\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}dx-\frac{3(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{4/3}}{10d}$$

$$\Big\downarrow \ 459$$

$$\frac{6}{5}c\left(\frac{6}{7}c\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}dx-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{7d\sqrt[3]{c+dx}}\right)-\frac{3(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{4/3}}{10d}$$

$$\Big\downarrow \ 458$$

$$\frac{6}{5}c\left(-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{7d\sqrt[3]{c+dx}}-\frac{9c\left(c^2-d^2x^2\right)^{4/3}}{14d(c+dx)^{4/3}}\right)-\frac{3(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{4/3}}{10d}$$

input
```
Int[(c + d*x)^(5/3)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-3*(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(4/3))/(10*d) + (6*c*((-9*c*(c^2 - d^2
*x^2)^(4/3))/(14*d*(c + d*x)^(4/3)) - (3*(c^2 - d^2*x^2)^(4/3))/(7*d*(c +
d*x)^(1/3))))/5
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{3(-dx+c)(7d^2x^2+26cdx+37c^2)(-d^2x^2+c^2)^{\frac{1}{3}}}{70d(dx+c)^{\frac{1}{3}}}$	52
orering	$-\frac{3(-dx+c)(7d^2x^2+26cdx+37c^2)(-d^2x^2+c^2)^{\frac{1}{3}}}{70d(dx+c)^{\frac{1}{3}}}$	52
risch	$-\frac{3\left((-d^2x^2+c^2)^2\right)^{\frac{1}{3}}\left(\frac{(d^2x^2-c^2)^2}{(dx+c)^2}\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}(-7d^3x^3-19c\,d^2x^2-11c^2dx+37c^3)(-dx+c)}{70(-d^2x^2+c^2)^{\frac{2}{3}}\left((d^2x^2-c^2)^2\right)^{\frac{1}{3}}d\left((dx-c)^2\right)^{\frac{1}{3}}}$	132

input
```
int((d*x+c)^(5/3)*(-d^2*x^2+c^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/70*(-d*x+c)*(7*d^2*x^2+26*c*d*x+37*c^2)*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^
(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx=\frac{3\,(7\,d^3x^3+19\,cd^2x^2+11\,c^2dx-37\,c^3)(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{70\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(5/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
3/70*(7*d^3*x^3 + 19*c*d^2*x^2 + 11*c^2*d*x - 37*c^3)*(-d^2*x^2 + c^2)^(1/
3)*(d*x + c)^(2/3)/(d^2*x + c*d)
```

Sympy [F]

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx=\int \sqrt[3]{-(-c+dx)\,(c+dx)}(c+dx)^{\frac{5}{3}}\,dx$$

input
```
integrate((d*x+c)**(5/3)*(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(5/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx=\frac{3\,(7\,d^3x^3+19\,cd^2x^2+11\,c^2dx-37\,c^3)(dx+c)(-dx+c)^{\frac{1}{3}}}{70\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(5/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
3/70*(7*d^3*x^3 + 19*c*d^2*x^2 + 11*c^2*d*x - 37*c^3)*(d*x + c)*(-d*x + c)
^(1/3)/(d^2*x + c*d)
```

Giac [F]

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^{\frac{5}{3}}\,dx$$

input
```
integrate((d*x+c)^(5/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{(c^2-d^2\,x^2)^{1/3}\left(\frac{57\,c\,x^2\,(c+d\,x)^{2/3}}{70}-\frac{111\,c^3\,(c+d\,x)^{2/3}}{70\,d^2}+\frac{3\,d\,x^3\,(c+d\,x)^{2/3}}{10}+\frac{33\,c^2\,x\,(c+d\,x)^{2/3}}{70\,d}\right)}{x+\frac{c}{d}}$$

input
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(5/3),x)
```

output
```
((c^2 - d^2*x^2)^(1/3)*((57*c*x^2*(c + d*x)^(2/3))/70 - (111*c^3*(c + d*x)
^(2/3))/(70*d^2) + (3*d*x^3*(c + d*x)^(2/3))/10 + (33*c^2*x*(c + d*x)^(2/3
))/(70*d)))/(x + c/d)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.43

$$\int (c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{3(-dx+c)^{\frac{1}{3}}\left(7d^3x^3+19c\,d^2x^2+11c^2dx-37c^3\right)}{70d}$$

input
```
int((d*x+c)^(5/3)*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
(3*(c - d*x)**(1/3)*( - 37*c**3 + 11*c**2*d*x + 19*c*d**2*x**2 + 7*d**3*x*
*3))/(70*d)
```

3.274 $\int (c+dx)^{2/3} \sqrt[3]{c^2-d^2x^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int (c+dx)^{2/3} \sqrt[3]{c^2-d^2x^2} \, dx = -\frac{9c(c^2-d^2x^2)^{4/3}}{14d(c+dx)^{4/3}} - \frac{3(c^2-d^2x^2)^{4/3}}{7d\sqrt[3]{c+dx}}$$

output
```
-9/14*c*(-d^2*x^2+c^2)^(4/3)/d/(d*x+c)^(4/3)-3/7*(-d^2*x^2+c^2)^(4/3)/d/(d
*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int (c+dx)^{2/3} \sqrt[3]{c^2-d^2x^2} \, dx = \frac{3\sqrt[3]{c^2-d^2x^2}(-5c^2+3cdx+2d^2x^2)}{14d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(3*(c^2 - d^2*x^2)^(1/3)*(-5*c^2 + 3*c*d*x + 2*d^2*x^2))/(14*d*(c + d*x)^(
1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{2/3} \sqrt[3]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \; 459$$

$$\frac{6}{7}c\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}dx-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{7d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \; 458$$

$$-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{7d\sqrt[3]{c+dx}}-\frac{9c\left(c^2-d^2x^2\right)^{4/3}}{14d(c+dx)^{4/3}}$$

input
```
Int[(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(-9*c*(c^2 - d^2*x^2)^(4/3))/(14*d*(c + d*x)^(4/3)) - (3*(c^2 - d^2*x^2)^(
4/3))/(7*d*(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))    Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(-dx+c)(2dx+5c)\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{14d(dx+c)^{\frac{1}{3}}}$	41
orering	$-\frac{3(-dx+c)(2dx+5c)\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{14d(dx+c)^{\frac{1}{3}}}$	41
risch	$-\frac{3\left(\left(-d^2x^2+c^2\right)^2\right)^{\frac{1}{3}}\left(\frac{\left(d^2x^2-c^2\right)^2}{(dx+c)^2}\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-2d^2x^2-3cdx+5c^2\right)(-dx+c)}{14\left(-d^2x^2+c^2\right)^{\frac{2}{3}}\left(\left(d^2x^2-c^2\right)^2\right)^{\frac{1}{3}}d\left((dx-c)^2\right)^{\frac{1}{3}}}$	121

input
```
int((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/14*(-d*x+c)*(2*d*x+5*c)*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{3\,(2\,d^2x^2+3\,cdx-5\,c^2)(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{14\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
3/14*(2*d^2*x^2 + 3*c*d*x - 5*c^2)*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)/
(d^2*x + c*d)
```

Sympy [F]

$$\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}\,dx = \int \sqrt[3]{-(-c+dx)(c+dx)}(c+dx)^{\frac{2}{3}}\,dx$$

input
```
integrate((d*x+c)**(2/3)*(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

$$\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{3(2d^2x^2+3cdx-5c^2)(dx+c)(-dx+c)^{\frac{1}{3}}}{14(d^2x+cd)}$$

input
```
integrate((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
3/14*(2*d^2*x^2 + 3*c*d*x - 5*c^2)*(d*x + c)*(-d*x + c)^(1/3)/(d^2*x + c*d
)
```

Giac [F]

$$\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}\,dx = \int (-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{(c^2-d^2\,x^2)^{1/3}\left(\frac{3\,x^2\,(c+d\,x)^{2/3}}{7}-\frac{15\,c^2\,(c+d\,x)^{2/3}}{14\,d^2}+\frac{9\,c\,x\,(c+d\,x)^{2/3}}{14\,d}\right)}{x+\frac{c}{d}}$$

input
```
int((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(2/3),x)
```

output
```
((c^2 - d^2*x^2)^(1/3)*((3*x^2*(c + d*x)^(2/3))/7 - (15*c^2*(c + d*x)^(2/3
))/(14*d^2) + (9*c*x*(c + d*x)^(2/3))/(14*d)))/(x + c/d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

$$\int (c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}\,dx = \frac{3(-dx+c)^{\frac{1}{3}}\,(2d^2x^2+3cdx-5c^2)}{14d}$$

input
```
int((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(1/3),x)
```

output
```
(3*(c - d*x)**(1/3)*( - 5*c**2 + 3*c*d*x + 2*d**2*x**2))/(14*d)
```

3.275 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 32

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}} \, dx = -\frac{3(c^2-d^2x^2)^{4/3}}{4d(c+dx)^{4/3}}$$

output
```
-3/4*(-d^2*x^2+c^2)^(4/3)/d/(d*x+c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}} \, dx = -\frac{3(c^2-d^2x^2)^{4/3}}{4d(c+dx)^{4/3}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(1/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(4/3))/(4*d*(c + d*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}\,dx$$

$$\Big\downarrow \text{458}$$

$$-\frac{3\left(c^2-d^2x^2\right)^{4/3}}{4d(c+dx)^{4/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(1/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(4/3))/(4*d*(c + d*x)^(4/3))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
gosper	$-\frac{3(-dx+c)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{1}{3}}}{4d(dx+c)^{\frac{1}{3}}}$	33
orering	$-\frac{3(-dx+c)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{1}{3}}}{4d(dx+c)^{\frac{1}{3}}}$	33
risch	$-\frac{3\left(\left(-d^{2}x^{2}+c^{2}\right)^{2}\right)^{\frac{1}{3}}\left(\frac{\left(d^{2}x^{2}-c^{2}\right)^{2}}{(dx+c)^{2}}\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}(-dx+c)^{2}}{4\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}\left(\left(d^{2}x^{2}-c^{2}\right)^{2}\right)^{\frac{1}{3}}d\left((dx-c)^{2}\right)^{\frac{1}{3}}}$	104

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-d*x+c)*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}\,dx=\frac{3\left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}(dx-c)}{4\left(d^2x+cd\right)}$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
3/4*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*(d*x - c)/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}\,dx=\int \frac{\sqrt[3]{-\left(-c+dx\right)\left(c+dx\right)}}{\sqrt[3]{c+dx}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c)**(1/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x)**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}\,dx = \frac{3\,(dx-c)(-dx+c)^{\frac{1}{3}}}{4\,d}$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
3/4*(d*x - c)*(-d*x + c)^(1/3)/d
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}\,dx = \frac{(c^2-d^2\,x^2)^{1/3}\,\left(\frac{3\,x}{4}-\frac{3\,c}{4\,d}\right)}{(c+d\,x)^{1/3}}$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(1/3),x)
```

output
```
((c^2 - d^2*x^2)^(1/3)*((3*x)/4 - (3*c)/(4*d)))/(c + d*x)^(1/3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}\,dx=\frac{3(-dx+c)^{\frac{1}{3}}\,(dx-c)}{4d}$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(1/3),x)
```

output
```
(3*(c - d*x)**(1/3)*( - c + d*x))/(4*d)
```

3.276 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 251

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx = \frac{3\sqrt[3]{c^2-d^2x^2}}{d\sqrt[3]{c+dx}} - \frac{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt[3]{c^2-d^2x^2}\arctan\left(\frac{\sqrt[3]{c}+2^{2/3}\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{d\sqrt[3]{c-dx}\sqrt[3]{c+dx}} - \frac{\sqrt[3]{c}\sqrt[3]{c^2-d^2x^2}\log(c+dx)}{2^{2/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}} + \frac{3\sqrt[3]{c}\sqrt[3]{c^2-d^2x^2}\log\left(\sqrt[3]{2}\sqrt[3]{c}-\sqrt[3]{c-dx}\right)}{2^{2/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}}$$

output
```
3*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(1/3)-2^(1/3)*3^(1/2)*c^(1/3)*(-d^2*x^2+c
^2)^(1/3)*arctan(1/3*(c^(1/3)+2^(2/3)*(-d*x+c)^(1/3))*3^(1/2)/c^(1/3))/d/(
-d*x+c)^(1/3)/(d*x+c)^(1/3)-1/2*c^(1/3)*(-d^2*x^2+c^2)^(1/3)*ln(d*x+c)*2^(
1/3)/d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)+3/2*c^(1/3)*(-d^2*x^2+c^2)^(1/3)*ln(2^
(1/3)*c^(1/3)-(-d*x+c)^(1/3))*2^(1/3)/d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx = \frac{\frac{6\sqrt[3]{c^2-d^2x^2}}{\sqrt[3]{c+dx}}+2\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c+dx}}{\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/3}\sqrt[3]{c^2-d^2x^2}}\right)+2\sqrt[3]{2}\sqrt[3]{c}\log\left(-2\right.}{}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(4/3),x]
```

output
```
((6*(c^2 - d^2*x^2)^(1/3))/(c + d*x)^(1/3) + 2*2^(1/3)*Sqrt[3]*c^(1/3)*Arc
Tan[(Sqrt[3]*c^(1/3)*(c + d*x)^(1/3))/(c^(1/3)*(c + d*x)^(1/3) + 2^(2/3)*(
c^2 - d^2*x^2)^(1/3))] + 2*2^(1/3)*c^(1/3)*Log[-2*c^(1/3)*(c + d*x)^(1/3)
+ 2^(2/3)*(c^2 - d^2*x^2)^(1/3)] - 2^(1/3)*c^(1/3)*Log[2*c^(2/3)*(c + d*x)
^(2/3) + 2^(2/3)*c^(1/3)*(c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3) + 2^(1/3)*(
c^2 - d^2*x^2)^(2/3)])/(2*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.78, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {474, 473, 27, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\left(\frac{dx}{c}+1\right)^{4/3}}dx}{c\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{473}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\int \frac{c\sqrt[3]{c^2-cdx}}{c+dx}dx}{c\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\int \frac{\sqrt[3]{c^2-cdx}}{c+dx}dx}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(2c^2\int \frac{1}{(c+dx)\left(c^2-cdx\right)^{2/3}}dx+\frac{3\sqrt[3]{c^2-cdx}}{d}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \text{69}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(2c^2\left(-\frac{3\int \frac{1}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}}d\sqrt[3]{c^2-cdx}}{2\ 2^{2/3}c^{4/3}d}-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cd}}{2\sqrt[3]{2}c^{2/3}d}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(2c^2\left(-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-}\right.}{2\ 2^{2/3}c^{4/3}d}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(2c^2\left(\frac{3\int \frac{1}{-\left(c^2-cdx\right)^{2/3}-3}d\left(\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}\right)+\frac{3\sqrt[3]{c}}{}\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(2c^2\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1}{\sqrt{3}}\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}\right)+\frac{3\sqrt[3]{c^2-cdx}}{d}\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(4/3),x]
```

output
```
((c^2 - d^2*x^2)^(4/3)*((3*(c^2 - c*d*x)^(1/3))/d + 2*c^2*(-((Sqrt[3]*ArcT
an[(1 + (2^(2/3)*(c^2 - c*d*x)^(1/3))/c^(2/3))/Sqrt[3]])/(2^(2/3)*c^(4/3)*
d)) - Log[c + d*x]/(2*2^(2/3)*c^(4/3)*d) + (3*Log[2^(1/3)*c^(2/3) - (c^2 -
 c*d*x)^(1/3)])/(2*2^(2/3)*c^(4/3)*d))))/((c + d*x)^(1/3)*(1 + (d*x)/c)*(c
^2 - c*d*x)^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(4/3),x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx =$$
$$-\frac{2\sqrt{3}2^{\frac{1}{3}}(dx+c)c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}c^{\frac{2}{3}}+\sqrt{3}(cdx+c^2)}{3(cdx+c^2)}\right)+2^{\frac{1}{3}}(dx+c)c^{\frac{1}{3}}\log\left(\frac{2^{\frac{2}{3}}(dx+c)c^{\frac{2}{3}}+2^{\frac{1}{3}}(-d^2x^2}{}\right.}{}$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*2^(1/3)*(d*x + c)*c^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(-d^
2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*c^(2/3) + sqrt(3)*(c*d*x + c^2))/(c*d*x
 + c^2)) + 2^(1/3)*(d*x + c)*c^(1/3)*log((2^(2/3)*(d*x + c)*c^(2/3) + 2^(1
/3)*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*c^(1/3) + (-d^2*x^2 + c^2)^(2/3
)*(d*x + c)^(1/3))/(d*x + c)) - 2*2^(1/3)*(d*x + c)*c^(1/3)*log(-(2^(1/3)*
(d*x + c)*c^(1/3) - (-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) - 6
*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx = \int \frac{\sqrt[3]{-(-c+dx)(c+dx)}}{(c+dx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c)**(4/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(4/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx = \int \frac{(c^2-d^2\,x^2)^{1/3}}{(c+d\,x)^{4/3}}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(4/3),x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}\,c+(dx+c)^{\frac{1}{3}}\,dx}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(4/3),x)
```

output
```
int((c**2 - d**2*x**2)**(1/3)/((c + d*x)**(1/3)*c + (c + d*x)**(1/3)*d*x),
x)
```

3.277 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 254

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx = -\frac{\sqrt[3]{c^2-d^2x^2}}{d(c+dx)^{4/3}}+\frac{\sqrt[3]{c^2-d^2x^2}\arctan\left(\frac{\sqrt[3]{c}+2^{2/3}\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{2^{2/3}\sqrt{3}c^{2/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}}+\frac{\sqrt[3]{c^2-d^2x^2}\log(c+dx)}{6\ 2^{2/3}c^{2/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}}-\frac{\sqrt[3]{c^2-d^2x^2}\log\left(\sqrt[3]{2}\sqrt[3]{c}-\sqrt[3]{c-dx}\right)}{2\ 2^{2/3}c^{2/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}}$$

output
```
-(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(4/3)+1/6*(-d^2*x^2+c^2)^(1/3)*arctan(1/3*
(c^(1/3)+2^(2/3)*(-d*x+c)^(1/3))*3^(1/2)/c^(1/3))*2^(1/3)*3^(1/2)/c^(2/3)/
d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)+1/12*(-d^2*x^2+c^2)^(1/3)*ln(d*x+c)*2^(1/3)
/c^(2/3)/d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)-1/4*(-d^2*x^2+c^2)^(1/3)*ln(2^(1/3
)*c^(1/3)-(-d*x+c)^(1/3))*2^(1/3)/c^(2/3)/d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx = \frac{-\frac{12\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}} - \frac{2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c+dx}}{\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/3}\sqrt[3]{c^2-d^2x^2}}\right)}{c^{2/3}} - \frac{2\sqrt[3]{2}\log\left(-2\sqrt[3]{c}\sqrt[3]{c+\right.}{c^2}}{1}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(7/3),x]
```

output
```
((-12*(c^2 - d^2*x^2)^(1/3))/(c + d*x)^(4/3) - (2*2^(1/3)*Sqrt[3]*ArcTan[(
Sqrt[3]*c^(1/3)*(c + d*x)^(1/3))/(c^(1/3)*(c + d*x)^(1/3) + 2^(2/3)*(c^2 -
 d^2*x^2)^(1/3))])/c^(2/3) - (2*2^(1/3)*Log[-2*c^(1/3)*(c + d*x)^(1/3) + 2
^(2/3)*(c^2 - d^2*x^2)^(1/3)])/c^(2/3) + (2^(1/3)*Log[2*c^(2/3)*(c + d*x)^
(2/3) + 2^(2/3)*c^(1/3)*(c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3) + 2^(1/3)*(c
^2 - d^2*x^2)^(2/3)])/c^(2/3))/(12*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {474, 473, 27, 51, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx$$

$$\Big\downarrow \ 474$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\left(\frac{dx}{c}+1\right)^{7/3}}dx}{c^2\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 473$$

$$\frac{(c^2-d^2x^2)^{4/3}\int \frac{c^2\sqrt[3]{c^2-cdx}}{(c+dx)^2}dx}{c^2\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)(c^2-cdx)^{4/3}}$$

$\downarrow$ 27

$$\frac{(c^2-d^2x^2)^{4/3}\int \frac{\sqrt[3]{c^2-cdx}}{(c+dx)^2}dx}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)(c^2-cdx)^{4/3}}$$

$\downarrow$ 51

$$\frac{(c^2-d^2x^2)^{4/3}\left(-\frac{1}{3}c\int \frac{1}{(c+dx)(c^2-cdx)^{2/3}}dx-\frac{\sqrt[3]{c^2-cdx}}{d(c+dx)}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)(c^2-cdx)^{4/3}}$$

$\downarrow$ 69

$$\frac{(c^2-d^2x^2)^{4/3}\left(-\frac{1}{3}c\left(-\frac{3\int \frac{1}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}}d\sqrt[3]{c^2-cdx}}{2\ 2^{2/3}c^{4/3}d}-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-c}}{2\sqrt[3]{2}c^{2/3}d}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)(c^2-cdx)^{4/3}}$$

$\downarrow$ 16

$$\frac{(c^2-d^2x^2)^{4/3}\left(-\frac{1}{3}c\left(-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2}\right.}{2\ 2^{2/3}c^{4/3}d}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)(c^2-cdx)^{4/3}}$$

$\downarrow$ 1082

$$\frac{(c^2-d^2x^2)^{4/3}\left(-\frac{1}{3}c\left(\frac{3\int \frac{1}{-\left(c^2-cdx\right)^{2/3}-3}d\left(\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}\right)-\frac{\sqrt[3]{c}}{d(}\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)(c^2-cdx)^{4/3}}$$

$\downarrow$ 217

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{3}c\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1}{\sqrt{3}}\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}\right)-\frac{\sqrt[3]{c^2-cdx}}{d(c+dx)}\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(7/3),x]
```

output
```
((c^2 - d^2*x^2)^(4/3)*(-((c^2 - c*d*x)^(1/3)/(d*(c + d*x))) - (c*(-((Sqrt
[3]*ArcTan[(1 + (2^(2/3)*(c^2 - c*d*x)^(1/3))/c^(2/3))/Sqrt[3]])/(2^(2/3)*
c^(4/3)*d)) - Log[c + d*x]/(2*2^(2/3)*c^(4/3)*d) + (3*Log[2^(1/3)*c^(2/3)
- (c^2 - c*d*x)^(1/3)])/(2*2^(2/3)*c^(4/3)*d)))/3))/((c + d*x)^(1/3)*(1 +
(d*x)/c)*(c^2 - c*d*x)^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{1}{3}}}{\left(d x +c \right)^{\frac{7}{3}}}d x$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(7/3),x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx = \frac{4^{\frac{2}{3}}(d^2x^2+2\,cdx+c^2)(-c^2)^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(-d^2x^2+c^2)^{\frac{1}{3}}(-c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+2\,(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}c-2\cdot 4^{\frac{1}{3}}\,(}{dx+c}\right.}{}$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
1/24*(4^(2/3)*(d^2*x^2 + 2*c*d*x + c^2)*(-c^2)^(2/3)*log((4^(2/3)*(-d^2*x^
2 + c^2)^(1/3)*(-c^2)^(2/3)*(d*x + c)^(2/3) + 2*(-d^2*x^2 + c^2)^(2/3)*(d*
x + c)^(1/3)*c - 2*4^(1/3)*(c*d*x + c^2)*(-c^2)^(1/3))/(d*x + c)) - 2*4^(2
/3)*(d^2*x^2 + 2*c*d*x + c^2)*(-c^2)^(2/3)*log(-(4^(2/3)*(-c^2)^(2/3)*(d*x
 + c) - 2*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*c)/(d*x + c)) - 24*(-d^2*
x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*c^2 + 12*sqrt(1/3)*(c*d^2*x^2 + 2*c^2*d*x
 + c^3)*sqrt(-4^(1/3)*(-c^2)^(1/3))*arctan(1/2*sqrt(1/3)*(4^(2/3)*(-d^2*x^
2 + c^2)^(1/3)*(-c^2)^(2/3)*(d*x + c)^(2/3) - 4^(1/3)*(c*d*x + c^2)*(-c^2)
^(1/3))*sqrt(-4^(1/3)*(-c^2)^(1/3))/(c^2*d*x + c^3)))/(c^2*d^3*x^2 + 2*c^3
*d^2*x + c^4*d)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx = \int \frac{\sqrt[3]{-(-c+dx)\,(c+dx)}}{(c+dx)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c)**(7/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x)**(7/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(7/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx=\int \frac{(c^2-d^2\,x^2)^{1/3}}{(c+d\,x)^{7/3}}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(7/3),x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(7/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}\,c^2+2\,(dx+c)^{\frac{1}{3}}\,cdx+(dx+c)^{\frac{1}{3}}\,d^2x^2}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(7/3),x)
```

output
```
int((c**2 - d**2*x**2)**(1/3)/((c + d*x)**(1/3)*c**2 + 2*(c + d*x)**(1/3)*
c*d*x + (c + d*x)**(1/3)*d**2*x**2),x)
```

3.278 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 294

$$\begin{aligned}\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx = & -\frac{\sqrt[3]{c^2-d^2x^2}}{2d(c+dx)^{7/3}}+\frac{\sqrt[3]{c^2-d^2x^2}}{12cd(c+dx)^{4/3}} \\ & + \frac{\sqrt[3]{c^2-d^2x^2}\arctan\left(\frac{\sqrt[3]{c}+2^{2/3}\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{6\ 2^{2/3}\sqrt{3}c^{5/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}} \\ & + \frac{\sqrt[3]{c^2-d^2x^2}\log(c+dx)}{36\ 2^{2/3}c^{5/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}} - \frac{\sqrt[3]{c^2-d^2x^2}\log\left(\sqrt[3]{2}\sqrt[3]{c}-\sqrt[3]{c-dx}\right)}{12\ 2^{2/3}c^{5/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}}\end{aligned}$$

output
```
-1/2*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(7/3)+1/12*(-d^2*x^2+c^2)^(1/3)/c/d/(d
*x+c)^(4/3)+1/36*(-d^2*x^2+c^2)^(1/3)*arctan(1/3*(c^(1/3)+2^(2/3)*(-d*x+c)
^(1/3))*3^(1/2)/c^(1/3))*2^(1/3)*3^(1/2)/c^(5/3)/d/(-d*x+c)^(1/3)/(d*x+c)^
(1/3)+1/72*(-d^2*x^2+c^2)^(1/3)*ln(d*x+c)*2^(1/3)/c^(5/3)/d/(-d*x+c)^(1/3)
/(d*x+c)^(1/3)-1/24*(-d^2*x^2+c^2)^(1/3)*ln(2^(1/3)*c^(1/3)-(-d*x+c)^(1/3)
)*2^(1/3)/c^(5/3)/d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx = \frac{\frac{6c^{2/3}(-5c+dx)\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}} - 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c+dx}}{\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/3}\sqrt[3]{c^2-d^2x^2}}\right) - 2\sqrt[3]{2}\log}{}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(10/3),x]
```

output
```
((6*c^(2/3)*(-5*c + d*x)*(c^2 - d^2*x^2)^(1/3))/(c + d*x)^(7/3) - 2*2^(1/3
)*Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*(c + d*x)^(1/3))/(c^(1/3)*(c + d*x)^(1/3
) + 2^(2/3)*(c^2 - d^2*x^2)^(1/3))] - 2*2^(1/3)*Log[-2*c^(1/3)*(c + d*x)^(
1/3) + 2^(2/3)*(c^2 - d^2*x^2)^(1/3)] + 2^(1/3)*Log[2*c^(2/3)*(c + d*x)^(2
/3) + 2^(2/3)*c^(1/3)*(c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3) + 2^(1/3)*(c^2
 - d^2*x^2)^(2/3)])/(72*c^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {474, 473, 27, 51, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\left(\frac{dx}{c}+1\right)^{10/3}}dx}{c^3\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{473}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\int \frac{c^3\sqrt[3]{c^2-cdx}}{(c+dx)^3}dx}{c^3\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\int \frac{\sqrt[3]{c^2-cdx}}{(c+dx)^3}dx}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 51$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{6}c\int \frac{1}{(c+dx)^2\left(c^2-cdx\right)^{2/3}}dx-\frac{\sqrt[3]{c^2-cdx}}{2d(c+dx)^2}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 52$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{6}c\left(\frac{\int \frac{1}{(c+dx)\left(c^2-cdx\right)^{2/3}}dx}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)-\frac{\sqrt[3]{c^2-cdx}}{2d(c+dx)^2}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 69$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{6}c\left(\frac{-\frac{3\int \frac{1}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}}d\sqrt[3]{c^2-cdx}}{2\ 2^{2/3}c^{4/3}d}-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}}{3c}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 16$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{6}c\left(\frac{-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cd\right.}}{2\ 2^{2/3}c^{4/3}d}}{3c}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 1082$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{6}c\left(\frac{\frac{3\int \frac{1}{-\left(c^2-cdx\right)^{2/3}-3}d\left(\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)\ldots\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$\big\downarrow$ 217

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{6}c\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1}{\sqrt{3}}\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)-\ldots\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(10/3),x]
```

output
```
((c^2 - d^2*x^2)^(4/3)*(-1/2*(c^2 - c*d*x)^(1/3)/(d*(c + d*x)^2) - (c*(-1/
2*(c^2 - c*d*x)^(1/3)/(c^2*d*(c + d*x)) + (-((Sqrt[3]*ArcTan[(1 + (2^(2/3)
*(c^2 - c*d*x)^(1/3))/c^(2/3))/Sqrt[3]])/(2^(2/3)*c^(4/3)*d)) - Log[c + d*
x]/(2*2^(2/3)*c^(4/3)*d) + (3*Log[2^(1/3)*c^(2/3) - (c^2 - c*d*x)^(1/3)])/
(2*2^(2/3)*c^(4/3)*d))/(3*c)))/6))/((c + d*x)^(1/3)*(1 + (d*x)/c)*(c^2 - c
*d*x)^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{10}{3}}}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(10/3),x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(10/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx = \frac{4^{\frac{2}{3}}(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3)(-c^2)^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}\left(-d^2x^2+c^2\right)^{\frac{1}{3}}\left(-c^2\right)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+2\left(-d^2x^2+c^2\right)^{\frac{2}{3}}(c}{dx+c}\right.}{}$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(10/3),x, algorithm="fricas")
```

output
```
1/144*(4^(2/3)*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-c^2)^(2/3)*log(
(4^(2/3)*(-d^2*x^2 + c^2)^(1/3)*(-c^2)^(2/3)*(d*x + c)^(2/3) + 2*(-d^2*x^2
 + c^2)^(2/3)*(d*x + c)^(1/3)*c - 2*4^(1/3)*(c*d*x + c^2)*(-c^2)^(1/3))/(d
*x + c)) - 2*4^(2/3)*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-c^2)^(2/3
)*log(-(4^(2/3)*(-c^2)^(2/3)*(d*x + c) - 2*(-d^2*x^2 + c^2)^(1/3)*(d*x + c
)^(2/3)*c)/(d*x + c)) + 12*sqrt(1/3)*(c*d^3*x^3 + 3*c^2*d^2*x^2 + 3*c^3*d*
x + c^4)*sqrt(-4^(1/3)*(-c^2)^(1/3))*arctan(1/2*sqrt(1/3)*(4^(2/3)*(-d^2*x
^2 + c^2)^(1/3)*(-c^2)^(2/3)*(d*x + c)^(2/3) - 4^(1/3)*(c*d*x + c^2)*(-c^2
)^(1/3))*sqrt(-4^(1/3)*(-c^2)^(1/3))/(c^2*d*x + c^3)) + 12*(c^2*d*x - 5*c^
3)*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3))/(c^3*d^4*x^3 + 3*c^4*d^3*x^2 +
3*c^5*d^2*x + c^6*d)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx = \int \frac{\sqrt[3]{-(-c+dx)\,(c+dx)}}{(c+dx)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c)**(10/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x)**(10/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(10/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(10/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{(dx+c)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(10/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx=\int \frac{\left(c^2-d^2\,x^2\right)^{1/3}}{(c+d\,x)^{10/3}}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(10/3),x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(10/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}\,c^3+3\,(dx+c)^{\frac{1}{3}}\,c^2dx+3\,(dx+c)^{\frac{1}{3}}\,c\,d^2x^2+(dx+c)^{\frac{1}{3}}\,d^3x^3}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(10/3),x)
```

output
```
int((c**2 - d**2*x**2)**(1/3)/((c + d*x)**(1/3)*c**3 + 3*(c + d*x)**(1/3)*
c**2*d*x + 3*(c + d*x)**(1/3)*c*d**2*x**2 + (c + d*x)**(1/3)*d**3*x**3),x)
```

3.279 $\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 329

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx = -\frac{\sqrt[3]{c^2-d^2x^2}}{3d(c+dx)^{10/3}}+\frac{\sqrt[3]{c^2-d^2x^2}}{36cd(c+dx)^{7/3}} + \frac{5\sqrt[3]{c^2-d^2x^2}}{216c^2d(c+dx)^{4/3}}+\frac{5\sqrt[3]{c^2-d^2x^2}\arctan\left(\frac{\sqrt[3]{c}+2^{2/3}\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{108\ 2^{2/3}\sqrt{3}c^{8/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}} + \frac{5\sqrt[3]{c^2-d^2x^2}\log(c+dx)}{648\ 2^{2/3}c^{8/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}}-\frac{5\sqrt[3]{c^2-d^2x^2}\log\left(\sqrt[3]{2}\sqrt[3]{c}-\sqrt[3]{c-dx}\right)}{216\ 2^{2/3}c^{8/3}d\sqrt[3]{c-dx}\sqrt[3]{c+dx}}$$

output
```
-1/3*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(10/3)+1/36*(-d^2*x^2+c^2)^(1/3)/c/d/(
d*x+c)^(7/3)+5/216*(-d^2*x^2+c^2)^(1/3)/c^2/d/(d*x+c)^(4/3)+5/648*(-d^2*x^
2+c^2)^(1/3)*arctan(1/3*(c^(1/3)+2^(2/3)*(-d*x+c)^(1/3))*3^(1/2)/c^(1/3))*
2^(1/3)*3^(1/2)/c^(8/3)/d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)+5/1296*(-d^2*x^2+c^
2)^(1/3)*ln(d*x+c)*2^(1/3)/c^(8/3)/d/(-d*x+c)^(1/3)/(d*x+c)^(1/3)-5/432*(-
d^2*x^2+c^2)^(1/3)*ln(2^(1/3)*c^(1/3)-(-d*x+c)^(1/3))*2^(1/3)/c^(8/3)/d/(-
d*x+c)^(1/3)/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx=\frac{-\frac{6c^{2/3}(61c^2-16cdx-5d^2x^2)\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{10/3}}-10\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c+dx}}{\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/3}\sqrt[3]{c^2-d^2x^2}}\right)}{}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(13/3),x]
```

output
```
((-6*c^(2/3)*(61*c^2 - 16*c*d*x - 5*d^2*x^2)*(c^2 - d^2*x^2)^(1/3))/(c + d
*x)^(10/3) - 10*2^(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*(c + d*x)^(1/3))/(
c^(1/3)*(c + d*x)^(1/3) + 2^(2/3)*(c^2 - d^2*x^2)^(1/3))] - 10*2^(1/3)*Log
[-2*c^(1/3)*(c + d*x)^(1/3) + 2^(2/3)*(c^2 - d^2*x^2)^(1/3)] + 5*2^(1/3)*L
og[2*c^(2/3)*(c + d*x)^(2/3) + 2^(2/3)*c^(1/3)*(c + d*x)^(1/3)*(c^2 - d^2*
x^2)^(1/3) + 2^(1/3)*(c^2 - d^2*x^2)^(2/3)])/(1296*c^(8/3)*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {474, 473, 27, 51, 52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{\sqrt[3]{c^2-d^2x^2}}{\left(\frac{dx}{c}+1\right)^{13/3}}dx}{c^4\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{473}$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\int \frac{c^4\sqrt[3]{c^2-cdx}}{(c+dx)^4}dx}{c^4\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\int \frac{\sqrt[3]{c^2-cdx}}{(c+dx)^4}dx}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 51$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{9}c\int \frac{1}{(c+dx)^3\left(c^2-cdx\right)^{2/3}}dx-\frac{\sqrt[3]{c^2-cdx}}{3d(c+dx)^3}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 52$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{9}c\left(\frac{5\int \frac{1}{(c+dx)^2\left(c^2-cdx\right)^{2/3}}dx}{12c}-\frac{\sqrt[3]{c^2-cdx}}{4c^2d(c+dx)^2}\right)-\frac{\sqrt[3]{c^2-cdx}}{3d(c+dx)^3}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 52$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{9}c\left(\frac{5\left(\frac{\int \frac{1}{(c+dx)\left(c^2-cdx\right)^{2/3}}dx}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{12c}-\frac{\sqrt[3]{c^2-cdx}}{4c^2d(c+dx)^2}\right)-\frac{\sqrt[3]{c^2-cdx}}{3d(c+dx)^3}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$$\Big\downarrow \; 69$$

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{9}c\left(\frac{5\left(\frac{-\frac{3\int \frac{1}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}}d\sqrt[3]{c^2-cdx}}{2\ 2^{2/3}c^{4/3}d}-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-c}}{2\sqrt[3]{2}c^{2/3}d}}{3c}\right.}{12c}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

$\downarrow$ 16

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{9}c\left(\frac{5\left(\frac{-\frac{3\int\frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-c\ldots}\right)}{2\ 2^{2/3}c^{4/3}d}\ldots}{3c}\right)\ldots}{12c}\ldots\right)\ldots\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}\ldots}$$

$\downarrow$ 1082

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{9}c\left(\frac{5\left(\frac{\frac{3\int\frac{1}{-\left(c^2-cdx\right)^{2/3}-3}d\left(\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cd\ldots}}{2c^2d(c+dx)}\ldots\right)}{12c}\ldots\right)\ldots\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}\ldots}$$

$\downarrow$ 217

$$\frac{\left(c^2-d^2x^2\right)^{4/3}\left(-\frac{1}{9}c\left(\frac{5\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1}{\sqrt{3}}\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{12c}-\frac{\sqrt[3]{\cdots}}{4\cdots}\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)\left(c^2-cdx\right)^{4/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/3)/(c + d*x)^(13/3),x]
```

output
```
((c^2 - d^2*x^2)^(4/3)*(-1/3*(c^2 - c*d*x)^(1/3)/(d*(c + d*x)^3) - (c*(-1/
4*(c^2 - c*d*x)^(1/3)/(c^2*d*(c + d*x)^2) + (5*(-1/2*(c^2 - c*d*x)^(1/3)/(
c^2*d*(c + d*x)) + (-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(c^2 - c*d*x)^(1/3))/c
^(2/3))/Sqrt[3]])/(2^(2/3)*c^(4/3)*d)) - Log[c + d*x]/(2*2^(2/3)*c^(4/3)*d
) + (3*Log[2^(1/3)*c^(2/3) - (c^2 - c*d*x)^(1/3)])/(2*2^(2/3)*c^(4/3)*d))/
(3*c)))/(12*c)))/9))/((c + d*x)^(1/3)*(1 + (d*x)/c)*(c^2 - c*d*x)^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{13}{3}}}dx$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(13/3),x)
```

output
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(13/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx = \frac{5\cdot 4^{\frac{2}{3}}(d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4)(-c^2)^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(-d^2x^2+c^2)^{\frac{1}{3}}(-c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(13/3),x, algorithm="fricas")
```

output
```
1/2592*(5*4^(2/3)*(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4
)*(-c^2)^(2/3)*log((4^(2/3)*(-d^2*x^2 + c^2)^(1/3)*(-c^2)^(2/3)*(d*x + c)^
(2/3) + 2*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(1/3)*c - 2*4^(1/3)*(c*d*x + c^
2)*(-c^2)^(1/3))/(d*x + c)) - 10*4^(2/3)*(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^
2*x^2 + 4*c^3*d*x + c^4)*(-c^2)^(2/3)*log(-(4^(2/3)*(-c^2)^(2/3)*(d*x + c)
 - 2*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*c)/(d*x + c)) + 60*sqrt(1/3)*(
c*d^4*x^4 + 4*c^2*d^3*x^3 + 6*c^3*d^2*x^2 + 4*c^4*d*x + c^5)*sqrt(-4^(1/3)
*(-c^2)^(1/3))*arctan(1/2*sqrt(1/3)*(4^(2/3)*(-d^2*x^2 + c^2)^(1/3)*(-c^2)
^(2/3)*(d*x + c)^(2/3) - 4^(1/3)*(c*d*x + c^2)*(-c^2)^(1/3))*sqrt(-4^(1/3)
*(-c^2)^(1/3))/(c^2*d*x + c^3)) + 12*(5*c^2*d^2*x^2 + 16*c^3*d*x - 61*c^4)
*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3))/(c^4*d^5*x^4 + 4*c^5*d^4*x^3 + 6*
c^6*d^3*x^2 + 4*c^7*d^2*x + c^8*d)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx = \int \frac{\sqrt[3]{-(-c+dx)(c+dx)}}{(c+dx)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/3)/(d*x+c)**(13/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/3)/(c + d*x)**(13/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{3}}}{(dx+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(13/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(13/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{(dx+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(13/3),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/3)/(d*x + c)^(13/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx=\int \frac{\left(c^2-d^2\,x^2\right)^{1/3}}{\left(c+d\,x\right)^{13/3}}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(13/3),x)
```

output
```
int((c^2 - d^2*x^2)^(1/3)/(c + d*x)^(13/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{13/3}}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}c^4+4\,(dx+c)^{\frac{1}{3}}c^3dx+6\,(dx+c)^{\frac{1}{3}}c^2d^2x^2+4\,(dx+c)^{\frac{1}{3}}c\,d^3x^3+(dx+c}$$

input
```
int((-d^2*x^2+c^2)^(1/3)/(d*x+c)^(13/3),x)
```

output
```
int((c**2 - d**2*x**2)**(1/3)/((c + d*x)**(1/3)*c**4 + 4*(c + d*x)**(1/3)*
c**3*d*x + 6*(c + d*x)**(1/3)*c**2*d**2*x**2 + 4*(c + d*x)**(1/3)*c*d**3*x
**3 + (c + d*x)**(1/3)*d**4*x**4),x)
```

3.280 $\int (c+dx)^{2/3} \left(c^2-d^2x^2\right)^{2/3} dx$

Optimal result

Integrand size = 26, antiderivative size = 303

$$\int (c+dx)^{2/3} \left(c^2-d^2x^2\right)^{2/3} dx = \frac{8c^2(c^2-d^2x^2)^{2/3}}{27d\sqrt[3]{c+dx}} - \frac{4c(c^2-d^2x^2)^{5/3}}{9d(c+dx)^{4/3}} - \frac{(c^2-d^2x^2)^{5/3}}{3d\sqrt[3]{c+dx}} + \frac{32c^3(c^2-d^2x^2)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c+dx}}\right)}{27\sqrt{3}d(c-dx)^{2/3}(c+dx)^{2/3}} + \frac{16c^3(c^2-d^2x^2)^{2/3}\log(c+dx)}{81d(c-dx)^{2/3}(c+dx)^{2/3}} + \frac{16c^3(c^2-d^2x^2)^{2/3}\log\left(1+\frac{\sqrt[3]{c-dx}}{\sqrt[3]{c+dx}}\right)}{27d(c-dx)^{2/3}(c+dx)^{2/3}}$$

output

```
8/27*c^2*(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)^(1/3)-4/9*c*(-d^2*x^2+c^2)^(5/3)/d
/(d*x+c)^(4/3)-1/3*(-d^2*x^2+c^2)^(5/3)/d/(d*x+c)^(1/3)-32/81*c^3*(-d^2*x^
2+c^2)^(2/3)*arctan(-1/3*3^(1/2)+2/3*(-d*x+c)^(1/3)*3^(1/2)/(d*x+c)^(1/3))
*3^(1/2)/d/(-d*x+c)^(2/3)/(d*x+c)^(2/3)+16/81*c^3*(-d^2*x^2+c^2)^(2/3)*ln(
d*x+c)/d/(-d*x+c)^(2/3)/(d*x+c)^(2/3)+16/27*c^3*(-d^2*x^2+c^2)^(2/3)*ln(1+
(-d*x+c)^(1/3)/(d*x+c)^(1/3))/d/(-d*x+c)^(2/3)/(d*x+c)^(2/3)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.66

$$\int (c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\,dx =$$
$$-\frac{\frac{3(13c^2-12cdx-9d^2x^2)(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}+32\sqrt{3}c^3\arctan\left(\frac{\sqrt{3}(c+dx)^{2/3}}{(c+dx)^{2/3}-2\sqrt[3]{c^2-d^2x^2}}\right)-32c^3\log\left((c+dx)^{2/3}+\sqrt[3]{c^2-\right.}{81d}$$

input
```
Integrate[(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
-1/81*((3*(13*c^2 - 12*c*d*x - 9*d^2*x^2)*(c^2 - d^2*x^2)^(2/3))/(c + d*x)
^(1/3) + 32*Sqrt[3]*c^3*ArcTan[(Sqrt[3]*(c + d*x)^(2/3))/((c + d*x)^(2/3)
- 2*(c^2 - d^2*x^2)^(1/3))] - 32*c^3*Log[(c + d*x)^(2/3) + (c^2 - d^2*x^2)
^(1/3)] + 16*c^3*Log[(c + d*x)^(4/3) - (c + d*x)^(2/3)*(c^2 - d^2*x^2)^(1/
3) + (c^2 - d^2*x^2)^(2/3)])/d
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {474, 473, 60, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{(c+dx)^{2/3}\int \left(\frac{dx}{c}+1\right)^{2/3}\left(c^2-d^2x^2\right)^{2/3}dx}{\left(\frac{dx}{c}+1\right)^{2/3}}$$

$$\Big\downarrow \text{473}$$

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{5/3}\int \left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}dx}{\left(\frac{dx}{c}+1\right)^{7/3}\left(c^2-cdx\right)^{5/3}}$$

$\big\downarrow$ 60

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{5/3}\left(\frac{4}{9}c^2\int\frac{\left(\frac{dx}{c}+1\right)^{4/3}}{\sqrt[3]{c^2-cdx}}dx+\frac{c\left(c^2-cdx\right)^{2/3}\left(\frac{dx}{c}+1\right)^{7/3}}{3d}\right)}{\left(\frac{dx}{c}+1\right)^{7/3}\left(c^2-cdx\right)^{5/3}}$$

$\big\downarrow$ 60

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{5/3}\left(\frac{4}{9}c^2\left(\frac{4}{3}\int\frac{\sqrt[3]{\frac{dx}{c}+1}}{\sqrt[3]{c^2-cdx}}dx-\frac{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}{2cd}\right)+\frac{c\left(c^2-cdx\right)^{2/3}\left(\frac{dx}{c}+1\right)^{7/3}}{3d}\right)}{\left(\frac{dx}{c}+1\right)^{7/3}\left(c^2-cdx\right)^{5/3}}$$

$\big\downarrow$ 60

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{5/3}\left(\frac{4}{9}c^2\left(\frac{4}{3}\left(\frac{2}{3}\int\frac{1}{\left(\frac{dx}{c}+1\right)^{2/3}\sqrt[3]{c^2-cdx}}dx-\frac{\sqrt[3]{\frac{dx}{c}+1}\left(c^2-cdx\right)^{2/3}}{cd}\right)-\frac{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/}}{2cd}\right.\right.}{\left(\frac{dx}{c}+1\right)^{7/3}\left(c^2-cdx\right)^{5/3}}$$

$\big\downarrow$ 72

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{5/3}\left(\frac{4}{9}c^2\left(\frac{4}{3}\left(\frac{2}{3}\left(\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c^2-cdx}}{\sqrt{3}c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}\right)}{d}+\frac{3\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c^2-cdx}}{c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}+1\right)}{2d}+\frac{\sqrt[3]{c}}{}\right.\right.\right.\right.}{\left(\frac{dx}{c}+1\right)^{7/3}\left(c^2-cdx\right)^{5/3}}$$

input

```
Int[(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(2/3),x]
```

output
```
((c + d*x)^(2/3)*(c^2 - d^2*x^2)^(5/3)*((c*(1 + (d*x)/c)^(7/3)*(c^2 - c*d*
x)^(2/3))/(3*d) + (4*c^2*(-1/2*((1 + (d*x)/c)^(4/3)*(c^2 - c*d*x)^(2/3))/(
c*d) + (4*(-(((1 + (d*x)/c)^(1/3)*(c^2 - c*d*x)^(2/3))/(c*d)) + (2*((Sqrt[
3]*c^(1/3)*ArcTan[1/Sqrt[3] - (2*(c^2 - c*d*x)^(1/3))/(Sqrt[3]*c^(2/3)*(1
+ (d*x)/c)^(1/3))]])/d + (c^(1/3)*Log[1 + (d*x)/c])/(2*d) + (3*c^(1/3)*Log[
1 + (c^2 - c*d*x)^(1/3)/(c^(2/3)*(1 + (d*x)/c)^(1/3))])/(2*d)))/3))/3))/9)
)/((1 + (d*x)/c)^(7/3)*(c^2 - c*d*x)^(5/3))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (dx+c)^{\frac{2}{3}} \left(-d^2x^2+c^2\right)^{\frac{2}{3}} dx$$

input
```
int((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.94

$$\int (c+dx)^{2/3} \left(c^2-d^2x^2\right)^{2/3} dx =$$

$$-\frac{32\sqrt{3}(c^3dx+c^4)\arctan\left(\frac{2\sqrt{3}(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+\sqrt{3}(d^2x^2-c^2)}{3(d^2x^2-c^2)}\right)-3(9d^2x^2+12cdx-13c^2)(-d^2x^2+c^2)^{\frac{2}{3}}}{}$$

input
```
integrate((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-1/81*(32*sqrt(3)*(c^3*d*x + c^4)*arctan(1/3*(2*sqrt(3)*(-d^2*x^2 + c^2)^(
2/3)*(d*x + c)^(2/3) + sqrt(3)*(d^2*x^2 - c^2))/(d^2*x^2 - c^2)) - 3*(9*d^
2*x^2 + 12*c*d*x - 13*c^2)*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3) + 16*(c^
3*d*x + c^4)*log((d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(4/3) +
 (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) - 32*(c^3*d*x +
c^4)*log(-(d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^
2 - c^2)))/(d^2*x + c*d)
```

Sympy [F]

$$\int (c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\,dx=\int \left(-(-c+dx)\,(c+dx)\right)^{\frac{2}{3}}(c+dx)^{\frac{2}{3}}\,dx$$

input
```
integrate((d*x+c)**(2/3)*(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)*(c + d*x)**(2/3), x)
```

Maxima [F]

$$\int (c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}\,dx$$

input
```
integrate((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.99

$$\int (c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\,dx=$$

$$-\frac{144\sqrt{3}c^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)-\left(16\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3\left(2\left(\frac{2c}{dx+c}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
-1/324*(144*sqrt(3)*c^3*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) -
1)) - (16*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) - 1)) +
3*(2*(2*c/(d*x + c) - 1)^(8/3) - 35*(2*c/(d*x + c) - 1)^(5/3) - (2*c/(d*x
+ c) - 1)^(2/3))*(d*x + c)^3/c^3 + 8*log((2*c/(d*x + c) - 1)^(2/3) - (2*c/
(d*x + c) - 1)^(1/3) + 1) - 16*log(abs((2*c/(d*x + c) - 1)^(1/3) + 1)))*c^
3 + 72*c^3*log((2*c/(d*x + c) - 1)^(2/3) - (2*c/(d*x + c) - 1)^(1/3) + 1)
- 144*c^3*log(abs((2*c/(d*x + c) - 1)^(1/3) + 1)) + 54*(2*c^3*(2*c/(d*x +
c) - 1)^(5/3) - c^3*(2*c/(d*x + c) - 1)^(2/3))*(d*x + c)^2/c^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{2/3} \left(c^2-d^2x^2\right)^{2/3} \, dx = \int \left(c^2-d^2\,x^2\right)^{2/3}\,{\left(c+d\,x\right)}^{2/3} \,dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(2/3),x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(2/3), x)
```

Reduce [F]

$$\int (c+dx)^{2/3} \left(c^2-d^2x^2\right)^{2/3} \, dx = \frac{9(dx+c)^{\frac{2}{3}} \left(-d^2x^2+c^2\right)^{\frac{2}{3}} c+3(dx+c)^{\frac{2}{3}} \left(-d^2x^2+c^2\right)^{\frac{2}{3}} dx+16\left(\int \frac{(dx+c)^{\frac{2}{3}} x}{\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}dx\right) c\, d^2}{9d}$$

input
```
int((d*x+c)^(2/3)*(-d^2*x^2+c^2)^(2/3),x)
```

output
```
(9*(c + d*x)**(2/3)*(c**2 - d**2*x**2)**(2/3)*c + 3*(c + d*x)**(2/3)*(c**2
 - d**2*x**2)**(2/3)*d*x + 16*int(((c + d*x)**(2/3)*(c**2 - d**2*x**2)**(2
/3)*x)/(c**2 - d**2*x**2),x)*c*d**2)/(9*d)
```

3.281 $\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{\sqrt[3]{c+dx}}\,dx = \frac{c\left(c^2-d^2x^2\right)^{2/3}}{3d\sqrt[3]{c+dx}} - \frac{\left(c^2-d^2x^2\right)^{5/3}}{2d(c+dx)^{4/3}} + \frac{4c^2\left(c^2-d^2x^2\right)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}d(c-dx)^{2/3}(c+dx)^{2/3}} + \frac{2c^2\left(c^2-d^2x^2\right)^{2/3}\log(c+dx)}{9d(c-dx)^{2/3}(c+dx)^{2/3}} + \frac{2c^2\left(c^2-d^2x^2\right)^{2/3}\log\left(1+\frac{\sqrt[3]{c-dx}}{\sqrt[3]{c+dx}}\right)}{3d(c-dx)^{2/3}(c+dx)^{2/3}}$$

output
```
1/3*c*(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)^(1/3)-1/2*(-d^2*x^2+c^2)^(5/3)/d/(d*x
+c)^(4/3)-4/9*c^2*(-d^2*x^2+c^2)^(2/3)*arctan(-1/3*3^(1/2)+2/3*(-d*x+c)^(1
/3)*3^(1/2)/(d*x+c)^(1/3))*3^(1/2)/d/(-d*x+c)^(2/3)/(d*x+c)^(2/3)+2/9*c^2*
(-d^2*x^2+c^2)^(2/3)*ln(d*x+c)/d/(-d*x+c)^(2/3)/(d*x+c)^(2/3)+2/3*c^2*(-d^
2*x^2+c^2)^(2/3)*ln(1+(-d*x+c)^(1/3)/(d*x+c)^(1/3))/d/(-d*x+c)^(2/3)/(d*x+
c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.80

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}\,dx = \frac{-\frac{3c(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}+\frac{9dx(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}-8\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}(c+dx)^{2/3}}{(c+dx)^{2/3}-2\sqrt[3]{c^2-d^2x^2}}\right)+8c^2\log}{}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(1/3),x]
```

output
```
((-3*c*(c^2 - d^2*x^2)^(2/3))/(c + d*x)^(1/3) + (9*d*x*(c^2 - d^2*x^2)^(2/
3))/(c + d*x)^(1/3) - 8*Sqrt[3]*c^2*ArcTan[(Sqrt[3]*(c + d*x)^(2/3))/((c +
 d*x)^(2/3) - 2*(c^2 - d^2*x^2)^(1/3))] + 8*c^2*Log[d*((c + d*x)^(2/3) + (
c^2 - d^2*x^2)^(1/3))] - 4*c^2*Log[(c + d*x)^(4/3) - (c + d*x)^(2/3)*(c^2
- d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3)])/(18*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {474, 473, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{\frac{dx}{c}+1}}dx}{\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{473}$$

$$\frac{\left(c^2-d^2x^2\right)^{5/3}\int \sqrt[3]{\frac{dx}{c}+1}\left(c^2-cdx\right)^{2/3}dx}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{5/3}}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(c^2-d^2x^2\right)^{5/3}\left(\frac{2}{3}c^2\int \frac{\sqrt[3]{\frac{dx}{c}+1}}{\sqrt[3]{c^2-cdx}}dx+\frac{c\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}{2d}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{5/3}}$$

$$\Big\downarrow \; 60$$

$$\frac{\left(c^2-d^2x^2\right)^{5/3}\left(\frac{2}{3}c^2\left(\frac{2}{3}\int \frac{1}{\left(\frac{dx}{c}+1\right)^{2/3}\sqrt[3]{c^2-cdx}}dx-\frac{\sqrt[3]{\frac{dx}{c}+1}\left(c^2-cdx\right)^{2/3}}{cd}\right)+\frac{c\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}{2d}\right)}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{5/3}}$$

$$\Big\downarrow \; 72$$

$$\frac{\left(c^2-d^2x^2\right)^{5/3}\left(\frac{2}{3}c^2\left(\frac{2}{3}\left(\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c^2-cdx}}{\sqrt{3}c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}\right)}{d}+\frac{3\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c^2-cdx}}{c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}+1\right)}{2d}+\frac{\sqrt[3]{c}\log\left(\frac{dx}{c}+1\right)}{2d}\right)-\cdots\right.\right.}{\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{5/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(1/3),x]
```

output
```
((c^2 - d^2*x^2)^(5/3)*((c*(1 + (d*x)/c)^(4/3)*(c^2 - c*d*x)^(2/3))/(2*d)
+ (2*c^2*(-(((1 + (d*x)/c)^(1/3)*(c^2 - c*d*x)^(2/3))/(c*d)) + (2*((Sqrt[3
]*c^(1/3)*ArcTan[1/Sqrt[3] - (2*(c^2 - c*d*x)^(1/3))/(Sqrt[3]*c^(2/3)*(1 +
 (d*x)/c)^(1/3))])/d + (c^(1/3)*Log[1 + (d*x)/c])/(2*d) + (3*c^(1/3)*Log[1
 + (c^2 - c*d*x)^(1/3)/(c^(2/3)*(1 + (d*x)/c)^(1/3))])/(2*d)))/3))/3))/((c
 + d*x)^(1/3)*(1 + (d*x)/c)^(4/3)*(c^2 - c*d*x)^(5/3))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(-d^2x^2 + c^2)^{\frac{2}{3}}}{(dx + c)^{\frac{1}{3}}}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(1/3),x)
```

output
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.03

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}\,dx =$$
$$-\frac{8\sqrt{3}(c^2dx+c^3)\arctan\left(\frac{2\sqrt{3}(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+\sqrt{3}(d^2x^2-c^2)}{3(d^2x^2-c^2)}\right)-3(-d^2x^2+c^2)^{\frac{2}{3}}(3dx-c)(dx+c)^{\frac{2}{3}}+4(c}{18}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
-1/18*(8*sqrt(3)*(c^2*d*x + c^3)*arctan(1/3*(2*sqrt(3)*(-d^2*x^2 + c^2)^(2
/3)*(d*x + c)^(2/3) + sqrt(3)*(d^2*x^2 - c^2))/(d^2*x^2 - c^2)) - 3*(-d^2*
x^2 + c^2)^(2/3)*(3*d*x - c)*(d*x + c)^(2/3) + 4*(c^2*d*x + c^3)*log((d^2*
x^2 - c^2 - (-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(4/3) + (-d^2*x^2 + c^2)^(2/3
)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) - 8*(c^2*d*x + c^3)*log(-(d^2*x^2 - c^
2 - (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)))/(d^2*x + c*d
)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}\,dx = \int \frac{(-(-c+dx)(c+dx))^{\frac{2}{3}}}{\sqrt[3]{c+dx}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**(1/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.57

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}\,dx=$$

$$-\frac{8\sqrt{3}c^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)+4c^3\log\left(\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}-\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}+1\right)-8c^3\log\left(\left|\left(\frac{2c}{dx+\cdots}\right.\right.\right.}{18\,cd}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
-1/18*(8*sqrt(3)*c^3*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) - 1))
 + 4*c^3*log((2*c/(d*x + c) - 1)^(2/3) - (2*c/(d*x + c) - 1)^(1/3) + 1) -
8*c^3*log(abs((2*c/(d*x + c) - 1)^(1/3) + 1)) + 3*(2*c^3*(2*c/(d*x + c) -
1)^(5/3) - c^3*(2*c/(d*x + c) - 1)^(2/3))*(d*x + c)^2/c^2)/(c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}\,dx=\int \frac{(c^2-d^2\,x^2)^{2/3}}{(c+d\,x)^{1/3}}\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(1/3),x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^{\frac{1}{3}}}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(1/3),x)
```

output
```
int((c**2 - d**2*x**2)**(2/3)/(c + d*x)**(1/3),x)
```

3.282 $\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{4/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 222

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{4/3}}\,dx = \frac{(c^2-d^2x^2)^{2/3}}{d\sqrt[3]{c+dx}} + \frac{4c(c^2-d^2x^2)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c+dx}}\right)}{\sqrt{3}d(c-dx)^{2/3}(c+dx)^{2/3}} + \frac{2c(c^2-d^2x^2)^{2/3}\log(c+dx)}{3d(c-dx)^{2/3}(c+dx)^{2/3}} + \frac{2c(c^2-d^2x^2)^{2/3}\log\left(1+\frac{\sqrt[3]{c-dx}}{\sqrt[3]{c+dx}}\right)}{d(c-dx)^{2/3}(c+dx)^{2/3}}$$

output
```
(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)^(1/3)-4/3*c*(-d^2*x^2+c^2)^(2/3)*arctan(-1/
3*3^(1/2)+2/3*(-d*x+c)^(1/3)*3^(1/2)/(d*x+c)^(1/3))*3^(1/2)/d/(-d*x+c)^(2/
3)/(d*x+c)^(2/3)+2/3*c*(-d^2*x^2+c^2)^(2/3)*ln(d*x+c)/d/(-d*x+c)^(2/3)/(d*
x+c)^(2/3)+2*c*(-d^2*x^2+c^2)^(2/3)*ln(1+(-d*x+c)^(1/3)/(d*x+c)^(1/3))/d/(
-d*x+c)^(2/3)/(d*x+c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{4/3}}\,dx = \frac{\frac{3(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}} - 4\sqrt{3}c\arctan\left(\frac{\sqrt{3}(c+dx)^{2/3}}{(c+dx)^{2/3}-2\sqrt[3]{c^2-d^2x^2}}\right) + 4c\log\left(d\left((c+dx)^{2/3}+\sqrt[3]{c\right.\right.}{3d}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(4/3),x]
```

output
```
((3*(c^2 - d^2*x^2)^(2/3))/(c + d*x)^(1/3) - 4*Sqrt[3]*c*ArcTan[(Sqrt[3]*(
c + d*x)^(2/3))/((c + d*x)^(2/3) - 2*(c^2 - d^2*x^2)^(1/3))] + 4*c*Log[d*(
(c + d*x)^(2/3) + (c^2 - d^2*x^2)^(1/3))] - 2*c*Log[(c + d*x)^(4/3) - (c +
 d*x)^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3)])/(3*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {474, 473, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{4/3}}\,dx$$

$$\Big\downarrow \; 474$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{(c^2-d^2x^2)^{2/3}}{\left(\frac{dx}{c}+1\right)^{4/3}}dx}{c\sqrt[3]{c+dx}}$$

$$\Big\downarrow \; 473$$

$$\frac{(c^2-d^2x^2)^{5/3}\int \frac{(c^2-cdx)^{2/3}}{\left(\frac{dx}{c}+1\right)^{2/3}}dx}{c\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}(c^2-cdx)^{5/3}}$$

$$\Big\downarrow \; 60$$

$$\frac{(c^2-d^2x^2)^{5/3}\left(\frac{4}{3}c^2\int \frac{1}{\left(\frac{dx}{c}+1\right)^{2/3}\sqrt[3]{c^2-cdx}}dx+\frac{c\sqrt[3]{\frac{dx}{c}+1}\,(c^2-cdx)^{2/3}}{d}\right)}{c\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}(c^2-cdx)^{5/3}}$$

$$\Big\downarrow \; 72$$

$$\frac{(c^2-d^2x^2)^{5/3}\left(\frac{4}{3}c^2\left(\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c^2-cdx}}{\sqrt{3}c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}\right)}{d}+\frac{3\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c^2-cdx}}{c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}+1\right)}{2d}+\frac{\sqrt[3]{c}\log\left(\frac{dx}{c}+1\right)}{2d}\right)+\frac{c\sqrt[3]{\cdots}}{\cdots}\right)}{c\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}(c^2-cdx)^{5/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(4/3),x]
```

output
```
((c^2 - d^2*x^2)^(5/3)*((c*(1 + (d*x)/c)^(1/3)*(c^2 - c*d*x)^(2/3))/d + (4
*c^2*((Sqrt[3]*c^(1/3)*ArcTan[1/Sqrt[3] - (2*(c^2 - c*d*x)^(1/3))/(Sqrt[3]
*c^(2/3)*(1 + (d*x)/c)^(1/3))])/d + (c^(1/3)*Log[1 + (d*x)/c])/(2*d) + (3*
c^(1/3)*Log[1 + (c^2 - c*d*x)^(1/3)/(c^(2/3)*(1 + (d*x)/c)^(1/3))])/(2*d))
)/3))/(c*(c + d*x)^(1/3)*(1 + (d*x)/c)^(4/3)*(c^2 - c*d*x)^(5/3))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{3}}}{\left(d x +c \right)^{\frac{4}{3}}}d x$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(4/3),x)
```

output
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(4/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.18

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{4/3}}\,dx = -\frac{4\sqrt{3}(cdx+c^2)\arctan\left(\frac{2\sqrt{3}(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+\sqrt{3}(d^2x^2-c^2)}{3(d^2x^2-c^2)}\right)+2(cdx+c^2)\log\left(\frac{d^2x^2-c^2-(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{4}{3}}}{d^2x^2-c^2}\right.}{3(d^2x+}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
-1/3*(4*sqrt(3)*(c*d*x + c^2)*arctan(1/3*(2*sqrt(3)*(-d^2*x^2 + c^2)^(2/3)
*(d*x + c)^(2/3) + sqrt(3)*(d^2*x^2 - c^2))/(d^2*x^2 - c^2)) + 2*(c*d*x +
c^2)*log((d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(4/3) + (-d^2*x
^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) - 4*(c*d*x + c^2)*log(-(
d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) -
 3*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{4/3}}\, dx = \int \frac{(-(-c + dx)(c + dx))^{\frac{2}{3}}}{(c + dx)^{\frac{4}{3}}}\, dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**(4/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{4/3}}\, dx = \int \frac{(-d^2x^2 + c^2)^{\frac{2}{3}}}{(dx + c)^{\frac{4}{3}}}\, dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^(4/3), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{4/3}} dx =$$
$$-\frac{\left(4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)-\frac{3(dx+c)\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}}{c}+2\log\left(\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}-\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}+1\right)\right.}{3d}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
-1/3*(4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) - 1)) - 3*
(d*x + c)*(2*c/(d*x + c) - 1)^(2/3)/c + 2*log((2*c/(d*x + c) - 1)^(2/3) -
(2*c/(d*x + c) - 1)^(1/3) + 1) - 4*log(abs((2*c/(d*x + c) - 1)^(1/3) + 1))
)*c/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{4/3}} dx = \int \frac{(c^2 - d^2 x^2)^{2/3}}{(c + d x)^{4/3}} dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(4/3),x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{4/3}}\,dx = \int \frac{(-d^2x^2 + c^2)^{\frac{2}{3}}}{(dx + c)^{\frac{1}{3}}\,c + (dx + c)^{\frac{1}{3}}\,dx}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(4/3),x)
```

output
```
int((c**2 - d**2*x**2)**(2/3)/((c + d*x)**(1/3)*c + (c + d*x)**(1/3)*d*x),
x)
```

3.283 $\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{7/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 224

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{7/3}}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{2d(c+dx)^{4/3}} - \frac{\sqrt{3}(c^2-d^2x^2)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c+dx}}\right)}{d(c-dx)^{2/3}(c+dx)^{2/3}} - \frac{(c^2-d^2x^2)^{2/3}\log(c+dx)}{2d(c-dx)^{2/3}(c+dx)^{2/3}} - \frac{3(c^2-d^2x^2)^{2/3}\log\left(1+\frac{\sqrt[3]{c-dx}}{\sqrt[3]{c+dx}}\right)}{2d(c-dx)^{2/3}(c+dx)^{2/3}}$$

output
```
-3/2*(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)^(4/3)+3^(1/2)*(-d^2*x^2+c^2)^(2/3)*arc
tan(-1/3*3^(1/2)+2/3*(-d*x+c)^(1/3)*3^(1/2)/(d*x+c)^(1/3))/d/(-d*x+c)^(2/3
)/(d*x+c)^(2/3)-1/2*(-d^2*x^2+c^2)^(2/3)*ln(d*x+c)/d/(-d*x+c)^(2/3)/(d*x+c
)^(2/3)-3/2*(-d^2*x^2+c^2)^(2/3)*ln(1+(-d*x+c)^(1/3)/(d*x+c)^(1/3))/d/(-d*
x+c)^(2/3)/(d*x+c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.77

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{7/3}}\,dx = \frac{-\frac{3(c^2-d^2x^2)^{2/3}}{(c+dx)^{4/3}}+2\sqrt{3}\arctan\left(\frac{\sqrt{3}(c+dx)^{2/3}}{(c+dx)^{2/3}-2\sqrt[3]{c^2-d^2x^2}}\right)-2\log\left(d\left((c+dx)^{2/3}+\sqrt[3]{c\right.\right.}{2d}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(7/3),x]
```

output
```
((-3*(c^2 - d^2*x^2)^(2/3))/(c + d*x)^(4/3) + 2*Sqrt[3]*ArcTan[(Sqrt[3]*(c
 + d*x)^(2/3))/((c + d*x)^(2/3) - 2*(c^2 - d^2*x^2)^(1/3))] - 2*Log[d*((c
+ d*x)^(2/3) + (c^2 - d^2*x^2)^(1/3))] + Log[(c + d*x)^(4/3) - (c + d*x)^(
2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3)])/(2*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {474, 473, 57, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{7/3}}\,dx$$

$$\Big\downarrow \; 474$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{(c^2-d^2x^2)^{2/3}}{\left(\frac{dx}{c}+1\right)^{7/3}}dx}{c^2\sqrt[3]{c+dx}}$$

$$\Big\downarrow \; 473$$

$$\frac{(c^2-d^2x^2)^{5/3}\int \frac{(c^2-cdx)^{2/3}}{\left(\frac{dx}{c}+1\right)^{5/3}}dx}{c^2\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}(c^2-cdx)^{5/3}}$$

$$\Big\downarrow \; 57$$

$$\frac{\left(c^2-d^2x^2\right)^{5/3}\left(c^2\left(-\int \frac{1}{\left(\frac{dx}{c}+1\right)^{2/3}\sqrt[3]{c^2-cdx}}dx\right)-\frac{3c\left(c^2-cdx\right)^{2/3}}{2d\left(\frac{dx}{c}+1\right)^{2/3}}\right)}{c^2\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{5/3}}$$

$$\Big\downarrow \; 72$$

$$\frac{\left(c^2-d^2x^2\right)^{5/3}\left(-\left(c^2\left(\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c^2-cdx}}{\sqrt{3}c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}\right)}{d}+\frac{3\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c^2-cdx}}{c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}+1\right)}{2d}+\frac{\sqrt[3]{c}\log\left(\frac{dx}{c}+1\right)}{2d}\right)\right)-\right.}{c^2\sqrt[3]{c+dx}\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{5/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(7/3),x]
```

output
```
((c^2 - d^2*x^2)^(5/3)*((-3*c*(c^2 - c*d*x)^(2/3))/(2*d*(1 + (d*x)/c)^(2/3
)) - c^2*((Sqrt[3]*c^(1/3)*ArcTan[1/Sqrt[3] - (2*(c^2 - c*d*x)^(1/3))/(Sqr
t[3]*c^(2/3)*(1 + (d*x)/c)^(1/3))])/d + (c^(1/3)*Log[1 + (d*x)/c])/(2*d) +
 (3*c^(1/3)*Log[1 + (c^2 - c*d*x)^(1/3)/(c^(2/3)*(1 + (d*x)/c)^(1/3))])/(2
*d))))/(c^2*(c + d*x)^(1/3)*(1 + (d*x)/c)^(4/3)*(c^2 - c*d*x)^(5/3))
```

Defintions of rubi rules used

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^{\frac{7}{3}}}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(7/3),x)
```

output
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.32

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{7/3}}\,dx = \frac{2\sqrt{3}(d^2x^2+2\,cdx+c^2)\arctan\left(\frac{2\sqrt{3}(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+\sqrt{3}(d^2x^2-c^2)}{3\,(d^2x^2-c^2)}\right)+(d^2x^2+2\,cd$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*(d^2*x^2 + 2*c*d*x + c^2)*arctan(1/3*(2*sqrt(3)*(-d^2*x^2 +
 c^2)^(2/3)*(d*x + c)^(2/3) + sqrt(3)*(d^2*x^2 - c^2))/(d^2*x^2 - c^2)) +
(d^2*x^2 + 2*c*d*x + c^2)*log((d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(1/3)*(d*x
 + c)^(4/3) + (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) - 2
*(d^2*x^2 + 2*c*d*x + c^2)*log(-(d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(2/3)*(d
*x + c)^(2/3))/(d^2*x^2 - c^2)) - 3*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3)
)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{7/3}}\,dx=\int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{3}}}{(c+dx)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**(7/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x)**(7/3), x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{7/3}}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^(7/3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.46

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{7/3}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)-3\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}+\log\left(\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}-\left(\frac{2}{dx}\right.\right.}{2d}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) - 1)) - 3*(
2*c/(d*x + c) - 1)^(2/3) + log((2*c/(d*x + c) - 1)^(2/3) - (2*c/(d*x + c)
- 1)^(1/3) + 1) - 2*log(abs((2*c/(d*x + c) - 1)^(1/3) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{7/3}}\,dx = \int \frac{\left(c^2-d^2\,x^2\right)^{2/3}}{\left(c+d\,x\right)^{7/3}}\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(7/3),x)
```

output
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(7/3), x)
```

Reduce [F]

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{7/3}}\,dx = \int \frac{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}{\left(dx+c\right)^{\frac{1}{3}}c^2+2\left(dx+c\right)^{\frac{1}{3}}cdx+\left(dx+c\right)^{\frac{1}{3}}d^2x^2}dx$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(7/3),x)
```

output
```
int((c**2 - d**2*x**2)**(2/3)/((c + d*x)**(1/3)*c**2 + 2*(c + d*x)**(1/3)*
c*d*x + (c + d*x)**(1/3)*d**2*x**2),x)
```

3.284 $\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{10/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{10/3}}\,dx = -\frac{3(c^2-d^2x^2)^{5/3}}{10cd(c+dx)^{10/3}}$$

output
```
-3/10*(-d^2*x^2+c^2)^(5/3)/c/d/(d*x+c)^(10/3)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{10/3}}\,dx = -\frac{3(c^2-d^2x^2)^{5/3}}{10cd(c+dx)^{10/3}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(10/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(5/3))/(10*c*d*(c + d*x)^(10/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c+dx)^{10/3}}\,dx$$

$$\Big\downarrow \text{460}$$

$$-\frac{3(c^2 - d^2x^2)^{5/3}}{10cd(c+dx)^{10/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(10/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(5/3))/(10*c*d*(c + d*x)^(10/3))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
gosper	$-\frac{3(-dx+c)\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}{10(dx+c)^{\frac{7}{3}}cd}$	36
orering	$-\frac{3(-dx+c)\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}{10(dx+c)^{\frac{7}{3}}cd}$	36

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(10/3),x,method=_RETURNVERBOSE)
```

output
```
-3/10/(d*x+c)^(7/3)*(-d*x+c)/c/d*(-d^2*x^2+c^2)^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{10/3}} \, dx = \frac{3 \left(-d^2x^2 + c^2\right)^{\frac{2}{3}} (dx + c)^{\frac{2}{3}} (dx - c)}{10 \left(cd^4x^3 + 3 \, c^2d^3x^2 + 3 \, c^3d^2x + c^4d\right)}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(10/3),x, algorithm="fricas")
```

output
```
3/10*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3)*(d*x - c)/(c*d^4*x^3 + 3*c^2*d
^3*x^2 + 3*c^3*d^2*x + c^4*d)
```

Sympy [F]

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{10/3}} \, dx = \int \frac{\left(-(-c + dx)(c + dx)\right)^{\frac{2}{3}}}{(c + dx)^{\frac{10}{3}}} \, dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**(10/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x)**(10/3), x)
```

Maxima [F]

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{10/3}}\,dx = \int \frac{(-d^2x^2 + c^2)^{\frac{2}{3}}}{(dx + c)^{\frac{10}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(10/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^(10/3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{10/3}}\,dx = -\frac{3\left(\frac{2c}{dx+c} - 1\right)^{\frac{5}{3}}}{10\,cd}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(10/3),x, algorithm="giac")
```

output
```
-3/10*(2*c/(d*x + c) - 1)^(5/3)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{10/3}}\,dx = -\frac{(c^2 - d^2\,x^2)^{2/3}\left(\frac{3}{10\,d^3} - \frac{3\,x}{10\,c\,d^2}\right)}{x^2\,(c + d\,x)^{1/3} + \frac{c^2\,(c+d\,x)^{1/3}}{d^2} + \frac{2\,c\,x\,(c+d\,x)^{1/3}}{d}}$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(10/3),x)
```

output
```
-((c^2 - d^2*x^2)^(2/3)*(3/(10*d^3) - (3*x)/(10*c*d^2)))/(x^2*(c + d*x)^(1
/3) + (c^2*(c + d*x)^(1/3))/d^2 + (2*c*x*(c + d*x)^(1/3))/d)
```

Reduce [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{(c^2 - d^2x^2)^{2/3}}{(c + dx)^{10/3}}\, dx = \frac{3(-dx + c)^{\frac{2}{3}}\,(dx - c)}{10\,(dx + c)^{\frac{5}{3}}\, cd}$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(10/3),x)
```

output
```
(3*(c - d*x)**(2/3)*( - c + d*x))/(10*(c + d*x)**(2/3)*c*d*(c + d*x))
```

3.285 $\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx = -\frac{3(c^2-d^2x^2)^{5/3}}{16cd(c+dx)^{13/3}} - \frac{9(c^2-d^2x^2)^{5/3}}{160c^2d(c+dx)^{10/3}}$$

output
```
-3/16*(-d^2*x^2+c^2)^(5/3)/c/d/(d*x+c)^(13/3)-9/160*(-d^2*x^2+c^2)^(5/3)/c
^2/d/(d*x+c)^(10/3)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx = -\frac{3(c-dx)(13c+3dx)\left(c^2-d^2x^2\right)^{2/3}}{160c^2d(c+dx)^{10/3}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(13/3),x]
```

output
```
(-3*(c - d*x)*(13*c + 3*d*x)*(c^2 - d^2*x^2)^(2/3))/(160*c^2*d*(c + d*x)^(
10/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{3\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{10/3}}dx}{16c}-\frac{3(c^2-d^2x^2)^{5/3}}{16cd(c+dx)^{13/3}}$$

$$\Big\downarrow \text{460}$$

$$-\frac{9(c^2-d^2x^2)^{5/3}}{160c^2d(c+dx)^{10/3}}-\frac{3(c^2-d^2x^2)^{5/3}}{16cd(c+dx)^{13/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(13/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(5/3))/(16*c*d*(c + d*x)^(13/3)) - (9*(c^2 - d^2*x^2)^
(5/3))/(160*c^2*d*(c + d*x)^(10/3))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(-dx+c)(3dx+13c)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}{160(dx+c)^{\frac{10}{3}}c^{2}d}$	44
orering	$-\frac{3(-dx+c)(3dx+13c)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}{160(dx+c)^{\frac{10}{3}}c^{2}d}$	44

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(13/3),x,method=_RETURNVERBOSE)
```

output
```
-3/160*(-d*x+c)*(3*d*x+13*c)*(-d^2*x^2+c^2)^(2/3)/(d*x+c)^(10/3)/c^2/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx = \frac{3\,(3\,d^2x^2+10\,cdx-13\,c^2)(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{160\,(c^2d^5x^4+4\,c^3d^4x^3+6\,c^4d^3x^2+4\,c^5d^2x+c^6d)}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(13/3),x, algorithm="fricas")
```

output
```
3/160*(3*d^2*x^2 + 10*c*d*x - 13*c^2)*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/
3)/(c^2*d^5*x^4 + 4*c^3*d^4*x^3 + 6*c^4*d^3*x^2 + 4*c^5*d^2*x + c^6*d)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx=\int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{3}}}{(c+dx)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**(13/3),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/3)/(c + d*x)**(13/3), x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(13/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^(13/3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx=-\frac{3\left(5\left(\frac{2\,c}{dx+c}-1\right)^{\frac{8}{3}}+8\left(\frac{2\,c}{dx+c}-1\right)^{\frac{5}{3}}\right)}{160\,c^2d}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(13/3),x, algorithm="giac")
```

output
```
-3/160*(5*(2*c/(d*x + c) - 1)^(8/3) + 8*(2*c/(d*x + c) - 1)^(5/3))/(c^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx = \frac{(c^2-d^2\,x^2)^{2/3}\left(\frac{3\,x}{16\,c\,d^3}-\frac{39}{160\,d^4}+\frac{9\,x^2}{160\,c^2\,d^2}\right)}{x^3\,(c+d\,x)^{1/3}+\frac{c^3\,(c+d\,x)^{1/3}}{d^3}+\frac{3\,c\,x^2\,(c+d\,x)^{1/3}}{d}+\frac{3\,c^2\,x\,(c+d\,x)^{1/3}}{d^2}}$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(13/3),x)
```

output
```
((c^2 - d^2*x^2)^(2/3)*((3*x)/(16*c*d^3) - 39/(160*d^4) + (9*x^2)/(160*c^2
*d^2)))/(x^3*(c + d*x)^(1/3) + (c^3*(c + d*x)^(1/3))/d^3 + (3*c*x^2*(c + d
*x)^(1/3))/d + (3*c^2*x*(c + d*x)^(1/3))/d^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{13/3}}\,dx = \frac{3(-dx+c)^{\frac{2}{3}}\,(3d^2x^2+10cdx-13c^2)}{160\,(dx+c)^{\frac{2}{3}}\,c^2d\,(d^2x^2+2cdx+c^2)}$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(13/3),x)
```

output
```
(3*(c - d*x)**(2/3)*( - 13*c**2 + 10*c*d*x + 3*d**2*x**2))/(160*(c + d*x)*
*(2/3)*c**2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.286 $\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 106

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = -\frac{3(c^2-d^2x^2)^{5/3}}{22cd(c+dx)^{16/3}} - \frac{9(c^2-d^2x^2)^{5/3}}{176c^2d(c+dx)^{13/3}} - \frac{27(c^2-d^2x^2)^{5/3}}{1760c^3d(c+dx)^{10/3}}$$

output
```
-3/22*(-d^2*x^2+c^2)^(5/3)/c/d/(d*x+c)^(16/3)-9/176*(-d^2*x^2+c^2)^(5/3)/c
^2/d/(d*x+c)^(13/3)-27/1760*(-d^2*x^2+c^2)^(5/3)/c^3/d/(d*x+c)^(10/3)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = -\frac{3(c-dx)\,(c^2-d^2x^2)^{2/3}\,(119c^2+48cdx+9d^2x^2)}{1760c^3d(c+dx)^{13/3}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(16/3),x]
```

output
```
(-3*(c - d*x)*(c^2 - d^2*x^2)^(2/3)*(119*c^2 + 48*c*d*x + 9*d^2*x^2))/(176
0*c^3*d*(c + d*x)^(13/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{16/3}}dx$$

$$\Big\downarrow \text{ 461}$$

$$\frac{3\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{13/3}}dx}{11c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{22cd(c+dx)^{16/3}}$$

$$\Big\downarrow \text{ 461}$$

$$\frac{3\left(\frac{3\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{10/3}}dx}{16c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{16cd(c+dx)^{13/3}}\right)}{11c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{22cd(c+dx)^{16/3}}$$

$$\Big\downarrow \text{ 460}$$

$$\frac{3\left(-\frac{9\left(c^2-d^2x^2\right)^{5/3}}{160c^2d(c+dx)^{10/3}}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{16cd(c+dx)^{13/3}}\right)}{11c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{22cd(c+dx)^{16/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(16/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(5/3))/(22*c*d*(c + d*x)^(16/3)) + (3*((-3*(c^2 - d^2*
x^2)^(5/3))/(16*c*d*(c + d*x)^(13/3)) - (9*(c^2 - d^2*x^2)^(5/3))/(160*c^2
*d*(c + d*x)^(10/3))))/(11*c)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{3(-dx+c)\left(9d^{2}x^{2}+48cdx+119c^{2}\right)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}{1760(dx+c)^{\frac{13}{3}}c^{3}d}$	55
orering	$-\frac{3(-dx+c)\left(9d^{2}x^{2}+48cdx+119c^{2}\right)\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}{1760(dx+c)^{\frac{13}{3}}c^{3}d}$	55

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(16/3),x,method=_RETURNVERBOSE)
```

output
```
-3/1760*(-d*x+c)*(9*d^2*x^2+48*c*d*x+119*c^2)*(-d^2*x^2+c^2)^(2/3)/(d*x+c)
^(13/3)/c^3/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = \frac{3\,(9\,d^3x^3+39\,cd^2x^2+71\,c^2dx-119\,c^3)(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{1760\,(c^3d^6x^5+5\,c^4d^5x^4+10\,c^5d^4x^3+10\,c^6d^3x^2+5\,c^7d^2x+c^8d)}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(16/3),x, algorithm="fricas")
```

output
```
3/1760*(9*d^3*x^3 + 39*c*d^2*x^2 + 71*c^2*d*x - 119*c^3)*(-d^2*x^2 + c^2)^
(2/3)*(d*x + c)^(2/3)/(c^3*d^6*x^5 + 5*c^4*d^5*x^4 + 10*c^5*d^4*x^3 + 10*c
^6*d^3*x^2 + 5*c^7*d^2*x + c^8*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = \text{Timed out}$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**(16/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^{\frac{16}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(16/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^(16/3), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = -\frac{3\left(20\left(\frac{2c}{dx+c}-1\right)^{\frac{11}{3}}+55\left(\frac{2c}{dx+c}-1\right)^{\frac{8}{3}}+44\left(\frac{2c}{dx+c}-1\right)^{\frac{5}{3}}\right)}{1760\,c^3d}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(16/3),x, algorithm="giac")
```

output
```
-3/1760*(20*(2*c/(d*x + c) - 1)^(11/3) + 55*(2*c/(d*x + c) - 1)^(8/3) + 44
*(2*c/(d*x + c) - 1)^(5/3))/(c^3*d)
```

Mupad [B] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = \frac{(c^2-d^2\,x^2)^{2/3}\left(\frac{213\,x}{1760\,c\,d^4}-\frac{357}{1760\,d^5}+\frac{117\,x^2}{1760\,c^2\,d^3}+\frac{27\,x^3}{1760\,c^3\,d^2}\right)}{x^4\,(c+d\,x)^{1/3}+\frac{c^4\,(c+d\,x)^{1/3}}{d^4}+\frac{6\,c^2\,x^2\,(c+d\,x)^{1/3}}{d^2}+\frac{4\,c\,x^3\,(c+d\,x)^{1/3}}{d}+\frac{4\,c^3\,x\,(c+d\,x)^{1/3}}{d^3}}$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(16/3),x)
```

output
```
((c^2 - d^2*x^2)^(2/3)*((213*x)/(1760*c*d^4) - 357/(1760*d^5) + (117*x^2)/
(1760*c^2*d^3) + (27*x^3)/(1760*c^3*d^2)))/(x^4*(c + d*x)^(1/3) + (c^4*(c
+ d*x)^(1/3))/d^4 + (6*c^2*x^2*(c + d*x)^(1/3))/d^2 + (4*c*x^3*(c + d*x)^(
1/3))/d + (4*c^3*x*(c + d*x)^(1/3))/d^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{16/3}}\,dx = \frac{3(-dx+c)^{\frac{2}{3}}\,(9d^3x^3+39c\,d^2x^2+71c^2dx-119c^3)}{1760\,(dx+c)^{\frac{2}{3}}\,c^3d\,(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(16/3),x)
```

output
```
(3*(c - d*x)**(2/3)*( - 119*c**3 + 71*c**2*d*x + 39*c*d**2*x**2 + 9*d**3*x
**3))/(1760*(c + d*x)**(2/3)*c**3*d*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3))
```

3.287 $\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx = -\frac{3(c^2-d^2x^2)^{5/3}}{28cd(c+dx)^{19/3}}-\frac{27(c^2-d^2x^2)^{5/3}}{616c^2d(c+dx)^{16/3}} -\frac{81(c^2-d^2x^2)^{5/3}}{4928c^3d(c+dx)^{13/3}}-\frac{243(c^2-d^2x^2)^{5/3}}{49280c^4d(c+dx)^{10/3}}$$

output
```
-3/28*(-d^2*x^2+c^2)^(5/3)/c/d/(d*x+c)^(19/3)-27/616*(-d^2*x^2+c^2)^(5/3)/
c^2/d/(d*x+c)^(16/3)-81/4928*(-d^2*x^2+c^2)^(5/3)/c^3/d/(d*x+c)^(13/3)-243
/49280*(-d^2*x^2+c^2)^(5/3)/c^4/d/(d*x+c)^(10/3)
```

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.50

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx = -\frac{3(c-dx)\,(c^2-d^2x^2)^{2/3}\,(2831c^3+1503c^2dx+513cd^2x^2+81d^3x^3)}{49280c^4d(c+dx)^{16/3}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(19/3),x]
```

output
```
(-3*(c - d*x)*(c^2 - d^2*x^2)^(2/3)*(2831*c^3 + 1503*c^2*d*x + 513*c*d^2*x
^2 + 81*d^3*x^3))/(49280*c^4*d*(c + d*x)^(16/3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{19/3}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{9\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{16/3}}dx}{28c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{28cd(c+dx)^{19/3}}$$

$$\Big\downarrow \text{461}$$

$$\frac{9\left(\frac{3\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{13/3}}dx}{11c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{22cd(c+dx)^{16/3}}\right)}{28c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{28cd(c+dx)^{19/3}}$$

$$\Big\downarrow \text{461}$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{\left(c^2-d^2x^2\right)^{2/3}}{(c+dx)^{10/3}}dx}{16c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{16cd(c+dx)^{13/3}}\right)}{11c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{22cd(c+dx)^{16/3}}\right)}{28c}-\frac{3\left(c^2-d^2x^2\right)^{5/3}}{28cd(c+dx)^{19/3}}$$

$$\Big\downarrow \text{460}$$

$$\frac{9\left(\frac{3\left(-\frac{9(c^2-d^2x^2)^{5/3}}{160c^2d(c+dx)^{10/3}}-\frac{3(c^2-d^2x^2)^{5/3}}{16cd(c+dx)^{13/3}}\right)}{11c}-\frac{3(c^2-d^2x^2)^{5/3}}{22cd(c+dx)^{16/3}}\right)}{28c}-\frac{3(c^2-d^2x^2)^{5/3}}{28cd(c+dx)^{19/3}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/3)/(c + d*x)^(19/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(5/3))/(28*c*d*(c + d*x)^(19/3)) + (9*((-3*(c^2 - d^2*
x^2)^(5/3))/(22*c*d*(c + d*x)^(16/3)) + (3*((-3*(c^2 - d^2*x^2)^(5/3))/(16
*c*d*(c + d*x)^(13/3)) - (9*(c^2 - d^2*x^2)^(5/3))/(160*c^2*d*(c + d*x)^(1
0/3))))/(11*c)))/(28*c)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{3(-dx+c)\left(81d^3x^3+513c\,d^2x^2+1503c^2dx+2831c^3\right)\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}{49280(dx+c)^{\frac{16}{3}}c^4d}$	66
orering	$-\frac{3(-dx+c)\left(81d^3x^3+513c\,d^2x^2+1503c^2dx+2831c^3\right)\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}{49280(dx+c)^{\frac{16}{3}}c^4d}$	66

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(19/3),x,method=_RETURNVERBOSE)
```

output
```
-3/49280*(-d*x+c)*(81*d^3*x^3+513*c*d^2*x^2+1503*c^2*d*x+2831*c^3)*(-d^2*x
^2+c^2)^(2/3)/(d*x+c)^(16/3)/c^4/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx = \frac{3\,(81\,d^4x^4+432\,cd^3x^3+990\,c^2d^2x^2+1328\,c^3dx-2831\,c^4)(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{49280\,(c^4d^7x^6+6\,c^5d^6x^5+15\,c^6d^5x^4+20\,c^7d^4x^3+15\,c^8d^3x^2+6\,c^9d^2x+c^{10}d)}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(19/3),x, algorithm="fricas")
```

output
```
3/49280*(81*d^4*x^4 + 432*c*d^3*x^3 + 990*c^2*d^2*x^2 + 1328*c^3*d*x - 283
1*c^4)*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3)/(c^4*d^7*x^6 + 6*c^5*d^6*x^5
 + 15*c^6*d^5*x^4 + 20*c^7*d^4*x^3 + 15*c^8*d^3*x^2 + 6*c^9*d^2*x + c^10*d
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx = \text{Timed out}$$

input
```
integrate((-d**2*x**2+c**2)**(2/3)/(d*x+c)**(19/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{3}}}{(dx+c)^{\frac{19}{3}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(19/3),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/3)/(d*x + c)^(19/3), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.52

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx = -\frac{3\left(220\left(\frac{2c}{dx+c}-1\right)^{\frac{14}{3}}+840\left(\frac{2c}{dx+c}-1\right)^{\frac{11}{3}}+1155\left(\frac{2c}{dx+c}-1\right)^{\frac{8}{3}}+616\left(\frac{2c}{dx+c}-1\right)^{\frac{5}{3}}\right)}{49280\,c^4d}$$

input
```
integrate((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(19/3),x, algorithm="giac")
```

output
```
-3/49280*(220*(2*c/(d*x + c) - 1)^(14/3) + 840*(2*c/(d*x + c) - 1)^(11/3)
+ 1155*(2*c/(d*x + c) - 1)^(8/3) + 616*(2*c/(d*x + c) - 1)^(5/3))/(c^4*d)
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.13

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx = \frac{(c^2-d^2\,x^2)^{2/3}\left(\frac{249\,x}{3080\,c\,d^5}-\frac{8493}{49280\,d^6}+\frac{27\,x^2}{448\,c^2\,d^4}+\frac{81\,x^3}{3080\,c^3\,d^3}+\frac{243\,x^4}{49280\,c^4\,d^2}\right)}{x^5\,(c+d\,x)^{1/3}+\frac{c^5\,(c+d\,x)^{1/3}}{d^5}+\frac{10\,c^2\,x^3\,(c+d\,x)^{1/3}}{d^2}+\frac{10\,c^3\,x^2\,(c+d\,x)^{1/3}}{d^3}+\frac{5\,c\,x^4\,(c+d\,x)^{1/3}}{d}+\frac{5\,c^4}{}}$$

input
```
int((c^2 - d^2*x^2)^(2/3)/(c + d*x)^(19/3),x)
```

output
```
((c^2 - d^2*x^2)^(2/3)*((249*x)/(3080*c*d^5) - 8493/(49280*d^6) + (27*x^2)
/(448*c^2*d^4) + (81*x^3)/(3080*c^3*d^3) + (243*x^4)/(49280*c^4*d^2)))/(x^
5*(c + d*x)^(1/3) + (c^5*(c + d*x)^(1/3))/d^5 + (10*c^2*x^3*(c + d*x)^(1/3
))/d^2 + (10*c^3*x^2*(c + d*x)^(1/3))/d^3 + (5*c*x^4*(c + d*x)^(1/3))/d +
(5*c^4*x*(c + d*x)^(1/3))/d^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int \frac{(c^2-d^2x^2)^{2/3}}{(c+dx)^{19/3}}\,dx = \frac{3(-dx+c)^{\frac{2}{3}}\left(81d^4x^4+432c\,d^3x^3+990c^2d^2x^2+1328c^3dx-2831c^4\right)}{49280\,(dx+c)^{\frac{2}{3}}\,c^4d\left(d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4\right)}$$

input
```
int((-d^2*x^2+c^2)^(2/3)/(d*x+c)^(19/3),x)
```

output
```
(3*(c - d*x)**(2/3)*( - 2831*c**4 + 1328*c**3*d*x + 990*c**2*d**2*x**2 + 4
32*c*d**3*x**3 + 81*d**4*x**4))/(49280*(c + d*x)**(2/3)*c**4*d*(c**4 + 4*c
**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4))
```

3.288 $\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{4c(c^2-d^2x^2)^{2/3}}{3d\sqrt[3]{c+dx}} - \frac{(c+dx)^{2/3}\,(c^2-d^2x^2)^{2/3}}{2d} + \frac{8c^2\sqrt[3]{c-dx}\sqrt[3]{c+dx}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c+dx}}\right)}{3\sqrt{3}d\sqrt[3]{c^2-d^2x^2}} + \frac{4c^2\sqrt[3]{c-dx}\sqrt[3]{c+dx}\log(c+dx)}{9d\sqrt[3]{c^2-d^2x^2}} + \frac{4c^2\sqrt[3]{c-dx}\sqrt[3]{c+dx}\log\left(1+\frac{\sqrt[3]{c-dx}}{\sqrt[3]{c+dx}}\right)}{3d\sqrt[3]{c^2-d^2x^2}}$$

output

```
-4/3*c*(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)^(1/3)-1/2*(d*x+c)^(2/3)*(-d^2*x^2+c^
2)^(2/3)/d-8/9*c^2*(-d*x+c)^(1/3)*(d*x+c)^(1/3)*arctan(-1/3*3^(1/2)+2/3*(-
d*x+c)^(1/3)*3^(1/2)/(d*x+c)^(1/3))*3^(1/2)/d/(-d^2*x^2+c^2)^(1/3)+4/9*c^2
*(-d*x+c)^(1/3)*(d*x+c)^(1/3)*ln(d*x+c)/d/(-d^2*x^2+c^2)^(1/3)+4/3*c^2*(-d
*x+c)^(1/3)*(d*x+c)^(1/3)*ln(1+(-d*x+c)^(1/3)/(d*x+c)^(1/3))/d/(-d^2*x^2+c
^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \frac{-\frac{3(11c+3dx)(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}} + 16\sqrt{3}c^2\arctan\left(\frac{1-\frac{2\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{2/3}}}{\sqrt{3}}\right) + 16c^2\log\left(d\left((c+dx)^{2/3}\right.\right.}{18c}$$

input
```
Integrate[(c + d*x)^(5/3)/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
((-3*(11*c + 3*d*x)*(c^2 - d^2*x^2)^(2/3))/(c + d*x)^(1/3) + 16*Sqrt[3]*c^
2*ArcTan[(1 - (2*(c^2 - d^2*x^2)^(1/3))/(c + d*x)^(2/3))/Sqrt[3]] + 16*c^2
*Log[d*((c + d*x)^(2/3) + (c^2 - d^2*x^2)^(1/3))] - 8*c^2*Log[(c + d*x)^(4
/3) - (c + d*x)^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3)])/(18*
d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {474, 473, 60, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx$$

$$\Big\downarrow \; 474$$

$$\frac{c(c+dx)^{2/3}\int \frac{\left(\frac{dx}{c}+1\right)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}dx}{\left(\frac{dx}{c}+1\right)^{2/3}}$$

$$\Big\downarrow \; 473$$

$$\frac{c(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\int \frac{\left(\frac{dx}{c}+1\right)^{4/3}}{\sqrt[3]{c^2-cdx}}dx}{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{c(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\left(\frac{4}{3}\int \frac{\sqrt[3]{\frac{dx}{c}+1}}{\sqrt[3]{c^2-cdx}}dx-\frac{\left(\frac{dx}{c}+1\right)^{4/3}(c^2-cdx)^{2/3}}{2cd}\right)}{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{c(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\left(\frac{4}{3}\left(\frac{2}{3}\int \frac{1}{\left(\frac{dx}{c}+1\right)^{2/3}\sqrt[3]{c^2-cdx}}dx-\frac{\sqrt[3]{\frac{dx}{c}+1}(c^2-cdx)^{2/3}}{cd}\right)-\frac{\left(\frac{dx}{c}+1\right)^{4/3}(c^2-cdx)^{2/3}}{2cd}\right)}{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}$$

$$\Big\downarrow \text{ 72}$$

$$\frac{c(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\left(\frac{4}{3}\left(\frac{2}{3}\left(\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c^2-cdx}}{\sqrt{3}c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}\right)}{d}+\frac{3\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c^2-cdx}}{c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}+1\right)}{2d}+\frac{\sqrt[3]{c}\log\left(\frac{dx}{c}\right.}{2d}\right.\right.\right.}{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}$$

input
```
Int[(c + d*x)^(5/3)/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
(c*(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(2/3)*(-1/2*((1 + (d*x)/c)^(4/3)*(c^2 -
 c*d*x)^(2/3))/(c*d) + (4*(-(((1 + (d*x)/c)^(1/3)*(c^2 - c*d*x)^(2/3))/(c*
d)) + (2*((Sqrt[3]*c^(1/3)*ArcTan[1/Sqrt[3] - (2*(c^2 - c*d*x)^(1/3))/(Sqr
t[3]*c^(2/3)*(1 + (d*x)/c)^(1/3))])/d + (c^(1/3)*Log[1 + (d*x)/c])/(2*d) +
 (3*c^(1/3)*Log[1 + (c^2 - c*d*x)^(1/3)/(c^(2/3)*(1 + (d*x)/c)^(1/3))])/(2
*d)))/3))/3))/((1 + (d*x)/c)^(4/3)*(c^2 - c*d*x)^(2/3))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(dx+c)^{\frac{5}{3}}}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx =$$
$$-\frac{16\sqrt{3}(c^2dx+c^3)\arctan\left(\frac{2\sqrt{3}(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+\sqrt{3}(d^2x^2-c^2)}{3(d^2x^2-c^2)}\right)+3\left(-d^2x^2+c^2\right)^{\frac{2}{3}}(3\,dx+11\,c)(dx+c)^{\frac{2}{3}}+}{}$$

input
```
integrate((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
-1/18*(16*sqrt(3)*(c^2*d*x + c^3)*arctan(1/3*(2*sqrt(3)*(-d^2*x^2 + c^2)^(
2/3)*(d*x + c)^(2/3) + sqrt(3)*(d^2*x^2 - c^2))/(d^2*x^2 - c^2)) + 3*(-d^2
*x^2 + c^2)^(2/3)*(3*d*x + 11*c)*(d*x + c)^(2/3) + 8*(c^2*d*x + c^3)*log((
d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(4/3) + (-d^2*x^2 + c^2)^
(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) - 16*(c^2*d*x + c^3)*log(-(d^2*x^2
 - c^2 - (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)))/(d^2*x
+ c*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(c+dx)^{\frac{5}{3}}}{\sqrt[3]{-(-c+dx)\,(c+dx)}}\,dx$$

input
```
integrate((d*x+c)**(5/3)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral((c + d*x)**(5/3)/(-(-c + d*x)*(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^{\frac{5}{3}}}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(5/3)/(-d^2*x^2 + c^2)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx =$$

$$-\frac{16\sqrt{3}c^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)+8c^3\log\left(\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}-\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}+1\right)-16c^3\log\left(\left|\left(\cdots\right.\right.\right.}{18cd}$$

input
```
integrate((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
-1/18*(16*sqrt(3)*c^3*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) - 1)
) + 8*c^3*log((2*c/(d*x + c) - 1)^(2/3) - (2*c/(d*x + c) - 1)^(1/3) + 1) -
 16*c^3*log(abs((2*c/(d*x + c) - 1)^(1/3) + 1)) + 3*(4*c^3*(2*c/(d*x + c)
- 1)^(5/3) + 7*c^3*(2*c/(d*x + c) - 1)^(2/3))*(d*x + c)^2/c^2)/(c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx=\int \frac{(c+d\,x)^{5/3}}{(c^2-d^2\,x^2)^{1/3}}\,dx$$

input
```
int((c + d*x)^(5/3)/(c^2 - d^2*x^2)^(1/3),x)
```

output
```
int((c + d*x)^(5/3)/(c^2 - d^2*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx=\left(\int \frac{(dx+c)^{\frac{2}{3}}}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)c+\left(\int \frac{(dx+c)^{\frac{2}{3}}\,x}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx\right)d$$

input
```
int((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((c + d*x)**(2/3)/(c**2 - d**2*x**2)**(1/3),x)*c + int(((c + d*x)**(2/3
)*x)/(c**2 - d**2*x**2)**(1/3),x)*d
```

3.289 $\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 222

$$\begin{aligned}\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = &-\frac{\left(c^2-d^2x^2\right)^{2/3}}{d\sqrt[3]{c+dx}}\\&+\frac{2c\sqrt[3]{c-dx}\sqrt[3]{c+dx}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c+dx}}\right)}{\sqrt{3}d\sqrt[3]{c^2-d^2x^2}}\\&+\frac{c\sqrt[3]{c-dx}\sqrt[3]{c+dx}\log(c+dx)}{3d\sqrt[3]{c^2-d^2x^2}}+\frac{c\sqrt[3]{c-dx}\sqrt[3]{c+dx}\log\left(1+\frac{\sqrt[3]{c-dx}}{\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c^2-d^2x^2}}\end{aligned}$$

output
```
-(-d^2*x^2+c^2)^(2/3)/d/(d*x+c)^(1/3)-2/3*c*(-d*x+c)^(1/3)*(d*x+c)^(1/3)*a
rctan(-1/3*3^(1/2)+2/3*(-d*x+c)^(1/3)*3^(1/2)/(d*x+c)^(1/3))*3^(1/2)/d/(-d
^2*x^2+c^2)^(1/3)+1/3*c*(-d*x+c)^(1/3)*(d*x+c)^(1/3)*ln(d*x+c)/d/(-d^2*x^2
+c^2)^(1/3)+c*(-d*x+c)^(1/3)*(d*x+c)^(1/3)*ln(1+(-d*x+c)^(1/3)/(d*x+c)^(1/
3))/d/(-d^2*x^2+c^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.76

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \frac{-\frac{3(c^2-d^2x^2)^{2/3}}{\sqrt[3]{c+dx}} + 2\sqrt{3}c\arctan\left(\frac{1-\frac{2\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{2/3}}}{\sqrt{3}}\right) + 2c\log\left(d\left((c+dx)^{2/3} + \sqrt[3]{c^2-d^2 \cdots}\right)\right)}{3d}$$

input
```
Integrate[(c + d*x)^(2/3)/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
((-3*(c^2 - d^2*x^2)^(2/3))/(c + d*x)^(1/3) + 2*Sqrt[3]*c*ArcTan[(1 - (2*(
c^2 - d^2*x^2)^(1/3))/(c + d*x)^(2/3))/Sqrt[3]] + 2*c*Log[d*((c + d*x)^(2/
3) + (c^2 - d^2*x^2)^(1/3))] - c*Log[(c + d*x)^(4/3) - (c + d*x)^(2/3)*(c^
2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3)])/(3*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {474, 473, 60, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \ 474$$

$$\frac{(c+dx)^{2/3}\int \frac{\left(\frac{dx}{c}+1\right)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}dx}{\left(\frac{dx}{c}+1\right)^{2/3}}$$

$$\Big\downarrow \ 473$$

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\int \frac{\sqrt[3]{\frac{dx}{c}+1}}{\sqrt[3]{c^2-cdx}}dx}{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}$$

$$\Big\downarrow \text{60}$$

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\left(\frac{2}{3}\int \frac{1}{\left(\frac{dx}{c}+1\right)^{2/3}\sqrt[3]{c^2-cdx}}dx-\frac{\sqrt[3]{\frac{dx}{c}+1}\left(c^2-cdx\right)^{2/3}}{cd}\right)}{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}$$

$$\Big\downarrow \text{72}$$

$$\frac{(c+dx)^{2/3}\left(c^2-d^2x^2\right)^{2/3}\left(\frac{2}{3}\left(\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c^2-cdx}}{\sqrt{3}c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}\right)}{d}+\frac{3\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c^2-cdx}}{c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}+1\right)}{2d}+\frac{\sqrt[3]{c}\log\left(\frac{dx}{c}+1\right)}{2d}\right.\right.}{\left(\frac{dx}{c}+1\right)^{4/3}\left(c^2-cdx\right)^{2/3}}$$

input
```
Int[(c + d*x)^(2/3)/(c^2 - d^2*x^2)^(1/3),x]
```

output
```
((c + d*x)^(2/3)*(c^2 - d^2*x^2)^(2/3)*(-(((1 + (d*x)/c)^(1/3)*(c^2 - c*d*
x)^(2/3))/(c*d)) + (2*((Sqrt[3]*c^(1/3)*ArcTan[1/Sqrt[3] - (2*(c^2 - c*d*x
)^(1/3))/(Sqrt[3]*c^(2/3)*(1 + (d*x)/c)^(1/3))]])/d + (c^(1/3)*Log[1 + (d*x
)/c])/(2*d) + (3*c^(1/3)*Log[1 + (c^2 - c*d*x)^(1/3)/(c^(2/3)*(1 + (d*x)/c
)^(1/3))])/(2*d)))/3))/((1 + (d*x)/c)^(4/3)*(c^2 - c*d*x)^(2/3))
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(dx+c)^{\frac{2}{3}}}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{2\sqrt{3}(cdx+c^2)\arctan\left(\frac{2\sqrt{3}(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+\sqrt{3}(d^2x^2-c^2)}{3(d^2x^2-c^2)}\right)+(cdx+c^2)\log\left(\frac{d^2x^2-c^2-(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{4}{3}}+}{d^2x^2-c^2}\right.}{3(d^2x+}$$

input
```
integrate((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(3)*(c*d*x + c^2)*arctan(1/3*(2*sqrt(3)*(-d^2*x^2 + c^2)^(2/3)
*(d*x + c)^(2/3) + sqrt(3)*(d^2*x^2 - c^2))/(d^2*x^2 - c^2)) + (c*d*x + c^
2)*log((d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(4/3) + (-d^2*x^2
 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) - 2*(c*d*x + c^2)*log(-(d^
2*x^2 - c^2 - (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x^2 - c^2)) + 3
*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(c+dx)^{\frac{2}{3}}}{\sqrt[3]{-(-c+dx)(c+dx)}}\,dx$$

input
```
integrate((d*x+c)**(2/3)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral((c + d*x)**(2/3)/(-(-c + d*x)*(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^{\frac{2}{3}}}{(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(2/3)/(-d^2*x^2 + c^2)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{\left(2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)+\frac{3(dx+c)\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}}{c}+\log\left(\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}-\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}+1\right)-}{3d}$$

input
```
integrate((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
-1/3*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) - 1)) + 3*
(d*x + c)*(2*c/(d*x + c) - 1)^(2/3)/c + log((2*c/(d*x + c) - 1)^(2/3) - (2
*c/(d*x + c) - 1)^(1/3) + 1) - 2*log(abs((2*c/(d*x + c) - 1)^(1/3) + 1)))*
c/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(c+d\,x)^{2/3}}{(c^2-d^2\,x^2)^{1/3}}\,dx$$

input
```
int((c + d*x)^(2/3)/(c^2 - d^2*x^2)^(1/3),x)
```

output
```
int((c + d*x)^(2/3)/(c^2 - d^2*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{2/3}}{\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^{\frac{2}{3}}}{(-d^2x^2+c^2)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int((c + d*x)**(2/3)/(c**2 - d**2*x**2)**(1/3),x)
```

3.290 $\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 191

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx = \frac{\sqrt{3}\sqrt[3]{c-dx}\sqrt[3]{c+dx}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c+dx}}\right)}{d\sqrt[3]{c^2-d^2x^2}} + \frac{\sqrt[3]{c-dx}\sqrt[3]{c+dx}\log(c+dx)}{2d\sqrt[3]{c^2-d^2x^2}} + \frac{3\sqrt[3]{c-dx}\sqrt[3]{c+dx}\log\left(1+\frac{\sqrt[3]{c-dx}}{\sqrt[3]{c+dx}}\right)}{2d\sqrt[3]{c^2-d^2x^2}}$$

output
```
-3^(1/2)*(-d*x+c)^(1/3)*(d*x+c)^(1/3)*arctan(-1/3*3^(1/2)+2/3*(-d*x+c)^(1/
3)*3^(1/2)/(d*x+c)^(1/3))/d/(-d^2*x^2+c^2)^(1/3)+1/2*(-d*x+c)^(1/3)*(d*x+c
)^(1/3)*ln(d*x+c)/d/(-d^2*x^2+c^2)^(1/3)+3/2*(-d*x+c)^(1/3)*(d*x+c)^(1/3)*
ln(1+(-d*x+c)^(1/3)/(d*x+c)^(1/3))/d/(-d^2*x^2+c^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{2/3}}}{\sqrt{3}}\right)+2\log\left(d\left((c+dx)^{2/3}+\sqrt[3]{c^2-d^2x^2}\right)\right)-\log\left((c+dx)^{4/3}-(c+dx)^{2/}\right.}{2d}$$

input
```
Integrate[1/((c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 - (2*(c^2 - d^2*x^2)^(1/3))/(c + d*x)^(2/3))/Sqrt[3]]
 + 2*Log[d*((c + d*x)^(2/3) + (c^2 - d^2*x^2)^(1/3))] - Log[(c + d*x)^(4/3
) - (c + d*x)^(2/3)*(c^2 - d^2*x^2)^(1/3) + (c^2 - d^2*x^2)^(2/3)])/(2*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {474, 473, 72}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx$$

$$\Big\downarrow \ 474$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{1}{\sqrt[3]{\frac{dx}{c}+1}\sqrt[3]{c^2-d^2x^2}}dx}{\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 473$$

$$\frac{\left(c^2-d^2x^2\right)^{2/3}\int \frac{1}{\left(\frac{dx}{c}+1\right)^{2/3}\sqrt[3]{c^2-cdx}}dx}{\sqrt[3]{c+dx}\sqrt[3]{\frac{dx}{c}+1}\left(c^2-cdx\right)^{2/3}}$$

$$\Big\downarrow \text{72}$$

$$\frac{\left(c^2-d^2x^2\right)^{2/3}\left(\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c^2-cdx}}{\sqrt{3}c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}\right)}{d}+\frac{3\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c^2-cdx}}{c^{2/3}\sqrt[3]{\frac{dx}{c}+1}}+1\right)}{2d}+\frac{\sqrt[3]{c}\log\left(\frac{dx}{c}+1\right)}{2d}\right)}{\sqrt[3]{c+dx}\sqrt[3]{\frac{dx}{c}+1}\left(c^2-cdx\right)^{2/3}}$$

input
```
Int[1/((c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
((c^2 - d^2*x^2)^(2/3)*((Sqrt[3]*c^(1/3)*ArcTan[1/Sqrt[3] - (2*(c^2 - c*d*
x)^(1/3))/(Sqrt[3]*c^(2/3)*(1 + (d*x)/c)^(1/3))])/d + (c^(1/3)*Log[1 + (d*
x)/c])/(2*d) + (3*c^(1/3)*Log[1 + (c^2 - c*d*x)^(1/3)/(c^(2/3)*(1 + (d*x)/
c)^(1/3))])/(2*d)))/((c + d*x)^(1/3)*(1 + (d*x)/c)^(1/3)*(c^2 - c*d*x)^(2/
3))
```

Defintions of rubi rules used

rule 72
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-d/b, 3]}, Simp[Sqrt[3]*(q/d)*ArcTan[1/Sqrt[3] - 2*q*((a + b*
x)^(1/3)/(Sqrt[3]*(c + d*x)^(1/3)))], x] + (Simp[3*(q/(2*d))*Log[q*((a + b*
x)^(1/3)/(c + d*x)^(1/3)) + 1], x] + Simp[(q/(2*d))*Log[c + d*x], x])] /; F
reeQ[{a, b, c, d}, x] && NegQ[d/b]
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{1}{(dx+c)^{\frac{1}{3}}\,(-d^2x^2+c^2)^{\frac{1}{3}}}\,dx$$

input
```
int(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+\sqrt{3}(d^2x^2-c^2)}{3\,(d^2x^2-c^2)}\right)+\log\left(\frac{d^2x^2-c^2-(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{4}{3}}+(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{d^2x^2-c^2}\right)}{2\,d}$$

input
```
integrate(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/
3) + sqrt(3)*(d^2*x^2 - c^2))/(d^2*x^2 - c^2)) + log((d^2*x^2 - c^2 - (-d^
2*x^2 + c^2)^(1/3)*(d*x + c)^(4/3) + (-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3
))/(d^2*x^2 - c^2)) - 2*log(-(d^2*x^2 - c^2 - (-d^2*x^2 + c^2)^(2/3)*(d*x
+ c)^(2/3))/(d^2*x^2 - c^2)))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-(-c+dx)(c+dx)}\sqrt[3]{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/3)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(1/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx =$$
$$-\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}-1\right)\right)+\log\left(\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}-\left(\frac{2c}{dx+c}-1\right)^{\frac{1}{3}}+1\right)-2\log\left(\left|\left(\frac{2c}{dx+c}-1\right.\right.\right.}{2d}$$

input
```
integrate(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*c/(d*x + c) - 1)^(1/3) - 1)) + lo
g((2*c/(d*x + c) - 1)^(2/3) - (2*c/(d*x + c) - 1)^(1/3) + 1) - 2*log(abs((
2*c/(d*x + c) - 1)^(1/3) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\left(c^2-d^2\,x^2\right)^{1/3}\,\left(c+d\,x\right)^{1/3}}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(1/3)),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{3}}\,(-d^2x^2+c^2)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)),x)
```

3.291 $\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx=-\frac{3(c^2-d^2x^2)^{2/3}}{4cd(c+dx)^{4/3}}$$

output
```
-3/4*(-d^2*x^2+c^2)^(2/3)/c/d/(d*x+c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx=-\frac{3(c^2-d^2x^2)^{2/3}}{4cd(c+dx)^{4/3}}$$

input
```
Integrate[1/((c + d*x)^(4/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3))/(4*c*d*(c + d*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx$$

$$\Big\downarrow \; 460$$

$$-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{4cd(c+dx)^{4/3}}$$

input
```
Int[1/((c + d*x)^(4/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3))/(4*c*d*(c + d*x)^(4/3))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
gosper	$-\frac{3(-dx+c)}{4(dx+c)^{\frac{1}{3}} cd(-d^2x^2+c^2)^{\frac{1}{3}}}$	36
orering	$-\frac{3(-dx+c)}{4(dx+c)^{\frac{1}{3}} cd(-d^2x^2+c^2)^{\frac{1}{3}}}$	36

input
```
int(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4/(d*x+c)^(1/3)*(-d*x+c)/c/d/(-d^2*x^2+c^2)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3\,(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{4\,(cd^3x^2+2\,c^2d^2x+c^3d)}$$

input
```
integrate(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/4*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2/3)/(c*d^3*x^2 + 2*c^2*d^2*x + c^3
*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-\left(-c+dx\right)\left(c+dx\right)}\left(c+dx\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(4/3)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(4/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(4/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}}{4\,cd}$$

input
```
integrate(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
-3/4*(2*c/(d*x + c) - 1)^(2/3)/(c*d)
```

Mupad [B] (verification not implemented)

Time = 7.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3\,(c^2-d^2\,x^2)^{2/3}}{4\,c^2\,d\,(c+d\,x)^{1/3}+4\,c\,d^2\,x\,(c+d\,x)^{1/3}}$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(4/3)),x)
```

output
```
-(3*(c^2 - d^2*x^2)^(2/3))/(4*c^2*d*(c + d*x)^(1/3) + 4*c*d^2*x*(c + d*x)^
(1/3))
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{3}}\,(-d^2x^2+c^2)^{\frac{1}{3}}\,c+(dx+c)^{\frac{1}{3}}\,(-d^2x^2+c^2)^{\frac{1}{3}}\,dx}dx$$

input
```
int(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)*c + (c + d*x)**(1/3)*(c*
*2 - d**2*x**2)**(1/3)*d*x),x)
```

3.292 $\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{10cd(c+dx)^{7/3}}-\frac{9(c^2-d^2x^2)^{2/3}}{40c^2d(c+dx)^{4/3}}$$

output
```
-3/10*(-d^2*x^2+c^2)^(2/3)/c/d/(d*x+c)^(7/3)-9/40*(-d^2*x^2+c^2)^(2/3)/c^2
/d/(d*x+c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(7c+3dx)\,(c^2-d^2x^2)^{2/3}}{40c^2d(c+dx)^{7/3}}$$

input
```
Integrate[1/((c + d*x)^(7/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(7*c + 3*d*x)*(c^2 - d^2*x^2)^(2/3))/(40*c^2*d*(c + d*x)^(7/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{3\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}dx}{10c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{10cd(c+dx)^{7/3}}$$

$$\Big\downarrow \text{460}$$

$$-\frac{9\left(c^2-d^2x^2\right)^{2/3}}{40c^2d(c+dx)^{4/3}}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{10cd(c+dx)^{7/3}}$$

input
```
Int[1/((c + d*x)^(7/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3))/(10*c*d*(c + d*x)^(7/3)) - (9*(c^2 - d^2*x^2)^(
2/3))/(40*c^2*d*(c + d*x)^(4/3))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{3(-dx+c)(3dx+7c)}{40(dx+c)^{\frac{4}{3}}c^{2}d(-d^{2}x^{2}+c^{2})^{\frac{1}{3}}}$	44
orering	$-\frac{3(-dx+c)(3dx+7c)}{40(dx+c)^{\frac{4}{3}}c^{2}d(-d^{2}x^{2}+c^{2})^{\frac{1}{3}}}$	44

input
```
int(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/40*(-d*x+c)*(3*d*x+7*c)/(d*x+c)^(4/3)/c^2/d/(-d^2*x^2+c^2)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx=-\frac{3\,(-d^2x^2+c^2)^{\frac{2}{3}}(3\,dx+7\,c)(dx+c)^{\frac{2}{3}}}{40\,(c^2d^4x^3+3\,c^3d^3x^2+3\,c^4d^2x+c^5d)}$$

input
```
integrate(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/40*(-d^2*x^2 + c^2)^(2/3)*(3*d*x + 7*c)*(d*x + c)^(2/3)/(c^2*d^4*x^3 +
3*c^3*d^3*x^2 + 3*c^4*d^2*x + c^5*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-(-c+dx)(c+dx)}\,(c+dx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(7/3)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(7/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(7/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3\left(\frac{2c}{dx+c}-1\right)^{\frac{5}{3}}}{20\,c^2d} - \frac{3\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}}{8\,c^2d}$$

input
```
integrate(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
-3/20*(2*c/(d*x + c) - 1)^(5/3)/(c^2*d) - 3/8*(2*c/(d*x + c) - 1)^(2/3)/(c
^2*d)
```

Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{\left(c^2-d^2\,x^2\right)^{2/3}\,\left(\frac{21}{40\,c\,d^3}+\frac{9\,x}{40\,c^2\,d^2}\right)}{x^2\,{\left(c+d\,x\right)}^{1/3}+\frac{c^2\,{\left(c+d\,x\right)}^{1/3}}{d^2}+\frac{2\,c\,x\,{\left(c+d\,x\right)}^{1/3}}{d}}$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(7/3)),x)
```

output
```
-((c^2 - d^2*x^2)^(2/3)*(21/(40*c*d^3) + (9*x)/(40*c^2*d^2)))/(x^2*(c + d*
x)^(1/3) + (c^2*(c + d*x)^(1/3))/d^2 + (2*c*x*(c + d*x)^(1/3))/d)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{3}}\,(-d^2x^2+c^2)^{\frac{1}{3}}\,c^2+2\,(dx+c)^{\frac{1}{3}}\,(-d^2x^2+c^2)^{\frac{1}{3}}\,cdx+(dx+c)^{\frac{1}{3}}}$$

input
```
int(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)*c**2 + 2*(c + d*x)**(1/3
)*(c**2 - d**2*x**2)**(1/3)*c*d*x + (c + d*x)**(1/3)*(c**2 - d**2*x**2)**(
1/3)*d**2*x**2),x)
```

3.293 $\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 106

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx=-\frac{3(c^2-d^2x^2)^{2/3}}{16cd(c+dx)^{10/3}}-\frac{9(c^2-d^2x^2)^{2/3}}{80c^2d(c+dx)^{7/3}}-\frac{27(c^2-d^2x^2)^{2/3}}{320c^3d(c+dx)^{4/3}}$$

output
```
-3/16*(-d^2*x^2+c^2)^(2/3)/c/d/(d*x+c)^(10/3)-9/80*(-d^2*x^2+c^2)^(2/3)/c^
2/d/(d*x+c)^(7/3)-27/320*(-d^2*x^2+c^2)^(2/3)/c^3/d/(d*x+c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx=-\frac{3(c^2-d^2x^2)^{2/3}(41c^2+30cdx+9d^2x^2)}{320c^3d(c+dx)^{10/3}}$$

input
```
Integrate[1/((c + d*x)^(10/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3)*(41*c^2 + 30*c*d*x + 9*d^2*x^2))/(320*c^3*d*(c +
 d*x)^(10/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \ 461$$

$$\frac{3\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}dx}{8c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{16cd(c+dx)^{10/3}}$$

$$\Big\downarrow \ 461$$

$$\frac{3\left(\frac{3\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}dx}{10c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{10cd(c+dx)^{7/3}}\right)}{8c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{16cd(c+dx)^{10/3}}$$

$$\Big\downarrow \ 460$$

$$\frac{3\left(-\frac{9\left(c^2-d^2x^2\right)^{2/3}}{40c^2d(c+dx)^{4/3}}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{10cd(c+dx)^{7/3}}\right)}{8c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{16cd(c+dx)^{10/3}}$$

input
```
Int[1/((c + d*x)^(10/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3))/(16*c*d*(c + d*x)^(10/3)) + (3*((-3*(c^2 - d^2*
x^2)^(2/3))/(10*c*d*(c + d*x)^(7/3)) - (9*(c^2 - d^2*x^2)^(2/3))/(40*c^2*d
*(c + d*x)^(4/3))))/(8*c)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{3(-dx+c)(9d^2x^2+30cdx+41c^2)}{320(dx+c)^{\frac{7}{3}}c^3d(-d^2x^2+c^2)^{\frac{1}{3}}}$	55
orering	$-\frac{3(-dx+c)(9d^2x^2+30cdx+41c^2)}{320(dx+c)^{\frac{7}{3}}c^3d(-d^2x^2+c^2)^{\frac{1}{3}}}$	55

input
```
int(1/(d*x+c)^(10/3)/(-d^2*x^2+c^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/320*(-d*x+c)*(9*d^2*x^2+30*c*d*x+41*c^2)/(d*x+c)^(7/3)/c^3/d/(-d^2*x^2+
c^2)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3\,(9\,d^2x^2+30\,cdx+41\,c^2)(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{320\,(c^3d^5x^4+4\,c^4d^4x^3+6\,c^5d^3x^2+4\,c^6d^2x+c^7d)}$$

input
```
integrate(1/(d*x+c)^(10/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/320*(9*d^2*x^2 + 30*c*d*x + 41*c^2)*(-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(2
/3)/(c^3*d^5*x^4 + 4*c^4*d^4*x^3 + 6*c^5*d^3*x^2 + 4*c^6*d^2*x + c^7*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx=\int \frac{1}{\sqrt[3]{-(-c+dx)(c+dx)}\,(c+dx)^{\frac{10}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(10/3)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(10/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx=\int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{10}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(10/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(10/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx=-\frac{3\left(5\left(\frac{2c}{dx+c}-1\right)^{\frac{8}{3}}+16\left(\frac{2c}{dx+c}-1\right)^{\frac{5}{3}}\right)}{320\,c^3d}-\frac{3\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}}{16\,c^3d}$$

input
```
integrate(1/(d*x+c)^(10/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
-3/320*(5*(2*c/(d*x + c) - 1)^(8/3) + 16*(2*c/(d*x + c) - 1)^(5/3))/(c^3*d
) - 3/16*(2*c/(d*x + c) - 1)^(2/3)/(c^3*d)
```

Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{\left(c^2-d^2\,x^2\right)^{2/3}\left(\frac{123}{320\,c\,d^4}+\frac{9\,x}{32\,c^2\,d^3}+\frac{27\,x^2}{320\,c^3\,d^2}\right)}{x^3\,\left(c+d\,x\right)^{1/3}+\frac{c^3\,\left(c+d\,x\right)^{1/3}}{d^3}+\frac{3\,c\,x^2\,\left(c+d\,x\right)^{1/3}}{d}+\frac{3\,c^2\,x\,\left(c+d\,x\right)^{1/3}}{d^2}}$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(10/3)),x)
```

output
```
-((c^2 - d^2*x^2)^(2/3)*(123/(320*c*d^4) + (9*x)/(32*c^2*d^3) + (27*x^2)/(
320*c^3*d^2)))/(x^3*(c + d*x)^(1/3) + (c^3*(c + d*x)^(1/3))/d^3 + (3*c*x^2
*(c + d*x)^(1/3))/d + (3*c^2*x*(c + d*x)^(1/3))/d^2)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^3+3\,(dx+c)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^2dx+3\,(dx+$$

input
```
int(1/(d*x+c)^(10/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)*c**3 + 3*(c + d*x)**(1/3
)*(c**2 - d**2*x**2)**(1/3)*c**2*d*x + 3*(c + d*x)**(1/3)*(c**2 - d**2*x**
2)**(1/3)*c*d**2*x**2 + (c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)*d**3*x*
*3),x)
```

3.294 $\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}}{22cd(c+dx)^{13/3}} - \frac{27(c^2-d^2x^2)^{2/3}}{352c^2d(c+dx)^{10/3}} - \frac{81(c^2-d^2x^2)^{2/3}}{1760c^3d(c+dx)^{7/3}} - \frac{243(c^2-d^2x^2)^{2/3}}{7040c^4d(c+dx)^{4/3}}$$

output
```
-3/22*(-d^2*x^2+c^2)^(2/3)/c/d/(d*x+c)^(13/3)-27/352*(-d^2*x^2+c^2)^(2/3)/
c^2/d/(d*x+c)^(10/3)-81/1760*(-d^2*x^2+c^2)^(2/3)/c^3/d/(d*x+c)^(7/3)-243/
7040*(-d^2*x^2+c^2)^(2/3)/c^4/d/(d*x+c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.46

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3(c^2-d^2x^2)^{2/3}(689c^3+639c^2dx+351cd^2x^2+81d^3x^3)}{7040c^4d(c+dx)^{13/3}}$$

input
```
Integrate[1/((c + d*x)^(13/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3)*(689*c^3 + 639*c^2*d*x + 351*c*d^2*x^2 + 81*d^3*
x^3))/(7040*c^4*d*(c + d*x)^(13/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {461, 461, 461, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{461}$$

$$\frac{9\int \frac{1}{(c+dx)^{10/3}\sqrt[3]{c^2-d^2x^2}}dx}{22c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{22cd(c+dx)^{13/3}}$$

$$\Big\downarrow \text{461}$$

$$\frac{9\left(\frac{3\int \frac{1}{(c+dx)^{7/3}\sqrt[3]{c^2-d^2x^2}}dx}{8c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{16cd(c+dx)^{10/3}}\right)}{22c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{22cd(c+dx)^{13/3}}$$

$$\Big\downarrow \text{461}$$

$$\frac{9\left(\frac{3\left(\frac{3\int \frac{1}{(c+dx)^{4/3}\sqrt[3]{c^2-d^2x^2}}dx}{10c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{10cd(c+dx)^{7/3}}\right)}{8c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{16cd(c+dx)^{10/3}}\right)}{22c}-\frac{3\left(c^2-d^2x^2\right)^{2/3}}{22cd(c+dx)^{13/3}}$$

$$\Big\downarrow \text{460}$$

$$\frac{9\left(\frac{3\left(-\frac{9(c^2-d^2x^2)^{2/3}}{40c^2d(c+dx)^{4/3}}-\frac{3(c^2-d^2x^2)^{2/3}}{10cd(c+dx)^{7/3}}\right)}{8c}-\frac{3(c^2-d^2x^2)^{2/3}}{16cd(c+dx)^{10/3}}\right)}{22c}-\frac{3(c^2-d^2x^2)^{2/3}}{22cd(c+dx)^{13/3}}$$

input
```
Int[1/((c + d*x)^(13/3)*(c^2 - d^2*x^2)^(1/3)),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(2/3))/(22*c*d*(c + d*x)^(13/3)) + (9*((-3*(c^2 - d^2*
x^2)^(2/3))/(16*c*d*(c + d*x)^(10/3)) + (3*((-3*(c^2 - d^2*x^2)^(2/3))/(10
*c*d*(c + d*x)^(7/3)) - (9*(c^2 - d^2*x^2)^(2/3))/(40*c^2*d*(c + d*x)^(4/3
))))/(8*c)))/(22*c)
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 461
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(2*b*c*(n + p + 1))), x] + Simp[Simpl
ify[n + 2*p + 2]/(2*c*(n + p + 1))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && ILtQ[Simp
lify[n + 2*p + 2], 0] && (LtQ[n, -1] || GtQ[n + p, 0])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{3(-dx+c)\left(81d^3x^3+351c\,d^2x^2+639c^2dx+689c^3\right)}{7040(dx+c)^{\frac{10}{3}}c^4d\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}$	66
orering	$-\frac{3(-dx+c)\left(81d^3x^3+351c\,d^2x^2+639c^2dx+689c^3\right)}{7040(dx+c)^{\frac{10}{3}}c^4d\left(-d^2x^2+c^2\right)^{\frac{1}{3}}}$	66

input
```
int(1/(d*x+c)^(13/3)/(-d^2*x^2+c^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/7040*(-d*x+c)*(81*d^3*x^3+351*c*d^2*x^2+639*c^2*d*x+689*c^3)/(d*x+c)^(1
0/3)/c^4/d/(-d^2*x^2+c^2)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3\,(81\,d^3x^3+351\,cd^2x^2+639\,c^2dx+689\,c^3)(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}}{7040\,(c^4d^6x^5+5\,c^5d^5x^4+10\,c^6d^4x^3+10\,c^7d^3x^2+5\,c^8d^2x+c^9d)}$$

input
```
integrate(1/(d*x+c)^(13/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="fricas")
```

output
```
-3/7040*(81*d^3*x^3 + 351*c*d^2*x^2 + 639*c^2*d*x + 689*c^3)*(-d^2*x^2 + c
^2)^(2/3)*(d*x + c)^(2/3)/(c^4*d^6*x^5 + 5*c^5*d^5*x^4 + 10*c^6*d^4*x^3 +
10*c^7*d^3*x^2 + 5*c^8*d^2*x + c^9*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{\sqrt[3]{-(-c+dx)\,(c+dx)}\,(c+dx)^{\frac{13}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(13/3)/(-d**2*x**2+c**2)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(1/3)*(c + d*x)**(13/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{13}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(13/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(13/3)), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{3\left(40\left(\frac{2c}{dx+c}-1\right)^{\frac{11}{3}}+165\left(\frac{2c}{dx+c}-1\right)^{\frac{8}{3}}+264\left(\frac{2c}{dx+c}-1\right)^{\frac{5}{3}}\right)}{7040\,c^4d} - \frac{3\left(\frac{2c}{dx+c}-1\right)^{\frac{2}{3}}}{32\,c^4d}$$

input
```
integrate(1/(d*x+c)^(13/3)/(-d^2*x^2+c^2)^(1/3),x, algorithm="giac")
```

output
```
-3/7040*(40*(2*c/(d*x + c) - 1)^(11/3) + 165*(2*c/(d*x + c) - 1)^(8/3) + 2
64*(2*c/(d*x + c) - 1)^(5/3))/(c^4*d) - 3/32*(2*c/(d*x + c) - 1)^(2/3)/(c^
4*d)
```

Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = -\frac{\left(c^2-d^2\,x^2\right)^{2/3}\left(\frac{2067}{7040\,c\,d^5}+\frac{1917\,x}{7040\,c^2\,d^4}+\frac{1053\,x^2}{7040\,c^3\,d^3}+\frac{243\,x^3}{7040\,c^4\,d^2}\right)}{x^4\left(c+d\,x\right)^{1/3}+\frac{c^4\,{\left(c+d\,x\right)}^{1/3}}{d^4}+\frac{6\,c^2\,x^2\,{\left(c+d\,x\right)}^{1/3}}{d^2}+\frac{4\,c\,x^3\,{\left(c+d\,x\right)}^{1/3}}{d}+\frac{4\,c^3\,x\,{\left(c+d\,x\right)}^{1/3}}{d^3}}$$

input
```
int(1/((c^2 - d^2*x^2)^(1/3)*(c + d*x)^(13/3)),x)
```

output
```
-((c^2 - d^2*x^2)^(2/3)*(2067/(7040*c*d^5) + (1917*x)/(7040*c^2*d^4) + (10
53*x^2)/(7040*c^3*d^3) + (243*x^3)/(7040*c^4*d^2)))/(x^4*(c + d*x)^(1/3) +
 (c^4*(c + d*x)^(1/3))/d^4 + (6*c^2*x^2*(c + d*x)^(1/3))/d^2 + (4*c*x^3*(c
 + d*x)^(1/3))/d + (4*c^3*x*(c + d*x)^(1/3))/d^3)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{13/3}\sqrt[3]{c^2-d^2x^2}}\,dx = \int \frac{}{(dx+c)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^4+4\left(dx+c\right)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{1}{3}}c^3dx+6\left(dx+\right.}$$

input
```
int(1/(d*x+c)^(13/3)/(-d^2*x^2+c^2)^(1/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)*c**4 + 4*(c + d*x)**(1/3
)*(c**2 - d**2*x**2)**(1/3)*c**3*d*x + 6*(c + d*x)**(1/3)*(c**2 - d**2*x**
2)**(1/3)*c**2*d**2*x**2 + 4*(c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)*c*
d**3*x**3 + (c + d*x)**(1/3)*(c**2 - d**2*x**2)**(1/3)*d**4*x**4),x)
```

3.295 $\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{486c^3\sqrt[3]{c^2-d^2x^2}}{35d\sqrt[3]{c+dx}} - \frac{81c^2(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{35d} - \frac{27c(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{35d} - \frac{3(c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}}{10d}$$

output
```
-486/35*c^3*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(1/3)-81/35*c^2*(d*x+c)^(2/3)*(
-d^2*x^2+c^2)^(1/3)/d-27/35*c*(d*x+c)^(5/3)*(-d^2*x^2+c^2)^(1/3)/d-3/10*(d
*x+c)^(8/3)*(-d^2*x^2+c^2)^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\sqrt[3]{c^2-d^2x^2}(403c^3+111c^2dx+39cd^2x^2+7d^3x^3)}{70d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(11/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3)*(403*c^3 + 111*c^2*d*x + 39*c*d^2*x^2 + 7*d^3*x^
3))/(70*d*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{459}$$

$$\frac{9}{5}c\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}}{10d}$$

$$\Big\downarrow \text{459}$$

$$\frac{9}{5}c\left(\frac{12}{7}c\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{7d}\right)-\frac{3(c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}}{10d}$$

$$\Big\downarrow \text{459}$$

$$\frac{9}{5}c\left(\frac{12}{7}c\left(\frac{3}{2}c\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{4d}\right)-\frac{3(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{7d}\right)-\frac{3(c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}}{10d}$$

$$\Big\downarrow \text{458}$$

$$\frac{9}{5}c\left(\frac{12}{7}c\left(-\frac{9c\sqrt[3]{c^2-d^2x^2}}{2d\sqrt[3]{c+dx}}-\frac{3(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{4d}\right)-\frac{3(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{7d}\right)-\frac{3(c+dx)^{8/3}\sqrt[3]{c^2-d^2x^2}}{10d}$$

input
```
Int[(c + d*x)^(11/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c + d*x)^(8/3)*(c^2 - d^2*x^2)^(1/3))/(10*d) + (9*c*((-3*(c + d*x)^(5
/3)*(c^2 - d^2*x^2)^(1/3))/(7*d) + (12*c*((-9*c*(c^2 - d^2*x^2)^(1/3))/(2*
d*(c + d*x)^(1/3)) - (3*(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(1/3))/(4*d)))/7))
/5
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{3(-dx+c)\left(7d^{3}x^{3}+39c\,d^{2}x^{2}+111c^{2}dx+403c^{3}\right)\left(dx+c\right)^{\frac{2}{3}}}{70d\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}$	63
orering	$-\frac{3(-dx+c)\left(7d^{3}x^{3}+39c\,d^{2}x^{2}+111c^{2}dx+403c^{3}\right)\left(dx+c\right)^{\frac{2}{3}}}{70d\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}$	63
risch	$-\frac{3\left(\left(-d^{2}x^{2}+c^{2}\right)^{2}\right)^{\frac{1}{3}}\left(\frac{\left(d^{2}x^{2}-c^{2}\right)^{2}}{\left(dx+c\right)^{2}}\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{2}{3}}\left(7d^{3}x^{3}+39c\,d^{2}x^{2}+111c^{2}dx+403c^{3}\right)\left(-dx+c\right)}{70\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}\left(\left(d^{2}x^{2}-c^{2}\right)^{2}\right)^{\frac{1}{3}}d\left(\left(dx-c\right)^{2}\right)^{\frac{1}{3}}}$	132

input
```
int((d*x+c)^(11/3)/(-d^2*x^2+c^2)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/70*(-d*x+c)*(7*d^3*x^3+39*c*d^2*x^2+111*c^2*d*x+403*c^3)*(d*x+c)^(2/3)/
d/(-d^2*x^2+c^2)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\,(7\,d^3x^3+39\,cd^2x^2+111\,c^2dx+403\,c^3)(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{70\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(11/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-3/70*(7*d^3*x^3 + 39*c*d^2*x^2 + 111*c^2*d*x + 403*c^3)*(-d^2*x^2 + c^2)^
(1/3)*(d*x + c)^(2/3)/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(c+dx)^{\frac{11}{3}}}{(-(-c+dx)\,(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)**(11/3)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral((c + d*x)**(11/3)/(-(-c + d*x)*(c + d*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.40

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3\,(7\,d^4x^4+32\,cd^3x^3+72\,c^2d^2x^2+292\,c^3dx-403\,c^4)}{70\,(-dx+c)^{\frac{2}{3}}d}$$

input
```
integrate((d*x+c)^(11/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
3/70*(7*d^4*x^4 + 32*c*d^3*x^3 + 72*c^2*d^2*x^2 + 292*c^3*d*x - 403*c^4)/(
(-d*x + c)^(2/3)*d)
```

Giac [F]

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^{\frac{11}{3}}}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^(11/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(11/3)/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 7.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{(c^2-d^2\,x^2)^{1/3}\left(\frac{1209\,c^3\,(c+d\,x)^{2/3}}{70\,d^2}+\frac{117\,c\,x^2\,(c+d\,x)^{2/3}}{70}+\frac{3\,d\,x^3\,(c+d\,x)^{2/3}}{10}+\frac{333\,c^2\,x\,(c+d\,x)^{2/3}}{70\,d}\right)}{x+\frac{c}{d}}$$

input
```
int((c + d*x)^(11/3)/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
-((c^2 - d^2*x^2)^(1/3)*((1209*c^3*(c + d*x)^(2/3))/(70*d^2) + (117*c*x^2*
(c + d*x)^(2/3))/70 + (3*d*x^3*(c + d*x)^(2/3))/10 + (333*c^2*x*(c + d*x)^
(2/3))/(70*d)))/(x + c/d)
```

Reduce [F]

$$\int \frac{(c+dx)^{11/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{-3(-dx+c)^{\frac{1}{3}}c^3+\left(\int \frac{(dx+c)^{\frac{2}{3}}x^3}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)d^4+3\left(\int \frac{(dx+c)^{\frac{2}{3}}x^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)c\,d^3+3\left(\int \frac{(dx+c)}{(-d^2x^2+\cdots}\right.}{d}$$

input
```
int((d*x+c)^(11/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 3*(c - d*x)**(1/3)*c**3 + int(((c + d*x)**(2/3)*x**3)/(c**2 - d**2*x**
2)**(2/3),x)*d**4 + 3*int(((c + d*x)**(2/3)*x**2)/(c**2 - d**2*x**2)**(2/3
),x)*c*d**3 + 3*int(((c + d*x)**(2/3)*x)/(c**2 - d**2*x**2)**(2/3),x)*c**2
*d**2)/d
```

3.296 $\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{54c^2\sqrt[3]{c^2-d^2x^2}}{7d\sqrt[3]{c+dx}} - \frac{9c(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{7d} - \frac{3(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{7d}$$

output
```
-54/7*c^2*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(1/3)-9/7*c*(d*x+c)^(2/3)*(-d^2*x
^2+c^2)^(1/3)/d-3/7*(d*x+c)^(5/3)*(-d^2*x^2+c^2)^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.50

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\sqrt[3]{c^2-d^2x^2}(22c^2+5cdx+d^2x^2)}{7d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(8/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3)*(22*c^2 + 5*c*d*x + d^2*x^2))/(7*d*(c + d*x)^(1/
3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {459, 459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx$$

$$\Big\downarrow \ 459$$

$$\frac{12}{7}c\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{7d}$$

$$\Big\downarrow \ 459$$

$$\frac{12}{7}c\left(\frac{3}{2}c\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{4d}\right)-\frac{3(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{7d}$$

$$\Big\downarrow \ 458$$

$$\frac{12}{7}c\left(-\frac{9c\sqrt[3]{c^2-d^2x^2}}{2d\sqrt[3]{c+dx}}-\frac{3(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{4d}\right)-\frac{3(c+dx)^{5/3}\sqrt[3]{c^2-d^2x^2}}{7d}$$

input
```
Int[(c + d*x)^(8/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c + d*x)^(5/3)*(c^2 - d^2*x^2)^(1/3))/(7*d) + (12*c*((-9*c*(c^2 - d^2
*x^2)^(1/3))/(2*d*(c + d*x)^(1/3)) - (3*(c + d*x)^(2/3)*(c^2 - d^2*x^2)^(1
/3))/(4*d)))/7
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{3(-dx+c)(d^2x^2+5cdx+22c^2)(dx+c)^{\frac{2}{3}}}{7d(-d^2x^2+c^2)^{\frac{2}{3}}}$	51
orering	$-\frac{3(-dx+c)(d^2x^2+5cdx+22c^2)(dx+c)^{\frac{2}{3}}}{7d(-d^2x^2+c^2)^{\frac{2}{3}}}$	51
risch	$-\frac{3\left(\left(-d^2x^2+c^2\right)^2\right)^{\frac{1}{3}}\left(\frac{\left(d^2x^2-c^2\right)^2}{(dx+c)^2}\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}(d^2x^2+5cdx+22c^2)(-dx+c)}{7(-d^2x^2+c^2)^{\frac{2}{3}}\left(\left(d^2x^2-c^2\right)^2\right)^{\frac{1}{3}}d\left(\left(dx-c\right)^2\right)^{\frac{1}{3}}}$	120

input
```
int((d*x+c)^(8/3)/(-d^2*x^2+c^2)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/7*(-d*x+c)*(d^2*x^2+5*c*d*x+22*c^2)*(d*x+c)^(2/3)/d/(-d^2*x^2+c^2)^(2/3
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\,(d^2x^2+5\,cdx+22\,c^2)(-d^2x^2+c^2)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{7\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(8/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-3/7*(d^2*x^2 + 5*c*d*x + 22*c^2)*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)/(
d^2*x + c*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(c+dx)^{\frac{8}{3}}}{(-(-c+dx)\,(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)**(8/3)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral((c + d*x)**(8/3)/(-(-c + d*x)*(c + d*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3\,(d^3x^3+4\,cd^2x^2+17\,c^2dx-22\,c^3)}{7\,(-dx+c)^{\frac{2}{3}}d}$$

input
```
integrate((d*x+c)^(8/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
3/7*(d^3*x^3 + 4*c*d^2*x^2 + 17*c^2*d*x - 22*c^3)/((-d*x + c)^(2/3)*d)
```

Giac [F]

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^{\frac{8}{3}}}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^(8/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(8/3)/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{(c^2-d^2\,x^2)^{1/3}\left(\frac{3\,x^2\,(c+d\,x)^{2/3}}{7}+\frac{66\,c^2\,(c+d\,x)^{2/3}}{7\,d^2}+\frac{15\,c\,x\,(c+d\,x)^{2/3}}{7\,d}\right)}{x+\frac{c}{d}}$$

input
```
int((c + d*x)^(8/3)/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
-((c^2 - d^2*x^2)^(1/3)*((3*x^2*(c + d*x)^(2/3))/7 + (66*c^2*(c + d*x)^(2/
3))/(7*d^2) + (15*c*x*(c + d*x)^(2/3))/(7*d)))/(x + c/d)
```

Reduce [F]

$$\int \frac{(c+dx)^{8/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{-3(-dx+c)^{\frac{1}{3}}c^2+\left(\int \frac{(dx+c)^{\frac{2}{3}}x^2}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)d^3+2\left(\int \frac{(dx+c)^{\frac{2}{3}}x}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)c\,d^2}{d}$$

input
```
int((d*x+c)^(8/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 3*(c - d*x)**(1/3)*c**2 + int(((c + d*x)**(2/3)*x**2)/(c**2 - d**2*x**
2)**(2/3),x)*d**3 + 2*int(((c + d*x)**(2/3)*x)/(c**2 - d**2*x**2)**(2/3),x
)*c*d**2)/d
```

3.297 $\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 66

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{9c\sqrt[3]{c^2-d^2x^2}}{2d\sqrt[3]{c+dx}}-\frac{3(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{4d}$$

output
```
-9/2*c*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(1/3)-3/4*(d*x+c)^(2/3)*(-d^2*x^2+c^
2)^(1/3)/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3(7c+dx)\sqrt[3]{c^2-d^2x^2}}{4d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(7*c + d*x)*(c^2 - d^2*x^2)^(1/3))/(4*d*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {459, 458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx$$

$$\Big\downarrow \ 459$$

$$\frac{3}{2}c\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}dx-\frac{3(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{4d}$$

$$\Big\downarrow \ 458$$

$$-\frac{9c\sqrt[3]{c^2-d^2x^2}}{2d\sqrt[3]{c+dx}}-\frac{3(c+dx)^{2/3}\sqrt[3]{c^2-d^2x^2}}{4d}$$

input
```
Int[(c + d*x)^(5/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-9*c*(c^2 - d^2*x^2)^(1/3))/(2*d*(c + d*x)^(1/3)) - (3*(c + d*x)^(2/3)*(c
^2 - d^2*x^2)^(1/3))/(4*d)
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 459
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[2*c*
(Simplify[n + p]/(n + 2*p + 1))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && IGtQ[Simplif
y[n + p], 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{3(-dx+c)(dx+7c)(dx+c)^{\frac{2}{3}}}{4d(-d^2x^2+c^2)^{\frac{2}{3}}}$	40
orering	$-\frac{3(-dx+c)(dx+7c)(dx+c)^{\frac{2}{3}}}{4d(-d^2x^2+c^2)^{\frac{2}{3}}}$	40
risch	$-\frac{3\left((-d^2x^2+c^2)^2\right)^{\frac{1}{3}}\left(\frac{(d^2x^2-c^2)^2}{(dx+c)^2}\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}(dx+7c)(-dx+c)}{4(-d^2x^2+c^2)^{\frac{2}{3}}\left((d^2x^2-c^2)^2\right)^{\frac{1}{3}}d\left((dx-c)^2\right)^{\frac{1}{3}}}$	109

input
```
int((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-d*x+c)*(d*x+7*c)*(d*x+c)^(2/3)/d/(-d^2*x^2+c^2)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\,(-d^2x^2+c^2)^{\frac{1}{3}}(dx+7\,c)(dx+c)^{\frac{2}{3}}}{4\,(d^2x+cd)}$$

input
```
integrate((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-3/4*(-d^2*x^2 + c^2)^(1/3)*(d*x + 7*c)*(d*x + c)^(2/3)/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(c+dx)^{\frac{5}{3}}}{(-(-c+dx)(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)**(5/3)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral((c + d*x)**(5/3)/(-(-c + d*x)*(c + d*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.47

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3\,(d^2x^2+6\,cdx-7\,c^2)}{4\,(-dx+c)^{\frac{2}{3}}d}$$

input
```
integrate((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
3/4*(d^2*x^2 + 6*c*d*x - 7*c^2)/((-d*x + c)^(2/3)*d)
```

Giac [F]

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^{\frac{5}{3}}}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/3)/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{(c^2-d^2\,x^2)^{1/3}\left(\frac{21\,c\,(c+d\,x)^{2/3}}{4\,d^2}+\frac{3\,x\,(c+d\,x)^{2/3}}{4\,d}\right)}{x+\frac{c}{d}}$$

input
```
int((c + d*x)^(5/3)/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
-((c^2 - d^2*x^2)^(1/3)*((21*c*(c + d*x)^(2/3))/(4*d^2) + (3*x*(c + d*x)^(
2/3))/(4*d)))/(x + c/d)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{-3(-dx+c)^{\frac{1}{3}}\,c+\left(\int \frac{(dx+c)^{\frac{2}{3}}\,x}{(-d^2x^2+c^2)^{\frac{2}{3}}}dx\right)d^2}{d}$$

input
```
int((d*x+c)^(5/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 3*(c - d*x)**(1/3)*c + int(((c + d*x)**(2/3)*x)/(c**2 - d**2*x**2)**(2
/3),x)*d**2)/d
```

3.298 $\int \frac{(c+dx)^{2/3}}{\left(c^2-d^2x^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 30

$$\int \frac{(c+dx)^{2/3}}{\left(c^2-d^2x^2\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{c^2-d^2x^2}}{d\sqrt[3]{c+dx}}$$

output
```
-3*(-d^2*x^2+c^2)^(1/3)/d/(d*x+c)^(1/3)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^{2/3}}{\left(c^2-d^2x^2\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{c^2-d^2x^2}}{d\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(2/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3))/(d*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {458}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 458$$

$$-\frac{3\sqrt[3]{c^2-d^2x^2}}{d\sqrt[3]{c+dx}}$$

input
```
Int[(c + d*x)^(2/3)/(c^2 - d^2*x^2)^(2/3),x]
```

output
```
(-3*(c^2 - d^2*x^2)^(1/3))/(d*(c + d*x)^(1/3))
```

Defintions of rubi rules used

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{3(-dx+c)(dx+c)^{\frac{2}{3}}}{d(-d^2x^2+c^2)^{\frac{2}{3}}}$	33
orering	$-\frac{3(-dx+c)(dx+c)^{\frac{2}{3}}}{d(-d^2x^2+c^2)^{\frac{2}{3}}}$	33
risch	$-\frac{3(-dx+c)\left(\left(-d^2x^2+c^2\right)^2\right)^{\frac{1}{3}}\left(\frac{\left(d^2x^2-c^2\right)^2}{(dx+c)^2}\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{d\left((dx-c)^2\right)^{\frac{1}{3}}(-d^2x^2+c^2)^{\frac{2}{3}}\left((d^2x^2-c^2)^2\right)^{\frac{1}{3}}}$	102

input
```
int((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3*(-d*x+c)*(d*x+c)^(2/3)/d/(-d^2*x^2+c^2)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\left(-d^2x^2+c^2\right)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{d^2x+cd}$$

input
```
integrate((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-3*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)/(d^2*x + c*d)
```

Sympy [F]

$$\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(c+dx)^{\frac{2}{3}}}{\left(-\left(-c+dx\right)\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)**(2/3)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral((c + d*x)**(2/3)/(-(-c + d*x)*(c + d*x))**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \frac{3\,(dx-c)}{(-dx+c)^{\frac{2}{3}}d}$$

input
```
integrate((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
3*(d*x - c)/((-d*x + c)^(2/3)*d)
```

Giac [F]

$$\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{(dx+c)^{\frac{2}{3}}}{(-d^2x^2+c^2)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(2/3)/(-d^2*x^2 + c^2)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3\,(c^2-d^2\,x^2)^{1/3}}{d\,(c+d\,x)^{1/3}}$$

input
```
int((c + d*x)^(2/3)/(c^2 - d^2*x^2)^(2/3),x)
```

output
```
-(3*(c^2 - d^2*x^2)^(1/3))/(d*(c + d*x)^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

$$\int \frac{(c+dx)^{2/3}}{(c^2-d^2x^2)^{2/3}}\,dx = -\frac{3(-dx+c)^{\frac{1}{3}}}{d}$$

input
```
int((d*x+c)^(2/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
( - 3*(c - d*x)**(1/3))/d
```

3.299 $\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 225

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx = -\frac{\sqrt{3}(c-dx)^{2/3}(c+dx)^{2/3}\arctan\left(\frac{\sqrt[3]{c}+2^{2/3}\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{2^{2/3}c^{2/3}d\,(c^2-d^2x^2)^{2/3}} - \frac{(c-dx)^{2/3}(c+dx)^{2/3}\log(c+dx)}{2\,2^{2/3}c^{2/3}d\,(c^2-d^2x^2)^{2/3}} + \frac{3(c-dx)^{2/3}(c+dx)^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{c}-\sqrt[3]{c-dx}\right)}{2\,2^{2/3}c^{2/3}d\,(c^2-d^2x^2)^{2/3}}$$

output
```
-1/2*3^(1/2)*(-d*x+c)^(2/3)*(d*x+c)^(2/3)*arctan(1/3*(c^(1/3)+2^(2/3)*(-d*
x+c)^(1/3))*3^(1/2)/c^(1/3))*2^(1/3)/c^(2/3)/d/(-d^2*x^2+c^2)^(2/3)-1/4*(-
d*x+c)^(2/3)*(d*x+c)^(2/3)*ln(d*x+c)*2^(1/3)/c^(2/3)/d/(-d^2*x^2+c^2)^(2/3
)+3/4*(-d*x+c)^(2/3)*(d*x+c)^(2/3)*ln(2^(1/3)*c^(1/3)-(-d*x+c)^(1/3))*2^(1
/3)/c^(2/3)/d/(-d^2*x^2+c^2)^(2/3)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c+dx}}{\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/3}\sqrt[3]{c^2-d^2x^2}}\right)+2\log\left(-2\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/\cdots}\right.}{}$$

input
```
Integrate[1/((c + d*x)^(1/3)*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*(c + d*x)^(1/3))/(c^(1/3)*(c + d*x)^(1/
3) + 2^(2/3)*(c^2 - d^2*x^2)^(1/3))] + 2*Log[-2*c^(1/3)*(c + d*x)^(1/3) +
2^(2/3)*(c^2 - d^2*x^2)^(1/3)] - Log[2*c^(2/3)*(c + d*x)^(2/3) + 2^(2/3)*c
^(1/3)*(c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3) + 2^(1/3)*(c^2 - d^2*x^2)^(2/
3)])/(2*2^(2/3)*c^(2/3)*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {474, 473, 27, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{1}{\sqrt[3]{\frac{dx}{c}+1}(c^2-d^2x^2)^{2/3}}dx}{\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{473}$$

$$\frac{\sqrt[3]{c^2-d^2x^2}\int \frac{c}{(c+dx)(c^2-cdx)^{2/3}}dx}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\downarrow$ 27

$$\frac{c\sqrt[3]{c^2-d^2x^2}\int \frac{1}{(c+dx)(c^2-cdx)^{2/3}}dx}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\downarrow$ 69

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(-\frac{3\int \frac{1}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}}d\sqrt[3]{c^2-cdx}}{2\ 2^{2/3}c^{4/3}d}-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\text{lo}}{2}\right.}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\downarrow$ 16

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\downarrow$ 1082

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{3\int \frac{1}{-\left(c^2-cdx\right)^{2/3}-3}d\left(\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\downarrow$ 217

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1}{\sqrt{3}}\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

input
```
Int[1/((c + d*x)^(1/3)*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(c*(c^2 - d^2*x^2)^(1/3)*(-((Sqrt[3]*ArcTan[(1 + (2^(2/3)*(c^2 - c*d*x)^(1
/3))/c^(2/3))/Sqrt[3]])/(2^(2/3)*c^(4/3)*d)) - Log[c + d*x]/(2*2^(2/3)*c^(
4/3)*d) + (3*Log[2^(1/3)*c^(2/3) - (c^2 - c*d*x)^(1/3)])/(2*2^(2/3)*c^(4/3
)*d)))/((c + d*x)^(1/3)*(c^2 - c*d*x)^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{\frac{1}{3}}\left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx = -\frac{4\sqrt{3}c\sqrt{4^{\frac{1}{3}}(c^2)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(4^{\frac{2}{3}}(-d^2x^2+c^2)^{\frac{1}{3}}(c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+4^{\frac{1}{3}}(cdx+c^2)(c^2)^{\frac{1}{3}}\right)\sqrt{4^{\frac{1}{3}}(c^2)^{\frac{1}{3}}}}{6(c^2dx+c^3)}\right)+4^{\frac{2}{3}}(c^2)^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(-d^2x^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-1/8*(4*sqrt(3)*c*sqrt(4^(1/3)*(c^2)^(1/3))*arctan(1/6*sqrt(3)*(4^(2/3)*(-
d^2*x^2 + c^2)^(1/3)*(c^2)^(2/3)*(d*x + c)^(2/3) + 4^(1/3)*(c*d*x + c^2)*(
c^2)^(1/3))*sqrt(4^(1/3)*(c^2)^(1/3))/(c^2*d*x + c^3)) + 4^(2/3)*(c^2)^(2/
3)*log((4^(2/3)*(-d^2*x^2 + c^2)^(1/3)*(c^2)^(2/3)*(d*x + c)^(2/3) + 2*(-d
^2*x^2 + c^2)^(2/3)*(d*x + c)^(1/3)*c + 2*4^(1/3)*(c*d*x + c^2)*(c^2)^(1/3
))/(d*x + c)) - 2*4^(2/3)*(c^2)^(2/3)*log(-(4^(2/3)*(c^2)^(2/3)*(d*x + c)
- 2*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*c)/(d*x + c)))/(c^2*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{\left(-\left(-c+dx\right)\left(c+dx\right)\right)^{\frac{2}{3}}\sqrt[3]{c+dx}}\,dx$$

input
```
integrate(1/(d*x+c)**(1/3)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(2/3)*(c + d*x)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx=\int \frac{1}{(c^2-d^2\,x^2)^{2/3}\,(c+d\,x)^{1/3}}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(1/3)),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{c+dx}\,(c^2-d^2x^2)^{2/3}}\,dx=\int \frac{1}{(dx+c)^{\frac{1}{3}}\,(-d^2x^2+c^2)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^(1/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x***2)**(2/3)),x)
```

3.300 $\int \frac{1}{(c+dx)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 260

$$\begin{aligned}\int \frac{1}{(c+dx)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx = &-\frac{\sqrt[3]{c^2-d^2x^2}}{2cd(c+dx)^{4/3}}\\ &-\frac{(c-dx)^{2/3}(c+dx)^{2/3}\arctan\left(\frac{\sqrt[3]{c}+2^{2/3}\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{2^{2/3}\sqrt{3}c^{5/3}d\left(c^2-d^2x^2\right)^{2/3}}\\ &-\frac{(c-dx)^{2/3}(c+dx)^{2/3}\log(c+dx)}{6\ 2^{2/3}c^{5/3}d\left(c^2-d^2x^2\right)^{2/3}}\\ &+\frac{(c-dx)^{2/3}(c+dx)^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{c}-\sqrt[3]{c-dx}\right)}{2\ 2^{2/3}c^{5/3}d\left(c^2-d^2x^2\right)^{2/3}}\end{aligned}$$

output

```
-1/2*(-d^2*x^2+c^2)^(1/3)/c/d/(d*x+c)^(4/3)-1/6*(-d*x+c)^(2/3)*(d*x+c)^(2/
3)*arctan(1/3*(c^(1/3)+2^(2/3)*(-d*x+c)^(1/3))*3^(1/2)/c^(1/3))*2^(1/3)*3^
(1/2)/c^(5/3)/d/(-d^2*x^2+c^2)^(2/3)-1/12*(-d*x+c)^(2/3)*(d*x+c)^(2/3)*ln(
d*x+c)*2^(1/3)/c^(5/3)/d/(-d^2*x^2+c^2)^(2/3)+1/4*(-d*x+c)^(2/3)*(d*x+c)^(
2/3)*ln(2^(1/3)*c^(1/3)-(-d*x+c)^(1/3))*2^(1/3)/c^(5/3)/d/(-d^2*x^2+c^2)^(
2/3)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^{4/3}(c^2-d^2x^2)^{2/3}}\,dx = \frac{-\frac{6c^{2/3}\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{4/3}}+2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c+dx}}{\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/3}\sqrt[3]{c^2-d^2x^2}}\right)+2}{}$$

input
```
Integrate[1/((c + d*x)^(4/3)*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
((-6*c^(2/3)*(c^2 - d^2*x^2)^(1/3))/(c + d*x)^(4/3) + 2*2^(1/3)*Sqrt[3]*Ar
cTan[(Sqrt[3]*c^(1/3)*(c + d*x)^(1/3))/(c^(1/3)*(c + d*x)^(1/3) + 2^(2/3)*
(c^2 - d^2*x^2)^(1/3))] + 2*2^(1/3)*Log[-2*c^(1/3)*(c + d*x)^(1/3) + 2^(2/
3)*(c^2 - d^2*x^2)^(1/3)] - 2^(1/3)*Log[2*c^(2/3)*(c + d*x)^(2/3) + 2^(2/3
)*c^(1/3)*(c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3) + 2^(1/3)*(c^2 - d^2*x^2)^
(2/3)])/(12*c^(5/3)*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {474, 473, 27, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^{4/3}(c^2-d^2x^2)^{2/3}}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{1}{\left(\frac{dx}{c}+1\right)^{4/3}(c^2-d^2x^2)^{2/3}}dx}{c\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{473}$$

$$\frac{\sqrt[3]{c^2-d^2x^2}\int \frac{c^2}{(c+dx)^2(c^2-cdx)^{2/3}}dx}{c\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\Big\downarrow$ 27

$$\frac{c\sqrt[3]{c^2-d^2x^2}\int \frac{1}{(c+dx)^2(c^2-cdx)^{2/3}}dx}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\Big\downarrow$ 52

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{\int \frac{1}{(c+dx)\left(c^2-cdx\right)^{2/3}}dx}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\Big\downarrow$ 69

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{-\frac{3\int \frac{1}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}}d\sqrt[3]{c^2-cdx}}{2\ 2^{2/3}c^{4/3}d}-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+ \ldots}{2\ 2^{2/3}c \ldots}}{3c} \ldots\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\Big\downarrow$ 16

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{\ldots}}{2c \ldots} \ldots\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\Big\downarrow$ 1082

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{\frac{3\int \frac{1}{-\left(c^2-cdx\right)^{2/3}-3}d\left(\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$\Big\downarrow$ 217

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1}{\sqrt{3}}\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\,2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\,2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

input
```
Int[1/((c + d*x)^(4/3)*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(c*(c^2 - d^2*x^2)^(1/3)*(-1/2*(c^2 - c*d*x)^(1/3)/(c^2*d*(c + d*x)) + (-(
(Sqrt[3]*ArcTan[(1 + (2^(2/3)*(c^2 - c*d*x)^(1/3))/c^(2/3))/Sqrt[3]])/(2^(
2/3)*c^(4/3)*d)) - Log[c + d*x]/(2*2^(2/3)*c^(4/3)*d) + (3*Log[2^(1/3)*c^(
2/3) - (c^2 - c*d*x)^(1/3)])/(2*2^(2/3)*c^(4/3)*d))/(3*c)))/((c + d*x)^(1/
3)*(c^2 - c*d*x)^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{1}{(dx+c)^{\frac{4}{3}} \left(-d^{2}x^{2}+c^{2}\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)^{4/3}(c^2-d^2x^2)^{2/3}}\,dx =$$

$$-\frac{4^{\frac{2}{3}}(d^2x^2+2\,cdx+c^2)(c^2)^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(-d^2x^2+c^2)^{\frac{1}{3}}(c^2)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+2\,(-d^2x^2+c^2)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}c+2\cdot 4^{\frac{1}{3}}(cdx+c^2)(c^2)^{\frac{1}{3}}}{dx+c}\right)-2\cdot 4^{\frac{2}{3}}}{}$$

input
```
integrate(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-1/24*(4^(2/3)*(d^2*x^2 + 2*c*d*x + c^2)*(c^2)^(2/3)*log((4^(2/3)*(-d^2*x^
2 + c^2)^(1/3)*(c^2)^(2/3)*(d*x + c)^(2/3) + 2*(-d^2*x^2 + c^2)^(2/3)*(d*x
 + c)^(1/3)*c + 2*4^(1/3)*(c*d*x + c^2)*(c^2)^(1/3))/(d*x + c)) - 2*4^(2/3
)*(d^2*x^2 + 2*c*d*x + c^2)*(c^2)^(2/3)*log(-(4^(2/3)*(c^2)^(2/3)*(d*x + c
) - 2*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3)*c)/(d*x + c)) + 12*(-d^2*x^2
+ c^2)^(1/3)*(d*x + c)^(2/3)*c^2 + 12*sqrt(1/3)*(c*d^2*x^2 + 2*c^2*d*x + c
^3)*sqrt(4^(1/3)*(c^2)^(1/3))*arctan(1/2*sqrt(1/3)*(4^(2/3)*(-d^2*x^2 + c^
2)^(1/3)*(c^2)^(2/3)*(d*x + c)^(2/3) + 4^(1/3)*(c*d*x + c^2)*(c^2)^(1/3))*
sqrt(4^(1/3)*(c^2)^(1/3))/(c^2*d*x + c^3)))/(c^3*d^3*x^2 + 2*c^4*d^2*x + c
^5*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{4/3}(c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-(-c+dx)(c+dx))^{\frac{2}{3}}(c+dx)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(4/3)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(2/3)*(c + d*x)**(4/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx=\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(4/3)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx=\int \frac{1}{\left(c^2-d^2\,x^2\right)^{2/3}\,\left(c+d\,x\right)^{4/3}}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(4/3)),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{4/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx = \int \frac{1}{(dx+c)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c+(dx+c)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{2}{3}}dx}dx$$

input
```
int(1/(d*x+c)^(4/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x**2)**(2/3)*c + (c + d*x)**(1/3)*(c*
*2 - d**2*x**2)**(2/3)*d*x),x)
```

3.301 $\int \frac{1}{(c+dx)^{7/3}(c^2-d^2x^2)^{2/3}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\begin{aligned}\int \frac{1}{(c+dx)^{7/3}(c^2-d^2x^2)^{2/3}} \, dx = & -\frac{\sqrt[3]{c^2-d^2x^2}}{4cd(c+dx)^{7/3}} - \frac{5\sqrt[3]{c^2-d^2x^2}}{24c^2d(c+dx)^{4/3}} \\ & - \frac{5(c-dx)^{2/3}(c+dx)^{2/3}\arctan\left(\frac{\sqrt[3]{c}+2^{2/3}\sqrt[3]{c-dx}}{\sqrt{3}\sqrt[3]{c}}\right)}{12\ 2^{2/3}\sqrt{3}c^{8/3}d\left(c^2-d^2x^2\right)^{2/3}} \\ & - \frac{5(c-dx)^{2/3}(c+dx)^{2/3}\log(c+dx)}{72\ 2^{2/3}c^{8/3}d\left(c^2-d^2x^2\right)^{2/3}} \\ & + \frac{5(c-dx)^{2/3}(c+dx)^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{c}-\sqrt[3]{c-dx}\right)}{24\ 2^{2/3}c^{8/3}d\left(c^2-d^2x^2\right)^{2/3}}\end{aligned}$$

output

```
-1/4*(-d^2*x^2+c^2)^(1/3)/c/d/(d*x+c)^(7/3)-5/24*(-d^2*x^2+c^2)^(1/3)/c^2/
d/(d*x+c)^(4/3)-5/72*(-d*x+c)^(2/3)*(d*x+c)^(2/3)*arctan(1/3*(c^(1/3)+2^(2
/3)*(-d*x+c)^(1/3))*3^(1/2)/c^(1/3))*2^(1/3)*3^(1/2)/c^(8/3)/d/(-d^2*x^2+c
^2)^(2/3)-5/144*(-d*x+c)^(2/3)*(d*x+c)^(2/3)*ln(d*x+c)*2^(1/3)/c^(8/3)/d/(
-d^2*x^2+c^2)^(2/3)+5/48*(-d*x+c)^(2/3)*(d*x+c)^(2/3)*ln(2^(1/3)*c^(1/3)-(
-d*x+c)^(1/3))*2^(1/3)/c^(8/3)/d/(-d^2*x^2+c^2)^(2/3)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.86

$$\int \frac{1}{(c+dx)^{7/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx = \frac{-\frac{6c^{2/3}(11c+5dx)\sqrt[3]{c^2-d^2x^2}}{(c+dx)^{7/3}}+10\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}\sqrt[3]{c+dx}}{\sqrt[3]{c}\sqrt[3]{c+dx}+2^{2/3}\sqrt[3]{c^2-d}}\right.}{}$$

input

```
Integrate[1/((c + d*x)^(7/3)*(c^2 - d^2*x^2)^(2/3)),x]
```

output

```
((-6*c^(2/3)*(11*c + 5*d*x)*(c^2 - d^2*x^2)^(1/3))/(c + d*x)^(7/3) + 10*2^
(1/3)*Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*(c + d*x)^(1/3))/(c^(1/3)*(c + d*x)^
(1/3) + 2^(2/3)*(c^2 - d^2*x^2)^(1/3))] + 10*2^(1/3)*Log[-2*c^(1/3)*(c + d
*x)^(1/3) + 2^(2/3)*(c^2 - d^2*x^2)^(1/3)] - 5*2^(1/3)*Log[2*c^(2/3)*(c +
d*x)^(2/3) + 2^(2/3)*c^(1/3)*(c + d*x)^(1/3)*(c^2 - d^2*x^2)^(1/3) + 2^(1/
3)*(c^2 - d^2*x^2)^(2/3)])/(144*c^(8/3)*d)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {474, 473, 27, 52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^{7/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx$$

$$\Big\downarrow \ 474$$

$$\frac{\sqrt[3]{\frac{dx}{c}+1}\int \frac{1}{\left(\frac{dx}{c}+1\right)^{7/3}\left(c^2-d^2x^2\right)^{2/3}}dx}{c^2\sqrt[3]{c+dx}}$$

$$\Big\downarrow \ 473$$

$$\frac{\sqrt[3]{c^2-d^2x^2}\int \frac{c^3}{(c+dx)^3\left(c^2-cdx\right)^{2/3}}dx}{c^2\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$$\Big\downarrow \; 27$$

$$\frac{c\sqrt[3]{c^2-d^2x^2}\int \frac{1}{(c+dx)^3(c^2-cdx)^{2/3}}dx}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$$\Big\downarrow \; 52$$

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{5\int \frac{1}{(c+dx)^2\left(c^2-cdx\right)^{2/3}}dx}{12c}-\frac{\sqrt[3]{c^2-cdx}}{4c^2d(c+dx)^2}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$$\Big\downarrow \; 52$$

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{5\left(\frac{\int \frac{1}{(c+dx)\left(c^2-cdx\right)^{2/3}}dx}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{12c}-\frac{\sqrt[3]{c^2-cdx}}{4c^2d(c+dx)^2}\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$$\Big\downarrow \; 69$$

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{5\left(\frac{-\frac{3\int \frac{1}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}}d\sqrt[3]{c^2-cdx}}{2\ 2^{2/3}c^{4/3}d}-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(\cdots}{2\ 2^{2/\cdots}}\cdots}{3c}\cdots\right)}{12c}\cdots\right)}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$$\Big\downarrow \; 16$$

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{5\left(\frac{-\frac{3\int \frac{1}{2^{2/3}c^{4/3}+\sqrt[3]{2}\sqrt[3]{c^2-cdx}c^{2/3}+\left(c^2-cdx\right)^{2/3}}d\sqrt[3]{c^2-cdx}}{2\sqrt[3]{2}c^{2/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\cdots\right)}{12c}\right.}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{5\left(\frac{\frac{3\int \frac{1}{-\left(c^2-cdx\right)^{2/3}-3}d\left(\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\ 2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\ 2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{12c}-\cdots\right.}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

$$\Big\downarrow \text{217}$$

$$\frac{c\sqrt[3]{c^2-d^2x^2}\left(\frac{5\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2^{2/3}\sqrt[3]{c^2-cdx}}{c^{2/3}}+1}{\sqrt{3}}\right)}{2^{2/3}c^{4/3}d}-\frac{\log(c+dx)}{2\,2^{2/3}c^{4/3}d}+\frac{3\log\left(\sqrt[3]{2}c^{2/3}-\sqrt[3]{c^2-cdx}\right)}{2\,2^{2/3}c^{4/3}d}}{3c}-\frac{\sqrt[3]{c^2-cdx}}{2c^2d(c+dx)}\right)}{12c}-\frac{\sqrt[3]{c^2-cd}}{4c^2d(c+dx)}\right.}{\sqrt[3]{c+dx}\sqrt[3]{c^2-cdx}}$$

input
```
Int[1/((c + d*x)^(7/3)*(c^2 - d^2*x^2)^(2/3)),x]
```

output
```
(c*(c^2 - d^2*x^2)^(1/3)*(-1/4*(c^2 - c*d*x)^(1/3)/(c^2*d*(c + d*x)^2) + (
5*(-1/2*(c^2 - c*d*x)^(1/3)/(c^2*d*(c + d*x)) + (-((Sqrt[3]*ArcTan[(1 + (2
^(2/3)*(c^2 - c*d*x)^(1/3))/c^(2/3))/Sqrt[3]])/(2^(2/3)*c^(4/3)*d)) - Log[
c + d*x]/(2*2^(2/3)*c^(4/3)*d) + (3*Log[2^(1/3)*c^(2/3) - (c^2 - c*d*x)^(1
/3)])/(2*2^(2/3)*c^(4/3)*d))/(3*c)))/(12*c)))/((c + d*x)^(1/3)*(c^2 - c*d*
x)^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [F]

$$\int \frac{1}{(dx+c)^{\frac{7}{3}}\left(-d^2x^2+c^2\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(2/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.36

$$\int \frac{1}{(c+dx)^{7/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx =$$

$$-\frac{5\cdot 4^{\frac{2}{3}}(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3)(c^2)^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}\left(-d^2x^2+c^2\right)^{\frac{1}{3}}\left(c^2\right)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}+2\left(-d^2x^2+c^2\right)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}c+2\cdot 4^{\frac{1}{3}}\left(cdx+c^2\right)\left(c\right.}{dx+c}\right.}{}$$

input
```
integrate(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="fricas")
```

output
```
-1/288*(5*4^(2/3)*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(c^2)^(2/3)*lo
g((4^(2/3)*(-d^2*x^2 + c^2)^(1/3)*(c^2)^(2/3)*(d*x + c)^(2/3) + 2*(-d^2*x^
2 + c^2)^(2/3)*(d*x + c)^(1/3)*c + 2*4^(1/3)*(c*d*x + c^2)*(c^2)^(1/3))/(d
*x + c)) - 10*4^(2/3)*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(c^2)^(2/3
)*log(-(4^(2/3)*(c^2)^(2/3)*(d*x + c) - 2*(-d^2*x^2 + c^2)^(1/3)*(d*x + c)
^(2/3)*c)/(d*x + c)) + 60*sqrt(1/3)*(c*d^3*x^3 + 3*c^2*d^2*x^2 + 3*c^3*d*x
 + c^4)*sqrt(4^(1/3)*(c^2)^(1/3))*arctan(1/2*sqrt(1/3)*(4^(2/3)*(-d^2*x^2
+ c^2)^(1/3)*(c^2)^(2/3)*(d*x + c)^(2/3) + 4^(1/3)*(c*d*x + c^2)*(c^2)^(1/
3))*sqrt(4^(1/3)*(c^2)^(1/3))/(c^2*d*x + c^3)) + 12*(5*c^2*d*x + 11*c^3)*(
-d^2*x^2 + c^2)^(1/3)*(d*x + c)^(2/3))/(c^4*d^4*x^3 + 3*c^5*d^3*x^2 + 3*c^
6*d^2*x + c^7*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^{7/3} (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-(-c+dx)(c+dx))^{\frac{2}{3}} (c+dx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)**(7/3)/(-d**2*x**2+c**2)**(2/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c + d*x))**(2/3)*(c + d*x)**(7/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^{7/3} (c^2-d^2x^2)^{2/3}}\,dx = \int \frac{1}{(-d^2x^2+c^2)^{\frac{2}{3}} (dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^2*x^2 + c^2)^(2/3)*(d*x + c)^(7/3)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)^{7/3} (c^2-d^2x^2)^{2/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(2/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^{7/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx=\int \frac{1}{\left(c^2-d^2\,x^2\right)^{2/3}\,\left(c+d\,x\right)^{7/3}}\,dx$$

input
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(7/3)),x)
```

output
```
int(1/((c^2 - d^2*x^2)^(2/3)*(c + d*x)^(7/3)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^{7/3}\left(c^2-d^2x^2\right)^{2/3}}\,dx=\int \frac{1}{(dx+c)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{2}{3}}c^2+2\,(dx+c)^{\frac{1}{3}}\left(-d^2x^2+c^2\right)^{\frac{2}{3}}cdx+(dx+$$

input
```
int(1/(d*x+c)^(7/3)/(-d^2*x^2+c^2)^(2/3),x)
```

output
```
int(1/((c + d*x)**(1/3)*(c**2 - d**2*x**2)**(2/3)*c**2 + 2*(c + d*x)**(1/3
)*(c**2 - d**2*x**2)**(2/3)*c*d*x + (c + d*x)**(1/3)*(c**2 - d**2*x**2)**(
2/3)*d**2*x**2),x)
```

3.302 $\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx=\frac{2\sqrt[4]{c^2-d^2x^2}}{d}+\frac{2c^2\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{dx}{c}\right),2\right)}{d\,(c^2-d^2x^2)^{3/4}}$$

output
```
2*(-d^2*x^2+c^2)^(1/4)/d+2*c^2*(1-d^2*x^2/c^2)^(3/4)*InverseJacobiAM(1/2*a
rcsin(d*x/c),2^(1/2))/d/(-d^2*x^2+c^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx=\frac{2c^2-2d^2x^2+cdx\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{d\,(c^2-d^2x^2)^{3/4}}$$

input
```
Integrate[(c - d*x)/(c^2 - d^2*x^2)^(3/4),x]
```

output
```
(2*c^2 - 2*d^2*x^2 + c*d*x*(1 - (d^2*x^2)/c^2)^(3/4)*Hypergeometric2F1[1/2
, 3/4, 3/2, (d^2*x^2)/c^2])/(d*(c^2 - d^2*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {455, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 455$$

$$c\int \frac{1}{(c^2-d^2x^2)^{3/4}}dx+\frac{2\sqrt[4]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 231$$

$$\frac{c\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}}dx}{(c^2-d^2x^2)^{3/4}}+\frac{2\sqrt[4]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 230$$

$$\frac{2c^2\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{dx}{c}\right),2\right)}{d\,(c^2-d^2x^2)^{3/4}}+\frac{2\sqrt[4]{c^2-d^2x^2}}{d}$$

input
```
Int[(c - d*x)/(c^2 - d^2*x^2)^(3/4),x]
```

output
```
(2*(c^2 - d^2*x^2)^(1/4))/d + (2*c^2*(1 - (d^2*x^2)/c^2)^(3/4)*EllipticF[A
rcSin[(d*x)/c]/2, 2])/(d*(c^2 - d^2*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int \frac{-dx+c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{3}{4}}}dx$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(3/4),x)
```

output
```
int((-d*x+c)/(-d^2*x^2+c^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx = \int -\frac{dx-c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(3/4),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/4)/(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx = -d\left(\begin{cases} \frac{x^2}{2(c^2)^{\frac{3}{4}}} & \text{for } d^2=0 \\ -\frac{2\sqrt[4]{c^2-d^2x^2}}{d^2} & \text{otherwise} \end{cases}\right) + \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{\sqrt{c}}$$

input
```
integrate((-d*x+c)/(-d**2*x**2+c**2)**(3/4),x)
```

output
```
-d*Piecewise((x**2/(2*(c**2)**(3/4)), Eq(d**2, 0)), (-2*(c**2 - d**2*x**2)
**(1/4)/d**2, True)) + x*hyper((1/2, 3/4), (3/2,), d**2*x**2*exp_polar(2*I
*pi)/c**2)/sqrt(c)
```

Maxima [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx = \int -\frac{dx-c}{(-d^2x^2+c^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(3/4),x, algorithm="maxima")
```

output
```
-integrate((d*x - c)/(-d^2*x^2 + c^2)^(3/4), x)
```

Giac [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx = \int -\frac{dx-c}{(-d^2x^2+c^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(3/4),x, algorithm="giac")
```

output
```
integrate(-(d*x - c)/(-d^2*x^2 + c^2)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx=\frac{2\,(c^2-d^2\,x^2)^{1/4}}{d}+\frac{c\,x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{(c^2-d^2\,x^2)^{3/4}}$$

input
```
int((c - d*x)/(c^2 - d^2*x^2)^(3/4),x)
```

output
```
(2*(c^2 - d^2*x^2)^(1/4))/d + (c*x*(1 - (d^2*x^2)/c^2)^(3/4)*hypergeom([1/
2, 3/4], 3/2, (d^2*x^2)/c^2))/(c^2 - d^2*x^2)^(3/4)
```

Reduce [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{3/4}}\,dx=\frac{2(-d^2x^2+c^2)^{\frac{1}{4}}+\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{3}{4}}}dx\right)cd}{d}$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(3/4),x)
```

output
```
(2*(c**2 - d**2*x**2)**(1/4) + int((c**2 - d**2*x**2)**(1/4)/(c**2 - d**2*
x**2),x)*c*d)/d
```

3.303 $\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx=\frac{2\sqrt[4]{c^2-d^2x^2}}{d}+\frac{2c^2\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{dx}{c}\right),2\right)}{d\left(c^2-d^2x^2\right)^{3/4}}$$

output
```
2*(-d^2*x^2+c^2)^(1/4)/d+2*c^2*(1-d^2*x^2/c^2)^(3/4)*InverseJacobiAM(1/2*a
rcsin(d*x/c),2^(1/2))/d/(-d^2*x^2+c^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.93 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx =-\frac{2\sqrt[4]{2}(c-dx)\left(1+\frac{dx}{c}\right)^{3/4}\sqrt[4]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{9}{4},\frac{c-dx}{2c}\right)}{5d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/4)/(c + d*x),x]
```

output
```
(-2*2^(1/4)*(c - d*x)*(1 + (d*x)/c)^(3/4)*(c^2 - d^2*x^2)^(1/4)*Hypergeome
tric2F1[3/4, 5/4, 9/4, (c - d*x)/(2*c)])/(5*d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {493, 27, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx$$

$$\Big\downarrow \; 493$$

$$\frac{\int \frac{cd}{(c^2-d^2x^2)^{3/4}}dx}{d}+\frac{2\sqrt[4]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 27$$

$$c\int \frac{1}{(c^2-d^2x^2)^{3/4}}dx+\frac{2\sqrt[4]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 231$$

$$\frac{c\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}}dx}{(c^2-d^2x^2)^{3/4}}+\frac{2\sqrt[4]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 230$$

$$\frac{2c^2\left(1-\frac{d^2x^2}{c^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{dx}{c}\right),2\right)}{d\,(c^2-d^2x^2)^{3/4}}+\frac{2\sqrt[4]{c^2-d^2x^2}}{d}$$

input
```
Int[(c^2 - d^2*x^2)^(1/4)/(c + d*x),x]
```

output
```
(2*(c^2 - d^2*x^2)^(1/4))/d + (2*c^2*(1 - (d^2*x^2)/c^2)^(3/4)*EllipticF[A
rcSin[(d*x)/c]/2, 2])/(d*(c^2 - d^2*x^2)^(3/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

Maple [F]

$$\int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{1}{4}}}{d x +c}d x$$

input
```
int((-d^2*x^2+c^2)^(1/4)/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^(1/4)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{4}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/4)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/4)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\sqrt[4]{-(-c+dx)\,(c+dx)}}{c+dx}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/4)/(d*x+c),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/4)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{4}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/4)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/4)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{4}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/4)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/4)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(c^2-d^2\,x^2)^{1/4}}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/4)/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^(1/4)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{4}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(1/4)/(d*x+c),x)
```

output
```
int((c**2 - d**2*x**2)**(1/4)/(c + d*x),x)
```

3.304 $\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 223

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \frac{3\sqrt[4]{3}\sqrt{2+ex}\sqrt[4]{4-e^2x^2}}{2e} - \frac{\sqrt[4]{3}(4-e^2x^2)^{5/4}}{2e\sqrt{2+ex}} - \frac{3\sqrt[4]{3}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt{2}e} + \frac{3\sqrt[4]{3}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt{2}e} - \frac{3\sqrt[4]{3}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{\sqrt{2}e}$$

output

```
3/2*3^(1/4)*(e*x+2)^(1/2)*(-e^2*x^2+4)^(1/4)/e-1/2*3^(1/4)*(-e^2*x^2+4)^(5
/4)/e/(e*x+2)^(1/2)-3/2*3^(1/4)*arctan(1-2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1
/4))*2^(1/2)/e+3/2*3^(1/4)*arctan(1+2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*
2^(1/2)/e-3/2*3^(1/4)*arctanh(2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4)/(1+(e*x
+2)^(1/2)/(-e*x+2)^(1/2)))*2^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx$$
$$=\frac{\sqrt[4]{3}\left((1+ex)\sqrt{2+ex}\sqrt[4]{4-e^2x^2}-3\sqrt{2}\arctan\left(\frac{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}{2+ex-\sqrt{4-e^2x^2}}\right)-3\sqrt{2}\text{arctanh}\left(\frac{2+ex+\sqrt{4-e^2x^2}}{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}\right)\right)}{2e}$$

input
```
Integrate[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
(3^(1/4)*((1 + e*x)*Sqrt[2 + e*x]*(4 - e^2*x^2)^(1/4) - 3*Sqrt[2]*ArcTan[(
Sqrt[4 + 2*e*x]*(4 - e^2*x^2)^(1/4))/(2 + e*x - Sqrt[4 - e^2*x^2])] - 3*Sq
rt[2]*ArcTanh[(2 + e*x + Sqrt[4 - e^2*x^2])/(Sqrt[4 + 2*e*x]*(4 - e^2*x^2)
^(1/4))]))/(2*e)
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {456, 60, 27, 60, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex+2}\sqrt[4]{12-3e^2x^2}\,dx$$
$$\Big\downarrow \; 456$$
$$\int \sqrt[4]{6-3ex}(ex+2)^{3/4}dx$$
$$\Big\downarrow \; 60$$
$$\frac{3}{2}\int \frac{\sqrt[4]{3}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}dx-\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$
$$\Big\downarrow \; 27$$

$$\frac{3}{2}\sqrt[4]{3}\int \frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}dx-\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\Big\downarrow$ 60

$$\frac{3}{2}\sqrt[4]{3}\left(\int \frac{1}{(2-ex)^{3/4}\sqrt[4]{ex+2}}dx+\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\Big\downarrow$ 73

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\int \frac{1}{\sqrt[4]{ex+2}}d\sqrt[4]{2-ex}}{e}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\Big\downarrow$ 770

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\int \frac{1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{e}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\Big\downarrow$ 755

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{\sqrt{2-ex}+1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\Big\downarrow$ 1476

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{\cdots}{\sqrt{2-ex}+\cdots}\cdots\right.\right.}{e}\right.$$

$$\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\Big\downarrow$ 1082

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{2-ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{2-ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int\frac{1-\sqrt{2-}}{3-ex}\right.}{e}\right.$$

$$\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\downarrow$ 217

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\int\frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)\right.}{e}\right.$$

$$\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\downarrow$ 1479

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)\right.}{e}\right.$$

$$\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\downarrow$ 25

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.\right.}{e}\right.$$

$$\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\downarrow$ 27

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)+\right.}{e}\right.$$

$$\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

$\downarrow$ 1103

$$\frac{3}{2}\sqrt[4]{3}\left(\frac{\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-}}{\sqrt[4]{ex+}}\right.}{2\sqrt{2}}\right.\right.}{e}\right.$$

$$\frac{\sqrt[4]{3}(2-ex)^{5/4}(ex+2)^{3/4}}{2e}$$

input
```
Int[Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
-1/2*(3^(1/4)*(2 - e*x)^(5/4)*(2 + e*x)^(3/4))/e + (3*3^(1/4)*(((2 - e*x)^
(1/4)*(2 + e*x)^(3/4))/e - (4*((-(ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2
+ e*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1
/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[2 - e*x] - (Sqrt[2]*(2 - e*x)^(1/4))/
(2 + e*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[2 - e*x] + (Sqrt[2]*(2 - e*x)^(1/4
))/(2 + e*x)^(1/4)]/(2*Sqrt[2]))/2))/e))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \sqrt{ex+2}\,\left(-3e^{2}x^{2}+12\right)^{\frac{1}{4}}dx$$

input
```
int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x)
```

output
```
int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \frac{\left(-3\,e^{2}x^{2}+12\right)^{\frac{1}{4}}\sqrt{ex+2}(ex+1)-6\left(\frac{3}{4}\right)^{\frac{1}{4}}\arctan\left(\frac{3\,ex+4\,\left(\frac{3}{4}\right)^{\frac{3}{4}}\left(-3\,e^{2}x^{2}+12\right)^{\frac{1}{4}}\sqrt{ex+2}+6}{3\,(ex+2)}\right)-6\left(\frac{3}{4}\right)^{\frac{1}{4}}\arctan\left(-\right.}{}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
1/2*((-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*(e*x + 1) - 6*(3/4)^(1/4)*arcta
n(1/3*(3*e*x + 4*(3/4)^(3/4)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2) + 6)/(e
*x + 2)) - 6*(3/4)^(1/4)*arctan(-1/3*(3*e*x - 4*(3/4)^(3/4)*(-3*e^2*x^2 +
12)^(1/4)*sqrt(e*x + 2) + 6)/(e*x + 2)) - 3*(3/4)^(1/4)*log((sqrt(3)*(e*x
+ 2) + 2*(3/4)^(1/4)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2) + sqrt(-3*e^2*x
^2 + 12))/(e*x + 2)) + 3*(3/4)^(1/4)*log((sqrt(3)*(e*x + 2) - 2*(3/4)^(1/4
)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2) + sqrt(-3*e^2*x^2 + 12))/(e*x + 2)
))/e
```

Sympy [F]

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \sqrt[4]{3}\int \sqrt{ex+2}\sqrt[4]{-e^2x^2+4}\,dx$$

input
```
integrate((e*x+2)**(1/2)*(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(1/4)*Integral(sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4), x)
```

Maxima [F]

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \int \left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}\,dx$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \int \left(12-3\,e^2\,x^2\right)^{1/4}\,\sqrt{e\,x+2}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(1/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{2+ex}\sqrt[4]{12-3e^2x^2}\,dx = \frac{3^{\frac{1}{4}}\left(\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}\,ex+4\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}-3\left(\int \frac{\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}x}{e^2x^2-4}dx\right)e^2\right)}{2e}$$

input
```
int((e*x+2)^(1/2)*(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(3**(1/4)*(sqrt(e*x + 2)*( - e**2*x**2 + 4)**(1/4)*e*x + 4*sqrt(e*x + 2)*(
 - e**2*x**2 + 4)**(1/4) - 3*int((sqrt(e*x + 2)*( - e**2*x**2 + 4)**(1/4)*
x)/(e**2*x**2 - 4),x)*e**2))/(2*e)
```

3.305 $\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \frac{\sqrt[4]{3}\sqrt{2+ex}\sqrt[4]{4-e^2x^2}}{e} - \frac{\sqrt{2}\sqrt[4]{3}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{e} + \frac{\sqrt{2}\sqrt[4]{3}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{e} - \frac{\sqrt{2}\sqrt[4]{3}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{e}$$

output
```
3^(1/4)*(e*x+2)^(1/2)*(-e^2*x^2+4)^(1/4)/e-3^(1/4)*arctan(1-2^(1/2)*(e*x+2
)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e+3^(1/4)*arctan(1+2^(1/2)*(e*x+2)^(1/4)/(
-e*x+2)^(1/4))*2^(1/2)/e-3^(1/4)*arctanh(2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1
/4)/(1+(e*x+2)^(1/2)/(-e*x+2)^(1/2)))*2^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \frac{\sqrt[4]{3}\left(\sqrt{2+ex}\sqrt[4]{4-e^2x^2}-\sqrt{2}\arctan\left(\frac{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}{2+ex-\sqrt{4-e^2x^2}}\right)-\sqrt{2}\text{arctanh}\left(\frac{2+ex+\sqrt{4-e^2x^2}}{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}\right)\right)}{e}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(1/4)/Sqrt[2 + e*x],x]
```

output
```
(3^(1/4)*(Sqrt[2 + e*x]*(4 - e^2*x^2)^(1/4) - Sqrt[2]*ArcTan[(Sqrt[4 + 2*e
*x]*(4 - e^2*x^2)^(1/4))/(2 + e*x - Sqrt[4 - e^2*x^2])] - Sqrt[2]*ArcTanh[
(2 + e*x + Sqrt[4 - e^2*x^2])/(Sqrt[4 + 2*e*x]*(4 - e^2*x^2)^(1/4))]))/e
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {456, 60, 27, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{ex+2}}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{ex+2}}dx$$

$$\Big\downarrow \text{60}$$

$$3\int \frac{1}{3^{3/4}(2-ex)^{3/4}\sqrt[4]{ex+2}}dx+\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}$$

$$\Big\downarrow \text{27}$$

$$\sqrt[4]{3}\int \frac{1}{(2-ex)^{3/4}\sqrt[4]{ex+2}}dx+\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}$$

$\Big\downarrow$ 73

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\sqrt[4]{3}\int \frac{1}{\sqrt[4]{ex+2}}d\sqrt[4]{2-ex}}{e}$$

$\Big\downarrow$ 770

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\sqrt[4]{3}\int \frac{1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{e}$$

$\Big\downarrow$ 755

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\sqrt[4]{3}\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{\sqrt{2-ex}+1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}$$

$\Big\downarrow$ 1476

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\sqrt[4]{3}\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)\right)}{e}$$

$\Big\downarrow$ 1082

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}$$

$\Big\downarrow$ 217

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-\frac{4\sqrt[4]{3}\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)\right)}{e}$$

$\Big\downarrow$ 1479

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right.\right.}{e}$$

$\Big\downarrow$ 25

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\right.\right.}{e}$$

$\Big\downarrow$ 27

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right.\right.}{e}$$

$\Big\downarrow$ 1103

$$\frac{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}{e}-$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right.}{2\sqrt{2}}\right.\right.}{e}$$

input
```
Int[(12 - 3*e^2*x^2)^(1/4)/Sqrt[2 + e*x],x]
```

output
```
(3^(1/4)*(2 - e*x)^(1/4)*(2 + e*x)^(3/4))/e - (4*3^(1/4)*((-(ArcTan[1 - (S
qrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(2
 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[2 - e*x] -
 (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[2 - e*x
] + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/(2*Sqrt[2]))/2))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(-3e^2x^2+12)^{\frac{1}{4}}}{\sqrt{ex+2}}dx$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x)
```

output
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = -\frac{2\left(\frac{3}{4}\right)^{\frac{1}{4}}\arctan\left(\frac{3\,ex+4\left(\frac{3}{4}\right)^{\frac{3}{4}}(-3\,e^2x^2+12)^{\frac{1}{4}}\sqrt{ex+2}+6}{3\,(ex+2)}\right)+2\left(\frac{3}{4}\right)^{\frac{1}{4}}\arctan\left(-\frac{3\,ex-4\left(\frac{3}{4}\right)^{\frac{3}{4}}(-3\,e^2x^2+12)^{\frac{1}{4}}\sqrt{ex+2}+6}{3\,(ex+2)}\right)+$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x, algorithm="fricas")
```

output
```
-(2*(3/4)^(1/4)*arctan(1/3*(3*e*x + 4*(3/4)^(3/4)*(-3*e^2*x^2 + 12)^(1/4)*
sqrt(e*x + 2) + 6)/(e*x + 2)) + 2*(3/4)^(1/4)*arctan(-1/3*(3*e*x - 4*(3/4)
^(3/4)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2) + 6)/(e*x + 2)) + (3/4)^(1/4)
*log((sqrt(3)*(e*x + 2) + 2*(3/4)^(1/4)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x +
 2) + sqrt(-3*e^2*x^2 + 12))/(e*x + 2)) - (3/4)^(1/4)*log((sqrt(3)*(e*x +
2) - 2*(3/4)^(1/4)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2) + sqrt(-3*e^2*x^2
 + 12))/(e*x + 2)) - (-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2))/e
```

Sympy [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \sqrt[4]{3}\int \frac{\sqrt[4]{-e^2x^2+4}}{\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(1/2),x)
```

output
```
3**(1/4)*Integral((-e**2*x**2 + 4)**(1/4)/sqrt(e*x + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(1/4)/sqrt(e*x + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx =$$
$$-\frac{3^{\frac{1}{4}}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{4}{ex+2}-1\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{4}{ex+2}-1\right)^{\frac{1}{4}}\right)\right)+\sqrt{2}\,l\ldots\right.}{\ldots}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*3^(1/4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(4/(e*x + 2) - 1)^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(4/(e*x + 2) - 1)^(1/
4))) + sqrt(2)*log(sqrt(2)*(4/(e*x + 2) - 1)^(1/4) + sqrt(4/(e*x + 2) - 1)
 + 1) - sqrt(2)*log(-sqrt(2)*(4/(e*x + 2) - 1)^(1/4) + sqrt(4/(e*x + 2) -
1) + 1) - 2*(e*x + 2)*(4/(e*x + 2) - 1)^(1/4))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \int \frac{\left(12-3\,e^2\,x^2\right)^{1/4}}{\sqrt{e\,x+2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(1/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{\sqrt{2+ex}}\,dx = \frac{3^{\frac{1}{4}}\left((ex+2)^{\frac{3}{4}}\,(-ex+2)^{\frac{1}{4}}-\left(\int \frac{(ex+2)^{\frac{3}{4}}(-ex+2)^{\frac{1}{4}}}{e^2x^2-4}dx\right)e\right)}{e}$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(1/2),x)
```

output
```
(3**(1/4)*((e*x + 2)**(3/4)*( - e*x + 2)**(1/4) - int(((e*x + 2)**(3/4)*(
- e*x + 2)**(1/4))/(e**2*x**2 - 4),x)*e))/e
```

3.306 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = -\frac{4\sqrt[4]{3}\sqrt[4]{4-e^2x^2}}{e\sqrt{2+ex}}+\frac{\sqrt{2}\sqrt[4]{3}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{e} - \frac{\sqrt{2}\sqrt[4]{3}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{e}+\frac{\sqrt{2}\sqrt[4]{3}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{e}$$

output

```
-4*3^(1/4)*(-e^2*x^2+4)^(1/4)/e/(e*x+2)^(1/2)+3^(1/4)*arctan(1-2^(1/2)*(e*
x+2)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e-3^(1/4)*arctan(1+2^(1/2)*(e*x+2)^(1/4
)/(-e*x+2)^(1/4))*2^(1/2)/e+3^(1/4)*arctanh(2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)
^(1/4)/(1+(e*x+2)^(1/2)/(-e*x+2)^(1/2)))*2^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \frac{\sqrt[4]{3}\left(-4\sqrt[4]{4-e^2x^2}+\sqrt{4+2ex}\arctan\left(\frac{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}{2+ex-\sqrt{4-e^2x^2}}\right)+\sqrt{4+2ex}\,\mathrm{arctanh}\left(\ldots\right)\right)}{e\sqrt{2+ex}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(3/2),x]
```

output
```
(3^(1/4)*(-4*(4 - e^2*x^2)^(1/4) + Sqrt[4 + 2*e*x]*ArcTan[(Sqrt[4 + 2*e*x]
*(4 - e^2*x^2)^(1/4))/(2 + e*x - Sqrt[4 - e^2*x^2])] + Sqrt[4 + 2*e*x]*Arc
Tanh[(2 + e*x + Sqrt[4 - e^2*x^2])/(Sqrt[4 + 2*e*x]*(4 - e^2*x^2)^(1/4))])
)/(e*Sqrt[2 + e*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {456, 57, 27, 73, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(ex+2)^{3/2}}\,dx$$

$$\Big\downarrow \ 456$$

$$\int \frac{\sqrt[4]{6-3ex}}{(ex+2)^{5/4}}dx$$

$$\Big\downarrow \ 57$$

$$-3\int \frac{1}{3^{3/4}(2-ex)^{3/4}\sqrt[4]{ex+2}}dx-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \ 27$$

$$-\sqrt[4]{3}\int \frac{1}{(2-ex)^{3/4}\sqrt[4]{ex+2}}dx-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{4\sqrt[4]{3}\int \frac{1}{\sqrt[4]{ex+2}}d\sqrt[4]{2-ex}}{e}-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \textbf{770}$$

$$\frac{4\sqrt[4]{3}\int \frac{1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{e}-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \textbf{755}$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{\sqrt{2-ex}+1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \textbf{1476}$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)\right)}{e}$$
$$\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-$$
$$\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)\right)}{e}-\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$\downarrow$ 1479

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\cdots\right)\right)}{e}$$

$$\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$\downarrow$ 25

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\cdots\right)\right)}{e}$$

$$\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$\downarrow$ 27

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right.}{e}$$

$$\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{4\sqrt[4]{3}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2}}{\sqrt[4]{ex}}\right.}{2\sqrt{2}}\right.}{e}$$

$$\frac{4\sqrt[4]{3}\sqrt[4]{2-ex}}{e\sqrt[4]{ex+2}}$$

input
```
Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(3/2),x]
```

output
```
(-4*3^(1/4)*(2 - e*x)^(1/4))/(e*(2 + e*x)^(1/4)) + (4*3^(1/4)*((-(ArcTan[1
 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[
2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2])/2 + (-1/2*Log[1 + Sqrt[2 - e
*x] - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2] + Log[1 + Sqrt[2
- e*x] + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/(2*Sqrt[2]))/2))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(-3e^2x^2 + 12)^{\frac{1}{4}}}{(ex + 2)^{\frac{3}{2}}}dx$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x)
```

output
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \frac{2\left(\frac{3}{4}\right)^{\frac{1}{4}}(ex+2)\arctan\left(\frac{3\,ex+4\left(\frac{3}{4}\right)^{\frac{3}{4}}(-3\,e^2x^2+12)^{\frac{1}{4}}\sqrt{ex+2}+6}{3\,(ex+2)}\right)+2\left(\frac{3}{4}\right)^{\frac{1}{4}}(ex+2)\arctan\left(-\right.$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x, algorithm="fricas")
```

output
```
(2*(3/4)^(1/4)*(e*x + 2)*arctan(1/3*(3*e*x + 4*(3/4)^(3/4)*(-3*e^2*x^2 + 1
2)^(1/4)*sqrt(e*x + 2) + 6)/(e*x + 2)) + 2*(3/4)^(1/4)*(e*x + 2)*arctan(-1
/3*(3*e*x - 4*(3/4)^(3/4)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2) + 6)/(e*x
+ 2)) + (3/4)^(1/4)*(e*x + 2)*log((sqrt(3)*(e*x + 2) + 2*(3/4)^(1/4)*(-3*e
^2*x^2 + 12)^(1/4)*sqrt(e*x + 2) + sqrt(-3*e^2*x^2 + 12))/(e*x + 2)) - (3/
4)^(1/4)*(e*x + 2)*log((sqrt(3)*(e*x + 2) - 2*(3/4)^(1/4)*(-3*e^2*x^2 + 12
)^(1/4)*sqrt(e*x + 2) + sqrt(-3*e^2*x^2 + 12))/(e*x + 2)) - 4*(-3*e^2*x^2
+ 12)^(1/4)*sqrt(e*x + 2))/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \sqrt[4]{3}\int \frac{\sqrt[4]{-e^2x^2+4}}{ex\sqrt{ex+2}+2\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(3/2),x)
```

output
```
3**(1/4)*Integral((-e**2*x**2 + 4)**(1/4)/(e*x*sqrt(e*x + 2) + 2*sqrt(e*x
+ 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{1}{4}}}{(ex+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \int \frac{(12-3\,e^2\,x^2)^{1/4}}{(e\,x+2)^{3/2}}\,dx$$

input
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(3/2),x)
```

output
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{3/2}}\,dx = \int \frac{(-3e^2x^2+12)^{\frac{1}{4}}}{(ex+2)^{\frac{3}{2}}}dx$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x)
```

output
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(3/2),x)
```

3.307 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx=-\frac{\sqrt[4]{3}(4-e^2x^2)^{5/4}}{5e(2+ex)^{5/2}}$$

output
```
-1/5*3^(1/4)*(-e^2*x^2+4)^(5/4)/e/(e*x+2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx=-\frac{\sqrt[4]{3}(4-e^2x^2)^{5/4}}{5e(2+ex)^{5/2}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(5/2),x]
```

output
```
-1/5*(3^(1/4)*(4 - e^2*x^2)^(5/4))/(e*(2 + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {456, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(ex+2)^{5/2}}dx$$
$$\Big\downarrow \text{ 456}$$
$$\int \frac{\sqrt[4]{6-3ex}}{(ex+2)^{9/4}}dx$$
$$\Big\downarrow \text{ 48}$$
$$-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{5e(ex+2)^{5/4}}$$

input
```
Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(5/2),x]
```

output
```
-1/5*(3^(1/4)*(2 - e*x)^(5/4))/(e*(2 + e*x)^(5/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{(ex-2)\left(-3e^{2}x^{2}+12\right)^{\frac{1}{4}}}{5(ex+2)^{\frac{3}{2}}e}$	30
orering	$\frac{(ex-2)\left(-3e^{2}x^{2}+12\right)^{\frac{1}{4}}}{5(ex+2)^{\frac{3}{2}}e}$	30

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(e*x-2)/(e*x+2)^(3/2)/e*(-3*e^2*x^2+12)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \frac{\left(-3\,e^{2}x^{2}+12\right)^{\frac{1}{4}}\sqrt{ex+2}(ex-2)}{5\,\left(e^{3}x^{2}+4\,e^{2}x+4\,e\right)}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x, algorithm="fricas")
```

output
```
1/5*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)*(e*x - 2)/(e^3*x^2 + 4*e^2*x + 4
*e)
```

Sympy [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \sqrt[4]{3}\int \frac{\sqrt[4]{-e^{2}x^{2}+4}}{e^{2}x^{2}\sqrt{ex+2}+4ex\sqrt{ex+2}+4\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(5/2),x)
```

output
```
3**(1/4)*Integral((-e**2*x**2 + 4)**(1/4)/(e**2*x**2*sqrt(e*x + 2) + 4*e*x
*sqrt(e*x + 2) + 4*sqrt(e*x + 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{\left(ex+2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = -\frac{3^{\frac{1}{4}}\left(\frac{4}{ex+2}-1\right)^{\frac{5}{4}}}{5\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x, algorithm="giac")
```

output
```
-1/5*3^(1/4)*(4/(e*x + 2) - 1)^(5/4)/e
```

Mupad [B] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx = \frac{\left(\frac{x}{5\,e}-\frac{2}{5\,e^2}\right)\,\left(12-3\,e^2\,x^2\right)^{1/4}}{\frac{2\,\sqrt{e\,x+2}}{e}+x\,\sqrt{e\,x+2}}$$

input
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(5/2),x)
```

output
```
((x/(5*e) - 2/(5*e^2))*(12 - 3*e^2*x^2)^(1/4))/((2*(e*x + 2)^(1/2))/e + x*
(e*x + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{5/2}}\,dx=\frac{(-ex+2)^{\frac{1}{4}}\,3^{\frac{1}{4}}(ex-2)}{5\,(ex+2)^{\frac{5}{4}}\,e}$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2),x)
```

output
```
(( - e*x + 2)**(1/4)*3**(1/4)*(e*x - 2))/(5*(e*x + 2)**(1/4)*e*(e*x + 2))
```

3.308 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx=-\frac{(4-e^2x^2)^{5/4}}{3\ 3^{3/4}e(2+ex)^{7/2}}-\frac{(4-e^2x^2)^{5/4}}{15\ 3^{3/4}e(2+ex)^{5/2}}$$

output
```
-1/9*(-e^2*x^2+4)^(5/4)*3^(1/4)/e/(e*x+2)^(7/2)-1/45*3^(1/4)*(-e^2*x^2+4)^
(5/4)/e/(e*x+2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx=-\frac{(7+ex)\left(4(2+ex)-(2+ex)^2\right)^{5/4}}{15\ 3^{3/4}e(2+ex)^{7/2}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(7/2),x]
```

output
```
-1/15*((7 + e*x)*(4*(2 + e*x) - (2 + e*x)^2)^(5/4))/(3^(3/4)*e*(2 + e*x)^(
7/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {456, 55, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(ex+2)^{7/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{\sqrt[4]{6-3ex}}{(ex+2)^{13/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{9}\int \frac{\sqrt[4]{3}\sqrt[4]{2-ex}}{(ex+2)^{9/4}}dx-\frac{(2-ex)^{5/4}}{3\ 3^{3/4}e(ex+2)^{9/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[4]{2-ex}}{(ex+2)^{9/4}}dx}{3\ 3^{3/4}}-\frac{(2-ex)^{5/4}}{3\ 3^{3/4}e(ex+2)^{9/4}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{(2-ex)^{5/4}}{15\ 3^{3/4}e(ex+2)^{5/4}}-\frac{(2-ex)^{5/4}}{3\ 3^{3/4}e(ex+2)^{9/4}}$$

input
```
Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(7/2),x]
```

output
```
-1/3*(2 - e*x)^(5/4)/(3^(3/4)*e*(2 + e*x)^(9/4)) - (2 - e*x)^(5/4)/(15*3^(
3/4)*e*(2 + e*x)^(5/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{(ex-2)(ex+7)\left(-3e^{2}x^{2}+12\right)^{\frac{1}{4}}}{45(ex+2)^{\frac{5}{2}}e}$	35
orering	$\frac{(ex-2)(ex+7)\left(-3e^{2}x^{2}+12\right)^{\frac{1}{4}}}{45(ex+2)^{\frac{5}{2}}e}$	35

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*(e*x-2)*(e*x+7)*(-3*e^2*x^2+12)^(1/4)/(e*x+2)^(5/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \frac{(e^2x^2+5\,ex-14)(-3\,e^2x^2+12)^{\frac{1}{4}}\sqrt{ex+2}}{45\,(e^4x^3+6\,e^3x^2+12\,e^2x+8\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x, algorithm="fricas")
```

output
```
1/45*(e^2*x^2 + 5*e*x - 14)*(-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)/(e^4*x^3
 + 6*e^3*x^2 + 12*e^2*x + 8*e)
```

Sympy [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \sqrt[4]{3}\int \frac{\sqrt[4]{-e^2x^2+4}}{e^3x^3\sqrt{ex+2}+6e^2x^2\sqrt{ex+2}+12ex\sqrt{ex+2}+8\sqrt{ex+2}}\,dx$$

input
```
integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(7/2),x)
```

output
```
3**(1/4)*Integral((-e**2*x**2 + 4)**(1/4)/(e**3*x**3*sqrt(e*x + 2) + 6*e**
2*x**2*sqrt(e*x + 2) + 12*e*x*sqrt(e*x + 2) + 8*sqrt(e*x + 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{1}{4}}}{(ex+2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \frac{\left(12-3\,e^2\,x^2\right)^{1/4}\,\left(e^2\,x^2+5\,e\,x-14\right)}{45\,e\,\left(e\,x+2\right)^{5/2}}$$

input
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(7/2),x)
```

output
```
((12 - 3*e^2*x^2)^(1/4)*(5*e*x + e^2*x^2 - 14))/(45*e*(e*x + 2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{7/2}}\,dx = \frac{(-ex+2)^{\frac{1}{4}}\,3^{\frac{1}{4}}\left(e^2x^2+5ex-14\right)}{45\,(ex+2)^{\frac{1}{4}}\,e\left(e^2x^2+4ex+4\right)}$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2),x)
```

output
```
(( - e*x + 2)**(1/4)*3**(1/4)*(e**2*x**2 + 5*e*x - 14))/(45*(e*x + 2)**(1/
4)*e*(e**2*x**2 + 4*e*x + 4))
```

3.309 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = -\frac{\sqrt[4]{3}(4-e^2x^2)^{5/4}}{13e(2+ex)^{9/2}} - \frac{2(4-e^2x^2)^{5/4}}{39\ 3^{3/4}e(2+ex)^{7/2}} - \frac{2(4-e^2x^2)^{5/4}}{195\ 3^{3/4}e(2+ex)^{5/2}}$$

output
```
-1/13*3^(1/4)*(-e^2*x^2+4)^(5/4)/e/(e*x+2)^(9/2)-2/117*(-e^2*x^2+4)^(5/4)*
3^(1/4)/e/(e*x+2)^(7/2)-2/585*3^(1/4)*(-e^2*x^2+4)^(5/4)/e/(e*x+2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = \frac{(-2+ex)\sqrt[4]{4-e^2x^2}(73+18ex+2e^2x^2)}{195\ 3^{3/4}e(2+ex)^{7/2}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(9/2),x]
```

output
```
((-2 + e*x)*(4 - e^2*x^2)^(1/4)*(73 + 18*e*x + 2*e^2*x^2))/(195*3^(3/4)*e*
(2 + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {456, 55, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(ex+2)^{9/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{\sqrt[4]{6-3ex}}{(ex+2)^{17/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2}{13}\int \frac{\sqrt[4]{3}\sqrt[4]{2-ex}}{(ex+2)^{13/4}}dx-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{13e(ex+2)^{13/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{13}\sqrt[4]{3}\int \frac{\sqrt[4]{2-ex}}{(ex+2)^{13/4}}dx-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{13e(ex+2)^{13/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{2}{13}\sqrt[4]{3}\left(\frac{1}{9}\int \frac{\sqrt[4]{2-ex}}{(ex+2)^{9/4}}dx-\frac{(2-ex)^{5/4}}{9e(ex+2)^{9/4}}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{13e(ex+2)^{13/4}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2}{13}\sqrt[4]{3}\left(-\frac{(2-ex)^{5/4}}{45e(ex+2)^{5/4}}-\frac{(2-ex)^{5/4}}{9e(ex+2)^{9/4}}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{13e(ex+2)^{13/4}}$$

input
```
Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(9/2),x]
```

output
```
-1/13*(3^(1/4)*(2 - e*x)^(5/4))/(e*(2 + e*x)^(13/4)) + (2*3^(1/4)*(-1/9*(2
 - e*x)^(5/4)/(e*(2 + e*x)^(9/4)) - (2 - e*x)^(5/4)/(45*e*(2 + e*x)^(5/4))
))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{(ex-2)(2e^2x^2+18ex+73)(-3e^2x^2+12)^{\frac{1}{4}}}{585(ex+2)^{\frac{7}{2}}e}$	44
orering	$\frac{(ex-2)(2e^2x^2+18ex+73)(-3e^2x^2+12)^{\frac{1}{4}}}{585(ex+2)^{\frac{7}{2}}e}$	44

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/585*(e*x-2)*(2*e^2*x^2+18*e*x+73)*(-3*e^2*x^2+12)^(1/4)/(e*x+2)^(7/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = \frac{(2\,e^3x^3+14\,e^2x^2+37\,ex-146)(-3\,e^2x^2+12)^{\frac{1}{4}}\sqrt{ex+2}}{585\,(e^5x^4+8\,e^4x^3+24\,e^3x^2+32\,e^2x+16\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x, algorithm="fricas")
```

output
```
1/585*(2*e^3*x^3 + 14*e^2*x^2 + 37*e*x - 146)*(-3*e^2*x^2 + 12)^(1/4)*sqrt
(e*x + 2)/(e^5*x^4 + 8*e^4*x^3 + 24*e^3*x^2 + 32*e^2*x + 16*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = \int \frac{(-3\,e^2x^2+12)^{\frac{1}{4}}}{(ex+2)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = -\frac{3^{\frac{1}{4}}\left(45\left(\frac{4}{ex+2}-1\right)^{\frac{13}{4}}+130\left(\frac{4}{ex+2}-1\right)^{\frac{9}{4}}+117\left(\frac{4}{ex+2}-1\right)^{\frac{5}{4}}\right)}{9360\,e}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x, algorithm="giac")
```

output
```
-1/9360*3^(1/4)*(45*(4/(e*x + 2) - 1)^(13/4) + 130*(4/(e*x + 2) - 1)^(9/4)
 + 117*(4/(e*x + 2) - 1)^(5/4))/e
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = \frac{\left(12-3\,e^2\,x^2\right)^{1/4}\left(2\,e^3\,x^3+14\,e^2\,x^2+37\,e\,x-146\right)}{585\,e\,\left(e\,x+2\right)^{7/2}}$$

input
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(9/2),x)
```

output
```
((12 - 3*e^2*x^2)^(1/4)*(37*e*x + 14*e^2*x^2 + 2*e^3*x^3 - 146))/(585*e*(e
*x + 2)^(7/2))
```

Reduce [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{9/2}}\,dx = \int \frac{(-3e^2x^2+12)^{\frac{1}{4}}}{(ex+2)^{\frac{9}{2}}}dx$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x)
```

output
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(9/2),x)
```

3.310 $\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = -\frac{\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{17e(2+ex)^{11/2}}-\frac{3\sqrt[4]{3}\left(4-e^2x^2\right)^{5/4}}{221e(2+ex)^{9/2}} -\frac{2\left(4-e^2x^2\right)^{5/4}}{221\ 3^{3/4}e(2+ex)^{7/2}}-\frac{2\left(4-e^2x^2\right)^{5/4}}{1105\ 3^{3/4}e(2+ex)^{5/2}}$$

output
```
-1/17*3^(1/4)*(-e^2*x^2+4)^(5/4)/e/(e*x+2)^(11/2)-3/221*3^(1/4)*(-e^2*x^2+
4)^(5/4)/e/(e*x+2)^(9/2)-2/663*(-e^2*x^2+4)^(5/4)*3^(1/4)/e/(e*x+2)^(7/2)-
2/3315*3^(1/4)*(-e^2*x^2+4)^(5/4)/e/(e*x+2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = \frac{(-2+ex)\sqrt[4]{4-e^2x^2}\left(341+109ex+22e^2x^2+2e^3x^3\right)}{1105\ 3^{3/4}e(2+ex)^{9/2}}$$

input
```
Integrate[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(11/2),x]
```

output
```
((-2 + e*x)*(4 - e^2*x^2)^(1/4)*(341 + 109*e*x + 22*e^2*x^2 + 2*e^3*x^3))/
(1105*3^(3/4)*e*(2 + e*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {456, 55, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(ex+2)^{11/2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{\sqrt[4]{6-3ex}}{(ex+2)^{21/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{17}\int \frac{\sqrt[4]{3}\sqrt[4]{2-ex}}{(ex+2)^{17/4}}dx-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{17e(ex+2)^{17/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{17}\sqrt[4]{3}\int \frac{\sqrt[4]{2-ex}}{(ex+2)^{17/4}}dx-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{17e(ex+2)^{17/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{17}\sqrt[4]{3}\left(\frac{2}{13}\int \frac{\sqrt[4]{2-ex}}{(ex+2)^{13/4}}dx-\frac{(2-ex)^{5/4}}{13e(ex+2)^{13/4}}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{17e(ex+2)^{17/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{3}{17}\sqrt[4]{3}\left(\frac{2}{13}\left(\frac{1}{9}\int \frac{\sqrt[4]{2-ex}}{(ex+2)^{9/4}}dx-\frac{(2-ex)^{5/4}}{9e(ex+2)^{9/4}}\right)-\frac{(2-ex)^{5/4}}{13e(ex+2)^{13/4}}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{17e(ex+2)^{17/4}}$$

$$\Big\downarrow \text{48}$$

$$\frac{3}{17}\sqrt[4]{3}\left(\frac{2}{13}\left(-\frac{(2-ex)^{5/4}}{45e(ex+2)^{5/4}}-\frac{(2-ex)^{5/4}}{9e(ex+2)^{9/4}}\right)-\frac{(2-ex)^{5/4}}{13e(ex+2)^{13/4}}\right)-\frac{\sqrt[4]{3}(2-ex)^{5/4}}{17e(ex+2)^{17/4}}$$

input
```
Int[(12 - 3*e^2*x^2)^(1/4)/(2 + e*x)^(11/2),x]
```

output
```
-1/17*(3^(1/4)*(2 - e*x)^(5/4))/(e*(2 + e*x)^(17/4)) + (3*3^(1/4)*(-1/13*(
2 - e*x)^(5/4)/(e*(2 + e*x)^(13/4)) + (2*(-1/9*(2 - e*x)^(5/4)/(e*(2 + e*x
)^(9/4)) - (2 - e*x)^(5/4)/(45*e*(2 + e*x)^(5/4))))/13))/17
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.37

method	result	size
gosper	$\frac{(ex-2)\left(2e^{3}x^{3}+22e^{2}x^{2}+109ex+341\right)\left(-3e^{2}x^{2}+12\right)^{\frac{1}{4}}}{3315(ex+2)^{\frac{9}{2}}e}$	52
orering	$\frac{(ex-2)\left(2e^{3}x^{3}+22e^{2}x^{2}+109ex+341\right)\left(-3e^{2}x^{2}+12\right)^{\frac{1}{4}}}{3315(ex+2)^{\frac{9}{2}}e}$	52

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/3315*(e*x-2)*(2*e^3*x^3+22*e^2*x^2+109*e*x+341)*(-3*e^2*x^2+12)^(1/4)/(e
*x+2)^(9/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = \frac{(2\,e^4x^4+18\,e^3x^3+65\,e^2x^2+123\,ex-682)(-3\,e^2x^2+12)^{\frac{1}{4}}\sqrt{ex+2}}{3315\,(e^6x^5+10\,e^5x^4+40\,e^4x^3+80\,e^3x^2+80\,e^2x+32\,e)}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x, algorithm="fricas")
```

output
```
1/3315*(2*e^4*x^4 + 18*e^3*x^3 + 65*e^2*x^2 + 123*e*x - 682)*(-3*e^2*x^2 +
 12)^(1/4)*sqrt(e*x + 2)/(e^6*x^5 + 10*e^5*x^4 + 40*e^4*x^3 + 80*e^3*x^2 +
 80*e^2*x + 32*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((-3*e**2*x**2+12)**(1/4)/(e*x+2)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = \int \frac{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}{\left(ex+2\right)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x, algorithm="maxima")
```

output
```
integrate((-3*e^2*x^2 + 12)^(1/4)/(e*x + 2)^(11/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = \frac{(12-3\,e^2\,x^2)^{1/4}\left(\frac{41\,x}{1105\,e^4}-\frac{682}{3315\,e^5}+\frac{2\,x^4}{3315\,e}+\frac{6\,x^3}{1105\,e^2}+\frac{x^2}{51\,e^3}\right)}{\frac{16\,\sqrt{e\,x+2}}{e^4}+x^4\,\sqrt{e\,x+2}+\frac{32\,x\,\sqrt{e\,x+2}}{e^3}+\frac{8\,x^3\,\sqrt{e\,x+2}}{e}+\frac{24\,x^2\,\sqrt{e\,x+2}}{e^2}}$$

input
```
int((12 - 3*e^2*x^2)^(1/4)/(e*x + 2)^(11/2),x)
```

output
```
((12 - 3*e^2*x^2)^(1/4)*((41*x)/(1105*e^4) - 682/(3315*e^5) + (2*x^4)/(331
5*e) + (6*x^3)/(1105*e^2) + x^2/(51*e^3)))/((16*(e*x + 2)^(1/2))/e^4 + x^4
*(e*x + 2)^(1/2) + (32*x*(e*x + 2)^(1/2))/e^3 + (8*x^3*(e*x + 2)^(1/2))/e
+ (24*x^2*(e*x + 2)^(1/2))/e^2)
```

Reduce [F]

$$\int \frac{\sqrt[4]{12-3e^2x^2}}{(2+ex)^{11/2}}\,dx = \int \frac{(-3e^2x^2+12)^{\frac{1}{4}}}{(ex+2)^{\frac{11}{2}}}dx$$

input
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x)
```

output
```
int((-3*e^2*x^2+12)^(1/4)/(e*x+2)^(11/2),x)
```

3.311 $\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 258

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{5\ 3^{3/4}(4-e^2x^2)^{3/4}}{2e\sqrt{2+ex}} - \frac{3^{3/4}\sqrt{2+ex}(4-e^2x^2)^{3/4}}{2e} - \frac{(2+ex)^{3/2}\left(4-e^2x^2\right)^{3/4}}{3\sqrt[4]{3}e} - \frac{5\ 3^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt{2}e} + \frac{5\ 3^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt{2}e} + \frac{5\ 3^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{\sqrt{2}e}$$

output
```
-5/2*3^(3/4)*(-e^2*x^2+4)^(3/4)/e/(e*x+2)^(1/2)-1/2*3^(3/4)*(e*x+2)^(1/2)*
(-e^2*x^2+4)^(3/4)/e-1/9*(e*x+2)^(3/2)*(-e^2*x^2+4)^(3/4)*3^(3/4)/e-5/2*3^
(3/4)*arctan(1-2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e+5/2*3^(3/4)
*arctan(1+2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e+5/2*3^(3/4)*arct
anh(2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4)/(1+(e*x+2)^(1/2)/(-e*x+2)^(1/2)))
*2^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.66

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{-(4-e^2x^2)^{3/4}\,(71+17ex+2e^2x^2)-45\sqrt{4+2ex}\arctan\left(\frac{\sqrt{4+2ex}\,\sqrt[4]{4-e^2x^2}}{2+ex-\sqrt{4-e^2x^2}}\right)+45}{6\sqrt[4]{3}e\sqrt{2+ex}}$$

input
```
Integrate[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
(-((4 - e^2*x^2)^(3/4)*(71 + 17*e*x + 2*e^2*x^2)) - 45*Sqrt[4 + 2*e*x]*Arc
Tan[(Sqrt[4 + 2*e*x]*(4 - e^2*x^2)^(1/4))/(2 + e*x - Sqrt[4 - e^2*x^2])] +
 45*Sqrt[4 + 2*e*x]*ArcTanh[(2 + e*x + Sqrt[4 - e^2*x^2])/(Sqrt[4 + 2*e*x]
*(4 - e^2*x^2)^(1/4))])/(6*3^(1/4)*e*Sqrt[2 + e*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {456, 60, 27, 60, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{5/2}}{\sqrt[4]{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(ex+2)^{9/4}}{\sqrt[4]{6-3ex}}dx$$

$$\Big\downarrow \text{60}$$

$$3\int \frac{(ex+2)^{5/4}}{\sqrt[4]{3}\sqrt[4]{2-ex}}dx-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow \text{27}$$

$$3^{3/4}\int \frac{(ex+2)^{5/4}}{\sqrt[4]{2-ex}}dx-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow \text{60}$$

$$3^{3/4}\left(\frac{5}{2}\int \frac{\sqrt[4]{ex+2}}{\sqrt[4]{2-ex}}dx-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2e}\right)-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow \text{60}$$

$$3^{3/4}\left(\frac{5}{2}\left(\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{3/4}}dx-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2e}\right)-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow \text{73}$$

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\int \frac{\sqrt{2-ex}}{(ex+2)^{3/4}}d\sqrt[4]{2-ex}}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2e}\right)-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow \text{854}$$

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\int \frac{\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2e}\right)-\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow \text{826}$$

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\int \frac{\sqrt{2-ex}+1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)-\frac{(2-ex)^{3/4}(ex+2}{2e}\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow \text{1476}$$

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\cdots\right.}{e}\right.\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$\downarrow$ 1082

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\cdots\right.\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$\downarrow$ 217

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{\cdots}}{e}\right.\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$\downarrow$ 1479

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-\cdots}}{\sqrt[4]{ex+\cdots}}\right.}{\sqrt{2}}\cdots\right.\right.}{e}\right.\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow 25$$

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-\cdots}}{\sqrt[4]{ex+\cdots}}\right.}{\sqrt{2}}\cdots\right.\right.}{e}\right.\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$$\Big\downarrow 27$$

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right.}{\sqrt{2}}\right.\right.}{e}\right.\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

$\downarrow$ 1103

$$3^{3/4}\left(\frac{5}{2}\left(-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2-e\right.}}{}\right.\right.}{e}\right.\right.$$

$$\frac{(2-ex)^{3/4}(ex+2)^{9/4}}{3\sqrt[4]{3}e}$$

input
```
Int[(2 + e*x)^(5/2)/(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
-1/3*((2 - e*x)^(3/4)*(2 + e*x)^(9/4))/(3^(1/4)*e) + 3^(3/4)*(-1/2*((2 - e
*x)^(3/4)*(2 + e*x)^(5/4))/e + (5*(-(((2 - e*x)^(3/4)*(2 + e*x)^(1/4))/e)
- (4*((-(ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2]) +
ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2])/2 + (Log[1
+ Sqrt[2 - e*x] - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/(2*Sqrt[2]) -
 Log[1 + Sqrt[2 - e*x] + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/(2*Sqr
t[2]))/2))/e))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(ex+2)^{\frac{5}{2}}}{(-3e^2x^2+12)^{\frac{1}{4}}}dx$$

input
```
int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.26

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx =$$

$$-\frac{(2\,e^2x^2+17\,ex+71)(-3\,e^2x^2+12)^{\frac{3}{4}}\sqrt{ex+2}+90\left(\frac{27}{4}\right)^{\frac{1}{4}}(ex+2)\arctan\left(\frac{27\,e^2x^2+4\left(\frac{27}{4}\right)^{\frac{3}{4}}(-3\,e^2x^2+12)^{\frac{3}{4}}\sqrt{ex+}}{27\,(e^2x^2-4)}\right.}{}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/18*((2*e^2*x^2 + 17*e*x + 71)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2) + 9
0*(27/4)^(1/4)*(e*x + 2)*arctan(1/27*(27*e^2*x^2 + 4*(27/4)^(3/4)*(-3*e^2*
x^2 + 12)^(3/4)*sqrt(e*x + 2) - 108)/(e^2*x^2 - 4)) + 90*(27/4)^(1/4)*(e*x
 + 2)*arctan(-1/27*(27*e^2*x^2 - 4*(27/4)^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sq
rt(e*x + 2) - 108)/(e^2*x^2 - 4)) + 45*(27/4)^(1/4)*(e*x + 2)*log((3*sqrt(
3)*(e^2*x^2 - 4) + 2*(27/4)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2) -
3*sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)) - 45*(27/4)^(1/4)*(e*x +
 2)*log((3*sqrt(3)*(e^2*x^2 - 4) - 2*(27/4)^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*
sqrt(e*x + 2) - 3*sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)))/(e^2*x
+ 2*e)
```

Sympy [F]

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{3}{4}}\left(\int \frac{4\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx+\int \frac{4ex\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx+\int \frac{e^2x^2\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx\right)}{3}$$

input
```
integrate((e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(3/4)*(Integral(4*sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x) + Integral(
4*e*x*sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x) + Integral(e**2*x**2*sqrt(
e*x + 2)/(-e**2*x**2 + 4)**(1/4), x))/3
```

Maxima [F]

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{(ex+2)^{\frac{5}{2}}}{(-3\,e^2x^2+12)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate((e*x + 2)^(5/2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{(e\,x+2)^{5/2}}{(12-3\,e^2\,x^2)^{1/4}}\,dx$$

input
```
int((e*x + 2)^(5/2)/(12 - 3*e^2*x^2)^(1/4),x)
```

output
```
int((e*x + 2)^(5/2)/(12 - 3*e^2*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(2+ex)^{5/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{\left(8(ex+2)^{\frac{1}{4}}(-ex+2)^{\frac{1}{4}}e^2x^2 - 32(ex+2)^{\frac{1}{4}}(-ex+2)^{\frac{1}{4}} + 20\sqrt{-ex+2}\left(\int \frac{\sqrt{ex+2}}{(-e^2x^2+4}\right.\right.}{15\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(8*(e*x + 2)**(1/4)*( - e*x + 2)**(1/4)*e**2*x**2 - 32*(e*x + 2)**(1/4)*(
- e*x + 2)**(1/4) + 20*sqrt( - e*x + 2)*int(sqrt(e*x + 2)/( - e**2*x**2 +
4)**(1/4),x)*e + 5*sqrt( - e*x + 2)*int((sqrt(e*x + 2)*x**2)/( - e**2*x**2
 + 4)**(1/4),x)*e**3)/(5*sqrt( - e*x + 2)*3**(1/4)*e)
```

3.312 $\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 223

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{5(4-e^2x^2)^{3/4}}{2\sqrt[4]{3}e\sqrt{2+ex}} - \frac{\sqrt{2+ex}(4-e^2x^2)^{3/4}}{2\sqrt[4]{3}e} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{\sqrt{2}\sqrt[4]{3}e}$$

output
```
-5/6*3^(3/4)*(-e^2*x^2+4)^(3/4)/e/(e*x+2)^(1/2)-1/6*3^(3/4)*(e*x+2)^(1/2)*
(-e^2*x^2+4)^(3/4)/e-5/6*3^(3/4)*arctan(1-2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(
1/4))*2^(1/2)/e+5/6*3^(3/4)*arctan(1+2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))
*2^(1/2)/e+5/6*3^(3/4)*arctanh(2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4)/(1+(e*
x+2)^(1/2)/(-e*x+2)^(1/2)))*2^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.72

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{-\left((7+ex)\left(4-e^2x^2\right)^{3/4}\right)-5\sqrt{4+2ex}\arctan\left(\frac{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}{2+ex-\sqrt{4-e^2x^2}}\right)+5\sqrt{4+2ex}\mathrm{ar}}{2\sqrt[4]{3}e\sqrt{2+ex}}$$

input
```
Integrate[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
(-((7 + e*x)*(4 - e^2*x^2)^(3/4)) - 5*Sqrt[4 + 2*e*x]*ArcTan[(Sqrt[4 + 2*e
*x]*(4 - e^2*x^2)^(1/4))/(2 + e*x - Sqrt[4 - e^2*x^2])] + 5*Sqrt[4 + 2*e*x
]*ArcTanh[(2 + e*x + Sqrt[4 - e^2*x^2])/(Sqrt[4 + 2*e*x]*(4 - e^2*x^2)^(1/
4))])/(2*3^(1/4)*e*Sqrt[2 + e*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {456, 60, 27, 60, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex+2)^{3/2}}{\sqrt[4]{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(ex+2)^{5/4}}{\sqrt[4]{6-3ex}}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{5}{2}\int \frac{\sqrt[4]{ex+2}}{\sqrt[4]{3}\sqrt[4]{2-ex}}dx-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{\sqrt[4]{ex+2}}{\sqrt[4]{2-ex}}dx}{2\sqrt[4]{3}}-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 60

$$\frac{5\left(\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{3/4}}dx-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)}{2\sqrt[4]{3}}-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 73

$$\frac{5\left(-\frac{4\int \frac{\sqrt{2-ex}}{(ex+2)^{3/4}}d\sqrt[4]{2-ex}}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)}{2\sqrt[4]{3}}-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 854

$$\frac{5\left(-\frac{4\int \frac{\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)}{2\sqrt[4]{3}}-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 826

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\int \frac{\sqrt{2-ex}+1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)}{2\sqrt[4]{3}}-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 1476

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+\frac{1}{2}\int \frac{1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\frac{(2-e}{}\right.}{2\sqrt[4]{3}}-\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 1082

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right.}{2\sqrt[4]{3}}$$

$$\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\downarrow$ 217

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{e}\right)}{2\sqrt[4]{3}}-$$

$$\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\downarrow$ 1479

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\arctan\right.\right.}{e}\right.}{2\sqrt[4]{3}}$$

$$\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 25

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.}{e}\right.}{2\sqrt[4]{3}}$$

$$\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 27

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan}{}\right.}{e}\right.}{2\sqrt[4]{3}}$$

$$\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

$\Big\downarrow$ 1103

$$\frac{5\left(-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{2\sqrt{2}}\right)}{e}\right.}{2\sqrt[4]{3}}$$

$$\frac{(2-ex)^{3/4}(ex+2)^{5/4}}{2\sqrt[4]{3}e}$$

input
```
Int[(2 + e*x)^(3/2)/(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
-1/2*((2 - e*x)^(3/4)*(2 + e*x)^(5/4))/(3^(1/4)*e) + (5*(-(((2 - e*x)^(3/4
)*(2 + e*x)^(1/4))/e) - (4*((-(ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e
*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)
]/Sqrt[2])/2 + (Log[1 + Sqrt[2 - e*x] - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x
)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[2 - e*x] + (Sqrt[2]*(2 - e*x)^(1/4))/(
2 + e*x)^(1/4)]/(2*Sqrt[2]))/2))/e))/(2*3^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(ex+2)^{\frac{3}{2}}}{(-3e^2x^2+12)^{\frac{1}{4}}}dx$$

input
```
int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.42

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{10 \cdot 12^{\frac{3}{4}}(ex+2)\arctan\left(\frac{3\,e^2x^2+12^{\frac{1}{4}}\,(-3\,e^2x^2+12)^{\frac{3}{4}}\,\sqrt{ex+2}-12}{3\,(e^2x^2-4)}\right)+10 \cdot 12^{\frac{3}{4}}(ex+2)\arctan\left(-\frac{3\,e^2x^2-12^{\frac{1}{4}}\,(-3\,e^2x^2+\ldots}{3\,(e^2x^2-\ldots}\right.}{}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/24*(10*12^(3/4)*(e*x + 2)*arctan(1/3*(3*e^2*x^2 + 12^(1/4)*(-3*e^2*x^2
+ 12)^(3/4)*sqrt(e*x + 2) - 12)/(e^2*x^2 - 4)) + 10*12^(3/4)*(e*x + 2)*arc
tan(-1/3*(3*e^2*x^2 - 12^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2) - 12)
/(e^2*x^2 - 4)) + 5*12^(3/4)*(e*x + 2)*log((12^(3/4)*(-3*e^2*x^2 + 12)^(3/
4)*sqrt(e*x + 2) + 6*sqrt(3)*(e^2*x^2 - 4) - 6*sqrt(-3*e^2*x^2 + 12)*(e*x
+ 2))/(e^2*x^2 - 4)) - 5*12^(3/4)*(e*x + 2)*log(-(12^(3/4)*(-3*e^2*x^2 + 1
2)^(3/4)*sqrt(e*x + 2) - 6*sqrt(3)*(e^2*x^2 - 4) + 6*sqrt(-3*e^2*x^2 + 12)
*(e*x + 2))/(e^2*x^2 - 4)) + 4*(-3*e^2*x^2 + 12)^(3/4)*(e*x + 7)*sqrt(e*x
+ 2))/(e^2*x + 2*e)
```

Sympy [F]

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{3}{4}}\left(\int \frac{2\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx + \int \frac{ex\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx\right)}{3}$$

input
```
integrate((e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(3/4)*(Integral(2*sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x) + Integral(
e*x*sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x))/3
```

Maxima [F]

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{(ex+2)^{\frac{3}{2}}}{(-3\,e^2x^2+12)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate((e*x + 2)^(3/2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.83

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx =$$
$$-\frac{3^{\frac{3}{4}}\left((ex+2)^2\left(5\left(\frac{4}{ex+2}-1\right)^{\frac{7}{4}}+9\left(\frac{4}{ex+2}-1\right)^{\frac{3}{4}}\right)+20\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{4}{ex+2}-1\right)^{\frac{1}{4}}\right)\right)+20\sqrt{2}\,\mathrm{ar}\right.}{}$$

input
```
integrate((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
-1/24*3^(3/4)*((e*x + 2)^2*(5*(4/(e*x + 2) - 1)^(7/4) + 9*(4/(e*x + 2) - 1
)^(3/4)) + 20*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(4/(e*x + 2) - 1)^(1
/4))) + 20*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(4/(e*x + 2) - 1)^(1/4
))) - 10*sqrt(2)*log(sqrt(2)*(4/(e*x + 2) - 1)^(1/4) + sqrt(4/(e*x + 2) -
1) + 1) + 10*sqrt(2)*log(-sqrt(2)*(4/(e*x + 2) - 1)^(1/4) + sqrt(4/(e*x +
2) - 1) + 1))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{(e\,x+2)^{3/2}}{(12-3\,e^2\,x^2)^{1/4}}\,dx$$

input
```
int((e*x + 2)^(3/2)/(12 - 3*e^2*x^2)^(1/4),x)
```

output
```
int((e*x + 2)^(3/2)/(12 - 3*e^2*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(2+ex)^{3/2}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{2\left((ex+2)^{\frac{1}{4}}\,(-ex+2)^{\frac{1}{4}}\,e^2x^2 - 4(ex+2)^{\frac{1}{4}}\,(-ex+2)^{\frac{1}{4}} + 5\sqrt{-ex+2}\left(\int \frac{\sqrt{ex+2}}{(-e^2x^2+4)^{\frac{1}{4}}}\right.\right.}{15\sqrt{-ex+2}\,e}$$

input
```
int((e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(2*((e*x + 2)**(1/4)*( - e*x + 2)**(1/4)*e**2*x**2 - 4*(e*x + 2)**(1/4)*(
- e*x + 2)**(1/4) + 5*sqrt( - e*x + 2)*int(sqrt(e*x + 2)/( - e**2*x**2 + 4
)**(1/4),x)*e))/(5*sqrt( - e*x + 2)*3**(1/4)*e)
```

3.313 $\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{(4-e^2x^2)^{3/4}}{\sqrt[4]{3}e\sqrt{2+ex}} - \frac{\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt[4]{3}e} + \frac{\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt[4]{3}e} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{\sqrt[4]{3}e}$$

output
```
-1/3*3^(3/4)*(-e^2*x^2+4)^(3/4)/e/(e*x+2)^(1/2)-1/3*3^(3/4)*arctan(1-2^(1/
2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e+1/3*3^(3/4)*arctan(1+2^(1/2)*(e
*x+2)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e+1/3*3^(3/4)*arctanh(2^(1/2)*(e*x+2)^
(1/4)/(-e*x+2)^(1/4)/(1+(e*x+2)^(1/2)/(-e*x+2)^(1/2)))*2^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{-(4-e^2x^2)^{3/4}-\sqrt{4+2ex}\arctan\left(\frac{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}{2+ex-\sqrt{4-e^2x^2}}\right)+\sqrt{4+2ex}\,\text{arctanh}\left(\frac{2+ex+\sqrt{4-e^2x^2}}{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}\right)}{\sqrt[4]{3}e\sqrt{2+ex}}$$

input
```
Integrate[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
(-(4 - e^2*x^2)^(3/4) - Sqrt[4 + 2*e*x]*ArcTan[(Sqrt[4 + 2*e*x]*(4 - e^2*x
^2)^(1/4))/(2 + e*x - Sqrt[4 - e^2*x^2])] + Sqrt[4 + 2*e*x]*ArcTanh[(2 + e
*x + Sqrt[4 - e^2*x^2])/(Sqrt[4 + 2*e*x]*(4 - e^2*x^2)^(1/4))])/(3^(1/4)*e
*Sqrt[2 + e*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {456, 60, 27, 73, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex+2}}{\sqrt[4]{12-3e^2x^2}}dx$$

$$\Big\downarrow \ 456$$

$$\int \frac{\sqrt[4]{ex+2}}{\sqrt[4]{6-3ex}}dx$$

$$\Big\downarrow \ 60$$

$$\int \frac{1}{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{3/4}}dx-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{3/4}}dx}{\sqrt[4]{3}} - \frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{73}$$

$$-\frac{4\int \frac{\sqrt{2-ex}}{(ex+2)^{3/4}}d\sqrt[4]{2-ex}}{\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{854}$$

$$-\frac{4\int \frac{\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{826}$$

$$-\frac{4\left(\frac{1}{2}\int \frac{\sqrt{2-ex}+1}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}} - \frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}} + \frac{1}{2}\int \frac{1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right) - \frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}} - \frac{\int \frac{1}{-\sqrt{2-ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right) - \frac{1}{2}\int \frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e} - \frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{217}$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{2-ex}}{3-ex}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 1479

$$-\frac{4\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\right.}{\sqrt[4]{3}e}$$

$$\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 25

$$-\frac{4\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\right.}{\sqrt[4]{3}e}$$

$$\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$\Big\downarrow$ 27

$$-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}{\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1}d\frac{\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}\right.\right.}{\sqrt[4]{3}e}$$

$$\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 1103$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2-ex}-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2-ex}+\frac{\sqrt{2}\sqrt[4]{2-}}{\sqrt[4]{ex}}\right.}{2\sqrt{2}}\right.\right.}{\sqrt[4]{3}e}$$

$$\frac{(2-ex)^{3/4}\sqrt[4]{ex+2}}{\sqrt[4]{3}e}$$

input
```
Int[Sqrt[2 + e*x]/(12 - 3*e^2*x^2)^(1/4),x]
```

output
```
-(((2 - e*x)^(3/4)*(2 + e*x)^(1/4))/(3^(1/4)*e)) - (4*((-(ArcTan[1 - (Sqrt
[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2]) + ArcTan[1 + (Sqrt[2]*(2 -
e*x)^(1/4))/(2 + e*x)^(1/4)]/Sqrt[2])/2 + (Log[1 + Sqrt[2 - e*x] - (Sqrt[2
]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/(2*Sqrt[2]) - Log[1 + Sqrt[2 - e*x] +
(Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)]/(2*Sqrt[2]))/2))/(3^(1/4)*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{\sqrt{ex+2}}{(-3e^2x^2+12)^{\frac{1}{4}}}dx$$

input
```
int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(146) = 292$.

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{2\cdot 12^{\frac{3}{4}}(ex+2)\arctan\left(\frac{3\,e^2x^2+12^{\frac{1}{4}}\,(-3\,e^2x^2+12)^{\frac{3}{4}}\sqrt{ex+2}-12}{3\,(e^2x^2-4)}\right)+2\cdot 12^{\frac{3}{4}}(ex+2)\arctan\left(-\frac{3\,e^2x^2-12^{\frac{1}{4}}\,(-3\,e^2x^2+}{3\,(e^2x^2-}\right.}{}$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/12*(2*12^(3/4)*(e*x + 2)*arctan(1/3*(3*e^2*x^2 + 12^(1/4)*(-3*e^2*x^2 +
 12)^(3/4)*sqrt(e*x + 2) - 12)/(e^2*x^2 - 4)) + 2*12^(3/4)*(e*x + 2)*arcta
n(-1/3*(3*e^2*x^2 - 12^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2) - 12)/(
e^2*x^2 - 4)) + 12^(3/4)*(e*x + 2)*log((12^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*s
qrt(e*x + 2) + 6*sqrt(3)*(e^2*x^2 - 4) - 6*sqrt(-3*e^2*x^2 + 12)*(e*x + 2)
)/(e^2*x^2 - 4)) - 12^(3/4)*(e*x + 2)*log(-(12^(3/4)*(-3*e^2*x^2 + 12)^(3/
4)*sqrt(e*x + 2) - 6*sqrt(3)*(e^2*x^2 - 4) + 6*sqrt(-3*e^2*x^2 + 12)*(e*x
+ 2))/(e^2*x^2 - 4)) + 4*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2))/(e^2*x + 2
*e)
```

Sympy [F]

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{3}{4}}\int \frac{\sqrt{ex+2}}{\sqrt[4]{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate((e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(3/4)*Integral(sqrt(e*x + 2)/(-e**2*x**2 + 4)**(1/4), x)/3
```

Maxima [F]

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{\sqrt{ex+2}}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + 2)/(-3*e^2*x^2 + 12)^(1/4), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx =$$
$$-\frac{3^{\frac{3}{4}}\left(2\,(ex+2)\left(\frac{4}{ex+2}-1\right)^{\frac{3}{4}}+2\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\left(\frac{4}{ex+2}-1\right)^{\frac{1}{4}}\right)\right)+2\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-\right.\right.\right.}{}$$

input
```
integrate((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
-1/6*3^(3/4)*(2*(e*x + 2)*(4/(e*x + 2) - 1)^(3/4) + 2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2) + 2*(4/(e*x + 2) - 1)^(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2*(4/(e*x + 2) - 1)^(1/4))) - sqrt(2)*log(sqrt(2)*(4/(e*x +
 2) - 1)^(1/4) + sqrt(4/(e*x + 2) - 1) + 1) + sqrt(2)*log(-sqrt(2)*(4/(e*x
 + 2) - 1)^(1/4) + sqrt(4/(e*x + 2) - 1) + 1))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx=\int \frac{\sqrt{e\,x+2}}{\left(12-3\,e^2\,x^2\right)^{1/4}}\,dx$$

input
```
int((e*x + 2)^(1/2)/(12 - 3*e^2*x^2)^(1/4),x)
```

output
```
int((e*x + 2)^(1/2)/(12 - 3*e^2*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{2+ex}}{\sqrt[4]{12-3e^2x^2}}\,dx=\frac{\left(\int \frac{\sqrt{ex+2}}{\left(-e^2x^2+4\right)^{\frac{1}{4}}}dx\right)3^{\frac{3}{4}}}{3}$$

input
```
int((e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
int(sqrt(e*x + 2)/( - e**2*x**2 + 4)**(1/4),x)/3**(1/4)
```

3.314 $\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt[4]{3}e} + \frac{\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}}\right)}{\sqrt[4]{3}e} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{2+ex}}{\sqrt[4]{2-ex}\left(1+\frac{\sqrt{2+ex}}{\sqrt{2-ex}}\right)}\right)}{\sqrt[4]{3}e}$$

output
```
-1/3*3^(3/4)*arctan(1-2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e+1/3*
3^(3/4)*arctan(1+2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4))*2^(1/2)/e+1/3*3^(3/
4)*arctanh(2^(1/2)*(e*x+2)^(1/4)/(-e*x+2)^(1/4)/(1+(e*x+2)^(1/2)/(-e*x+2)^
(1/2)))*2^(1/2)/e
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{\sqrt{2}\left(-\arctan\left(\frac{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}{2+ex-\sqrt{4-e^2x^2}}\right)+\operatorname{arctanh}\left(\frac{2+ex+\sqrt{4-e^2x^2}}{\sqrt{4+2ex}\sqrt[4]{4-e^2x^2}}\right)\right)}{\sqrt[4]{3}e}$$

input
```
Integrate[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
(Sqrt[2]*(-ArcTan[(Sqrt[4 + 2*e*x]*(4 - e^2*x^2)^(1/4))/(2 + e*x - Sqrt[4
- e^2*x^2])] + ArcTanh[(2 + e*x + Sqrt[4 - e^2*x^2])/(Sqrt[4 + 2*e*x]*(4 -
 e^2*x^2)^(1/4))]))/(3^(1/4)*e)
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.26, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {456, 73, 27, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex+2}\sqrt[4]{12-3e^2x^2}}\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{\sqrt[4]{6-3ex}(ex+2)^{3/4}}dx$$

$$\Big\downarrow \text{73}$$

$$-\frac{4\int \frac{3^{3/4}\sqrt{6-3ex}}{(3ex+6)^{3/4}}d\sqrt[4]{6-3ex}}{3e}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{\sqrt{6-3ex}}{(3ex+6)^{3/4}}d\sqrt[4]{6-3ex}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 854$$

$$-\frac{4\int \frac{\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 826$$

$$-\frac{4\left(\frac{1}{2}\int \frac{\sqrt{6-3ex}+1}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}-\frac{1}{2}\int\frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 1476$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+\frac{1}{2}\int\frac{1}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)-\frac{1}{2}\int\frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 1082$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\sqrt{6-3ex}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\sqrt{6-3ex}-1}d\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 217$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\sqrt{6-3ex}}{7-3ex}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt[4]{3}e}$$

$$\Big\downarrow \; 1479$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3e\cdots}}{\sqrt[4]{3ex+6}}\cdots\right)}{\sqrt{2}}\cdots\right)\cdots\right)}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\cdots\right)}{\sqrt{2}}\cdots\right)\cdots\right)}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}{\sqrt{6-3ex}+\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1}d\frac{\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-\cdots}}{\sqrt[4]{3ex+\cdots}}\cdots\right)}{\sqrt{2}}\cdots\right)\cdots\right)}{\sqrt[4]{3}e}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{4\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{6-3ex}-\frac{\sqrt{2}\sqrt[4]{6-3ex}}{\sqrt[4]{3ex+6}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{6-3ex}+\sqrt{\cdots}\right)}{2\cdots}\cdots\right)\cdots\right)}{\sqrt[4]{3}e}$$

input
```
Int[1/(Sqrt[2 + e*x]*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
(-4*((-(ArcTan[1 - (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3*e*x)^(1/4)]/Sqrt[2])
 + ArcTan[1 + (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3*e*x)^(1/4)]/Sqrt[2])/2 +
(Log[1 + Sqrt[6 - 3*e*x] - (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3*e*x)^(1/4)]/
(2*Sqrt[2]) - Log[1 + Sqrt[6 - 3*e*x] + (Sqrt[2]*(6 - 3*e*x)^(1/4))/(6 + 3
*e*x)^(1/4)]/(2*Sqrt[2]))/2))/(3^(1/4)*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{1}{\sqrt{ex+2}\,(-3e^2x^2+12)^{\frac{1}{4}}}dx$$

input
```
int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(119) = 238$.

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{2\cdot 12^{\frac{3}{4}}\arctan\left(\frac{3\,e^2x^2+12^{\frac{1}{4}}\left(-3\,e^2x^2+12\right)^{\frac{3}{4}}\sqrt{ex+2}-12}{3\,(e^2x^2-4)}\right)+2\cdot 12^{\frac{3}{4}}\arctan\left(-\frac{3\,e^2x^2-12^{\frac{1}{4}}\left(-3\,e^2x^2+12\right)^{\frac{3}{4}}\sqrt{ex+2}-12}{3\,(e^2x^2-4)}\right)+}{}$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/12*(2*12^(3/4)*arctan(1/3*(3*e^2*x^2 + 12^(1/4)*(-3*e^2*x^2 + 12)^(3/4)
*sqrt(e*x + 2) - 12)/(e^2*x^2 - 4)) + 2*12^(3/4)*arctan(-1/3*(3*e^2*x^2 -
12^(1/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2) - 12)/(e^2*x^2 - 4)) + 12^(
3/4)*log((12^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2) + 6*sqrt(3)*(e^2*
x^2 - 4) - 6*sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)) - 12^(3/4)*lo
g(-(12^(3/4)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2) - 6*sqrt(3)*(e^2*x^2 -
4) + 6*sqrt(-3*e^2*x^2 + 12)*(e*x + 2))/(e^2*x^2 - 4)))/e
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{3}{4}}\int \frac{1}{\sqrt{ex+2}\sqrt[4]{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate(1/(e*x+2)**(1/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(3/4)*Integral(1/(sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4)), x)/3
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{1}{\left(-3\,e^2x^2+12\right)^{\frac{1}{4}}\sqrt{ex+2}}\,dx$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*e^2*x^2 + 12)^(1/4)*sqrt(e*x + 2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{3^{\frac{3}{4}}\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(\frac{4}{ex+2}-1\right)^{\frac{1}{4}}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(\frac{4}{ex+2}-1\right)^{\frac{1}{4}}\right)\right)-\sqrt{2}\,\mathrm{l}\right.}{6\,e}$$

input
```
integrate(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
-1/6*3^(3/4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(4/(e*x + 2) - 1)^
(1/4))) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(4/(e*x + 2) - 1)^(1/
4))) - sqrt(2)*log(sqrt(2)*(4/(e*x + 2) - 1)^(1/4) + sqrt(4/(e*x + 2) - 1)
 + 1) + sqrt(2)*log(-sqrt(2)*(4/(e*x + 2) - 1)^(1/4) + sqrt(4/(e*x + 2) -
1) + 1))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{1}{\left(12-3\,e^2\,x^2\right)^{1/4}\,\sqrt{e\,x+2}}\,dx$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(1/2)),x)
```

output
```
int(1/((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{2+ex}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{1}{4}}\left(3\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}\,ex-6\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}-(ex+2)^{\frac{1}{4}}\,(-ex+2)^{\frac{3}{4}}\,\sqrt{-e^2x^2+4}\,\sqrt{9}\right)}{3\sqrt{-e^2x^2+4}\,\sqrt{3}\,e\,(ex-2)}$$

input
```
int(1/(e*x+2)^(1/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(3**(1/4)*(3*sqrt(e*x + 2)*( - e**2*x**2 + 4)**(1/4)*e*x - 6*sqrt(e*x + 2)
*( - e**2*x**2 + 4)**(1/4) - (e*x + 2)**(1/4)*( - e*x + 2)**(3/4)*sqrt( -
e**2*x**2 + 4)*sqrt(9)))/(3*sqrt( - e**2*x**2 + 4)*sqrt(3)*e*(e*x - 2))
```

3.315 $\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx=-\frac{(4-e^2x^2)^{3/4}}{3\sqrt[4]{3}e(2+ex)^{3/2}}$$

output
```
-1/9*(-e^2*x^2+4)^(3/4)*3^(3/4)/e/(e*x+2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx=-\frac{(4-e^2x^2)^{3/4}}{3\sqrt[4]{3}e(2+ex)^{3/2}}$$

input
```
Integrate[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/3*(4 - e^2*x^2)^(3/4)/(3^(1/4)*e*(2 + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {456, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex+2)^{3/2}\sqrt[4]{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{\sqrt[4]{6-3ex}(ex+2)^{7/4}}dx$$

$$\Big\downarrow \text{48}$$

$$-\frac{(2-ex)^{3/4}}{3\sqrt[4]{3}e(ex+2)^{3/4}}$$

input
```
Int[1/((2 + e*x)^(3/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/3*(2 - e*x)^(3/4)/(3^(1/4)*e*(2 + e*x)^(3/4))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{ex-2}{3\sqrt{ex+2}\,e(-3e^2x^2+12)^{\frac{1}{4}}}$	30
orering	$\frac{ex-2}{3\sqrt{ex+2}\,e(-3e^2x^2+12)^{\frac{1}{4}}}$	30

input
```
int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*(e*x-2)/(e*x+2)^(1/2)/e/(-3*e^2*x^2+12)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{(-3\,e^2x^2+12)^{\frac{3}{4}}\sqrt{ex+2}}{9\,(e^3x^2+4\,e^2x+4\,e)}$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/9*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)/(e^3*x^2 + 4*e^2*x + 4*e)
```

Sympy [F]

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{3}{4}}\int \frac{1}{ex\sqrt{ex+2}\sqrt[4]{-e^2x^2+4}+2\sqrt{ex+2}\sqrt[4]{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate(1/(e*x+2)**(3/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(3/4)*Integral(1/(e*x*sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4) + 2*sqrt(e*
x + 2)*(-e**2*x**2 + 4)**(1/4)), x)/3
```

Maxima [F]

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{1}{(-3\,e^2x^2+12)^{\frac{1}{4}}(ex+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{\left(12-3\,e^2\,x^2\right)^{3/4}}{9\,e\,\left(e\,x+2\right)^{3/2}}$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(3/2)),x)
```

output
```
-(12 - 3*e^2*x^2)^(3/4)/(9*e*(e*x + 2)^(3/2))
```

Reduce [F]

$$\int \frac{1}{(2+ex)^{3/2}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{1}{4}}\left(3\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}+\sqrt{-e^2x^2+4}\,\sqrt{9}\left(\int \frac{(ex+2)^{\frac{1}{4}}(-ex+2)^{\frac{3}{4}}x^2}{e^4x^4-8e^2x^2+16}dx\right)e}{3\sqrt{-e^2x^2+4}\,\sqrt{3}\,e}$$

input
```
int(1/(e*x+2)^(3/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(3**(1/4)*(3*sqrt(e*x + 2)*( - e**2*x**2 + 4)**(1/4) + sqrt( - e**2*x**2 +
 4)*sqrt(9)*int(((e*x + 2)**(1/4)*( - e*x + 2)**(3/4)*x**2)/(e**4*x**4 - 8
*e**2*x**2 + 16),x)*e**3))/(3*sqrt( - e**2*x**2 + 4)*sqrt(3)*e)
```

3.316 $\int \frac{1}{(2+ex)^{5/2} \sqrt[4]{12-3e^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{1}{(2+ex)^{5/2} \sqrt[4]{12-3e^2x^2}} \, dx = -\frac{(4-e^2x^2)^{3/4}}{7\sqrt[4]{3}e(2+ex)^{5/2}} - \frac{(4-e^2x^2)^{3/4}}{21\sqrt[4]{3}e(2+ex)^{3/2}}$$

output
```
-1/21*(-e^2*x^2+4)^(3/4)*3^(3/4)/e/(e*x+2)^(5/2)-1/63*(-e^2*x^2+4)^(3/4)*3
^(3/4)/e/(e*x+2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{(2+ex)^{5/2} \sqrt[4]{12-3e^2x^2}} \, dx = -\frac{(5+ex)\left(4-e^2x^2\right)^{3/4}}{21\sqrt[4]{3}e(2+ex)^{5/2}}$$

input
```
Integrate[1/((2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/21*((5 + e*x)*(4 - e^2*x^2)^(3/4))/(3^(1/4)*e*(2 + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {456, 55, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex+2)^{5/2}\sqrt[4]{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{\sqrt[4]{6-3ex}(ex+2)^{11/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{7}\int \frac{1}{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{7/4}}dx-\frac{(2-ex)^{3/4}}{7\sqrt[4]{3}e(ex+2)^{7/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{7/4}}dx}{7\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{7\sqrt[4]{3}e(ex+2)^{7/4}}$$

$$\Big\downarrow \text{48}$$

$$-\frac{(2-ex)^{3/4}}{21\sqrt[4]{3}e(ex+2)^{3/4}}-\frac{(2-ex)^{3/4}}{7\sqrt[4]{3}e(ex+2)^{7/4}}$$

input
```
Int[1/((2 + e*x)^(5/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/7*(2 - e*x)^(3/4)/(3^(1/4)*e*(2 + e*x)^(7/4)) - (2 - e*x)^(3/4)/(21*3^(
1/4)*e*(2 + e*x)^(3/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{(ex-2)(ex+5)}{21(ex+2)^{\frac{3}{2}}e(-3e^{2}x^{2}+12)^{\frac{1}{4}}}$	35
orering	$\frac{(ex-2)(ex+5)}{21(ex+2)^{\frac{3}{2}}e(-3e^{2}x^{2}+12)^{\frac{1}{4}}}$	35

input
```
int(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/21*(e*x-2)*(e*x+5)/(e*x+2)^(3/2)/e/(-3*e^2*x^2+12)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{(-3\,e^2x^2+12)^{\frac{3}{4}}(ex+5)\sqrt{ex+2}}{63\,(e^4x^3+6\,e^3x^2+12\,e^2x+8\,e)}$$

input
```
integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/63*(-3*e^2*x^2 + 12)^(3/4)*(e*x + 5)*sqrt(e*x + 2)/(e^4*x^3 + 6*e^3*x^2
 + 12*e^2*x + 8*e)
```

Sympy [F]

$$\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{3}{4}}\int \frac{1}{e^2x^2\sqrt{ex+2}\sqrt[4]{-e^2x^2+4}+4ex\sqrt{ex+2}\sqrt[4]{-e^2x^2+4}+4\sqrt{ex+2}\sqrt[4]{-e^2x^2+4}}\,dx}{3}$$

input
```
integrate(1/(e*x+2)**(5/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(3/4)*Integral(1/(e**2*x**2*sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4) + 4*e
**x*sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4) + 4*sqrt(e*x + 2)*(-e**2*x**2 + 4
)**(1/4)), x)/3
```

Maxima [F]

$$\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{1}{(-3\,e^2x^2+12)^{\frac{1}{4}}(ex+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{3^{\frac{3}{4}}\left(3\left(\frac{4}{ex+2}-1\right)^{\frac{7}{4}}+7\left(\frac{4}{ex+2}-1\right)^{\frac{3}{4}}\right)}{252\,e}$$

input
```
integrate(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
-1/252*3^(3/4)*(3*(4/(e*x + 2) - 1)^(7/4) + 7*(4/(e*x + 2) - 1)^(3/4))/e
```

Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{\left(\frac{x}{63\,e^2}+\frac{5}{63\,e^3}\right)\left(12-3\,e^2\,x^2\right)^{3/4}}{\frac{4\,\sqrt{e\,x+2}}{e^2}+x^2\,\sqrt{e\,x+2}+\frac{4\,x\,\sqrt{e\,x+2}}{e}}$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(5/2)),x)
```

output
```
-((x/(63*e^2) + 5/(63*e^3))*(12 - 3*e^2*x^2)^(3/4))/((4*(e*x + 2)^(1/2))/e
^2 + x^2*(e*x + 2)^(1/2) + (4*x*(e*x + 2)^(1/2))/e)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{1}{(2+ex)^{5/2}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}\,3^{\frac{1}{4}}(ex-2)}{5\sqrt{-e^2x^2+4}\,\sqrt{3}\,e\,(e^2x^2+4ex+4)}$$

input
```
int(1/(e*x+2)^(5/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(sqrt(e*x + 2)*( - e**2*x**2 + 4)**(1/4)*3**(1/4)*(e*x - 2))/(5*sqrt( - e*
*2*x**2 + 4)*sqrt(3)*e*(e**2*x**2 + 4*e*x + 4))
```

3.317 $\int \frac{1}{(2+ex)^{7/2} \sqrt[4]{12-3e^2x^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{1}{(2+ex)^{7/2} \sqrt[4]{12-3e^2x^2}} \, dx = -\frac{(4-e^2x^2)^{3/4}}{11\sqrt[4]{3}e(2+ex)^{7/2}} - \frac{2(4-e^2x^2)^{3/4}}{77\sqrt[4]{3}e(2+ex)^{5/2}} - \frac{2(4-e^2x^2)^{3/4}}{231\sqrt[4]{3}e(2+ex)^{3/2}}$$

output
```
-1/33*(-e^2*x^2+4)^(3/4)*3^(3/4)/e/(e*x+2)^(7/2)-2/231*(-e^2*x^2+4)^(3/4)*
3^(3/4)/e/(e*x+2)^(5/2)-2/693*(-e^2*x^2+4)^(3/4)*3^(3/4)/e/(e*x+2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{1}{(2+ex)^{7/2} \sqrt[4]{12-3e^2x^2}} \, dx = -\frac{(4-e^2x^2)^{3/4} (41+14ex+2e^2x^2)}{231\sqrt[4]{3}e(2+ex)^{7/2}}$$

input
```
Integrate[1/((2 + e*x)^(7/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/231*((4 - e^2*x^2)^(3/4)*(41 + 14*e*x + 2*e^2*x^2))/(3^(1/4)*e*(2 + e*x
)^(7/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {456, 55, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex+2)^{7/2}\sqrt[4]{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{\sqrt[4]{6-3ex}(ex+2)^{15/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{2}{11}\int \frac{1}{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{11/4}}dx-\frac{(2-ex)^{3/4}}{11\sqrt[4]{3}e(ex+2)^{11/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{11/4}}dx}{11\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{11\sqrt[4]{3}e(ex+2)^{11/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{2\left(\frac{1}{7}\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{7/4}}dx-\frac{(2-ex)^{3/4}}{7e(ex+2)^{7/4}}\right)}{11\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{11\sqrt[4]{3}e(ex+2)^{11/4}}$$

$$\Big\downarrow \text{48}$$

$$\frac{2\left(-\frac{(2-ex)^{3/4}}{21e(ex+2)^{3/4}}-\frac{(2-ex)^{3/4}}{7e(ex+2)^{7/4}}\right)}{11\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{11\sqrt[4]{3}e(ex+2)^{11/4}}$$

input
```
Int[1/((2 + e*x)^(7/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/11*(2 - e*x)^(3/4)/(3^(1/4)*e*(2 + e*x)^(11/4)) + (2*(-1/7*(2 - e*x)^(3
/4)/(e*(2 + e*x)^(7/4)) - (2 - e*x)^(3/4)/(21*e*(2 + e*x)^(3/4))))/(11*3^(
1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{(ex-2)(2e^2x^2+14ex+41)}{231(ex+2)^{\frac{5}{2}}e(-3e^2x^2+12)^{\frac{1}{4}}}$	44
orering	$\frac{(ex-2)(2e^2x^2+14ex+41)}{231(ex+2)^{\frac{5}{2}}e(-3e^2x^2+12)^{\frac{1}{4}}}$	44

input
```
int(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/231*(e*x-2)*(2*e^2*x^2+14*e*x+41)/(e*x+2)^(5/2)/e/(-3*e^2*x^2+12)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

$$\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{(2\,e^2x^2+14\,ex+41)(-3\,e^2x^2+12)^{\frac{3}{4}}\sqrt{ex+2}}{693\,(e^5x^4+8\,e^4x^3+24\,e^3x^2+32\,e^2x+16\,e)}$$

input
```
integrate(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/693*(2*e^2*x^2 + 14*e*x + 41)*(-3*e^2*x^2 + 12)^(3/4)*sqrt(e*x + 2)/(e^
5*x^4 + 8*e^4*x^3 + 24*e^3*x^2 + 32*e^2*x + 16*e)
```

Sympy [F]

$$\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{3^{\frac{3}{4}}\int \frac{1}{e^3x^3\sqrt{ex+2}\,\sqrt[4]{-e^2x^2+4}+6e^2x^2\sqrt{ex+2}\,\sqrt[4]{-e^2x^2+4}+12ex\sqrt{ex+2}\,\sqrt[4]{-e^2x^2+4}}}{3}$$

input
```
integrate(1/(e*x+2)**(7/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
3**(3/4)*Integral(1/(e**3*x**3*sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4) + 6*e
**2*x**2*sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4) + 12*e*x*sqrt(e*x + 2)*(-e*
*2*x**2 + 4)**(1/4) + 8*sqrt(e*x + 2)*(-e**2*x**2 + 4)**(1/4)), x)/3
```

Maxima [F]

$$\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{1}{(-3\,e^2x^2+12)^{\frac{1}{4}}(ex+2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(7/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{\left(12-3\,e^2\,x^2\right)^{3/4}\left(\frac{2\,x}{99\,e^3}+\frac{41}{693\,e^4}+\frac{2\,x^2}{693\,e^2}\right)}{\frac{8\,\sqrt{e\,x+2}}{e^3}+x^3\,\sqrt{e\,x+2}+\frac{12\,x\,\sqrt{e\,x+2}}{e^2}+\frac{6\,x^2\,\sqrt{e\,x+2}}{e}}$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(7/2)),x)
```

output
```
-((12 - 3*e^2*x^2)^(3/4)*((2*x)/(99*e^3) + 41/(693*e^4) + (2*x^2)/(693*e^2
)))/((8*(e*x + 2)^(1/2))/e^3 + x^3*(e*x + 2)^(1/2) + (12*x*(e*x + 2)^(1/2)
)/e^2 + (6*x^2*(e*x + 2)^(1/2))/e)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{1}{(2+ex)^{7/2}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}\,3^{\frac{1}{4}}(e^2x^2+5ex-14)}{45\sqrt{-e^2x^2+4}\,\sqrt{3}\,e\,(e^3x^3+6e^2x^2+12ex+8)}$$

input
```
int(1/(e*x+2)^(7/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(sqrt(e*x + 2)*( - e**2*x**2 + 4)**(1/4)*3**(1/4)*(e**2*x**2 + 5*e*x - 14)
)/(45*sqrt( - e**2*x**2 + 4)*sqrt(3)*e*(e**3*x**3 + 6*e**2*x**2 + 12*e*x +
 8))
```

3.318 $\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{\left(4-e^2x^2\right)^{3/4}}{15\sqrt[4]{3}e(2+ex)^{9/2}} - \frac{\left(4-e^2x^2\right)^{3/4}}{55\sqrt[4]{3}e(2+ex)^{7/2}} - \frac{2\left(4-e^2x^2\right)^{3/4}}{385\sqrt[4]{3}e(2+ex)^{5/2}} - \frac{2\left(4-e^2x^2\right)^{3/4}}{1155\sqrt[4]{3}e(2+ex)^{3/2}}$$

output
```
-1/45*(-e^2*x^2+4)^(3/4)*3^(3/4)/e/(e*x+2)^(9/2)-1/165*(-e^2*x^2+4)^(3/4)*
3^(3/4)/e/(e*x+2)^(7/2)-2/1155*(-e^2*x^2+4)^(3/4)*3^(3/4)/e/(e*x+2)^(5/2)-
2/3465*(-e^2*x^2+4)^(3/4)*3^(3/4)/e/(e*x+2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.40

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{\left(4-e^2x^2\right)^{3/4}\left(159+69ex+18e^2x^2+2e^3x^3\right)}{1155\sqrt[4]{3}e(2+ex)^{9/2}}$$

input
```
Integrate[1/((2 + e*x)^(9/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/1155*((4 - e^2*x^2)^(3/4)*(159 + 69*e*x + 18*e^2*x^2 + 2*e^3*x^3))/(3^(
1/4)*e*(2 + e*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {456, 55, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex+2)^{9/2}\sqrt[4]{12-3e^2x^2}}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{1}{\sqrt[4]{6-3ex}(ex+2)^{19/4}}dx$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{5}\int \frac{1}{\sqrt[4]{3}\sqrt[4]{2-ex}(ex+2)^{15/4}}dx-\frac{(2-ex)^{3/4}}{15\sqrt[4]{3}e(ex+2)^{15/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{15/4}}dx}{5\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{15\sqrt[4]{3}e(ex+2)^{15/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{\frac{2}{11}\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{11/4}}dx-\frac{(2-ex)^{3/4}}{11e(ex+2)^{11/4}}}{5\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{15\sqrt[4]{3}e(ex+2)^{15/4}}$$

$$\Big\downarrow \text{55}$$

$$\frac{\frac{2}{11}\left(\frac{1}{7}\int \frac{1}{\sqrt[4]{2-ex}(ex+2)^{7/4}}dx-\frac{(2-ex)^{3/4}}{7e(ex+2)^{7/4}}\right)-\frac{(2-ex)^{3/4}}{11e(ex+2)^{11/4}}}{5\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{15\sqrt[4]{3}e(ex+2)^{15/4}}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{2}{11}\left(-\frac{(2-ex)^{3/4}}{21e(ex+2)^{3/4}}-\frac{(2-ex)^{3/4}}{7e(ex+2)^{7/4}}\right)-\frac{(2-ex)^{3/4}}{11e(ex+2)^{11/4}}}{5\sqrt[4]{3}}-\frac{(2-ex)^{3/4}}{15\sqrt[4]{3}e(ex+2)^{15/4}}$$

input
```
Int[1/((2 + e*x)^(9/2)*(12 - 3*e^2*x^2)^(1/4)),x]
```

output
```
-1/15*(2 - e*x)^(3/4)/(3^(1/4)*e*(2 + e*x)^(15/4)) + (-1/11*(2 - e*x)^(3/4
)/(e*(2 + e*x)^(11/4)) + (2*(-1/7*(2 - e*x)^(3/4)/(e*(2 + e*x)^(7/4)) - (2
 - e*x)^(3/4)/(21*e*(2 + e*x)^(3/4))))/11)/(5*3^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.37

method	result	size
gosper	$\frac{(ex-2)(2e^3x^3+18e^2x^2+69ex+159)}{1155(ex+2)^{\frac{7}{2}}e(-3e^2x^2+12)^{\frac{1}{4}}}$	52
orering	$\frac{(ex-2)(2e^3x^3+18e^2x^2+69ex+159)}{1155(ex+2)^{\frac{7}{2}}e(-3e^2x^2+12)^{\frac{1}{4}}}$	52

input
```
int(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/1155*(e*x-2)*(2*e^3*x^3+18*e^2*x^2+69*e*x+159)/(e*x+2)^(7/2)/e/(-3*e^2*x
^2+12)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{(2\,e^3x^3+18\,e^2x^2+69\,ex+159)(-3\,e^2x^2+12)^{\frac{3}{4}}\sqrt{ex+2}}{3465\,(e^6x^5+10\,e^5x^4+40\,e^4x^3+80\,e^3x^2+80\,e^2x+32\,e)}$$

input
```
integrate(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="fricas")
```

output
```
-1/3465*(2*e^3*x^3 + 18*e^2*x^2 + 69*e*x + 159)*(-3*e^2*x^2 + 12)^(3/4)*sq
rt(e*x + 2)/(e^6*x^5 + 10*e^5*x^4 + 40*e^4*x^3 + 80*e^3*x^2 + 80*e^2*x + 3
2*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+2)**(9/2)/(-3*e**2*x**2+12)**(1/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = \int \frac{1}{(-3\,e^2x^2+12)^{\frac{1}{4}}(ex+2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*e^2*x^2 + 12)^(1/4)*(e*x + 2)^(9/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{3^{\frac{3}{4}}\left(77\left(\frac{4}{ex+2}-1\right)^{\frac{15}{4}}+315\left(\frac{4}{ex+2}-1\right)^{\frac{11}{4}}+495\left(\frac{4}{ex+2}-1\right)^{\frac{7}{4}}+385\left(\frac{4}{ex+2}-1\right)^{\frac{3}{4}}\right)}{221760\,e}$$

input
```
integrate(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x, algorithm="giac")
```

output
```
-1/221760*3^(3/4)*(77*(4/(e*x + 2) - 1)^(15/4) + 315*(4/(e*x + 2) - 1)^(11
/4) + 495*(4/(e*x + 2) - 1)^(7/4) + 385*(4/(e*x + 2) - 1)^(3/4))/e
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = -\frac{\left(12-3\,e^2\,x^2\right)^{3/4}\left(\frac{23\,x}{1155\,e^4}+\frac{53}{1155\,e^5}+\frac{2\,x^3}{3465\,e^2}+\frac{2\,x^2}{385\,e^3}\right)}{\frac{16\,\sqrt{e\,x+2}}{e^4}+x^4\,\sqrt{e\,x+2}+\frac{32\,x\,\sqrt{e\,x+2}}{e^3}+\frac{8\,x^3\,\sqrt{e\,x+2}}{e}+\frac{24\,x^2\,\sqrt{e\,x+2}}{e^2}}$$

input
```
int(1/((12 - 3*e^2*x^2)^(1/4)*(e*x + 2)^(9/2)),x)
```

output
```
-((12 - 3*e^2*x^2)^(3/4)*((23*x)/(1155*e^4) + 53/(1155*e^5) + (2*x^3)/(346
5*e^2) + (2*x^2)/(385*e^3)))/((16*(e*x + 2)^(1/2))/e^4 + x^4*(e*x + 2)^(1/
2) + (32*x*(e*x + 2)^(1/2))/e^3 + (8*x^3*(e*x + 2)^(1/2))/e + (24*x^2*(e*x
 + 2)^(1/2))/e^2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{1}{(2+ex)^{9/2}\sqrt[4]{12-3e^2x^2}}\,dx = \frac{\sqrt{ex+2}\,(-e^2x^2+4)^{\frac{1}{4}}\,3^{\frac{1}{4}}(2e^3x^3+14e^2x^2+37ex-146)}{585\sqrt{-e^2x^2+4}\,\sqrt{3}\,e\,(e^4x^4+8e^3x^3+24e^2x^2+32ex+16)}$$

input
```
int(1/(e*x+2)^(9/2)/(-3*e^2*x^2+12)^(1/4),x)
```

output
```
(sqrt(e*x + 2)*( - e**2*x**2 + 4)**(1/4)*3**(1/4)*(2*e**3*x**3 + 14*e**2*x
**2 + 37*e*x - 146))/(585*sqrt( - e**2*x**2 + 4)*sqrt(3)*e*(e**4*x**4 + 8*
e**3*x**3 + 24*e**2*x**2 + 32*e*x + 16))
```

3.319 $\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/5} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/5} \, dx = \frac{5(c^2-d^2x^2)^{22/5} \text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{22}{5},\frac{27}{5},\frac{c+dx}{2c}\right)}{11\ 2^{3/5} c^4 d \left(\frac{c-dx}{c}\right)^{22/5}}$$

output
```
5/22*(-d^2*x^2+c^2)^(22/5)*hypergeom([-2/5, 22/5],[27/5],1/2*(d*x+c)/c)*2^
(2/5)/c^4/d/((-d*x+c)/c)^(22/5)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(67) = 134.

Time = 8.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/5} \, dx = \frac{(c^2-d^2x^2)^{2/5}\left(5\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}(-41c^4+34c^2d^2x^2+7d^4x^4)+168c^3dx\,\text{Hypergeometric2F}\right.}{168d\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}$$

input
```
Integrate[(c + d*x)^3*(c^2 - d^2*x^2)^(2/5),x]
```

output
```
((c^2 - d^2*x^2)^(2/5)*(5*(1 - (d^2*x^2)/c^2)^(2/5)*(-41*c^4 + 34*c^2*d^2*
x^2 + 7*d^4*x^4) + 168*c^3*d*x*Hypergeometric2F1[-2/5, 1/2, 3/2, (d^2*x^2)
/c^2] + 168*c*d^3*x^3*Hypergeometric2F1[-2/5, 3/2, 5/2, (d^2*x^2)/c^2]))/(
168*d*(1 - (d^2*x^2)/c^2)^(2/5))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/5} dx$$

$$\Big\downarrow \text{473}$$

$$\frac{c^2 \left(c^2-d^2x^2\right)^{7/5} \int (c-dx)^{2/5} \left(\frac{dx}{c}+1\right)^{17/5}dx}{(c-dx)^{7/5} \left(\frac{dx}{c}+1\right)^{7/5}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{40\ 2^{2/5} c^2 \left(c^2-d^2x^2\right)^{7/5} \text{Hypergeometric2F1}\left(-\frac{17}{5},\frac{7}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{7d \left(\frac{dx}{c}+1\right)^{7/5}}$$

input
```
Int[(c + d*x)^3*(c^2 - d^2*x^2)^(2/5),x]
```

output
```
(-40*2^(2/5)*c^2*(c^2 - d^2*x^2)^(7/5)*Hypergeometric2F1[-17/5, 7/5, 12/5,
 (c - d*x)/(2*c)])/(7*d*(1 + (d*x)/c)^(7/5))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int (dx+c)^3 \left(-d^2x^2+c^2\right)^{\frac{2}{5}} dx$$

input
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(2/5),x)
```

output
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(2/5),x)
```

Fricas [F]

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/5} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{5}} (dx+c)^3 dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(2/5),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(-d^2*x^2 + c^2)^(2/5),
 x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 661, normalized size of antiderivative = 9.87

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/5} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(-d**2*x**2+c**2)**(2/5),x)
```

output
```
c**(19/5)*x*hyper((-2/5, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
 c**(9/5)*d**2*x**3*hyper((-2/5, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)
/c**2) + 3*c**2*d*Piecewise((x**2*(c**2)**(2/5)/2, Eq(d**2, 0)), (-5*(c**2
 - d**2*x**2)**(7/5)/(14*d**2), True)) + d**3*Piecewise((25*c**(44/5)*(-1
+ d**2*x**2/c**2)**(2/5)*exp(2*I*pi/5)/(-168*c**4*d**4 + 168*c**2*d**6*x**
2) - 25*c**(44/5)/(-168*c**4*d**4 + 168*c**2*d**6*x**2) - 15*c**(34/5)*d**
2*x**2*(-1 + d**2*x**2/c**2)**(2/5)*exp(2*I*pi/5)/(-168*c**4*d**4 + 168*c*
*2*d**6*x**2) + 25*c**(34/5)*d**2*x**2/(-168*c**4*d**4 + 168*c**2*d**6*x**
2) - 45*c**(24/5)*d**4*x**4*(-1 + d**2*x**2/c**2)**(2/5)*exp(2*I*pi/5)/(-1
68*c**4*d**4 + 168*c**2*d**6*x**2) + 35*c**(14/5)*d**6*x**6*(-1 + d**2*x**
2/c**2)**(2/5)*exp(2*I*pi/5)/(-168*c**4*d**4 + 168*c**2*d**6*x**2), Abs(d*
*2*x**2/c**2) > 1), (25*c**(44/5)*(1 - d**2*x**2/c**2)**(2/5)/(-168*c**4*d
**4 + 168*c**2*d**6*x**2) - 25*c**(44/5)/(-168*c**4*d**4 + 168*c**2*d**6*x
**2) - 15*c**(34/5)*d**2*x**2*(1 - d**2*x**2/c**2)**(2/5)/(-168*c**4*d**4
+ 168*c**2*d**6*x**2) + 25*c**(34/5)*d**2*x**2/(-168*c**4*d**4 + 168*c**2*
d**6*x**2) - 45*c**(24/5)*d**4*x**4*(1 - d**2*x**2/c**2)**(2/5)/(-168*c**4
*d**4 + 168*c**2*d**6*x**2) + 35*c**(14/5)*d**6*x**6*(1 - d**2*x**2/c**2)*
*(2/5)/(-168*c**4*d**4 + 168*c**2*d**6*x**2), True))
```

Maxima [F]

$$\int (c+dx)^3 \left(c^2-d^2x^2\right)^{2/5} \, dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{5}} (dx+c)^3 \, dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(2/5),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)*(d*x + c)^3, x)
```

Giac [F]

$$\int (c+dx)^3\left(c^2-d^2x^2\right)^{2/5}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}(dx+c)^3\,dx$$

input
```
integrate((d*x+c)^3*(-d^2*x^2+c^2)^(2/5),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)*(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\left(c^2-d^2x^2\right)^{2/5}\,dx=\int \left(c^2-d^2\,x^2\right)^{2/5}\,(c+d\,x)^3\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/5)*(c + d*x)^3,x)
```

output
```
int((c^2 - d^2*x^2)^(2/5)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3\left(c^2-d^2x^2\right)^{2/5}\,dx=\frac{7616\left(-d^2x^2+c^2\right)^{\frac{3}{5}}\left(\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{3}{5}}}dx\right)c^5d-11685c^6+1960c^5dx+21375c^4d^2x^2+5600c^3}{9576\left(-d^2x^2+c^2\right)^{\frac{3}{5}}d}$$

input
```
int((d*x+c)^3*(-d^2*x^2+c^2)^(2/5),x)
```

output
```
(7616*(c**2 - d**2*x**2)**(3/5)*int((c**2 - d**2*x**2)**(2/5)/(c**2 - d**2
*x**2),x)*c**5*d - 11685*c**6 + 1960*c**5*d*x + 21375*c**4*d**2*x**2 + 560
0*c**3*d**3*x**3 - 7695*c**2*d**4*x**4 - 7560*c*d**5*x**5 - 1995*d**6*x**6
)/(9576*(c**2 - d**2*x**2)**(3/5)*d)
```

3.320 $\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} \, dx = \frac{5\ 2^{2/5} \left(c^2-d^2x^2\right)^{17/5} \text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{17}{5},\frac{22}{5},\frac{c+dx}{2c}\right)}{17 c^3 d \left(\frac{c-dx}{c}\right)^{17/5}}$$

output
```
5/17*2^(2/5)*(-d^2*x^2+c^2)^(17/5)*hypergeom([-2/5, 17/5],[22/5],1/2*(d*x+
c)/c)/c^3/d/((-d*x+c)/c)^(17/5)
```

Mathematica [A] (verified)

Time = 8.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} \, dx = \frac{\left(c^2-d^2x^2\right)^{2/5} \left(-15c(c^2-d^2x^2)\left(1-\frac{d^2x^2}{c^2}\right)^{2/5} + 21c^2dx \, \text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},\frac{d^2}{c}\right.\right.}{21d\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}$$

input
```
Integrate[(c + d*x)^2*(c^2 - d^2*x^2)^(2/5),x]
```

output
```
((c^2 - d^2*x^2)^(2/5)*(-15*c*(c^2 - d^2*x^2)*(1 - (d^2*x^2)/c^2)^(2/5) +
21*c^2*d*x*Hypergeometric2F1[-2/5, 1/2, 3/2, (d^2*x^2)/c^2] + 7*d^3*x^3*Hy
pergeometric2F1[-2/5, 3/2, 5/2, (d^2*x^2)/c^2]))/(21*d*(1 - (d^2*x^2)/c^2)
^(2/5))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} dx$$

$$\Big\downarrow \; 473$$

$$\frac{c\left(c^2-d^2x^2\right)^{7/5} \int (c-dx)^{2/5} \left(\frac{dx}{c}+1\right)^{12/5} dx}{(c-dx)^{7/5} \left(\frac{dx}{c}+1\right)^{7/5}}$$

$$\Big\downarrow \; 79$$

$$-\frac{20\ 2^{2/5} c\left(c^2-d^2x^2\right)^{7/5} \text{Hypergeometric2F1}\left(-\frac{12}{5},\frac{7}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{7d\left(\frac{dx}{c}+1\right)^{7/5}}$$

input
```
Int[(c + d*x)^2*(c^2 - d^2*x^2)^(2/5),x]
```

output
```
(-20*2^(2/5)*c*(c^2 - d^2*x^2)^(7/5)*Hypergeometric2F1[-12/5, 7/5, 12/5, (
c - d*x)/(2*c)])/(7*d*(1 + (d*x)/c)^(7/5))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int (dx + c)^2 \left(-d^2x^2 + c^2\right)^{\frac{2}{5}} dx$$

input
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(2/5),x)
```

output
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(2/5),x)
```

Fricas [F]

$$\int (c + dx)^2 \left(c^2 - d^2x^2\right)^{2/5} dx = \int \left(-d^2x^2 + c^2\right)^{\frac{2}{5}} (dx + c)^2 dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(2/5),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(-d^2*x^2 + c^2)^(2/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.64

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} dx = c^{\frac{14}{5}} x\,{}_2F_1\left(\begin{matrix} -\frac{2}{5}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + \frac{c^{\frac{4}{5}} d^2 x^3\,{}_2F_1\left(\begin{matrix} -\frac{2}{5}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{3} + 2cd\left(\begin{cases} \frac{x^2(c^2)^{\frac{2}{5}}}{2} & \text{for } d^2=0 \\ -\frac{5(c^2-d^2x^2)^{\frac{7}{5}}}{14d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**2*(-d**2*x**2+c**2)**(2/5),x)
```

output
```
c**(14/5)*x*hyper((-2/5, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
 c**(4/5)*d**2*x**3*hyper((-2/5, 3/2), (5/2,), d**2*x**2*exp_polar(2*I*pi)
/c**2)/3 + 2*c*d*Piecewise((x**2*(c**2)**(2/5)/2, Eq(d**2, 0)), (-5*(c**2
- d**2*x**2)**(7/5)/(14*d**2), True))
```

Maxima [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{5}} (dx+c)^2\, dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(2/5),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)*(d*x + c)^2, x)
```

Giac [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{5}} (dx+c)^2 \, dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^(2/5),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)*(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} dx = \int \left(c^2-d^2\,x^2\right)^{2/5} (c+d\,x)^2 \, dx$$

input
```
int((c^2 - d^2*x^2)^(2/5)*(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^(2/5)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^{2/5} dx = \frac{224(-d^2x^2+c^2)^{\frac{3}{5}} \left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{3}{5}}} dx\right) c^4 d - 285c^5 + 175c^4dx + 570c^3d^2x^2 - 70c^2d^3x^3 - 28}{399\left(-d^2x^2+c^2\right)^{\frac{3}{5}} d}$$

input
```
int((d*x+c)^2*(-d^2*x^2+c^2)^(2/5),x)
```

output
```
(224*(c**2 - d**2*x**2)**(3/5)*int((c**2 - d**2*x**2)**(2/5)/(c**2 - d**2*
x**2),x)*c**4*d - 285*c**5 + 175*c**4*d*x + 570*c**3*d**2*x**2 - 70*c**2*d
**3*x**3 - 285*c*d**4*x**4 - 105*d**5*x**5)/(399*(c**2 - d**2*x**2)**(3/5)
*d)
```

3.321 $\int (c+dx) \left(c^2-d^2x^2\right)^{2/5} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/5} \, dx = \frac{5(c^2-d^2x^2)^{12/5} \operatorname{Hypergeometric2F1}\left(-\frac{2}{5},\frac{12}{5},\frac{17}{5},\frac{c+dx}{2c}\right)}{6\ 2^{3/5} c^2 d \left(\frac{c-dx}{c}\right)^{12/5}}$$

output
```
5/12*(-d^2*x^2+c^2)^(12/5)*hypergeom([-2/5, 12/5],[17/5],1/2*(d*x+c)/c)*2^
(2/5)/c^2/d/((-d*x+c)/c)^(12/5)
```

Mathematica [A] (verified)

Time = 7.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/5} \, dx = -\frac{5(c^2-d^2x^2)^{7/5}}{14d} + \frac{cx(c^2-d^2x^2)^{2/5} \operatorname{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}$$

input
```
Integrate[(c + d*x)*(c^2 - d^2*x^2)^(2/5),x]
```

output
```
(-5*(c^2 - d^2*x^2)^(7/5))/(14*d) + (c*x*(c^2 - d^2*x^2)^(2/5)*Hypergeomet
ric2F1[-2/5, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(2/5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/5}dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(c^2-d^2x^2\right)^{2/5}dx-\frac{5\left(c^2-d^2x^2\right)^{7/5}}{14d}$$

$$\Big\downarrow \text{238}$$

$$\frac{c\left(c^2-d^2x^2\right)^{2/5}\int \left(1-\frac{d^2x^2}{c^2}\right)^{2/5}dx}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}-\frac{5\left(c^2-d^2x^2\right)^{7/5}}{14d}$$

$$\Big\downarrow \text{237}$$

$$\frac{cx\left(c^2-d^2x^2\right)^{2/5}\text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}-\frac{5\left(c^2-d^2x^2\right)^{7/5}}{14d}$$

input
```
Int[(c + d*x)*(c^2 - d^2*x^2)^(2/5),x]
```

output

```
(-5*(c^2 - d^2*x^2)^(7/5))/(14*d) + (c*x*(c^2 - d^2*x^2)^(2/5)*Hypergeomet
ric2F1[-2/5, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(2/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int (dx+c)\left(-d^2x^2+c^2\right)^{\frac{2}{5}}dx$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(2/5),x)
```

output
```
int((d*x+c)*(-d^2*x^2+c^2)^(2/5),x)
```

Fricas [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/5}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(2/5),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/5)*(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/5}\,dx = c^{\frac{9}{5}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{5},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + d\left(\begin{cases}\frac{x^2(c^2)^{\frac{2}{5}}}{2} & \text{for } d^2=0\\ -\frac{5(c^2-d^2x^2)^{\frac{7}{5}}}{14d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(-d**2*x**2+c**2)**(2/5),x)
```

output
```
c**(9/5)*x*hyper((-2/5, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
d*Piecewise((x**2*(c**2)**(2/5)/2, Eq(d**2, 0)), (-5*(c**2 - d**2*x**2)**(
7/5)/(14*d**2), True))
```

Maxima [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{2/5}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(2/5),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/5} \, dx = \int \left(-d^2x^2+c^2\right)^{\frac{2}{5}} (dx+c) \, dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(2/5),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)*(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/5} \, dx = \frac{c\,x\,\left(c^2-d^2\,x^2\right)^{2/5}\,{}_2\mathrm{F}_1\left(-\frac{2}{5},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{2/5}} - \frac{5\,\left(c^2-d^2\,x^2\right)^{7/5}}{14\,d}$$

input
```
int((c^2 - d^2*x^2)^(2/5)*(c + d*x),x)
```

output
```
(c*x*(c^2 - d^2*x^2)^(2/5)*hypergeom([-2/5, 1/2], 3/2, (d^2*x^2)/c^2))/(1
- (d^2*x^2)/c^2)^(2/5) - (5*(c^2 - d^2*x^2)^(7/5))/(14*d)
```

Reduce [F]

$$\int (c+dx) \left(c^2-d^2x^2\right)^{2/5} \, dx = \frac{56\left(-d^2x^2+c^2\right)^{\frac{3}{5}} \left(\int \frac{1}{\left(-d^2x^2+c^2\right)^{\frac{3}{5}}}dx\right) c^3d - 45c^4 + 70c^3dx + 90c^2d^2x^2 - 70c\,d^3x^3 - 45d^4x^4}{126\left(-d^2x^2+c^2\right)^{\frac{3}{5}} d}$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(2/5),x)
```

output
```
(56*(c**2 - d**2*x**2)**(3/5)*int((c**2 - d**2*x**2)**(2/5)/(c**2 - d**2*x
**2),x)*c**3*d - 45*c**4 + 70*c**3*d*x + 90*c**2*d**2*x**2 - 70*c*d**3*x**
3 - 45*d**4*x**4)/(126*(c**2 - d**2*x**2)**(3/5)*d)
```

3.322 $\int \left(c^2-d^2x^2\right)^{2/5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=\frac{x\left(c^2-d^2x^2\right)^{2/5}\text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}$$

output
```
x*(-d^2*x^2+c^2)^(2/5)*hypergeom([-2/5, 1/2],[3/2],d^2*x^2/c^2)/(1-d^2*x^2
/c^2)^(2/5)
```

Mathematica [A] (verified)

Time = 7.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=\frac{x\left(c^2-d^2x^2\right)^{2/5}\text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/5),x]
```

output
```
(x*(c^2 - d^2*x^2)^(2/5)*Hypergeometric2F1[-2/5, 1/2, 3/2, (d^2*x^2)/c^2])
/(1 - (d^2*x^2)/c^2)^(2/5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2-d^2x^2\right)^{2/5} dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(c^2-d^2x^2\right)^{2/5}\int \left(1-\frac{d^2x^2}{c^2}\right)^{2/5} dx}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(c^2-d^2x^2\right)^{2/5} \text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{2/5}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/5),x]
```

output
```
(x*(c^2 - d^2*x^2)^(2/5)*Hypergeometric2F1[-2/5, 1/2, 3/2, (d^2*x^2)/c^2])
/(1 - (d^2*x^2)/c^2)^(2/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}dx$$

input
```
int((-d^2*x^2+c^2)^(2/5),x)
```

output
```
int((-d^2*x^2+c^2)^(2/5),x)
```

Fricas [F]

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.56

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=c^{\frac{4}{5}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{5},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{d^2x^2e^{2i\pi}}{c^2}\right)$$

input
```
integrate((-d**2*x**2+c**2)**(2/5),x)
```

output
```
c**(4/5)*x*hyper((-2/5, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)
```

Maxima [F]

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5), x)
```

Giac [F]

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5), x)
```

Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=\frac{x\,\left(c^2-d^2\,x^2\right)^{2/5}\,{}_2\mathrm{F}_1\left(-\frac{2}{5},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{2/5}}$$

input
```
int((c^2 - d^2*x^2)^(2/5),x)
```

output
```
(x*(c^2 - d^2*x^2)^(2/5)*hypergeom([-2/5, 1/2], 3/2, (d^2*x^2)/c^2))/(1 -
(d^2*x^2)/c^2)^(2/5)
```

Reduce [F]

$$\int \left(c^2-d^2x^2\right)^{2/5}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{2}{5}}dx$$

input
```
int((-d^2*x^2+c^2)^(2/5),x)
```

output
```
int((c**2 - d**2*x**2)**(2/5),x)
```

3.323 $\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx=\frac{5(c^2-d^2x^2)^{2/5}\,\text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{2}{5},\frac{7}{5},\frac{c+dx}{2c}\right)}{2^{3/5}d\left(\frac{c-dx}{c}\right)^{2/5}}$$

output
```
5/2*(-d^2*x^2+c^2)^(2/5)*hypergeom([-2/5, 2/5],[7/5],1/2*(d*x+c)/c)*2^(2/5
)/d/((-d*x+c)/c)^(2/5)
```

Mathematica [A] (verified)

Time = 7.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx = -\frac{5(c-dx)\left(1+\frac{dx}{c}\right)^{3/5}(c^2-d^2x^2)^{2/5}\,\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{7}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{7\ 2^{3/5}d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/5)/(c + d*x),x]
```

output
```
(-5*(c - d*x)*(1 + (d*x)/c)^(3/5)*(c^2 - d^2*x^2)^(2/5)*Hypergeometric2F1[
3/5, 7/5, 12/5, (c - d*x)/(2*c)])/(7*2^(3/5)*d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx$$

$$\Big\downarrow \text{473}$$

$$\frac{(c^2-d^2x^2)^{7/5}\int \frac{(c-dx)^{2/5}}{\left(\frac{dx}{c}+1\right)^{3/5}}dx}{c^2(c-dx)^{7/5}\left(\frac{dx}{c}+1\right)^{7/5}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{5(c^2-d^2x^2)^{7/5}\,\text{Hypergeometric2F1}\left(\frac{3}{5},\frac{7}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{7\;2^{3/5}c^2d\left(\frac{dx}{c}+1\right)^{7/5}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/5)/(c + d*x),x]
```

output
```
(-5*(c^2 - d^2*x^2)^(7/5)*Hypergeometric2F1[3/5, 7/5, 12/5, (c - d*x)/(2*c
)])/(7*2^(3/5)*c^2*d*(1 + (d*x)/c)^(7/5))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/5)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx=\int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{5}}}{c+dx}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/5)/(d*x+c),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/5)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)/(d*x + c), x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx=\int \frac{{\left(c^2-d^2\,x^2\right)}^{2/5}}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/5)/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^(2/5)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{c+dx}\,dx=\frac{4(-d^2x^2+c^2)^{\frac{3}{5}}\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{3}{5}}}dx\right)cd+5c^2-5d^2x^2}{4\left(-d^2x^2+c^2\right)^{\frac{3}{5}}d}$$

input
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c),x)
```

output
```
(4*(c**2 - d**2*x**2)**(3/5)*int((c**2 - d**2*x**2)**(2/5)/(c**2 - d**2*x*
*2),x)*c*d + 5*c**2 - 5*d**2*x**2)/(4*(c**2 - d**2*x**2)**(3/5)*d)
```

3.324 $\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx = -\frac{5\,2^{2/5}c\left(\frac{c-dx}{c}\right)^{3/5}\text{Hypergeometric2F1}\left(-\frac{3}{5},-\frac{2}{5},\frac{2}{5},\frac{c+dx}{2c}\right)}{3d\,(c^2-d^2x^2)^{3/5}}$$

output
```
-5/3*2^(2/5)*c*((-d*x+c)/c)^(3/5)*hypergeom([-3/5, -2/5],[2/5],1/2*(d*x+c)
/c)/d/(-d^2*x^2+c^2)^(3/5)
```

Mathematica [A] (verified)

Time = 8.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx = -\frac{5(c-dx)\left(1+\frac{dx}{c}\right)^{3/5}(c^2-d^2x^2)^{2/5}\,\text{Hypergeometric2F1}\left(\frac{7}{5},\frac{8}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{14\,2^{3/5}cd(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/5)/(c + d*x)^2,x]
```

output
```
(-5*(c - d*x)*(1 + (d*x)/c)^(3/5)*(c^2 - d^2*x^2)^(2/5)*Hypergeometric2F1[
7/5, 8/5, 12/5, (c - d*x)/(2*c)])/(14*2^(3/5)*c*d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 473$$

$$\frac{(c^2-d^2x^2)^{7/5}\int \frac{(c-dx)^{2/5}}{\left(\frac{dx}{c}+1\right)^{8/5}}dx}{c^3(c-dx)^{7/5}\left(\frac{dx}{c}+1\right)^{7/5}}$$

$$\Big\downarrow \; 79$$

$$-\frac{5(c^2-d^2x^2)^{7/5}\,\text{Hypergeometric2F1}\left(\frac{7}{5},\frac{8}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{14\;2^{3/5}c^3d\left(\frac{dx}{c}+1\right)^{7/5}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/5)/(c + d*x)^2,x]
```

output
```
(-5*(c^2 - d^2*x^2)^(7/5)*Hypergeometric2F1[7/5, 8/5, 12/5, (c - d*x)/(2*c
)])/(14*2^(3/5)*c^3*d*(1 + (d*x)/c)^(7/5))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{(dx+c)^2}dx$$

input
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c)^2,x)
```

output
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{(dx+c)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/5)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx=\int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{5}}}{(c+dx)^2}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/5)/(d*x+c)**2,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/5)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{(dx+c)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^2}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{(dx+c)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2 - d^2x^2)^{2/5}}{(c + dx)^2} \, dx = \int \frac{(c^2 - d^2 x^2)^{2/5}}{(c + d\,x)^2} \, dx$$

input
```
int((c^2 - d^2*x^2)^(2/5)/(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^(2/5)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(c^2 - d^2x^2)^{2/5}}{(c + dx)^2} \, dx = \frac{-4(-d^2x^2 + c^2)^{\frac{3}{5}} \left(\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}} x}{-d^3x^3 - c\,d^2x^2 + c^2dx + c^3} dx\right) d^2 - 5c + 5dx}{5\,(-d^2x^2 + c^2)^{\frac{3}{5}}\, d}$$

input
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c)^2,x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(3/5)*int(((c**2 - d**2*x**2)**(2/5)*x)/(c**3 +
c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*d**2 - 5*c + 5*d*x)/(5*(c**2 - d**2
*x**2)**(3/5)*d)
```

3.325 $\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx = -\frac{5c^2\left(\frac{c-dx}{c}\right)^{8/5}\text{Hypergeometric2F1}\left(-\frac{8}{5},-\frac{2}{5},-\frac{3}{5},\frac{c+dx}{2c}\right)}{4\ 2^{3/5}d\left(c^2-d^2x^2\right)^{8/5}}$$

output
```
-5/8*c^2*((-d*x+c)/c)^(8/5)*hypergeom([-8/5, -2/5],[-3/5],1/2*(d*x+c)/c)*2
^(2/5)/d/(-d^2*x^2+c^2)^(8/5)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.18

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx = -\frac{5(c-dx)\left(1+\frac{dx}{c}\right)^{3/5}\left(c^2-d^2x^2\right)^{2/5}\text{Hypergeometric2F1}\left(\frac{7}{5},\frac{13}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{28\ 2^{3/5}c^2d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(2/5)/(c + d*x)^3,x]
```

output
```
(-5*(c - d*x)*(1 + (d*x)/c)^(3/5)*(c^2 - d^2*x^2)^(2/5)*Hypergeometric2F1[
7/5, 13/5, 12/5, (c - d*x)/(2*c)])/(28*2^(3/5)*c^2*d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c^2-d^2x^2\right)^{2/5}}{(c+dx)^3}dx$$

$$\Big\downarrow \text{473}$$

$$\frac{\left(c^2-d^2x^2\right)^{7/5}\int \frac{(c-dx)^{2/5}}{\left(\frac{dx}{c}+1\right)^{13/5}}dx}{c^4(c-dx)^{7/5}\left(\frac{dx}{c}+1\right)^{7/5}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{5\left(c^2-d^2x^2\right)^{7/5}\text{Hypergeometric2F1}\left(\frac{7}{5},\frac{13}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{28\ 2^{3/5}c^4d\left(\frac{dx}{c}+1\right)^{7/5}}$$

input
```
Int[(c^2 - d^2*x^2)^(2/5)/(c + d*x)^3,x]
```

output
```
(-5*(c^2 - d^2*x^2)^(7/5)*Hypergeometric2F1[7/5, 13/5, 12/5, (c - d*x)/(2*
c)])/(28*2^(3/5)*c^4*d*(1 + (d*x)/c)^(7/5))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{5}}}{\left(d x +c \right)^{3}}d x$$

input
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c)^3,x)
```

output
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx=\int \frac{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{2}{5}}}{\left(d x +c\right)^{3}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/5)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3),
 x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx = \int \frac{(-(-c+dx)\,(c+dx))^{\frac{2}{5}}}{(c+dx)^3}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(2/5)/(d*x+c)**3,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(2/5)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(2/5)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(2/5)/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx=\int \frac{{\left(c^2-d^2\,x^2\right)}^{2/5}}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((c^2 - d^2*x^2)^(2/5)/(c + d*x)^3,x)
```

output
```
int((c^2 - d^2*x^2)^(2/5)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^{2/5}}{(c+dx)^3}\,dx=\frac{-4(-d^2x^2+c^2)^{\frac{3}{5}}\left(\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c\,d^2-4(-d^2x^2+c^2)^{\frac{3}{5}}\left(\int \frac{(-d^2x^2}{-d^4x^4-2c\,d^3}\right.}{10\,(-d^2x^2+c^2)^{\frac{3}{5}}\,d\,(dx+c)}$$

input
```
int((-d^2*x^2+c^2)^(2/5)/(d*x+c)^3,x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(3/5)*int(((c**2 - d**2*x**2)**(2/5)*x)/(c**4 +
2*c**3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c*d**2 - 4*(c**2 - d**2*x**2)**
(3/5)*int(((c**2 - d**2*x**2)**(2/5)*x)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3
 - d**4*x**4),x)*d**3*x - 5*c + 5*d*x)/(10*(c**2 - d**2*x**2)**(3/5)*d*(c
+ d*x))
```

3.326 $\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 67

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx = -\frac{5(c^2-d^2x^2)^{7/5}\,\text{Hypergeometric2F1}\left(\frac{7}{5},\frac{13}{5},\frac{12}{5},\frac{c-dx}{2c}\right)}{28\;2^{3/5}c^4d\left(\frac{c+dx}{c}\right)^{7/5}}$$

output
```
-5/56*(-d^2*x^2+c^2)^(7/5)*hypergeom([7/5, 13/5],[12/5],1/2*(-d*x+c)/c)*2^
(2/5)/c^4/d/((d*x+c)/c)^(7/5)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 157 vs. 2(67) = 134.

Time = 10.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.34

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx = \frac{-5(c^5+2c^3d^2x^2)+12c^2dx(c^2-d^2x^2)\left(1-\frac{d^2x^2}{c^2}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{5},\frac{3}{2}\right.}{12c^3d\,(c^2-}$$

input
```
Integrate[(c - d*x)^3/(c^2 - d^2*x^2)^(13/5),x]
```

output
```
(-5*(c^5 + 2*c^3*d^2*x^2) + 12*c^2*d*x*(c^2 - d^2*x^2)*(1 - (d^2*x^2)/c^2)
^(3/5)*Hypergeometric2F1[1/2, 13/5, 3/2, (d^2*x^2)/c^2] + 12*d^3*x^3*(c^2
- d^2*x^2)*(1 - (d^2*x^2)/c^2)^(3/5)*Hypergeometric2F1[3/2, 13/5, 5/2, (d^
2*x^2)/c^2])/(12*c^3*d*(c^2 - d^2*x^2)^(8/5))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx$$

$$\Big\downarrow \; 473$$

$$\frac{c^2(c+dx)^{8/5}\left(1-\frac{dx}{c}\right)^{8/5}\int \frac{\left(1-\frac{dx}{c}\right)^{2/5}}{(c+dx)^{13/5}}dx}{(c^2-d^2x^2)^{8/5}}$$

$$\Big\downarrow \; 79$$

$$-\frac{5c^2\left(1-\frac{dx}{c}\right)^{8/5}\text{Hypergeometric2F1}\left(-\frac{8}{5},-\frac{2}{5},-\frac{3}{5},\frac{c+dx}{2c}\right)}{4\,2^{3/5}d\,(c^2-d^2x^2)^{8/5}}$$

input
```
Int[(c - d*x)^3/(c^2 - d^2*x^2)^(13/5),x]
```

output
```
(-5*c^2*(1 - (d*x)/c)^(8/5)*Hypergeometric2F1[-8/5, -2/5, -3/5, (c + d*x)/
(2*c)])/(4*2^(3/5)*d*(c^2 - d^2*x^2)^(8/5))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{(-dx+c)^3}{(-d^2x^2+c^2)^{\frac{13}{5}}}dx$$

input
```
int((-d*x+c)^3/(-d^2*x^2+c^2)^(13/5),x)
```

output
```
int((-d*x+c)^3/(-d^2*x^2+c^2)^(13/5),x)
```

Fricas [F]

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx = \int -\frac{(dx-c)^3}{(-d^2x^2+c^2)^{\frac{13}{5}}}\,dx$$

input
```
integrate((-d*x+c)^3/(-d^2*x^2+c^2)^(13/5),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(2/5)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3),
 x)
```

Sympy [F]

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx = -\int \left(-\frac{c^3}{c^4(c^2-d^2x^2)^{\frac{3}{5}} - 2c^2d^2x^2(c^2-d^2x^2)^{\frac{3}{5}} + d^4x^4(c^2-d^2x^2)^{\frac{3}{5}}}\right)dx - \int \frac{d^3x^3}{c^4(c^2-d^2x^2)^{\frac{3}{5}} - 2c^2d^2x^2(c^2-d^2x^2)^{\frac{3}{5}} + d^4x^4(c^2-d^2x^2)^{\frac{3}{5}}}\,dx - \int \left(-\frac{3cd^2x^2}{c^4(c^2-d^2x^2)^{\frac{3}{5}} - 2c^2d^2x^2(c^2-d^2x^2)^{\frac{3}{5}} + d^4x^4(c^2-d^2x^2)^{\frac{3}{5}}}\right)dx - \int \frac{3c^2dx}{c^4(c^2-d^2x^2)^{\frac{3}{5}} - 2c^2d^2x^2(c^2-d^2x^2)^{\frac{3}{5}} + d^4x^4(c^2-d^2x^2)^{\frac{3}{5}}}\,dx$$

input
```
integrate((-d*x+c)**3/(-d**2*x**2+c**2)**(13/5),x)
```

output
```
-Integral(-c**3/(c**4*(c**2 - d**2*x**2)**(3/5) - 2*c**2*d**2*x**2*(c**2 -
 d**2*x**2)**(3/5) + d**4*x**4*(c**2 - d**2*x**2)**(3/5)), x) - Integral(d
**3*x**3/(c**4*(c**2 - d**2*x**2)**(3/5) - 2*c**2*d**2*x**2*(c**2 - d**2*x
**2)**(3/5) + d**4*x**4*(c**2 - d**2*x**2)**(3/5)), x) - Integral(-3*c*d**
2*x**2/(c**4*(c**2 - d**2*x**2)**(3/5) - 2*c**2*d**2*x**2*(c**2 - d**2*x**
2)**(3/5) + d**4*x**4*(c**2 - d**2*x**2)**(3/5)), x) - Integral(3*c**2*d*x
/(c**4*(c**2 - d**2*x**2)**(3/5) - 2*c**2*d**2*x**2*(c**2 - d**2*x**2)**(3
/5) + d**4*x**4*(c**2 - d**2*x**2)**(3/5)), x)
```

Maxima [F]

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx = \int -\frac{(dx-c)^3}{(-d^2x^2+c^2)^{\frac{13}{5}}}\,dx$$

input
```
integrate((-d*x+c)^3/(-d^2*x^2+c^2)^(13/5),x, algorithm="maxima")
```

output
```
-integrate((d*x - c)^3/(-d^2*x^2 + c^2)^(13/5), x)
```

Giac [F]

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx=\int -\frac{(dx-c)^3}{(-d^2x^2+c^2)^{\frac{13}{5}}}\,dx$$

input
```
integrate((-d*x+c)^3/(-d^2*x^2+c^2)^(13/5),x, algorithm="giac")
```

output
```
integrate(-(d*x - c)^3/(-d^2*x^2 + c^2)^(13/5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx=\int \frac{(c-d\,x)^3}{(c^2-d^2\,x^2)^{13/5}}\,dx$$

input
```
int((c - d*x)^3/(c^2 - d^2*x^2)^(13/5),x)
```

output
```
int((c - d*x)^3/(c^2 - d^2*x^2)^(13/5), x)
```

Reduce [F]

$$\int \frac{(c-dx)^3}{(c^2-d^2x^2)^{13/5}}\,dx=\frac{-4(-d^2x^2+c^2)^{\frac{3}{5}}\left(\int \frac{(-d^2x^2+c^2)^{\frac{2}{5}}}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c^2d-4(-d^2x^2+c^2)^{\frac{3}{5}}\left(\int \frac{(-d^2x}{-d^4x^4-2c\,d}\right.}{6\,(-d^2x^2+c^2)^{\frac{3}{5}}\,d\,(dx+c)}$$

input
```
int((-d*x+c)^3/(-d^2*x^2+c^2)^(13/5),x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(3/5)*int((c**2 - d**2*x**2)**(2/5)/(c**4 + 2*c*
*3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c**2*d - 4*(c**2 - d**2*x**2)**(3/5
)*int((c**2 - d**2*x**2)**(2/5)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*
x**4),x)*c*d**2*x - 5*c + 5*d*x)/(6*(c**2 - d**2*x**2)**(3/5)*d*(c + d*x))
```

3.327 $\int \frac{c-dx}{(c^2-d^2x^2)^{5/6}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\int \frac{c-dx}{(c^2-d^2x^2)^{5/6}}\,dx = \frac{3\sqrt[6]{c^2-d^2x^2}}{d} - \frac{3^{3/4}\sqrt[3]{c}\sqrt[6]{c^2-d^2x^2}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{c^2-d^2x^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\ldots}{c^{2/3}-\left(1+\sqrt{3}\right)\ldots}\right.\right.}{2d^2x\sqrt{-\frac{\sqrt[3]{c^2-d^2x^2}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3*(-d^2*x^2+c^2)^(1/6)/d-1/2*3^(3/4)*c^(1/3)*(-d^2*x^2+c^2)^(1/6)*(c^(2/3)
-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^
2)^(2/3))/(c^(2/3)-(1+3^(1/2))*(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*InverseJacob
iAM(arccos((c^(2/3)-(1-3^(1/2))*(-d^2*x^2+c^2)^(1/3))/(c^(2/3)-(1+3^(1/2))
*(-d^2*x^2+c^2)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/d^2/x/(-(-d^2*x^2+c^2)^(1
/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/(c^(2/3)-(1+3^(1/2))*(-d^2*x^2+c^2)^(1/
3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.23

$$\int \frac{c-dx}{(c^2-d^2x^2)^{5/6}}\,dx = \frac{3c^2-3d^2x^2+cdx\left(1-\frac{d^2x^2}{c^2}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{d\,(c^2-d^2x^2)^{5/6}}$$

input
```
Integrate[(c - d*x)/(c^2 - d^2*x^2)^(5/6),x]
```

output
```
(3*c^2 - 3*d^2*x^2 + c*d*x*(1 - (d^2*x^2)/c^2)^(5/6)*Hypergeometric2F1[1/2
, 5/6, 3/2, (d^2*x^2)/c^2])/(d*(c^2 - d^2*x^2)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {455, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{(c^2-d^2x^2)^{5/6}}dx$$

$$\Big\downarrow \text{455}$$

$$c\int \frac{1}{(c^2-d^2x^2)^{5/6}}dx+\frac{3\sqrt[6]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \text{236}$$

$$\frac{c\int \frac{1}{\left(\frac{d^2x^2}{c^2-d^2x^2}+1\right)^{2/3}}d\frac{x}{\sqrt{c^2-d^2x^2}}}{\sqrt[3]{\frac{c^2}{c^2-d^2x^2}}\sqrt[3]{c^2-d^2x^2}}+\frac{3\sqrt[6]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \text{234}$$

$$\frac{3c\sqrt{\frac{d^2x^2}{c^2-d^2x^2}}\sqrt[6]{c^2-d^2x^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(c^2-d^2x^2\right)^{3/2}}-1}}d\sqrt[3]{\frac{d^2x^2}{c^2-d^2x^2}+1}}{2d^2x\sqrt[3]{\frac{c^2}{c^2-d^2x^2}}}+\frac{3\sqrt[6]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 760$$

$$\frac{3\sqrt[6]{c^2-d^2x^2}}{d}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}c\sqrt{\frac{d^2x^2}{c^2-d^2x^2}}\sqrt[6]{c^2-d^2x^2}\left(1-\sqrt[3]{\frac{d^2x^2}{c^2-d^2x^2}+1}\right)\sqrt{\frac{\frac{x^2}{c^2-d^2x^2}+\sqrt[3]{\frac{d^2x^2}{c^2-d^2x^2}+1}+1}{\left(-\sqrt[3]{\frac{d^2x^2}{c^2-d^2x^2}+1}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{d^2x\sqrt[3]{\frac{c^2}{c^2-d^2x^2}}\sqrt{\frac{x^3}{(c^2-d^2x^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{\frac{d^2x^2}{c^2-d^2x^2}+1}}{\left(-\sqrt[3]{\frac{d^2x^2}{c^2-d^2x^2}+1}-\sqrt{3}+1\right)^2}}}$$

input
```
Int[(c - d*x)/(c^2 - d^2*x^2)^(5/6),x]
```

output
```
(3*(c^2 - d^2*x^2)^(1/6))/d - (3^(3/4)*Sqrt[2 - Sqrt[3]]*c*Sqrt[(d^2*x^2)/
(c^2 - d^2*x^2)]*(c^2 - d^2*x^2)^(1/6)*(1 - (1 + (d^2*x^2)/(c^2 - d^2*x^2)
)^(1/3))*Sqrt[(1 + x^2/(c^2 - d^2*x^2) + (1 + (d^2*x^2)/(c^2 - d^2*x^2))^(
1/3))/(1 - Sqrt[3] - (1 + (d^2*x^2)/(c^2 - d^2*x^2))^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (1 + (d^2*x^2)/(c^2 - d^2*x^2))^(1/3))/(1 - Sqrt[3] -
 (1 + (d^2*x^2)/(c^2 - d^2*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(d^2*x*(c^2/(c^
2 - d^2*x^2))^(1/3)*Sqrt[-1 + x^3/(c^2 - d^2*x^2)^(3/2)]*Sqrt[-((1 - (1 +
(d^2*x^2)/(c^2 - d^2*x^2))^(1/3))/(1 - Sqrt[3] - (1 + (d^2*x^2)/(c^2 - d^2
*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{-dx+c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{5}{6}}}dx$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(5/6),x)
```

output
```
int((-d*x+c)/(-d^2*x^2+c^2)^(5/6),x)
```

Fricas [F]

$$\int \frac{c-dx}{\left(c^2-d^2x^2\right)^{5/6}}\,dx=\int -\frac{dx-c}{\left(-d^2x^2+c^2\right)^{\frac{5}{6}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(5/6),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/6)/(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.20

$$\int \frac{c-dx}{\left(c^2-d^2x^2\right)^{5/6}}\,dx=-d\left(\begin{cases}\frac{x^2}{2(c^2)^{\frac{5}{6}}} & \text{for } d^2=0\\ -\frac{3\sqrt[6]{c^2-d^2x^2}}{d^2} & \text{otherwise}\end{cases}\right)+\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)}{c^{\frac{2}{3}}}$$

input
```
integrate((-d*x+c)/(-d**2*x**2+c**2)**(5/6),x)
```

output
```
-d*Piecewise((x**2/(2*(c**2)**(5/6)), Eq(d**2, 0)), (-3*(c**2 - d**2*x**2)
**(1/6)/d**2, True)) + x*hyper((1/2, 5/6), (3/2,), d**2*x**2*exp_polar(2*I
*pi)/c**2)/c**(2/3)
```

Maxima [F]

$$\int \frac{c-dx}{\left(c^2-d^2x^2\right)^{5/6}}\,dx=\int -\frac{dx-c}{\left(-d^2x^2+c^2\right)^{\frac{5}{6}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(5/6),x, algorithm="maxima")
```

output
```
-integrate((d*x - c)/(-d^2*x^2 + c^2)^(5/6), x)
```

Giac [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{5/6}}\,dx = \int -\frac{dx-c}{(-d^2x^2+c^2)^{\frac{5}{6}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(5/6),x, algorithm="giac")
```

output
```
integrate(-(d*x - c)/(-d^2*x^2 + c^2)^(5/6), x)
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.21

$$\int \frac{c-dx}{(c^2-d^2x^2)^{5/6}}\,dx = \frac{3\,(c^2-d^2\,x^2)^{1/6}}{d} + \frac{c\,x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{5/6}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{(c^2-d^2\,x^2)^{5/6}}$$

input
```
int((c - d*x)/(c^2 - d^2*x^2)^(5/6),x)
```

output
```
(3*(c^2 - d^2*x^2)^(1/6))/d + (c*x*(1 - (d^2*x^2)/c^2)^(5/6)*hypergeom([1/
2, 5/6], 3/2, (d^2*x^2)/c^2))/(c^2 - d^2*x^2)^(5/6)
```

Reduce [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{5/6}}\,dx = \frac{-(-d^2x^2+c^2)^{\frac{5}{6}} + (-d^2x^2+c^2)^{\frac{2}{3}}\left(\int \frac{(-d^2x^2+c^2)^{\frac{7}{6}}}{d^4x^4-2c^2d^2x^2+c^4}dx\right)cd}{(-d^2x^2+c^2)^{\frac{2}{3}}\,d}$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(5/6),x)
```

output
```
( - (c**2 - d**2*x**2)**(5/6) + (c**2 - d**2*x**2)**(2/3)*int((c**2 - d**2
*x**2)**(7/6)/(c**4 - 2*c**2*d**2*x**2 + d**4*x**4),x)*c*d)/((c**2 - d**2*
x**2)**(2/3)*d)
```

3.328 $\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 323

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx = \frac{3\sqrt[6]{c^2-d^2x^2}}{d} - \frac{3^{3/4}\sqrt[3]{c}\sqrt[6]{c^2-d^2x^2}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)\sqrt{\frac{c^{4/3}+c^{2/3}\sqrt[3]{c^2-d^2x^2}+(c^2-d^2x^2)^{2/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{c^2-d^2x^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-(1-}{c^{2/3}-(1+}\right.\right.}{2d^2x\sqrt{-\frac{\sqrt[3]{c^2-d^2x^2}\left(c^{2/3}-\sqrt[3]{c^2-d^2x^2}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{c^2-d^2x^2}\right)^2}}}$$

output
```
3*(-d^2*x^2+c^2)^(1/6)/d-1/2*3^(3/4)*c^(1/3)*(-d^2*x^2+c^2)^(1/6)*(c^(2/3)
-(-d^2*x^2+c^2)^(1/3))*((c^(4/3)+c^(2/3)*(-d^2*x^2+c^2)^(1/3)+(-d^2*x^2+c^
2)^(2/3))/(c^(2/3)-(1+3^(1/2))*(-d^2*x^2+c^2)^(1/3))^2)^(1/2)*InverseJacob
iAM(arccos((c^(2/3)-(1-3^(1/2))*(-d^2*x^2+c^2)^(1/3))/(c^(2/3)-(1+3^(1/2))
*(-d^2*x^2+c^2)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/d^2/x/(-(-d^2*x^2+c^2)^(1
/3)*(c^(2/3)-(-d^2*x^2+c^2)^(1/3))/(c^(2/3)-(1+3^(1/2))*(-d^2*x^2+c^2)^(1/
3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.93 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx = -\frac{3\sqrt[6]{2}(c-dx)\left(1+\frac{dx}{c}\right)^{5/6}\sqrt[6]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{13}{6},\frac{c-dx}{2c}\right)}{7d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/6)/(c + d*x),x]
```

output
```
(-3*2^(1/6)*(c - d*x)*(1 + (d*x)/c)^(5/6)*(c^2 - d^2*x^2)^(1/6)*Hypergeome
tric2F1[5/6, 7/6, 13/6, (c - d*x)/(2*c)])/(7*d*(c + d*x))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx$$

$$\Big\downarrow \; 473$$

$$\frac{\left(c^2-d^2x^2\right)^{7/6}\int \frac{\sqrt[6]{c-dx}}{\left(\frac{dx}{c}+1\right)^{5/6}}dx}{c^2(c-dx)^{7/6}\left(\frac{dx}{c}+1\right)^{7/6}}$$

$$\Big\downarrow \; 79$$

$$-\frac{3\sqrt[6]{2}\left(c^2-d^2x^2\right)^{7/6}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{13}{6},\frac{c-dx}{2c}\right)}{7c^2d\left(\frac{dx}{c}+1\right)^{7/6}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/6)/(c + d*x),x]
```

output
```
(-3*2^(1/6)*(c^2 - d^2*x^2)^(7/6)*Hypergeometric2F1[5/6, 7/6, 13/6, (c - d
*x)/(2*c)])/(7*c^2*d*(1 + (d*x)/c)^(7/6))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{6}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(1/6)/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^(1/6)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{6}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/6)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/6)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\sqrt[6]{-\left(-c+dx\right)\left(c+dx\right)}}{c+dx}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/6)/(d*x+c),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/6)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{6}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/6)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/6)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{6}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/6)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/6)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(c^2-d^2\,x^2)^{1/6}}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/6)/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^(1/6)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sqrt[6]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^{\frac{1}{6}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(1/6)/(d*x+c),x)
```

output
```
int((c**2 - d**2*x**2)**(1/6)/(c + d*x),x)
```

3.329 $\int (c+dx) \left(c^2-d^2x^2\right)^{5/8} dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int (c+dx) \left(c^2-d^2x^2\right)^{5/8} dx = \frac{8\ 2^{5/8}(c^2-d^2x^2)^{21/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{21}{8},\frac{29}{8},\frac{c+dx}{2c}\right)}{21c^2d\left(\frac{c-dx}{c}\right)^{21/8}}$$

output
```
8/21*2^(5/8)*(-d^2*x^2+c^2)^(21/8)*hypergeom([-5/8, 21/8],[29/8],1/2*(d*x+
c)/c)/c^2/d/((-d*x+c)/c)^(21/8)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (c+dx) \left(c^2-d^2x^2\right)^{5/8} dx = -\frac{4(c^2-d^2x^2)^{13/8}}{13d} + \frac{cx(c^2-d^2x^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{5/8}}$$

input
```
Integrate[(c + d*x)*(c^2 - d^2*x^2)^(5/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(13/8))/(13*d) + (c*x*(c^2 - d^2*x^2)^(5/8)*Hypergeome
tric2F1[-5/8, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(5/8)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(c^2-d^2x^2\right)^{5/8}dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(c^2-d^2x^2\right)^{5/8}dx-\frac{4\left(c^2-d^2x^2\right)^{13/8}}{13d}$$

$$\Big\downarrow \text{238}$$

$$\frac{c\left(c^2-d^2x^2\right)^{5/8}\int \left(1-\frac{d^2x^2}{c^2}\right)^{5/8}dx}{\left(1-\frac{d^2x^2}{c^2}\right)^{5/8}}-\frac{4\left(c^2-d^2x^2\right)^{13/8}}{13d}$$

$$\Big\downarrow \text{237}$$

$$\frac{cx\left(c^2-d^2x^2\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{5/8}}-\frac{4\left(c^2-d^2x^2\right)^{13/8}}{13d}$$

input
```
Int[(c + d*x)*(c^2 - d^2*x^2)^(5/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(13/8))/(13*d) + (c*x*(c^2 - d^2*x^2)^(5/8)*Hypergeome
tric2F1[-5/8, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int (dx+c)\left(-d^2x^2+c^2\right)^{\frac{5}{8}}dx$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(5/8),x)
```

output
```
int((d*x+c)*(-d^2*x^2+c^2)^(5/8),x)
```

Fricas [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{5/8}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{5}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(5/8),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(5/8)*(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (c+dx)\left(c^2-d^2x^2\right)^{5/8}\,dx = c^{\frac{9}{4}}x\,{}_2F_1\left(\begin{matrix} -\frac{5}{8},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + d\left(\begin{cases} \frac{x^2(c^2)^{\frac{5}{8}}}{2} & \text{for } d^2=0 \\ -\frac{4(c^2-d^2x^2)^{\frac{13}{8}}}{13d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)*(-d**2*x**2+c**2)**(5/8),x)
```

output
```
c**(9/4)*x*hyper((-5/8, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
d*Piecewise((x**2*(c**2)**(5/8)/2, Eq(d**2, 0)), (-4*(c**2 - d**2*x**2)**(
13/8)/(13*d**2), True))
```

Maxima [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{5/8}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{5}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(5/8),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(5/8)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{5/8}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{5}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(5/8),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(5/8)*(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int (c+dx)\left(c^2-d^2x^2\right)^{5/8}\,dx=\frac{c\,x\,\left(c^2-d^2\,x^2\right)^{5/8}\,{}_2\mathrm{F}_1\left(-\frac{5}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{5/8}}-\frac{4\,\left(c^2-d^2\,x^2\right)^{13/8}}{13\,d}$$

input
```
int((c^2 - d^2*x^2)^(5/8)*(c + d*x),x)
```

output
```
(c*x*(c^2 - d^2*x^2)^(5/8)*hypergeom([-5/8, 1/2], 3/2, (d^2*x^2)/c^2))/(1
- (d^2*x^2)/c^2)^(5/8) - (4*(c^2 - d^2*x^2)^(13/8))/(13*d)
```

Reduce [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{5/8}\,dx=\frac{-4(-d^2x^2+c^2)^{\frac{5}{8}}c^2+4(-d^2x^2+c^2)^{\frac{5}{8}}d^2x^2+13\left(\int (-d^2x^2+c^2)^{\frac{5}{8}}dx\right)cd}{13d}$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(5/8),x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(5/8)*c**2 + 4*(c**2 - d**2*x**2)**(5/8)*d**2*x*
*2 + 13*int((c**2 - d**2*x**2)**(5/8),x)*c*d)/(13*d)
```

3.330 $\int (c+dx) (c^2-d^2x^2)^{3/8} dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int (c+dx) (c^2-d^2x^2)^{3/8} dx = \frac{8\,2^{3/8}(c^2-d^2x^2)^{19/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{19}{8},\frac{27}{8},\frac{c+dx}{2c}\right)}{19c^2d\left(\frac{c-dx}{c}\right)^{19/8}}$$

output
```
8/19*2^(3/8)*(-d^2*x^2+c^2)^(19/8)*hypergeom([-3/8, 19/8],[27/8],1/2*(d*x+
c)/c)/c^2/d/((-d*x+c)/c)^(19/8)
```

Mathematica [A] (verified)

Time = 9.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (c+dx) (c^2-d^2x^2)^{3/8} dx = -\frac{4(c^2-d^2x^2)^{11/8}}{11d} + \frac{cx(c^2-d^2x^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{3/8}}$$

input
```
Integrate[(c + d*x)*(c^2 - d^2*x^2)^(3/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(11/8))/(11*d) + (c*x*(c^2 - d^2*x^2)^(3/8)*Hypergeome
tric2F1[-3/8, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(3/8)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(c^2-d^2x^2\right)^{3/8}dx$$

$$\Big\downarrow \; 455$$

$$c\int \left(c^2-d^2x^2\right)^{3/8}dx-\frac{4\left(c^2-d^2x^2\right)^{11/8}}{11d}$$

$$\Big\downarrow \; 238$$

$$\frac{c\left(c^2-d^2x^2\right)^{3/8}\int \left(1-\frac{d^2x^2}{c^2}\right)^{3/8}dx}{\left(1-\frac{d^2x^2}{c^2}\right)^{3/8}}-\frac{4\left(c^2-d^2x^2\right)^{11/8}}{11d}$$

$$\Big\downarrow \; 237$$

$$\frac{cx\left(c^2-d^2x^2\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\left(1-\frac{d^2x^2}{c^2}\right)^{3/8}}-\frac{4\left(c^2-d^2x^2\right)^{11/8}}{11d}$$

input
```
Int[(c + d*x)*(c^2 - d^2*x^2)^(3/8),x]
```

output

```
(-4*(c^2 - d^2*x^2)^(11/8))/(11*d) + (c*x*(c^2 - d^2*x^2)^(3/8)*Hypergeome
tric2F1[-3/8, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int (dx+c)\left(-d^2x^2+c^2\right)^{\frac{3}{8}}dx$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(3/8),x)
```

output
```
int((d*x+c)*(-d^2*x^2+c^2)^(3/8),x)
```

Fricas [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{3/8}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{3}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(3/8),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(3/8)*(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (c+dx)\left(c^2-d^2x^2\right)^{3/8}\,dx = c^{\frac{7}{4}}x\,{}_2F_1\left(\begin{matrix}-\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + d\left(\begin{cases}\frac{x^2(c^2)^{\frac{3}{8}}}{2} & \text{for } d^2=0\\ -\frac{4(c^2-d^2x^2)^{\frac{11}{8}}}{11d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(-d**2*x**2+c**2)**(3/8),x)
```

output
```
c**(7/4)*x*hyper((-3/8, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
d*Piecewise((x**2*(c**2)**(3/8)/2, Eq(d**2, 0)), (-4*(c**2 - d**2*x**2)**(
11/8)/(11*d**2), True))
```

Maxima [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^{3/8}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{3}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(3/8),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(3/8)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx) \left(c^2-d^2x^2\right)^{3/8} \, dx = \int \left(-d^2x^2+c^2\right)^{\frac{3}{8}}(dx+c) \, dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(3/8),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(3/8)*(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 6.61 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int (c+dx) \left(c^2-d^2x^2\right)^{3/8} \, dx = \frac{c\,x\,\left(c^2-d^2\,x^2\right)^{3/8}\,{}_2\mathrm{F}_1\left(-\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{3/8}}-\frac{4\,\left(c^2-d^2\,x^2\right)^{11/8}}{11\,d}$$

input
```
int((c^2 - d^2*x^2)^(3/8)*(c + d*x),x)
```

output
```
(c*x*(c^2 - d^2*x^2)^(3/8)*hypergeom([-3/8, 1/2], 3/2, (d^2*x^2)/c^2))/(1
- (d^2*x^2)/c^2)^(3/8) - (4*(c^2 - d^2*x^2)^(11/8))/(11*d)
```

Reduce [F]

$$\int (c+dx) \left(c^2-d^2x^2\right)^{3/8} \, dx = \frac{-4\left(-d^2x^2+c^2\right)^{\frac{3}{8}}c^2+4\left(-d^2x^2+c^2\right)^{\frac{3}{8}}d^2x^2+11\left(\int \left(-d^2x^2+c^2\right)^{\frac{3}{8}}dx\right)cd}{11d}$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(3/8),x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(3/8)*c**2 + 4*(c**2 - d**2*x**2)**(3/8)*d**2*x*
*2 + 11*int((c**2 - d**2*x**2)**(3/8),x)*c*d)/(11*d)
```

3.331 $\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx=\frac{8\sqrt[8]{2}(c^2-d^2x^2)^{17/8}\operatorname{Hypergeometric2F1}\left(-\frac{1}{8},\frac{17}{8},\frac{25}{8},\frac{c+dx}{2c}\right)}{17c^2d\left(\frac{c-dx}{c}\right)^{17/8}}$$

output
```
8/17*2^(1/8)*(-d^2*x^2+c^2)^(17/8)*hypergeom([-1/8, 17/8],[25/8],1/2*(d*x+
c)/c)/c^2/d/((-d*x+c)/c)^(17/8)
```

Mathematica [A] (verified)

Time = 8.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx=-\frac{4(c^2-d^2x^2)^{9/8}}{9d}+\frac{cx\sqrt[8]{c^2-d^2x^2}\operatorname{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[8]{1-\frac{d^2x^2}{c^2}}}$$

input
```
Integrate[(c + d*x)*(c^2 - d^2*x^2)^(1/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(9/8))/(9*d) + (c*x*(c^2 - d^2*x^2)^(1/8)*Hypergeometr
ic2F1[-1/8, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(1/8)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx$$

$$\Big\downarrow \text{455}$$

$$c\int \sqrt[8]{c^2-d^2x^2}dx-\frac{4\left(c^2-d^2x^2\right)^{9/8}}{9d}$$

$$\Big\downarrow \text{238}$$

$$\frac{c\sqrt[8]{c^2-d^2x^2}\int \sqrt[8]{1-\frac{d^2x^2}{c^2}}dx}{\sqrt[8]{1-\frac{d^2x^2}{c^2}}}-\frac{4\left(c^2-d^2x^2\right)^{9/8}}{9d}$$

$$\Big\downarrow \text{237}$$

$$\frac{cx\sqrt[8]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[8]{1-\frac{d^2x^2}{c^2}}}-\frac{4\left(c^2-d^2x^2\right)^{9/8}}{9d}$$

input
```
Int[(c + d*x)*(c^2 - d^2*x^2)^(1/8),x]
```

output

```
(-4*(c^2 - d^2*x^2)^(9/8))/(9*d) + (c*x*(c^2 - d^2*x^2)^(1/8)*Hypergeometr
ic2F1[-1/8, 1/2, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (dx+c)\left(-d^2x^2+c^2\right)^{\frac{1}{8}}dx$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(1/8),x)
```

output
```
int((d*x+c)*(-d^2*x^2+c^2)^(1/8),x)
```

Fricas [F]

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx=\int \left(-d^2x^2+c^2\right)^{\frac{1}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(1/8),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/8)*(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx = c^{\frac{5}{4}}x_2F_1\left(\begin{matrix}-\frac{1}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)+d\left(\begin{cases}\frac{x^2\sqrt[8]{c^2}}{2} & \text{for } d^2=0\\ -\frac{4(c^2-d^2x^2)^{\frac{9}{8}}}{9d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(-d**2*x**2+c**2)**(1/8),x)
```

output
```
c**(5/4)*x*hyper((-1/8, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
d*Piecewise((x**2*(c**2)**(1/8)/2, Eq(d**2, 0)), (-4*(c**2 - d**2*x**2)**(
9/8)/(9*d**2), True))
```

Maxima [F]

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(1/8),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/8)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx = \int \left(-d^2x^2+c^2\right)^{\frac{1}{8}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^(1/8),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/8)*(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx = \frac{c\,x\,\left(c^2-d^2\,x^2\right)^{1/8}\,{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^{1/8}}-\frac{4\,\left(c^2-d^2\,x^2\right)^{9/8}}{9\,d}$$

input
```
int((c^2 - d^2*x^2)^(1/8)*(c + d*x),x)
```

output
```
(c*x*(c^2 - d^2*x^2)^(1/8)*hypergeom([-1/8, 1/2], 3/2, (d^2*x^2)/c^2))/(1
- (d^2*x^2)/c^2)^(1/8) - (4*(c^2 - d^2*x^2)^(9/8))/(9*d)
```

Reduce [F]

$$\int (c+dx)\sqrt[8]{c^2-d^2x^2}\,dx = \frac{-4(-d^2x^2+c^2)^{\frac{1}{8}}\,c^2+4(-d^2x^2+c^2)^{\frac{1}{8}}\,d^2x^2+9\left(\int (-d^2x^2+c^2)^{\frac{1}{8}}\,dx\right)cd}{9d}$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^(1/8),x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(1/8)*c**2 + 4*(c**2 - d**2*x**2)**(1/8)*d**2*x*
*2 + 9*int((c**2 - d**2*x**2)**(1/8),x)*c*d)/(9*d)
```

3.332 $\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx=\frac{4\,2^{7/8}(c^2-d^2x^2)^{15/8}\operatorname{Hypergeometric2F1}\left(\frac{1}{8},\frac{15}{8},\frac{23}{8},\frac{c+dx}{2c}\right)}{15c^2d\left(\frac{c-dx}{c}\right)^{15/8}}$$

output
```
4/15*2^(7/8)*(-d^2*x^2+c^2)^(15/8)*hypergeom([1/8, 15/8],[23/8],1/2*(d*x+c
)/c)/c^2/d/((-d*x+c)/c)^(15/8)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx=-\frac{4(c^2-d^2x^2)^{7/8}}{7d}+\frac{cx\sqrt[8]{1-\frac{d^2x^2}{c^2}}\operatorname{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[8]{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)/(c^2 - d^2*x^2)^(1/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(7/8))/(7*d) + (c*x*(1 - (d^2*x^2)/c^2)^(1/8)*Hypergeo
metric2F1[1/8, 1/2, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(1/8)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{455}$$

$$c\int \frac{1}{\sqrt[8]{c^2-d^2x^2}}dx-\frac{4\left(c^2-d^2x^2\right)^{7/8}}{7d}$$

$$\Big\downarrow \text{238}$$

$$\frac{c\sqrt[8]{1-\frac{d^2x^2}{c^2}}\int \frac{1}{\sqrt[8]{1-\frac{d^2x^2}{c^2}}}dx}{\sqrt[8]{c^2-d^2x^2}}-\frac{4\left(c^2-d^2x^2\right)^{7/8}}{7d}$$

$$\Big\downarrow \text{237}$$

$$\frac{cx\sqrt[8]{1-\frac{d^2x^2}{c^2}}\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{\sqrt[8]{c^2-d^2x^2}}-\frac{4\left(c^2-d^2x^2\right)^{7/8}}{7d}$$

input
```
Int[(c + d*x)/(c^2 - d^2*x^2)^(1/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(7/8))/(7*d) + (c*x*(1 - (d^2*x^2)/c^2)^(1/8)*Hypergeo
metric2F1[1/8, 1/2, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \frac{dx+c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{1}{8}}}dx$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(1/8),x)
```

output
```
int((d*x+c)/(-d^2*x^2+c^2)^(1/8),x)
```

Fricas [F]

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx=\int \frac{dx+c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{1}{8}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(1/8),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(7/8)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx = c^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix} \frac{1}{8},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + d\left(\begin{cases} \frac{x^2}{2\sqrt[8]{c^2}} & \text{for } d^2=0 \\ -\frac{4(c^2-d^2x^2)^{\frac{7}{8}}}{7d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)/(-d**2*x**2+c**2)**(1/8),x)
```

output
```
c**(3/4)*x*hyper((1/8, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + d
*Piecewise((x**2/(2*(c**2)**(1/8)), Eq(d**2, 0)), (-4*(c**2 - d**2*x**2)**
(7/8)/(7*d**2), True))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx = \int \frac{dx+c}{(-d^2x^2+c^2)^{\frac{1}{8}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(1/8),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(1/8), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx = \int \frac{dx+c}{(-d^2x^2+c^2)^{\frac{1}{8}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(1/8),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(1/8), x)
```

Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx = \frac{c\,x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{1/8}\,{}_2\mathrm{F}_1\left(\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(c^2-d^2\,x^2\right)^{1/8}}-\frac{4\,\left(c^2-d^2\,x^2\right)^{7/8}}{7\,d}$$

input
```
int((c + d*x)/(c^2 - d^2*x^2)^(1/8),x)
```

output
```
(c*x*(1 - (d^2*x^2)/c^2)^(1/8)*hypergeom([1/8, 1/2], 3/2, (d^2*x^2)/c^2))/
(c^2 - d^2*x^2)^(1/8) - (4*(c^2 - d^2*x^2)^(7/8))/(7*d)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt[8]{c^2-d^2x^2}}\,dx = \frac{4(-d^2x^2+c^2)^{\frac{5}{8}}\,c^2-4(-d^2x^2+c^2)^{\frac{5}{8}}\,d^2x^2+5(-d^2x^2+c^2)^{\frac{3}{4}}\left(\int \frac{\sqrt{-d^2x^2+c^2}}{(-d^2x^2+c^2)^{\frac{5}{8}}}dx\right)cd}{5\left(-d^2x^2+c^2\right)^{\frac{3}{4}}d}$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(1/8),x)
```

output
```
(4*(c**2 - d**2*x**2)**(5/8)*c**2 - 4*(c**2 - d**2*x**2)**(5/8)*d**2*x**2
+ 5*(c**2 - d**2*x**2)**(3/4)*int(sqrt(c**2 - d**2*x**2)/(c**2 - d**2*x**2
)**(5/8),x)*c*d)/(5*(c**2 - d**2*x**2)**(3/4)*d)
```

3.333 $\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx=\frac{4\;2^{5/8}(c^2-d^2x^2)^{13/8}\,\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{13}{8},\frac{21}{8},\frac{c+dx}{2c}\right)}{13c^2d\left(\frac{c-dx}{c}\right)^{13/8}}$$

output
```
4/13*2^(5/8)*(-d^2*x^2+c^2)^(13/8)*hypergeom([3/8, 13/8],[21/8],1/2*(d*x+c
)/c)/c^2/d/((-d*x+c)/c)^(13/8)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx=-\frac{4(c^2-d^2x^2)^{5/8}}{5d}+\frac{cx\left(1-\frac{d^2x^2}{c^2}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{(c^2-d^2x^2)^{3/8}}$$

input
```
Integrate[(c + d*x)/(c^2 - d^2*x^2)^(3/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(5/8))/(5*d) + (c*x*(1 - (d^2*x^2)/c^2)^(3/8)*Hypergeo
metric2F1[3/8, 1/2, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(3/8)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx$$

$$\Big\downarrow \; 455$$

$$c\int \frac{1}{(c^2-d^2x^2)^{3/8}}dx-\frac{4\left(c^2-d^2x^2\right)^{5/8}}{5d}$$

$$\Big\downarrow \; 238$$

$$\frac{c\left(1-\frac{d^2x^2}{c^2}\right)^{3/8}\int \frac{1}{\left(1-\frac{d^2x^2}{c^2}\right)^{3/8}}dx}{(c^2-d^2x^2)^{3/8}}-\frac{4\left(c^2-d^2x^2\right)^{5/8}}{5d}$$

$$\Big\downarrow \; 237$$

$$\frac{cx\left(1-\frac{d^2x^2}{c^2}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{(c^2-d^2x^2)^{3/8}}-\frac{4\left(c^2-d^2x^2\right)^{5/8}}{5d}$$

input
```
Int[(c + d*x)/(c^2 - d^2*x^2)^(3/8),x]
```

output
```
(-4*(c^2 - d^2*x^2)^(5/8))/(5*d) + (c*x*(1 - (d^2*x^2)/c^2)^(3/8)*Hypergeo
metric2F1[3/8, 1/2, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int \frac{dx + c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{3}{8}}}dx$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(3/8),x)
```

output
```
int((d*x+c)/(-d^2*x^2+c^2)^(3/8),x)
```

Fricas [F]

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx = \int \frac{dx + c}{\left(-d^{2} x^{2}+c^{2}\right)^{\frac{3}{8}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(3/8),x, algorithm="fricas")
```

output
```
integral(-(-d^2*x^2 + c^2)^(5/8)/(d*x - c), x)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx = \sqrt[4]{c}x\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)+d\left(\begin{cases}\frac{x^2}{2(c^2)^{\frac{3}{8}}} & \text{for } d^2=0\\ -\frac{4(c^2-d^2x^2)^{\frac{5}{8}}}{5d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)/(-d**2*x**2+c**2)**(3/8),x)
```

output
```
c**(1/4)*x*hyper((3/8, 1/2), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + d
*Piecewise((x**2/(2*(c**2)**(3/8)), Eq(d**2, 0)), (-4*(c**2 - d**2*x**2)**
(5/8)/(5*d**2), True))
```

Maxima [F]

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx = \int \frac{dx+c}{(-d^2x^2+c^2)^{\frac{3}{8}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(3/8),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(3/8), x)
```

Giac [F]

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx = \int \frac{dx+c}{(-d^2x^2+c^2)^{\frac{3}{8}}}\,dx$$

input
```
integrate((d*x+c)/(-d^2*x^2+c^2)^(3/8),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(-d^2*x^2 + c^2)^(3/8), x)
```

Mupad [B] (verification not implemented)

Time = 6.70 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx = \frac{c\,x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{3/8}\,{}_2\mathrm{F}_1\left(\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{(c^2-d^2\,x^2)^{3/8}} - \frac{4\,(c^2-d^2\,x^2)^{5/8}}{5\,d}$$

input
```
int((c + d*x)/(c^2 - d^2*x^2)^(3/8),x)
```

output
```
(c*x*(1 - (d^2*x^2)/c^2)^(3/8)*hypergeom([3/8, 1/2], 3/2, (d^2*x^2)/c^2))/
(c^2 - d^2*x^2)^(3/8) - (4*(c^2 - d^2*x^2)^(5/8))/(5*d)
```

Reduce [F]

$$\int \frac{c+dx}{(c^2-d^2x^2)^{3/8}}\,dx = \frac{4(-d^2x^2+c^2)^{\frac{3}{8}}\,c^2 - 4(-d^2x^2+c^2)^{\frac{3}{8}}\,d^2x^2 + 5(-d^2x^2+c^2)^{\frac{3}{4}}\left(\int \frac{1}{(-d^2x^2+c^2)^{\frac{3}{8}}}dx\right)cd}{5\,(-d^2x^2+c^2)^{\frac{3}{4}}\,d}$$

input
```
int((d*x+c)/(-d^2*x^2+c^2)^(3/8),x)
```

output
```
(4*(c**2 - d**2*x**2)**(3/8)*c**2 - 4*(c**2 - d**2*x**2)**(3/8)*d**2*x**2
+ 5*(c**2 - d**2*x**2)**(3/4)*int((c**2 - d**2*x**2)**(1/4)/(c**2 - d**2*x
**2)**(5/8),x)*c*d)/(5*(c**2 - d**2*x**2)**(3/4)*d)
```

3.334 $\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx = -\frac{4\sqrt[8]{2}(c^2-d^2x^2)^{9/8}\,\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{9}{8},\frac{17}{8},\frac{c-dx}{2c}\right)}{9c^2d\left(\frac{c+dx}{c}\right)^{9/8}}$$

output
```
-4/9*2^(1/8)*(-d^2*x^2+c^2)^(9/8)*hypergeom([7/8, 9/8],[17/8],1/2*(-d*x+c)
/c)/c^2/d/((d*x+c)/c)^(9/8)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx = \frac{4c^2-4d^2x^2+cdx\left(1-\frac{d^2x^2}{c^2}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{d\left(c^2-d^2x^2\right)^{7/8}}$$

input
```
Integrate[(c - d*x)/(c^2 - d^2*x^2)^(7/8),x]
```

output
```
(4*c^2 - 4*d^2*x^2 + c*d*x*(1 - (d^2*x^2)/c^2)^(7/8)*Hypergeometric2F1[1/2
, 7/8, 3/2, (d^2*x^2)/c^2])/(d*(c^2 - d^2*x^2)^(7/8))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx$$

$$\Big\downarrow \; 455$$

$$c\int \frac{1}{(c^2-d^2x^2)^{7/8}}dx+\frac{4\sqrt[8]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 238$$

$$\frac{c\left(1-\frac{d^2x^2}{c^2}\right)^{7/8}\int \frac{1}{\left(1-\frac{d^2x^2}{c^2}\right)^{7/8}}dx}{(c^2-d^2x^2)^{7/8}}+\frac{4\sqrt[8]{c^2-d^2x^2}}{d}$$

$$\Big\downarrow \; 237$$

$$\frac{cx\left(1-\frac{d^2x^2}{c^2}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},\frac{d^2x^2}{c^2}\right)}{(c^2-d^2x^2)^{7/8}}+\frac{4\sqrt[8]{c^2-d^2x^2}}{d}$$

input
```
Int[(c - d*x)/(c^2 - d^2*x^2)^(7/8),x]
```

output
```
(4*(c^2 - d^2*x^2)^(1/8))/d + (c*x*(1 - (d^2*x^2)/c^2)^(7/8)*Hypergeometri
c2F1[1/2, 7/8, 3/2, (d^2*x^2)/c^2])/(c^2 - d^2*x^2)^(7/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int \frac{-dx + c}{(-d^2x^2 + c^2)^{\frac{7}{8}}} dx$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(7/8),x)
```

output
```
int((-d*x+c)/(-d^2*x^2+c^2)^(7/8),x)
```

Fricas [F]

$$\int \frac{c - dx}{(c^2 - d^2x^2)^{7/8}} dx = \int -\frac{dx - c}{(-d^2x^2 + c^2)^{\frac{7}{8}}} dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(7/8),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/8)/(d*x + c), x)
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx = -d\left(\begin{cases} \frac{x^2}{2(c^2)^{\frac{7}{8}}} & \text{for } d^2=0 \\ -\frac{4\sqrt[8]{c^2-d^2x^2}}{d^2} & \text{otherwise} \end{cases}\right) + \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2},\frac{7}{8} \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{c^{\frac{3}{4}}}$$

input
```
integrate((-d*x+c)/(-d**2*x**2+c**2)**(7/8),x)
```

output
```
-d*Piecewise((x**2/(2*(c**2)**(7/8)), Eq(d**2, 0)), (-4*(c**2 - d**2*x**2)
**(1/8)/d**2, True)) + x*hyper((1/2, 7/8), (3/2,), d**2*x**2*exp_polar(2*I
*pi)/c**2)/c**(3/4)
```

Maxima [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx = \int -\frac{dx-c}{(-d^2x^2+c^2)^{\frac{7}{8}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(7/8),x, algorithm="maxima")
```

output
```
-integrate((d*x - c)/(-d^2*x^2 + c^2)^(7/8), x)
```

Giac [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx = \int -\frac{dx-c}{(-d^2x^2+c^2)^{\frac{7}{8}}}\,dx$$

input
```
integrate((-d*x+c)/(-d^2*x^2+c^2)^(7/8),x, algorithm="giac")
```

output
```
integrate(-(d*x - c)/(-d^2*x^2 + c^2)^(7/8), x)
```

Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx = \frac{4\,(c^2-d^2\,x^2)^{1/8}}{d} + \frac{c\,x\,\left(1-\frac{d^2\,x^2}{c^2}\right)^{7/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{8};\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{(c^2-d^2\,x^2)^{7/8}}$$

input
```
int((c - d*x)/(c^2 - d^2*x^2)^(7/8),x)
```

output
```
(4*(c^2 - d^2*x^2)^(1/8))/d + (c*x*(1 - (d^2*x^2)/c^2)^(7/8)*hypergeom([1/
2, 7/8], 3/2, (d^2*x^2)/c^2))/(c^2 - d^2*x^2)^(7/8)
```

Reduce [F]

$$\int \frac{c-dx}{(c^2-d^2x^2)^{7/8}}\,dx = \frac{-4(-d^2x^2+c^2)^{\frac{7}{8}}+5(-d^2x^2+c^2)^{\frac{3}{4}}\left(\int \frac{(-d^2x^2+c^2)^{\frac{3}{4}}}{(-d^2x^2+c^2)^{\frac{5}{8}}c^2-(-d^2x^2+c^2)^{\frac{5}{8}}d^2x^2}dx\right)cd}{5\,(-d^2x^2+c^2)^{\frac{3}{4}}\,d}$$

input
```
int((-d*x+c)/(-d^2*x^2+c^2)^(7/8),x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(7/8) + 5*(c**2 - d**2*x**2)**(3/4)*int((c**2 -
d**2*x**2)**(3/4)/((c**2 - d**2*x**2)**(5/8)*c**2 - (c**2 - d**2*x**2)**(5
/8)*d**2*x**2),x)*c*d)/(5*(c**2 - d**2*x**2)**(3/4)*d)
```

3.335 $\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx=\frac{8\sqrt[8]{2}\sqrt[8]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{8},\frac{9}{8},\frac{c+dx}{2c}\right)}{d\sqrt[8]{\frac{c-dx}{c}}}$$

output
```
8*2^(1/8)*(-d^2*x^2+c^2)^(1/8)*hypergeom([-1/8, 1/8],[9/8],1/2*(d*x+c)/c)/
d/((-d*x+c)/c)^(1/8)
```

Mathematica [A] (verified)

Time = 8.94 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx = -\frac{4\sqrt[8]{2}(c-dx)\left(1+\frac{dx}{c}\right)^{7/8}\sqrt[8]{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{9}{8},\frac{17}{8},\frac{c-dx}{2c}\right)}{9d(c+dx)}$$

input
```
Integrate[(c^2 - d^2*x^2)^(1/8)/(c + d*x),x]
```

output
```
(-4*2^(1/8)*(c - d*x)*(1 + (d*x)/c)^(7/8)*(c^2 - d^2*x^2)^(1/8)*Hypergeome
tric2F1[7/8, 9/8, 17/8, (c - d*x)/(2*c)])/(9*d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx$$
$$\Big\downarrow \; 473$$
$$\frac{\left(c^2-d^2x^2\right)^{9/8}\int \frac{\sqrt[8]{c-dx}}{\left(\frac{dx}{c}+1\right)^{7/8}}dx}{c^2(c-dx)^{9/8}\left(\frac{dx}{c}+1\right)^{9/8}}$$
$$\Big\downarrow \; 79$$
$$-\frac{4\sqrt[8]{2}\left(c^2-d^2x^2\right)^{9/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{9}{8},\frac{17}{8},\frac{c-dx}{2c}\right)}{9c^2d\left(\frac{dx}{c}+1\right)^{9/8}}$$

input
```
Int[(c^2 - d^2*x^2)^(1/8)/(c + d*x),x]
```

output
```
(-4*2^(1/8)*(c^2 - d^2*x^2)^(9/8)*Hypergeometric2F1[7/8, 9/8, 17/8, (c - d
*x)/(2*c)])/(9*c^2*d*(1 + (d*x)/c)^(9/8))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^{\frac{1}{8}}}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^(1/8)/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^(1/8)/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx = \int \frac{(-d^2x^2+c^2)^{\frac{1}{8}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/8)/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^(1/8)/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\sqrt[8]{-\left(-c+dx\right)\left(c+dx\right)}}{c+dx}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**(1/8)/(d*x+c),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(1/8)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{8}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/8)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/8)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(-d^2x^2+c^2\right)^{\frac{1}{8}}}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^(1/8)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(1/8)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx=\int \frac{\left(c^2-d^2\,x^2\right)^{1/8}}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/8)/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^(1/8)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sqrt[8]{c^2-d^2x^2}}{c+dx}\,dx = \frac{-4(-d^2x^2+c^2)^{\frac{7}{8}}+5(-d^2x^2+c^2)^{\frac{3}{4}}\left(\int \frac{(-d^2x^2+c^2)^{\frac{3}{4}}}{(-d^2x^2+c^2)^{\frac{5}{8}}c^2-(-d^2x^2+c^2)^{\frac{5}{8}}d^2x^2}dx\right)cd}{5\left(-d^2x^2+c^2\right)^{\frac{3}{4}}d}$$

input
```
int((-d^2*x^2+c^2)^(1/8)/(d*x+c),x)
```

output
```
( - 4*(c**2 - d**2*x**2)**(7/8) + 5*(c**2 - d**2*x**2)**(3/4)*int((c**2 -
d**2*x**2)**(3/4)/((c**2 - d**2*x**2)**(5/8)*c**2 - (c**2 - d**2*x**2)**(5
/8)*d**2*x**2),x)*c*d)/(5*(c**2 - d**2*x**2)**(3/4)*d)
```

3.336 $\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 \, dx = \frac{8b^3c^3(c+dx)^{4+n}}{d(4+n)} - \frac{12b^3c^2(c+dx)^{5+n}}{d(5+n)} + \frac{6b^3c(c+dx)^{6+n}}{d(6+n)} - \frac{b^3(c+dx)^{7+n}}{d(7+n)}$$

output
```
8*b^3*c^3*(d*x+c)^(4+n)/d/(4+n)-12*b^3*c^2*(d*x+c)^(5+n)/d/(5+n)+6*b^3*c*(
d*x+c)^(6+n)/d/(6+n)-b^3*(d*x+c)^(7+n)/d/(7+n)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.74

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 \, dx = \frac{b^3(c+dx)^{4+n}\left(\frac{8c^3}{4+n} - \frac{12c^2(c+dx)}{5+n} + \frac{6c(c+dx)^2}{6+n} - \frac{(c+dx)^3}{7+n}\right)}{d}$$

input
```
Integrate[(c + d*x)^n*(b*c^2 - b*d^2*x^2)^3,x]
```

output
```
(b^3*(c + d*x)^(4 + n)*((8*c^3)/(4 + n) - (12*c^2*(c + d*x))/(5 + n) + (6*
c*(c + d*x)^2)/(6 + n) - (c + d*x)^3/(7 + n)))/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bc^2-bd^2x^2\right)^3 (c+dx)^n\,dx$$

$$\Big\downarrow \text{ 456}$$

$$\int (bc-bdx)^3(c+dx)^{n+3}dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(8b^3c^3(c+dx)^{n+3}-12b^3c^2(c+dx)^{n+4}+6b^3c(c+dx)^{n+5}-b^3(c+dx)^{n+6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8b^3c^3(c+dx)^{n+4}}{d(n+4)}-\frac{12b^3c^2(c+dx)^{n+5}}{d(n+5)}+\frac{6b^3c(c+dx)^{n+6}}{d(n+6)}-\frac{b^3(c+dx)^{n+7}}{d(n+7)}$$

input
```
Int[(c + d*x)^n*(b*c^2 - b*d^2*x^2)^3,x]
```

output
```
(8*b^3*c^3*(c + d*x)^(4 + n))/(d*(4 + n)) - (12*b^3*c^2*(c + d*x)^(5 + n))
/(d*(5 + n)) + (6*b^3*c*(c + d*x)^(6 + n))/(d*(6 + n)) - (b^3*(c + d*x)^(7
 + n))/(d*(7 + n))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.89

method	result
gosper	$\frac{b^3(dx+c)^{4+n}(-d^3n^3x^3+3c\,d^2n^3x^2-15d^3n^2x^3-3c^2d\,n^3x+51c\,d^2n^2x^2-74d^3n\,x^3+c^3n^3-57c^2d\,n^2x+276c\,d^2n\,x^2-120d^3x^3}{d(n^4+22n^3+179n^2+638n+840)}$
orering	$\frac{(-d^3n^3x^3+3c\,d^2n^3x^2-15d^3n^2x^3-3c^2d\,n^3x+51c\,d^2n^2x^2-74d^3n\,x^3+c^3n^3-57c^2d\,n^2x+276c\,d^2n\,x^2-120d^3x^3+21c^3n^2-35}{d(n^4+22n^3+179n^2+638n+840)(-dx+c)^3}$
risch	$\frac{b^3(-d^7n^3x^7-c\,d^6n^3x^6-15d^7n^2x^7+3c^2d^5n^3x^5-9c\,d^6n^2x^6-74n\,d^7x^7+3c^3d^4n^3x^4+57c^2d^5n^2x^5-20cn\,d^6x^6-120d^7x^7-3c^4}{}$
norman	$\frac{b^3c^6(n^3+27n^2+254n+840)x\,e^{n\ln(dx+c)}}{n^4+22n^3+179n^2+638n+840}+\frac{b^3c^7(n^3+21n^2+152n+384)e^{n\ln(dx+c)}}{d(n^4+22n^3+179n^2+638n+840)}-\frac{b^3d^6x^7e^{n\ln(dx+c)}}{7+n}-\frac{b^3cn\,d^5x^6e^{n}}{n^2+13n}$
parallelrisch	$-\frac{-3x^4(dx+c)^nb^3c^3d^4n^3-306x^5(dx+c)^nb^3c^2d^5n-39x^4(dx+c)^nb^3c^3d^4n^2+3x^3(dx+c)^nb^3c^4d^3n^3-96x^4(dx+c)^nb^3c^3d^4n+}{}$

input
```
int((d*x+c)^n*(-b*d^2*x^2+b*c^2)^3,x,method=_RETURNVERBOSE)
```

output
```
b^3/d*(d*x+c)^(4+n)/(n^4+22*n^3+179*n^2+638*n+840)*(-d^3*n^3*x^3+3*c*d^2*n
^3*x^2-15*d^3*n^2*x^3-3*c^2*d*n^3*x+51*c*d^2*n^2*x^2-74*d^3*n*x^3+c^3*n^3-
57*c^2*d*n^2*x+276*c*d^2*n*x^2-120*d^3*x^3+21*c^3*n^2-354*c^2*d*n*x+480*c*
d^2*x^2+152*c^3*n-696*c^2*d*x+384*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(96) = 192$.

Time = 0.11 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.21

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 dx = \frac{(b^3c^7n^3+21\,b^3c^7n^2+152\,b^3c^7n+384\,b^3c^7-(b^3d^7n^3+15\,b^3d^7n^2+74\,b^3d^7n+120\,b^3d^7)x^7-(b^3cd^6n^3+$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2)^3,x, algorithm="fricas")
```

output
```
(b^3*c^7*n^3 + 21*b^3*c^7*n^2 + 152*b^3*c^7*n + 384*b^3*c^7 - (b^3*d^7*n^3
 + 15*b^3*d^7*n^2 + 74*b^3*d^7*n + 120*b^3*d^7)*x^7 - (b^3*c*d^6*n^3 + 9*b
^3*c*d^6*n^2 + 20*b^3*c*d^6*n)*x^6 + 3*(b^3*c^2*d^5*n^3 + 19*b^3*c^2*d^5*n
^2 + 102*b^3*c^2*d^5*n + 168*b^3*c^2*d^5)*x^5 + 3*(b^3*c^3*d^4*n^3 + 13*b^
3*c^3*d^4*n^2 + 32*b^3*c^3*d^4*n)*x^4 - 3*(b^3*c^4*d^3*n^3 + 23*b^3*c^4*d^
3*n^2 + 162*b^3*c^4*d^3*n + 280*b^3*c^4*d^3)*x^3 - 3*(b^3*c^5*d^2*n^3 + 17
*b^3*c^5*d^2*n^2 + 76*b^3*c^5*d^2*n)*x^2 + (b^3*c^6*d*n^3 + 27*b^3*c^6*d*n
^2 + 254*b^3*c^6*d*n + 840*b^3*c^6*d)*x)*(d*x + c)^n/(d*n^4 + 22*d*n^3 + 1
79*d*n^2 + 638*d*n + 840*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2242 vs. $2(80) = 160$.

Time = 1.35 (sec) , antiderivative size = 2242, normalized size of antiderivative = 23.35

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**n*(-b*d**2*x**2+b*c**2)**3,x)
```

output
```
Piecewise((b**3*c**6*c**n*x, Eq(d, 0)), (-3*b**3*c**3*log(c/d + x)/(3*c**3
*d + 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*d**4*x**3) - 8*b**3*c**3/(3*c**3*d
+ 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*d**4*x**3) - 9*b**3*c**2*d*x*log(c/d +
 x)/(3*c**3*d + 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*d**4*x**3) - 18*b**3*c**
2*d*x/(3*c**3*d + 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*d**4*x**3) - 9*b**3*c*
d**2*x**2*log(c/d + x)/(3*c**3*d + 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*d**4*
x**3) - 18*b**3*c*d**2*x**2/(3*c**3*d + 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*
d**4*x**3) - 3*b**3*d**3*x**3*log(c/d + x)/(3*c**3*d + 9*c**2*d**2*x + 9*c
*d**3*x**2 + 3*d**4*x**3), Eq(n, -7)), (6*b**3*c**3*log(c/d + x)/(c**2*d +
 2*c*d**2*x + d**3*x**2) + 13*b**3*c**3/(c**2*d + 2*c*d**2*x + d**3*x**2)
+ 12*b**3*c**2*d*x*log(c/d + x)/(c**2*d + 2*c*d**2*x + d**3*x**2) + 21*b**
3*c**2*d*x/(c**2*d + 2*c*d**2*x + d**3*x**2) + 6*b**3*c*d**2*x**2*log(c/d
+ x)/(c**2*d + 2*c*d**2*x + d**3*x**2) + 3*b**3*c*d**2*x**2/(c**2*d + 2*c*
d**2*x + d**3*x**2) - b**3*d**3*x**3/(c**2*d + 2*c*d**2*x + d**3*x**2), Eq
(n, -6)), (-24*b**3*c**3*log(c/d + x)/(2*c*d + 2*d**2*x) - 50*b**3*c**3/(2
*c*d + 2*d**2*x) - 24*b**3*c**2*d*x*log(c/d + x)/(2*c*d + 2*d**2*x) - 24*b
**3*c**2*d*x/(2*c*d + 2*d**2*x) + 9*b**3*c*d**2*x**2/(2*c*d + 2*d**2*x) -
b**3*d**3*x**3/(2*c*d + 2*d**2*x), Eq(n, -5)), (8*b**3*c**3*log(c/d + x)/d
 - 7*b**3*c**2*x + 2*b**3*c*d*x**2 - b**3*d**2*x**3/3, Eq(n, -4)), (b**3*c
**7*n**3*(c + d*x)**n/(d*n**4 + 22*d*n**3 + 179*d*n**2 + 638*d*n + 840*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(96) = 192$.

Time = 0.05 (sec) , antiderivative size = 480, normalized size of antiderivative = 5.00

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 \, dx = \frac{(dx+c)^{n+1}b^3c^6}{d(n+1)} - \frac{3\left((n^2+3n+2)d^3x^3+(n^2+n)cd^2x^2-2c^2dnx+2c^3\right)(dx+c)^n b^3c^4}{(n^3+6n^2+11n+6)d} + \frac{3\left((n^4+10n^3+35n^2+50n+24)d^5x^5+(n^4+6n^3+11n^2+6n)cd^4x^4-4(n^3+3n^2+2n)c^2d^3x^3\right.}{(n^5+15n^4+85n^3+225n^2+274n+120)d} - \frac{\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)d^7x^7+(n^6+15n^5+85n^4+225n^3+274\right.}{}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2)^3,x, algorithm="maxima")
```

output
```
(d*x + c)^(n + 1)*b^3*c^6/(d*(n + 1)) - 3*((n^2 + 3*n + 2)*d^3*x^3 + (n^2
+ n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*b^3*c^4/((n^3 + 6*n^2 +
11*n + 6)*d) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^5*x^5 + (n^4 + 6*n
^3 + 11*n^2 + 6*n)*c*d^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*c^2*d^3*x^3 + 12*(n^2
 + n)*c^3*d^2*x^2 - 24*c^4*d*n*x + 24*c^5)*(d*x + c)^n*b^3*c^2/((n^5 + 15*
n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*d) - ((n^6 + 21*n^5 + 175*n^4 + 735*
n^3 + 1624*n^2 + 1764*n + 720)*d^7*x^7 + (n^6 + 15*n^5 + 85*n^4 + 225*n^3
+ 274*n^2 + 120*n)*c*d^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*c
^2*d^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*c^3*d^4*x^4 - 120*(n^3 + 3*n^
2 + 2*n)*c^4*d^3*x^3 + 360*(n^2 + n)*c^5*d^2*x^2 - 720*c^6*d*n*x + 720*c^7
)*(d*x + c)^n*b^3/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n
^2 + 13068*n + 5040)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(96) = 192$.

Time = 0.13 (sec) , antiderivative size = 635, normalized size of antiderivative = 6.61

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 \, dx = -\frac{(dx+c)^n b^3 d^7 n^3 x^7 + (dx+c)^n b^3 c d^6 n^3 x^6 + 15\,(dx+c)^n b^3 d^7 n^2 x^7 - 3\,(dx+c)^n b^3 c^2 d^5 n^3 x^5 + 9\,(dx+c}{}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2)^3,x, algorithm="giac")
```

output
```
-((d*x + c)^n*b^3*d^7*n^3*x^7 + (d*x + c)^n*b^3*c*d^6*n^3*x^6 + 15*(d*x +
c)^n*b^3*d^7*n^2*x^7 - 3*(d*x + c)^n*b^3*c^2*d^5*n^3*x^5 + 9*(d*x + c)^n*b
^3*c*d^6*n^2*x^6 + 74*(d*x + c)^n*b^3*d^7*n*x^7 - 3*(d*x + c)^n*b^3*c^3*d^
4*n^3*x^4 - 57*(d*x + c)^n*b^3*c^2*d^5*n^2*x^5 + 20*(d*x + c)^n*b^3*c*d^6*
n*x^6 + 120*(d*x + c)^n*b^3*d^7*x^7 + 3*(d*x + c)^n*b^3*c^4*d^3*n^3*x^3 -
39*(d*x + c)^n*b^3*c^3*d^4*n^2*x^4 - 306*(d*x + c)^n*b^3*c^2*d^5*n*x^5 + 3
*(d*x + c)^n*b^3*c^5*d^2*n^3*x^2 + 69*(d*x + c)^n*b^3*c^4*d^3*n^2*x^3 - 96
*(d*x + c)^n*b^3*c^3*d^4*n*x^4 - 504*(d*x + c)^n*b^3*c^2*d^5*x^5 - (d*x +
c)^n*b^3*c^6*d*n^3*x + 51*(d*x + c)^n*b^3*c^5*d^2*n^2*x^2 + 486*(d*x + c)^
n*b^3*c^4*d^3*n*x^3 - (d*x + c)^n*b^3*c^7*n^3 - 27*(d*x + c)^n*b^3*c^6*d*n
^2*x + 228*(d*x + c)^n*b^3*c^5*d^2*n*x^2 + 840*(d*x + c)^n*b^3*c^4*d^3*x^3
 - 21*(d*x + c)^n*b^3*c^7*n^2 - 254*(d*x + c)^n*b^3*c^6*d*n*x - 152*(d*x +
 c)^n*b^3*c^7*n - 840*(d*x + c)^n*b^3*c^6*d*x - 384*(d*x + c)^n*b^3*c^7)/(
d*n^4 + 22*d*n^3 + 179*d*n^2 + 638*d*n + 840*d)
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.71

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3 \, dx = (c+d\,x)^n \left(\frac{b^3\,c^6\,x\,\left(n^3+27\,n^2+254\,n+840\right)}{n^4+22\,n^3+179\,n^2+638\,n+840} + \frac{b^3\,c^7\,\left(n^3+21\,n^2+152\,n+384\right)}{d\,\left(n^4+22\,n^3+179\,n^2+638\,n+840\right)} - \frac{b^3\,d^6\,x^7\,\left(n^3+15\,n^2+74\,n+120\right)}{n^4+22\,n^3+179\,n^2+638\,n+840} + \frac{3\,b^3\,c^2\,d^4\,x^5\,\left(n^3+19\,n^2+102\,n+168\right)}{n^4+22\,n^3+179\,n^2+638\,n+840} - \frac{3\,b^3\,c^4\,d^2\,x^3\,\left(n^3+23\,n^2+162\,n+280\right)}{n^4+22\,n^3+179\,n^2+638\,n+840} - \frac{b^3\,c\,d^5\,n\,x^6\,\left(n^2+9\,n+20\right)}{n^4+22\,n^3+179\,n^2+638\,n+840} - \frac{3\,b^3\,c^5\,d\,n\,x^2\,\left(n^2+17\,n+76\right)}{n^4+22\,n^3+179\,n^2+638\,n+840} + \frac{3\,b^3\,c^3\,d^3\,n\,x^4\,\left(n^2+13\,n+32\right)}{n^4+22\,n^3+179\,n^2+638\,n+840}\right)$$

input
```
int((b*c^2 - b*d^2*x^2)^3*(c + d*x)^n,x)
```

output
```
(c + d*x)^n*((b^3*c^6*x*(254*n + 27*n^2 + n^3 + 840))/(638*n + 179*n^2 + 2
2*n^3 + n^4 + 840) + (b^3*c^7*(152*n + 21*n^2 + n^3 + 384))/(d*(638*n + 17
9*n^2 + 22*n^3 + n^4 + 840)) - (b^3*d^6*x^7*(74*n + 15*n^2 + n^3 + 120))/(
638*n + 179*n^2 + 22*n^3 + n^4 + 840) + (3*b^3*c^2*d^4*x^5*(102*n + 19*n^2
 + n^3 + 168))/(638*n + 179*n^2 + 22*n^3 + n^4 + 840) - (3*b^3*c^4*d^2*x^3
*(162*n + 23*n^2 + n^3 + 280))/(638*n + 179*n^2 + 22*n^3 + n^4 + 840) - (b
^3*c*d^5*n*x^6*(9*n + n^2 + 20))/(638*n + 179*n^2 + 22*n^3 + n^4 + 840) -
(3*b^3*c^5*d*n*x^2*(17*n + n^2 + 76))/(638*n + 179*n^2 + 22*n^3 + n^4 + 84
0) + (3*b^3*c^3*d^3*n*x^4*(13*n + n^2 + 32))/(638*n + 179*n^2 + 22*n^3 + n
^4 + 840))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.65

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^3\,dx$$
$$= \frac{(dx+c)^n b^3(-d^7n^3x^7 - c\,d^6n^3x^6 - 15d^7n^2x^7 + 3c^2d^5n^3x^5 - 9c\,d^6n^2x^6 - 74d^7n\,x^7 + 3c^3d^4n^3x^4 + 57c^2d^5}{}$$

input
```
int((d*x+c)^n*(-b*d^2*x^2+b*c^2)^3,x)
```

output
```
((c + d*x)**n*b**3*(c**7*n**3 + 21*c**7*n**2 + 152*c**7*n + 384*c**7 + c**
6*d*n**3*x + 27*c**6*d*n**2*x + 254*c**6*d*n*x + 840*c**6*d*x - 3*c**5*d**
2*n**3*x**2 - 51*c**5*d**2*n**2*x**2 - 228*c**5*d**2*n*x**2 - 3*c**4*d**3*
n**3*x**3 - 69*c**4*d**3*n**2*x**3 - 486*c**4*d**3*n*x**3 - 840*c**4*d**3*
x**3 + 3*c**3*d**4*n**3*x**4 + 39*c**3*d**4*n**2*x**4 + 96*c**3*d**4*n*x**
4 + 3*c**2*d**5*n**3*x**5 + 57*c**2*d**5*n**2*x**5 + 306*c**2*d**5*n*x**5
+ 504*c**2*d**5*x**5 - c*d**6*n**3*x**6 - 9*c*d**6*n**2*x**6 - 20*c*d**6*n
*x**6 - d**7*n**3*x**7 - 15*d**7*n**2*x**7 - 74*d**7*n*x**7 - 120*d**7*x**
7))/(d*(n**4 + 22*n**3 + 179*n**2 + 638*n + 840))
```

3.337 $\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 70

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2 \, dx = \frac{4b^2c^2(c+dx)^{3+n}}{d(3+n)} - \frac{4b^2c(c+dx)^{4+n}}{d(4+n)} + \frac{b^2(c+dx)^{5+n}}{d(5+n)}$$

output
```
4*b^2*c^2*(d*x+c)^(3+n)/d/(3+n)-4*b^2*c*(d*x+c)^(4+n)/d/(4+n)+b^2*(d*x+c)^
(5+n)/d/(5+n)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2 \, dx = \frac{b^2(c+dx)^{3+n}\left(\frac{4c^2}{3+n} - \frac{4c(c+dx)}{4+n} + \frac{(c+dx)^2}{5+n}\right)}{d}$$

input
```
Integrate[(c + d*x)^n*(b*c^2 - b*d^2*x^2)^2,x]
```

output
```
(b^2*(c + d*x)^(3 + n)*((4*c^2)/(3 + n) - (4*c*(c + d*x))/(4 + n) + (c + d
*x)^2/(5 + n)))/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bc^2 - bd^2x^2)^2 (c+dx)^n \, dx$$

$$\Big\downarrow \text{456}$$

$$\int (bc-bdx)^2(c+dx)^{n+2}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(4b^2c^2(c+dx)^{n+2} - 4b^2c(c+dx)^{n+3} + b^2(c+dx)^{n+4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4b^2c^2(c+dx)^{n+3}}{d(n+3)} - \frac{4b^2c(c+dx)^{n+4}}{d(n+4)} + \frac{b^2(c+dx)^{n+5}}{d(n+5)}$$

input

```
Int[(c + d*x)^n*(b*c^2 - b*d^2*x^2)^2,x]
```

output

```
(4*b^2*c^2*(c + d*x)^(3 + n))/(d*(3 + n)) - (4*b^2*c*(c + d*x)^(4 + n))/(d
*(4 + n)) + (b^2*(c + d*x)^(5 + n))/(d*(5 + n))
```

Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

method	result
gosper	$\frac{b^2(dx+c)^{3+n}\left(d^2n^2x^2-2cd\,n^2x+7d^2n\,x^2+c^2n^2-18cdnx+12d^2x^2+11c^2n-36cdx+32c^2\right)}{d(n^3+12n^2+47n+60)}$
orering	$\frac{\left(d^2n^2x^2-2cd\,n^2x+7d^2n\,x^2+c^2n^2-18cdnx+12d^2x^2+11c^2n-36cdx+32c^2\right)(dx+c)(dx+c)^n\left(-b\,x^2d^2+b\,c^2\right)^2}{d(n^3+12n^2+47n+60)(-dx+c)^2}$
risch	$\frac{b^2\left(d^5n^2x^5+c\,d^4n^2x^4+7n\,d^5x^5-2c^2d^3n^2x^3+3cn\,d^4x^4+12d^5x^5-2c^3d^2n^2x^2-22c^2d^3n\,x^3+c^4d\,n^2x-14c^3d^2n\,x^2-40c^2d^3x^3-\right.}{(4+n)(5+n)d(3+n)}$
norman	$\frac{b^2d^4x^5\mathrm{e}^{n\ln(dx+c)}}{5+n}+\frac{b^2c^4\left(n^2+15n+60\right)x\,\mathrm{e}^{n\ln(dx+c)}}{n^3+12n^2+47n+60}+\frac{b^2c^5\left(n^2+11n+32\right)\mathrm{e}^{n\ln(dx+c)}}{d(n^3+12n^2+47n+60)}+\frac{b^2cn\,d^3x^4\mathrm{e}^{n\ln(dx+c)}}{n^2+9n+20}-\frac{2b^2c^2}{}$
parallelrisch	$\frac{x^5(dx+c)^nb^2c\,d^5n^2+7x^5(dx+c)^nb^2c\,d^5n+x^4(dx+c)^nb^2c^2d^4n^2+12x^5(dx+c)^nb^2c\,d^5+3x^4(dx+c)^nb^2c^2d^4n-2x^3(dx+c)^nb^2c}{}$

input
```
int((d*x+c)^n*(-b*d^2*x^2+b*c^2)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/d*(d*x+c)^(3+n)/(n^3+12*n^2+47*n+60)*(d^2*n^2*x^2-2*c*d*n^2*x+7*d^2*n*
x^2+c^2*n^2-18*c*d*n*x+12*d^2*x^2+11*c^2*n-36*c*d*x+32*c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(70) = 140$.

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.16

$$\int (c+dx)^n\left(bc^2-bd^2x^2\right)^2\,dx$$
$$=\frac{(b^2c^5n^2+11\,b^2c^5n+32\,b^2c^5+(b^2d^5n^2+7\,b^2d^5n+12\,b^2d^5)x^5+(b^2cd^4n^2+3\,b^2cd^4n)x^4-2\,(b^2c^2d^3n^2+}{dn^3+12\,dn^2+}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2)^2,x, algorithm="fricas")
```

output
```
(b^2*c^5*n^2 + 11*b^2*c^5*n + 32*b^2*c^5 + (b^2*d^5*n^2 + 7*b^2*d^5*n + 12
*b^2*d^5)*x^5 + (b^2*c*d^4*n^2 + 3*b^2*c*d^4*n)*x^4 - 2*(b^2*c^2*d^3*n^2 +
 11*b^2*c^2*d^3*n + 20*b^2*c^2*d^3)*x^3 - 2*(b^2*c^3*d^2*n^2 + 7*b^2*c^3*d
^2*n)*x^2 + (b^2*c^4*d*n^2 + 15*b^2*c^4*d*n + 60*b^2*c^4*d)*x)*(d*x + c)^n
/(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(58) = 116$.

Time = 0.77 (sec) , antiderivative size = 978, normalized size of antiderivative = 13.97

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**n*(-b*d**2*x**2+b*c**2)**2,x)
```

output
```
Piecewise((b**2*c**4*c**n*x, Eq(d, 0)), (b**2*c**2*log(c/d + x)/(c**2*d +
2*c*d**2*x + d**3*x**2) + 2*b**2*c**2/(c**2*d + 2*c*d**2*x + d**3*x**2) +
2*b**2*c*d*x*log(c/d + x)/(c**2*d + 2*c*d**2*x + d**3*x**2) + 4*b**2*c*d*x
/(c**2*d + 2*c*d**2*x + d**3*x**2) + b**2*d**2*x**2*log(c/d + x)/(c**2*d +
 2*c*d**2*x + d**3*x**2), Eq(n, -5)), (-4*b**2*c**2*log(c/d + x)/(c*d + d*
*2*x) - 7*b**2*c**2/(c*d + d**2*x) - 4*b**2*c*d*x*log(c/d + x)/(c*d + d**2
*x) - 2*b**2*c*d*x/(c*d + d**2*x) + b**2*d**2*x**2/(c*d + d**2*x), Eq(n, -
4)), (4*b**2*c**2*log(c/d + x)/d - 3*b**2*c*x + b**2*d*x**2/2, Eq(n, -3)),
 (b**2*c**5*n**2*(c + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) + 11*b*
*2*c**5*n*(c + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) + 32*b**2*c**5
*(c + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) + b**2*c**4*d*n**2*x*(c
 + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) + 15*b**2*c**4*d*n*x*(c +
d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) + 60*b**2*c**4*d*x*(c + d*x)*
*n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) - 2*b**2*c**3*d**2*n**2*x**2*(c +
d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) - 14*b**2*c**3*d**2*n*x**2*(c
 + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) - 2*b**2*c**2*d**3*n**2*x*
*3*(c + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) - 22*b**2*c**2*d**3*n
*x**3*(c + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) - 40*b**2*c**2*d**
3*x**3*(c + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) + b**2*c*d**4*n**
2*x**4*(c + d*x)**n/(d*n**3 + 12*d*n**2 + 47*d*n + 60*d) + 3*b**2*c*d**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(70) = 140$.

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.46

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2 \, dx = \frac{(dx+c)^{n+1}b^2c^4}{d(n+1)} - \frac{2\,((n^2+3\,n+2)d^3x^3+(n^2+n)cd^2x^2-2\,c^2dnx+2\,c^3)(dx+c)^n b^2c^2}{(n^3+6\,n^2+11\,n+6)d} + \frac{((n^4+10\,n^3+35\,n^2+50\,n+24)d^5x^5+(n^4+6\,n^3+11\,n^2+6\,n)cd^4x^4-4\,(n^3+3\,n^2+2\,n)c^2d^3x^3+}{(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)d}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2)^2,x, algorithm="maxima")
```

output
```
(d*x + c)^(n + 1)*b^2*c^4/(d*(n + 1)) - 2*((n^2 + 3*n + 2)*d^3*x^3 + (n^2
+ n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*b^2*c^2/((n^3 + 6*n^2 +
11*n + 6)*d) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^5*x^5 + (n^4 + 6*n^3
 + 11*n^2 + 6*n)*c*d^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*c^2*d^3*x^3 + 12*(n^2 +
 n)*c^3*d^2*x^2 - 24*c^4*d*n*x + 24*c^5)*(d*x + c)^n*b^2/((n^5 + 15*n^4 +
85*n^3 + 225*n^2 + 274*n + 120)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(70) = 140$.

Time = 0.13 (sec) , antiderivative size = 336, normalized size of antiderivative = 4.80

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2 dx = \frac{(dx+c)^n b^2 d^5 n^2 x^5 + (dx+c)^n b^2 c d^4 n^2 x^4 + 7\,(dx+c)^n b^2 d^5 n x^5 - 2\,(dx+c)^n b^2 c^2 d^3 n^2 x^3 + 3\,(dx+c)^n b^2 c}{}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2)^2,x, algorithm="giac")
```

output
```
((d*x + c)^n*b^2*d^5*n^2*x^5 + (d*x + c)^n*b^2*c*d^4*n^2*x^4 + 7*(d*x + c)
^n*b^2*d^5*n*x^5 - 2*(d*x + c)^n*b^2*c^2*d^3*n^2*x^3 + 3*(d*x + c)^n*b^2*c
*d^4*n*x^4 + 12*(d*x + c)^n*b^2*d^5*x^5 - 2*(d*x + c)^n*b^2*c^3*d^2*n^2*x^
2 - 22*(d*x + c)^n*b^2*c^2*d^3*n*x^3 + (d*x + c)^n*b^2*c^4*d*n^2*x - 14*(d
*x + c)^n*b^2*c^3*d^2*n*x^2 - 40*(d*x + c)^n*b^2*c^2*d^3*x^3 + (d*x + c)^n
*b^2*c^5*n^2 + 15*(d*x + c)^n*b^2*c^4*d*n*x + 11*(d*x + c)^n*b^2*c^5*n + 6
0*(d*x + c)^n*b^2*c^4*d*x + 32*(d*x + c)^n*b^2*c^5)/(d*n^3 + 12*d*n^2 + 47
*d*n + 60*d)
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2\,dx=(c+d\,x)^n\left(\frac{b^2\,c^5\,(n^2+11\,n+32)}{d\,(n^3+12\,n^2+47\,n+60)}+\frac{b^2\,d^4\,x^5\,(n^2+7\,n+12)}{n^3+12\,n^2+47\,n+60}+\frac{b^2\,c^4\,x\,(n^2+15\,n+60)}{n^3+12\,n^2+47\,n+60}-\frac{2\,b^2\,c^2\,d^2\,x^3\,(n^2+11\,n+20)}{n^3+12\,n^2+47\,n+60}+\frac{b^2\,c\,d^3\,n\,x^4\,(n+3)}{n^3+12\,n^2+47\,n+60}-\frac{2\,b^2\,c^3\,d\,n\,x^2\,(n+7)}{n^3+12\,n^2+47\,n+60}\right)$$

input
```
int((b*c^2 - b*d^2*x^2)^2*(c + d*x)^n,x)
```

output
```
(c + d*x)^n*((b^2*c^5*(11*n + n^2 + 32))/(d*(47*n + 12*n^2 + n^3 + 60)) +
(b^2*d^4*x^5*(7*n + n^2 + 12))/(47*n + 12*n^2 + n^3 + 60) + (b^2*c^4*x*(15
*n + n^2 + 60))/(47*n + 12*n^2 + n^3 + 60) - (2*b^2*c^2*d^2*x^3*(11*n + n^
2 + 20))/(47*n + 12*n^2 + n^3 + 60) + (b^2*c*d^3*n*x^4*(n + 3))/(47*n + 12
*n^2 + n^3 + 60) - (2*b^2*c^3*d*n*x^2*(n + 7))/(47*n + 12*n^2 + n^3 + 60))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.61

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)^2\,dx=\frac{(dx+c)^n\,b^2(d^5n^2x^5+c\,d^4n^2x^4+7d^5n\,x^5-2c^2d^3n^2x^3+3c\,d^4n\,x^4+12d^5x^5-2c^3d^2n^2x^2-22c^2d^3n\,x^3+}{d\,(n^3+12n^2+47n+60)}$$

input
```
int((d*x+c)^n*(-b*d^2*x^2+b*c^2)^2,x)
```

output
```
((c + d*x)**n*b**2*(c**5*n**2 + 11*c**5*n + 32*c**5 + c**4*d*n**2*x + 15*c
**4*d*n*x + 60*c**4*d*x - 2*c**3*d**2*n**2*x**2 - 14*c**3*d**2*n*x**2 - 2*
c**2*d**3*n**2*x**3 - 22*c**2*d**3*n*x**3 - 40*c**2*d**3*x**3 + c*d**4*n**
2*x**4 + 3*c*d**4*n*x**4 + d**5*n**2*x**5 + 7*d**5*n*x**5 + 12*d**5*x**5))
/(d*(n**3 + 12*n**2 + 47*n + 60))
```

3.338 $\int (c+dx)^n \left(bc^2-bd^2x^2\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right) dx = \frac{2bc(c+dx)^{2+n}}{d(2+n)} - \frac{b(c+dx)^{3+n}}{d(3+n)}$$

output
```
2*b*c*(d*x+c)^(2+n)/d/(2+n)-b*(d*x+c)^(3+n)/d/(3+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right) dx = \frac{b(c+dx)^{2+n}(c(4+n)-d(2+n)x)}{d(2+n)(3+n)}$$

input
```
Integrate[(c + d*x)^n*(b*c^2 - b*d^2*x^2),x]
```

output
```
(b*(c + d*x)^(2 + n)*(c*(4 + n) - d*(2 + n)*x))/(d*(2 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bc^2-bd^2x^2\right)(c+dx)^n\,dx$$

$$\Big\downarrow \text{456}$$

$$\int (bc-bdx)(c+dx)^{n+1}dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(2bc(c+dx)^{n+1}-b(c+dx)^{n+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2bc(c+dx)^{n+2}}{d(n+2)}-\frac{b(c+dx)^{n+3}}{d(n+3)}$$

input
```
Int[(c + d*x)^n*(b*c^2 - b*d^2*x^2),x]
```

output
```
(2*b*c*(c + d*x)^(2 + n))/(d*(2 + n)) - (b*(c + d*x)^(3 + n))/(d*(3 + n))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result
gosper	$\frac{b(dx+c)^{2+n}(-dnx+cn-2dx+4c)}{d(n^2+5n+6)}$
orering	$\frac{(-dnx+cn-2dx+4c)(dx+c)(dx+c)^n(-b\,x^2d^2+b\,c^2)}{d(n^2+5n+6)(-dx+c)}$
risch	$\frac{b(-d^3n\,x^3-c\,d^2n\,x^2-2d^3x^3+c^2dnx+c^3n+6c^2dx+4c^3)(dx+c)^n}{d(3+n)(2+n)}$
norman	$\frac{b\,c^2(6+n)x\,\mathrm{e}^{n\ln(dx+c)}}{n^2+5n+6}+\frac{b\,c^3(4+n)\mathrm{e}^{n\ln(dx+c)}}{d(n^2+5n+6)}-\frac{b\,d^2x^3\mathrm{e}^{n\ln(dx+c)}}{3+n}-\frac{bcnd\,x^2\mathrm{e}^{n\ln(dx+c)}}{n^2+5n+6}$
parallelrisch	$-\frac{x^3(dx+c)^nb\,d^3n+2x^3(dx+c)^nb\,d^3+x^2(dx+c)^nbc\,d^2n-x(dx+c)^nb\,c^2dn-6x(dx+c)^nb\,c^2d-(dx+c)^nb\,c^3n-4(dx+c)^nb\,c^3}{d(n^2+5n+6)}$

input
```
int((d*x+c)^n*(-b*d^2*x^2+b*c^2),x,method=_RETURNVERBOSE)
```

output
```
b/d*(d*x+c)^(2+n)/(n^2+5*n+6)*(-d*n*x+c*n-2*d*x+4*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.98

$$\int(c+dx)^n\left(bc^2-bd^2x^2\right)\,dx$$
$$=-\frac{(bcd^2nx^2-bc^3n-4\,bc^3+(bd^3n+2\,bd^3)x^3-(bc^2dn+6\,bc^2d)x)(dx+c)^n}{dn^2+5\,dn+6\,d}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2),x, algorithm="fricas")
```

output
```
-(b*c*d^2*n*x^2 - b*c^3*n - 4*b*c^3 + (b*d^3*n + 2*b*d^3)*x^3 - (b*c^2*d*n
 + 6*b*c^2*d)*x)*(d*x + c)^n/(d*n^2 + 5*d*n + 6*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(32) = 64$.

Time = 0.40 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.88

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)\,dx$$

$$= \begin{cases} bc^2c^nx & \text{f} \\ -\frac{bc\log\left(\frac{c}{d}+x\right)}{cd+d^2x} - \frac{2bc}{cd+d^2x} - \frac{bdx\log\left(\frac{c}{d}+x\right)}{cd+d^2x} & \text{f} \\ \frac{2bc\log\left(\frac{c}{d}+x\right)}{d} - bx & \text{f} \\ \frac{bc^3n(c+dx)^n}{dn^2+5dn+6d} + \frac{4bc^3(c+dx)^n}{dn^2+5dn+6d} + \frac{bc^2dnx(c+dx)^n}{dn^2+5dn+6d} + \frac{6bc^2dx(c+dx)^n}{dn^2+5dn+6d} - \frac{bcd^2nx^2(c+dx)^n}{dn^2+5dn+6d} - \frac{bd^3nx^3(c+dx)^n}{dn^2+5dn+6d} - \frac{2bd^3x^3(c+dx)^n}{dn^2+5dn+6d} & \text{o} \end{cases}$$

input
```
integrate((d*x+c)**n*(-b*d**2*x**2+b*c**2),x)
```

output
```
Piecewise((b*c**2*c**n*x, Eq(d, 0)), (-b*c*log(c/d + x)/(c*d + d**2*x) - 2
*b*c/(c*d + d**2*x) - b*d*x*log(c/d + x)/(c*d + d**2*x), Eq(n, -3)), (2*b*
c*log(c/d + x)/d - b*x, Eq(n, -2)), (b*c**3*n*(c + d*x)**n/(d*n**2 + 5*d*n
 + 6*d) + 4*b*c**3*(c + d*x)**n/(d*n**2 + 5*d*n + 6*d) + b*c**2*d*n*x*(c +
 d*x)**n/(d*n**2 + 5*d*n + 6*d) + 6*b*c**2*d*x*(c + d*x)**n/(d*n**2 + 5*d*
n + 6*d) - b*c*d**2*n*x**2*(c + d*x)**n/(d*n**2 + 5*d*n + 6*d) - b*d**3*n*
x**3*(c + d*x)**n/(d*n**2 + 5*d*n + 6*d) - 2*b*d**3*x**3*(c + d*x)**n/(d*n
**2 + 5*d*n + 6*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(42) = 84$.

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.21

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)\,dx$$
$$=\frac{(dx+c)^{n+1}bc^2}{d(n+1)}-\frac{((n^2+3\,n+2)d^3x^3+(n^2+n)cd^2x^2-2\,c^2dnx+2\,c^3)(dx+c)^nb}{(n^3+6\,n^2+11\,n+6)d}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2),x, algorithm="maxima")
```

output
```
(d*x + c)^(n + 1)*b*c^2/(d*(n + 1)) - ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)
*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*b/((n^3 + 6*n^2 + 11*n + 6)*
d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.98

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right)\,dx=$$
$$-\frac{(dx+c)^nbd^3nx^3+(dx+c)^nbcd^2nx^2+2\,(dx+c)^nbd^3x^3-(dx+c)^nbc^2dnx-(dx+c)^nbc^3n-6\,(dx}{dn^2+5\,dn+6\,d}$$

input
```
integrate((d*x+c)^n*(-b*d^2*x^2+b*c^2),x, algorithm="giac")
```

output
```
-((d*x + c)^n*b*d^3*n*x^3 + (d*x + c)^n*b*c*d^2*n*x^2 + 2*(d*x + c)^n*b*d^
3*x^3 - (d*x + c)^n*b*c^2*d*n*x - (d*x + c)^n*b*c^3*n - 6*(d*x + c)^n*b*c^
2*d*x - 4*(d*x + c)^n*b*c^3)/(d*n^2 + 5*d*n + 6*d)
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.14

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right) dx = (c+d\,x)^n\left(\frac{b\,c^2\,x\,(n+6)}{n^2+5\,n+6}+\frac{b\,c^3\,(n+4)}{d\,(n^2+5\,n+6)} - \frac{b\,d^2\,x^3\,(n+2)}{n^2+5\,n+6}-\frac{b\,c\,d\,n\,x^2}{n^2+5\,n+6}\right)$$

input
```
int((b*c^2 - b*d^2*x^2)*(c + d*x)^n,x)
```

output
```
(c + d*x)^n*((b*c^2*x*(n + 6))/(5*n + n^2 + 6) + (b*c^3*(n + 4))/(d*(5*n +
 n^2 + 6)) - (b*d^2*x^3*(n + 2))/(5*n + n^2 + 6) - (b*c*d*n*x^2)/(5*n + n^
2 + 6))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

$$\int (c+dx)^n \left(bc^2-bd^2x^2\right) dx = \frac{(dx+c)^n\, b(-d^3n\,x^3-c\,d^2n\,x^2-2d^3x^3+c^2dnx+c^3n+6c^2dx+4c^3)}{d\,(n^2+5n+6)}$$

input
```
int((d*x+c)^n*(-b*d^2*x^2+b*c^2),x)
```

output
```
((c + d*x)**n*b*(c**3*n + 4*c**3 + c**2*d*n*x + 6*c**2*d*x - c*d**2*n*x**2
 - d**3*n*x**3 - 2*d**3*x**3))/(d*(n**2 + 5*n + 6))
```

3.339 $\int \frac{(c+dx)^n}{bc^2-bd^2x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 41

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx=\frac{(c+dx)^n \operatorname{Hypergeometric2F1}\left(1,n,1+n,\frac{c+dx}{2c}\right)}{2bcdn}$$

output
```
1/2*(d*x+c)^n*hypergeom([1, n],[1+n],1/2*(d*x+c)/c)/b/c/d/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx =\frac{(c+dx)^n \left(2c(1+n)+n(c+dx) \operatorname{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c+dx}{2c}\right)\right)}{4bc^2dn(1+n)}$$

input
```
Integrate[(c + d*x)^n/(b*c^2 - b*d^2*x^2),x]
```

output
```
((c + d*x)^n*(2*c*(1 + n) + n*(c + d*x)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (c + d*x)/(2*c)]))/(4*b*c^2*d*n*(1 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {456, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}dx$$
$$\Big\downarrow \text{456}$$
$$\int \frac{(c+dx)^{n-1}}{bc-bdx}dx$$
$$\Big\downarrow \text{78}$$
$$\frac{(c+dx)^n \text{Hypergeometric2F1}\left(1,n,n+1,\frac{c+dx}{2c}\right)}{2bcdn}$$

input
```
Int[(c + d*x)^n/(b*c^2 - b*d^2*x^2),x]
```

output
```
((c + d*x)^n*Hypergeometric2F1[1, n, 1 + n, (c + d*x)/(2*c)])/(2*b*c*d*n)
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [F]

$$\int \frac{(dx+c)^n}{-b\,x^2 d^2+b\,c^2}dx$$

input
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2),x)
```

output
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx = \int -\frac{(dx+c)^n}{bd^2x^2-bc^2}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2),x, algorithm="fricas")
```

output
```
integral(-(d*x + c)^n/(b*d^2*x^2 - b*c^2), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx = -\frac{\int \frac{(c+dx)^n}{-c^2+d^2x^2}\,dx}{b}$$

input
```
integrate((d*x+c)**n/(-b*d**2*x**2+b*c**2),x)
```

output
```
-Integral((c + d*x)**n/(-c**2 + d**2*x**2), x)/b
```

Maxima [F]

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx=\int -\frac{(dx+c)^n}{bd^2x^2-bc^2}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2),x, algorithm="maxima")
```

output
```
-integrate((d*x + c)^n/(b*d^2*x^2 - b*c^2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx=\int -\frac{(dx+c)^n}{bd^2x^2-bc^2}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^n/(b*d^2*x^2 - b*c^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx=\int \frac{(c+d\,x)^n}{b\,c^2-b\,d^2\,x^2}\,dx$$

input
```
int((c + d*x)^n/(b*c^2 - b*d^2*x^2),x)
```

output
```
int((c + d*x)^n/(b*c^2 - b*d^2*x^2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n}{bc^2-bd^2x^2}\,dx = \frac{\int \frac{(dx+c)^n}{-d^2x^2+c^2}dx}{b}$$

input
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2),x)
```

output
```
int((c + d*x)**n/(c**2 - d**2*x**2),x)/b
```

3.340 $\int \frac{(c+dx)^n}{\left(bc^2-bd^2x^2\right)^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{(c+dx)^n}{\left(bc^2-bd^2x^2\right)^2} \, dx = -\frac{(c+dx)^{-1+n} \text{Hypergeometric2F1}\left(2,-1+n,n,\frac{c+dx}{2c}\right)}{4b^2c^2d(1-n)}$$

output
```
-1/4*(d*x+c)^(-1+n)*hypergeom([2, -1+n],[n],1/2*(d*x+c)/c)/b^2/c^2/d/(1-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int \frac{(c+dx)^n}{\left(bc^2-bd^2x^2\right)^2} \, dx$$
$$= \frac{(c+dx)^n \left(4c\left(\frac{1}{n}+\frac{c}{(-1+n)(c+dx)}\right)+\frac{2(c+dx)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c+dx}{2c}\right)}{1+n}+\frac{(c+dx)\text{Hypergeometric2F1}\left(2,1+n,2+\right.}{1+n}}{16b^2c^4d}$$

input
```
Integrate[(c + d*x)^n/(b*c^2 - b*d^2*x^2)^2,x]
```

output
```
((c + d*x)^n*(4*c*(n^(-1) + c/((-1 + n)*(c + d*x))) + (2*(c + d*x)*Hyperge
ometric2F1[1, 1 + n, 2 + n, (c + d*x)/(2*c)])/(1 + n) + ((c + d*x)*Hyperge
ometric2F1[2, 1 + n, 2 + n, (c + d*x)/(2*c)])/(1 + n)))/(16*b^2*c^4*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {456, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^2}dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(c+dx)^{n-2}}{(bc-bdx)^2}dx$$

$$\Big\downarrow \text{78}$$

$$-\frac{(c+dx)^{n-1}\text{Hypergeometric2F1}\left(2,n-1,n,\frac{c+dx}{2c}\right)}{4b^2c^2d(1-n)}$$

input
```
Int[(c + d*x)^n/(b*c^2 - b*d^2*x^2)^2,x]
```

output
```
-1/4*((c + d*x)^(-1 + n)*Hypergeometric2F1[2, -1 + n, n, (c + d*x)/(2*c)])
/(b^2*c^2*d*(1 - n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [F]

$$\int \frac{(dx+c)^n}{\left(-b\,x^2d^2+b\,c^2\right)^2}dx$$

input
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2)^2,x)
```

output
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^2}\,dx = \int \frac{(dx+c)^n}{(bd^2x^2-bc^2)^2}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^n/(b^2*d^4*x^4 - 2*b^2*c^2*d^2*x^2 + b^2*c^4), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^2}\,dx = \frac{\int \frac{(c+dx)^n}{c^4-2c^2d^2x^2+d^4x^4}\,dx}{b^2}$$

input
```
integrate((d*x+c)**n/(-b*d**2*x**2+b*c**2)**2,x)
```

output
```
Integral((c + d*x)**n/(c**4 - 2*c**2*d**2*x**2 + d**4*x**4), x)/b**2
```

Maxima [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^2}\,dx = \int \frac{(dx+c)^n}{(bd^2x^2-bc^2)^2}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n/(b*d^2*x^2 - b*c^2)^2, x)
```

Giac [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^2}\,dx = \int \frac{(dx+c)^n}{(bd^2x^2-bc^2)^2}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^n/(b*d^2*x^2 - b*c^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^2}\,dx=\int \frac{(c+d\,x)^n}{(b\,c^2-b\,d^2\,x^2)^2}\,dx$$

input
```
int((c + d*x)^n/(b*c^2 - b*d^2*x^2)^2,x)
```

output
```
int((c + d*x)^n/(b*c^2 - b*d^2*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^2}\,dx=\frac{\int \frac{(dx+c)^n}{d^4x^4-2c^2d^2x^2+c^4}dx}{b^2}$$

input
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2)^2,x)
```

output
```
int((c + d*x)**n/(c**4 - 2*c**2*d**2*x**2 + d**4*x**4),x)/b**2
```

3.341 $\int \frac{(c+dx)^n}{\left(bc^2-bd^2x^2\right)^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int \frac{(c+dx)^n}{\left(bc^2-bd^2x^2\right)^3} \, dx = -\frac{(c+dx)^{-2+n} \text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{c+dx}{2c}\right)}{8b^3c^3d(2-n)}$$

output
```
-1/8*(d*x+c)^(-2+n)*hypergeom([3, -2+n],[-1+n],1/2*(d*x+c)/c)/b^3/c^3/d/(2
-n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. $2(49) = 98$.

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.18

$$\int \frac{(c+dx)^n}{\left(bc^2-bd^2x^2\right)^3} \, dx$$
$$= \frac{(c+dx)^n \left(\frac{12c}{n} + \frac{8c^3}{(-2+n)(c+dx)^2} + \frac{12c^2}{(-1+n)(c+dx)} + \frac{6(c+dx) \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{c+dx}{2c}\right)}{1+n} + \frac{3(c+dx) \text{Hypergeometr}}{1}\right.}{64b^3c^6d}$$

input
```
Integrate[(c + d*x)^n/(b*c^2 - b*d^2*x^2)^3,x]
```

output
```
((c + d*x)^n*((12*c)/n + (8*c^3)/((-2 + n)*(c + d*x)^2) + (12*c^2)/((-1 +
n)*(c + d*x)) + (6*(c + d*x)*Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*x)/
(2*c)])/(1 + n) + (3*(c + d*x)*Hypergeometric2F1[2, 1 + n, 2 + n, (c + d*x
)/(2*c)])/(1 + n) + ((c + d*x)*Hypergeometric2F1[3, 1 + n, 2 + n, (c + d*x
)/(2*c)])/(1 + n)))/(64*b^3*c^6*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {456, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^3}\,dx$$
$$\Big\downarrow \text{ 456}$$
$$\int \frac{(c+dx)^{n-3}}{(bc-bdx)^3}dx$$
$$\Big\downarrow \text{ 78}$$
$$-\frac{(c+dx)^{n-2}\,\text{Hypergeometric2F1}\left(3,n-2,n-1,\frac{c+dx}{2c}\right)}{8b^3c^3d(2-n)}$$

input
```
Int[(c + d*x)^n/(b*c^2 - b*d^2*x^2)^3,x]
```

output
```
-1/8*((c + d*x)^(-2 + n)*Hypergeometric2F1[3, -2 + n, -1 + n, (c + d*x)/(2
*c)])/(b^3*c^3*d*(2 - n))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [F]

$$\int \frac{(dx+c)^n}{(-b\,x^2d^2+b\,c^2)^3}dx$$

input
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2)^3,x)
```

output
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2)^3,x)
```

Fricas [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^3}\,dx = \int -\frac{(dx+c)^n}{(bd^2x^2-bc^2)^3}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2)^3,x, algorithm="fricas")
```

output
```
integral(-(d*x + c)^n/(b^3*d^6*x^6 - 3*b^3*c^2*d^4*x^4 + 3*b^3*c^4*d^2*x^2
 - b^3*c^6), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^3}\,dx = -\frac{\int \frac{(c+dx)^n}{-c^6+3c^4d^2x^2-3c^2d^4x^4+d^6x^6}\,dx}{b^3}$$

input
```
integrate((d*x+c)**n/(-b*d**2*x**2+b*c**2)**3,x)
```

output
```
-Integral((c + d*x)**n/(-c**6 + 3*c**4*d**2*x**2 - 3*c**2*d**4*x**4 + d**6
*x**6), x)/b**3
```

Maxima [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^3}\,dx = \int -\frac{(dx+c)^n}{(bd^2x^2-bc^2)^3}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2)^3,x, algorithm="maxima")
```

output
```
-integrate((d*x + c)^n/(b*d^2*x^2 - b*c^2)^3, x)
```

Giac [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^3}\,dx = \int -\frac{(dx+c)^n}{(bd^2x^2-bc^2)^3}\,dx$$

input
```
integrate((d*x+c)^n/(-b*d^2*x^2+b*c^2)^3,x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^n/(b*d^2*x^2 - b*c^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^3}\,dx=\int \frac{(c+d\,x)^n}{(b\,c^2-b\,d^2\,x^2)^3}\,dx$$

input
```
int((c + d*x)^n/(b*c^2 - b*d^2*x^2)^3,x)
```

output
```
int((c + d*x)^n/(b*c^2 - b*d^2*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(c+dx)^n}{(bc^2-bd^2x^2)^3}\,dx=\frac{\int \frac{(dx+c)^n}{-d^6x^6+3c^2d^4x^4-3c^4d^2x^2+c^6}dx}{b^3}$$

input
```
int((d*x+c)^n/(-b*d^2*x^2+b*c^2)^3,x)
```

output
```
int((c + d*x)**n/(c**6 - 3*c**4*d**2*x**2 + 3*c**2*d**4*x**4 - d**6*x**6),
x)/b**3
```

3.342 $\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} \, dx = -\frac{2^{\frac{5}{2}+n}(c+dx)^n \left(1+\frac{dx}{c}\right)^{-\frac{5}{2}-n} \left(c^2-d^2x^2\right)^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2}-n,\frac{7}{2},\frac{c-dx}{2c}\right)}{5cd}$$

output
```
-1/5*2^(5/2+n)*(d*x+c)^n*(1+d*x/c)^(-5/2-n)*(-d^2*x^2+c^2)^(5/2)*hypergeom
([5/2, -3/2-n],[7/2],1/2*(-d*x+c)/c)/c/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.54 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.30

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} \, dx = -\frac{2^n(c+dx)^n \left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-2n} \left(d^3x^3\sqrt{c-dx}\sqrt{c+dx}\left(\frac{1}{2}+\frac{dx}{2c}\right)^n \text{AppellF1}\left(3,-\frac{1}{2},-\frac{1}{2}-n,4,\frac{dx}{c},-\frac{dx}{c}\right)+2c^2(\cdots}{3d\sqrt{1-\frac{dx}{c}}}$$

input
```
Integrate[(c + d*x)^n*(c^2 - d^2*x^2)^(3/2),x]
```

output
```
-1/3*(2^n*(c + d*x)^n*(1 + (d*x)/c)^(-1/2 - 2*n)*(d^3*x^3*Sqrt[c - d*x]*Sq
rt[c + d*x]*(1/2 + (d*x)/(2*c))^n*AppellF1[3, -1/2, -1/2 - n, 4, (d*x)/c,
-((d*x)/c)] + 2*c^2*(c - d*x)*Sqrt[2 - (2*d*x)/c]*(1 + (d*x)/c)^n*Sqrt[c^2
 - d^2*x^2]*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (c - d*x)/(2*c)]))/(d*Sq
rt[1 - (d*x)/c])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2-d^2x^2\right)^{3/2}(c+dx)^n\,dx$$

$$\Big\downarrow \text{474}$$

$$(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}\int \left(\frac{dx}{c}+1\right)^n\left(c^2-d^2x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{473}$$

$$\frac{\left(c^2-d^2x^2\right)^{5/2}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n-\frac{5}{2}}\int \left(\frac{dx}{c}+1\right)^{n+\frac{3}{2}}\left(c^2-cdx\right)^{3/2}dx}{\left(c^2-cdx\right)^{5/2}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{n+\frac{5}{2}}\left(c^2-d^2x^2\right)^{5/2}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n-\frac{5}{2}}\text{Hypergeometric2F1}\left(\frac{5}{2},-n-\frac{3}{2},\frac{7}{2},\frac{c-dx}{2c}\right)}{5cd}$$

input
```
Int[(c + d*x)^n*(c^2 - d^2*x^2)^(3/2),x]
```

output
```
-1/5*(2^(5/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(-5/2 - n)*(c^2 - d^2*x^2)^(5/
2)*Hypergeometric2F1[5/2, -3/2 - n, 7/2, (c - d*x)/(2*c)])/(c*d)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (dx+c)^n \left(-d^2x^2+c^2\right)^{\frac{3}{2}} dx$$

input
```
int((d*x+c)^n*(-d^2*x^2+c^2)^(3/2),x)
```

output
```
int((d*x+c)^n*(-d^2*x^2+c^2)^(3/2),x)
```

Fricas [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} \, dx = \int \left(-d^2x^2+c^2\right)^{\frac{3}{2}} (dx+c)^n \, dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(d^2*x^2 - c^2)*sqrt(-d^2*x^2 + c^2)*(d*x + c)^n, x)
```

Sympy [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} \, dx = \int \left(-(-c+dx)\,(c+dx)\right)^{\frac{3}{2}} (c+dx)^n \, dx$$

input
```
integrate((d*x+c)**n*(-d**2*x**2+c**2)**(3/2),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**(3/2)*(c + d*x)**n, x)
```

Maxima [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} \, dx = \int \left(-d^2x^2+c^2\right)^{\frac{3}{2}} (dx+c)^n \, dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^(3/2)*(d*x + c)^n, x)
```

Giac [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} dx = \int \left(-d^2x^2+c^2\right)^{\frac{3}{2}} (dx+c)^n \, dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^(3/2)*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} dx = \int \left(c^2-d^2\,x^2\right)^{3/2} (c+d\,x)^n \, dx$$

input
```
int((c^2 - d^2*x^2)^(3/2)*(c + d*x)^n,x)
```

output
```
int((c^2 - d^2*x^2)^(3/2)*(c + d*x)^n, x)
```

Reduce [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^{3/2} dx = -\left(\int (dx+c)^n \sqrt{-d^2x^2+c^2}\, x^2 dx\right) d^2 + \left(\int (dx+c)^n \sqrt{-d^2x^2+c^2} dx\right) c^2$$

input
```
int((d*x+c)^n*(-d^2*x^2+c^2)^(3/2),x)
```

output
```
- int((c + d*x)**n*sqrt(c**2 - d**2*x**2)*x**2,x)*d**2 + int((c + d*x)**n
*sqrt(c**2 - d**2*x**2),x)*c**2
```

3.343 $\int (c+dx)^n\sqrt{c^2-d^2x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int (c+dx)^n\sqrt{c^2-d^2x^2}\,dx = -\frac{2^{\frac{3}{2}+n}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-\frac{3}{2}-n}(c^2-d^2x^2)^{3/2}\operatorname{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-n,\frac{5}{2},\frac{c-dx}{2c}\right)}{3cd}$$

output
```
-1/3*2^(3/2+n)*(d*x+c)^n*(1+d*x/c)^(-3/2-n)*(-d^2*x^2+c^2)^(3/2)*hypergeom
([3/2, -1/2-n],[5/2],1/2*(-d*x+c)/c)/c/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int (c+dx)^n\sqrt{c^2-d^2x^2}\,dx = -\frac{2^{\frac{3}{2}+n}(c-dx)(c+dx)^n\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-n}\sqrt{c^2-d^2x^2}\operatorname{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-n,\frac{5}{2},\frac{c-dx}{2c}\right)}{3d}$$

input
```
Integrate[(c + d*x)^n*Sqrt[c^2 - d^2*x^2],x]
```

output
```
-1/3*(2^(3/2 + n)*(c - d*x)*(c + d*x)^n*(1 + (d*x)/c)^(-1/2 - n)*Sqrt[c^2
- d^2*x^2]*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (c - d*x)/(2*c)])/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c^2-d^2x^2}(c+dx)^n\,dx$$

$$\Big\downarrow \text{ 474}$$

$$(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}\int \left(\frac{dx}{c}+1\right)^n\sqrt{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{ 473}$$

$$\frac{\left(c^2-d^2x^2\right)^{3/2}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n-\frac{3}{2}}\int \left(\frac{dx}{c}+1\right)^{n+\frac{1}{2}}\sqrt{c^2-cdx}dx}{\left(c^2-cdx\right)^{3/2}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{2^{n+\frac{3}{2}}\left(c^2-d^2x^2\right)^{3/2}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n-\frac{3}{2}}\text{Hypergeometric2F1}\left(\frac{3}{2},-n-\frac{1}{2},\frac{5}{2},\frac{c-dx}{2c}\right)}{3cd}$$

input
```
Int[(c + d*x)^n*Sqrt[c^2 - d^2*x^2],x]
```

output
```
-1/3*(2^(3/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(-3/2 - n)*(c^2 - d^2*x^2)^(3/
2)*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (c - d*x)/(2*c)])/(c*d)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (dx+c)^n \sqrt{-d^2x^2+c^2}dx$$

input
```
int((d*x+c)^n*(-d^2*x^2+c^2)^(1/2),x)
```

output
```
int((d*x+c)^n*(-d^2*x^2+c^2)^(1/2),x)
```

Fricas [F]

$$\int (c+dx)^n \sqrt{c^2-d^2x^2}\,dx = \int \sqrt{-d^2x^2+c^2}(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-d^2*x^2 + c^2)*(d*x + c)^n, x)
```

Sympy [F]

$$\int (c+dx)^n \sqrt{c^2-d^2x^2}\,dx = \int \sqrt{-(-c+dx)(c+dx)}(c+dx)^n\,dx$$

input
```
integrate((d*x+c)**n*(-d**2*x**2+c**2)**(1/2),x)
```

output
```
Integral(sqrt(-(-c + d*x)*(c + d*x))*(c + d*x)**n, x)
```

Maxima [F]

$$\int (c+dx)^n \sqrt{c^2-d^2x^2}\,dx = \int \sqrt{-d^2x^2+c^2}(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-d^2*x^2 + c^2)*(d*x + c)^n, x)
```

Giac [F]

$$\int (c+dx)^n\sqrt{c^2-d^2x^2}\,dx=\int \sqrt{-d^2x^2+c^2}(dx+c)^n\,dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-d^2*x^2 + c^2)*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^n\sqrt{c^2-d^2x^2}\,dx=\int \sqrt{c^2-d^2\,x^2}\,(c+d\,x)^n\,dx$$

input
```
int((c^2 - d^2*x^2)^(1/2)*(c + d*x)^n,x)
```

output
```
int((c^2 - d^2*x^2)^(1/2)*(c + d*x)^n, x)
```

Reduce [F]

$$\int (c+dx)^n\sqrt{c^2-d^2x^2}\,dx=\int (dx+c)^n\sqrt{-d^2x^2+c^2}dx$$

input
```
int((d*x+c)^n*(-d^2*x^2+c^2)^(1/2),x)
```

output
```
int((c + d*x)**n*sqrt(c**2 - d**2*x**2),x)
```

3.344 $\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx = -\frac{2^{\frac{1}{2}+n}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-n}\sqrt{c^2-d^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{c-dx}{2c}\right)}{cd}$$

output
```
-2^(1/2+n)*(d*x+c)^n*(1+d*x/c)^(-1/2-n)*(-d^2*x^2+c^2)^(1/2)*hypergeom([1/
2, 1/2-n],[3/2],1/2*(-d*x+c)/c)/c/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx = -\frac{2^{\frac{1}{2}+n}(c-dx)(c+dx)^n\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{c-dx}{2c}\right)}{d\sqrt{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)^n/Sqrt[c^2 - d^2*x^2],x]
```

output
```
-((2^(1/2 + n)*(c - d*x)*(c + d*x)^n*(1 + (d*x)/c)^(1/2 - n)*Hypergeometri
c2F1[1/2, 1/2 - n, 3/2, (c - d*x)/(2*c)])/(d*Sqrt[c^2 - d^2*x^2]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{474}$$

$$(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}\int \frac{\left(\frac{dx}{c}+1\right)^n}{\sqrt{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{473}$$

$$\frac{\sqrt{c^2-d^2x^2}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n-\frac{1}{2}}\int \frac{\left(\frac{dx}{c}+1\right)^{n-\frac{1}{2}}}{\sqrt{c^2-cdx}}dx}{\sqrt{c^2-cdx}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{n+\frac{1}{2}}\sqrt{c^2-d^2x^2}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n-\frac{1}{2}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{c-dx}{2c}\right)}{cd}$$

input
```
Int[(c + d*x)^n/Sqrt[c^2 - d^2*x^2],x]
```

output
```
-((2^(1/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(-1/2 - n)*Sqrt[c^2 - d^2*x^2]*Hy
pergeometric2F1[1/2, 1/2 - n, 3/2, (c - d*x)/(2*c)])/(c*d))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}}dx$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(1/2),x)
```

output
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx=\int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-d^2*x^2 + c^2)*(d*x + c)^n/(d^2*x^2 - c^2), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx=\int \frac{(c+dx)^n}{\sqrt{-\left(-c+dx\right)\left(c+dx\right)}}\,dx$$

input
```
integrate((d*x+c)**n/(-d**2*x**2+c**2)**(1/2),x)
```

output
```
Integral((c + d*x)**n/sqrt(-(-c + d*x)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx=\int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n/sqrt(-d^2*x^2 + c^2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^n/sqrt(-d^2*x^2 + c^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx = \int \frac{(c+d\,x)^n}{\sqrt{c^2-d^2\,x^2}}\,dx$$

input
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(1/2),x)
```

output
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n}{\sqrt{c^2-d^2x^2}}\,dx = \int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}}dx$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(1/2),x)
```

output
```
int((c + d*x)**n/sqrt(c**2 - d**2*x**2),x)
```

3.345 $\int \frac{(c+dx)^n}{\left(c^2-d^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{(c+dx)^n}{\left(c^2-d^2x^2\right)^{3/2}}\,dx=\frac{2^{-\frac{1}{2}+n}(c+dx)^n\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{c-dx}{2c}\right)}{cd\sqrt{c^2-d^2x^2}}$$

output
```
2^(-1/2+n)*(d*x+c)^n*(1+d*x/c)^(1/2-n)*hypergeom([-1/2, 3/2-n],[1/2],1/2*(
-d*x+c)/c)/c/d/(-d^2*x^2+c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^n}{\left(c^2-d^2x^2\right)^{3/2}}\,dx=\frac{2^{-\frac{1}{2}+n}(c+dx)^n\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{c-dx}{2c}\right)}{cd\sqrt{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)^n/(c^2 - d^2*x^2)^(3/2),x]
```

output
```
(2^(-1/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(1/2 - n)*Hypergeometric2F1[-1/2,
3/2 - n, 1/2, (c - d*x)/(2*c)])/(c*d*Sqrt[c^2 - d^2*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{474}$$

$$(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}\int \frac{\left(\frac{dx}{c}+1\right)^n}{(c^2-d^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{473}$$

$$\frac{\sqrt{c^2-cdx}(c+dx)^n\left(\frac{dx}{c}+1\right)^{\frac{1}{2}-n}\int \frac{\left(\frac{dx}{c}+1\right)^{n-\frac{3}{2}}}{(c^2-cdx)^{3/2}}dx}{\sqrt{c^2-d^2x^2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{n-\frac{1}{2}}(c+dx)^n\left(\frac{dx}{c}+1\right)^{\frac{1}{2}-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{c-dx}{2c}\right)}{cd\sqrt{c^2-d^2x^2}}$$

input
```
Int[(c + d*x)^n/(c^2 - d^2*x^2)^(3/2),x]
```

output
```
(2^(-1/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(1/2 - n)*Hypergeometric2F1[-1/2,
3/2 - n, 1/2, (c - d*x)/(2*c)])/(c*d*Sqrt[c^2 - d^2*x^2])
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(3/2),x)
```

output
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(3/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{3/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-d^2*x^2 + c^2)*(d*x + c)^n/(d^4*x^4 - 2*c^2*d^2*x^2 + c^4),
 x)
```

Sympy [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{3/2}}\,dx = \int \frac{(c+dx)^n}{(-(-c+dx)(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**n/(-d**2*x**2+c**2)**(3/2),x)
```

output
```
Integral((c + d*x)**n/(-(-c + d*x)*(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{3/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n/(-d^2*x^2 + c^2)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{3/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^n/(-d^2*x^2 + c^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{3/2}}\,dx = \int \frac{(c+d\,x)^n}{(c^2-d^2\,x^2)^{3/2}}\,dx$$

input
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(3/2),x)
```

output
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{3/2}}\,dx = \int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}\,c^2-\sqrt{-d^2x^2+c^2}\,d^2x^2}dx$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(3/2),x)
```

output
```
int((c + d*x)**n/(sqrt(c**2 - d**2*x**2)*c**2 - sqrt(c**2 - d**2*x**2)*d**
2*x**2),x)
```

3.346 $\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx=\frac{2^{-\frac{3}{2}+n}(c+dx)^n\left(1+\frac{dx}{c}\right)^{\frac{3}{2}-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{c-dx}{2c}\right)}{3cd\,(c^2-d^2x^2)^{3/2}}$$

output
```
1/3*2^(-3/2+n)*(d*x+c)^n*(1+d*x/c)^(3/2-n)*hypergeom([-3/2, 5/2-n],[-1/2],
1/2*(-d*x+c)/c)/c/d/(-d^2*x^2+c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx=\frac{2^{-\frac{3}{2}+n}(c+dx)^n\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{c-dx}{2c}\right)}{(3c^3d-3c^2d^2x)\sqrt{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)^n/(c^2 - d^2*x^2)^(5/2),x]
```

output
```
(2^(-3/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(1/2 - n)*Hypergeometric2F1[-3/2,
5/2 - n, -1/2, (c - d*x)/(2*c)])/((3*c^3*d - 3*c^2*d^2*x)*Sqrt[c^2 - d^2*x
^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}dx$$

$$\Big\downarrow \ 474$$

$$(c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} \int \frac{\left(\frac{dx}{c}+1\right)^n}{(c^2-d^2x^2)^{5/2}}dx$$

$$\Big\downarrow \ 473$$

$$\frac{(c^2-cdx)^{3/2}(c+dx)^n \left(\frac{dx}{c}+1\right)^{\frac{3}{2}-n} \int \frac{\left(\frac{dx}{c}+1\right)^{n-\frac{5}{2}}}{(c^2-cdx)^{5/2}}dx}{(c^2-d^2x^2)^{3/2}}$$

$$\Big\downarrow \ 79$$

$$\frac{2^{n-\frac{3}{2}}(c+dx)^n \left(\frac{dx}{c}+1\right)^{\frac{3}{2}-n} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{c-dx}{2c}\right)}{3cd\,(c^2-d^2x^2)^{3/2}}$$

input
```
Int[(c + d*x)^n/(c^2 - d^2*x^2)^(5/2),x]
```

output
```
(2^(-3/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(3/2 - n)*Hypergeometric2F1[-3/2,
5/2 - n, -1/2, (c - d*x)/(2*c)])/(3*c*d*(c^2 - d^2*x^2)^(3/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(5/2),x)
```

output
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(5/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-d^2*x^2 + c^2)*(d*x + c)^n/(d^6*x^6 - 3*c^2*d^4*x^4 + 3*c^
4*d^2*x^2 - c^6), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx = \int \frac{(c+dx)^n}{(-(-c+dx)\,(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**n/(-d**2*x**2+c**2)**(5/2),x)
```

output
```
Integral((c + d*x)**n/(-(-c + d*x)*(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n/(-d^2*x^2 + c^2)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^n/(-d^2*x^2 + c^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx = \int \frac{(c+d\,x)^n}{(c^2-d^2\,x^2)^{5/2}}\,dx$$

input
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(5/2),x)
```

output
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{5/2}}\,dx = \int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}\,c^4-2\sqrt{-d^2x^2+c^2}\,c^2d^2x^2+\sqrt{-d^2x^2+c^2}\,d^4x^4}dx$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(5/2),x)
```

output
```
int((c + d*x)**n/(sqrt(c**2 - d**2*x**2)*c**4 - 2*sqrt(c**2 - d**2*x**2)*c
**2*d**2*x**2 + sqrt(c**2 - d**2*x**2)*d**4*x**4),x)
```

3.347 $\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}} \, dx = \frac{2^{-\frac{5}{2}+n}(c+dx)^n \left(1+\frac{dx}{c}\right)^{\frac{5}{2}-n} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{c-dx}{2c}\right)}{5cd\,(c^2-d^2x^2)^{5/2}}$$

output
```
1/5*2^(-5/2+n)*(d*x+c)^n*(1+d*x/c)^(5/2-n)*hypergeom([-5/2, 7/2-n],[-3/2],
1/2*(-d*x+c)/c)/c/d/(-d^2*x^2+c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}} \, dx = \frac{2^{-\frac{5}{2}+n}(c+dx)^n \left(1+\frac{dx}{c}\right)^{\frac{1}{2}-n} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{c-dx}{2c}\right)}{5c^3d(c-dx)^2\sqrt{c^2-d^2x^2}}$$

input
```
Integrate[(c + d*x)^n/(c^2 - d^2*x^2)^(7/2),x]
```

output
```
(2^(-5/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(1/2 - n)*Hypergeometric2F1[-5/2,
7/2 - n, -3/2, (c - d*x)/(2*c)])/(5*c^3*d*(c - d*x)^2*Sqrt[c^2 - d^2*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{474}$$

$$(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}\int \frac{\left(\frac{dx}{c}+1\right)^n}{(c^2-d^2x^2)^{7/2}}dx$$

$$\Big\downarrow \text{473}$$

$$\frac{(c^2-cdx)^{5/2}(c+dx)^n\left(\frac{dx}{c}+1\right)^{\frac{5}{2}-n}\int \frac{\left(\frac{dx}{c}+1\right)^{n-\frac{7}{2}}}{(c^2-cdx)^{7/2}}dx}{(c^2-d^2x^2)^{5/2}}$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{n-\frac{5}{2}}(c+dx)^n\left(\frac{dx}{c}+1\right)^{\frac{5}{2}-n}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{c-dx}{2c}\right)}{5cd\,(c^2-d^2x^2)^{5/2}}$$

input
```
Int[(c + d*x)^n/(c^2 - d^2*x^2)^(7/2),x]
```

output
```
(2^(-5/2 + n)*(c + d*x)^n*(1 + (d*x)/c)^(5/2 - n)*Hypergeometric2F1[-5/2,
7/2 - n, -3/2, (c - d*x)/(2*c)])/(5*c*d*(c^2 - d^2*x^2)^(5/2))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && !Gt
Q[a, 0] && !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] && !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{7}{2}}}dx$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(7/2),x)
```

output
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(7/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-d^2*x^2 + c^2)*(d*x + c)^n/(d^8*x^8 - 4*c^2*d^6*x^6 + 6*c^4
*d^4*x^4 - 4*c^6*d^2*x^2 + c^8), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}}\,dx = \int \frac{(c+dx)^n}{(-(-c+dx)\,(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)**n/(-d**2*x**2+c**2)**(7/2),x)
```

output
```
Integral((c + d*x)**n/(-(-c + d*x)*(c + d*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n/(-d^2*x^2 + c^2)^(7/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}}\,dx = \int \frac{(dx+c)^n}{(-d^2x^2+c^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^n/(-d^2*x^2+c^2)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^n/(-d^2*x^2 + c^2)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}}\,dx = \int \frac{(c+d\,x)^n}{(c^2-d^2\,x^2)^{7/2}}\,dx$$

input
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(7/2),x)
```

output
```
int((c + d*x)^n/(c^2 - d^2*x^2)^(7/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n}{(c^2-d^2x^2)^{7/2}}\,dx = \int \frac{(dx+c)^n}{\sqrt{-d^2x^2+c^2}\,c^6-3\sqrt{-d^2x^2+c^2}\,c^4d^2x^2+3\sqrt{-d^2x^2+c^2}\,c^2d^4x^4-\sqrt{-d^2x^2+c^2}}$$

input
```
int((d*x+c)^n/(-d^2*x^2+c^2)^(7/2),x)
```

output
```
int((c + d*x)**n/(sqrt(c**2 - d**2*x**2)*c**6 - 3*sqrt(c**2 - d**2*x**2)*c
**4*d**2*x**2 + 3*sqrt(c**2 - d**2*x**2)*c**2*d**4*x**4 - sqrt(c**2 - d**2
*x**2)*d**6*x**6),x)
```

3.348 $\int (c+dx)^2 \left(1-\frac{d^2x^2}{c^2}\right)^p dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int (c+dx)^2 \left(1-\frac{d^2x^2}{c^2}\right)^p dx = \frac{2^p c^3 \left(\frac{c+dx}{c}\right)^{3+p} \text{Hypergeometric2F1}\left(-p,3+p,4+p,\frac{c+dx}{2c}\right)}{d(3+p)}$$

output
```
2^p*c^3*((d*x+c)/c)^(3+p)*hypergeom([-p, 3+p],[4+p],1/2*(d*x+c)/c)/d/(3+p)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.72

$$\int (c+dx)^2 \left(1-\frac{d^2x^2}{c^2}\right)^p dx = -\frac{c^3\left(1-\frac{d^2x^2}{c^2}\right)^{1+p}}{d(1+p)} + c^2x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right) + \frac{1}{3}d^2x^3 \,\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{d^2x^2}{c^2}\right)$$

input
```
Integrate[(c + d*x)^2*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
-((c^3*(1 - (d^2*x^2)/c^2)^(1 + p))/(d*(1 + p))) + c^2*x*Hypergeometric2F1
[1/2, -p, 3/2, (d^2*x^2)/c^2] + (d^2*x^3*Hypergeometric2F1[3/2, -p, 5/2, (
d^2*x^2)/c^2])/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {472, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(1-\frac{d^2x^2}{c^2}\right)^p \, dx$$

$$\Big\downarrow \text{472}$$

$$c\left(\frac{c-dx}{c}\right)^{p+1}\left(\frac{1}{c}-\frac{dx}{c^2}\right)^{-p-1}\int \left(\frac{1}{c}-\frac{dx}{c^2}\right)^p\left(\frac{dx}{c}+1\right)^{p+2}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{c^3 2^{p+2}\left(\frac{c-dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(-p-2,p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)}$$

input
```
Int[(c + d*x)^2*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
-((2^(2 + p)*c^3*((c - d*x)/c)^(1 + p)*Hypergeometric2F1[-2 - p, 1 + p, 2
+ p, (c - d*x)/(2*c)])/(d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 472
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^(p + 1)*c^(n - 1)*(((c - d*x)/c)^(p + 1)/(a/c + b*(x/d))^(p + 1))   Int[(
1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && GtQ[a, 0] &&  !(
IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

method	result
meijerg	$\frac{d^{2} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{2},-p\right],\left[\frac{5}{2}\right],\frac{x^{2} d^{2}}{c^{2}}\right)}{3}+c d \,x^{2} \operatorname{hypergeom}\left(\left[1,-p\right],\left[2\right],\frac{x^{2} d^{2}}{c^{2}}\right)+c^{2} x \operatorname{hypergeom}\left(\left[\frac{1}{2},-p\right],\right.$

input
```
int((d*x+c)^2*(1-1/c^2*x^2*d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*x^3*hypergeom([3/2,-p],[5/2],1/c^2*x^2*d^2)+c*d*x^2*hypergeom([1,-
p],[2],1/c^2*x^2*d^2)+c^2*x*hypergeom([1/2,-p],[3/2],1/c^2*x^2*d^2)
```

Fricas [F]

$$\int (c+dx)^2\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)^2\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)^2*(1-d^2*x^2/c^2)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(-(d^2*x^2 - c^2)/c^2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.32

$$\int (c+dx)^2\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx$$
$$= c^2x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + 2cd\left(\begin{cases}\frac{x^2}{2} & \text{for } d^2=0\\ -\frac{c^2\left(\begin{cases}\frac{\left(1-\frac{d^2x^2}{c^2}\right)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(1-\frac{d^2x^2}{c^2}\right) & \text{otherwise}\end{cases}\right)}{2d^2} & \text{otherwise}\end{cases}\right) + \frac{d^2x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right)}{3}$$

input
```
integrate((d*x+c)**2*(1-d**2*x**2/c**2)**p,x)
```

output
```
c**2*x*hyper((1/2, -p), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + 2*c*d*
Piecewise((x**2/2, Eq(d**2, 0)), (-c**2*Piecewise(((1 - d**2*x**2/c**2)**(
p + 1)/(p + 1), Ne(p, -1)), (log(1 - d**2*x**2/c**2), True))/(2*d**2), Tru
e)) + d**2*x**3*hyper((3/2, -p), (5/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)
/3
```

Maxima [F]

$$\int (c+dx)^2 \left(1-\frac{d^2x^2}{c^2}\right)^p \, dx = \int (dx+c)^2 \left(-\frac{d^2x^2}{c^2}+1\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(1-d^2*x^2/c^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(-d^2*x^2/c^2 + 1)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(1-\frac{d^2x^2}{c^2}\right)^p \, dx = \int (dx+c)^2 \left(-\frac{d^2x^2}{c^2}+1\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(1-d^2*x^2/c^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(-d^2*x^2/c^2 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(1-\frac{d^2x^2}{c^2}\right)^p \, dx = \int \left(1-\frac{d^2\,x^2}{c^2}\right)^p (c+d\,x)^2 \, dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p*(c + d*x)^2,x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx = \frac{-4(-d^2x^2+c^2)^p\,c^3p^2-8(-d^2x^2+c^2)^p\,c^3p-3(-d^2x^2+c^2)^p\,c^3+3(-d^2x^2+c^2)^p\,c^2dpx+3(-d^2x^2+c^2 \cdots}{\cdots}$$

input
```
int((d*x+c)^2*(1-d^2*x^2/c^2)^p,x)
```

output
```
( - 4*(c**2 - d**2*x**2)**p*c**3*p**2 - 8*(c**2 - d**2*x**2)**p*c**3*p - 3
*(c**2 - d**2*x**2)**p*c**3 + 3*(c**2 - d**2*x**2)**p*c**2*d*p*x + 3*(c**2
 - d**2*x**2)**p*c**2*d*x + 4*(c**2 - d**2*x**2)**p*c*d**2*p**2*x**2 + 8*(
c**2 - d**2*x**2)**p*c*d**2*p*x**2 + 3*(c**2 - d**2*x**2)**p*c*d**2*x**2 +
 2*(c**2 - d**2*x**2)**p*d**3*p**2*x**3 + 3*(c**2 - d**2*x**2)**p*d**3*p*x
**3 + (c**2 - d**2*x**2)**p*d**3*x**3 + 16*int((c**2 - d**2*x**2)**p/(4*c*
*2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x*
*2),x)*c**4*d*p**5 + 80*int((c**2 - d**2*x**2)**p/(4*c**2*p**2 + 8*c**2*p
+ 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x**2),x)*c**4*d*p**4
+ 140*int((c**2 - d**2*x**2)**p/(4*c**2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*
p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x**2),x)*c**4*d*p**3 + 100*int((c**2 -
d**2*x**2)**p/(4*c**2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2
*p*x**2 - 3*d**2*x**2),x)*c**4*d*p**2 + 24*int((c**2 - d**2*x**2)**p/(4*c*
*2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x*
*2),x)*c**4*d*p)/(c**(2*p)*d*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.349 $\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\frac{2^pc^2\left(\frac{c+dx}{c}\right)^{2+p}\text{Hypergeometric2F1}\left(-p,2+p,3+p,\frac{c+dx}{2c}\right)}{d(2+p)}$$

output
```
2^p*c^2*((d*x+c)/c)^(2+p)*hypergeom([-p, 2+p],[3+p],1/2*(d*x+c)/c)/d/(2+p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=-\frac{c^2\left(1-\frac{d^2x^2}{c^2}\right)^{1+p}}{2d(1+p)}+cx\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)$$

input
```
Integrate[(c + d*x)*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
-1/2*(c^2*(1 - (d^2*x^2)/c^2)^(1 + p))/(d*(1 + p)) + c*x*Hypergeometric2F1
[1/2, -p, 3/2, (d^2*x^2)/c^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {455, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p dx$$

$$\Big\downarrow \text{ 455}$$

$$c\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx-\frac{c^2\left(1-\frac{d^2x^2}{c^2}\right)^{p+1}}{2d(p+1)}$$

$$\Big\downarrow \text{ 237}$$

$$cx\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)-\frac{c^2\left(1-\frac{d^2x^2}{c^2}\right)^{p+1}}{2d(p+1)}$$

input
```
Int[(c + d*x)*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
-1/2*(c^2*(1 - (d^2*x^2)/c^2)^(1 + p))/(d*(1 + p)) + c*x*Hypergeometric2F1
[1/2, -p, 3/2, (d^2*x^2)/c^2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

method	result	size
meijerg	$\frac{d\,x^{2}\,\mathrm{hypergeom}\left(\left[1,-p\right],\left[2\right],\frac{x^{2} d^{2}}{c^{2}}\right)}{2}+c x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},-p\right],\left[\frac{3}{2}\right],\frac{x^{2} d^{2}}{c^{2}}\right)$	47

input
```
int((d*x+c)*(1-1/c^2*x^2*d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2*hypergeom([1,-p],[2],1/c^2*x^2*d^2)+c*x*hypergeom([1/2,-p],[3/2]
,1/c^2*x^2*d^2)
```

Fricas [F]

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)*(1-d^2*x^2/c^2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(-(d^2*x^2 - c^2)/c^2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx$$
$$=cx\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{d^2x^2e^{2i\pi}}{c^2}\right)+d\left(\begin{cases}\frac{x^2}{2} & \text{for}\ d^2=0\\ -\frac{c^2\left(\begin{cases}\frac{\left(1-\frac{d^2x^2}{c^2}\right)^{p+1}}{p+1} & \text{for}\ p\neq -1\\ \log\left(1-\frac{d^2x^2}{c^2}\right) & \text{otherwise}\end{cases}\right)}{2d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(1-d**2*x**2/c**2)**p,x)
```

output
```
c*x*hyper((1/2, -p), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) + d*Piecewi
se((x**2/2, Eq(d**2, 0)), (-c**2*Piecewise(((1 - d**2*x**2/c**2)**(p + 1)/
(p + 1), Ne(p, -1)), (log(1 - d**2*x**2/c**2), True))/(2*d**2), True))
```

Maxima [F]

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)*(1-d^2*x^2/c^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(-d^2*x^2/c^2 + 1)^p, x)
```

Giac [F]

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)*(1-d^2*x^2/c^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(-d^2*x^2/c^2 + 1)^p, x)
```

Mupad [B] (verification not implemented)

Time = 7.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx = c\,x\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)-\frac{\left(c^2-d^2\,x^2\right)\,\left(1-\frac{d^2\,x^2}{c^2}\right)^p}{2\,d\,\left(p+1\right)}$$

input
```
int((1 - (d^2*x^2)/c^2)^p*(c + d*x),x)
```

output
```
c*x*hypergeom([1/2, -p], 3/2, (d^2*x^2)/c^2) - ((c^2 - d^2*x^2)*(1 - (d^2*
x^2)/c^2)^p)/(2*d*(p + 1))
```

Reduce [F]

$$\int (c+dx)\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx$$
$$= \frac{-2(-d^2x^2+c^2)^p\,c^2p-(-d^2x^2+c^2)^p\,c^2+2(-d^2x^2+c^2)^p\,cdpx+2(-d^2x^2+c^2)^p\,cdx+2(-d^2x^2+c^2)^p}{}$$

input
```
int((d*x+c)*(1-d^2*x^2/c^2)^p,x)
```

output
```
( - 2*(c**2 - d**2*x**2)**p*c**2*p - (c**2 - d**2*x**2)**p*c**2 + 2*(c**2
- d**2*x**2)**p*c*d*p*x + 2*(c**2 - d**2*x**2)**p*c*d*x + 2*(c**2 - d**2*x
**2)**p*d**2*p*x**2 + (c**2 - d**2*x**2)**p*d**2*x**2 + 8*int((c**2 - d**2
*x**2)**p/(2*c**2*p + c**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c**3*d*p**3 + 1
2*int((c**2 - d**2*x**2)**p/(2*c**2*p + c**2 - 2*d**2*p*x**2 - d**2*x**2),
x)*c**3*d*p**2 + 4*int((c**2 - d**2*x**2)**p/(2*c**2*p + c**2 - 2*d**2*p*x
**2 - d**2*x**2),x)*c**3*d*p)/(2*c**(2*p)*d*(2*p**2 + 3*p + 1))
```

3.350 $\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)$$

output
```
x*hypergeom([1/2, -p],[3/2],d^2*x^2/c^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)$$

input
```
Integrate[(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
x*Hypergeometric2F1[1/2, -p, 3/2, (d^2*x^2)/c^2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx$$

$$\Big\downarrow \text{ 237}$$

$$x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)$$

input
```
Int[(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
x*Hypergeometric2F1[1/2, -p, 3/2, (d^2*x^2)/c^2]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{2},-p\right],\left[\frac{3}{2}\right],\frac{x^2d^2}{c^2}\right)$	21

input
```
int((1-1/c^2*x^2*d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/2,-p],[3/2],1/c^2*x^2*d^2)
```

Fricas [F]

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx = \int \left(-\frac{d^2x^2}{c^2}+1\right)^p dx$$

input
```
integrate((1-d^2*x^2/c^2)^p,x, algorithm="fricas")
```

output
```
integral((-(d^2*x^2 - c^2)/c^2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx = x_2F_1\left(\begin{matrix} \frac{1}{2},-p \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)$$

input
```
integrate((1-d**2*x**2/c**2)**p,x)
```

output
```
x*hyper((1/2, -p), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)
```

Maxima [F]

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx = \int \left(-\frac{d^2x^2}{c^2}+1\right)^p dx$$

input
```
integrate((1-d^2*x^2/c^2)^p,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p, x)
```

Giac [F]

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx = \int \left(-\frac{d^2x^2}{c^2}+1\right)^p dx$$

input
```
integrate((1-d^2*x^2/c^2)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p, x)
```

Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)$$

input
```
int((1 - (d^2*x^2)/c^2)^p,x)
```

output
```
x*hypergeom([1/2, -p], 3/2, (d^2*x^2)/c^2)
```

Reduce [F]

$$\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx$$
$$= \frac{(-d^2x^2+c^2)^p x + 4\left(\int \frac{(-d^2x^2+c^2)^p}{-2d^2p\,x^2-d^2x^2+2c^2p+c^2}dx\right)c^2p^2 + 2\left(\int \frac{(-d^2x^2+c^2)^p}{-2d^2p\,x^2-d^2x^2+2c^2p+c^2}dx\right)c^2p}{c^{2p}(2p+1)}$$

input
```
int((1-d^2*x^2/c^2)^p,x)
```

output
```
((c**2 - d**2*x**2)**p*x + 4*int((c**2 - d**2*x**2)**p/(2*c**2*p + c**2 -
2*d**2*p*x**2 - d**2*x**2),x)*c**2*p**2 + 2*int((c**2 - d**2*x**2)**p/(2*c
**2*p + c**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c**2*p)/(c**(2*p)*(2*p + 1))
```

3.351 $\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx = \frac{2^p\left(\frac{c+dx}{c}\right)^p \text{Hypergeometric2F1}\left(-p,p,1+p,\frac{c+dx}{2c}\right)}{dp}$$

output
```
2^p*((d*x+c)/c)^p*hypergeom([p, -p],[p+1],1/2*(d*x+c)/c)/d/p
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx = -\frac{2^{-1+p}(c-dx)\left(1+\frac{dx}{c}\right)^{-p}\left(1-\frac{d^2x^2}{c^2}\right)^p \text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{c-dx}{2c}\right)}{cd(1+p)}$$

input
```
Integrate[(1 - (d^2*x^2)/c^2)^p/(c + d*x),x]
```

output
```
-((2^(-1 + p)*(c - d*x)*(1 - (d^2*x^2)/c^2)^p*Hypergeometric2F1[1 - p, 1 +
 p, 2 + p, (c - d*x)/(2*c)])/(c*d*(1 + p)*(1 + (d*x)/c)^p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {472, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx$$

$$\Big\downarrow \text{472}$$

$$\frac{\left(\frac{c-dx}{c}\right)^{p+1}\left(\frac{1}{c}-\frac{dx}{c^2}\right)^{-p-1}\int \left(\frac{1}{c}-\frac{dx}{c^2}\right)^p\left(\frac{dx}{c}+1\right)^{p-1}dx}{c^2}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-1}\left(\frac{c-dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(1-p,p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)}$$

input
```
Int[(1 - (d^2*x^2)/c^2)^p/(c + d*x),x]
```

output
```
-((2^(-1 + p)*((c - d*x)/c)^(1 + p)*Hypergeometric2F1[1 - p, 1 + p, 2 + p,
 (c - d*x)/(2*c)])/(d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 472
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^(p + 1)*c^(n - 1)*(((c - d*x)/c)^(p + 1)/(a/c + b*(x/d))^(p + 1))   Int[(
1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && GtQ[a, 0] &&  !(
IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{\left(1-\frac{x^2 d^2}{c^2}\right)^p}{dx+c}dx$$

input
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c),x)
```

output
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx=\int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{dx+c}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-(d^2*x^2 - c^2)/c^2)^p/(d*x + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 318, normalized size of antiderivative = 7.76

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx$$
$$= \begin{cases} \frac{0^p\log\left(-1+\frac{d^2x^2}{c^2}\right)}{2d}+\frac{0^p\operatorname{acoth}\left(\frac{dx}{c}\right)}{d}+\frac{c^{1-2p}d^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)}+\frac{dx^2\Gamma(p)\Gamma(1-p){}_3F_2\left(2,1\ldots\right.}{2c^2\Gamma(-p)\Gamma\ldots} \\ \frac{0^p\log\left(1-\frac{d^2x^2}{c^2}\right)}{2d}+\frac{0^p\operatorname{atanh}\left(\frac{dx}{c}\right)}{d}+\frac{c^{1-2p}d^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p,\frac{1}{2}-p\\ \frac{3}{2}-p\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)}+\frac{dx^2\Gamma(p)\Gamma(1-p){}_3F_2\left(2,1,\ldots\right.}{2c^2\Gamma(-p)\Gamma\ldots} \end{cases}$$

input
```
integrate((1-d**2*x**2/c**2)**p/(d*x+c),x)
```

output
```
Piecewise((0**p*log(-1 + d**2*x**2/c**2)/(2*d) + 0**p*acoth(d*x/c)/d + c**
(1 - 2*p)*d**(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*
hyper((1 - p, 1/2 - p), (3/2 - p,), c**2/(d**2*x**2))/(2*gamma(3/2 - p)*ga
mma(p + 1)) + d*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), d*
*2*x**2*exp_polar(2*I*pi)/c**2)/(2*c**2*gamma(-p)*gamma(p + 1)), Abs(d**2*
x**2/c**2) > 1), (0**p*log(1 - d**2*x**2/c**2)/(2*d) + 0**p*atanh(d*x/c)/d
 + c**(1 - 2*p)*d**(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2
 - p)*hyper((1 - p, 1/2 - p), (3/2 - p,), c**2/(d**2*x**2))/(2*gamma(3/2 -
 p)*gamma(p + 1)) + d*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2,
2), d**2*x**2*exp_polar(2*I*pi)/c**2)/(2*c**2*gamma(-p)*gamma(p + 1)), Tru
e))
```

Maxima [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{dx+c}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{dx+c}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx = \int \frac{\left(1-\frac{d^2\,x^2}{c^2}\right)^p}{c+d\,x}\,dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x),x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{c+dx}\,dx = \frac{\int \frac{(-d^2x^2+c^2)^p}{dx+c}dx}{c^{2p}}$$

input
```
int((1-d^2*x^2/c^2)^p/(d*x+c),x)
```

output
```
int((c**2 - d**2*x**2)**p/(c + d*x),x)/c**(2*p)
```

3.352 $\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx = -\frac{2^p\left(\frac{c+dx}{c}\right)^{-1+p}\text{Hypergeometric2F1}\left(-1+p,-p,p,\frac{c+dx}{2c}\right)}{cd(1-p)}$$

output
```
-2^p*((d*x+c)/c)^(-1+p)*hypergeom([-p, -1+p],[p],1/2*(d*x+c)/c)/c/d/(1-p)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx = -\frac{2^{-2+p}(c-dx)\left(1+\frac{dx}{c}\right)^{-p}\left(1-\frac{d^2x^2}{c^2}\right)^p\text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{c-dx}{2c}\right)}{c^2d(1+p)}$$

input
```
Integrate[(1 - (d^2*x^2)/c^2)^p/(c + d*x)^2,x]
```

output
```
-((2^(-2 + p)*(c - d*x)*(1 - (d^2*x^2)/c^2)^p*Hypergeometric2F1[2 - p, 1 +
 p, 2 + p, (c - d*x)/(2*c)])/(c^2*d*(1 + p)*(1 + (d*x)/c)^p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {472, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}dx$$
$$\Big\downarrow \text{472}$$
$$\frac{\left(\frac{c-dx}{c}\right)^{p+1}\left(\frac{1}{c}-\frac{dx}{c^2}\right)^{-p-1}\int \left(\frac{1}{c}-\frac{dx}{c^2}\right)^p\left(\frac{dx}{c}+1\right)^{p-2}dx}{c^3}$$
$$\Big\downarrow \text{79}$$
$$-\frac{2^{p-2}\left(\frac{c-dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{c-dx}{2c}\right)}{cd(p+1)}$$

input
```
Int[(1 - (d^2*x^2)/c^2)^p/(c + d*x)^2,x]
```

output
```
-((2^(-2 + p)*((c - d*x)/c)^(1 + p)*Hypergeometric2F1[2 - p, 1 + p, 2 + p,
 (c - d*x)/(2*c)])/(c*d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 472
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^(p + 1)*c^(n - 1)*(((c - d*x)/c)^(p + 1)/(a/c + b*(x/d))^(p + 1))   Int[(
1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && GtQ[a, 0] &&  !(
IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{\left(1-\frac{x^2 d^2}{c^2}\right)^p}{(dx+c)^2}dx$$

input
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^2,x)
```

output
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{(dx+c)^2}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-(d^2*x^2 - c^2)/c^2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx = \int \frac{\left(-\left(-1+\frac{dx}{c}\right)\left(1+\frac{dx}{c}\right)\right)^p}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((1-d**2*x**2/c**2)**p/(d*x+c)**2,x)
```

output
```
Integral((-(-1 + d*x/c)*(1 + d*x/c))**p/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx = \int \frac{\left(1-\frac{d^2\,x^2}{c^2}\right)^p}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^2,x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^2}\,dx$$
$$= \frac{-(-d^2x^2+c^2)^p - 2\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right) c\,d^2p - 2\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right) d^3px}{c^{2p}d\,(dx+c)}$$

input
```
int((1-d^2*x^2/c^2)^p/(d*x+c)^2,x)
```

output
```
( - (c**2 - d**2*x**2)**p - 2*int(((c**2 - d**2*x**2)**p*x)/(c**3 + c**2*d
*x - c*d**2*x**2 - d**3*x**3),x)*c*d**2*p - 2*int(((c**2 - d**2*x**2)**p*x
)/(c**3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*d**3*p*x)/(c**(2*p)*d*(c
+ d*x))
```

3.353 $\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx=-\frac{2^p\left(\frac{c+dx}{c}\right)^{-2+p}\text{Hypergeometric2F1}\left(-2+p,-p,-1+p,\frac{c+dx}{2c}\right)}{c^2d(2-p)}$$

output
```
-2^p*((d*x+c)/c)^(-2+p)*hypergeom([-p, -2+p],[-1+p],1/2*(d*x+c)/c)/c^2/d/(
2-p)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.43

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx = -\frac{2^{-3+p}(c-dx)\left(1+\frac{dx}{c}\right)^{-p}\left(1-\frac{d^2x^2}{c^2}\right)^p\text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{c-dx}{2c}\right)}{c^3d(1+p)}$$

input
```
Integrate[(1 - (d^2*x^2)/c^2)^p/(c + d*x)^3,x]
```

output
```
-((2^(-3 + p)*(c - d*x)*(1 - (d^2*x^2)/c^2)^p*Hypergeometric2F1[3 - p, 1 +
 p, 2 + p, (c - d*x)/(2*c)])/(c^3*d*(1 + p)*(1 + (d*x)/c)^p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {472, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}dx$$

$$\Big\downarrow \text{472}$$

$$\frac{\left(\frac{c-dx}{c}\right)^{p+1}\left(\frac{1}{c}-\frac{dx}{c^2}\right)^{-p-1}\int \left(\frac{1}{c}-\frac{dx}{c^2}\right)^p\left(\frac{dx}{c}+1\right)^{p-3}dx}{c^4}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-3}\left(\frac{c-dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{c-dx}{2c}\right)}{c^2d(p+1)}$$

input
```
Int[(1 - (d^2*x^2)/c^2)^p/(c + d*x)^3,x]
```

output
```
-((2^(-3 + p)*((c - d*x)/c)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p,
 (c - d*x)/(2*c)])/(c^2*d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 472
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^(p + 1)*c^(n - 1)*(((c - d*x)/c)^(p + 1)/(a/c + b*(x/d))^(p + 1))   Int[(
1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && GtQ[a, 0] && !(
IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{\left(1-\frac{x^2 d^2}{c^2}\right)^p}{(dx+c)^3}dx$$

input
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^3,x)
```

output
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{(dx+c)^3}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-(d^2*x^2 - c^2)/c^2)^p/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3
), x)
```

Sympy [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx=\int \frac{\left(-\left(-1+\frac{dx}{c}\right)\left(1+\frac{dx}{c}\right)\right)^p}{\left(c+dx\right)^3}\,dx$$

input
```
integrate((1-d**2*x**2/c**2)**p/(d*x+c)**3,x)
```

output
```
Integral((-(-1 + d*x/c)*(1 + d*x/c))**p/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx=\int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{\left(dx+c\right)^3}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx=\int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{\left(dx+c\right)^3}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx=\int \frac{\left(1-\frac{d^2\,x^2}{c^2}\right)^p}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^3,x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^3}\,dx = \frac{-(-d^2x^2+c^2)^p-2\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c^2d^2p-4\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c\,d^3px-2\left(\int \frac{}{-d^4}\right.}{2c^{2p}d\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int((1-d^2*x^2/c^2)^p/(d*x+c)^3,x)
```

output
```
( - (c**2 - d**2*x**2)**p - 2*int(((c**2 - d**2*x**2)**p*x)/(c**4 + 2*c**3
*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c**2*d**2*p - 4*int(((c**2 - d**2*x**
2)**p*x)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c*d**3*p*x - 2
*int(((c**2 - d**2*x**2)**p*x)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x
**4),x)*d**4*p*x**2)/(2*c**(2*p)*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.354 $\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx = \frac{2^p \left(\frac{c-dx}{c}\right)^{-3-p} \left(c^2-d^2x^2\right)^{3+p} \text{Hypergeometric2F1}\left(-p,3+p,4+p,\frac{c+dx}{2c}\right)}{c^3 d (3+p)}$$

output
```
2^p*((-d*x+c)/c)^(-3-p)*(-d^2*x^2+c^2)^(3+p)*hypergeom([-p, 3+p],[4+p],1/2
*(d*x+c)/c)/c^3/d/(3+p)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx = \frac{\left(c^2-d^2x^2\right)^p \left(1-\frac{d^2x^2}{c^2}\right)^{-p} \left(-3c\left(c^2-d^2x^2\right)\left(1-\frac{d^2x^2}{c^2}\right)^p + 3c^2 d(1+p)x \,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d}{}\right.\right.}{3d(1+p)}$$

input
```
Integrate[(c + d*x)^2*(c^2 - d^2*x^2)^p,x]
```

output
```
((c^2 - d^2*x^2)^p*(-3*c*(c^2 - d^2*x^2)*(1 - (d^2*x^2)/c^2)^p + 3*c^2*d*(
1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (d^2*x^2)/c^2] + d^3*(1 + p)*x^3*
Hypergeometric2F1[3/2, -p, 5/2, (d^2*x^2)/c^2]))/(3*d*(1 + p)*(1 - (d^2*x^
2)/c^2)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{ 473}$$

$$c(c-dx)^{-p-1}\left(\frac{dx}{c}+1\right)^{-p-1}\left(c^2-d^2x^2\right)^{p+1}\int (c-dx)^p\left(\frac{dx}{c}+1\right)^{p+2}dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{c2^{p+2}\left(\frac{dx}{c}+1\right)^{-p-1}\left(c^2-d^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p-2,p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)}$$

input
```
Int[(c + d*x)^2*(c^2 - d^2*x^2)^p,x]
```

output
```
-((2^(2 + p)*c*(1 + (d*x)/c)^(-1 - p)*(c^2 - d^2*x^2)^(1 + p)*Hypergeometr
ic2F1[-2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int (dx+c)^2 \left(-d^2x^2+c^2\right)^p dx$$

input
```
int((d*x+c)^2*(-d^2*x^2+c^2)^p,x)
```

output
```
int((d*x+c)^2*(-d^2*x^2+c^2)^p,x)
```

Fricas [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx = \int (dx+c)^2 \left(-d^2x^2+c^2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(-d^2*x^2 + c^2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.80

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx = c^2c^{2p}x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right) + 2cd\left(\begin{cases} \frac{x^2(c^2)^p}{2} & \text{for } d^2=0 \\ -\frac{\begin{cases} \frac{(c^2-d^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log\left(c^2-d^2x^2\right) & \text{otherwise} \end{cases}}{2d^2} & \text{otherwise} \end{cases}\right) + \frac{c^{2p}d^2x^3\,{}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| \frac{d^2x^2e^{2i\pi}}{c^2}\right)}{3}$$

input
```
integrate((d*x+c)**2*(-d**2*x**2+c**2)**p,x)
```

output
```
c**2*c**(2*p)*x*hyper((1/2, -p), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)
 + 2*c*d*Piecewise((x**2*(c**2)**p/2, Eq(d**2, 0)), (-Piecewise((((c**2 - d
**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(c**2 - d**2*x**2), True))/(2*
d**2), True)) + c**(2*p)*d**2*x**3*hyper((3/2, -p), (5/2,), d**2*x**2*exp_
polar(2*I*pi)/c**2)/3
```

Maxima [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx = \int (dx+c)^2\left(-d^2x^2+c^2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(-d^2*x^2 + c^2)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx = \int (dx+c)^2 \left(-d^2x^2+c^2\right)^p \, dx$$

input
```
integrate((d*x+c)^2*(-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(-d^2*x^2 + c^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx = \int \left(c^2-d^2\,x^2\right)^p\,(c+d\,x)^2 \, dx$$

input
```
int((c^2 - d^2*x^2)^p*(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(c^2-d^2x^2\right)^p \, dx$$
$$= \frac{-4(-d^2x^2+c^2)^p\, c^3p^2 - 8(-d^2x^2+c^2)^p\, c^3p - 3(-d^2x^2+c^2)^p\, c^3 + 3(-d^2x^2+c^2)^p\, c^2dpx + 3(-d^2x^2+c^{}}{}$$

input
```
int((d*x+c)^2*(-d^2*x^2+c^2)^p,x)
```

output

```
( - 4*(c**2 - d**2*x**2)**p*c**3*p**2 - 8*(c**2 - d**2*x**2)**p*c**3*p - 3
*(c**2 - d**2*x**2)**p*c**3 + 3*(c**2 - d**2*x**2)**p*c**2*d*p*x + 3*(c**2
 - d**2*x**2)**p*c**2*d*x + 4*(c**2 - d**2*x**2)**p*c*d**2*p**2*x**2 + 8*(
c**2 - d**2*x**2)**p*c*d**2*p*x**2 + 3*(c**2 - d**2*x**2)**p*c*d**2*x**2 +
 2*(c**2 - d**2*x**2)**p*d**3*p**2*x**3 + 3*(c**2 - d**2*x**2)**p*d**3*p*x
**3 + (c**2 - d**2*x**2)**p*d**3*x**3 + 16*int((c**2 - d**2*x**2)**p/(4*c*
*2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x*
*2),x)*c**4*d*p**5 + 80*int((c**2 - d**2*x**2)**p/(4*c**2*p**2 + 8*c**2*p
+ 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x**2),x)*c**4*d*p**4
+ 140*int((c**2 - d**2*x**2)**p/(4*c**2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*
p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x**2),x)*c**4*d*p**3 + 100*int((c**2 -
d**2*x**2)**p/(4*c**2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2
*p*x**2 - 3*d**2*x**2),x)*c**4*d*p**2 + 24*int((c**2 - d**2*x**2)**p/(4*c*
*2*p**2 + 8*c**2*p + 3*c**2 - 4*d**2*p**2*x**2 - 8*d**2*p*x**2 - 3*d**2*x*
*2),x)*c**4*d*p)/(d*(4*p**3 + 12*p**2 + 11*p + 3))
```

3.355 $\int (c+dx) \left(c^2-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int (c+dx) \left(c^2-d^2x^2\right)^p \, dx = \frac{2^p \left(\frac{c-dx}{c}\right)^{-2-p} \left(c^2-d^2x^2\right)^{2+p} \text{Hypergeometric2F1}\left(-p,2+p,3+p,\frac{c+dx}{2c}\right)}{c^2 d(2+p)}$$

output
```
2^p*((-d*x+c)/c)^(-2-p)*(-d^2*x^2+c^2)^(2+p)*hypergeom([-p, 2+p],[3+p],1/2
*(d*x+c)/c)/c^2/d/(2+p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int (c+dx) \left(c^2-d^2x^2\right)^p \, dx = -\frac{\left(c^2-d^2x^2\right)^{1+p}}{2d(1+p)} + cx\left(c^2-d^2x^2\right)^p \left(1 - \frac{d^2x^2}{c^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)$$

input
```
Integrate[(c + d*x)*(c^2 - d^2*x^2)^p,x]
```

output
```
-1/2*(c^2 - d^2*x^2)^(1 + p)/(d*(1 + p)) + (c*x*(c^2 - d^2*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^p
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {455, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx) \left(c^2-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{455}$$

$$c\int \left(c^2-d^2x^2\right)^p dx-\frac{\left(c^2-d^2x^2\right)^{p+1}}{2d(p+1)}$$

$$\Big\downarrow \text{238}$$

$$c\left(c^2-d^2x^2\right)^p\left(1-\frac{d^2x^2}{c^2}\right)^{-p}\int \left(1-\frac{d^2x^2}{c^2}\right)^p dx-\frac{\left(c^2-d^2x^2\right)^{p+1}}{2d(p+1)}$$

$$\Big\downarrow \text{237}$$

$$cx\left(c^2-d^2x^2\right)^p\left(1-\frac{d^2x^2}{c^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)-\frac{\left(c^2-d^2x^2\right)^{p+1}}{2d(p+1)}$$

input
```
Int[(c + d*x)*(c^2 - d^2*x^2)^p,x]
```

output

```
-1/2*(c^2 - d^2*x^2)^(1 + p)/(d*(1 + p)) + (c*x*(c^2 - d^2*x^2)^p*Hypergeo
metric2F1[1/2, -p, 3/2, (d^2*x^2)/c^2])/(1 - (d^2*x^2)/c^2)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

Maple [F]

$$\int (dx + c) \left(-d^2x^2 + c^2\right)^p dx$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^p,x)
```

output
```
int((d*x+c)*(-d^2*x^2+c^2)^p,x)
```

Fricas [F]

$$\int (c + dx) \left(c^2 - d^2x^2\right)^p \, dx = \int (dx + c) \left(-d^2x^2 + c^2\right)^p dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(-d^2*x^2 + c^2)^p, x)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int (c+dx)\left(c^2-d^2x^2\right)^p\,dx = cc^{2p}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\middle|\frac{d^2x^2e^{2i\pi}}{c^2}\right) + d\left(\begin{cases}\frac{x^2(c^2)^p}{2} & \text{for } d^2=0\\ -\frac{\begin{cases}\frac{(c^2-d^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log\left(c^2-d^2x^2\right) & \text{otherwise}\end{cases}}{2d^2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(-d**2*x**2+c**2)**p,x)
```

output
```
c*c**(2*p)*x*hyper((1/2, -p), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2) +
d*Piecewise((x**2*(c**2)**p/2, Eq(d**2, 0)), (-Piecewise(((c**2 - d**2*x**
2)**(p + 1)/(p + 1), Ne(p, -1)), (log(c**2 - d**2*x**2), True))/(2*d**2),
True))
```

Maxima [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^p\,dx = \int (dx+c)\left(-d^2x^2+c^2\right)^p\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(-d^2*x^2 + c^2)^p, x)
```

Giac [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^p\,dx=\int (dx+c)\left(-d^2x^2+c^2\right)^p\,dx$$

input
```
integrate((d*x+c)*(-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(-d^2*x^2 + c^2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int (c+dx)\left(c^2-d^2x^2\right)^p\,dx=\frac{c\,x\,\left(c^2-d^2\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^p}-\frac{\left(c^2-d^2\,x^2\right)^{p+1}}{2\,d\,(p+1)}$$

input
```
int((c^2 - d^2*x^2)^p*(c + d*x),x)
```

output
```
(c*x*(c^2 - d^2*x^2)^p*hypergeom([1/2, -p], 3/2, (d^2*x^2)/c^2))/(1 - (d^2
*x^2)/c^2)^p - (c^2 - d^2*x^2)^(p + 1)/(2*d*(p + 1))
```

Reduce [F]

$$\int (c+dx)\left(c^2-d^2x^2\right)^p\,dx$$
$$=\frac{-2(-d^2x^2+c^2)^p\,c^2p-(-d^2x^2+c^2)^p\,c^2+2(-d^2x^2+c^2)^p\,cdpx+2(-d^2x^2+c^2)^p\,cdx+2(-d^2x^2+c^2)^p}{}$$

input
```
int((d*x+c)*(-d^2*x^2+c^2)^p,x)
```

output
```
( - 2*(c**2 - d**2*x**2)**p*c**2*p - (c**2 - d**2*x**2)**p*c**2 + 2*(c**2
- d**2*x**2)**p*c*d*p*x + 2*(c**2 - d**2*x**2)**p*c*d*x + 2*(c**2 - d**2*x
**2)**p*d**2*p*x**2 + (c**2 - d**2*x**2)**p*d**2*x**2 + 8*int((c**2 - d**2
*x**2)**p/(2*c**2*p + c**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c**3*d*p**3 + 1
2*int((c**2 - d**2*x**2)**p/(2*c**2*p + c**2 - 2*d**2*p*x**2 - d**2*x**2),
x)*c**3*d*p**2 + 4*int((c**2 - d**2*x**2)**p/(2*c**2*p + c**2 - 2*d**2*p*x
**2 - d**2*x**2),x)*c**3*d*p)/(2*d*(2*p**2 + 3*p + 1))
```

3.356 $\int (c^2 - d^2x^2)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 70

$$\int (c^2 - d^2x^2)^p \, dx = -\frac{2^p \left(\frac{c+dx}{c}\right)^{-1-p} (c^2 - d^2x^2)^{1+p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, \frac{c-dx}{2c}\right)}{cd(1+p)}$$

output
```
-2^p*((d*x+c)/c)^(-1-p)*(-d^2*x^2+c^2)^(p+1)*hypergeom([-p, p+1],[2+p],1/2
*(-d*x+c)/c)/c/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int (c^2 - d^2x^2)^p \, dx = x(c^2 - d^2x^2)^p \left(1 - \frac{d^2x^2}{c^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, \frac{d^2x^2}{c^2}\right)$$

input
```
Integrate[(c^2 - d^2*x^2)^p,x]
```

output
```
(x*(c^2 - d^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (d^2*x^2)/c^2])/(1 -
(d^2*x^2)/c^2)^p
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c^2-d^2x^2\right)^p \, dx$$
$$\Big\downarrow \text{ 238}$$
$$\left(c^2-d^2x^2\right)^p \left(1-\frac{d^2x^2}{c^2}\right)^{-p} \int \left(1-\frac{d^2x^2}{c^2}\right)^p dx$$
$$\Big\downarrow \text{ 237}$$
$$x\left(c^2-d^2x^2\right)^p \left(1-\frac{d^2x^2}{c^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{d^2x^2}{c^2}\right)$$

input
```
Int[(c^2 - d^2*x^2)^p,x]
```

output
```
(x*(c^2 - d^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (d^2*x^2)/c^2])/(1 -
(d^2*x^2)/c^2)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(-d^2x^2+c^2\right)^p dx$$

input
```
int((-d^2*x^2+c^2)^p,x)
```

output
```
int((-d^2*x^2+c^2)^p,x)
```

Fricas [F]

$$\int \left(c^2-d^2x^2\right)^p \, dx = \int \left(-d^2x^2+c^2\right)^p dx$$

input
```
integrate((-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

$$\int \left(c^2-d^2x^2\right)^p \, dx = c^{2p}x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{d^2x^2e^{2i\pi}}{c^2}\right)$$

input
```
integrate((-d**2*x**2+c**2)**p,x)
```

output
```
c**(2*p)*x*hyper((1/2, -p), (3/2,), d**2*x**2*exp_polar(2*I*pi)/c**2)
```

Maxima [F]

$$\int \left(c^2-d^2x^2\right)^p\,dx=\int \left(-d^2x^2+c^2\right)^p\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p, x)
```

Giac [F]

$$\int \left(c^2-d^2x^2\right)^p\,dx=\int \left(-d^2x^2+c^2\right)^p\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

$$\int \left(c^2-d^2x^2\right)^p\,dx=\frac{x\,\left(c^2-d^2\,x^2\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ \frac{d^2\,x^2}{c^2}\right)}{\left(1-\frac{d^2\,x^2}{c^2}\right)^p}$$

input
```
int((c^2 - d^2*x^2)^p,x)
```

output
```
(x*(c^2 - d^2*x^2)^p*hypergeom([1/2, -p], 3/2, (d^2*x^2)/c^2))/(1 - (d^2*x
^2)/c^2)^p
```

Reduce [F]

$$\int \left(c^2-d^2x^2\right)^p\,dx$$
$$=\frac{(-d^2x^2+c^2)^p\,x+4\left(\int \frac{(-d^2x^2+c^2)^p}{-2d^2p\,x^2-d^2x^2+2c^2p+c^2}dx\right)c^2p^2+2\left(\int \frac{(-d^2x^2+c^2)^p}{-2d^2p\,x^2-d^2x^2+2c^2p+c^2}dx\right)c^2p}{2p+1}$$

input
```
int((-d^2*x^2+c^2)^p,x)
```

output
```
((c**2 - d**2*x**2)**p*x + 4*int((c**2 - d**2*x**2)**p/(2*c**2*p + c**2 -
2*d**2*p*x**2 - d**2*x**2),x)*c**2*p**2 + 2*int((c**2 - d**2*x**2)**p/(2*c
**2*p + c**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c**2*p)/(2*p + 1)
```

3.357 $\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx = \frac{2^p\left(\frac{c-dx}{c}\right)^{-p}(c^2-d^2x^2)^p \text{Hypergeometric2F1}\left(-p,p,1+p,\frac{c+dx}{2c}\right)}{dp}$$

output
```
2^p*(-d^2*x^2+c^2)^p*hypergeom([p, -p],[p+1],1/2*(d*x+c)/c)/d/p/(((-d*x+c)
/c)^p)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

$$\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx = -\frac{2^{-1+p}(c-dx)\left(1+\frac{dx}{c}\right)^{-p}(c^2-d^2x^2)^p \text{Hypergeometric2F1}\left(1-p,1+p,2+p,\frac{c-dx}{2c}\right)}{cd(1+p)}$$

input
```
Integrate[(c^2 - d^2*x^2)^p/(c + d*x),x]
```

output
```
-((2^(-1 + p)*(c - d*x)*(c^2 - d^2*x^2)^p*Hypergeometric2F1[1 - p, 1 + p,
2 + p, (c - d*x)/(2*c)])/(c*d*(1 + p)*(1 + (d*x)/c)^p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx$$

$$\Big\downarrow \ 473$$

$$\frac{(c-dx)^{-p-1}\left(\frac{dx}{c}+1\right)^{-p-1}(c^2-d^2x^2)^{p+1}\int (c-dx)^p\left(\frac{dx}{c}+1\right)^{p-1}dx}{c^2}$$

$$\Big\downarrow \ 79$$

$$-\frac{2^{p-1}\left(\frac{dx}{c}+1\right)^{-p-1}(c^2-d^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(1-p,p+1,p+2,\frac{c-dx}{2c}\right)}{c^2d(p+1)}$$

input
```
Int[(c^2 - d^2*x^2)^p/(c + d*x),x]
```

output
```
-((2^(-1 + p)*(1 + (d*x)/c)^(-1 - p)*(c^2 - d^2*x^2)^(1 + p)*Hypergeometri
c2F1[1 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c^2*d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^p}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c),x)
```

output
```
int((-d^2*x^2+c^2)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx=\int \frac{(-d^2x^2+c^2)^p}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^p/(d*x + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.11 (sec) , antiderivative size = 318, normalized size of antiderivative = 5.48

$$\int \frac{(c^2 - d^2x^2)^p}{c + dx}\,dx$$

$$= \begin{cases} \frac{0^p \log\left(-1+\frac{d^2x^2}{c^2}\right)}{2d} + \frac{0^p \operatorname{acoth}\left(\frac{dx}{c}\right)}{d} + \frac{cd^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p, \frac{1}{2}-p \\ \frac{3}{2}-p\end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{c^{2p}dx^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2, 1, \\ 2\end{matrix}\right.}{2c^2\Gamma(-p)\Gamma(p} \\ \frac{0^p \log\left(1-\frac{d^2x^2}{c^2}\right)}{2d} + \frac{0^p \operatorname{atanh}\left(\frac{dx}{c}\right)}{d} + \frac{cd^{2p-2}px^{2p-1}e^{i\pi p}\Gamma(p)\Gamma\left(\frac{1}{2}-p\right){}_2F_1\left(\begin{matrix}1-p, \frac{1}{2}-p \\ \frac{3}{2}-p\end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{2\Gamma\left(\frac{3}{2}-p\right)\Gamma(p+1)} + \frac{c^{2p}dx^2\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2, 1, 1 \\ 2,\end{matrix}\right.}{2c^2\Gamma(-p)\Gamma(p+} \end{cases}$$

input
```
integrate((-d**2*x**2+c**2)**p/(d*x+c),x)
```

output
```
Piecewise((0**p*log(-1 + d**2*x**2/c**2)/(2*d) + 0**p*acoth(d*x/c)/d + c*d
**(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((1 -
p, 1/2 - p), (3/2 - p,), c**2/(d**2*x**2))/(2*gamma(3/2 - p)*gamma(p + 1))
 + c**(2*p)*d*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), d**2
*x**2*exp_polar(2*I*pi)/c**2)/(2*c**2*gamma(-p)*gamma(p + 1)), Abs(d**2*x*
*2/c**2) > 1), (0**p*log(1 - d**2*x**2/c**2)/(2*d) + 0**p*atanh(d*x/c)/d +
 c*d**(2*p - 2)*p*x**(2*p - 1)*exp(I*pi*p)*gamma(p)*gamma(1/2 - p)*hyper((
1 - p, 1/2 - p), (3/2 - p,), c**2/(d**2*x**2))/(2*gamma(3/2 - p)*gamma(p +
 1)) + c**(2*p)*d*x**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2),
d**2*x**2*exp_polar(2*I*pi)/c**2)/(2*c**2*gamma(-p)*gamma(p + 1)), True))
```

Maxima [F]

$$\int \frac{(c^2 - d^2x^2)^p}{c + dx}\,dx = \int \frac{(-d^2x^2 + c^2)^p}{dx + c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx = \int \frac{(-d^2x^2+c^2)^p}{dx+c}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx = \int \frac{(c^2-d^2\,x^2)^p}{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^p/(c + d*x),x)
```

output
```
int((c^2 - d^2*x^2)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^p}{c+dx}\,dx = \int \frac{(-d^2x^2+c^2)^p}{dx+c}dx$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c),x)
```

output
```
int((c**2 - d**2*x**2)**p/(c + d*x),x)
```

3.358 $\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2} \, dx = -\frac{2^p c \left(\frac{c-dx}{c}\right)^{1-p} (c^2-d^2x^2)^{-1+p} \text{Hypergeometric2F1}\left(-1+p,-p,p,\frac{c+dx}{2c}\right)}{d(1-p)}$$

output
```
-2^p*c*((-d*x+c)/c)^(1-p)*(-d^2*x^2+c^2)^(-1+p)*hypergeom([-p, -1+p],[p],1
/2*(d*x+c)/c)/d/(1-p)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2} \, dx = -\frac{2^{-2+p}(c-dx)\left(1+\frac{dx}{c}\right)^{-p} (c^2-d^2x^2)^p \text{Hypergeometric2F1}\left(2-p,1+p,2+p,\frac{c-dx}{2c}\right)}{c^2d(1+p)}$$

input
```
Integrate[(c^2 - d^2*x^2)^p/(c + d*x)^2,x]
```

output
```
-((2^(-2 + p)*(c - d*x)*(c^2 - d^2*x^2)^p*Hypergeometric2F1[2 - p, 1 + p,
2 + p, (c - d*x)/(2*c)])/(c^2*d*(1 + p)*(1 + (d*x)/c)^p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 473$$

$$\frac{(c-dx)^{-p-1}\left(\frac{dx}{c}+1\right)^{-p-1}(c^2-d^2x^2)^{p+1}\int (c-dx)^p\left(\frac{dx}{c}+1\right)^{p-2}dx}{c^3}$$

$$\Big\downarrow \; 79$$

$$-\frac{2^{p-2}\left(\frac{dx}{c}+1\right)^{-p-1}(c^2-d^2x^2)^{p+1}\,\text{Hypergeometric2F1}\left(2-p,p+1,p+2,\frac{c-dx}{2c}\right)}{c^3d(p+1)}$$

input
```
Int[(c^2 - d^2*x^2)^p/(c + d*x)^2,x]
```

output
```
-((2^(-2 + p)*(1 + (d*x)/c)^(-1 - p)*(c^2 - d^2*x^2)^(1 + p)*Hypergeometri
c2F1[2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c^3*d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{(-d^2x^2 + c^2)^p}{(dx + c)^2} dx$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^2,x)
```

output
```
int((-d^2*x^2+c^2)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(c^2 - d^2x^2)^p}{(c + dx)^2} dx = \int \frac{(-d^2x^2 + c^2)^p}{(dx + c)^2} dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2}\,dx = \int \frac{(-(-c+dx)\,(c+dx))^p}{(c+dx)^2}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**p/(d*x+c)**2,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2}\,dx = \int \frac{(-d^2x^2+c^2)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2}\,dx = \int \frac{(-d^2x^2+c^2)^p}{(dx+c)^2}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2}\,dx=\int \frac{{\left(c^2-d^2\,x^2\right)}^p}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((c^2 - d^2*x^2)^p/(c + d*x)^2,x)
```

output
```
int((c^2 - d^2*x^2)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^2}\,dx = \frac{-(-d^2x^2+c^2)^p-2\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right)c\,d^2p-2\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right)d^3px}{d\,(dx+c)}$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^2,x)
```

output
```
( - (c**2 - d**2*x**2)**p - 2*int(((c**2 - d**2*x**2)**p*x)/(c**3 + c**2*d
*x - c*d**2*x**2 - d**3*x**3),x)*c*d**2*p - 2*int(((c**2 - d**2*x**2)**p*x
)/(c**3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*d**3*p*x)/(d*(c + d*x))
```

3.359 $\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3} dx = -\frac{2^p c^2 \left(\frac{c-dx}{c}\right)^{2-p} (c^2-d^2x^2)^{-2+p} \text{Hypergeometric2F1}\left(-2+p,-p,-1+p,\frac{c+dx}{2c}\right)}{d(2-p)}$$

output
```
-2^p*c^2*((-d*x+c)/c)^(2-p)*(-d^2*x^2+c^2)^(-2+p)*hypergeom([-p, -2+p],[-1
+p],1/2*(d*x+c)/c)/d/(2-p)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3} dx = -\frac{2^{-3+p}(c-dx) \left(1+\frac{dx}{c}\right)^{-p} (c^2-d^2x^2)^p \text{Hypergeometric2F1}\left(3-p,1+p,2+p,\frac{c-dx}{2c}\right)}{c^3 d(1+p)}$$

input
```
Integrate[(c^2 - d^2*x^2)^p/(c + d*x)^3,x]
```

output
```
-((2^(-3 + p)*(c - d*x)*(c^2 - d^2*x^2)^p*Hypergeometric2F1[3 - p, 1 + p,
2 + p, (c - d*x)/(2*c)])/(c^3*d*(1 + p)*(1 + (d*x)/c)^p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2 - d^2x^2)^p}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{473}$$

$$\frac{(c-dx)^{-p-1}\left(\frac{dx}{c}+1\right)^{-p-1}(c^2-d^2x^2)^{p+1}\int (c-dx)^p\left(\frac{dx}{c}+1\right)^{p-3}dx}{c^4}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-3}\left(\frac{dx}{c}+1\right)^{-p-1}(c^2-d^2x^2)^{p+1}\text{Hypergeometric2F1}\left(3-p,p+1,p+2,\frac{c-dx}{2c}\right)}{c^4d(p+1)}$$

input
```
Int[(c^2 - d^2*x^2)^p/(c + d*x)^3,x]
```

output
```
-((2^(-3 + p)*(1 + (d*x)/c)^(-1 - p)*(c^2 - d^2*x^2)^(1 + p)*Hypergeometri
c2F1[3 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c^4*d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^p}{(dx+c)^3}dx$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^3,x)
```

output
```
int((-d^2*x^2+c^2)^p/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3}\,dx=\int \frac{(-d^2x^2+c^2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^p/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3}\,dx = \int \frac{(-(-c+dx)(c+dx))^p}{(c+dx)^3}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**p/(d*x+c)**3,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3}\,dx = \int \frac{(-d^2x^2+c^2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3}\,dx = \int \frac{(-d^2x^2+c^2)^p}{(dx+c)^3}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3}\,dx = \int \frac{(c^2-d^2\,x^2)^p}{(c+d\,x)^3}\,dx$$

input
```
int((c^2 - d^2*x^2)^p/(c + d*x)^3,x)
```

output
```
int((c^2 - d^2*x^2)^p/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^3}\,dx = \frac{-(-d^2x^2+c^2)^p-2\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c^2d^2p-4\left(\int \frac{(-d^2x^2+c^2)^p x}{-d^4x^4-2c\,d^3x^3+2c^3dx+c^4}dx\right)c\,d^3px-2\left(\int \frac{}{-d^4}\right.}{2d\,(d^2x^2+2cdx+c^2)}$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^3,x)
```

output
```
( - (c**2 - d**2*x**2)**p - 2*int(((c**2 - d**2*x**2)**p*x)/(c**4 + 2*c**3
*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c**2*d**2*p - 4*int(((c**2 - d**2*x**
2)**p*x)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x**4),x)*c*d**3*p*x - 2
*int(((c**2 - d**2*x**2)**p*x)/(c**4 + 2*c**3*d*x - 2*c*d**3*x**3 - d**4*x
**4),x)*d**4*p*x**2)/(2*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.360 $\int dx(c^2-d^2x^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int dx\left(c^2-d^2x^2\right)^p\,dx=-\frac{\left(c^2-d^2x^2\right)^{1+p}}{2d(1+p)}$$

output
```
-1/2*(-d^2*x^2+c^2)^(p+1)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int dx\left(c^2-d^2x^2\right)^p\,dx=-\frac{\left(c^2-d^2x^2\right)^{1+p}}{2d(1+p)}$$

input
```
Integrate[d*x*(c^2 - d^2*x^2)^p,x]
```

output
```
-1/2*(c^2 - d^2*x^2)^(1 + p)/(d*(1 + p))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int dx\left(c^2-d^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{27}$$

$$d\int x\left(c^2-d^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{241}$$

$$-\frac{\left(c^2-d^2x^2\right)^{p+1}}{2d(p+1)}$$

input
```
Int[d*x*(c^2 - d^2*x^2)^p,x]
```

output
```
-1/2*(c^2 - d^2*x^2)^(1 + p)/(d*(1 + p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{\left(-d^{2} x^{2}+c^{2}\right)^{p+1}}{2 d (p+1)}$	27
default	$-\frac{\left(-d^{2} x^{2}+c^{2}\right)^{p+1}}{2 d (p+1)}$	27
gosper	$-\frac{(d x+c)(-d x+c)\left(-d^{2} x^{2}+c^{2}\right)^{p}}{2 d (p+1)}$	36
orering	$-\frac{(d x+c)(-d x+c)\left(-d^{2} x^{2}+c^{2}\right)^{p}}{2 d (p+1)}$	36
risch	$-\frac{\left(-d^{2} x^{2}+c^{2}\right)\left(-d^{2} x^{2}+c^{2}\right)^{p}}{2 d (p+1)}$	37
parallelrisch	$\frac{x^{2}\left(-d^{2} x^{2}+c^{2}\right)^{p} d^{2}-\left(-d^{2} x^{2}+c^{2}\right)^{p} c^{2}}{2 d (p+1)}$	52
norman	$-\frac{c^{2} \mathrm{e}^{p \ln \left(-d^{2} x^{2}+c^{2}\right)}}{2 d (p+1)}+\frac{d\, x^{2} \mathrm{e}^{p \ln \left(-d^{2} x^{2}+c^{2}\right)}}{2+2 p}$	58

input
```
int(d*x*(-d^2*x^2+c^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d^2*x^2+c^2)^(p+1)/d/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int dx\left(c^2-d^2x^2\right)^p\,dx=\frac{(d^2x^2-c^2)(-d^2x^2+c^2)^p}{2\,(dp+d)}$$

input
```
integrate(d*x*(-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
1/2*(d^2*x^2 - c^2)*(-d^2*x^2 + c^2)^p/(d*p + d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.71

$$\int dx(c^2 - d^2x^2)^p \, dx = d \left(\begin{cases} \frac{x^2}{2c^2} & \text{for } d = 0 \wedge p = -1 \\ \frac{x^2(c^2)^p}{2} & \text{for } d = 0 \\ -\frac{\log\left(-\frac{c}{d}+x\right)}{2d^2} - \frac{\log\left(\frac{c}{d}+x\right)}{2d^2} & \text{for } p = -1 \\ -\frac{c^2(c^2-d^2x^2)^p}{2d^2p+2d^2} + \frac{d^2x^2(c^2-d^2x^2)^p}{2d^2p+2d^2} & \text{otherwise} \end{cases} \right)$$

input
```
integrate(d*x*(-d**2*x**2+c**2)**p,x)
```

output
```
d*Piecewise((x**2/(2*c**2), Eq(d, 0) & Eq(p, -1)), (x**2*(c**2)**p/2, Eq(d
, 0)), (-log(-c/d + x)/(2*d**2) - log(c/d + x)/(2*d**2), Eq(p, -1)), (-c**
2*(c**2 - d**2*x**2)**p/(2*d**2*p + 2*d**2) + d**2*x**2*(c**2 - d**2*x**2)
**p/(2*d**2*p + 2*d**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int dx(c^2 - d^2x^2)^p \, dx = -\frac{(-d^2x^2 + c^2)^{p+1}}{2\,d(p+1)}$$

input
```
integrate(d*x*(-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
-1/2*(-d^2*x^2 + c^2)^(p + 1)/(d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int dx(c^2-d^2x^2)^p\,dx=-\frac{(-d^2x^2+c^2)^{p+1}}{2\,d(p+1)}$$

input
```
integrate(d*x*(-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
-1/2*(-d^2*x^2 + c^2)^(p + 1)/(d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int dx(c^2-d^2x^2)^p\,dx=-\frac{(c^2-d^2\,x^2)^{p+1}}{2\,d\,(p+1)}$$

input
```
int(d*x*(c^2 - d^2*x^2)^p,x)
```

output
```
-(c^2 - d^2*x^2)^(p + 1)/(2*d*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int dx(c^2-d^2x^2)^p\,dx=\frac{(-d^2x^2+c^2)^p\,(d^2x^2-c^2)}{2d\,(p+1)}$$

input
```
int(d*x*(-d^2*x^2+c^2)^p,x)
```

output
```
((c**2 - d**2*x**2)**p*( - c**2 + d**2*x**2))/(2*d*(p + 1))
```

3.361 $\int \left(-c(c^2-d^2x^2)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 28

$$\int \left(-c\left(c^2-d^2x^2\right)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx=-\frac{\left(c^2-d^2x^2\right)^{1+p}}{2d(1+p)}$$

output
```
-1/2*(-d^2*x^2+c^2)^(p+1)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(-c\left(c^2-d^2x^2\right)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx=-\frac{\left(c^2-d^2x^2\right)^{1+p}}{2d(1+p)}$$

input
```
Integrate[-(c*(c^2 - d^2*x^2)^p) + (c + d*x)*(c^2 - d^2*x^2)^p,x]
```

output
```
-1/2*(c^2 - d^2*x^2)^(1 + p)/(d*(1 + p))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left((c+dx)\left(c^2-d^2x^2\right)^p-c\left(c^2-d^2x^2\right)^p\right)\,dx$$

$$\Big\downarrow \, 2009$$

$$-\frac{\left(c^2-d^2x^2\right)^{p+1}}{2d(p+1)}$$

input
```
Int[-(c*(c^2 - d^2*x^2)^p) + (c + d*x)*(c^2 - d^2*x^2)^p,x]
```

output
```
-1/2*(c^2 - d^2*x^2)^(1 + p)/(d*(1 + p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{(dx+c)(-dx+c)(-d^{2}x^{2}+c^{2})^{p}}{2d(p+1)}$	36
risch	$-\frac{(-d^{2}x^{2}+c^{2})(-d^{2}x^{2}+c^{2})^{p}}{2d(p+1)}$	37
parallelrisch	$\frac{x^{2}(-d^{2}x^{2}+c^{2})^{p}d^{3}-(-d^{2}x^{2}+c^{2})^{p}c^{2}d}{2d^{2}(p+1)}$	53
norman	$-\frac{c^{2}e^{p\ln(-d^{2}x^{2}+c^{2})}}{2d(p+1)}+\frac{d\,x^{2}e^{p\ln(-d^{2}x^{2}+c^{2})}}{2+2p}$	58
orering	$-\frac{(dx+c)(-dx+c)(-c(-d^{2}x^{2}+c^{2})^{p}+(dx+c)(-d^{2}x^{2}+c^{2})^{p})}{2d^{2}(p+1)x}$	63

input
```
int(-c*(-d^2*x^2+c^2)^p+(d*x+c)*(-d^2*x^2+c^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x+c)*(-d*x+c)/d/(p+1)*(-d^2*x^2+c^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \left(-c(c^2-d^2x^2)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx=\frac{(d^2x^2-c^2)(-d^2x^2+c^2)^p}{2\,(dp+d)}$$

input
```
integrate(-c*(-d^2*x^2+c^2)^p+(d*x+c)*(-d^2*x^2+c^2)^p,x, algorithm="frica
s")
```

output
```
1/2*(d^2*x^2 - c^2)*(-d^2*x^2 + c^2)^p/(d*p + d)
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \left(-c\left(c^2-d^2x^2\right)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx$$
$$= d\left(\begin{cases} \frac{x^2(c^2)^p}{2} & \text{for } d^2=0 \\ -\frac{\begin{cases} \frac{(c^2-d^2x^2)^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log\left(c^2-d^2x^2\right) & \text{otherwise} \end{cases}}{2d^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(-c*(-d**2*x**2+c**2)**p+(d*x+c)*(-d**2*x**2+c**2)**p,x)
```

output
```
d*Piecewise((x**2*(c**2)**p/2, Eq(d**2, 0)), (-Piecewise(((c**2 - d**2*x**
2)**(p + 1)/(p + 1), Ne(p, -1)), (log(c**2 - d**2*x**2), True))/(2*d**2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \left(-c\left(c^2-d^2x^2\right)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx = \frac{(d^2x^2-c^2)e^{(p\log(dx+c)+p\log(-dx+c))}}{2\,d(p+1)}$$

input
```
integrate(-c*(-d^2*x^2+c^2)^p+(d*x+c)*(-d^2*x^2+c^2)^p,x, algorithm="maxim
a")
```

output
```
1/2*(d^2*x^2 - c^2)*e^(p*log(d*x + c) + p*log(-d*x + c))/(d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(-c\left(c^2-d^2x^2\right)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx=-\frac{\left(-d^2x^2+c^2\right)^{p+1}}{2\,d(p+1)}$$

input
```
integrate(-c*(-d^2*x^2+c^2)^p+(d*x+c)*(-d^2*x^2+c^2)^p,x, algorithm="giac"
)
```

output
```
-1/2*(-d^2*x^2 + c^2)^(p + 1)/(d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(-c\left(c^2-d^2x^2\right)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx=-\frac{\left(c^2-d^2\,x^2\right)^{p+1}}{2\,d\,\left(p+1\right)}$$

input
```
int((c^2 - d^2*x^2)^p*(c + d*x) - c*(c^2 - d^2*x^2)^p,x)
```

output
```
-(c^2 - d^2*x^2)^(p + 1)/(2*d*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \left(-c\left(c^2-d^2x^2\right)^p+(c+dx)\left(c^2-d^2x^2\right)^p\right)\,dx=\frac{\left(-d^2x^2+c^2\right)^p\left(d^2x^2-c^2\right)}{2d\left(p+1\right)}$$

input
```
int(-c*(-d^2*x^2+c^2)^p+(d*x+c)*(-d^2*x^2+c^2)^p,x)
```

output
```
((c**2 - d**2*x**2)**p*( - c**2 + d**2*x**2))/(2*d*(p + 1))
```

3.362 $\int (c+dx)^{3/2} \left(1-\frac{d^2x^2}{c^2}\right)^p dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int (c+dx)^{3/2} \left(1-\frac{d^2x^2}{c^2}\right)^p dx = \frac{2^{1+p}c^2\sqrt{c+dx}\left(1+\frac{dx}{c}\right)^{2+p} \text{Hypergeometric2F1}\left(-p,\frac{5}{2}+p,\frac{7}{2}+p,\frac{c+dx}{2c}\right)}{d(5+2p)}$$

output
```
2^(p+1)*c^2*(d*x+c)^(1/2)*(1+d*x/c)^(2+p)*hypergeom([-p, 5/2+p],[7/2+p],1/
2*(d*x+c)/c)/d/(5+2*p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.94 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.67

$$\int (c+dx)^{3/2} \left(1-\frac{d^2x^2}{c^2}\right)^p dx = \frac{\sqrt{c+dx}\left(1-\frac{dx}{c}\right)^{-p}\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-2p}\left(d^2(1+p)x^2\left(\frac{c-dx}{c^2}\right)^p(c+dx)^p\left(1+\frac{dx}{c}\right)^p \text{AppellF1}(2,-\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
(Sqrt[c + d*x]*(1 + (d*x)/c)^(-1/2 - 2*p)*(d^2*(1 + p)*x^2*((c - d*x)/c^2)
^p*(c + d*x)^p*(1 + (d*x)/c)^p*AppellF1[2, -1/2 - p, -p, 3, -((d*x)/c), (d
*x)/c] - 2^(3/2 + p)*c*(c - d*x)*(1 - (d^2*x^2)/c^2)^(2*p)*Hypergeometric2
F1[-1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)]))/(2*d*(1 + p)*(1 - (d*x)/c)^p
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {474, 456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p dx$$

$$\Big\downarrow \text{ 474}$$

$$\frac{c\sqrt{c+dx}\int \left(\frac{dx}{c}+1\right)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p dx}{\sqrt{\frac{dx}{c}+1}}$$

$$\Big\downarrow \text{ 456}$$

$$\frac{c\sqrt{c+dx}\int \left(1-\frac{dx}{c}\right)^p\left(\frac{dx}{c}+1\right)^{p+\frac{3}{2}} dx}{\sqrt{\frac{dx}{c}+1}}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{c^2 2^{p+\frac{3}{2}}\sqrt{c+dx}\left(1-\frac{dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(-p-\frac{3}{2},p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)\sqrt{\frac{dx}{c}+1}}$$

input
```
Int[(c + d*x)^(3/2)*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
-((2^(3/2 + p)*c^2*Sqrt[c + d*x]*(1 - (d*x)/c)^(1 + p)*Hypergeometric2F1[-
3/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)*Sqrt[1 + (d*x)/c]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(1-\frac{x^2d^2}{c^2}\right)^p dx$$

input
```
int((d*x+c)^(3/2)*(1-1/c^2*x^2*d^2)^p,x)
```

output
```
int((d*x+c)^(3/2)*(1-1/c^2*x^2*d^2)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(-(d^2*x^2 - c^2)/c^2)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int \left(-\left(-1+\frac{dx}{c}\right)\left(1+\frac{dx}{c}\right)\right)^p(c+dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(1-d**2*x**2/c**2)**p,x)
```

output
```
Integral((-(-1 + d*x/c)*(1 + d*x/c))**p*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(-d^2*x^2/c^2 + 1)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(-d^2*x^2/c^2 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int \left(1-\frac{d^2\,x^2}{c^2}\right)^p\,(c+d\,x)^{3/2}\,dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p*(c + d*x)^(3/2),x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\frac{32\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,c^2p^2+48\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,c^2p+6\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,c}{}$$

input
```
int((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x)
```

output
```
(2*(16*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c**2*p**2 + 24*sqrt(c + d*x)*(c
**2 - d**2*x**2)**p*c**2*p + 3*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c**2 +
4*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c*d*p*x + 6*sqrt(c + d*x)*(c**2 - d*
*2*x**2)**p*c*d*x + 4*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*d**2*p*x**2 + 3*
sqrt(c + d*x)*(c**2 - d**2*x**2)**p*d**2*x**2 + 512*int((sqrt(c + d*x)*(c*
*2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c**2 - 16*d**2*p**2*x
**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**5 + 2048*int((sqrt(c
+ d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c**2 - 16*d
**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**4 + 2912*in
t((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c
**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**3
 + 1728*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**
2*p + 15*c**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2
*d**2*p**2 + 360*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2
 + 32*c**2*p + 15*c**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2
),x)*c**2*d**2*p))/(c**(2*p)*d*(16*p**2 + 32*p + 15))
```

3.363 $\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 66

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\frac{2^{1+p}c^2\sqrt{c+dx}\left(1+\frac{dx}{c}\right)^{2+p}\text{Hypergeometric2F1}\left(-p,\frac{5}{2}+p,\frac{7}{2}+p,\frac{c+dx}{2c}\right)}{d(5+2p)}$$

output
```
2^(p+1)*c^2*(d*x+c)^(1/2)*(1+d*x/c)^(2+p)*hypergeom([-p, 5/2+p],[7/2+p],1/
2*(d*x+c)/c)/d/(5+2*p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.67

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\frac{\sqrt{c+dx}\left(1-\frac{dx}{c}\right)^{-p}\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-2p}\left(d^2(1+p)x^2\left(\frac{c-dx}{c^2}\right)^p(c+dx)^p\left(1+\frac{dx}{c}\right)^p\text{AppellF1}\left(2,-\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
(Sqrt[c + d*x]*(1 + (d*x)/c)^(-1/2 - 2*p)*(d^2*(1 + p)*x^2*((c - d*x)/c^2)
^p*(c + d*x)^p*(1 + (d*x)/c)^p*AppellF1[2, -1/2 - p, -p, 3, -((d*x)/c), (d
*x)/c] - 2^(3/2 + p)*c*(c - d*x)*(1 - (d^2*x^2)/c^2)^(2*p)*Hypergeometric2
F1[-1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)]))/(2*d*(1 + p)*(1 - (d*x)/c)^p
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {474, 456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p dx$$

$$\Big\downarrow \ 474$$

$$\frac{c\sqrt{c+dx}\int \left(\frac{dx}{c}+1\right)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p dx}{\sqrt{\frac{dx}{c}+1}}$$

$$\Big\downarrow \ 456$$

$$\frac{c\sqrt{c+dx}\int \left(1-\frac{dx}{c}\right)^p\left(\frac{dx}{c}+1\right)^{p+\frac{3}{2}}dx}{\sqrt{\frac{dx}{c}+1}}$$

$$\Big\downarrow \ 79$$

$$-\frac{c^2 2^{p+\frac{3}{2}}\sqrt{c+dx}\left(1-\frac{dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(-p-\frac{3}{2},p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)\sqrt{\frac{dx}{c}+1}}$$

input
```
Int[(c + d*x)^(3/2)*(1 - (d^2*x^2)/c^2)^p,x]
```

output
```
-((2^(3/2 + p)*c^2*Sqrt[c + d*x]*(1 - (d*x)/c)^(1 + p)*Hypergeometric2F1[-
3/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)*Sqrt[1 + (d*x)/c]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(1-\frac{x^2 d^2}{c^2}\right)^p dx$$

input
```
int((d*x+c)^(3/2)*(1-1/c^2*x^2*d^2)^p,x)
```

output
```
int((d*x+c)^(3/2)*(1-1/c^2*x^2*d^2)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(-(d^2*x^2 - c^2)/c^2)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int \left(-\left(-1+\frac{dx}{c}\right)\left(1+\frac{dx}{c}\right)\right)^p (c+dx)^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(1-d**2*x**2/c**2)**p,x)
```

output
```
Integral((-(-1 + d*x/c)*(1 + d*x/c))**p*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(-d^2*x^2/c^2 + 1)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int (dx+c)^{\frac{3}{2}}\left(-\frac{d^2x^2}{c^2}+1\right)^p\,dx$$

input
```
integrate((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(-d^2*x^2/c^2 + 1)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\int \left(1-\frac{d^2\,x^2}{c^2}\right)^p\,(c+d\,x)^{3/2}\,dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p*(c + d*x)^(3/2),x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\left(1-\frac{d^2x^2}{c^2}\right)^p\,dx=\frac{32\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,c^2p^2+48\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,c^2p+6\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,c}{}$$

input
```
int((d*x+c)^(3/2)*(1-d^2*x^2/c^2)^p,x)
```

output
```
(2*(16*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c**2*p**2 + 24*sqrt(c + d*x)*(c
**2 - d**2*x**2)**p*c**2*p + 3*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c**2 +
4*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c*d*p*x + 6*sqrt(c + d*x)*(c**2 - d*
*2*x**2)**p*c*d*x + 4*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*d**2*p*x**2 + 3*
sqrt(c + d*x)*(c**2 - d**2*x**2)**p*d**2*x**2 + 512*int((sqrt(c + d*x)*(c*
*2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c**2 - 16*d**2*p**2*x
**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**5 + 2048*int((sqrt(c
+ d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c**2 - 16*d
**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**4 + 2912*in
t((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c
**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**3
 + 1728*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**
2*p + 15*c**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2
*d**2*p**2 + 360*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2
 + 32*c**2*p + 15*c**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2
),x)*c**2*d**2*p))/(c**(2*p)*d*(16*p**2 + 32*p + 15))
```

3.364 $\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx=\frac{2^{1+p}c\left(1+\frac{dx}{c}\right)^{1+p}\text{Hypergeometric2F1}\left(-p,\frac{1}{2}+p,\frac{3}{2}+p,\frac{c+dx}{2c}\right)}{d(1+2p)\sqrt{c+dx}}$$

output
```
2^(p+1)*c*(1+d*x/c)^(p+1)*hypergeom([-p, 1/2+p],[3/2+p],1/2*(d*x+c)/c)/d/(
1+2*p)/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx =\frac{2^{-\frac{1}{2}+p}(-c+dx)\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-p}\left(1-\frac{d^2x^2}{c^2}\right)^p\text{Hypergeometric2F1}\left(\frac{1}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{d(1+p)\sqrt{c+dx}}$$

input
```
Integrate[(1 - (d^2*x^2)/c^2)^p/Sqrt[c + d*x],x]
```

output
```
(2^(-1/2 + p)*(-c + d*x)*(1 + (d*x)/c)^(1/2 - p)*(1 - (d^2*x^2)/c^2)^p*Hyp
ergeometric2F1[1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)*Sqrt[c
+ d*x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {474, 456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt{\frac{dx}{c}+1}\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{\frac{dx}{c}+1}}dx}{\sqrt{c+dx}}$$

$$\Big\downarrow \text{456}$$

$$\frac{\sqrt{\frac{dx}{c}+1}\int \left(1-\frac{dx}{c}\right)^p\left(\frac{dx}{c}+1\right)^{p-\frac{1}{2}}dx}{\sqrt{c+dx}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{c2^{p-\frac{1}{2}}\sqrt{\frac{dx}{c}+1}\left(1-\frac{dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)\sqrt{c+dx}}$$

input
```
Int[(1 - (d^2*x^2)/c^2)^p/Sqrt[c + d*x],x]
```

output
```
-((2^(-1/2 + p)*c*(1 - (d*x)/c)^(1 + p)*Sqrt[1 + (d*x)/c]*Hypergeometric2F
1[1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)*Sqrt[c + d*x]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\left(1-\frac{x^2 d^2}{c^2}\right)^p}{\sqrt{dx+c}}dx$$

input
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^(1/2),x)
```

output
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((-(d^2*x^2 - c^2)/c^2)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx = \int \frac{\left(-\left(-1+\frac{dx}{c}\right)\left(1+\frac{dx}{c}\right)\right)^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((1-d**2*x**2/c**2)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((-(-1 + d*x/c)*(1 + d*x/c))**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx=\int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx=\int \frac{\left(1-\frac{d^2\,x^2}{c^2}\right)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^(1/2),x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\sqrt{c+dx}}\,dx=\frac{2\sqrt{dx+c}\,(-d^2x^2+c^2)^p+4\left(\int \frac{\sqrt{dx+c}\,(-d^2x^2+c^2)^p x}{-d^2x^2+c^2}dx\right)d^2p}{c^{2p}d}$$

input
```
int((1-d^2*x^2/c^2)^p/(d*x+c)^(1/2),x)
```

output
```
(2*(sqrt(c + d*x)*(c**2 - d**2*x**2)**p + 2*int((sqrt(c + d*x)*(c**2 - d**
2*x**2)**p*x)/(c**2 - d**2*x**2),x)*d**2*p))/(c**(2*p)*d)
```

3.365 $\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx = -\frac{2^{1+p}\left(1+\frac{dx}{c}\right)^p \text{Hypergeometric2F1}\left(-\frac{1}{2}+p,-p,\frac{1}{2}+p,\frac{c+dx}{2c}\right)}{d(1-2p)\sqrt{c+dx}}$$

output
```
-2^(p+1)*(1+d*x/c)^p*hypergeom([-p, -1/2+p],[1/2+p],1/2*(d*x+c)/c)/d/(1-2*
p)/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.50

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx = \frac{2^{-\frac{3}{2}+p}(-c+dx)\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-p}\left(1-\frac{d^2x^2}{c^2}\right)^p \text{Hypergeometric2F1}\left(\frac{3}{2}-p,1+p,2+p,\frac{c-}{2}\right.}{cd(1+p)\sqrt{c+dx}}$$

input
```
Integrate[(1 - (d^2*x^2)/c^2)^p/(c + d*x)^(3/2),x]
```

output
```
(2^(-3/2 + p)*(-c + d*x)*(1 + (d*x)/c)^(1/2 - p)*(1 - (d^2*x^2)/c^2)^p*Hyp
ergeometric2F1[3/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c*d*(1 + p)*Sqrt[
c + d*x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {474, 456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt{\frac{dx}{c}+1}\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{\left(\frac{dx}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx}}$$

$$\Big\downarrow \text{456}$$

$$\frac{\sqrt{\frac{dx}{c}+1}\int \left(1-\frac{dx}{c}\right)^p\left(\frac{dx}{c}+1\right)^{p-\frac{3}{2}}dx}{c\sqrt{c+dx}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-\frac{3}{2}}\sqrt{\frac{dx}{c}+1}\left(1-\frac{dx}{c}\right)^{p+1}\text{Hypergeometric2F1}\left(\frac{3}{2}-p,p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)\sqrt{c+dx}}$$

input
```
Int[(1 - (d^2*x^2)/c^2)^p/(c + d*x)^(3/2),x]
```

output
```
-((2^(-3/2 + p)*(1 - (d*x)/c)^(1 + p)*Sqrt[1 + (d*x)/c]*Hypergeometric2F1[
3/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)*Sqrt[c + d*x]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{\left(1-\frac{x^2d^2}{c^2}\right)^p}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^(3/2),x)
```

output
```
int((1-1/c^2*x^2*d^2)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-(d^2*x^2 - c^2)/c^2)^p/(d^2*x^2 + 2*c*d*x + c^2),
 x)
```

Sympy [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\left(-\left(-1+\frac{dx}{c}\right)\left(1+\frac{dx}{c}\right)\right)^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-d**2*x**2/c**2)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((-(-1 + d*x/c)*(1 + d*x/c))**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx = \int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx=\int \frac{\left(-\frac{d^2x^2}{c^2}+1\right)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1-d^2*x^2/c^2)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2/c^2 + 1)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx=\int \frac{\left(1-\frac{d^2\,x^2}{c^2}\right)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^(3/2),x)
```

output
```
int((1 - (d^2*x^2)/c^2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\left(1-\frac{d^2x^2}{c^2}\right)^p}{(c+dx)^{3/2}}\,dx=\frac{-2\sqrt{dx+c}\,(-d^2x^2+c^2)^p-4\left(\int \frac{\sqrt{dx+c}\,(-d^2x^2+c^2)^p x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right)c\,d^2p-4\left(\int \frac{\sqrt{dx+c}\,(-d^2x^2+c^2)}{-d^3x^3-c\,d^2x^2+c^2dx}\right.}{c^{2p}d\,(dx+c)}$$

input
```
int((1-d^2*x^2/c^2)^p/(d*x+c)^(3/2),x)
```

output
```
(2*( - sqrt(c + d*x)*(c**2 - d**2*x**2)**p - 2*int((sqrt(c + d*x)*(c**2 -
d**2*x**2)**p*x)/(c**3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*c*d**2*p -
 2*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(c**3 + c**2*d*x - c*d**2*x
**2 - d**3*x**3),x)*d**3*p*x))/(c**(2*p)*d*(c + d*x))
```

3.366 $\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx = -\frac{2^{\frac{3}{2}+p}\sqrt{c+dx}\left(1+\frac{dx}{c}\right)^{-\frac{3}{2}-p}\left(c^2-d^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{d(1+p)}$$

output
```
-2^(3/2+p)*(d*x+c)^(1/2)*(1+d*x/c)^(-3/2-p)*(-d^2*x^2+c^2)^(p+1)*hypergeom
([p+1, -3/2-p],[2+p],1/2*(-d*x+c)/c)/d/(p+1)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.96 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.22

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx = \frac{2^{-1+p}\sqrt{c+dx}\left(1-\frac{dx}{c}\right)^{-p}\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-2p}\left(d^2(1+p)x^2(c-dx)^p(c+dx)^p\left(\frac{1}{2}+\frac{dx}{2c}\right)^p \text{AppellF1}\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*(c^2 - d^2*x^2)^p,x]
```

output
```
(2^(-1 + p)*Sqrt[c + d*x]*(1 + (d*x)/c)^(-1/2 - 2*p)*(d^2*(1 + p)*x^2*(c -
 d*x)^p*(c + d*x)^p*(1/2 + (d*x)/(2*c))^p*AppellF1[2, -p, -1/2 - p, 3, (d*
x)/c, -((d*x)/c)] - 2*Sqrt[2]*c*(c - d*x)*(c^2 - d^2*x^2)^p*(1 - (d^2*x^2)
/c^2)^p*Hypergeometric2F1[-1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)]))/(d*(1
 + p)*(1 - (d*x)/c)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{474}$$

$$\frac{c\sqrt{c+dx}\int \left(\frac{dx}{c}+1\right)^{3/2} \left(c^2-d^2x^2\right)^p dx}{\sqrt{\frac{dx}{c}+1}}$$

$$\Big\downarrow \text{473}$$

$$c\sqrt{c+dx}\left(\frac{dx}{c}+1\right)^{-p-\frac{3}{2}} \left(c^2-cdx\right)^{-p-1} \left(c^2-d^2x^2\right)^{p+1} \int \left(\frac{dx}{c}+1\right)^{p+\frac{3}{2}} \left(c^2-cdx\right)^p dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p+\frac{3}{2}}\sqrt{c+dx}\left(\frac{dx}{c}+1\right)^{-p-\frac{3}{2}} \left(c^2-d^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(-p-\frac{3}{2},p+1,p+2,\frac{c-dx}{2c}\right)}{d(p+1)}$$

input
```
Int[(c + d*x)^(3/2)*(c^2 - d^2*x^2)^p,x]
```

output
```
-((2^(3/2 + p)*Sqrt[c + d*x]*(1 + (d*x)/c)^(-3/2 - p)*(c^2 - d^2*x^2)^(1 +
 p)*Hypergeometric2F1[-3/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)
))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \left(-d^2x^2+c^2\right)^p \, dx$$

input
```
int((d*x+c)^(3/2)*(-d^2*x^2+c^2)^p,x)
```

output
```
int((d*x+c)^(3/2)*(-d^2*x^2+c^2)^p,x)
```

Fricas [F]

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx=\int (dx+c)^{\frac{3}{2}} \left(-d^2x^2+c^2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)^(3/2)*(-d^2*x^2 + c^2)^p, x)
```

Sympy [F]

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx=\int (-(-c+dx)(c+dx))^p (c+dx)^{\frac{3}{2}} \, dx$$

input
```
integrate((d*x+c)**(3/2)*(-d**2*x**2+c**2)**p,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p*(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx=\int (dx+c)^{\frac{3}{2}} \left(-d^2x^2+c^2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)*(-d^2*x^2 + c^2)^p, x)
```

Giac [F]

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx = \int (dx+c)^{\frac{3}{2}} \left(-d^2x^2+c^2\right)^p \, dx$$

input
```
integrate((d*x+c)^(3/2)*(-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*(-d^2*x^2 + c^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx = \int \left(c^2-d^2\,x^2\right)^p\,(c+d\,x)^{3/2} \, dx$$

input
```
int((c^2 - d^2*x^2)^p*(c + d*x)^(3/2),x)
```

output
```
int((c^2 - d^2*x^2)^p*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2} \left(c^2-d^2x^2\right)^p \, dx = \frac{32\sqrt{dx+c}\left(-d^2x^2+c^2\right)^p c^2p^2+48\sqrt{dx+c}\left(-d^2x^2+c^2\right)^p c^2p+6\sqrt{dx+c}\left(-d^2x^2+c^2\right)^p c^2}{}$$

input
```
int((d*x+c)^(3/2)*(-d^2*x^2+c^2)^p,x)
```

output
```
(2*(16*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c**2*p**2 + 24*sqrt(c + d*x)*(c
**2 - d**2*x**2)**p*c**2*p + 3*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c**2 +
4*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c*d*p*x + 6*sqrt(c + d*x)*(c**2 - d*
*2*x**2)**p*c*d*x + 4*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*d**2*p*x**2 + 3*
sqrt(c + d*x)*(c**2 - d**2*x**2)**p*d**2*x**2 + 512*int((sqrt(c + d*x)*(c*
*2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c**2 - 16*d**2*p**2*x
**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**5 + 2048*int((sqrt(c
+ d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c**2 - 16*d
**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**4 + 2912*in
t((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**2*p + 15*c
**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2*d**2*p**3
 + 1728*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2 + 32*c**
2*p + 15*c**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2),x)*c**2
*d**2*p**2 + 360*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(16*c**2*p**2
 + 32*c**2*p + 15*c**2 - 16*d**2*p**2*x**2 - 32*d**2*p*x**2 - 15*d**2*x**2
),x)*c**2*d**2*p))/(d*(16*p**2 + 32*p + 15))
```

3.367 $\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx = -\frac{2^{\frac{1}{2}+p}\sqrt{c+dx}\left(1+\frac{dx}{c}\right)^{-\frac{3}{2}-p}\left(c^2-d^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(-\frac{1}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{cd(1+p)}$$

output
```
-2^(1/2+p)*(d*x+c)^(1/2)*(1+d*x/c)^(-3/2-p)*(-d^2*x^2+c^2)^(p+1)*hypergeom
([p+1, -1/2-p],[2+p],1/2*(-d*x+c)/c)/c/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx = \frac{2^{\frac{1}{2}+p}(-c+dx)\sqrt{c+dx}\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-p}\left(c^2-d^2x^2\right)^{p}\text{Hypergeometric2F1}\left(-\frac{1}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{d(1+p)}$$

input
```
Integrate[Sqrt[c + d*x]*(c^2 - d^2*x^2)^p,x]
```

output
```
(2^(1/2 + p)*(-c + d*x)*Sqrt[c + d*x]*(1 + (d*x)/c)^(-1/2 - p)*(c^2 - d^2*
x^2)^p*Hypergeometric2F1[-1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 +
 p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt{c+dx}\int \sqrt{\frac{dx}{c}+1}\left(c^2-d^2x^2\right)^p dx}{\sqrt{\frac{dx}{c}+1}}$$

$$\Big\downarrow \text{473}$$

$$\sqrt{c+dx}\left(\frac{dx}{c}+1\right)^{-p-\frac{3}{2}}\left(c^2-cdx\right)^{-p-1}\left(c^2-d^2x^2\right)^{p+1}\int \left(\frac{dx}{c}+1\right)^{p+\frac{1}{2}}\left(c^2-cdx\right)^p dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p+\frac{1}{2}}\sqrt{c+dx}\left(\frac{dx}{c}+1\right)^{-p-\frac{3}{2}}\left(c^2-d^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p-\frac{1}{2},p+1,p+2,\frac{c-dx}{2c}\right)}{cd(p+1)}$$

input
```
Int[Sqrt[c + d*x]*(c^2 - d^2*x^2)^p,x]
```

output
```
-((2^(1/2 + p)*Sqrt[c + d*x]*(1 + (d*x)/c)^(-3/2 - p)*(c^2 - d^2*x^2)^(1 +
 p)*Hypergeometric2F1[-1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c*d*(1 +
p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \sqrt{dx+c}\,(-d^2x^2+c^2)^p\,dx$$

input
```
int((d*x+c)^(1/2)*(-d^2*x^2+c^2)^p,x)
```

output
```
int((d*x+c)^(1/2)*(-d^2*x^2+c^2)^p,x)
```

Fricas [F]

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx=\int \sqrt{dx+c}\left(-d^2x^2+c^2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-d^2*x^2 + c^2)^p, x)
```

Sympy [F]

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx=\int \left(-\left(-c+dx\right)\left(c+dx\right)\right)^p\sqrt{c+dx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(-d**2*x**2+c**2)**p,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p*sqrt(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx=\int \sqrt{dx+c}\left(-d^2x^2+c^2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*(-d^2*x^2 + c^2)^p, x)
```

Giac [F]

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx=\int \sqrt{dx+c}\left(-d^2x^2+c^2\right)^p\,dx$$

input
```
integrate((d*x+c)^(1/2)*(-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*(-d^2*x^2 + c^2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx=\int {\left(c^2-d^2\,x^2\right)}^p\,\sqrt{c+d\,x}\,dx$$

input
```
int((c^2 - d^2*x^2)^p*(c + d*x)^(1/2),x)
```

output
```
int((c^2 - d^2*x^2)^p*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\left(c^2-d^2x^2\right)^p\,dx$$
$$=\frac{8\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,cp+2\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,c+2\sqrt{dx+c}\,(-d^2x^2+c^2)^p\,dx+64\Big(\int \frac{\sqrt{dx+c}\,(-}{-4d^2p\,x^2-3c}}{d\,(4p\,-}$$

input
```
int((d*x+c)^(1/2)*(-d^2*x^2+c^2)^p,x)
```

output
```
(2*(4*sqrt(c + d*x)*(c**2 - d**2*x**2)**p*c*p + sqrt(c + d*x)*(c**2 - d**2
*x**2)**p*c + sqrt(c + d*x)*(c**2 - d**2*x**2)**p*d*x + 32*int((sqrt(c + d
*x)*(c**2 - d**2*x**2)**p*x)/(4*c**2*p + 3*c**2 - 4*d**2*p*x**2 - 3*d**2*x
**2),x)*c*d**2*p**3 + 40*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(4*c*
*2*p + 3*c**2 - 4*d**2*p*x**2 - 3*d**2*x**2),x)*c*d**2*p**2 + 12*int((sqrt
(c + d*x)*(c**2 - d**2*x**2)**p*x)/(4*c**2*p + 3*c**2 - 4*d**2*p*x**2 - 3*
d**2*x**2),x)*c*d**2*p))/(d*(4*p + 3))
```

3.368 $\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx = -\frac{2^{-\frac{1}{2}+p}\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-p}(c^2-d^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{cd(1+p)\sqrt{c+dx}}$$

output
```
-2^(-1/2+p)*(1+d*x/c)^(-1/2-p)*(-d^2*x^2+c^2)^(p+1)*hypergeom([p+1, 1/2-p]
,[2+p],1/2*(-d*x+c)/c)/c/d/(p+1)/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx = \frac{2^{-\frac{1}{2}+p}(-c+dx)\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-p}(c^2-d^2x^2)^p\,\text{Hypergeometric2F1}\left(\frac{1}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{d(1+p)\sqrt{c+dx}}$$

input
```
Integrate[(c^2 - d^2*x^2)^p/Sqrt[c + d*x],x]
```

output
```
(2^(-1/2 + p)*(-c + d*x)*(1 + (d*x)/c)^(1/2 - p)*(c^2 - d^2*x^2)^p*Hyperge
ometric2F1[1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p)*Sqrt[c + d*
x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \ 474$$

$$\frac{\sqrt{\frac{dx}{c}+1}\int \frac{(c^2-d^2x^2)^p}{\sqrt{\frac{dx}{c}+1}}dx}{\sqrt{c+dx}}$$

$$\Big\downarrow \ 473$$

$$\frac{\left(\frac{dx}{c}+1\right)^{-p-\frac{1}{2}}(c^2-cdx)^{-p-1}(c^2-d^2x^2)^{p+1}\int \left(\frac{dx}{c}+1\right)^{p-\frac{1}{2}}(c^2-cdx)^p\,dx}{\sqrt{c+dx}}$$

$$\Big\downarrow \ 79$$

$$-\frac{2^{p-\frac{1}{2}}\left(\frac{dx}{c}+1\right)^{-p-\frac{1}{2}}(c^2-d^2x^2)^{p+1}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,p+1,p+2,\frac{c-dx}{2c}\right)}{cd(p+1)\sqrt{c+dx}}$$

input
```
Int[(c^2 - d^2*x^2)^p/Sqrt[c + d*x],x]
```

output
```
-((2^(-1/2 + p)*(1 + (d*x)/c)^(-1/2 - p)*(c^2 - d^2*x^2)^(1 + p)*Hypergeom
etric2F1[1/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c*d*(1 + p)*Sqrt[c + d*
x]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^p}{\sqrt{dx+c}}dx$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^(1/2),x)
```

output
```
int((-d^2*x^2+c^2)^p/(d*x+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(-d^2x^2+c^2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^p/sqrt(d*x + c), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(-(-c+dx)\,(c+dx))^p}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**p/(d*x+c)**(1/2),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx=\int \frac{(-d^2x^2+c^2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(-d^2x^2+c^2)^p}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx = \int \frac{(c^2-d^2\,x^2)^p}{\sqrt{c+d\,x}}\,dx$$

input
```
int((c^2 - d^2*x^2)^p/(c + d*x)^(1/2),x)
```

output
```
int((c^2 - d^2*x^2)^p/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^p}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{dx+c}\,(-d^2x^2+c^2)^p+4\left(\int \frac{\sqrt{dx+c}\,(-d^2x^2+c^2)^p x}{-d^2x^2+c^2}dx\right)d^2p}{d}$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^(1/2),x)
```

output
```
(2*(sqrt(c + d*x)*(c**2 - d**2*x**2)**p + 2*int((sqrt(c + d*x)*(c**2 - d**
2*x**2)**p*x)/(c**2 - d**2*x**2),x)*d**2*p))/d
```

3.369 $\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2^{-\frac{3}{2}+p}\left(1+\frac{dx}{c}\right)^{-\frac{1}{2}-p}(c^2-d^2x^2)^{1+p}\,\text{Hypergeometric2F1}\left(\frac{3}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{c^2d(1+p)\sqrt{c+dx}}$$

output
```
-2^(-3/2+p)*(1+d*x/c)^(-1/2-p)*(-d^2*x^2+c^2)^(p+1)*hypergeom([p+1, 3/2-p]
,[2+p],1/2*(-d*x+c)/c)/c^2/d/(p+1)/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx = -\frac{2^{-\frac{3}{2}+p}(c-dx)\left(1+\frac{dx}{c}\right)^{\frac{1}{2}-p}(c^2-d^2x^2)^{p}\,\text{Hypergeometric2F1}\left(\frac{3}{2}-p,1+p,2+p,\frac{c-dx}{2c}\right)}{cd(1+p)\sqrt{c+dx}}$$

input
```
Integrate[(c^2 - d^2*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
-((2^(-3/2 + p)*(c - d*x)*(1 + (d*x)/c)^(1/2 - p)*(c^2 - d^2*x^2)^p*Hyperg
eometric2F1[3/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c*d*(1 + p)*Sqrt[c +
 d*x]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{474}$$

$$\frac{\sqrt{\frac{dx}{c}+1}\int \frac{(c^2-d^2x^2)^p}{\left(\frac{dx}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx}}$$

$$\Big\downarrow \text{473}$$

$$\frac{\left(\frac{dx}{c}+1\right)^{-p-\frac{1}{2}}\left(c^2-cdx\right)^{-p-1}\left(c^2-d^2x^2\right)^{p+1}\int \left(\frac{dx}{c}+1\right)^{p-\frac{3}{2}}\left(c^2-cdx\right)^p dx}{c\sqrt{c+dx}}$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{p-\frac{3}{2}}\left(\frac{dx}{c}+1\right)^{-p-\frac{1}{2}}\left(c^2-d^2x^2\right)^{p+1}\text{Hypergeometric2F1}\left(\frac{3}{2}-p,p+1,p+2,\frac{c-dx}{2c}\right)}{c^2d(p+1)\sqrt{c+dx}}$$

input
```
Int[(c^2 - d^2*x^2)^p/(c + d*x)^(3/2),x]
```

output
```
-((2^(-3/2 + p)*(1 + (d*x)/c)^(-1/2 - p)*(c^2 - d^2*x^2)^(1 + p)*Hypergeom
etric2F1[3/2 - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c^2*d*(1 + p)*Sqrt[c +
d*x]))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \frac{(-d^2x^2+c^2)^p}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^(3/2),x)
```

output
```
int((-d^2*x^2+c^2)^p/(d*x+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx=\int \frac{(-d^2x^2+c^2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*(-d^2*x^2 + c^2)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx=\int \frac{(-(-c+dx)\,(c+dx))^p}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d**2*x**2+c**2)**p/(d*x+c)**(3/2),x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p/(c + d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx=\int \frac{(-d^2x^2+c^2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(-d^2x^2+c^2)^p}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d^2*x^2+c^2)^p/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx = \int \frac{(c^2-d^2\,x^2)^p}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((c^2 - d^2*x^2)^p/(c + d*x)^(3/2),x)
```

output
```
int((c^2 - d^2*x^2)^p/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c^2-d^2x^2)^p}{(c+dx)^{3/2}}\,dx = \frac{-2\sqrt{dx+c}\,(-d^2x^2+c^2)^p - 4\left(\int \frac{\sqrt{dx+c}\,(-d^2x^2+c^2)^p x}{-d^3x^3-c\,d^2x^2+c^2dx+c^3}dx\right) c\,d^2p - 4\left(\int \frac{\sqrt{dx+c}\,(-d^2x^2+c^2)}{-d^3x^3-c\,d^2x^2+c^2dx}\right.}{d\,(dx+c)}$$

input
```
int((-d^2*x^2+c^2)^p/(d*x+c)^(3/2),x)
```

output
```
(2*( - sqrt(c + d*x)*(c**2 - d**2*x**2)**p - 2*int((sqrt(c + d*x)*(c**2 -
d**2*x**2)**p*x)/(c**3 + c**2*d*x - c*d**2*x**2 - d**3*x**3),x)*c*d**2*p -
 2*int((sqrt(c + d*x)*(c**2 - d**2*x**2)**p*x)/(c**3 + c**2*d*x - c*d**2*x
**2 - d**3*x**3),x)*d**3*p*x))/(d*(c + d*x))
```

3.370 $\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = -\frac{2^{n+p}(c+dx)^n \left(1+\frac{dx}{c}\right)^{-1-n-p} \left(c^2-d^2x^2\right)^{1+p} \text{Hypergeometric2F1}\left(-n-p,1+p,2+p,\frac{c-dx}{2c}\right)}{cd(1+p)}$$

output
```
-2^(n+p)*(d*x+c)^n*(1+d*x/c)^(-1-n-p)*(-d^2*x^2+c^2)^(p+1)*hypergeom([p+1,
 -n-p],[2+p],1/2*(-d*x+c)/c)/c/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = \frac{2^{n+p}(-c+dx)(c+dx)^n \left(1+\frac{dx}{c}\right)^{-n-p} \left(c^2-d^2x^2\right)^p \text{Hypergeometric2F1}\left(-n-p,1+p,2+p,\frac{c-dx}{2c}\right)}{d(1+p)}$$

input
```
Integrate[(c + d*x)^n*(c^2 - d^2*x^2)^p,x]
```

output
```
(2^(n + p)*(-c + d*x)*(c + d*x)^n*(1 + (d*x)/c)^(-n - p)*(c^2 - d^2*x^2)^p
*Hypergeometric2F1[-n - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(d*(1 + p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \; 474$$

$$(c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} \int \left(\frac{dx}{c}+1\right)^n \left(c^2-d^2x^2\right)^p dx$$

$$\Big\downarrow \; 473$$

$$(c+dx)^n \left(c^2-cdx\right)^{-p-1} \left(c^2-d^2x^2\right)^{p+1} \left(\frac{dx}{c}+1\right)^{-n-p-1} \int \left(\frac{dx}{c}+1\right)^{n+p} \left(c^2-cdx\right)^p dx$$

$$\Big\downarrow \; 79$$

$$-\frac{2^{n+p}(c+dx)^n \left(c^2-d^2x^2\right)^{p+1} \left(\frac{dx}{c}+1\right)^{-n-p-1} \text{Hypergeometric2F1}\left(-n-p,p+1,p+2,\frac{c-dx}{2c}\right)}{cd(p+1)}$$

input
```
Int[(c + d*x)^n*(c^2 - d^2*x^2)^p,x]
```

output
```
-((2^(n + p)*(c + d*x)^n*(1 + (d*x)/c)^(-1 - n - p)*(c^2 - d^2*x^2)^(1 + p
)*Hypergeometric2F1[-n - p, 1 + p, 2 + p, (c - d*x)/(2*c)])/(c*d*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && !Gt
Q[a, 0] && !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] && !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (dx+c)^n \left(-d^2x^2+c^2\right)^p dx$$

input
```
int((d*x+c)^n*(-d^2*x^2+c^2)^p,x)
```

output
```
int((d*x+c)^n*(-d^2*x^2+c^2)^p,x)
```

Fricas [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int (-d^2x^2+c^2)^p (dx+c)^n \, dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^p*(d*x + c)^n, x)
```

Sympy [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int (-(-c+dx)(c+dx))^p (c+dx)^n \, dx$$

input
```
integrate((d*x+c)**n*(-d**2*x**2+c**2)**p,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p*(c + d*x)**n, x)
```

Maxima [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int (-d^2x^2+c^2)^p (dx+c)^n \, dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p*(d*x + c)^n, x)
```

Giac [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int (-d^2x^2+c^2)^p (dx+c)^n \, dx$$

input
```
integrate((d*x+c)^n*(-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int \left(c^2-d^2\,x^2\right)^p \,(c+d\,x)^n \, dx$$

input
```
int((c^2 - d^2*x^2)^p*(c + d*x)^n,x)
```

output
```
int((c^2 - d^2*x^2)^p*(c + d*x)^n, x)
```

Reduce [F]

$$\int (c+dx)^n \left(c^2-d^2x^2\right)^p \, dx = \frac{(dx+c)^n \left(-d^2x^2+c^2\right)^p cn + 2(dx+c)^n \left(-d^2x^2+c^2\right)^p cp + (dx+c)^n \left(-d^2x^2+c^2\right)^p dnx + 4\Big(\int \frac{(d}{-d^2n\,x^2-}}{}$$

input
```
int((d*x+c)^n*(-d^2*x^2+c^2)^p,x)
```

output
```
((c + d*x)**n*(c**2 - d**2*x**2)**p*c*n + 2*(c + d*x)**n*(c**2 - d**2*x**2
)**p*c*p + (c + d*x)**n*(c**2 - d**2*x**2)**p*d*n*x + 4*int(((c + d*x)**n*
(c**2 - d**2*x**2)**p*x)/(c**2*n + 2*c**2*p + c**2 - d**2*n*x**2 - 2*d**2*
p*x**2 - d**2*x**2),x)*c*d**2*n**2*p + 12*int(((c + d*x)**n*(c**2 - d**2*x
**2)**p*x)/(c**2*n + 2*c**2*p + c**2 - d**2*n*x**2 - 2*d**2*p*x**2 - d**2*
x**2),x)*c*d**2*n*p**2 + 4*int(((c + d*x)**n*(c**2 - d**2*x**2)**p*x)/(c**
2*n + 2*c**2*p + c**2 - d**2*n*x**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c*d**2
*n*p + 8*int(((c + d*x)**n*(c**2 - d**2*x**2)**p*x)/(c**2*n + 2*c**2*p + c
**2 - d**2*n*x**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c*d**2*p**3 + 4*int(((c
+ d*x)**n*(c**2 - d**2*x**2)**p*x)/(c**2*n + 2*c**2*p + c**2 - d**2*n*x**2
 - 2*d**2*p*x**2 - d**2*x**2),x)*c*d**2*p**2)/(d*n*(n + 2*p + 1))
```

3.371 $\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx = \frac{2^{n+p}(c-dx)^n \left(1-\frac{dx}{c}\right)^{-1-n-p} \left(c^2-d^2x^2\right)^{1+p} \text{Hypergeometric2F1}\left(-n-p,1+p,2+p,\frac{c+dx}{2c}\right)}{cd(1+p)}$$

output
```
2^(n+p)*(-d*x+c)^n*(1-d*x/c)^(-1-n-p)*(-d^2*x^2+c^2)^(p+1)*hypergeom([p+1,
 -n-p],[2+p],1/2*(d*x+c)/c)/c/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx = \frac{2^{n+p}(c-dx)^n(c+dx) \left(1-\frac{dx}{c}\right)^{-n-p} \left(c^2-d^2x^2\right)^p \text{Hypergeometric2F1}\left(-n-p,1+p,2+p,\frac{c+dx}{2c}\right)}{d(1+p)}$$

input
```
Integrate[(c - d*x)^n*(c^2 - d^2*x^2)^p,x]
```

output
```
(2^(n + p)*(c - d*x)^n*(c + d*x)*(1 - (d*x)/c)^(-n - p)*(c^2 - d^2*x^2)^p*
Hypergeometric2F1[-n - p, 1 + p, 2 + p, (c + d*x)/(2*c)])/(d*(1 + p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{474}$$

$$(c-dx)^n \left(1-\frac{dx}{c}\right)^{-n} \int \left(1-\frac{dx}{c}\right)^n \left(c^2-d^2x^2\right)^p dx$$

$$\Big\downarrow \text{473}$$

$$(c-dx)^n \left(c^2+cdx\right)^{-p-1} \left(c^2-d^2x^2\right)^{p+1} \left(1-\frac{dx}{c}\right)^{-n-p-1} \int \left(1-\frac{dx}{c}\right)^{n+p} \left(c^2+dxc\right)^p dx$$

$$\Big\downarrow \text{79}$$

$$\frac{2^{n+p}(c-dx)^n \left(c^2-d^2x^2\right)^{p+1} \left(1-\frac{dx}{c}\right)^{-n-p-1} \text{Hypergeometric2F1}\left(-n-p,p+1,p+2,\frac{c+dx}{2c}\right)}{cd(p+1)}$$

input
```
Int[(c - d*x)^n*(c^2 - d^2*x^2)^p,x]
```

output
```
(2^(n + p)*(c - d*x)^n*(1 - (d*x)/c)^(-1 - n - p)*(c^2 - d^2*x^2)^(1 + p)*
Hypergeometric2F1[-n - p, 1 + p, 2 + p, (c + d*x)/(2*c)])/(c*d*(1 + p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (-dx+c)^n \left(-d^2x^2+c^2\right)^p dx$$

input
```
int((-d*x+c)^n*(-d^2*x^2+c^2)^p,x)
```

output
```
int((-d*x+c)^n*(-d^2*x^2+c^2)^p,x)
```

Fricas [F]

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int (-d^2x^2+c^2)^p(-dx+c)^n \, dx$$

input
```
integrate((-d*x+c)^n*(-d^2*x^2+c^2)^p,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + c^2)^p*(-d*x + c)^n, x)
```

Sympy [F]

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int \left(-\left(-c+dx\right)\left(c+dx\right)\right)^p (c-dx)^n \, dx$$

input
```
integrate((-d*x+c)**n*(-d**2*x**2+c**2)**p,x)
```

output
```
Integral((-(-c + d*x)*(c + d*x))**p*(c - d*x)**n, x)
```

Maxima [F]

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx = \int (-d^2x^2+c^2)^p(-dx+c)^n \, dx$$

input
```
integrate((-d*x+c)^n*(-d^2*x^2+c^2)^p,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + c^2)^p*(-d*x + c)^n, x)
```

Giac [F]

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx=\int \left(-d^2x^2+c^2\right)^p(-dx+c)^n \, dx$$

input
```
integrate((-d*x+c)^n*(-d^2*x^2+c^2)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + c^2)^p*(-d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx=\int \left(c^2-d^2\,x^2\right)^p\,(c-d\,x)^n \, dx$$

input
```
int((c^2 - d^2*x^2)^p*(c - d*x)^n,x)
```

output
```
int((c^2 - d^2*x^2)^p*(c - d*x)^n, x)
```

Reduce [F]

$$\int (c-dx)^n \left(c^2-d^2x^2\right)^p \, dx = \frac{-(-dx+c)^n \left(-d^2x^2+c^2\right)^p cn-2(-dx+c)^n \left(-d^2x^2+c^2\right)^p cp+(-dx+c)^n \left(-d^2x^2+c^2\right)^p dnx-4\left(\int\right.}{}$$

input
```
int((-d*x+c)^n*(-d^2*x^2+c^2)^p,x)
```

output

```
( - (c - d*x)**n*(c**2 - d**2*x**2)**p*c*n - 2*(c - d*x)**n*(c**2 - d**2*x
**2)**p*c*p + (c - d*x)**n*(c**2 - d**2*x**2)**p*d*n*x - 4*int(((c - d*x)*
*n*(c**2 - d**2*x**2)**p*x)/(c**2*n + 2*c**2*p + c**2 - d**2*n*x**2 - 2*d*
*2*p*x**2 - d**2*x**2),x)*c*d**2*n**2*p - 12*int(((c - d*x)**n*(c**2 - d**
2*x**2)**p*x)/(c**2*n + 2*c**2*p + c**2 - d**2*n*x**2 - 2*d**2*p*x**2 - d*
*2*x**2),x)*c*d**2*n*p**2 - 4*int(((c - d*x)**n*(c**2 - d**2*x**2)**p*x)/(
c**2*n + 2*c**2*p + c**2 - d**2*n*x**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c*d
**2*n*p - 8*int(((c - d*x)**n*(c**2 - d**2*x**2)**p*x)/(c**2*n + 2*c**2*p
+ c**2 - d**2*n*x**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c*d**2*p**3 - 4*int((
(c - d*x)**n*(c**2 - d**2*x**2)**p*x)/(c**2*n + 2*c**2*p + c**2 - d**2*n*x
**2 - 2*d**2*p*x**2 - d**2*x**2),x)*c*d**2*p**2)/(d*n*(n + 2*p + 1))
```

3.372 $\int (1+dx)^{3-p} \left(1-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int (1+dx)^{3-p} \left(1-d^2x^2\right)^p \, dx = -\frac{8(1-dx)^{1+p}}{d(1+p)} + \frac{12(1-dx)^{2+p}}{d(2+p)} - \frac{6(1-dx)^{3+p}}{d(3+p)} + \frac{(1-dx)^{4+p}}{d(4+p)}$$

output
```
-8*(-d*x+1)^(p+1)/d/(p+1)+12*(-d*x+1)^(2+p)/d/(2+p)-6*(-d*x+1)^(3+p)/d/(3+
p)+(-d*x+1)^(4+p)/d/(4+p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int (1+dx)^{3-p} \left(1-d^2x^2\right)^p \, dx = \frac{(-1+dx)(1+dx)^{-p} \left(1-d^2x^2\right)^p \left(6(2+dx)(p+dpx)^2+(p+dpx)^3+6\left(15+11dx+5d^2x^2+d^3x^3\right)+\right.}{d(1+p)(2+p)(3+p)(4+p)}$$

input
```
Integrate[(1 + d*x)^(3 - p)*(1 - d^2*x^2)^p,x]
```

output
```
((-1 + d*x)*(1 - d^2*x^2)^p*(6*(2 + d*x)*(p + d*p*x)^2 + (p + d*p*x)^3 + 6
*(15 + 11*d*x + 5*d^2*x^2 + d^3*x^3) + p*(53 + 93*d*x + 51*d^2*x^2 + 11*d^
3*x^3)))/(d*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(1 + d*x)^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx+1)^{3-p} \left(1-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \ 456$$

$$\int (dx+1)^3(1-dx)^p dx$$

$$\Big\downarrow \ 53$$

$$\int \left(8(1-dx)^p - 12(1-dx)^{p+1} + 6(1-dx)^{p+2} - (1-dx)^{p+3}\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{8(1-dx)^{p+1}}{d(p+1)} + \frac{12(1-dx)^{p+2}}{d(p+2)} - \frac{6(1-dx)^{p+3}}{d(p+3)} + \frac{(1-dx)^{p+4}}{d(p+4)}$$

input
```
Int[(1 + d*x)^(3 - p)*(1 - d^2*x^2)^p,x]
```

output
```
(-8*(1 - d*x)^(1 + p))/(d*(1 + p)) + (12*(1 - d*x)^(2 + p))/(d*(2 + p)) -
(6*(1 - d*x)^(3 + p))/(d*(3 + p)) + (1 - d*x)^(4 + p)/(d*(4 + p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(80) = 160$.

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

method	result
gosper	$\frac{(dx-1)\left(-d^2x^2+1\right)^p(dx+1)^{-p}\left(d^3p^3x^3+6d^3p^2x^3+11d^3p\,x^3+3d^2p^3x^2+6d^3x^3+24d^2p^2x^2+51d^2x^2p+3d\,p^3x+30d^2x^2+30d\,p^2x+93\right.}{d(p^4+10p^3+35p^2+50p+24)}$
orering	$\frac{\left(-d^2x^2+1\right)^p(dx+1)^{3-p}\left(d^3p^3x^3+6d^3p^2x^3+11d^3p\,x^3+3d^2p^3x^2+6d^3x^3+24d^2p^2x^2+51d^2x^2p+3d\,p^3x+30d^2x^2+30d\,p^2x+93pxd+\right.}{(dx+1)^3d(p^4+10p^3+35p^2+50p+24)}$

input
```
int((d*x+1)^(3-p)*(-d^2*x^2+1)^p,x,method=_RETURNVERBOSE)
```

output
```
1/d*(d*x-1)*(-d^2*x^2+1)^p*(d*x+1)^(-p)/(p^4+10*p^3+35*p^2+50*p+24)*(d^3*p
^3*x^3+6*d^3*p^2*x^3+11*d^3*p*x^3+3*d^2*p^3*x^2+6*d^3*x^3+24*d^2*p^2*x^2+5
1*d^2*p*x^2+3*d*p^3*x+30*d^2*x^2+30*d*p^2*x+93*d*p*x+p^3+66*d*x+12*p^2+53*
p+90)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(80) = 160$.

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.65

$$\int (1+dx)^{3-p}\left(1-d^2x^2\right)^p\,dx$$
$$= \frac{((d^4p^3 + 6\,d^4p^2 + 11\,d^4p + 6\,d^4)x^4 + 2\,(d^3p^3 + 9\,d^3p^2 + 20\,d^3p + 12\,d^3)x^3 - p^3 + 6\,(d^2p^2 + 7\,d^2p + 6\,d^2)x}{dp^4 + 10\,dp^3 + (d^4p^4 + 10\,d^4p^3 + 35\,d^4p^2 + 50\,d^4p + 24\,d^4)x^3 + 35\,dp^2 + 3\,(d^3p^4 + 10\,d^3p^3 + 35\,d^3p^2 +}$$

input
```
integrate((d*x+1)^(3-p)*(-d^2*x^2+1)^p,x, algorithm="fricas")
```

output
```
((d^4*p^3 + 6*d^4*p^2 + 11*d^4*p + 6*d^4)*x^4 + 2*(d^3*p^3 + 9*d^3*p^2 + 2
0*d^3*p + 12*d^3)*x^3 - p^3 + 6*(d^2*p^2 + 7*d^2*p + 6*d^2)*x^2 - 12*p^2 -
 2*(d*p^3 + 9*d*p^2 + 20*d*p - 12*d)*x - 53*p - 90)*(-d^2*x^2 + 1)^p*(d*x
+ 1)^(-p + 3)/(d*p^4 + 10*d*p^3 + (d^4*p^4 + 10*d^4*p^3 + 35*d^4*p^2 + 50*
d^4*p + 24*d^4)*x^3 + 35*d*p^2 + 3*(d^3*p^4 + 10*d^3*p^3 + 35*d^3*p^2 + 50
*d^3*p + 24*d^3)*x^2 + 50*d*p + 3*(d^2*p^4 + 10*d^2*p^3 + 35*d^2*p^2 + 50*
d^2*p + 24*d^2)*x + 24*d)
```

Sympy [F]

$$\int (1+dx)^{3-p}\left(1-d^2x^2\right)^p\,dx = \int \left(-(dx-1)\,(dx+1)\right)^p (dx+1)^{3-p}\,dx$$

input
```
integrate((d*x+1)**(3-p)*(-d**2*x**2+1)**p,x)
```

output
```
Integral((-(d*x - 1)*(d*x + 1))**p*(d*x + 1)**(3 - p), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int (1+dx)^{3-p}\left(1-d^2x^2\right)^p\,dx$$
$$=\frac{((p^3+6\,p^2+11\,p+6)d^4x^4+2\,(p^3+9\,p^2+20\,p+12)d^3x^3+6\,(p^2+7\,p+6)d^2x^2-p^3-2\,(p^3+9\,p^2+}{(p^4+10\,p^3+35\,p^2+50\,p+24)d}$$

input
```
integrate((d*x+1)^(3-p)*(-d^2*x^2+1)^p,x, algorithm="maxima")
```

output
```
((p^3 + 6*p^2 + 11*p + 6)*d^4*x^4 + 2*(p^3 + 9*p^2 + 20*p + 12)*d^3*x^3 +
6*(p^2 + 7*p + 6)*d^2*x^2 - p^3 - 2*(p^3 + 9*p^2 + 20*p - 12)*d*x - 12*p^2
 - 53*p - 90)*(-d*x + 1)^p/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*d)
```

Giac [F]

$$\int (1+dx)^{3-p}\left(1-d^2x^2\right)^p\,dx=\int \left(-d^2x^2+1\right)^p(dx+1)^{-p+3}\,dx$$

input
```
integrate((d*x+1)^(3-p)*(-d^2*x^2+1)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p + 3), x)
```

Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.54

$$\begin{aligned}\int (1+dx)^{3-p}\left(1-d^2x^2\right)^p\,dx=&\frac{(1-d^2\,x^2)^p\,(d\,x+1)^3\,(3\,d\,x-p+d\,p\,x+3)}{d\,(d\,x+1)^p\,\left(p+3\right)\,\left(p+4\right)}\\&-\frac{96\,(1-d^2\,x^2)^p}{d\,(d\,x+1)^p\,\left(p+1\right)\,\left(p+2\right)\,\left(p+3\right)\,\left(p+4\right)}\\&-\frac{12\,p\,(1-d^2\,x^2)^p\,(d\,x+1)^2}{d\,(d\,x+1)^p\,\left(p+2\right)\,\left(p+3\right)\,\left(p+4\right)}\\&-\frac{48\,p\,(1-d^2\,x^2)^p\,(d\,x+1)}{d\,(d\,x+1)^p\,\left(p+1\right)\,\left(p+2\right)\,\left(p+3\right)\,\left(p+4\right)}\end{aligned}$$

input
```
int((1 - d^2*x^2)^p*(d*x + 1)^(3 - p),x)
```

output
```
((1 - d^2*x^2)^p*(d*x + 1)^3*(3*d*x - p + d*p*x + 3))/(d*(d*x + 1)^p*(p +
3)*(p + 4)) - (96*(1 - d^2*x^2)^p)/(d*(d*x + 1)^p*(p + 1)*(p + 2)*(p + 3)*
(p + 4)) - (12*p*(1 - d^2*x^2)^p*(d*x + 1)^2)/(d*(d*x + 1)^p*(p + 2)*(p +
3)*(p + 4)) - (48*p*(1 - d^2*x^2)^p*(d*x + 1))/(d*(d*x + 1)^p*(p + 1)*(p +
 2)*(p + 3)*(p + 4))
```

Reduce [F]

$$\int (1+dx)^{3-p} \left(1-d^2x^2\right)^p \,dx = \int \frac{\left(-d^2x^2+1\right)^p}{(dx+1)^p}dx + \left(\int \frac{\left(-d^2x^2+1\right)^p x^3}{(dx+1)^p}dx\right) d^3 + 3\left(\int \frac{\left(-d^2x^2+1\right)^p x^2}{(dx+1)^p}dx\right) d^2 + 3\left(\int \frac{\left(-d^2x^2+1\right)^p x}{(dx+1)^p}dx\right) d$$

input
```
int((d*x+1)^(3-p)*(-d^2*x^2+1)^p,x)
```

output
```
int(( - d**2*x**2 + 1)**p/(d*x + 1)**p,x) + int((( - d**2*x**2 + 1)**p*x**
3)/(d*x + 1)**p,x)*d**3 + 3*int((( - d**2*x**2 + 1)**p*x**2)/(d*x + 1)**p,
x)*d**2 + 3*int((( - d**2*x**2 + 1)**p*x)/(d*x + 1)**p,x)*d
```

3.373 $\int (1+dx)^{2-p} \left(1-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 61

$$\int (1+dx)^{2-p} \left(1-d^2x^2\right)^p \, dx = -\frac{4(1-dx)^{1+p}}{d(1+p)} + \frac{4(1-dx)^{2+p}}{d(2+p)} - \frac{(1-dx)^{3+p}}{d(3+p)}$$

output
```
-4*(-d*x+1)^(p+1)/d/(p+1)+4*(-d*x+1)^(2+p)/d/(2+p)-(-d*x+1)^(3+p)/d/(3+p)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int (1+dx)^{2-p} \left(1-d^2x^2\right)^p \, dx = \frac{(-1+dx)(1+dx)^{-p} \left(1-d^2x^2\right)^p \left((p+dpx)^2+2(7+4dx+d^2x^2)+p(7+10dx+3d^2x^2)\right)}{d(1+p)(2+p)(3+p)}$$

input
```
Integrate[(1 + d*x)^(2 - p)*(1 - d^2*x^2)^p,x]
```

output
```
((-1 + d*x)*(1 - d^2*x^2)^p*((p + d*p*x)^2 + 2*(7 + 4*d*x + d^2*x^2) + p*(
7 + 10*d*x + 3*d^2*x^2)))/(d*(1 + p)*(2 + p)*(3 + p)*(1 + d*x)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx+1)^{2-p}\left(1-d^2x^2\right)^p\,dx$$

$$\Big\downarrow \ 456$$

$$\int (dx+1)^2(1-dx)^p dx$$

$$\Big\downarrow \ 53$$

$$\int \left(4(1-dx)^p-4(1-dx)^{p+1}+(1-dx)^{p+2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4(1-dx)^{p+1}}{d(p+1)}+\frac{4(1-dx)^{p+2}}{d(p+2)}-\frac{(1-dx)^{p+3}}{d(p+3)}$$

input
```
Int[(1 + d*x)^(2 - p)*(1 - d^2*x^2)^p,x]
```

output
```
(-4*(1 - d*x)^(1 + p))/(d*(1 + p)) + (4*(1 - d*x)^(2 + p))/(d*(2 + p)) - (
1 - d*x)^(3 + p)/(d*(3 + p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

method	result	size
gosper	$\frac{(dx-1)\left(-d^2x^2+1\right)^p (dx+1)^{-p} \left(d^2p^2x^2+3d^2x^2p+2d^2x^2+2d\,p^2x+10pxd+8dx+p^2+7p+14\right)}{d\left(p^3+6p^2+11p+6\right)}$	97
orering	$\frac{(dx-1)\left(d^2p^2x^2+3d^2x^2p+2d^2x^2+2d\,p^2x+10pxd+8dx+p^2+7p+14\right)(dx+1)^{-p+2}\left(-d^2x^2+1\right)^p}{(dx+1)^2 d\left(p^3+6p^2+11p+6\right)}$	106

input
```
int((d*x+1)^(-p+2)*(-d^2*x^2+1)^p,x,method=_RETURNVERBOSE)
```

output
```
1/d*(d*x-1)*(-d^2*x^2+1)^p*(d*x+1)^(-p)/(p^3+6*p^2+11*p+6)*(d^2*p^2*x^2+3*
d^2*p*x^2+2*d^2*x^2+2*d*p^2*x+10*d*p*x+8*d*x+p^2+7*p+14)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(61) = 122.

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.92

$$\int (1+dx)^{2-p}\left(1-d^2x^2\right)^p\,dx$$
$$=\frac{((d^3p^2+3\,d^3p+2\,d^3)x^3+(d^2p^2+7\,d^2p+6\,d^2)x^2-p^2-(dp^2+3\,dp-6\,d)x-7\,p-14)(-d^2x^2+1)^p(d}{dp^3+6\,dp^2+(d^3p^3+6\,d^3p^2+11\,d^3p+6\,d^3)x^2+11\,dp+2\,(d^2p^3+6\,d^2p^2+11\,d^2p+6\,d^2)x+}$$

input
```
integrate((d*x+1)^(2-p)*(-d^2*x^2+1)^p,x, algorithm="fricas")
```

output
```
((d^3*p^2 + 3*d^3*p + 2*d^3)*x^3 + (d^2*p^2 + 7*d^2*p + 6*d^2)*x^2 - p^2 -
 (d*p^2 + 3*d*p - 6*d)*x - 7*p - 14)*(-d^2*x^2 + 1)^p*(d*x + 1)^(-p + 2)/(
d*p^3 + 6*d*p^2 + (d^3*p^3 + 6*d^3*p^2 + 11*d^3*p + 6*d^3)*x^2 + 11*d*p +
2*(d^2*p^3 + 6*d^2*p^2 + 11*d^2*p + 6*d^2)*x + 6*d)
```

Sympy [F]

$$\int (1+dx)^{2-p}\left(1-d^2x^2\right)^p\,dx=\int\left(-(dx-1)\,(dx+1)\right)^p(dx+1)^{2-p}\,dx$$

input
```
integrate((d*x+1)**(2-p)*(-d**2*x**2+1)**p,x)
```

output
```
Integral((-(d*x - 1)*(d*x + 1))**p*(d*x + 1)**(2 - p), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int (1+dx)^{2-p}\left(1-d^2x^2\right)^p\,dx$$
$$=\frac{((p^2+3\,p+2)d^3x^3+(p^2+7\,p+6)d^2x^2-(p^2+3\,p-6)dx-p^2-7\,p-14)(-dx+1)^p}{(p^3+6\,p^2+11\,p+6)d}$$

input
```
integrate((d*x+1)^(2-p)*(-d^2*x^2+1)^p,x, algorithm="maxima")
```

output
```
((p^2 + 3*p + 2)*d^3*x^3 + (p^2 + 7*p + 6)*d^2*x^2 - (p^2 + 3*p - 6)*d*x -
 p^2 - 7*p - 14)*(-d*x + 1)^p/((p^3 + 6*p^2 + 11*p + 6)*d)
```

Giac [F]

$$\int (1+dx)^{2-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-d^2x^2+1\right)^p (dx+1)^{-p+2} \, dx$$

input
```
integrate((d*x+1)^(2-p)*(-d^2*x^2+1)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p + 2), x)
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.36

$$\int (1+dx)^{2-p} \left(1-d^2x^2\right)^p \, dx = \frac{(1-d^2\,x^2)^p\,(d\,x+1)^2\,(2\,d\,x-p+d\,p\,x+2)}{d\,(d\,x+1)^p\,(p+2)\,(p+3)} - \frac{16\,(1-d^2\,x^2)^p}{d\,(d\,x+1)^p\,(p+1)\,(p+2)\,(p+3)} - \frac{8\,p\,(1-d^2\,x^2)^p\,(d\,x+1)}{d\,(d\,x+1)^p\,(p+1)\,(p+2)\,(p+3)}$$

input
```
int((1 - d^2*x^2)^p*(d*x + 1)^(2 - p),x)
```

output
```
((1 - d^2*x^2)^p*(d*x + 1)^2*(2*d*x - p + d*p*x + 2))/(d*(d*x + 1)^p*(p + 
2)*(p + 3)) - (16*(1 - d^2*x^2)^p)/(d*(d*x + 1)^p*(p + 1)*(p + 2)*(p + 3))
 - (8*p*(1 - d^2*x^2)^p*(d*x + 1))/(d*(d*x + 1)^p*(p + 1)*(p + 2)*(p + 3))
```

Reduce [F]

$$\int (1+dx)^{2-p}\left(1-d^2x^2\right)^p\,dx = \int \frac{(-d^2x^2+1)^p}{(dx+1)^p}dx + \left(\int \frac{(-d^2x^2+1)^p\,x^2}{(dx+1)^p}dx\right)d^2 + 2\left(\int \frac{(-d^2x^2+1)^p\,x}{(dx+1)^p}dx\right)d$$

input
```
int((d*x+1)^(2-p)*(-d^2*x^2+1)^p,x)
```

output
```
int(( - d**2*x**2 + 1)**p/(d*x + 1)**p,x) + int((( - d**2*x**2 + 1)**p*x**
2)/(d*x + 1)**p,x)*d**2 + 2*int((( - d**2*x**2 + 1)**p*x)/(d*x + 1)**p,x)*
d
```

3.374 $\int (1+dx)^{1-p} \left(1-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int (1+dx)^{1-p} \left(1-d^2x^2\right)^p \, dx = -\frac{2(1-dx)^{1+p}}{d(1+p)} + \frac{(1-dx)^{2+p}}{d(2+p)}$$

output
```
-2*(-d*x+1)^(p+1)/d/(p+1)+(-d*x+1)^(2+p)/d/(2+p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int (1+dx)^{1-p} \left(1-d^2x^2\right)^p \, dx = \frac{(-1+dx)(1+dx)^{-p}(3+p+dx+dpx)\left(1-d^2x^2\right)^p}{d(1+p)(2+p)}$$

input
```
Integrate[(1 + d*x)^(1 - p)*(1 - d^2*x^2)^p,x]
```

output
```
((-1 + d*x)*(3 + p + d*x + d*p*x)*(1 - d^2*x^2)^p)/(d*(1 + p)*(2 + p)*(1 +
 d*x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {456, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx+1)^{1-p} \left(1-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{456}$$

$$\int (dx+1)(1-dx)^p dx$$

$$\Big\downarrow \text{53}$$

$$\int \left(2(1-dx)^p-(1-dx)^{p+1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-dx)^{p+2}}{d(p+2)}-\frac{2(1-dx)^{p+1}}{d(p+1)}$$

input
```
Int[(1 + d*x)^(1 - p)*(1 - d^2*x^2)^p,x]
```

output
```
(-2*(1 - d*x)^(1 + p))/(d*(1 + p)) + (1 - d*x)^(2 + p)/(d*(2 + p))
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

method	result
gosper	$\frac{(dx-1)\left(-d^{2}x^{2}+1\right)^{p}(dx+1)^{-p}(pxd+dx+p+3)}{d\left(p^{2}+3p+2\right)}$
orering	$\frac{\left(-d^{2}x^{2}+1\right)^{p}(dx+1)^{1-p}(pxd+dx+p+3)(dx-1)}{(dx+1)d\left(p^{2}+3p+2\right)}$
parallelrisch	$\frac{x^{2}\left(-d^{2}x^{2}+1\right)^{p}(dx+1)^{1-p}d^{2}p+x^{2}\left(-d^{2}x^{2}+1\right)^{p}(dx+1)^{1-p}d^{2}+2(dx+1)^{1-p}\left(-d^{2}x^{2}+1\right)^{p}xd-\left(-d^{2}x^{2}+1\right)^{p}(dx+1)^{1-p}p-3(d}{(dx+1)d\left(p^{2}+3p+2\right)}$

input
```
int((d*x+1)^(1-p)*(-d^2*x^2+1)^p,x,method=_RETURNVERBOSE)
```

output
```
1/d*(d*x-1)*(-d^2*x^2+1)^p*(d*x+1)^(-p)/(p^2+3*p+2)*(d*p*x+d*x+p+3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int (1+dx)^{1-p}\left(1-d^{2}x^{2}\right)^{p}\,dx=\frac{\left((d^{2}p+d^{2})x^{2}+2\,dx-p-3\right)(-d^{2}x^{2}+1)^{p}(dx+1)^{-p+1}}{dp^{2}+3\,dp+(d^{2}p^{2}+3\,d^{2}p+2\,d^{2})x+2\,d}$$

input
```
integrate((d*x+1)^(1-p)*(-d^2*x^2+1)^p,x, algorithm="fricas")
```

output
```
((d^2*p + d^2)*x^2 + 2*d*x - p - 3)*(-d^2*x^2 + 1)^p*(d*x + 1)^(-p + 1)/(d
*p^2 + 3*d*p + (d^2*p^2 + 3*d^2*p + 2*d^2)*x + 2*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(27) = 54$.

Time = 0.54 (sec) , antiderivative size = 369, normalized size of antiderivative = 9.22

$$\int (1+dx)^{1-p} (1-d^2x^2)^p \, dx$$

$$= \begin{cases} x \\ 0^p 0^{1-p} x \\ \frac{dx \log\left(x-\frac{1}{d}\right)}{d^2x-d} - \frac{\log\left(x-\frac{1}{d}\right)}{d^2x-d} - \frac{2}{d^2x-d} \\ -x - \frac{2\log\left(x-\frac{1}{d}\right)}{d} \\ \frac{d^2px^2(dx+1)^{1-p}(-d^2x^2+1)^p}{d^2p^2x+3d^2px+2d^2x+dp^2+3dp+2d} + \frac{d^2x^2(dx+1)^{1-p}(-d^2x^2+1)^p}{d^2p^2x+3d^2px+2d^2x+dp^2+3dp+2d} + \frac{2dx(dx+1)^{1-p}(-d^2x^2+1)^p}{d^2p^2x+3d^2px+2d^2x+dp^2+3dp+2d} - \frac{p(dx+1)^{1-p}(\cdots}{d^2p^2x+3d^2px+2d^2\cdots} \end{cases}$$

input
```
integrate((d*x+1)**(1-p)*(-d**2*x**2+1)**p,x)
```

output
```
Piecewise((x, Eq(d, 0)), (0**p*0**(1 - p)*x, Eq(d, -1/x)), (d*x*log(x - 1/
d)/(d**2*x - d) - log(x - 1/d)/(d**2*x - d) - 2/(d**2*x - d), Eq(p, -2)),
(-x - 2*log(x - 1/d)/d, Eq(p, -1)), (d**2*p*x**2*(d*x + 1)**(1 - p)*(-d**2
*x**2 + 1)**p/(d**2*p**2*x + 3*d**2*p*x + 2*d**2*x + d*p**2 + 3*d*p + 2*d)
 + d**2*x**2*(d*x + 1)**(1 - p)*(-d**2*x**2 + 1)**p/(d**2*p**2*x + 3*d**2*
p*x + 2*d**2*x + d*p**2 + 3*d*p + 2*d) + 2*d*x*(d*x + 1)**(1 - p)*(-d**2*x
**2 + 1)**p/(d**2*p**2*x + 3*d**2*p*x + 2*d**2*x + d*p**2 + 3*d*p + 2*d) -
 p*(d*x + 1)**(1 - p)*(-d**2*x**2 + 1)**p/(d**2*p**2*x + 3*d**2*p*x + 2*d*
*2*x + d*p**2 + 3*d*p + 2*d) - 3*(d*x + 1)**(1 - p)*(-d**2*x**2 + 1)**p/(d
**2*p**2*x + 3*d**2*p*x + 2*d**2*x + d*p**2 + 3*d*p + 2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int (1+dx)^{1-p} \left(1-d^2x^2\right)^p \, dx = \frac{(d^2(p+1)x^2 + 2\,dx - p - 3)(-dx+1)^p}{(p^2+3\,p+2)d}$$

input
```
integrate((d*x+1)^(1-p)*(-d^2*x^2+1)^p,x, algorithm="maxima")
```

output
```
(d^2*(p + 1)*x^2 + 2*d*x - p - 3)*(-d*x + 1)^p/((p^2 + 3*p + 2)*d)
```

Giac [F]

$$\int (1+dx)^{1-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-d^2x^2+1\right)^p (dx+1)^{-p+1} \, dx$$

input
```
integrate((d*x+1)^(1-p)*(-d^2*x^2+1)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p + 1), x)
```

Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int (1+dx)^{1-p} \left(1-d^2x^2\right)^p \, dx$$
$$= \frac{\left(1-d^2\,x^2\right)^p \left(\frac{2\,x\,(d\,x+1)^{1-p}}{d\,(p^2+3\,p+2)} - \frac{(d\,x+1)^{1-p}\,(p+3)}{d^2\,(p^2+3\,p+2)} + \frac{x^2\,(d\,x+1)^{1-p}\,(p+1)}{p^2+3\,p+2}\right)}{x+\frac{1}{d}}$$

input
```
int((1 - d^2*x^2)^p*(d*x + 1)^(1 - p),x)
```

output
```
((1 - d^2*x^2)^p*((2*x*(d*x + 1)^(1 - p))/(d*(3*p + p^2 + 2)) - ((d*x + 1)
^(1 - p)*(p + 3))/(d^2*(3*p + p^2 + 2)) + (x^2*(d*x + 1)^(1 - p)*(p + 1))/
(3*p + p^2 + 2)))/(x + 1/d)
```

Reduce [F]

$$\int (1+dx)^{1-p}\left(1-d^2x^2\right)^p\,dx=\int \frac{(-d^2x^2+1)^p}{(dx+1)^p}dx+\left(\int \frac{(-d^2x^2+1)^p\,x}{(dx+1)^p}dx\right)d$$

input
```
int((d*x+1)^(1-p)*(-d^2*x^2+1)^p,x)
```

output
```
int(( - d**2*x**2 + 1)**p/(d*x + 1)**p,x) + int((( - d**2*x**2 + 1)**p*x)/
(d*x + 1)**p,x)*d
```

3.375 $\int (1+dx)^{-p} \left(1-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 20

$$\int (1+dx)^{-p} \left(1-d^2x^2\right)^p \, dx = -\frac{(1-dx)^{1+p}}{d(1+p)}$$

output
```
-(-d*x+1)^(p+1)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int (1+dx)^{-p} \left(1-d^2x^2\right)^p \, dx = \frac{(-1+dx)(1+dx)^{-p} \left(1-d^2x^2\right)^p}{d(1+p)}$$

input
```
Integrate[(1 - d^2*x^2)^p/(1 + d*x)^p,x]
```

output
```
((-1 + d*x)*(1 - d^2*x^2)^p)/(d*(1 + p)*(1 + d*x)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx+1)^{-p}\left(1-d^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{456}$$

$$\int (1-dx)^p dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{(1-dx)^{p+1}}{d(p+1)}$$

input
```
Int[(1 - d^2*x^2)^p/(1 + d*x)^p,x]
```

output
```
-((1 - d*x)^(1 + p)/(d*(1 + p)))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

method	result
gosper	$\frac{(dx-1)\left(-d^{2}x^{2}+1\right)^{p}(dx+1)^{-p}}{d(p+1)}$
orering	$\frac{(dx-1)\left(-d^{2}x^{2}+1\right)^{p}(dx+1)^{-p}}{d(p+1)}$
parallelrisch	$\frac{\left(x\left(-d^{2}x^{2}+1\right)^{p}d-\left(-d^{2}x^{2}+1\right)^{p}\right)(dx+1)^{-p}}{d(p+1)}$
norman	$\left(\frac{x\,\mathrm{e}^{p\ln\left(-d^{2}x^{2}+1\right)}}{p+1}-\frac{\mathrm{e}^{p\ln\left(-d^{2}x^{2}+1\right)}}{d(p+1)}\right)\mathrm{e}^{-\ln(dx+1)p}$
risch	$\frac{(dx-1)(dx-1)^{p}\mathrm{e}^{-\frac{ip\pi\left(-\mathrm{csgn}(i(dx-1)(dx+1))^{3}-\mathrm{csgn}(i(dx-1)(dx+1))^{2}\mathrm{csgn}(i(dx-1))-\mathrm{csgn}(i(dx-1)(dx+1))^{2}\mathrm{csgn}(i(dx+1))+\mathrm{csgn}(i(d\ldots\right.}{2}}}{d(p+1)}$

input
```
int((-d^2*x^2+1)^p/((d*x+1)^p),x,method=_RETURNVERBOSE)
```

output
```
(d*x-1)/d/(p+1)*(-d^2*x^2+1)^p/((d*x+1)^p)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int(1+dx)^{-p}\left(1-d^{2}x^{2}\right)^{p}\,dx=\frac{(dx-1)(-d^{2}x^{2}+1)^{p}}{(dp+d)(dx+1)^{p}}$$

input
```
integrate((-d^2*x^2+1)^p/((d*x+1)^p),x, algorithm="fricas")
```

output
```
(d*x - 1)*(-d^2*x^2 + 1)^p/((d*p + d)*(d*x + 1)^p)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(14) = 28$.

Time = 3.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.80

$$\int (1+dx)^{-p} \left(1-d^2x^2\right)^p \, dx = \begin{cases} x & \text{for } d = 0 \wedge (d = 0 \vee p = -1) \\ -\frac{\log\left(x-\frac{1}{d}\right)}{d} & \text{for } p = -1 \\ \frac{dx(-d^2x^2+1)^p}{dp(dx+1)^p+d(dx+1)^p} - \frac{(-d^2x^2+1)^p}{dp(dx+1)^p+d(dx+1)^p} & \text{otherwise} \end{cases}$$

input
```
integrate((-d**2*x**2+1)**p/((d*x+1)**p),x)
```

output
```
Piecewise((x, Eq(d, 0) & (Eq(d, 0) | Eq(p, -1))), (-log(x - 1/d)/d, Eq(p,
-1)), (d*x*(-d**2*x**2 + 1)**p/(d*p*(d*x + 1)**p + d*(d*x + 1)**p) - (-d**
2*x**2 + 1)**p/(d*p*(d*x + 1)**p + d*(d*x + 1)**p), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (1+dx)^{-p} \left(1-d^2x^2\right)^p \, dx = \frac{(dx-1)(-dx+1)^p}{d(p+1)}$$

input
```
integrate((-d^2*x^2+1)^p/((d*x+1)^p),x, algorithm="maxima")
```

output
```
(d*x - 1)*(-d*x + 1)^p/(d*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.50

$$\int (1+dx)^{-p}\left(1-d^2x^2\right)^p\,dx = \frac{\frac{dxe^{(p\log(dx+1)+p\log(-dx+1))}}{(dx+1)^p} - \frac{e^{(p\log(dx+1)+p\log(-dx+1))}}{(dx+1)^p}}{dp+d}$$

input
```
integrate((-d^2*x^2+1)^p/((d*x+1)^p),x, algorithm="giac")
```

output
```
(d*x*e^(p*log(d*x + 1) + p*log(-d*x + 1))/(d*x + 1)^p - e^(p*log(d*x + 1)
+ p*log(-d*x + 1))/(d*x + 1)^p)/(d*p + d)
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int (1+dx)^{-p}\left(1-d^2x^2\right)^p\,dx = \frac{(1-d^2\,x^2)^p\,(d\,x-1)}{d\,(d\,x+1)^p\,(p+1)}$$

input
```
int((1 - d^2*x^2)^p/(d*x + 1)^p,x)
```

output
```
((1 - d^2*x^2)^p*(d*x - 1))/(d*(d*x + 1)^p*(p + 1))
```

Reduce [F]

$$\int (1+dx)^{-p}\left(1-d^2x^2\right)^p\,dx = \int \frac{(-d^2x^2+1)^p}{(dx+1)^p}dx$$

input
```
int((-d^2*x^2+1)^p/((d*x+1)^p),x)
```

output
```
int(( - d**2*x**2 + 1)**p/(d*x + 1)**p,x)
```

3.376 $\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = -\frac{(1-dx)^{1+p} \operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{1}{2}(1-dx)\right)}{2d(1+p)}$$

output
```
-1/2*(-d*x+1)^(p+1)*hypergeom([1, p+1],[2+p],-1/2*d*x+1/2)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = -\frac{(1-dx)^{1+p} \operatorname{Hypergeometric2F1}\left(1,1+p,2+p,\frac{1}{2}(1-dx)\right)}{2d+2dp}$$

input
```
Integrate[(1 + d*x)^(-1 - p)*(1 - d^2*x^2)^p,x]
```

output
```
-(((1 - d*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (1 - d*x)/2])/(2*d
 + 2*d*p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {456, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx+1)^{-p-1} \left(1-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(1-dx)^p}{dx+1}dx$$

$$\Big\downarrow \text{78}$$

$$-\frac{(1-dx)^{p+1} \text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{1}{2}(1-dx)\right)}{2d(p+1)}$$

input
```
Int[(1 + d*x)^(-1 - p)*(1 - d^2*x^2)^p,x]
```

output
```
-1/2*((1 - d*x)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (1 - d*x)/2])/(
d*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [F]

$$\int (dx+1)^{-p-1} \left(-d^2x^2+1\right)^p dx$$

input
```
int((d*x+1)^(-p-1)*(-d^2*x^2+1)^p,x)
```

output
```
int((d*x+1)^(-p-1)*(-d^2*x^2+1)^p,x)
```

Fricas [F]

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-d^2x^2+1\right)^p (dx+1)^{-p-1} \, dx$$

input
```
integrate((d*x+1)^(-1-p)*(-d^2*x^2+1)^p,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 1), x)
```

Sympy [F]

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-(dx-1)(dx+1)\right)^p (dx+1)^{-p-1} \, dx$$

input
```
integrate((d*x+1)**(-1-p)*(-d**2*x**2+1)**p,x)
```

output
```
Integral((-(d*x - 1)*(d*x + 1))**p*(d*x + 1)**(-p - 1), x)
```

Maxima [F]

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = \int (-d^2x^2+1)^p (dx+1)^{-p-1} \, dx$$

input
```
integrate((d*x+1)^(-1-p)*(-d^2*x^2+1)^p,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 1), x)
```

Giac [F]

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = \int (-d^2x^2+1)^p (dx+1)^{-p-1} \, dx$$

input
```
integrate((d*x+1)^(-1-p)*(-d^2*x^2+1)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = \int \frac{(1-d^2\,x^2)^p}{(d\,x+1)^{p+1}} \, dx$$

input
```
int((1 - d^2*x^2)^p/(d*x + 1)^(p + 1),x)
```

output
```
int((1 - d^2*x^2)^p/(d*x + 1)^(p + 1), x)
```

Reduce [F]

$$\int (1+dx)^{-1-p} \left(1-d^2x^2\right)^p \, dx = \int \frac{(-d^2x^2+1)^p}{(dx+1)^p \, dx + (dx+1)^p} dx$$

input
```
int((d*x+1)^(-1-p)*(-d^2*x^2+1)^p,x)
```

output
```
int(( - d**2*x**2 + 1)**p/((d*x + 1)**p*d*x + (d*x + 1)**p),x)
```

3.377 $\int (1+dx)^{-2-p} (1-d^2x^2)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int (1+dx)^{-2-p} (1-d^2x^2)^p \, dx = -\frac{(1-dx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{1}{2}(1-dx)\right)}{4d(1+p)}$$

output
```
-1/4*(-d*x+1)^(p+1)*hypergeom([2, p+1],[2+p],-1/2*d*x+1/2)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (1+dx)^{-2-p} (1-d^2x^2)^p \, dx = -\frac{(1-dx)^{1+p} \text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{1}{2}(1-dx)\right)}{4d+4dp}$$

input
```
Integrate[(1 + d*x)^(-2 - p)*(1 - d^2*x^2)^p,x]
```

output
```
-(((1 - d*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (1 - d*x)/2])/(4*d
 + 4*d*p))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {456, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx+1)^{-p-2} \left(1-d^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(1-dx)^p}{(dx+1)^2}dx$$

$$\Big\downarrow \text{78}$$

$$-\frac{(1-dx)^{p+1} \text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{1}{2}(1-dx)\right)}{4d(p+1)}$$

input
```
Int[(1 + d*x)^(-2 - p)*(1 - d^2*x^2)^p,x]
```

output
```
-1/4*((1 - d*x)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, (1 - d*x)/2])/(
d*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [F]

$$\int (dx+1)^{-p-2} \left(-d^2x^2+1\right)^p dx$$

input
```
int((d*x+1)^(-p-2)*(-d^2*x^2+1)^p,x)
```

output
```
int((d*x+1)^(-p-2)*(-d^2*x^2+1)^p,x)
```

Fricas [F]

$$\int (1+dx)^{-2-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-d^2x^2+1\right)^p (dx+1)^{-p-2} \, dx$$

input
```
integrate((d*x+1)^(-2-p)*(-d^2*x^2+1)^p,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 2), x)
```

Sympy [F]

$$\int (1+dx)^{-2-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-(dx-1)(dx+1)\right)^p (dx+1)^{-p-2} \, dx$$

input
```
integrate((d*x+1)**(-2-p)*(-d**2*x**2+1)**p,x)
```

output
```
Integral((-(d*x - 1)*(d*x + 1))**p*(d*x + 1)**(-p - 2), x)
```

Maxima [F]

$$\int (1+dx)^{-2-p} \left(1-d^2x^2\right)^p \, dx = \int (-d^2x^2+1)^p (dx+1)^{-p-2} \, dx$$

input
```
integrate((d*x+1)^(-2-p)*(-d^2*x^2+1)^p,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 2), x)
```

Giac [F]

$$\int (1+dx)^{-2-p} \left(1-d^2x^2\right)^p \, dx = \int (-d^2x^2+1)^p (dx+1)^{-p-2} \, dx$$

input
```
integrate((d*x+1)^(-2-p)*(-d^2*x^2+1)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+dx)^{-2-p} \left(1-d^2x^2\right)^p \, dx = \int \frac{(1-d^2\,x^2)^p}{(d\,x+1)^{p+2}} \, dx$$

input
```
int((1 - d^2*x^2)^p/(d*x + 1)^(p + 2),x)
```

output
```
int((1 - d^2*x^2)^p/(d*x + 1)^(p + 2), x)
```

Reduce [F]

$$\int (1+dx)^{-2-p}\left(1-d^2x^2\right)^p\,dx=\int \frac{\left(-d^2x^2+1\right)^p}{\left(dx+1\right)^p d^2x^2+2\left(dx+1\right)^p dx+\left(dx+1\right)^p}dx$$

input
```
int((d*x+1)^(-2-p)*(-d^2*x^2+1)^p,x)
```

output
```
int(( - d**2*x**2 + 1)**p/((d*x + 1)**p*d**2*x**2 + 2*(d*x + 1)**p*d*x + (
d*x + 1)**p),x)
```

3.378 $\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx = -\frac{(1-dx)^{1+p} \text{Hypergeometric2F1}\left(3,1+p,2+p,\frac{1}{2}(1-dx)\right)}{8d(1+p)}$$

output
```
-1/8*(-d*x+1)^(p+1)*hypergeom([3, p+1],[2+p],-1/2*d*x+1/2)/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx = -\frac{(1-dx)^{1+p} \text{Hypergeometric2F1}\left(3,1+p,2+p,\frac{1}{2}(1-dx)\right)}{8d+8dp}$$

input
```
Integrate[(1 + d*x)^(-3 - p)*(1 - d^2*x^2)^p,x]
```

output
```
-(((1 - d*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (1 - d*x)/2])/(8*d
 + 8*d*p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {456, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx+1)^{-p-3}\left(1-d^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{456}$$

$$\int \frac{(1-dx)^p}{(dx+1)^3}dx$$

$$\Big\downarrow \text{78}$$

$$-\frac{(1-dx)^{p+1}\text{Hypergeometric2F1}\left(3,p+1,p+2,\frac{1}{2}(1-dx)\right)}{8d(p+1)}$$

input
```
Int[(1 + d*x)^(-3 - p)*(1 - d^2*x^2)^p,x]
```

output
```
-1/8*((1 - d*x)^(1 + p)*Hypergeometric2F1[3, 1 + p, 2 + p, (1 - d*x)/2])/(
d*(1 + p))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

Maple [F]

$$\int (dx+1)^{-p-3} \left(-d^2x^2+1\right)^p dx$$

input
```
int((d*x+1)^(-p-3)*(-d^2*x^2+1)^p,x)
```

output
```
int((d*x+1)^(-p-3)*(-d^2*x^2+1)^p,x)
```

Fricas [F]

$$\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-d^2x^2+1\right)^p (dx+1)^{-p-3} \, dx$$

input
```
integrate((d*x+1)^(-3-p)*(-d^2*x^2+1)^p,x, algorithm="fricas")
```

output
```
integral((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 3), x)
```

Sympy [F]

$$\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx = \int \left(-(dx-1)(dx+1)\right)^p (dx+1)^{-p-3} \, dx$$

input
```
integrate((d*x+1)**(-3-p)*(-d**2*x**2+1)**p,x)
```

output
```
Integral((-(d*x - 1)*(d*x + 1))**p*(d*x + 1)**(-p - 3), x)
```

Maxima [F]

$$\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx = \int (-d^2x^2+1)^p (dx+1)^{-p-3} \, dx$$

input
```
integrate((d*x+1)^(-3-p)*(-d^2*x^2+1)^p,x, algorithm="maxima")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 3), x)
```

Giac [F]

$$\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx = \int (-d^2x^2+1)^p (dx+1)^{-p-3} \, dx$$

input
```
integrate((d*x+1)^(-3-p)*(-d^2*x^2+1)^p,x, algorithm="giac")
```

output
```
integrate((-d^2*x^2 + 1)^p*(d*x + 1)^(-p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (1+dx)^{-3-p} \left(1-d^2x^2\right)^p \, dx = \int \frac{(1-d^2\,x^2)^p}{(d\,x+1)^{p+3}} \, dx$$

input
```
int((1 - d^2*x^2)^p/(d*x + 1)^(p + 3),x)
```

output
```
int((1 - d^2*x^2)^p/(d*x + 1)^(p + 3), x)
```

Reduce [F]

$$\int (1+dx)^{-3-p}\left(1-d^2x^2\right)^p\,dx$$
$$=\int \frac{(-d^2x^2+1)^p}{(dx+1)^p\,d^3x^3+3\,(dx+1)^p\,d^2x^2+3\,(dx+1)^p\,dx+(dx+1)^p}dx$$

input
```
int((d*x+1)^(-3-p)*(-d^2*x^2+1)^p,x)
```

output
```
int(( - d**2*x**2 + 1)**p/((d*x + 1)**p*d**3*x**3 + 3*(d*x + 1)**p*d**2*x*
*2 + 3*(d*x + 1)**p*d*x + (d*x + 1)**p),x)
```

3.379 $\int (d+ex)^{-5-2p} \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 199

$$\int (d+ex)^{-5-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{(d+ex)^{-5-2p} \left(d^2-e^2x^2\right)^{1+p}}{2de(4+p)} - \frac{3(d+ex)^{-3-2p} \left(d^2-e^2x^2\right)^{1+p}}{4d^3e(2+p)(3+p)(4+p)} - \frac{3(d+ex)^{-2(1+p)} \left(d^2-e^2x^2\right)^{1+p}}{8d^4e(1+p)(2+p)(3+p)(4+p)} - \frac{3(d+ex)^{-2(2+p)} \left(d^2-e^2x^2\right)^{1+p}}{4d^2e(3+p)(4+p)}$$

output
```
-1/2*(e*x+d)^(-5-2*p)*(-e^2*x^2+d^2)^(p+1)/d/e/(4+p)-3/4*(e*x+d)^(-3-2*p)*
(-e^2*x^2+d^2)^(p+1)/d^3/e/(2+p)/(3+p)/(4+p)-3/8*(-e^2*x^2+d^2)^(p+1)/d^4/
e/(p+1)/(2+p)/(3+p)/(4+p)/((e*x+d)^(2*p+2))-3/4*(-e^2*x^2+d^2)^(p+1)/d^2/e
/(3+p)/(4+p)/((e*x+d)^(4+2*p))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.41

$$\int (d+ex)^{-5-2p}\left(d^2-e^2x^2\right)^p\,dx = -\frac{2^{-5-p}(d-ex)(d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^p\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(1+p,5+p,2+p,\frac{d-ex}{2d}\right)}{d^5e(1+p)}$$

input
```
Integrate[(d + e*x)^(-5 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(-5 - p)*(d - e*x)*(1 + (e*x)/d)^p*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[1 + p, 5 + p, 2 + p, (d - e*x)/(2*d)])/(d^5*e*(1 + p)*(d + e*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {474, 473, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-5}\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{474}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{2p}\int \left(\frac{ex}{d}+1\right)^{-2p-5}\left(d^2-e^2x^2\right)^p dx}{d^5}$$

$$\Big\downarrow \text{473}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\int \left(\frac{ex}{d}+1\right)^{-p-5}\left(d^2-dex\right)^p dx}{d^5}$$

$$\Big\downarrow \text{55}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{3\int\left(\frac{ex}{d}+1\right)^{-p-4}(d^2-dex)^p dx}{2(p+4)}-\frac{\left(\frac{ex}{d}+1\right)^{-p-4}(d^2-dex)^{p+1}}{2de(p+4)}\right)}{d^5}$$

$$\Big\downarrow \; 55$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{3\left(\frac{\int\left(\frac{ex}{d}+1\right)^{-p-3}\left(d^2-dex\right)^p dx}{p+3}-\frac{\left(\frac{ex}{d}+1\right)^{-p-3}\left(d^2-dex\right)^{p+1}}{2de(p+3)}\right)}{2(p+4)}-\frac{\left(\frac{ex}{d}\right.}{}\right.}{d^5}$$

$$\Big\downarrow \; 55$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{3\left(\frac{\frac{\int\left(\frac{ex}{d}+1\right)^{-p-2}\left(d^2-dex\right)^p dx}{2(p+2)}-\frac{\left(\frac{ex}{d}+1\right)^{-p-2}\left(d^2-dex\right)^{p+1}}{2de(p+2)}}{p+3}-\frac{\left(\frac{ex}{d}+1\right.}{}\right.}{2(p+4)}\right.}{d^5}$$

$$\Big\downarrow \; 48$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{3\left(\frac{-\frac{\left(d^2-dex\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-2}}{2de(p+2)}-\frac{\left(d^2-dex\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}}{4de(p+1)(p+2)}}{p+3}-\frac{\left(\frac{ex}{d}+\right.}{}\right.}{2(p+4)}\right.}{d^5}$$

input
```
Int[(d + e*x)^(-5 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
((1 + (e*x)/d)^(-1 + p)*(d^2 - d*e*x)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*(-1
/2*((1 + (e*x)/d)^(-4 - p)*(d^2 - d*e*x)^(1 + p))/(d*e*(4 + p)) + (3*(-1/2
*((1 + (e*x)/d)^(-3 - p)*(d^2 - d*e*x)^(1 + p))/(d*e*(3 + p)) + (-1/2*((1
+ (e*x)/d)^(-2 - p)*(d^2 - d*e*x)^(1 + p))/(d*e*(2 + p)) - ((1 + (e*x)/d)^
(-1 - p)*(d^2 - d*e*x)^(1 + p))/(4*d*e*(1 + p)*(2 + p)))/(3 + p)))/(2*(4 +
 p))))/(d^5*(d + e*x)^(2*p))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

method	result
gosper	$-\frac{(-ex+d)(ex+d)^{-4-2p}\left(-e^2x^2+d^2\right)^p\left(4d^3p^3+6d^2e\,p^2x+6d\,e^2x^2p+3e^3x^3+30d^3p^2+30d^2epx+15d\,e^2x^2+68d^3p+33d^2ex+45d^3\right)}{8d^4e(p^4+10p^3+35p^2+50p+24)}$
orering	$-\frac{(ex+d)(-ex+d)\left(4d^3p^3+6d^2e\,p^2x+6d\,e^2x^2p+3e^3x^3+30d^3p^2+30d^2epx+15d\,e^2x^2+68d^3p+33d^2ex+45d^3\right)(ex+d)^{-5-2p}\left(-e^2x^2\right.}{8d^4(p^4+10p^3+35p^2+50p+24)e}$

input
```
int((e*x+d)^(-5-2*p)*(-e^2*x^2+d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^4/e*(-e*x+d)*(e*x+d)^(-4-2*p)*(-e^2*x^2+d^2)^p/(p^4+10*p^3+35*p^2+5
0*p+24)*(4*d^3*p^3+6*d^2*e*p^2*x+6*d*e^2*p*x^2+3*e^3*x^3+30*d^3*p^2+30*d^2
*e*p*x+15*d*e^2*x^2+68*d^3*p+33*d^2*e*x+45*d^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.15

$$\int (d+ex)^{-5-2p}\left(d^2-e^2x^2\right)^p\,dx$$
$$=\frac{(3\,e^5x^5-4\,d^5p^3-30\,d^5p^2-68\,d^5p-45\,d^5+3\,(2\,de^4p+5\,de^4)x^4+6\,(d^2e^3p^2+5\,d^2e^3p+5\,d^2e^3)x^3+2\,(}{8\,(d^4ep^4+10\,d^4ep^3+3}$$

input
```
integrate((e*x+d)^(-5-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
1/8*(3*e^5*x^5 - 4*d^5*p^3 - 30*d^5*p^2 - 68*d^5*p - 45*d^5 + 3*(2*d*e^4*p
 + 5*d*e^4)*x^4 + 6*(d^2*e^3*p^2 + 5*d^2*e^3*p + 5*d^2*e^3)*x^3 + 2*(2*d^3
*e^2*p^3 + 15*d^3*e^2*p^2 + 31*d^3*e^2*p + 15*d^3*e^2)*x^2 - 3*(2*d^4*e*p^
2 + 10*d^4*e*p + 11*d^4*e)*x)*(-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 5)/(d^4
*e*p^4 + 10*d^4*e*p^3 + 35*d^4*e*p^2 + 50*d^4*e*p + 24*d^4*e)
```

Sympy [F]

$$\int (d+ex)^{-5-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \left(-\left(-d+ex\right)\left(d+ex\right)\right)^p\left(d+ex\right)^{-2p-5}\,dx$$

input
```
integrate((e*x+d)**(-5-2*p)*(-e**2*x**2+d**2)**p,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p*(d + e*x)**(-2*p - 5), x)
```

Maxima [F]

$$\int (d+ex)^{-5-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \left(-e^2x^2+d^2\right)^p(ex+d)^{-2p-5}\,dx$$

input
```
integrate((e*x+d)^(-5-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 5), x)
```

Giac [F]

$$\int (d+ex)^{-5-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \left(-e^2x^2+d^2\right)^p(ex+d)^{-2p-5}\,dx$$

input
```
integrate((e*x+d)^(-5-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 5), x)
```

Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.60

$$\int (d+ex)^{-5-2p} \left(d^2-e^2x^2\right)^p \, dx = \left(d^2-e^2\,x^2\right)^p \left(\frac{3\,e^4\,x^5}{8\,d^4\,{\left(d+e\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} - \frac{3\,x\,\left(2\,p^2+10\,p+11\right)}{8\,{\left(d+e\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} - \frac{d\,\left(4\,p^3+30\,p^2+68\,p+45\right)}{8\,e\,{\left(d+e\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} + \frac{3\,e^2\,x^3\,\left(p^2+5\,p+5\right)}{4\,d^2\,{\left(d+e\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} + \frac{e\,x^2\,\left(2\,p^3+15\,p^2+31\,p+15\right)}{4\,d\,{\left(d+e\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)} + \frac{3\,e^3\,x^4\,\left(2\,p+5\right)}{8\,d^3\,{\left(d+e\,x\right)}^{2\,p+5}\,\left(p^4+10\,p^3+35\,p^2+50\,p+24\right)}\right)$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p + 5),x)
```

output
```
(d^2 - e^2*x^2)^p*((3*e^4*x^5)/(8*d^4*(d + e*x)^(2*p + 5)*(50*p + 35*p^2 +
 10*p^3 + p^4 + 24)) - (3*x*(10*p + 2*p^2 + 11))/(8*(d + e*x)^(2*p + 5)*(5
0*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (d*(68*p + 30*p^2 + 4*p^3 + 45))/(8*e
*(d + e*x)^(2*p + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (3*e^2*x^3*(5*
p + p^2 + 5))/(4*d^2*(d + e*x)^(2*p + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 2
4)) + (e*x^2*(31*p + 15*p^2 + 2*p^3 + 15))/(4*d*(d + e*x)^(2*p + 5)*(50*p
+ 35*p^2 + 10*p^3 + p^4 + 24)) + (3*e^3*x^4*(2*p + 5))/(8*d^3*(d + e*x)^(2
*p + 5)*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [F]

$$\int (d+ex)^{-5-2p} \left(d^2-e^2x^2\right)^p \, dx$$
$$= \int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2p} d^5 + 5 (ex+d)^{2p} d^4 ex + 10 (ex+d)^{2p} d^3 e^2 x^2 + 10 (ex+d)^{2p} d^2 e^3 x^3 + 5 (ex+d)^{2p} d e^4 x^4 +}$$

input
```
int((e*x+d)^(-5-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((d**2 - e**2*x**2)**p/((d + e*x)**(2*p)*d**5 + 5*(d + e*x)**(2*p)*d**4
*e*x + 10*(d + e*x)**(2*p)*d**3*e**2*x**2 + 10*(d + e*x)**(2*p)*d**2*e**3*
x**3 + 5*(d + e*x)**(2*p)*d*e**4*x**4 + (d + e*x)**(2*p)*e**5*x**5),x)
```

3.380 $\int (d+ex)^{-4-2p} \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 142

$$\int (d+ex)^{-4-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{(d+ex)^{-3-2p}\left(d^2-e^2x^2\right)^{1+p}}{2d^2e(2+p)(3+p)} - \frac{(d+ex)^{-2(1+p)}\left(d^2-e^2x^2\right)^{1+p}}{4d^3e(1+p)(2+p)(3+p)} - \frac{(d+ex)^{-2(2+p)}\left(d^2-e^2x^2\right)^{1+p}}{2de(3+p)}$$

output
```
-1/2*(e*x+d)^(-3-2*p)*(-e^2*x^2+d^2)^(p+1)/d^2/e/(2+p)/(3+p)-1/4*(-e^2*x^2
+d^2)^(p+1)/d^3/e/(p+1)/(2+p)/(3+p)/((e*x+d)^(2*p+2))-1/2*(-e^2*x^2+d^2)^(
p+1)/d/e/(3+p)/((e*x+d)^(4+2*p))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int (d+ex)^{-4-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{2^{-4-p}(d-ex)(d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^p\left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(1+p,4+p,2+p,\frac{d-ex}{2d}\right)}{d^4e(1+p)}$$

input
```
Integrate[(d + e*x)^(-4 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(-4 - p)*(d - e*x)*(1 + (e*x)/d)^p*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[1 + p, 4 + p, 2 + p, (d - e*x)/(2*d)])/(d^4*e*(1 + p)*(d + e*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {474, 473, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-4}\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{ 474}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{2p}\int\left(\frac{ex}{d}+1\right)^{-2(p+2)}\left(d^2-e^2x^2\right)^p dx}{d^4}$$

$$\Big\downarrow \text{ 473}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\int\left(\frac{ex}{d}+1\right)^{-p-4}\left(d^2-dex\right)^p dx}{d^4}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{\int\left(\frac{ex}{d}+1\right)^{-p-3}\left(d^2-dex\right)^p dx}{p+3}-\frac{\left(\frac{ex}{d}+1\right)^{-p-3}\left(d^2-dex\right)^{p+1}}{2de(p+3)}\right)}{d^4}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{\frac{\int\left(\frac{ex}{d}+1\right)^{-p-2}\left(d^2-dex\right)^p dx}{2(p+2)}-\frac{\left(\frac{ex}{d}+1\right)^{-p-2}\left(d^2-dex\right)^{p+1}}{2de(p+2)}}{p+3}-\frac{\left(\frac{ex}{d}+1\right)\dots}{}\right.}{d^4}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{(d+ex)^{-2p}\left(\frac{ex}{d}+1\right)^{p-1}\left(d^2-dex\right)^{-p-1}\left(d^2-e^2x^2\right)^{p+1}\left(\frac{-\frac{\left(d^2-dex\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-2}}{2de(p+2)}-\frac{\left(d^2-dex\right)^{p+1}\left(\frac{ex}{d}+1\right)^{-p-1}}{4de(p+1)(p+2)}}{p+3}-\frac{\left(\frac{ex}{d}+1\right.}{}\right.}{d^4}$$

input
```
Int[(d + e*x)^(-4 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
((1 + (e*x)/d)^(-1 + p)*(d^2 - d*e*x)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*(-1
/2*((1 + (e*x)/d)^(-3 - p)*(d^2 - d*e*x)^(1 + p))/(d*e*(3 + p)) + (-1/2*((
1 + (e*x)/d)^(-2 - p)*(d^2 - d*e*x)^(1 + p))/(d*e*(2 + p)) - ((1 + (e*x)/d
)^(-1 - p)*(d^2 - d*e*x)^(1 + p))/(4*d*e*(1 + p)*(2 + p)))/(3 + p)))/(d^4*
(d + e*x)^(2*p))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{(-ex+d)(ex+d)^{-3-2p}\left(-e^{2} x^{2}+d^{2}\right)^{p}\left(2d^{2}p^{2}+2depx+e^{2}x^{2}+8d^{2}p+4dex+7d^{2}\right)}{4d^{3}e(2+p)(p^{2}+4p+3)}$	93
orering	$-\frac{\left(-e^{2} x^{2}+d^{2}\right)^{p}(ex+d)^{-4-2p}\left(2d^{2}p^{2}+2depx+e^{2}x^{2}+8d^{2}p+4dex+7d^{2}\right)(ex+d)(-ex+d)}{4e(p^{2}+4p+3)(2+p)d^{3}}$	98

input
```
int((e*x+d)^(-4-2*p)*(-e^2*x^2+d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d^3/e/(2+p)*(-e*x+d)*(e*x+d)^(-3-2*p)/(p^2+4*p+3)*(-e^2*x^2+d^2)^p*(2
*d^2*p^2+2*d*e*p*x+e^2*x^2+8*d^2*p+4*d*e*x+7*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08

$$\int (d+ex)^{-4-2p}\left(d^{2}-e^{2}x^{2}\right)^{p}\,dx$$
$$=\frac{\left(e^{4}x^{4}-2\,d^{4}p^{2}-8\,d^{4}p-7\,d^{4}+2\,(de^{3}p+2\,de^{3})x^{3}+2\,(d^{2}e^{2}p^{2}+4\,d^{2}e^{2}p+3\,d^{2}e^{2})x^{2}-2\,(d^{3}ep+2\,d^{3}e)x\right.}{4\,(d^{3}ep^{3}+6\,d^{3}ep^{2}+11\,d^{3}ep+6\,d^{3}e)}$$

input
```
integrate((e*x+d)^(-4-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
1/4*(e^4*x^4 - 2*d^4*p^2 - 8*d^4*p - 7*d^4 + 2*(d*e^3*p + 2*d*e^3)*x^3 + 2
*(d^2*e^2*p^2 + 4*d^2*e^2*p + 3*d^2*e^2)*x^2 - 2*(d^3*e*p + 2*d^3*e)*x)*(-
e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 4)/(d^3*e*p^3 + 6*d^3*e*p^2 + 11*d^3*e*
p + 6*d^3*e)
```

Sympy [F]

$$\int (d+ex)^{-4-2p}\left(d^2-e^2x^2\right)^p\,dx=\int (-(-d+ex)\,(d+ex))^p\,(d+ex)^{-2p-4}\,dx$$

input
```
integrate((e*x+d)**(-4-2*p)*(-e**2*x**2+d**2)**p,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p*(d + e*x)**(-2*p - 4), x)
```

Maxima [F]

$$\int (d+ex)^{-4-2p}\left(d^2-e^2x^2\right)^p\,dx=\int (-e^2x^2+d^2)^p(ex+d)^{-2\,p-4}\,dx$$

input
```
integrate((e*x+d)^(-4-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 4), x)
```

Giac [F]

$$\int (d+ex)^{-4-2p}\left(d^2-e^2x^2\right)^p\,dx=\int (-e^2x^2+d^2)^p(ex+d)^{-2\,p-4}\,dx$$

input
```
integrate((e*x+d)^(-4-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 4), x)
```

Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.55

$$\int (d+ex)^{-4-2p} \left(d^2-e^2x^2\right)^p \, dx = \left(d^2-e^2\,x^2\right)^p \left(\frac{e^3\,x^4}{4\,d^3\,\left(d+e\,x\right)^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)} - \frac{d\,\left(2\,p^2+8\,p+7\right)}{4\,e\,\left(d+e\,x\right)^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)} - \frac{x\,\left(p+2\right)}{2\,\left(d+e\,x\right)^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)} + \frac{e\,x^2\,\left(p^2+4\,p+3\right)}{2\,d\,\left(d+e\,x\right)^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)} + \frac{e^2\,x^3\,\left(p+2\right)}{2\,d^2\,\left(d+e\,x\right)^{2\,p+4}\,\left(p^3+6\,p^2+11\,p+6\right)}\right)$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p + 4),x)
```

output
```
(d^2 - e^2*x^2)^p*((e^3*x^4)/(4*d^3*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 + p^
3 + 6)) - (d*(8*p + 2*p^2 + 7))/(4*e*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 + p
^3 + 6)) - (x*(p + 2))/(2*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 + p^3 + 6)) +
(e*x^2*(4*p + p^2 + 3))/(2*d*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 + p^3 + 6))
 + (e^2*x^3*(p + 2))/(2*d^2*(d + e*x)^(2*p + 4)*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [F]

$$\int (d+ex)^{-4-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \frac{\left(-e^2x^2+d^2\right)^p}{\left(ex+d\right)^{2p} d^4+4\left(ex+d\right)^{2p} d^3ex+6\left(ex+d\right)^{2p} d^2e^2x^2+4\left(ex+d\right)^{2p} d\,e^3x^3+\left(ex+d\right)^{2p} e^4x^4}dx$$

input
```
int((e*x+d)^(-4-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((d**2 - e**2*x**2)**p/((d + e*x)**(2*p)*d**4 + 4*(d + e*x)**(2*p)*d**3
*e*x + 6*(d + e*x)**(2*p)*d**2*e**2*x**2 + 4*(d + e*x)**(2*p)*d*e**3*x**3
+ (d + e*x)**(2*p)*e**4*x**4),x)
```

3.381 $\int (d+ex)^{-3-2p} \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (d+ex)^{-3-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{(d+ex)^{-3-2p} \left(d^2-e^2x^2\right)^{1+p}}{2de(2+p)} - \frac{(d+ex)^{-2(1+p)} \left(d^2-e^2x^2\right)^{1+p}}{4d^2e(1+p)(2+p)}$$

output
```
-1/2*(e*x+d)^(-3-2*p)*(-e^2*x^2+d^2)^(p+1)/d/e/(2+p)-1/4*(-e^2*x^2+d^2)^(p
+1)/d^2/e/(p+1)/(2+p)/((e*x+d)^(2*p+2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int (d+ex)^{-3-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{2^{-3-p}(d-ex)(d+ex)^{-2p} \left(1+\frac{ex}{d}\right)^p \left(d^2-e^2x^2\right)^p \text{Hypergeometric2F1}\left(1+p,3+p,2+p,\frac{d-ex}{2d}\right)}{d^3e(1+p)}$$

input
```
Integrate[(d + e*x)^(-3 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(-3 - p)*(d - e*x)*(1 + (e*x)/d)^p*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[1 + p, 3 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e*(1 + p)*(d + e*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {474, 473, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-3} \left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{474}$$

$$\frac{(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{2p} \int \left(\frac{ex}{d}+1\right)^{-2p-3} \left(d^2-e^2x^2\right)^p dx}{d^3}$$

$$\Big\downarrow \text{473}$$

$$\frac{(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-dex\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \int \left(\frac{ex}{d}+1\right)^{-p-3} \left(d^2-dex\right)^p dx}{d^3}$$

$$\Big\downarrow \text{55}$$

$$\frac{(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-dex\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \left(\frac{\int \left(\frac{ex}{d}+1\right)^{-p-2} \left(d^2-dex\right)^p dx}{2(p+2)}-\frac{\left(\frac{ex}{d}+1\right)^{-p-2} \left(d^2-dex\right)^{p+1}}{2de(p+2)}\right)}{d^3}$$

$$\Big\downarrow \text{48}$$

$$\frac{(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-dex\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \left(-\frac{\left(d^2-dex\right)^{p+1} \left(\frac{ex}{d}+1\right)^{-p-2}}{2de(p+2)}-\frac{\left(d^2-dex\right)^{p+1} \left(\frac{ex}{d}+1\right)^{-p-1}}{4de(p+1)(p+2)}\right)}{d^3}$$

input
```
Int[(d + e*x)^(-3 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
((1 + (e*x)/d)^(-1 + p)*(d^2 - d*e*x)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*(-1
/2*((1 + (e*x)/d)^(-2 - p)*(d^2 - d*e*x)^(1 + p))/(d*e*(2 + p)) - ((1 + (e
*x)/d)^(-1 - p)*(d^2 - d*e*x)^(1 + p))/(4*d*e*(1 + p)*(2 + p))))/(d^3*(d +
 e*x)^(2*p))
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

method	result
gosper	$-\frac{(-ex+d)(ex+d)^{-2p-2}(-e^2x^2+d^2)^p(2dp+ex+3d)}{4d^2e(p^2+3p+2)}$
orering	$-\frac{(ex+d)(-ex+d)(2dp+ex+3d)(ex+d)^{-3-2p}(-e^2x^2+d^2)^p}{4d^2(p^2+3p+2)e}$
parallelrisch	$\frac{x^3(-e^2x^2+d^2)^p(ex+d)^{-3-2p}e^4+2x^2(-e^2x^2+d^2)^p(ex+d)^{-3-2p}d\,e^3p+3x^2(-e^2x^2+d^2)^p(ex+d)^{-3-2p}d\,e^3-x(-e^2x^2+d^2)}{4d^2(p^2+3p+2)e^2}$

input
```
int((e*x+d)^(-3-2*p)*(-e^2*x^2+d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d^2/e*(-e*x+d)*(e*x+d)^(-2*p-2)/(p^2+3*p+2)*(-e^2*x^2+d^2)^p*(2*d*p+e
*x+3*d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int (d+ex)^{-3-2p}\left(d^2-e^2x^2\right)^p\,dx = \frac{(e^3x^3-2\,d^3p-d^2ex-3\,d^3+(2\,de^2p+3\,de^2)x^2)(-e^2x^2+d^2)^p(ex+d)^{-2\,p-3}}{4\,(d^2ep^2+3\,d^2ep+2\,d^2e)}$$

input
```
integrate((e*x+d)^(-3-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
1/4*(e^3*x^3 - 2*d^3*p - d^2*e*x - 3*d^3 + (2*d*e^2*p + 3*d*e^2)*x^2)*(-e^
2*x^2 + d^2)^p*(e*x + d)^(-2*p - 3)/(d^2*e*p^2 + 3*d^2*e*p + 2*d^2*e)
```

Sympy [F]

$$\int (d+ex)^{-3-2p}\left(d^2-e^2x^2\right)^p\,dx=\int (-(-d+ex)\,(d+ex))^p\,(d+ex)^{-2p-3}\,dx$$

input
```
integrate((e*x+d)**(-3-2*p)*(-e**2*x**2+d**2)**p,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p*(d + e*x)**(-2*p - 3), x)
```

Maxima [F]

$$\int (d+ex)^{-3-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \left(-e^2x^2+d^2\right)^p(ex+d)^{-2\,p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 3), x)
```

Giac [F]

$$\int (d+ex)^{-3-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \left(-e^2x^2+d^2\right)^p(ex+d)^{-2\,p-3}\,dx$$

input
```
integrate((e*x+d)^(-3-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.64

$$\int (d+ex)^{-3-2p}\left(d^2-e^2x^2\right)^p\,dx=-\left(d^2-e^2\,x^2\right)^p\left(\frac{x}{4\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}+\frac{d\,(2\,p+3)}{4\,e\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}-\frac{e^2\,x^3}{4\,d^2\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}-\frac{e\,x^2\,(2\,p+3)}{4\,d\,(d+e\,x)^{2\,p+3}\,(p^2+3\,p+2)}\right)$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p + 3),x)
```

output
```
-(d^2 - e^2*x^2)^p*(x/(4*(d + e*x)^(2*p + 3)*(3*p + p^2 + 2)) + (d*(2*p +
3))/(4*e*(d + e*x)^(2*p + 3)*(3*p + p^2 + 2)) - (e^2*x^3)/(4*d^2*(d + e*x)
^(2*p + 3)*(3*p + p^2 + 2)) - (e*x^2*(2*p + 3))/(4*d*(d + e*x)^(2*p + 3)*(
3*p + p^2 + 2)))
```

Reduce [F]

$$\int (d+ex)^{-3-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \frac{(-e^2x^2+d^2)^p}{(ex+d)^{2p}\,d^3+3\,(ex+d)^{2p}\,d^2ex+3\,(ex+d)^{2p}\,d\,e^2x^2+(ex+d)^{2p}\,e^3x^3}dx$$

input
```
int((e*x+d)^(-3-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((d**2 - e**2*x**2)**p/((d + e*x)**(2*p)*d**3 + 3*(d + e*x)**(2*p)*d**2
*e*x + 3*(d + e*x)**(2*p)*d*e**2*x**2 + (d + e*x)**(2*p)*e**3*x**3),x)
```

3.382 $\int (d+ex)^{-2-2p} \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 42

$$\int (d+ex)^{-2-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{(d+ex)^{-2(1+p)} \left(d^2-e^2x^2\right)^{1+p}}{2de(1+p)}$$

output
```
-1/2*(-e^2*x^2+d^2)^(p+1)/d/e/(p+1)/((e*x+d)^(2*p+2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int (d+ex)^{-2-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{(d-ex)(d+ex)^{-1-2p} \left(d^2-e^2x^2\right)^p}{2de(1+p)}$$

input
```
Integrate[(d + e*x)^(-2 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-1/2*((d - e*x)*(d + e*x)^(-1 - 2*p)*(d^2 - e^2*x^2)^p)/(d*e*(1 + p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-2}\left(d^2-e^2x^2\right)^p\,dx$$

$$\Big\downarrow \text{460}$$

$$-\frac{(d+ex)^{-2(p+1)}\left(d^2-e^2x^2\right)^{p+1}}{2de(p+1)}$$

input
```
Int[(d + e*x)^(-2 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-1/2*(d^2 - e^2*x^2)^(1 + p)/(d*e*(1 + p)*(d + e*x)^(2*(1 + p)))
```

Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{(-ex+d)(ex+d)^{-1-2p}\left(-e^2x^2+d^2\right)^p}{2de(p+1)}$	45
orering	$-\frac{(ex+d)(-ex+d)(ex+d)^{-2p-2}\left(-e^2x^2+d^2\right)^p}{2de(p+1)}$	50
parallelrisch	$\frac{x^2\left(-e^2x^2+d^2\right)^p(ex+d)^{-2p-2}e^3-\left(-e^2x^2+d^2\right)^p(ex+d)^{-2p-2}d^2e}{2e^2(p+1)d}$	78

input
```
int((e*x+d)^(-2*p-2)*(-e^2*x^2+d^2)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/e/(p+1)*(-e*x+d)*(e*x+d)^(-1-2*p)*(-e^2*x^2+d^2)^p
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int (d+ex)^{-2-2p}\left(d^2-e^2x^2\right)^p\,dx=\frac{(e^2x^2-d^2)(-e^2x^2+d^2)^p(ex+d)^{-2\,p-2}}{2\,(dep+de)}$$

input
```
integrate((e*x+d)^(-2-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
1/2*(e^2*x^2 - d^2)*(-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 2)/(d*e*p + d*e)
```

Sympy [F]

$$\int (d+ex)^{-2-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \left(-(-d+ex)\,(d+ex)\right)^p(d+ex)^{-2p-2}\,dx$$

input
```
integrate((e*x+d)**(-2-2*p)*(-e**2*x**2+d**2)**p,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p*(d + e*x)**(-2*p - 2), x)
```

Maxima [F]

$$\int (d+ex)^{-2-2p} \left(d^2-e^2x^2\right)^p \, dx = \int (-e^2x^2+d^2)^p(ex+d)^{-2\,p-2}\, dx$$

input
```
integrate((e*x+d)^(-2-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 2), x)
```

Giac [F]

$$\int (d+ex)^{-2-2p} \left(d^2-e^2x^2\right)^p \, dx = \int (-e^2x^2+d^2)^p(ex+d)^{-2\,p-2}\, dx$$

input
```
integrate((e*x+d)^(-2-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 2), x)
```

Mupad [B] (verification not implemented)

Time = 6.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int (d+ex)^{-2-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{\left(d^2-e^2\,x^2\right)^p\,\left(\frac{d}{2}-\frac{e\,x}{2}\right)}{d\,e\,\left(p+1\right)\,{\left(d+e\,x\right)}^{2\,p+1}}$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p + 2),x)
```

output
```
-((d^2 - e^2*x^2)^p*(d/2 - (e*x)/2))/(d*e*(p + 1)*(d + e*x)^(2*p + 1))
```

Reduce [F]

$$\int (d+ex)^{-2-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2p} d^2+2\,(ex+d)^{2p} dex+(ex+d)^{2p} e^2x^2} dx$$

input
```
int((e*x+d)^(-2-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((d**2 - e**2*x**2)**p/((d + e*x)**(2*p)*d**2 + 2*(d + e*x)**(2*p)*d*e*
x + (d + e*x)**(2*p)*e**2*x**2),x)
```

3.383 $\int (d+ex)^{-1-2p} \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 80

$$\int (d+ex)^{-1-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{2^{-1-p}(d+ex)^{-2p} \left(1+\frac{ex}{d}\right)^{-1+p} \left(d^2-e^2x^2\right)^{1+p} \text{Hypergeometric2F1}\left(1+p,1+p,2+p,\frac{d-ex}{2d}\right)}{d^2e(1+p)}$$

output
```
-2^(-1-p)*(1+e*x/d)^(-1+p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, p+1],[2+p]
,1/2*(-e*x+d)/d)/d^2/e/(p+1)/((e*x+d)^(2*p))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int (d+ex)^{-1-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{2^{-1-p}(d-ex)(d+ex)^{-2p} \left(1+\frac{ex}{d}\right)^{p} \left(d^2-e^2x^2\right)^{p} \text{Hypergeometric2F1}\left(1+p,1+p,2+p,\frac{d-ex}{2d}\right)}{de(1+p)}$$

input
```
Integrate[(d + e*x)^(-1 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(-1 - p)*(d - e*x)*(1 + (e*x)/d)^p*(d^2 - e^2*x^2)^p*Hypergeometric2F
1[1 + p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d*e*(1 + p)*(d + e*x)^(2*p)))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p-1} \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \; 474$$

$$\frac{(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{2p} \int \left(\frac{ex}{d}+1\right)^{-2p-1} \left(d^2-e^2x^2\right)^p dx}{d}$$

$$\Big\downarrow \; 473$$

$$\frac{(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-dex\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \int \left(\frac{ex}{d}+1\right)^{-p-1} \left(d^2-dex\right)^p dx}{d}$$

$$\Big\downarrow \; 79$$

$$-\frac{2^{-p-1}(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-e^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(p+1,p+1,p+2,\frac{d-ex}{2d}\right)}{d^2e(p+1)}$$

input
```
Int[(d + e*x)^(-1 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(-1 - p)*(1 + (e*x)/d)^(-1 + p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometri
c2F1[1 + p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e*(1 + p)*(d + e*x)^(2*p)
))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (ex+d)^{-1-2p}\left(-e^2x^2+d^2\right)^p dx$$

input
```
int((e*x+d)^(-1-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((e*x+d)^(-1-2*p)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (d+ex)^{-1-2p} (d^2-e^2x^2)^p \, dx = \int (-e^2x^2+d^2)^p (ex+d)^{-2p-1} \, dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 1), x)
```

Sympy [F]

$$\int (d+ex)^{-1-2p} (d^2-e^2x^2)^p \, dx = \int (-(-d+ex)(d+ex))^p (d+ex)^{-2p-1} \, dx$$

input
```
integrate((e*x+d)**(-1-2*p)*(-e**2*x**2+d**2)**p,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p*(d + e*x)**(-2*p - 1), x)
```

Maxima [F]

$$\int (d+ex)^{-1-2p} (d^2-e^2x^2)^p \, dx = \int (-e^2x^2+d^2)^p (ex+d)^{-2p-1} \, dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 1), x)
```

Giac [F]

$$\int (d+ex)^{-1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p (ex+d)^{-2\,p-1} \, dx$$

input
```
integrate((e*x+d)^(-1-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \frac{\left(d^2-e^2\,x^2\right)^p}{\left(d+e\,x\right)^{2\,p+1}} \, dx$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p + 1),x)
```

output
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p + 1), x)
```

Reduce [F]

$$\int (d+ex)^{-1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2p}\, d+(ex+d)^{2p}\, ex} dx$$

input
```
int((e*x+d)^(-1-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((d**2 - e**2*x**2)**p/((d + e*x)**(2*p)*d + (d + e*x)**(2*p)*e*x),x)
```

3.384 $\int (d+ex)^{-2p}\left(d^2-e^2x^2\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int (d+ex)^{-2p}\left(d^2-e^2x^2\right)^p\,dx = -\frac{2^{-p}(d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^{-1+p}\left(d^2-e^2x^2\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{d-ex}{2d}\right)}{de(1+p)}$$

output
```
-(1+e*x/d)^(-1+p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p, p+1],[2+p],1/2*(-e*x+
d)/d)/(2^p)/d/e/(p+1)/((e*x+d)^(2*p))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int (d+ex)^{-2p}\left(d^2-e^2x^2\right)^p\,dx = \frac{2^{-p}(-d+ex)(d+ex)^{-2p}\left(1+\frac{ex}{d}\right)^p\left(d^2-e^2x^2\right)^p\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{d-ex}{2d}\right)}{e(1+p)}$$

input
```
Integrate[(d^2 - e^2*x^2)^p/(d + e*x)^(2*p),x]
```

output
```
((-d + e*x)*(1 + (e*x)/d)^p*(d^2 - e^2*x^2)^p*Hypergeometric2F1[p, 1 + p,
2 + p, (d - e*x)/(2*d)])/(2^p*e*(1 + p)*(d + e*x)^(2*p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{-2p} \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{474}$$

$$(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{2p} \int \left(\frac{ex}{d}+1\right)^{-2p} \left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{473}$$

$$(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-dex\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \int \left(\frac{ex}{d}+1\right)^{-p} \left(d^2-dex\right)^p dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{-p}(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-e^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{d-ex}{2d}\right)}{de(p+1)}$$

input
```
Int[(d^2 - e^2*x^2)^p/(d + e*x)^(2*p),x]
```

output
```
-(((1 + (e*x)/d)^(-1 + p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[p, 1 +
 p, 2 + p, (d - e*x)/(2*d)])/(2^p*d*e*(1 + p)*(d + e*x)^(2*p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int \left(-e^{2} x^{2}+d^{2}\right)^{p} \left(e x +d \right)^{-2 p}d x$$

input
```
int((-e^2*x^2+d^2)^p/((e*x+d)^(2*p)),x)
```

output
```
int((-e^2*x^2+d^2)^p/((e*x+d)^(2*p)),x)
```

Fricas [F]

$$\int (d+ex)^{-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2\,p}}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/((e*x+d)^(2*p)),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p/(e*x + d)^(2*p), x)
```

Sympy [F]

$$\int (d+ex)^{-2p}\left(d^2-e^2x^2\right)^p\,dx=\int (-(-d+ex)\,(d+ex))^p\,(d+ex)^{-2p}\,dx$$

input
```
integrate((-e**2*x**2+d**2)**p/((e*x+d)**(2*p)),x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p/(d + e*x)**(2*p), x)
```

Maxima [F]

$$\int (d+ex)^{-2p}\left(d^2-e^2x^2\right)^p\,dx=\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2\,p}}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/((e*x+d)^(2*p)),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^(2*p), x)
```

Giac [F]

$$\int (d+ex)^{-2p} \left(d^2-e^2x^2\right)^p \,dx = \int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2\,p}}\,dx$$

input
```
integrate((-e^2*x^2+d^2)^p/((e*x+d)^(2*p)),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p/(e*x + d)^(2*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{-2p} \left(d^2-e^2x^2\right)^p \,dx = \int \frac{\left(d^2-e^2\,x^2\right)^p}{(d+e\,x)^{2\,p}}\,dx$$

input
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p),x)
```

output
```
int((d^2 - e^2*x^2)^p/(d + e*x)^(2*p), x)
```

Reduce [F]

$$\int (d+ex)^{-2p} \left(d^2-e^2x^2\right)^p \,dx = \int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2p}}dx$$

input
```
int((-e^2*x^2+d^2)^p/((e*x+d)^(2*p)),x)
```

output
```
int((d**2 - e**2*x**2)**p/(d + e*x)**(2*p),x)
```

3.385 $\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 26, antiderivative size = 77

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = -\frac{2^{1-p}(d+ex)^{-2p} \left(1+\frac{ex}{d}\right)^{-1+p} \left(d^2-e^2x^2\right)^{1+p} \text{Hypergeometric2F1}\left(-1+p,1+p,2+p,\frac{d-ex}{2d}\right)}{e(1+p)}$$

output
```
-2^(1-p)*(1+e*x/d)^(-1+p)*(-e^2*x^2+d^2)^(p+1)*hypergeom([p+1, -1+p],[2+p]
,1/2*(-e*x+d)/d)/e/(p+1)/((e*x+d)^(2*p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.79

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = \frac{(d+ex)^{-2p} \left(1+\frac{ex}{d}\right)^p \left(e^2x^2(d-ex)^p(d+ex)^p \left(1-\frac{ex}{d}\right)^{-p} \text{AppellF1}\left(2,-p,p,3,\frac{ex}{d},-\frac{ex}{d}\right) - \frac{2^{1-p}d(d-ex)(d}{\cdots}\right.}{2e}$$

input
```
Integrate[(d + e*x)^(1 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
((1 + (e*x)/d)^p*((e^2*x^2*(d - e*x)^p*(d + e*x)^p*AppellF1[2, -p, p, 3, (
e*x)/d, -((e*x)/d)])/(1 - (e*x)/d)^p - (2^(1 - p)*d*(d - e*x)*(d^2 - e^2*x
^2)^p*Hypergeometric2F1[p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(1 + p)))/(2*e*
(d + e*x)^(2*p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {474, 473, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{ 474}$$

$$d(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{2p} \int \left(\frac{ex}{d}+1\right)^{1-2p} \left(d^2-e^2x^2\right)^p dx$$

$$\Big\downarrow \text{ 473}$$

$$d(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-dex\right)^{-p-1} \left(d^2-e^2x^2\right)^{p+1} \int \left(\frac{ex}{d}+1\right)^{1-p} \left(d^2-dex\right)^p dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{2^{1-p}(d+ex)^{-2p} \left(\frac{ex}{d}+1\right)^{p-1} \left(d^2-e^2x^2\right)^{p+1} \text{Hypergeometric2F1}\left(p-1,p+1,p+2,\frac{d-ex}{2d}\right)}{e(p+1)}$$

input
```
Int[(d + e*x)^(1 - 2*p)*(d^2 - e^2*x^2)^p,x]
```

output
```
-((2^(1 - p)*(1 + (e*x)/d)^(-1 + p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric
2F1[-1 + p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(e*(1 + p)*(d + e*x)^(2*p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 473
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^(n - 1)*((a + b*x^2)^(p + 1)/((1 + d*(x/c))^(p + 1)*(a/c + (b*x)/d)^(p +
1)))   Int[(1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) &&  !Gt
Q[a, 0] &&  !(IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

rule 474
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(1 + d
*(x/c))^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c^2 +
 a*d^2, 0] &&  !(IntegerQ[n] || GtQ[c, 0])
```

Maple [F]

$$\int (ex+d)^{1-2p} \left(-e^2x^2+d^2\right)^p dx$$

input
```
int((e*x+d)^(1-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((e*x+d)^(1-2*p)*(-e^2*x^2+d^2)^p,x)
```

Fricas [F]

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p (ex+d)^{-2\,p+1} \, dx$$

input
```
integrate((e*x+d)^(1-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p + 1), x)
```

Sympy [F]

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \left(-\left(-d+ex\right)\left(d+ex\right)\right)^p (d+ex)^{1-2p} \, dx$$

input
```
integrate((e*x+d)**(1-2*p)*(-e**2*x**2+d**2)**p,x)
```

output
```
Integral((-(-d + e*x)*(d + e*x))**p*(d + e*x)**(1 - 2*p), x)
```

Maxima [F]

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p (ex+d)^{-2\,p+1} \, dx$$

input
```
integrate((e*x+d)^(1-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p + 1), x)
```

Giac [F]

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+d^2\right)^p (ex+d)^{-2\,p+1} \, dx$$

input
```
integrate((e*x+d)^(1-2*p)*(-e^2*x^2+d^2)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + d^2)^p*(e*x + d)^(-2*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = \int \left(d^2-e^2\,x^2\right)^p \, (d+e\,x)^{1-2\,p} \, dx$$

input
```
int((d^2 - e^2*x^2)^p*(d + e*x)^(1 - 2*p),x)
```

output
```
int((d^2 - e^2*x^2)^p*(d + e*x)^(1 - 2*p), x)
```

Reduce [F]

$$\int (d+ex)^{1-2p} \left(d^2-e^2x^2\right)^p \, dx = \left(\int \frac{\left(-e^2x^2+d^2\right)^p}{(ex+d)^{2p}}dx\right) d+\left(\int \frac{\left(-e^2x^2+d^2\right)^p x}{(ex+d)^{2p}}dx\right) e$$

input
```
int((e*x+d)^(1-2*p)*(-e^2*x^2+d^2)^p,x)
```

output
```
int((d**2 - e**2*x**2)**p/(d + e*x)**(2*p),x)*d + int(((d**2 - e**2*x**2)*
*p*x)/(d + e*x)**(2*p),x)*e
```

3.386 $\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx = \frac{4^p(2+ex)^{1+p+q} \text{Hypergeometric2F1}\left(-p,1+p+q,2+p+q,\frac{1}{4}(2+ex)\right)}{e(1+p+q)}$$

output
```
4^p*(e*x+2)^(1+p+q)*hypergeom([-p, 1+p+q],[2+p+q],1/4*e*x+1/2)/e/(1+p+q)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx = \frac{4^{p+q}(-2+ex)(2+ex)^{-p} \left(4-e^2x^2\right)^p \text{Hypergeometric2F1}\left(1+p,-p-q,2+p,\frac{1}{4}(2-ex)\right)}{e(1+p)}$$

input
```
Integrate[(2 + e*x)^q*(4 - e^2*x^2)^p,x]
```

output
```
(4^(p + q)*(-2 + e*x)*(4 - e^2*x^2)^p*Hypergeometric2F1[1 + p, -p - q, 2 +
 p, (2 - e*x)/4])/(e*(1 + p)*(2 + e*x)^p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4-e^2x^2\right)^p (ex+2)^q \, dx$$

$$\Big\downarrow \text{456}$$

$$\int (2-ex)^p(ex+2)^{p+q}dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{4^{p+q}(2-ex)^{p+1} \text{Hypergeometric2F1}\left(p+1,-p-q,p+2,\frac{1}{4}(2-ex)\right)}{e(p+1)}$$

input
```
Int[(2 + e*x)^q*(4 - e^2*x^2)^p,x]
```

output
```
-((4^(p + q)*(2 - e*x)^(1 + p)*Hypergeometric2F1[1 + p, -p - q, 2 + p, (2
- e*x)/4])/(e*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [F]

$$\int (ex+2)^q \left(-e^2x^2+4\right)^p dx$$

input
```
int((e*x+2)^q*(-e^2*x^2+4)^p,x)
```

output
```
int((e*x+2)^q*(-e^2*x^2+4)^p,x)
```

Fricas [F]

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx = \int \left(-e^2x^2+4\right)^p (ex+2)^q \, dx$$

input
```
integrate((e*x+2)^q*(-e^2*x^2+4)^p,x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + 4)^p*(e*x + 2)^q, x)
```

Sympy [F]

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx = \int \left(-(ex-2)\,(ex+2)\right)^p (ex+2)^q \, dx$$

input
```
integrate((e*x+2)**q*(-e**2*x**2+4)**p,x)
```

output
```
Integral((-(e*x - 2)*(e*x + 2))**p*(e*x + 2)**q, x)
```

Maxima [F]

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx = \int (-e^2x^2+4)^p(ex+2)^q \, dx$$

input
```
integrate((e*x+2)^q*(-e^2*x^2+4)^p,x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + 4)^p*(e*x + 2)^q, x)
```

Giac [F]

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx = \int (-e^2x^2+4)^p(ex+2)^q \, dx$$

input
```
integrate((e*x+2)^q*(-e^2*x^2+4)^p,x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + 4)^p*(e*x + 2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx = \int \left(4-e^2\,x^2\right)^p\,\left(e\,x+2\right)^q \,dx$$

input
```
int((4 - e^2*x^2)^p*(e*x + 2)^q,x)
```

output
```
int((4 - e^2*x^2)^p*(e*x + 2)^q, x)
```

Reduce [F]

$$\int (2+ex)^q \left(4-e^2x^2\right)^p \, dx$$

$$= \frac{(ex+2)^q \left(-e^2x^2+4\right)^p eqx + 4(ex+2)^q \left(-e^2x^2+4\right)^p p + 2(ex+2)^q \left(-e^2x^2+4\right)^p q - 16\left(\int \frac{(ex+2)^q (}{2e^2p\,x^2+e^2q\,x^2}\right.}{}$$

input
```
int((e*x+2)^q*(-e^2*x^2+4)^p,x)
```

output
```
((e*x + 2)**q*( - e**2*x**2 + 4)**p*e*q*x + 4*(e*x + 2)**q*( - e**2*x**2 +
 4)**p*p + 2*(e*x + 2)**q*( - e**2*x**2 + 4)**p*q - 16*int(((e*x + 2)**q*(
 - e**2*x**2 + 4)**p*x)/(2*e**2*p*x**2 + e**2*q*x**2 + e**2*x**2 - 8*p - 4
*q - 4),x)*e**2*p**3 - 24*int(((e*x + 2)**q*( - e**2*x**2 + 4)**p*x)/(2*e*
*2*p*x**2 + e**2*q*x**2 + e**2*x**2 - 8*p - 4*q - 4),x)*e**2*p**2*q - 8*in
t(((e*x + 2)**q*( - e**2*x**2 + 4)**p*x)/(2*e**2*p*x**2 + e**2*q*x**2 + e*
*2*x**2 - 8*p - 4*q - 4),x)*e**2*p**2 - 8*int(((e*x + 2)**q*( - e**2*x**2
+ 4)**p*x)/(2*e**2*p*x**2 + e**2*q*x**2 + e**2*x**2 - 8*p - 4*q - 4),x)*e*
*2*p*q**2 - 8*int(((e*x + 2)**q*( - e**2*x**2 + 4)**p*x)/(2*e**2*p*x**2 +
e**2*q*x**2 + e**2*x**2 - 8*p - 4*q - 4),x)*e**2*p*q)/(e*q*(2*p + q + 1))
```

3.387 $\int (2-ex)^{-q} \left(4-e^2x^2\right)^{p+q} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 49

$$\int (2-ex)^{-q} \left(4-e^2x^2\right)^{p+q} \, dx = -\frac{4^{p+q}(2-ex)^{1+p} \text{Hypergeometric2F1}\left(1+p,-p-q,2+p,\frac{1}{4}(2-ex)\right)}{e(1+p)}$$

output
```
-4^(p+q)*(-e*x+2)^(p+1)*hypergeom([p+1, -p-q],[2+p],-1/4*e*x+1/2)/e/(p+1)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int (2-ex)^{-q} \left(4-e^2x^2\right)^{p+q} \, dx = \frac{4^p(2-ex)^{-p-q}(2+ex) \left(4-e^2x^2\right)^{p+q} \text{Hypergeometric2F1}\left(-p,1+p+q,2+p+q,\frac{1}{4}(2+ex)\right)}{e(1+p+q)}$$

input
```
Integrate[(4 - e^2*x^2)^(p + q)/(2 - e*x)^q,x]
```

output
```
(4^p*(2 - e*x)^(-p - q)*(2 + e*x)*(4 - e^2*x^2)^(p + q)*Hypergeometric2F1[
-p, 1 + p + q, 2 + p + q, (2 + e*x)/4])/(e*(1 + p + q))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {456, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-ex)^{-q} \left(4-e^2x^2\right)^{p+q} \, dx$$

$$\Big\downarrow \text{ 456}$$

$$\int (2-ex)^p (ex+2)^{p+q} dx$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{4^{p+q} (2-ex)^{p+1} \text{Hypergeometric2F1}\left(p+1,-p-q,p+2,\frac{1}{4}(2-ex)\right)}{e(p+1)}$$

input
```
Int[(4 - e^2*x^2)^(p + q)/(2 - e*x)^q,x]
```

output
```
-((4^(p + q)*(2 - e*x)^(1 + p)*Hypergeometric2F1[1 + p, -p - q, 2 + p, (2
- e*x)/4])/(e*(1 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

Maple [F]

$$\int \left(-e^{2} x^{2}+4\right)^{p+q} \left(-e x +2\right)^{-q}d x$$

input
```
int((-e^2*x^2+4)^(p+q)/((-e*x+2)^q),x)
```

output
```
int((-e^2*x^2+4)^(p+q)/((-e*x+2)^q),x)
```

Fricas [F]

$$\int (2-ex)^{-q} \left(4-e^2x^2\right)^{p+q} \, dx = \int \frac{\left(-e^{2} x^{2} + 4\right)^{p + q}}{\left(-e x + 2\right)^{q}} \,d x$$

input
```
integrate((-e^2*x^2+4)^(p+q)/((-e*x+2)^q),x, algorithm="fricas")
```

output
```
integral((-e^2*x^2 + 4)^(p + q)/(-e*x + 2)^q, x)
```

Sympy [F]

$$\int (2-ex)^{-q} \left(4-e^2x^2\right)^{p+q} \, dx = \int \left(- \left(e x - 2\right) \left(e x + 2\right)\right)^{p + q} \left(- e x + 2\right)^{- q}\, dx$$

input
```
integrate((-e**2*x**2+4)**(p+q)/((-e*x+2)**q),x)
```

output
```
Integral((-(e*x - 2)*(e*x + 2))**(p + q)/(-e*x + 2)**q, x)
```

Maxima [F]

$$\int (2-ex)^{-q}\left(4-e^2x^2\right)^{p+q}\,dx=\int \frac{\left(-e^2x^2+4\right)^{p+q}}{\left(-ex+2\right)^{q}}\,dx$$

input
```
integrate((-e^2*x^2+4)^(p+q)/((-e*x+2)^q),x, algorithm="maxima")
```

output
```
integrate((-e^2*x^2 + 4)^(p + q)/(-e*x + 2)^q, x)
```

Giac [F]

$$\int (2-ex)^{-q}\left(4-e^2x^2\right)^{p+q}\,dx=\int \frac{\left(-e^2x^2+4\right)^{p+q}}{\left(-ex+2\right)^{q}}\,dx$$

input
```
integrate((-e^2*x^2+4)^(p+q)/((-e*x+2)^q),x, algorithm="giac")
```

output
```
integrate((-e^2*x^2 + 4)^(p + q)/(-e*x + 2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-ex)^{-q}\left(4-e^2x^2\right)^{p+q}\,dx=\int \frac{\left(4-e^2\,x^2\right)^{p+q}}{\left(2-e\,x\right)^{q}}\,dx$$

input
```
int((4 - e^2*x^2)^(p + q)/(2 - e*x)^q,x)
```

output
```
int((4 - e^2*x^2)^(p + q)/(2 - e*x)^q, x)
```

Reduce [F]

$$\int (2-ex)^{-q}\left(4-e^2x^2\right)^{p+q}\,dx=\int \frac{(-e^2x^2+4)^{p+q}}{(-ex+2)^q}dx$$

input
```
int((-e^2*x^2+4)^(p+q)/((-e*x+2)^q),x)
```

output
```
int(( - e**2*x**2 + 4)**(p + q)/( - e*x + 2)**q,x)
```

3.388 $\int (2-ex)^p(2+ex)^{p+q}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int (2-ex)^p(2+ex)^{p+q}\,dx = \frac{4^p(2+ex)^{1+p+q}\,\text{Hypergeometric2F1}\left(-p,1+p+q,2+p+q,\frac{1}{4}(2+ex)\right)}{e(1+p+q)}$$

output
```
4^p*(e*x+2)^(1+p+q)*hypergeom([-p, 1+p+q],[2+p+q],1/4*e*x+1/2)/e/(1+p+q)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int (2-ex)^p(2+ex)^{p+q}\,dx = -\frac{4^{p+q}(2-ex)^{1+p}\,\text{Hypergeometric2F1}\left(1+p,-p-q,2+p,\frac{1}{4}(2-ex)\right)}{e(1+p)}$$

input
```
Integrate[(2 - e*x)^p*(2 + e*x)^(p + q),x]
```

output
```
-((4^(p + q)*(2 - e*x)^(1 + p)*Hypergeometric2F1[1 + p, -p - q, 2 + p, (2
- e*x)/4])/(e*(1 + p)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2-ex)^p(ex+2)^{p+q}\,dx$$

$$\Big\downarrow \; 79$$

$$-\frac{4^{p+q}(2-ex)^{p+1}\,\text{Hypergeometric2F1}\left(p+1,-p-q,p+2,\frac{1}{4}(2-ex)\right)}{e(p+1)}$$

input
```
Int[(2 - e*x)^p*(2 + e*x)^(p + q),x]
```

output

```
-((4^(p + q)*(2 - e*x)^(1 + p)*Hypergeometric2F1[1 + p, -p - q, 2 + p, (2
- e*x)/4])/(e*(1 + p)))
```


Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Maple [F]

$$\int (-ex+2)^p (ex+2)^{p+q} \, dx$$

input
```
int((-e*x+2)^p*(e*x+2)^(p+q),x)
```

output
```
int((-e*x+2)^p*(e*x+2)^(p+q),x)
```

Fricas [F]

$$\int (2-ex)^p(2+ex)^{p+q} \, dx = \int (ex+2)^{p+q}(-ex+2)^p \, dx$$

input
```
integrate((-e*x+2)^p*(e*x+2)^(p+q),x, algorithm="fricas")
```

output
```
integral((e*x + 2)^(p + q)*(-e*x + 2)^p, x)
```

Sympy [F]

$$\int (2-ex)^p(2+ex)^{p+q} \, dx = \int (-ex+2)^p (ex+2)^{p+q} \, dx$$

input
```
integrate((-e*x+2)**p*(e*x+2)**(p+q),x)
```

output
```
Integral((-e*x + 2)**p*(e*x + 2)**(p + q), x)
```

Maxima [F]

$$\int (2-ex)^p(2+ex)^{p+q}\,dx = \int (ex+2)^{p+q}(-ex+2)^p\,dx$$

input
```
integrate((-e*x+2)^p*(e*x+2)^(p+q),x, algorithm="maxima")
```

output
```
integrate((e*x + 2)^(p + q)*(-e*x + 2)^p, x)
```

Giac [F]

$$\int (2-ex)^p(2+ex)^{p+q}\,dx = \int (ex+2)^{p+q}(-ex+2)^p\,dx$$

input
```
integrate((-e*x+2)^p*(e*x+2)^(p+q),x, algorithm="giac")
```

output
```
integrate((e*x + 2)^(p + q)*(-e*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (2-ex)^p(2+ex)^{p+q}\,dx = \int (2-e\,x)^p\,(e\,x+2)^{p+q}\,dx$$

input
```
int((2 - e*x)^p*(e*x + 2)^(p + q),x)
```

output
```
int((2 - e*x)^p*(e*x + 2)^(p + q), x)
```

Reduce [F]

$$\int (2-ex)^p(2+ex)^{p+q}\,dx$$

$$= \frac{(ex+2)^{p+q}(-ex+2)^p eqx + 4(ex+2)^{p+q}(-ex+2)^p p + 2(ex+2)^{p+q}(-ex+2)^p q - 16\Big(\int \frac{(ex+2)^p}{2e^2px^2+e^2qx}}{}$$

input
```
int((-e*x+2)^p*(e*x+2)^(p+q),x)
```

output
```
((e*x + 2)**(p + q)*( - e*x + 2)**p*e*q*x + 4*(e*x + 2)**(p + q)*( - e*x +
 2)**p*p + 2*(e*x + 2)**(p + q)*( - e*x + 2)**p*q - 16*int(((e*x + 2)**(p
+ q)*( - e*x + 2)**p*x)/(2*e**2*p*x**2 + e**2*q*x**2 + e**2*x**2 - 8*p - 4
*q - 4),x)*e**2*p**3 - 24*int(((e*x + 2)**(p + q)*( - e*x + 2)**p*x)/(2*e*
*2*p*x**2 + e**2*q*x**2 + e**2*x**2 - 8*p - 4*q - 4),x)*e**2*p**2*q - 8*in
t(((e*x + 2)**(p + q)*( - e*x + 2)**p*x)/(2*e**2*p*x**2 + e**2*q*x**2 + e*
*2*x**2 - 8*p - 4*q - 4),x)*e**2*p**2 - 8*int(((e*x + 2)**(p + q)*( - e*x
+ 2)**p*x)/(2*e**2*p*x**2 + e**2*q*x**2 + e**2*x**2 - 8*p - 4*q - 4),x)*e*
*2*p*q**2 - 8*int(((e*x + 2)**(p + q)*( - e*x + 2)**p*x)/(2*e**2*p*x**2 +
e**2*q*x**2 + e**2*x**2 - 8*p - 4*q - 4),x)*e**2*p*q)/(e*q*(2*p + q + 1))
```

3.389 $\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx = -\frac{4^p(6-3bx)^{4+p} \operatorname{Hypergeometric2F1}\left(-p,4+p,5+p,\frac{1}{4}(2-bx)\right)}{3b(4+p)}$$

output
```
-1/3*4^p*(-3*b*x+6)^(4+p)*hypergeom([-p, 4+p],[5+p],-1/4*b*x+1/2)/b/(4+p)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(45) = 90.

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.76

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx = \frac{72\left(12-3b^2x^2\right)^{1+p}}{b(1+p)} - \frac{3\left(12-3b^2x^2\right)^{2+p}}{2b(2+p)} + 2^{3+2p}3^{3+p}x \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{b^2x^2}{4}\right) + 2^{1+2p}3^{3+p}b^2x^3 \operatorname{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{b^2x^2}{4}\right)$$

input
```
Integrate[(6 - 3*b*x)^3*(12 - 3*b^2*x^2)^p,x]
```

output
```
(72*(12 - 3*b^2*x^2)^(1 + p))/(b*(1 + p)) - (3*(12 - 3*b^2*x^2)^(2 + p))/(
2*b*(2 + p)) + 2^(3 + 2*p)*3^(3 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (b^
2*x^2)/4] + 2^(1 + 2*p)*3^(3 + p)*b^2*x^3*Hypergeometric2F1[3/2, -p, 5/2,
(b^2*x^2)/4]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {472, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx$$

$$\Big\downarrow \text{472}$$

$$9\ 2^{p+3}(bx+2)^{-p-1}(3bx+6)^{p+1}\int \left(1-\frac{bx}{2}\right)^{p+3}(bx+2)^p dx$$

$$\Big\downarrow \text{79}$$

$$-\frac{9\ 2^{3p+4}\left(1-\frac{bx}{2}\right)^{p+4}(bx+2)^{-p-1}(3bx+6)^{p+1}\text{Hypergeometric2F1}\left(-p,p+4,p+5,\frac{1}{4}(2-bx)\right)}{b(p+4)}$$

input
```
Int[(6 - 3*b*x)^3*(12 - 3*b^2*x^2)^p,x]
```

output
```
(-9*2^(4 + 3*p)*(1 - (b*x)/2)^(4 + p)*(2 + b*x)^(-1 - p)*(6 + 3*b*x)^(1 +
p)*Hypergeometric2F1[-p, 4 + p, 5 + p, (2 - b*x)/4])/(b*(4 + p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 472
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^(p + 1)*c^(n - 1)*(((c - d*x)/c)^(p + 1)/(a/c + b*(x/d))^(p + 1))   Int[(
1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && GtQ[a, 0] &&  !(
IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(41) = 82$.

Time = 3.00 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.04

method	result
meijerg	$-3^{3+p}2^{-2+2p}b^3x^4\,\text{hypergeom}\left(\left[2,-p\right],\left[3\right],\frac{b^2x^2}{4}\right)+2^{2p+1}3^{3+p}b^2x^3\,\text{hypergeom}\left(\left[\frac{3}{2},-p\right],\left[\frac{5}{2}\right],\frac{b^2x}{4}\right.$

input
```
int((-3*b*x+6)^3*(-3*b^2*x^2+12)^p,x,method=_RETURNVERBOSE)
```

output
```
-3^(3+p)*2^(-2+2*p)*b^3*x^4*hypergeom([2,-p],[3],1/4*b^2*x^2)+2^(2*p+1)*3^
(3+p)*b^2*x^3*hypergeom([3/2,-p],[5/2],1/4*b^2*x^2)-2^(2*p+1)*3^(4+p)*b*x^
2*hypergeom([1,-p],[2],1/4*b^2*x^2)+2^(3+2*p)*3^(3+p)*x*hypergeom([1/2,-p]
,[3/2],1/4*b^2*x^2)
```

Fricas [F]

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx = \int -27 \, (bx-2)^3 \left(-3 \, b^2x^2+12\right)^p \, dx$$

input
```
integrate((-3*b*x+6)^3*(-3*b^2*x^2+12)^p,x, algorithm="fricas")
```

output
```
integral(-27*(b^3*x^3 - 6*b^2*x^2 + 12*b*x - 8)*(-3*b^2*x^2 + 12)^p, x)
```

Sympy [A] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 434, normalized size of antiderivative = 9.64

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx$$
$$= 54 \cdot 12^p b^2 x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix} \middle| \frac{b^2x^2e^{2i\pi}}{4}\right) + 216 \cdot 12^p x {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{b^2x^2e^{2i\pi}}{4}\right)$$
$$-27b^3 \left(\begin{cases} \frac{12^p x^4}{4} & \text{for}\: b=0 \\ \frac{b^2x^2\log\left(x-\frac{2}{b}\right)}{18b^6x^2-72b^4} + \frac{b^2x^2\log\left(x+\frac{2}{b}\right)}{18b^6x^2-72b^4} - \frac{4\log\left(x-\frac{2}{b}\right)}{18b^6x^2-72b^4} - \frac{4\log\left(x+\frac{2}{b}\right)}{18b^6x^2-72b^4} - \frac{4}{18b^6x^2-72b^4} & \text{for}\: p=-2 \\ -\frac{x^2}{6b^2} - \frac{2\log\left(x-\frac{2}{b}\right)}{3b^4} - \frac{2\log\left(x+\frac{2}{b}\right)}{3b^4} & \text{for}\: p=-1 \\ \frac{b^4px^4(-3b^2x^2+12)^p}{2b^4p^2+6b^4p+4b^4} + \frac{b^4x^4(-3b^2x^2+12)^p}{2b^4p^2+6b^4p+4b^4} - \frac{4b^2px^2(-3b^2x^2+12)^p}{2b^4p^2+6b^4p+4b^4} - \frac{16(-3b^2x^2+12)^p}{2b^4p^2+6b^4p+4b^4} & \text{otherwise} \end{cases}\right)$$
$$-324b \left(\begin{cases} \frac{12^p x^2}{2} & \text{for}\: b^2=0 \\ -\frac{\begin{cases} \frac{(-3b^2x^2+12)^{p+1}}{p+1} & \text{for}\: p\neq -1 \\ \log\left(-3b^2x^2+12\right) & \text{otherwise} \end{cases}}{6b^2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((-3*b*x+6)**3*(-3*b**2*x**2+12)**p,x)
```

output
```
54*12**p*b**2*x**3*hyper((3/2, -p), (5/2,), b**2*x**2*exp_polar(2*I*pi)/4)
 + 216*12**p*x*hyper((1/2, -p), (3/2,), b**2*x**2*exp_polar(2*I*pi)/4) - 2
7*b**3*Piecewise((12**p*x**4/4, Eq(b, 0)), (b**2*x**2*log(x - 2/b)/(18*b**
6*x**2 - 72*b**4) + b**2*x**2*log(x + 2/b)/(18*b**6*x**2 - 72*b**4) - 4*lo
g(x - 2/b)/(18*b**6*x**2 - 72*b**4) - 4*log(x + 2/b)/(18*b**6*x**2 - 72*b*
*4) - 4/(18*b**6*x**2 - 72*b**4), Eq(p, -2)), (-x**2/(6*b**2) - 2*log(x -
2/b)/(3*b**4) - 2*log(x + 2/b)/(3*b**4), Eq(p, -1)), (b**4*p*x**4*(-3*b**2
*x**2 + 12)**p/(2*b**4*p**2 + 6*b**4*p + 4*b**4) + b**4*x**4*(-3*b**2*x**2
 + 12)**p/(2*b**4*p**2 + 6*b**4*p + 4*b**4) - 4*b**2*p*x**2*(-3*b**2*x**2
+ 12)**p/(2*b**4*p**2 + 6*b**4*p + 4*b**4) - 16*(-3*b**2*x**2 + 12)**p/(2*
b**4*p**2 + 6*b**4*p + 4*b**4), True)) - 324*b*Piecewise((12**p*x**2/2, Eq
(b**2, 0)), (-Piecewise(((-3*b**2*x**2 + 12)**(p + 1)/(p + 1), Ne(p, -1)),
 (log(-3*b**2*x**2 + 12), True))/(6*b**2), True))
```

Maxima [F]

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx = \int -27 \, (bx-2)^3 \left(-3 \, b^2x^2+12\right)^p \, dx$$

input
```
integrate((-3*b*x+6)^3*(-3*b^2*x^2+12)^p,x, algorithm="maxima")
```

output
```
-27*integrate((b*x - 2)^3*(-3*b^2*x^2 + 12)^p, x)
```

Giac [F]

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx = \int -27 \, (bx-2)^3 \left(-3 \, b^2x^2+12\right)^p \, dx$$

input
```
integrate((-3*b*x+6)^3*(-3*b^2*x^2+12)^p,x, algorithm="giac")
```

output
```
integrate(-27*(b*x - 2)^3*(-3*b^2*x^2 + 12)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx = -\int \left(12-3\,b^2\,x^2\right)^p \left(3\,b\,x-6\right)^3 dx$$

input
```
int(-(12 - 3*b^2*x^2)^p*(3*b*x - 6)^3,x)
```

output
```
-int((12 - 3*b^2*x^2)^p*(3*b*x - 6)^3, x)
```

Reduce [F]

$$\int (6-3bx)^3 \left(12-3b^2x^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((-3*b*x+6)^3*(-3*b^2*x^2+12)^p,x)
```

output
```
(27*( - 4*( - 3*b**2*x**2 + 12)**p*b**4*p**3*x**4 - 12*( - 3*b**2*x**2 + 1
2)**p*b**4*p**2*x**4 - 11*( - 3*b**2*x**2 + 12)**p*b**4*p*x**4 - 3*( - 3*b
**2*x**2 + 12)**p*b**4*x**4 + 24*( - 3*b**2*x**2 + 12)**p*b**3*p**3*x**3 +
 84*( - 3*b**2*x**2 + 12)**p*b**3*p**2*x**3 + 84*( - 3*b**2*x**2 + 12)**p*
b**3*p*x**3 + 24*( - 3*b**2*x**2 + 12)**p*b**3*x**3 - 32*( - 3*b**2*x**2 +
 12)**p*b**2*p**3*x**2 - 160*( - 3*b**2*x**2 + 12)**p*b**2*p**2*x**2 - 216
*( - 3*b**2*x**2 + 12)**p*b**2*p*x**2 - 72*( - 3*b**2*x**2 + 12)**p*b**2*x
**2 - 64*( - 3*b**2*x**2 + 12)**p*b*p**3*x - 144*( - 3*b**2*x**2 + 12)**p*
b*p**2*x + 16*( - 3*b**2*x**2 + 12)**p*b*p*x + 96*( - 3*b**2*x**2 + 12)**p
*b*x + 192*( - 3*b**2*x**2 + 12)**p*p**3 + 832*( - 3*b**2*x**2 + 12)**p*p*
*2 + 1040*( - 3*b**2*x**2 + 12)**p*p + 336*( - 3*b**2*x**2 + 12)**p - 1024
*int(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x
**2 - 16*p**2 - 32*p - 12),x)*b*p**6 - 8192*int(( - 3*b**2*x**2 + 12)**p/(
4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b
*p**5 - 24320*int(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x*
*2 + 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b*p**4 - 33280*int(( - 3*b**2*x
**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 3
2*p - 12),x)*b*p**3 - 20736*int(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2
 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b*p**2 - 4608*int
(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x*...
```

3.390 $\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3} dx = \frac{12^{3+p}(2+bx)^{1+p}\,\text{Hypergeometric2F1}\left(-3-p,1+p,2+p,\frac{1}{4}(2+bx)\right)}{b(1+p)}$$

output
```
12^(3+p)*(b*x+2)^(p+1)*hypergeom([p+1, -3-p],[2+p],1/4*b*x+1/2)/b/(p+1)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3} dx = \frac{72(12-3b^2x^2)^{1+p}}{b(1+p)} - \frac{3(12-3b^2x^2)^{2+p}}{2b(2+p)} + 2^{3+2p}3^{3+p}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{b^2x^2}{4}\right) + 2^{1+2p}3^{3+p}b^2x^3\,\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},\frac{b^2x^2}{4}\right)$$

input
```
Integrate[(12 - 3*b^2*x^2)^(3 + p)/(2 + b*x)^3,x]
```

output
```
(72*(12 - 3*b^2*x^2)^(1 + p))/(b*(1 + p)) - (3*(12 - 3*b^2*x^2)^(2 + p))/(
2*b*(2 + p)) + 2^(3 + 2*p)*3^(3 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, (b^
2*x^2)/4] + 2^(1 + 2*p)*3^(3 + p)*b^2*x^3*Hypergeometric2F1[3/2, -p, 5/2,
(b^2*x^2)/4]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{472, 79\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(12-3b^2x^2\right)^{p+3}}{(bx+2)^3}dx$$

$$\Big\downarrow \text{472}$$

$$2^p3^{p+4}(6-3bx)^{-p-4}(2-bx)^{p+4}\int (6-3bx)^{p+3}\left(\frac{bx}{2}+1\right)^pdx$$

$$\Big\downarrow \text{79}$$

$$-\frac{2^{2p}3^{p+3}(2-bx)^{p+4}\text{Hypergeometric2F1}\left(-p,p+4,p+5,\frac{1}{4}(2-bx)\right)}{b(p+4)}$$

input
```
Int[(12 - 3*b^2*x^2)^(3 + p)/(2 + b*x)^3,x]
```

output
```
-((2^(2*p)*3^(3 + p)*(2 - b*x)^(4 + p)*Hypergeometric2F1[-p, 4 + p, 5 + p,
 (2 - b*x)/4])/(b*(4 + p)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 472
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^(p + 1)*c^(n - 1)*(((c - d*x)/c)^(p + 1)/(a/c + b*(x/d))^(p + 1))   Int[(
1 + d*(x/c))^(n + p)*(a/c + (b/d)*x)^p, x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[n] || GtQ[c, 0]) && GtQ[a, 0] &&  !(
IntegerQ[n] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Maple [F]

$$\int \frac{\left(-3b^2x^2+12\right)^{3+p}}{\left(bx+2\right)^3}dx$$

input
```
int((-3*b^2*x^2+12)^(3+p)/(b*x+2)^3,x)
```

output
```
int((-3*b^2*x^2+12)^(3+p)/(b*x+2)^3,x)
```

Fricas [F]

$$\int \frac{\left(12-3b^2x^2\right)^{3+p}}{(2+bx)^3}\,dx=\int \frac{\left(-3\,b^2x^2+12\right)^{p+3}}{\left(bx+2\right)^3}\,dx$$

input
```
integrate((-3*b^2*x^2+12)^(3+p)/(b*x+2)^3,x, algorithm="fricas")
```

output
```
integral((-3*b^2*x^2 + 12)^(p + 3)/(b^3*x^3 + 6*b^2*x^2 + 12*b*x + 8), x)
```

Sympy [F]

$$\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3}\,dx = \int \frac{(-3(bx-2)(bx+2))^{p+3}}{(bx+2)^3}\,dx$$

input
```
integrate((-3*b**2*x**2+12)**(3+p)/(b*x+2)**3,x)
```

output
```
Integral((-3*(b*x - 2)*(b*x + 2))**(p + 3)/(b*x + 2)**3, x)
```

Maxima [F]

$$\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3}\,dx = \int \frac{(-3\,b^2x^2+12)^{p+3}}{(bx+2)^3}\,dx$$

input
```
integrate((-3*b^2*x^2+12)^(3+p)/(b*x+2)^3,x, algorithm="maxima")
```

output
```
integrate((-3*b^2*x^2 + 12)^(p + 3)/(b*x + 2)^3, x)
```

Giac [F]

$$\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3}\,dx = \int \frac{(-3\,b^2x^2+12)^{p+3}}{(bx+2)^3}\,dx$$

input
```
integrate((-3*b^2*x^2+12)^(3+p)/(b*x+2)^3,x, algorithm="giac")
```

output
```
integrate((-3*b^2*x^2 + 12)^(p + 3)/(b*x + 2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3}\,dx=\int \frac{{\left(12-3\,b^2\,x^2\right)}^{p+3}}{{\left(b\,x+2\right)}^3}\,dx$$

input
```
int((12 - 3*b^2*x^2)^(p + 3)/(b*x + 2)^3,x)
```

output
```
int((12 - 3*b^2*x^2)^(p + 3)/(b*x + 2)^3, x)
```

Reduce [F]

$$\int \frac{(12-3b^2x^2)^{3+p}}{(2+bx)^3}\,dx=\text{Too large to display}$$

input
```
int((-3*b^2*x^2+12)^(3+p)/(b*x+2)^3,x)
```

output
```
(27*( - 4*( - 3*b**2*x**2 + 12)**p*b**4*p**3*x**4 - 12*( - 3*b**2*x**2 + 1
2)**p*b**4*p**2*x**4 - 11*( - 3*b**2*x**2 + 12)**p*b**4*p*x**4 - 3*( - 3*b
**2*x**2 + 12)**p*b**4*x**4 + 24*( - 3*b**2*x**2 + 12)**p*b**3*p**3*x**3 +
 84*( - 3*b**2*x**2 + 12)**p*b**3*p**2*x**3 + 84*( - 3*b**2*x**2 + 12)**p*
b**3*p*x**3 + 24*( - 3*b**2*x**2 + 12)**p*b**3*x**3 - 32*( - 3*b**2*x**2 +
 12)**p*b**2*p**3*x**2 - 160*( - 3*b**2*x**2 + 12)**p*b**2*p**2*x**2 - 216
*( - 3*b**2*x**2 + 12)**p*b**2*p*x**2 - 72*( - 3*b**2*x**2 + 12)**p*b**2*x
**2 - 64*( - 3*b**2*x**2 + 12)**p*b*p**3*x - 144*( - 3*b**2*x**2 + 12)**p*
b*p**2*x + 16*( - 3*b**2*x**2 + 12)**p*b*p*x + 96*( - 3*b**2*x**2 + 12)**p
*b*x + 192*( - 3*b**2*x**2 + 12)**p*p**3 + 832*( - 3*b**2*x**2 + 12)**p*p*
*2 + 1040*( - 3*b**2*x**2 + 12)**p*p + 336*( - 3*b**2*x**2 + 12)**p - 1024
*int(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x
**2 - 16*p**2 - 32*p - 12),x)*b*p**6 - 8192*int(( - 3*b**2*x**2 + 12)**p/(
4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b
*p**5 - 24320*int(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x*
*2 + 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b*p**4 - 33280*int(( - 3*b**2*x
**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 3
2*p - 12),x)*b*p**3 - 20736*int(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2
 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b*p**2 - 4608*int
(( - 3*b**2*x**2 + 12)**p/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x*...
```

3.391 $\int (6-3bx)^{3+p}(2+bx)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int (6-3bx)^{3+p}(2+bx)^p\,dx = \frac{12^{3+p}(2+bx)^{1+p}\,\text{Hypergeometric2F1}\left(-3-p,1+p,2+p,\frac{1}{4}(2+bx)\right)}{b(1+p)}$$

output
```
12^(3+p)*(b*x+2)^(p+1)*hypergeom([p+1, -3-p],[2+p],1/4*b*x+1/2)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int (6-3bx)^{3+p}(2+bx)^p\,dx = -\frac{4^p(6-3bx)^{4+p}\,\text{Hypergeometric2F1}\left(-p,4+p,5+p,\frac{1}{12}(6-3bx)\right)}{3b(4+p)}$$

input
```
Integrate[(6 - 3*b*x)^(3 + p)*(2 + b*x)^p,x]
```

output
```
-1/3*(4^p*(6 - 3*b*x)^(4 + p)*Hypergeometric2F1[-p, 4 + p, 5 + p, (6 - 3*b
*x)/12])/(b*(4 + p))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (6-3bx)^{p+3}(bx+2)^p\,dx$$

$$\Big\downarrow 79$$

$$-\frac{4^p(6-3bx)^{p+4}\,\text{Hypergeometric2F1}\left(-p,p+4,p+5,\frac{1}{4}(2-bx)\right)}{3b(p+4)}$$

input
```
Int[(6 - 3*b*x)^(3 + p)*(2 + b*x)^p,x]
```

output
```
-1/3*(4^p*(6 - 3*b*x)^(4 + p)*Hypergeometric2F1[-p, 4 + p, 5 + p, (2 - b*x
)/4])/(b*(4 + p))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Maple [F]

$$\int (-3bx+6)^{3+p} (bx+2)^p \, dx$$

input
```
int((-3*b*x+6)^(3+p)*(b*x+2)^p,x)
```

output
```
int((-3*b*x+6)^(3+p)*(b*x+2)^p,x)
```

Fricas [F]

$$\int (6-3bx)^{3+p}(2+bx)^p \, dx = \int (bx+2)^p(-3\,bx+6)^{p+3} \, dx$$

input
```
integrate((-3*b*x+6)^(3+p)*(b*x+2)^p,x, algorithm="fricas")
```

output
```
integral((b*x + 2)^p*(-3*b*x + 6)^(p + 3), x)
```

Sympy [F]

$$\int (6-3bx)^{3+p}(2+bx)^p \, dx = \int (-3bx+6)^{p+3} (bx+2)^p \, dx$$

input
```
integrate((-3*b*x+6)**(3+p)*(b*x+2)**p,x)
```

output
```
Integral((-3*b*x + 6)**(p + 3)*(b*x + 2)**p, x)
```

Maxima [F]

$$\int (6-3bx)^{3+p}(2+bx)^p\,dx = \int (bx+2)^p(-3\,bx+6)^{p+3}\,dx$$

input
```
integrate((-3*b*x+6)^(3+p)*(b*x+2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x + 2)^p*(-3*b*x + 6)^(p + 3), x)
```

Giac [F]

$$\int (6-3bx)^{3+p}(2+bx)^p\,dx = \int (bx+2)^p(-3\,bx+6)^{p+3}\,dx$$

input
```
integrate((-3*b*x+6)^(3+p)*(b*x+2)^p,x, algorithm="giac")
```

output
```
integrate((b*x + 2)^p*(-3*b*x + 6)^(p + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (6-3bx)^{3+p}(2+bx)^p\,dx = \int (b\,x+2)^p\,(6-3\,b\,x)^{p+3}\,dx$$

input
```
int((b*x + 2)^p*(6 - 3*b*x)^(p + 3),x)
```

output
```
int((b*x + 2)^p*(6 - 3*b*x)^(p + 3), x)
```

Reduce [F]

$$\int (6-3bx)^{3+p}(2+bx)^p\,dx = \text{Too large to display}$$

input
```
int((-3*b*x+6)^(3+p)*(b*x+2)^p,x)
```

output
```
(27*( - 4*(b*x + 2)**p*( - 3*b*x + 6)**p*b**4*p**3*x**4 - 12*(b*x + 2)**p*
( - 3*b*x + 6)**p*b**4*p**2*x**4 - 11*(b*x + 2)**p*( - 3*b*x + 6)**p*b**4*
p*x**4 - 3*(b*x + 2)**p*( - 3*b*x + 6)**p*b**4*x**4 + 24*(b*x + 2)**p*( -
3*b*x + 6)**p*b**3*p**3*x**3 + 84*(b*x + 2)**p*( - 3*b*x + 6)**p*b**3*p**2
*x**3 + 84*(b*x + 2)**p*( - 3*b*x + 6)**p*b**3*p*x**3 + 24*(b*x + 2)**p*(
- 3*b*x + 6)**p*b**3*x**3 - 32*(b*x + 2)**p*( - 3*b*x + 6)**p*b**2*p**3*x*
*2 - 160*(b*x + 2)**p*( - 3*b*x + 6)**p*b**2*p**2*x**2 - 216*(b*x + 2)**p*
( - 3*b*x + 6)**p*b**2*p*x**2 - 72*(b*x + 2)**p*( - 3*b*x + 6)**p*b**2*x**
2 - 64*(b*x + 2)**p*( - 3*b*x + 6)**p*b*p**3*x - 144*(b*x + 2)**p*( - 3*b*
x + 6)**p*b*p**2*x + 16*(b*x + 2)**p*( - 3*b*x + 6)**p*b*p*x + 96*(b*x + 2
)**p*( - 3*b*x + 6)**p*b*x + 192*(b*x + 2)**p*( - 3*b*x + 6)**p*p**3 + 832
*(b*x + 2)**p*( - 3*b*x + 6)**p*p**2 + 1040*(b*x + 2)**p*( - 3*b*x + 6)**p
*p + 336*(b*x + 2)**p*( - 3*b*x + 6)**p - 1024*int(((b*x + 2)**p*( - 3*b*x
 + 6)**p)/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 32*p
 - 12),x)*b*p**6 - 8192*int(((b*x + 2)**p*( - 3*b*x + 6)**p)/(4*b**2*p**2*
x**2 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b*p**5 - 2432
0*int(((b*x + 2)**p*( - 3*b*x + 6)**p)/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 +
 3*b**2*x**2 - 16*p**2 - 32*p - 12),x)*b*p**4 - 33280*int(((b*x + 2)**p*(
- 3*b*x + 6)**p)/(4*b**2*p**2*x**2 + 8*b**2*p*x**2 + 3*b**2*x**2 - 16*p**2
 - 32*p - 12),x)*b*p**3 - 20736*int(((b*x + 2)**p*( - 3*b*x + 6)**p)/(4...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#    "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file